The Rat Race Revisited: Mobility and Wage Inequality

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Abstract

In labour markets with high mobility, firms can free-ride on screening by their competitors. Firms screen talent by a task difficulty, as in Akerlof (1976). In equilibrium, highly skilled workers work at high-productivity firms. If other firms can poach these workers, they can exploit the revealed information on skill. This increases their willingness to pay for skilled labour, raising the outside option of talented workers. This increases wages, as well as the difficulty of the task. High-productivity firms need to require a high difficulty, in order to deter less talented workers. Thus, existing wage differentiation is exacerbated by labour mobility.

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This paper argues that in industries with high returns to skill, wage differentials are exacerbated by employee mobility. Firms have technologies with varying marginal returns to skill and screen potential employees through features of the contract, such as the difficulty level of their work. This has been dubbed the *rat race* (Akerlof, 1976). Firms with the highest returns to skill will be willing to offer the highest wages to skilled workers. This means that the most skilled workers end up at those firms. In these cases, employment status becomes a signal of quality. If workers can easily move between jobs, other firms can use this signal to obtain skilled labour without needing to screen, free-riding on the screening efforts of other firms. This makes it harder to retain skilled workers and raises the premium these workers demand. Firms increase wages in order to prevent poaching.

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Ex ante, however, they still need to screen their workers, and as pay rises, these screening features become more pronounced.

This paper aims to model the high-potential labour market in skill-intensive industries. Particularly industries like banking, law, consulting, or (information) technology are prone to the dynamic in this paper. The rat race model in this paper predicts a pronounced vertical differentiation between firms, in which the top firms are more productive, hire the most talented workers and pay a sizeable wage premium.[1]

Workers with similar qualifications can often exhibit unobservable differences in skills that have a sizeable impact on the bottom line of their employer. Employers screen their employees by letting them engage in a rat race, rewarding especially difficult tasks or longer work hours. In this rat race, the skilled employees will end up taking inefficiently difficult tasks or long hours in order to show their skill. As Landers, Rebitzer, and Taylor (1996) show, lawyers at elite law firms engage in such a rat race. Anecdotal evidence strongly suggest that investment banking analysts work inefficiently long hours, as was on display when a group of juniors openly complained about their jobs' long hours, demanding jobs, and low work satisfaction.[2] As the top firms hire the most skilled workers, this rat race become more pronounced for precisely those workers.[3]

The main innovation in this paper is that the competition for workers intensifies when firms can easily poach each others' workers, as they can free-ride on the screening of other firms. This free-riding would exacerbate wage inequality and the rat race. The parallel developments in international integration, standardization of educational standards, and information technology have arguably decreased barriers to mobility for those working in high-skill industries. This is indeed what workers themselves say. When interviewed, a trader from the City of London put it this way:[4]:

> "In the end the bank is like a shell. You need a place to trade from, this is how we saw our bank. Sometimes an entire team can be poached and go from one bank to another. There's no loyalty either way."

Firms admit that poaching is a concern. Interestingly, after the aforementioned complaints by junior investment bankers, executives decided to raise junior pay in order to keep an-

[1] Indeed, Rebitzer and Taylor (1995) find that a significant employer-size wage effect is present for law firms.

[2] cf. among others the Financial Times article by Morris, Kinder, and Franklin (2021). Surveys also show that despite a reduction in deal volume, work hours have not decreased since last year (Brownstein, 2022).

[3] Zhang, Chen, Gong, Wang, Ding, Xiao, and Hui (2020) show that overtime work is more prevalent in large tech firms in China than in small ones.

[4] cf. the article in the Guardian's "Voices of Finance" series by Joris Luyendijk (2013).

alysts from leaving, but not to reduce working hours, showing that ex ante screening remained an important concern. In a similar move, three large consulting firms recently raised starting salaries, with industry insiders citing the need to "retain high-calibre people".[5] Evidence suggests that an increased potential for mobility raises the wages of managers (Garmaise, 2011).

Free-riding on information generated by labour contracts has been studied in a number of papers (Milgrom and Oster, 1987; Ricart i Costa, 1988; Waldman, 1984). These papers all argue that in order to prevent poaching of skilled workers, firms hide their employees' skills by assigning talented employees to inefficiently low-level tasks[6]. This would generally predict that whenever poaching becomes more of a concern, there would be a levelling of task difficulty between workers, as well as a decrease in wage inequality between various workers. In contrast to those papers, this one provides a mechanism through which mobility on the labour market would increase both the wage differential between workers of various skill levels, and the amount of work highly skilled workers have to perform. In this way, the current paper could go some way towards explaining how, whereas barriers to mobility have arguably decreased, and highly educated workers have become more mobile (Bauer and Bender, 2004), wage inequality has increased (Piketty and Saez, 2006), and top earners have started to work ever longer hours (Kuhn and Lozano, 2008).

The model, introduced in section 1, studies the effect of mobility by comparing two different labour markets. As a benchmark, it analyses a *low mobility* model with no scope for poaching and then compares that to a *high mobility* model in which firms can poach each others' workers. The model with no poaching is a fairly standard rat race model. The effect of mobility is captured by the difference between the two models. Each firm has a vacancy for a single worker. This gives positive assortative matching between firms and workers. Larger firms have to offer their matched workers an attractive enough contract to outbid their smaller competitor. In order to make sure workers of lower skill do not accept their offers, they set a *task difficulty* as a screening device. This leads to gradually higher wages and more difficult tasks moving up the matching. Every firm is then certain of her worker's ability.

Once the possibility of poaching is introduced, firms can use the information revealed by employment contracts in order to learn the ability of the workers employed at larger firms. Ex post, they would be willing to employ those workers without screening, raising the utility they would be willing to offer these workers. This means that larger firms will have to make the terms they offer their own workers even better in order to preclude poaching, offering higher wages. Ex ante, however, screening is still necessary, so that also

[5] cf. the article by O'Dwyer (2022)

[6] in a different context, de Garidel-Thoron (2005) provides a similar reasoning for insurance markets.

the task levels increase with respect to the low mobility case.

As Section 1 shows, various equilibria arise under different parameter conditions, some of which are characterized by higher wage inequality and more difficult tasks. Next, Section 2 discusses the results of the model and relates them to past empirical and theoretical findings, and offers some empirical predictions. Section 3 features a number of extensions: a first extension revisits the basic model, but with a more general production function. A second extension embeds the labour market into a repeated game framework in order to make some features of the model endogenous. A third extension studies a stylized model in which mobility costs are captured by a continuous parameter. Proofs are in the appendix.

1 The Model

The model is a simple version of the Akerlof (1976) rat race model. There are M firms of *sizes* $I_1 > I_2 > \ldots > I_M$, and $N > M$ *workers* of privately known skill types ϑ_n with $\vartheta_1 > \vartheta_2 > \vartheta_3 > \ldots > \vartheta_{N-1} > \vartheta_N > 0$.[7] Each firm has a single vacancy for which it can employ a single worker at a contract (w, e), where w denotes the wage the firm pays to the worker, and e denotes a verifiable difficulty level, or *task*, the firm demands of the worker. From being employed at a contract (w, e), a type ϑ worker obtains a utility of

$$u(w, e|\vartheta) = w - \frac{e}{\vartheta}. \tag{1}$$

Saliently, workers with higher skill are more resilient to difficult tasks, meaning the task can be used as a screening device. Workers who decide to enjoy an outside option rather than to work for a firm receive a utility of $\underline{u}$.

A firm of size I, when hiring a worker of type ϑ at a contract (w, e), obtains a net profit of

$$\Pi(w, e, \vartheta) = \vartheta I - w, \tag{2}$$

The level of e has no bearing on the firm's profits. The extension in section 3.1 addresses a more general profit function for the firm, which is allowed to be increasing in e. As is shown, the main results carry over. When not hiring any worker, the firm has a profit of zero.

The firm's size I and the worker's skill ϑ are complements. For firm m, I_m is referred to as the size of the firm,[8]. The interpretation is much wider: for consultancies one can think of the client base, for financial firms of the size of assets available for proprietary

[7] For convenience, the worker of type ϑ_n will occasionally be referred to as worker n. It should be noted that n is the worker's private information.

[8] This terminology is in line with Gabaix and Landier (2008); Terviö (2008)

trading, or for technology firms of the quality of laboratories, facilities and intellectual property. Firms' sizes are public information.

This model will be analyzed in a *low mobility* version and a *high mobility* version. The comparison between these two versions serves to illustrate the effect of mobility. As will be shown, mobility has the potential to increase wage inequality and the rat race effect. As the low-mobility market is very close to already existing models, the bulk of this section will be devoted to the high mobility market. Mobility is in this sense captured by a parameter that can take only the two values "high" and "low". The extension in section 3.3 shows how inequality varies if mobility is captured by a continuous parameter, and shows how wage inequality and task difficulty increase as moving jobs becomes easier.

First Best

If all firms can observe the workers' types, the market features an efficient allocation of workers: as firm size and worker talent are complements, the largest firm employs the most skilled worker, the second-largest firm employs the second-most skilled worker, etc. All workers n with $n > M$ are self-employed. Furthermore, as there is no need to screen, all contracts are efficient and feature a task level equal to zero. For each worker-firm pairing, the wage will be endogenously determined by the competitive pressure from smaller firms. This is stated in the following proposition.

Proposition 1. *If skill is publicly observable, each firm m hires the worker n with $n = m$. Firm M hires worker M at a wage of $\underline{u}$. For each $m < M$, firm m employs the worker of type ϑ_m at a wage of $\underline{u} + \sum_{i=1}^{M-m} I_{m+i} \left(\vartheta_{m+i-1} - \vartheta_{m+i}\right)$. All firms set a task level of $e = 0$.*

As there is no need for screening, all firms can set the efficient level of effort, $e = 0$. Given this, the largest firm profits the most from hiring the ϑ_1-worker, so she is willing to outbid firm 2, and hires the ϑ_1-worker exactly at the wage that would make firm 2 prefer hiring the ϑ_2-worker. Firm 2 outbids firm 3 in a similar manner for the worker of type ϑ_2, and so on. Ultimately firm M can hire worker $n = M$ at his reservation utility.

1.1 The Low-Mobility Labour Market

We first study the low-mobility market, in which there is no scope for mobility at all. This serves as a benchmark in order to assess the effect of mobility. The low-mobility market is modelled as a simultaneous move game, in which all firms simultaneously offer their contracts. Workers decide whether to apply for employment at one of the firms, or to remain self-employed.

This low-mobility labour market is a standard rat-race model. In this version, an equilibrium with positive assortative matching always exists, in which each firm outbids smaller firms for her own matched worker, at a contract that would not be acceptable to workers of lower ability. The utility that each firm m has to offer her matched worker must be high enough that the next-largest firm, $m+1$, would prefer hiring *her* next worker, rather than outbidding firm m for the worker of type ϑ_m.

In equilibrium, firm m hires the worker of type ϑ_m at a contract (w_m, e_m). Contract variables are defined by a system of recursive equations. Each can be found by thinking from the point of view of the smaller firm, $m+1$. She makes a profit of $\Pi_{m+1} = \vartheta_{m+1} I_{m+1} - w_{m+1}$, giving her worker a utility of $u_{m+1} = w_{m+1} - \frac{e_{m+1}}{\vartheta_{m+1}}$. If she considers hiring the worker with ϑ_m, she would have to offer a contract that is certain to hire worker ϑ_m. For any given utility $\tilde{u}$ that she wants to offer worker ϑ_m, she has to make sure to offer a contract $(\tilde{w}, \tilde{e})$ that is unattractive to worker ϑ_{m+1}, i.e. the contract must satisfy the screening conditions

$$\tilde{w} - \frac{\tilde{e}}{\vartheta_m} \geq \tilde{u}$$
$$\tilde{w} - \frac{\tilde{e}}{\vartheta_{m+1}} \leq u_{m+1}$$

The cheapest way to do so is to offer the contract with $\tilde{e} = \vartheta_m \vartheta_{m+1} \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}}$, and $\tilde{w} = u_{m+1} + \vartheta_m \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}}$.

This determines a minimum utility that serves as a "market price" for the worker of type ϑ_m. Firm m now needs to make sure to offer worker m a utility that is so high that firm $m+1$ would rather stick to hiring worker $m+1$ at the contract (w_{m+1}, e_{m+1}), giving that u_m must be the smallest $\tilde{u}$ for which

$$\vartheta_m I_{m+1} - \left(u_{m+1} + \vartheta_m \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}} \right) \geq \Pi_{m+1}. \tag{3}$$

Firm m then needs to offer a contract that provides a utility u_m to the worker of type ϑ_m, in order to outcompete firm $m+1$, while simultaneously making sure the worker of type ϑ_{m+1} prefers to remain employed at firm $m+1$.

Noting that firm M does not have any competitors wanting to poach the worker of type ϑ_M and can hire this worker for a price of $\underline{u}$, the recursion can be solved, yielding the solution characterized in the following proposition.

Proposition 2. *In the low-mobility labour market, each firm m employs a worker of type ϑ_m, with workers of type ϑ_n for $n > M$ remaining self-employed. For $m = 1, 2, \ldots, M$, the contracts are (w_m, e_m) with $w_M = \underline{u}$ and for $m < M$*

$$w_m = \underline{u} + \sum_{i=1}^{M-m} \left(\vartheta_{M-i} - \vartheta_{M-i+1} \right) I_{M-i+1}, \tag{4}$$

and with $e_M = 0$ *and for* $m < M$

$$e_m = \sum_{i=1}^{M-m} \vartheta_{M-i} \left(\vartheta_{M-i} - \vartheta_{M-i+1}\right) I_{M-i+1}, \quad (5)$$

This equilibrium always exists.

The low-mobility labour market always features positive assortative matching. Each firm offers her matched worker the market price, while making sure that the next-best worker prefers staying employed at his matched firm. The market price is determined by the next-largest firm's potential profits from hiring a slightly more skilled worker.

The equilibrium exists, because larger firms would always be willing to outbid smaller firms, whereas smaller firms remain unable to outbid the larger ones. In order to keep lower ability workers from accepting these contracts, they set a task level just high enough so that only their matching worker type would accept it. This is the classic Akerlof (1976) result: firms set higher task levels than first best in order to screen workers.

1.2 High Mobility

In the high mobility labour market, workers are allowed to switch employers after signing their first contract, but before starting work.[9] This is modelled in a stylized manner in this paper. The contracting happens in two rounds: in the first round, all firms offer contracts simultaneously. As before, workers apply to one or more of the firms, or decide to remain self-employed and receive $\underline{u}$. If multiple workers accept the contract from a particular firm, one of these workers will be assigned at random, with equal probabilities.[10]

After the first round, all firms observe the contracts signed by all the workers. They form beliefs about the types of the workers. Next, in a second round of offering, each firm can fire her own worker and offer a poaching contract to one of the workers who is employed at that time, replacing the worker she hired in the first round. Each of these workers can then decide to switch to the poaching employer, or stay with their original employer. After this decision, workers start working and utilities and profits are realized. The timing of contract offers serves as a reduced form of a bidding game between the various firms, with offers and counteroffers. Such a bidding game is explored in more detail in section 3.2.[11]

[9] The bidding game precedes the working stage. This can be justified by the stylized fact that most labour market mobility happens in the early stages of careers (Topel and Ward, 1992).

[10] The way in which the random assignment formally works in the case when multiple workers apply to multiple firms is specified in the appendix, section A.2

[11] By allowing for more different strategies ex-post, one could imagine that a firm that knows the type of her worker ex-post, but employs him at an inefficiently high effort level, could decide to change the

In order to solve this model we look for a weak perfect Bayesian equilibrium. When considering the set of contracts in the low-mobility labour market, poaching presents a problem in sustaining this set of contracts: in the second round, firms can learn the skill of the workers at other firms and then poach these workers without needing to engage in costly screening. Typically, a smaller firm, say m, could now be able to offer worker ϑ_{m-1} a higher utility than u_{m-1} ex post, as a direct offer to the worker she now knows to be of type ϑ_{m-1} does not need to satisfy the constraint that screens out lower-skilled workers. This increases the competition each firm feels from smaller firms.

