Income inequality and the German export surplus

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Abstract

We investigate the contribution of the increase in German (DE) income inequality to the German export surplus increase and the decline of the natural rate of interest in the Euro Area in an open economy model with rich and non-rich households. Rich households have Capitalist Spirit type Preferences (CSP) over their wealth and thus save out of an increase in their permanent income. Simulating the increase in DE income inequality over the 1992-2016 period generates a decline of the EA natural rate of interest rate of about 1 p.p. and an increase of the DE net-export-to-GDP ratio of about 3 p.p..

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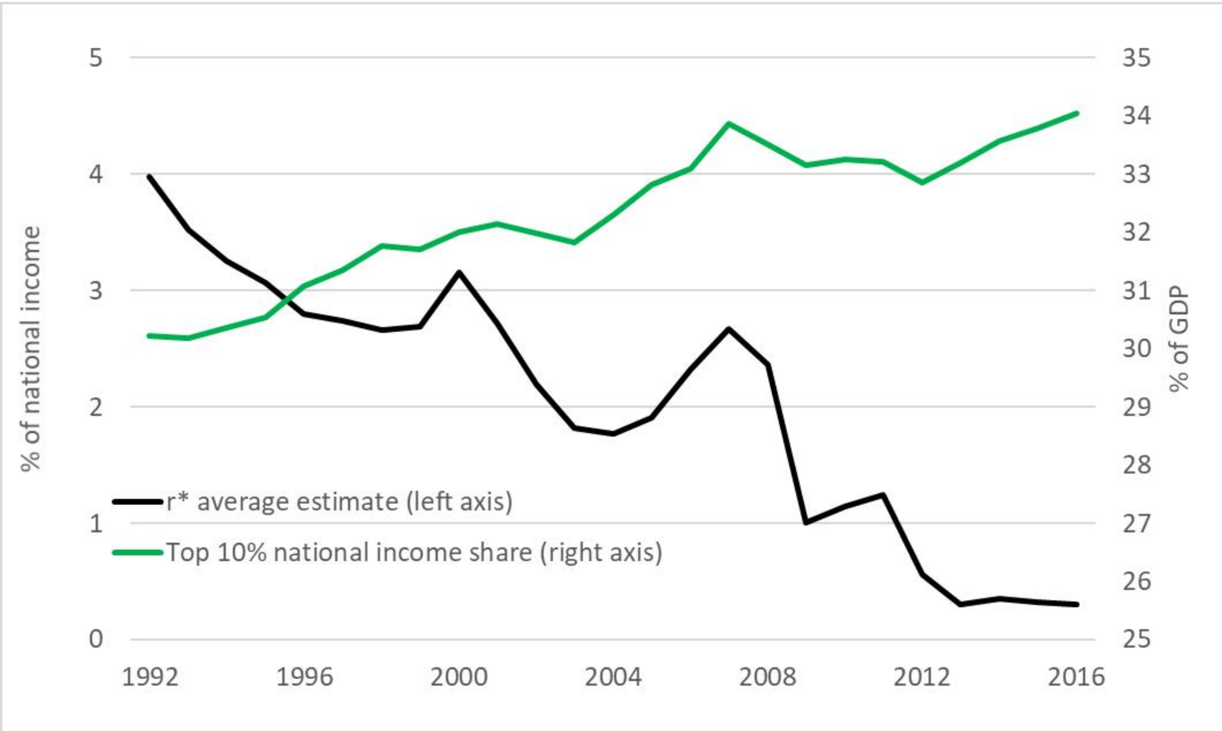
1 Introduction

This paper examines the relationship between income inequality, the natural interest rate and trade imbalances in the Euro Area. Our motivation is summarized by the following stylized facts. Firstly, since the early 1990s until the mid-2010s, real interest rates in the Euro Area have declined, and this has been reflected in declining estimates of the so called "natural rate of interest", i.e. the real interest rate consistent with a closed output gap and stable inflation. At the same time top-end income inequality as measured by the income share of the top 10% richest households has increased significantly (see Figure 1). Secondly, the increase in the top 10% income share has in fact been concentrated in Germany, while it has remained essentially flat in the Rest of the Euro Area (REA). Finally, the German export surplus has increased over the same period, while the REA export surplus has remained flat (see Figure 2).

The goal of this paper is to connect these trends in a macroeconomic model which builds on Rannenberg (2023), who links, inter alia, the decline of r* and the increase in inequality in the US economy in a model with "rich" households (representing the top 10% of the income distribution) with "Capitalist Spirit" type Preferences (CSP) over their wealth, and "non-rich" households without CSP. At the microeconomic level, CSP allows to mach rich households' large marginal propensity to save out of permanent income changes (see the empirical evidence of Dynan et al. (2004) and Kumhof et al. (2015) and the model of Kumhof et al. (2015)). At the macroeconomic level, this feature implies that an increase in the income of the rich at the expense of the non-rich will ceteris paribus cause an increase in aggregate saving. Rannenberg (2023) feeds the empirically observed increase in inequality into the model and finds a decline of the natural interest rate and an increase in non-rich household indebtedness in line with empirical estimates (see Straub (2017) and Mian et al. (2020) for other contributions also generating a link between income inequality and the natural rate of interest using CSP)).

In this paper, we extend the approach of Rannenberg (2023) to a New Keynesian open economy model of Germany (DE) and the REA, and (ultimately) also the rest of the world (ROW). Rich households have CSP, i.e. they derive utility from their domestic and foreign assets, and are the owners of firms and holders of government bonds and financial intermediary deposits. Non-rich

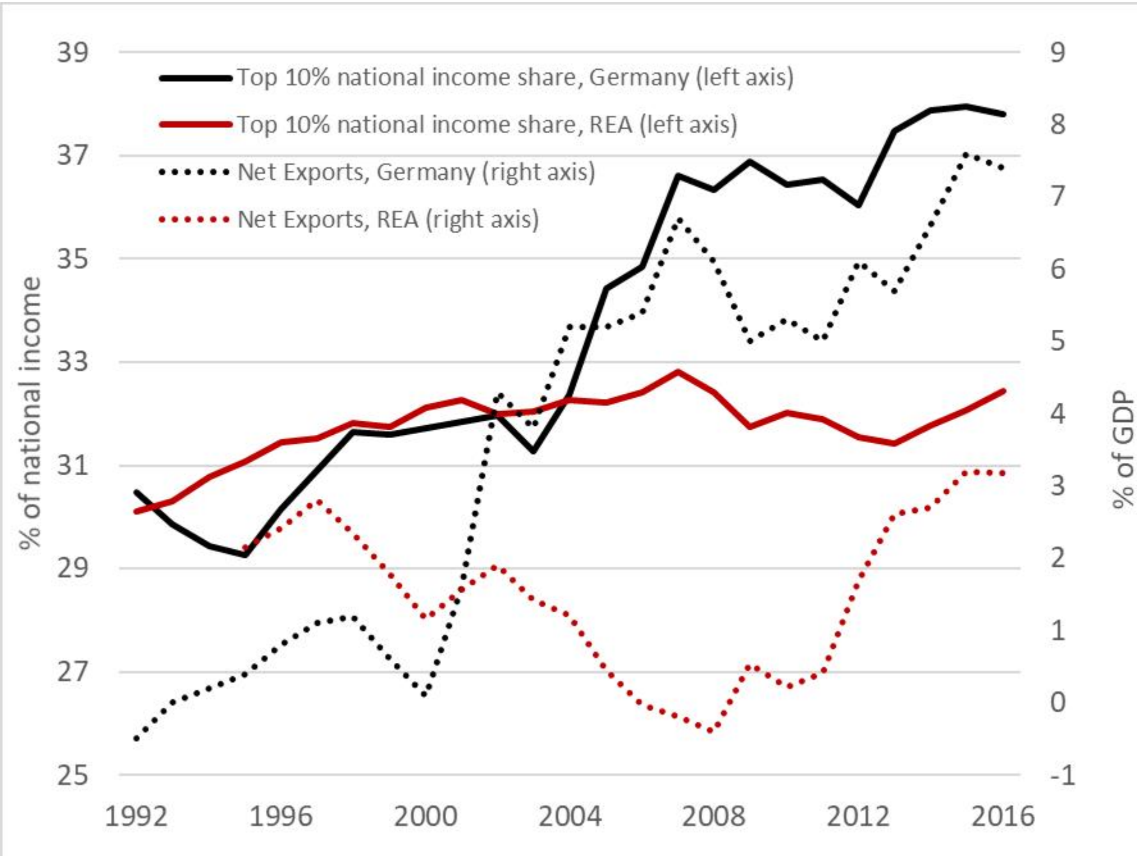
Figure 1: The top 10% national income share and r* in the Euro Area



Note: "r* average estimate" is the simple average of the Euro Area natural rate estimates reported in Brand et al. (2018): Brand and Mazelis (2019), Holston et al. (2017), Gerali and Neri (2019), Haavio et al. (2017), Fiorentini et al. (2018). "Top 10% national income share" is the share of the top 10% of households in net national income from the World Inequality Database (WID), see Alvaredo et al. (2020). We compute the Euro Area average using constant Dollar PPP national income as country weights.

households earn labor income and can borrow from rich households via financial intermediaries, subject to a simple borrowing friction which generates an upward sloping loan supply curve. In the model, income inequality may rise due to higher wage dispersion caused by an increase in the relative human capital of rich households, or a decline in the labor share in national income caused by an increase in the price markup of firms, which are owned by rich households. We find that without CSP, in response to a permanent increase in inequality, DE rich household (non-rich household) consumption jumps (drops) close to its new steady-state level, with little further adjustment and only small and transitory effects on the other variables and the REA economy. By contrast with CSP, as in Rannenberg (2023), rich households increase their consumption on impact

Figure 2: DE and REA net exports and Top 10% national income share



by much less than the increase in their permanent income, implying that total domestic demand declines. Furthermore, the associated decline in labor demand lowers the real wage and inflation. The decline in DE inflation triggers a decline of the real policy interest rate at the Euro Area level, which persistently stimulates REA consumption and inflation. As a result, DE imports decline and exports increase. As a consequence, the export surplus increases persistently, while non-rich borrowing increases in both DE and the REA. If calibrated to cause the same medium to long-run effect on the bottom 90% national income share and after a short transitory period, both potential drivers of rising inequality have quantitatively very similar effects on all variables except the labor share.

Using the aforementioned wage dispersion and price markup shocks, we replicate the path of the labor share and the path of the bottom 90% national income share over the 1992-2016 period in the model with CSP. We can replicate much of the increase of DE net exports with respect to the REA, including more than two thirds of the peak increase, with the total increase accumulating to about 3 percentage points of GDP by 2016. The simulation also replicates about two thirds of the decline the of the (average) empirical estimate of the natural interest rate displayed in Figure 1, and explains three fifths of the increase of the DE bottom 90% debt-to-income ratio observed over this period.

Our model results are robust to allowing for physical capital as an additional asset owned by rich households, and adding a ROW block and taking into account the increase in ROW inequality in the historical simulation. Specifically, an increase in DE inequality has a somewhat larger effect on DE net exports in the three region than in the two region model, with the increase in net exports being roughly equally split between REA trade and ROW trade. Furthermore, the Euro depreciates due to the lower EA interest rate. While the top 10% national income share in the ROW has increased considerably, taking this trend into account has only a small effect on the simulated path of DE net exports. The reason is that in our model, an increase in ROW inequality has only a small and transitory effect on the DE export surplus, compared to a DE inequality increase. The reason for this weaker effect is that the ROW real interest rate declines much more than the DE real interest rate, while ROW inflation does not decline at all. The reason is that ROW has its own central bank targeting ROW inflation while DE does not. As a result, ROW domestic demand declines only little, and the real exchange rate of DE w.r.t. ROW responds less due to a lack of disinflation in ROW.

There is an extensive literature using structural models to explore possible sources of the persistent DE export surplus. Two papers consider the role of the income distribution, but do not attempt to quantitatively link the long-run trends of inequality, natural rate of interest and trade imbalances, and do not consider the role of labor income inequality explicitly. Specifically, Gruening et al. (2015) investigate the effect of rising inequality in DE and UK, using a stylized small open economy RBC model where rich households have CSP. Their focus is on distributed versus retained firm profits to capture differences between these economies, and they do not endogenize the physical capital stock. Hoffmann et al. (2021) investigate effects of a transitory wage-push shock on DE net exports in estimated DSGE model of DE, REA and ROW, using demeaned data on inter alia real wage growth and GDP growth, and thus eliminating the trend of the labor share from consideration. In their historical shock decomposition, they find only a small effect of wage push shocks on net exports. In their simulation exercises, they find that only very large wage increases lead to significantly lower net exports. Their model does not allow for CSP and wage dispersion, and they doe not discuss the effect of price markup shocks on net exports.

Schoen and Staehler (2020) and Ruppert and Staehler (2022) simulate the effect of aging as well as tax, pension, and labor market reforms on the DE current account using OLG models of DE, REA and ROW. They attribute an increase in the current account balance of about 1-3 percentage points on average over the 2000-2020 period to aging as the most important driver. The effect of the German labor market and tax reforms have been mostly found to have small or marginal effects on DE net exports (see Hochmuth et al. (2019), Ruppert and Staehler (2022), Gadatsch et al. (2016b) and Kollmann et al. (2015)).

Furthermore, Kollmann et al. (2015) estimate a "private saving shock", i.e. a decline in the subjective rate of time preference of DE unconstrained (as opposed to liquidity constrained rule-of-thumb) households to be the main driver of the post 2000 increase in DE net exports in their estimated DSGE model of DE, REA and ROW. This finding is consistent with our simulation where net exports increase due to higher saving by rich households as a result of rising inequality. Interestingly, both the magnitude and the time profile of the net export increase, which Kollmann et al. (2015) attribute to the private shock, correspond closely to our simulated net export increase.

Finally, an empirical literature using cross country panels has examined the relationship between inequality and the current account more generally. Behringer and van Treeck (2018) find that a decline in the labor share improves the current account, while an increase in the top 5% personal income share worsens it. Kumhof et al. (2022) find that the effect of the top 5% national income share on the current account depends on the degree of stock market capitalization.

The remainder of this paper is structured as follows: Section 2 develops the model, Section 3 discusses the calibration and Section 4 the baseline simulation results. Section 5 discusses the effect of allowing for physical capital and incorporating the increase in ROW inequality in the analysis.

2 Model

The model consists of two regions, Germany (DE) and the Rest of the Euro Area (REA). Their economic sizes are denoted as $size^{DE}$ and $size^{REA}$. DE variables and DE holdings of REA assets have the superscript DE. Model equations derived for the DE region are identical to those of the REA region unless otherwise mentioned. The households in the economy have a total mass of 1, with the share of rich households denoted as pop_r. This share is identical in both regions. Rich households are assumed to be the model counterparts of top 10% of households in the income distribution. They are the sole owners of firms, in line with the evidence reported in Schroeder et al. (2020), and they are the only holders of other assets. Throughout, household choice variables are headed with a $\tilde{}$, and are expressed in per-member-of-the household terms. The corresponding per-capita-of-the-total-population variables $X^{DE}_{r,t}$ are calculated as $X^{DE}_{r,t} = pop_r \tilde{X}^{DE}_{r,t}$ for rich households and $X_{n,t} = (1 - pop_r)\,\tilde{X}_{n,t}$ for non-rich households. Liabilities held by foreigners are expressed per-capita of the domestic population. In particular total REA assets held by DE rich households per-capita of the DE population are given by

$$\tilde{b}^{DE}_{REA,t} pop_r = b^{DE}_{REA,t} \frac{size^{REA}}{size^{DE}} \tag{1}$$

2.1 Rich households

Rich households derive utility from consumption $\tilde{C}^{DE}_{r,t}$ and disutility from labor $\tilde{N}^{DE}_{r,t}$. Furthermore, in the model with Capitalist Spirit Preferences (CSP), they also derive utility from their their real asset holdings $\tilde{b}^{DE}_{r,dom,t} + q^{DE}_{REA,t} \tilde{b}^{DE}_{REA,t}$, where $\tilde{b}^{DE}_{r,dom,t}$ and $\tilde{b}^{DE}_{REA,t}$ denote holdings of domestic and REA assets respectively, while $q^{DE}_{REA,t}$ denotes the DE real exchange rate, i.e. the price of a unit of REA consumption expressed in DE consumption units. Hence the objective of rich households is given by

$$\sum_{i=0}^{\infty} \frac{\left(\beta^{DE}_r\right)^i}{pop_r} \left[\frac{\left(pop_r \tilde{C}^{DE}_{r,t+i}\right)^{1-\sigma^{DE}}}{1-\sigma^{DE}} - \frac{\chi^{DE}_{N,r}}{1+\eta^{DE}} \left(pop_r \tilde{N}^{DE}_{r,t+i}\right)^{1+\eta^{DE}} + \frac{\chi^{DE}_{b,r}}{1-\sigma^{DE}_{b,r}} \left(pop_r \left(\tilde{b}^{DE}_{r,dom,t} + q^{DE}_{REA,t} \tilde{b}^{DE}_{REA,t}\right)\right)^{1-\sigma^{DE}_{b,r}} \right] \tag{2}$$

with $\chi^{DE}_{N,r}, \sigma^{DE}, \eta^{DE} > 0$ and $\chi^{DE}_{b,r} \geq 0$. Note that presence of pop_r is merely for normalization purposes without any effect on the results. Throughout we will refer to the model with $\chi^{DE}_{b,r} > 0$ as the CSP model, while the model with $\chi^{DE}_{b,r} = 0$ is the NOCSP model. Apart from income derived from their asset holdings, rich households also earn labor income and profit income from their ownership of firms. A rich households budget constraint is given by

$$\tilde{b}^{DE}_{r,dom,t} + \tilde{C}^{DE}_{r,t} + q^{DE}_{REA,t}\tilde{b}^{DE}_{REA,t} = \frac{R^{DE}_{t-1}}{\Pi^{DE}_{t}}\tilde{b}^{DE}_{r,dom,t-1} + \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t}\tilde{b}_{REA,t-1} exp\left(-D^{RP}\psi_{rp}\widehat{NFA}^{DE}_{t-1}\right)$$
$$+ w^{DE}_{r,t}\tilde{N}^{DE}_{r,t} - \tilde{T}^{DE}_{r,t} + \tilde{\Xi}^{DE}_{t}$$

where R^{DE}_t, $\Pi^{DE}_t, w^{DE}_{r,t}, \tilde{T}^{DE}_{r,t}, \tilde{\Xi}^{DE}_t, R^{REA}_t$ and $\Pi_{REA,t}$ denote the nominal interest rate on DE assets, the DE inflation rate, the rich household real wage, lump sum taxes and profits of firms (the later three in real terms), and the REA nominal interest and inflation rates. The term $exp\left(-D_{rp}\psi_{rp}\widehat{NFA}^{DE}_{t}\right)$ represents an ad hoc intermediation cost on lending to REA which depends negatively on the DE net -foreign-asset-to-annual-GDP ratio, with ψ_{rp} and D_{rp} denoting the elasticity and a dummy variable which can take values 0 and 1. The first order conditions w.r.t. $\tilde{b}^{DE}_{r,dom,t}$, $\tilde{b}^{DE}_{REA,t}$ and $\tilde{C}^{DE}_{r,t}$ are given by

$$\Lambda^{DE}_{r,t} = \beta^{DE}_{r} E_t \left\{ \frac{R^{DE}_t}{\Pi^{DE}_{t+1}} \Lambda^{DE}_{r,t+1} \right\} + \chi^{DE}_{b,r} \left(b^{DE}_{r,t}\right)^{-\sigma^{DE}_{b,r}} \tag{3}$$

$$\Lambda^{DE}_{r,t} = \beta^{DE}_{r} E_t \left\{ \frac{R^{DE}_{REA,t} exp\left(-D^{RP}\psi_{rp}\widehat{NFA}^{DE}_{t}\right)}{\Pi^{REA}_{t+1}} \frac{q^{DE}_{REA,t+1}}{q^{DE}_{REA,t}} \Lambda^{DE}_{r,t+1} \right\} + \chi^{DE}_{b,r} \left(b^{DE}_{r,t}\right)^{-\sigma^{DE}_{b,r}} \tag{4}$$

$$\Lambda^{DE}_{r,t} = \left(C^{DE}_{r,t}\right)^{-\sigma^{DE}} \tag{5}$$

where we have already converted all choice variables into their per-capita-of-the population counterparts.

As explained in Rannenberg (2023) and Rannenberg (2021), if $\chi^{DE}_{b,r} > 0$, $\chi^{DE}_{b,r}\left(b^{DE}_{r,t}\right)^{-\sigma^{DE}_{b,r}}$ represents an extra marginal benefit from saving over and above the utility associated with the future consumption opportunity saving entails (represented by $\beta^{DE}_r E_t \left\{ \frac{R^{DE}_t}{\Pi^{DE}_{t+1}} \Lambda^{DE}_{r,t+1} \right\}$). Thus CSP weakens the effect of an increase in permanent income and thus a decline in $\Lambda_{r,t+1}^{DE}$ on $\Lambda_{r,t}^{DE}$, since the two become less than proportional. To gain some intuition, compare the bond market equilibrium in the CSP and NOCSP case, assuming that the economy is initially in the steady state. The presence of the extra benefit $\chi_{b,r}^{DE}\left(b_{r,t}^{DE}\right)^{-\sigma_{b,r}^{DE}}$ with CSP implies that, for the bond market to clear, the present value $\beta_r^{DE}\frac{R_t}{\Pi_{t+1}}$ which the household attaches to $\Lambda_{r,t+1}^{DE}$ -the net effect of the reward of waiting and the household's impatience- has to be smaller than in the NOCSP case. This extra benefit thus reduces the importance it attaches to a decline in $\Lambda_{r,t+1}^{DE}$. Furthermore, this weakening of intertemporal consumption smoothing compounds the more distant in time the anticipated future consumption increase is located, as $\Lambda_{r,t+1}^{DE}$ is no longer proportional to $\Lambda_{r,t+2}^{DE}$ either, and so on and so forth. As a result, with CSP, a one percent permanent increase in saver household income will, ceteris paribus, not cause a one percent increase in consumption, but instead a lower increase in both saving and consumption.

Standard open economy models need an endogenous risk premium or an equivalent assumption in order to induce stationarity of net foreign assets (see Schmitt-Grohe and Uribe (2003)). Therefore in the NOCSP model we assume $D_{rp} = 1$. By contrast, in the CSP model we eliminate the ad-hoc risk premium ($D_{rp} = 0$) as under our calibration CSP is sufficient to render net foreign assets stationary. CSP has a stabilizing effect on net foreign assets because it implies that rich household consumption depends positively on their real assets $b_{r,t}^{DE}$, implying a positive relationship between foreign asset holdings and consumption.

We assume that DE rich households invest in REA but not vice versa. The reason is that under our assumption that domestic and REA assets are perfect substitutes and with $D_{rp} = 0$, the portfolios of DE and REA rich households would not be determined if we allowed REA households to invest in DE assets. To see this, combine equations (3) and (4), and use the law of motion of the real exchange rate $q_{REA,t}^{DE} = \frac{\Pi_t^{REA,t}}{\Pi_t^{DE}} q_{REA,t-1}^{DE}$, which yields

$$R_t^{DE} = R_{REA,t}^{DE} \tag{6}$$

If we allowed REA households to invest in DE assets, the REA FOCs would yield the same equation, meaning that we would have one redundant equation and thus fewer independent equations than variables, and that the foreign asset holdings of DE and REA households would not both be pinned down. To eliminate this degree of freedom we therefore assume that REA households invest only in REA assets.

