Country- and sector-specific trade liberalization, directed technical change, and long-run growth

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Abstract

To examine how country- and sector-specific trade liberalization affects long-run growth, we extend Acemoglu's directed technical change model to include two asymmetric countries trading differentiated machines that augment either skilled or unskilled labor. Country 1's skill premium increases with import liberalization in the skilled-labor-augmenting machine sector by country 2 but decreases with that by country 1 iff the elasticity of substitution across the two factors is larger than one. In contrast, liberalization raises the balanced growth rate for any liberalizing country or sector. These results obtained analytically around a symmetric balanced growth path (BGP) are robust around a factual BGP.

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1 Introduction

Does trade liberalization raise economic growth? Since Rodriguez and Rodrik (2000) criticized the first generation of empirical work on this issue in the 1990s for its weak research designs, the second generation of empirical research during the 2000s to 2010s has used both more recent observations (including a big wave of liberalization events in the 1990s) and more reliable identification strategies to find that liberalization does raise growth on average (e.g., Wacziarg and Welch, 2008; Estevadeordal and Taylor, 2013; see Irwin, 2019, for a review). This is followed by the theoretical literature on trade and endogenous growth featuring the Melitz (2003)-type intra-industry reallocations across heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Dinopoulos and Unel, 2011; Sampson, 2016; Ourens, 2016; Naito, 2017, 2019, 2021; Impullitti and Licandro, 2018; Fukuda, 2019; Perla et al., 2021; Akcigit et al., 2021). A general conclusion from this literature is that trade liberalization is likely to raise long-run growth, although the former may partly lower the latter because the liberalization-induced selection of less productive domestic firms makes it more difficult for a potential entrant to survive (e.g., Baldwin and Robert-Nicoud, 2008).

Unfortunately, this does not mean that the problem is solved completely. In the above theoretical literature, all papers but Naito (2017, 2019, 2021) and Akcigit et al. (2021) assume symmetric countries, and all papers consider trade costs in a single differentiated good sector. In reality, however, trade costs are different across countries and sectors. The latest estimates of bilateral trade costs by WTO (2021), using the gravity-based method of Egger et al. (2021), report that in 2018 high-income economies' all-inclusive global trade costs relative to domestic ones in agriculture, manufacturing, and services amounted to 3.3, 2.7, and 3.9 (ad valorem equivalent of 230%, 170%, and 290%), whereas those in lower-income economies were 3.6, 2.9, and 4.6 (ad valorem equivalent of 260%, 190%, and 360%), respectively. This suggests that the long-run growth effect of reducing trade costs in services in lower-income economies might be quite different from that in manufacturing in high-income economies. By introducing country- and sector-specific trade costs, we could develop a finer theoretical framework that leads to a better empirical specification of the trade-growth relationship. The purpose of this paper is to examine analytically how country- and sector-specific trade liberalization affects long-run global growth.

To this end, we extend the directed technical change (DTC) model of Acemoglu (2002) to include two possibly asymmetric countries. Acemoglu's (2002) DTC model has one final good produced from two intermediate goods (e.g., services and manufacturing), each of which is produced from a sector-specific factor (e.g., skilled labor for services, or unskilled labor for manufacturing) and a variety of differentiated machines that specifically augment the factor.[1] It is true that there have been some attempts to construct two-country DTC models (e.g., Acemoglu, 2002, 2003; Gancia and Bonfiglioli, 2008; Chu et al., 2015; Acemoglu et al., 2015). However, all of them assume specialized R&D activities (i.e., the North specializing in innovation and the South specializing in imitation or offshoring), and allow for only two extreme trade status, namely either autarky or free trade. To enable us to analyze the effects of incremental and asymmetric trade liberalization, we consider two countries, both of which innovate new machines, and trade only differentiated machines subject to country- and sector-specific iceberg import trade costs. This allows us to compare our results with the standard models of endogenous technical change with two countries and one tradable differentiated good sector mentioned in the first paragraph. For R&D, we choose the lab-equipment specification (where the final good is used as the R&D input) over the knowledge-driven one (where labor and public knowledge are used as the R&D inputs). Although it is more difficult to characterize an equilibrium with the lab-equipment specification than the knowledge-driven one especially in an asymmetric two-country setting, the former accommodates richer

[1] Cravino and Sotelo (2019) find that services sectors such as finance and insurance, real estate, health, and education are more skilled-labor-intensive than non-service sectors such as agriculture, manufacturing, and mining.

general equilibrium interactions through the relative price of the final goods of the two countries (e.g., Naito, 2019, 2021).

Due to the technical difficulty of considering two countries, two differentiated machine sectors, and two factors at the same time, we make a few simplifying assumptions in doing the (local) hat algebra of Jones (1965). First, we focus on a balanced growth path (BGP), where all wages are constant (with the unskilled wage of the last country normalized to one), and the numbers of machines for all machine sectors and for all countries grow at a common constant rate (i.e., the balanced growth rate). Second, machine firms have homogeneous technologies within each sector. Third, we evaluate the long-run effects of a change in country- and sector-specific import trade cost around a symmetric BGP, where all exogenous variables are the same across countries, and trade and entry costs are originally the same across machine sectors. For generality, the last two assumptions will be relaxed later.

We obtain the following main results. Let τ_{kj}^{i} denote country j's iceberg import trade cost factor from country k for i-augmenting machines, where $i = S, L$ for skilled and unskilled labor, respectively, and $j, k = 1, 2, k \neq j$. First, country j's relative technology (i.e., relative number of S- to L-augmenting machines) is increasing in $\tau_{kj}^{S}/\tau_{kj}^{L}$ but decreasing in $\tau_{jk}^{S}/\tau_{jk}^{L}$, and so is its skill premium (i.e., relative wage of skilled to unskilled labor) if and only if the (derived) elasticity of substitution across the two factors is larger than one. For example, suppose that τ_{kj}^{S} increases. Since this shifts country j's demand for S-augmenting machines from imported to domestic sources, entry into machine sector S becomes relatively more profitable, thereby inducing more entry there. Moreover, when demands for the two factors are elastic, the induced relatively S-augmenting technical change is S-biased in Acemoglu's (2002) sense (i.e., the increase in country j's relative number of S- to L-augmenting machines increases its relative value of marginal product of S to L), which increases country j's skill premium. We can also understand that the effects of an increase in τ_{jk}^{S} on country j's relative technology and skill premium work in the opposite directions of τ_{kj}^{S} because it shrinks country j's export market for S-augmenting machines. This implies that, although a country's skill premium increases with import liberalization in S-augmenting machine sector of its trading partner, the country's skill premium decreases with import liberalization in its own S-augmenting machine sector. In contrast to the literature on trade liberalization and skill premium in static quantitative trade models (e.g., Epifani and Gancia, 2008; Parro, 2013; Burstein and Vogel, 2017; Cravino and Sotelo, 2019), where global trade liberalization increases countries' skill premiums, we show that country- and sector-specific trade liberalization may not increase a country's skill premium.

Second, an increase in τ_{kj}^{i}, for any $i = S, L$ and for any $j, k = 1, 2, k \neq j$, decreases the balanced growth rate. It can be shown that country j's growth rate (of the number of i-augmenting machines that is common across machine sectors on a BGP) is decreasing in its "autarkiness" function in each machine sector i. This in turn is increasing in country j's import trade cost for i-augmenting machines τ_{kj}^{i} (because country j's imports of i-augmenting machines become relatively more expensive then) and the relative number of L-augmenting machines in country j to country k (because country j imports relatively less L-augmenting machines then). An increase in τ_{kj}^{i}, ceteris paribus, pulls down country j's growth rate. By importing relatively less L-augmenting machines, country k also grows more slowly. Even after taking everything else (including countries' relative technologies, skill premiums, relative final good price, relative unskilled wage, etc.) into account, the balanced growth rate decreases compared with the old BGP. In spite of the different effects of country- and sector-specific reductions in import trade costs on countries' skill premiums, they have qualitatively the same positive effect on long-run growth. This result contributes to the literature on trade and endogenous growth with homogeneous firms (e.g., Rivera-Batiz and Romer, 1991a, 1991b; Baldwin and Forslid, 1999) and heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Dinopoulos and Unel, 2011; Sampson, 2016; Ourens, 2016; Naito, 2017, 2019, 2021; Impullitti and Licandro, 2018; Fukuda, 2019; Perla et al., 2021; Akcigit et al., 2021) by showing that the positive effect of trade liberalization on long-run growth is robust even if it occurs in only one of the two countries and only one of the two machine sectors, and regardless of the elasticity of factor substitution.

Moreover, to examine how robust the above analytical results obtained around a symmetric BGP are to more realistic situations, we apply the exact hat algebra of Dekle et al. (2008) to a factual BGP. It turns out that the qualitative results around a symmetric BGP are mostly robust even around a factual BGP. In particular, the positive effect of any country- and sector-specific trade liberalization on long-run growth remains valid.

Finally, to see the implications of reallocations across heterogeneous machine firms for the long-run effects of country- and sector-specific trade liberalization, we extend our model to allow for firm heterogeneity following Melitz (2003) with a Pareto distribution (e.g., Arkolakis et al., 2012). The resulting formulas for the long-run skill premium and growth effects of country- and sector-specific trade cost changes around a symmetric BGP with heterogeneous firms turn out to be exactly the same as those with homogeneous firms. Moreover, even starting from a factual BGP, our results obtained with heterogeneous firms are qualitatively the same as those with homogeneous firms.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 characterizes a BGP. Section 4 examines analytically the long-run effects of country- and sector-specific trade cost changes around a symmetric BGP. Section 5 complements section 4 by doing some counterfactual simulations around a factual BGP. Section 6 considers heterogeneous machine firms. Section 7 concludes.

2 The model

We extend the directed technical change model of Acemoglu (2002) to include two countries trading differentiated machines. There are two countries, $j = 1, 2$. Each country j is endowed with two factors, $i = S, L$ for skilled and unskilled labor, respectively. There are two machine sectors also indexed by i, each of which supplies specifically i-augmenting machines. Factor i is combined only with i-augmenting machines to produce good i, that is, the i-intensive intermediate good. Goods S and L are used to produce the final good. The final good is used for consumption, variable input to produce each machine, and R&D (i.e., fixed input to create a new differentiated machine). The machines are produced under monopolistic competition, whereas all other goods are produced under perfect competition. Only the machines are tradable. Due to the input specificity, the index i denotes three things at the same time: a factor, a factor-augmenting machine sector, and a factor-intensive intermediate good sector.

2.1 Households

The representative household in country j solves the following problem:

$$\begin{aligned}
\max: U_j &= \int_0^\infty \ln C_{jt} \exp(-\rho t) dt, \\
\text{s.t.}: \dot{A}_{jt} &= r_{jt} A_{jt} + w_{jt}^S S_j + w_{jt}^L L_j - E_{jt}; \dot{A}_{jt} \equiv dA_{jt}/dt, E_{jt} \equiv q_{jt}^Y C_{jt}, \\
\text{given}: &\{r_{jt}, w_{jt}^S, w_{jt}^L, q_{jt}^Y\}_{t=0}^\infty, A_{j0},
\end{aligned}$$

where $t \in [0, \infty)$ is time (omitted whenever no confusion arises), U_j is country j's welfare, C_j is country j's consumption, ρ is the subjective discount rate,[2] A_j is country j's asset, r_j is country j's interest rate, w_j^i is country j's wage rate of factor i, S_j is country j's supply of skilled labor, L_j is country j's supply of unskilled labor, E_j is country j's consumption expenditure, and q_j^Y is country j's

[2]Parameters without index j are common to all countries. Similarly, parameters without index i are common to all factors, machine sectors, and/or intermediate good sectors.

price of the final good.[3] The second line represents the budget constraint. Dynamic optimization with respect to E_j implies the Euler equation:

$$\dot{E}_{jt}/E_{jt} = r_{jt} - \rho.$$

2.2 Final good firms

The representative final good firm in country j solves the following problem:

$$\begin{aligned}
\max : \pi_j^Y &= q_j^Y Y_j - q_j^S D_j^S - q_j^L D_j^L, \\
\text{s.t.} : Y_j &= [\alpha (D_j^S)^{(\varepsilon-1)/\varepsilon} + (1-\alpha)(D_j^L)^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)}; \alpha \in (0,1), \varepsilon > 0, \\
\text{given} &: q_j^Y, q_j^S, q_j^L,
\end{aligned}$$

where π_j^Y is the profit of country j's representative final good firm, Y_j is country j's supply of the final good, q_j^i is country j's price of good i, D_j^i is country j's demand for good i, α is a parameter governing the expenditure share of good S, and ε is the elasticity of substitution across the two intermediate goods. The second line represents country j's production function of the final good. Profit maximization requires that the price of the final good is equal to its unit cost, implying that the maximized profit is zero:

$$q_j^Y = c_j^Y(q_j^S, q_j^L); c_j^Y(q_j^S, q_j^L) \equiv [\alpha^\varepsilon (q_j^S)^{1-\varepsilon} + (1-\alpha)^\varepsilon (q_j^L)^{1-\varepsilon}]^{1/(1-\varepsilon)} \Rightarrow q_j^Y Y_j = q_j^S D_j^S + q_j^L D_j^L,$$

where $c_j^Y(q_j^S, q_j^L)$ is country j's unit cost function of the final good. Country j's expenditure share of good S is defined as:

$$\beta_j \equiv q_j^S D_j^S/(c_j^Y Y_j) = \alpha^\varepsilon (q_j^S)^{1-\varepsilon}/[\alpha^\varepsilon (q_j^S)^{1-\varepsilon} + (1-\alpha)^\varepsilon (q_j^L)^{1-\varepsilon}] \in (0,1).$$

It can be easily verified that β_j is decreasing in q_j^S/q_j^L, the relative price of good S to L, if and only if $\varepsilon > 1$, because the relative demand for good S to L decreases more than the relative price increase.

