# Fair Gatekeeping in Digital Ecosystems*

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*Abstract.* Do users receive their fair contribution to digital ecosystems? The frequent accusations of excessive platform fees and self-preferencing leveled at dominant gatekeepers raise the issue of the standard gatekeepers should be held to. The paper provides a framework to explain business strategies and assess regulatory proposals. It stresses the key role played by the zero lower bounds on core and app prices in the setting of privately and socially optimal platform fees. Finally, it derives a simple rule for regulating access conditions and analyses its implementation.

*Keyworks.* Ecosystems, fair access, price and non-price foreclosure, platforms, zero lower bounds.

*JEL numbers.* L12, L4.

## 1 Introduction

Platforms – the gatekeepers of the digital economy – control sellers', app developers' and advertisers' access to their consumers. They do so through consumers' use of their "core services", which, under EU law for instance, may signify a search engine, a marketplace, a social network, an app-store or a video-sharing platform. Recent regulatory guidelines, including the EU Digital Market Act (DMA) and the proposed American Innovation and Choice Act and Open App Markets Act, echo earlier public-utility regulations and the antitrust treatment of essential facilities; they require that business users be given equal access to the core services and receive a fair share of their contribution to the ecosystem. They reflect two key concerns.[1]

First, platforms operate markets, but also compete in them; there have been many allegations

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[1]See Online Appendix A for a brief description of the antitrust cases related to access conditions and of the regulatory proposals.

that platforms engage in self-preferencing. Relatedly, even pure platform players (like Airbnb or Booking, which operate markets, but do not compete in them) may enter "sweet deals" with specific business users. Guidelines therefore require platforms to offer a level playing field among all business users, internal or external. Second, the magnitude of the merchant fees has always raised eyebrows, from the 2 or 3% levied by numerous payment systems to the 30% demanded by Apple's app-store or Google Play. A number of ongoing antitrust cases (Epic Games v. Apple; Apple v. Spotify) concern 3rd party apps trying to circumvent app-store fees they deem unfairly high. The DMA's suggestion that access condition be fair, reasonable and non-discriminatory (FRAND) leaves open the question of what "fair and reasonable" conceptually means, even putting aside the measurement issue.

This "fairness" demand raises several questions. When are platform business users expected to enjoy equal access to the gatekeeper's core service under laissez faire, or conversely is the platform likely to engage in self-preferencing? Can we design good rules to monitor fees charged to merchants and advertisers by digital platforms such as Apple, Amazon, Booking, Google or Facebook? Should platforms be prohibited from entering complementary segments, i.e., will they continue living from fees or will they charge consumers for core services? Looking ahead, the answer to these questions will determine the business model of our largest firms, and in part the growth of our economies.

This paper provides a first answer to the question of how antitrust authorities can enable users to get their fair share of their contribution to a digital ecosystem. While there is a rich and important literature on pure-player and hybrid platforms, which we will later review, this literature mostly ignores two key features of the digital economy: Both core services (search, marketplaces, etc.) and digital goods (let's call them apps) are often offered to consumers for free, implying that platforms and apps would like to pay for usage but are prevented from doing so by consumer arbitrage.[2]

As of July 2022, 97% and 94% of apps in Google Play and Apple's app-store were freely available.[3] These include some of the most common 3rd party apps (e.g., PayPal, Dropbox), as well as the competing in-house apps by Apple and Google (e.g., Apple Pay and Google Pay, iCloud and Google Drive, respectively). On the core side, most digital platforms, such as the major app-stores, e-commerce platforms, search engines and social networks, grant free access to consumers. As is well known, a zero price on the consumer side is not to be taken as granted, as it hinges on the platform's ability to monetize the consumer's data or the other sides of the market – namely, 3rd party sellers and advertisers.[4] One of our main contributions is precisely to show how a downward price constraint (which for convenience we label the "zero lower bound", ZLB) either on core services ("core ZLB") or on applications ("app ZLB") is essential to the

[2]Negative prices would attract fake consumers (say, bots) who would not buy on the platform, nor be receptive to advertising, and supply meaningless data.

[3]See, e.g., https://www.statista.com/statistics/263797/number-of-applications-for-mobile-phones/.

[4]For instance, after its ads-business has been severely hurt by Apple's ad-tracking changes on iOS, and following the recent broader pullback in digital ad-spending, Meta created a new division with the aim of building paid products across Facebook, Instagram and WhatsApp. See https://www.theverge.com/2022/8/31/23331342/meta-plans-paid-features-facebook-instagram-whatsapp.

understanding of platforms' business strategies and desirable policy interventions.

Our basic model considers a two-sided platform that connects sellers of digital goods with consumers patronizing the platform. The platform levies per-unit access charges on 3rd party app sales (our results are robust to ad-valorem fees), can charge a non-negative access price to consumers, and operates as a hybrid marketplace: It sells its own in-house apps in competition with the 3rd party sellers' ones. A key feature is that apps' developers may receive ancillary benefits from attracting a consumer, such as advertising revenues (content providers), data collection (most apps), or ancillary services and add-ons (Dropbox); we capture these per-consumer benefits by a number $b > 0$. In the digital, low-or-zero-marginal-cost world, ancillary benefits imply that the app's opportunity cost of supplying a customer is negative, raising the issue that competitive prices are infeasible. The app ZLB then forces the developer of an inferior app to settle for a complimentary use of the app.

To understand why these ZLB-based departures from existing theory matter, it is useful to return to the old Chicago School critique of foreclosure theory, which can be stated for the platform context in the following way: "*Aside from efficiency motives, a platform (the monopoly segment) has no incentive to foreclose a 3rd party app (an independent player in the competitive market): A rich ecosystem benefits consumers in two ways, product variety and enhanced competition, and allows the platform to raise its consumer price to extract the associated increase in consumer surplus.*"

*(a) Impact of the access charge.*

To make our point in the starkest possible form, Section 2 develops the simplest possible model of complementary core and app segments, with zero marginal costs. We show that, if none of the two ZLBs is binding, the Chicago School's "rich ecosystem argument" prevails. Furthermore, the access charge levied by the platform on 3rd party sellers' sales is neutral: It has no allocative or redistributive consequences. Novel insights emerge when either the app ZLB or the core ZLB bind:

*App supranormal profit.* For per-consumer access charges $a$ smaller than the ancillary benefit ($a < b$), superior 3rd party app developers obtain a supranormal profit, i.e. they receive more than the value they create. The reason is that they do not feel the full competitive pressure from the in-house (or other inferior) apps that are constrained by the app ZLB. Because its wholesale activity is insufficiently profitable, the platform is then tempted to engage in self-preferencing and to foreclose the 3rd party app.

*Impossibility to cash in on a rich ecosystem.* A binding ZLB on the core product price means that a zero price for consumer access to the platform is already too high; the platform would like to subsidize consumers' participation in the first place and therefore may not want to raise the core price above zero, even to reflect a richer ecosystem as the Chicago School argument would have it. We show that the core ZLB is binding when the access charge is high (but not so high that the 3rd party apps exit the market); a high access charge implies high app prices, making it necessary for the platform to stop charging for the core product in order to maintain the consumers on the platform. The $3^{\text{rd}}$ party app is then squeezed, in that it receives less than its contribution to the ecosystem.

The analysis also unveils a deep connection between the ZLBs, fairness, and Baumol and Willig's "Efficient Component Pricing Rule" (known through its acronym ECPR). The latter defines a cap for the access charge equal to the vertically integrated firm's equilibrium mark-up in the competitive segment. The region in which the platform wishes to foreclose the $3^{\text{rd}}$ party app ($a < b$) coincides with the region in which the access fee lies below the ECPR level and the $3^{\text{rd}}$ party app receives more than its contribution to the ecosystem. The competitive neutrality region exhibits an equality between the access charge and the ECPR level, and a fair reward for the $3^{\text{rd}}$ party app. Finally, the squeeze region has an access charge above the ECPR level and an unfair compensation of the $3^{\text{rd}}$ party app.

*(b) Pigouvian regulation.*

In the basic model, these two ZLBs do not play out in the same circumstances (as we will see, the two ZLBs may bind simultaneously in extensions of the basic model). The app ZLB arises for low access fees, when the platform has too little skin in the game when providing access. The core ZLB operates for high access fees, which entails squeeze of the superior $3^{\text{rd}}$ party sellers, who then receive only a fraction of their contribution to the ecosystem. Such a margin squeeze is profitable for the platform and emerges under laissez faire, i.e. when the platform chooses the access fee: The platform optimal access charge fully squeezes the $3^{\text{rd}}$ party app. Capping the access charge to any level in the competitive neutrality region (i.e., the intermediate region where no ZLB binds) is thus needed to ensure that $3^{\text{rd}}$ party developers appropriate the social value of their innovations, while at the same time preventing self-preferencing.

Yet, provided the $3^{\text{rd}}$ party apps already exist, all outcomes in which they are not foreclosed are ex-post equivalent from a social welfare standpoint, since the margin squeeze has purely redistributional effects. The unregulated outcome is socially efficient, so that one may argue that there is little scope for regulation besides fairness concerns.

To refine the optimal access charge regulation, Section 3 introduces additional desiderata:

*(i) Investment incentives:* To have proper incentives to invest (i.e., to develop the app if and only if the expected development cost is smaller than the contribution to the ecosystem), the $3^{\text{rd}}$ party app developer must receive its fair contribution to the ecosystem. In contrast, a squeezed $3^{\text{rd}}$ party seller has a suboptimal incentive to develop its app in the first place; and the impact of the access charge on the richness of the ecosystem is accounted for by the platform, but incompletely so. Therefore, letting $v$ denote the value of the in-house app for the consumer, from an app investment incentive viewpoint the optimal access charge must belong to the range $a \in [b, b + v]$, over which there is no squeeze (Section 3.1). This condition holds whether foreclosure is monitored or not.

*(ii) Avoidance of double marginalization:* When consumers' demand for the platform's in-house app is elastic, the higher the access charge, the larger the negative externality of the $3^{\text{rd}}$ party

app developer's pricing decision onto the platform owner. Even when the core ZLB binds, welfare-neutrality obtains as long as $a \geq b$ is not too large. This is because the $3^{\text{rd}}$ party app does not pass through increased fees to avoid losing customers. However, as the app developer's margin is correspondingly squeezed, proper investment incentives require that $a = b$ (Section 3.2).

*(iii) Efficient choice of app:* When consumers prefer the $3^{\text{rd}}$ party app but differ with respect to their preference for it (are heterogeneous in their willingness to pay for the $3^{\text{rd}}$ party app relative to the in-house app), the lower the price difference between the two apps, the more consumers will use the superior $3^{\text{rd}}$ party app, and so the more efficient the allocation. Because cheap access ($a < b$) is not an option when non-price foreclosure is feasible, $a = b$ is optimal (Section 3.3).

Combining all the desiderata requires that, when foreclosure is not monitored, the access charge be at the Pigouvian level: The regulated access fee should coincide with the ancillary benefit associated with app distribution: $\hat{a} = b$.[5] The reason why this can be interpreted as a Pigouvian access charge is that the $3^{\text{rd}}$ party app "steals" $b$ from the in-house app when taking a consumer away from it, thereby setting $\hat{a} = b$ gives the independent seller incentives to internalize this externality.

Section 4 considers platform competition. Section 4.1 shows that the core ZLB binds as platforms compete to become the bottleneck of access to consumers. As in the case of a monopoly platform, competing platforms fully squeeze the $3^{\text{rd}}$ party apps, and the core ZLB prevents the corresponding profits from being passed through to consumers. Fierce platform competition incentivises platforms to price in-house apps aggressively (i.e., below the opportunity cost). This constrains superior $3^{\text{rd}}$ party sellers' pricing, preventing their prices to increase in response to higher access charges, whenever, individually, they cannot affect the allocation of single-homing consumers across platforms. As a result, platform competition does not reduce the desirability of regulating/capping access charges.

Section 4.2 demonstrates that platform competition and app-store competition work very differently. While Section 4.1 showed that platform competition is very merchant unfriendly (too much so), Section 4.2 demonstrates that app-store competition is very merchant friendly (too much so). If, following recent regulatory prescriptions, designated platforms must give access to external app-stores, creating app-store rivalry that itself precludes any access charge, there is no levy along the value chain on merchants, who are then empowered. Superior apps make supranormal profits. Consumers surplus is the same under platform competition and under app-store competition, so app-store competition transfers rents from platforms to merchants. Overall, we should not expect competition to remove inefficiencies in the digital world of ZLBs.

Section 5 turns to the implementation of the Pigouvian rule, according to which the access

[5] If foreclosure can be monitored, then setting the access charge below the Pigouvian level is equally good to prevent double marginalization, as long as $a$ is not too low (so as to avoid an inefficiently large price for the core), and would strictly improve welfare on the front of consumers' efficient choice of app ($a = 0$ maximizes the share of consumers who patronize the superior app).

charge should be equal to the ancillary benefit. Can ancillary benefits be measured? In principle, merchant and advertising fees might be measurable, although the platform may try to prevent the regulator from observing $b$ (subsidiaries abroad in charge of advertising, bundling with other services, etc.), which implies that some interventions may be needed on that front to secure measurability. It may be more difficult to value data and consumer lock-in benefits.

Section 5.1 shows that, under non-monitoring of foreclosure, the relevant information cannot be elicited from the platform, even if the regulator knows the distribution of ancillary benefits among the various app categories. The latter's incentive indeed is to charge high access charges to low ancillary-benefit markets, so as to profitably squeeze superior sellers in these markets, and foreclose superior apps in markets where ancillary benefits are higher (where it is constrained to set lower fees, for the distribution of access charges to mimic that of benefits). Section 5.2 in contrast shows that the information about the ancillary benefits can be obtained from app developers, provided the platform can deny them access (which prevents the app developers from being too greedy). Section 5.3 then discusses the robustness of this important, and perhaps counterintuitive, insight. It shows that an appeal procedure can be a very useful complement to the previous elicitation rule, even if the court's measurement of the ancillary benefits is imprecise.

Section 6 concludes. Omitted proofs and additional material can be found in the Online Appendix.

**Relevant literature.** There is a large literature on foreclosure practices and the essential facility doctrine (e.g., Hart and Tirole, 1990; Rey and Tirole, 2007), and on access pricing for one-sided markets (e.g., Laffont and Tirole, 1994) and for telecom and payment card markets (e.g., Armstrong, 1998; Laffont et al., 1998; Rochet and Tirole, 2002, 2011). The literature on access to regulated bottlenecks showed that the regulation of access is needed, as a vertically integrated incumbent has little incentive to provide access to competitors. A celebrated rule, the ECPR (or Baumol-Willig) rule states that the access charge should be no greater than the vertically integrated monopolist's lost margin in the competitive retail segment. Its properties are analysed in Laffont and Tirole (1994); obviously it just connects two prices and says little about their absolute level. Another classic implication of the theoretical analysis is that an access mark-up does not necessarily mean that competitors are disadvantaged, as the mark-up increases the opportunity cost of the vertically integrated firm and its rivals alike.[6]

Three papers study the regulation of platform fees when the consumer and the merchant multi-home and can transact through multiple channels: the platform and another channel (direct purchases, other platforms, other payment methods in the case of a payment platform). Because the consumer chooses the channel, the welfare analysis is naturally grounded in the externalities associated with this choice.

[6]This is important because marginal-cost pricing of access is not the right welfare benchmark. It jeopardizes the recovery of fixed costs for the essential infrastructure owner and it further incentivises foreclosure ("self-preferencing" in modern parlance), requiring heavy investment in regulatory capacity: The vertically integrated firm cannot make money by selling access and therefore must make its money on the competitive segment.

Two papers suppose that the merchant offers the same price regardless of the channel (there is a most-favored-nation, MFN, clause); the merchant's revenue from a sale is then channel-independent, which does not mean that its markup is. The merchant may enjoy a convenience benefit from the platform channel, as in Rochet and Tirole (2011): A card payment may dominate cash and cheque in terms of expediency, fraud prevention, accounting, or absence of hold up. The socially optimal access charge corrects for externalities of consumer channel choice upon merchants, and the socially optimal access charge (which in payment networks is at least partially passed through by issuers to consumers) is equal to the merchant benefit from a card usage;[7] this internalization principle is the so-called *tourist test*. In Gomes and Mantovani (2021), the platform creates an informational and a convenience benefit for consumers; in particular, the platform offers products that they were unaware of. This improved-opportunities benefit of the platform is of course internalized by consumers. But, consumers' access to the platform being free, they do not directly reward the platform for it, which is a problem if the platform is created only if sufficiently profitable. The platform however can charge consumers indirectly through the competing merchants' access charge, then passed through to consumers. Gomes and Mantovani show that, provided the presence of the platform does not increase aggregate sales, the welfare-maximizing access fee equals the sum of these two benefits.[8]

Alternatively, there may be no MFN and so prices are lower on the platform, which displays tougher merchant competition than the direct sale channel. The consumer may then choose to transact through the platform not because they prefer this channel, but because the latter lowers merchants' markups, at least in part a redistributive effect (Wang and Wright, 2022). The privately optimal fee may now fall short of the socially efficient one, which equals the platform's marginal cost of implementing the transaction plus the amount by which the platform, by intensifying seller competition, decreases the merchants' margins. Again, the merchants' pass-through of the access charge is key to restoring proper consumer incentives.

In contrast with these three papers, which hinge on consumers' choice of channel to interact with merchants, we assume that consumers single-home, whether there is platform competition of not: the platform is a "gatekeeper". The set of potential externalities under consideration is then rather different: (a) a vertically integrated gatekeeping platform may use non-price instruments to prevent consumers from accessing the best product; (b) the platform may jeopardize the existence of superior 3$^{rd}$ party apps by squeezing them through a high access charge; (c) the 3$^{rd}$ party app enjoys supranormal profit when the app ZLB binds.[9] The welfare-maximizing access charge is then equal to the opportunity cost for the platform of letting 3$^{rd}$ party sellers serve consumers, rather than as the benefits it brings to one or both sides of the market.

A number of recent papers examine platforms' incentive to vertically integrate, and the welfare effect of this vertical integration, in the presence of foreclosure and/or imitation concerns: see Etro (2021b, 2022a), Anderson and Bedre-Defolie (2021), Gutiérrez (2022), Hagiu et al. (2022)

[7] This socially optimal access charge is nonetheless smaller that the platform's preferred one.

[8] As the platform's profit-maximizing fee always exceeds this level, it is again optimal to cap platform fees.

[9] We also look at (d) externalities stemming from double marginalization.

and Zennyo (2022). Yet, these works, as the ones on platform fees' regulation, assume non-negative opportunity costs (i.e., rule out an app ZLB) and do not consider access pricing on the consumer side.[10] To be certain, one may argue that the widespread assumption that platforms grant free access to consumers in these papers reflects a core ZLB. However, they do not connect the validity of the underlying assumption with the level of seller access charges: They just assume that the access charge choice – whether by the platform or the regulator – is unconstrained, which we show cannot be the case.

Another closely related contribution to our paper is Choi and Jeon (2021). They show that tying may help a firm circumvent a non-negative price constraint in the tied (complementary) product market that prevents it from squeezing superior sellers in that market. Zero lower bounds do not usually emerge in standard models (e.g., Choi and Stefanadis, 2001, Carlton and Waldman, 2002), which assume that the tied market involves a positive marginal cost.[11] Unlike in this literature on tying, which does not consider access pricing, in our paper margin squeeze of superior 3rd party sellers by the platform does not necessarily occur via below-cost pricing in the tied (competitive) good market, but primarily via fees' extraction: In this case, it is the core ZLB, rather than the ZLB in the tied market (the app ZLB in our terminology), that binds.

In our model, below-cost pricing by the platform in the competitive segment may occur in equilibrium when allowing for heterogeneous consumers' value brought about by the 3rd party app (see Section 3.3). The mechanism is similar to the one at play in Chen and Rey (2012), who provide a rationale for loss leading by large retailers who face competition by more efficient, smaller, retailers, in a model where consumers have heterogeneous shopping costs.[12]

# 2 Impact of the access charge

## 2.1 Basic framework

Consider a two-sided platform (e.g., an app-store) that connects sellers of *digital goods* (hereafter, "apps") with consumers. Digital goods are assumed to entail negligible marginal costs of production and distribution. Rather, their usage by consumers entails several benefits for the app providers, such as advertising revenues, fees collected from merchants selling their products

[10]By considering access pricing both on consumer and seller side, our work relates to the literature on optimal pricing by two-sided platforms pioneered by Caillaud and Jullien (2003) and Rochet and Tirole (2003), though we abstract from cross-group externalities. This literature however is not concerned with hybrid platforms and mostly ignores ZLB constraints. In other papers on hybrid platforms, including Etro (2022b) and Padilla et al. (2022), app-stores are bundled with physical devices, so that consumers are always charged a positive price.

[11]For an earlier work on the effects of tying in two-sided markets where ZLB constraints may bind, see Amelio and Jullien (2012). They show that, in situations where a platform would like to set negative prices on one side of the market, tying serves as a mechanism to introduce implicit subsidies on that side. As a result, it can raise participation on both sides and benefit consumers.

[12]By pricing the competitive good below cost, and raising the price for the monopolized good (that is, consumers' access price) accordingly, the platform: (i) maintains the total price charged to consumers with low (extra-)willingness to pay (wtp) for the 3rd party app (corresponding to one-stop shoppers in Chen-Rey), who buy the in-house app; (ii) increases the margin earned on those with higher wtp, who buy the 3rd party app (Chen-Rey's multi-stop shoppers) in the monopolized segment; (iii) induces 3rd party sellers to reduce their prices (hence, squeezes their margin).

through the app, or consumers' data that can be monetized. Hence, their opportunity cost is negative. Sellers face a zero lower bound constraint because negative prices are subject to arbitrage: Bots and uninterested consumers may take advantage of the payment for usage, and yet bring no profit for merchants and advertisers and provide valueless data.

As described in Figure 1, the platform: (i) is a gatekeeping platform, and charges consumers a fixed access price; (ii) adopts an agency business model (app providers pay access fees for distributing their apps and set their prices); and (iii) operates a hybrid marketplace, i.e. also distributes its in-house app in the app-store.

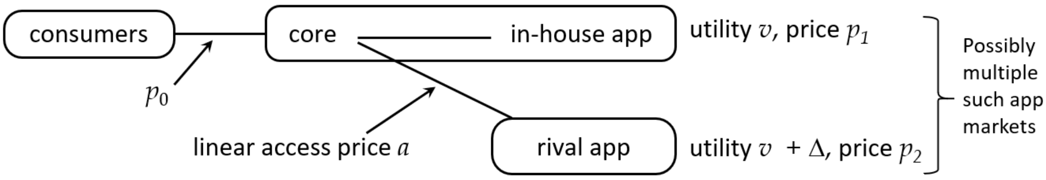


Figure 1: Two-sided market.

Two apps, one in-house and another one supplied by a 3rd party, compete for the platform's customers (there may more generally be multiple such app submarkets that depend on access to the platform). Consumers have unit demand, and benefit $v \geq 0$ (resp. $v + \Delta$, with $\Delta > 0$) when using the platform's in-house app (resp. the 3rd party app).[13] We assume that the platform brings no per-se value to consumers (independently of the consumption in the competitive segment). This assumption, which will be later relaxed, is a good approximation for app-stores (Apple's App Store and Google's Play Store), OTAs and other reservation systems (Booking, Airbnb and Uber), and e-commerce platforms (Amazon and eBay).