Regarding the labor market, we assume that each rich household supplies a labor variety j in a monopolistically competitive labor market where it faces a demand curve

$$\tilde{N}^{DE}_{r,t}(j) = \tilde{N}^{DE}_{r,t}\left(\frac{W^{DE}_{r,t}(j)}{W_{r,t}}\right)^{-\epsilon^{DE}_N} \tag{7}$$

where $\epsilon^{DE}_N > 1$ denotes the elasticity of demand and $W^{DE}_{r,t}(j)$ the nominal wage paid to labor variety j. Households are subject to nominal rigidities in the form of wage adjustment costs $W^{DE}_{r,t}\tilde{N}^{DE}_{r,t}\frac{\xi^{DE}_H}{2}\left(\frac{W^{DE}_{r,t}(j)}{W^{DE}_{r,t-1}(j)}\frac{1}{\Pi^{DE}_{ind,t}}-1\right)^2$ where Π^{DE}_{ind} denotes the amount of "cost-less" wage inflation. The FOC w.r.t. to $W_{r,t}(j)$ is given by

$$\begin{aligned}
&\epsilon^{DE}_N\chi^{DE}_{N,r}\left(\tilde{N}^{DE}_{r,t}\left(\frac{W^{DE}_{r,t}(j)}{W^{DE}_{r,t}}\right)^{-\epsilon^{DE}_N}\right)^{\eta^{DE}}\tilde{N}^{DE}_{r,t}\left(\frac{W^{DE}_{r,t}(j)}{W^{DE}_{r,t}}\right)^{-\epsilon^{DE}_N-1}\frac{1}{W^{DE}_{r,t}}+\Lambda^{DE}_{r,t}\left(1-\epsilon^{DE}_N\right)\frac{1}{P^{DE}_t}\tilde{N}^{DE}_{r,t}\frac{W^{DE}_{r,t}(j)}{(W_{r,t})^{-\epsilon^{DE}_N}}^{-\epsilon^{DE}_N}\\
&-\Lambda^{DE}_{r,t}\frac{W^{DE}_{r,t}}{P^{DE}_t}\tilde{N}^{DE}_{r,t}\xi^{DE}_H\left(\frac{W^{DE}_{r,t}(j)}{W^{DE}_{r,t-1}(j)}\frac{1}{\Pi^{DE}_{ind}}-1\right)\frac{1}{W^{DE}_{r,t-1}(j)}\frac{1}{\Pi^{DE}_{ind}}\\
&+\beta^{DE}_r E_t\left\{\Lambda^{DE}_{r,t+1}\frac{W_{r,t+1}}{P_{t+1}}\tilde{N}_{r,t+1}\xi_H\left(\frac{W(j)_{r,t+1}}{W(j)_{,t}}\frac{1}{\Pi_{ind}}-1\right)\frac{W(j)_{r,t+1}}{W(j)^2_{,t}}\frac{1}{\Pi_{ind}}\right\}\\
&=0
\end{aligned}$$

Using the fact that in equilibrium all rich households set the same wage, i.e. $W_{r,t}(j) = W_{r,t}$, and re-arranging yields

$$\left(\frac{\Pi^{DE}_{W,r,t}}{\Pi^{DE}_{ind}}-1\right)\frac{\Pi^{DE}_{W,r,t}}{\Pi^{DE}_{ind}} = \kappa^{DE}_w\left(\mu^{DE}_w\frac{\chi^{DE}_{N,r}\left(\frac{N^{DE}_{r,t}}{pop_r}\right)^{\eta}}{\Lambda^{DE}_{r,t}w_{r,t}}-1\right)+\beta_r E_t\left\{\frac{\Lambda_{r,t+1}}{\Lambda_{r,t}}\frac{N_{r,t+1}}{N_{r,t}}\left(\frac{\Pi_{W,r,t+1}}{\Pi_{ind}}-1\right)\frac{\Pi^2_{W,r,t+1}}{\Pi_{t+1}\Pi_{ind}}\right\} \tag{8}$$

$$w^{DE}_{r,t} = \frac{\Pi^{DE}_{W,r,t}}{\Pi^{DE}_t}w^{DE}_{r,t-1} \tag{9}$$

where we have used $\tilde{X}_{r,t}^{DE} = \frac{X_{r,t}}{pop_r}$, $\mu_w^{DE} \equiv \frac{\epsilon_N^{DE}}{\epsilon_N^{DE}-1}$ and $\kappa_w^{DE} \equiv \frac{\epsilon_N^{DE}-1}{\xi_H^{DE}}$. If we were to linearize the wage Phillips Curve, κ_w^{DE} would denote the coefficient on the percentage deviation of the marginal rate of substitution between consumption and leisure.

2.2 Non-rich households

Non-rich households, who represent the bottom 90%, derive utility from consumption $\tilde{C}_{n,t}^{DE}$ and disutility from labor $\tilde{N}_{n,t}^{DE}$:

$$\sum_{i=0}^{\infty} \frac{\left(\beta_n^{DE}\right)^i}{1-pop_r} \left[\frac{\left(\left(1-pop_r\right)\tilde{C}_{n,t+i}^{DE}\right)^{1-\sigma^{DE}}}{1-\sigma^{DE}} - \frac{\chi_{N,n}^{DE}}{1+\eta^{DE}} \left(\left(1-pop_r\right)\tilde{N}_{n,t+i}^{DE}\right)^{1+\eta} \right] \tag{10}$$

They fund their consumption via labor income and borrowing $\tilde{l}_{n,t}^{DE}$ at interest rate $R_{L,t}^{DE}$, and pay lump sum taxes $\tilde{T}_{n,t}^{DE}$, implying that their budget constraint is given by

$$\tilde{C}_{n,t}^{DE} = \tilde{l}_{n,t}^{DE} - \frac{R_{L,t-1}^{DE}}{\Pi_t^{DE}} \tilde{l}_{n,t-1}^{DE} + w_{n,t}^{DE}\tilde{N}_{n,t}^{DE} - \tilde{T}_{n,t}^{DE} \tag{11}$$

We assume that there is a financial intermediation cost $f\left(\frac{\tilde{l}_{n,t}^{DE}}{w_{n,t}^{DE}\frac{N_{n,t}^{DE}}{1-pop_r}}\right)$ which depends positively on the debt-to-income ratio of non-rich households and for which the financial intermediary needs to be compensated. Hence the loan rate is positively related to the housholds' debt-to-income ratio:

$$\frac{R_{L,t}^{DE}}{R_t^{DE}} = \left(1 + f\left(\frac{\tilde{l}_{n,t}}{w_{n,t}^{DE}\frac{N_{n,t}^{DE}}{1-pop_r}}\right)\right) \tag{12}$$

The households first order conditions w.r.t. borrowing consumption are given by are

$$\Lambda_{n,t}^{DE} = \beta_n^{DE} E_t \left\{ \Lambda_{n,t+1}^{DE} \left(\frac{R_{L,t}^{DE} + f'\left(\frac{l_{n,t}^{DE}}{w_{n,t}^{DE}N_{n,t}^{DE}}\right)\frac{l_{n,t}^{DE}}{w_{n,t}^{DE}N_{n,t}^{DE}} R_t^{DE}}{\Pi_{t+1}^{DE}} \right) \right\} \tag{13}$$

$$\Lambda_{n,t}^{DE} = \left(C_{n,t}^{DE}\right)^{-\sigma} \tag{14}$$

Note that the $f'\left(\frac{l_{n,t}^{DE}}{w_{n,t}^{DE}N_{n,t}^{DE}}\right)\frac{l_{n,t}^{DE}}{w_{n,t}^{DE}N_{n,t}^{DE}}R_t^{DE}$ represent the effect of an increase in borrowing on the debt burden of the household resulting from the fact that their loan rate increases.

2.3 Firms

2.3.1 Labor assemblers

Perfectly competitive labor assemblers produce a homogeneous labor input N_t^{DE} out of rich and non-rich household labor and sell it at price w_t^{DE} to intermediate goods firms. Their production technology is given by

$$N_t^{DE} = z_{r,t}^{DE}N_{r,t}^{DE} + z_{n,t}^{DE}N_{n,t}^{DE} \tag{15}$$

where $z_{r,t}^{DE}$ and $z_{n,t}^{DE}$ denote the labor productivities of rich and non-rich households, respectively. The labor assemblers first order conditions are

$$w_{n,t}^{DE} = w_t^{DE}z_{n,t}^{DE} \tag{16}$$

$$w_{r,t}^{DE} = w_t^{DE}z_{r,t}^{DE} \tag{17}$$

We will use a mean preserving shock to those labor productivities in order to capture increases in labor income inequality, i.e. changes in income inequality which do not result from a change in the labor share. Specifically, we assume that $z_{n,t}^{DE}$ and $z_{r,t}^{DE}$ are determined as follows:

$$\omega_{n,t}^{DE} = \omega_n^{DE} + d_{n,t}^{DE} \tag{18}$$

$$\omega_{n,t}^{DE} \equiv \frac{w_t^{DE}z_{n,t}^{DE}N_{n,t}^{DE}}{w_t^{DE}z_{r,t}^{DE}N_{r,t}^{DE} + w_t^{DE}z_{n,t}^{DE}N_{n,t}^{DE}} \tag{19}$$

$$1 = \frac{z_{n,t}^{DE}N_{n,t}^{DE} + z_{r,t}^{DE}N_{r,t}^{DE}}{N_{r,t}^{DE} + N_{n,t}^{DE}} \tag{20}$$

where $\omega_{n,t}^{DE}$ denotes the labor income share of non-rich households, $d_{n,t}^{DE}$ denotes a shock to this share. Equation (20) fixes the average labor productivity across the two household types at 1 and thus forces $z_{r,t}^{DE}$ and $z_{n,t}^{DE}$ to adjust accordingly.

2.3.2 Final goods firms

Final goods firms produce a final output DD_t^{DE} by combing domestically produced goods $DD_{H,t}^{DE}$ and imported goods $DD_{F,t}^{DE}$ using a CES technology:

$$DD_t^{DE} = \left[\left(\phi_H^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_H^{DE} DD_{H,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} + \left(1-\phi_H^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_F^{DE} DD_{F,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} \right]^{\frac{\lambda_m^{DE}}{\lambda_m^{DE}-1}} \tag{21}$$

where A_H^{DE} and AA_F^{DE} are constants used for normalization purposes, ϕ_H^{DE} denotes the (quasi)-share of domestically produced goods in the consumption basked and λ_m^{DE} denotes the elasticity of substitution between domestically and foreign produced goods. Final goods firms purchase their inputs at prices $P_{H,t}^{DE}$ and $P_{F,t}^{DE}$ from domestic and foreign intermediate goods firms, respectively, and sell their production to domestic households at price P_t^{DE}. Hence their objective is given by

$$P_t \left[\left(\phi_H^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_H^{DE} DD_{H,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} + \left(1-\phi_H^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_F^{DE} DD_{F,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} \right]^{\frac{\lambda_m^{DE}}{\lambda_m^{DE}-1}} - P_{H,t}^{DE} DD_{H,t}^{DE} - P_{F,t}^{DE} DD_{F,t}^{DE}$$

which gives rise to the following FOCs

$$A_H^{DE} DD_{H,t}^{DE} = DD_t^{DE} \phi_H^{DE} \left(\frac{A_H^{DE}}{p_{H,t}}\right)^{\lambda_m^{DE}} \tag{22}$$

$$A_F^{DE} DD_{F,t}^{DE} = DD_t^{DE} \left(1-\phi_H^{DE}\right) \left(\frac{A_F^{DE}}{p_{F,t}}\right)^{\lambda_m^{DE}} \tag{23}$$

$$1 = \left[\phi_H^{DE} \left(\frac{p_{H,t,}^{DE}}{A_H^{DE}}\right)^{1-\lambda_m^{DE}} + \left(1-\phi_H^{DE}\right) \left(\frac{p_{F,t,}^{DE}}{A_F^{DE}}\right)^{1-\lambda_m^{DE}} \right] \tag{24}$$

where $p_{H,t}^{DE} \equiv \frac{P_{H,t}^{DE}}{P_t^{DE}}$ and $p_{F,t} \equiv \frac{P_{F,t}^{DE}}{P_t}$. The total demand for final goods firms out is given by

$$DD_t^{DE} = C_{r,t}^{DE} + C_{n,t}^{DE} + \frac{\frac{R_{t-1}^{DE}}{\Pi_t^{DE}} f\left(\frac{l_{n,t-1}^{DE}}{w_{n,t-1}^{DE} N_{n,t-1}}\right)}{p_{H,t,}^{DE}} l_{n,t-1}^{DE} \tag{25}$$

where the final term represents the resource cost associated with the borrowing friction.

2.3.3 Intermediate goods firms

Home produced goods Y_t are produced by a home goods assembler operating under perfect competition who combines a continuum of varieties $Y_t^{DE}(j)$ purchased at price $P_{H,t}^{DE}(j)$ using a CES technology:

$$Y_t^{DE} = \left(\int_0^1 \left(Y_t^{DE}(j)^{\frac{\epsilon_{H,t}^{DE}-1}{\epsilon_{H,t}^{DE}}} \right) dj \right)^{\frac{\epsilon_{H,t}^{DE}}{\epsilon_{H,t-1}^{DE}}}$$

Their optimal demand for variety j is given by

$$Y_t^{DE}(j) = Y_t^{DE} \left(\frac{P_{H,t}^{DE}(j)}{P_{H,t}^{DE}} \right)^{-\epsilon_{H,t}^{DE}} \tag{26}$$

There is a continuum of monopolistically competitive firms owned by rich households, each producing a variety j from the domestic CES basket of goods Y_t^{DE}. They set prices subject to Rotemberg (1982)-type quadratic price adjustment costs:

$$AC_t^{DE}(j) = Y_t^{DE} \frac{\xi_{H,t}^{DE}}{2} \left(\frac{P_{H,t}^{DE}(j)}{P_{H,t-1}^{DE}(j)} \frac{1}{\Pi_{ind}^{DE}} - 1 \right)^2 \tag{27}$$

where $\xi_{H,t}^{DE} > 0$ denotes the adjustment cost curvature and Π_{ind}^{DE} denotes the rate of price change for which adjustment costs are zero. We assume that price adjustment costs are only "private" costs, i.e. they are rebated lump sum to rich households. Retailers employ labor using the technology:

$$Y_t^{DE}(j) = \left(N_t^{DE}(j) \right)^{1-\alpha_K} \tag{28}$$

The first order condition of the intermediate goods firm with respect to its price $P_{H,t}^{DE}(j)$ results in the price Phillips Curve and the first order condition with respect to labor results in a marginal cost curve:

$$\left(\frac{\Pi_{H,t}^{DE}}{\Pi_{ind}^{DE}}-1\right)\frac{\Pi_{H,t}^{DE}}{\Pi_{ind}^{DE}}=\frac{\epsilon_{H,t}^{DE}-1}{\xi_{H,t}^{DE}}\left(\mu_t^{DE}\frac{mc_t^{DE}}{p_{H,t,}^{DE}}-1\right)+\beta_r^{DE}E_t\left\{\frac{p_{H,t+1}^{DE}}{p_{H,t,}^{DE}}\frac{\Lambda_{r,t+1}^{DE}}{\Lambda_{r,t}^{DE}}\frac{Y_{t+1}^{DE}}{D_{H,t}}\left(\frac{\Pi_{H,t+1}^{DE}}{\Pi_{ind}^{DE}}-1\right)\frac{\Pi_{H,t+1}^{DE}}{\Pi_{ind,t+1}^{DE}}\right\} \tag{29}$$

$$w_t^{DE}=mc_t^{DE}\left(1-\alpha_K^{DE}\right)\frac{Y_t^{DE}}{N_t^{DE}} \tag{30}$$

where mc_t^{DE} denotes real marginal costs of production, $\mu_t^{DE}\equiv\frac{\epsilon_{H,t}^{DE}}{\epsilon_{H,t}^{DE}-1}$ denotes the price markup firms would charge in the absence of price adjustment costs, and $\kappa_t^{DE}\equiv\frac{\epsilon_{H,t}^{DE}-1}{\xi_{H,t}^{DE}}$ denotes the marginal cost coefficient that would apply if we would linearize the Phillips Curve.[1] We assume producer currency pricing of exports, implying that the price of exported domestic goods in REA $p_{F,t}^{REA}$ is given by

$$p_{F,t}^{REA}=q_{DE,t}^{REA}p_{H,t,}^{DE} \tag{31}$$

Total DE output equals the sum of goods demanded by domestic final goods' firms $DD_{H,t}^{DE}$ and REA final goods' firms $\frac{size_{REA}}{size_{DE}}DD_{F,t}^{REA}$ (expressed per inhabitant of DE) as well as government expenditure, which we assume consists of domestically produced goods only:

$$Y_t^{DE}=DD_{H,t}^{DE}+EX_t^{DE}+G_t^{DE} \tag{32}$$

Finally, the price parkup in the absence of price adjustment costs μ_t^{DE} is given by

$$\mu_t^{DE}=\mu^{DE}+d_{\mu,t}^{DE} \tag{33}$$

where we will later use $d_{\mu,t}^{DE}$ to replicate the path of the trend labor share.

[1] We will use an increase price markup μ_t^{DE} in the labor share. Below we fix κ_t^{DE} and hence assume that any impact of a change in μ_t^{DE} on the marginal cost coefficient κ_t^{DE} is offset by an appropriate change in the price adjustment cost curvature $\xi_{H,t}^{DE}$.

2.4 Government

The fiscal authority plays a passive role. Specifically, it holds government debt- and government demand-to-GDP ratios constant at targets $Target^{DE}_{bgov2GDP}$ and $Target^{DE}_{G2GDP}$, and sets the share of non-rich households in the total tax burden T^{DE}_t equal to their pre-tax national income share $NIS^{DE}_{n,t}$. Hence the equations governing the fiscal authority are given by

$$b^{DE}_{gov,t} = \frac{R^{DE}_{t-1}}{\Pi^{DE}_t} b^{DE}_{gov,t-1} + p^{DE}_{H,t,} G^{DE}_t - T^{DE}_{r,t} - T^{DE}_{n,t} \tag{34}$$

$$Target^{DE}_{bgov2GDP} = \frac{b^{DE}_{gov,t}}{4Y^{DE}_t} \tag{35}$$

$$Target^{DE}_{G2GDP} = \frac{G^{DE}_t}{Y^{DE}_t} \tag{36}$$

$$Target^{DE}_{T_n 2T_t} = \frac{T^{DE}_{n,t}}{T^{DE}_{r,t} + T^{DE}_{n,t}} \tag{37}$$

$$Target^{DE}_{T_n 2T_t} = NIS^{DE}_{n,t} \tag{38}$$

$$NI^{DE}_t \equiv p^{DE}_{H,t,} Y^{DE}_t + FI^{DE}_t \tag{39}$$

$$NIS^{DE}_{n,t} \equiv \frac{w^{DE}_{n,t} N^{DE}_{n,t} - \left(\frac{R^{DE}_{L,t-1}}{\Pi^{DE}_t} - 1\right) l^{DE}_{n,t-1}}{\left(1 + \frac{b^{DE}_{gov,t}}{NI^{DE}_t}\left(\frac{R^{DE}_{t-1}}{\Pi^{DE}_t} - 1\right)\right) NI^{DE}_t} \tag{40}$$

where NI^{DE}_t and FI^{DE}_t denote national income and net foreign income (from holding of foreign assets or having foreign liabilities).[2]

2.5 Monetary Policy

Monetary policy sets the DE interest rate R^{DE}_t, which with CSP equals R^{REA}_t in equilibrium (see equation (6)). We assume that the central bank pursues a perfect inflation target at the EA level, i.e.

$$\Pi^{EA}_t = \Pi^{EA}_{target} \tag{41}$$

with

$$\Pi^{EA}_t = \frac{size_{DE}}{size_{DE} + size_{REA}} \Pi^{DE}_t + \frac{size_{REA}}{size_{DE} + size_{REA}} \Pi^{REA}_t \tag{42}$$

[2]Applying the the WID definition of the pre-tax national income share to our model, its model counterpart is given by $NIS^{DE}_{n,t} \equiv \frac{w^{DE}_{n,t} N^{DE}_{n,t} - \left(\frac{R^{DE}_{L,t-1}}{\Pi^{DE}_t} - 1\right) l^{DE}_{n,t-1} - NIS^{DE}_{n,t} b^{DE}_{gov,t}\left(\frac{R^{DE}_{t-1}}{\Pi^{DE}_t} - 1\right)}{NI^{DE}_t}$, (see Alvaredo et al. (2020)). The term -$NIS^{DE}_{n,t} b^{DE}_{gov,t}\left(\frac{R^{DE}_{t-1}}{\Pi^{DE}_t} - 1\right)$ represents the share of the primary factor income of the government (which is negative since there is government debt) allocated to non-rich households for the computation of $NIS^{DE}_{n,t}$. Solving for $NIS^{DE}_{n,t}$ yields equation (40).

Under this assumption, the EA real interest rate may be interpreted as the natural rate of interest. Regarding other variables, assuming that the central bank follows a Taylor type interest feedback rule instead of the perfect inflation target does not materially affect our main results.

2.6 Equilibrium

Combing the total demand for home produced goods (32) and the zero profit condition of final goods firms $DD_t^{DE} = p_{H,t,}^{DE} DD_{H,t}^{DE} + p_{F,t,}^{DE} DD_{F,t}^{DE}$ yields the familiar GDP expenditure equation:

$$Y_t^{DE} = \frac{DD_t^{DE}}{p_{H,t,}^{DE}} + G_t^{DE} + EX_t^{DE} - IM_t^{DE} \tag{43}$$

with

$$IM_t^{DE} = \frac{p_{F,t,}^{DE}}{p_{H,t,}^{DE}} DD_{F,t}^{DE} \tag{44}$$

Total domestic assets held by rich households are given by are given by

$$b_{r,dom,t}^{DE} = l_{n,t}^{DE} + b_{r,gov,t}^{DE} \tag{45}$$

Government bond market clearance requires

$$b_{gov,t}^{DE} = b_{r,gov,t}^{DE} + b_{F,t}^{DE} \tag{46}$$

where $b_{F,t}^{DE}$ denotes foreign holdings of DE government bonds. Since DE invest in REA assets but not vice versa, we have

$$b_{F,t}^{DE} = 0 \tag{47}$$

$$b_{F,t}^{REA} = b_{REA,t}^{DE} \tag{48}$$

Net foreign income results from the holding of REA assets by DE rich households:

$$FI_t^{DE} = \left(\frac{R_{t-1}^{REA}}{\Pi_{REA,t}} - 1\right) q_{REA,t}^{DE} \frac{size^{REA}}{size^{DE}} b_{REA,t-1}^{DE} \tag{49}$$

$$FI_t^{REA} = -\left(\frac{R_{t-1}^{REA}}{\Pi_{REA,t}} - 1\right) b_{REA,t-1}^{DE} \tag{50}$$

Furthermore, combining the budget constraints of DE rich and non-rich households (in per-capita-of-the-population terms), equation (45), the definition of firms profits, equations (12) and (25) yields the law of motion of DE net foreign assets (see Appendix (B.1) for details):

$$q_{REA,t}^{DE}\left(\frac{size_{REA}}{size_{DE}} b_{REA,t}^{DE}\right) = p_{H,t}^{DE}\left(EX_t^{DE} - IM_t^{DE}\right) + \frac{R_{t-1}^{REA}}{\Pi_{REA,t}} q_{REA,t}^{DE}\left(\frac{size_{REA}}{size_{DE}} b_{REA,t-1}^{DE}\right) \tag{51}$$

The exports of DE are REA's imports and vice versa:

$$EX_t^{DE} = \frac{size^{REA}}{size^{DE}} DD_{F,t}^{REA} \tag{52}$$

$$EX_t^{REA} = \frac{size^{DE} DD_{F,t}^{DE}}{size^{REA}} \tag{53}$$

For a complete overview of the model equations see Appendix B.2.

