STICKY INFORMATION AND THE TAYLOR PRINCIPLE
PRELIMINARY AND INCOMPLETE

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ABSTRACT. This paper presents determinacy bounds on monetary policy derived in the frequency domain. These bounds confirm results for standard Calvo sticky price New Keynesian models obtained in the time domain, but provide bounds for models like those under sticky information previously unobtainable. We find that the restrictions are independent of parameters outside the monetary policy rule and more restrictive than sticky price bounds. These results provide conservative restrictions on monetary policy rules for policy makers in the absence of model certainty.

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1. INTRODUCTION

We implement the sticky information model of Mankiw and Reis (2002) in the frequency domain which enables it to be expressed in a fully recursive manner. To our knowledge, this is the first to do so and enables us to characterize its stability properties, closing a gap in the literature. Specifically we examine the consequences for New Keynesian policy recommendations in the form of determinacy bounds on the monetary authority's policy rule and show that the Taylor principle, a more than one-for-one response of the nominal interest rate in response to inflation, holds in a stricter sense than in the standard sticky price framework.

The sticky information posits a vertical long-run Phillips curve, even out of equilibrium, whereas the sticky-price model, in contrast, imposes a systematic relationship between

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inflation and output, stable even in the long run.[1] This systematic relationship widens the parameter spaces of monetary policy's Taylor rule associated with unique equilibria under sticky prices with, for example, a reaction of the nominal interest rate to the output gap serving as a substitute for a reaction to inflation and allowing the (direct) response to inflation to be less than one while still adhering to the Taylor principle. Woodford (2003, pp. 254–255), "... indeed, a large enough [response to] *either* [the output gap or inflation] suffices to guarantee determinacy."[2] As is shown here, the long-run verticality of Mankiw and Reis's (2002) sticky-information Phillips curve precludes such a substitutability and the bounds for determinacy depend only on the reaction of monetary policy to nominal variables.

Solving linear rational expectation (LRE) models means to replace agents' "expectations" with the mathematical counterpart. The challenge thereby is that the assumption of rational expectations leads to the necessity of finding a fixed point of the reduced model which matches the agents' forecasts of the model. One method to do so is to partition the system into stable and unstable dynamics of the model (see Klein, 2000; Söderlind, 1999; Sims, 2001) and then pin down the initial conditions. However, as LRE models often feature multiple equilibria they attain determinacy only for certain parameter values (see Blanchard and Kahn, 1980). Nonetheless, these solution methods are only applicable for a certain class of linear models. Applying these methods to models featuring lagged expectations would require the system to specifically include the lagged expectation as additional state variables. This is very inconvenient for a model that features a potentially infinite sum of lagged expectations of variables as it would make models unnecessary large and lagged expectations would need to be truncated giving rise to possibly inaccurate solutions (Andres et al., 2005). Another method to solve LRE models is the method of undetermined coefficients (Taylor, 1986). Here the solution to the model is presented in terms of the state variables. Meyer-Gohde (2010) eliminates the state-space expansion and truncation requirement of lagged LRE models by extending the method proposed by Taylor (1986) with the Generalized Schur Decomposition.

However, these methods are usually presented in the time-domain and miss useful model insights and frequency implications. Working in the frequency domain allows the researcher to examine a models' performance over different frequencies. Watson (1993)

[1]See, e.g., Woodford (2003, p. 254) or Galí (2008, p. 78).

[2]Emphasis in the original.

stresses the importance of model spectra as "informative diagnostics" and Diebold et al. (1998) assess the performance of models based on estimated spectral density functions. Further, the advantage of solving macroeconomic models in the frequency domain is that these methods are not model-specific which makes them applicable to a wide range of economic models.

Our paper adds to two strands of literature. First, it adds to the literature on sticky information in macroeconomic models which has large supporting evidence in explaining inflation dynamics and agents' information inattention (Mankiw and Reis, 2002; Branch, 2007; Gomes, 2009; Mertens and Nason, 2020; Angeletos et al., 2021; Bellemare et al., 2020; Jang, 2020; Bouchaud et al., 2019; Link et al., 2023; Reis, 2022; Dupor et al., 2010; Bürgi and Ortiz, 2022; Coibion and Gorodnichenko, 2015; Andrade and Le Bihan, 2013; Cornand and Hubert, 2022; Korenok, 2008; Chou et al., 2023; Link et al., 2023; Reis, 2020; Eggertsson and Garga, 2019, amongst others). However, methods which have been proposed to solve that model numerically (see Menz and Vogel, 2009; Meyer-Gohde, 2010; Reis, 2009; Trabandt, 2007) are either inefficient or only consider a finite number of lagged expectations and therefore miss useful insights.