Let $x = 1$ if consumers buy the 3rd party app and $x = 0$ if they buy the in-house one. The platform levies a unit access charge $a \geq 0$ on apps distributed by the 3rd party provider (Online Appendix C.2 shows that our insights are unchanged considering instead ad-valorem access charges). In the following, $b \geq 0$ denotes the unit benefit accruing to the app provider (advertising, value of data, consumer lock-in), $p_0$ the consumers' access price to the app-store, and $p_1$ (resp. $p_2$) the price of the in-house app (resp. the 3rd party app). The platform's profit when the consumers patronize the platform can be written as the profit, $p_0 + a$, it would make as a pure platform, plus (if $x = 0$) the extra-profit, $p_1 + b - a$, it would obtain by capturing the app market as well:

$$\pi_1 \equiv p_0 + a + (1 - x)(p_1 + b - a),$$

while the 3rd party app developer's profit is:

$$\pi_2 \equiv x(p_2 + b - a).$$

We first assume that non-price foreclosure is impossible. We then allow the platform to make

[13] The case of an inferior 3rd party app ($\Delta < 0$) is uninteresting as it would play no role in this framework.

its competitor less attractive (i.e., to engage in self-preferencing), and to pick, at no cost, any 3rd party app's competitive advantage (or disadvantage if negative) $\delta \leq \Delta$. We will employ "foreclosure" and "self-preferencing" indifferently in our context. By contrast, $\delta = \Delta$ is the only option for the platform when a regulator can and does monitor non-price foreclosure. Throughout the analysis in this section, we take the access charge $a$ as given (whether set by regulation or by the platform) and consider simultaneous pricing choices. The timing is given in Figure 2.

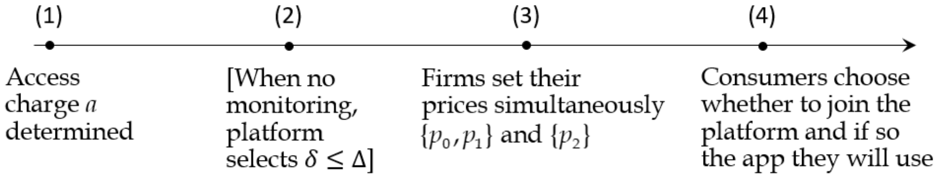


Figure 2: Timing.

Before moving forward to the equilibrium analysis, we introduce some useful definitions:

*Definition (competitive neutrality).* The access charge $a$ is competitively neutral in a range $[\underline{a}, \bar{a}]$ if, in this range, (i) the platform has no incentive to use non-price instruments to foreclose the 3rd party app (even if it can), and (ii) the equilibrium profits, $\pi_1^*(a)$ and $\pi_2^*(a)$, and the allocation $x$ are independent of $a$ over the range.

*Definition (fairness and squeeze).* The 3rd party app developer receives its fair share of its contribution to the ecosystem if $\pi_2^*(a) = \Delta$. The 3rd party app developer is squeezed if (i) the platform does not foreclose it ($\delta = \Delta$), but (ii) $\pi_2^*(a) < \Delta$.

*Definition (zero lower bounds).* The app zero lower bound (ZLB) binds if $p_1^* = 0$. The core ZLB binds if $p_0^* = 0$.

*Definition (ECPR level).* The access charge is below (equal to, above) the Baumol-Willig efficient component pricing rule level if $a$ is smaller than (equal to, higher than) the unit profit, $p_1 + b$, lost by the platform when the 3rd party app attracts a consumer.

Finally, the simultaneity of price choices gives rise to a multiplicity of equilibria, as is familiar from Nash demand games. We look for an equilibrium in which the platform is "pivotal" whenever unconstrained by the core ZLB:

*Definition (platform pivotality).* An equilibrium of the price subgame exhibits platform pivotality if *(a)* the in-house and 3rd party apps play the Bertrand equilibrium $\{p_1^*, p_2^*\}$ in undominated strategies of the pure app-pricing game that would prevail were the consumers already present on the platform; and *(b)* at those app prices, consumers are strictly willing to join the platform when the consumer membership price is nil ($p_0 = 0$).

Intuitively, in a platform pivotality equilibrium, the 3rd party app does not feel responsible for attracting consumers to the platform. To see what pivotality implies, suppose that the consumers' participation on the platform is a foregone conclusion. The app providers have the same opportunity cost $a-b$ and do not charge below this level in an equilibrium in undominated strategies.

- If this opportunity cost is non-negative ($a-b \geq 0$), the standard Bertrand equilibrium in undominated strategies has $p_1^* = a-b$ and $p_2^* = (a-b)+\Delta$. The pivotality condition requires that the access charge not be so high that the core ZLB binds, i.e., that the consumers' utility be non-negative when access to the platform is free; using the fact that consumers are in equilibrium indifferent between the in-house and 3$^{\text{rd}}$ party app, platform pivotality requires $a-b \leq v$.
- When the opportunity cost is negative ($a-b<0$), then the platform app cannot charge an app price $p_1$ below 0 due to the app ZLB and therefore sets $p_1^* = 0$ while the 3$^{\text{rd}}$ party app is priced at $p_2^* = \Delta$. The consumers obtain surplus $v > 0$ and so the platform pivotality condition is automatically satisfied.

When $a > b+v$, the core ZLB necessarily binds and so the the 3$^{\text{rd}}$ party app must set its price so as to attract consumers to the platform, which requires picking $p_2^* = v+\Delta < (a-b)+\Delta$.

## 2.2 Absence of foreclosure

The analysis in Section 2.1 proves part *(i)* of Proposition 1 below. In the tradition of Nash (1950), Proposition 1 *(ii)* shows that, under a further and reasonable condition, this equilibrium is actually the robust one, in the sense that when there is small uncertainty about the consumers' willingnesses to join the platform (there is a smooth distribution $F(\tilde{v})$ over types, converging to a spike at $v$), the equilibrium with heterogeneous types converges to the equilibrium in which the platform is pivotal. The proof of Proposition 1 *(ii)* can be found in Online Appendix C.4, where we also show that our main results are robust when considering a pivotal, *large*, app.

**Proposition 1** (retail prices in the absence of foreclosure)**.** *Suppose that the platform cannot use non-price strategies to foreclose the 3$^{rd}$ party app.*

*(i)* Equilibrium: *Under platform pivotality, the app ZLB binds ($p_1^* = 0$) if and only if $a < b$. Furthermore,*

- *when $a < b$,*

$$\begin{cases} p_1^* = 0 \quad \textit{and} \quad p_2^* = \Delta, \\ \textit{Supranormal app profit:} \quad \pi_2^*(a) = \Delta + (b-a) > \Delta. \end{cases}$$

- *when $b \leq a \leq b+v$*

$$\begin{cases} p_1^* = a-b \quad \textit{and} \quad p_2^* = p_1^* + \Delta, \\ \textit{Fair reward:} \quad \pi_2^*(a) = \Delta. \end{cases}$$

- *when $b+v < a \leq b+v+\Delta$:*

$$\begin{cases} p_1^* = a-b \quad \textit{and} \quad p_2^* = v+\Delta < p_1^* + \Delta, \\ \textit{Squeeze:} \quad \pi_2^*(a) = b+v+\Delta-a < \Delta. \end{cases}$$

*(ii)* Robustness: *A necessary and sufficient condition for the platform pivotality equilibrium*

*to coincide with the limit of the equilibrium when the smooth distribution of consumers' willingnesses to pay, $F(\tilde{v})$, converges to a spike at $v$ is:*

$$v \geq \Delta + b.$$

When $a \leq b$ and in the absence of foreclosure, the superior 3rd party app does not feel the full competitive pressure from the in-house app, that is constrained by the app ZLB ($p_1^* = 0$). The 3rd party app then makes supranormal profit ($\pi_2^*(a) > \Delta$).

When $a \in (b, b+v)$, the app ZLB does not bind. A change in the access charge increases one-for-one the 3rd party app's marginal cost and raises, also one-for-one, the platform's opportunity cost of supplying the app in-house. Charging the consumer or the 3rd party app for access are perfect substitutes: Hence, the neutrality result.[14] This competitive neutrality region is the counterpart of the Chicago School theory that a dominant firm does not benefit from tying. It shows that the Chicago School conclusion requires an access price that exceeds the benefit of attracting consumers on the app. The neutrality region exhibits the familiar "seesaw property" of two-sided-market theory, in which an increase in the merchant fee translates (in our case one-for-one) into a decrease in the consumer fee.[15]

When $a \in [b+v, b+v+\Delta]$, the core ZLB binds ($p_0^* = 0$), and the 3rd party app is constrained by the consumers' willingness to pay for the app (as $a - b + \Delta \geq v + \Delta$). The access charge is no longer neutral, as the 3rd party app developer must absorb its increase to keep customers. Hence, the 3rd party seller's margin is squeezed ($\pi_2^*(a) < \Delta$), and the platform appropriates part of the 3rd party app's contribution to the ecosystem.

*The link between ECPR and the ZLBs.* The equilibrium characterization unveils a deep connection between the ZLBs and Baumol and Willig's ECPR rule:

**Corollary 1** (ECPR)**.** *The access charge is*

- *below the ECPR level ($a < p_1^* - (-b)$) if and only if the app ZLB binds ($a < b$);*
- *at the ECPR level ($a = p_1^* - (-b)$) if and only if no ZLB binds ($b \leq a \leq b+v$);*
- *above the ECPR level ($a > p_1^* - (-b)$) if and only if the core ZLB binds ($a > b+v$).*

*Evidence on ZLBs.* The basic model has the virtue of simplicity while capturing the main forces of the gatekeeping environment. Its simplicity may imply overly strong restrictions, though. Indeed, at first sight, the evidence on ZLBs seem to run counter two subordinate implications of the basic theory. First, the latter predicts that the core and app ZLBs cannot bind simultaneously, while the data shows that frequently $p_0 = p_1 = 0$. The latter coexistence of binding ZLBs can however be rationalized in a variety of ways that are consistent with the

[14]For the relaxation of the one-for-one substitution assumption, see Gans and King (2005).

[15]Sullivan (2022) studies the impact of US city regulations that have capped the fee paid by restaurants to food delivery platforms to 15% (instead of the customary 30%). In the absence of MFN clauses, this cap resulted in a 9-22% increase in fees paid by consumers; while neutrality did not prevail for reasons studied by Sullivan, the pattern is a clear seesaw one.

model. Indeed, Section 3.2 and 4 will show that it arises when the demand for the core product is downward-sloping, or when there are competing platforms, respectively.

The second subordinate implication of the basic theory is that superior apps should fetch a positive price ($p_2 > 0$); this however does not need to be the case when the demand for the app is itself downward sloping; in this case and even ignoring traditional explanations for low prices such as introductory pricing or installed base building in the presence of network externalities, the equilibrium price of superior apps can be zero: see Section 3.3.

Moreover, many apps adopt the so-called freemium model – i.e., a basic version (with limited functionalities) is made available for free, while consumers are charged a positive price for the premium version (which includes all functionalities). The freemium model obtains in our framework if (i) the basic versions of both in-house and 3$^{\text{rd}}$ party apps provide the same value to consumers; (ii) there is consumer lock-in and no commitment to premium versions' prices (the ancillary benefit from distributing the basic app then stems from the extraction of consumer value for the proprietary premium version); and (iii) the platform levies ad-valorem fees on 3$^{\text{rd}}$ party sales. The same conclusions, including the desirability of access charge regulation, applies to the freemium environment: See Online Appendix C.3.

### 2.3 Foreclosure

Let us augment the strategy space by letting the platform choose a non-price foreclosure strategy (the platform is left unmonitored). Without loss of generality, we can assume that $\delta = \Delta$ (no foreclosure) or $\delta = -v$ (full foreclosure). Intuitively, the platform's choice determines which among the in-house and 3$^{\text{rd}}$ party apps the consumers will select. In the former case, making the 3$^{\text{rd}}$ party app worthless involves no loss of generality. In the latter case, picking $\delta < \Delta$ creates social waste and reduces the platform's ability to monetize its ecosystem.

**Proposition 2** (foreclosure)**.** *Left unmonitored, the platform engages in non-price foreclosure if and only if* $a < b$.

*Proof of Proposition 2.* When foreclosing, only the total price $p_0 + p_1$ matters, and so we can assume w.l.o.g. that $p_1 = 0$. The platform can achieve profit $v + b$. We interpret this profit as the value created by the platform's existence. When not foreclosing, the platform makes profit $\pi_1^*(a) = v + a$ if there is no squeeze (and more when there is a squeeze, which is the case for $a > b + v$). So, the platform wants to foreclose if and only if $a < b$. ■

In a nutshell, for $a < b$, the app ZLB binds and the platform does not have enough skin in the game to want to give access to its rival. Figure 3 indicates the platform's and the 3$^{\text{rd}}$ party app developer's profit, with and without foreclosure.

### 2.4 Platform-optimal, welfare-optimal, and fair access pricing

We define social welfare $W$ as the sum of consumer net surplus $S$ and the firms' profit: $W = S + \sum_{i=1,2} \pi_i$.

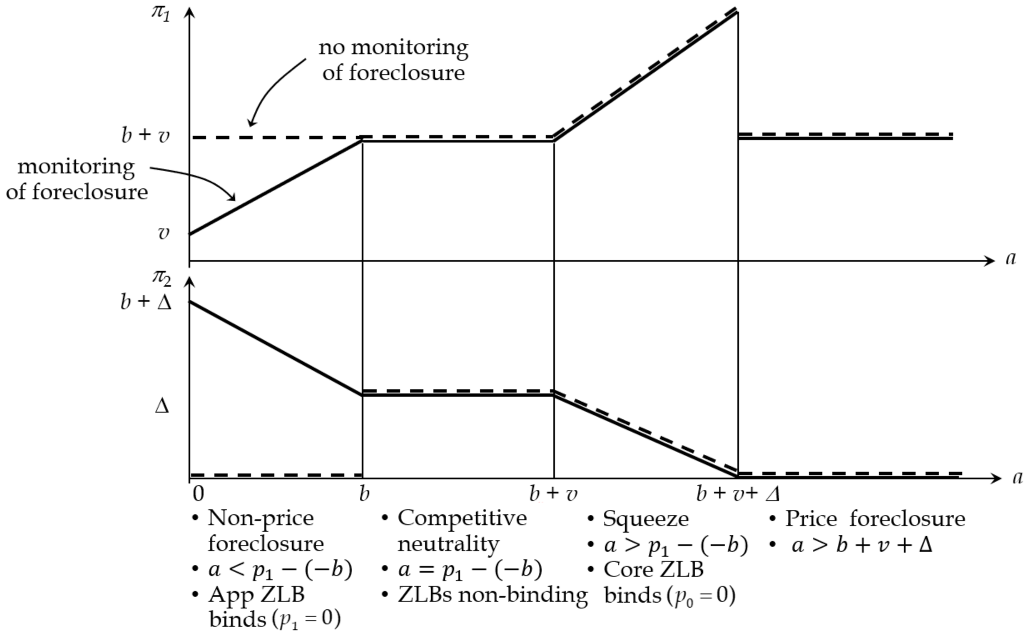


Figure 3: The dashed lines represent the profits when non-price foreclosure is feasible and the full lines the profits when it is not. They differ only when $a < b$.

**Proposition 3** (optimal access charges)**.** *The equilibrium of the game is such that:*

*(i) (Welfare-optimal access charges.) Any access charge such that the 3rd party app is not foreclosed maximizes ex-post social welfare:* $a \in [b, b + v + \Delta]$ *if non-price foreclosure cannot be monitored,* $a \in [0, b + v + \Delta]$ *under monitoring of self-preferencing;*

*(ii) (Profit-maximizing access charge.) Platform's profit is maximized at the extractive access charge* $a^* = b + v + \Delta$*, implying* $\pi_2^*(a^*) = 0$*;*

*(iii) (Fair access charges.) The independent developer receives a fair reward for its contribution to the ecosystem if and only if* $a \in [b, b + v]$*.*

*Proof of Proposition 3.* In this basic model, consumer surplus is always extracted by the platform through the access price,[16] and $W^* = \pi_1^*(a) + \pi_2^*(a) = b + v + \Delta x$ is maximized whenever there is no price or non-price foreclosure, so that $x = 1$, from which *(i)* follows. For $a = a^*$: $\pi_1^*(a^*) = W^*$, which establishes *(ii)*. Finally, the result in *(iii)* follows from the equilibrium profit $\pi_2^*(a)$ given in Proposition 1. ■

Because, for any $a$ in the competitive neutrality or squeeze regions, the platform has no incentives to foreclose the 3rd party app, whether or not self-preferencing can be monitored does not affect welfare.

Any access charge in the competitive neutrality region is such that the independent developer receives a fair reward for its contribution to the ecosystem. However, an unregulated platform would find it optimal to set a strictly higher access charge, such that the independent seller is fully squeezed.

[16] This will not be the case when the platform faces a downward-sloping demand: see Section 3.2.

## 2.5 Simple extensions

*Physical goods.* While our analysis mostly focuses on platforms' intermediating sales of digital goods (e.g., app-stores, search engines), it is also applicable to platforms' hosting sellers of physical goods (e.g., e-commerce) or services (e.g., OTAs or ride-hailing platforms) that entail positive marginal costs.[17] Let $\gamma_a \in (0, v_a)$ denote this unit cost. With ancillary benefit (repeat sales or consumer lock-in) $b > 0$, the net opportunity cost is then $\tilde{b} \equiv b - \gamma_a$. If this opportunity cost is non-negative, the foreclosure region becomes $a \in [0, \tilde{b})$; otherwise it is empty.

**Observation 1.** *The previous analysis carries over to physical goods, whenever their net opportunity cost is non-negative ($0 \leq \gamma_a < b$). Otherwise, there is no foreclosure for any access charge.*

In any case, the fair levels of the access charge $a \in [\tilde{b}, \tilde{b}+v]$ always lie below the platform-optimal level $a^* = \tilde{b} + v + \Delta$.

*Asymmetries in benefits from app distribution.* Throughout the paper, we assume that platform and 3rd party seller(s) reap the same benefit $b$ from app distribution. This need not be the case in reality: First, the platform may obtain a share of benefits – e.g., from data[18] – when the 3rd party app is sold; second, the benefits from app distribution may depend on app-specific and/or provider-specific features.[19]

To accommodate both cases, suppose that platform and 3rd party seller obtain side benefits $b_1(x)$ and $b_2(x)$, respectively, depending on which app is purchased ($x \in \{0, 1\}$), with $b_1(0) > b_1(1) \geq 0$, and $b_2(1) > b_2(0) = 0$: Each firm obtains higher benefits when its app is distributed; and while the platform can get some benefits if consumers buy the 3rd party app, the converse looks implausible. Let

$$\tilde{b}_1 \equiv b_1(0) - b_1(1) > 0, \quad \text{and} \quad \tilde{b}_2 \equiv b_2(1).$$

In this model, the platform prefers selling its own app rather than distributing the competing one if and only if $p_1 + b_1(0) \leq a + b_1(1)$. Hence,

$$p_1^* = \max(0, a - \tilde{b}_1) \quad \text{and} \quad p_2^* = \min\{p_1^* + \Delta, v + \Delta\}.$$

Suppose the 3rd party app brings a positive value, defined as the sum of the value $\Delta$ of its innovation and the (positive or negative) extra-benefits $(\tilde{b}_2 - \tilde{b}_1)$ its app distribution generates,

[17] We here keep assuming that the platform is a digital good that provides no per se value. The case in which the platform is (bundled with) a physical device with positive stand-alone value and production cost is examined in Section 4.2.

[18] For instance, the platform may have unique or shared access to some data generated by consumers' usage of 3rd party apps on its app-store – e.g., by processing in-app payments.

[19] On the one hand, $b$ may positively depend on app quality – e.g., more user engagement, associated to higher-quality apps, generates more data and ad-revenues; on the other hand, compared with smaller independent developers, the platform may have more bargaining power vis-à-vis advertisers, or extract more value from user data from any app, as these can be combined with other data on the same consumers obtained from other services it offers.

compared with the in-house app: $\tilde{b}_1 - \tilde{b}_2 \le \Delta$.[20] The foregoing analysis then goes through provided that $b$ is replaced by $\tilde{b}_1$, which measures the extra-benefit for the platform deriving from its in-house app compared to 3rd party app distribution. That is, for all $a \in [\tilde{b}_1, \tilde{b}_1 + v]$: *(i)* no ZLB binds; *(ii)* the platform obtains the same profit as under foreclosure: $\pi_1^*(\tilde{b}_1) = v + b_1(0)$; and *(iii)* the 3rd party seller appropriates the social value generated by its app, which constitutes its contribution to the ecosystem, and can thus be regarded as its fair compensation.

**Observation 2.** *The previous analysis carries over for general ancillary benefits, provided that the 3rd party app creates net value ($\tilde{b}_1 - \tilde{b}_2 \le \Delta$).*

Existing (GDPR, DMA) and forthcoming regulations aim at restricting the use of data, thereby reducing the ancillary benefits (as we noted, there are of course other sources of ancillary benefits). How does it affect the platform's incentive to foreclose? A uniform decrease in $b$, say because data sets cannot be combined or resold, reduces the incentive for foreclosure, keeping the access charge a constant; in contrast, the above analysis shows that the end of current arrangements in which the platform shares data with its apps would increase the incentive for foreclosure (by reducing $b_1(1)$ and so increasing $\tilde{b}_1$); put differently, such a move would have to be accompanied with augmented regulatory monitoring.

*Platform business model.* We take as granted throughout that the platform operates as a hybrid marketplace. Its business model can be easily endogenised in equilibrium. Suppose that in the status quo both apps in the marketplace are developed by 3rd party sellers (pure-player platform). Then, asymmetric Bertrand competition still implies that the low-value app is priced at $\max\{a - b, 0\}$. As app prices are as above, consumers' access price $p_0^*$, and platform's and superior seller's profits, are also unchanged, under the assumption of no-foreclosure.

From our analysis it then follows that, for all $a < b$, the platform has incentives to vertically integrate by acquiring the low-value seller at a negligible cost (given that the latter makes zero profits in equilibrium), and practice self-preferencing to foreclose the superior seller. For larger values of the access charge, the platform has no incentives to vertically integrate.[21]

Hence, absent access charge regulation, the platform could optimally operate as a pure marketplace and squeeze the superior seller's profit setting $a^* = b + v + \Delta$. Capping the access charge to any $a \in [b, b + v]$ prevents the platform from engaging in socially harmful vertical integration (combined with self-preferencing), and guarantees a fair reward to the innovative developer.[22]

**Observation 3.** *The previous analysis carries over when the low-value app is owned by a 3rd*

[20] If, on the contrary, $\tilde{b}_1 - \tilde{b}_2 > \Delta$, then at any access charge for which the 3rd party app is viable, the platform would foreclose it. Intuitively, the platform must trade off two inefficiencies: the lower ability of the 3rd party app to generate ancillary benefits (no foreclosure) and the consumers' lower demand for the in-house app (foreclosure).