The analysis below will illustrate the two outcomes in terms of how larger firms deal with this increased competition. Firms can keep their matched worker, but in that case, they need to increase the utility that they offer to their respective matched workers to a point that they outbid smaller competitors, even if these competitors were to exploit the ex post information. They raise the utility by offering a higher wage, but, since screening is still necessary ex ante, they also need to increase the difficulty of the task. As workers need to be compensated for the more difficult task, there is a multiplier effect that raises wages more than utilities.

In contrast to the benchmark, it might be that raising wages in order to keep the matched worker becomes prohibitively expensive. In this case, a firm can decide to offer a contract that also attracts lower quality workers, pooling together with smaller firms. In this case the expected productivity of her worker is lower, but she can hire him at a lower wage.

Depending on parameters, equilibria can feature several pools of firms hiring from the same pool of workers, or single firms matched to their corresponding worker. As will become clear, the number of candidate equilibria for a general number of firms becomes far too large to enumerate and analyze, which is why the analysis below focuses entirely on the case for three firms, which already gives all of the important features of potential equilibria, and provides clear predictions as to the effects on both wage inequality and the rat race.

1.2.1 Equilibria for Three Firms

This section explicitly explores the various equilibria that are possible, and under which conditions they prevail. These equilibria are then compared to the benchmark outcomes.

contract to the optimal one, or fire and rehire that worker. This is exogenously assumed not to be allowed in this version of the model. Section 3.2, however, provides an endogenous reason for firms not to engage in this type of behaviour: in a repeated game, firms need to maintain a reputation for only employing their own type of worker.

This section is restricted to analyzing the potential equilibria for $M = 3$. For each of these equilibria, the equilibrium contract variables are compared to the low-mobility outcome in order to find the effect of mobility. Section 1.2.2 then finds which equilibrium arises under which parameters.

In equilibrium, each firm wants to make sure her worker does not get poached, meaning the equilibria are characterized by posterior expected productivities associated to each contract, where no firm has an incentive to poach the worker knowing this productivity. Firms can match one to one with workers, but doing so in a poach-proof manner can become very expensive. In this case, a firm could opt to hire the next best worker. In equilibrium, this leads a firm to offer the same contract as the next smaller firm. If the competitive pressure from the firm that is two sizes smaller is also too large, she will opt to offer the same contract as the next two firms. For three firms, this leads to four candidate equilibria:

1. a *fully matched* equilibrium, in which each firm is matched one-to-one with the worker of the corresponding type,
2. a *pooled at the bottom* equilibrium, with the largest firm surely employing the most skilled worker, and the two other firms both employing the second- and third-most skilled worker with equal probabilities,
3. a *pooled at the top* equilibrium, with the two largest firms offering the same contract, and the two most skilled workers working at either firm with the same probability. The third-most able worker surely works for the third-largest firm, and
4. a *fully pooled* equilibrium, with all firms employing each of the top three workers, all with equal probability.

The Fully Matched Equilibrium In a fully matched equilibrium, each firm is matched to the corresponding worker, and will have to preclude poaching by smaller firms. In this case, firm 3 employs worker 3 and can do so at a wage of $\underline{u}$ and an effort level of zero. This means she makes a profit of $\Pi_3^M = \vartheta_3 I_3 - \underline{u}$. Ex post, she knows that the worker at firm 2 is of type ϑ_2, meaning she would be willing to poach at a salary $\tilde{w}$ such that

$$\vartheta_2 I_3 - \tilde{w} = \vartheta_3 I_3 - \underline{u}.$$

For firm 2, this means that in order to retain worker 2, she must offer a utility of at least $\overline{u}_2 = \underline{u} + (\vartheta_2 - \vartheta_3) I_3$. Ex ante, however, she must make sure that workers of type ϑ_3 or lower do not want to accept employment, so that she must set a contract $\left(w_2^M, e_M^2\right)$

satisfying the joint constraints for retaining worker 2 and screening out worker 3.

$$\begin{aligned} w_2^M - \frac{e_2^M}{\vartheta_2} &\geq \underline{u} + (\vartheta_2 - \vartheta_3) I_3 \\ w_2^M - \frac{e_2^M}{\vartheta_3} &\leq \underline{u}, \end{aligned}$$

giving

$$\begin{aligned} w_2^M &= \underline{u} + \vartheta_2 I_3 \\ e_2^M &= \vartheta_2 \vartheta_3 I_3. \end{aligned}$$

It can readily be seen that both the wage and the effort level are higher than in the low mobility benchmark.

Firm 1 similarly needs to retain the worker of type ϑ_1 against poaching by the smaller firms. As firm 2 is the most willing to poach, firm 2 decides the utility that firm 1 needs to offer worker 1 in order to retain him. Firm 1 must make sure ex ante to screen out workers of type ϑ_2 and lower, giving the contracts

$$\begin{aligned} w_1^M &= \underline{u} + \vartheta_1 I_2 + \vartheta_2 \left(1 + \frac{\vartheta_3}{\vartheta_1 - \vartheta_2}\right) I_3 \\ e_1^M &= \vartheta_1 \vartheta_2 \left(I_2 + \frac{\vartheta_3}{\vartheta_1 - \vartheta_2} I_3\right) \end{aligned}$$

Both e_1^M and the wage differential $w_1^M - w_2^M$ are higher than in the low mobility benchmark. Each firm has to preclude poaching by raising the wage of their workers, but then also needs to set a higher task level in order to discourage lower ability workers.

Pooled at the Bottom Next, we study an equilibrium in which only the largest firm separates itself from the other two. In this type of equilibrium, firm 1 hires the ϑ_1-type with certainty, whereas firms 2 and 3 both hire the ϑ_2 and ϑ_3 types with equal probability. These last two firms can do so at a wage of $\underline{u}$ and a task level of zero, as they do not need face any competition for these workers. They thus make profits of respectively

$$\Pi_m = \mathbf{E}(\vartheta | \vartheta_3 \leq \vartheta \leq \vartheta_2) I_m - \underline{u}$$

for $m \in \{2, 3\}$. Both know that the worker of type ϑ_1 works for firm 1 and potentially have an interest in poaching this worker. Firm 2 gains the most by poaching, so that the market price for the ϑ_1-type is determined by the utility that firm 2 would be willing to offer the ϑ_1-type. This utility $\overline{u}^B$ satisfies

$$\vartheta_1 I_2 - \overline{u}^B = \mathbf{E}(\vartheta | \vartheta_3 \leq \vartheta \leq \vartheta_2) I_2 - \underline{u},$$

giving

$$\overline{u}^B = \underline{u} + \left(\vartheta_1 - \frac{\vartheta_2 + \vartheta_3}{2}\right) I_2.$$

In order to screen, firm 1 must offer a contract such that type ϑ_1 obtains a utility of $\overline{u}^B$, but also such that workers of type ϑ_2 or lower would rather take the contract at firm 2 or 3. This means that firm 1 needs to set an effort level at least such that

$$e\left(\frac{1}{\vartheta_2}-\frac{1}{\vartheta_1}\right)=\overline{u}^B-\underline{u},$$

giving

$$e=e_1^B:=\vartheta_1\vartheta_2\frac{\vartheta_1-\frac{\vartheta_2+\vartheta_3}{2}}{\vartheta_1-\vartheta_2}I_2.$$

The corresponding wage then equals

$$w=w_1^B:=\underline{u}+\vartheta_1\frac{\vartheta_1-\frac{\vartheta_2+\vartheta_3}{2}}{\vartheta_1-\vartheta_2}I_2$$

Comparing the wage for the ϑ_1-type to the low mobility benchmark, one finds that

$$\begin{aligned}w_1^B&=\underline{u}+\vartheta_1\left(1+\frac{1}{2}\frac{\vartheta_2-\vartheta_3}{\vartheta_1-\vartheta_2}\right)I_2\\&>\underline{u}+\vartheta_1 I_2\\&>\underline{u}+\left(\vartheta_1-\vartheta_2\right)I_2+\left(\vartheta_2-\vartheta_3\right)I_3,\end{aligned}$$

so that the highest earner earns more than she would in the low mobility market.

In this equilibrium, all wage earners, except for one, make $\underline{u}$, whereas the ϑ_1-type earns a wage that is larger than the average wage of the top two earners in the benchmark. This gives that inequality is larger in this equilibrium than in the low mobility case.

When considering the rat race effect, it is immediate that the ϑ_2-type both works and earns less than in the benchmark. As can be seen from a direct comparison of e_1^B to the benchmark, the task difficulty is also higher for worker 1, meaning that top earners have a more difficult task than in the benchmark.

Pooled at the Top A candidate for equilibrium would also be the one in which firms 1 and 2 both offer the same contract. This contract should be acceptable to types ϑ_1 and ϑ_2, but not to type ϑ_3. In this case, firm 3 employs type ϑ_3 without needing to screen, at a contract $(\underline{u},0)$, and has a profit of

$$\Pi_3=\vartheta_3 I_3-\underline{u}$$

Firm 3's ex-post poaching incentives would then determine the wage needed to keep workers 1 and 2 working for the contract that the two largest firms offer. If firm 3 were to decide to poach the worker from either of the two larger firms, she can expect a productivity of $\mathbf{E}(\vartheta|\vartheta\geq\vartheta_2)$. This means that in order to outbid firm 3, firms 1 and 2 must offer a utility of at least

$$\overline{u}^T:=\underline{u}+\left(\frac{\vartheta_1+\vartheta_2}{2}-\vartheta_3\right)I_3$$

The contract offered by firms 1 and 2 must be acceptable to the ϑ_2-type, but not to the ϑ_3-type, meaning it must specify an effort of

$$e^T := \frac{\vartheta_2\vartheta_3}{\vartheta_2 - \vartheta_3}\left(\overline{u}^T - \underline{u}\right)$$
$$= \vartheta_2\vartheta_3 \frac{\frac{\vartheta_1+\vartheta_2}{2} - \vartheta_3}{\vartheta_2 - \vartheta_3} I_3,$$

giving a wage of

$$w^T = \underline{u} + \vartheta_2 \frac{\frac{\vartheta_1+\vartheta_2}{2} - \vartheta_3}{\vartheta_2 - \vartheta_3} I_3,$$

This equilibrium presupposes that the two top firms do not poach from one another. Both, however, would have an incentive to do so, as ex-post they know they can hire a worker of the same posterior expected productivity, but at the cheaper contract with a task level of zero, and a wage a tiny bit higher than $w^T - \frac{e^T}{\vartheta_1}$. This means that this cannot be a weak perfect Bayesian equilibrium.

The reason this equilibrium is nevertheless interesting to mention, is that for reasons outside of the current model, such an arrangement might still arise. If reputation concerns such as those in the extension in Section 3.2 are taken into account, both firms have an incentive to keep the difficulty high enough in order not to attract lower quality workers in subsequent periods. In this case, such an equilibrium can be sustained. Alternatively, groups of firms might collude not to poach each other's workers, which happened, for example, among some of the top information technology firms in Silicon Valley.[12]

Fully Pooled In the fully pooled equilibrium, all three firms offer the same contract and get one of the first three workers with equal probability. They can do so setting a task level of zero and a wage of $\underline{u}$. Each of the respective firms m makes a profit of

$$\Pi_m^F = \mathbf{E}(\vartheta|\vartheta \geq \vartheta_3)I_m - \underline{u}$$

In this equilibrium, there is no inequality, and no rat race. If the firms are close enough together in size, the increased competition makes it unfeasible for firms to separate from one another, leading to a levelling of wages.

1.2.2 Finding the Equilibrium

The possible equilibria are thus the fully matched and the pooled at the bottom equilibrium on the one hand, both featuring higher inequality than the benchmark, and the fully pooled equilibrium on the other, with no inequality. Finding which equilibrium prevails under

[12]See, for example, the TechCrunch article by Constine (2012), citing no-poaching agreements between seven firms, including Apple, Google, and Intel.

which set of parameters boils down to finding the preferences of the firms over the different equilibria.

In order to derive sufficient conditions for one of the more unequal equilibria to prevail, the incentives of the largest firm play an important part. This largest firm can take a leading role. If the match between the top firm and the top worker is very valuable, she can decide to offer a matching contract, making her certain to hire the most talented workers. On the other hand, this is precisely what exposes her to poaching. This gives the following condition.

Proposition 3. *If*

$$\frac{2}{3}\frac{I_1}{I_2} \geq \frac{\vartheta_1}{\vartheta_1 - \vartheta_2}, \tag{6}$$

the fully matched or the pooled at the bottom equilibrium is a weak perfect Bayesian equilibrium. In this case, no other equilibrium is possible.

Condition (6) contains the intuition that the match between worker 1 and firm 1 is valuable. The left hand side is large precisely when I_1 is large with respect to I_2, and the right hand side is small precisely when ϑ_1 is large compared to ϑ_2. This condition is equivalent to stating that firm 1 prefers the pooled at the bottom equilibrium to the fully pooled one. This implies that whenever firms 2 and 3 offer the non-screening contract, firm 1's best response is to offer a poach-proof screening contract to worker 1. As is shown in the appendix, if firm 2 prefers the fully matched equilibrium, then so does firm 1, meaning that in those cases, the fully matched equilibrium will arise.

Condition 6 is sufficient. Some more conditions can be derived under which some specific equilibria uniquely occur, and some under which multiple equilibria might occur.

Proposition 4. *If*

$$\vartheta_1 I_2 + \frac{\vartheta_2}{\vartheta_1 - \vartheta_2}\left(\vartheta_1 - \vartheta_2 + \vartheta_3\right) I_3 \leq \frac{2\vartheta_1 - \vartheta_2 + \vartheta_3}{3} I_1 \leq \vartheta_1 \left(1 + \frac{\vartheta_2 - \vartheta_3}{\vartheta_1 - \vartheta_2}\right) I_2 \tag{7}$$

and

$$\frac{1}{2}\left(\vartheta_2 + \vartheta_3 - \frac{1}{2}(\vartheta_1 - \vartheta_2)\right) I_2 \geq \vartheta_2 I_3, \tag{8}$$

then both the fully matched and the fully pooled equilibrium are weak perfect Bayesian equilibria.

This represents an intermediate case in terms of the value of the match between firm 1 and worker 1, in which various equilibria are possible. Furthermore, firm 3 needs to be relatively small with respect to firm 2 in order for the above conditions to hold.[13]

[13]Note that if collusion between the top two firms were possible, this is also when we would expect them to engage in it and pool together.

If the match between firm 1 and worker 1 is less valuable, ultimately only the fully pooled equilibrium remains, as is stated in the following proposition.

Proposition 5. *If*

$$\frac{2\vartheta_1 - \vartheta_2 + \vartheta_3}{3} I_1 < \vartheta_1 I_2 + \frac{\vartheta_2}{\vartheta_1 - \vartheta_2} (\vartheta_1 - \vartheta_2 + \vartheta_3) I_3 \tag{9}$$

or, if both

$$\frac{2}{3} I_1 < \frac{\vartheta_1}{\vartheta_1 - \vartheta_2} I_2 \tag{10}$$

and

$$\frac{1}{2} \left(\vartheta_2 + \vartheta_3 - \frac{1}{2}(\vartheta_1 - \vartheta_2) \right) I_2 < \vartheta_2 I_3, \tag{11}$$

then the fully pooled equilibrium is the unique weak perfect Bayesian equilibrium.

1.2.3 Remark on Equilibria with a General Number of Firms

As per the remark on the non-existence of a "pooled at the top" equilibrium, there cannot be pools of firms offering the same contract if such a contract features a positive task level. This means that the only equilibria possible can be those in which the top firms all assortatively match to their own workers, whereas the bottom firms all pool together. Such an equilibrium would be characterized by a firm $\underline{m}$ such that all firms $\underline{m} < m \leq M$ offer the contract $(\underline{u}, 0)$ and employ the workers $\underline{m} < n \leq M$ with equal probability.