2.3 Intermediate good firms

The representative firm in sector L (i.e., L-intensive intermediate good sector) in country j solves the following problem (the same applies to sector S by replacing L with S):

$$\begin{aligned}
\max : \pi_j^L &= q_j^L Y_j^L - \int_{\Phi_j^L} p_j^L(\phi) x_j^L(\phi) d\phi - w_j^L L_j^D, \\
\text{s.t.} : Y_j^L &= [X_j^L/(1-1/\sigma)]^{1-1/\sigma} [L_j^D/(1/\sigma)]^{1/\sigma}; \sigma > 1, \\
: X_j^L &= \left(\int_{\Phi_j^L} x_j^L(\phi)^{(\sigma-1)/\sigma} d\phi\right)^{\sigma/(\sigma-1)}, \\
\text{given} &: q_j^L, \{p_j^L(\phi)\}_{\phi \in \Phi_j^L}, w_j^L,
\end{aligned}$$

[3]Unlike Acemoglu (2002) normalizing the price of the final good to one, we will instead choose country 2's unskilled labor as the numeraire. This is because, even if the price of the final good of one country is normalized to one, that of all other countries must anyway be determined endogenously as long as we allow for asymmetric countries. Of course, the choice of the numeraire does not matter for our analysis.

where π_j^L is the profit of country j's representative firm in sector L, Y_j^L is country j's supply of good L, Φ_j^L is the set of L-augmenting machines available to country j, $p_j^L(\phi)$ is country j's demand price of variety $\phi \in \Phi_j^L$ of L-augmenting machine,[4] $x_j^L(\phi)$ is country j's demand for variety ϕ of L-augmenting machine, L_j^D is country j's demand for unskilled labor, X_j^L is country j's quantity index of L-augmenting machines, and σ is the elasticity of substitution across any two varieties of L-augmenting machines. The second and third lines represent country j's production functions of good L and the quantity index of L-augmenting machines, respectively.[5] The first-order condition for profit maximization, implying a zero profit, is given by:

$$q_j^L = c_j^L(P_j^L, w_j^L); c_j^L(P_j^L, w_j^L) \equiv (P_j^L)^{1-1/\sigma}(w_j^L)^{1/\sigma}, P_j^L \equiv (\int_{\Phi_j^L} p_j^L(\phi)^{1-\sigma} d\phi)^{1/(1-\sigma)}$$
$$\Rightarrow q_j^L Y_j^L = \int_{\Phi_j^L} p_j^L(\phi) x_j^L(\phi) d\phi + w_j^L L_j^D,$$

where $c_j^L(P_j^L, w_j^L)$ is country j's unit cost function of good L, and P_j^L is country j's price index of L-augmenting machines (i.e., country j's unit cost function of X_j^L). Country j's conditional demands for variety ϕ of L-augmenting machine, quantity index of L-augmenting machines, and unskilled labor, are given by, respectively:

$$\begin{aligned} x_j^L(\phi) &= p_j^L(\phi)^{-\sigma}(P_j^L)^{\sigma} X_j^L, \\ P_j^L X_j^L &= (1-1/\sigma) q_j^L Y_j^L, \\ w_j^L L_j^D &= (1/\sigma) q_j^L Y_j^L. \end{aligned}$$

The last two equations mean that country j's representative firm in sector L spends constant fractions $1 - 1/\sigma$ and $1/\sigma$ of its revenue for L-augmenting machines and unskilled labor, respectively.

2.4 Machine firms

The firm producing variety ϕ of i-augmenting machine in country j and selling to country $k = 1, 2$ solves the following problem:

$$\begin{aligned} \max : \pi_{jk}^i(\phi) &= p_{jk}^i(\phi) y_{jk}^i(\phi) - q_j^Y y_{jk}^i(\phi), \\ \text{s.t.} : y_{jk}^i(\phi) &= \tau_{jk}^i x_{jk}^i(\phi); \tau_{jk}^i \geq 1, \tau_{jj}^i = 1, \\ : x_{jk}^i(\phi) &= (\tau_{jk}^i p_{jk}^i(\phi))^{-\sigma}(P_k^i)^{\sigma} X_k^i, \\ \text{given} : &\ q_j^Y, P_k^i, X_k^i, \end{aligned}$$

where $\pi_{jk}^i(\phi)$ is the firm's profit, $p_{jk}^i(\phi)$ is the firm's supply price, $y_{jk}^i(\phi)$ is the firm's supply, τ_{jk}^i is the iceberg trade cost factor of delivering one unit of a variety of i-augmenting machine from country j to country k, and $x_{jk}^i(\phi)$ is country k's demand for the firm's variety. Assuming that producing one unit of any variety requires one unit of the final good, any firm's marginal cost is given by q_j^Y. The second

[4] We omit j and L from ϕ just for notational simplicity.

[5] Combining the two production functions gives $Y_j^L = A_j^L(\int_{\Phi_j^L} x_j^L(\phi)^{(\sigma-1)/\sigma} d\phi)(L_j^D)^{1/\sigma}$, where $A_j^L \equiv [1/(1 - 1/\sigma)]^{1-1/\sigma}[1/(1/\sigma)]^{1/\sigma}$. The combined production function is equivalent to Eq. (5) of Acemoglu (2002) up to a proportionality constant. Our decomposition highlights the role of the machine price index P_j^L, which includes the demand prices of imported varieties. The proportionality constant A_j^L is added to simplify the resulting unit cost function $c_j^L(P_j^L, w_j^L)$.

and third lines represent the market-clearing condition for the firm's variety and the conditional demand function for the firm's variety (following from subsection 2.3), respectively.

The profit-maximizing pricing formula is derived as:

$$(p^i_{jk}(\phi) - q^Y_j)/p^i_{jk}(\phi) = 1/\sigma \Leftrightarrow p^i_{jk}(\phi) = q^Y_j/(1-1/\sigma)\forall\phi\forall i\forall k.$$

Since all machine firms in country j face a common marginal cost q^Y_j for all varieties ϕ and for all machine sectors i, and a common constant demand elasticity σ for all destination countries k, their profit-maximizing supply prices are common to all ϕ, i, and k. This allows us to omit ϕ from now on.

The revenue, profit, and value (and hence no-arbitrage condition) of a firm producing a variety of i-augmenting machine in country j and selling to country k are given by, respectively:

$$\begin{aligned}
e^i_{jk} &\equiv p^i_{jk}y^i_{jk} = [\tau^i_{jk}q^Y_j/(1-1/\sigma)]^{1-\sigma}(P^i_k)^\sigma X^i_k,\\
\pi^i_{jk} &= e^i_{jk}/\sigma = [\tau^i_{jk}q^Y_j/(1-1/\sigma)]^{1-\sigma}(P^i_k)^\sigma X^i_k/\sigma,\\
v^i_{jkt} &\equiv \int_t^\infty \pi^i_{jks}\exp(-\int_t^s r_{ju}du)ds \Rightarrow \dot{v}^i_{jkt} = r_{jt}v^i_{jkt} - \pi^i_{jkt}.
\end{aligned}$$

Suppose that an entrant has to spend κ^i_j units of the final good to create a new variety of i-augmenting machine in country j. Variations in κ^i_j across machine sectors and source countries reflect differences in the difficulty of R&D. The free entry condition requires that the sum of firm values for both domestic and export markets is equal to the fixed R&D cost:

$$\textstyle\sum_k v^i_{jk} = q^Y_j\kappa^i_j.$$

Let n^i_j denote the number of entrants of i-augmenting machines in country j. Since each entrant can sell its unique variety to all countries with no additional fixed cost, n^i_j also represents the number of i-augmenting machines country j sells to country k.

Finally, dividing $p^i_{jk} = q^Y_j/(1-1/\sigma)$ by itself with j and $k(\neq j)$ interchanged, country j's terms of trade for i-augmenting machines is derived as:

$$p^i_{jk}/p^i_{kj} = q^Y_j/q^Y_k\forall i\forall j,k,k\neq j.$$

This implies that country j's terms of trade in machine sector i is proportional to country j's price of the final good relative to country k. Unlike the closed-economy model of Acemoglu (2002), the relative price of the final good is determined through rich general equilibrium interactions in the present two-country model.

2.5 Markets

Country j's market-clearing conditions for the asset, skilled labor, unskilled labor, machines, intermediate goods, and final good are given by, respectively:

$$
\begin{aligned}
A_j &= \sum_i n_j^i \sum_k v_{jk}^i = q_j^Y \sum_i n_j^i \kappa_j^i, j = 1, 2, \\
S_j &= S_j^D, j = 1, 2, \\
L_j &= L_j^D, j = 1, 2, \\
y_{jk}^i &= \tau_{jk}^i x_{jk}^i, i = S, L; j, k = 1, 2, \\
Y_j^i &= D_j^i, i = S, L; j = 1, 2, \\
Y_j &= C_j + \sum_i \kappa_j^i \dot{n}_j^i + \sum_i \sum_k n_j^i y_{jk}^i, j = 1, 2.
\end{aligned}
$$

Walras' law for country j is obtained as:[6]

$$
\begin{aligned}
0 = {} & w_j^S (S_j^D - S_j) + w_j^L (L_j^D - L_j) + \sum_i q_j^i (D_j^i - Y_j^i) + q_j^Y (C_j + \sum_i \kappa_j^i \dot{n}_j^i + \sum_i \sum_k n_j^i y_{jk}^i - Y_j) \\
& + \sum_i \sum_k n_k^i p_{kj}^i \tau_{kj}^i x_{kj}^i - \sum_i \sum_k n_j^i p_{jk}^i y_{jk}^i.
\end{aligned}
$$

Combining this with its market-clearing conditions, we obtain:

$$
\begin{aligned}
\sum_i \sum_k E_{jk}^i &= \sum_i \sum_k E_{kj}^i = \sum_i P_j^i X_j^i; E_{jk}^i \equiv n_j^i e_{jk}^i, \\
\sum_i E_{jk}^i &= \sum_i E_{kj}^i, k \neq j,
\end{aligned}
$$

where E_{jk}^i is country j's revenue of selling i-augmenting machines to country k, or country k's expenditure for buying i-augmenting machines from country j. If $k \neq j$, then E_{jk}^i represents country j's value of exports for i-augmenting machines to country k, or country k's value of imports for i-augmenting machines from country j. The first line is country j's national budget constraint, meaning that country j's total revenue of selling machines to all destinations is equal to its total expenditure for buying machines from all sources. Subtracting country j's domestic revenue and expenditure from the first line, we obtain country j's balance of trade in the second line, showing that country j's total value of exports is equal to its total value of imports.

3 Balanced growth path

Since our model has two factors at which technical change is directed, and two (possibly asymmetric) countries between which varieties of machines are traded, it is technically very difficult to characterize an equilibrium. To make things manageable, we focus on a balanced growth path (BGP) defined below.

From now on, let country 2's unskilled labor be the numeraire:

$$
w_2^L \equiv 1.
$$

Suppose that, for all $t \geq 0$, the world economy is on a BGP, where all variables grow at constant (including zero) rates. We also assume that all wages are constant, and n_j^i grows at a common constant rate for all i and j, on a BGP:

[6] Time differentiating country j's asset market-clearing condition, and using its no-arbitrage conditions and free entry conditions, we obtain $\dot{A}_j = q_j^Y \sum_i \kappa_j^i \dot{n}_j^i + r_j A_j - \sum_i \sum_k n_j^i \pi_{jk}^i$. Combining this with country j's budget constraint and zero-profit conditions for intermediate and final good sectors gives country j's Walras' law.

$$\gamma_j^S = \gamma_j^L \equiv \gamma_j; \gamma_j^i \equiv \dot{n}_j^i / n_j^i, \tag{1}$$

$$\gamma_1 = \gamma_2 \equiv \gamma. \tag{2}$$

Eq. (1) means that, in each country j, the numbers of S- and L-augmenting machines grow at a common constant rate γ_j, which we call country j's growth rate. Eq. (2) states that both countries' growth rates are equalized at γ, the balanced growth rate. Eqs. (1) and (2) are interpreted as intersectoral and international balanced growth conditions, respectively.

3.1 Growth equation

First of all, country j's growth rate is expressed as (see Appendix A for derivation):

$$\gamma_j^* = (1 - 1/\sigma)[(s_j\omega_j^* + 1)/(\kappa_j\nu_j^* + 1)](L_j/\kappa_j^L)/(p_j^{e*}/w_j^{L*}) - \rho; \tag{3}$$
$$s_j \equiv S_j/L_j, \omega_j \equiv w_j^S/w_j^L, \kappa_j \equiv \kappa_j^S/\kappa_j^L, \nu_j \equiv n_j^S/n_j^L, p_j^e \equiv n_j^L q_j^Y,$$

where an asterisk indicates a BGP, s_j is country j's relative supply of skilled to unskilled labor, ω_j is country j's skill premium (i.e., relative wage of skilled to unskilled labor), κ_j is country j's relative fixed R&D cost of S- to L-augmenting machine, ν_j is what Acemoglu (2002) calls country j's "relative technology" (i.e., relative number of S- to L-augmenting machines), and p_j^e is country j's "price of entry", that is, its price of the final good as the fixed R&D input for L-augmenting machines, adjusted for its continual decrease due to economic growth (as will be apparent in subsection 3.3). It is assumed that ρ is sufficiently small that γ_j^* is positive. The growth equation (3) indicates that country j's growth rate γ_j^* is increasing in its total wage income $w_j^{L*}L_j(s_j\omega_j^* + 1) = w_j^{S*}S_j + w_j^{L*}L_j$ but is decreasing in its total R&D cost $p_j^{e*}\kappa_j^L(\kappa_j\nu_j^* + 1) = q_j^Y(n_j^S\kappa_j^S + n_j^L\kappa_j^L)$, just as current is directly proportional to voltage but is inversely proportional to resistance.[7] Since all wages are constant, and n_j^S and n_j^L grow at the same constant rate γ_j^*, both ω_j^* and ν_j^* are constant on a BGP. Then Eq. (3) ensures that p_j^{e*} is constant on a BGP.

Eq. (3) implies that we cannot determine γ_j^* until we know the BGP values of $w_j^{L*}, p_j^{e*}, \nu_j^*$, and ω_j^*. In the following subsections, we will see how they are determined.

3.2 Balanced trade condition

The revenue share of i-augmenting machines country j sells to country k is given by:

$$\lambda_{jk}^i \equiv E_{jk}^i / \textstyle\sum_l E_{jl}^i; \sum_k \lambda_{jk}^i = 1. \tag{4}$$

The expenditure share of i-augmenting machines country j buys from country k is defined as:[8]

$$\zeta_{kj}^i \equiv E_{kj}^i / \textstyle\sum_l E_{lj}^i; \sum_k \zeta_{kj}^i = 1. \tag{5}$$

Using countries' import expenditure shares, country 1's (and also 2's) balanced trade condition $E_{12}^S + E_{12}^L = E_{21}^S + E_{21}^L$ is rewritten as (see Appendix A for derivation):

[7] Country j's asset market-clearing condition implies that $p_j^{e*}\kappa_j^L(\kappa_j\nu_j^* + 1) = q_j^Y(n_j^S\kappa_j^S + n_j^L\kappa_j^L) = A_j$.