3 Calibration

We divide the model parameters into two groups. The first group (see Table 1a.)), comprising 29 parameters, is calibrated such that the steady-state of the model matches an identical number of selected target values (see Table 2), which, with the exception of the real exchange rate, net foreign assets and hours, are all empirical targets, and are described in more detail in Appendix A. The superscript $J = \{DE, REA\}$ indexes the geographic region. 15 of the empirical targets are are "great ratio" type targets (i.e. for both countries the loan rate spread, the government-demand-to-GDP-ratio, the government-debt-to-annual-GDP ratio, the bottom-90%–debt-to-annual income ratio, the bottom-90%-national-income share, the labor share in national income, and for Germany the real interest rate, inflation and the ratio of imports from the REA to GDP) which are calculated as averages over the period over which we will simulate the historical increase in inequality (i.e. 1992-2016), or the longest available subperiod. Furthermore, we set the intertemporal elasticity of substitution of households to the mean estimate reported in the meta-analysis of Havranek (2015).

The model with CSP requires two additional targets to pin down the two CSP related parameters ($\chi_{b,r}^J$, $\sigma_{b,r}^J$). Following Rannenberg (2023), Rannenberg (2021) and Rannenberg (2019)), the first target is an estimate of the steady-state "discounting wedge" $\theta_r^J \equiv \beta_r^J \frac{R^J}{\Pi^J}$ of rich households. Note that $\theta_r^J < 1$ implies a smaller value of β_r^J than in the NOCSP case (which corresponds to $\theta = 1$), given the unchanged target for the real interest rate.[3] We set $\theta_r^J = 0.97$, close to the choice of Rannenberg (2019), who obtains evidence on θ by drawing on 34 empirical estimates of the (time-varying) nominal individual discount rate which the household applies to future nominal income streams (i.e. $\frac{\Pi}{\beta}$ in the steady-state) and then combines them with a measure of the respectively appropriate safe interest rate to compute an estimate of θ.[4] Among those studies used in Rannenberg (2019), the contributions of Pleeter and Warner (2001) and Harrison et al. (2002) and Harrison et al. (2005) are of particular relevance in our context, because they report estimates for rich or educated households. The values of θ_r^J obtained from these studies are between 0.95-0.97. Following Mian et al. (2020), our second target specific to the CSP model is an estimate of the Marginal Propensity to Consume (MPC) out of wealth of the top 10% of households in the wealth distribution. We take target for Germany from Garbinti et al. (2020) for Germany, and compute the REA target as a weighted average of the estimates for France, Belgium, Spain and Italy reported by Garbinti et al. (2020) and Arrondel et al. (2019). Our approach thus differs from Kumhof et al. (2015) and Rannenberg (2023), who target the marginal propensity to save out of permanent income increases, because we are not aware of DE or REA estimates of the rich households' permanent income Marginal Propensity to Save (MPS).

In the NOCSP model, we set the elasticity of the risk premium DE households earn on REA assets with respect to DE net foreign assets to the smallest value for which the Blanchard and Kahn (1980) conditions are still met. Turning to the second group of parameters (see Table 1b.)), which we calibrate directly, we set the price and wage Phillips Curve slopes κ_π^J and κ_w^J to the estimates of

[3]Conditional on an assumption for $\sigma_{b,r}^J$, the steady state relationship implied by the Euler equation (3) allows to back out $\chi_{b,r}^J$ as $\chi_{b,r}^J = \left(1 - \theta_r^J\right) \Lambda_r^J \left(b_r^J\right)^{-\sigma_{b,r}^J}$

[4]See Rannenberg (2019) for more details on how to estimate θ using the evidence on individual discount rates.

Gadatsch et al. (2016a) and therefore also adopt their calibration of the labor disutility curvature. We set the elasticity of output w.r.t. labor $1-\alpha_K^J$ to the value also used in the model with physical capital. We set the price elasticity of the CES consumption basket in line with the estimates of Osbat and Corbo (2013). Without loss of generality, we set the wage markup to 1.5, and set $\Pi_{ind}^{DE} = \Pi_{ind}^{DE} = \Pi^{DE}$, implying that wage and price adjustment costs are zero in the steady-state. Finally, the calibration of the share of rich households in the total population pop_r^J follows from our assumption that these households represent the top 10% of the population.

All simulations in this paper are performed using the Newton-type solver for deterministic non-linear simulations as implemented in Dynare 4.6.3. (see Adjemian et al. (2011) and Juillard (1996)).[5]

[5]This solver treats the deterministic simulation as a system of simultaneous equations in n endogenous variables in T periods.

Table 1: Calibration of the two region model

a.) Parameters implied by empirical targets

Name	**Parameter**	**DE**	**REA**
Rich household discount factor	β_r^J	0.96	0.96
Non-rich household discount factor	β_n^J	0.99	0.99
Rich household labor disutility weight	$\chi_{N,r}^J$	37851.50	19249.89
Non-rich household labor disutility weight	$\chi_{N,n}^J$	617100.27	682887.64
Consumption utility curvature	σ^J	2.00	2.00
Borrowing friction	ϕ_d^J	0.00	0.00
Price markup	μ^J	1.03	1.08
Non-rich labor income share	ω_n^J	0.90	0.95
Central bank inflation target	Π_{target}^{EA}	1.01	
Gov.-demand-to-GDP ratio target	$G2GDP_{target}^J$	0.21	0.24
Gov. debt-to-GDP ratio target	$bgov2GDP_{target}^J$	0.65	0.85
Country size	$size^J$	33.33	66.67
Consumption basket, home bias	ϕ_H^J	0.82	0.91
Rich household safe asset utility weight	$\chi_{b,r}^J$	0/0.04	0/0.07
Safe asset utility curvature	$\sigma_{b,r}^J$	NaN/0.21	NaN/0.76

b.) Other parameters

Name	**Parameter**	**DE**	**REA**
Labor disutility curvature	η^J	10.00	10.00
Elasticity of output w.r.t. labor	$1-\alpha_K^J$	0.80	0.80
Wage markup	μ_W^J	1.50	1.50
Price Phillips curve slope	κ_π^J	0.05	0.04
Wage Phillips curve slope	κ_w^J	0.05	0.04
Non-costly price/ wage inflation	Π_{ind}^J	1.01	1.01
Consumption basket, price elasticity	λ_m^J	2.50	2.50
Rich household population share	pop_r^J	0.10	0.10
Elasticity of risk premium w.r.t. NFA	ψ_{rp}	0.00001/0	

Note: This table displays the calibration of the two region model. Parameters listed under "Implied by empirical targets" are set to match the targets listed in Table 2. Whenever two values are reported, the first (second) corresponds to the NOCSP (CSP) case.

Table 2: Calibration targets for the two region model

Target	Model counterpart	a.) Domestic variables DE	REA
Intertemp. elasticity of substitution	$\frac{1}{\sigma^{DE}}$	0.50	0.50
Hours rich	N_r^J	0.33	0.33
Hours non-rich	N_n^J	0.33	0.33
Inflation, APR	$\left(\Pi^J - 1\right) * 400$	2.02	
Real short-term interest rate, APR	$\left(\frac{R^J}{\Pi^J} - 1\right) * 400$	1.30	
Spread loan rate over risk-free rate, APR	$\left(\frac{R_L^J}{R^J} - 1\right) * 400$	2.20	2.30
Government demand-to-GDP-ratio, %	$\left(\frac{G^J}{Y^J}\right) * 100$	21.00	24.00
Government debt-to-annual-GDP ratio, %	$\left(\frac{b_{gov}^J}{4Y^J}\right) * 100$	65.00	65.00
Bottom-90%-debt-to-annual-income ratio, %	$\left(\frac{d_n^J}{4w_n^J N_n^J}\right) * 100$	54.00	80.00
Bottom-90%-national-income share, %	$NIS_n^J 100$	68.31	67.54
Labor share in national income, %	$\left(\frac{w^J N^J}{NI^J}\right) * 100$	78.00	74.00
Share in EA national income, PPP %	$size^J$	33.33	0.00
MPC out of wealth of the top 10%, %	See note below for details	0/0.60	0/1.40
Rich discounting wedge	θ_r^J	1.0/0.97	1.0/0.97

Target	Model counterpart	b.) Foreign assets and trade flows
Net foreign assets DE	b_{REA}^{DE}	0.00
DE real exchange rate w.r.t. REA	q^{DE}	1.00
German imports-from-REA-to-GDP ratio, %	$\frac{p_F^{DE} DD_F^{DE}}{p_H^{DE} Y^{DE}}$	14.00

Note: This table displays the targets used to pin down the first group of parameters listed in 1. Note that on top of the target listed in the table, we also impose without loss of generality that the size of the Euro Area economy equals 100. The number of targets is then exactly equal to the number of parameters. Note that empty cells mean that the respective value is not targeted, i.e. it is endogenously determined. Whenever two values are reported, the first (second) corresponds to the NOCSP (CSP) case. APR=Annual Percentage Rate. For details on the source of the targets see the discussion in the text and Appendix A. We compute the Marginal Propensity to Consume (MPC) out of Wealth in the model from a microsimulation of a one-off exogenous increase in rich-household wealth, see Rannenberg (2023), Appendix C, for etails. We compute the MPC as the ratio of the first year average increase in consumption to the first year average increase in wealth: $MPCW_{r,1Y}^J = \frac{\sum_{t=1}^4 \left(C_{r,t}^J - C_{r,0}^J\right)}{\sum_{t=1}^4 \left(b_{r,t}^J - b_{r,0}^J\right)} * 100$.

4 Baseline results

4.1 A one-off permanent increase in income inequality

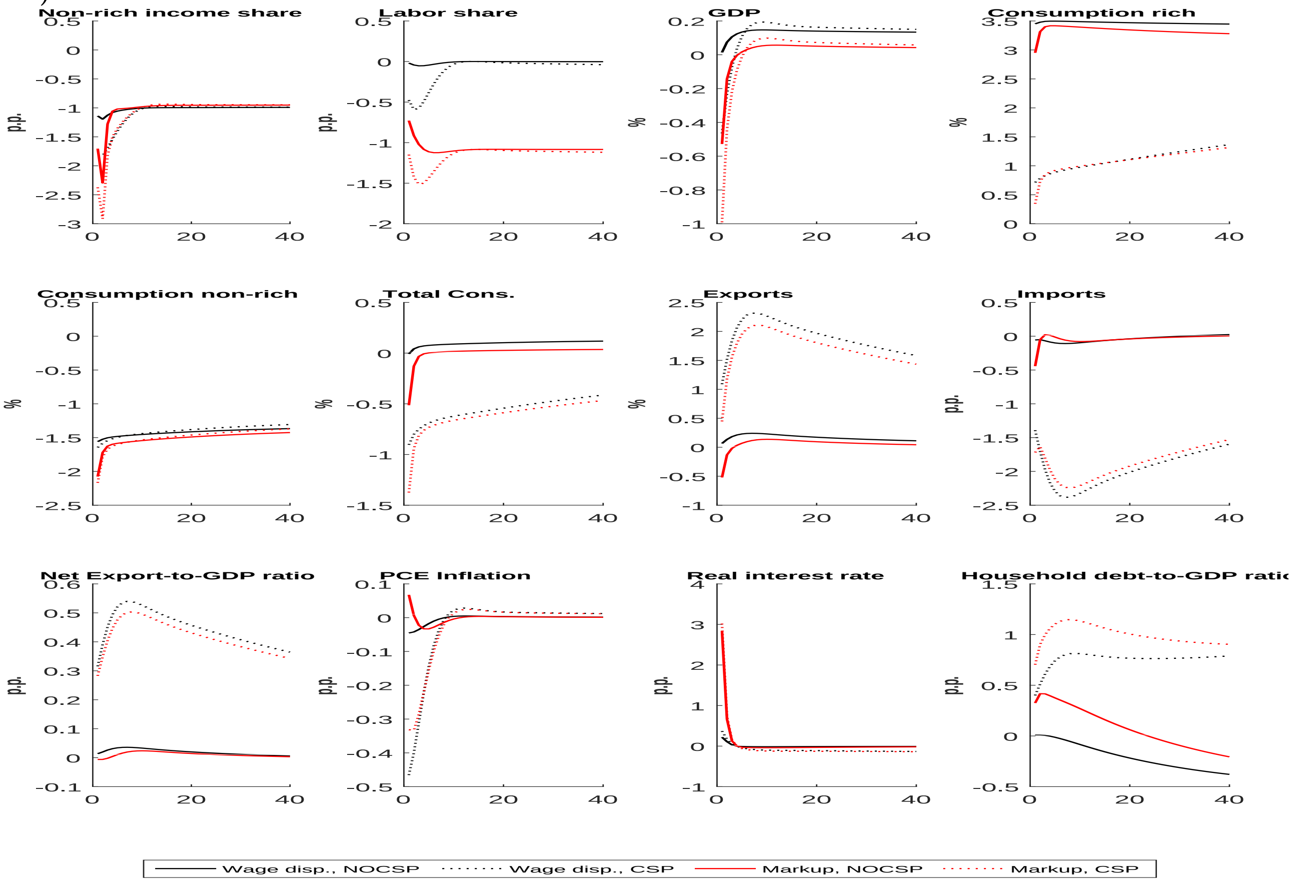
This section describes the effect of an increase in inequality driven either by an increase in the top 10% labor income share or an increase in the price markup, with and without CSP. Both shocks are normalized such that they reduce the non-rich national income share by about 1 p.p. in the long run. We start by describing the effect of the wage inequality shock (a decline in $d^{DE}_{n,t}$). Without CSP (solid black line), rich households increase their consumption on impact by approximately the magnitude of their permanent income change, and keep it subsequently stable. Non-rich households lower their consumption by approximately the decrease of their permanent income. They also permanently lower their borrowing, implying that the household debt-to-GDP ratio declines permanently, because at their initial debt level, their debt-to-income ratio and thus their cost of borrowing are higher than before the shock. There is only a marginal effect on the other variables. By contrast, with CSP, the on-impact consumption increase of rich households equals only a quarter of the increase observed without CSP, implying that total domestic demand declines (see also the flow chart Figure 4 for a graphical illustration of the transmission with CSP). This decline has a direct negative effect on imports, which increases net exports. Furthermore, the decline in labor demand and non-rich household consumption lowers the real wage (note the temporary decline in the labor share) and thus inflation, thus stimulating both exports and imports. The decline in DE inflation triggers a monetary loosening at the EA level. In order to keep EA inflation on target, the central bank persistently lowers the interest rate, implying that the real interest rate declines persistently in DE and REA. In DE, the lower real interest rate incentivizes households to postpone the decline in their consumption somewhat, causing their borrowing to increase. In the REA, the eventual real interest rate decline stimulates the consumption of REA households, which increases REA inflation. Both of these developments also contribute to the improvement in DE net exports. Part of the REA non-rich household consumption increase is funded by additional borrowing, implying that the household-debt-to-GDP ratio increases persistently.

An increase in the price markup (i.e. in $d^{DE}_{\mu,t}$) differs from the labor inequality shock in that it -in itself- causes inflationary pressure, and that, relatedly, it permanently lowers the labor share.

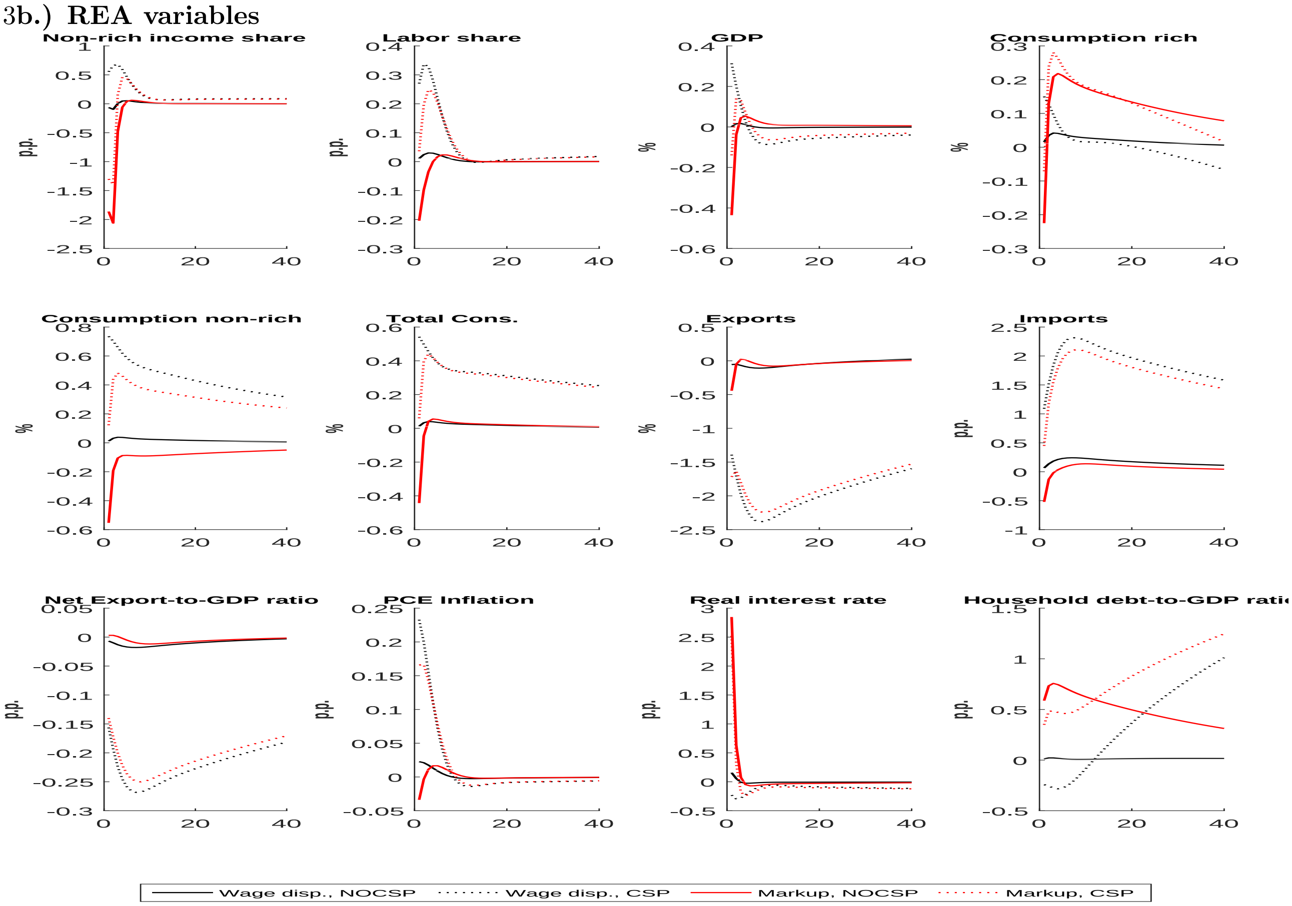
In the model without CSP (red solid line), this aspect is reflected in an increase in DE inflation, which triggers a very temporary monetary tightening at the EA level, reflected in an increase in the nominal and real interest rate. This tightening causes a negative spillover to REA aggregate demand and thus inflation. By contrast, with CSP, the situation is reversed in that DE inflation drops in response to the price markup shock due to the extra decline in demand caused by the redistribution of income from non-rich to rich households, as already described above, while REA inflation briefly increases due to the stimulus provided by the highly persistent drop in the real interest rate starting in quarter 3. In contrast to the aforementioned short-run differences between the responses to wage inequality shock and the price markup shocks, the medium to long-run response are remarkably close for both shocks. This result is in line with the findings of Rannenberg (2023) in a closed economy context.

Figure 3: One-off increase in German inequality

3a.) DE variables

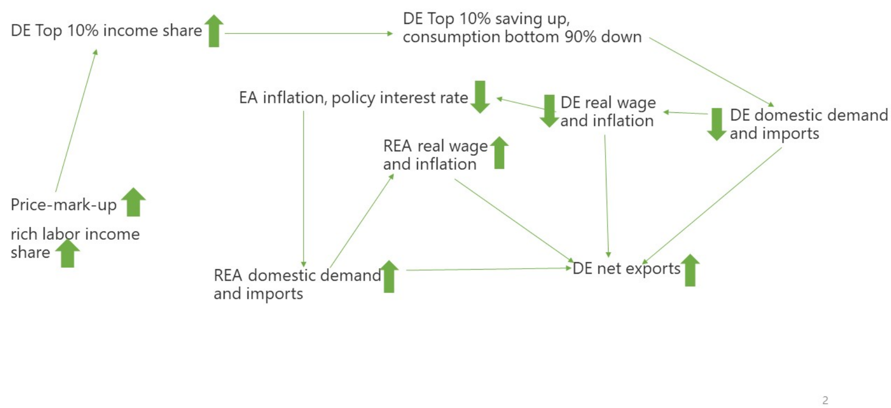


3b.) REA variables



Note: This figure displays the effects of an increase in the DE top 10% national income share in the baseline two region model stemming from two different sources. "Wage disp." indicates decline in the bottom 90% share in total labor income (a decline in $d_{n,t}^{DE}$). "Markup" indicates and increase in the price markup of firms (an increase in $d_{\mu,t}^{DE}$). PCE=Personal Consumption Expenditure.