For firm $\underline{m}$, as the lowest firm which matches to her own worker, the contract must be poach-proof in the sense that firm $\underline{m}+1$ does not want to poach the worker from $\underline{m}$. The worker with $n = \underline{m}$ must thus get a utility of $\overline{u}_{\underline{m}}$ such that

$$\vartheta_{\underline{m}} I_{\underline{m}+1} - \overline{u}_{\underline{m}} \leq \mathbf{E} \left(\vartheta_n | \underline{m} < n \leq M \right) I_{\underline{m}+1} - \underline{u}. \tag{12}$$

At the same time, it must be so that this is the utility the worker $n = \underline{m}$ receives, but firm $\underline{m}$ still needs to screen out workers of types below $\vartheta_{\underline{m}}$, giving that $\left(w_{\underline{m}}, e_{\underline{m}}\right)$ solves $\min_{(w,e)} w$ subject to

$$w - \frac{e}{\vartheta_{\underline{m}}} \geq \overline{u}_{\underline{m}}$$
$$w - \frac{e}{\vartheta_{\underline{m}+1}} \leq \underline{u},$$

giving

$$w_{\underline{m}} = \underline{u} + \vartheta_{\underline{m}} \frac{\vartheta_{\underline{m}} - \mathbf{E} \left(\vartheta_n | \underline{m} < n \leq M \right)}{\vartheta_{\underline{m}} - \vartheta_{\underline{m}+1}} I_{\underline{m}+1}$$
$$e_{\underline{m}} = \vartheta_{\underline{m}} \vartheta_{\underline{m}+1} \frac{\vartheta_{\underline{m}} - \mathbf{E} \left(\vartheta_n | \underline{m} < n \leq M \right)}{\vartheta_{\underline{m}} - \vartheta_{\underline{m}+1}} I_{\underline{m}+1}$$

Then for $m < \underline{m}$, the contracts can be found through backwards recursion, given that the contracts must be poach-proof in the sense that

$$\vartheta_m I_{m+1} - \overline{u}_m \leq \vartheta_{m+1} I_{m+1} - w_{m+1},$$

and should satisfy the screening constraints

$$w_m - \frac{e_m}{\vartheta_m} \geq \overline{u}_m$$
$$w_m - \frac{e_m}{\vartheta_{m+1}} \leq \overline{u}_{m+1},$$

leading to the recursion

$$w_m = w_{m+1} + \vartheta_m I_{m+1} + \frac{e_{m+1}}{\vartheta_m \vartheta_{m+1}} \quad (13)$$

$$e_m = \vartheta_m \vartheta_{m+1} I_{m+1} + \frac{\vartheta_m e_{m+1}}{\vartheta_m \vartheta_{m+1}}. \quad (14)$$

Solving this recursion then gives the equilibrium contracts. This gives $m+1$ potential equilibria, with $\underline{m} = 0$ corresponding to a fully pooled one and $\underline{m} = M$ corresponding to a fully matched one.

It should be noted that the set of equilibria is rather restricted due to the feature of the model that firms in the same pool could poach from each other, as mentioned in the discussion of the "pooled at the top" equilibrium above. If collusion or reputation concerns preclude such poaching, any contiguous partition of the set of firms can define an equilibrium, with firms in each subset in the partition offering the same contract and hiring from the same pool of workers.[14]

2 Discussion and Empirical Implications

Other papers have addressed the potential for free-riding on information conveyed by labour contracts. Milgrom and Oster (1987) and Ricart i Costa (1988) both feature firms learning about their managers' skill through observing performance.[15] Absent mobility, promoting these managers would be beneficial to the firm, but promotion can then signal skill to competitors. These papers argue that firms would underemploy talented workers by not assigning them to the tasks in which they are most productive. The partially and fully pooling equilibria share features with the outcome in these papers: the types of better workers are hidden, and these workers are not necessarily assigned to the place where they

[14] Also note that this would be exponential in the number of firms, rather than linear.

[15] The same mechanism is at play in a non-labour context in De Garidel-Thoron (2005), which models an insurance market in which insurers learn about their own clients' riskiness through observing accidents. Sharing this information allows competitors to lure away low-risk clients.

would be most productive. The difference here is that in the aforementioned papers, workers are underemployed despite their types being known to the employer, whereas here the firms choose not to precisely learn the workers' types.

The interesting observation, however, lies in the complementary mechanism to the one in Milgrom and Oster (1987) and Ricart i Costa (1988). Both in those papers, and in this one, contracts generate information on which competitors can free-ride by poaching talented workers. The difference lies in the way incumbent employers deal with this kind of poaching. In the matched equilibria in this paper, employers prevent this type of poaching by making the highly skilled workers' job so lucrative that competitors will not want to poach. The only way to do so while preserving screening is by *overemploying* them. In the aforementioned papers, the incumbents' response was always to *underemploy* talented workers. As stated in the introduction, the aforementioned papers would predict talented workers being assigned to low-level tasks, as well as a general levelling of wages. This paper finds that free-riding can generate inefficiently *high*, rather than *low* task levels for skilled workers, as well as an increased wage differential between types.

An increase in wage differentials for high-skilled workers may well have contributed to a more general, secular increase in wage inequality, as has been documented over the past three to four decades. Card, Heining, and Kline (2013) describe the increased earnings inequality in West-Germany from 1985 through 2008. They find that much of the increase can be explained by the assortative assignment between workers and employers. Song, Price, Guvenen, Bloom, and Von Wachter (2019) use the full sample of US employees from 1978 until 2005 and find, among other things, that two thirds of the increase in wage inequality stems from between-firm dispersion. Katz et al. (1999) review the extant literature and document a number of interesting facts about wage inequality that are in line with this paper. To mention a few, the interquartile range of wages has increased drastically between the 70's and the 90's, with an even more drastic increase for the top one percent of wage earners. Furthermore, the wage differences have increased also within groups of workers with similar observable characteristics, even within the same industry. Abowd, Kramarz, and Margolis (1999) follow a sample of one million French workers. They find that much of the variation remained after correcting for observables. In line with this paper's predictions, they find that firms with high-paid workers are more productive, but not necessarily more profitable.

As argued above, the rat race per se does not increase inequality with respect to the first best. As will be shown in section 3.1, wage differences in a rat race model can even be lower than in the first best. It is the potential for poaching alone that generates wage differentials above and beyond the first best.

Moreover, this paper predicts an exacerbation of the Akerlof (1976) rat race dynamic, with high earners being assigned inefficiently difficult tasks. In this light, Kuhn and Lozano (2008) document an interesting stylized fact: over the past half century, the top earners in the US have started to have ever longer workweeks. This seems to stand in stark contrast to how things were in previous centuries, when the lowest earners made the longest hours (cf. e.g. Voth, 2001). The model in this paper proposes a mechanism that simultaneously explains the increase in working hours for top earners *and* the increase in inequality. This would also provide an alternative explanation for the association between inequality and hours worked as found by Bell and Freeman (2001): rather than high inequality causing long hours, both are jointly determined.

Mobility could have been instrumental in this shift. Indeed, for the United States, Rhode and Strumpf (2003) examine worker mobility over the period from 1850 through 1990, and document a secular decrease in mobility costs. Johnson, Lavetti, and Lipsitz (2020), as well as Starr, Prescott, and Bishara (2021) proxy for the potential of mobility by exploiting differences in the enforceability of non-compete clauses, and find that higher mobility is associated with higher wages, in line with the predictions of this paper.

Concerning overemployment and increased inequality, section 1.2.2 predicts when we should expect mobility to exacerbate these problems. When the pooling at the bottom, or fully separating equilibrium prevails, inequality increases, and top earners have higher inefficient task levels. As found above, the equilibria with increased inequality are more likely to prevail if $\frac{I_1}{I_2}$ and $\frac{\vartheta_1}{\vartheta_2}$ are large, that is if worker skill or complementary firm inputs are particularly important. The main prediction deriving from this is that precisely in those sectors where the rat race effect and wage inequality already abound, these are amplified by mobility.

This gives the following empirical implications

Skill differences: in sectors in which the skills of top employees have particularly strong effects, an increase in mobility increases both the rat race effect and inequality

Firm sizes: in sectors that are dominated by one or a small number of particularly large or profitable firms, increased mobility increases the rat race and inequality.

Amplification: in those sectors in which the rat race and wage inequality are already particularly pronounced, these are exacerbated by an increase in mobility.

It is important to realize that for the predicted effects to occur, it is the mere *potential* for mobility that matters. Workers do not actually have to switch jobs more often, they should just have the possibility to do so. As a secular trend, it can be argued that the potential for mobility among top earners has increased through increased international cooperation, standardization of educational standards, and improvements in information and communication technology. The challenge is to identify individual cross-sectional measures or events that could serve as proxies for the potential of mobility.

A potential measure could be the enforceability of non-compete agreements and garden leave clauses. These clauses often are meant serve to protect client relationships or trade secrets, but they also hinder mobility and reduce workers' ex-post bargaining position vis-à-vis their employers. The enforceability of these clauses differs between different US States (Bishara, 2010). Two recent studies (Johnson, Lavetti, and Lipsitz, 2020; Starr, Prescott, and Bishara, 2021; Garmaise, 2011) have shown that higher enforceability of non-compete clauses lead to lower wages, which is in line with the mechanism outlined in this paper. The current paper predicts that, more specifically, lower enforceability of non-compete contracts would raise wages particularly for high earners, increase the use of screening devices (such as hours worked) in skill-intensive industries, and raise wage inequality.[16]

This paper is different from those that propose dynamic models in which inequality evolves over time. Postel-Vinay and Robin (2002) and Harris and Holmstrom (1982) both propose dynamic models with symmetric information. Both predict inequality to evolve over time across different worker types and predict that mobility influences this inequality. There are some predictions that set the current paper apart from these. Specifically, this paper would predict that an increase in the potential for mobility should already entail a dispersion of starting wages, rather than a divergence of wage paths over the course of workers' careers. News media and practitioners' publications do report on a steady increase in entry-level salaries at the top tier firms in consultancy and investment banking[17], but so far this issue has received little attention in the academic literature. Furthermore, the aforementioned papers are silent on the rat race effect, giving no predictions pertaining to the increase in work hours or job difficulty.

Other papers with asymmetric information propose static models or three-period models. In Inderst and Wambach (2002), firms with a limited number of vacancies offer a menu of screening contracts, but then have to hire all applying workers, without the "cream-skimming" allowed by the one-to-one matching between workers and firms in this paper.

[16] Another potential development that seems particularly interesting for the types of industries considered in this paper is the transition from partnerships to public firms, which has been well documented, for example, for investment banks (cf. e.g. Morrison and Wilhelm Jr, 2008). The shift from workers being either partners or aspiring partners to being mere employees should have greatly affected employee mobility. The problem here, though, is to see to which extent this transition happened for reasons exogenous to increased mobility.

[17] Employee retention is often mentioned explicitly as a reason for the recent increase in junior salaries. See, for example, the SHRM report by Anderson (2021), addressing entry salary increases in law, private equity, and banking, or the Bloomberg article by Biekert (2021).

Inderst (2001) models a similar matching between workers and firms, but studies the effect of bargaining power, without the free-riding in this paper. A converse to the model in this paper is the one by Greenwald (1986): there, rather than employment status being a positive signal, separations are a negative signal, leading to adverse selection in the secondary labour market. If more workers quit for personal reasons, this mitigates the adverse selection problem.

3 Extensions

3.1 Generalized Production Function

This section covers an extension of the model that generalizes the profit function of firms to also potentially include an effect of e on the firm's bottom line. For simplicity the analysis is restricted to the case with $M = 2$. The results from the previous section carry over: equilibria arise featuring positive assortative matching with overemployment of the skilled worker and increased income inequality. Alternatively, equilibria arise in which both firms pool on the same contract to attract the top two workers with equal probability.

The productivity of the worker is generalized in the following manner: a firm of size I, when hiring a worker of type ϑ, at a contract (w, e), obtains a net profit of

$$\Pi(w, e, \vartheta) = \pi(e, \vartheta)I - w, \tag{15}$$

where the function $\pi(\cdot, \cdot)$ is increasing in both arguments and concave in e, and satisfies a weak single crossing condition[18]: for $\vartheta > \vartheta'$,

$$\frac{\partial}{\partial e}\pi(e, \vartheta) \geq \frac{\partial}{\partial e}\pi(e, \vartheta'). \tag{16}$$

Furthermore, for all ϑ, $\lim_{e\to\infty} \frac{\partial}{\partial e}\pi(e, \vartheta)I_1 < \frac{1}{\vartheta}$. The worker's preferences remain unchanged: a worker of type ϑ still has a utility of

$$u(w, e|\vartheta) = w - \frac{e}{\vartheta}$$

when employed at the contract (w, e)[19] and $\underline{u}$ when unemployed.

The appendix, section B, features a monopsonist version of this model that might be useful to study first in order to better understand this version of the model.

[18]assuming a strong single crossing condition with strict inequality, but making π independent of ϑ would correspond to the "private values" setup of Inderst (2001). Note that the analysis below would remain valid for this case.

[19]A more general dependence on e and ϑ can be subsumed by substituting the appropriate expressions into the firm's production function. As can be seen from the analysis below, the reasoning remains the same if the utility function were still separable, but concave in w. Linearity in w is assumed only for tractability reasons.

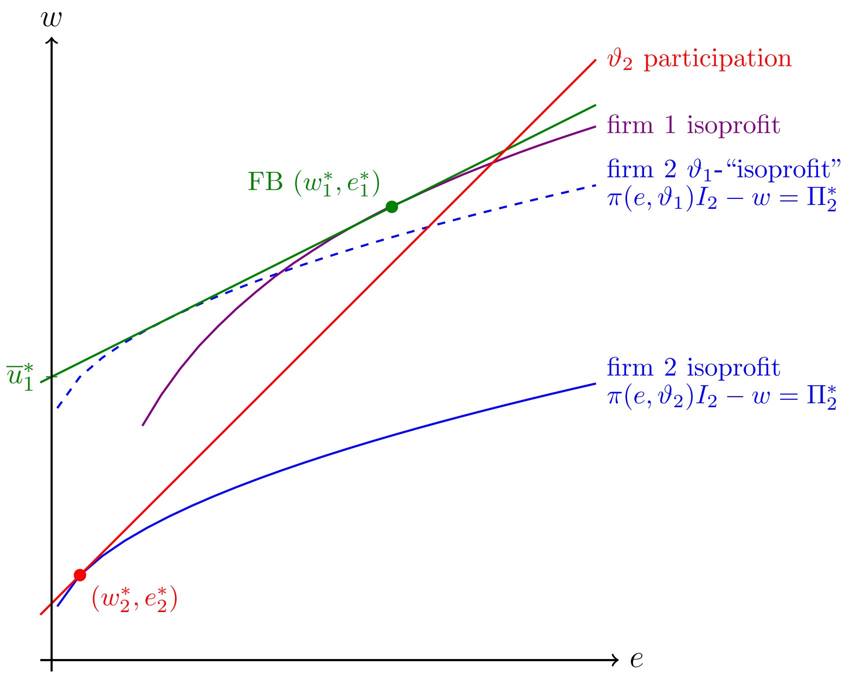

Figure 1: Labour market with firms of sizes $I_1 > I_2$: first best

3.1.1 Benchmarks

First best if the type of workers is public information, firms 1 and 2 are matched to workers 1 and 2 respectively, as firm 1 is always willing to bid more for the highest ability worker than firm 2. The first best is illustrated in figure 1. In this figure, the indifference curves for the workers are straight lines, with the ϑ_2 workers' steeper than those for the ϑ_1 worker. Each firm has two different sets of isoprofit curves: those when employing a ϑ_1 worker, and those for employing a ϑ_2 worker.