Further, this paper contributes to the debate on optimal monetary policy. Specifically, it studies the effectiveness of the Taylor rule in the sticky price model and shows the effectiveness of alternative monetary policy rule specifications with a smaller emphasis on inflation targeting which recently has received new attention (e.g. Sumner, 2014; Beckworth and Hendrickson, 2020; Sims, 2013; Cúrdia et al., 2015; Garín et al., 2016; Cúrdia et al., 2011; Lyu et al., 2023; Levrero, 2023; Boehm and House, 2019; Billi, 2020; Svensson, 2020; Budianto et al., 2023) and clarifies some misconceptions on Taylor rules in the sticky price model.

This paper is structured as follows. In Section 2 we first introduce the solution of economic models in frequency domain following Whiteman (1983). Next, section 3 provides frequency domain representations of the sticky price and sticky information Phillips curves and shows how the latter can be expressed recursively in the frequency domain. In Section 4 we address the existence and uniqueness conditions under a standard Taylor rule via the frequency domain approach for both the sticky price and information models. Afterwards, Section 5 analyzes the implications of alternative monetary policy rules in the sticky information model. Lastly we conclude.

2. EXISTENCE AND UNIQUENESS: A FREQUENCY DOMAIN PERSPECTIVE

In this section we review the approach of Whiteman (1983) that we will use and extend to establish determinacy results for the sticky-information model in later sections. Whiteman (1983) assumes that exogenous driving processes are mean zero, linearly regular covariance stationary stochastic processes with known Wold representations, expectations are computed using the Wiener-Kolmogorov formulas, and solutions will be sought in the space spanned by time-independent square-summable linear combinations of the process fundamental for the driving process.

Let ϵ_t be this fundamental process, then solutions are of the form $y_t = \sum_{j=0}^{\infty} y_j \epsilon_{t-j}$ with $\sum_{j=0}^{\infty} y_j^2 < \infty$. The Wiener-Kolmogorov prediction formula gives us $E_t[y_{t+n}] = E_t\left[\sum_{j=0}^{\infty} y_j \epsilon_{t-j+n}\right] = \sum_{j=0}^{\infty} y_{j+n}\epsilon_{t-j}$. Following, e.g., Sargent (1987, ch. XI) the Riesz-Fischer Theorem gives an equivalence (a one-to-one and onto transformation) between the space of squared summable sequences $\sum_{j=0}^{\infty} y_j^2 < \infty$ and the space of analytic functions in unit disk $y(z)$ corresponding to the z-transform of the sequence, $y(z) = \sum_{j=0}^{\infty} y_j z^j$.

The Wiener-Kolmogorov prediction formula of "plussing" gives the frequency domain version

$$\mathscr{Z}\{E_t[x_{t+1}]\} = [\frac{X(z)}{z}]_+ = \frac{1}{z}(X(z) - X(0)) \tag{1}$$

where $_+$ is the annihilation operator, see Sargent (1987) and Hamilton (1994).

We briefly demonstrate the requirement of analyticity of the z-transform in the frequency domain in relation to known requirements in the time domain in order to establish intuition. Consider first an autoregressive process of order 1, an AR(1) process:

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2)$$

which can be rewritten as

$$y_t = \sum_{j=0}^{\infty} L^j y_j \epsilon_t.$$

The AR(1) process in the frequency domain is given by applying the z-transform:

$$\begin{aligned} y(z) &= \rho y(z) + 1 \\ y(z) &= (1 - \rho z)^{-1} \end{aligned}$$

$\epsilon(z)$ is analytic inside unit disk, y(z) analytic inside the unite disk if $|\rho| < 1$ and determines the solution to the autoregressive process.

Now consider a forward-looking process:

$$y_t = \alpha E_t y_{t+1} + \epsilon_t$$

where by the forecast can be rewritten in terms of deviations from the driving process:

$$E_t y_{t+1} = y_{t+1} - y_0 \epsilon_{t+1} = \frac{1}{L}(\sum_{j=0}^{\infty} L^j y_j - y_0)\epsilon_t$$

In the frequency domain the forward-looking process is described by:

$$y(z) = \alpha \frac{1}{z}(y(z) - y_0) + 1$$

To determine a solution we solve for $y(z)$:

$$\begin{aligned} (1 - z\frac{1}{\alpha})y(z) &= y_0 - \frac{z}{\alpha} \\ y(z) &= (1 - \frac{1}{\alpha}z)^{-1}(y_0 - \frac{z}{\alpha}) \end{aligned}$$

whereby y_0 is not determined yet. If $|\alpha| < 1$, then for $z = \alpha$ there is a removable singularity inside the unit disk and we can solve for a boundary condition on y_0:

$$\lim_{z\to\infty}(1 - z\frac{1}{\alpha})y(z) = 0 \quad \Rightarrow \quad y_0 = 1$$

