Contagious Stablecoins?

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Abstract

In an overlapping-generations model, we study competition between stablecoins that are pegged to a stable currency. Stablecoins are issued by coalescing agents, are backed by long-term assets and can either be redeemed with the issuer or traded in a secondary market. When an issuer limits redemption and sticks to an investment rule, its coin is truly stable in an idiosyncratic sense—it is invulnerable to runs and always trades at the pegged price. Competition between issuers, however, opens the door to a coordination problem in which an issuer must pay interest on its coin when other issuers do so too. As a consequence, the economy can be inefficient and unstable. The efficient allocation can be implemented as a unique equilibrium when regulation prevents the issuance of contagious interest-bearing stablecoins.

Keywords: Stablecoins, currency competition, free banking, private money, digital money.

JEL Classification: E4, E5, G1, G2

1 Introduction

With the newest attempt to construct a private money in the form of stablecoins, we have entered yet another episode in which it is tested whether competing private monies can serve as effective payment and saving instruments. Stablecoins are a digital form of privately created money and, in contrast to cryptocurrencies like Bitcoin and Ethereum, aim at maintaining a stable value against a national currency or a basket of national currencies. The five largest stablecoins by market capitalization to date, including Tether (USDT) and USDC, are all pegged one-to-one to the US dollar.

The history of free-banking episodes is full of examples of competing privately-produced monies that were pegged to national currencies but still faced severe difficulties to serve as a stable medium of exchange.[1] From a theoretical point of view it is also still an open question whether and under which set of rules private money competition can deliver stable and socially desirable outcomes. In light of the recent stablecoin rage, three questions are particularly relevant. First, how should an individual stablecoin be designed so that it is truly stable, i.e., always trades at the peg, and do we require regulation for this? Second, does competition between issuers make it more or less likely for stablecoins to be truly stable? Third, how should a system of competing stablecoins be designed so that poorly designed coins do not harm well-designed ones, i.e., how can contagion from poorly to well-designed coins be avoided?

To answer these questions, we develop a model featuring the issuance of stablecoins as well as competition between issuers. Stablecoins in the model are backed by long-term assets, are pegged to a stable currency, are traded in the secondary market, and can be used by investors to insure against idiosyncratic consumption risk when insurance markets are absent.[2] We derive four main insights from our model.

First, an individual stablecoin can be made both truly stable and attractive relative to currency by means of a simple investment rule. This rule stipulates that at each point in time, a fraction of the issuer's disposable resources is invested in long-term assets with stable returns that dominate currency. Resources earmarked for redemption thus have

[1] See, e.g., Gorton and Zhang (2021) on the 1837–1862 free-banking episode in the US.

[2] We do not consider algorithmic stablecoins (such as Terra/Luna, which crashed in May 2022) or stablecoins backed by other cryptoassets (such as DAI). See Cao, Dai, Kou, Li and Yang (2021) for a model on stablecoin design based on option pricing and smart contracts implemented on the Ethereum platform. Mayer (2022) develop a model that rationalizes dual-token structures such as Terra/Luna.

to be limited to respect the investment rule.[3] Since these coins are traded in a secondary market, there is no reason for a run and if a run occurs accidentally, there are no losses for agents who desire to consume. Such coins are called *micro-well-designed* as they are run-proof if the coin was alone in the market.

Second, a monopolistic issuer of a micro-well-designed stablecoin implements the efficient allocation by issuing, in an initial coin offering (ICO), a zero-interest stablecoin at a discount. This arrangement provides insurance against idiosyncratic consumption risk while still allowing investors to enjoy, on average, the return on long-term assets. A coin that is micro-well-designed, pays zero interest and is issued at a discount is called *macro-well-designed* as it implements the efficient allocation.

Third, competition between issuers opens the door to a coordination problem, even when all stablecoins are micro-well-designed. The reason is that an issuer has to match the returns of other coins or else it becomes subject to runs. If a coin is micro-well-designed but pays a positive interest rate, it is contagious to other coins to offer the same returns. Hence, competition of issues of micro-well-designed coins may not lead to macro-well-designed coins. In fact, besides the efficient allocation, a continuum of other, inefficient allocations featuring interest-bearing coins are equilibria.

Fourth, ruling out interest payments on stablecoins leaves the efficient allocation as the unique equilibrium even when issuers compete. This insight provides a rationale for policies that prevent the payment of interest on stablecoins. In the US, for instance, interest-bearing stablecoins would bring the issuer under banking or securities market regulation, which strongly discourages issuers to pay interest. The EU is even set to prohibit interest-bearing stablecoins (Read and Diefenbach, 2022). Our paper provides a rationale for such types of regulation.

Model summary and detailed results. We derive our insights in a continuous-time, infinite-horizon model with overlapping generations of agents that represent stablecoin investors. Agents exert effort to produce a homogeneous good at birth—representing market entry and initial investment—and consume at a random time of death—representing disinvestment and market exit, for instance to buy consumption goods or cryptoassets on an online platform—, which is idiosyncratic risk. Goods are perfectly storable, so

[3]The rule may be implemented using blockchain technology, as arged by Cong, Li and Wang (2022).

we interpret them as stable currency, and can also be invested in scalable, long-term investment opportunities that we call *trees*. Once planted, a tree gestates a deterministic amount of goods (*fruit*) at a stochastic point in time, which is idiosyncratic risk. A tree is destroyed after gestation and the resulting fruit can be consumed, stored or used to plant new trees. We assume that the expected return on a tree equals agents' rate of time preference.

In the efficient Arrow-Debreu allocation there is no storage and investment is such that the aggregate stock of trees stays constant over time, so that consumption is financed from the fruit left after replacing destroyed trees. Further, agents' consumption levels are independent from the point in time at which they die, and thus there is perfect insurance against random death while agents still earn, on average, the rate of time preference on their provided effort. Particularly, those who die early earn a high annualized return on their effort, whereas those who die late earn a low annualized return.

The insurance arrangement breaks down in a decentralized economy where agents trade claims on trees in a secondary market. Particularly, this market allows agents to earn the rate of time preference at each point in time. Those who die early therefore earn the same annualized return on their initial effort as those who die late, so that consumption increases with the time of death. This lack of insurance is inefficient, which is a standard finding in the banking literature following Diamond and Dybvig (1983).

The stablecoin economy is characterized by the fact that individual agents can undertake storage on their own but can neither plant trees themselves nor trade claims on trees. Instead, agents of a particular generation coalesce to form *coin issuers*, which can plant trees, store goods, issue coins to its members in an ICO, and redeem existing coins subject to a sequential service constraint and private information of death. Existing coins can be traded among the agents in a competitive secondary market.

We consider first a version of the stablecoin economy with a single generation of agents that coalesce as an issuer, representing a platform with only a single but tradeable stablecoin. The Arrow-Debreu allocation materializes as an equilibrium when the expected lifetime of an agent exceeds that of a tree, as the Arrow-Debreu allocation then requires no inter-generational transfers. By issuing stablecoins in the ICO at a discount, investing the proceeds in planting trees, maintaining a time-invariant peg with goods after the ICO, and paying zero-interest, the issuer replicates exactly the Arrow-Debreu insurance arrangement—those who die early (late) earn a high (low) annualized return by holding the coin, and consumption is independent of the time of death.

Further, the single issuer earns exactly enough fruit from its gestating trees to meet redemption by all dying agents and to keep the amount of trees per outstanding coin constant. When only dying agents redeem their coins, non-dying agents therefore have no incentive to redeem their coins or sell them in the secondary market—they know that the peg can be maintained so their outside option—storage—earns the same return as the coin. Fruit from the gestating trees is however insufficient to service all redemption requests at the peg when all non-dying agents run to ask for redemption. When the issuer then redeems as much as possible at the peg, gestating trees cannot be replaced so the stock of trees per coin contracts. This reduces resources available for future redemption and thereby makes the run self-fulfilling—agents know the peg cannot be maintained and coins start to trade at a discount in the secondary market.

The run can be prevented by limiting the funds earmarked for redemption, reminsicent of the suspension of convertibility advocated by Diamond and Dybvig (1983). More precisely, the issuer should ensure that after meeting redemption requests up to the earmark, it has enough resources left to keep the trees per outstanding coin constant. Adhering to such an *investment rule* is in the issuer's own interest because it avoids a deviation from the efficient allocation even when a run happens by accident, as dying agents can also trade the coin at the peg in the secondary market. In this sense, stablecoins are micro-well-designed as long as limiting redemptions is in line with regulation.

Competition among coins arises in the stablecoin economy with overlapping generations, representing a platform with multiple tradable stablecoins since each generation's members can trade other issuers' coins. Each generation has an incentive to issue a micro-well-designed coin as this avoids runs if the coin were the only one in the economy. Whether a generation can issue a macro-well-designed stablecoin, i.e., a zero-interest micro-well-designed coin that is issued to its members at a discount and thus consistent with the efficient allocation, depends on the return that agents can earn by trading other coins.

Particularly, when an issuer expects others to issue an interest-bearing micro-well-designed stablecoin, it has to pay at least that interest on its own coin or else be subject to a run. In this sense, micro-well-designed stablecoins with interest payments are *conta-*

gious. As a consequence, issuers face a coordination problem and there is a continuum of equilibria, among which only one implements the efficient allocation. Besides inefficient equilibria with stationary returns, we obtain macroeconomic instability in the sense of perfect-foresight equilibria with cyclical and non-stationary dynamics as well as equilibria that randomly switch between steady states. A prohibition of interest payments on micro-well-designed coints can implement the efficient allocation as the unique equilibrium.