[8] With only one machine sector, $\sum_l E_{jl}^i = \sum_l E_{lj}^i$ and $E_{jk}^i = E_{kj}^i, k \neq j$ imply that $\lambda_{jk}^i = \zeta_{kj}^i \forall j, k$. This is not always true in the present setting with two machine sectors.

$$L_2(\zeta_{12}^S s_2\omega_2 + \zeta_{12}^L) = w_1^L L_1(\zeta_{21}^S s_1\omega_1 + \zeta_{21}^L), \tag{6}$$

where the left- and right-hand sides of Eq. (6) correspond to country 2's and 1's total values of imports, respectively. As in Krugman (1980), the balanced trade condition (6) determines w_1^L, country 1's unskilled wage (measured in terms of country 2's unskilled labor). The next two subsections will explain, respectively, how the four import expenditure shares and two skill premiums are expressed.

3.3 Prices

Using a machine firm's profit maximizing pricing formula $p_{jk}^i = q_j^Y/(1-1/\sigma)$ with j and k interchanged, country j's price index of i-augmenting machines $P_j^i = (\int_{\Phi_j^i} p_j^i(\phi)^{1-\sigma}d\phi)^{1/(1-\sigma)}$ is rewritten as:

$$\begin{aligned} P_j^i &= \{\textstyle\sum_k n_k^i[\tau_{kj}^i q_k^Y/(1-1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = (n_j^i)^{1/(1-\sigma)} q_j^Y m_j^i/(1-1/\sigma); \\ m_j^i &\equiv [\textstyle\sum_k (n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}. \end{aligned} \tag{7}$$

Eq. (7) means that P_j^i is decreasing in n_j^i but is proportional to $q_j^Y m_j^i/(1-1/\sigma)$, country j's average demand price of i-augmenting machines. The new function $m_j^i = [1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}, k \neq j$, is interpreted as country j's "autarkiness" in machine sector i. In autarky (i.e., $n_k^i/n_j^i \to 0$ and/or $\tau_{kj}^i q_k^Y/q_j^Y \to \infty$), m_j^i takes its maximum $m_j^i = 1$, implying that country j's average demand price of i-augmenting machines is equal to $q_j^Y/(1-1/\sigma)$, the supply price of country j's domestic firm producing a variety of i-augmenting machine. Compared with autarky, the more varieties of i-augmenting machines country j imports (i.e., the larger n_k^i/n_j^i is), and/or the cheaper imported varieties are (i.e., the smaller $\tau_{kj}^i q_k^Y/q_j^Y$ is), the smaller m_j^i and hence $q_j^Y m_j^i/(1-1/\sigma)$ become.

The autarkiness function m_j^i is directly related to country j's domestic expenditure share in machine sector i:[9]

$$\zeta_{jj}^i = (m_j^i)^{\sigma-1}. \tag{8}$$

Eq. (8) makes intuitive sense: the smaller m_j^i is, the more open country j is in machine sector i in terms of its import expenditure share $\zeta_{kj}^i = 1-\zeta_{jj}^i, k \neq j$.

Substituting Eq. (7) into the first-order condition for profit maximization in intermediate good sector i: $q_j^i = c_j^i(P_j^i, w_j^i) = (P_j^i)^{1-1/\sigma}(w_j^i)^{1/\sigma}$, country j's price of good i is solved as:

$$q_j^i = (w_j^i/n_j^i)^{1/\sigma}[q_j^Y m_j^i/(1-1/\sigma)]^{(\sigma-1)/\sigma}. \tag{9}$$

Moreover, substituting Eq. (9) into the first-order condition for profit maximization in the final good sector $q_j^Y = c_j^Y(q_j^S, q_j^L) = q_j^L c_j^Y(q_j^S/q_j^L, 1)$, we obtain $q_j^Y = (w_j^L/n_j^L)^{1/\sigma}[q_j^Y m_j^L/(1-1/\sigma)]^{(\sigma-1)/\sigma} \times c_j^Y((\omega_j/\nu_j)^{1/\sigma}(m_j^S/m_j^L)^{(\sigma-1)/\sigma}, 1)$, which is solved for q_j^Y as:

$$q_j^Y = (w_j^L/n_j^L)[m_j^L/(1-1/\sigma)]^{\sigma-1} c_j^Y((\omega_j/\nu_j)^{1/\sigma}(m_j^S/m_j^L)^{(\sigma-1)/\sigma}, 1)^{\sigma}. \tag{10}$$

Eq. (10) indicates that a continual increase in n_j^L at a constant rate γ_j, ceteris paribus, continues to decrease q_j^Y at the rate γ_j. This is why we have to consider the adjusted price of entry $p_j^e = n_j^L q_j^Y$. Eq.

[9] Substituting $e_{jk}^i = [\tau_{jk}^i q_j^Y/(1-1/\sigma)]^{1-\sigma}(P_k^i)^\sigma X_k^i$ and $E_{jk}^i = n_j^i e_{jk}^i$ into Eq. (5), we obtain $\zeta_{kj}^i = (n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}/[1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}], k \neq j$. Combining this with $\zeta_{jj}^i + \zeta_{kj}^i = 1, k \neq j$ and $m_j^i = [1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}, k \neq j$ gives Eq. (8).

(10) immediately implies two other prices:

$$p_j^e/w_j^L = [m_j^L/(1-1/\sigma)]^{\sigma-1} c_j^Y((\omega_j/\nu_j)^{1/\sigma}(m_j^S/m_j^L)^{(\sigma-1)/\sigma}, 1)^\sigma, \tag{11}$$

$$q_1^Y/q_2^Y = (w_1^L/\chi)(m_1^L/m_2^L)^{\sigma-1} c_1^Y((\omega_1/\nu_1)^{1/\sigma}(m_1^S/m_1^L)^{(\sigma-1)/\sigma}, 1)^\sigma / c_2^Y((\omega_2/\nu_2)^{1/\sigma}(m_2^S/m_2^L)^{(\sigma-1)/\sigma}, 1)^\sigma; \tag{12}$$

$$\chi \equiv n_1^L/n_2^L,$$

where χ is the relative number of L-augmenting machines in country 1 to country 2. Eq. (11) represents country j's price of entry measured in terms of its unskilled labor p_j^e/w_j^L, which is negatively related to its growth rate γ_j in Eq. (3). Eq. (12) shows the relative price of the final good in country 1 to country 2, which appears in the autarkiness function m_j^i in Eq. (7). Without skilled labor (i.e., $\alpha = 0$), we would have $c_j^Y((\omega_j/\nu_j)^{1/\sigma}(m_j^S/m_j^L)^{(\sigma-1)/\sigma}, 1)^\sigma = 1$, and then Eq. (11) were simplified to $p_j^e/w_j^L = [m_j^L/(1-1/\sigma)]^{\sigma-1}$. In fact, with skilled labor (i.e., $\alpha > 0$), p_j^e/w_j^L depends not just on m_j^L but also on ω_j, ν_j, and m_j^S. Similarly, q_1^Y/q_2^Y depends on $w_1^L, \omega_j, \nu_j, m_j^i$, and χ. Since $w_1^{L*}, \omega_j^*, \nu_j^*$, and p_j^{e*} are constant on a BGP, m_j^{i*} is constant from Eq. (11). Also, χ^* is constant from Eq. (2). Therefore, $(q_1^Y/q_2^Y)^*$ is constant on a BGP from Eq. (12).

3.4 Relative technology and skill premium

Country j's relative technology ν_j is determined by taking the ratio of the free entry condition $\sum_k v_{jk}^i = q_j^Y \kappa_j^i$ for the two machine sectors as (see Appendix A for derivation):

$$\kappa_j = s_j(\omega_j^*/\nu_j^*)\delta_j^*; \tag{13}$$

$$\delta_j^* \equiv [\zeta_{jj}^{S*} + \zeta_{jk}^{S*}(L_k/L_j)(w_k^{L*}/w_j^{L*})(s_k/s_j)(\omega_k^*/\omega_j^*)]/[\zeta_{jj}^{L*} + \zeta_{jk}^{L*}(L_k/L_j)(w_k^{L*}/w_j^{L*})], k \neq j.$$

Eq. (13) is what Acemoglu (2002) calls country j's "technology market clearing condition". Its left- and right-hand sides show country j's relative fixed R&D cost and relative firm value of S- to L-augmenting machine, respectively. In autarky (i.e., $\zeta_{jj}^{i*} = 1 \forall i \forall j$), we have $\delta_j^* = 1$, and then Eq. (13) reduces to the one in Acemoglu (2002).[10] This highlights that the extra term δ_j^* measures country j's relative firm value of S- to L-augmenting machine, compared with autarky. We simply call δ_j^* country j's "relative profitability" in machine sector S to L. For example, if country j's decreased domestic market due to imports (i.e., a decrease in ζ_{jj}^{S*}) is more than compensated by its increased export market (i.e., an increase in $\zeta_{jk}^{S*}(L_k/L_j)(w_k^{L*}/w_j^{L*})(s_k/s_j)(\omega_k^*/\omega_j^*)$) in machine sector S relative to L, then more firms enter machine sector S relative to L (i.e., ν_j^* increases). Since w_j^{L*} and ω_j^* are constant, and ζ_{kj}^{i*} is constant from Eq. (8), δ_j^* is constant on a BGP.

Country j's relative factor market-clearing condition is given by (see Appendix A for derivation):

$$s_j = [\alpha/(1-\alpha)]^\varepsilon \omega_j^{-\psi} \nu_j^{\psi-1}(m_j^S/m_j^L)^{(1-\sigma)(\psi-1)}; \psi \equiv 1 + (\varepsilon-1)/\sigma > 1 - 1/\sigma > 0, \tag{14}$$

where the left- and right-hand sides of Eq. (14) represent S_j/L_j and S_j^D/L_j^D, country j's relative supply and demand of skilled to unskilled labor, respectively. In autarky (i.e., $m_j^i = 1 \forall i \forall j$), Eq. (14) is equivalent to Eq. (18) of Acemoglu (2002). Since a 1% increase in country j's skill premium ω_j decreases S_j^D/L_j^D by ψ%, the parameter ψ is interpreted as the (derived) elasticity of substitution across the two

[10] In Acemoglu (2002), using his Eqs. (12) and (18), his technology market clearing condition (20) is rewritten as $\eta_L/\eta_Z = (Z/L)(w_Z/w_L)/(N_Z/N_L)$, which is exactly the same as our Eq. (13) for $\delta_j^* = 1$.

factors.[11] As in Acemoglu (2002), an increase in country j's relative technology ν_j increases S_j^D/L_j^D if and only if $\psi > 1 \Leftrightarrow \varepsilon > 1$, that is, demands for the two factors (and also demands for the two intermediate goods) are elastic. What is new in the present paper is the term $(m_j^S/m_j^L)^{(1-\sigma)(\psi-1)}$, which is equal to one in autarky (i.e., $m_j^i = 1 \forall i \forall j$). For example, a decrease in m_j^S/m_j^L (possibly due to greater trade liberalization in machine sector S relative to L) increases S_j^D/L_j^D if and only if $\psi > 1 \Leftrightarrow \varepsilon > 1$ just because it makes good S cheaper relative to good L (see Eq. (9)).

From country j's technology market-clearing condition (13) and relative factor market-clearing condition (14), its relative technology and skill premium are solved as, respectively:

$$\nu_j^* = [\alpha/(1-\alpha)]^\varepsilon \kappa_j^{-\psi} s_j^{\psi-1} (m_j^{S*}/m_j^{L*})^{(1-\sigma)(\psi-1)} \delta_j^{*\psi}, \tag{15}$$

$$\omega_j^* = [\alpha/(1-\alpha)]^\varepsilon \kappa_j^{1-\psi} s_j^{\psi-2} (m_j^{S*}/m_j^{L*})^{(1-\sigma)(\psi-1)} \delta_j^{*\psi-1}. \tag{16}$$

In autarky (i.e., $m_j^{i*} = 1 \forall i \forall j$ and $\delta_j^* = 1 \forall j$), Eqs. (15) and (16) reproduce Eqs. (21) and (22) of Acemoglu (2002), respectively. In particular, his "strong induced-bias hypothesis", stating that an increase in s_j paradoxically increases ω_j, is true if and only if $\psi > 2 \Leftrightarrow \varepsilon > \sigma + 1$. With international trade, country j's relative technology and skill premium depend on its relative autarkiness m_j^{S*}/m_j^{L*} and relative profitability δ_j^*, which in turn depend on many endogenous variables.

3.5 Revised growth equation

Using the results in subsections 3.3 and 3.4, we derive country j's growth equation that is comparable to that of Acemoglu (2002, p. 793). Using Eq. (13), $c_j^Y(q_j^S, q_j^L) = [\alpha^\varepsilon (q_j^S)^{1-\varepsilon} + (1-\alpha)^\varepsilon (q_j^L)^{1-\varepsilon}]^{1/(1-\varepsilon)}$, and $\kappa_j/s_j = (\kappa_j^S/\kappa_j^L)/(S_j/L_j) = (L_j/\kappa_j^L)/(S_j/\kappa_j^S)$, Eq. (11) is rewritten as:

$$\begin{aligned} p_j^{e*}/w_j^{L*} &= [m_j^{L*}/(1-1/\sigma)]^{\sigma-1}(L_j/\kappa_j^L) R_j^{*-1}; \\ R_j^* &\equiv c_j^Y((S_j/\kappa_j^S)^{-1/\sigma}\delta_j^{*-1/\sigma}(m_j^{S*}/m_j^{L*})^{(\sigma-1)/\sigma}, (L_j/\kappa_j^L)^{-1/\sigma})^{-\sigma} \\ &\equiv [\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1}(m_j^{S*}/m_j^{L*})^{(1-\sigma)(\psi-1)}\delta_j^{*\psi-1} + (1-\alpha)^\varepsilon (L_j/\kappa_j^L)^{\psi-1}]^{1/(\psi-1)}, \end{aligned} \tag{17}$$

where R_j^* is called country j's "resource function", which aggregates its two factors L_j/κ_j^L and S_j/κ_j^S, adjusting for its relative autarkiness m_j^{S*}/m_j^{L*} and relative profitability δ_j^*.[12] Since m_j^{i*} and δ_j^* are constant, R_j^* is constant on a BGP. Using Eqs. (13) and (17) to eliminate $\kappa_j \nu_j^*$ and p_j^{e*}/w_j^{L*} from Eq. (3), we obtain:

$$\gamma_j^* = (1-1/\sigma)^\sigma [(s_j\omega_j^* + 1)/(s_j\omega_j^*\delta_j^* + 1)] R_j^* (m_j^{L*})^{1-\sigma} - \rho. \tag{18}$$

The revised growth equation (18) includes two special cases. First, in autarky (i.e., $m_j^{i*} = 1 \forall i \forall j$ and $\delta_j^* = 1 \forall j$), Eq. (18) reduces to $\gamma_j^* = (1-1/\sigma)^\sigma [\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1} + (1-\alpha)^\varepsilon (L_j/\kappa_j^L)^{\psi-1}]^{1/(\psi-1)} - \rho$. This is essentially the same as the growth equation of Acemoglu (2002, p. 793). Compared with autarky, international trade affects country j's growth rate through $\omega_j^*, \delta_j^*, R_j^*$, and m_j^{L*}. Second, without skilled labor (i.e., $\alpha = 0$), we would have $\omega_j^* = 0$ and $R_j^* = L_j/\kappa_j^L$, and thus Eq. (18) is simplified to $\gamma_j^* = (1-1/\sigma)^\sigma (L_j/\kappa_j^L)(m_j^{L*})^{1-\sigma} - \rho = (1-1/\sigma)^\sigma L_j/(\kappa_j^L \zeta_{jj}^{L*}) - \rho$. This is country j's ACR (Arkolakis–Costinot–Rodriguez-Clare) formula (e.g., Arkolakis et al., 2012) for long-run growth: country j grows

[11] In Acemoglu (2002), this is denoted by σ. Since we already used σ to denote the elasticity of substitution across varieties of machines, we use a different symbol ψ here.