[21] Note that there is no scope for acquiring the high-value app given that the sum of platform's and high-value app provider's profit is always (excluding the uninteresting Pareto-dominated price-foreclosure region) $b + v + \Delta$, which coincides with the vertically integrated platform's profit.

[22] Hence, access charge regulation is likely to outperform outright prohibitions of vertical integration on the ground of fairness, given that: (i) if fees are left unregulated, the platform can optimally engage in margin squeeze of sellers, even if it is not vertically integrated; and (ii) vertical integration in some cases benefits consumers – e.g., through increased product variety and/or lower prices.

*party provider, if either the access charge is not regulated or foreclosure is monitored. The pure-player platform gains from becoming hybrid by purchasing the low-value app if and only if $a < b$ and foreclosure is not monitored.*

*Location of value creation in ecosystem.* We assumed that the ecosystem's value is created solely by its app(s), which is a reasonable approximation for app-stores and many other platforms; by contrast, consumers attach per-se value to Google's search engine or Facebook's social networks, which are also core products. More generally, let $v_a$ and $v_c$ denote the values of the (in-house) app and the core, and $v \equiv v_a + v_c$ denote the total value. The 3$^{\text{rd}}$ party app brings value $v_a + \Delta$ to the consumer.

When the core brings value per se ($v_c > 0$), consumers can decide to patronize the ecosystem while dispensing with the app (*unbundling*). Because it is still the case that $p_1^* + \Delta \leq p_2^*$, a consumer in the ecosystem buys the (3$^{\text{rd}}$ party) app if and only if $p_2^* \leq v_a + \Delta$. To illustrate the difference it makes, consider the extreme case in which $v \equiv v_c$. For $a < b$, the platform still wants to foreclose the 3$^{\text{rd}}$ party app. For $a > b$, the 3$^{\text{rd}}$ party app cannot appropriate its contribution $\Delta$ to the ecosystem without losing its customers: $\pi_2^*(a) = \Delta + b - a < \Delta$. More generally, the competitive neutrality region corresponds to $a \in [b, b + v_a]$. Moreover, the core ZLB never binds with homogeneous consumers as the platform always extracts the value $v_c$ of the core: $p_0^* = v_c + v_a + \Delta - p_2^* \geq v_c$.[23]

**Observation 4.** *The lower the value $v_a$ created by the in-house app, the smaller the competitive neutrality region:*

- *For $a - b \leq v_a$, app prices and profits are the same as in Proposition 1 (with $v_a$ in place of $v$);*
- *The squeeze region corresponds to $a \in (v_a + b, a^*]$, where $a^* \equiv v_a + b + \Delta$ is the platform optimal fee;*
- *The platform charges consumers $v_c$ on top of the core price characterized in Proposition 1: The possibility of consumer unbundling creates a $v_c$-lower bound constraint.*

*When the in-house app is valueless ($v_a = 0$), the only access charge that (i) discourages foreclosure and (ii) provides a fair reward to the 3$^{rd}$ party app is $a = b$.*[24]

## 2.6 Summing up

The results of the basic model, in the more general case where $v \equiv v_a + v_c$, are illustrated in Table 1.

[23] The core ZLB may bind if consumers have heterogeneous willingness to pay for the core (see the remark at the end of Section 3.2). This is a fortiori true when, as in the case of search engines and social networks, the platform reaps ancillary benefits from consumers' participation even when they do not purchase apps.

[24] Even when there is no in-house app (hence, no foreclosure concerns), nor 3$^{\text{rd}}$ party competitor to the high-value app, $a = b$ is the unique level of the access charge ensuring that the superior 3$^{\text{rd}}$ party app developer receives a fair reward for its contribution to the ecosystem.

| 0 | $b$ | $b + v_a$ | $b + v_a + \Delta$ access price $a$ |
|---|---|---|---|
| **Non-price foreclosure** (access provider does not have sufficient skin in the game) | **Competitive neutrality** (Chicago School's "rich ecosystem argument" holds) | **Squeeze** | **Price foreclosure** |
| • $a$ below ECPR level<br>• Supranormal app profit (above $\Delta$) if no foreclosure<br>• App ZLB binds | • $a$ at ECPR level<br>• Fair reward (at $\Delta$)<br>• ZLBs do not bind | • $a$ above ECPR level<br>• Infranormal app profit (below $\Delta$)<br>• Core ZLB binds if $v_c = 0$ | • $a$ way above ECPR level |
| Self preferencing if no regulatory monitoring | Efficient ex-post allocation: $a$ neutral | Efficient ex-post allocation: $a$ squeezes 3rd party app | Superior 3rd party app out of the market |

Table 1: Access pricing's delicate balancing act.

## 3 Pigouvian regulation

In the simple model of Section 2,[25] all outcomes in which there is no foreclosure are equivalent from a total welfare standpoint, since consumer surplus is entirely captured by the platform via the consumer price $p_0$, and 3rd party seller's margin squeeze has only redistributional effects. Hence, even the extractive unregulated outcome ($a = a^*$) is socially efficient, so that one may argue that there is little scope for regulation.

In this section, we first show that this conclusion is unwarranted if the introduction of a superior 3rd party app in the marketplace is endogenous, and depends on the independent developer's incentives to invest in product innovation (Section 3.1). We then extend the model allowing for heterogeneous consumers' valuations for the core (Section 3.2) and for the extra quality of the 3rd party app (Section 3.3), deriving a Pigouvian principle that underlies optimal access charge regulation in more general environments. Concretely, we show that the regulated access fee should coincide with the ancillary benefit associated with app distribution:

$$\hat{a} = b.$$

The reason why this can be interpreted as the Pigouvian level of the access charge is that the 3rd party app "steals" $b$ from the in-house app when taking a consumer away from it, thereby setting $\hat{a} = b$ gives the independent seller incentives to internalize this externality.

The app ZLB prevents the platform from fully appropriating $b$ for all $a < b$. For these values of the access charge, the only way for the platform to appropriate this benefit is to foreclose the 3rd party app. The Pigouvian level of the access charge makes the platform indifferent between

[25] Let us return to the case in which the value is created in the competitive segment ($v_a = v$); the analysis generalises if the core also contributes to the ecosystem's value.

in-house app and 3rd party app distribution to any consumer, and so also indifferent between engaging or not in self-preferencing. As we shall see, capping, by regulation, the access charge to that level allows the other ecosystem participants (i.e., independent developers and consumers) to fully reap the benefits from the presence of superior apps.

### 3.1 Endogenous innovation

Suppose that, absent innovation, both the platform's in-house app and the 3rd party app bring value $v$ to consumers. In this case, the 3rd party seller makes zero profit, whereas the platform obtains $b + v$. Upon observing the access conditions (i.e., $a$ and $\delta$), the independent developer decides whether to sunk a cost $c > 0$ to introduce a superior version of the app, which brings an extra-value $\Delta$ to consumers. Suppose that the development cost $c$ is distributed according to a smooth cdf $G(c)$, with density $g(c)$ and monotone hazard rate, and its realization is privately observed by the developer. Proposition 1 and 2 then imply:

**Proposition 4** (fairness and innovation)**.** *With privately observed app developers' costs, welfare is maximized if and only if the access charge a lies in* $[b, b+v]$*, levels that are strictly lower than the platform's profit maximizing level* $a^*$*.*

Squeezed 3rd party sellers have a suboptimal incentive to develop their apps. The impact of the access charge on the richness of the ecosystem is accounted for by the platform, but incompletely so. As a result, an inefficiently low amount of innovation takes place under laissez faire. Capping, by regulation, the access charge to any level in the competitive neutrality region (i.e., $a \in [b, b + v]$) is needed to maximize social welfare. Considering independent developers' innovation incentives unveils a natural link between fair access pricing and welfare maximization.

*Competing sellers and imitation.* Access charges $a \in [b, b + v]$ also prevent socially inefficient innovations or imitation which arise when $a < b$ if foreclosure can be monitored. First, the 3rd party app's supranormal profit that arises when the access charge is low attracts investments whose cost $c$ exceeds the contribution, $\Delta$, to the platform's ecosystem. Second, assuming that the superior 3rd party app enters, other 3rd party apps could imitate it and, provided they have a tiny advantage, make a profit when $a < b$. When $a \geq b$ by contrast, such me-too, high-value apps would compete à la Bertrand with the innovator: Both apps would fetch $a - b \geq 0$, and imitation would yield zero profits. For $a \in [b, b + v]$, any 3rd party seller has the right incentives to innovate also because it does not fear expropriation of investment by imitators or latecomer competitors.

*Fair remuneration of innovations.* Lastly, let us discuss the notion of "fairness". The first point to make is that the call for reward $\Delta$ was made from the point of view of welfare maximization. A consumer standard might seem to lead to a social demand for some "taxation" of innovation in the form of a squeeze on profits, provided that the increase in access charge is passed through to consumers. But there is zero pass through in this model (the squeeze region coincides with the core ZLB one), and so the consumer standard and welfare maximization lead to the same conclusion on the innovation front. This conclusion is robust to consumer downward sloping demand for the platform (see Online Appendix C.1) and to platform competition (Section 4.1): whenever the core ZLB binds there is zero pass-through to consumers; and, whenever it does not bind, there is limited pass-through.

The second point relates to the "excessive innovation" that arises when the access charge lies below the ancillary benefit ($a < b$) and foreclosure can be monitored. One may be suspicious of worries about excessive innovation. Yet, this possibility is natural in the digital economy. A me-too innovation on the app segment, bringing along a small improvement $\epsilon$ in app quality, allows the innovator to corner the app market, engendering profit $(b - a)$. Indeed it is easy to envision a free-entry-into-the-app-market equilibrium in which the undue profit is dissipated by competition for this rent.[26]

### 3.2 Elastic platform demand

Assume that consumers' willingness to pay for the in-house app, $v \geq 0$, has wide support, and is distributed according to a smooth cdf $F(v)$ with density $f(v)$ on $\mathbb{R}^+$, and monotone (inverse) hazard rate $\rho(v) \equiv [1 - F(v)]/f(v)$.[27] If there is no $3^{\text{rd}}$ party app (or if the $3^{\text{rd}}$ party app is foreclosed), both the core and app ZLBs bind whenever

$$\arg\max_{p_0+p_1} (p_0 + p_1 + b)[1 - F(p_0 + p_1)] \leq 0 \iff b \geq \rho(0).$$

As this is a novel feature of this model compared to the foregoing analysis (where at most one ZLB binds),[28] in the remainder of this section we restrict attention to this region of parameters.[29] To ensure that the platform does not foreclose the $3^{\text{rd}}$ party app for all values of the access charge, we also assume that the rival app's (type-independent) quality advantage is $\Delta \geq b$.

Suppose that the app market is served by the $3^{\text{rd}}$ party app (there is no foreclosure). As the platform's opportunity cost of letting the $3^{\text{rd}}$ party app serve consumers is $a - b$, the in-house

[26] Suppose that there are $n$ innovators, that $a = 0$ (so that the app ZLB binds), and that for any $\epsilon \geq 0$, any innovator can bring along an innovation of size $\epsilon \in [0, b]$ (with $\epsilon = 0$ for the in-house app), at increasing and convex cost $C(\epsilon)$ (with $C(0) = 0$ and $C(b) > b$). In the mixed-strategy equilibrium in which any innovator's cumulative distribution of innovation size is $Z(\epsilon)$, such that $Z^{n-1}(\epsilon)b \equiv C(\epsilon)$, the rent $b$ is fully wasted.

[27] We assume that the platform is per-se valueless to all consumers (or offers them a known value). The analysis for the case in which consumers have heterogeneous valuations $v_c$ for the core (and homogeneous valuations $v_a$ and $\Delta$ for the apps) is provided in Online Appendix C.1.

[28] Can platforms or apps escape a ZLB through enhanced quality? Consider for example a platform facing a ZLB and the case in which there is only an in-house app (the same reasoning holds with a superior $3^{\text{rd}}$ party party app). Suppose that consumers' value for the platform service is distributed according to $F(v_c + v_a)$; and that the quality of the core product can be increased at per consumer cost $c \geq 1$ (if $c < 1$, the platform could create infinite value and monetize it); so starting at level $v_c^0$ the platform can deliver value $v_c$ at per-consumer cost $c(v_c - v_c^0)$. The platform solves

$$\max[p_0 + b - c(v_c - v_c^0)][1 - F(p_0 - (v_c - v_c^0))].$$

Then, the core ZLB binds and yet no quality improvement is made if and only if $b > \rho(0)$. When to the contrary the improvement in value costs a fixed, increasing and convex $K(v_c - v_c^0)$, the condition becomes the conjunction of $b > \rho(0)$ and $K^{'}(0) \geq 1 - F(0)$.

[29] For $b < \rho(0)$, the results are as in the base model: Competitive neutrality arises for any $a \geq b$ not too large, such that the core ZLB does not bind and the platform offsets the increase in price by the $3^{\text{rd}}$ party seller, as $a$ grows larger, through a reduction in the core price. Details are in Online Appendix B.2.

app is again priced at $p_1^* = \max\{a-b, 0\}$. Hence, whenever the core ZLB binds, the 3rd party seller solves

$$\begin{cases} \max_{p_2 \geq 0} [p_2 - (a-b)][1 - F(p_2 - \Delta)] \\ \text{s.t.} \quad p_2 \leq \max\{a-b, 0\} + \Delta \end{cases}$$

Ignoring the constraint, the first-order condition is

$$\frac{\partial \pi_2}{\partial p_2} \propto \rho(p_2 - \Delta) - [p_2 - (a-b)].$$

Because all consumers have a non-negative valuation ($F(0) = 0$), there is no point charging $p_2 < \Delta$. Can one have a corner solution at $\Delta$? The answer is yes, provided that the access charge not be too much above the ancillary benefit:

$$p_2^* = \Delta \iff \Delta - (a-b) \geq \rho(0).$$

In particular the 3rd party app provider is squeezed if and only if $a > b$. A consequence of this characterization is that the platform's profit is $[1 - F(0)]a$ whenever $a \leq b + [\Delta - \rho(0)]$ and $[1 - F(0)]b$ when it forecloses the 3rd party app. So foreclosure is again optimal if and only if $a < b$.

For larger values of the access charge ($a > b + \Delta - \rho(0)$), the 3rd party seller's problem admits an interior solution $p_2^*(a) \in (\Delta, a - b + \Delta)$. In particular, the monotone hazard rate assumption implies

$$0 < \frac{\mathrm{d}p_2^*}{\mathrm{d}a} < 1,$$

and so increases in the access charge generate double marginalization, thereby inefficiently reducing demand. The platform optimal access charge solves $\max_a a[1 - F(p_2^*(a) - \Delta)]$, and so

$$a^* \geq b + \Delta - \rho(0),$$

which trades off the gains from margin squeeze and the losses in demand due to double marginalization.

To summarize, we have:

**Proposition 5** (double marginalization)**.** *Suppose that consumers have heterogeneous valuations for the apps, distributed for the in-house app according to $F(v)$ with support $\mathbb{R}^+$ (with a quality advantage $\Delta$ for the 3rd party app), that the core ZLB binds under foreclosure ($b \geq \rho(0)$), and that $\Delta \geq b$. Then, regardless of whether foreclosure is monitored or not:*

- *Any access charge $a \in [b, b - \rho(0) + \Delta]$ maximizes consumer surplus and social welfare.*[30]

[30] For $a \in [\rho(0), b)$, as the 3rd party app extracts the extra value it brings to consumers ($p_2^* = \Delta$), a consumer surplus standard would view foreclosure with indifference. That is, even if foreclosure cannot be monitored, the set of consumer surplus maximizing access charges is $a \in [\rho(0), b - \rho(0) + \Delta]$. This coincides with the set of welfare optimal access charges if foreclosure can be monitored: lower access charges imply $p_0^* > 0$ and $p_0^* + p_2^* > \Delta$, and so inefficiently reduce demand.

- *These access charges lie weakly below the platform's profit maximizing level:* $a^* \geq b - \rho(0) + \Delta$ *(strictly so if and only if* $b + \Delta < \rho(0)[3 + \rho(0)^2 f'(0)]$*).*
- *The 3*[rd] *party app developer receives a fair reward for its contribution to the ecosystem if and only if* $\hat{a} = b$*: Any higher access charge leads to a squeeze, any lower charge to a supranormal profit or foreclosure.*

The last bullet point in the Proposition implies that capping the access charge at the Pigouvian level is needed to maximize welfare when considering endogenous innovation.

### 3.3 Heterogeneous quality valuations

This section introduces heterogeneity with respect to the perceived extra quality of the 3[rd] party app.[31] Suppose that all consumers have the same $v$, but $\Delta$ is distributed according to a smooth cdf $H(\Delta)$, with support $\mathbb{R}^+$, density $h(\Delta)$ and a monotone hazard rate.

For $p_0 + p_1 \leq v$, all consumers buy one app. Since a consumer with type $\Delta$ prefers the 3[rd] party app if and only if $\Delta \geq p_2 - p_1$, firms' profits are

$$\pi_1 = p_0 + a + H(p_2 - p_1)(p_1 + b - a),$$

and

$$\pi_2 = [1 - H(p_2 - p_1)](p_2 + b - a).$$

Since $\pi_1$ is increasing in $p_0$, the platform optimally sets $p_0 = v - p_1 \in [0, v]$, so that all consumers buy one app, and those buying the in-house app are left with no surplus. By doing so, it achieves a higher profit compared with the one attainable setting any larger prices so that $p_0 + p_1 > v$. For any given $a$, the equilibrium app prices and firms' profits are as follows:

**Lemma 1.** *Non-price foreclosure is optimal for the platform if and only if* $a < b$*. There are two thresholds* $(\underline{a}, \overline{a})$*, with* $b < \underline{a} < b + v < \overline{a}$*, such that:*

- *For* $a \leq \underline{a}$*, the app ZLB binds:* $p_1^* = 0 \leq p_2^*$*, with* $p_2^*$ *being increasing in* $a$*, and* $p_0^* = v$*.*
- *For* $a \in (\underline{a}, \overline{a})$*:* $0 < p_1^* < a - b < p_2^*$*, and* $p_0^* = v - p_1^* > 0$*, with* $(p_2^* - p_1^*)$ *and firms' profits being constant when* $a$ *varies.*
- *For* $a \geq \overline{a}$*, the core ZLB binds* ($p_0^* = 0$) *and* $p_1^* = v < p_2^*$*, with* $p_2^*$ *being increasing in* $a$*. The platform's profit is inverted U-shaped in* $a$*.*

By pricing the in-house app below the opportunity cost ($p_1^* < a - b$), and raising the consumer price of access accordingly, the platform maintains the total price ($p_0^* + p_1^* = v$) charged to consumers with low (extra-)willingness to pay for the 3[rd] party app, who buy the in-house app, and increases the margin earned on those with stronger preference for the 3[rd] party app, who patronize the superior app developer. By doing so, the platform earns strictly more than under

[31] One might think of the 3[rd] party app as adding a functionality relative to the in-house one. Consumers value this extra functionality diversely.

foreclosure for all $a > b$.[32] As app prices are strategic complements, this behaviour also induces the 3rd party seller to reduce its price, and hence squeezes its margin.

Below-cost pricing implies that the app ZLB binds also for $a \in [b, \underline{a}]$. In this range, as any increase in fees is passed through by the 3rd party seller to consumers, some of them switch to the inferior in-house app as $a$ grows larger: This harms them and induces a welfare loss from their misallocation. For $a \in (\underline{a}, \overline{a})$, no ZLB binds, and the access charge is competitively neutral: $a$ is passed through one-for-one to consumers by both app providers, and the platform correspondingly reduces consumers' access price to still ensure their participation. Yet, the platform has incentives to set a larger access charge, $a^* > \overline{a}$, such that the core ZLB binds and high unit fees are collected on an inefficiently low amount of 3rd party app's sales.

**Proposition 6** (efficient choice of app)**.** *Suppose consumers have heterogeneous valuations for the 3rd party app, and non-price foreclosure cannot be monitored. Then, consumer surplus and social welfare are maximized at* $\hat{a} = b < a^*$.

By encouraging an excessive consumption of the in-house app, platform's below-cost pricing in the competitive segment, which emerges in equilibrium for all $a > b$, harms both consumers and the 3rd party seller. Thus, under no monitoring of non-price foreclosure, even neglecting fairness considerations, optimal access charge regulation must follow a Pigouvian principle.[33]

### 3.4 Summing up

Table 2 summarizes the impact of the various desiderata on the level of the desired access charge $\hat{a}$.

| **Objective** | No monitoring of foreclosure | Monitoring of foreclosure |
|---|---|---|
| App innovation incentive | $\hat{a} \in [b, b+v]$ | |
| Avoidance of double marginalization (for $\Delta \geq b > \rho(0)$) | $\hat{a} \in [b, b+\Delta-\rho(0)]$ | $\hat{a} \in [\rho(0), b+\Delta-\rho(0)]$ |
| Efficient choice of app | $\hat{a} = b$ | $\hat{a} = 0$ |

Table 2: Welfare maximizing access charges.

[32] Below-cost pricing is not feasible for $a < b$ because of the app ZLB. In this case, the platform collects $p_0^* = v$, which coincides with the access price under foreclosure, from all consumers, plus $b$ from consumers purchasing the in-house app and $a$ from those purchasing the 3rd party app, and so foreclosure is optimal. The platform is thus indifferent between foreclosing or not the superior 3rd party app if and only if $a = b$.

[33] The above results imply that, if non-price foreclosure could be monitored, then consumer surplus and social welfare would be maximized by granting free access to the marketplace (i.e., for $a = 0$), though this would raise fairness concerns on the platform side, and excessive innovation on the app side.

# 4 Contested bottlenecks

Does platform competition improve the lot of consumers? That of developers? Does competition eliminate the scope for access charge regulation? To address these questions, this section returns to the framework of Section 2 and considers multiple competing platforms, single-homing on the consumer side, and multi-homing on the 3rd party app-developer side. This is consistent with evidence that consumers usually use one device, where they access only one app-store (Apple Store on iPhone, Google's Play Store on Android-powered devices, though recent regulations aim to induce app-store competition: see Section 4.2),[34] whereas most common apps are available on both Apple's and Google's stores (Bresnahan et al., 2015).

We suppose that 3rd party sellers take consumers' demand on all the platforms they patronize as given when setting their prices (monopolistic competition, as in Etro, 2021a, 2022b, and Jeon and Rey, 2023). This would be the case with a continuum of (independent) app markets, because the presence (and price) of an individual (superior) 3rd party app in one market has a negligible impact on consumers' overall utility from access to each platform, and so cannot affect their platform choice. Nonetheless, to ease exposition, we keep using the notation of a one-app market throughout.[35]

## 4.1 Platform competition

Consider $N \geq 2$ (symmetric) competing platforms, indexed by $i$, and let $U^i \equiv u^i - p_0^i$ denote consumers' net value from access to platform $i$'s ecosystem, where $u^i \equiv \max\{v - p_1^i,\ v + \Delta - p_2^i,\ 0\}$, and $(p_0^i, p_1^i, p_2^i)$ are consumers' access price, in-house and 3rd party app prices on platform $i$, respectively. To analyse platform competition in the starkest way, we consider perfect competition. That is, we suppose that all consumers patronize the platform offering the highest net value $U^i$. As a tie breaking condition, we assume that platforms offering the same utility split equally the demand, though this does not affect the main results. The timing is the same as with a single platform: (i) The platforms select their access conditions $(a^i, \delta^i)$;[36] (ii) The platforms and the apps select their prices $\{p_0^i, p_1^i\}$ and $p_2^i$; (iii) Consumers choose their platform, and their app on that platform.