Figure 4: Transmission of an inequality increase



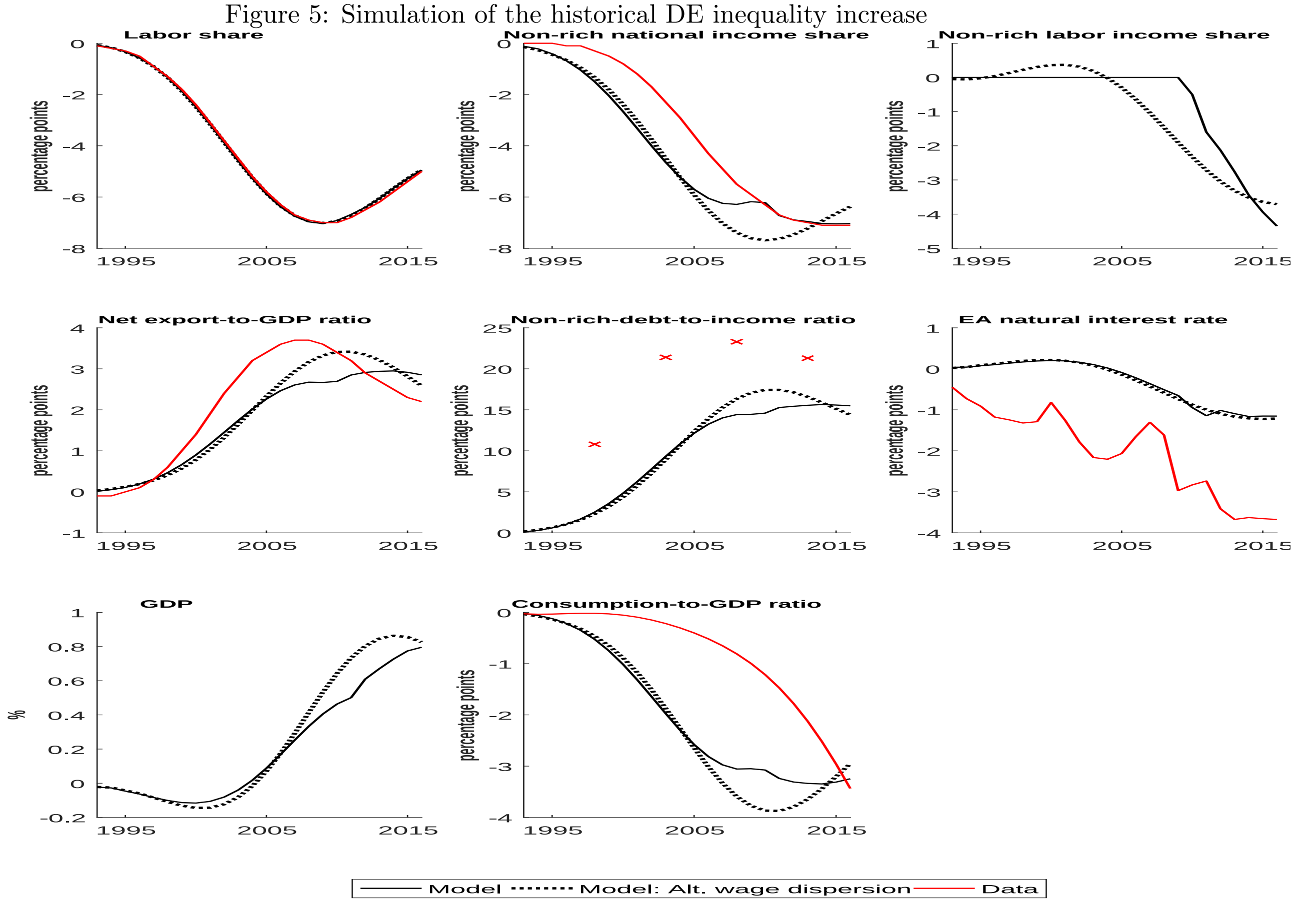
4.2 Simulation of the historical inequality increase

In this section, we perform a simulation which attempts to match the paths of the trend labor share and the bottom 90% national income share from 1992 to 2016, using the model with CSP. Both of these measures decline roughly in parallel until about 2010. Afterwards the bottom 90% income share continues to decline, while the labor share recovers somewhat. (See Figure 5, red line). We match the path of the labor share by setting the path of the price markup shock of the intermediate goods firms $d_{\mu,t}^{DE}$ accordingly, following Caballero et al. (2017), Farhi and Gourio (2018) and Rannenberg (2023), who adopt this approach for the US case.[6] To pin down the wage inequality shock $d_{n,t}^{DE}$ we consider two alternative approaches. In the first approach, we set it to match the empirical decline of the top 10% national income share occurring on top of the decline already replicated by the price markup increase. For the second, we calibrate $d_{n,t}^{DE}$ directly, by setting it equal to the evolution of an estimate of the share of the top 10% income-richest households in total labor income, obtained from the German household survey Socioeconomic Panel (SOEP, Wagner et al. (2007), see Appendix A for further details). Since we back out the two shocks to match long-run trends, we assume that households expect the shocks to be permanent once they occur, but assume that the shocks are not anticipated.

[6] An obvious alternative might seem to be a negative wage mark-up shock. However, even a permanent wage mark-up shock cannot *permanently* affect the labor share in this class of models.

Figure 5 displays the result of the simulation. The simulation tracks the path of the labor share exactly, while the non-rich national income share declines somewhat faster than in the data for most of the simulation period. For our first approach of Starting in 2010, we activate the labor income inequality shock $d_{n,t}^{DE}$ to match the subsequent further decline in the bottom 90% national income share (see the first row, third panel of Figure 5, the black solid line labeled "Model").

Figure 5: Simulation of the historical DE inequality increase



Note: This graph displays two simulations of the historical increase in inequality in the two region model using the price markup shock $d_{\mu,t}^{DE}$ and the labor income inequality shock $d_{n,t}^{DE}$. Both simulations target the path of the trend labor share using a sequence of price markup shocks $d_{\mu,t}^{DE}$. The line labeled "Model" indicates that we activate $d_{n,t}^{DE}$ in 2010 to match the subsequent decline in the bottom 90% national income share. The line labeled "Model: Alt. wage dispersion shocks" indicates that $d_{n,t}^{DE}$ is set equal to the evolution of an estimate of the share of the top 10% income-richest households in total labor income, obtained from the German Socioeconomic Panel (Wagner et al. (2007)). Net-export-to-GDP ratio refers to DE net exports with respect to the REA. See the note below Figure 1 for the source of the natural rates estimates. From all annual data series except the natural rate estimate, we remove fluctuations with an amplitude of 2 to 16 years using the asymmetric full sample band pass filter of Christiano and Fitzgerald (2003), assuming a unit root with drift. We subtract the respective 1992 value from all reported series.

The simulated increase in inequality replicates a large part of the trend net-export-to-GDP ratio with respect to the REA observed in the data. At its peak reached in 2008, the empirical value exceeds its 1992 value by 3.7 p.p., while the model simulated increase equals 2.7 p.p, which rises further to 2.9 p.p. later on. Furthermore, by 2016, the simulated natural interest rate has declined by 1.2 p.p., compared to 3.7 p.p. in the data. Furthermore, the model replicates about 60% of the peak increase of the non-rich household debt-to-income ratio.

Our results strengthen if we adopt the alternative approach to setting $d_{n,t}^{DE}$. According to the SOEP estimate, the decline in the bottom 90% share in labor income began earlier then what is implied by our first approach to setting $d_{n,t}^{DE}$ (compare the black dotted to the black solid line in the rightmost panel in the first row of Figure 5), namely in 2005. Therefore, once we set $d_{n,t}^{DE}$ to target the SOEP data, the post 2005 trajectory of net exports is higher than in the first simulation, with the net exports increase peaking at 3.4 percentage points in 2011.

5 Extensions

5.1 Physical capital

We now extend the model to allow rich households to invest in physical capital. The production technology of intermediate goods firms now becomes a CES aggregate of capital and labor:

$$Y_t^{DE}(j) = \left[\left(1 - \alpha_K^{DE}\right) \left(A_N^{DE} N_t^{DE}(j)\right)^{\frac{\epsilon-1}{\epsilon}} + \alpha_K^{DE} \left(A_K^{DE} K_t^{DE}(j)\right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $K_t(j)$ denotes capital services used by intermediate goods firm j, $A_N^{DE}, A_K^{DE} > 0$ denote normalization constants, and $\alpha_K^{DE}, \epsilon > 0$ denote the quasi capital share and the elasticity of substitution between capital and labor, respectively. DE rich households now maximize

$$\sum_{i=0}^{\infty} \frac{\beta_r^i}{pop_r} \left[\frac{\left(pop_r \tilde{C}_{r,t+i}^{DE}\right)^{1-\sigma^{DE}}}{1-\sigma} - \frac{\chi_N^{DE}}{1+\eta} \left(pop_r \tilde{N}_{r,t+i}^{DE}\right)^{1+\eta} + \left(\begin{array}{c} \frac{\chi_{b,r}^{DE}}{1-\sigma_{b,r}^{DE}} \frac{1}{pop_r} \left(pop_r \left(\tilde{b}_{r,dom,t+i}^{DE} + q_{REA,t}^{DE} \tilde{b}_{REA,t+i}^{DE}\right)\right)^{1-\sigma_{b,r}^{DE}} \\ + \frac{\chi_{r,K}^{DE}}{1-\sigma_{r,K}^{DE}} \left(pop_r p_{K,t+i} \tilde{K}_{t+i}^{DE}\right)^{1-\sigma_{K,r}^{DE}} \end{array} \right) \right]$$

subject to

$$\tilde{b}^{DE}_{r,dom,t} + \tilde{C}^{DE}_{r,t} + q^{DE}_{REA,t}\tilde{b}^{DE}_{REA,t} = \frac{R^{DE}_{t-1}}{\Pi^{dE}_{t}}\tilde{b}^{DE}_{r,dom,t-1} + \frac{R^{REA}_{t-1}}{\Pi_{REA,t}}q^{DE}_{REA,t}\tilde{b}^{DE}_{REA,t-1} + w^{DE}_{r,t}\tilde{N}^{DE}_{r,t} - \tilde{T}^{DE}_{r,t}$$

$$+ \tilde{\Xi}^{DE}_{t} + r^{DE}_{K,t}\tilde{K}^{DE}_{t-1} - \left(\tilde{I}^{DE}_{t} + \tilde{K}^{DE}_{t-1}\Phi\left(\frac{\tilde{I}^{DE}_{t}}{\tilde{K}^{DE}_{t-1}} - \delta^{DE}\right)\right) \tag{54}$$

where $r_{K,t}$, $\tilde{I}_t$ and δ denote the capital rental, investment and the depreciation rate, respectively, while $\Phi\left(\right)$ denote convex capital adjustment costs, with

$$\Phi\left(\frac{\tilde{I}^{DE}_{t}}{\tilde{K}^{DE}_{t-1}} - \delta^{DE}\right) = \frac{\epsilon^{DE}{}_{I}}{2}\left(\frac{I^{DE}_{t}}{K^{DE}_{t-1}} - \delta^{DE}\right)^{2} \tag{55}$$

while the capital accumulation equation is given by

$$\tilde{K}^{DE}_{t} = \left(1 - \delta^{DE}\right)\tilde{K}^{DE}_{t-1} + \tilde{I}^{DE}_{t} \tag{56}$$

As a result, there are now two additional first order conditions, with respect to capital and investment:

$$p^{DE}_{K,t} = E_t\left\{\beta^{DE}_{r}\frac{\Lambda^{DE}_{r,t+1}}{\Lambda^{DE}_{r,t}}\left[r^{DE}_{K,t+1} + \frac{I^{DE}_{t+1}}{K^{DE}_{t}}\Phi'\left(\frac{I^{DE}_{t+1}}{K^{DE}_{t}} - \delta^{DE}\right) - \Phi\left(\frac{I^{DE}_{t+1}}{K^{DE}_{t}} - \delta^{DE}\right) + \left(1 - \delta^{DE}\right)p^{DE}_{K,t+1}\right] + p^{DE}_{K,t}\frac{\chi^{DE}_{r,K}\left(p^{DE}_{K,t}K^{DE}_{t}\right)^{-\sigma^{DE}_{K,r}}}{\Lambda^{DE}_{r,t}}\right\} \tag{57}$$

$$p^{DE}_{K,t} = 1 + \Phi'\left(\frac{I^{DE}_{t}}{K^{DE}_{t-1}} - \delta^{DE}\right) \tag{58}$$

Table 3: Calibration of the two region model with capital

a.) Parameters implied by empirical targets

Name	Parameter	DE	REA
Rich household discount factor	β_r^J	0.96	0.96
Non-rich household discount factor	β_n^J	0.99	0.99
Rich household labor disutility weight	$\chi_{N,r}^J$	19394.47	9923.55
Non-rich household labor disutility weight	$\chi_{N,n}^J$	317783.81	353643.88
Consumption utility curvature	σ^J	2.00	2.00
Borrowing friction	ϕ_d^J	0.00	0.00
Price markup	μ^J	1.16	1.21
Non-rich labor income share	ω_n^J	0.90	0.95
Central bank inflation target	Π_{target}^{EA}	1.01	
Gov.-demand-to-GDP ratio target	$G2GDP_{target}^J$	0.21	0.24
Gov. debt-to-GDP ratio target	$bgov2GDP_{target}^J$	0.65	0.85
Country size	$size^J$	33.33	66.67
Consumption basket, home bias	ϕ_H^J	0.82	0.91
Elasticity of output w.r.t. capital	α_K^J	0.21	0.21
Rich household safe asset utility weight	$\chi_{b,r}^J$	0/0.01	0/0.03
Safe asset utility curvature	$\sigma_{b,r}^J$	NaN/0.20	NaN/0.82
Rich household capital utility weight	$\chi_{K,r}^J$	0/9.08	0/188.28
Capital utility curvature	$\sigma_{K,r}^J$	4.50	6.50

b.) Other parameters

Name	Parameter	DE	REA
Labor disutility curvature	η^J	10.00	10.00
Wage markup	μ_W^J	1.50	1.50
Price Phillips curve slope	κ_π^J	0.05	0.04
Wage Phillips curve slope	κ_w^J	0.05	0.04
Non-costly price/ wage inflation	Π_{ind}^J	1.01	1.01
Consumption basket, price elasticity	λ_m^J	2.50	2.50
Rich household population share	pop_r^J	0.10	0.10
Elasticity of risk premium w.r.t. NFA	ψ_{rp}	0.00001/0	
Depreciation rate	δ^J	0.025	0.025
Capital adjustment cost curvature	ϵ_I^J	7.00	7.00
Elasticity of substitution capital/ labor	ϵ^J	0.30	0.30

Note: This table displays the calibration of the two region mode with capitall. Parameters listed under "Implied by empirical targets" are set to match the targets listed in Table 4. Whenever two values are reported, the first (second) corresponds to the NOCSP (CSP) case.

Table 4: Calibration targets for the two region model with capital

		a.) Domestic variables	
Target	**Model counterpart**	**DE**	**REA**
Intertemp. elasticity of substitution	$\frac{1}{\sigma^{DE}}$	0.50	0.50
Hours rich	N_r^J	0.33	0.33
Hours non-rich	N_n^J	0.33	0.33
Inflation, APR	$(\Pi^J - 1) * 400$	2.02	
Real short-term interest rate, APR	$\left(\frac{R^J}{\Pi^J} - 1\right) * 400$	1.30	
Spread loan rate over risk-free rate, APR	$\left(\frac{R_L^J}{R^J} - 1\right) * 400$	2.20	2.30
Government demand-to-GDP-ratio, %	$\left(\frac{G^J}{Y^J}\right) * 100$	21.00	24.00
Government debt-to-annual-GDP ratio, %	$\left(\frac{b_{gov}^J}{4Y^J}\right) * 100$	65.00	65.00
Bottom-90%-debt-to-annual-income ratio, %	$\left(\frac{d_n^J}{4w_n^J N_n^J}\right) * 100$	54.00	80.00
Bottom-90%-national-income share, %	$NIS_n^J 100$	68.22	67.44
Labor share in national income, %	$\left(\frac{w^J N^J}{NI^J}\right) * 100$	78.00	74.00
Share in EA national income, PPP %	$size^J$	33.33	
Private-investment-to-GDP ratio, %	$\left(\frac{I^J}{Y^J}\right) * 100$	12.60	12.10
MPC out of wealth of the top 10%, %	See note below for details	0/0.60	0/1.40
Rich discounting wedge	θ_r^J	1.0/0.97	1.0/0.97
Spread costs of capital over risk free rate, APR	$100 * \left(\left(\frac{r_K^{DE} - \delta^{DE} + 1}{\frac{R^{DE}}{\Pi^{DE}}}\right)^4 - 1\right)$	0/3.10	0/3.20
Effect of gov. debt-to-GDP ratio on bond yield, APR	See note below for details	0/0.03-0.06p.p	

Target	**Model counterpart**	**b.) Foreign assets and trade flows**
Net foreign assets DE	b_{REA}^{DE}	0.00
DE real exchange rate w.r.t. REA	q^{DE}	1.00
German imports-from-REA-to-GDP ratio, %	$\frac{p_F^{DE} DD_F^{DE}}{p_H^{DE} Y^{DE}}$	14.00

Note: Unless otherwise mentioned, for details on the empirical targets and their sources see the note below Table 2. The target regarding the effect of an increase in the government debt to GDP ratio relates to the effect of an increase in the five year ahead Euro Area wide government-debt-to-GDP ratio on the ex-ante real 5-year-ahead 10-year forward Treasury rate. To obtain the model counterpart, we simulate the effect of a permanent expected increase in the government debt-to-GDP ratio target in the fith year from today and compute $\frac{d\left(E_t\left\{\frac{\sum_{j=1}^{40} 4\left(\hat{R}_{t+16+j} - \hat{\Pi}_{t+17+j}\right)}{40}\right\}\right)}{d\left(\hat{b}_{G,t+17}\frac{b_G}{4Y}\right)}$.

Hence there are four additional parameters for each country, namely $\chi_{r,K}^J$, $\sigma_{r,K}^J$, δ^J and ϵ_I^J (see Table 3). We set the depreciation rate δ^J to the standard value of 0.025 and ϵ_I^J to the estimate of Cummins et al. (2006). We calibrate $\chi_{r,K}^J$ and $\sigma_{r,K}^J$ by adopting two additional empirical targets.

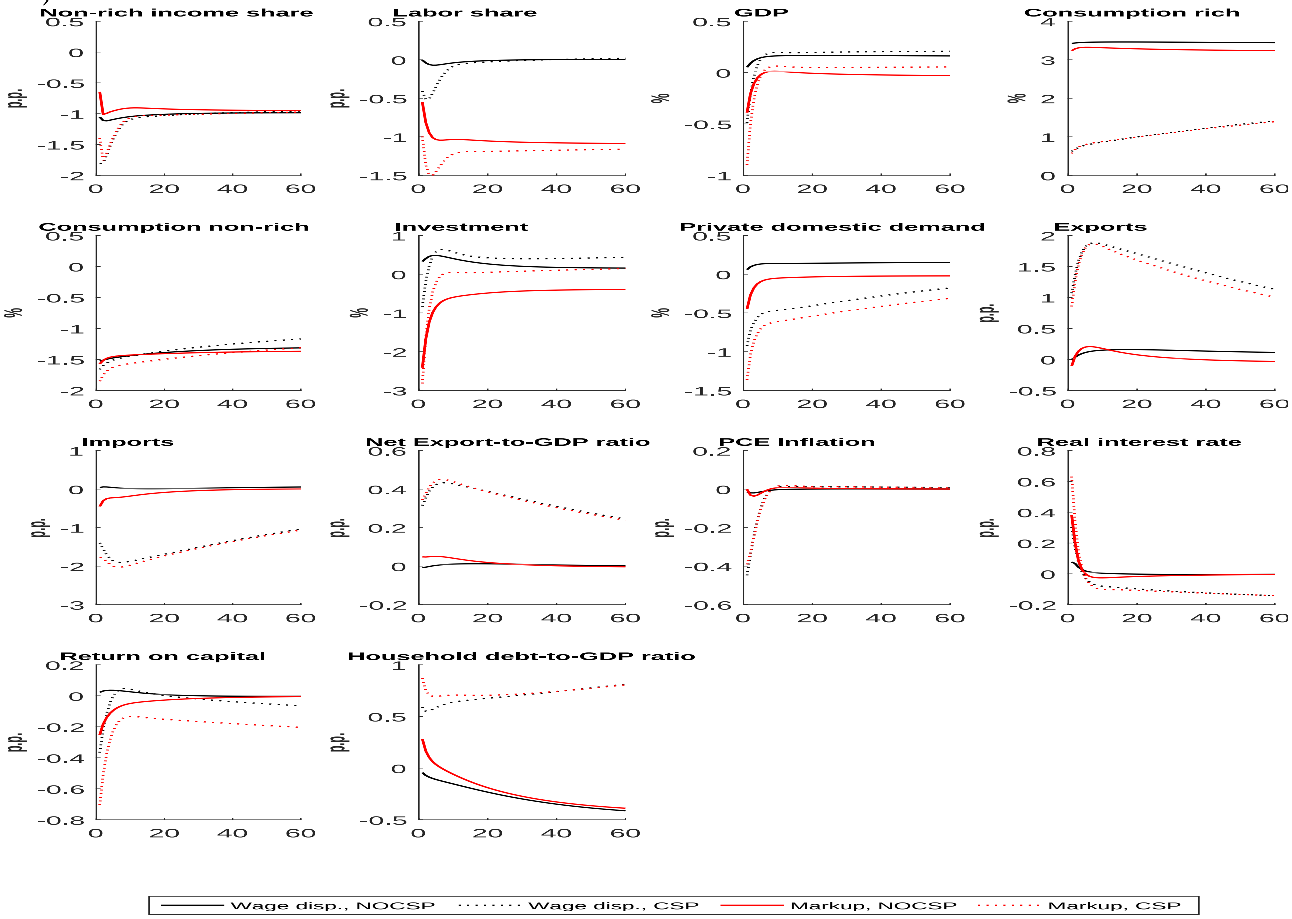
The first is an estimate of the average spread between the costs of capital and the risk free rate, whose model counterpart equals $100 * \left(\left(\frac{r_K^J - \delta^J + 1}{\frac{R^J}{\Pi^J}} \right)^4 - 1 \right)$. We compute this target as a weighted average of the the spread of the corporate borrowing rate over the risk free rate and an estimate of the equity premium reported by Aswath Damodaran, where the weights are the shares of debt and equity in total non-financial corporate funding (see Appendix A for details). The second additional target we adopt is the effect of an increase in the supply of government bonds on the risk free rate. We are not aware of estimates of this effect for the Euro Area. Therefore we draw on estimates for the US, namely Gale and Orszag (2004), Engen and Hubbard (2005) and Laubach (2009). Specifically, we target the effect of an increase of the five year ahead Euro Area wide government-debt-to-GDP ratio on the ex-ante real 5-year-ahead 10-year forward Treasury rate estimated by these authors. In practice, in the model, this object is closely linked to the value of $\sigma_{b,r}^J$.

The effect of an increase in inequality is overall very similar to the model without physical capital (see Figure 6). The main difference is that the improvement of DE net exports is somewhat lower (by about one 5th for an increase in wage dispersion and one 10th for a markup increase), because unlike total consumption, investment does not decline persistently, implying that overall domestic demand declines somewhat less than in the baseline case. The reason that investment increases eventually is that rich households now invest part of their additional savings in physical capital, as they derive utility from their capital stock. However, the increase in investment is considerably smaller in case of a markup increase, which in itself lowers the demand for capital of the intermediate goods firm.