In the first best, firm 2 employs worker 2 at the contract (w_2^*, e_2^*), such that either the firm's ϑ_2-isoprofit curve is tangent to the ϑ_2-type's participation constraint, or, if $\frac{\partial}{\partial e}\pi(0,\vartheta_2)I_2 < \frac{1}{\vartheta_2}$, the contract at $(\underline{u}, 0)$. Call the profit at this point Π_2^*. The utility that needs to be paid to the ϑ_1-type is determined by those contracts that would give a profit of Π_2^* if firm 2 were to employ the ϑ_1-type. In figure 1, this corresponds to the dashed blue isoprofit curve.

In order to make sure that she outbids firm 2, firm 1 needs to give a utility to worker 1 of at least $\overline{u}_1^*$, which is given by the dual problem

$$\overline{u}_1^* = \max_{w,e>0} \{u(w,e|\vartheta_2)\,|\pi(e,\vartheta_1)I_2 - w \geq \Pi_2^*\}\,, \quad (17)$$

as illustrated in figure 1. Call the corresponding contract $(\tilde{w}, \tilde{e})$ The first best contract offered by firm 1 is then given by

$$(w_1^*, e_1^*) = \underset{(w,e)}{\text{argmax}} \left\{ \pi(e, \vartheta_1) I_1 - w \,\middle|\, w - \frac{e}{\vartheta_1} \geq \overline{u}_1 \right\}. \tag{18}$$

Low mobility as in Akerlof (1976), screening potentially creates a rat race, even if no poaching is present. The point of this paper is that poaching exacerbates this rat race. In order to illustrate this, first consider a low mobility labour market model. There is asymmetric information on skill and simultaneous, one-time contract offers from both firms. This is illustrated in figure 2. Consider again an equilibrium in which firm 2 employs worker 2. She can do so at the first best contract.

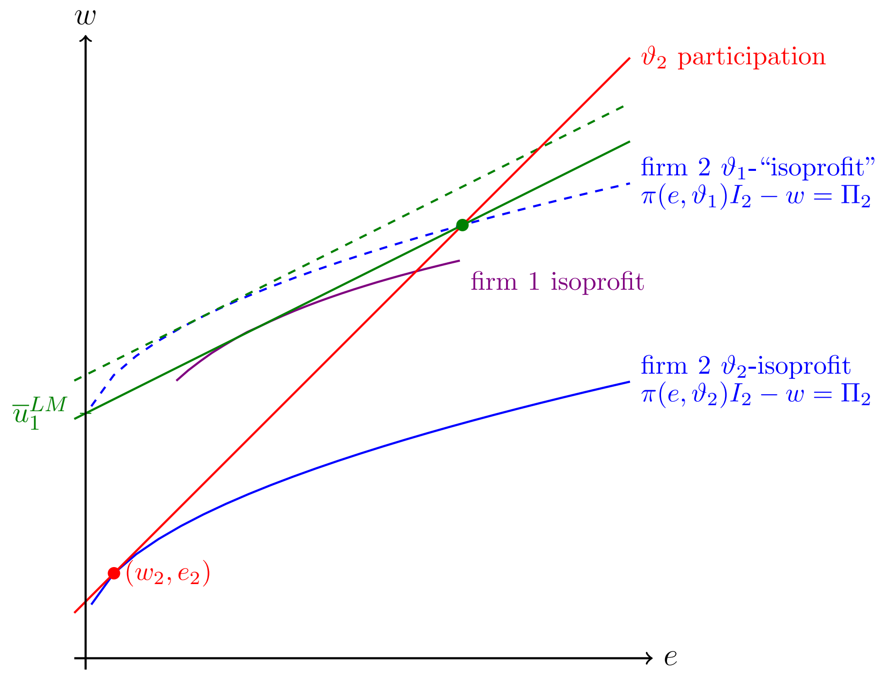

Figure 2: Low mobility labour market with firms of sizes $I_1 > I_2$

Now, again consider the contracts that would give her a profit of Π_2^*. Consider the case, as in the illustration, that the solution to problem (17) lies to the upper left of worker 2's participation constraint. In this case, firm 2 could not offer that solution to hire worker 1, as it would attract worker 2. The best she can offer is the contract that gives her a profit of Π_2^*, while screening out worker 2. In the figure, this contract would lie at the intersection of the ϑ_2 participation constraint and the dashed isoprofit curve. The utility that the ϑ_1-worker would enjoy from this contract is now the best firm 2 can offer. This serves as the new "market price" $\overline{u}_1^{LM}$ for worker 1.

Firm 1 needs to offer the best possible contract that guarantees to worker 1 a utility of at least $\overline{u}_1^{LM}$ in order to be competitive. As illustrated, the contract maximizing profits given this utility, could be preferred by worker 2 to the contract (w_2^*, e_2^*). In that case, she needs instead to find the best contract that also screens out worker 2. This is given by the point where the ϑ_1-type's indifference curve at a utility of $\overline{u}_1^{LM}$ intersects worker 2's participation constraint. This gives a second-best contract of

$$\left(w_1^{LM}, e_1^{LM}\right) = \left(\underline{u} + \vartheta_1 \frac{\overline{u}_1^{LM} - \underline{u}}{\vartheta_1 - \vartheta_2}, \vartheta_1 \vartheta_2 \frac{\overline{u}_1^{LM} - \underline{u}}{\vartheta_1 - \vartheta_2}\right) \tag{19}$$

3.1.2 High mobility

Matched equilibrium in an equilibrium with positive assortative matching, worker 1 reveals his type by accepting firm 1's offer. Firm 2 now perfectly learns this worker's type and can exploit that by poaching firm 1's worker. As she does not need to screen the worker anymore, this raises the utility she is able to offer the worker of type ϑ_1, and thus this worker's market price. Firm 1, however, does not have this information yet when she offers the contract and thus still needs to screen. This exacerbates the rat race and increases wage inequality between the two types.

The situation in this case is illustrated in figure 3. If, in equilibrium, firm 1 employs worker 1 at a contract that only he would accept, firm 2 perfectly learns that this worker is of type ϑ_1. She would now offer the same contract $(\tilde{w}, \tilde{e})$ that we saw in the first best, specifically to worker 1. As she is certain of the worker's type, she does not need to worry about screening. This raises worker 1's minimum utility to $\overline{u}_1^{HM} = \overline{u}_1^*$.

In order to preclude poaching, firm 1 needs to offer a utility of at least $\overline{u}_1^{HM}$. As she does so in the first round, however, she still needs to screen. If, as in the illustration, (w_1^*, e_1^*) is preferred by worker 2 to (w_2^*, e_2^*), firm 1 then needs to offer the contract

$$\left(w_1^{M}, e_1^{M}\right) := \left(\underline{u} + \vartheta_1 \frac{\overline{u}_1^{M} - \underline{u}}{\vartheta_1 - \vartheta_2}, \vartheta_1 \vartheta_2 \frac{\overline{u}_1^{M} - \underline{u}}{\vartheta_1 - \vartheta_2}\right) \tag{20}$$

which is the contract for which the constraint that preclude poaching, and the constraint that screens out worker 2 are both binding. Define firm 1's profit in this case as Π_1^M. It can be seen both from the illustrations and from the expressions that $w_1^M > w_1^{LM}$ and that $e_1^M > e_1^{LM}$.

The task level and wage in this type of equilibrium are strictly higher than in the low-mobility benchmark as long as the first best is not the solution to the high mobility labour market, i.e. if

$$u\left(w_1^*, e_1^* \middle| \vartheta_2\right) > \underline{u} \tag{21}$$

The following definitions summarize the above

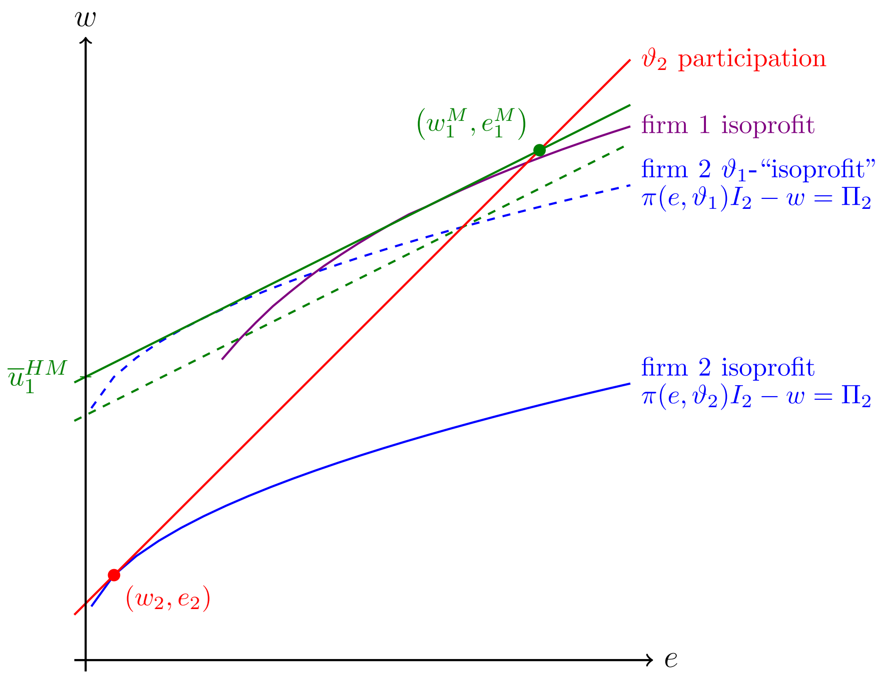

Figure 3: High mobility labour market with firms of sizes $I_1 > I_2$ and overprovision of task difficulty

Definition 1. Let $\overline{u}_1^M$ be the maximum between $\underline{u}$ and

$$\max\left\{ w - \frac{e}{\vartheta_2} \middle| \pi(e,\vartheta_2)I_2 - w > \Pi_2^* \right\} \tag{22}$$

The *matched equilibrium* consists of firm 1 offering the contract (w_1^M, e_1^M) that solves

$$\max_{w,e} \{\pi(e,\vartheta)I_1 - w\}$$

subject to

$$w - \frac{e}{\vartheta_2} \geq \overline{u}_1^M$$
$$w - \frac{e}{\vartheta_1} \leq \underline{u}.$$

Firm 1's profit under these contracts is called Π_1^M.

The conditions under which this equilibrium exists are stated in proposition 7 further below. Note that the underlying assumption for this type of equilibrium is that whenever a smaller firm *could* outbid the large firm, she *would* do so. A potential problem with this is that low-skill workers might mimic high-skill ones and accept employment at the large firm on unfavourable terms, only to accept a competing offer from the small firm afterwards. This, however, cannot be part of a sequentially rational equilibrium. The beliefs and strategies supporting this equilibrium are specified in the proofs in the appendix.

Furthermore, note that in this equilibrium, firm 1 is certain to hire the worker of type ϑ_1. She would then have an incentive ex post to change the contract to one that is more profitable. This is explicitly not allowed in this model. Section 3.2 provides an endogenous reason for this: if the firm has a reputation to maintain, she will keep the contract so that only a ϑ_1 worker would accept, lest workers of lower skill are to apply in subsequent periods.

Pooled equilibrium positive assortative matching becomes more expensive because of the potential for free-riding and poaching. If it is too expensive, the larger firm might prefer not to hire the best worker. In this case, she can offer a contract that both workers ϑ_1 and ϑ_2 would accept at a utility of at least $\underline{u}$. Firm 2's best response in this case would be to also do so. Thus, both firms hire worker 1 or 2 with equal probability. As in Milgrom and Oster (1987) and Ricart i Costa (1988), the larger firm chooses not to reveal the worker's type, in order to preclude poaching. The difference is that in this pooled equilibrium, the largest firm chooses not to learn the worker's type in the first place, and the worker's type remains unknown to anyone except for the worker throughout the game.

For a contract in this equilibrium, say (w^P, e^P), both workers will have to prefer it to staying self-employed, which is equivalent to

$$w^P - \frac{e^P}{\vartheta_2} \geq \underline{u}. \tag{23}$$

At such a contract, firm $i \in \{1, 2\}$ has an expected profit of

$$\frac{\pi\left(e^P, \vartheta_1\right) + \pi\left(e^P, \vartheta_2\right)}{2} I_i - w^P.$$

Optimizing this subject to (23) might yield different solutions for $i = 1$ and $i = 2$. Call the respective solutions $\left(w_i^P, e_i^P\right)$ for $i = 1, 2$. If $\frac{1}{2}\vartheta_2\left(\frac{\partial}{\partial e}\pi(0, \vartheta_1) + \frac{\partial}{\partial e}\pi(0, \vartheta_2)\right) I_1 < 1$, the solution is the $(w_i^P, e_i^P) = (\underline{u}, 0)$ for both i. Defining the corresponding maximum profit $\overline{\Pi}_i^P := \left(\pi(e_i^P, \vartheta_1) + \pi(e_i^P, \vartheta_2)\right) - w_i^P$, this allows to write down the following proposition

Proposition 6. *If $\overline{\Pi}_1^P > \Pi_1^M$, then there exists an equilibrium such that the two firms $i = 1, 2$ offer the contracts (w_i^P, e_i^P). Workers 1 and 2 apply to both firms and are employed at either with equal probability.*

The following proposition establishes when the matched equilibrium exists

Proposition 7. *If*

$$\Pi_1^M \geq \overline{\Pi}_1^P, \tag{24}$$

then the matched equilibrium is the unique weak perfect Bayesian equilibrium of the high-mobility labour market.

In the matched equilibrium, firm 2 offers the contract (w_2^*, e_2^*). The right hand side of condition (24) represents the profits firm 1 would make by also offering this contract, inducing a pooled equilibrium.

For the purpose of this paper, it is interesting to see when this equilibrium leads to higher inequality. In line with the analysis above, we have the following result

Corollary 8. *If condition* (24) *is satisfied, and*

$$u\left(w_1^*, e_1^* | \vartheta_2\right) > \underline{u}, \tag{25}$$

the weak perfect Bayesian equilibrium of the mobile labour market exhibits a strictly higher wage and task level for worker 1 *than the low-mobility benchmark.*

3.2 Repeated Game Model

In section 1 the larger firm was at a disadvantage: she was certain to employ more highly skilled workers, but still had to employ them at a screening contract. In a sense, the larger firm has to commit to following through on this contract, even though it would be better for both her and the worker to change the contract ex post. In this section, the model is embedded in a repeated game setting in order to model a more open bidding game between the firms. In the high mobility market, the large firm endogenously has a reason to follow through on the screening contract, namely a reputation concern. She has to develop and maintain a reputation for only offering this type of contract, in order to make sure that in subsequent periods only good workers seek employment with her.

For simplicity it is assumed that there are two firms, $i = 1, 2$, of sizes $I_1 > I_2$ Both firms are infinitely lived and maximize the sum of discounted future dividends, with the same discount factor δ. At the beginning of each period t, a large number N of workers is born, of types $\vartheta_1 > \vartheta_2 > \ldots > \vartheta_N$. At the end of each period, workers who are not employed enjoy a reservation utility $\overline{u}$ and workers employed at a contract (w, e) enjoy a utility of

$$u(w, e|\vartheta) = w - \frac{e}{\vartheta},$$

after which they leave the game. A firm of size I employing a worker of type ϑ at a contract (w, e) generates an end-of-period dividend of

$$\Delta(w, e, \vartheta) = \vartheta I - w.$$

For simplicity, the profit is assumed not to depend on the task difficulty.

Crucially, at the beginning of each period, workers can observe the entire history of traded contracts of each firm. At the end of each period firms consume their dividend.

There are multiple rounds of contracting at the beginning of each period. Workers are born and observe the contracting history of both firms. In a first contracting round, firms post contract offers to all workers, upon which workers decide to accept one of the contracts or reject all of them. If multiple workers accept a given contract, one of them gets chosen at random.