Relation to the literature. Our paper relates to three strands of the literature. First, Gorton and Zhang (2021) emphasize that the new world of stablecoins resembles earlier free-banking episodes. They stress that experience from the latest free-banking episode in the US—the 1837–1862 wildcat-banking era—suggests that unregulated competition among stablecoins could result in fluctuating coin values, as coins might not always be accepted at par, and is likely to produce runs. The idea of free banking has a long history and has been subject to enduring analyses and debates (Friedman, 1960 and 1969; Klein, 1974; Hayek, 1976; King, 1983; White, 1984; Friedman and Schwartz, 1986).[4] Many of the theoretical studies on free banking are relevant for competition between stablecoins as well. Gersbach (1998), for instance, shows that there is a tendency to overissue banknotes (or coins) since an individual bank's circulating notes increase the liquidity costs of all other banks in the system too. Other contributions on modeling private money competition include Cavalcanti and Wallace (1999b,1999a), Cavalcanti, Erosa and Temzelides (1999, 2005), Williamson (1999), Aghion, Bolton and Dewatripont (2000), Berentsen (2006), and Martin and Schreft (2006). Their findings on whether a free banking system has the potential to deliver efficient outcomes are mixed.

Second, with the rise of cryptocurrencies and digital money, there is a growing literature on the risks, optimal design and regulation of these new assets. Stablecoins are at the center of interest because they promise to be a stable store of value and means of payment. Most policy papers focus on explaining what stablecoins actually are and what their effects on the financial and monetary system may be (e.g., Adachi, Cominetta, Kaufmann and van der Kraaij, 2020; Bordo, 2021; U.S. Department of the Treasury, 2021; Bains et al., 2022; Catalini, de Gortari and Shah, 2022; Bruce, Odinet and Tosato,

[4] For reviews of the literature, see Goodhart (1988), Selgin (1988), Selgin and White (1994) or, more recently, White (2014).

2023). They also offer proposals for regulation such as requiring issuers to comply with standard banking laws or, even more drastically, simply regulating them out of existence and instituting a central-bank digital currency (CBDC) instead.[5]

Our goal is to investigate how to design the market for stablecoins that can implement efficient allocations. Related to this question but within a different framework, Fernández-Villaverde and Sanches (2019) analyze whether competition in private digital currencies can be compatible with price stability and efficient money supply. Li and Mayer (2021) develop a dynamic model of stablecoin management and argue that sufficient reserve holdings by the issuers are key for stability. Brunnermeier, James and Landau (2021), Rogoff and You (2023), and Guennewig (2022) study issuance of digital money by retailers, in particular large online platforms such as Amazon or Alibaba. In this context, issuers typically have an incentive to limit interoperability between coins so as to bind consumers to their platform and exploit intra-platform transaction data to earn informational rents. Cong *et al.* (2022) study a token-based platform economy where underinvestment in real productivity can be avoided by using blockchain technology that allows for commitment to predetermined investment schedules, which relates to our finding that a micro-well-designed stablecoin requires the issuer to stick to an investment rule. We also provide a rationale for the prohibition of interest payments on holding stable coins.

Third, regarding the role of secondary markets, competition among stablecoin issuers is closely related to the classical theme developed by Jacklin (1987) and the recent analysis by Rogoff and You (2023) on the role of tradable versus non-tradable tokens. Jacklin (1987) argues that the deposit contract in the Diamond and Dybvig (1983) model would offer inferior risk-sharing opportunities if these deposits were also tradable in a secondary market. Rogoff and You (2023) show that from the perspective of a large online retailer who issues tokens solely for on-platform purchases, it may be beneficial to make these tokens non-tradable. The tokens in their context, however, resemble loyalty points, as offered by hotel chains and airlines, or a videogame-specific currency. Goldstein, Gupta and Sverchkov (2022) show that the tradablity of general-purpose coins is an essential feature to ensure that issuing platforms maintain competitive pricing and do not engage in rent seeking. In our model, where rent-seeking is not an issue, secondary markets allow

[5] Likewise, the 1837–1862 wildcat-banking era was ended when the government introduced a uniform national currency and taxed all private money out of existence (Gorton and Zhang, 2021).

agents to sell micro-well-designed coins at par even when, in case of an accidental run, the coin cannot be redeemed with the issuer. We also find, however, that the tradability of multiple micro-well-designed coins in secondary markets opens the door to a coordination problem which undermines efficiency. Limiting tradability can thus also be advantageous, albeit for different reasons than identified by Rogoff and You (2023).

Outline. Section 2 introduces the model environment. Section 3 studies an Arrow-Debreu setting and characterizes an efficient allocation. Section 4 considers a decentralized economy with tradable long-term assets. Section 5 introduces the stablecoin economy, where agents coalesce to issue coins backed by long-term assets and trade the coins in a secondary market. To start with, we study a single generation represented by one coin issuer in Section 6. Section 7 considers overlapping generations and competition among issuers. Section 8 concludes. Proofs are relegated to the Appendix.

2 The model

Time, denoted by $t \geq 0$, is continuous and the horizon is infinite. For any stock variable X, we interpret X_t as a state at time t. For any variable Y we let Y_t^+ be the right hand limit of Y_t and $\dot{Y}_t$ the right-hand derivative of Y_t w.r.t. to t.

The economy is inhabited by finitely lived agents that we label as *investors*. A unit mass of investors is born at time $t = 0$. Incumbent investors die at Poisson rate δ and new investors are born at Poisson rate δ, so that there is a unit mass of investors alive at any point in time. There is a single, perfectly divisible and storable good available in the economy which we interpret as a stable currency.[6] The good can be produced from the investors' labor effort at a one-to-one rate and the aggregate stock of goods stored is denoted by S_t. There are also scalable but risky long-term investment opportunities, which we label as *trees* in the spirit of Lucas (1978). At any date t, goods can be used to plant new trees at a one-to-one rate. There is idiosyncratic investment risk in the following sense: a tree generates a *fruit* of y goods with Poisson arrival rate ϕ and the tree is destroyed exogenously after generating fruit. The expected real return r from planting a tree is $r = \phi(y - 1)$. We let A_t denote the stock of living trees and we let

[6]Interpreting a fiat currency as a real good is natural when we think about our model as describing a small part of an economy.

$I_t \geq 0$ denote the planting of new trees. The law of motion for A_t reads

$$\dot{A}_t = I_t - \phi A_t. \tag{1}$$

Prefereces. Preferences of an investor born at time t are represented by the function

$$\mathcal{U}_t = -h_t + \mathbb{E}\left\{e^{-\rho T} u(c_{T,t})\right\}, \tag{2}$$

where $h_t \geq 0$ is the investors's labor effort at time t, T is the time period at which the investor dies, $c_{T,t}$ is the investor's consumption at the moment of death, ρ is the rate of time preference, and $u : \mathbb{R}_+ \mapsto \mathbb{R}$ is concave with the standard Inada properties. Time of death T is random and follows from a Poisson distribution with arrival rate δ. We interpret birth and the associated labor effort as respectively market entry and initial investment. Death and the associated consumption represent market exit and disinvestment, which constitute idiosyncratic risk to have a role for risk-sharing arrangements. To have a stationary efficient allocation, we assume that y is such that $r = \rho$.[7]

Resource constraints. When the economy starts at time $t = 0$, there is a unit mass of investors devoting labor effort h_0. There is no consumption yet, so the goods produced from labor are used for (i) planting trees or (ii) storage. Thus,

$$h_0 = A_0^+ + S_0^+. \tag{3}$$

Consumption, the production of goods from labor, and the amount of fruit produced from trees become flows at time $t > 0$. Aggregate flow consumption is

$$c_t = \int_0^t \delta^2 e^{-\delta(t-\tau)} c_{t,\tau} \mathrm{d}\tau + \delta e^{-\delta t} c_{t,0} \tag{4}$$

and the aggregate flow labor supply is δh_t. The aggregate resource constraint reads as

$$(\rho + \phi) A_t + \delta h_t = c_t + I_t + \dot{S}_t. \tag{5}$$

[7]Our assumption that assets earn the rate-of-time preference is natural when we think about our model as describing a small part of an economy.

3 Arrow-Debreu allocation

We study first an Arrow-Debreu setting in which the market for time-t good opens already at $t = 0$. Let p_t denote the Arrow-Debreu price of the time-t good.[8] The availability of storage implies $p_\tau \leq p_t \ \forall \tau \geq t$, i.e., prices cannot increase over time. Furthermore, we have $S_t^+ > 0 \Rightarrow \dot{p}_t = 0$, i.e., storage is undertaken only when prices remain constant. Investors' ability to plant trees implies

$$p_t \geq V_t \equiv \int_t^\infty (\rho + \phi) e^{-\phi(\tau - t)} p_\tau \mathrm{d}\tau \ \forall t \tag{6}$$

and furthermore $I_t > 0 \Rightarrow p_t = V_t$, i.e., a tree is planted only when the present value of its fruit, denoted by V_t, equals the cost of planting p_t. Note that $\dot{V}_t = -(\rho + \phi)p_t + \phi V_t$. With $p_t \geq V_t$, this implies

$$-\rho V_t \geq \dot{V}_t, \quad \text{with “=” if } I_t > 0. \tag{7}$$

An investor born at time t chooses $\{h_t, (c_{\tau,t})_{\tau=t}^\infty\}$ to maximize

$$-h_t + \int_t^\infty \delta e^{-(\rho+\delta)(\tau-t)} u(c_{\tau,t}) \mathrm{d}\tau \tag{8}$$

subject to the budget constraint

$$\int_t^\infty \delta e^{-\delta(\tau-t)} p_\tau c_{\tau,t} \mathrm{d}\tau \leq p_t h_t. \tag{9}$$