[12] Without skilled labor (i.e., $\alpha = 0$), we would have $R_j^* = L_j/\kappa_j^L$, and then Eq. (17) would reduce to $p_j^{e*}/w_j^{L*} = [m_j^{L*}/(1-1/\sigma)]^{\sigma-1}$ as we saw in subsection 3.3.

faster if and only if it becomes more open (i.e., ζ_{jj}^{L*} decreases). Such a simple formula does not apply in the present model with both unskilled and skilled labor.

3.6 Summary

The system determining a BGP is summarized as follows. Considering country j's revised growth equation (18), the international balanced growth condition (2) is given by:

$$\gamma_1^* = \gamma_2^* \equiv \gamma^* \Leftrightarrow [(s_1\omega_1^*+1)/(s_1\omega_1^*\delta_1^*+1)]R_1^*(m_1^{L*})^{1-\sigma} = [(s_2\omega_2^*+1)/(s_2\omega_2^*\delta_2^*+1)]R_2^*(m_2^{L*})^{1-\sigma}.$$

With Eq. (8), the balanced trade condition (6) is expressed as:

$$L_2[(1-(m_2^{S*})^{\sigma-1})s_2\omega_2^* + 1 - (m_2^{L*})^{\sigma-1}] = w_1^{L*}L_1[(1-(m_1^{S*})^{\sigma-1})s_1\omega_1^* + 1 - (m_1^{L*})^{\sigma-1}].$$

From Eq. (7), the autarkiness functions $m_j^i = [1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}, k \neq j$ for the two countries and two machine sectors are given by:

$$\begin{aligned}
m_1^{S*} &= [1+(\nu_2^*/\nu_1^*)(1/\chi^*)(\tau_{21}^S(q_2^Y/q_1^Y)^*)^{1-\sigma}]^{1/(1-\sigma)},\\
m_2^{S*} &= [1+(\nu_1^*/\nu_2^*)\chi^*(\tau_{12}^S(q_1^Y/q_2^Y)^*)^{1-\sigma}]^{1/(1-\sigma)},\\
m_1^{L*} &= [1+(1/\chi^*)(\tau_{21}^L(q_2^Y/q_1^Y)^*)^{1-\sigma}]^{1/(1-\sigma)},\\
m_2^{L*} &= [1+\chi^*(\tau_{12}^L(q_1^Y/q_2^Y)^*)^{1-\sigma}]^{1/(1-\sigma)}.
\end{aligned}$$

Using Eq. (17), the relative price of the final good (12) is rewritten as:

$$(q_1^Y/q_2^Y)^* = [(L_1/\kappa_1^L)/(L_2/\kappa_2^L)](w_1^{L*}/\chi^*)(m_1^{L*}/m_2^{L*})^{\sigma-1}R_2^*/R_1^*.$$

Substituting Eq. (8) into country j's relative profitability function in Eq (13), we obtain:

$$\begin{aligned}
\delta_1^* &= \frac{(m_1^{S*})^{\sigma-1} + (1-(m_2^{S*})^{\sigma-1})(L_2/L_1)(1/w_1^{L*})(s_2/s_1)(\omega_2^*/\omega_1^*)}{(m_1^{L*})^{\sigma-1} + (1-(m_2^{L*})^{\sigma-1})(L_2/L_1)(1/w_1^{L*})},\\
\delta_2^* &= \frac{(m_2^{S*})^{\sigma-1} + (1-(m_1^{S*})^{\sigma-1})(L_1/L_2)w_1^{L*}(s_1/s_2)(\omega_1^*/\omega_2^*)}{(m_2^{L*})^{\sigma-1} + (1-(m_1^{L*})^{\sigma-1})(L_1/L_2)w_1^{L*}}.
\end{aligned}$$

From Eqs. (15), (16), and (17), countries' relative technologies, skill premiums, and resource functions are given by, respectively:

$$\begin{aligned}
\nu_1^* &= [\alpha/(1-\alpha)]^\varepsilon \kappa_1^{-\psi} s_1^{\psi-1}(m_1^{S*}/m_1^{L*})^{(1-\sigma)(\psi-1)}\delta_1^{*\psi},\\
\nu_2^* &= [\alpha/(1-\alpha)]^\varepsilon \kappa_2^{-\psi} s_2^{\psi-1}(m_2^{S*}/m_2^{L*})^{(1-\sigma)(\psi-1)}\delta_2^{*\psi}.
\end{aligned}$$

$$\begin{aligned}
\omega_1^* &= [\alpha/(1-\alpha)]^\varepsilon \kappa_1^{1-\psi} s_1^{\psi-2}(m_1^{S*}/m_1^{L*})^{(1-\sigma)(\psi-1)}\delta_1^{*\psi-1},\\
\omega_2^* &= [\alpha/(1-\alpha)]^\varepsilon \kappa_2^{1-\psi} s_2^{\psi-2}(m_2^{S*}/m_2^{L*})^{(1-\sigma)(\psi-1)}\delta_2^{*\psi-1}.
\end{aligned}$$

$$R_1^* = [\alpha^\varepsilon (S_1/\kappa_1^S)^{\psi-1}(m_1^{S*}/m_1^{L*})^{(1-\sigma)(\psi-1)}\delta_1^{*\psi-1} + (1-\alpha)^\varepsilon (L_1/\kappa_1^L)^{\psi-1}]^{1/(\psi-1)},$$
$$R_2^* = [\alpha^\varepsilon (S_2/\kappa_2^S)^{\psi-1}(m_2^{S*}/m_2^{L*})^{(1-\sigma)(\psi-1)}\delta_2^{*\psi-1} + (1-\alpha)^\varepsilon (L_2/\kappa_2^L)^{\psi-1}]^{1/(\psi-1)}.$$

These fifteen equations determine fifteen variables, namely $\chi^*, w_1^{L*}, m_1^{S*}, m_2^{S*}, m_1^{L*}, m_2^{L*}, (q_1^Y/q_2^Y)^*$, $\delta_1^*, \delta_2^*, \nu_1^*, \nu_2^*, \omega_1^*, \omega_2^*, R_1^*, R_2^*$. Once these variables are determined, the balanced growth rate $\gamma^* \equiv \gamma_2^* = \gamma_1^*$ is determined from Eq. (18) for $j = 2$.

3.7 Long-run welfare

Before analyzing the long-run effects of changes in country- and sector-specific import trade costs, we derive country j's long-run welfare. Noting that country j's consumption expenditure is constant on a BGP (see Eq. (76)), and that its price of the final good decreases at the balanced growth rate γ^*: $q_{jt}^Y = p_j^{e*}/n_{jt}^L = q_j^{Y*}\exp(-\gamma^* t)$, where $q_j^{Y*} \equiv q_{j0}^Y$ is evaluated at the initial period of a BGP, country j's long-run welfare (expressed in flow terms) is given by (see Appendix A for derivation):[13]

$$\begin{aligned}
\rho U_j &= \ln E_j^* - \ln q_j^{Y*} + (1/\rho)\gamma^* = \ln K_j + \ln W_j^* + \ln \eta^* + (1/\rho)\gamma^*; \qquad (19)\\
K_j &\equiv n_{j0}^L (1-1/\sigma)^{\sigma-1}\kappa_j^L,\\
W_j^* &\equiv (m_j^{L*})^{1-\sigma} R_j^* (s_j\omega_j^* + 1),\\
\eta^* &\equiv (1-1/\sigma)\rho/(\rho+\gamma^*) + 1 > 1.
\end{aligned}$$

Eq. (19) states that country j's long-run welfare (expressed in flow terms) is increasing in $C_{j0} = E_j^*/q_j^{Y*}$, consumption in the initial period of a BGP, and γ^*, the balanced growth rate. The former is rewritten as $C_{j0} = E_j^*/q_j^{Y*} = K_j W_j^* \eta^*$. The term $K_j W_j^*$ is equal to $(w_j^{L*}/q_j^{Y*})L_j(s_j\omega_j^* + 1)$, country j's total real wage income. The function η^* indicates the composition of country j's total income: the first and second terms in the definition of η^* correspond to the interest income from asset and the total wage income, respectively. Since country j's asset is equal to its total R&D cost (see footnote 7), which is negatively related to its growth rate (see Eq. (3)), η^* is decreasing in γ^*.

Let $\widehat{x} \equiv d\ln x \equiv dx/x$, where a hat over x represents the logarithmic change, or the rate of change, in x. The differentiated form of Eq. (19) is:

$$\begin{aligned}
\rho dU_j &= \widehat{W}_j^* + \Gamma^* d\gamma^*; \qquad (20)\\
\Gamma^* &\equiv -[(1-1/\sigma)\rho/(\rho+\gamma^*)^2]/\eta^* + 1/\rho\\
&= [1/(\rho\eta^*)][(1-1/\sigma)\rho\gamma^*/(\rho+\gamma^*)^2 + 1] > 0.
\end{aligned}$$

An increase in γ^* increases ρU_j directly, but it indirectly decreases ρU_j through a decrease in the interest income from asset. Since the direct effect of growth on country j's long-run welfare is always stronger than its indirect effect, country j always gains from faster long-run growth. However, the total long-run welfare effects of changes in country- and sector-specific import trade costs are unclear until their effects on country j's total real wage income are determined.

[13] ρU_j expresses country j's long-run welfare in flow terms because $\int_0^\infty \rho U_j \exp(-\rho t)dt = \rho U_j(1/\rho) = U_j$.

4 Long-run effects of country- and sector-specific trade cost changes I: hat algebra around a symmetric BGP

We examine how small changes in country- and sector-specific import trade costs affect long-run growth and welfare using the (local) hat algebra of Jones (1965). However, as summarized in subsection 3.6, our model has so many endogenous variables that we cannot derive general conclusions analytically. To obtain some meaningful analytical results, we make the following assumption in this section:

Assumption 1

At an old BGP, all exogenous variables are the same across countries, and τ^i_{jk}, κ^i_j are the same across machine sectors:

$$
\begin{aligned}
&S_j = S, L_j = L \Rightarrow s_j = S/L \equiv s\forall j,\\
&\tau^i_{jk} = \tau \in (1,\infty)\forall i\forall j, k, k \neq j,\\
&\kappa^i_j = \kappa\forall i\forall j \Rightarrow \kappa_j = \kappa/\kappa = 1\forall j.
\end{aligned}
$$

Under Assumption 1, the two countries are symmetric, and the autarkiness functions take the same value for all machine sectors, at an old BGP:[14]

$$
\begin{aligned}
\chi &= 1,\\
w^L_1 &= w^L_2 \equiv 1,\\
m^i_j &= (1+\tau^{1-\sigma})^{1/(1-\sigma)} \equiv m\forall i\forall j,\\
q^Y_1/q^Y_2 &= 1,\\
\delta_j &= [\zeta^S_{jj} + \zeta^S_{kj}(L/L)(1/1)(s/s)(\omega_j/\omega_j)]/[\zeta^L_{jj} + \zeta^L_{kj}(L/L)(1/1)] = 1\forall j,\\
\nu_j &= [\alpha/(1-\alpha)]^{\varepsilon} s^{\psi-1} \equiv \nu\forall j,\\
\omega_j &= [\alpha/(1-\alpha)]^{\varepsilon} s^{\psi-2} \equiv \omega\forall j,\\
R_j &= [\alpha^{\varepsilon}(S/\kappa)^{\psi-1} + (1-\alpha)^{\varepsilon}(L/\kappa)^{\psi-1}]^{1/(\psi-1)} \equiv R\forall j,
\end{aligned}
$$

where we omit asterisks just for notational simplicity. By evaluating the logarithmically differentiated form of the BGP system at the symmetric old BGP defined above, we can focus on the main mechanism at work when country- and sector-specific import trade costs are changed. From now on, we assume that τ^i_{jk} is the only exogenous variable that can be changed.

4.1 Relative profitability, skill premium, relative technology, and resource function

Noting that country j's relative profitability δ_j depends on countries' skill premiums $\omega_k/\omega_j, k \neq j$ from Eq. (13), whereas ω_j depends on δ_j from Eq. (16), we solve for $\widehat{\delta}_j$ and $\widehat{\omega}_j$ as (see Appendix A for derivations):[15]

[14] In deriving $\delta_j = 1$, we use the fact that $\zeta^i_{jk} = \zeta^i_{kj} = 1 - \zeta^i_{jj}, k \neq j$ at the symmetric old BGP.

[15] The effects of w^L_k/w^L_j on the numerator and denominator of δ_j in Eq. (13) are canceled out with each other at the symmetric old BGP.

$$\begin{aligned}
\widehat{\delta}_j &= (\sigma-1)\lambda[(\widehat{m}_j^S-\widehat{m}_j^L)-(\widehat{m}_k^S-\widehat{m}_k^L)]-(1-\lambda)(\widehat{\omega}_j-\widehat{\omega}_k)\\
&= (1/A)(\sigma-1)[\lambda+(\psi-1)(1-\lambda)][(\widehat{m}_j^S-\widehat{m}_j^L)-(\widehat{m}_k^S-\widehat{m}_k^L)], \qquad (21)\\
\widehat{\omega}_j &= (\psi-1)[(1-\sigma)(\widehat{m}_j^S-\widehat{m}_j^L)+\widehat{\delta}_j]\\
&= -[(\psi-1)/A](\sigma-1)\{\psi(1-\lambda)(\widehat{m}_j^S-\widehat{m}_j^L)+[\lambda+(\psi-1)(1-\lambda)](\widehat{m}_k^S-\widehat{m}_k^L)\}, k\neq j; \qquad (22)\\
\lambda &\equiv 1/(1+\tau^{1-\sigma})=\zeta_{jj}^i=\lambda_{jj}^i\in(1/2,1)\forall i\forall j,\\
A &\equiv 1+2(\psi-1)(1-\lambda)>\lambda+(\psi-1)(1-\lambda)=2\lambda-1+\psi(1-\lambda)>0,
\end{aligned}$$

where country j's domestic revenue share in machine sector i turns out to be equal to the corresponding domestic expenditure share at the symmetric old BGP, which is given by $1/(1+\tau^{1-\sigma})\equiv\lambda$. This means that a constant λ represents a common domestic revenue or expenditure share for all countries and machine sectors at the symmetric old BGP. The range of a common trade cost $\tau\in(1,\infty)$ implies that $\lambda\in(1/2,1)$.