This gives the solution to our process in the frequency domain as

$$y(z) = \frac{1 - \frac{z}{\alpha}}{1 - \frac{z}{\alpha}} = 1.$$

In the time domain the equivalent unique stationary solution is given by:

$$y_t = \epsilon_t$$

Finally, consider a backward and forward looking process:

$$a E_t y_{t+1} + b y_t + c y_{t-1} + \epsilon_t = 0$$

The process can be presented in the frequency domain as:

$$a\frac{1}{z}(y(z) - y_0) + b y(z) + c z y(z) + 1 = 0$$

Rearranging allows us to determine the solution to this model:

$$\begin{aligned} a(y(z) - y_0) + bzy(z) + cz^2 y(z) + z &= 0 \\ (a + bz + z^2)y(z) &= a y_0 - z \\ (a - a(\lambda_1 + \lambda_2)z + a\lambda_1\lambda_2 z^2)y(z) &= a y_0 - z \\ (1 - \lambda_1 z)(1 - \lambda_2 z)y(z) &= y_0 - \frac{z}{a} \end{aligned}$$

which again depends on the initial condition on y_0. Consider now the following possible conditions to determine if a solution is unique or not: If $|\lambda_1|, |\lambda_2| < 1$, y_0 is not uniquely determined and if $|\lambda_1|, |\lambda_2| > 1$, y_0 is also not uniquely determined. If however, $|\lambda_2| < 1 < |\lambda_1|$, the boundary condition on y_0 is given by:

$$\lim_{z\to\infty}(1-\lambda_1 z)(1-\lambda_2 z)y(z) = 0 = y_0 - \frac{1}{\lambda_1 a} \quad \Rightarrow \quad y_0 \quad = \quad \frac{1}{\lambda_1 a}$$

which then determines the unique solution for the process on $y(z)$:

$$y(z) = \frac{1}{1-\lambda_1 z}\frac{1}{1-\lambda_2 z}\frac{1}{a}\left(\frac{1}{\lambda_1} - z\right) \quad = \quad \frac{1}{1-\lambda_2 z}\frac{1}{\lambda_1 a} = \frac{1}{\lambda_1 a}\frac{1}{1-\lambda_2 z}$$

The solution stated in the time domain would give us:

$$y_t \quad = \quad \lambda_2 y_{t-1} + \frac{1}{\lambda_1 a}\epsilon_t$$

giving us the famed Blanchard and Kahn (1980) conditions. Hence, deriving the conditions in either time or frequency domain doesn't alter the model in itself but allows to determine unique solutions and boundary conditions which were previously unobtained eventually.

3. Phillips Curves in the Frequency Domain

In this section, we review two Phillips curves and present their frequency domain equivalents. While this merely provides an alternative representation for the canonical sticky price Phillips curve, we show that the frequency domain provides a more fundamental perspective on the sticky information Phillips curve. In contrast to the sticky price Phillips curve, whose infinite regress of forward-looking price setting behavior can be represented recursively in the time domain, the sticky information Phillips curve has an infinite regress of price plans or lagged expectations that cannot be expressed recursively in the time domain, precluding the application of standard DSGE techniques to assess determinacy. We prove in the following, however, that the sticky information Phillips curve does have a recursive representation in the frequency domain, which will enable the application of the techniques reviewed in the previous section.

We begin with the standard New Keynesian (NK) sticky-price Phillips curve with Calvo (1983)-style overlapping contracts. Up to first order, in log-deviations and abstracting from exogenous driving processes, the Phillips Curve is given by[3]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{2}$$

[3] See, eg., Woodford (2003, p. 246) or Galí (2008, p. 49).

where y_t is the output gap, π_t inflation. Expressing this in the frequency domain, gives the following representation of the NKPC

$$\pi(z) = \beta\frac{1}{z}(\pi(z) - \pi_0) + \kappa y(z) \tag{3}$$

Sticky information models implement probabilistic contracts of predetermined prices in the vein of Fischer (1977) with the Calvo (1983) mechanism.[4] Mankiw and Reis's (2002) version, the sticky-information model, yields the following aggregate supply equation

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})] \tag{4}$$

where y_t is the output gap, π_t inflation, $\xi > 0$ measures the degree of strategic complementarities, and $0 < 1-\lambda < 1$ is the probability of an information update. The infinite regress of lagged expectations precludes a recursive representation in the time domain.

Whiteman (1983) calls lagged expectations ($E_{t-i}[x_t],\ i > 0$) "withholding equations" and the Wiener-Kolmogorov prediction formula (1) provides the representation

$$\mathscr{Z}\{E_{t-i}[x_t]\} = z^i[\frac{X(z)}{z^i}]_+ = X(z) - \sum_{j=0}^{i} X^j(0)z^j \tag{5}$$

where $X^j(0)$ is the $j'th$ derivative of $X(z)$ evaluated at the origin. These withholding equations by themselves are not sufficient to solve models like those involving the sticky information Phillips curve (4), as they involve an *infinite* number of withholding equations.[5]

Using (5), the sticky information Phillips curve (4) can be expressed as

$$\pi(z) = \frac{1-\lambda}{\lambda}\xi y(z) + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i\left[\pi(z) - \sum_{j=0}^{i}\pi^j(0)z^j + \xi(1-z)\left(y(z) - \sum_{j=0}^{i} y^j(0)z^j\right)\right] \tag{6}$$

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[4]See Bénassy (2002, Ch. 10), Mankiw and Reis (2002), and Devereux and Yetman (2003).