Equation (9) states that the expected consumption expenditure of the investor, i.e., taking into account the probability distribution over possible times of death, cannot exceed the value of the investor's labor effort at birth. In this sense, the Arrow-Debreu environment allows the investor to take out actuarially fair insurance against death. We can substitute (9) into (8) to find that the investor chooses $(c_{\tau,t})_{\tau=t}^\infty$ to maximize

$$\int_t^\infty \delta e^{-(\rho+\delta)(\tau-t)} \left[u(c_{\tau,t}) - e^{\rho(\tau-t)} \frac{p_\tau}{p_t} c_{\tau,t} \right] \mathrm{d}\tau, \tag{10}$$

[8] Here, the level of p_t is meaningless—only the relative prices matter and the level of p_t is indeterminate.

so that the optimal consumption and labor choices satisfy

$$c_{\tau,t} = u'^{-1}\left(\frac{e^{\rho(\tau-t)}p_\tau}{p_t}\right) \quad \text{and} \quad h_t = \int_t^\infty \delta e^{-\delta(\tau-t)}\frac{p_\tau}{p_t}u'^{-1}\left(\frac{e^{\rho(\tau-t)}p_\tau}{p_t}\right)\mathrm{d}\tau. \tag{11}$$

Aggregate consumption at time $t > 0$ is thus given by

$$c_t = \int_0^t \delta^2 e^{-\delta(t-\tau)}u'^{-1}\left(\frac{e^{\rho(t-\tau)}p_t}{p_\tau}\right)\mathrm{d}\tau + \delta e^{-\delta t}u'^{-1}\left(\frac{e^{\rho t}p_t}{p_0}\right). \tag{12}$$

We now characterize the allocation in which there is no storage and only planting of trees, which turns out to be an Arrow-Debreu equilibrium. We have $I_t > 0\ \forall t$, so $\dot{p}_t = -\rho p_t\ \forall t$, i.e., prices decline at the rate $r = \rho$, which also implies $S_t = 0\ \forall t$. For notational convenience, we define $c^\star \equiv u'^{-1}(1)$. Using $\frac{e^{\rho(\tau-t)}p_\tau}{p_t} = 1$, expressions for individual consumption, individual labor supply, and aggregate consumption simplify to

$$c_{\tau,t} = c^\star, \quad h_t = \frac{\delta c^\star}{\rho+\delta}, \quad \text{and} \quad c_t = \delta c^\star. \tag{13}$$

From the aggregate resource constraints, we find that the stock of trees develops according to

$$\dot{A}_t = (\rho+\phi)A_t - \delta c^\star + \frac{\delta^2 c^\star}{\rho+\delta}, \quad \text{with } A_0^+ = \frac{\delta c^\star}{\rho+\delta} \quad \Rightarrow \quad \frac{\dot{A}_t}{A_t} = \rho\left(1 - \frac{A_0^+}{A_t}\right). \tag{14}$$

Clearly, this implies $A_t = A_0^+\ \forall t > 0$, which in turn requires $I_t > 0\ \forall t$—we found an Arrow-Debreu equilibrium. By the first welfare theorem, the resulting allocation is Pareto efficient. By assuming that u is logarithmic we can furthermore prove uniqueness, but this is not necessary for our results.

Lemma 1. *With $u(c) = \ln(c)$, there is a unique Arrow-Debreu equilibrium.*

Implementing the Arrow-Debreu allocation may require transfers between generations of investors. Suppose that every generation has its own stock of trees. Let $A_{t,T}$ denote the stock of trees owned by generation T at time $t \geq T$, where we measure the stock of trees relative to the mass of investors born at time T. This definition implies $A_t = A_{t,0} + \int_0^t \delta A_{t,T}\mathrm{d}T$. Generation T's stock of trees at time $t \geq T$ per investor still alive at time $t \geq T$, denoted by $a_{t,T}$, is

$$a_{t,T} = A_{t,T}e^{\delta(t-T)}. \tag{15}$$

Without storage, the law of motion for $A_{t,T}$ reads

$$\dot{A}_{t,T} = \rho A_{t,T} - \delta e^{-\delta(t-T)} c_{t,T}, \quad \text{where } A^{+}_{T,T} = h_T. \tag{16}$$

The allocation requires intergenerational transfers at time t if and only if there exists a $t > T$ for which $\dot{A}_{t,T} < -\phi A_{t,T}$. In words, at time t, generation T wants to reduce its stock of trees at a rate greater than the gestation of fruit, which is only feasible when the implicit ownership of trees is transferred to a different generation of investors.

Using (13) in (16), we obtain

$$\dot{A}_{t,T} = \rho A_{t,T} - (\rho+\delta) A^{+}_{T,T} e^{-\delta(t-T)} \quad \Rightarrow \quad \frac{\dot{a}_{t,T}}{a_{t,T}} = (\rho+\delta)\left(1 - \frac{a^{+}_{T,T}}{a_{t,T}}\right), \tag{17}$$

so clearly, $a_{t,T} = a^{+}_{T,T}$ $\forall t > T$. This implies directly that $\dot{A}_{t,T} = -\delta A_{t,T}$, i.e., the stock of trees owned by generation T diminishes at a rate δ. Thus, implementing the Arrow-Debreu allocation requires inter-generational transfers if and only if $\delta > \phi$—a tree has a longer expected lifetime than an investor.

4 A decentralized economy with tradable trees

Consider a decentralized economy in which investors can undertake storage and plant trees by themselves. There are no markets for time-t goods where investors can trade in advance, but trees, which are identical once planted, can be sold in a continuously-open secondary market for v_t goods.[9] We rule out short-selling.

We have $v_t \leq 1$ in any possible equilibrium, as $v_t > 1$ allows for arbitrage by planting trees and immediately selling them. Trees are only planted if they cannot be acquired more cheaply in the secondary market, so $I_t > 0 \Rightarrow v_t = 1$. The real return from holding a tree from time t to time $t+\varepsilon$, with $\varepsilon > 0$ but infinitesimally small, satisfies

$$r^{+}_{v,t} v_t = \rho + \phi(1 - v_t) + \dot{v}_t. \tag{18}$$

All trees would be offered for sale when $r^{+}_{v,t} < 0$ because storage is available as an alternative investment technology, so we must have $A_t > 0 \Rightarrow r^{+}_{v,t} \geq 0$. Likewise, trees

[9]Interpreting continuous time as the limit of discrete time, we consider v_t as an ex-dividend price.

cannot dominate storage whenever storage is undertaken, so $S_t^+ > 0 \Rightarrow r_{v,t}^+ \leq 1$.

All incumbent investors can save at an effective real return of $\max\{r_{v,t}, 0\}$ due to the possibility of storing goods and trading trees. Assuming that the process $(\max\{r_{v,t}, 0\})_{t=0}^{\infty}$ is integrable, an investor born at time t consumes

$$c_{\tau,t} = e^{\int_t^\tau \max\{r_{v,s},0\}\mathrm{d}s} h_t \tag{19}$$

upon death at time τ. It follows that h_t is chosen to maximize

$$\int_t^\infty \delta e^{-(\rho+\delta)(\tau-t)} u\left(e^{\int_t^\tau \max\{r_{v,s},0\}\mathrm{d}s} h_t\right) \mathrm{d}\tau - h_t. \tag{20}$$

The optimal choice for h_t thus follows uniquely from the first-order condition

$$0 = \int_t^\infty \delta e^{\int_t^\tau (\max\{r_{v,s},0\}-\rho-\delta)\mathrm{d}s} u'\left(e^{\int_t^\tau \max\{r_{v,s},0\}\mathrm{d}s} h_t\right) \mathrm{d}\tau - 1 \tag{21}$$

and aggregate consumption at time t satisfies

$$c_t = \int_0^t \delta^2 e^{\int_\tau^t (\max\{r_{v,s},0\}-\delta)\mathrm{d}s} h_\tau \mathrm{d}\tau + \delta e^{\int_0^t (\max\{r_{v,s},0\}-\delta)\mathrm{d}s} h_0. \tag{22}$$

We show there is a decentralized equilibrium in which $I_t > 0\ \forall t$. This implies $v_t = 1\ \forall t$, so that $r_{v,t} = \rho > 0\ \forall t > 0$ and $S_t = 0\ \forall t > 0$. From the aggregate resource constraint (5), we find that the stock of trees develops according to

$$\dot{A}_t = \rho A_t - \int_0^t \delta^2 e^{(\rho-\delta)(t-\tau)} h_\tau \mathrm{d}\tau - \delta e^{(\rho-\delta)t} h_0 + \delta h_t, \quad \text{with } A_0^+ = h_0. \tag{23}$$

We have that $h_t = h_0 = A_0^+\ \forall t$ when $r_{v,t}$ is time-invariant, so

$$\dot{A}_t = \rho A_t + A_0^+ \left(\delta - \delta e^{(\rho-\delta)t} - \int_0^t \delta^2 e^{(\rho-\delta)(t-\tau)} \mathrm{d}\tau\right). \tag{24}$$

Consider again $A_{t,T}$—trees owned at time t by the generation born at time T. It satisfies

$$\dot{A}_{t,T} = \rho A_{t,T} - \delta e^{(\rho-\delta)(t-T)} A_{T,T}^+ \quad \Rightarrow \quad A_{t,T} = A_{T,T}^+ e^{(\rho-\delta)(t-T)}. \tag{25}$$

Inter-generational transfers are needed when $\exists\ t > T$ such that $\dot{A}_{t,T}/A_{t,T} < -\phi$, which is the case if and only if $\rho + \phi < \delta$. The decentralized economy thus relies less on inter-generational transfers compared to the Arrow-Debreu economy.