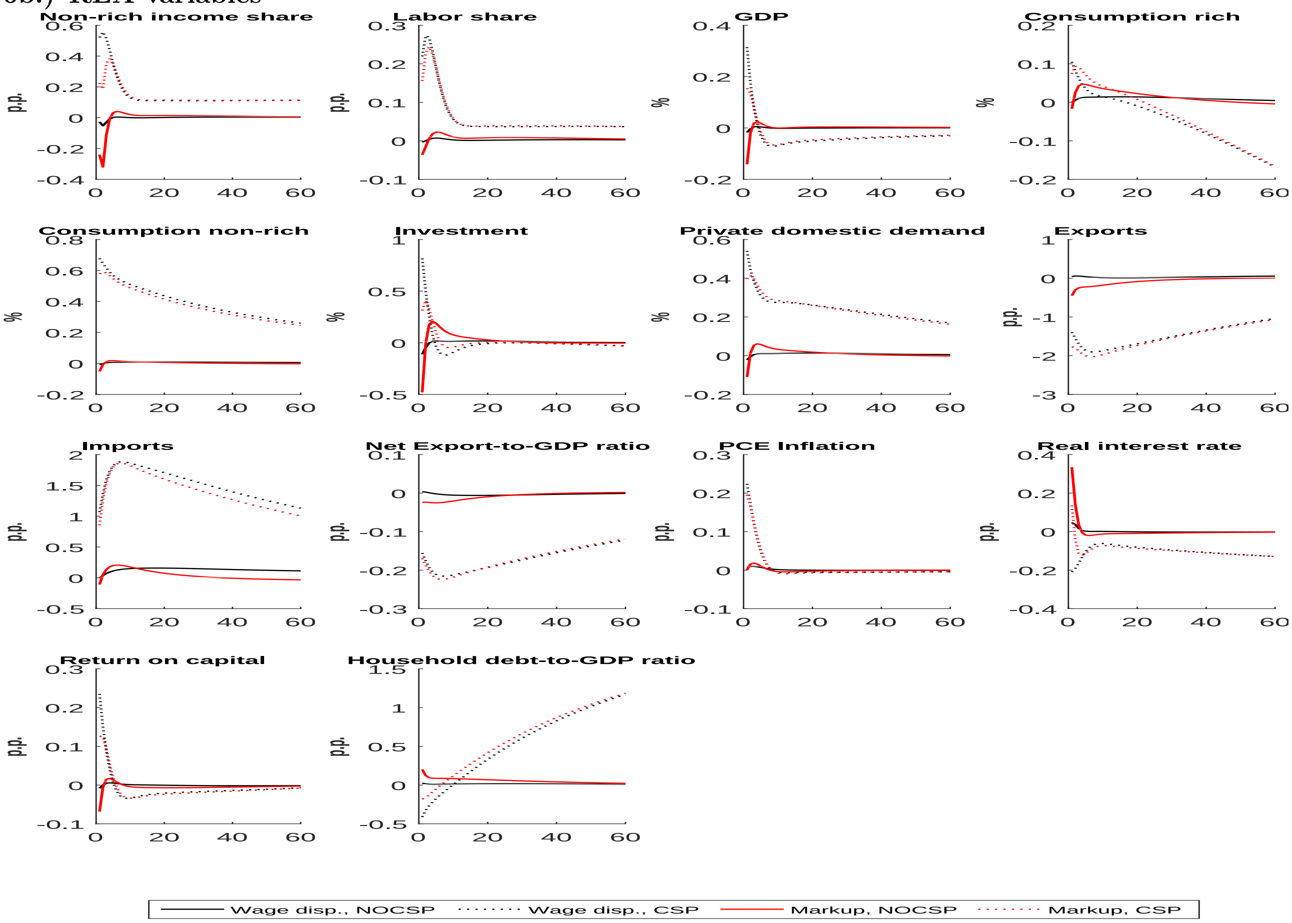
Correspondingly, when simulating the historically observed increase in inequality, the overall increase in net exports is also somewhat reduced. Still, the model is able to reproduce about 61% of the peak increase in the export surplus w.r.t. the REA.

Figure 6: Model with capital: one-off increase in German inequality

6a.) DE variables

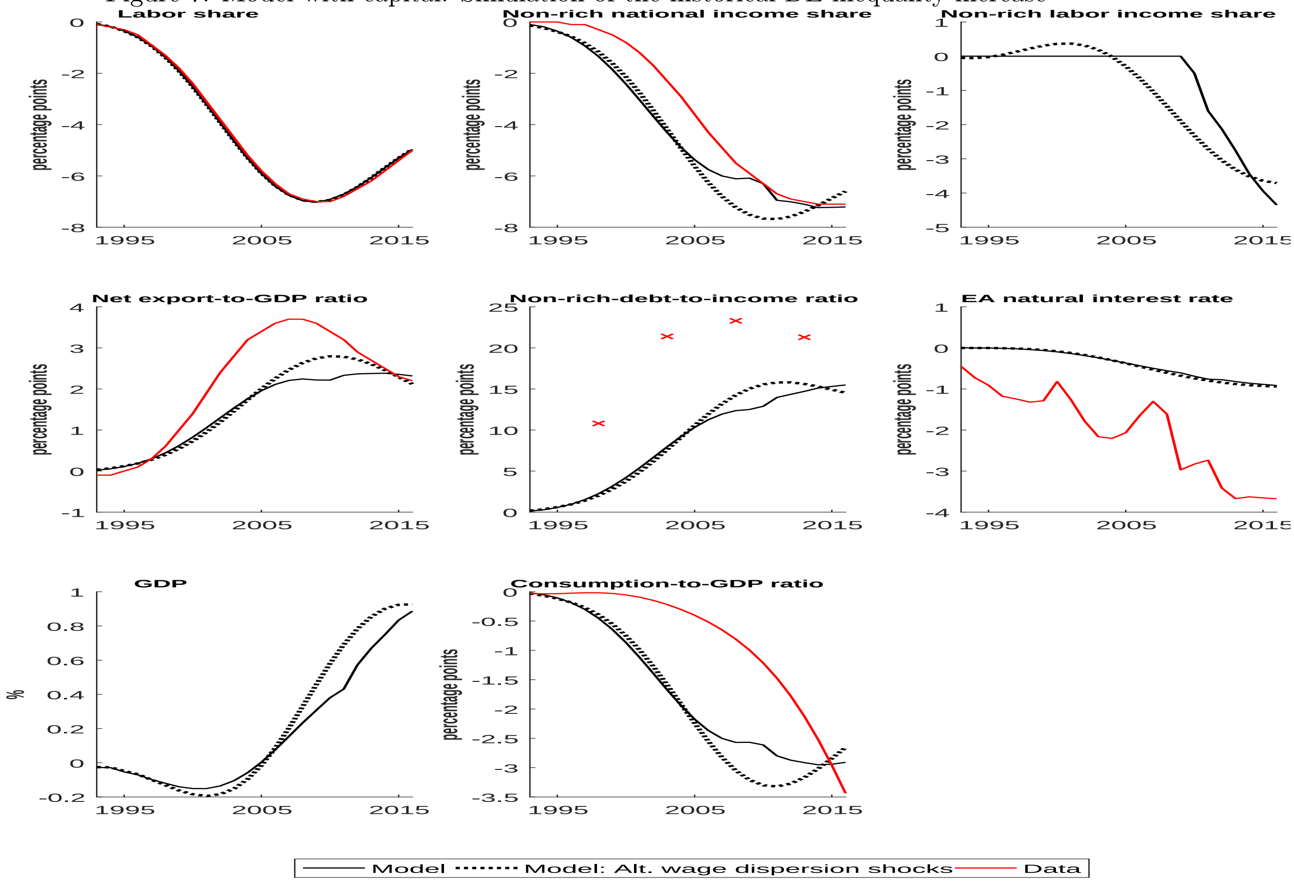


6b.) REA variables



Note: See the note below Figure 3 for details on the meaning of the labels.

Figure 7: Model with capital: Simulation of the historical DE inequality increase



Note: This graph displays the effect of simulating the historical increase in inequality in the two region model with capital. For details regarding the simulation setup and information on the data displayed see the note below Figure 5.

5.2 Incorporating the inequality increase in the rest of the world (ROW)

We now extend the model to include a rest of the world (ROW) block. The DE final consumption good is now a CES basket of the products of all three regions:

$$DD_t^{DE} = \left[\left(\phi_H^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_H^{DE} DD_{H,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} + \left(\phi_{REA}^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_{REA}^{DE} DD_{REA,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} + \left(1-\phi_H^{DE}-\phi_{REA}^{DE}\right)^{\frac{1}{\lambda_m^{DE}}} \left(A_{ROW}^{DE} DD_{ROW,t}^{DE}\right)^{\frac{\lambda_m^{DE}-1}{\lambda_m^{DE}}} \right]$$

and analogously for the other regions. Regarding international capital flows, we continue to assume that rich households (now including those in ROW) treat DE bonds and REA bonds as perfect substitutes, but that they have separate preferences over ROW bonds. Furthermore, we now assume that in each region, rich households invest in assets of all countries. As discussed above, the assumption that DE and REA bonds are perfect substitutes implies that cross-country claims between DE and REA are not pinned down by the households optimal choices (see Section2.1). We therefore assume that the ratio of REA's holdings of DE bonds to ROW bonds remains constant at its steady-state value. Similarly, we also assume that the ratio of ROW's holdings of DE to REA bonds is constant, as it would be equally indeterminate. Furthermore, we assume that the safe asset curvature is identical across safe asset classes, implying that an income increase does not affect the shares of EA and ROW assets in the rich household's portfolio. The utility of rich DE households is thus be given by

$$E_t \left\{ \sum_{i=0}^{\infty} \frac{(\beta_r^{DE})^i}{pop_r} \left[\frac{\left(pop_r \tilde{C}_{r,t+i}^{DE}\right)^{1-\sigma^{DE}}}{1-\sigma^{DE}} - \frac{\chi_{N,r}^{DE}}{1+\eta^{DE}} \left(pop_r \tilde{N}_{r,t+i}^{DE}\right)^{1+\eta^{DE}} + \left(\begin{array}{c} \frac{\chi_{b,r}^{DE}}{1-\sigma_{b,r}^{DE}} \left(pop_r \left(\tilde{b}_{r,dom,t+i}^{DE} + q_{REA,t+i}^{DE} \tilde{b}_{REA,t+i}^{DE}\right)\right)^{1-\sigma_{b,r}^{DE}} \\ + \frac{\chi_{b_{ROW},r}^{DE}}{1-\sigma_{b_{ROW},r}^{DE}} \left(pop_r q_{ROW,t+i}^{DE} \tilde{b}_{ROW,t+i}^{DE}\right)^{1-\sigma_{r,b}^{DE}} \end{array} \right) \right] \tag{59}$$

while the budget constraint is given by

$$\begin{aligned} \tilde{b}_{r,dom,t}^{DE} + \tilde{C}_{r,t}^{DE} + q_{REA,t}^{DE} \tilde{b}_{REA,t}^{DE} + q_{ROW,t}^{DE} \tilde{b}_{ROW,t}^{DE} &= w_{r,t}^{DE} \tilde{N}_{r,t}^{DE} - \tilde{T}_{r,t}^{DE} + \tilde{\Xi}_t^{DE} \\ &+ \frac{R_{t-1}^{DE}}{\Pi_t^{DE}} \tilde{b}_{r,dom,t-1}^{DE} + \frac{R_{t-1}^{REA}}{\Pi_{REA,t}} q_{REA,t}^{DE} \tilde{b}_{REA,t-1}^{DE} + \frac{R_{t-1}^{ROW}}{\Pi_{ROW,t}} q_{ROW,t}^{DE} \tilde{b}_{ROW,t-1}^{DE} \end{aligned}$$

The utility of REA households is analogous, while the utility of ROW households is given by

$$E_t \left\{ \sum_{i=0}^{\infty} \frac{\left(\beta_r^{ROW}\right)^i}{pop_r} \left[\begin{array}{c} \frac{\left(pop_r \tilde{C}_{r,t+i}^{ROW}\right)^{1-\sigma^{ROW}}}{1-\sigma} - \frac{\chi_{N,r}^{ROW}}{1+\eta^{ROW}} \left(pop_r \tilde{N}_{r,t+i}^{ROW}\right)^{1+\eta^{ROW}} \\ + \left(\begin{array}{c} \frac{\chi_{b,r}^{ROW}}{1-\sigma_{b,r}^{ROW}} \left(pop_r \tilde{b}_{r,dom,t+i}^{ROW}\right)^{1-\sigma_{b,r}^{ROW}} \\ + \frac{\chi_{b_{EA},r}^{ROW}}{1-\sigma_{b_{EA},r}^{ROW}} \left(pop_r \left(q_{REA,t}^{ROW} \left(\frac{size^{REA}}{size^{ROW}} \tilde{b}_{REA,t}^{ROW}\right) + q_{DE,t}^{ROW} \left(\frac{size^{DE}}{size^{ROW}} \tilde{b}_{DE,t}^{ROW}\right)\right)\right)^{1-\sigma_{r,b}^{ROW}} \end{array} \right) \end{array} \right] \right\} \tag{60}$$

The FOCs w.r.t. domestic assets and the FOC of DE rich households w.r.t. REA assets are all identical to the two region model. Compared to the two region model, DE rich households have an additional first order condition, with respect to holdings of ROW assets $\tilde{b}_{ROW,t+i}^{DE}$:

$$\Lambda_{r,t}^{DE} = \beta_r^{DE} E_t \left\{ \frac{R_{ROW,t}}{\Pi_{t+1}^{ROW,t}} \frac{q_{ROW,t+1}^{DE}}{q_{ROW,t}^{DE}} \Lambda_{r,t+1}^{DE} \right\} + \chi_{b_{ROW},r}^{DE} \left(q_{ROW,t}^{DE} \left(\frac{size^{ROW}}{size^{DE}} b_{ROW,t}^{DE} \right) \right)^{\sigma_{r,b}^{DE}} \tag{61}$$

The optimal REA and ROW rich foreign asset demands are given by

$$\Lambda_{r,t}^{REA} = \beta_r^{REA} E_t \left\{ \frac{R_{ROW,t}}{\Pi_{t+1}^{ROW,t}} \frac{q_{ROW,t+1}^{REA}}{q_{ROW,t}^{REA}} \Lambda_{r,t+1}^{REA} \right\} + \chi_{b_{ROW},r}^{REA} \left(q_{ROW,t}^{REA} \left(\frac{size^{ROW}}{size^{REA}} b_{ROW,t}^{REA} \right) \right)^{-\sigma_{b,r}^{REA}} \tag{62}$$

$$\Lambda_{r,t}^{ROW} = \beta_r^{ROW} E_t \left\{ \frac{R_{DE,t}}{\Pi_{t+1}^{DE,t}} \frac{q_{DE,t+1}^{ROW}}{q_{DE,t}^{ROW}} \Lambda_{r,t+1}^{ROW} \right\} + \chi_{b_{EA},r}^{ROW} \left(q_{REA,t}^{ROW} \left(\frac{size^{REA}}{size^{ROW}} b_{REA,t}^{ROW} \right) + q_{DE,t}^{ROW} \left(\frac{size^{DE}}{size^{ROW}} b_{DE,t}^{ROW} \right) \right)^{-\sigma_{b,r}^{ROW}} \tag{63}$$

$$q_{DE,t}^{REA} \left(\frac{size^{DE}}{size^{REA}} * b_{DE,t}^{REA} \right) = \chi_{b_{DE}}^{REA} q_{ROW,t}^{REA} \left(\frac{size^{ROW}}{size^{REA}} * b_{ROW,t}^{REA} \right) \tag{64}$$

$$q_{DE,t}^{ROW} \left(\frac{size^{DE}}{size^{ROW}} * b_{DE,t}^{ROW} \right) = \chi_{b_{DE}}^{ROW} q_{REA,t}^{ROW} \left(\frac{size^{REA}}{size^{ROW}} * b_{REA,t}^{ROW} \right) \tag{65}$$

where the first two equations are the FOCs of the REA rich w.r.t. ROW assets and the FOC of the ROW rich w.r.t. DE assets, respectively. The final two equations pin down the demand of REA for DE assets and the portfolio composition of ROW, reflecting the aforementioned consequences of the fact that DE and REA assets are perfect substitutes. The full set of model equations can be obtained from Appendix C.

To calibrate the additional parameters arising in the three region model, we impose additional empirical targets for the steady-state of the model, reported in Table 6. Calibrating the ROW regions is inherently difficult, which is why some of the targets are calculated based on US data only. These include the household loan rate spread, the bottom-90%-debt-to-annual-income ratio and the target related to rich household's saving behavior. For the later, we use rich households marginal propensity to save out of permanent income changes as estimated by Dynan et al. (2004), following Kumhof et al. (2015).

To pin down the steady-state trade flows and thus the weights in the CES consumption basket, we assume that DE and REA NFA are zero (implying zero ROW NFA and zero net exports in all regions), we target the empirical Euro Area and DE import-to-GDP ratio, the DE imports-from-REA-to-GDP ratio, and assume that steady-state trade between DE and REA is balanced and that the real exchange rates between DE and REA and DE and ROW equal one. To pin down cross-country/region financial claims, we set an empirical target for the share of total DE liabilities held by foreigners, obtained from the annual sector accounts.[7] Furthermore, we impose that net

[7]Specifically, the numerator is (from Eurostat, national accounts, DE annual sector accounts) consolidated total

claims of DE on ROW are zero, the share of DE foreign liabilities held by ROW corresponds to the ROW share in total DE imports, and that the share of DE claims in REA foreign liabilities equals the share of REA imports from DE in REA total imports. We rely on the latter two assumptions because we are not aware of a data source which would allow us to compute say the share of REA and ROW liabilities held by foreigners as we do for DE.[8] We therefore opt for a simple approach, which links cross country holdings we need to pin down to the trade flows on which we do have data.

Finally, we assume the same real interest rate in all regions because the NOCSP model cannot accommodate cross-country differences in real interest rates without modifications.

We draw the wage and price Phillips Curve slopes κ_w^{ROW} and κ_π^{ROW} from evidence for the US over the sample period by Rannenberg (2021), and adopt the and the same labor disutility curvature and elasticity of output with respect to labor $\left(1-\alpha_K^J\right)$ as for DE and REA.

assets of the "Rest of the World" sector. The denominator is total liabilities of the"Total economy" sector.

[8]For instance, while Eurostat does publish the annual sector accounts we rely on for DE (see the previous footnote) also for an EA aggregate, unfortunately, the total assets of the EA "Rest of the World" sector is simply the sum of the total assets of the "Rest of the World" sector across all EA member states. Hence the EA "Rest of the World" total assets include claims of EA members on each other, on top of the claims of extra-Euro Area economies on the EA, which is what we would be interested in.

Table 5: Calibration three region model

		DE	REA	ROW
a.) Parameters implied by empirical targets				
Name	**Parameter**	**DE**	**REA**	**ROW**
Rich household discount factor	β_r^J	0.96	0.96	0.96
Non-rich household discount factor	β_n^J	0.99	0.99	0.99
Rich household labor disutility weight	$\chi_{N,r}^J$	37851.50	19249.89	53719.23
Non-rich household labor disutility weight	$\chi_{N,n}^J$	617100.27	682887.64	756279.44
Consumption utility curvature	σ^J	2.00	2.00	2.00
Borrowing friction	ϕ_d^J	0.0025	0.0018	0.0009
Price markup	μ^J	1.03	1.08	1.01
Non-rich labor income share	ω_n^J	0.90	0.95	0.81
Central bank inflation target	$\Pi_{target}^{EA}, \Pi_{target}^{ROW},$	1.01		1.01
Gov.-demand-to-GDP ratio target	$G2GDP_{target}^J$	0.21	0.24	0.24
Gov. debt-to-GDP ratio target	$bgov2GDP_{target}^J$	0.65	0.85	0.64
Country size	$size^J$	5.50	13.75	80.75
Consumption basket, home bias	ϕ_H^J	0.62	0.64	0.94
Consumption basket, DE goods weight	ϕ_{DE}^{REA}		0.07	
Consumption basket, REA goods weight	ϕ_{REA}^J	0.18		0.05
Rich household domestic currency asset weight	$\chi_{b,r}^J$	0/0.04	0/0.06	0/0.06
Rich household foreign currency asset weight	$\chi_{b_{ROW},r}^{DE}; \chi_{b_{ROW},r}^{REA}; \chi_{b_{EA},r}^{ROW}$	0/0.025	0/0.015	0/0.001
Safe asset utility curvature	$\sigma_{b,r}^J$	NaN/0.20	NaN/0.79	NaN/1.20
(Claims on DE)/(Claims on ROW)	$\chi_{b_{DE}}^{ROW}$		0.26	
(Claims on DE)/(Claims on REA)	$\chi_{b_{DE}}^{REA}$			0.30

b.) Other parameters				
Name	**Parameter**	**DE**	**REA**	**ROW**
η^J	10.00	10.00	10.00	
$1-\alpha_K^J$	0.80	0.80	0.80	
μ_W^J	1.50	1.50	1.50	
κ_π^J	0.05	0.04	0.01	
κ_w^J	0.05	0.04	0.01	
Π_{ind}^J	1.01	1.01	1.01	
λ_m^J	2.50	2.50	2.50	
pop_r^J	0.10	0.10	0.10	
Elasticity of risk premium w.r.t. NFA	ψ_{rp}		0.00200/0	

Note: This table displays the calibration of the three region model.

Table 6: Calibration targets in the three region model

a.) Domestic variables

Target	Model counterpart	DE	REA	ROW
Intertemp. elasticity of substitution	$\frac{1}{\sigma^{DE}}$	0.50	0.50	0.50
Hours rich	N_r^J	0.33	0.33	0.33
Hours non-rich	N_n^J	0.33	0.33	0.33
Inflation, APR	$(\Pi^J - 1) * 400$	2.02		2.02
Real short-term interest rate, APR	$\left(\frac{R^J}{\Pi^J} - 1\right) * 400$	1.30		1.30
Spread loan rate over risk-free rate, APR	$\left(\frac{R_L^J}{R^J} - 1\right) * 400$	2.20	2.30	1.69
Government demand-to-GDP-ratio, %	$\left(\frac{G^J}{Y^J}\right) * 100$	21.00	24.00	24.00
Government debt-to-annual-GDP ratio, %	$\left(\frac{b_{gov}^J}{4Y^J}\right) * 100$	65.00	85.00	64.00
Bottom-90%-debt-to-annual-income ratio, %	$\left(\frac{d_n^J}{4w_n^J N_n^J}\right) * 100$	54.00	80.00	118.00
Bottom-90%-national-income share, %	$NIS_n^J 100$	68.31	67.54	61.23
Labor share in national income, %	$\left(\frac{w^J N^J}{NI^J}\right) * 100$	78.00	74.00	79.00
Share in world national income, PPP %	$size^J$	5.50	13.75	
MPC out of wealth of the top 10%, %	See note below for details	0/0.60	0/1.40	0/
Rich discounting wedge	θ_r^J	1.0/0.97	1.0/0.97	1.0/0.97

b.) Foreign assets and trade flows

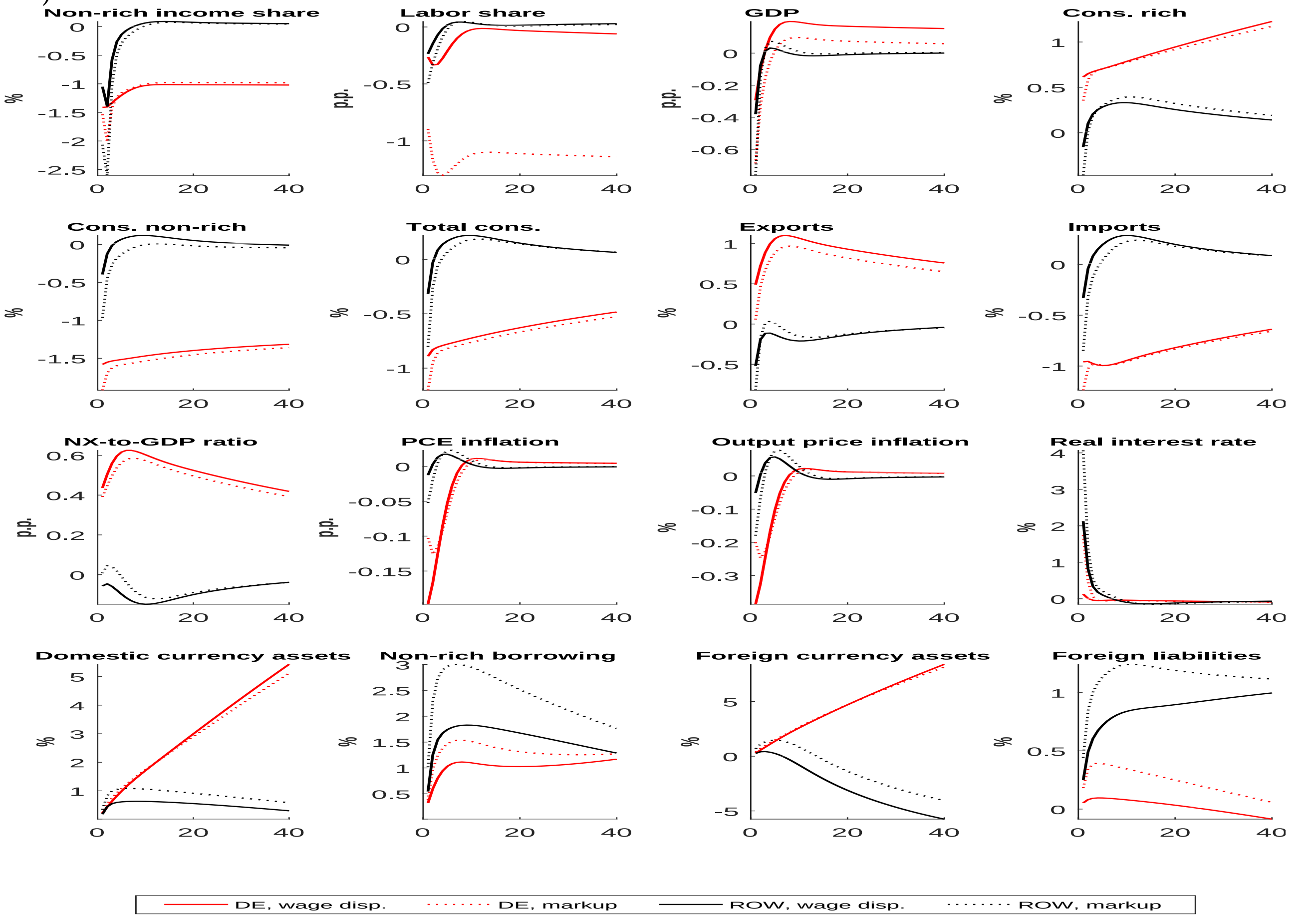
Target	Model counterpart	
Net foreign assets DE and REA	NFA^{DE}, NFA^{REA}	0.00
DE net claims w.r.t. REA	$q_{REA}^{DE}\left(size^{REA}/size^{DE} * b_{REA}^{DE}\right) - b_{DE}^{REA}$	0.00
DE share of foreign claims in total liabilities, %	$\left(\frac{b_F^{DE}}{b_{gov}^{DE}+d_n^{DE}}\right) * 100$	27.00
DE real exchange rates w.r.t. REA and ROW	$q_{REA}^{DE}, q_{ROW}^{DE}$	1.00
DE import-to-GDP ratio, %	$\left(\frac{IM^{DE}}{Y^{DE}}\right) * 100$	30.00
DE import-from-REA-to-GDP ratio, %	$\left(\frac{IM_{REA}^{DE}}{Y^{DE}}\right) * 100$	14.00
DE net exports w.r.t. REA	$\frac{size_{REA}}{size^{DE}} * DD_{DE}^{REA} - \frac{p_{REA}^{DE} DD_{REA}^{DE}}{p_H^{DE}}$	0.00
EA import-to-GDP ratio, %	$\left(\frac{IM^{EA}}{Y^{EA}}\right) * 100$	20.00