Perfect competition implies that in equilibrium all platforms offer the same net utility $U^* = v$ to consumers,[37] and that the core ZLB binds. Whenever the access charge on platform $i$ is

[34] Similarly, most consumers primarily use a single search engine (e.g., Evans, 2008).

[35] If, on the contrary, the price set by the superior 3rd party app could shift consumers' demand across platforms, then, under perfect platform competition (that, for simplicity, we will assume throughout this section), in equilibrium platforms would grant free access to their marketplaces ($a^* = 0$) and make zero profits, whilst the 3rd party app would make a supranormal profit $\Delta + b$ (i.e., $p_0^* = p_1^* = 0$ and $p_2^* = \Delta$ on all platforms). Also the ability to foreclose would not help platforms in this environment. In a nutshell, any platform would have incentives to grant access to the superior 3rd party seller, and to induce it to price its app at the lowest price, so as to increase the attractiveness of its ecosystem.

[36] It is straightforward to see that, in this simple model, whether these first-stage decisions are observed by rival platforms is immaterial to the results.

[37] An equilibrium featuring $U^* = v$ exists for all values of parameters, but need not be unique. A sufficient condition for uniqueness is $\Delta < (N-1)b$. Yet, the regulatory implications of the analysis are robust: see Online Appendix B.5 for the details.

such that $a^i \leq b$, since a platform optimally prices its app at zero under platform monopoly, a fortiori it must do so under competition. However, the presence of platform competition acts as a commitment device for platforms to price their in-house app at zero even if $a^i > b$. Indeed, because in equilibrium consumers are indifferent between the in-house and the 3rd party app, any $p_1^i > 0$ would give room for platform $i$ to undercut its rivals. By the same reasoning, the core price $p_0^i$ must be equal to 0. The analysis is similar to that with a monopoly platform in which the core ZLB binds; indeed, it is optimal for each platform to fully squeeze the 3rd party app: $a^* = b + \Delta$. The only difference with the monopoly platform case is a transfer of value $v$ from the platform to the consumers.

**Proposition 7** (platform competition)**.** *Consider $N \geq 2$ identical competing platforms, $i \in \{1, ..., N\}$.*

(i) Laissez-faire. *In the laissez-faire equilibrium, both ZLBs are binding ($p_0^i = p_1^i = 0$) and $p_2^i = \Delta$. All platforms select access charge $a^* = b + \Delta$ and make profit $(b + \Delta)/N$ each. The core ZLB prevents platforms' total profit $b + \Delta$ from being competed away. Consumers receive net surplus $v$ each, and 3rd party apps are fully squeezed.*

(ii) Access charge regulation. *A regulator concerned with fairness optimally sets $\hat{a} = b < a^*$, yielding per-platform profit $b/N$ and 3rd party app profit $\Delta$. Consumers still receive net surplus $v$ each.*

The laissez-faire result, illustrated in Figure 4 aligns with the conventional wisdom in platform economics[38] that the multi-homing side does not benefit from platform competition, while the single-homing one (the competitive bottleneck) does, because the platform is the gatekeeper for users on the single-homing side: Platform competition allows consumers to get positive net surplus $v$. Yet, in our framework perfectly competing platforms earn, in total, positive profits $b + \Delta$ under laissez faire. The first component of this unit profit is the benefit from app distribution, which can be appropriated also through foreclosure; the second component is the value brought about by superior sellers, which is extracted through access charges. Both revenues are not competed away by price competition because of the core ZLB.

Thus, enforcing the Pigouvian rule through a cap on access charges is needed to guarantee proper incentives to invest by independent app developers, even in the presence of fierce inter-platform competition.[39,40]

[38] See Caillaud and Jullien (2003), Armstrong (2006), Armstrong and Wright (2007) and, more recently, Teh et al. (2023). Only Armstrong and Wright (2007) explore the implications of a ZLB constraint on the access price charged to the single-homing side, which competing platforms would like to subsidize.

[39] Note that we have assumed that the 3rd party app already exists. If not, the prospect of being fully squeezed will discourage it from entering, even for a small entry cost. To remedy this, platforms may voluntarily cap their access charges. Assuming that such a commitment is feasible, it still would not bring about a fair access charge. As long as app developers have negligible multi-homing costs, 3rd party app entry is a public good from the point of view of platforms and free riding would be expected (this would necessarily be the case if $\Delta$ were random): see Jeon and Rey (2023).

[40] This result hinges on the assumption that platforms are vertically integrated in the app segment: If also the low-value apps are offered by 3rd party providers, then $a^* = b$ would prevail in the laissez-faire equilibrium, which would eliminate the scope for regulation. The reason is that (as $p_1^i = \max\{a^i - b, 0\}$ and $p_2^i = \min\{p_1^i + \Delta, v + \Delta\}$) the superior app is priced at $p_2^i = \Delta$ for all $a^i \leq b$, whilst any larger access charge implies $p_2^i > \Delta$ and so $U^i < v$

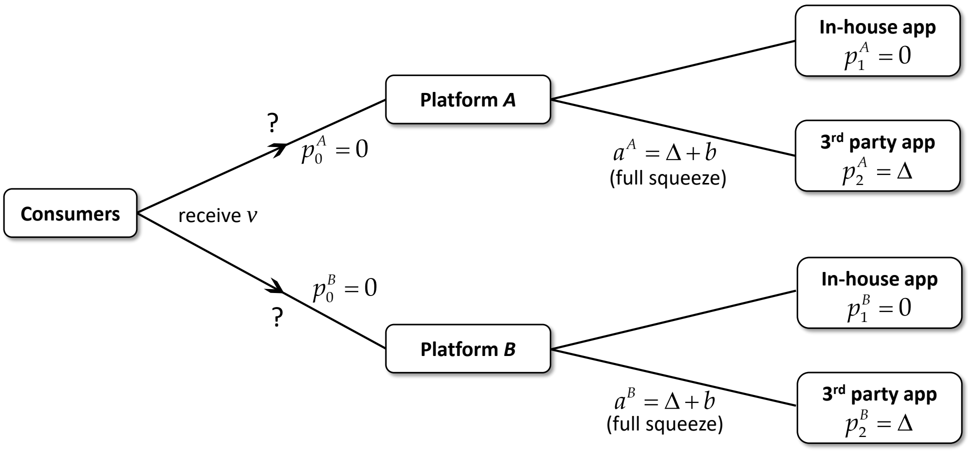


Figure 4: Laissez faire equilibrium with competing platforms under consumer single-homing.

*Platform viability and entry.* Assume that the social welfare function is $U + \alpha\Pi$, where $\Pi$ is total profit (platforms and apps) and $\alpha \in (0, 1)$ is the weight on industry profits relative to consumer surplus.[41] Suppose that there is free entry into the platform segment, with entry cost $J$.[42] Suppose further that non-price foreclosure cannot be monitored, and so the access charge must be no lower than $b$.

The socially optimal number of platforms is at most two, because extra platforms beyond $N = 2$ do not alter the consumer surplus and variable profit, and add entry costs and so are necessarily suboptimal if $N > 2$. Given that, with $N = 3$, each entrant makes $\frac{b}{3}$ under the Pigouvian rule, and higher profits under laissez-faire, for all $J \leq \frac{b}{3}$ there is always too much entry into the platform segment, which, without monitoring of non-price foreclosure (or a ban of the hybrid platform model), cannot be prevented by access charge regulation.

Under a monopoly platform, the consumers obtain no surplus ($U = 0$). A second entrant increases consumer surplus by $v$, at the expense of platform total profit, but also entails a socially wasteful entry cost $J$. Formally, $U + \alpha\Pi = v + \alpha(b + \Delta - 2J)$ under duopoly and $U + \alpha\Pi = \alpha(b + v + \Delta - J)$ under monopoly. Hence, a duopoly is preferred to monopoly if and only if $(1 - \alpha)v \geq \alpha J$, or $J \leq \frac{1-\alpha}{\alpha} v$.

Thus, assuming that the access charge is set, by regulation, at the Pigouvian level, for $J \in (\max\{\frac{1-\alpha}{\alpha} v, \frac{b}{3}\}, \frac{b}{2}]$ there is again too much entry, as two platforms enter but it would be optimal to have one.[43] If, on the contrary, $J \in (\frac{b}{2}, \min\{\frac{1-\alpha}{\alpha} v, b + v\}]$, then spurring the welfare-maximizing second entry requires setting the access charge above the Pigouvian level. Similarly,

and no customer for platform $i$.

[41]Under a social welfare standard ($\alpha = 1$), platform competition just entails socially wasteful duplicative entry costs: welfare maximization dictates $N = 1$. On the contrary, under a consumer surplus standard ($\alpha = 0$), as a monopolist brings zero net value to consumers, entry by any number $N \geq 2$ of platforms would be optimal (i.e., there is never excessive entry in equilibrium from consumers' standpoint).

[42]To rule out uninteresting coordination problems, we suppose that platforms' entry decisions are sequential. As a tie breaking condition, we assume that a platform enters in case of indifference.

[43]The region of parameters where excessive entry prevails of course expands when platforms are free to set access charges, as the absence of regulation increases their profits.

if $J \in (b+v, b+v+\Delta]$, there is a potential trade-off between the first platform's viability, which requires a squeeze in the app's profit, and app viability, which calls for staying away from the squeeze region to obtain the proper level of innovation.

**Observation 5.** *Because of the core ZLB preventing profits from being competed away by competition, socially excessive entry prevails when the entry cost is too low and foreclosure cannot be monitored. By contrast, for high entry costs, setting access charges above the Pigouvian level is desirable to spur platform entry, if no other instrument is available (as we saw, $a > b$ introduces distortions).*

This analysis suggests that, while access charge regulation is an effective instrument to achieve fairness, thereby promoting efficient entry and investment decisions in the app segment, it may not be a jack of all trades and take on extra tasks such as ensuring contestability of the core segment.

## 4.2 App-store competition on a device

The DMA and the proposed Open App Markets Act require Apple and Google to guarantee $3^{\text{rd}}$ party app-stores access to their respective devices.[44] Does the availability of competing app-stores on a single device eliminate the scope for access charge regulation?

Consider a (vertically integrated) monopoly device manufacturer, hereafter denoted by $M$.[45] As above, its device brings value $v_c$ to consumers and is produced at marginal cost $\gamma_c > 0$. Let $p^M$ denote its price. On its app-store, whose access is priced at $p_0^M$,[46] consumers can find its in-house app valued $v_a$ and a superior $3^{\text{rd}}$ party app valued $v_a + \Delta$, at prices $p_1^M$ and $p_2^M$ respectively. $M$'s in-house app-store faces competition by $3^{\text{rd}}$ party app-stores, indexed by $j$ and priced at $p_0^j$, where consumers can find the respective in-house apps, bringing value $v_a$, at prices $p_1^j$, and the same, multi-homing $3^{\text{rd}}$ party app available on $M$'s store at prices $p_2^j$.

Suppose consumers multi-home across app-stores that can be accessed for free (which is always the case in equilibrium).[47] Then the superior $3^{\text{rd}}$ party app would optimally serve all consumers on the least expensive platform: App-stores de facto engage in Bertrand competition for the $3^{\text{rd}}$

[44] The regulatory texts are silent as to the relative access conditions (see Online Appendix A). However, it looks hard to reconcile the guarantee-of-access request with the possibility of charging for access. Moreover, (i) the Open App Markets Act explicitly requires to allow consumers to install $3^{\text{rd}}$ party app-stores "through means other than the platform's app-store", which prevents the platform from controlling (and, a fortiori, taxing) access; and (ii) app-stores do not charge consumers positive access prices (consistently with the core ZLB being binding in our model), and so the platform would obtain no revenues anyway under ad-valorem fees. On this ground, in what follows we assume that $3^{\text{rd}}$ party app-stores must be given free access to the platform.

[45] As we have seen, competing device manufacturers end up making zero profits in equilibrium even absent app-store competition on each device, which would make it pointless to introduce further competition.

[46] Unlike in the previous case, when multiple app-store compete for consumers on the same device, its vertically integrated manufacturer is forced to unbundle its two core products (the device and the app-store), charging two different prices. In what follows, we refer to the app-stores as the core.

[47] If instead consumers always single-home (because of, e.g., memory constraints, each downloads at most one app-store), then, as all app-stores are equally constrained by the core ZLBs, $p_0^M \geq 0$ and $p_0^j \geq 0$, the analysis is as in Section 4.1 (with the only difference that the monopoly manufacturer appropriates consumer surplus charging $p^M = v$ for the device). Pigouvian regulation is thus still needed to fairly reward the superior $3^{\text{rd}}$ party app provider.

party app, which dissipates their profits – i.e., $a^* = 0$ in equilibrium (Figure 5).

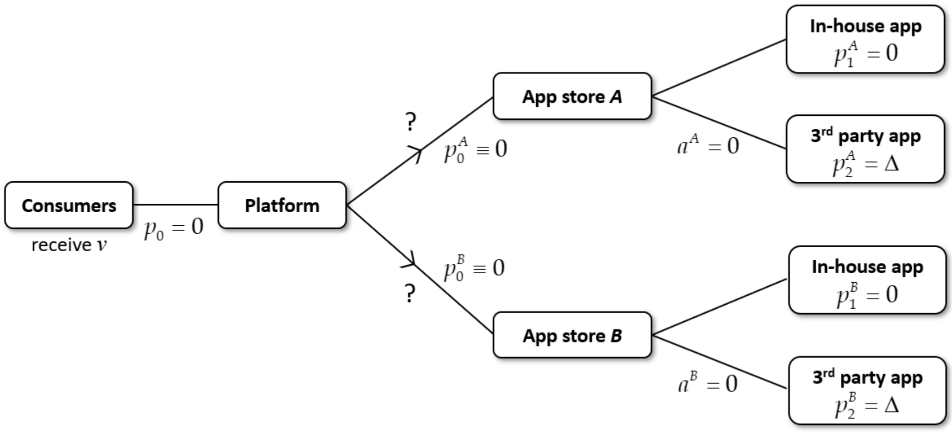


Figure 5: Laissez faire equilibrium with competing app-stores.

**Observation 6.** *Suppose that the regulator mandates app-store competition on each device, with app-stores enjoying free access to the device. Suppose further that consumers can multi-home on app-stores on their device. Bertrand competition among app-stores induces them to charge nothing for consumer access to the app-store and to also levy no app-store access charge on the 3rd party app, as the latter steers the consumer to the lowest access charge app-store, that is the most profitable for the superior app. The 3rd party app then makes supranormal profit $\Delta + b$. The Pigouvian access charge ($\hat{a} = b$) is needed to ensure fairness and avoid over-entry in the app market.*[48]

# 5 Implementation

While theoretically appealing, the Pigouvian access pricing rule may not be easy to implement in practice. The socially optimal fee is set at a level that makes the platform indifferent between engaging or not in self-preferencing. This indifference makes it unlikely that the platform engages in foreclosure for a small error in estimating the ancillary benefit;[49] but larger errors will require a costly monitoring of foreclosure activities.

How difficult is it to estimate the ancillary benefit? In practice, app categories differ substantially in terms of the benefit their distribution generates. For instance, there are data-poor and data-rich markets – e.g., social media and food delivery apps sell much more personal data to 3rd party advertisers than videoconferencing apps.[50] The industry has private information about these values that is hardly available to the regulator. Somehow the regulator needs to elicit the value of the benefit $b$ from the industry. Sections 5.1 and 5.2 consider schemes that aim at

[48]This conclusion would be supported also by a consumer surplus standard in a model where consumers have heterogeneous valuations $v_c$ for the device, because $M$ could react to the reduced profitability of the app-store due to competition by increasing the device price $p^M$.

[49]This is because the platform may face a small probability of being sued for foreclosure, or may want to avoid building a bad reputation vis-à-vis 3rd party developers.

[50]See `https://www.pcloud.com/it/invasive-apps`.

obtaining the information from the platform and from the 3rd party developer, respectively.

### 5.1 Eliciting the information from the platform

We here restrict attention to an elicitation of ancillary benefits from the platform. Suppose that the latter gives access to a continuum of mass 1 of (independent and heterogeneous) app markets, indexed by $k \in [0,1]$. In each market, there is a in-house platform app, valued $v^k \geq 0$ by all consumers, and a superior 3rd party app, valued $v^k + \Delta^k$, with $\Delta^k \geq 0$. We assume that each 3rd party developer sells in only one market. The ancillary benefit is denoted $b^k$.[51]

To examine how the regulator's limited knowledge of market-specific ancillary benefits affects access charge regulation in the simplest possible model, we consider the best-case scenario in which the regulator knows their cumulative distribution $K(b)$ in the population of apps.[52] Of course, in order for the Pigouvian access charge to be implemented in all markets, the distribution of (observed) access charges must equal the distribution of benefits.

**Proposition 8** (impossibility of elicitation from the platform). *Suppose the regulator knows the distribution $K(b)$ of ancillary benefits and lets the platform choose $(a^k)_{k\in[0,1]}$ subject to the constraint that the distribution of access charges mimics that of benefits (i.e., follows $K(a)$). Then, if non-price foreclosure cannot be monitored, setting $a^k = b^k$ for all $k \in [0,1]$ is not incentive-compatible for the platform.*

If non-price foreclosure cannot be monitored, the Pigouvian rule is not implementable in all markets even if the regulator knows the distribution of $b$, so that it can require that $K(a) = K(b)$. The reason is that, rather than charging $a^k = b^k$ in all markets, the platform can profitably charge higher fees in markets where $b$ is lower, so as to squeeze 3rd party developers' margins in these markets, and foreclose developers in markets where $b$ is higher, where it is constrained to set lower fees, which allows it not to lose profits in these markets. Thus, under no monitoring of non-price foreclosure, market-specific fees cannot be enforced under asymmetric information, and the regulator faces a trade-off between preventing foreclosure of developers in high-$b$ markets and allowing margin squeeze (hence, dampening innovation incentives) of sellers in low-$b$ ones.

### 5.2 Eliciting the information from app developers

The previous impossibility result hinged on the assumption that fee setting is delegated to the platform. The next proposition reverses the roles in access fee setting:

**Proposition 9** (information light regulation). *Regardless of the regulator's information, the Pigouvian rule can be implemented in all markets by letting 3rd party app developers pick their*

[51] The results of the baseline model easily extend to each app market. In particular, any $a^k \in [b^k, b^k + v^k]$ is a fair access charge, whilst the platform's profit is maximized at the extractive access charge $a^{k*} = b^k + v^k + \Delta^k$. Hence, if $b^k$ were observed by the regulator, whilst $v^k$ were not, and were distributed with a cdf $L(v^k)$ such that $L(v^k) > 0$ for all $v^k > 0$, then the regulator could guarantee fair access pricing in all app markets by imposing $\hat{a}^k = b^k$.

[52] We can keep assuming that the regulator does not observe $(v^k, \Delta^k)$. Whether or not it knows their distribution is immaterial to the results.

*access charge subject to the threat of foreclosure.*

*Proof of Proposition 9.* If foreclosure is not monitored, then choosing $a^k \in [b^k, b^k + v^k]$ is optimal for 3rd party sellers, as they are foreclosed for $a^k < b^k$, and squeezed for $a^k > b^k + v^k$. ■

If, on the contrary, the platform were not able to foreclose the 3rd party app (e.g., under monitoring of self-preferencing, or because it is not vertically integrated), then independent sellers would choose $a^k = 0$: This would overincentivise 3rd party apps to enter and would further expropriate the platform's investments, thereby dampening its innovation incentives.[53] Yet, this is not the case in the presence of competing 3rd party apps:

*Multiple 3rd party apps.* Let us assume that the platform has no in-house app and that there are, say, two 3rd party apps, app 1 with value $v$ and app 2 with value $v + \Delta$. Can the Pigouvian outcome still be implemented by letting each app provider pick its own access charge? Suppose that the timing goes as follows: (i) the inferior and superior 3rd party apps propose access charges $a_1$ and $a_2$; (ii) the platform gives access to no, one or the two app providers; (iii) the platform and the (non foreclosed) app providers set their prices $\{p_0, p_1, p_2\}$; (iv) consumers take their consumption choice.

Suppose that the platform gives access to a single app. It will do so only with an app that can make money and thus serve the market. Thus, we can without loss of generality focus on proposed access charges $a_1 \in [0, b + v]$ and $a_2 \in [0, b + v + \Delta]$. App 1, if selected alone, chooses $p_1 = v$ and makes profit $v + b - a_1$. App 2, if selected alone, chooses $p_2 = v + \Delta$ and makes profit $v + b + \Delta - a_2$. In either case, the consumer surplus from the app is extracted and so $p_0 = 0$. Thus, when foreclosing, the platform is better off foreclosing the lower access charge app. Next, let us look at what happens when there is no foreclosure.

Consider $a_2 \leq a_1 + \Delta$, so that, if none of the apps is foreclosed, the superior app takes the app market, which must be the case in equilibrium.[54] The prices are $p_1 = \max\{a_1 - b, 0\}$ and $p_2 = p_1 + \Delta$. Because the price $p_2$ would be even higher in the absence of app 1, there is no point foreclosing app 1. Does the platform want to foreclose the superior app? A necessary and sufficient condition for this is

$$p_0 + a_2 < a_1 \iff a_1 - a_2 > v - \max\{a_1 - b, 0\}.$$

The Bertrand equilibrium in access prices has consumers patronizing solely app 2 and the app

[53] Moreover, the ability to foreclose ensures that the platform keeps controlling access to consumers, which protects them from fraudulent apps. Under monitoring of foreclosure, and provided the regulator knows the distribution $K(b)$ of benefits, the Pigouvian rule can also be implemented by delegating fee setting to the platform under the constraint that $K(a) = K(b)$.

[54] An equilibrium where $a_2 > a_1 + \Delta$, and so the inferior app drives the superior one out of the market, cannot exist because the superior app would always profitably deviate and match the offer of the inferior one. By proposing $a_2 = a_1$ (say, plus $\epsilon$), the platform has no incentives to foreclose it: The condition for no foreclosure, $a_1 - a_2 \leq v - \max\{a_1 - b, 0\}$ (see below), yields $0 \leq v - \max\{a_1 - b, 0\}$, which holds for all $a_1 \in [0, b + v]$. The superior app then wins consumers and obtains strictly positive profits.

providers selecting access charges:[55]

$$a_1^* = a_2^* = b + v.$$

Note that, for these specific access charges, the platform is indifferent between selecting an app provider and letting both operate; in all cases the platform selects $p_0 = 0$ and receives $b + v$. If app 2 raises $a_2$ above $b + v$, it lowers its profit; if it decreases $a_2$ below $b + v$, it is foreclosed by the platform as $a_1 > a_2$. As for app 1, an increase in $a_1$ makes it non viable, while a reduction in $a_1$ keeps its profit at 0.