An interesting thing about Eq. (21) is that the logarithmic change in country j's relative profitability δ_j is proportional to a "difference in differences" in logarithmic changes in autarkiness across machine sectors and countries. For example, if machine sector S becomes more closed relative to L in country j relative to k, then it becomes more profitable to enter machine sector S relative to L in country j due to a relatively larger domestic market for S-augmenting machines.

Eq. (22) shows that country j's skill premium ω_j is decreasing in relative autarkiness in machine sector S to L of both countries in the case of elastic factor demands: $\psi>1$. In that case, an increase in $m_k^S/m_k^L, k\neq j$ decreases country j's relative profitability δ_j, which necessarily decreases its skill premium ω_j. An increase in m_j^S/m_j^L directly decreases ω_j, but it indirectly increases ω_j through an increase in δ_j. The direct effect always outweighs the indirect effect.

Logarithmically differentiating country j's relative technology (15), and substituting Eq. (21) into it, we obtain:

$$\begin{aligned}
\widehat{\nu}_j &= (1-\sigma)(\psi-1)(\widehat{m}_j^S-\widehat{m}_j^L)+\psi\widehat{\delta}_j\\
&= -(1/A)(\sigma-1)\{A(\psi-1)(\widehat{m}_j^S-\widehat{m}_j^L)-\psi[\lambda+(\psi-1)(1-\lambda)][(\widehat{m}_j^S-\widehat{m}_j^L)-(\widehat{m}_k^S-\widehat{m}_k^L)]\}, k\neq j.
\end{aligned} \qquad (23)$$

As we can see from Eqs. (15) and (16), Eq. (23) is similar to Eq. (22). However, the total effect of m_j^S/m_j^L on ν_j is ambiguous because its indirect effect through δ_j is more pronounced than on ω_j.

The logarithmically differentiated form of country j's resource function R_j in Eq. (17) is given by (see Appendix A for derivation):

$$\begin{aligned}
\widehat{R}_j &= \beta[(1-\sigma)(\widehat{m}_j^S-\widehat{m}_j^L)+\widehat{\delta}_j]=[\beta/(\psi-1)]\widehat{\omega}_j\\
&= -(\beta/A)(\sigma-1)\{\psi(1-\lambda)(\widehat{m}_j^S-\widehat{m}_j^L)+[\lambda+(1-\lambda)(\psi-1)](\widehat{m}_k^S-\widehat{m}_k^L)\}, k\neq j; \qquad (24)\\
\beta_j &= \frac{\alpha^\varepsilon(S_j/\kappa_j^S)^{\psi-1}(m_j^S/m_j^L)^{(1-\sigma)(\psi-1)}\delta_j^{\psi-1}}{\alpha^\varepsilon(S_j/\kappa_j^S)^{\psi-1}(m_j^S/m_j^L)^{(1-\sigma)(\psi-1)}\delta_j^{\psi-1}+(1-\alpha)^\varepsilon(L_j/\kappa_j^L)^{\psi-1}}\\
&= \frac{\alpha^\varepsilon(S/\kappa)^{\psi-1}}{\alpha^\varepsilon(S/\kappa)^{\psi-1}+(1-\alpha)^\varepsilon(L/\kappa)^{\psi-1}}\equiv\beta\forall j.
\end{aligned}$$

In Eq. (24), the elasticity of R_j with respect to its endogenous part $(m_j^S/m_j^L)^{1-\sigma}\delta_j$ is equal to β_j, country j's expenditure share of good S, originally defined in subsection 2.2. At the symmetric old BGP, this is equal to a constant β defined above. Moreover, comparing Eq. (17) with Eq. (16), we can say that country j's skill premium is a sufficient statistic for its resource function.

Eqs. (21) to (24) reveal that country j's relative profitability, skill premium, relative technology, and resource function depend on the differences in countries' logarithmic changes in autarkiness across machine sectors $\widehat{m}_j^S - \widehat{m}_j^L$ and $\widehat{m}_k^S - \widehat{m}_k^L, k \neq j$, and the difference in differences $(\widehat{m}_j^S - \widehat{m}_j^L) - (\widehat{m}_k^S - \widehat{m}_k^L)$. We next see how they are expressed in terms of logarithmic changes in trade costs.

First of all, logarithmically differentiating $m_j^i = [1 + (n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}, k \neq j$ in Eq. (7), and using $\zeta_{kj}^i = (n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}/\sum_l (n_l^i/n_j^i)(\tau_{lj}^i q_l^Y/q_j^Y)^{1-\sigma}$ (see footnote 9), $\widehat{m}_j^i$ is calculated as:

$$\widehat{m}_1^S = (1-\lambda)\{\widehat{\tau}_{21}^S - [\widehat{q}_1^Y - \widehat{q}_2^Y - \widehat{\chi}/(\sigma-1)] + (\widehat{\nu}_1 - \widehat{\nu}_2)/(\sigma-1)\}, \tag{25}$$

$$\widehat{m}_2^S = (1-\lambda)[\widehat{\tau}_{12}^S + \widehat{q}_1^Y - \widehat{q}_2^Y - \widehat{\chi}/(\sigma-1) - (\widehat{\nu}_1 - \widehat{\nu}_2)/(\sigma-1)], \tag{26}$$

$$\widehat{m}_1^L = (1-\lambda)\{\widehat{\tau}_{21}^L - [\widehat{q}_1^Y - \widehat{q}_2^Y - \widehat{\chi}/(\sigma-1)]\}, \tag{27}$$

$$\widehat{m}_2^L = (1-\lambda)[\widehat{\tau}_{12}^L + \widehat{q}_1^Y - \widehat{q}_2^Y - \widehat{\chi}/(\sigma-1)], \tag{28}$$

If we take country 1's autarkiness in machine sector S: $m_1^S = [1+(\nu_2/\nu_1)(1/\chi)(\tau_{21}^S q_2^Y/q_1^Y)^{1-\sigma}]^{1/(1-\sigma)}$, for example, it is increasing in the corresponding import trade cost τ_{21}^S, decreasing in country 1's terms of trade q_1^Y/q_2^Y, and decreasing in country 1's relative number of imported varieties in that sector $n_2^S/n_1^S = (\nu_2/\nu_1)(1/\chi)$. Also, the elasticity of m_1^S with respect to τ_{21}^S is $1-\lambda$, a common export revenue or import expenditure share at the symmetric old BGP.

Using Eqs. (25) to (28), together with Eq. (23), we can solve for $\widehat{m}_1^S - \widehat{m}_1^L, \widehat{m}_2^S - \widehat{m}_2^L$, and $(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)$ as (see Appendix A for derivations):

$$(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L) = [(1-\lambda)/(2\lambda-1)^2]A[(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]. \tag{29}$$

$$\widehat{m}_1^S - \widehat{m}_1^L = (1-\lambda)\{\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L + [A + \psi(2\lambda-1)][(1-\lambda)/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}, \tag{30}$$

$$\widehat{m}_2^S - \widehat{m}_2^L = (1-\lambda)\{\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L - [A + \psi(2\lambda-1)][(1-\lambda)/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}. \tag{31}$$

Eq. (29) shows an interesting result: the "difference in differences" in logarithmic changes in autarkiness across machine sectors and countries is proportional to the "difference in differences" in logarithmic changes in the corresponding import trade costs. As expected from Eqs. (25) to (28), when import becomes more costly in machine sector S relative to L in country 1 relative to 2 (i.e., $(\tau_{21}^S/\tau_{21}^L)/(\tau_{12}^S/\tau_{12}^L)$ increases), machine sector S becomes more closed relative to L in country 1 relative to 2 (i.e., $(m_1^S/m_1^L)/(m_2^S/m_2^L)$ increases). Moreover, this creates a multiplier effect: by making machine sector S more profitable relative to L in country 1 relative to 2 (i.e., increasing δ_1/δ_2), it induces more firms to enter machine sector S relative to L in country 1 relative to 2 (i.e., increases ν_1/ν_2). By making country 1 import relatively less varieties in machine sector S relative to L, this further causes machine sector S to be more closed relative to L in country 1 relative to 2 (i.e., increases $(m_1^S/m_1^L)/(m_2^S/m_2^L)$), and so on. Eqs. (30) and (31) follow from a similar logic.

Having solved for $\widehat{m}_1^S - \widehat{m}_1^L, \widehat{m}_2^S - \widehat{m}_2^L$, and $(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)$ in terms of logarithmic changes in trade costs, we can also solve for $\widehat{\delta}_j, \widehat{\omega}_j, \widehat{\nu}_j$, and $\widehat{R}_j$ in terms of logarithmic changes in trade costs:

$$\widehat{\delta}_j = (\sigma-1)[\lambda+(\psi-1)(1-\lambda)][(1-\lambda)/(2\lambda-1)^2][(\widehat{\tau}_{kj}^S-\widehat{\tau}_{kj}^L)-(\widehat{\tau}_{jk}^S-\widehat{\tau}_{jk}^L)], \tag{32}$$

$$\widehat{R}_j = [\beta(\sigma-1)(1-\lambda)/(2\lambda-1)][(1-\lambda)(\widehat{\tau}_{kj}^S-\widehat{\tau}_{kj}^L)-\lambda(\widehat{\tau}_{jk}^S-\widehat{\tau}_{jk}^L)], \tag{33}$$

$$\widehat{\omega}_j = [(\psi-1)/\beta]\widehat{R}_j = [(\psi-1)(\sigma-1)(1-\lambda)/(2\lambda-1)][(1-\lambda)(\widehat{\tau}_{kj}^S-\widehat{\tau}_{kj}^L)-\lambda(\widehat{\tau}_{jk}^S-\widehat{\tau}_{jk}^L)], \tag{34}$$

$$\begin{aligned}\widehat{\nu}_j &= [(\sigma-1)(1-\lambda)/(2\lambda-1)^2] \\ &\quad\times\{\lambda A(\widehat{\tau}_{kj}^S-\widehat{\tau}_{kj}^L)-\{(1-\lambda)(2\lambda-1)+\psi[1-2\lambda(1-\lambda)]\}(\widehat{\tau}_{jk}^S-\widehat{\tau}_{jk}^L)\}, k\neq j,\end{aligned} \tag{35}$$

where $1-2\lambda(1-\lambda)>0$ follows from $\lambda\in(1/2,1)$. From Eqs. (32), (33), (34), and (35), we obtain the following proposition:

Proposition 1. *Around the symmetric old BGP, country j's relative profitability δ_j, resource function R_j, and relative technology ν_j are increasing in τ_{kj}^S/τ_{kj}^L but decreasing in $\tau_{jk}^S/\tau_{jk}^L, k\neq j$. Moreover, country j's skill premium ω_j is increasing in τ_{kj}^S/τ_{kj}^L but decreasing in $\tau_{jk}^S/\tau_{jk}^L, k\neq j$ if and only if $\psi>1$.*

An increase in τ_{21}^S/τ_{21}^L increases $(m_1^S/m_1^L)/(m_2^S/m_2^L)$ (see Eq. (29)), which makes machine sector S more profitable relative to L in country 1 (see Eq. (21)). The increase in δ_1 is so strong that it not only increases country 1's resource function R_1 (see Eq. (33)), but it also induces more entry into machine sector S relative to L in country 1 (see Eq. (35)). The effect of an increase in δ_1 on country 1's relative demand for skilled labor and hence its skill premium ω_1 depend on whether demands for the two factors (and also demands for the two intermediate goods) are elastic or not. Overall, as long as $\psi>1$, δ_j, R_j, ν_j, and ω_j tend to move in the same direction.

An implication of Proposition 1 for long-run growth is that, from the revised growth equation (18), an increase in $\tau_{kj}^S/\tau_{kj}^L, k\neq j$, at least partly, increases country j's growth rate γ_j through an increase in its resource function R_j. This might sound that a country can raise its growth rate by raising its import trade cost specifically for machine sector S and encouraging entry into that sector. However, we cannot determine the long-run growth effects of changes in trade costs until we take changes in $q_1^Y/q_2^Y, w_1^L$, and χ into account.

4.2 Balanced growth rate

Using Eqs. (2), (24), and $s_j\omega_j\delta_j/(s_j\omega_j\delta_j+1)=s_j\omega_j/(s_j\omega_j+1)=\beta_j$ at the symmetric old BGP (see Eq. (80)), the differentiated form of country j's revised growth equation (18) is given by:

$$\begin{aligned}d\gamma_j &= (\rho+\gamma)[\beta\widehat{\omega}_j-\beta(\widehat{\omega}_j+\widehat{\delta}_j)+\widehat{R}_j+(1-\sigma)\widehat{m}_j^L] \\ &= -(\sigma-1)(\rho+\gamma)[\beta(\widehat{m}_j^S-\widehat{m}_j^L)+\widehat{m}_j^L].\end{aligned} \tag{36}$$

In Eq. (36), the effects of ω_j on the numerator and denominator of $(s_j\omega_j+1)/(s_j\omega_j\delta_j+1)$ are canceled out with each other at the symmetric old BGP. Also, the effect of δ_j on the denominator of $(s_j\omega_j+1)/(s_j\omega_j\delta_j+1)$ is canceled out by the second term in the right-hand side of $\widehat{R}_j=\beta[(1-\sigma)(\widehat{m}_j^S-\widehat{m}_j^L)+\widehat{\delta}_j]$ in Eq. (24), thereby leaving only its first term $\beta(1-\sigma)(\widehat{m}_j^S-\widehat{m}_j^L)$. Eq. (36) implies that the possibility argued at the end of subsection 4.1 that an increase in $\tau_{kj}^S, k\neq j$ directly increases its γ_j through an increase in R_j seems to be wrong. Similarly, an increase in $\tau_{kj}^L, k\neq j$, ceteris paribus, tends to decrease γ_j because $\beta\in(0,1)$.