In a next round of contracting, firms observe the contracts traded. Then they can choose to fire the worker currently employed with them (if any) and offer a contract to any specific subset of the workers. If an employed worker receives an offer, he can choose to renege on his old contract and work for a different firm. Again, workers choose to accept or reject and if multiple workers accept, a random one is chosen. This continues until neither firm wants to make any other offer. After the bidding stage is over, workers start working for their employer, after which dividends and utilities are realized.[20]

The aim is to prove the existence of the type of assortatively matched equilibrium found above. Before, the large firm was exogenously committed to providing the contract she offered at first. Note that in such a matched equilibrium, firm 1 needs to offer worker 1 a contract giving at least the poach-proof utility

$$\overline{u}^{HM} := \underline{u} + (\vartheta_1 - \vartheta_2) I_2,$$

In order to screen out lower type workers, she would then offer the contract

$$\begin{aligned} \overline{w}^{HM} &= \underline{u} + \vartheta_1 I_2 \\ \overline{e}^{HM} &= \vartheta_1 \vartheta_2 I_2. \end{aligned}$$

Once the large firm knows that the worker who works for her is of high ability, she can change the contract ex-post. Instead of trading the inefficient contract $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$, she can change the contract to, for example $\left(\overline{u}^{HM}, 0\right)$, giving the same utility, but with a lower wage and a task level of zero. Such a contract change could, however, not be part of an equilibrium, as this would attract lower-ability workers ex-ante.

Because the game repeats itself, there is a reputation concern for the large firm: if she ever changes a contract offer to one that workers of ability lower than ϑ_1 would accept, all workers in the future might believe that at the end of the bidding game, she will offer a contract which they would prefer to either working for firm 2 or being self-employed. All workers then accept any offer from the large firm in the future.

In order for this set of strategies to be part of a stationary equilibrium, it must be that at any time the large firm prefers hiring a good worker at the contract $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$, in

[20]The bidding war precedes the working stage. This can be justified by the stylized fact that most labour market mobility happens in the early stages of careers (Topel and Ward, 1992).

the current and each subsequent period to hiring the best worker at a contract $\left(\overline{u}^{HM},0\right)$ in the current period and hiring less skilled workers at each subsequent period. From following the former of these strategies, the firm will obtain a dividend in the current and each subsequent period of

$$\Delta_{t+\tau} = \vartheta_1 I_1 - \overline{w}^{HM} \tag{26}$$

for $\tau = 0, 1, 2, \ldots$, giving a period-t discounted sum of dividends of

$$\sum_{\tau=0}^{\infty} \delta^{\tau} \Delta_{t+\tau} = \frac{1}{1-\delta}\left(\vartheta_1 I_1 - \overline{w}^{HM}\right). \tag{27}$$

Filling in the expression for $\overline{w}^{HM}$ gives a discounted sum of dividends

$$\frac{1}{1-\delta}\left(\vartheta_1\left(I_1 - I_2\right) - \underline{u}\right) \tag{28}$$

By following the latter strategy, the large firm can offer the ϑ_1 worker, once she is certain she hired him, a contract with $e = 0$ and $w = \overline{u}^{HM}$, giving a period-t dividend of $\vartheta_1 I_1 - \overline{u}^{HM}$. In all of the subsequent periods, however, she will hire lower ability workers and can thus expect a dividend of $\mathbf{E}\vartheta I_1 - \underline{u}$. This means that with this strategy, the firm can get a discounted sum of dividends

$$\vartheta_1 I_1 - \overline{u}^{HM} + \sum_{\tau=1}^{\infty} \delta^{\tau}\left(\mathbf{E}\vartheta I_1 - \underline{u}\right), \tag{29}$$

which equals

$$\vartheta_1 I_1 - \overline{u}^{HM} + \frac{\delta}{1-\delta}\left(\mathbf{E}\vartheta I_1 - \underline{u}\right), \tag{30}$$

which can be rewritten by filling in $\overline{u}^{HM} = \underline{u} + \left(\vartheta_1 - \vartheta_2\right) I_2$

$$\vartheta_1 I_1 - \left(\vartheta_1 - \vartheta_2\right) I_2 + \frac{\delta}{1-\delta}\mathbf{E}\vartheta I_1 - \frac{1}{1-\delta}\underline{u} \tag{31}$$

This means that in the presumptive stationary equilibrium, the large firm would not have an incentive to deviate from her strategy of keeping the offer $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$ whenever

$$\frac{1}{1-\delta}\vartheta_1\left(I_1 - I_2\right) \geq \vartheta_1 I_1 - \left(\vartheta_1 - \vartheta_2\right) I_2 + \frac{\delta}{1-\delta}\mathbf{E}\vartheta I_1, \tag{32}$$

which can be rewritten as

$$\frac{\vartheta_1}{\vartheta_2} \geq \frac{\mathbf{E}\vartheta}{\vartheta_2}\frac{I_1}{I_1 - I_2} + \frac{1-\delta}{\delta}\frac{I_2}{I_1 - I_2}, \tag{33}$$

or, alternatively, as

$$\frac{I_1}{I_2} \geq \frac{\vartheta_1 + \frac{1-\delta}{\delta}\vartheta_2}{\vartheta_1 - \mathbf{E}\vartheta} \tag{34}$$

Condition (33) thus guarantees both that the large firm would prefer to keep up its reputation for offering the $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$-contract, and to outbid the smaller firm. This means that (33) is a sufficient condition for the existence of a stationary equilibrium in which the large firm offers $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$, as is stated in the following proposition.

Proposition 9. *If condition* (33) *is satisfied, there exists a stationary equilibrium in which the large firm always employs the good worker at a contract* $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$, *with*

$$\overline{w}^{HM} = \underline{u} + \vartheta_1 I_2, \quad \textit{and}$$
$$\overline{e}^{HM} = \vartheta_2 \vartheta_1 I_2.$$

In every period, the equilibrium exhibits positive assortative matching, as before. Before, however, the large firm's commitment to follow through on the inefficient screening contract $\left(\overline{w}^{HM}, \overline{e}^{HM}\right)$ was an exogenous feature of the game. In the stationary equilibrium this commitment is endogenously enforced by the behaviour of future prospective employees.

3.3 Switching Costs

In this version of the model, ex-post poaching comes at a cost. This can be thought of as a relocation cost, a cost of a notice period, or the costs of fighting a non-compete clause in court. This cost is modelled to be a utility cost to the switching worker, but would have to be borne by the poaching firm in case of poaching. If the cost is high enough, the model corresponds to the low-mobility benchmark. If the cost goes to zero, the outcome converges to the high mobility labour market.

For simplicity, it is again assumed that there are only two firms, of sizes $I_1 > I_2$ and $N > 2$ workers of abilities $\vartheta_1 > \vartheta_2 > \ldots > \vartheta_N$. There is one period, which starts with two rounds of contracting. In the first round of contracting, both firms post contract offers. Workers then accept these contracts or decide to remain self-employed. In the second round, firms can again fire their workers, and offer a contract to a worker employed at the other firm. This time around, workers who stay with their first employer at a contract (w, e) obtain a utility of $w - \frac{e}{\vartheta}$. Workers who leave the first employer to work for a poaching firm at a contract $(\tilde{w}, \tilde{e})$ enjoy a utility of $\tilde{w} - \frac{\tilde{e}}{\vartheta} - c$, where $c \geq 0$ is called the *switching cost.* Firms again have the simplified profit function of

$$\Pi_i(w, e|\vartheta) = \vartheta I_i - w$$

Note that the model with $c = 0$ corresponds to the high mobility labour market from the main model. As will be shown, if c is large enough, this corresponds to the low mobility labour market from before. In order to solve this model, we again start reasoning from an equilibrium with positive assortative matching and reason from the point of view of firm 2 considering poaching the worker from firm 1, whom she now knows to be of type ϑ_1. If this worker enjoys a utility of $\tilde{u}$, the poaching contract $(\tilde{w}, \tilde{e})$ would have to such that

$$\tilde{w} - \frac{\tilde{e}}{\vartheta_1} - c \geq \tilde{u}.$$

The optimal poaching contract would thus be $(\tilde{u}+c,0)$. As the following lemma states, if the cost is too high, poaching becomes irrelevant and the market is equivalent to the low mobility benchmark from before.

Lemma 10. *If*

$$c \geq \vartheta_2 \frac{\vartheta_1-\vartheta_2}{\vartheta_1} I_2, \tag{35}$$

Firm 1 *employs the worker of type* ϑ_1 *and firm* 2 *employs the worker of type* ϑ_2, *at respective contracts*

$$(w_1,e_1) = (\underline{u}+(\vartheta_1-\vartheta_2)\,I_2, \vartheta_2\,(\vartheta_1-\vartheta_2)\,I_2) \tag{36}$$

$$(w_2,e_2) = (\underline{u},0) \tag{37}$$

If condition 35 is not satisfied and there is positive assortative matching, the utility $\overline{u}(c)$ offered by firm 1 to the worker of type ϑ_1 should just be large enough so that firm 2 would rather keep the worker of type ϑ_2 than poach, i.e.

$$\vartheta_1 I_2 - (\overline{u}(c)+c) = \vartheta_2 I_2 - \underline{u}, \tag{38}$$

i.e.

$$\overline{u}(c) = \underline{u} + (\vartheta_1-\vartheta_2)\,I_2 - c. \tag{39}$$

In order to find the equilibrium as a function of c, there are two cases to distinguish. It could either be that in the high mobility labour market from before, firm 1 prefers an equilibrium with matching, or that firm 1 prefers to pool for the top two workers. In the former case, the equilibrium features positive assortative matching for all c.

Proposition 11. *If*

$$\frac{\vartheta_1-\vartheta_2}{2} I_1 \geq \vartheta_1 I_2, \tag{40}$$

the equilibrium features positive assortative matching for all c. *Firm* 2 *employs the worker of type* ϑ_1 *at a contract* $(\underline{u},0)$. *Firm* 1 *employs the worker of type* ϑ_1 *at a contract* $(w_1(c),e_1(c))$, *with*

$$w_1(c) = \underline{u} + \vartheta_1\left(I_2 - \frac{c}{\vartheta_1-\vartheta_2}\right) \tag{41}$$

$$e_1(c) = \vartheta_1\vartheta_2\left(I_2 - \frac{c}{\vartheta_1-\vartheta_2}\right) \tag{42}$$

whenever $c \leq \vartheta_2 \frac{\vartheta_1-\vartheta_2}{\vartheta_1} I_2$, *and*

$$(w_1(c),e_1(c)) = (\underline{u}+(\vartheta_1-\vartheta_2)\,I_2, \vartheta_2\,(\vartheta_1-\vartheta_2)\,I_2) \tag{43}$$

whenever $c > \vartheta_2 \frac{\vartheta_1-\vartheta_2}{\vartheta_1} I_2$.

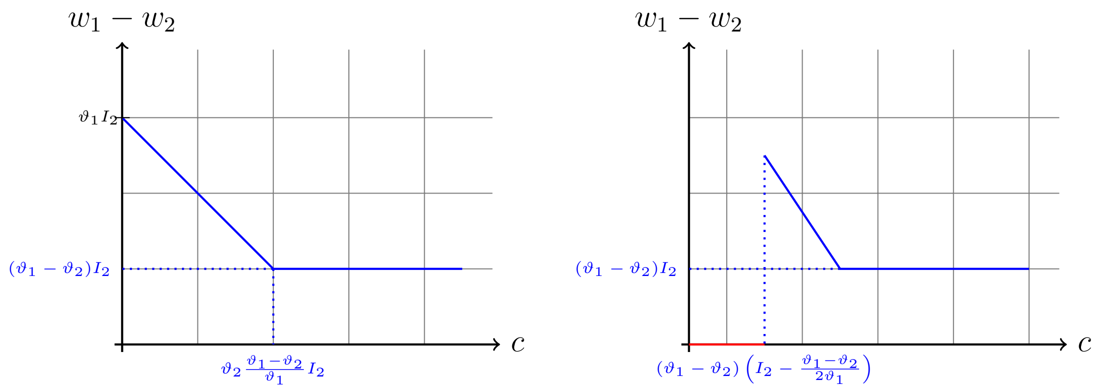

Figure 4: Wage differential as a function of c when the equilibrium with $c = 0$ is matched (left) or pooled (right).

In this case, wage inequality and the task level are at first strictly decreasing in c, up to the point where both become constant.

If the high mobility equilibrium features pooling, the equilibrium will be pooled for c up to a certain value. If c is high enough, the competitive pressure from the smaller firm will be less intense, making it preferable to propose a matched contract again.

Proposition 12. *If*

$$\frac{\vartheta_1 - \vartheta_2}{2} I_1 \leq \vartheta_1 I_2, \tag{44}$$

then for

$$c < (\vartheta_1 - \vartheta_2)\left(I_2 - \frac{\vartheta_1 - \vartheta_2}{2\vartheta_1}\right), \tag{45}$$

both firms offer a contract $(\underline{u}, 0)$ and employ the workers with ϑ_1 and ϑ_2 with equal probability. For larger c, the contracts and labour assignment are as in the previous proposition.

In this case, for low values of c, there would be no wage inequality and no rat race, after which there is a jump once the matched equilibrium becomes more attractive. After this, inequality decreases in c, until it flattens out at the low mobility level.

4 Conclusion

This paper finds that firms can free-ride on the screening effort of others. In a simple labour market model, this exacerbates both the inefficiency generated by screening and the wage inequality between workers of different talents. This seems to be a general feature of an economy with complementarity between firm and worker quality, with positive assortative one-to-one matching between firms and workers.

This paper predicts that fewer barriers to mobility mean more wage inequality and more inefficient effort provision. This inequality will be between workers at different firms. Contrary to the predictions of most dynamic models (Harris and Holmstrom, 1982; Postel-Vinay and Robin, 2002; Cabrales, Calvó-Armengol, and Pavoni, 2008), workers do not need to be mobile in order for inequality to increase. A mere decrease in mobility costs, without any increase in actual mobility, suffices.

This paper models a partial equilibrium, which seems appropriate when modelling a specific sector, which attracts only a small part of the population. When one is more concerned with overall inequality, it would be interesting to extend this model to one where workers also consume the good they produce, to assess the welfare effects of the increased inequality.

It should be noted that the high ability workers prefer the high inequality equilibria. Despite the inefficiency, there is no Pareto ranking between the high and the low mobility markets: the high mobility market partially features a transfer of wealth from the large firm to its workers. This means that we can expect the top employees to prefer the high mobility labour market and to resist any political or institutional barriers to mobility, whereas shareholders might be inclined to support these. These incentives were on clear display when several Silicon Valley firms turned out to have secret no-poaching agreements and were subsequently sued by tech employees. It is interesting to see that this happened in the state with the lowest enforceability of non-compete clauses in the entire United States.

The increased inequality can be seen as a negative effect of increased mobility. Other papers note how the potential for mobility has a bright side as well. For example, the possibility to easily switch jobs might create career motivations, providing incentives for workers, as has been argued theoretically (Bar-Isaac and Lévy, 2019), and empirically (Kempf, 2020). Mobility can also have effects on both investment, innovation, and firm entry, as is shown among others by Jeffers (2019). Rather than arguing against employee mobility altogether, I hope to add to the discussion by highlighting a previously unstudied effect of labour mobility.

The screening feature of the contract can take the form of hours worked, but it can also take on other shapes in different contexts. As several papers (e.g. Bijlsma, Boone, and Zwart, 2018) argue, banks can screen their workers through inefficiently convex bonus contracts, in which case higher mobility might lead to more risk-taking.

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A Proofs

A.1 Derivation of the Benchmarks for Multiple Firms

This part derives the benchmark models mentioned at the beginning of section 1

A.1.1 First Best

This benchmark consists of a labour market in which all firms can observe the workers' types. As can be expected, this market features an efficient allocation of workers: as firm size and worker talent are complements, the largest firm employs the most skilled worker, the second-largest firm employs the second-most skilled worker, etc. All workers n with $n > M$ are self-employed. Furthermore, as there is no need to screen, all contracts are efficient and feature a task level equal to zero. For each worker-firm pairing, the wage will be endogenously determined by the competitive pressure from smaller firms. Now I can prove proposition 1.