Recall that $A_t = A_{t,0} + \delta \int_0^t A_{t,T}\mathrm{d}T$. With $A^+_{T,T} = A^+_{0,0} = A^+_0 \ \forall T$, we thus obtain

$$A_t = A_0^+ \left[e^{(\rho-\delta)t} + \delta \int_0^t e^{(\rho-\delta)(t-\tau)}\mathrm{d}\tau \right] = A_0 \frac{\rho e^{(\rho-\delta)t} - \delta}{\rho - \delta} \quad \Rightarrow \quad \frac{\dot{A}_t}{A_t} = \frac{\rho(\rho-\delta)e^{(\rho-\delta)t}}{\rho e^{(\rho-\delta)t} - \delta}. \tag{26}$$

We have $\dot{A}_t/A_t \geq 0$ with $\lim_{t\to\infty}[\dot{A}_t/A_t] = \rho - \delta$ when $\rho \geq \delta$, and $\dot{A}_t/A_t > 0$ with $\lim_{t\to\infty} A_t = \frac{\delta}{\delta-\rho}A_0$ and $\lim_{t\to\infty}[\dot{A}_t/A_t] = 0$ when $\rho < \delta$. Summarizing, the stock of trees always expands, so that $I_t > 0 \ \forall t$—we found a decentralized equilibrium, for which the growth rate of the stock of trees converges to $\max\{0, \rho - \delta\}$. By assuming that u is logarithmic we can again prove uniqueness, but this is not crucial.

Lemma 2. *With $u(c) = \ln(c)$, there is a unique decentralized equilibrium.*

The decentralized equilibrium allocation is socially inefficient—every investor is worse off compared to the Arrow-Debreu allocation. This follows from the fact that individual consumption $c_{\tau,t}$ increases at a rate ρ with time of death τ in the decentralized allocation, while prices in the Arrow-Debreu allocation declined at the rate ρ. The decentralized equilibrium allocation $\{h_t, (c_{\tau,t})_{\tau=t}^{\infty}\}$ therefore respects the budget constraint for any investor in the Arrow-Debreu equilibrium, but is never chosen by an investor in the Arrow-Debreu equilibrium.

Proposition 1. *The decentralized allocation characterized above is socially inefficient.*

The inefficiency relates to the actuarially fair insurance against death which is available in the Arrow-Debreu economy, so that investors choose time-invariant consumption. Insurance is, however, infeasible in the decentralized economy. Investors would then namely claim death immediately after birth and invest the proceeds in the secondary market to earn the return ρ, so as to increase future consumption. This is reminiscent of Jacklin's (1987) finding.

5 Preliminaries for a stablecoin economy

In what follows we construct a trading arrangement featuring coins to implement the Arrow-Debreu allocation when insurance markets are absent as examined in the last section.

We assume that investors can claim birth only at the time of actual birth and that trees are illiquid, whereas death is unobserveable and coins are tradable in a secondary market. The investors born at time T coalesce to issue a coin. They form an *issuer* that plants trees and issues a prefectly divisible coin in an initial coin offering (ICO) to the member investors and *only* to the member investors. We assume that $\phi \geq \delta$ so that the Arrow-Debreu allocation requires only intra-generational trade. Also, $\phi \geq \delta$ implies that issuers invest in assets which have an expected maturity that is shorter than the expected time until an investor needs to redeem coins for consumption. This is quite realistic given that stablecoin issuers invest mostly in short-term assets.

We interpret the stablecoin version of our model as supported by a digital platform, for instance a mobile-phone app or an online portal. Investors can use the platform to hold a currency (goods in our model), to create and hold stablecoins, to trade a stablecoin for other stablecoins or currency, and to pay for consumption. Consumption could represent the actual purchase of consumption goods, for instance with online retailers, but also the purchase of cryptoassets like bitcoin. We implicitly assume that those who sell consumption goods or cryptoassets prefer to hold physical currency. When sellers accept the stablecoin in payment, they would then immediately convert it into currency and move this currency off the platform. The implicit assumption follows from the fact that the investors in our model consume goods, where we interpret goods as currency.

The observability of birth and the unobservability of death imply a form of limited record keeping by the platform. Particularly, investors can claim a need to consume at any time, which would imply that currency is moved off the platform and stored by the investor itself until actual death arises. However, moving currency off the platform is observed, so that the investor can be excluded from using this currency to participate again in an ICO.[10]

6 Monopolistic issuance of stablecoins

In this section we consider a setup in which investors hold and trade only their own coin, i.e., a single generation that operates in isolation. This represents a platform with only a single stablecoin.

[10] In case of pseudo-anonymity, one can argue that investors find it too costly to move fiat currency off the platform and then on the platform again under a different identity.

A unit mass of investors is born at time $t = 0$, after which the pool of investors gradually declines at a rate δ—no new investors are born. The Arrow-Debreau allocation we seek to implement is

$$c_{t,0} = c^\star, \quad c_t = \delta e^{-\delta t} c^\star, \quad h_0 = \frac{\delta c^\star}{\rho + \delta}, \quad A_t = \frac{e^{-\delta t} \delta c^\star}{\rho + \delta}, \quad \text{and} \quad I_t = (\phi - \delta) A_t. \tag{27}$$

D_0^+ units of the coin are minted in an ICO by the issuer, which seeks to maximize its members' utility. The ICO's proceeds are used to plant A_0^+ trees. We set $A_0^+ = D_0^+$, so that the ICO price of the coin is normalized to one. The issuer allows investors to redeem a coin for x_t goods at times $t > 0$, where x_t represents a pegged conversion rate, subject to a sequential service constraint. With A_t trees owned by the issuer, the flow payment to cover redemption is at most $(\phi + \rho) A_t$.

Suppose that the issuer commits to use the full amount $(\phi + \rho) A_t$ for redemption when necessary. When a fraction χ_t of the coins D_t in circulation are offered for redemption, the arrival rate of a successful redemption from the perspective of the investor is

$$\alpha_t = \frac{(\phi + \rho) A_t}{x_t \chi_t D_t}. \tag{28}$$

To implement the Arrow-Debreu allocation, we let $x_t = \frac{\delta + \rho}{\delta} \frac{A_t}{D_t}$. When only dying investors redeem their coins, they can then do so with probability one. We then furthermore have that $\dot{A}_t = -\delta A_t$ when the issuer uses the fruit left after redemption for planting new trees—the dynamic development of A_t is the same as in the Arrow-Debreu allocation when only the dying investors redeem.

At all times $t > 0$ the coin can be traded among investors in a secondary market, i.e., on the platform, at price q_t.[11] We focus on equilibria in which $x_t \leq q_t$, as otherwise not even the dying investors would redeem their coins with the issuer. Dying investors are indifferent between selling their coins in the secondary market or redeeming them when $x_t = q_t$, in which case we assume that all dying investors choose to redeem. All investors attempt to have their coins redeemed—a run takes place—when $x_t > q_t$. A run features

[11] Interpreting continuous time as the limit of discrete time, we consider q_t as a cum-dividend price, i.e., including the option value of redeeming the coin at time t.

$\chi_t = 1$ by definition and our assumption that $x_t = \frac{\delta+\rho}{\delta}\frac{A_t}{D_t}$ implies

$$\alpha_t = \delta\frac{\phi+\rho}{\delta+\rho}. \tag{29}$$

The return from holding the coin therefore satisfies

$$r_t^+ q_t = \delta\frac{\phi+\rho}{\delta+\rho}\max\{0, x_t - q_t\} + \dot{q}_t \tag{30}$$

and the law of motion for the stock of coins reads as

$$\dot{D}_t = \begin{cases} -\delta\frac{\phi+\rho}{\delta+\rho}D_t & \text{if } q_t < x_t \\ -\delta D_t & \text{if } q_t = x_t \end{cases}. \tag{31}$$

The issuer cannot plant new trees during a run since all fruit is used for redemption. Thus

$$\dot{A}_t = \begin{cases} -\phi A_t & \text{if } q_t < x_t \\ -\delta A_t & \text{if } q_t = x_t \end{cases}, \tag{32}$$

i.e., during a run the stock of trees diminishes faster since we assumed $\phi \geq \delta$. When a run does not takes place, so that $q_t = x_t\ \forall t > 0$, we have

$$r_t^+ = \frac{\dot{q}_t}{q_t} = \frac{\dot{x}_t}{x_t} = \frac{\dot{A}_t}{A_t} - \frac{\dot{D}_t}{D_t} = 0, \tag{33}$$

where the second equality uses $q_t = x_t$, the third equality follows from the specification of x_t, and the fourth equality uses $\dot{D}_t/D_t = -\delta$ when there is no run. Furthermore, since we have normalized $A_0^+ = D_0^+$, we have $x_0^+ = \frac{\delta+\rho}{\delta}$. Because the ICO price of a coin is one, the investors choose D_0^+ to maximize

$$\int_0^\infty \delta e^{-(\delta+\rho)t} u\left(\frac{\delta+\rho}{\delta}D_0^+\right) - D_0^+. \tag{34}$$

So, $A_0^+ = D_0^+ = \frac{\delta c^\star}{\rho+\delta}$ and $c_{t,0} = c^\star\ \forall t > 0$. The Arrow-Debreu allocation is implemented and although investors can store goods, they have no incentive to do so. First, the real return from holding the coin at all times $t > 0$ is zero, i.e., it is not dominated by storage. Second, the investor strictly prefers to hold the coin rather than invest in storage at time $t = 0$ because $x_0^+ > 1$, i.e., the real return from holding the coin from time $t = 0$ to time

$t = \varepsilon$, where $\varepsilon > 0$ is infinitesimally small, approaches infinity.