Note: This table displays the targets used to pin down the first group of parameters listed in 5. We need three additional restrictions to pin down the parameters listed in 5a.). Firstly, without loss of generality, we impose that the size of the world economy equals 100. Furthermore, we assume that the share of DE foreign liabilities b_F^{DE} held by ROW equals the share of ROW in total DE imports (i.e. $\frac{b_{DE}^{ROW}}{b_F^{DE}} = \frac{\left(\frac{IM_{ROW}^{DE}}{Y^{DE}}\right)}{\left(\frac{IM^{DE}}{Y^{DE}}\right)}$), and that the share of DE claims in REA foreign liabilities b_F^{REA} equals the share of REA imports from DE in REA total imports (i.e. $\frac{b_{REA}^{DE}}{b_F^{REA}} = \frac{\left(\frac{IM_{DE}^{REA}}{Y^{REA}}\right)}{\left(\frac{IM^{REA}}{Y^{REA}}\right)}$). Adopting these three restrictions then implies that the number of targets is exactly equal to the number of parameters to be pinned down. Note that empty cells mean that the respective value is not targeted, i.e. it is endogenously determined.

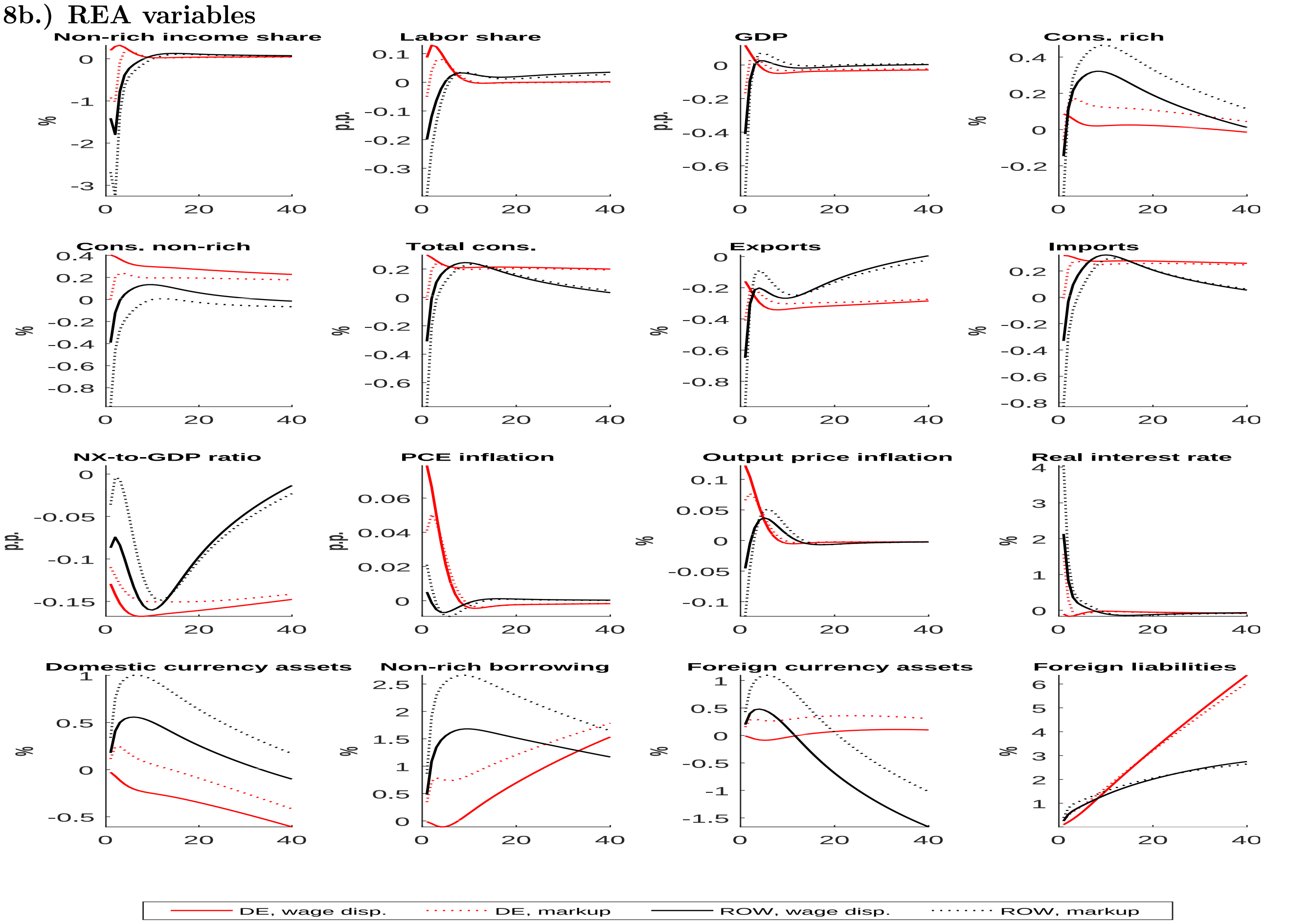
As can be obtained from Figure 8a.), the effect of an inequality increase on DE variables in the three region model with CSP is close to the two region model, with the effect on DE net exports being somewhat larger but similarly persistent. However, about half of the DE net export increase now originates from trade with ROW (see Figure 10), meaning that part of the increase in DE rich household saving flows to ROW. As a result, the decline of the real interest rate in DE and REA is smaller, while the real interest rate declines slightly but persistently in ROW. Furthermore, the EA exchange rate depreciates somewhat due to the persistently lower interest rate in EA than in ROW.

Figure 8: Effect of DE and ROW inequality increase - DE and REA variables

8a.) DE variables

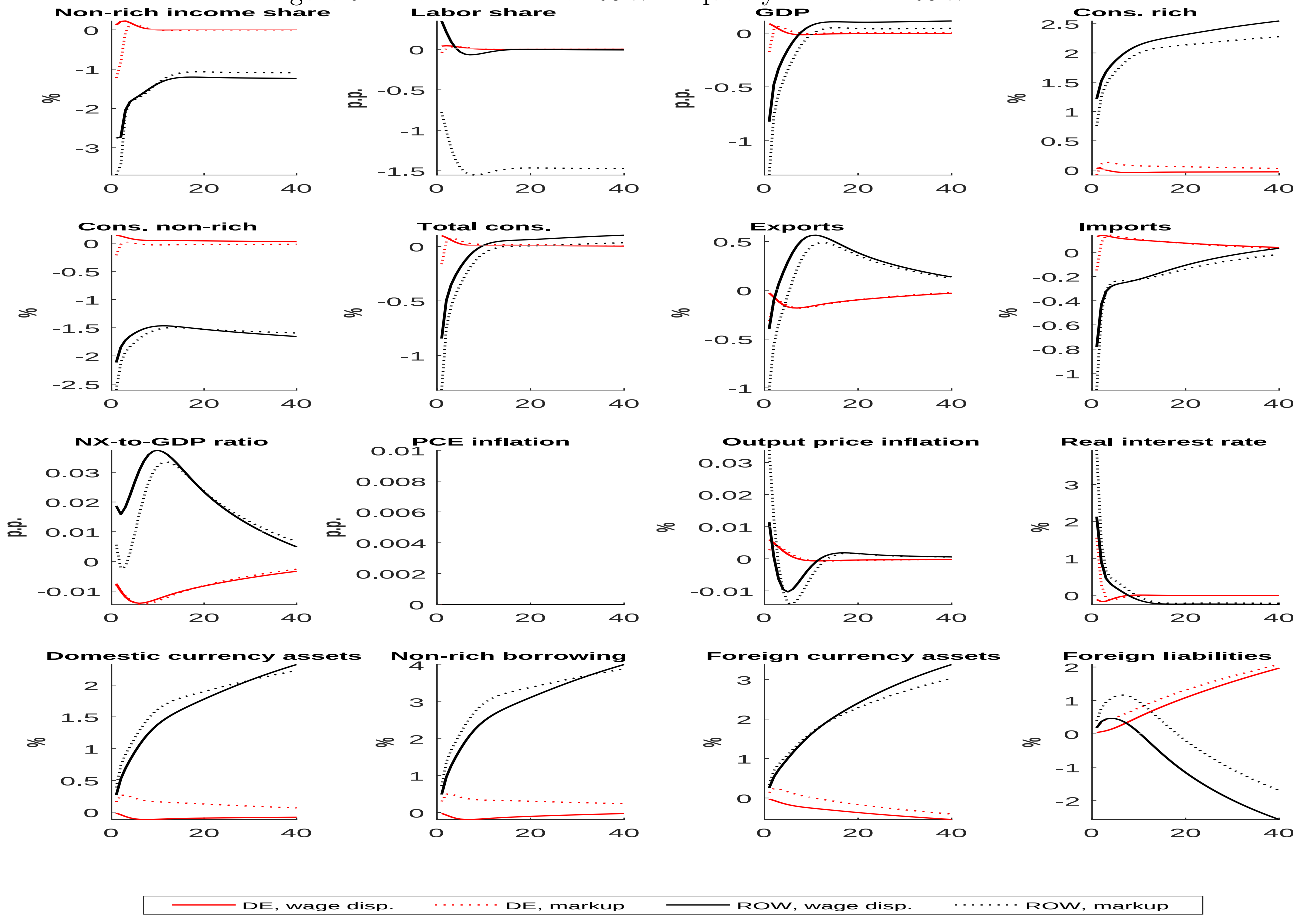


8b.) REA variables



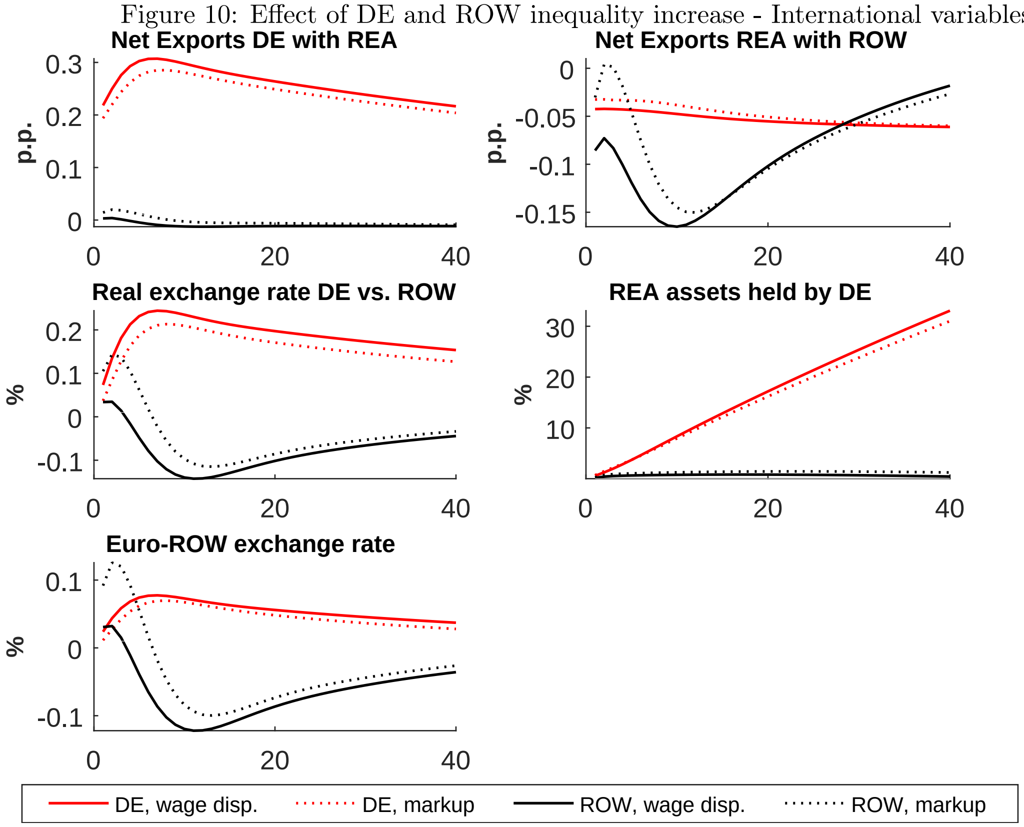
Note: These figures displays the effect of a permanent increase in DE (red lines) and ROW (black lines) inequality in the three region model with CSP. Solid lines display a decline in the non-rich labor income share (a decline in $d_{n,t}^{DE}$) while dotted lines display an increase in the markup shock $d_{\mu,t}$. Both shocks are calibrated to cause a long-run decline in the non-rich national income share of 1 percentage point.

Figure 9: Effect of DE and ROW inequality increase - ROW variables



Note: See the note below Figure 8 for details.

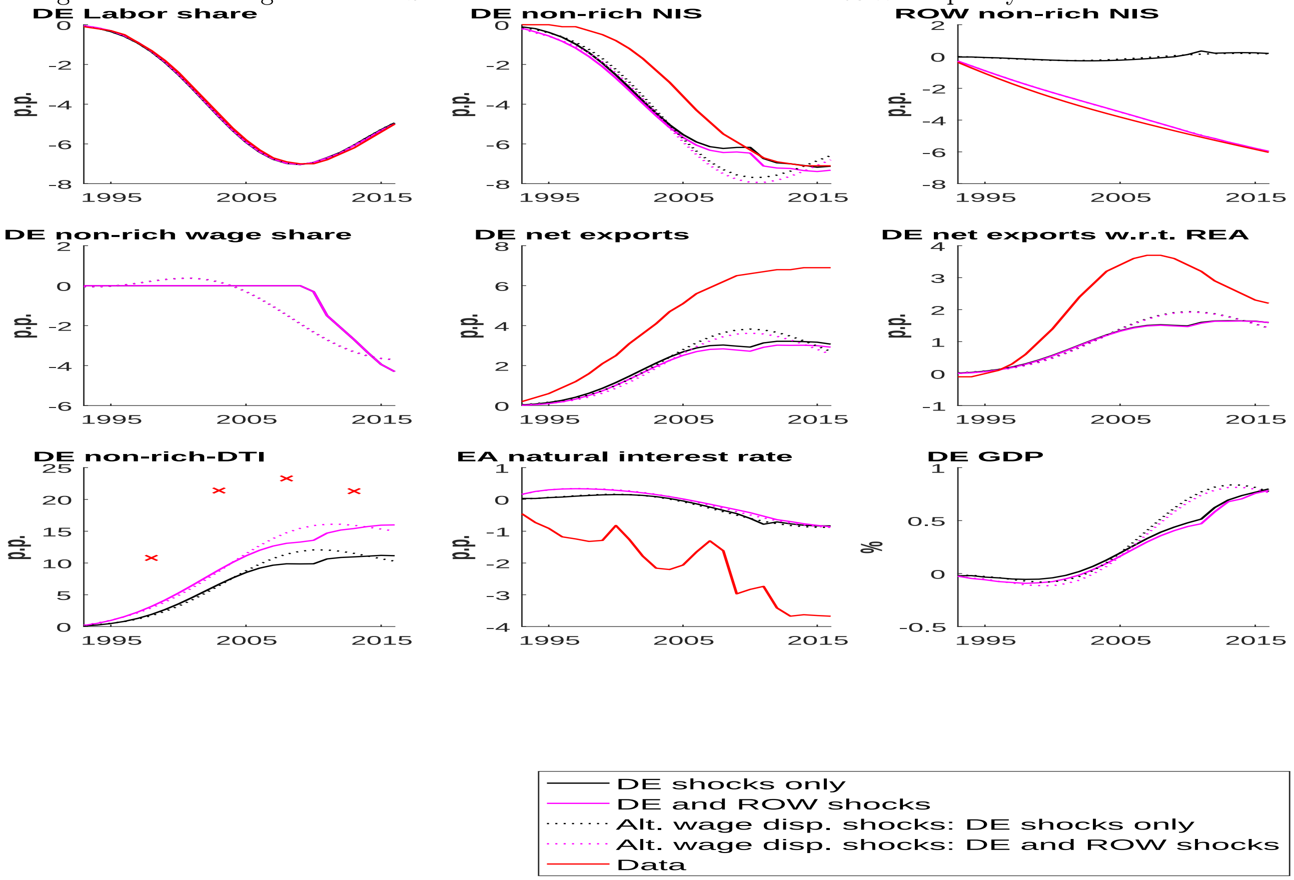
Figure 10: Effect of DE and ROW inequality increase - International variables



Note: See the note below Figure 8 for details.

An increase in ROW inequality lowers ROW consumption and imports due to the increase in rich household saving. In order to meet its inflation target, the central bank correspondingly lowers the nominal and real interest rate (see Figure 9). The persistently lower ROW interest rate triggers a depreciation of the exchange rate (see Figure 10), thus making domestic production more competitive. As a result, the German export surplus decreases (see Figure 8a.)). However, the decline in DE net export caused by an ROW inequality increase is much smaller and less persistent than the rise caused by a DE inequality increase. The reason is the stronger decline of the ROW interest rate (i.e. stronger than the decline of the DE real interest rate caused by a DE inequality increase). The stronger decline of the ROW real interest rate arises because unlike DE, ROW has its own monetary policy and targets inflation at the ROW level. The stronger real interest rate decline implies that the decline of total ROW consumption and imports is not persistent, unlike the decline in DE consumption and imports in response to a DE inequality increase. The perfect inflation target at the ROW level also implies that ROW inflation does not decline in response to an ROW inequality increase, while DE inflation does decline in response to a DE inequality increase. As a result, the real exchange rate of DE w.r.t. ROW responds by less when ROW inequality increases, even though the nominal exchange rate actually responds more strongly.

Figure 11: Three region model: Simulation of the historical DE and ROW inequality increase



Note: This graph displays the effect of simulating the historical increase in inequality in the three region model. The line labeled "Model: DE shocks only" features only DE markup and wage inequality shocks, see the note below Figure 5. The line labeled "Model: DE and ROW shocks" adds on top of that a sequence of ROW price markup shocks $d^{DE}_{\mu,t}$ such that the simulation matches the path of the ROW bottom 90% national income share, on top of the other targets. For the meaning of the "Alt. wage disp shocks" label see the note below Figure 5. NIS: National Income Share.

We now repeat the simulations of the historical German inequality increase. Furthermore, in a second set of simulations, we feed on top of that the increase in the ROW trend top 10% income share into the model. We match the increase in the ROW top 10% income share using the price markup shock, consistent with the decline of the US labor share since the end of the 1990s and the evidence in Bauluz et al. (2022).[9] As can be obtained from Figure 11, the effect of the increase of German inequality on the Net-export-to-GDP ratio is larger than in the two region model, with an about 0.7 p.p. higher peak (see the black solid line). However, the effect on net-exports with respect to the REA is smaller than in the two region model, as part of the total increase in DE net exports is now with respect to the ROW. Taking into account the increase in ROW inequality lowers the peak of the net-export-to GDP ratio only little, consistent with the discussion in the previous section. However, the ROW inequality does have a substantial effect on the bottom 90% debt-to-income ratio, which increases almost twice as much as for an increase in DE inequality only.