[5]Tan and Walker (2015, p. 99) claim that their method can be "easily adapted" to models like the sticky information model using withholding equations by "replacing E_t with E_{t-j} for any finite j." This is misleading or incomplete, as the sticky information model involves lagged information that reaches back past any finite j.

The infinite sums in (6) can be resolved by noting that:

$$\sum_{i=0}^{\infty}\lambda^i\left[x(z)-\sum_{j=0}^{i}x_jz^j\right]=\frac{1}{1-\lambda}x(z)-\sum_{i=0}^{\infty}\lambda^i\sum_{j=0}^{i}x_jz^j \tag{7}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}x_jz^j\lambda^i \tag{8}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\lambda^i x_jz^j\lambda^j \tag{9}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\frac{1}{1-\lambda}\lambda^i x_jz^j\lambda^j \tag{10}$$

$$=\frac{1}{1-\lambda}(x(z)-x(\lambda z)) \tag{11}$$

Further, in the time domain we can derive a recursive representation of the lagged expectations of the endogenous variables in (4) as:

$$(1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[x_t],\quad x_t=\left(\sum_{j=0}^{\infty}x_jz^j\right)\epsilon_t \tag{12}$$

$$=(1-\lambda)\left(E_{t-1}[x_t]+\lambda E_{t-2}[x_t]+\lambda^2E_{t-3}[x_t]+\ldots\right) \tag{13}$$

Applying the Wiener-Kolmogorov prediction formula to the lagged expectations (5), equation (13) gives the frequency domain representation as:

$$\begin{aligned}
&(1-\lambda)\left(x(z)-x_0+\lambda(x(z)-x_0-zx_1)+\lambda^2(x(z)-x_0-zx_1-z^2x_2)+\ldots\right)\\
=\;&(1-\lambda)\left(x(z)+\lambda x(z)+\lambda^2x(z)+\ldots-x_0-\lambda x_0-\lambda^2x_0\ldots-\lambda zx_1-\lambda^2zx_1\ldots-\lambda^2zx_2\ldots\right)\\
=\;&(1-\lambda)((1+\lambda+\lambda^2+\ldots)x(z)-(1+\lambda+\lambda^2+\ldots)x_0-\lambda z(1+\lambda+\lambda^2+\ldots)x_1\\
&-\lambda^2z^2(1+\lambda+\lambda^2+\ldots)x_2-\ldots)\\
=\;&(1-\lambda)\left(\frac{1}{1-\lambda}x(z)-\frac{1}{1-\lambda}x_0-\frac{\lambda z}{1-\lambda}x_1-\frac{\lambda^2z^2}{1-\lambda}x_2-\ldots\right)\\
=\;&x(z)-\sum_{j=0}^{\infty}\lambda^iz^jx_j=x(z)-x(\lambda z)
\end{aligned}$$

Hence, the lagged expectations in (13) can be transformed from the time into the frequency domain as:

$$(1-\lambda)\sum_{j=0}^{\infty}\lambda^iE_{t-i-1}[x_{t-1}]=(1-\lambda)\left(\frac{z}{1-\lambda}x(z)-\frac{\lambda z}{1-\lambda}x_0-\frac{(\lambda z)^2}{1-\lambda}x_1-\ldots\right)=zx(z)-\lambda zx(\lambda z)$$

The original sticky information Phillips from curve in the time domain based on (13) is given by:

$$\pi_t=\xi\frac{1-\lambda}{\lambda}y_t+(1-\lambda)\sum_{j=0}^{\infty}\lambda^jE_{t-j-1}[\pi_t+\xi(y_t-y_{t-1})] \tag{14}$$

Applying the z-transform we get the following representation of the Phillips curve (14):

$$\begin{aligned}
\pi(z) &= \xi(\frac{1-\lambda}{\lambda}y(z)+\pi(z)-\pi(\lambda z)+\xi(1-z)y(z)-\xi(1-\lambda z)y(\lambda z) \\
0 &= \xi(\frac{1-\lambda}{\lambda}+1)y(z)-\pi(z)-\xi z y(z)-\xi(1-\lambda z)y(\lambda z) \\
&= \xi(\frac{\xi}{\lambda}z)y(z)=\pi(\lambda z)+\xi(1-\lambda z)y(\lambda z) \\
\xi(1-\lambda z)y(z) &= \lambda\pi(z\lambda)+\xi\lambda(1-\lambda z)y(\lambda z)
\end{aligned}$$

yielding the representation of the Phillips curve of the sticky information in the frequency domain

$$\xi\left(\frac{1}{\lambda}-z\right)y(z)=\pi(\lambda z)+\xi(1-\lambda z)y(\lambda z) \tag{15}$$

which can now be used to determine uniqueness and boundary conditions for the sticky information model derived in the frequency domain.