Because the issuer does not limit redemption, there can be unexpected runs at any time $T > 0$. Suppose first that $x_t = q_t \; \forall t < T$. Then, consider what happens when the secondary market price drops discontinuously and unexpectedly at time T, so that we have $q_T < x_T$. The aggregate resource constraint implies

$$\dot{S}_T = (\alpha_T x_T - \delta q_T)\, D_T, \tag{35}$$

where δD_T is the flow of coins offered for sale by dying investors who want to consume and $\alpha_T x_T D_T$ is the flow of fruit paid by the issuer. Given the secondary market price q_T, the difference between fruit paid by the issuer and consumption is therefore $(\alpha_T x_T - \delta q_T)\, D_T$, which indicates the change in storage. Using the characterization of α_t, we can write

$$\dot{S}_T = \delta \left(\frac{\phi + \rho}{\delta + \rho} \frac{x_T}{q_T} - 1 \right) q_T D_T. \tag{36}$$

Given that $x_T > q_T$ during a run and that $\phi \geq \delta$, it follows that $\dot{S}_T > 0$. Intuitively, the low secondary market price implies that dying investors, who are forced to sell in the secondary market, consume less than the fruit paid by the issuer. The excess supply of fruit is then stored since investors cannot plant trees on their own. Investors are unwilling to undertake storage when $r_T^+ > 1$, and investors would attempt to sell in the secondary market when $r_T^+ < 0$. Clearance of the secondary market therefore requires $r_T^+ = 0$, so

$$0 = \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{x_T}{q_T} - 1 \right) + \frac{\dot{q}_T}{q_T}. \tag{37}$$

At this point, it is convenient to define $\theta_t \equiv q_t / x_t$, i.e., the discount at which the coin trades in the secondary market. With $\theta_T < 1$ we thus have

$$\begin{aligned} 0 &= \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{1}{\theta_T} - 1 \right) + \frac{\dot{\theta}_T}{\theta_T} + \frac{\dot{x}_T}{x_T} & (38) \\ &= \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{1}{\theta_T} - 1 \right) + \frac{\dot{\theta}_T}{\theta_T} + \frac{\dot{A}_T}{A_T} - \frac{\dot{D}_T}{D_T} & (39) \\ &= \delta \frac{\phi + \rho}{\delta + \rho} \frac{1}{\theta_T} + \frac{\dot{\theta}_T}{\theta_T} - \phi & (40) \end{aligned}$$

where the second line uses that $\dot{x}_t / x_t = \dot{A}_t / A_t - \dot{D}_t / D_t \; \forall t$ and the third line uses that

$\dot{A}_t/A_t = -\phi$ and $\dot{D}_t/D_t = -\alpha_t$ during a run. Summarizing, as long as $\theta_t < 1$, it must be that

$$\dot{\theta}_t = \phi\theta_t - \delta\frac{\phi+\rho}{\delta+\rho}. \tag{41}$$

Because q_t cannot drop discontinuously with positive probability (if it does, at time $t-\varepsilon$ investors either start to run already or start to sell in the secondary market), it follows that starting from time T, we have

$$\theta_t = \min\left\{\theta_T e^{\phi(t-T)} + \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}\left(1-e^{\phi(t-T)}\right), 1\right\}. \tag{42}$$

There is a steady state at $\theta = \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho} < 1$ and at $\theta = 1$. Note that $\lim_{\phi\to\delta}\frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho} = 1$. For $\theta_T < \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}$, θ_t will contract at a rate that increases over time, so that at some $T' > T$, we have that $q_{T'} = 0$, which cannot be an equilibrium. For $\theta_T \in \left(\frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}, 1\right)$, θ_t grows at an increasing rate, until at some $T' > T$, we have $\theta_{T'} = 1$. There is no kink in the development of q_t—it can be verify that $\lim_{\varepsilon\to 0}\frac{\dot{q}_{T'-\varepsilon}}{q_{T'-\varepsilon}} = \lim_{\varepsilon\to 0}\frac{\dot{q}_{T'+\varepsilon}}{q_{T'+\varepsilon}} = 0$. Summarizing, at any time $T > 0$, a run can start unexpectedly when θ_T jumps from 1 into the interval $\left[\frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}, 1\right)$. The run either continues forever when $\theta_T = \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}$, or it lasts for a finite period of time when $\theta_T \in \left(\frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}, 1\right)$. The run furthermore implies that all investors are worse off, as the stock of trees contracts at an inefficiently high rate.

Proposition 2. *Unexpected runs exist when $\phi > \delta$ and are socially inefficient.*

The reason why runs exist is that the issuer cannot plant new trees during a run—the fruit from gestating trees is exhausted to cover redemption. This implies that x_t declines during the run, which explains why investors attempt to run: Anticipating that x_t is going to decline, investors attempt to redeem coins as they can use storage as an alternative. This suggest that the issuer should restrict redemption in such a way that it maintains investment at $I_t = (\phi-\delta)A_t$, even in case of a run. Coins with this property are called micro-well-designed coins.

In a run in which redemption is limited such that $I_t = (\phi-\delta)A_t$, one has $\alpha_t = \delta$. The rate at which coins are redeemed, from the perspective of the issuer, therefore equals δ no matter whether a run takes place or whether there is business as usual. The law of motion for coins therefore reads

$$\dot{D}_t = -\delta D_t. \tag{43}$$

From equation (35), based on the aggregate resource constraint, we obtain

$$\dot{S}_T = \delta(x_T - q_T)D_T, \tag{44}$$

so that as before, $\dot{S}_T > 0$ when there is a run. This implies that the return on coins should be zero, and this return satisfies

$$r_T^+ q_T = \delta(x_T - q_T) + \dot{q}_T \tag{45}$$

because investors can now redeem at arrival rate δ instead of $\delta\frac{\phi+\rho}{\delta+\rho}$. Imposing $r_T^+ = 0$ and defining θ as before, we have

$$\begin{aligned} 0 &= \delta\left(\frac{1}{\theta_T} - 1\right) + \frac{\dot{\theta}_T}{\theta_T} + \frac{\dot{x}_T}{x_T} && (46)\\ &= \delta\left(\frac{1}{\theta_T} - 1\right) + \frac{\dot{\theta}_T}{\theta_T} + \frac{\dot{A}_T}{A_T} - \frac{\dot{D}_T}{D_T} && (47)\\ &= \delta\left(\frac{1}{\theta_T} - 1\right) + \frac{\dot{\theta}_T}{\theta_T}, && (48)\end{aligned}$$

where we now used that $\dot{A}_t/A_t = -\delta$ and $\dot{D}_t/D_t = -\delta$. Starting from time T, we thus have

$$\theta_t = \min\left\{(\theta_T - 1)e^{\delta(t-T)} + 1, 1\right\}. \tag{49}$$

There is now only one steady state at $\theta = 1$, i.e., in which there is no run. If unexpectedly a run occurred at time T, we would have $\theta_T < 1$, so that θ_t starts to diminish at an increasing rate, until at some $T' > T$, we have $q_{T'} = 0$, which cannot be an equilibrium.

Investors have no reason to run because a run does not lead to a change in x_t. Although a run implies that a dying investor cannot redeem coins—only an infinitesimally small fraction of its coin holdings can be redeemed—, dying investors can sell their coins in the secondary market at a price $q_t = x_t$. The reason is exactly that the coin issuer restricts redemption in case of a run so that the dynamic development of the stock of trees is left unaffected. When a run would occur by accident, the combination of competitive pricing and limited redemption therefore imply that dying investors can still trade the coin in the secondary market at a price equal to the pegged conversion rate x_t.

7 Stablecoin competition

We now turn back to the baseline model where at every point in time, a new generation of investors is born. The stablecoin economy then represents a platform on which multiple micro-well-designed stablecoins[12] can be traded (see Section 5). We continue to assume that $\phi \geq \delta$—inter-generational transfers are not necessary to implement the Arrow-Debreu allocation.

7.1 Incumbent issuers

Consider first the problem of the incumbent issuers at time T. Recall that every issuer is a coalition of investors born at a particular date $s \in [0, T)$. Let the issuer representing investors born at time s have $D_{T,s}$ coins outstanding and let it own $A_{T,s} > 0$ trees, where these quanitities are expressed relative to the initial mass of investors born at time s. Furthermore, assume that $D_{T,s}$ is fully held by the investors born at time s and impose the normalization $D_{T,s} = A_{T,s}$. Finally, let $Z_{t,s}$ denote generation-s's holdings of other coins and/or storage at time $t \geq T$. We suppose that $Z_{T,s} = 0$ to represent a situation in which the incumbent investors initially only hold their own coin, which, in turn, is backed by a positive stock of trees.

The incumbent issuer maximizes the utility of the investors it represents, but has to take as given the return process $(r_t)_{t=T}^{\infty}$ that these investors can earn by storing goods or trading other coins, where the option of storage implies $r_t \geq 0 \ \forall t > T$. Particularly, $(c_{t,s}, A_{t,s}, Z_{t,s})_{t=T}^{\infty}$ is chosen to maximize

$$\int_T^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s}) \mathrm{d}t \tag{50}$$

subject to the law of motion

$$\dot{A}_{t,s} + \dot{Z}_{t,s} = \rho A_{t,s} + r_t Z_{t,s} - \delta e^{-\delta(t-s)} c_{t,s}, \tag{51}$$

the constraints $\dot{A}_{t,s} \geq -\phi A_{t,s}$ and $Z_{t,s} \geq 0$, the starting values $A_{T,s} > 0$ and $Z_{T,s} = 0$, and the incentive-feasibility constraint $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$. The intuition for the incentive-

[12]Each generation that coalesces to issue a coin has an incentive to issue a well-designed coin as this maximizes the expected utility of the members of the generation.

feasibility constraint is as follows. The issuer's coin is designed to provide consumption process $(c_{t,s})_{t=T}^{\infty}$ for the investors it represents. The effective return from holding the coin from time t to time $t+\varepsilon$, with $\varepsilon > 0$ infinitesimaly small, is therefore $\dot{c}_{t,s}/c_{t,s}$. But with the outside option of trading other coins and/or storage, investors can earn an effective real return $\max\{r_t^+, \dot{c}_{t,s}/c_{t,s}\}$. When $r_t^+ > \dot{c}_{t,s}/c_{t,s}$, the investors' outside option would thus dominate holding their own coin, so that they would deviate from the targeted process for consumption.