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[9] There are 150 countries in the WID for which both the top 10% national income share and constant US Dollar PPP national income are available. By contrast, the labor share in national income is available only for 37 countries, only 20 of which are part of ROW. In particular, the labor share is not available for China

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A Data sources used to set calibration targets and in the historical simulations

Whenever we calculate the target values from data, we calculate them as averages over the 1993-2016 period, unless otherwise mentioned. To compute the REA and ROW aggregates, we use constant Dollar PPP national income from the World Inequality Database (WID) to compute country weights, unless otherwise mentioned. The target values used to calibrate the parameters of the model are obtained/ computed as follows:

Target types and data series used for all model variants

- Intertemporal elasticity of substitution: The meta-analysis of Havranek (2015).
- Inflation: EA: The European Central Bank (ECB) inflation target. ROW: Federal Reserve Bank inflation target
- Average real interest rate: EA: Short term interest rate minus private consumption deflator inflation, obtained from the Area Wide Model (AWM) database, see Fagan et al. (2005).
- Government-demand-to-GDP ratio: Government demand is the sum government consumption and government investment.
  - DE: AMECO.
  - REA: AMECO, Government demand and GDP are calculated as "Euro Area - 12 countries" minus DE. Data available from 1995 onward.
- DE exports to to the REA: Bundesbank database, "German foreign trade: exports / At current prices, flows / Aggregate / all goods / Euro area (Member States and Institutions of the Euro Area) changing composition / Calendar and seasonally adjusted" [BBDA1.M.DE.Y.EX.S.A.U2.A.V.ABA.A.]. We use this series because the Eurostat series "Exports and imports by Member States of the EU/third countries" starts only in 2008. Since the Bundesbank series comprises only goods exports, we scale it up using the ratio of total DE exports to total DE goods exports from Eurostat, as *total* exports of goods and services and *total* goods exports are available from 1991. From 2008 inwards the series is very close to the Eurostat series on total exports to the REA just mentioned.
- DE imports from the REA: Same procedure as for imports. The corresponding Bundesbank database series is "German foreign trade: imports / At current prices, flows / Aggregate / all goods / Euro area (Member States and Institutions of the Euro Area) changing composition / Calendar and seasonally adjusted", BBDA1.M.DE.Y.IM.S.A.U2.A.V.ABA.A.
- Average spread of the household loan rate over risk free rate:

  - DE and REA: Loan rate: ECB Statistical Data Warehouse, Bank interest rates, DE and EA, loans, weighted average of loans for consumption (new business), lending for house purchases (new business), other lending (new business). The weights are the corresponding "new business" volumes. We also use these volumes as weights to compute REA series from the DE and EA series. Safe interest rate: German 10 year treasury bonds, Eurostat, EMU convergence criterion bond yields. We calculate the average spread over the 2010-2016 period because this is roughly the period over which we have data on the bottom 90% debt-to-income ratio for the EA.
  - ROW: Loan rate: "30-Year Fixed Rate Mortgage Average in the United States [MORTGAGE30US], Freddie Mac, Primary Mortgage Market Survey, retrieved from FRED". Since this measure of the loan rate refers only to mortgage loans, unlike the loan rate measure we use for the EA regions, we adjust this measure using the average spread between our computed EA average loan rate and the EA interest rate on lending for house purchases. Risk free rate: 30-Year US Treasury Constant Maturity Rate, downloaded from FRED.

- Government debt-to-annual-GDP ratio:
  - DE and REA: AMECO, Gross public debt, linked series, and GDP at current prices. The REA aggregate is the sum of all REA countries for which data was available over the period of interest, namely Belgium, Ireland, Greece, Spain, France, Italy, Luxembourg, Malta, Netherlands, Austria and Portugal.
  - ROW: Government-debt-to-annual-GDP ratio for individual countries: IMF World Economic Outlook (WEO) database, except for the United States, where we use "Gross Federal Debt as Percent of Gross Domestic Product [GFDGDPA188S], U.S. Office of Management and Budget and Federal Reserve Bank of St. Louis, retrieved from FRED", because the IMF data is available only from the year 2000. We included all non-EA countries for which data is available from 1995 onward. We do not use pre-1995 data because the data for China becomes available only in 1995.

- Debt-to-annual-income ratio (DTI) of the bottom 90% of the income distribution, target values used in the calibration:

  - DE and REA: Household Finance and Consumption Survey (HFCS). We compute the DTI by dividing total debt of the bottom 90% of households by total gross income of the bottom 90%. The REA aggregate was computed from the country debt-to-income ratios using standard HFCS weights. We use data from the 2010, 2014 and 2017 HFCS waves, i.e. all waves available within or close to our period of interest.
  - ROW: We use the US as a proxy, and compute the value from the survey of consumer finances (SCF), Historical tables, based on public data, estimates in nominal dollars. We compute the average income of the bottom 90% from Table 1 89-98 and Table 1 01-19. Average debt of the bottom 90%: For each income percentile reported in Table 13, we compute average total debt as (Mean value of debt holdings of families holding any debt)*(Percentage of family holding any debt). We then average over the bottom 90%.

- DE Debt-to-annual-income ratio (DTI) of the bottom 90% of the income distribution, time series used to validate the historical simulation: Income, Receipts and expenditure survey (EVS) of the German statistical office. We compute the DTI by dividing total debt of the bottom 90% of households by total net income (EF60) of the bottom 90%. Total debt is the sum of consumer credit (EF612) and mortgages (EF590). We use this series to evaluate the corresponding model simulation result because (unlike the HFCS) it is available from 1993 onward, in five year intervals.

- Bottom 90% national income share: WID. We use constant Dollar PPP national income as weights to compute the REA and ROW aggregates.

  - REA aggregate: Austria, Belgium, Finland, France, Greece, Malta, Luxembourg, Netherlands,Italy, Portugal, Spain.
  - ROW aggregates: All countries for which both the bottom 90% national income share and constant Dollar PPP national income are available.[10]

[10]Specifically, we include Switzerland, Uruguay, Syrian Arab Republic, Guinea, Malawi, Mongolia, Slovakia, Zambia, Eritrea, Kenya, Panama, Guatemala, United Arab Emirates, Oceania, Sao Tome and Principe, Japan, Mozam-

- Alternative DE wage dispersion shock used in the historical simulation: We target the evolution of an estimate of the share of the top 10% income-richest households in total labor income, obtained the German Socioeconomic Panel, seeWagner et al. (2007)), specifically the SOEP-CORE variables from SOEPv35. The relevant income variables used after standard transformation are labor income (i11103) and pre-government income (i11101). pre-government income is defined as total gross household income excluding transfers.

- Labor share in national income: WID. We use constant Dollar PPP national income as weights to compute the REA and ROW aggregates.

  - REA aggregate: Austria, Belgium, Finland, France, Greece, Luxembourg, Netherlands, Italy, Portugal, Spain, calumniated over 1995-2016.

  - ROW aggregate: Australia, Canada, Czech Republic, China, Japan, Kazakhstan, Kyrgyzstan, Mexico, Niger, Norway, Sweden, United Kingdom, United States. For China, the WID does not report the labor share because data on mixed income is missing. The labor share in the WID is defined as $\frac{Compensation of + employees + 0.7 * \text{mixed income}}{national\ income - taxes on products and production}$. To proxy the Chinese labor share, we proxy the Chinese share of mixed income in $national\ income - taxes on products and production$ as the average world mixed income share.

- Discounting wedge θ^J_r: See Rannenberg (2019).

- Marginal Propensity to Consume out of Wealth (MPCW) of the top 10% of households:

  - DE: Garbinti et al. (2020) for Germany.

bique, Tajikistan, Nigeria, China, Ethiopia, Jordan, Korea, Bahamas, El Salvador, Cote d'Ivoire, Namibia, South Sudan, Cambodia, Mauritania, Russian Federation, Mauritius, Bulgaria, Palestine, Hungary, Lesotho, Bolivia, Suriname, Uzbekistan, Lebanon, Bosnia and Herzegovina, Egypt, Cameroon, India, Bahrain, Canada, Saudi Arabia, Algeria, Honduras, Trinidad and Tobago, South Africa, Bhutan, Gabon, Costa Rica, Rwanda, Uganda, Montenegro, Nepal, Haiti, Somalia, Dominican Republic, Kuwait, Kosovo, Belarus, Croatia, Afghanistan, Azerbaijan, Guinea-Bissau, Paraguay, Togo, Ghana, Sri Lanka, Nicaragua, Yemen, Gambia, USA, Chad, Zimbabwe, Seychelles, Romania, Belize, Burkina Faso, Maldives, Mali, New Zealand, Burundi, Norway, Liberia, Peru, Benin, Tanzania, Moldova, Central African Republic, Bangladesh, Chile, Colombia, North Macedonia, Turkmenistan, Thailand, Ecuador, Israel, United Kingdom, Taiwan, Slovenia, Ukraine, Turkey, Guyana, Serbia, Brunei Darussalam, Viet Nam, Cuba, Qatar, Swaziland, Myanmar, Cabo Verde, Djibouti, Iran, Timor-Leste, Sweden, Lao PDR, Sierra Leone, Malaysia, Jamaica, Niger, Albania, Kazakhstan, Philippines, Iraq, Oman, Libya, Cyprus, Argentina, Brazil, Pakistan, Angola, Tunisia, Sudan, Mexico, Comoros, Iceland, Zanzibar, Czech Republic, Botswana, Morocco, Kyrgyzstan, Venezuela, Armenia, Australia, Congo, Singapore, Denmark, Equatorial Guinea, Senegal, Madagascar, Congo, Poland, Papua New Guinea, Indonesia.

- REA: weighted average of the estimates for France, Belgium, Spain and Italy by Garbinti et al. (2020) and Arrondel et al. (2019).

Targets specific to the model with physical capital:

- Private non-residential-investment-to-GDP ratio: Eurostat, Sector accounts, non-financial transactions, General government, Gross fixed capital formation (GFCF), and Cross-classification of gross fixed capital formation by industry and by asset (flows). We estimate private non-residential investment by using the industry "Public administration, defense, education, human health and social work activities" as a proxy the government sector as far as the construction of dwellings is concerned. This assumptions allows us to calculate private non-residential GFCF as (GFCF: Total economy, total fixed assets)-[(GFCF: general government) - (GFCF: "Public administration, defense, education, human health and social work activities, dwellings)] - (GFCF: Total economy, dwellings).

- Spread of net capital rental over risk free rate $400*\left(r_K^J - \delta^J - \left(\frac{R^J}{\Pi^J} - 1\right)\right)$: We estimate $400*\left(r_K^J - \delta^J\right)$ as $400*\left(r_K^J - \delta^J\right) = \left(\frac{E^J}{TL^J}\right)\left(ERP^J + 10YGOV^{DE}\right) + \left(1 - \left(\frac{E^J}{TL^J}\right)\right)(CorpLoanrate)$,with $\left(\frac{E^J}{TL^J}\right)$:Share of equity in total corporate liabilities (including equity), ERP : Equity risk premium, $10YGOV^{DE}$: German 10 year government bond rate.

  - REA aggregate: weighted average of the Spreads for Greece, Spain, France, Italy, Finland, because these were the only REA countries outside Germany for which we could find corporate or enterprise debt interest rates going back to at least 1995.
  - $\left(\frac{E^J}{TL^J}\right)$: From Eurostat, Financial balance sheets, consolidated, Non-financial corporations , E^J : Equity and investment fund shares. TL^J: Total financial assets/liabilities.
  - $10YGOV^{DE}$: See above.
  - ERP^J: We use the estimates of Aswath Damodaran, who estimates the ERP based as a combination of the US ERP and a country risk premium estimated based on CDS spreads. See Damodaran (2022) and Damodaran (2021) for details. We downloaded the estimates from https://pages.stern.nyu.edu/∼adamodar/pc/datasets/ctryprem.xlsx and his data archive.

- *CorpLoanrate*:
  * Germany: Industrial Bonds, secondary market, BIS databank.
  * Greece, Spain, Italy, France, Finland 2003-2016: ECB Statistical Data Warehouse (SDW) Bank interest rates - loans to corporations (outstanding amounts), all maturities.
  * Greece: 1999-2003: Backward extension using the BIS Databank series "bank loans, long-term, enterprise, over 1 year variable rate". Adjusted for the average difference between the two series during the period where both are available (i.e. 31/01/2003-31/01/2004). 1995-1998: Backward extension using the BIS Databank series, "bank loans, short term, new". We perform the same adjustment as over the 1999-2003 period.
  * Spain: 1995-2002: Backward extension using the BIS Databank series "Medium-Term Bank Credits (3 months - 3 years)". Adjusted for the average difference between the two series during the period where both are available (i.e. 31/01/2003-31/03/2003).
  * Italy: 1995-2002: Backward extension using the BIS Databank series "Bank short-term loans, average rate (large sample)". Adjusted for the average difference between the two series during the period where both are available (i.e. 31/01/2003-31/03/2004).
  * France: 1995-2002: Backward extension using the BIS Databank series "Bank overdraft rate, credits of all sizes, average rate". Adjusted for the average difference between the two series in 2002.
  * Finland: 1995-2003: Backward extension using the BIS Databank series "Bank lending, average rate, deposit banks". Adjusted for the average difference between the two series during the period where both are available (i.e. 31/01/2003-30/06/2003).

Targets and series specific to the three region model:

- DE: Share of foreign claims in total DE liabilities: Eurostat, national accounts, annual sector accounts. Numerator: Sector: Rest of the World, total assets, consolidated. Denominator: Sector: Total economy, total liabilities, consolidated. Average over 1995-2016.

- Euro Area extra-Euro-Area-import-to-GDP ratio: Hoekstra and van der Helm (2010).
- DE Export-to-GDP ratio: Eurostat.

B Model Appendix: Two region model

B.1 Derivation of resource constraint and net foreign asset law of motion

The aggregate resource constraint (or GDP expenditure equation) is derived as follows. The zero profit condition of final goods firms is given by

$$DD_t^{DE} = p_{H,t,}^{DE} DD_{H,t}^{DE} + p_{F,t,}^{DE} DD_{F,t}^{DE} \tag{66}$$

Combining (66) with the equilibrium in the final goods market (32) and the definition of imports yields yields

$$\begin{aligned} Y_t^{DE} &= DD_{H,t}^{DE} + EX_t^{DE} + G_t^{DE} \\ &= \frac{DD_t^{DE} - p_{F,t,}^{DE} DD_{F,t}^{DE}}{p_{H,t,}^{DE}} + EX_t^{DE} + G_t^{DE} \\ &= \frac{DD_t^{DE}}{p_{H,t,}^{DE}} + G_t^{DE} + EX_t^{DE} - IM_t^{DE} \end{aligned} \tag{67}$$

The law of motion of net foreign assets is derived as follows. Expressing the the budget constraints of rich and non-rich households in per-capita terms, using also (1), yields

$$\begin{aligned} b_{r,dom,t} + C_{r,t}^{DE} &= \frac{R_{t-1}^{DE}}{\Pi_t^{DE}} b_{r,dom,t-1} + w_{r,t}^{DE} N_{r,t}^{DE} - T_{r,t}^{DE} + \Xi_t + \frac{size_{REA}}{size_{DE}} * \frac{R_{t-1}^{REA}}{\Pi_{REA,t}} q_{REA,t}^{DE} b_{REA,t-1} - \frac{size_{REA}}{size_{DE}} * q_{REA,t}^{DE} b_{REA,t} \\ -l_{n,t} + C_{n,t}^{DE} &= -\frac{R_{L,t-1}}{\Pi_t^{DE}} l_{n,t-1} + w_{n,t}^{DE} N_{n,t}^{DE} - T_{n,t}^{DE} \end{aligned}$$

Adding these equations and using (45) and (46):

$$b^{DE}_{gov,t} + C_t = \frac{R^{DE}_{t-1}}{\Pi^{DE}_t} b^{DE}_{gov,t-1} + \frac{\left(R^{DE}_{t-1} - R_{L,t-1}\right)}{\Pi^{DE}_t} l_{n,t-1} + w^{DE}_t N^{DE}_t - T_t + \Xi_t$$
$$+ \frac{size_{REA}}{size_{DE}} * \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t} b_{REA,t-1} - \frac{size_{REA}}{size_{DE}} * q^{DE}_{REA,t} b_{REA,t}$$

or, using the definition of $R^{DE}_{L,t}$

$$b^{DE}_{gov,t} + C_t = \frac{R^{DE}_{t-1}}{\Pi^{DE}_t} b^{DE}_{gov,t-1} - R^{DE}_{t-1} f\left(\frac{l_{n,t-1}}{w^{DE}_{n,t-1} N_{n,t-1}}\right) l_{n,t-1} + w^{DE}_t N^{DE}_t - T_t + \Xi_t$$
$$+ \frac{size_{REA}}{size_{DE}} * \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t} b_{REA,t-1} - \frac{size_{REA}}{size_{DE}} * q^{DE}_{REA,t} b_{REA,t}$$

Use the definition of intermediate firm profits: $\Xi_t \equiv p^{DE}_{H,t,} \left(EX^{DE}_t + DD^{DE}_{H,t} + G^{DE}_t\right) - w^{DE}_t N^{DE}_t$ and (34)

$$C_t = p^{DE}_{H,t} EX^{DE}_t + p^{DE}_{H,t,} DD^{DE}_{H,t} - R^{DE}_{t-1} f\left(\frac{l_{n,t-1}}{w^{DE}_{n,t-1} N_{n,t-1}}\right) + \frac{size_{REA}}{size_{DE}} * \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t} b_{REA,t-1} - \frac{size_{REA}}{size_{DE}} * q^{DE}_{REA,t} b_{REA,t}$$

Using (66) yields

$$C_t = p^{DE}_{H,t} EX^{DE}_t + p^{DE}_{H,t,} \frac{DD^{DE}_t - p^{DE}_{F,t,} DD^{DE}_{F,t}}{p^{DE}_{H,t,}} - R^{DE}_{t-1} f\left(\frac{l_{n,t-1}}{w^{DE}_{n,t-1} N_{n,t-1}}\right) + \frac{size_{REA}}{size_{DE}} * \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t} b_{REA,t-1} - \frac{size_{REA}}{size_{DE}} * q^{DE}_{REA,t} b_{REA,t}$$

or, using (25)

$$q^{DE}_{REA,t}\left(\frac{size_{REA}}{size_{DE}} * b_{REA,t}\right) = p^{DE}_{H,t}\left(EX^{DE}_t - IM^{DE}_t\right) + \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t}\left(\frac{size_{REA}}{size_{DE}} * b_{REA,t-1}\right) \tag{68}$$

the real net foreign asset accumulation equation.

B.2 Full set of model equations

$J = \{DE, REA\}$ indexes the country, i.e. equations with the J superscript appear once with $J = DE$ and once with $J = REA$.

B.2.1 Demand

$$\Lambda_{r,t}^J = \beta_r E_t \left\{ \frac{R_t}{\Pi_{t+1}^J} \Lambda_{r,t+1}^J \right\} + \chi_{b,r}^J \left(b_{r,t}^J \right)^{-\sigma_{b,r}} \tag{69}$$

$$\Lambda_{r,t}^J = \left(C_{r,t}^J \right)^{-\sigma} \tag{70}$$

$$b_{r,t}^J = l_{n,t}^J + b_{r,gov,t}^J + b_{r,dcfa,t}^J \tag{71}$$

$$\Lambda_{n,t}^J = \beta_n^J E_t \left\{ \Lambda_{n,t+1}^J \left(\frac{R_{L,t}^J + f' \left(\frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \right) \frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} R_t^J}{\Pi_{t+1}^J} \right) \right\} \tag{72}$$

$$R_{L,t}^J = \left(1 + f^J \left(\frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \right) \right) R_t^J \tag{73}$$

$$\Lambda_{n,t}^J = \left(C_{n,t}^J \right)^{-\sigma} \tag{74}$$

$$-l_{n,t}^J + C_{n,t}^J = -\frac{R_{L,t-1}^J}{\Pi_t^J} l_{n,t-1}^J + w_{n,t}^J N_{n,t}^J - T_{n,t}^J \tag{75}$$

$$b_{gov,t}^J = b_{r,gov,t}^J + b_{F,t}^J \tag{76}$$

$$DD_t^J = C_{r,t}^J + C_{n,t}^J + \frac{\frac{R_{t-1}^J}{\Pi_t^J} f \left(\frac{l_{n,t-1}^J}{w_{n,t-1}^J N_{n,t-1}} \right)}{p_{H,t,}^J} l_{n,t-1}^J \tag{77}$$

$$Y_t^J = \frac{DD_t^J}{p_{H,t,}^J} + G_t^J + EX_t^J - IM_t^J \tag{78}$$

$$A_H^J DD_{H,t}^J = DD_t^J \phi_H^J \left(\frac{p_{H,t,}^J}{A_H^J} \right)^{-\lambda_m^J} \tag{79}$$

$$f^J \left(\frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \right) = \phi_l^J \frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \tag{80}$$

$$\frac{df^J \left(\frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \right)}{d \left(\frac{l_{n,t}^J}{w_{n,t}^J N_{n,t}^J} \right)} = \phi_l^J \tag{81}$$

where $b_{r,dcfa,t}^J$ denotes rich household domestic currency foreign assets. Note that in the NOCSP model, $\chi_{b,r}^J = 0$. Loosely speaking, 14 Variables determined in this block: $\Lambda_{r,t}^J, C_{r,t}^J, b_{r,t}^J, \Lambda_{n,t}^J, C_{n,t}^J, R_{L,t}^J, l_{n,t}^J, DD_t^J, Y_t^J, DD$

B.2.2 Supply

Labor market, production, income distribution, price setting:

$$\left(\frac{\Pi^J_{W,r,t}}{\Pi^J_{ind,t}}-1\right)\frac{\Pi^J_{W,r,t}}{\Pi^J_{ind,t}}=\kappa^J_w\left(\mu_w\frac{\chi^J_{N,r}\left(N^J_{r,t}\right)^{\eta^J}}{\Lambda^J_{r,t}w^J_{r,t}}-1\right)+\beta_r E_t\left\{\frac{\Lambda^J_{r,t+1}}{\Lambda^J_{r,t}}\frac{N^J_{r,t+1}}{N^J_{r,t}}\left(\frac{\Pi^J_{W,r,t+1}}{\Pi^J_{ind,t+1}}-1\right)\frac{\left(\Pi^J_{W,r,t+1}\right)^2}{\Pi^J_{t+1}\Pi^J_{ind,t+1}}\right\} \tag{82}$$

$$w^J_{r,t}=\frac{\Pi^J_{W,r,t}}{\Pi^J_t}w_{r,t-1} \tag{83}$$

$$\left(\frac{\Pi^J_{W,n,t}}{\Pi^J_{ind}}-1\right)\frac{\Pi^J_{W,n,t}}{\Pi^J_{ind,t}}=\kappa^J_w\left(\mu_w\frac{\chi_{N,n}\left(N^J_{n,t}\right)^{\eta^J}}{\Lambda^J_{n,t}w^J_{n,t}}-1\right)+\beta_n E_t\left\{\frac{\Lambda^J_{n,t+1}}{\Lambda^J_{n,t}}\frac{N^J_{n,t+1}}{N^J_{n,t}}\left(\frac{\Pi^J_{W,n,t+1}}{\Pi^J_{ind}}-1\right)\frac{\left(\Pi^J_{W,n,t+1}\right)^2}{\Pi^J_{t+1}\Pi^J_{ind}}\right\} \tag{84}$$

$$w^J_{n,t}=\frac{\Pi^J_{W,n,t}}{\Pi^J_t}w^J_{n,t-1} \tag{85}$$

$$w^J_{n,t}=w^J_t z^J_{n,t} \tag{86}$$

$$w^J_{r,t}=w^J_t z^J_{r,t} \tag{87}$$

$$1=\frac{z^J_{n,t}N^J_{n,t}+z^J_{r,t}N^J_{r,t}}{N^J_{r,t}+N^J_{n,t}} \tag{88}$$

$$\omega^J_{n,t}=\frac{z^J_{n,t}N^J_{n,t}}{z^J_{r,t}N^J_{r,t}+z^J_{n,t}N^J_{n,t}} \tag{89}$$

$$\omega^J_{n,t}=\omega^J_n+d^J_{n,t} \tag{90}$$

$$w^J_t=mc^J_t\left(1-\alpha_K\right)\frac{Y^J_t}{N^J_t} \tag{91}$$

$$N^J_t=z^J_{n,t}N^J_{n,t}+z^J_{r,t}N^J_{r,t} \tag{92}$$

$$Y^J_t=A\left(N^J_t\right)^{1-\alpha_K} \tag{93}$$

12 endogenous variables determined in this block. loosely speaking: $\Pi^J_{W,r,t}, w^J_{r,t}, \Pi^J_{W,n,t}, w^J_{n,t}, N^J_t, mc^J_t, z^J_{n,t}, z^J_{r,t}, N^J_{r,t}, N^J_{n,t}, w^J_t, \omega^J_{n,t}$. Exogenous variable: $d^J_{n,t}$.