After endogenizing logarithmic changes in $q_1^Y/q_2^Y, w_1^L$, and χ, the amount of change in the balanced growth rate γ is solved as (see Appendix A for derivation):

$$d\gamma = -(1/2)(\sigma - 1)(\rho + \gamma)(1 - \lambda)[\beta(\widehat{\tau}_{kj}^S + \widehat{\tau}_{jk}^S) + (1 - \beta)(\widehat{\tau}_{kj}^L + \widehat{\tau}_{jk}^L)], k \neq j. \tag{37}$$

Eq. (37) immediately implies that:

$$\partial\gamma/\partial \ln \tau_{kj}^S = \partial\gamma/\partial \ln \tau_{jk}^S = -(1/2)(\sigma - 1)(\rho + \gamma)(1 - \lambda)\beta < 0,$$
$$\partial\gamma/\partial \ln \tau_{kj}^L = \partial\gamma/\partial \ln \tau_{jk}^L = -(1/2)(\sigma - 1)(\rho + \gamma)(1 - \lambda)(1 - \beta) < 0, k \neq j.$$

Proposition 2. *Around the symmetric old BGP, an increase in τ_{jk}^i, for any $i = S, L$ and for any $j, k = 1, 2, k \neq j$, decreases the balanced growth rate.*

The amount of change in the balanced growth rate $d\gamma$ is determined by the differentiated form of the international balanced growth condition (2): $d\gamma_1 = d\gamma_2 \equiv d\gamma$, where $d\gamma_j$ is given by Eq. (36). Suppose that χ increases. On the one hand, this directly decreases country 1's growth rate γ_1 by making that country more closed in machine sector L (see Eq. (27)). On the other hand, the increase in χ creates country 1's trade surplus, which is cleared by an increase in its unskilled wage w_1^L (see Eq. (94)). Since this increases country 1's terms of trade q_1^Y/q_2^Y (see Eq. (93)), it makes country 1 more open in machine sector L (see Eq. (27)), partly contributing to its faster growth (see Eq. (36)). However, such indirect, general equilibrium effect through w_1^L and q_1^Y/q_2^Y is always weaker than the direct negative effect of χ on γ_1.[16] Therefore, γ_1 is decreasing in χ. Similarly, country 2's growth rate γ_2 is increasing in χ. This ensures that the BGP value of χ is unique if exists.[17]

Suppose that country j increases its import trade cost in machine sector i. This makes country j more closed in machine sector i (see Eqs. (25) to (28)), thereby pulling down country j's growth rate γ_j (see Eq. (36)). For the international balanced growth condition (2) to be restored, country j's relative number of L-augmenting machines $n_j^L/n_k^L, k \neq j$ (i.e., χ for $j = 1$; $1/\chi$ for $j = 2$) decreases. This increases country j's growth rate γ_j but decreases country k's growth rate γ_k until they are equalized. On a new BGP, the balanced growth rate necessarily decreases, compared with the symmetric old BGP.

Proposition 2 has important implications. First, even country- and sector-specific trade liberalization raises long-run growth. In the literature on two-country models of trade and endogenous growth with homogeneous firms (e.g., Rivera-Batiz and Romer, 1991a, 1991b; Baldwin and Forslid, 1999) and heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Dinopoulos and Unel, 2011; Sampson, 2016; Ourens, 2016; Naito, 2017, 2019, 2021; Impullitti and Licandro, 2018; Fukuda, 2019; Perla et al., 2021; Akcigit et al., 2021), only a few papers consider country-specific trade liberalization (e.g., Baldwin and Forslid, 1999; Naito, 2017, 2019, 2021; Akcigit et al., 2021), and none of them studies country- and sector-specific trade liberalization. In a directed technical change model of Acemoglu (2002) extended to two countries, this paper makes a strong argument that trade liberalization is growth-enhancing even if it occurs in only one of the two countries and only one of the two differentiated good sectors. Second, the positive long-run growth effect of country- and sector-specific trade liberalization is independent of the elasticity of substitution across the two factors. The most important result of Acemoglu (2002), known as the strong induced-bias hypothesis, is that an increase in a country's relative supply of skilled labor paradoxically increases the country's skill premium if and only if the elasticity of substitution across the two factors is larger than two (under the lab-equipment specification for R&D), so that the induced entry into the skilled-labor-augmenting machine sector shifts out the relative demand curve for skilled labor more than its relative supply curve. Proposition 2 states that there is a positive relationship between

[16] Since $\partial \ln(q_1^Y/q_2^Y)/\partial \ln \chi = 1/(2\sigma - 1) < 1/(\sigma - 1)$ from Eq. (95), $\widehat{q}_1^Y - \widehat{q}_2^Y - \widehat{\chi}/(\sigma - 1)$ in Eq. (27) is decreasing in $\widehat{\chi}$.

[17] Although imposing the two balanced growth conditions (1) and (2) rules out transitional dynamics, the argument in this paragraph suggests a sort of stability: since $\dot{\chi}/\chi = \gamma_1 - \gamma_2$ is decreasing in χ, we have $\dot{\chi}/\chi > 0$ if and only if $\chi < \chi^*$, where χ^* satisfies Eq. (2): $0 = \gamma_1^* - \gamma_2^*$. This implies that χ approaches χ^* whenever $\chi \neq \chi^*$.

trade liberalization in any country in any machine sector and long-run growth, regardless of whether the strong induced-bias hypothesis is true or not, or whether demands for the two factors (and also demands for the two intermediate goods) are elastic or not.

4.3 Total real wage income and long-run welfare

From Eq. (20), we know that country j's long-run welfare (expressed in flow terms) ρU_j is increasing in the balanced growth rate γ and the endogenous part of country j's total real wage income W_j. Logarithmically differentiating $W_j = (m_j^L)^{1-\sigma} R_j(s_j\omega_j + 1)$ in Eq. (19), and using Eq. (24) and $s_j\omega_j/(s_j\omega_j+1) = \beta_j$ at the symmetric old BGP (see Eq. (80)), we obtain $\widehat{W}_j = -(\sigma-1)\widehat{m}_j^L + \psi\widehat{R}_j$, which is rewritten as:[18]

$$\begin{aligned}\widehat{W}_j = &(\sigma-1)(1-\lambda)\{-(1/2)(\widehat{\tau}_{kj}^L + \widehat{\tau}_{jk}^L)\\ &+ (1/2)[\beta/(2\lambda-1)^2]A[(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L) - (\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)]\\ &+ [\psi\beta/(2\lambda-1)][(1-\lambda)(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L) - \lambda(\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)]\}, k \neq j. \end{aligned} \tag{38}$$

Eq. (38) implies that:

$$\begin{aligned}&\partial \ln W_j/\partial \ln \tau_{kj}^S\\ &= (\sigma-1)(1-\lambda)\{(1/2)[\beta/(2\lambda-1)^2]A + [\psi\beta/(2\lambda-1)](1-\lambda)\} > 0,\\ &\partial \ln W_j/\partial \ln \tau_{kj}^L\\ &= (\sigma-1)(1-\lambda)\{-1/2 - (1/2)[\beta/(2\lambda-1)^2]A - [\psi\beta/(2\lambda-1)](1-\lambda)\} < 0,\\ &\partial \ln W_j/\partial \ln \tau_{jk}^S\\ &= (\sigma-1)(1-\lambda)\{-(1/2)[\beta/(2\lambda-1)^2]A - [\psi\beta/(2\lambda-1)]\lambda\} < 0,\\ &\partial \ln W_j/\partial \ln \tau_{jk}^L\\ &= (\sigma-1)(1-\lambda)\{-1/2 + (1/2)[\beta/(2\lambda-1)^2]A + [\psi\beta/(2\lambda-1)]\lambda\}, k \neq j.\end{aligned}$$

To understand Eq. (38) and the resulting elasticities, consider first the case without skilled labor (i.e., $\alpha = 0 \Rightarrow \beta = 0$). Then we only have $\partial \ln W_j/\partial \ln \tau_{kj}^L = \partial \ln W_j/\partial \ln \tau_{jk}^L = -(1/2)(\sigma-1)(1-\lambda) < 0$. This and Proposition 2 imply that an increase in τ_{kj}^L or τ_{jk}^L necessarily decreases country j's long-run welfare around the symmetric old BGP. This highlights that the presence of skilled labor as the second factor complicates the long-run welfare effects of import trade costs. In general, the signs of the partial elasticities of W_j with respect to import trade costs are the same as those of δ_j, R_j, and ν_j (and also ω_j for $\psi > 1$) as summarized in Proposition 1, except that the sign of $\partial \ln W_j/\partial \ln \tau_{jk}^L$ is ambiguous. This implies that the long-run welfare effects of τ_{kj}^S and τ_{jk}^L could be ambiguous because their effects on W_j could be positive in contrast to their negative long-run growth effects.

The long-run welfare effects of import trade costs are summarized in the following proposition:

Proposition 3. *Around the symmetric old BGP, an increase in τ_{kj}^L or $\tau_{jk}^S, k \neq j$ decreases country j's long-run welfare. Also, an increase in τ_{kj}^S or $\tau_{jk}^L, k \neq j$ decreases country j's long-run welfare if the common subjective discount rate ρ is sufficiently small.*

Proof. Γ in Eq. (20) is rewritten as:

[18] We use Eqs. (27), (28), (33), (95), and (96) to derive Eq. (38).

$$\Gamma = [1/(\rho\eta)][(1-1/\sigma)\rho\gamma/(\rho+\gamma)^2+1]$$
$$= [(1-1/\sigma)\gamma/(\rho+\gamma)^2+1/\rho]/[(1-1/\sigma)\rho/(\rho+\gamma)+1].$$

Since Eqs. (2), (18), and subsection 3.6 imply that the right-hand side of $\rho+\gamma = (1-1/\sigma)^{\sigma}[(s_j\omega_j+1)/(s_j\omega_j\delta_j+1)]R_j(m_j^L)^{1-\sigma}$ is independent of ρ, any change in ρ keeps $\rho+\gamma$ unchanged, implying that $\partial\gamma/\partial\rho = -1 < 0$. This ensures that Γ is decreasing in ρ. Moreover, we have $\lim_{\rho\to 0}\Gamma = \infty$.

Considering that all variables other than ρ or γ in Eqs. (37) and (38) are independent of ρ, even if the first term in the right-hand side of Eq. (20): $\rho dU_j = \widehat{W}_j + \Gamma d\gamma$ goes in the opposite direction of the second term, the latter outweighs the former if ρ is sufficiently small. This, together with Proposition 2, completes the proof. □

Starting from the symmetric old BGP, a decrease in country j's import trade cost in machine sector L or country $k(\neq j)$'s import trade cost in machine sector S necessarily increases country j's long-run welfare, whereas it could partly decrease country k's long-run welfare through a decrease in country k's total real wage income. However, as long as people are sufficiently patient, the long-run growth effect dominates, so that both countries gain from any country- and sector-specific trade liberalization.

5 Long-run effects of country- and sector-specific trade cost changes II: exact hat algebra around a factual BGP

To check the robustness of the analytical results obtained around a symmetric BGP in section 4 to more realistic situations with asymmetric countries and sectors, we do some counterfactual simulations around a factual BGP using the exact hat algebra of Dekle et al. (2008). Suppose that the value of a variable x changes from x to x', and let $\widetilde{x} \equiv x'/x$, where a tilde over x represents the relative change in x.[19] The advantage of the exact hat algebra is implementability: to solve for counterfactual relative changes in endogenous variables caused by counterfactual relative changes in exogenous variables, we only need some elasticities and factual share and rate variables, without estimating the old values of exogenous variables such as productivity or trade cost.

5.1 The model in relative changes

We express the model in relative changes in the same order as section 4 (see Appendix A for derivation):

$$\widetilde{\delta}_j = \delta_j^{-1}\frac{\zeta_{jj}^S(\widetilde{m}_j^S)^{\sigma-1} + (1-\zeta_{kk}^S(\widetilde{m}_k^S)^{\sigma-1})(\beta_k/\beta_j)(1/y_j)(\widetilde{w}_k^L/\widetilde{w}_j^L)(\widetilde{\omega}_k/\widetilde{\omega}_j)}{\zeta_{jj}^L(\widetilde{m}_j^L)^{\sigma-1} + (1-\zeta_{kk}^L(\widetilde{m}_k^L)^{\sigma-1})[(1-\beta_k)/(1-\beta_j)](1/y_j)(\widetilde{w}_k^L/\widetilde{w}_j^L)}; \tag{39}$$

$$\delta_j = \frac{\zeta_{jj}^S + (1-\zeta_{kk}^S)(\beta_k/\beta_j)(1/y_j)}{\zeta_{jj}^L + (1-\zeta_{kk}^L)[(1-\beta_k)/(1-\beta_j)](1/y_j)}, y_j \equiv (w_j^S S_j + w_j^L L_j)/(w_k^S S_k + w_k^L L_k), k \neq j,$$

$$\widetilde{\omega}_j = (\widetilde{m}_j^S/\widetilde{m}_j^L)^{(1-\sigma)(\psi-1)}\widetilde{\delta}_j^{\psi-1}, \tag{40}$$

$$\widetilde{\nu}_j = (\widetilde{m}_j^S/\widetilde{m}_j^L)^{(1-\sigma)(\psi-1)}\widetilde{\delta}_j^{\psi}, \tag{41}$$

$$\widetilde{R}_j = [\beta_j(\widetilde{m}_j^S/\widetilde{m}_j^L)^{(1-\sigma)(\psi-1)}\widetilde{\delta}_j^{\psi-1} + 1 - \beta_j]^{1/(\psi-1)}. \tag{42}$$

[19] Since we already used a hat to represent the logarithmic change, here we use a tilde to denote the relative change.