Regarding an issuer's desire to hold other coins, we find:

Lemma 3. *$Z_{t,s} = 0 \; \forall t > T$ is optimal only if $r_t < \rho \; \forall t > T$.*

The intuition is that when the return on other coins exceeds the fundamental return on trees $r = \rho$, no issuer is willing to plant new trees so that all incumbent issuers attempt to invest in other coins. This, however, violates the notion of a general equilibrium. In what follows we therefore focus on return processes $(r_t)_{t=T}^{\infty}$ which satisfy $r_t \in [0, \rho] \; \forall t > T$.

With $r_t \in [0, \rho] \; \forall t > T$, it turns out that the incentive feasibility constraint is always binding when the constraint $\dot{A}_{t,s} \geq -\phi A_{t,s}$ for the dynamic development of the stock of trees is always slack.

Lemma 4. *The optimal consumption process $(c_{t,s})_{t=T}^{\infty}$ satisfies $\dot{c}_{t,s}/c_{t,s} = r_t^+ \; \forall t \geq T$ when the constraint $\dot{A}_{t,s} \geq -\phi A_{t,s}$ is slack for all $t \geq T$.*

How is $c_{T,s}$ then determined? Defining $a_{t,s} \equiv A_{t,s} e^{\delta(t-s)}$, and focusing on an outcome in which $Z_{t,s} = 0 \; \forall t \geq T$, we have the differential equation

$$\dot{a}_{t,s} = (\rho + \delta)\, a_{t,s} - \delta c_{T,s} e^{\int_T^t r_s \mathrm{d}s}, \tag{52}$$

where we use that Lemma 4 implies $c_{t,s} = c_{T,s} e^{\int_T^t r_s \mathrm{d}s}$. The solution to the differential equation is

$$a_{t,s} = e^{(\rho+\delta)(t-T)} a_{T,s} - \delta c_{T,s} \int_T^t e^{(\rho+\delta)(t-\tau) + \int_T^\tau r_s \mathrm{d}s} \mathrm{d}\tau. \tag{53}$$

Optimal choices have to satisfy the transversality condition $\lim_{t\to\infty} \left\{ e^{-(\rho+\delta)(t-T)} a_{t,s} \right\} = 0$. Rewriting the equation above, we have

$$e^{-(\rho+\delta)(t-T)} a_{t,s} = a_{T,s} - \delta c_{T,s} \int_T^t e^{\int_T^\tau [r_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau. \tag{54}$$

The RHS of this equation is well behaved when $t \to \infty$ since $r_t \in [0, \rho]\ \forall t > T$. Imposing the transversality condition then yields

$$c_{T,s} = \frac{a_{T,s}}{\delta}\left[\int_T^\infty e^{\int_T^\tau [r_s - \rho - \delta]\mathrm{d}s}\mathrm{d}\tau\right]^{-1}. \tag{55}$$

It remains to verify that indeed $\dot{A}_{t,s} \geq -\phi A_{t,s}\ \forall t \geq T$. Using the definition for $a_{t,s}$, this is the same as showing that $\dot{a}_{t,s} \geq (-\phi+\delta)a_{t,s}\ \forall t \geq T$. Given that $\dot{a}_{t,s} = (\rho+\delta)a_{t,s} - \delta c_{t,s}$, we thus need

$$\frac{a_{t,s}}{\delta c_{t,s}} \geq \frac{1}{\phi+\rho}\ \forall t \geq T. \tag{56}$$

Using the equations for $c_{t,s}$, $a_{t,s}$, and the fact that $\frac{a_{T,s}}{\delta c_{T,s}} = \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau$, we find

$$\frac{a_{t,s}}{\delta c_{t,s}} = e^{(\rho+\delta)(t-T)}\frac{a_{T,s}}{\delta c_{T,s}}\frac{c_{T,s}}{c_{t,s}} - \frac{c_{T,s}}{c_{t,s}}\int_T^t e^{(\rho+\delta)(t-\tau)+\int_T^\tau r_s \mathrm{d}s}\mathrm{d}\tau \tag{57}$$

$$= e^{-\int_T^t r_s \mathrm{d}s} e^{(\rho+\delta)(t-T)}\frac{a_{T,s}}{\delta c_{T,s}} - e^{-\int_T^t r_s \mathrm{d}s}\int_T^t e^{(\rho+\delta)(t-\tau)+\int_T^\tau r_s \mathrm{d}s}\mathrm{d}\tau \tag{58}$$

$$= e^{-\int_T^t [r_s-\rho-\delta]\mathrm{d}s}\frac{a_{T,s}}{\delta c_{T,s}} - \int_T^t e^{-\int_\tau^t [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \tag{59}$$

$$= e^{-\int_T^t [r_s-\rho-\delta]\mathrm{d}s}\int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau - \int_T^t e^{-\int_\tau^t [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \tag{60}$$

$$= \int_T^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau - \int_T^t e^{-\int_\tau^t [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \tag{61}$$

$$= \int_t^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau, \tag{62}$$

which also shows that the issuer's policy is time consistent. We thus have

$$\dot{A}_{t,s} \geq -\phi A_{t,s} \Leftrightarrow \int_t^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \geq \frac{1}{\phi+\rho}. \tag{63}$$

Since $r_t \geq 0$ by construction, we have $\int_t^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \geq \frac{1}{\rho+\delta}\ \forall t \geq T$. Since we have assumed that $\phi \geq \delta$, it directly follows that $\dot{A}_{t,s} \geq -\phi A_{t,s}\ \forall t \geq T$.

How should the issuer's coin be designed to implement the allocations described above? At times $t \geq T$, the issuer stands ready to redeem a coin for $x_{t,s} > 0$ goods, subject to a sequential service constraint. With $A_{t,s}$ trees owned by the issuer at time t, the time-t flow payment by the issuer is at most $(\phi+\rho)A_{t,s}$. The allocation above, however, suggest that $\dot{A}_{t,s} = \rho A_{t,s} - \delta c_{t,s}e^{-\delta(t-s)}$, so to maintain this path, we assume that the issuer devotes at most

$$\delta c_{t,s} e^{-\delta(t-s)} = e^{-\delta(t-s)} \frac{\delta c_{t,s}}{a_{t,s}} \frac{a_{t,s}}{A_{t,s}} A_{t,s} \tag{64}$$

$$= \left[\int_t^\infty e^{\int_t^\tau [r_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau \right]^{-1} A_{t,s} \tag{65}$$

goods to cover redemption. Such a coin is called micro-well-designed, as it avoids runs on the coin if the coin were the only coin in the economy. This is shown next.

The sequential service constraint implies that if a fraction $\chi_{t,s}$ of the coins $D_{t,s}$ is offered for redemption, the arrival rate of a successful withdrawal from the perspective of an investor is

$$\alpha_{t,s} = \frac{\left[\int_t^\infty e^{\int_t^\tau [r_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau \right]^{-1} A_{t,s}}{x_{t,s} \chi_{t,s} D_{t,s}}. \tag{66}$$

To indeed implement the optimal consumption path we set

$$x_{t,s} = \frac{\left[\int_t^\infty e^{\int_t^\tau [r_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau \right]^{-1}}{\delta} \frac{A_{t,s}}{D_{t,s}}.$$

The coin can be traded in a secondary market at price $q_{t,s}$. As before, we focus on equilibria in which $x_{t,s} \leq q_{t,s}$. When $x_{t,s} = q_{t,s}$ all dying investors redeem their coins and when $x_{s,t} > q_{s,t}$ all investors, dying or not, attempt to have their coins redeemed—a run on coin s takes place. In case of a run, which features $\chi_{t,s} = 1$ by definition, the specification for $x_{t,s}$ implies $\alpha_{t,s} = \delta$. The return earned by holding the coin of issuer s at time t therefore satisfies the differential equation

$$r^+_{t,s} q_{t,s} = \delta \max\{0, x_{t,s} - q_{t,s}\} + \dot{q}_{t,s} \tag{67}$$

and, no matter whether a run takes place or not, we have

$$\dot{D}_{t,s} = -\delta D_{t,s} \quad \text{and} \quad \dot{A}_{t,s} = \left(\rho - \left[\int_t^\infty e^{\int_t^\tau [\tilde{r}_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau \right]^{-1} \right) A_{t,s}. \tag{68}$$

If there is no run at time t, the equilibrium return of holding coin s satisfies $r^+_{t,s} = r^+_t$. This follows from

$$r^+_{t,s} = \frac{\dot{q}_{t,s}}{q_{t,s}} = \frac{\dot{x}_{t,s}}{x_{t,s}} = \left[\int_t^\infty e^{\int_t^\tau [\tilde{r}_s - \rho - \delta] \mathrm{d}s} \mathrm{d}\tau \right]^{-1} + r^+_t - \rho - \delta + \frac{\dot{A}_{t,s}}{A_{t,s}} - \frac{\dot{D}_{t,s}}{D_{t,s}} = r^+_t, \tag{69}$$

where the second equality uses that $q_{t,s} = x_{t,s}$, the third equality follows from the specification of $x_{t,s}$, and the fourth equality uses that $\dot{D}_{t,s}/D_{t,s} = -\delta$ and $\dot{A}_{t,s}/A_{t,s} = \rho - \left[\int_t^\infty e^{\int_t^\tau [r_s - \rho - \delta]\mathrm{d}s}\mathrm{d}\tau\right]^{-1}$.