### 5.3 Adding appeals

While Proposition 9 is an encouraging result, one may argue that the platform could, through repeated play, commit to foreclose any app that chooses an access charge that it deems "too low" (i.e., way over $b$): A long-lived platform may thus still be able to operate margin squeezes. In a one-shot game, the platform provides access if and only if $a \geq b$, and the optimal offer for the app provider is $a = b$. The platform incurs no cost of denying access as long as $a \leq b + v$, and a limited cost beyond. So it may want to build a reputation for foreclosing unless the app provider accepts to be squeezed "sufficiently", i.e., proposes some $a \geq \tilde{a} > b + v$.

A further related caveat, in the spirit of Carlton and Waldman (2002), goes as follows. If the platform were concerned about future entry in the core segment, it could practice foreclosure so to make these 3$^{\text{rd}}$ party apps unviable (fostering their exit or dampening their incentives to invest in the first place), with the aim of depriving prospective entrants of potential high-quality participants to their ecosystems. This would discourage entry in the core segment, as an effective competitor would need to enter the app segment as well to provide a comparably rich ecosystem.

Confronted with the possibility that the platform either engages in reputation building or tries to erect app-barriers to entry into the core segment by foreclosing even after fair offers, the regulator may choose to empower individual app providers by letting them appeal to the authority if they perceive that they are unfairly treated by the platform, that is if they proposed a "decent" access charge and yet are foreclosed. We show how such a regulatory approach might be structured.

Consider a specific app market, with parameters $\{b, v, \Delta\}$ (we omit the superscript $k$ for notational convenience). Suppose that, while foreclosure is commonly observed, parameters are known to the parties, but not to the authority: The authority has no clue whether the proposed access charge is "fair" to the platform. Consider the following timing:

(1) The 3$^{\text{rd}}$ party app provider publicly offers to pay access charge $a$.

(2) The platform gives access to, or forecloses the 3$^{\text{rd}}$ party app provider.

(3) If denied access, the app provider chooses between exiting and appealing. An appeal imposes on the authority a verification cost $r > 0$; the authority then observes a garbled version of the ancillary benefit: $\tilde{b} = b + \varepsilon$, where the measurement error $\varepsilon$ is distributed

[55] The considered equilibrium is unique when restricting attention to equilibria in undominated prices.

according to smooth distribution $R(\varepsilon)$ on $\mathbb{R}$.

(4) If $\tilde{b} > a$, then the offer is perceived as unfair, and the plaintiff (the app provider) is still denied access and must reimburse the authority for the cost $r$; if $\tilde{b} < a$, then the offer is perceived as fair, and the defendant (the platform) must provide free access and pay the verification cost $r$.

Thus, upon appeal the plaintiff wins the case with probability $R(a - b)$ and the defendant wins with probability $1 - R(a - b)$.

An appeal is credible if and only if it is in the interest of the app provider to appeal, i.e., if and only if the proposed access charge, $a$, satisfies: $R(a - b)(b + \Delta) \geq [1 - R(a - b)]r$, or $a \geq a^\dagger$ for some $a^\dagger$. For example, for a distribution $R(\cdot)$ that is symmetric around 0 and if $r > b + \Delta$, then $a^\dagger > b$. Suppose that $a^\dagger > b$.

Next, we can look at the platform's incentive to foreclose. Either $a \geq a^\dagger > b$ and then foreclosure implies a loss for the platform whether the appeal succeeds or fails (as the platform's profit weakly exceeds the foreclosure payoff for any $a \geq b$); or $a < a^\dagger$, so that foreclosure is not appealed.

How effective is the threat of an appeal? Suppose that $R(v) = 1$; this is indeed the case if the size of measurement errors does not exceed the consumer value of the in-house app. Then $a^\dagger \leq b + v$ and so the 3rd party app provider can secure itself profit $\pi_2 = \Delta$, the same profit as under a fair access price and the absence of foreclosure. The platform cannot build a reputation for being tough, i.e., deny access for a proposed fee $a \in [b + v, \tilde{a})$, because, at any time it does so, the 3rd party developer appeals and, as it wins with probability one, obtains supranormal profits $\Delta + b > \Delta$ at the expense of the platform (who must also pay $r$).[56]

**Observation 7.** *Suppose a 3rd party seller can appeal the platform's foreclosure decision, and b can be measured in court, sinking a sufficiently high inspection cost, with sufficient precision (say, with a margin of error at most equal to the value of the in-house app). Then, the platform cannot build a reputation for being tough (i.e., will never deny access for a proposed fee* $a \in [b, b + v]$*), and so, as each 3rd party app provider can secure its contribution* $\Delta$ *to the ecosystem (i.e., avoid being squeezed) the Pigouvian rule is implemented by letting 3rd party sellers pick their access charge.*

The lower bound on the inspection cost may seem strange. It arises because punishments for "wrongful foreclosure" and "wrongful appeal" have been assumed equal to the inspection cost and therefore rather low if the latter is low; in particular, for low inspection costs the 3rd party app challenges too often. But it suffices to then impose a penalty beyond the inspection cost, exceeding (with probability one, given the information available to the regulator) $b + \Delta$, and/or offer less generous access conditions conditional on winning the appeal (e.g., $a = \tilde{b}$), such that $a^\dagger > b$.[57]

[56] More generally, this outcome arises whenever, for all $\tilde{a} > b + v$, the squeeze profit from proposing $\tilde{a}$ is less than the expected profit from proposing $b + v$, being foreclosed and appealing: $R(v)(b + \Delta) - [1 - R(v)]r \geq \Delta$. If this inequality is not satisfied, then the platform could optimally build a reputation for being tough, so as to induce future sellers to propose some $\tilde{a} > b + v$.

[57] Indeed, if $a^\dagger < b$ then, at least in a one-shot game, the 3rd party app would be able to extract a supranormal

# 6 Conclusion

Gatekeeping platforms control businesses' access to us. Novel to the literature, two zero lower bounds, on the pricing of apps and of core products, were shown to play a key role regarding business users' access to platforms. These non-negative-price constraints generate undesirable outcomes, respectively undue business-user profits and the concomitant incentive for non-price foreclosure for low access charges, and the squeezing of business users for high ones. The paper then stressed that laissez-faire – in the sense of a lack of interference with the platforms' preferred access policies – breeds unfair access conditions for these business users. Furthermore, we should not expect competition to solve the gatekeeping problem in the digital world of ZLBs. Indeed, the core price constraint prevents platform competition from disciplining access policies. We also showed that platform competition and app-store competition work very differently. While platform competition is too business user unfriendly, app-store competition is too business user friendly.

The overall picture is therefore a need for overseeing the terms and conditions offered by platforms to business users. In this we concur with recent regulatory developments. The latter however remain nebulous when it comes to specific recommendations, and the occasional invocation of the need for "fair, reasonable and non-discriminatory" terms is not helpful. The paper's second main insight relates to the social benefits of setting the access charge at the ancillary benefit associated with acquiring a customer. This level discourages non-price foreclosure and thereby spares intrusive assessment of whether access conditions are actually fair; it also provides app developers with a fair return and therefore a proper incentive to innovate; finally, it minimizes double marginalization conditional on intrusive regulation being infeasible or too costly.

Despite this clear theoretical message, meeting the empirical challenge of regulating platforms' access policies remains as difficult as it is essential. The task of answering whether a 20% or 30% merchant fee is appropriate is marred with asymmetric information. We made real progress on the question of how to implement the theoretical benchmark; but we feel that more work is necessary to properly tame the gatekeeping platforms while not preventing them from offering innovative services to consumers and businesses alike. This question should remain on our priorities.

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profit: For $a \in (a^\dagger, b)$ close enough to $b$, given the credible threat of appeal, the platform would not foreclose the $3^{rd}$ party app (the expected payoff from foreclosure, given appeal, $R(a-b)(v-r) + [1-R(a-b)](v+b)$, is less than the no-foreclosure profit $v+a$).

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# Online Appendix

## A Antitrust cases and regulation

### A.1 Antitrust cases

*Self-preferencing.* In 2017 the European Commission fined Google for giving an unfair advantage to its own shopping comparison service. The Commission argued that Google abused its market dominance as a search engine by systematically giving prominent placement to its own comparison shopping service, and demoting rival comparison shopping services in its search results. See `https://ec.europa.eu/commission/presscorner/detail/en/IP_17_1784` for the details.

In 2020, the same Commission launched an investigation into Amazon's Buy-Box, concerning the possible preferential treatment of Amazon's own retail offers and those of marketplace sellers that use Amazon's logistics and delivery services. See `https://ec.europa.eu/commission/presscorner/detail/en/ip_20_2077`.

*App-store commissions.* In August 2021, Apple removed Epic's Fortnite game from its app-store because it circumvented Apple's 30% fee by offering an external payment option. A federal judge in California in September 2021 ruled that Apple must allow developers to route customers to $3^{\text{rd}}$ party payment options in applications and not force them to pay the app-store's fees for in-app purchases. See, e.g., `https://www.forbes.com`.

In the same year, in the EU, following-up on a complaint by Spotify, the Commission informed Apple of its preliminary view that it distorted competition in the music streaming market as it abused its dominant position for the distribution of music streaming apps through its app-store. The Commission is concerned by the mandatory use of Apple's own in-app purchase mechanism imposed on music streaming app developers to distribute their apps via Apple's App Store. Executive Vice-President Margrethe Vestager said: "Our preliminary finding is that Apple is a gatekeeper to users of iPhones and iPads via the App Store. With Apple Music, Apple also competes with music streaming providers. By setting strict rules on the App store that disadvantage competing music streaming services, Apple deprives users of cheaper music streaming choices and distorts competition. This is done by charging high commission fees on each transaction in the App store for rivals and by forbidding them from informing their customers of alternative subscription options." See `https://ec.europa.eu/commission/presscorner/detail/en/ip_21_2061`.

In January 2022, the Competition Commission of India stated that the 30% commission Apple charges developers unfairly pushes up costs for both app makers and consumers, and also acts as a barrier to entry for new developers. See, e.g., `https://nypost.com/2022/01/03/apple-hit-with-antitrust-probe-in-india-over-app-store-fees/`.

## A.2 Regulatory proposals

*EU Digital Markets Act.* The DMA, which entered into force on November 1, 2022, has the purpose "to contribute to the proper functioning of the internal market by laying down harmonised rules ensuring for all businesses, contestable and fair markets in the digital sector across the Union where gatekeepers are present, to the benefit of business users and end users" (Article 1.1).

Inter alia, it is concerned with excessive access charges that gatekeeper platforms impose on retailers: In paragraph 62 (page L 265/16), it is stated: "Pricing or other general access conditions should be considered unfair if they lead to an imbalance of rights and obligations imposed on business users or confer an advantage on the gatekeeper which is disproportionate to the service provided by the gatekeeper to business users or lead to a disadvantage for business users in providing the same or similar services as the gatekeeper."

Vertically-integrated gatekeeper platforms also bring up concerns for non-price foreclosure practices: "Gatekeepers are often vertically integrated and offer certain products or services to end users through their own core platform services, or through a business user over which they exercise control which frequently leads to conflicts of interest. [...] When offering those products or services on the core platform service, gatekeepers can reserve a better position, in terms of ranking, and related indexing and crawling, for their own offering than that of the products or services of third parties also operating on that core platform service. [...] instances are those of software applications which are distributed through software application stores" (paragraph 51, L 265/13); "In such situations, the gatekeeper should not engage in any form of differentiated or preferential treatment in ranking on the core platform service, and related indexing and crawling, whether through legal, commercial or technical means, in favour of products or services it offers itself or through a business user which it controls." (paragraph 52, L 265/14).

Finally, the DMA promotes app-store competition: "To ensure contestability, the gatekeeper should furthermore allow the third-party [...] software application stores to prompt the end user to decide whether that service should become the default and enable that change to be carried out easily" (paragraph 50, L 265/13).

The full text is available at: `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32022R1925&from=EN`.

*US Innovation and Choice Act and Open App Markets Act.* The Innovation and Choice Act, passed in the Judiciary Committee on January 20, 2022, aims to prevent dominant hybrid platforms from self-preferencing their own products. Sec. 2(a) states: "It shall be unlawful for a [...] platform [...] to engage in any conduct [...] that advantages the platform operator's own products, services, or lines of business over those of another business user". The full text is available at: `https://www.congress.gov/bill/117th-congress/house-bill/3816/text`.

On February 3, 2022, the Senate Judiciary Committee advanced the legislation of the Open App Markets Act, which would prevent app-stores from requiring developers to directly use their in-app payment systems and mandate reduced fees for developers. Sec 3(a) states that platforms shall not: "(1) require developers to use or enable an in-app payment system owned or controlled by the covered company [the platform] or any of its business partners as a condition of the distribution of an app on an app store or accessible on an operating system; [...] take punitive action or otherwise impose less favorable terms and conditions against a developer for using or offering different pricing terms or conditions of sale through another in-app payment system or on another app store". Finally, it promotes app-store competition: "A covered company that controls the operating system or operating system configuration on which its app store operates shall allow and provide readily accessible means for users of that operating system (1) choose third-party apps or app stores as defaults for categories appropriate to the app or app store; (2) install third-party apps or app stores through means other than its app store; and (3) hide or delete apps or app stores provided or preinstalled by the app store owner or any of its business partners" (Sec. 3(d)). The full text is available at: `https://www.congress.gov/bill/117th-congress/senate-bill/2710/text`.

*Caps on access charges.* Many local governments in the US introduced caps on the commissions that food delivery platforms charge to restaurants after the beginning of the COVID-19 pandemic. Several jurisdictions then made their commission caps permanent. The major platforms (Uber Eats, Grubhub and DoorDash) typically charge a 30% fee, and most governments capped these commissions to 15%. See Sullivan (2022) for the details.

# B Omitted proofs

## B.1 Proof of Proposition 4

For $a < b$, either the anticipation of being foreclosed gives the 3$^{\text{rd}}$ party seller no incentive to innovate, or the supranormal profit obtained if foreclosure is monitored implies that some socially inefficient innovations – namely, those with development cost $c \in (\Delta, \Delta + b - a]$ – are undertaken. For $a \in [b, b+v]$, the innovation takes place if and only if $c \leq \pi_2^*(a) = \Delta$, i.e. whenever it is socially optimal, and, regardless of whether or not the superior app is developed, the platform earns profit $\pi_1 = b + v$. While this outcome is socially optimal, the platform has incentives to charge higher access fees, which inefficiently hamper innovation. Indeed, for any $a \geq b + v$, the 3$^{\text{rd}}$ party seller optimally innovates if and only if $c \leq v + \Delta + b - a$, so that the platform's expected profit is

$$\pi_1^*(a) = v + b + [a - (b + v)]G(v + \Delta + b - a).$$

Maximizing it with respect to $a$ then yields

$$a^* = b + v + \frac{G(v + \Delta + b - a^*)}{g(v + \Delta + b - a^*)},$$

with $a^* > b + v$. This implies that some socially optimal innovations – namely, those with development cost $c \in (v + \Delta + b - a^*, \Delta]$ – will not be undertaken. ■

## B.2 Proof of Proposition 5

Since the comparative advantage of the 3rd party app is still type-independent ($\Delta > 0$), asymmetric Bertrand competition again implies $p_1^* = \max\{a - b, 0\}$ and $p_2^* \leq p_1^* + \Delta$. Then,

$$\frac{\partial \pi_1}{\partial p_0} = -f(v^*)(p_0 + a) + 1 - F(v^*), \tag{B.1}$$

and

$$\frac{\partial \pi_2}{\partial p_2} = -f(v^*)(p_2 + b - a) + 1 - F(v^*), \tag{B.2}$$

where $v^* = p_0 + p_2 - \Delta$ is the cutoff consumer type. In what follows, let $\rho(v) \equiv [1 - F(v)]/f(v)$ denote the inverse hazard rate, assumed strictly decreasing, with $b \geq \rho(0)$ (so that the core ZLB binds under foreclosure). Hence, the foreclosure profit is $\pi^F \equiv [1 - F(0)]b = b$.

First consider $a < b$, so that $p_1^* = 0$ and $p_2^* \leq \Delta$. Then, (B.1) gives

$$p_0^* = 0 \iff a \geq \rho(0).$$

This is because $\frac{\partial \pi_1}{\partial p_0}|_{p_0=0} \leq 0 \iff a \geq \rho(p_2 - \Delta)$ and, if $p_0^* = 0$, the 3rd party app can serve all consumers setting $p_2^* = \Delta$. We then have $\pi_1^*(a) = [1 - F(0)]a = a < b = \pi^F$: Foreclosure is optimal for the platform.

In any equilibrium where $p_0^* > 0$ it must be $p_0 + a = \rho(v^*)$. We have to distinguish three cases:

- $p_2^* = 0$ if and only if $\frac{\partial \pi_2}{\partial p_2}|_{p_2=0} \leq 0$, which, from (B.2), gives

  $$b - a \geq \rho(v^*) = p_0 + a \iff p_0 \leq b - 2a.$$

  From (B.2) it follows that the latter inequality is satisfied if and only if

  $$b - a \geq \rho(b - 2a - \Delta),$$

  which can hold provided $b - 2a - \Delta > 0 \iff a < [b - \Delta]/2$, which is not consistent with the parametric restriction $\Delta \geq b$.

- $p_2^* = \Delta$ if and only if $\frac{\partial \pi_2}{\partial p_2}|_{p_2=\Delta} \geq 0$, which, from (B.2), gives

  $$b - a + \Delta \leq \rho(v^*) = p_0 + a \iff p_0 \leq b - 2a + \Delta.$$

  Given (B.1), the latter inequality is satisfied if and only if

  $$b - a + \Delta \geq \rho(b - 2a + \Delta).$$

In this equilibrium,

$$\pi_1^*(a) = \max_{p_0 \geq 0}[1 - F(p_0)](p_0 + a) < \max_{p_0 \geq 0}[1 - F(p_0)](p_0 + b) = \pi^F.$$

Note that $p_0^* > 0$ and $p_2^* = \Delta$ imply $p_0^* + p_2^* > \Delta$.

- In all other cases, $p_2^* \in (0, \Delta)$ and $(p_0^*, p_2^*)$ solve the system of FOCs $\frac{\partial \pi_1}{\partial p_0} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$, which can be rewritten as

$$\begin{cases} p_0 + a = p_2 - a + b \\ p_0 + a = \rho(2(p_0 + a) - b - \Delta) \end{cases}$$

The second equation implies that $p_0^* + a$ must be constant varying $a$. Then, the first equation implies that also $p_2^* - a$ is constant in $a$. As, then, also $p_0^* + p_2^*$ does not vary with $a$, firms' profit do not depend on $a$ in this equilibrium. Yet, the platform is again better off foreclosing the superior seller, given that we have seen that $\pi_1^*(a) < \pi_1^F$ as $p_2^* \to \Delta$. Moreover, in this equilibrium $p_0^* + p_2^* > \Delta$. To see this, recall that $p_0^* + p_2^* \geq \Delta$ in equilibrium, and suppose by contradiction that $p_2^* = \Delta - p_0^*$. Then, the first equation of the system would yield $p_0 = \frac{1}{2}(\Delta + b) - a$ which, substituted into the second equation, would give $\frac{1}{2}(\Delta + b) = \rho(0)$, which is not possible given $b \geq \Delta > \rho(0)$.

Hence, non-price foreclosure is optimal for all $a < b$.

We next turn to consider $a \geq b$, so that $p_1^* = a - b$ and $p_2^* \in (a - b, a - b + \Delta]$. In this case, (B.1) yields

$$p_0^* = 0 \iff a \geq \rho(p_2 - \Delta).$$

Since $p_2^* \geq \Delta$ and $a \geq b$, this condition always holds for all $b \geq \rho(0)$: The core ZLB binds for all $a \geq b$. Then, there are three cases to consider:

- $p_2^* = \Delta$ if and only if $\frac{\partial \pi_2}{\partial p_2}|_{p_2 = \Delta} \leq 0$. Imposing this condition, (B.2) gives

$$\Delta + b - a \geq \rho(0) \iff a \leq b + \Delta - \rho(0).$$

In this case, the platform's profit is increasing in $a$ (which implies $a^* \geq b + \Delta - \rho(0)$), and it is strictly better off not foreclosing the superior seller, as

$$\pi_1^*(a) = \max_{p_0 \geq 0}[1 - F(p_0)](p_0 + a) = a \geq b = \max_{p_0 \geq 0}[1 - F(p_0)](p_0 + b) = \pi^F,$$

with $\pi_1^*(a) = \pi^F$ at $a = b$ only.

- $p_2^* = a - b + \Delta$ if and only if $\frac{\partial \pi_2}{\partial p_2}|_{p_2 = a - b + \Delta} \geq 0$, which, using (B.2), gives

$$\Delta \leq \rho(a - b),$$

which is not compatible with the imposed parametric restriction $\Delta > \rho(0)$.[58]

- Otherwise (i.e., for $a > b + \Delta - \rho(0)$): $p_2^* \in (\Delta, a - b + \Delta)$ solves $\frac{\partial \pi_2}{\partial p_2} = 0$, with

$$\frac{\partial p_2^*(a)}{\partial a} = \frac{[f(p_2^*(a) - \Delta)]^2}{[1 - F(p_2^*(a) - \Delta)]f'(p_2^*(a) - \Delta) + 2[f(p_2^*(a) - \Delta)]^2} \in (0, 1).$$

Under mild restrictions on the derivatives of $f(\cdot)$, $\pi_1^*(a)$ is strictly concave in $a$,[59] and, at $a \to [b + \Delta - \rho(0)]^+$, since $p_2^* \to \Delta$:

$$\left.\frac{\partial \pi_1^*}{\partial a}\right|_{a \to [b+\Delta-\rho(0)]^+} > 0 \iff (b+\Delta-\rho(0)) \left.\frac{\partial p_2^*}{\partial a}\right|_{a \to [b+\Delta-\rho(0)]^+} < \rho(0) \iff b+\Delta < 3\rho(0)+\rho(0)^2 \frac{f'(0)}{f(0)},$$

which, as $F(0) = 0$, rewrites as $b + \Delta < \rho(0)[3 + \rho(0)^2 f'(0)]$. Under this condition, $a^* > b + \Delta - \rho(0)$.

Finally, note that for $b < \rho(0)$ there may be equilibria where the core ZLB does not bind, characterized by competitive neutrality:[60] Either $p_2^* = a - b + \Delta$, and so $p_0^* + a = \rho(p_0^* + a - b)$; or $p_2^* < a - b + \Delta$, such that (from $\frac{\partial \pi_1}{\partial p_0} = \frac{\partial \pi_2}{\partial p_2} = 0$) $p_2^* - a + b = \rho(2(p_2^* - a) + b - \Delta) = p_0^* + a$. In both cases, $(p_0^* + a)$ and $(p_2^* - a)$, hence also $p_0^* + p_2^*$, are constant when $a$ varies, which yields the neutrality result.

We now turn to the welfare analysis. Absent foreclosure, social welfare is given by

$$W^* = \int_{v \geq p_0^* + p_2^* - \Delta} (b + \Delta + v) dF(v).$$

Thus, it is decreasing in consumers' total price $p_0^* + p_2^*$, and access charges affect social welfare only through their impact on the equilibrium prices.