4. Existence and Uniqueness for Sticky Information

To assess the bounds on monetary policy, we will close the model using one of the two supply equations above with an IS equation

$$y_t = E_t y_{t+1} - \sigma R_t + \sigma E_t \pi_{t+1} \tag{16}$$

where R_t the nominal interest rate and monetary policy described by the following interest rate rule

$$R_t = \phi_\pi \pi_t + \phi_y y_t \tag{17}$$

Substituting the monetary policy rule and expressing it in the frequency domain gives

$$(1+\sigma\phi_y)zy(z)+\sigma\phi_\pi z\pi(z) = y(z)-y_0+\sigma(\pi(z)-\pi_0) \tag{18}$$

Notice that we are abstracting from shocks and these equations (along with either of the supply curves from the previous section) are entirely homogenous. Thus one solution, the fundamental solution is zero at all frequencies - an inability to rule out nonzero solutions is tantamount to not being able to rule out stable sunspot solutions - i.e. non-uniqueness or indeterminacy.

First, consider the standard sticky-price model. Hence,the model with (3)

$$\pi(z) = \beta\frac{1}{z}(\pi(z)-\pi_0)+\kappa y(z) \tag{19}$$

can be summarized in a matrix system as

$$\begin{bmatrix} -\beta & 0 \\ \sigma & 1 \end{bmatrix} \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} = \begin{bmatrix} -1 & \kappa \\ \sigma\phi_\pi & 1+\sigma\phi_y \end{bmatrix} z \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} + \begin{bmatrix} -\beta & 0 \\ \sigma & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ y_0 \end{bmatrix}$$

or equivalently,

$$(I_2 - zA) \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} = \begin{bmatrix} \pi_0 \\ y_0 \end{bmatrix} \tag{20}$$

where $A = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ \sigma(\phi_\pi - \frac{1}{\beta}) & 1+\frac{\sigma}{\beta}\kappa + \sigma\phi_y \end{bmatrix}$ is the matrix of coefficients. We summarize the condition for determinacy in the following

Theorem 1 (Sticky Price Determinacy)**.** *The sticky price model, given by (18), (3), with the Taylor rule (17), has a unique, stable equilibrium if and only if*

$$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$$

Proof. See the following □

To solve the system of equations in (20) we first, need to linearize the matrix A and then, using Klein's (2000) method, determine the boundary conditions for π_0 and y_0. Define ρ_i=$eig(A)$. Iff ρ_i, $i = 1,2$ there are two removable singularities. Decompose matrix A into its eigenvalues, and its eigenvector-matrix V as

$$A = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1} = V\Lambda V^{-1} \tag{21}$$

Following Klein (2000) we define

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = V^{-1} \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} \quad \text{for} \quad z = 0,1,2\dots \tag{22}$$

Substituting into our equation system (20) gives

$$\left(I_2 - zV\Lambda V^{-1}\right)V \begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = V \begin{bmatrix} w_0 \\ u_0 \end{bmatrix}$$

which can be rewritten and redefined as

$$(I_2 - z\Lambda) \begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = \begin{bmatrix} w_0 \\ u_0 \end{bmatrix} \tag{23}$$

Given our matrix of eigenvalues we can split the matrix system into two independent equations:

$$(1-z\rho_1)w(z) = w_0 \tag{24}$$

$$(1-z\rho_2)u(z) = u_0 \tag{25}$$

If both eigenvalues, $|\lambda_1|$ and $|\lambda_2|>1$, we can eliminate the singularity as:

$$\lim_{z\to 1/\lambda_1}(1-z\lambda_1)w(z)=0$$

and

$$\lim_{z\to 1/\lambda_2}(1-z\lambda_2)u(z)=0$$

pinning down the two conditions $w_0=0$ and $u_0=0$. From our definition (22) and equation (23) we can therefore deduce

$$\begin{bmatrix}\pi_0\\ y_0\end{bmatrix} = V\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