To demonstrate that there are no idiosyncratic runs on issuer s, suppose first that $x_{t,s} = q_{t,s}$ $\forall t \in [T, \tau)$. Then, consider what happens if at time τ the secondary market price $q_{\tau,s}$ drops unexpectedly so that we have $q_{\tau,s} < x_{\tau,s}$. The aggregate resource constraint implies

$$\dot{Z}_{\tau,s} = \delta \left(\frac{x_{\tau,s}}{q_{\tau,s}} - 1 \right) q_{\tau,s} D_{\tau,s}, \tag{70}$$

where $Z_{\tau,s}$ are generation s's holdings of other coins and/or storage. The return on $Z_{\tau,s}$ is, by definition, r_τ. Given that in a run $x_{\tau,s} > q_{\tau,s}$, it follows immediately that $\dot{Z}_{\tau,s} > 0$. Thus, generation s needs to invest in other coins and/or storage. With $r^+_{\tau,s} > r^+_\tau$, all incumbent investors prefer to hold coin s and with $r^+_{\tau,s} < r^+_\tau$, all households would attempt to sell coin s in the secondary market. Hence, clearance of the secondary market for coin s requires $r^+_{\tau,s} = r^+_\tau$ in case of a run. Thus,

$$r^+_\tau = \delta \left(\frac{x_{\tau,s}}{q_{\tau,s}} - 1 \right) + \frac{\dot{q}_{\tau,s}}{q_{\tau,s}}. \tag{71}$$

Defining $\theta_{t,s} \equiv q_{t,s}/x_{t,s}$, with $\theta_{\tau,s} < 1$, we have

$$\begin{aligned} r^+_\tau &= \delta \left(\frac{1}{\theta_{\tau,s}} - 1 \right) + \frac{\dot{\theta}_{\tau,s}}{\theta_{\tau,s}} + \frac{\dot{x}_{\tau,s}}{x_{\tau,s}} && (72)\\ &= \delta \left(\frac{1}{\theta_{\tau,s}} - 1 \right) + \frac{\dot{\theta}_{\tau,s}}{\theta_{\tau,s}} + \left[\int_\tau^\infty e^{\int_\tau^t [r_s - \rho - \delta]\mathrm{d}s}\mathrm{d}t \right]^{-1} + r^+_\tau - \rho - \delta + \frac{\dot{A}_{\tau,s}}{A_{\tau,s}} - \frac{\dot{D}_{\tau,s}}{D_{\tau,s}} && (73)\\ &= \delta \left(\frac{1}{\theta_{\tau,s}} - 1 \right) + \frac{\dot{\theta}_{\tau,s}}{\theta_{\tau,s}} + r^+_\tau. && (74) \end{aligned}$$

Starting from time τ, we find

$$\theta_{t,s} = \min \left\{ (\theta_{\tau,s} - 1)e^{\delta(t-\tau)} + 1, 1 \right\}. \tag{75}$$

As before, there is a unique steady state at $\theta = 1$. If $\theta_{\tau,s} < 1$, $\theta_{t,s}$ will decline at a rate that increases over time, so that at some $\tau' > \tau$, we will have $q_{\tau',s} = 0$, which cannot be an equilibrium. Thus, there is no run on issuer s.

However, as we will see when modeling the problem of new coin issuers, there is a continuum of processes $(r_t)_{t=T}^{\infty}$ which are consistent with an equilibrium and furthermore, the process can change unexpectedly.

7.2 New coin issuers

We next study the solution for an issuer respresenting a newborn generation at time T. The solution can be derived from the solution for an incumbent issuer, simply because the new issuer becomes incumbent at time $T+\varepsilon$. That is, the issuer selects a micro-well-designed coin and we only need to solve for $D^+_{T,T}$, the amount of coins issued in the ICO. We again use the normalization $D^+_{T,T} = A^+_{T,T}$ so that the ICO price is one. From the problem of the incumbent issuers, it is clear that the consumption path offered by the new issuer satisfies

$$c_{t,T} = \left[\delta \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s} \mathrm{d}\tau\right]^{-1} D^+_{T,T} e^{\int_T^t r_s \mathrm{d}s}, \tag{76}$$

i.e., once issued, the return $r_{t,T}$ earned by coin T mimics the return process $(r_t)_{t=T}^{\infty}$. We furthermore have $c_{t,T} = x_{t,T} = q_{t,T}\ \forall t > T$. The investors born at time T are willing to participate in the ICO if and only if

$$\lim_{\varepsilon\to 0} \frac{q_{T+\varepsilon,T} - D^+_{T,T}}{\varepsilon} \geq r^+_T D^+_{T,T}. \tag{77}$$

Given that $q_{t,T} = c_{t,T}\ \forall t > T$, equation (77) is satisfied if and only if

$$\delta \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s} \leq 1, \tag{78}$$

which holds true since the return process $(r_t)_{t=T}^{\infty}$ satisfies $r_t \in [0,\rho]\ \forall t > T$.

With the ICO indeed taking place and generating the consumption process as specified by (76), it follows that the new issuer chooses $D^+_{T,T}$ to maximize

$$\int_T^\infty \delta e^{-(\rho+\delta)(t-T)} u\left(\left[\delta \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s} \mathrm{d}\tau\right]^{-1} e^{\int_T^t r_s \mathrm{d}s} D^+_{T,T}\right) \mathrm{d}t - D^+_{T,T}. \tag{79}$$

There is a unique solution to this problem which follows from the first-order condition

$$0 = \int_T^\infty e^{\int_T^t [r_s-\rho-\delta]\mathrm{d}s} u'\left(\left[\delta \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau\right]^{-1} e^{\int_T^t r_s \mathrm{d}s} D_{T,T}^+\right)\mathrm{d}t - \int_T^\infty e^{\int_T^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau. \tag{80}$$

This shows that if a micro-well-designed coin wants to be stable, it needs to generate the return process $(r_t)_{t=T}^\infty$ which is exogenous to a single coin.

7.3 General equilibrium

To explore the consequences of competition among micro-well-designed coins, we focus on an equilibrium in which there is no need for inter-generational trade. This means that every ICO should succeed, i.e., each generation issues its own coin, and that $\dot{A}_{t,s} \geq -\phi A_{t,s}\ \forall s \in [0,t)\ \forall t$.

From the preceding analysis, it follows that an equilibrium, in which there is no need for inter-generational trade, is completely described by a process $(r_t)_{t=0}^\infty$ satisfying

$$r_t \in [0,\rho]\ \forall t > 0. \tag{81}$$

Every generation will then issue a coin and the return on any coin in circulation at time $t > 0$ is r_t. Otherwise, all agents would sell the coins with lower returns and switch to coins with higher returns.

We have $r_t \geq 0\ \forall t$ because storage is available as an outside option and furthermore, it implies $\int_t^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau \geq \frac{1}{\phi+\rho}$, so that indeed $\dot{A}_{t,s} \geq -\phi A_{t,s}\ \forall s \in [0,t)\ \forall t$. Also, $\int_t^\infty e^{\int_t^\tau [r_s-\rho-\delta]\mathrm{d}s}\mathrm{d}\tau$ is bounded from above by $1/\delta$ and from below by $1/(\delta+\rho)$, so that we have well-behaved solutions.

Summarizing, competition between coin issuers generates a continuum of perfect-foresight equilibria, which include cycles and non-stationary dynamics. With $r_t = 0\ \forall t$, the efficient Arrow-Debreu allocation is implemented and with $r_t = \rho\ \forall t$, the allocation is the same as in the inefficient decentralized equilibrium. Also, despite the fact that there are no runs, unexpected changes in the process $(r_t)_{t=0}^\infty$ can occur, as they are self-fulfilling.

Our findings imply that the entire economy may unexpectedly switch from the efficient Arrow-Debreu allocation to the inefficient decentralized equilibrium allocation. The reason is that incumbent coin issuers anticipate competition from future coin issuers. If the incumbent coin issuers expect the future issuers to issue a coin which is consistent with the Arrow-Debreu allocation, i.e., a macro-well-designed coin, the incumbent coin issuers also issue a macro-well-designed coin. However, if they expect that the future issuers will issue a micro-well-designed coin that pays a positive rate of return, the incumbent coin issuers have to change the design of their coins to prevent a run. In fact, they change the design of their coins in such a way that they can provide the same return as the competing coin. As a result, the equilibrium allocation in the stablecoin economy is driven away from the efficient Arrow-Debreu allocation because of a micro-well-designed yet contagious stablecoin.

More precisely, the result suggests that competition between issuers is a source of instability. There are overlapping generations of investors, and the members of a specific generation coalesce to act as an issuer. Because its members have the outside option to trade coins of other issuers in a secondary market, an issuer's ability to implement the efficient allocation by means of a macro-well-designed coin depends on the actions of all other coin issuers—issuers face a coordination game.

8 Conclusion

We have presented a simple model that allows to study how stablecoins should be designed, whether there is contagion when multiple stablecoins compete and how such contagion can be prevented. At the individual coin level, it seems to be straightforward. An investment rule and limited redemption avoids runs and guarantees the stability of the coin. At the macro level, micro-well-designed coins, however, are not enough as a coin paying interest is contagious for other coins. This provides a rationale for prohibiting interest rate payments on holding stablecoins.