[58] Whenever this equilibrium exists, the platform would be at best indifferent between foreclosing or not the superior seller:

$$\pi_1^* = \max_{p_0 \geq 0} [1 - F(p_0 + a - b)](p_0 + a) = \max_{\hat{p}_0 \geq a-b} [1 - F(\hat{p}_0)](\hat{p}_0 + b) \leq \pi^F.$$

Together with the results in the following point, this implies that if $\Delta < \rho(0) \leq b$ the platform has never (strict) incentives to give access to the superior seller.

[59] Formally,

$$\frac{\partial^2 \pi_1^*}{\partial a^2} = -af'(p_2^*(a) - \Delta)\left(\frac{\partial p_2^*(a)}{\partial a}\right)^2 - f(p_2^*(a) - \Delta)\left[2\frac{\partial p_2^*(a)}{\partial a} + a\frac{\partial^2 p_2^*(a)}{\partial a^2}\right],$$

where, using the implicit function theorem and the optimality condition for the 3rd party seller,

$$\frac{\partial^2 p_2^*(a)}{\partial a^2} = \frac{[1 - F(p_2^*(a) - \Delta)][2[f'(p_2^*(a) - \Delta)]^2 - f''(p_2^*(a) - \Delta)f(p_2^*(a) - \Delta)] - [f(p_2^*(a) - \Delta)]^2 f'(p_2^*(a) - \Delta)}{\left[\frac{[1-F(p_2^*(a)-\Delta)]}{f(p_2^*(a)-\Delta)} f'(p_2^*(a) - \Delta) + 2f(p_2^*(a) - \Delta)\right]^3}.$$

Using the definition of $\rho(\cdot)$ and simplifying, we find

$$\frac{\partial p_2^*(a)}{\partial a} < 0 \iff -2-a\frac{3f(p_2^*(a) - \Delta)f'(p_2^*(a) - \Delta) + \rho(p_2^*(a) - \Delta)[f''(p_2^*(a) - \Delta)f(p_2^*(a) - \Delta) - [f'(p_2^*(a) - \Delta)]^2]}{[\rho(p_2^*(a) - \Delta)f'(p_2^*(a) - \Delta) + 2f(p_2^*(a) - \Delta)]^2} < 0,$$

which is always satisfied if $f''(\cdot)f(\cdot) \geq [f'(\cdot)]^2$.

[60] It can be easily seen that these equilibria exist as long as $a$ is not too large, and the platform is indifferent between foreclosing or not the superior app if $p_2^* = a-b+\Delta$, and strictly better off not foreclosing it if $p_2^* < a-b+\Delta$.

If the 3rd party seller is foreclosed, $p_0^F = 0$ implies that[61]

$$W^F = b + \mathbb{E}[v],$$

which may be higher than $W^*$ if demand is inefficiently low in the no-foreclosure equilibrium.

For all $a \in [b, b + \Delta - \rho(0)]$, as the core ZLB binds and $p_2^* = \Delta$, consumers' demand is the same with or without foreclosure. As foreclosure reduces social surplus by $\Delta$, any such fee is preferable to $a < b$ under no-monitoring of foreclosure. If non-price foreclosure can be monitored, then all fees $a \in [\rho(0), b)$ are optimal as well, given that for any such $a$, in equilibrium $p_0^* + p_2^* = \Delta$ (by contrast, for lower fees $a < \rho(0)$, $p_0^* + p_2^* > \Delta$, and so social welfare is strictly lower). For $a > b + \Delta - \rho(0)$, the core ZLB keeps binding but $p_2^* > \Delta$ increases in $a$, which inefficiently reduces demand.

Yet, from the platform's viewpoint, this reduction in demand is more than compensated by the increased revenue from sellers' margin squeeze for all $a < a^*$. Hence, social welfare maximization requires imposing a cap on access charges.

Turning to consumer surplus, with foreclosure consumers get net value $v$, and so $S^F = \mathbb{E}[v]$. As, absent foreclosure, $p_0^* + p_2^* \geq \Delta$ for all $a < b$, $S^* \leq S^F$, with equality if and only if $a \in [\rho(0), b)$. The same net value $S^F$ is obtained for all $a \in [b, b + \Delta - \rho(0)]$, as the core ZLB binds and they purchase the high-value app at $p_2^* = \Delta$. For larger values of $a$, $p_2^* > \Delta$ and so $S^* < S^F$.

While consumer surplus and social welfare are maximized for a compact set of access charges, $\pi_2^*(a) = \Delta$ if and only if $\hat{a} = b$. Indeed, for $a < b$, either the platform forecloses it, or the 3rd party seller makes a supranormal profit $\pi_2^*(a) > \Delta$;[62] and any $a > b$ entails some margin squeeze, given that $p_2^* = \Delta$ for all $a \in [b, b + \Delta - \rho(0)]$, and $p_2^* < a - b + \Delta$ for larger values of $a$: and so $p_2^* + b - a < \Delta$, hence $\pi_2^*(a) < \Delta$. ■

### B.3 Proof of Lemma 1

Given app prices $(p_1, p_2)$, a consumer type $\Delta$ prefers the 3rd party app to the in-house app if and only if $\Delta \geq p_2 - p_1$. In this case, she buys that app (rather than forego consumption) if and only if $\Delta \geq p_0 + p_2 - v$, with $p_2 - p_1 \geq p_0 + p_2 - v$ if and only if $p_0 + p_1 \leq v$. Under the latter condition, any consumer $\Delta < p_2 - p_1$ prefers buying the in-house app rather than not accessing the app-store. Hence, if $p_0 + p_1 \leq v$, all consumers buy one app, and firms' profits are

$$\pi_1(p_0 + p_1 \leq v) = H(p_2 - p_1)(p_1 + b) + [1 - H(p_2 - p_1)]a + p_0,$$

and

$$\pi_2(p_0 + p_1 \leq v) = [1 - H(p_2 - p_1)](p_2 + b - a).$$

[61] Note that $v \geq 0$ implies $F(0) = 0$.

[62] As the 3rd party app creates an extra-value $\Delta$, the fact that the platform would prefer foreclosing it when $a < b$ clearly implies $\pi_2^*(a) > \Delta$.

If, on the contrary, $p_0+p_1>v$, then only the 3[rd] party app is bought in equilibrium, and firms' profits are

$$\pi_1(p_0+p_1>v)=[1-H(p_0+p_2-v)](p_0+a),$$

and

$$\pi_2(p_0+p_1>v)=[1-H(p_0+p_2-v)](p_2+b-a).$$

Since $\pi_1(p_0+p_1\leq v)$ is increasing in $p_0$, this profit is maximized at $p_0=v-p_1$ (with $0\leq p_1\leq v$, given the two ZLB constraints). As $p_0=v-p_1$ implies $p_0+p_2-v=p_2-p_1$, we have:

$$\pi_1(p_0+p_1\leq v)\equiv\pi_1^*(a)=[1-H(p_0+p_2-v)](p_0+a)+H(p_0+p_2-v)(b+v)>\pi_1(p_0+p_1>v).$$

Therefore, it is never optimal for the platform to set any larger prices such that $p_0+p_1>v$, and so $p_0^*=v-p_1$. In what follows, we characterize the equilibrium values for $p_1$ and $p_2$.

Denoting by $\rho_\Delta(\tilde{\Delta})\equiv[1-H(\tilde{\Delta})]/h(\tilde{\Delta})$ the inverse hazard rate, which is assumed decreasing, we have

$$\frac{\partial\pi_1}{\partial p_1}=-h(p_2-p_1)(p_1+b-a)-1+H(p_2-p_1)=0\iff a-b-p_1=\rho_\Delta(p_2-p_1),\quad\text{(B.3)}$$

and

$$\frac{\partial\pi_2}{\partial p_2}=-h(p_2-p_1)(p_2+b-a)+1-H(p_2-p_1)=0\iff p_2-(a-b)=\rho_\Delta(p_2-p_1).\quad\text{(B.4)}$$

Let us first consider an equilibrium where $p_1^*=v$. By (B.3), this is the case if and only if

$$\left.\frac{\partial\pi_1}{\partial p_1}\right|_{p_1=v}\geq0\iff a-b-v\geq\rho_\Delta(p_2-v)=p_2-(a-b)\iff p_2\leq2(a-b)-v,$$

where the equality uses (B.4). In turn, using again (B.4), we find

$$p_2\leq2(a-b)-v\iff a-b-v\geq\rho_\Delta(2(a-b-v)).\quad\text{(B.5)}$$

Since the LHS (RHS) of this inequality is increasing (decreasing) in $a$, there is a threshold $\overline{a}>b+v$ such that, for all $a\geq\overline{a}$, in equilibrium: $p_0^*=0$, $p_1^*=v$ and $p_2^*>v$ solves $p_2=a-b+\rho_\Delta(p_2-v)$. Note that $\overline{a}>b+v$ implies $p_1^*=v<a-b$.

Next, consider an equilibrium where $p_1^*=0\leq p_2^*$. By (B.3), this is the case if and only if

$$\left.\frac{\partial\pi_1}{\partial p_1}\right|_{p_1=0}\leq0\iff a-b\leq\rho_\Delta(p_2)\leq p_2-(a-b)\iff p_2\geq2(a-b),$$

where the second inequality uses (B.4), which holds with equality as long as $p_2>0$. Hence, in equilibrium $p_1^*(a)=p_2^*(a)=0$ if $\frac{\partial\pi_1}{\partial p_1}|_{p_1=p_2=0}\leq0$ and $\frac{\partial\pi_2}{\partial p_2}|_{p_1=p_2=0}\leq0$, which gives $a<b-\rho_\Delta(0)$. In turn, from (B.4),

$$p_2\geq2(a-b)\iff a-b\leq\rho_\Delta(2(a-b)),\quad\text{(B.6)}$$

which, by similar arguments of the above case, is satisfied if and only if $a \leq \underline{a}$, with $\underline{a} \in (b, \overline{a})$: $\underline{a} > b$, as otherwise the LHS is negative; and $\underline{a} < \overline{a}$, since the LHS (RHS) of (B.6) is smaller (larger) than the one of (B.5), and both are increasing (decreasing) in $a$.

Finally, consider an equilibrium where $p_2^* > p_1^* \in (0, \Delta)$. In this equilibrium, (B.3)-(B.4) imply

$$a - b - p_1 = \rho_\Delta(p_2 - p_1) = p_2 - (a - b) \iff p_1 + p_2 = 2(a - b). \tag{B.7}$$

Since prices have to be non-negative, this equilibrium can exist only for $a > b$. Moreover, as $p_2^* > p_1^*$, it must be $p_1^* < a - b < p_2^*$. Using (B.7), (B.3) rewrites as

$$a - p_1 - b = \rho_\Delta(2(a - p_1 - b)). \tag{B.8}$$

As the LHS (RHS) is decreasing (increasing) in $p_1$, this equilibrium exists if $a - b > \rho_\Delta(2(a - b))$ (which ensures $p_1^* > 0$) and $a - b - v < \rho_\Delta(2(a - b - v))$ (so that $p_1^* < v$), i.e. for $a \in (\underline{a}, \overline{a})$.

Summing up, we have the following equilibrium characterization:

- For $a \in [0, \underline{a}]$, with $\underline{a} > b$, in equilibrium $p_0^* = v$, and $p_1^* = 0 \leq p_2^*$. The platform's profit is

$$\pi_1^*(a) = H(p_2^*)(b - a) + a + v.$$

  For $a \in [0, b - \rho_\Delta(0)]$: $p_2^* = 0$, all consumers buy the 3rd party app (as $H(0) = 1$), and $\pi_1^*(a) = a + v < b + v \equiv \pi^F$. For $a \in (b - \rho_\Delta(0), \underline{a}]$, $p_2^* > 0$, and

$$\frac{\partial \pi_1^*}{\partial a} = h(p_2^*)\frac{\partial p_2^*}{\partial a}(b - a) - H(p_2^*) + 1 > 0 \iff \frac{\partial p_2^*}{\partial a}(a - b) < \rho_\Delta(p_2^*) = p_2^* - (a - b),$$

  which is satisfied for all $a < b$ whenever $\frac{\partial p_2^*}{\partial a} > 0$ (since $p_2^* > 0 > a - b$), and for $a \in [b, \underline{a}]$ as well if $\frac{\partial p_2^*}{\partial a} < 1$ (since $(a - b) \leq p_2^* - (a - b) \iff p_2^* \geq 2(a - b)$, which, as seen above, must be true for such an equilibrium to exist). Using (B.4) and the implicit function theorem,

$$\frac{\partial p_2^*}{\partial a} = \frac{h(p_2^*)}{2h(p_2^*) + h'(p_2^*)(p_2^* + b - a)} = \frac{1}{2 + \frac{h'(p_2^*)}{h(p_2^*)}\rho_\Delta(p_2^*)} \in (0, 1) \iff \rho_\Delta(p_2^*)\frac{h'(p_2^*)}{h(p_2^*)} > -1,$$

  where the latter inequality is equivalent to $\rho'_\Delta(p_2^*) < 0$, which we assumed at the beginning.[63] Therefore, we can conclude that $\frac{\partial \pi_1^*}{\partial a} > 0$ for all $a \in [0, \underline{a}]$. Given that $\pi_1^*(b) = \pi^F$, it then follows that non-price foreclosure is optimal if and only if $a < b$.

- For $a \in (\underline{a}, \overline{a})$, with $\overline{a} > b + v$, in equilibrium $0 < p_1^* < a - b < p_2^*$. From (B.8) it follows that $p_1^* - a$ is constant varying $a$. Since $p_1 + p_2 = 2(a - b) \iff p_1 - a = a - p_2 - 2b$, this implies that $a - p_2^*$ is constant in $a$ as well, and so also $p_2^* - p_1^*$ does not vary with $a$. This shows a neutrality result: $\pi_1^*(a) = [1 - H(p_2^* - p_1^*)](v - p_1^* - a) + H(p_2^* - p_1^*)(b + v)$ is

[63] Indeed, for any $x > 0$:

$$\rho'_\Delta(x) = -1 - \rho_\Delta(x)\frac{h'(x)}{h(x)}.$$

independent of $a$ in this range. However, $\pi_1^*(a) > \pi^F$ since $p_1^* < a - b$.

- For $a \geq \overline{a}$, in equilibrium $p_0^* = 0$ and $p_1^* = v < p_2^*$, so that the platform's profit is

$$\pi_1^*(a) = H(p_2^* - v)(b + v - a) + a.$$

We then have:

$$\frac{\partial \pi_1^*}{\partial a} = h(p_2^* - v)\frac{\partial p_2^*}{\partial a}(b + v - a) - H(p_2^* - v) + 1,$$

where, using the 3rd party seller's FOC and the implicit function theorem, as above, $\frac{\partial p_2^*}{\partial a} \in (0,1)$. Hence,

$$\frac{\partial \pi_1^*}{\partial a} > 0 \iff a - b - v < 2(p_2^* + b - a) + \frac{h'(p_2^* - v)}{h(p_2^* - v)}(p_2^* + b - a)^2. \tag{B.9}$$

Moreover, under mild restrictions on the derivatives of $h(\cdot)$, $\pi_1^*(a)$ is strictly concave in $a$ in this range of the access charge.[64] Finally, as $a \to \overline{a}$: $p_2^* = 2(a - b) - v$. Substituting into (B.9) and simplifying gives

$$\frac{h'(2(a - b - v))}{h(2(a - b - v))}(a - b - v) = \frac{h'(2(a - b - v))}{h(2(a - b - v))}\rho_\Delta(2(a - b - v)) > -1,$$

where the equality follows from the definition of $\overline{a}$. This inequality is always satisfied as it is equivalent to the assumption of decreasing inverse hazard rate. Therefore, the platform's equilibrium profit is still increasing at $a = \overline{a}$. As a consequence, since it is strictly concave, it must attain a maximum at some (finite) $a^* > \overline{a}$, and then converge to the foreclosure level.[65] ■

## B.4 Proof of Proposition 6

As $p_0^* + p_1^* = v$, consumers purchasing the platform's in-house app have zero surplus, and consumer surplus writes as

$$S^* = \int_{\Delta \geq p_2^* - p_1^*} [\Delta - (p_2^* - p_1^*)] dH(\Delta) > 0,$$

[64] Formally, we find

$$\frac{\partial^2 \pi_1^*}{\partial a^2} = h'(p_2^* - v)(a - b - v)\frac{1 - \frac{h'(p_2^* - v)}{h(p_2^* - v)}(1 + 2\frac{h'(p_2^* - v)}{h(p_2^* - v)}\rho_\Delta(p_2^* - v)) + \rho_\Delta(p_2^* - v)\frac{h''(p_2^* - v)}{h(p_2^* - v)}}{(2 + \frac{h'(p_2^* - v)}{h(p_2^* - v)}\rho_\Delta(p_2^* - v))^2} +$$
$$- \frac{2h(p_2^* - v)}{2 + \frac{h'(p_2^* - v)}{h(p_2^* - v)}\rho_\Delta(p_2^* - v)} < 0,$$

which holds provided $|h'(\cdot)|$ is not too large.

[65] This is because, as $a$ increases, $p_2^* > a - b$ grows larger, and so the demand of the 3rd party app $1 - H(p_2^* - v)$ goes down to zero, thereby $\pi_1^*(a) \to \pi^F$.

if there is no foreclosure, and $S^F = 0$ with foreclosure. Social welfare is given by

$$W^* = b + v + \int_{\Delta \geq p_2^* - p_1^*} \Delta dH(\Delta),$$

if there is no foreclosure, and $W^F = b + v$ with foreclosure.

Hence, both consumer surplus and social welfare are lower under foreclosure: If non-price foreclosure cannot be monitored, it must be that $a \geq b$. Moreover, they are both decreasing in the relative price $p_2^* - p_1^*$. The access charge thus affects $S^*$ and $W^*$ only through its impact on the equilibrium prices. Then:

- for $a \in [0, \underline{a}]$, as $p_2^*$ is increasing in $a$ (strictly so for $a > b - \rho_\Delta(0)$) and $p_1^* = 0$, $S^*$ and $W^*$ are decreasing in $a$ (strictly so for $a > b - \rho_\Delta(0)$);
- for $a \in (\underline{a}, \overline{a})$, $p_2^* - p_1^*$, and hence $S^*$ and $W^*$, are constant when $a$ varies;
- for $a \geq \overline{a}$, as $p_2^*$ is strictly increasing in $a$ and $p_1^* = v$, $S^*$ and $W^*$ are again strictly decreasing in $a$.

Hence, if monitoring no-price foreclosure is not feasible, the optimal access charge, both from a consumer-surplus and a total-welfare standpoint, is $\hat{a} = b$. If, on the contrary, non-price foreclosure could be monitored, then any $a \in [0, b - \rho_\Delta(0)]$ maximizes welfare. ■

## B.5 Proof of Proposition 7

In this appendix, besides proving Proposition 7, we derive conditions under which the considered equilibrium is unique and show that our regulatory implications are robust.

*Preliminaries.* To start with, we show that all equilibria of the game are such (i) that all platforms offer the same utility to consumers, and (ii) the core ZLB binds for all platforms:

**Claim 1.** *In all equilibria, $U^{1*} = \ldots = U^{N*} \equiv U^* \in [0, v]$, and $p_0^{1*} = \ldots = p_0^{N*} = 0$. Hence, $U^* = u^*$.*

*Proof.* First, as $u^i \equiv \max\{v - p_1^i,\ v + \Delta - p_2^i,\ 0\}$ and non-pivotal 3rd party sellers always charge $p_2^{i*} \geq \Delta$: $u^i \in [0, v]$ for all $i$. As $p_0 \geq 0$ and consumers' outside option is zero, also $U^i \in [0, v]$.

Second, take two platforms $i'$ and $i''$ and suppose they offer different utility levels to consumers $v \geq U^{i'} > U^{i''} \geq 0$, with $U^{i'} = \max_j\{U^j\}$. Then, platform $i''$ would face no demand and make zero profits. By foreclosing the 3rd party seller and setting prices $p_0^i + p_1^i \leq v - U^{i'}$, it would offer utility $U^{i''} \geq U^{i'}$ and make positive profit.

Third, given that all platforms offer utility $U^*$ in equilibrium, their profit writes as $\pi_1^i = \frac{1}{N}(p_0^i + p_1^i + b)$ with foreclosure, with $p_0^i + p_1^i = v - U^*$ and $\pi_1^i = \frac{1}{N}(p_0^i + a^i)$ without foreclosure, with $p_0^i = v + \Delta - p_2^* - U^*$. If $p_0^i > 0$ for some $i$, then this platform would find it optimally to deviate, charging a slightly lower access price to consumers to serve all demand. ■

*Equilibria with $U^* = v$: Existence and characterization.* We next characterize the equilibrium considered in Proposition 7:

**Claim 2.** *There always exist equilibria with $U^* = v$, such that, for any given $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$ (no foreclosure), in the continuation equilibrium $p_0^i = p_1^i = 0$ and $p_2^i = \Delta$.*

*Proof.* Whenever its rivals are expected to provide $U^* = v$ in equilibrium, any platform $i$ has no profitable deviations to $U^i \neq U^*$: offering $U^i < v$ drives its profit to zero, and, as shown above, it is never possible to provide $U^i > v$. As $U^* = v$ can always be provided by foreclosing the 3rd party app and setting $p_0^i = p_1^i = 0$, it follows that equilibria with $U^* = v$ always exist. We next characterize the corresponding subgame perfect equilibrium prices for any given $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$. Suppose, by contradiction, that in equilibrium $p_1^i > 0$. Then, as $p_2^{i*} = \min\{p_1^i + \Delta, v + \Delta\}$ in equilibrium: $u^i = \max\{v - p_1^i, 0\} < v$. Given that rival platforms offer higher value $U^* = v$ in equilibrium, the considered platform makes no profit. It has therefore a strictly profitable deviation: It can set $p_1^i = 0$ and thus, by selling its in-house app, offer value $U^i = v$ to consumers, so as to attract some of them and make positive profits (given that the marginal cost is negative). Anticipating this, the (non-foreclosed) 3rd party seller must set $p_2^{i*} = \Delta$ to sell its app. It optimally does so whenever selling its app yields positive profit (i.e., as long as $\Delta + b - a^i \geq 0$). Hence, in any equilibrium where $U^* = v$: $p_1^* = 0$, and hence $p_2^* = \Delta$, for all $a^i \in [0, b + \Delta]$. ■

*Equilibria with $U^* \in [0, v)$: Characterization.* We now consider the other equilibria of the game, providing existence conditions (supposing that platforms can freely set access charges):

**Claim 3.** *In any equilibrium with $U^* = u^* \in [0, v)$, absent regulation each platform would set $a^* = b + \Delta + v - u^*$, which fully squeezes the 3rd party seller it hosts. Any such equilibrium can exist if and only if $\Delta > (N-1)[v - u^* + b]$.*

*Proof.* Suppose there is an equilibrium where all platforms provide $u^* \in [0, v)$. Then, absent foreclosure, the joint profit of platform $i$ and the 3rd party seller it hosts cannot exceed $\pi_1^*(a) + \pi_2^*(a) = \frac{1}{N}(v + \Delta - u^* + b)$. Setting $a^* = b + \Delta + v - u^*$ allows platform $i$ to reap all this profit: At this level of the access charge, the 3rd party seller sets $p_2^* = v + \Delta - u^*$ and has unit margin $p_2^* + b - a^* = 0$. By foreclosing instead the 3rd party app, the platform could profitably offer slightly higher utility to consumers (by slightly reducing $p_1$, given that $u^* < v$) so as to serve all demand and get $(v - u^* + b)$. Hence, if platforms can freely set access charges, then an equilibrium with $u^* \in [0, v)$ exists if and only if $\frac{1}{N}(v + \Delta - u^* + b) > v - u^* + b$, which boils down to $\Delta > (N-1)(v - u^* + b)$. ■

*Equilibria with $u^* = v$: Uniqueness.* We derive a condition under which the equilibrium considered in Proposition 7 is unique:

**Claim 4.** *Suppose $\Delta < (N-1)b$. Then, for any $(a^i)_{i=1,\ldots,N}$, all platforms must offer $U^* = v$ in equilibrium.*

*Proof.* From the previous claim it follows that an equilibrium where platforms are free to set access charges and offer $u^* \in [0, v)$ does not exist if $\Delta < (N-1)(v - u^* + b)$. This holds for all

$u^* \in [0, v)$ if and only if $\Delta < (N-1)b$. This condition was obtained under the assumption that each platform is free to set access charges, and so optimally chooses the highest $a^i$ that satisfies the 3rd party app's participation constraint. As each platform $i$'s equilibrium profit is non-decreasing in $a^i$, whereas the deviation profit from implementing foreclosure does not depend on the access charge, it follows that $\Delta < (N-1)b$ is a sufficient condition for $u^* = v$ in equilibrium for all possible levels of access charges. This is because, given a candidate equilibrium with $u^* < v$, for any $a^i < a^*$ the platform would have stronger incentives to deviate by foreclosing the 3rd party app. ■

This result implies that, absent regulation, for $\Delta < (N-1)b$ the unique equilibrium is such that $a^* = \Delta + b$, $p_0^* = p_1^* = 0$ and $p_2^* = \Delta$.