The boundary conditions to our NKM are:

$$\pi_0=0, \qquad y_0=0 \tag{26}$$

The Schur-Cohn criteria can be applied to ascertain whether both eigenvalues, λ_1 and λ_2, indeed do lie outside the unit circle. These criteria, expressed in terms of A are $|det(A)|>1$ and $|tr(A)|<1+det(A)$. As

$$det(A)=\frac{1}{\beta}(1+\sigma\phi_y+\kappa\sigma\phi_\pi)>1$$

$$\text{tr}A = \frac{1}{\beta}+\frac{\sigma\kappa}{\beta}+1+\sigma\phi_y>1$$

The condition $|det(A)|>1$ necessarily holds and $|tr(A)|<1+det(A)$ holds if

$$1<\frac{1-\beta}{\kappa}\phi_y+\phi_\pi$$

Hence determinacy in the sticky price model demands

$$1<\frac{1-\beta}{\kappa}\phi_y+\phi_\pi$$

Turning to the sticky information model that was presented in the previous section, is given in the frequency domain by the Phillips curve (15)

$$\frac{\xi}{\lambda}y(z)=z\xi y(z)+\pi(\lambda z)+\xi(1-\lambda z)y(\lambda z) \tag{27}$$

and the IS curve equation with the interest rate rule (17)

$$(1+\sigma\phi_y)zy(z)+\sigma\phi_\pi z\pi(z)=y(z)-y_0+\sigma(\pi(z)-\pi_0) \tag{28}$$

We can summarize the determinacy in the following

Theorem 2 (Sticky Information Determinacy)**.** *The sticky information model, given by (18), (30), with the Taylor rule (17), has a unique, stable equilibrium if and only if*

$$\phi_\pi > 1$$

Proof. See the following □

At frequency zero, $z=0$, define $y(0)=y_0$, $\pi(0)=\pi_0$, the Phillips curve (15) is determined by

$$\xi\frac{1-\lambda}{\lambda}y_0=\pi_0 \tag{29}$$

which yields one initial condition. The remaining condition must follow from the system given by the Phillips curve (15)

$$\frac{\xi}{\lambda}y(z)=z\xi y(z)+\pi(\lambda z)+\xi(1-\lambda z)y(\lambda z) \tag{30}$$

and the IS curve equation with the interest rate rule (17)

$$(1+\sigma\phi_y)zy(z)+\sigma\phi_\pi z\pi(z)=y(z)-y_0+\sigma(\pi(z)-\pi_0) \tag{31}$$

The matrix system is determined by (29), (30) and (31) as

$$\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix}=\begin{bmatrix}\phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma}\\ 0 & \lambda\end{bmatrix}z\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix}+\begin{bmatrix}\frac{1-\lambda}{\lambda}\xi+\frac{1}{\sigma}\\ 0\end{bmatrix}y_0+\begin{bmatrix}-\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z)\\ \frac{\lambda}{\xi} & \lambda(1-\lambda z)\end{bmatrix}\begin{bmatrix}\pi(\lambda z)\\ y(\lambda z)\end{bmatrix}$$

If $[\pi(\lambda z), y(\lambda z)]'$ are analytic functions $\forall\ |z|<1$, then $[\pi(z), y(z)]'$ are analytic functions $\forall\ |z|<\frac{1}{\lambda}$ and as $0<\lambda<1$ for $|z|<1$. Similar to (20) the system of equations can be expressed as

$$(I_2-zA)\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix}=\begin{bmatrix}\frac{1-\lambda}{\lambda}\xi\\ 0\end{bmatrix}y_0+\begin{bmatrix}-\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z)\\ \frac{\lambda}{\xi} & \lambda(1-\lambda z)\end{bmatrix}\begin{bmatrix}\pi(\lambda z)\\ y(\lambda z)\end{bmatrix}$$

where $A = \begin{bmatrix} \phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma} \\ 0 & \lambda \end{bmatrix}$. The eigenvalues of matrix A are $\rho_1 = \phi_\pi, \rho_2 = \lambda$. The linearization of $A = V\Lambda V^{-1}$ where Λ is the matrix of eigenvalues. Following Klein (2000) the system can be decomposed into

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = V^{-1} \begin{bmatrix} \pi(z) \\ y(z) \end{bmatrix} \tag{32}$$

where $V = \begin{bmatrix} 1 & \frac{1+\sigma\phi_y-\lambda}{\sigma(\lambda-\phi_\pi)} \\ 0 & 1 \end{bmatrix}$ is the matrix of eigenvectors and $V^{-1} = \begin{bmatrix} 1 & -\frac{1+\sigma\phi_y-\lambda}{\sigma(\lambda-\phi_\pi)}. \\ 0 & 1 \end{bmatrix}$ its inverse.

At frequency $z = 0$, the system of equations can be expressed as

$$(I_2 - z\Lambda) \begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = \begin{bmatrix} \frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma} \\ 0 \end{bmatrix} u_0 + \begin{bmatrix} -\frac{\lambda}{\xi}(\frac{1}{\sigma} + v_{12}) & -\frac{\lambda}{\xi}(\frac{1}{\sigma} + v_{12})(\xi + v_{12})(1 - \frac{\xi\lambda}{\xi+v_{12}}z) \\ \frac{\lambda}{\xi} & \frac{\lambda}{\xi}(\frac{1}{\sigma} + v_{12})(1 - \frac{\xi\lambda}{\xi+v_{12}}z) \end{bmatrix} \begin{bmatrix} w(\lambda z) \\ u(\lambda z) \end{bmatrix} \tag{33}$$