There are several extensions that can be pursued with the current model. For instance, one could add a convenience yield to the holding of stablecoins if they can be used with low transaction costs to make payments and thus can serve as a medium of exchange and could compete with standard fiat currency provided by the government. Moreover, one might introduce agency conflicts when a private company issues and operates stablecoins on behalf of the generation of first holders of the stablecoin. Such potential agency conflicts could also necessitate standardized requirements an issuer of stablecoins has to fulfill in order to have the license for issuing stablecoins, similar to licenses of commercial banks. Finally, one could study how repeated issuance of stablecoins affect the result, i.e., we could allow a generation to issue further coins according to some predetermined plan, once they have started the issuance.

One can certainly agree with Gorton and Zhang (2021) that the new world of stablecoins posits the same problems as the earlier free-banking eras. Yet, knowledge about how to tackle these problems, the technical and financial infrastructure and our entire monetary system have evolved considerably. Hence, the search for stablecoins that compete and produce desirable results continues. The current paper may add a step in this search.

A Proofs

A.1 Discrete-time analogue

We consider the discrete version of our model with *dates* $t \in \{0, \Delta, 2\Delta, ...\}$. This is useful to prove some properties of our continuous-time model, which we interpret as the limit when Δ goes to zero.

The probability that an investor born at date t dies at date $\tau > t$ is $(1-e^{-\delta\Delta})e^{-\delta(\tau-t-\Delta)}$. Likewise, the probability that a tree planted at date t gestates at time $\tau > t$ is $(1 - e^{-\phi\Delta})e^{-\phi(\tau-t-\Delta)}$.

A unit mass of investors is born at $t = 0$ and a mass $(1 - e^{-\delta\Delta})$ of investors is born at $t \in \{\Delta, 2\Delta, 3\Delta, ...\}$ so that there is a unit mass of investors alive at all dates. To ensure that trees earn a fundamental return $r = \rho$, we impose $y(1 - e^{-\phi\Delta}) = e^{\rho\Delta}(1 - e^{-(\phi+\rho)\Delta})$. Aggregate consumption at date $t > 0$ is

$$c_t = \sum_{\tau=\Delta\ |\Delta}^{t-\Delta} (1 - e^{-\delta\Delta})^2 e^{-\delta(t-\Delta-\tau)} c_{t,\tau} + (1 - e^{-\delta\Delta}) e^{-\delta(t-\Delta)} c_{t,0} \tag{82}$$

with $c_{t,\tau}$ the date-t consumption by an investor born at date $\tau < t$.[13] The law motion for the stock of trees, defined as being alive at the beginning of time $t \geq 0$, reads as

$$A_{t+\Delta} = (1 - e^{-\phi\Delta})A_t + I_t, \quad \text{where } A_0 = 0, \tag{83}$$

and the aggregate resource constraints are

$$A_\Delta + S_\Delta = h_0 \quad \text{and} \quad c_t + S_{t+\Delta} + I_t \leq e^{\rho\Delta}(1 - e^{(\phi+\rho)\Delta})A_t + (1 - e^{-\delta\Delta})h_t \ \forall t > 0, \tag{84}$$

subject to the non-negativity constraints $S_{t+\Delta}, I_t \geq 0$, with h_t defined as the labor effort devoted by an investor born at date t.

A.2 Proof of Lemma 1

To be included ...

[13] There is no consumption at $t = 0$.

A.3 Proof of Lemma 2

To be included ...

A.4 Proof of Proposition 1

To be included ...

A.5 Proof of Proposition 2

To be included ...

A.6 Proof of Lemma 3

Other coins generate a higher return than trees when $r_t > \rho$, so it has to be that $r_t^+ > \rho \;\Rightarrow\; \dot{A}_{t,s} = -\phi A_{t,s}$, i.e., no new trees are planted by the incumbent issuer. If $Z_{t,s}^+ = 0$, we would then have

$$c_{t,s} \geq \frac{(\rho+\phi)A_{t,s}e^{\delta(t-s)}}{\delta} \quad \text{and} \quad \lim_{\varepsilon\to 0} c_{t+\varepsilon,s} \leq \lim_{\varepsilon\to 0} \frac{(\rho+\phi)A_{t,s}e^{(\delta-\phi)\varepsilon+\delta(t-s)}}{\delta}. \tag{85}$$

It follows that

$$\lim_{\varepsilon\to 0} \frac{c_{t+\varepsilon,s}-c_{t,s}}{\varepsilon} \leq c_{t,s} \lim_{\varepsilon\to 0} \frac{e^{(\delta-\phi)\varepsilon}-1}{\varepsilon}, \tag{86}$$

so $Z_{t,s}^+ = 0 \;\Rightarrow\; \dot{c}_{t,s}/c_{t,s} \leq \delta - \phi$. With $\phi \geq \delta$ and $r_t^+ > \rho$, this however implies that the constraint $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$ is violated. Thus $r_t^+ > \rho \;\Rightarrow\; Z_{t,s}^+ > 0$. □

A.7 Proof of Lemma 4

We define $a_{t,s} = A_{t,s}e^{-\delta(t-s)}$ and $z_{t,s} = Z_{t,s}e^{-\delta(t-s)}$ and consider the discrete-time analogue of the problem. The gross return on investors' outside option from date t to $t+\Delta$ is $e^{r_{\Delta+t}\Delta}$.

The incumbent issuer chooses the sequence $\{c_{\tau+T,s}, a_{\Delta+\tau+T}, z_{\Delta+\tau+T}\}_{\tau=0\,|\Delta}^{\infty}$ to maximize

$$\sum_{\tau=0\,|\Delta}^{\infty} e^{-(\rho+\delta)\tau} u(c_{\tau+T,s}) \tag{87}$$

subject to the law of motion

$$e^{-\delta\Delta}a_{\Delta+\tau+T,s} + e^{-\delta\Delta}z_{\Delta+\tau+T,s} = e^{\rho\Delta}a_{\tau+T,s} + e^{r_{\tau+T}\Delta}z_{\tau+T,s} - (1-e^{-\delta\Delta})c_{\tau+T,s} \tag{88}$$

the constraints $a_{\Delta+\tau+T,s} \geq e^{(-\phi+\delta)\Delta}a_{\tau+T,s}$, $z_{\Delta+\tau+T,s} \geq 0$, and

$$c_{\Delta+\tau+T,s} \geq c_{\tau+T,s}e^{r_{\Delta+\tau+T}\Delta}, \tag{89}$$

and the starting values $a_{T,s} > 0$ and $z_{T,s} = 0$.

Suppose that the constraint $a_{\Delta+\tau+T,s} \geq e^{(-\phi+\delta)\Delta}a_{\tau+T,s}$ is slack for all $\tau \geq 0$. It follows that we can assume $z_{\Delta+\tau+T,s} = 0\ \forall\tau \geq 0$, since accumulating other coins and/or storage does not yield a higher return than trees with $r_{\Delta+\tau+T} \in [0,\rho]\ \forall\tau \geq 0$. The law of motion then implies

$$(1-e^{-\delta\Delta})\sum_{\tau=0\ |\Delta}^{\infty} e^{-(\rho+\delta)\tau}c_{\tau+T,s} \leq a_{T,s}e^{\rho\Delta}, \tag{90}$$

where we use that $z_{T,s} = 0$ and $\lim_{\tau\to\infty} e^{-(\rho+\delta)\tau}a_{\tau+T,s} \geq 0$.

Let λ be the Lagrange multiplier associated with (90) and let $\mu_{\tau+T,s}e^{-(\rho+\delta)\tau}$ be the Lagrange multiplier associated with (89). The first-order condition for $c_{\tau+T,s}$ is

$$0 = u'(c_{\tau+T,s}) - \lambda(1-e^{-\delta\Delta}) - \mu_{\tau+T,s}e^{r_{\Delta+\tau+T}\Delta} + \mathbf{1}_{\{\tau>0\}}\mu_{\tau-\Delta+T,s}e^{(\rho+\delta)\Delta}. \tag{91}$$

Suppose $\exists\ \tau \in \{0,\Delta,2\Delta,...\}$ such that $c_{\Delta+\tau+T,s} > c_{\tau+T,s}e^{r_{\Delta+\tau+T}\Delta}$, so that complementary slackness implies $\mu_{\tau+T,s} = 0$ and $r_{\Delta+\tau+T} \geq 0$ implies $c_{\Delta+\tau+T,s} > c_{\tau+T,s}$. It follows that

$$0 = u'(c_{\tau+T,s}) - \lambda(1-e^{-\delta\Delta}) + \mathbf{1}_{\{\tau>0\}}\mu_{\tau-\Delta+T,s}e^{(\rho+\delta)\Delta}, \tag{92}$$

$$0 = u'(c_{\Delta+\tau+T,s}) - \lambda(1-e^{-\delta\Delta}) - \mu_{\Delta+\tau+T,s}e^{r_{2\Delta+\tau+t}\Delta}, \tag{93}$$

which can be combined to

$$0 = u'(c_{\Delta+\tau+T,s}) - u'(c_{\tau+T,s}) - \mathbf{1}_{\{\tau>0\}}\mu_{\tau-\Delta+T,s}e^{(\rho+\delta)\Delta} - \mu_{\Delta+\tau+T,s}e^{r_{2\Delta+\tau+t}\Delta}. \tag{94}$$

Since the multipliers are non-negative, it must be that $u'(c_{\Delta+\tau+T,s}) \geq u'(c_{\tau+T,s})$, but this contradicts $c_{\Delta+\tau+T,s} > c_{\tau+T,s}$. Thus, when the constraint $a_{\Delta+\tau+T,s} \geq e^{(-\phi+\delta)\Delta}a_{\Delta+T,s}$ is slack for all $\tau \geq 0$, we must have that the constraint $c_{\Delta+\tau+T,s} \geq c_{\tau+T,s}e^{r_{\Delta+\tau+T}\Delta}$ is binding for all $\tau \geq 0$.

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