Proof of Proposition 1. Since each firm can observe the worker's types, the best contract a firm of size I can give a worker of type ϑ, while guaranteeing a utility of $\tilde{u}$ is given by the problem

$$\max_{w,e>0} \vartheta I - w$$

subject to

$$w - \frac{e}{\vartheta} \geq \tilde{u}.$$

This gives the optimal contract $(\tilde{u}, 0)$, with corresponding profit $\vartheta I - \tilde{u}$

Now define, for $m = 1, 2, \ldots, M-1$, $w_m := \underline{u} + \sum_{i=1}^{M-m} I_{m+1} \left(\vartheta_{m+i-1} - \vartheta_{m+i}\right)$ and $w_M := \underline{u}$. Note that w_m can also be rewritten as

$$w_m = \underline{u} + \sum_{i=0}^{M-m-1} \left(\vartheta_{M-i-1} - \vartheta_{M-i}\right) I_{M-i}$$

Consider the following set of strategies: firm 1 offers the contract $(w_1, 0)$ to worker 1 and for $m = 2, 3, \ldots, M$ firm m offers $(w_{m-1}, 0)$ to worker $m-1$ and $(w_m, 0)$ to worker m. Then each worker n accepts the contract offered by firm n. It is immediate that none of the workers has an incentive to deviate.

Firms cannot offer their matched workers lower utilities, as this would lead them to choose the contract from the competitor, and given these utilities offer the optimal contract. They also do not want to offer higher utilities to the same worker.

Assume a firm m wants to outbid a larger firm $m' < m$ for a more talented worker. In that case, she would have to offer a utility of $w_{m'}$. Rather than her current profits of $\vartheta_m I_m - \underline{u} + \sum_{i=1}^{M-m} I_{m+1} (\vartheta_{m+i-1} - \vartheta_{m+i})$, she could then obtain at most

$$\begin{aligned}
\vartheta_{m'} I_m - w_{m'} &= \vartheta_{m'} I_m - \underline{u} - \sum_{i=0}^{M-m'-1} (\vartheta_{M-i-1} - \vartheta_{M-i}) I_{M-i} \\
&= \left(\vartheta_m + \sum_{\mu=m'}^{m-1} (\vartheta_\mu - \vartheta_{\mu+1}) \right) I_m - \underline{u} - \sum_{i=0}^{M-m'-1} (\vartheta_{M-i-1} - \vartheta_{M-i}) I_{M-i} \\
&= \vartheta_m I_m - w_m - \sum_{\mu=m'}^{m-1} (\vartheta_\mu - \vartheta_{\mu+1}) (I_{\mu+1} - I_m) \\
&\leq \vartheta_m I_m - w_m,
\end{aligned}$$

meaning she does not have an incentive to get a more talented worker at a competitive wage. Similarly, it can be found that she has no incentive to get a less talented worker. □

The intuition behind the proof is that since there is no need for screening, both firms can set the efficient level of effort, $e = 0$. Given this, the result is akin to Bertrand (1883) competition with heterogeneous marginal costs: since the largest firm profits the most from hiring the ϑ_1-worker, she is willing to outbid firm 2, and hires the ϑ_1-worker exactly at the wage that would make firm 2 prefer hiring the ϑ_2-worker. Firm 2 outbids firm 3 in a similar manner for the worker of type ϑ_2, and so on. Ultimately firm M can hire worker $n = M$ at his reservation utility.[21]

[21] Technically speaking, as in Bertrand (1883)-type competition with asymmetric costs, there are multiple equilibria in which, for example, firm $m+1$ offers a higher wage then she would be willing to pay for worker m, driving up the price that firm m has to pay. This means, however, that the worker m is indifferent between the contracts offered by firm m and firm $m+1$, yet if worker m were to choose the contract offered by firm $m+1$, firm $m+1$ would be strictly worse off. A large literature has been devoted to excluding this pathological type of equilibrium. Henceforth, when such multiplicity is possible, the least-wage equilibrium will be considered *the* equilibrium of the labour market.

A.1.2 The Low Mobility Labour Market

We define the low mobility labour market to be one in which each firm offers contracts precisely once, and upon accepting, workers will work for their chosen firm, according to the chosen contract terms, for the rest of the game.

The equilibrium studied is one in which the allocation of workers is the same as in the first best[22]: the largest firm wants to offer the most highly skilled worker a high enough wage so that the second-largest firm would prefer to hire a lower-skilled worker. In this way, each firm m will be able to hire worker m, but has to offer a high enough wage in order to make sure that the smaller firms $m+1, m+2, \dots$ do not want to outbid him. Since of all these firms, $m+1$ is the most willing to outbid, the focus can be on outcompeting firm $m+1$. This will then determine the worker's "market price", which is the endogenously determined utility the market would be willing and able to give him. This allows to set up the equilibrium contracts through a set of recursive equations.[23] In order to think about the competition between firm m and firm $m+1$, I start from the point of view of the smaller firm. In equilibrium, firm $m+1$ employs the worker of type ϑ_{m+1} at a contract (w_{m+1}, e_{m+1}) to get a profit of

$$\Pi_{m+1} = \vartheta_{m+1} I_{m+1} - w_{m+1}$$

This worker will get a utility of

$$u_{m+1} = w_{m+1} - \frac{e_{m+1}}{\vartheta_{m+1}}$$

which should correspond to the utility he would be able to get on the market. Now assume firm $m+1$ contemplates hiring a worker of type ϑ_m. If this worker can obtain a utility of $\tilde{u}$ on the market, the small firm, as seen in the analysis of the monopsonist firm, would need to pay a wage of

$$\tilde{w} = u_{m+1} + \vartheta_m \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}},$$

to be sure to hire the worker of type ϑ_m and not the one of type ϑ_{m+1}. She would do so if this is better than hiring a less talented worker, i.e. as long as

$$\vartheta_m I_- u_{m+1} - \vartheta_m \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}} > \vartheta_{m+1} I_{m+1} - u_{m+1} - \frac{e_{m+1}}{\vartheta_{m+1}},$$

which simplifies to

$$\vartheta_m \frac{\tilde{u} - u_{m+1}}{\vartheta_m - \vartheta_{m+1}} < (\vartheta_m - \vartheta_{m+1})\, I_{m+1} + \frac{e_{m+1}}{\vartheta_{m+1}}. \tag{46}$$

[22]it will be shown later that this is a feature of any equilibrium

[23]Note that the recursion goes backwards, starting at firm M and then backing out the contracts $M-1, M-2, \dots, 2, 1$.

Firm m will want to make sure to hire the ϑ_m-worker without the smaller firm outbidding her. This means that she has to offer a utility u_m to the good worker, which is the lowest level of $\tilde{u}$ for which condition (46) is not satisfied, i.e. she offers a utility such that

$$u_m - u_{m+1} = \frac{(\vartheta_m - \vartheta_{m+1})^2}{\vartheta_m} I_{m+1} + e_{m+1} \frac{\vartheta_m - \vartheta_{m+1}}{\vartheta_m \vartheta_{m+1}}. \tag{47}$$

If firm m wants to be sure to hire the good worker, she must give a screening contract that gives the ϑ_m-worker at least u_m, and that screens out the ϑ_{m+1}-worker, giving him at most u_{m+1}. Using the analysis from the monopsonist example, this means she must set an effort level of

$$e_m = \vartheta_{m+1} (\vartheta_m - \vartheta_{m+1}) I_{m+1} + e_{m+1}. \tag{48}$$

This defines a recursive equation for the task levels set in the contracts. As effort levels correspond to the utility increments between types by the identity (71) and wages can be derived from utilities and efforts by (72), this gives the recursive definition of all the contract variables.

In order to find the contracts, the only step still needed is to find the initial term of the recursion. In the presumptive equilibrium, firm M hires the ϑ_M-worker without any competition from other firms. This means that she can offer him his reservation utility $\underline{u}$ and without stipulating any positive task level, giving the contract $(w_M, e_M) = (\underline{u}, 0)$. This gives Proposition 2.

Proof of Proposition 2. Recall that in this equilibrium, each firm m employs a worker of type ϑ_m, with workers of type ϑ_n for $n > M$ remaining self-employed. For $m = 1, 2, \ldots, M$, the contracts are (w_m, e_m) with $w_M = \underline{u}$ and for $m < M$

$$w_m = \underline{u} + \sum_{i=1}^{M-m} (\vartheta_{M-i} - \vartheta_{M-i+1}) I_{M-i+1}, \tag{49}$$

and with $e_M = 0$ and for $m < M$

$$e_m = \sum_{i=1}^{M-m} \vartheta_{M-i} (\vartheta_{M-i} - \vartheta_{M-i+1}) I_{M-i+1}, \tag{50}$$

Call the profits each firm makes in this equilibrium Π_m

Note that the contracts are such that for each firm m, the profits she obtains are equal to the profits she would obtain from hiring the more talented worker $m-1$ at the contract

(w_{m-1}, e_{m-1}). This also shows that

$$\begin{aligned}
\Pi_m &= \vartheta_m I_m - w_m \\
&= \vartheta_m (I_m - I_{m+1}) + \vartheta_m I_{m+1} - w_m \\
&= \vartheta_m (I_m - I_{m+1}) + \vartheta_{m+1} I_{m+1} - w_{m+1} \\
&= (\vartheta_m - \vartheta_{m+1}) (I_m - I_{m+1}) + \vartheta_{m+1} I_m - w_{m+1} \\
&> \vartheta_{m+1} I_m - w_{m+1}.
\end{aligned}$$

So that in this equilibrium, she is better off then she would be by deviating and offering a competitive screening contract to worker $m + 1$. Note that the second term in the third line is Π_{m+1}, so that by induction, $\Pi_m > \vartheta_k I_m - w_k$ for all $m < k \leq M$. Similarly, it can be shown that $\Pi_m > \vartheta_k I_m - w_k$ for all $k < m - 1$. Firm m does not have an incentive to deviate by targeting any worker $k \neq m$. By definition of the contracts, every worker is best off picking his own contract. □

A.2 Proofs for Section 1.2

This part addresses the proofs of the general properties of equilibrium. First of all, there needs to be a formal description of how the labour market is cleared in case multiple workers apply to multiple firms:

Definition 2. Denote by ν_m the pool of workers applying to firm m. First, for $m = 1$, one of the workers in ν_m is chosen uniformly. This worker is then removed from the pools ν_m for all $m > 1$. Then the same procedure is repeated for $m = 2$, $m = 3$, and so on, until $m = M$.

This mechanism guarantees that if two workers both apply at the same two firms, both firms get either worker with probability one half, and both workers will have a probability one half of working at either of the two firms, and that there is no probability of one of these two firms ending up without a worker, or of one of these two workers being unemployed.

In order to make sure the subgame equilibria after the contract offers exhibit some degree of regularity, I assume the following tie-breaker:

Definition 3. All else equal, workers 1 through M prefer being employed and workers $M + 1$ through N prefer being self-employed.

It will be useful to start with the following lemmas.

Lemma 13. *There is no poaching in equilibrium.*

Proof. If firms poach, there has to be at least one firm left without a worker, making a profit of zero. As there are always unemployed workers in equilibrium, this firm would be better off employing this unemployed worker in the first round at $e = 0$. This would not induce poaching. □

This lemma entails that the assignment of workers to firms and the choice of contracts is given in the first round, making sure the model can be fully described in a static manner, with just the need that the contracts be *poach-proof*: no firm should have the incentive to ex-post to hire away a worker from a competitor.

Lemma 14. *If two firms, say m_1 and m_2, offer the same contract (w, e), it must be that the group of workers choosing firm m_1 and the group of workers choosing m_2 have the same conditional expected productivity.*

Proof. If not, the firm employing the lower expected productivity group of workers has an incentive to poach from the other firm at a contract $(w + \varepsilon, e)$, for some small enough ε. □

Functionally, the above lemma allows us to interpret the groups of workers at both firms to be the same. If the types are drawn according to a continuous distribution prior to the game, the probability of two different subsets of the workers having the same expected productivity is zero. From now on, it will be assumed that the group of workers applying to any number of firms offering the same contract is the same.

Lemma 15. *The posterior expected productivity of the workers working for a firm is non-decreasing in the size of the firm.*

Proof. If not, the larger firm would always be willing to poach a smaller firm's workers. □

Lemma 16. *The sets of workers applying to any firm ν_m for all m are* contiguous. *I.e. if $n_1, n_2 \in \nu_m$, with $n_1 \leq n_2$ then $n \in \nu_m$ for all n with $n_1 \leq n \leq n_2$*

Proof. This follows from the single-crossing property of worker's preferences. □

The lemmas above entail that we can partition the firms into subsets that offer the same contract. Note that since the workers' preferences satisfy a single crossing constraint, there will be a contiguous group of workers applying to each contract.

A.2.1 Proofs on Three Firm Equilibria

Now we can narrow down the equilibria to the three stipulated in the main text. We start with the following lemma:

Lemma 17. *Worker* 1 *always applies to firm* 1*. There is no equilibrium in which other workers also apply to firm* 1*, but worker* 1 *does not apply to any other firm.*

Proof. Every firm employs a worker and firm 1 always employs the pool of workers with the highest posterior expected productivity. Because of single crossing, any contract preferred by workers of type lower than ϑ_1, must also be preferred by worker 1.

Now assume there is an equilibrium with workers 1 and 2 applying to firm 1. In this case, worker 1's probability of being employed at firm 1 is at most one half. Firms smaller than firm 1 employ workers of type lower than ϑ_1, at contracts that they prefer to being self-employed, meaning (because of single crossing) that worker 1 also prefers these to being self employed. If worker 1 also applies to these contracts, she still has the same probability of being employed at firm 1, but the worker then adds the probability of being employed at another firm in case he is not employed at firm 1. □

In a similar vein, if there is an equilibrium in which only worker 1 applies to firm 1, then either worker 2 is the only one to apply to firm 2, or she also applies to firm 3.

Also note that in equilibrium, only the top three workers are employed, as otherwise firm 3 would always have an incentive to offer a contract that is acceptable only to the top three workers. This means the analysis can be restricted to the four equilibria enumerated in the main text. As argued there, the pooled at the top equilibrium is not an equilibrium, so all the analysis can be restricted to the fully pooled, the fully matched, and the pooled at the bottom equilibria.

For the proofs regarding which equilibrium prevails under which criteria, it will be useful to enumerate the profits of each of the firms in each of the respective equilibria. For $i = 1, 2, 3$, I denote by Π_i^S the profits in the separating equilibrium, by Π_i^B those in the pooling at the bottom equilibrium, and by Π_i^F those for the fully pooling one. Without loss of generality, it is assumed that $\underline{u} = 0$.

For the separating equilibrium the profits are

$$\Pi_1^M = \vartheta_1 I_1 - \vartheta_1 I_2 - \vartheta_2 \left(1 + \frac{\vartheta_3}{\vartheta_1 - \vartheta_2}\right) I_3 \tag{51a}$$

$$\Pi_2^M = \vartheta_2 I_2 - \vartheta_2 I_3 \tag{51b}$$

$$\Pi_3^M = \vartheta_3 I_3. \tag{51c}$$

For the pooling at the bottom equilibrium, the following profits are obtained

$$\Pi_1^B = \vartheta_1 I_1 - \vartheta_1 \left(1 + \frac{1}{2}\frac{\vartheta_2 - \vartheta_3}{\vartheta_1 - \vartheta_2}\right) I_2 \tag{52a}$$

$$\Pi_2^B = \frac{\vartheta_2 + \vartheta_3}{2} I_2 \tag{52b}$$

$$\Pi_3^B = \frac{\vartheta_2 + \vartheta_3}{2} I_3. \tag{52c}$$

Finally, for the fully pooling equilibrium, the profits for $i = 1, 2, 3$ equal

$$\Pi_i^F = \frac{\vartheta_1 + \vartheta_2 + \vartheta_3}{3} I_i. \tag{53}$$

The following series of lemmas provides comparisons between the various expressions above.