$$\widetilde{m}_1^S = \{\zeta_{11}^S + (1-\zeta_{11}^S)(\widetilde{\nu}_2/\widetilde{\nu}_1)(1/\widetilde{\chi})[\widetilde{\tau}_{21}^S/(\widetilde{q_1^Y/q_2^Y})]^{1-\sigma}\}^{1/(1-\sigma)}, \tag{43}$$
$$\widetilde{m}_2^S = \{\zeta_{22}^S + (1-\zeta_{22}^S)(\widetilde{\nu}_1/\widetilde{\nu}_2)\widetilde{\chi}[\widetilde{\tau}_{12}^S(\widetilde{q_1^Y/q_2^Y})]^{1-\sigma}\}^{1/(1-\sigma)}, \tag{44}$$
$$\widetilde{m}_1^L = \{\zeta_{11}^L + (1-\zeta_{11}^L)(1/\widetilde{\chi})[\widetilde{\tau}_{21}^L/(\widetilde{q_1^Y/q_2^Y})]^{1-\sigma}\}^{1/(1-\sigma)}, \tag{45}$$
$$\widetilde{m}_2^L = \{\zeta_{22}^L + (1-\zeta_{22}^L)\widetilde{\chi}[\widetilde{\tau}_{12}^L(\widetilde{q_1^Y/q_2^Y})]^{1-\sigma}\}^{1/(1-\sigma)}. \tag{46}$$

$$\widetilde{q_1^Y/q_2^Y} = (\widetilde{w}_1^L/\widetilde{\chi})(\widetilde{m}_1^L/\widetilde{m}_2^L)^{\sigma-1}\widetilde{R}_2/\widetilde{R}_1, \tag{47}$$
$$\begin{aligned}\widetilde{w}_1^L = &\left[\frac{(1-\zeta_{22}^S(\widetilde{m}_2^S)^{\sigma-1})\beta_2\widetilde{\omega}_2 + (1-\zeta_{22}^L(\widetilde{m}_2^L)^{\sigma-1})(1-\beta_2)}{(1-\zeta_{22}^S)\beta_2 + (1-\zeta_{22}^L)(1-\beta_2)}\right]\\ &\div\left[\frac{(1-\zeta_{11}^S(\widetilde{m}_1^S)^{\sigma-1})\beta_1\widetilde{\omega}_1 + (1-\zeta_{11}^L(\widetilde{m}_1^L)^{\sigma-1})(1-\beta_1)}{(1-\zeta_{11}^S)\beta_1 + (1-\zeta_{11}^L)(1-\beta_1)}\right],\end{aligned} \tag{48}$$
$$\widetilde{\gamma}_1 = \widetilde{\gamma}_2 = \widetilde{\gamma}; \tag{49}$$
$$\widetilde{\gamma}_j = \gamma_j^{-1}\{(\rho+\gamma_j)[(\beta_j\widetilde{\omega}_j + 1-\beta_j)(\beta_j\delta_j + 1-\beta_j)/(\beta_j\delta_j\widetilde{\omega}_j\widetilde{\delta}_j + 1-\beta_j)]\widetilde{R}_j(\widetilde{m}_j^L)^{1-\sigma} - \rho\}, \tag{50}$$

where $y_1 = (w_1^S S_1 + w_1^L L_1)/(w_2^S S_2 + w_2^L L_2)$ is country 1's GDP relative to country 2's (of course its inverse shows country 2's relative GDP). With parameters: $\rho, \sigma, \psi(\Rightarrow \varepsilon = 1 + \sigma(\psi - 1))$, data on a factual BGP: $(\beta_1, \beta_2, \zeta_{11}^S, \zeta_{22}^S, \zeta_{11}^L, \zeta_{22}^L, y_1, \gamma)$, and counterfactual relative changes in τ_{kj}^i: $(\widetilde{\tau}_{21}^S, \widetilde{\tau}_{21}^L, \widetilde{\tau}_{12}^S, \widetilde{\tau}_{12}^L)$ as inputs, the above system yields counterfactual relative changes in fifteen endogenous variables: $(\widetilde{\delta}_1, \widetilde{\delta}_2, \widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\nu}_1, \widetilde{\nu}_2, \widetilde{R}_1, \widetilde{R}_2, \widetilde{m}_1^S, \widetilde{m}_2^S, \widetilde{m}_1^L, \widetilde{m}_2^L, \widetilde{q_1^Y/q_2^Y}, \widetilde{w}_1^L, \widetilde{\chi})$ and the balanced growth rate $\widetilde{\gamma}$ as outputs. Once the model in relative changes is solved, the counterfactual relative changes in share and rate variables are calculated from (see Appendix A for derivation):

$$\widetilde{\beta}_j = \widetilde{\omega}_j/(\beta_j\widetilde{\omega}_j + 1 - \beta_j), \tag{51}$$
$$\widetilde{\zeta}_{jj}^i = (\widetilde{m}_j^i)^{\sigma-1}, \tag{52}$$
$$\widetilde{y}_1 = \widetilde{w}_1^L(\beta_1\widetilde{\omega}_1 + 1 - \beta_1)/(\beta_2\widetilde{\omega}_2 + 1 - \beta_2). \tag{53}$$

For the relative change in country j's long-run welfare, Eq. (19) implies that $\rho U_j(\widetilde{U}_j - 1) = \ln \widetilde{W}_j + \ln\widetilde{\eta} + (1/\rho)\gamma(\widetilde{\gamma} - 1)$. The problem is that calculating $\widetilde{U}_j$ requires calculating its factual BGP value U_j, which is unobservable without estimating the factual BGP values of exogenous variables. To avoid this problem, we calculate the absolute, not relative, change in U_j (expressed in flow terms): $\rho U_j(\widetilde{U}_j - 1) = \rho(U_j' - U_j) \equiv \Delta u_j$. This is given by (see Appendix A for derivation):

$$\begin{aligned}\Delta u_j &= \ln\widetilde{W}_j + \ln\widetilde{\eta} + (1/\rho)\gamma(\widetilde{\gamma} - 1);\\ \Delta u_j &\equiv \rho(U_j' - U_j),\\ \widetilde{W}_j &= (\widetilde{m}_j^L)^{1-\sigma}\widetilde{R}_j(\beta_j\widetilde{\omega}_j + 1 - \beta_j),\\ \widetilde{\eta} &= [(1-1/\sigma)\rho/(\rho+\gamma) + 1]^{-1}[(1-1/\sigma)\rho/(\rho+\gamma\widetilde{\gamma}) + 1].\end{aligned} \tag{54}$$

5.2 Methods

We do two sets of experiments. First, to verify the analytical results in section 4, we change one import trade cost at a time around a symmetric BGP. Second, to see how well the analytical results in section

4 apply to a factual world, we change one import trade cost at a time around a factual BGP.

The parameters are borrowed from other studies. The subjective discount rate is $\rho = 0.02$ from Acemoglu (2009). The elasticity of substitution across differentiated machines is $\sigma = 3.8$ from Bernard et al. (2003). For the elasticity of substitution across the two intermediate goods ε, we can back it out from $\varepsilon = 1 + \sigma(\psi - 1)$ once we obtain the elasticity of substitution between skilled and unskilled labor ψ. According to Acemoglu (2009, Chapter 15): "In the context of substitution between skilled and unskilled workers, an elasticity of substitution much higher than 2 is unlikely. Most estimates put the elasticity of substitution between 1.4 and 2." Taking its average as $\psi = 1.7$, ε is calculated as $\varepsilon = 1 + 3.8(1.7 - 1) = 3.66$.

For the data on a symmetric BGP, we arbitrarily set $\beta_1 = \beta_2 = 0.5, \zeta_{11}^S = \zeta_{22}^S = \zeta_{11}^L = \zeta_{22}^L = 0.9, y_1 = 1, \gamma = 0.01$. The data on a factual BGP are processed entirely from the World Development Indicators. We consider services as sector S, and agriculture and industry combined as sector L, following Cravino and Sotelo (2019), whereas we regard low and middle income countries combined as country 1, and high income countries as country 2. β_j is calculated from $\beta_j = q_j^S Y_j^S / q_j^Y Y_j = w_j^S S_j / (w_j^S S_j + w_j^L L_j)$ as country j's share of services value added in its GDP (see Eq. (80)). ζ_{jj}^i is obtained from $\zeta_{jj}^i = 1 - \zeta_{kj}^i, k \neq j$, where $\zeta_{kj}^L = E_{kj}^L / [(\sigma - 1)(w_j^L L_j)]$ and $\zeta_{kj}^S = E_{kj}^S / [(\sigma - 1)(w_j^S S_j)]$ from Eqs. (74) and (75), respectively. y_1 is simply the relative GDP of country 1 to country 2. γ is the annual growth rate of GDP per capita (at constant 2015 U.S. dollars) for countries 1 and 2 combined. We take the annual averages of these variables during 2010–2019. The result is: $\beta_1 = 0.515948, \beta_2 = 0.693663, \zeta_{11}^S = 0.968899, \zeta_{22}^S = 0.963665, \zeta_{11}^L = 0.826309, \zeta_{22}^L = 0.64393, y_1 = 0.537046, \gamma = 0.0174589$.

For counterfactual relative changes in iceberg trade costs, we have either $\widetilde{\tau}_{kj}^i = 0.9$ or $\widetilde{\tau}_{kj}^i = 1.1, k \neq j$, meaning that country j either decreases or increases its iceberg import trade cost factor for i-augmenting machines by 10%, respectively. This implies that we have sixteen counterfactual exercises in total (i.e., either $\widetilde{\tau}_{kj}^i = 0.9$ or $\widetilde{\tau}_{kj}^i = 1.1$, $i = S, L, j, k = 1, 2, k \neq j$, either around a symmetric BGP or around a factual BGP).

5.3 Results around a symmetric BGP

The long-run effects of relative changes in τ_{kj}^i around a symmetric BGP are summarized in Fig. 1 and the corresponding Table 1. In Fig. 1, the first, second, third, and fourth columns indicate the effects of $\widetilde{\tau}_{21}^S, \widetilde{\tau}_{21}^L, \widetilde{\tau}_{12}^S$, and $\widetilde{\tau}_{12}^L$, respectively.

Panel (a) shows $\widetilde{\gamma}$, the relative change in the balanced growth rate. The graphs of $\widetilde{\gamma}$ against $\widetilde{\tau}_{kj}^i$ are downward sloping for all four iceberg trade costs. This confirms Proposition 2.

In panel (b), the first and second rows illustrate $\widetilde{\omega}_1$ and $\widetilde{\omega}_2$, the relative changes in the skill premiums of countries 1 and 2, respectively. $\widetilde{\omega}_1$ is increasing in $\widetilde{\tau}_{21}^S$, decreasing in $\widetilde{\tau}_{21}^L$, decreasing in $\widetilde{\tau}_{12}^S$, and increasing in $\widetilde{\tau}_{12}^L$. Similarly, $\widetilde{\omega}_2$ is increasing in $\widetilde{\tau}_{12}^S$, decreasing in $\widetilde{\tau}_{12}^L$, decreasing in $\widetilde{\tau}_{21}^S$, and increasing in $\widetilde{\tau}_{21}^L$. This verifies Proposition 1 for skill premiums.

The first and second rows in panel (c) indicate Δu_1 and Δu_2, the absolute changes in the long-run welfare (expressed in flow terms) of countries 1 and 2, respectively. The patterns of the graphs of Δu_j against $\widetilde{\tau}_{kj}^i$ are qualitatively the same as those of $\widetilde{\omega}_j$ in panel (b). In particular, an increase in $\widetilde{\tau}_{kj}^S$ or $\widetilde{\tau}_{jk}^L, k \neq j$ from one increases Δu_j. This implies that the second statement of Proposition 3 is not observed for the conventional value of the subjective discount rate $\rho = 0.02$. Of course, it should theoretically be true for a sufficiently smaller ρ.

Overall, the first set of counterfactual exercises around a symmetric BGP confirms Propositions 1 to 3, thereby validating the exact hat algebra.

5.4 Results around a factual BGP

In the second set of counterfactual exercises, the only difference from the first one is an old BGP: we start from a factual BGP rather than a symmetric BGP. The results are summarized in Fig. 2 and the corresponding Table 2.

In panel (a) of Fig. 2, $\widetilde{\gamma}$ is decreasing in $\widetilde{\tau}^i_{kj}$ for all four iceberg trade costs, as in panel (a) of Fig. 1. This implies that any country- and sector-specific trade liberalization raises long-run growth even if we start from a factual BGP.

For skill premiums, panel (b) of Fig. 2 is qualitatively the same as panel (b) of Fig. 1, except that $\widetilde{\omega}_j$ is increasing in $\widetilde{\tau}^L_{kj}, k \neq j$. An increase in $\tau^L_{kj}, k \neq j$ increases m^L_j, country j's autarkiness in machine sector L, from Eq. (45) or (46), which partly increases its skill premium from Eq. (40) by making good S relatively cheaper (see Eq. (14)). Comparing the fourth column of Table 2 with that of Table 1, a 10% increase in τ^L_{21} increases m^L_1 by more around a factual BGP than around a symmetric BGP (i.e., $\widetilde{m}^L_1 = 1.01042$ versus $\widetilde{m}^L_1 = 1.00817$). Similarly, comparing the eighth column of Table 2 with that of Table 1, a 10% increase in τ^L_{12} increases m^L_2 by more around a factual BGP than around a symmetric BGP (i.e., $\widetilde{m}^L_2 = 1.02048$ versus $\widetilde{m}^L_2 = 1.00817$). These quantitative differences are responsible for the qualitative difference in the effect of $\tau^L_{kj}, k \neq j$ on ω_j.

For the long-run welfare, panel (c) of Fig. 2 is qualitatively the same as panel (c) of Fig. 1, except that Δu_2 is decreasing in $\widetilde{\tau}^L_{21}$. This is because the negative long-run growth effect of an increase in τ^L_{21} around a factual BGP is quantitatively larger than around a symmetric BGP (i.e., $\widetilde{\gamma} = 0.975125$ versus $\widetilde{\gamma} = 0.982164$), as we can see from the fourth columns of Table 2 and Table 1. Even if an increase in τ^L_{21} increases the skill premiums of both countries, it decreases the balanced growth rate enough to decrease the long-run welfare of both countries.

To sum up, Propositions 1 to 3 are mostly robust even around a factual BGP.

6 Heterogeneous machine firms

6.1 The model

In this section, we introduce firm heterogeneity in machine sectors in a standard way (see the Online Appendix for details). First, the marginal product of the final good in producing a variety of i-augmenting machine in country j is given by φ, which is a random variable drawn from a Pareto distribution $G(\varphi) \equiv 1-\varphi^{-\theta}; \varphi \in [1, \infty), \theta > \sigma - 1$, and the corresponding probability density function $g(\varphi) \equiv G'(\varphi) = \theta\varphi^{-\theta-1}$. Second, if a firm in country j sells to country k, it has to incur a one-time fixed marketing cost $q^Y_j \kappa^i_{jk}$, where κ^i_{jk} is the fixed marketing cost in terms of the final good.