Price setting:

$$\kappa\left(\mu_t^J\frac{mc_t^J}{p_{H,t,}^J}-1\right)$$
$$+\beta_r E_t\left\{\frac{p_{H,t+1}^J}{p_{H,t,}^J}\frac{\Lambda_{r,t+1}^J}{\Lambda_{r,t}^J}\frac{Y_{t+1}^J}{Y_t^J}\left(\frac{\Pi_{H,t+1}^J}{\Pi_{ind}^J}-1\right)\frac{\Pi_{H,t+1}^J}{\Pi_{ind}^J}\right\}=\left(\frac{\Pi_{H,t}^J}{\Pi_{ind}^J}-1\right)\frac{\Pi_{H,t}^J}{\Pi_{ind}^J} \tag{94}$$
$$\mu_t^J=\mu^J+d_{\mu,t}^J \tag{95}$$
$$p_{H,t,}^J=\frac{\Pi_{H,t}^J}{\Pi_t^J}p_{H,t-1,}^J \tag{96}$$

3 endogenous variables determined in this block: $\Pi_{H,t}^J,\mu_t^J,\Pi_t^J$. Exogenous variable: $d_{\mu,t}^J$.

B.2.3 Government

$$b_{gov,t}^J=\frac{R_{t-1}^J}{\Pi_t^J}b_{gov,t-1}^J+p_{H,t,}^J G_t^J-T_{r,t}^J-T_{n,t}^J \tag{97}$$
$$Target_{bgov2GDP}^J=\frac{b_{gov,t}^J}{4Y_t^J} \tag{98}$$
$$Target_{bgov2GDP}^J=\frac{G_t^J}{Y_t^J} \tag{99}$$
$$Target_{T_n2T_t}^J=\frac{T_{n,t}^J}{T_{r,t}^J+T_{n,t}^J} \tag{100}$$
$$Target_{T_n2T_t}^J=NIS_{n,t}^J \tag{101}$$
$$NI_t^J=p_{H,t,}^J Y_t^J+FI_t^J \tag{102}$$
$$NIS_{n,t}^J=\frac{w_{n,t}^J N_{n,t}^J-\left(\frac{R_{L,t-1}}{\Pi_t^J}-1\right)l_{n,t-1}}{\left(1+\frac{b_{gov,t}^J}{NI_t^J}\left(\frac{R_{t-1}^J}{\Pi_t^J}-1\right)\right)NI_t^J} \tag{103}$$

[11] 7 variables are determined in this block, loosely speaking: $T_{r,t}^J, b_{gov,t}^J, G_t^J, T_{n,t}^J, Target_{T_n2T_t}^J, NI_t^J, NIS_{n,t}^J$

B.2.4 International

Equations appearing only once

[11] The non-rich pre-tax national income share is defined as $NIS_{n,t}^J \equiv \frac{w_{n,t}^J N_{n,t}^J-\left(\frac{R_{L,t-1}}{\Pi_t^J}-1\right)l_{n,t-1}-NIS_{n,t}^J b_{gov,t}^J\left(\frac{R_{t-1}^J}{\Pi_t^J}-1\right)}{NI_t^J}$. The term $-NIS_{n,t}^J b_{gov,t}^J\left(\frac{R_{t-1}^J}{\Pi_t^J}-1\right)$ represents share of the primary factor income of the government (which is negative since there is government debt) allocated to non-rich households for the computation of $NIS_{n,t}^J$. Solving for $NIS_{n,t}^J$ yields equation (103).

EA Monetary Policy and demand for REA assets by DE

$$\Pi_t^{EA} = \Pi_{target}^{EA} \tag{104}$$

$$R_t^{DE} = R_t^{EA} \tag{105}$$

$$\Pi_t^{EA} = \frac{size_{DE}}{size_{DE} + size_{REA}} * \Pi_t^{DE} + \frac{size_{REA}}{size_{DE} + size_{REA}} \Pi_t^{REA} \tag{106}$$

$$\Lambda_{r,t}^{DE} = \beta_r^{DE} E_t \left\{ \frac{R_{REA,t} exp\left(-D^{RP} \psi_{rp} \widehat{NFA}_{t-1}^{DE}\right)}{\Pi_{t+1}^{REA}} \frac{q_{REA,t+1}^{DE}}{q_{REA,t}^{DE}} \Lambda_{r,t+1}^{DE} \right\} + \chi_{b,r}^{DE} \left(b_{r,t}^{DE}\right)^{-\sigma_{b,r}^{DE}} \tag{107}$$

Note: In the NOCSP model we have $D^{RP} = 1$, while with CSP $D^{RP} = 0$.

Export prices and real exchange rates

$$p_{F,t}^{DE} = q_{REA,t}^{DE} p_{H,t}^{REA} \tag{108}$$

$$p_{F,t}^{REA} = q_t^{REA} p_{H,t,}^{DE} \tag{109}$$

$$q_{REA,t}^{DE} = \frac{\Pi_t^{REA,t}}{\Pi_t^{DE}} q_{REA,t-1}^{DE} \tag{110}$$

$$q_t^{REA} = \frac{1}{q_{REA,t}^{DE}} \tag{111}$$

Exports

$$EX_t^{DE} = \left(\frac{size_{REA}}{size_{DE}}\right) * DD_{F,t}^{REA} \tag{112}$$

$$EX_t^{REA} = \frac{DD_{F,t}^{DE}}{\left(\frac{size_{REA}}{size_{DE}}\right)} \tag{113}$$

Foreign income

$$FI_t^{DE} = \left(\frac{R_{t-1}^{REA}}{\Pi_{REA,t}} - 1\right) q_{REA,t}^{DE} \left(\frac{size^{REA}}{size^{DE}}\right) b_{REA,t-1}^{DE} \tag{114}$$

$$FI_t^{REA} = -\left(\frac{R_{t-1}^{REA}}{\Pi_{REA,t}} - 1\right) b_{REA,t-1}^{DE} \tag{115}$$

Foreign assets

$$b^{DE}_{r,dcfa,t} = q^{DE}_{REA,t}\left(\frac{size^{REA}}{size^{DE}} * b^{DE}_{REA,t}\right) \tag{116}$$

$$b^{REA}_{r,dcfa,t} = 0 \tag{117}$$

$$b^{DE}_{F,t} = 0 \tag{118}$$

$$b^{REA}_{F,t} = b^{DE}_{REA,t} \tag{119}$$

$$q^{DE}_{REA,t}\left(\left(\frac{size_{REA}}{size_{DE}}\right) * b^{DE}_{REA,t}\right) = p^{DE}_{H,t}\left(EX^{DE}_t - IM^{DE}_t\right) \tag{120}$$

$$+ \frac{R^{REA}_{t-1}}{\Pi_{REA,t}} q^{DE}_{REA,t}\left(\left(\frac{size_{REA}}{size_{DE}}\right) * b^{DE}_{REA,t-1}\right) \tag{121}$$

where $dcfa$ 17 variables: $R^{DE}_t, R^{EA}_t, \Pi^{EA}_t, R_{REA,t}, EX^{DE}_t, EX^{REA}_t, FI_t{}^{DE}, FI^{REA}_t, p^{DE}_{F,t}, p^{REA}_{F,t}, b^{DE}_{r,dcfa,t}, b^{REA}_{r,dcfa,t}, b^{DE}_{F,t}, b^{R}_{F}$

Equations repeated for both DE and REA

$$IM^J_t = \frac{p^J_{F,t,}}{p^J_{H,t,}} DD^J_{F,t} \tag{122}$$

$$A^J_F DD^J_{F,t} = DD^J_t\left(1 - \phi^J_H\right)\left(\frac{p^J_{F,t,}}{A^J_F}\right)^{-\lambda^J_m} \tag{123}$$

$$\left[\phi^J_H\left(\frac{p^J_{H,t,}}{A^J_H}\right)^{1-\lambda^J_m} + \left(1 - \phi^J_H\right)\left(\frac{p^J_{F,t,}}{A^J_F}\right)^{1-\lambda^J_m}\right] = 1 \tag{124}$$

3 variables: $IM^J_t, DD^J_{F,t}, p^J_{H,t}$.

C Three region model

The three region model consists of DE, REA and the ROW, i.e. $J = \{DE, REA, ROW\}$. All equations listed in Sections (B.2.1)-(B.2.3) carry over to the three region model. The equations in Appendix B.2.4 are replaced by

Monetary Policy

$$\Pi_t^{EA} = \Pi_{target}^{EA} \tag{125}$$

$$R_t^{DE} = R_t^{EA} \tag{126}$$

$$\Pi_t^{EA} = \frac{size_{DE}}{size_{DE} + size_{REA}} * \Pi_t^{DE} + \frac{size_{REA}}{size_{DE} + size_{REA}} \Pi_t^{REA} \tag{127}$$

$$\Pi_t^{ROW} = \Pi_{target}^{ROW} \tag{128}$$

Exports and imports:

$$EX_t^{DE} = \frac{size^{REA}}{size^{DE}} * DD_{DE,t}^{REA} + \frac{size^{ROW}}{size^{DE}} * DD_{DE,t}^{ROW} \tag{129}$$

$$EX_t^{REA} = \frac{size^{DE}}{size^{REA}} DD_{REA,t}^{DE} + \frac{size^{ROW}}{size^{REA}} DD_{REA,t}^{ROW} \tag{130}$$

$$EX_t^{ROW} = \frac{size^{DE}}{size^{ROW}} DD_{ROW,t}^{DE} + \frac{size^{REA}}{size^{ROW}} DD_{ROW,t}^{REA} \tag{131}$$

$$IM_t^{DE} = \frac{p_{REA,t}^{DE} DD_{REA,t}^{DE} + p_{ROW,t}^{DE} DD_{ROW,t}^{DE}}{p_{H,t}^{DE}} \tag{132}$$

$$IM_t^{REA} = \frac{p_{DE,t}^{REA} DD_{DE,t}^{REA} + p_{ROW,t}^{REA} DD_{ROW,t}^{REA}}{p_{H,t}^{REA}} \tag{133}$$

$$IM_t^{ROW} = \frac{p_{REA,t}^{ROW} DD_{REA,t}^{ROW} + p_{DE,t}^{ROW} DD_{DE,t}^{ROW}}{p_{H,t}^{ROW}} \tag{134}$$

$$A_{REA}^{DE} DD_{REA,t}^{DE} = DD_t^{DE} \phi_{REA}^{DE} \left(\frac{A_{REA}^{DE}}{p_{REA,t}^{DE}} \right)^{\lambda_m^{DE}} \tag{135}$$

$$A_{ROW}^{DE} DD_{ROW,t}^{DE} = DD_t^{DE} \left(1 - \phi_H^{DE} - \phi_{REA}^{DE}\right) \left(\frac{A_{ROW}^{DE}}{p_{ROW,t}^{DE}} \right)^{\lambda_m^{DE}} \tag{136}$$

$$A_{DE}^{REA} DD_{DE,t}^{REA} = DD_t^{REA} \phi_{DE}^{REA} \left(\frac{A_{DE}^{REA}}{p_{DE,t}^{REA}} \right)^{\lambda_m^{REA}} \tag{137}$$

$$A_{ROW}^{REA} DD_{ROW,t}^{REA} = DD_t^{REA} \left(1 - \phi_H^{REA} - \phi_{DE}^{REA}\right) \left(\frac{A_{ROW}^{REA}}{p_{ROW,t}^{REA}} \right)^{\lambda_m^{REA}} \tag{138}$$

$$A_{REA}^{ROW} DD_{REA,t}^{ROW} = DD_t^{ROW} \phi_{REA}^{ROW} \left(\frac{A_{REA}^{ROW}}{p_{REA,t}^{ROW}} \right)^{\lambda_m^{ROW}} \tag{139}$$

$$A_{DE}^{ROW} DD_{DE,t}^{ROW} = DD_t^{ROW} \left(1 - \phi_H^{ROW} - \phi_{REA}^{ROW}\right) \left(\frac{A_{REA}^{ROW}}{p_{REA,t}^{ROW}} \right)^{\lambda_m^{ROW}} \tag{140}$$

Import prices, real exchange rates, real domestic output prices (price indices)

$$p^{DE}_{REA,t} = q^{DE}_{REA,t} p^{REA}_{H,t} \tag{141}$$

$$p^{DE}_{ROW,t} = q^{DE}_{ROW,t} p^{ROW}_{H,t} \tag{142}$$

$$p^{REA}_{DE,t} = q^{REA}_{DE,t} p^{DE}_{H,t} \tag{143}$$

$$p^{REA}_{ROW,t} = q^{REA}_{ROW,t} p^{ROW}_{H,t} \tag{144}$$

$$p^{ROW}_{REA,t} = q^{ROW}_{REA,t} p^{REA}_{H,t} \tag{145}$$

$$p^{ROW}_{DE,t} = q^{ROW}_{DE,t} \, p^{DE}_{H,t} \tag{146}$$

$$q^{DE}_{REA,t} = \frac{\Pi^{REA,t}_{t}}{\Pi^{DE}_{t}} q^{DE}_{REA,t-1} \tag{147}$$

$$q^{REA}_{DE,t} = \frac{1}{q^{DE}_{REA,t}} \tag{148}$$

$$q^{ROW}_{DE,t} = \frac{1}{q^{DE}_{ROW,t}} \tag{149}$$

$$q^{REA}_{ROW,t} = q^{REA}_{DE,t} q^{DE}_{ROW,t} \tag{150}$$

$$q^{ROW}_{REA,t} = \frac{1}{q^{REA}_{ROW,t}} \tag{151}$$

$$\frac{q^{DE}_{ROW,t}\Pi^{DE}_{t}}{q^{DE}_{ROW,t-1}\Pi^{ROW}_{t}} = \Delta S^{EA}_{ROW,t} \tag{152}$$

$$1 = \left[\phi^{DE}_{H} \left(\frac{p^{DE}_{H,t}}{A^{DE}_{H}} \right)^{1-\lambda^{DE}_{m}} + \phi^{DE}_{REA} \left(\frac{p^{DE}_{REA,t}}{A^{DE}_{REA}} \right)^{1-\lambda^{DE}_{m}} + \left(1 - \phi^{DE}_{H} - \phi^{DE}_{REA}\right) \left(\frac{p^{DE}_{ROW,t}}{A^{DE}_{ROW}} \right)^{1-\lambda^{DE}_{m}} \right] \tag{153}$$

$$1 = \left[\phi^{REA}_{H} \left(\frac{p^{REA}_{H,t}}{A^{REA}_{H}} \right)^{1-\lambda^{REA}_{m}} + \phi^{REA}_{DE} \left(\frac{p^{REA}_{DE,t}}{A^{REA}_{DE}} \right)^{1-\lambda^{REA}_{m}} + \left(1 - \phi^{REA}_{H} - \phi^{REA}_{DE}\right) \left(\frac{p^{REA}_{ROW,t}}{A^{REA}_{ROW}} \right)^{1-\lambda^{REA}_{m}} \right] \tag{154}$$

$$1 = \left[\phi^{ROW}_{H} \left(\frac{p^{ROW}_{H,t}}{A^{ROW}_{H}} \right)^{1-\lambda^{ROW}_{m}} + \phi^{ROW}_{REA} \left(\frac{p^{ROW}_{REA,t}}{A^{ROW}_{REA}} \right)^{1-\lambda^{ROW}_{m}} + \left(1 - \phi^{ROW}_{H} - \phi^{ROW}_{REA}\right) \left(\frac{p^{ROW}_{DE,t}}{A^{ROW}_{DE}} \right)^{1-\lambda^{ROW}_{m}} \right] \tag{155}$$

Foreign asset demands:

$$\Lambda_{r,t}^{DE} = \beta_r^{DE} E_t \left\{ \frac{R_{REA,t}}{\Pi_{t+1}^{REA}} \frac{q_{REA,t+1}^{DE}}{q_{REA,t}^{DE}} \Lambda_{r,t+1}^{DE} \right\} + \chi_{b,r}^{DE} \left(b_{r,t}^{DE} \right)^{-\sigma_{b,r}^{DE}} \tag{156}$$

$$\Lambda_{r,t}^{DE} = \beta_r^{DE} E_t \left\{ \frac{R_{ROW,t}}{\Pi_{t+1}^{ROW,t}} \frac{q_{ROW,t+1}^{DE}}{q_{ROW,t}^{DE}} \Lambda_{r,t+1}^{DE} \right\} + \chi_{b_{ROW},r}^{DE} \left(q_{ROW,t}^{DE} \left(\frac{size^{ROW}}{size^{DE}} * b_{ROW,t}^{DE} \right) \right)^{-\sigma_{b,r}^{DE}} \tag{157}$$

$$\Lambda_{r,t}^{REA} = \beta_r^{REA} E_t \left\{ \frac{R_{ROW,t}}{\Pi_{t+1}^{ROW,t}} \frac{q_{ROW,t+1}^{REA}}{q_{ROW,t}^{REA}} \Lambda_{r,t+1}^{REA} \right\} + \chi_{b_{ROW},r}^{REA} \left(q_{ROW,t}^{REA} \left(\frac{size^{ROW}}{size^{REA}} * b_{ROW,t}^{REA} \right) \right)^{-\sigma_{b,r}^{REA}} \tag{158}$$

$$\Lambda_{r,t}^{ROW} = \beta_r^{ROW} E_t \left\{ \frac{R_{DE,t}}{\Pi_{t+1}^{DE,t}} \frac{q_{DE,t+1}^{ROW}}{q_{DE,t}^{ROW}} \Lambda_{r,t+1}^{ROW} \right\} \tag{159}$$

$$+ \chi_{b_{EA},r}^{ROW} \left(q_{REA,t}^{ROW} \left(\frac{size^{REA}}{size^{ROW}} * b_{REA,t}^{ROW} \right) + q_{DE,t}^{ROW} \left(\frac{size^{DE}}{size^{ROW}} * b_{DE,t}^{ROW} \right) \right)^{-\sigma_{b,r}^{ROW}}$$

$$q_{DE,t}^{ROW} \left(\frac{size^{DE}}{size^{ROW}} * b_{DE,t}^{ROW} \right) = \chi_{b_{DE}}^{ROW} q_{REA,t}^{ROW} \left(\frac{size^{REA}}{size^{ROW}} * b_{REA,t}^{ROW} \right) \tag{160}$$

$$q_{DE,t}^{REA} \left(\frac{size^{DE}}{size^{REA}} * b_{DE,t}^{REA} \right) = \chi_{b_{DE}}^{REA} q_{ROW,t}^{REA} \left(\frac{size^{ROW}}{size^{REA}} * b_{ROW,t}^{REA} \right) \tag{161}$$

Domestic currency foreign asset holdings and foreign liabilities:

$$b_{r,dcfa,t}^{DE} = q_{REA,t}^{DE} \left(\frac{size^{REA}}{size^{DE}} * b_{REA,t}^{DE} \right)$$

$$b_{r,dcfa,t}^{REA} = q_{DE,t}^{REA} \left(\frac{size^{DE}}{size^{REA}} * b_{DE,t}^{REA} \right)$$

$$b_{r,dcfa,t}^{ROW} = 0$$

$$b_{F,t}^{DE} = b_{DE,t}^{ROW} + b_{DE,t}^{REA}$$

$$b_{F,t}^{REA} = b_{REA,t}^{DE} + b_{REA,t}^{ROW}$$

$$b_{F,t}^{ROW} = b_{ROW,t}^{DE} + b_{ROW,t}^{REA}$$

Law of motion of DE and REA net foreign assets:

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$$q^{DE}_{ROW,t}\left(\frac{size^{ROW}}{size^{DE}} * b^{DE}_{ROW,t}\right) + q^{DE}_{REA,t}\left(\frac{size^{REA}}{size^{DE}} * b^{DE}_{REA,t}\right) - b^{ROW}_{DE,t} - b^{REA}_{DE,t} = \tag{162}$$

$$p^{DE}_{H,t}\left(EX^{DE}_t - IM^{DE}_t\right) + \frac{R^{ROW}_{t-1}}{\Pi^{ROW}_t} q^{DE}_{ROW,t}\left(\frac{size^{ROW}}{size^{DE}} * b^{DE}_{ROW,t-1}\right)$$

$$+\frac{R^{REA}_{t-1}}{\Pi^{REA}_t} q^{DE}_{REA,t}\left(\frac{size^{REA}}{size^{DE}} * b^{DE}_{REA,t-1}\right) - \frac{R^{DE}_{t-1}}{\Pi^{DE}_t} b^{ROW}_{DE,t-1} - \frac{R^{DE}_{t-1}}{\Pi^{DE}_t} b^{REA}_{DE,t-1}$$

$$q^{REA}_{ROW,t}\left(\frac{size^{ROW}}{size^{REA}} * b^{REA}_{ROW,t}\right) + q^{REA}_{DE,t}\left(\frac{size^{DE}}{size^{REA}} * b^{REA}_{DE,t}\right) - b^{ROW}_{REA,t} - b^{DE}_{REA,t} = \tag{163}$$

$$p^{REA}_{H,t}\left(EX^{REA}_t - IM^{REA}_t\right) + \frac{R^{ROW}_{t-1}}{\Pi^{ROW}_t} q^{REA}_{ROW,t}\left(\frac{size^{ROW}}{size^{REA}} * b^{REA}_{ROW,t-1}\right)$$

$$+\frac{R^{DE}_{t-1}}{\Pi^{DE}_t} q^{REA}_{DE,t}\left(\frac{size^{DE}}{size^{REA}} * b^{REA}_{DE,t-1}\right) - \frac{R^{REA}_{t-1}}{\Pi^{REA}_t} b^{ROW}_{REA,t-1} - \frac{R^{REA}_{t-1}}{\Pi^{REA}_t} b^{DE}_{REA,t-1}$$

Foreign income:

$$FI^{DE}_t = \left(\frac{R^{ROW}_{t-1}}{\Pi_{ROW,t}} - 1\right) q^{DE}_{ROW,t}\frac{size^{ROW}}{size^{DE}} * b^{DE}_{ROW,t-1} + \left(\frac{R^{REA}_{t-1}}{\Pi_{REA,t}} - 1\right) q^{DE}_{REA,t}\frac{size^{REA}}{size^{DE}} * b^{DE}_{REA,t-1} - \left(\frac{R^{DE}_{t-1}}{\Pi_{DE,t}} - 1\right) b^{DE}_{F,t-1}$$

$$FI^{REA}_t = \left(\frac{R^{ROW}_{t-1}}{\Pi_{ROW,t}} - 1\right) q^{REA}_{ROW,t}\frac{size^{ROW}}{size^{REA}} * b^{REA}_{ROW,t-1} + \left(\frac{R^{DE}_{t-1}}{\Pi_{DE,t}} - 1\right) q^{REA}_{DE,t}\frac{size^{DE}}{size^{REA}} * b^{REA}_{DE,t-1} - \left(\frac{R^{REA}_{t-1}}{\Pi_{REA,t}} - 1\right) b^{REA}_{F,t-1}$$

$$FI^{ROW}_t = \left(\frac{R^{DE}_{t-1}}{\Pi_{DE,t}} - 1\right) q^{ROW}_{DE,t}\frac{size^{DE}}{size^{ROW}} * b^{ROW}_{DE,t-1} + \left(\frac{R^{REA}_{t-1}}{\Pi_{REA,t}} - 1\right) q^{ROW}_{REA,t}\frac{size^{REA}}{size^{ROW}} * b^{ROW}_{REA,t-1} - \left(\frac{R^{ROW}_{t-1}}{\Pi_{ROW,t}} - 1\right) b^{ROW}_{F,t-1}$$