The first equation is given by

$$(1 - z\phi_\pi)w(z) = \left(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\right)u_0 - \frac{\lambda}{\xi}\left(\frac{1}{\sigma} + u_{12}\right)w(\lambda z) - \frac{\lambda}{\xi}\left(\frac{1}{\sigma} + v_{12}\right)(\xi + v_{12})\left(1 - \frac{\lambda\xi}{\xi + v_{12}}z\right)u(\lambda z). \tag{34}$$

Iff $\phi_\pi > 1$, there is a removable singularity to provide the additional initial condition

$$\lim_{z \to \frac{1}{\phi_\pi}} (1 - z\phi_\pi)w(z) = 0 \tag{35}$$

which can be used together with (34):

$$\left(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\right)u_0 = \frac{\lambda}{\xi}\left(\frac{1}{\sigma} + v_{12}\right)\left(w\left(\frac{\lambda}{\phi_\pi}\right) + (\xi + v_{12})\left(1 - \frac{\lambda\xi}{\xi + v_{12}}\frac{1}{\phi_\pi}\right)u\left(\frac{\lambda}{\phi_\pi}\right)\right).$$

Thus, the system of equations becomes

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = \begin{bmatrix} (1 - \phi_\pi z)^{-1}(\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}) \\ 0 \end{bmatrix} u_0$$
$$+ \begin{bmatrix} -\frac{\lambda}{\xi}(\frac{1}{\sigma} + v_{12})((1 - \phi_\pi z)^{-1}) & -\frac{\lambda}{\xi}(\frac{1}{\sigma} + v_{12})(\xi + v_{12})(1 - \frac{\lambda\xi}{\xi+v_{12}}z(1 - \phi_\pi z)^{-1} \\ \frac{\lambda}{\xi}((1 - \phi_\pi z)^{-1}) & \frac{\lambda}{\xi}(\xi + v_{12})(1 - \frac{\lambda\xi}{\xi+v_{12}}z(1 - \lambda z)^{-1} \end{bmatrix} \begin{bmatrix} w(\lambda z) \\ u(\lambda z) \end{bmatrix}$$

or equivalently,

$$\begin{bmatrix} w(z) \\ u(z) \end{bmatrix} = B(z)u_0 + A(z) \begin{bmatrix} w(\lambda z) \\ u(\lambda z) \end{bmatrix}$$

Table 1 summarizes the results, namely that the Taylor principle, a more than one for one response of the nominal interest rate, is necessary in a strict sense for the sticky information model. In contrast, the sticky price model posits that a sufficiently strong reaction to real conditions, here the output gap, can substitute for a reaction to inflation.

	Taylor Rule	
	$R_t = \phi_\pi \pi_t$	$R_t = \phi_\pi \pi_t + \phi_y y_t$
Sticky Price	$\phi_\pi > 1$	$\frac{1-\beta}{\kappa}\phi_y + \phi_\pi > 1$
Sticky Info	$\phi_\pi > 1$	$\phi_\pi > 1$

TABLE 1. Determinacy Bounds on Monetary Policy

5. EXTENSIONS

Here we examine a more general form of the Taylor rule to capture different forms of interest rate rules. Consider the following general Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\phi_\pi(\alpha_\pi \pi_t + (1-\alpha_\pi)E_t\pi_{t+1}) + \phi_y(\alpha_y y_t + (1-\alpha_y)\Delta y_t)\right] \qquad (36)$$

$0 \geq \rho_R < 1$ allows for interest rate smoothing, α_π allows the rule to capture both contemporaneous ($\alpha_\pi = 1$) as well as future ($\alpha_\pi = 0$) interest rate targeting, and α_π enables us to examine output gap level ($\alpha_\pi = 1$) as well as output gap growth ($\alpha_\pi = 0$) targeting.

Theorem 3 (Sticky Information Determinacy and the General Taylor Rule)**.** *The sticky information model, given by (18), (30), with the general Taylor rule (36), has a unique, stable equilibrium if and only if*

$$\left|\frac{\rho_R\left(1-\phi_\pi\alpha_\pi\right)+\phi_\pi\alpha_\pi}{1-\left(1-\rho_R\right)\phi_\pi\left(1-\alpha_\pi\right)}\right| > 1$$

Proof. See the appendix. □

6. CONCLUSION

This paper has proposed a solution to the sticky information model of Mankiw and Reis (2002) by formulating it in the frequency domain and applying the z-transform proposed by Whiteman (1983). By doing so we bypass the need of expanding the model's state space or solving for an infinite sequence of undetermined $MA(\infty)$ coefficients, which is the standard approach to solve models with lagged expectations in the time domain, see, e.g., Mankiw and Reis (2002) and Meyer-Gohde (2010). The transformed model can then be solved with the methods of Klein (2000) and Whiteman (1983). Without the transformation of the model into the frequency domain, the conditions on monetary policy rules to ensure determinacy cannot be obtained for the sticky information model and therefore important implications for the stabilization of the economy are missed.