Lemma 18. *We have* $\Pi_1^B > \Pi_1^F$ *if and only if*

$$\frac{2}{3}\frac{I_1}{I_2} > \frac{\vartheta_1}{\vartheta_1 - \vartheta_2} \tag{54}$$

Lemma 19. *We have* $\Pi_1^B > \Pi_1^M$ *if and only if*

$$\frac{1}{2}\frac{I_2}{I_3} < \frac{\vartheta_2}{\vartheta_2 - \vartheta_3} - \frac{\vartheta_2}{\vartheta_1} \tag{55}$$

Lemma 20. *We have* $\Pi_2^B > \Pi_2^M$ *if and only if*

$$\frac{1}{2}\frac{I_2}{I_3} < \frac{\vartheta_2}{\vartheta_2 - \vartheta_3} \tag{56}$$

The proofs for the lemmas above follow directly from expressions (51) through (53).

The following corollary comes directly from the expressions above.

Corollary 21. *If* $\Pi_1^B > \Pi_1^M$, *then* $\Pi_2^B > \Pi_2^M$.

This allows us to prove Proposition 3

Proof of Proposition 3. If condition (6) holds, the fully pooled equilibrium cannot occur, as firm 1 would always have an incentive to deviate and offer the contract (w^M, e^M).

Now assume that condition (6) holds. If firm 2 prefers the fully matched equilibrium, then so does firm 1. All firms offer the contract $\left(w_i^M, e_i^M\right)$, and this constitutes an equilibrium.

If both firms 1 and 2 prefer the pooled at the bottom equilibrium, again, this one will be an equilibrium.

Now the only situation left to consider is when firm 2 prefers the pooled at the bottom equilibrium, and firm 1 prefers the fully matched one, i.e. when on top of condition (6), we have

$$\frac{\vartheta_2}{\vartheta_2-\vartheta_3}-\frac{\vartheta_2}{\vartheta_1}<\frac{1}{2}\frac{I_2}{I_3}\leq\frac{\vartheta_2}{\vartheta_2-\vartheta_3}$$

Now consider the pooled at the bottom equilibrium. Firm 1 offers the best poach-proof screening contract available, meaning she has no incentive to deviate. Consider in particular that if she deviates to offering the contract from the fully matched equilibrium, this would attract the ϑ_2-worker, since per definition

$$w_1^M-\frac{e_1^M}{\vartheta_2}=\underline{u}+(\vartheta_2-\vartheta_3)\,I_2,$$

which is larger than $\underline{u}$, meaning she would not get a profit of Π_1^M from doing so. □

Proof of Proposition 4. Note that condition (7) is equivalent to $\Pi_1^M>\Pi_1^F>\Pi_1^B$. This means that once in a fully pooled equilibrium, firm 1 has no incentive to unilaterally deviate to a poach-proof contract. Deviating to (w_1^M,e_1^M) would attract both of the top two workers in this case. Therefore the fully pooled equilibrium features no incentive to deviate.

Assume all firms offer the matched contracts (w_i^M,e_i^M). In this case, per definition, none of the workers has an incentive to deviate. Firm 1 has no incentive to deviate, as she is in her preferred equilibrium. By design, firm 2 has no incentive to poach ex post, and ex ante has no incentive to offer a contract that worker 1 prefers to (w_1^M,e_1^M). The remaining deviating strategy for firm 2 is to offer a contract such that worker 2 also applies to firm 1. This would always attract worker 3 or be open to poaching by firm 3, so out of these deviations, the best is to offer $(\underline{u},0)$. In this case, workers 1 and 2 would first apply to firm 1, with the remaining worker, as well as worker 3 applying to firm 2, giving firm 2 a profit of

$$\left(\frac{1}{4}\left(\vartheta_1+\vartheta_2\right)+\frac{1}{2}\vartheta_3\right)I_2-\underline{u}.$$

Condition (8) guarantees that this is not a profitable deviation. □

Proof of Proposition 5. Under condition (9), firm 1 prefers the fully pooled equilibrium to both other ones, so she would always have a profitable deviation offering the contract $(\underline{u},0)$ in any other equilibrium.

Alternatively, if both conditions (10) and (11) hold, the fully pooled equilibrium is an equilibrium by the reasoning from the previous proof. In the pooled at the bottom equilibrium, firm 1 would want to deviate to $(\underline{u},0)$, and in the matched equilibrium, firm 2 would want to deviate in the way described in the previous proof. □

A.3 Proofs from Section 3.1

I will use the following notation for the proofs in this subsection. Call the first best contract offered to the ϑ_2 worker by firm 2

$$(w_2^*, e_2^*) := \underset{w,e>0}{\operatorname{argmax}} \{\Pi_2(w, e, \vartheta_2) | u(w, e|\vartheta_2) \geq \underline{u}\} \tag{57}$$

and call Π_2^* the profit attained by firm 2 at this contract. This allows me to define the first best utility for worker 1, which is given by the dual problem of

$$\overline{u}_1^* := \max_{w,e>0} \{u(w, e|\vartheta_1)|\Pi_2(w, e, \vartheta_1) \geq \Pi_2^*\}. \tag{58}$$

Call the contract where this maximum is attained $(\tilde{w}_2, \tilde{e}_2)$.

Now recall the first best contract for ϑ_1 is given by

$$(w_1^*, e_1^*) := \underset{w,e>0}{\operatorname{argmax}} \{\Pi_1(w, e, \vartheta_1) | u(w, e|\vartheta_1) \geq \overline{u}_1^*\} \tag{59}$$

We have the following lemma

Lemma 22. *Necessarily,* $u\left(w_1^*, e_1^* \middle| \vartheta_2\right) \leq u\left(\tilde{w}_2, \tilde{e}_2 \middle| \vartheta_2\right)$

Proof. I start by showing that $e_1^* \geq \tilde{e}_2$. There are two cases to distinguish.

Case 1: if $e_1^* > 0$, then $\frac{\partial}{\partial e}\pi(e_1^*, \vartheta_1)I_1 = \frac{1}{\vartheta_1}$. In this case, either $\tilde{e}_2 = 0$, in which case the inequality is satisfied, or $\frac{\partial}{\partial e}\pi(\tilde{e}_2, \vartheta_1)I_2 = \frac{1}{\vartheta_1}$. As $\pi(\cdot, \cdot)$ is concave in its first argument, this implies that $\tilde{e}_2 < e_1^*$.

Case 2: if $e_1^* = 0$, then $\frac{\partial}{\partial e}\pi(0, \vartheta_1)I_1 < \frac{1}{\vartheta_1}$. In that case, also $\frac{\partial}{\partial e}\pi(0, \vartheta_1)I_2 < \frac{1}{\vartheta_1}$, giving $\tilde{e}_2 = 0$.

Knowing that $\tilde{e}_2 \leq e_1^*$ and that $w_1^* - \frac{e_1^*}{\vartheta_1} = \tilde{w} - \frac{\tilde{e}}{\vartheta_1} = \overline{u}_1^*$, we have that

$$\begin{aligned}
\tilde{w} - \frac{\tilde{e}}{\vartheta_2} &= \tilde{w} - \frac{\tilde{e}}{\vartheta_1} - \tilde{e}\left(\frac{1}{\vartheta_2} - \frac{1}{\vartheta_1}\right) \\
&= w_1^* - \frac{e_1^*}{\vartheta_1} - \tilde{e}\left(\frac{1}{\vartheta_2} - \frac{1}{\vartheta_1}\right) \\
&\geq w_1^* - \frac{e_1^*}{\vartheta_1} - e_1^*\left(\frac{1}{\vartheta_2} - \frac{1}{\vartheta_1}\right) \\
&= w_1^* - \frac{e_1^*}{\vartheta_2},
\end{aligned}$$

which concludes the proof. □

This has the following corollary

Corollary 23. *If* $u\left(w_1^*, e_1^* \middle| \vartheta_2\right) > \underline{u}$, *then also* $u\left(\tilde{w}_2, \tilde{e}_2 \middle| \vartheta_2\right) > \underline{u}$.

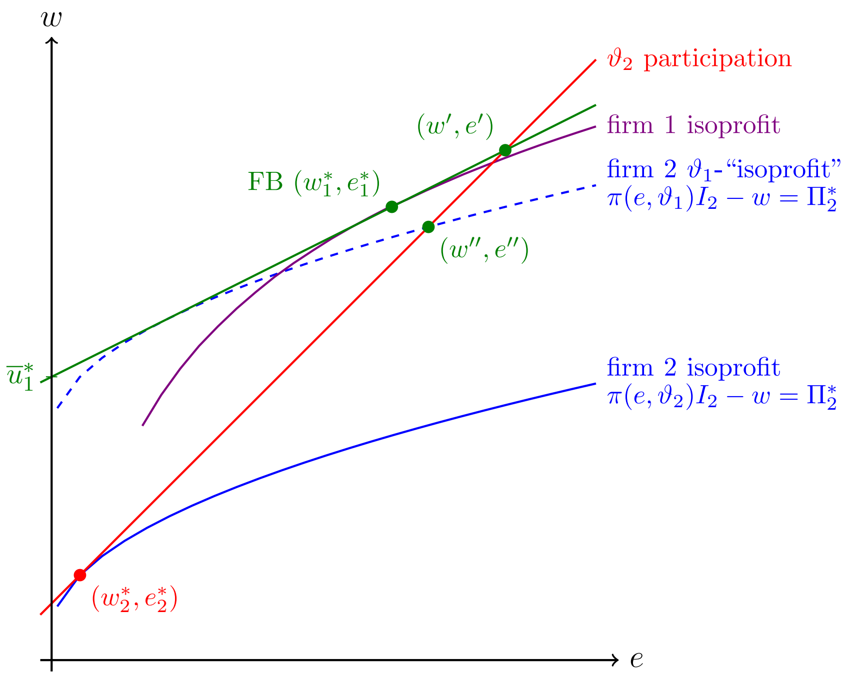

Figure 5: the points (w', e') and (w'', e'') illustrated for the case that $u\left(w_1^*, e_1^* \middle| \vartheta_2\right) > \underline{u}$.

Now define (w', e') as the crossing between the ϑ_1-indifference curve at $\overline{u}_1^*$ and the ϑ_2 participation constraint, i.e.

$$\begin{aligned} w' &:= \underline{u} + \vartheta_1 \frac{\overline{u}_1^{HM} - \underline{u}}{\vartheta_1 - \vartheta_2} \\ e' &:= \vartheta_1 \vartheta_2 \frac{\overline{u}_1^{HM} - \underline{u}}{\vartheta_1 - \vartheta_2}. \end{aligned} \tag{60}$$

Note that $u\left(w_1^*, e_1^* \middle| \vartheta_2\right) > \underline{u}$ is equivalent to $w_1^* < w'$ and $e_1^* < e'$.

Furthermore, we define w'' and e'' to be the point where the isoprofit curve of firm 2 obtaining Π_2^* from employing a ϑ_1 worker crosses the ϑ_2 indifference curve at a utility of $\underline{u}$, i.e. e'' is given by the equation

$$\pi(e'', \vartheta_1) I_2 - \frac{e''}{\vartheta_2} = \Pi_2^* + \underline{u} \tag{61}$$

and

$$w'' = \underline{u} + \frac{e''}{\vartheta_2}. \tag{62}$$

The utility of the ϑ_1 worker at this contract will be called u''.

We now have the following lemma giving the characteristics of the low mobility equilibrium.

Lemma 24. *Define the game in which both firms simultaneously offer contracts, only once, as the "low mobility labour market". Assume $u\left(\tilde{w}_2, \tilde{e}_2 \middle| \vartheta_2\right) > \underline{u}$. In this case, there* *is an equilibrium such that firm* 2 *employs the* ϑ_2 *worker at the contract* (w_2^*, e_2^*), *and firm* 1 *employs the* ϑ_1*-worker at a contract* $\left(w_1^{LM}, e_1^{LM}\right)$, *where*

$$u\left(w_1^{LM}, e_1^{LM}|\vartheta_1\right) = u'', \tag{63}$$

and $e_1^{LM} = \max\{e'', e_1^*\}$.

Proof. Take the following set of strategies: firm 2 offers a menu consisting of (w_2^*, e_2^*) and (w'', e''). Firm 1 offers only the contract $\left(w_1^{LM}, e_1^{LM}\right)$. The worker of type ϑ_1 chooses the contract from firm 1 and the worker of type ϑ_2 chooses the contract (w_2^*, e_2^*) from firm 2. All other workers remain unemployed.

First of all, by the definitions of the contracts, none of the workers has an incentive to deviate. Note that since $u\left(\tilde{w}_2, \tilde{e}_2|\,\vartheta_2\right) > \underline{u}$, the value u'' is the value for the solution of

$$\max_{w,e>0} u(w, e|\vartheta_1) \tag{64}$$

subject to

$$u(w, e|\vartheta_2) \leq \underline{u} \tag{65a}$$

$$\pi(e, \vartheta_1)I_2 - w \geq \Pi_2^* \tag{65b}$$

Also note that $\left(w_1^{LM}, e_1^{LM}|\vartheta_1\right)$ solves

$$\max_{w,e>0} \{\pi(e, \vartheta_1)I_1 - w\} \tag{66}$$

subject to

$$u(w, e|\vartheta_1) \geq u'' \tag{67a}$$

$$u(w, e|\vartheta_2) \leq \underline{u} \tag{67b}$$

Call the profit made at this contract Π_1^{LM}.

This entails that for firm 1, there is no way to hire the ϑ_1 worker at a higher profit, as this would necessarily entail a lower utility than u'' for this worker, who would then rather take the contract (w'', e'') from firm 2. Firm 1 also does not want to hire the ϑ_2 type worker. In order to see this, consider the largest profit firm 1 can make from hiring the ϑ_2 worker at a utility of $\underline{u}$:

$$\tilde{\Pi}_1 := \max\left\{\pi(e, \vartheta_2)I_1 - w|w - \frac{e}{\vartheta_2} = \underline{u}\right\}, \tag{68}$$

with the solution being attained at $(\tilde{w}_1, \tilde{e}_1)$. Note that in case $e_2^* < \tilde{e}_1 < e''$, we know that

$$\begin{aligned}\pi(\tilde{e}_1, \vartheta_2)I_2 - \left(\underline{u} + \frac{\tilde{e}_1}{\vartheta_2}\right) &\geq \Pi_2^* \\ &= \pi(e'', \vartheta_1)I_2 - \left(\underline{u} + \frac{e''}{\vartheta_2}\right).\end{aligned}$$

This gives that

$$\left(\pi(e'', \vartheta_1) - \pi(\tilde{e}_1, \vartheta_2)\right) I_1 > \left(\pi(e'', \vartheta_1) - \pi(\tilde{e}_1, \vartheta_2)\right) I_2$$
$$\geq \frac{1}{\vartheta_2}\left(e'' - \tilde{e}_1\right)$$

Meaning that $\pi(e'', \vartheta_1)I_1 - w'' > \pi(\tilde{e}_1, \vartheta_1)I_1 - \tilde{w}_1$. As the firm makes a profit of at least $\pi(e'', \vartheta_1)I_1 - w''$ in this equilibrium, this dominates hiring the ϑ_2 worker. If $\tilde{e}_1 > e''$, then also $e_1^{LM} > e''$, so $\left(w_1^{LM}, e_1^{LM}|\vartheta_1\right)$ gives the optimal ϑ_1-profit in a more relaxed problem than the one for $(\tilde{w}_1, \tilde{e}_1)$, so it is immediate that Π_1^{LM} is larger than $\tilde{\Pi}_1$.

Firm 2 also cannot make a higher profit, as any contract that surely attracts worker 1 without attracting worker 2, per definition, yields a lower profit than Π_2^*. Also per definition, firm 2 cannot give a contract to worker 2 at higher profit. □

Following the convention on Bertrand competition with asymmetric marginal costs, this least-cost equilibrium will be considered *the* equilibrium of the low-mobility labour market.

Proof of Proposition 6. Assume both firms offer their contracts (w_i^P, e_i^P). As worker 2 would get a utility of $\underline{u}$ from either contract, worker 1 would get a utility of *at least* $\underline{u}$ from either contract. It is thus an equilibrium for both workers to apply to both contracts.