In this equilibrium, without foreclosure, for all $a^i \in [0, b + \Delta]$ and $i = 1, \ldots, N$, firms' profits are $\pi_1^i(a^i) = \frac{1}{N} a^i$ and $\pi_2^i(a^i) = \frac{1}{N}(\Delta + b - a^i)$. Maximizing its profit subject to the 3rd party app's individual rationality constraint yields $a^i = b + \Delta$. By contrast, summing across platforms, $\sum_{i=1}^{N} \pi_2^i(a^i) = \Delta$ if and only if $a^i = b$ (at this level of access charge, the platform has not strict foreclosure incentives).

Finally, note that, for $\hat{a} = b$, it must be $p_1^* = 0$, and so $p_2^* = \Delta$ and $u^* = v$. Accordingly, $\pi_1^*(\hat{a}) = b$ and $\sum_{i=1}^{N} \pi_2^{i*}(\hat{a}) = \Delta$. Thus, for all values of the parameters, when access charges are set by regulation at the Pigouvian level, the game admits a unique equilibrium outcome, which features fairness.

### B.6 Proof of Proposition 8

Consumers' utility from accessing the app-store is

$$U \equiv \int_{k \in [0,1]} u^k \, dk - p_0,$$

with $p_0$ again denoting consumers' access price, and $u^k$ being the utility obtained from app market $k \in [0, 1]$:

$$u^k \equiv \max\{v^k - p_1^k,\ v^k + \Delta^k - p_2^k,\ 0\},$$

where $p_1^k$ and $p_2^k$ denote the prices for in-house app and 3rd party app, respectively, in the considered market $k$.

To start with, we show that the results of Section 2 extend to this setting with multiple heterogeneous app markets.

Asymmetric Bertrand competition in each app market, together with platform pivotality, yield

$$p_1^{k*} = \max\{a^k - b^k, 0\}, \quad p_2^{k*} = \min\{p_1^{k*} + \Delta^k, v^k + \Delta^k\},$$

whenever $a^k \leq b^k + v^k + \Delta^k$ (any larger access charge implies access-price foreclosure of the 3rd party seller). Then, $p_0$ is set so as to satisfy consumers' participation constraint with equality:

$$p_0^* = \int_{k\in[0,1]} u^{k*}\ dk.$$

Hence, denoting $x^k = 0$ ($x^k = 1$) if the in-house app (3rd party app) is sold in market $k$, platform's profit writes

$$\pi_1^* = p_0^* + \int_{\{k:\, x^k=0\}} (p_1^{k*} + b^k)\ dk + \int_{\{k:\, x^k=1\}} a^k\ dk = \int_{\{k:\, x^k=0\}} \pi_1^k(x^k=0)\ dk + \int_{\{k:\, x^k=1\}} \pi_1^k(x^k=1)\ dk,$$

where

$$\pi_1^k(x^k=0) \equiv v^k + b^k, \qquad \pi_1^k(x^k=1) \equiv v^k + \Delta^k - p_2^{k*} + a^k.$$

If not foreclosed, the 3rd party seller in market $k$ makes

$$\pi_2^{k*} = p_2^{k*} + b^k - a^k.$$

In any market $k$ where $a^k < b^k$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = \Delta^k$ and obtain utility $u^{k*} = v^k$. As this is the same utility that they would obtain under foreclosure and $p_1^{k*} = 0$, it follows that by foreclosing 3rd party rivals in any such market the platform can charge the same access price $p_0^*$ to consumers, but obtains higher unit revenues $b^k > a^k$.

In any market $k$ where $a^k \in [b^k, b^k + v^k]$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = a^k - b^k + \Delta^k < v^k + \Delta^k$ and obtain utility $u^{k*} = v^k - (a^k - b^k)$. From any such market, the platform obtains profit $\pi_1^k(x^k = 1) = v^k + b^k = \pi_1^k(x^k = 0)$, and so it is indifferent between foreclosing or not. The 3rd party seller gains $\pi_2^{k*} = \Delta^k$.

Finally, in any market $k$ where $a^k \in (b^k + v^k, b^k + v^k + \Delta^k]$, absent foreclosure, consumers purchase the 3rd party app at $p_2^{k*} = v^k + \Delta^k < a^k - b^k + \Delta^k$ and obtain utility $u^{k*} = 0$. From any such market, the platform obtains profit $\pi_1^k(x^k = 1) = a^k > b^k + v^k = \pi_1^k(x^k = 0)$, and so is strictly better off than under foreclosure. The 3rd party app is squeezed: $\pi_2^{k*} = v^k + \Delta^k + b^k - a^k < \Delta^k$. Hence, the profit maximizing fee in market $k$ is $a^* = v^k + \Delta^k + b^k$.

Suppose the regulator knows the distribution $K(b)$ of ancillary benefits and lets the platform choose $(a^k)_{k\in[0,1]}$ subject to the constraint that the distribution of access charges mimics that of benefits (i.e., follows $K(a)$). To show that setting $a^k = b^k$ is not incentive-compatible for the platform, take two markets $k'$ and $k''$ such that $v^{k'} = v^{k''} \equiv v \le b^{k''} - b^{k'} \le v + \Delta^{k'}$.[66] If the platform sets $a^k = b^k$ for $k \in \{k', k''\}$, it obtains profit $\pi_1^k = b^k + v$ from each of these markets (as $p_2^k = \Delta^k$, and so $u^k = v$ can be extracted through $p_0$). By setting instead $a^{k'} = b^{k''}$ and $a^{k''} = b^{k'}$, and foreclosing the 3rd party app in market $k''$, it still obtains profit $\pi^{k''} = v^{k''} + b^{k''}$ in the higher-$b$ market (as still $u^{k''} = v$), but now makes $\pi^{k'} = a^{k'} = b^{k''} > b^{k'} + v$ from the

[66] Given the assumption of a continuum of markets, the Pigouvian rule is not implemented in equilibrium in a positive-measure set of markets whenever the set of such $(k', k'')$ has positive measure: e.g., with homogeneous $v$ and $\Delta$, this is the case whenever $K(b)$ has positive density over some interval $[\underline{b}, \overline{b}]$, and $v \le \overline{b} - \underline{b} \le v + \Delta$.

lower-$b$ market (as $p_2^{k'} = v + \Delta^{k'}$ and so $u^{k'} = 0$).

Note that this deviation is not profitable if the platform cannot foreclose the 3rd party app in the high-$b$ market $k''$. Indeed, the deviation profit equals now $\pi^{k''} = v^{k''} + b^{k'}$ in market $k''$ (as $p_2^k = b^{k''} - b^{k'} + \Delta$, and so still $u^{k''} = v$), and so $\pi^{k''} + \pi^{k''} = b^{k'} + b^{k''} + v < b^{k'} + b^{k''} + 2v$ is lower than the equilibrium profit. This suffices to prove that, if foreclosure can be monitored, then, under the distributional constraint $K(a) = K(b)$, setting $a^k = b^k$ for all $k \in [0, 1]$ is incentive-compatible for the platform. ■

# C Further material

## C.1 Heterogeneous valuations for the core

In the models of Section 3.2 and 3.3, we have assumed that the platform is per-se valueless. This section introduces consumer heterogeneity with respect to the valuations $v_c \geq 0$ of the core.

Denoting by $F(v)$ the cdf of consumers' willingness to pay for the overall platform's services (core plus in-house app), heterogeneity in $v_c$ yields a demand $1 - F(p_0 + p_1 - v_a)$ under foreclosure, and $1 - F(p_0 + p_2 - v_a - \Delta)$ without foreclosure.

Assuming platform pivotality as in Section 2, we have:

**Lemma C.1.1.** *App prices in equilibrium are* $p_1^* = \max\{a - b, 0\}$ *and* $p_2^* = \min\{p_1^* + \Delta, v_a + \Delta\}$. *Moreover:*

- *for* $a < b$ *(i.e., when the app ZLB binds),* $p_0^*$ *is decreasing in* $a$*, and the platform is better off foreclosing the 3rd party app (if feasible);*
- *for* $a \in [b, b + v_a]$*, there is competitive neutrality:* $p_0^* + a$ *and* $p_2^* - a$ *are constant when* $a$ *varies, and the platform is indifferent between practising foreclosure or not;*
- *for* $a \in (b + v_a, b + v_a + \Delta]$*, the platform gains strictly more by squeezing the 3rd party app provider than by foreclosing it, and*
  - *for* $a \in (b + v_a, \rho(0))$*, if this region of parameters is non-empty,* $p_0^* > 0$ *is decreasing in* $a$*,*
  - *for* $a \in [\min\{b + v_a, \rho(0)\}, b + v_a + \Delta]$*, if this region of parameters is non-empty, the core ZLB binds.*

*Proof of Lemma C.1.1.* Under foreclosure, the platform can set $p_1 = 0$ and gets profit

$$\pi^F \equiv \max_{p_0 \geq 0} \, (p_0 + b)[1 - F(p_0 - v_a)].$$

Taking the first-order condition of the platform's problem gives

$$p_0^F + b = \rho(p_0^F - v_a),$$

and that the core ZLB is never binding under foreclosure when $v_c + v_a \geq 0$ for all consumers.

Without foreclosure, as consumers' valuations in the app segment are homogeneous, app prices $(p_1^*, p_2^*)$ are clearly as in the basic model. The demand is then $1 - F(p_0 + p_2^* - v_a - \Delta)$, and we have:

- For $a < b$, the platform's profit,

$$\max_{p_0 \geq 0} (p_0 + a)[1 - F(p_0 - v_a)] < \pi^F,$$

  is maximized at

$$p_0^* + a = \rho(p_0^* - v_a),$$

  from which (the RHS being decreasing in $p_0^*$ by assumption) it follows that $\frac{\partial p_0^*}{\partial a} \in (-1, 0)$.

- For $a \in [b, b + v_a]$,

$$\max_{p_0 \geq 0} (p_0 + a)[1 - F(p_0 + a - b - v_a)] = \max_{\tilde{p}_0 > 0} (\tilde{p}_0 + b)[1 - F(\tilde{p}_0 - v_a)] = \pi^F,$$

  where the equality follows because the core ZLB is not binding under foreclosure. In this case, there is competitive neutrality: $p_0^* + a$, as well as $p_2^* - a$, are constant when $a$ varies.

- for $a \in (b + v_a, b + v_a + \Delta]$,

$$\max_{p_0 \geq 0} (p_0 + a)[1 - F(p_0)] = \max_{\tilde{p}_0 > 0} (\tilde{p}_0 + b)[1 - F(\tilde{p}_0 - (a - b))] > \pi^F,$$

  given that $a - b > v_a$. The first-order condition for an interior optimum of the platform's problem gives

$$p_0^* + a = \rho(0),$$

  from which it follows that, in an interior optimum, $\frac{\partial p_0^*}{\partial a} \in (-1, 0)$, and also that, in this region of parameters, the core ZLB binds if and only if $a \geq \rho(0)$. ■

Suppose $b < \rho(0) - v_a$. Then, for $a \in (b + v_a, \rho(0))$ as in the basic model the $3^{\text{rd}}$ party seller is constrained by consumers' willingness to pay for its app, and so cannot pass through increased fees to consumers, but here the core ZLB is not yet binding. Therefore, as the access charge grows larger, the platform gains higher unit profits from the app-store (by squeezing the $3^{\text{rd}}$ party seller), which gives it an incentive to increase consumers' participation by reducing the access price ($\frac{\mathrm{d}p_0^*(a)}{\mathrm{d}a} < 0$ so long as the core ZLB does not bind). As a result, consumer total price $p_0^* + p_2^* = p_0^*(a) + v_a + \Delta$ is decreasing in $a$ in this range: Higher access charges in this case reduce, rather than exacerbate, double marginalization, and so, provided the superior $3^{\text{rd}}$ party already exists, the lowest among welfare-optimal access charges is $a = \rho(0) > b$. Otherwise, the Pigouvian rule is still welfare-optimal. In sum:

**Proposition C.1.1.** *Regardless of whether foreclosure is monitored or not:*

- *If $\rho(0) \leq b + v_a$, then any $a \geq b$ maximizes consumer surplus and social welfare;*
- *If instead $\rho(0) > b + v_a$, then consumer surplus and social welfare are maximized for any*

$a \geq \rho(0)$.

*Proof of Proposition C.1.1.* Both consumer surplus,

$$S = \int_{p_0+p_2-(v_a+\Delta)}^{\infty} (v_c + v_a + \Delta - (p_0 + p_2))dF(v_c),$$

and social welfare,

$$W = \int_{p_0+p_2-(v_a+\Delta)}^{\infty} (v_c + v_a + \Delta + b)dF(v_c),$$

are decreasing in the total price $p_0 + p_2$. From Lemma C.1.1, we have:

- for $a < b$, $p_0^* + p_2^* = p_0^* + \Delta$ is decreasing in $a$;
- for $a \in [b, b + v_a]$, $p_0^* + p_2^*$ is constant when $a$ varies;
- for $a \in (b + v_a, b + v_a + \Delta]$, $p_0^* + p_2^* = p_0^* + v_a + \Delta$ is decreasing in $a$ as long as the core ZLB does not bind, and constant in $a$ otherwise.

Hence, the welfare objectives are always maximized when the core ZLB binds, i.e. for $a \geq \rho(0)$. However, if $\rho(0) \leq b + v_a$, then the core ZLB always binds in the squeeze region, and so any $a \geq b$ is optimal. Lower values of the access charge are welfare-optimal as well, provided the core ZLB binds, only if foreclosure can be monitored. ■

If we allow some consumers to have an overall negative value from joining the platform (because of, e.g., privacy costs, or hassle of installing and learning how to use the platform or the apps), then the core ZLB may bind under foreclosure: precisely, it does so whenever $b \geq \rho(-v_a)$. In these cases, as seen above for the cases $b \geq \rho(0) - v_a$, the Pigouvian access charge is welfare-optimal.

If, however, the 3$^{\text{rd}}$ party seller internalizes the impact of its price on consumers' demand, then, provided the core ZLB binds under foreclosure ($b \geq \rho(-v_a)$), the analysis is analogous to the one provided in Section 3.2, with the exception that for $a < b + \Delta - \rho(-v_a)$ the 3$^{\text{rd}}$ party app is priced below $\Delta$ to spur demand, and so foreclosure need not be optimal for all $a < b$: The welfare maximizing access charge in this case would be below the Pigouvian level. A similar caveat also applies to the analysis of Section 3.2 if one allows for $v_a = v < 0$ for some consumers.

### C.2 Ad-valorem access charges

Throughout the paper we considered for simplicity linear (per-unit) access charges. Here we show that our results are robust when considering instead ad-valorem fees (which are more often employed in reality): For each app sold by the 3$^{\text{rd}}$ party seller at price $p_2$, the platform gets $tp_2$ and the seller $(1-t)p_2$, with $t \in [0, 1]$.

As in the base model, we restrict attention to the region of parameters where the platform is pivotal for consumers' participation, so that $p_2^* = p_1^* + \Delta$ whenever the core ZLB does not bind. It is shown below that the platform is again always pivotal if and only if $v \geq \Delta + b$. In what follows, to make things interesting we also posit $b < \Delta$ (as otherwise, under platform pivotality, the platform would optimally foreclose the 3rd party app for all $t \in [0,1]$).

**Lemma C.2.1.** *For any ad-valorem access charge $t \in [0,1]$, the equilibrium has the following features:*

- *For $t \in [0, \frac{b}{\Delta})$: $p_1^* = 0$, $p_2^* = \Delta$, and $p_0^* = v$; hence, $\pi_1^*(t) = v + t\Delta < b + v \equiv \pi^F$: the platform is better off foreclosing the 3rd party app; if foreclosure can be monitored, $\pi_2^*(t) = (1-t)\Delta + b > \Delta$.*
- *For $t \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$, $p_1^*(t) = \frac{t\Delta - b}{1-t}$, $p_2^*(t) = \frac{\Delta - b}{1-t}$, and $p_0^*(t) = v + \frac{b - t\Delta}{1-t}$; for any such $t$, $\pi_1^*(t) = \pi^F$ and $\pi_2^*(t) = \Delta$ (neutrality).*
- *For $t \in (\frac{b+v}{v+\Delta}, 1]$: $p_1^* = t(v+\Delta) - b$, $p_2^* = v + \Delta$, and $p_0^* = 0$; hence, $\pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$ (squeeze).*

*Proof of Lemma C.2.1.* In this setting, the platform prefers selling its own app as long as $tp_2 \leq b + p_1$. Hence, given the ZLB constraints, asymmetric Bertrand competition implies that the equilibrium app prices are such that

$$p_1^* = \max\{0, tp_2^* - b\} \quad \text{and} \quad p_2^* \leq p_1^* + \Delta,$$

with $p_2^* = p_1^* + \Delta$ whenever the platform is pivotal for consumers' participation. In this case, the equilibrium app prices are

$$\begin{cases} p_1^* = \frac{t\Delta - b}{1-t}, p_2 = \frac{\Delta - b}{1-t} & \text{if } t \geq b/\Delta \\ p_1^* = 0, p_2 = \Delta & \text{if } t < b/\Delta \end{cases}$$

Hence for $b \geq \Delta$, for all $t \in [0,1]$: $p_1^* = 0$ and $p_2^* = \Delta$ whenever the platform is pivotal:

$$p_0 + tp_2 > (1-t)p_2 + b.$$

As in this case $p_0^* = v$, platform pivotality requires $v + (2t-1)\Delta > b$, hence it is satisfied for all $t \in [0,1]$ whenever

$$v > b + \Delta,$$

which we assume throughout. In this region of parameters, the platform is always better off foreclosing, as

$$\pi_1^*(t) = v + t\Delta < b + v = \pi^F \iff t < b/\Delta,$$

which holds for all $t \in [0,1]$ when $b > \Delta$.

Suppose instead $b < \Delta$ (which is compatible with $v > b+\Delta$ if $v > 2b$). Then, for $t < b/\Delta \in (0,1)$, the analysis is unchanged: $p_0^* = v$, $p_1^* = 0$, $p_2^* = \Delta$, and foreclosure is optimal. For $t \geq b/\Delta$, whenever the platform is pivotal, $p_1^* = \frac{t\Delta - b}{1-t}, p_2^* = \frac{\Delta - b}{1-t}$ and so $p_0^* = v + \Delta - p_2^* = v + \frac{b - t\Delta}{1-t}$. Given these prices, the platform is pivotal for all $t$ if and only if $v > \Delta - b$, which is implied by $v > b + \Delta$. This is the equilibrium as long as the core ZLB does not bind, which requires $t < \frac{b+v}{v+\Delta}$. Hence, for $t \in [\frac{b}{\Delta}, \frac{b+v}{v+\Delta}]$: $\pi_1^*(t) = b + v = \pi^F$ and $\pi_2^*(t) = v$.

For $t > \frac{b+v}{v+\Delta}$, $p_0^* = 0$ hence $p_2^* = v + \Delta$, and so $p_1^* = t(v+\Delta) - b \in (v, p_2^* - \Delta)$. In this case, $\pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$. ■

The equilibrium characterization thus mirrors the one under unit fees: For low (high) values of the access charge, the app (core) ZLB binds, and the platform is strictly better off foreclosing (not foreclosing) the 3$^{\text{rd}}$ party app. For intermediate values of $t$, no ZLB binds, and the neutrality result holds. Accordingly, it is easy to derive the following results:

**Proposition C.2.1** (optimal access charges)**.** *The equilibrium of the game is such that:*

*(i) (Welfare-optimal access charges.) Any access charge such that the 3$^{rd}$ party app is not foreclosed maximizes ex-post social welfare:* $t \in [\frac{b}{\Delta}, 1]$ *if non-price foreclosure cannot be monitored,* $t \in [0,1]$ *under monitoring of self-preferencing;*

*(ii) (Profit-maximizing access charge.) Platform's profit is maximized at* $t^* = 1$*;*

*(iii) (Fair access charges.) The independent developer receives a fair reward for its contribution to the ecosystem if and only if* $\hat{t} \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$.

*Proof of Proposition C.2.1.* As consumer surplus is always extracted by the platform through the access price, social welfare is simply $W^* = \pi_1^*(t) + \pi_2^*(t) = b + v + \Delta x$, and so is maximized whenever there is no price or non-price foreclosure, so that $x = 1$, from which *(i)* follows. Platform's profit is continuous, non-decreasing in $t$ for $t \leq \frac{b+v}{v+\Delta}$, and strictly increasing for larger values of $t$, hence it is maximized at $t^* = 1$, which establishes *(ii)*. Finally, the result in *(iii)* follows from the equilibrium profit $\pi_2^*(t)$ given in Lemma 1. ■

Note that the lowest assess charge such that the platform has no incentives to practice self-preferencing, $\hat{t} = \frac{b}{\Delta}$, is such that $p_2^*(\hat{t}) = \Delta$, so that the platform obtains $\hat{t}p_2^*(\hat{t}) = b$ from distributing the 3$^{\text{rd}}$ party app. Hence, optimal access charge regulation still follows a Pigouvian principle: The superior seller must internalize that, for each app it distributes, it "steals" $b$ from the platform.

## C.3 Freemium apps

Many apps adopt a *freemium* model: A basic version (with limited functionalities) is made available for free, while consumers are charged a positive price for the premium version (which includes all functionalities). This outcome can be easily obtained by augmenting our model.