We also show that the solution derived in frequency domain is not model-specific and can be applied to a wide range on macroeconomic models. It therefore complements the solution methods on LRE models and proposes a well-suited alternative. It thereby adds to the ongoing research on solving macroeconomic models in the frequency domain and to the implementation of information frictions in economic models.

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Appendix

6.1. Additional results go here

Following Mankiw and Reis (2002) Section 3, consider alternatively

$$\Delta m_t = \pi_t + y_t - y_{t-1} \tag{A1}$$

where the money growth evolves according to an AR(1) process:

$$\Delta m_t = \rho \Delta m_{t-1} + \epsilon_t \tag{A2}$$

The frequency domain representation is:

$$\Delta m(z) = \frac{1}{1-\rho z} \tag{A3}$$

The sticky information Phillips curve is given by

$$\zeta(1-\lambda z)y(z) = \lambda\pi(\lambda z) + \zeta\lambda(1-\lambda z)y(\lambda z) \tag{A4}$$

The frequency representation of (A1) is

$$\Delta m(z) = \pi(z) + (1-z)y(z)$$

which can be rearranged to:

$$\pi(z) = -(1-z)y(z) + \Delta m(z)$$

Combining these equations gives

$$\begin{aligned}
\zeta(1-\lambda z)y(z) &= \lambda(-(1-\lambda z)y(\lambda z) + \Delta m(\lambda z) + \zeta\lambda(1-\lambda z)y(\lambda z) \\
\zeta(1-\lambda z)y(z) &= \lambda(\zeta-1)(1-\lambda z)y(\lambda z) + \Delta m(\lambda z)
\end{aligned}$$

Rearranging:

$$y(z) = \lambda\frac{\zeta-1}{\zeta}y(\lambda z) + \frac{1}{\zeta}\frac{\lambda}{\zeta}\frac{1}{1-\lambda z}\Delta m(\lambda z) \tag{A5}$$

A recursion in $y(z), y(\lambda z), \ldots$ gives:

$$\begin{aligned}
y(z) &= \frac{\lambda}{\zeta}\left(\frac{1}{1-\lambda z}\Delta m(\lambda z) + \lambda\frac{\zeta-1}{\zeta}\frac{1}{1-\lambda^2 z}\Delta m(\lambda^2 z) + \ldots\right) \\
&= \frac{\lambda}{\zeta}\left(\sum_{i=0}^{\infty}(\lambda\frac{\zeta-1}{\zeta})^i\frac{1}{1-\lambda^{i+1}z}\Delta m(\lambda^{i+1}z) + \lim_{i\to\infty}(\lambda\frac{\zeta-1}{\zeta})^i y(\lambda^i z)\right)
\end{aligned}$$

If $|\lambda\frac{\zeta-1}{\zeta}| < 1$ and $|\lambda(1-\frac{1}{\zeta}| < 1$

$$y(0) = \lambda\frac{\zeta-1}{\zeta}y(0) + \frac{\lambda}{\zeta}\Delta m(0)$$

where the derivative of $y(\lambda z)$ evaluated at the frequency z is

$$\frac{\partial y(\lambda z)}{\partial z} = \lambda y'(\lambda z) \quad , \frac{\partial^j y(\lambda z)}{\partial z^j} = \lambda^j y^j(\lambda z)$$

and:

$$y_j = \lambda^{j+1}\frac{\zeta-1}{\zeta}y_j + \frac{\lambda}{\zeta}\left(\frac{\partial}{\partial z}(\frac{1}{1-\lambda z}\Delta m(\lambda z))\right)$$

Then from (A5)

$$(1-\lambda z)y(z) = \lambda\frac{\zeta-1}{\zeta}(1-\lambda z)y(\lambda z) + \frac{\lambda}{\zeta}\Delta m(\lambda z)$$

Equivalently to fire, the first order derivative of $(1-\lambda z)y(z)$ is given by

$$\frac{\partial(1-\lambda z)y(z)}{\partial z} = -\lambda y(z) + (1-\lambda z)y^1(z)$$

The second order derivative

$$\begin{aligned}\frac{\partial^2(1-\lambda z)y(z)}{\partial z} &= -\lambda y^1(z) - \lambda y^1(z) + (1-\lambda z^2)y^2(z)\\ &= -2\lambda y^1(z) + (1-\lambda z)y^2(z)\end{aligned}$$

Moreover, the j'th derivative evaluated at frequency z^j is

$$\frac{1}{j!}\frac{\partial^j(1-\lambda z)y(z)}{\partial z^j} = -j\lambda y^{j-1}(z) + (1-\lambda z)y^j(z)$$

which evaluated at the origin is:

$$\frac{\partial^j y(z)}{\partial z^j}|_{z=0} = y_j - j!\lambda y_{j-1} + j!y_j$$