If either firm wants to offer worker 1 a specific contract that only he would accept, at a profit greater than Π_1^M, such a contract would per definition induce poaching. By assumption, a contract giving Π_1^M or less does not constitute a profitable deviation. □

Proof of Proposition 7. First, I show that an equilibrium exists as specified in the proposition. Consider the following set of strategies.

In the first round of contracting, firm 2 offers $(\underline{u}, 0)$. Worker 2 accepts this. In the second round of contracting, firm 2, observing the contract (w_1, e_1) between firm 1 and her employee, believes the worker is of skill ϑ_1 whenever $u(w_1, e_1|\vartheta_2) \leq \underline{u}$, and offers the poaching contract

$$\underset{w,e>0}{\text{argmax}} \left\{\pi(e, \vartheta_1)I_2 - w | u(w, e|\vartheta_1) \geq u(w_1, e_1|\vartheta_1) + \varepsilon\right\},$$

for some small ε, i.e. e equal to $\tilde{e}_2$ and

$$w = u(w_1, e_1|\vartheta_1) + \varepsilon + \frac{\tilde{e}_2}{\vartheta_1},$$

whenever both $u(w_1, e_1|\vartheta_2) \leq \underline{u}$ and $u(w_1, e_1|\vartheta_1) < \overline{u}_1^*$. This will certainly attract the ϑ_1 worker away. Believing the poached worker is of type ϑ_1, firm 2 will then expect to make a profit larger than Π_2^*, for ε small enough. When choosing not to poach, firm 2 can make at most Π_2^* from hiring worker 2, meaning poaching is sequentially rational given the beliefs.

If $u(w_1, e_1|\vartheta_1) \geq \overline{u}_1^*$, firm 2 decides not to poach. Indeed the profit from poaching is at most Π_2^* in this case, so that she is at least as well off choosing to employ the ϑ_2 worker at a contract (w_2^*, e_2^*) .

In the first round, If $u(w_1^*, e_1^*|\, \vartheta_2) > \underline{u}$, firm 1 offers the contract (w', e'), which is the optimal contract that satisfies non-participation for the ϑ_2-worker and at least a utility of u_1^* for the ϑ_1 worker. The ϑ_1 worker accepts this, and the ϑ_2 worker does not. Since this contract would not lead to a poaching offer, and a utility of at most $\underline{u}$, this is rational for worker 2. Given the fact that only the ϑ_1 worker accepts, firm 2's beliefs are consistent on the equilibrium path. It is also optimal for the ϑ_1 worker to accept this contract, as waiting for firm 2's offer without revealing his type will give him a utility strictly below u_1^*.

If $u(w_1^*, e_1^*|\, \vartheta_2) \leq \underline{u}$, and $\overline{u}_1^*$ she offers (w_1^*, e_1^*) and the analysis remains the same.

Lastly, firm 1 cannot hire the ϑ_1-worker at a lower utility, as this would induce poaching and leave the firm without a worker. As $u(w_1^*, e_1^*|\, \vartheta_2) > \underline{u}$, (w', e') is the optimal contract that does not attract the ϑ_2 worker.

Uniqueness follows from the fact that regardless of beliefs, firm 2 would never poach a worker at a higher utility than $\overline{u}_1^*$, and would always poach in an equilibrium in which the large firm employs the ϑ_1-worker at a lower utility. Condition (24) entails that the large firm prefers to offer a screening contract to ensure hiring the ϑ_1 worker, rather than deviate to pool with firm 2 for the top two workers. □

Corollary 8 follows directly from the proof above. It covers the case where $\tilde{e}_2 < e_1^* < e'$. It is interesting to also point out that if $\tilde{e}_2 < e' < e_1^*$, the high mobility equilibrium does not feature an increase in task difficulty with respect to the low mobility equilibrium, but does feature a higher wage for the ϑ_1 worker.

A.4 Proofs for Section 3.2

Proof of Proposition 9. Consider the following set of strategies. For the large firm: if a worker has at any point during the bidding game been employed at a contract (w, e) such that $u(w, e|\vartheta_1) > \underline{u} \geq u(w, e|\vartheta_2)$, and is currently employed at the small firm at a contract (w_2, e_2) such that $u(w_2, e_2|\vartheta_1) < \underline{u} + \frac{(\vartheta_1 - \vartheta_2)^2}{\vartheta_1} I_1$, poach with a contract $(\tilde{w}, \tilde{e})$ such that $u(\tilde{w}, \tilde{e}|\vartheta_1) = u(w_2, e_2|\vartheta_1) + \varepsilon$ for some small enough ε and $u(\tilde{w}, \tilde{e}|\vartheta_2) \geq \underline{u}$. Open by offering $(\overline{w}^{HM}, \overline{e}^{LM})$, do not change the offer afterwards.

For the smaller firm: if a worker is employed at the large firm at a contract (w,e) such that $\overline{u}^{HM} > u(w,e|\vartheta_1) > \underline{u} \geq u(w,e|\vartheta_2)$, poach with a contract $(\tilde{w},\tilde{e})$ such that $u(\tilde{w},\tilde{e}|\vartheta_1) = u(w,e|\vartheta_1) + \varepsilon$ for some small enough ε and $u(\tilde{w},\tilde{e}|\vartheta_2) \geq \underline{u}$. When employing a worker at a contract (w_2,e_2), if the larger firm tries to poach with an offer (w,e) such that $u(w,e|\vartheta_1) > u(w_2,e_2|\vartheta_1)$, and this worker does not accept, fire this worker and offer $(\underline{u},0)$ to a currently unemployed worker. Open by offering $(\underline{u},0)$ to all workers.

At time t, all workers observe the history of contracts $\left(w^i_\tau, e^i_\tau\right)$ actually signed by each firm i for all times $\tau < t$. If ever there is a $\tau < t$ such that a worker with $\vartheta < \vartheta_2$ would have $u\left(w^i_\tau, e^i_\tau|\vartheta\right) > \underline{u}$, all workers accept the contract from firm 1. If not, worker 1 accepts the contract from firm 1, worker 2 accepts the contract from firm 2, and all other workers remain self-employed.

As is derived in the main text, if condition (33) is satisfied, the large firm will not have an incentive to offer any other contract, as this will reduce her profits in the following periods. The small firm also does not want to outbid the large firm, and each worker chooses a best response strategy on the equilibrium path. □

A.5 Proofs for Section 3.3

Proof of Lemma 10. With the strategies mentioned, if firm 2 poaches, she would obtain at most $\vartheta_1 I_2 - w_1$. By condition (35), this is less than the $\vartheta_2 I_2 - \underline{u}$ she already obtains. Therefore she has no incentive to poach. Noting this, it suffices to observe that the market is thus equivalent to the low mobility labour market discussed before. This concludes the proof. □

Proof of Lemma 11. The case with c large has been covered by the previous lemma. Now if $c \leq \frac{\vartheta_1 - \vartheta_2}{\vartheta_1} I_2$, consider the following set of strategies and beliefs.

Any worker of type ϑ employed in the first stage at a contract (w,e), receiving a poaching offer $(\tilde{w},\tilde{e})$ such that $\tilde{w} - \frac{\tilde{e}}{\vartheta} - c > w - \frac{e}{\vartheta}$, switches. Receiving any other offer, he stays with his first-round employer. This is trivially a best response.

After observing the contracts, whenever firm 1 employs a worker at a contract (w,e) with $w - \frac{e}{\vartheta_2} \leq \underline{u}$, firm 2 believes that this worker is of type ϑ_1. Whenever firm 1 offers such a contract with $w - \frac{e}{\vartheta_1} < \underline{u} + (\vartheta_1 - \vartheta_2) I_2 - c$, firm one offers a poaching contract $(\tilde{w},\tilde{e})$ with $\tilde{e} = 0$ and $\tilde{w} = w - \frac{e}{\vartheta_1} + c + \varepsilon$ for some small enough epsilon. Given the beliefs, this would gives a posterior profit of

$$\vartheta_1 I_2 - \left(w - \frac{e}{\vartheta_1} + c + \varepsilon\right),$$

which, for ε small enough, is larger than $\vartheta_2 I_2 - \underline{u}$. If firm 2 employs the ϑ_2-worker at a wage $\underline{u}$, this is therefore a profitable deviation.

In the first stage, the firms offer the contracts specified in the proposition. Each worker chooses her matched firm. This makes firm 2's beliefs correct on the equilibrium path. Firm 2 cannot do better by attracting worker 1, by design.

If firm 1 decided to offer a contract that other workers than worker 1 accept, that would give a profit of at most $\frac{\vartheta_1+\vartheta_2}{2} I_1 - \underline{u}$, which, by condition (40), is not profitable. If she offers worker 1 a contract with a utility lower than $\underline{u} + (\vartheta_1 - \vartheta_2) I_2 - c$, this would induce poaching. The contract is the optimal one such that worker 1 gets at least this, and all workers with $\vartheta \leq \vartheta_2$ would not seek employment, and therefore firm 1 has no profitable deviation.

Uniqueness follows from the fact that in a pooled equilibrium, firm 1 would always be better off offering a poach-proof contract that only a worker of type ϑ_1 would accept, by condition (40). In a matched equilibrium such that worker 1 receives a utility below $\underline{u} + (\vartheta_1 - \vartheta_2) I_2 - c$, firm 2 would always poach. If worker 1 receives a contract other than $(w_1(c), e_1(c))$, yet with a utility of at least $\underline{u} + (\vartheta_1 - \vartheta_2) I_2 - c$, there is always a better one that is both poach-proof and screens out lower ability workers. □

Proof of Proposition 12. The cases for larger c are subsumed by the previous proposition. Now assume condition (45) is satisfied. In equilibrium, both firms believe the two employed workers are of the same posterior expected type, and have no incentive to poach.

Ex ante, neither firm has an incentive to offer a different contract that multiple workers would accept, as they would not be able to get a better expected type, and would not be able to offer a wage below $\underline{u}$.

Firm 1 does not have an incentive to deviate to a contract that only the ϑ_1-worker would accept, as any contract with a ϑ_1-utility below $\underline{u} + (\vartheta_1 - \vartheta_2) I_2 - c$, firm 2 would poach, and any such contract with a higher utility entails a wage of at least $\underline{u} + \vartheta_1 \left(I_2 - \frac{c}{\vartheta_1 - \vartheta_2}. \right)$ This would give a profit of at most

$$\vartheta_1 I_1 - \underline{u} - \vartheta_1 \left(I_2 - \frac{c}{\vartheta_1 - \vartheta_2} \right),$$

which by condition 45 is smaller than $\frac{1}{2}(\vartheta_1 + \vartheta_2) I_1 - \underline{u}$, and hence does not constitute a profitable deviation. □

B Primer: Monopsonist Firms

First, it will be useful to look at the following reduced model: there is only one firm, of size I, and two workers of different types with exogenously differing reservation utilities. The worker of type ϑ_1 has reservation utility $\underline{u}_1$, and the other workers has a reservation utility $\underline{u}_2$ The assumption is that $\underline{u}_1 > \underline{u}_2$. In the full model, this will arise endogenously from competition on the labour market. We also assume that the skill differential between the two workers justifies their different reservation utilities. Figure 6 illustrates this case. The red line represents the ϑ_2-workers participation constraint. The green line is less steep, but has a higher intercept, representing the ϑ_1-worker's participation constraint.

Full information First I address what happens when the firm has full information on worker types. The firm has to make sure that she offers enough utility to the ϑ_1-worker to participate, so her problem is

$$\max_{w,e>0} \left\{ \pi(e, \vartheta_1) I - w \right\}, \tag{69}$$

subject to

$$w - \frac{e}{\vartheta_1} \geq \underline{u}_1. \tag{70}$$

If $\frac{\partial}{\partial e}\pi(0, \vartheta_1) I_1 > \frac{1}{\vartheta_1}$ (as in the illustration), this has an internal solution at the point where the firm's isoprofit curve is tangent to the ϑ_1-worker's participation constraint. If not, the solution is to set no task, $e = 0$, and $w = \underline{u}_1$. As mentioned above, it is assumed that the firm prefers this to hiring worker 2.

Private information Assume the firm wants to hire the ϑ_1-type worker, while minimizing her wage costs. She thus has to set a wage and a task level that would be attractive for the best worker, and unattractive for the other ones, meaning her problem is the same as before, but now subject to

$$w - \frac{e}{\vartheta_1} \geq \underline{u}_1$$
$$w - \frac{e}{\vartheta_2} \leq \underline{u}_2.$$

The second constraint serves as a non-participation constraint for worker type ϑ_2. If the first best satisfies this non-participation constraint, it remains the solution here.

Consider, however, the case that the first best does not satisfy non-participation for the ϑ_2 worker. As in the illustration, this happens when the first best lies to the top left of the ϑ_2 participation constraint. In that case, the solution is that both constraints bind. This gives a task level

$$\tilde{e} = \vartheta_1 \vartheta_2 \frac{\underline{u}_1 - \underline{u}_2}{\vartheta_1 - \vartheta_2} \tag{71}$$

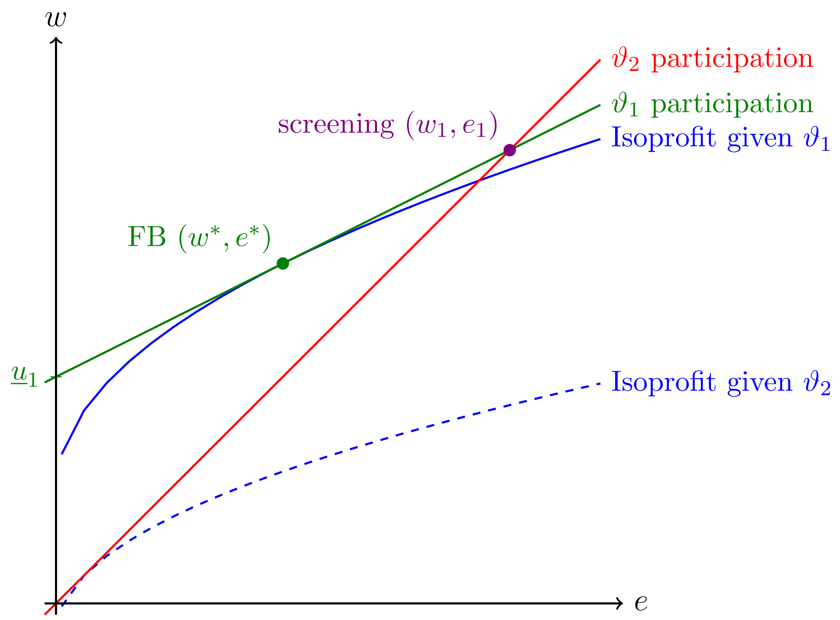

Figure 6: overprovision of effort with monopsony and $\underline{u}_1 > \underline{u}_2 = 0$

and a wage level

$$\begin{aligned}\tilde{w} &= \underline{u}_1 + \vartheta_2 \frac{\underline{u}_1 - \underline{u}}{\vartheta_1 - \vartheta_2} \\ &= \underline{u}_2 + \vartheta_1 \frac{\underline{u}_1 - \underline{u}}{\vartheta_1 - \vartheta_2}\end{aligned} \tag{72}$$

In this case, screening is costly. Setting a task level above the first best one is inefficient, yet necessary to separate the two types. In the special case with $\pi(e, \vartheta) = \vartheta$, the first best never satisfies non-participation for worker 2 and always gives the wage and task level above.

Note that in this case it is also necessary that employing the ϑ_1-worker at this contract is better than the alternative of hiring the ϑ_2-worker:

$$\pi(\tilde{e}, \vartheta_1)I - \tilde{w} \geq \max_{e>0} \left\{ \pi(e, \vartheta) - \left(\underline{u}_2 + \frac{e}{\vartheta_2} \right) \right\} \tag{73}$$

Otherwise, the firm would settle to hire the ϑ_2 worker at his reservation utility.