Suppose both firms sell a basic version and a premium version of their apps. The apps deliver the same value $v$ for the basic service; prices for the basic service are $p_1$ for the in-house app and $p_2$ for the 3$^{\text{rd}}$ party app. The premium service will add extra value, $V$ for the in-house app and $V + \Delta$ for the 3$^{\text{rd}}$ party app. Only those who have selected the basic service of a provider can enjoy its premium service (the two are complements).

There is no commitment as to the price of the premium service. The idea is that, while app providers can set a price for the current version of the premium service, they may start with no premium service or else will introduce new functionalities or new characters. And so the app providers charge the captive consumers' willingness to pay for the premium service. The consumers, anticipating that they will not receive surplus from the premium version, just compare the basic versions when selecting their app provider.

Suppose $v \geq 0$ follows a distribution $F(v)$ with support $\mathbb{R}^+$, with monotone (inverse) hazard rate $\rho(v)$, and that (absent foreclosure) the 3rd party app always corners consumers by setting $p_2 = 0$ (it cannot be undercut, but more on this below). The platform levies an ad-valorem tax $t \in [0, 1]$ on sales of the 3rd party app. Finally, to make our point in the simplest setting, we rule out exogenous ancillary benefits ($b = 0$).

**Proposition C.3.1.** *Suppose $V + \Delta > \rho(0)$. Then, both basic apps are priced at zero ("freemium apps") in equilibrium for all $t \in [0, 1]$, and the platform is strictly better off not foreclosing the 3rd party app for all $t \in (\frac{V}{V+\Delta}, 1]$.*

*Proof of Proposition C.3.1.* The 3rd party app solves[67]

$$\max_{p_2 \geq 0} [1 - F(p_0 + p_2)](1 - t)(p_2 + V + \Delta),$$

and so, for all $t \in [0, 1]$,

$$p_2^* = 0 \iff V + \Delta > \rho(p_0).$$

To demonstrate the possibility of the freemium (free basic apps) without-foreclosure outcome in equilibrium, we need to show that, for some $t \in [0, 1]$, the platform is strictly better off not foreclosing. In this case, the in-house app does not play any role, so that we can posit w.l.o.g. $p_1 = 0$.

So long as $p_2^* = 0$, no matter whether the 3rd party app is foreclosed or not, the platform solves

$$\max_{p_0 \geq 0} [1 - F(p_0)](b_1 + p_0),$$

with $b_1 = V$ under foreclosure, and $b_1 = t(V + \Delta)$ without foreclosure. Hence, foreclosure is not optimal whenever

$$t(V + \Delta) \geq V \iff t \geq \frac{V}{V + \Delta} \in (0, 1),$$

which concludes the proof. ∎

Given that the same ad-valorem fee is levied on both basic and premium app sales, the 3rd party app is willing to price its app at zero for all $t$ whenever the benefit that it obtains under no (or, more generally, limited) commitment from consumer lock-in is large enough. The platform is then better off not foreclosing whenever the unit tax revenue $t(V + \Delta)$ from the superior app sales exceeds the value $V$ from selling consumers the premium in-house app.

*Remark.* How strong is the assumption that the 3rd party app corners the market when $p_1 =$

[67] Given our focus on a "freemium equilibrium", the assumption that (absent foreclosure) the 3rd party app always corners consumers by setting $p_2 = 0$ allows us to neglect the constraint $p_2 \leq p_1$ for 3rd party app distribution.

$p_2 = 0$? After all, the consumers are indifferent and there would be no way for the 3rd party app to give them a bit more if the allocation of consumers to apps were different. But if the consumers' preferences for the premium version were heterogeneous, then the expected consumer net surplus from the upgrade (the relevant consideration if the consumers have no information about their preference for the premium version before trying the basic version) is (a) strictly positive, (b) under a reasonable assumption larger for the 3rd party app, implying that the only equilibrium allocation has all consumers go for the basic version of the 3rd party app (this extra surplus plays the role of $\Delta$ in our model). More formally, suppose that the distribution of $V$ is $L(V)$ for the in-house app and $L(V - \xi)$ for the 3rd party app, where $\xi > 0$. Then the in-house price of the upgrade, $P_1$, solves $\max_{P \geq 0}[1 - L(P)]$ while that for the 3rd party app, $P_2$, is obtained from $\max_{P \geq 0}[1 - L(P - \xi)]$. Under the monotone hazard rate condition for $L(V)$, we have $P_2 > P_1 > P_2 - \xi$ (the "net price" is smaller for the superior app) and so the expected net surplus is higher in the case of the 3rd party app.

The following proposition provides the regulatory implications of our analysis:

**Proposition C.3.2.** *For $V + \Delta > \rho(0)$, any access charge $t \geq \max\{\frac{\rho(0)}{V+\Delta}, \frac{V}{V+\Delta}\} \in (0, 1)$, such that the core ZLB binds and there is no foreclosure, maximizes consumer surplus and social welfare. The 3rd party app receives a fair compensation for its contribution $\Delta$ to the ecosystem if and only if $V \leq \rho(0)$ and $\hat{t} = \frac{V}{V+\Delta}$.*

*Proof of Proposition C.3.2.* As in the previously examined models with elastic demand for the platform, consumer surplus and social welfare are decreasing in the total price $p_0^* + p_2^*$. Provided the freemium equilibrium emerges, the core ZLB binds if and only if

$$\arg\max_{p_0 \geq 0}[1 - F(p_0)](t(V + \Delta) + p_0) \leq 0 \iff t \geq \frac{\rho(0)}{V + \Delta}.$$

Any such access charge is such that $p_0^* = p_2^* = 0$, and hence maximizes both welfare objectives, provided either foreclosure can be monitored, or the platform has no incentive to foreclose, i.e. $V \leq \rho(0)$. For $V > \rho(0)$, preventing foreclosure (which strictly increases social welfare) requires raising the access charge to $t \geq \frac{V}{V+\Delta} > \frac{\rho(0)}{V+\Delta}$.

Fairness requires

$$\pi_2^* = [1 - F(p_0)](1 - t)(V + \Delta) = \Delta \iff 1 - \frac{\Delta}{[1 - F(p_0)](V + \Delta)}.$$

Provided the core ZLB binds (i.e., $1 - F(p_0^*) = 1 - F(0) = 1$), this yields $t = \frac{V}{V+\Delta}$, at which the platform has no strict incentives to foreclose. If, however, $V > \rho(0)$, then fair compensation would require $t < \frac{V}{V+\Delta}$, which cannot be implemented without monitoring of foreclosure. ■

Fair compensation requires capping the access charge at the value such that the platform is indifferent between foreclosing or not the 3rd party app, i.e. obtains a unit revenue from 3rd party app distribution which exactly equals its opportunity cost from letting it serve consumers: $\hat{t}(V + \Delta) = V$. The Pigouvian principle is thus robust also when considering freemium apps.

## C.4 Pivotal, large "killer" app

This section complements the base analysis, considering cases where the 3rd party app delivers a large fraction of the ecosystem's value: $\Delta > v - b$, so that the platform need not always be pivotal for consumers' participation.

*Case 1: $b < \Delta$.* First suppose $\Delta > v + b$. The analysis of Section 2 shows that the platform cannot be pivotal alone. Neither can the 3rd party app developer be pivotal alone, otherwise the platform could increase $p_0$ without constraint. So it must be the case that margins are equal, that is that the stakes are equalized:

$$p_0^* + a = p_2^* - (a - b).$$

Because $p_0^* + p_2^* = v + \Delta$, such "co-pivotality" requires that

$$p_2^* = \frac{2a - b + v + \Delta}{2} \quad \text{and so} \quad p_0^* = \frac{v + \Delta - 2a + b}{2}. \tag{C.10}$$

The 3rd party app is then partially squeezed:

$$\pi_2^*(a) = p_2^* - (a - b) = \frac{b + v + \Delta}{2} \leq \Delta \quad \text{since} \quad \Delta \geq b + v.$$

This regime requires that $p_0^* > 0$, or $\Delta + v + b > 2a$. For $a \geq [\Delta + v + b]/2$, the platform charges $p_0^* = 0$ and the 3rd party app developer sets $p_2^* = v + \Delta$. Its profit is decreasing in $a$:

$$\pi_2^*(a) = p_2 - (a - b) = \Delta + v + b - a,$$

and, for $a > \Delta + v + b$, there is access-price foreclosure.

Finally, let us consider the incentive to foreclose. The platform gains from foreclosing if $a < (p_2^* - \Delta) + b \iff \Delta \leq v + b$ (whether $p_0 > 0$ or $p_0 = 0$), which is not the case. So the platform has no incentive to engage in non-price foreclosure. Figure 6 depicts firms' profits as function of the access charge when $\Delta > v + b$.

If, by contrast, $v - b < \Delta \leq v + b$, the results are like in the main analysis, except in part of the non-price foreclosure region. Specifically, let $\tilde{a}$ be defined by

$$\tilde{a} \equiv \frac{\Delta - (v - b)}{2} < b.$$

Then, the following results hold:

**Lemma C.4.1.** *Suppose $b < \Delta$. Then, if $v - b < \Delta \leq v + b$:*

- *For $a < \tilde{a}$, $p_0^*$ and $p_2^*$ are given by (C.10), and the platform is better off foreclosing the 3rd party app.*
- *For $a \geq \tilde{a}$ (with $\tilde{a} < b$), the results are as in Section 2.*

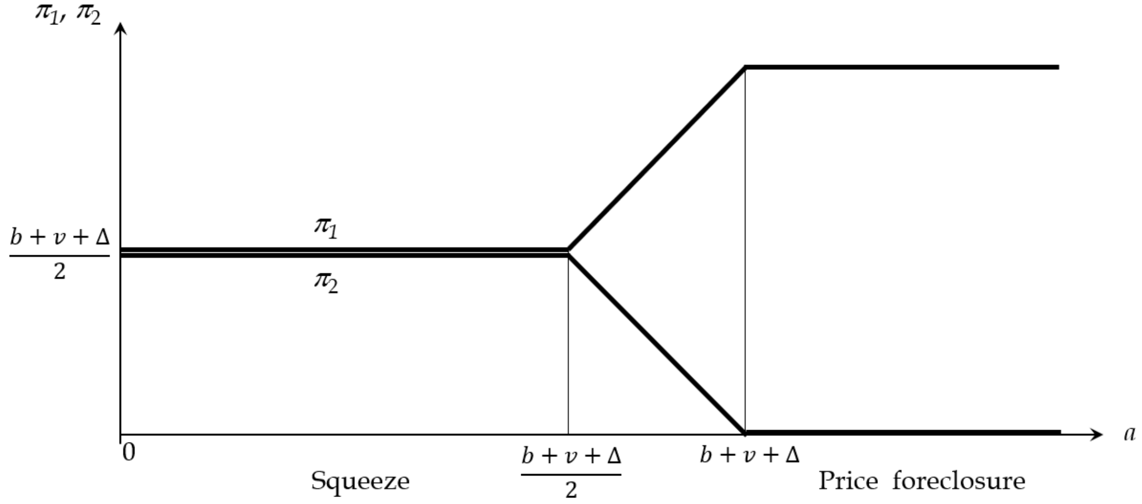


Figure 6: Profit for a pivotal, large "killer" app ($\Delta > v + b$).

*If $\Delta > v + b$, then, for all $a$: (i) the $3^{rd}$ party app developer is squeezed ($\pi_2^*(a) < \Delta$) and (ii) the platform has no incentive to foreclose.*

*Case 2:* $b > \Delta$. Define

$$\check{a} \equiv \frac{b - \Delta - v}{2} < \tilde{a},$$

with $\check{a} > 0$ if and only if $\Delta < b - v$. We have:

**Lemma C.4.2.** *Suppose $b > \Delta$. Then:*

- *For $a \leq \check{a}$: $p_0^* = v + \Delta$ and $p_2^* = 0$;*
- *For $a \in (\check{a}, \tilde{a})$: $p_0^*$ and $p_2^*$ are given by (C.10).*

*For all $a < \tilde{a}$, the platform is better off foreclosing the $3^{rd}$ party app.*

- *For $a \geq \tilde{a}$ (with $\tilde{a} < b$), the results are as in Section 2.*

*Summing up.* The regulatory implications of the main analysis are robust in the following sense:

**Proposition C.4.1.** *For all values of parameters, any access charge $a \in [b, b+v]$ maximizes the independent seller's reward for its contribution to the ecosystem.*

*Proof of Proposition C.4.1.* From Lemmata C.4.1 and C.4.2, we know that the results of Proposition 3 hold for all $\Delta \leq v + b$. Yet, also for $\Delta > v + b$, the platform would optimally set $a^* = \Delta + v + b$, and, as $[\Delta + v + b]/2 > v + b$, any access charge $a \in [b, b+v]$ still maximizes the superior seller's reward for its contribution to the ecosystem (though it makes less than $\Delta$). ■

In what follows, we derive the equilibria of the model under no foreclosure, for any given access charge $a$, for all possible values of parameters $(v, \Delta, b)$, which establishes the results in Proposition 1, and Lemmata C.4.1 and C.4.2.

*Equilibrium characterization.* Consider a family of distributions $F_\tau(\tilde{v})$, indexed by $\tau \in \mathbb{N}$, with density $f_\tau(\tilde{v})$, and decreasing inverse hazard rate $\rho_\tau(\tilde{v}) \equiv [1 - F_\tau(\tilde{v})]/f_\tau(\tilde{v})$, converging as $\tau \to \infty$

to a spike at some $v > 0$. The cutoff platform-consumer's type is

$$v^* = p_0 + \min\{p_2 - \Delta, p_1\}.$$

The two firms solve, respectively

$$\sup_{\{p_0 \geq 0,\ p_1 \geq 0\}} [1 - F_\tau(v^*)][p_0 + a + [p_1 - (a-b)]\mathbb{1}_{\{p_1 < p_2 - \Delta\}}],$$

and

$$\max_{\{p_2 \geq 0\}} [1 - F_\tau(v^*)][p_2 - (a-b)]\mathbb{1}_{\{p_2 - \Delta \leq p_1\}}.$$

We look for a pure-strategy equilibrium $\{p_0^*, p_1^*, p_2^*\}$. For $a$ and $p_0^*$ given, both app providers compete with (opportunity) marginal cost $a - b$. Hence, the platform and the 3$^{\text{rd}}$ party app engage in asymmetric Bertrand-Nash competition, and so:

$$p_1^* = \max\{a - b, 0\} \quad \text{and} \quad p_2^* \leq p_1^* + \Delta,$$

so that the in-house inferior app wins no consumers. The reason why the 3$^{\text{rd}}$ party app does not necessarily charge a markup equal to $\Delta$ over the in-house app's price is that it may also be concerned about the overall attractiveness of the ecosystem.

The FOCs are

$$\frac{\partial \pi_1}{\partial p_0} = 1 - F_\tau(v^*) - f_\tau(v^*)(p_0 + a) = 0 \quad \text{if } p_0 > 0, \text{ and } \leq 0 \text{ if } p_0 = 0,$$

and

$$\frac{\partial \pi_2}{\partial p_2} = 1 - F_\tau(v^*) - f_\tau(v^*)[p_2 - (a-b)] \begin{cases} = 0 & \text{if } \max\{a-b, 0\} < p_2 < p_1^* + \Delta, \\ \geq 0 & \text{if } p_2 = p_1^* + \Delta. \end{cases}$$

To start with, note that as $\tau \to \infty$ it must be the case that $v^* = p_0 + p_2 - \Delta \to v$. To see this, suppose by contradiction that there is a candidate equilibrium where $v^* < v$. Then, as $\tau \to \infty$: $F_\tau(v^*) = f_\tau(v^*) = 0$, which gives $\frac{\partial \pi_1}{\partial p_0} > 0$ and $\frac{\partial \pi_2}{\partial p_2} > 0$: Both firms would then have incentives to increase their prices, which destroys the candidate equilibrium. Similarly, consider a candidate equilibrium where $v^* > v$. Then, as $\tau \to \infty$: $F_\tau(v^*) = 1$, which implies that both firms make zero profits: Both firms would then have incentive to decrease their prices, which destroys the candidate equilibrium.[68]

First, consider $a < b$, so that it must be $p_1^* = 0$, and so $p_2^* \in [0, \Delta]$. In this region of parameters, in any equilibrium $p_0^* > 0$. This is because, if $p_0 = 0$, then $p_2^* \leq \Delta$ implies $v^* = p_2^* - \Delta \leq 0 < v$,

[68]There are equilibria where both $p_0$ and $p_2$ are very large so that (given the ZLB constraints) no firm can reduce enough its price to induce consumers' participation, and so no firm has, unilaterally, a strictly profitable deviation. However, these equilibria are Pareto-dominated and uninteresting, therefore we neglect them in what follows.

which, as explained above, cannot be the case in equilibrium. Next, $p_0^* > 0$ if and only if

$$\frac{\partial \pi_1}{\partial p_0} = 0 \iff p_0 + a = \rho_\tau(v^*). \tag{C.11}$$

We have to distinguish three cases:

- $p_2^* = \Delta$ if and only if

  $$\left.\frac{\partial \pi_2}{\partial p_2}\right|_{p_2=\Delta} \geq 0 \iff \Delta + b - a \leq \rho_\tau(v^*) = p_0 + a,$$

  where the equality uses (C.11). Since $v^* = v \iff p_0^* + p_2^* - \Delta = p_0^* = v$, this equilibrium exists if and only if

  $$a + v \geq \Delta + b - a \iff a \geq \tilde{a}. \tag{C.12}$$

- $p_2^* \in (0, \Delta)$ if and only if

  $$\frac{\partial \pi_2}{\partial p_0} = 0 \iff p_2 + b - a = \rho_\tau(v^*) = p_0 + a.$$

  Hence, this equilibrium is obtained from

  $$\begin{cases} p_0 + a = p_2 + b - a \\ p_0 + p_2 - \Delta = v \end{cases} \tag{C.13}$$

  This equilibrium exists if and only if $p_0^* > 0$ and $p_2 \in (0, \Delta)$, which gives

  $$\breve{a} < a < \tilde{a}. \tag{C.14}$$

- $p_2^* = 0$ if and only if

  $$\left.\frac{\partial \pi_2}{\partial p_2}\right|_{p_2=0} \leq 0 \iff b - a > \rho_\tau(p_0 - \Delta) = p_0 + a.$$

  Since $v^* = v \iff p_0^* = v + \Delta$, this equilibrium exists if and only if

  $$b - a \geq v + \Delta + a \iff a \leq \breve{a}. \tag{C.15}$$

Taken together, these results give:

- If $\Delta \leq v - b$, (C.12) holds for all $a \geq 0$, whereas (C.14) and (C.15) are violated for all $a \geq 0$. Hence, $p_0^* = v$ and $p_2^* = \Delta$ for all $a \in [0, b]$, so $\pi_1^*(a) = a + v < b + v \equiv \pi^F$.
- If $\Delta > v - b$:
  - for $a \in [0, \breve{a}]$: $p_0^* = v + \Delta$ and $p_2^* = 0$, so $\pi_1^*(a) = v + \Delta + a > \pi^F \iff a > b - \Delta$, with $b - \Delta < \breve{a} \iff \Delta > b + v$;

– for $a \in (\breve{a}, \tilde{a})$: $(p_0^*, p_2^*)$ solve (C.13), so $\pi_1^*(a) = \frac{b+\Delta+v}{2} > \pi^F \iff \Delta > b + v$;

– for $a \in [\tilde{a}, b)$: $p_0^* = v$ and $p_2^* = \Delta$, so $\pi_1^*(a) = a + v < \pi^F$.

Next, consider $a \geq b$, so that $p_1^* = a - b$ and $p_2^* \in (a - b, a - b + \Delta]$. Let us start from the case where $p_0^* > 0$. We have to distinguish two cases:

- $p_2^* = a - b + \Delta$ if and only if

$$\left.\frac{\partial \pi_2}{\partial p_2}\right|_{p_2=a-b+\Delta} \geq 0 \iff \Delta \leq \rho_\tau(v^*) = p_0 + a.$$

  Since $v^* = v \iff p_0^* = v - (a - b)$, this equilibrium exists if and only if

$$\Delta \leq b + v,$$

  and

$$p_0^* > 0 \iff a < b + v.$$

- $p_2^* \in (a - b, a - b + \Delta)$ if and only if $(p_0^*, p_2^*)$ solve system (C.13). This equilibrium exists if and only if $p_2^* < a - b + \Delta$, which gives

$$\Delta > b + v,$$

  and $p_0^* > 0$ and $p_2^* > 0$, which further requires

$$a < \frac{b + \Delta + v}{2}.$$

Note that, also when the platform is pivotal, for $a > b$ there may be equilibria where $p_1^* \in [0, a-b]$, and the platform correspondingly increases the access price $p_0$, so that $p_0^* + p_1^* \to v$. In what follows, we rule out these equilibria as $p_1 < a - b$ is not credible and the platform would lose money on the app if accepted.[69]

Finally, consider equilibria where $p_0^* = 0$. This is the case if and only if

$$\left.\frac{\partial \pi_1}{\partial p_0}\right|_{p_0=0} \leq 0 \iff a \geq \rho_\tau(v^*),$$

and

$$\left.\frac{\partial \pi_2}{\partial p_2}\right|_{p_0=0} \geq 0 \iff p_2 + b - a \leq \rho_\tau(v^*).$$

Hence, it must be

$$a \geq p_2^* + b - a.$$

In this equilibrium, $v^* = v \iff p_2^* = \Delta + v$. Since $p_2^* < a - b + \Delta$, this equilibrium exists if

[69]More formally, considering also a family of distributions $H_\tau(\tilde{\Delta})$, converging to a spike at $\Delta$ as $\tau \to \infty$, with $F_\tau(\cdot)$ and $H_\tau(\cdot)$ independent for all $\tau$, it is easy to show that, as $\tau \to \infty$: $p_1^* \to \max\{a - b, 0\}$.

and only if

$$a \geq b + v + \Delta - a \iff a \geq \frac{b + v + \Delta}{2},$$

and

$$\Delta + v \leq a - b + \Delta \iff a \geq b + v,$$

with

$$\frac{b + v + \Delta}{2} > b + v \iff \Delta > b + v.$$

Hence:

- If $\Delta \leq b + v$:
  - for $a \in [b, b + v)$: $p_0^* = v - (a - b), p_2^* = a - b + \Delta$, so $\pi_1^*(a) = b + v = \pi^F$;
  - for $a \in [b + v, b + v + \Delta]$: $p_0^* = 0, p_2^* = \Delta + v$, so $\pi_1^*(a) = a > \pi^F$.
- If $\Delta > b + v$:
  - for $a \in [b, \frac{b+v+\Delta}{2})$: $(p_0^*, p_2^*)$ solve (C.13), so $\pi_1^*(a) = \frac{b+v+\Delta}{2} > \pi^F$;
  - for $a \in [\frac{b+v+\Delta}{2}, b + v + \Delta]$: $p_0^* = 0, p_2^* = \Delta + v$, so $\pi_1^*(a) = a > \pi^F$.

Finally, for all $a > b + v + \Delta$, as in any equilibrium $p_0^* + p_2^* = \Delta + v > a - b$, it follows that there is access-price foreclosure. ■