On a BGP for all $t \geq 0$, the value of a firm indexed by φ producing a variety of i-augmenting machine in country j and selling to country k is given by $v^i_{jk0}(\varphi) = \pi^i_{jk0}(\varphi)/(\rho + \gamma^*) = e^i_{jk0}(\varphi)/[\sigma(\rho + \gamma^*)]$ (see Eq. (79) for its homogeneous firm counterpart), where the firm's revenue $e^i_{jk0}(\varphi) = \{\tau^i_{jk} q^Y_{j0}/[(1 - 1/\sigma)\varphi]\}^{1-\sigma}(P^i_{k0})^\sigma X^i_{k0}$ is increasing in φ. This allows us to define the cutoff productivity φ^{i*}_{jk} from the zero cutoff profit condition as in Melitz (2003):

$$v^i_{jk0}(\varphi^{i*}_{jk}) = q^Y_{j0}\kappa^i_{jk}.$$

Only firms whose productivity $\varphi \geq \varphi^{i*}_{jk}$ survive in market k (i.e., earns a nonnegative net firm value $v^i_{jk0}(\varphi) - q^Y_{j0}\kappa^i_{jk}$) with probability $1 - G(\varphi^{i*}_{jk}) = (\varphi^{i*}_{jk})^{-\theta}$. Calculating the expected net firm values, the free entry condition is now given by:[20]

[20]Combining the zero cutoff profit condition with $v^i_{jk0}(\varphi)/v^i_{jk0}(\varphi^{i*}_{jk}) = e^i_{jk0}(\varphi)/e^i_{jk0}(\varphi^{i*}_{jk}) = (\varphi/\varphi^{i*}_{jk})^{\sigma-1}$, $v^i_{jk0}(\varphi)$ is expressed as $v^i_{jk0}(\varphi) = (\varphi/\varphi^{i*}_{jk})^{\sigma-1} q^Y_{j0}\kappa^i_{jk}$. Then the expected gross and net firm values, conditional on survival, are

$$\sum_k \int_{\varphi^i_{jk}}^{\infty} (v^i_{jk0}(\varphi) - q^Y_{j0}\kappa^i_{jk})g(\varphi)d\varphi = q^Y_{j0}\kappa^i_j \Leftrightarrow [(\sigma-1)/\mu]\sum_k \kappa^i_{jk}(\varphi^{i*}_{jk})^{-\theta} = \kappa^i_j; \mu \equiv \theta - (\sigma-1) > 0. \quad (55)$$

The free entry condition (55) implies that φ^{i*}_{jj} and $\varphi^{i*}_{jk}, k \neq j$, country j's domestic and export cutoffs in machine sector i, respectively, always move in the opposite directions. For example, a decrease in φ^{i*}_{jk} means that country j's expected net firm value from exports increases. This should be compensated by a decrease in its expected net firm value from domestic sales, implying that φ^{i*}_{jj} increases.

Dividing the zero cutoff profit condition by itself for $j = k$, we obtain the relative cutoff condition:

$$\varphi^{i*}_{jk}/\varphi^{i*}_{kk} = \tau^i_{jk}(\kappa^i_{jk}/\kappa^i_{kk})^{1/(\sigma-1)}(q^Y_j/q^Y_k)^{*\sigma/(\sigma-1)}, k \neq j. \quad (56)$$

An increase in the left-hand side of Eq. (56) means that exporters from country $j (\neq k)$ become less competitive relative to country k's domestic firms in market k, in the sense that relatively less firms from country j survive in their export market k. This is more likely, when the variable and/or fixed trade costs become relatively larger, and/or country j's final good as the input becomes relatively more expensive.

For each machine sector $i = S, L$, the free entry condition (55) and the relative cutoff condition (56) for the two countries $j, k = 1, 2, k \neq j$ are solved for the four cutoffs (i.e., domestic and export cutoffs for each country) in terms of $\tau^i_{jk}, \kappa^i_{jk}/\kappa^i_{kk}$, and $(q^Y_j/q^Y_k)^*$.

Some parts of the homogeneous firm model are modified as follows. Country j's price index of i-augmenting machines (7) is now given by:

$$P^i_j = \{\sum_k n^i_k \int_{\varphi^i_{kj}}^{\infty} \{\tau^i_{kj}q^Y_k/[(1-1/\sigma)\varphi]\}^{1-\sigma}g(\varphi)d\varphi\}^{1/(1-\sigma)} = (n^i_j)^{1/(1-\sigma)}q^Y_j m^i_j/(1-1/\sigma); \quad (57)$$

$$m^i_j \equiv [(\theta/\mu)\sum_k (n^i_k/n^i_j)(\tau^i_{kj}q^Y_k/q^Y_j)^{1-\sigma}(\varphi^i_{kj})^{-\mu}]^{1/(1-\sigma)}.$$

Compared with Eq. (7), country j's autarkiness function m^i_j in Eq. (57) includes a new term $(\theta/\mu)(\varphi^i_{kj})^{-\mu} = \int_{\varphi^i_{kj}}^{\infty} \varphi^{\sigma-1}g(\varphi)d\varphi$, an aggregate productivity of country k's firms surviving in country j in machine sector i. An increase in φ^i_{kj}, on the one hand, increases the aggregate productivity by making the surviving firms stronger on average, but on the other hand it decreases the aggregate productivity by reducing the survival probability. Since the latter outweighs the former under $\mu = \theta - (\sigma-1) > 0$, an increase in φ^i_{kj} decreases $(\theta/\mu)(\varphi^i_{kj})^{-\mu}$ and thus increases m^i_j.

The definitions of revenue and expenditure shares in Eqs. (4) and (5), respectively, are unchanged, except that E^i_{jk} is redefined as $E^i_{jk} \equiv n^i_j \int_{\varphi^i_{jk}}^{\infty} e^i_{jk}(\varphi)g(\varphi)d\varphi$, where $e^i_{jk}(\varphi) = \{\tau^i_{jk}q^Y_j/[(1-1/\sigma)\varphi]\}^{1-\sigma}(P^i_k)^{\sigma}X^i_k$. Then country j's domestic expenditure share in machine sector i (8) is replaced by:

$$\zeta^i_{jj} = (\theta/\mu)(\varphi^i_{jj})^{-\mu}(m^i_j)^{\sigma-1}. \quad (58)$$

Finally, for country j's growth equation (3) and its revised version (18), the first term in their right-hand side is multiplied by $[(\sigma-1)/\theta](<1)$, which reflects the additional fixed marketing costs:

$$\gamma^*_j = [(\sigma-1)/\theta](1-1/\sigma)^{\sigma}[(s_j\omega^*_j+1)/(s_j\omega^*_j\delta^*_j+1)]R^*_j(m^{L*}_j)^{1-\sigma} - \rho. \quad (59)$$

All other parts of the homogeneous firm model apply to the heterogeneous firm model, except that

calculated as $\int_{\varphi^{i*}_{jk}}^{\infty} v^i_{jk0}(\varphi)[g(\varphi)/(1-G(\varphi^{i*}_{jk}))]d\varphi = (\theta/\mu)q^Y_{j0}\kappa^i_{jk}$ and $\int_{\varphi^{i*}_{jk}}^{\infty}(v^i_{jk0}(\varphi) - q^Y_{j0}\kappa^i_{jk})[g(\varphi)/(1-G(\varphi^{i*}_{jk}))]d\varphi = [(\sigma-1)/\mu]q^Y_{j0}\kappa^i_{jk}$, respectively. Multiplying the latter by the survival probability $1-G(\varphi^{i*}_{jk}) = (\varphi^{i*}_{jk})^{-\theta}$, substituting it into the first part of Eq. (55), and dividing it by q^Y_{j0}, we obtain the second part of Eq. (55).

m_j^i and ζ_{jj}^i are modified as Eqs. (57) and (58).

6.2 Long-run effects of country- and sector-specific trade cost changes I: hat algebra around a symmetric BGP

In this subsection, we apply the hat algebra around a symmetric BGP to the heterogeneous firm model. Just like section 4, we omit asterisks, and take τ_{jk}^i as the only changeable exogenous variable. In addition to Assumption 1, we also assume that the fixed marketing costs κ_{jk}^i are the same across machine sectors and countries, and that the common fixed marketing cost for exports κ_x is higher than that for domestic sales κ_d:

$$\kappa_{jj}^i = \kappa_d, \kappa_{jk}^i = \kappa_x > \kappa_d \forall i \forall j, k, k \neq j \Rightarrow \kappa_{jk}^i/\kappa_{jj}^i = \kappa_x/\kappa_d \in (1, \infty) \forall i \forall j, k, k \neq j.$$

Then Eqs. (55) and (56) imply that all cutoffs are the same across machine sectors and countries at the symmetric old BGP:

$$\begin{aligned}\kappa &= [(\sigma-1)/\mu](\kappa_d(\varphi_{jj}^i)^{-\theta} + \kappa_x(\varphi_{jk}^i)^{-\theta}),\\ \varphi_{jk}^i/\varphi_{kk}^i &= \tau(\kappa_x/\kappa_d)^{1/(\sigma-1)} = \varphi_{jk}^i/\varphi_{jj}^i, k \neq j\\ &\Rightarrow \varphi_{jj}^i = \varphi_d, \varphi_{jk}^i = \varphi_x > \varphi_d \forall i \forall j, k, k \neq j.\end{aligned}$$

All other endogenous variables at the symmetric old BGP are calculated in the same way as section 4, except that m_j^i and $\lambda_{jj}^i = \zeta_{jj}^i$ are replaced by, respectively:

$$\begin{aligned}m_j^i &= [(\theta/\mu)(\varphi_d^{-\mu} + \tau^{1-\sigma}\varphi_x^{-\mu})]^{1/(1-\sigma)} \equiv m \forall i \forall j,\\ \lambda_{jj}^i &= \zeta_{jj}^i = 1/[1+\tau^{-\theta}(\kappa_x/\kappa_d)^{-\mu/(\sigma-1)}] \equiv \lambda \in (1/2, 1) \forall i \forall j,\end{aligned}$$

where λ is again interpreted as a common domestic revenue or expenditure share for all countries and machine sectors at the symmetric old BGP, and is redefined as $\lambda \equiv 1/[1+\tau^{-\theta}(\kappa_x/\kappa_d)^{-\mu/(\sigma-1)}]$ (instead of $\lambda \equiv 1/(1+\tau^{1-\sigma})$ in the homogeneous firm model).

After long calculations, we obtain (see the Online Appendix for derivations):

$$\begin{aligned}\widehat{\delta}_j &= (1/A)[(1-\lambda)/(2\lambda-1)^2]\\ &\quad\times \{[\lambda + (\psi-1)(1-\lambda)][(\sigma-1)A + \mu\lambda] + \mu\psi\lambda(1-\lambda)\}[(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L) - (\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)], \quad (60)\\ \widehat{R}_j &= [\beta(\sigma-1)(1-\lambda)/(2\lambda-1)][(1-\lambda)(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L) - \lambda(\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)], \quad (61)\\ \widehat{\omega}_j &= [(\psi-1)/\beta]\widehat{R}_j = [(\psi-1)(\sigma-1)(1-\lambda)/(2\lambda-1)][(1-\lambda)(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L) - \lambda(\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)], \quad (62)\\ \widehat{\nu}_j &= (1/A)[(1-\lambda)/(2\lambda-1)^2]\{\lambda\{(\sigma-1)A^2 + \mu[2\lambda-1+2\psi(1-\lambda)]\}(\widehat{\tau}_{kj}^S - \widehat{\tau}_{kj}^L)\\ &\quad - \{(\sigma-1)A\{(1-\lambda)(2\lambda-1) + \psi[1-2\lambda(1-\lambda)]\} + \mu\lambda[2\lambda-1+2\psi(1-\lambda)]\}(\widehat{\tau}_{jk}^S - \widehat{\tau}_{jk}^L)\}, k \neq j. \quad (63)\end{aligned}$$

$$d\gamma = -(1/2)(\sigma-1)(\rho+\gamma)(1-\lambda)[\beta(\widehat{\tau}_{kj}^S + \widehat{\tau}_{jk}^S) + (1-\beta)(\widehat{\tau}_{kj}^L + \widehat{\tau}_{jk}^L)], k \neq j. \quad (64)$$

$$
\begin{aligned}
\widehat{W}_j &= (\sigma-1)(1-\lambda)\{-(1/2)(\widehat{\tau}^L_{kj}+\widehat{\tau}^L_{jk}) \\
&\quad + (1/2)[\beta/(2\lambda-1)^2][A+2\mu\lambda/(\sigma-1)][(\widehat{\tau}^S_{kj}-\widehat{\tau}^L_{kj})-(\widehat{\tau}^S_{jk}-\widehat{\tau}^L_{jk})] \\
&\quad + [\psi\beta/(2\lambda-1)][(1-\lambda)(\widehat{\tau}^S_{kj}-\widehat{\tau}^L_{kj})-\lambda(\widehat{\tau}^S_{jk}-\widehat{\tau}^L_{jk})]\}, k \neq j.
\end{aligned} \tag{65}
$$

Comparing Eqs. (60) to (63), (64), and (65) with Eqs. (32) to (35), (37), and (38), respectively, we see that considering the extensive margin effects (captured by the terms including μ) does not change Propositions 1 to 3 qualitatively. In particular, Eqs. (62) and (64) are the same as Eqs. (34) and (37), respectively. This implies that, as long as we start from the same symmetric old BGP, the long-run effects of trade liberalization on skill premiums and the balanced growth rate in the two models are the same, both qualitatively and quantitatively.

6.3 Long-run effects of country- and sector-specific trade cost changes II: exact hat algebra around a factual BGP

We can do counterfactual exercises around a factual BGP for the heterogeneous firm model in the same way as section 5. The relative change forms of the free entry condition (55) and the relative cutoff condition (56) are given by, respectively:[21]

$$
1 = \lambda^i_{jj}(\widetilde{\varphi}^i_{jj})^{-\theta} + (1-\lambda^i_{jj})(\widetilde{\varphi}^i_{jk})^{-\theta}; \tag{66}
$$

$$
\begin{aligned}
\lambda^S_{jj} &= \frac{\zeta^S_{jj}}{\zeta^S_{jj}+(1-\zeta^S_{kk})(\beta_k/\beta_j)(1/y_j)}, \\
\lambda^L_{jj} &= \frac{\zeta^L_{jj}}{\zeta^L_{jj}+(1-\zeta^L_{kk})[(1-\beta_k)/(1-\beta_j)](1/y_j)},
\end{aligned}
$$

$$
\widetilde{\varphi}^i_{jk}/\widetilde{\varphi}^i_{kk} = \widetilde{\tau}^i_{jk}(\widetilde{q^Y_j/q^Y_k})^{\sigma/(\sigma-1)}, k \neq j. \tag{67}
$$

Unlike the symmetric BGP, λ^i_{jj} is not equal to ζ^i_{jj}, but is calculated using the factual values of $\zeta^i_{jj}, \zeta^i_{kk}, \beta_j, \beta_k$, and y_j.[22] The relative change forms of other equations are the same as the homogeneous firm model, except that Eqs. (57) and (58) are used to calculate $\widetilde{m}^i_j$ and $\widetilde{\zeta}^i_{jj}$, respectively.

We use the same parameter values, data on symmetric and factual BGPs, and counterfactual trade cost shocks, as section 5. In addition, we set the Pareto shape parameter as $\theta = 3.4$ following Ghironi and Melitz (2005) and Bernard et al. (2007).[23]

Fig. 3 and the corresponding Table 3 summarize the results around a symmetric BGP. Compared with Fig. 1, Fig. 3 indicates that the long-run effects of country- and sector-specific trade cost changes for the heterogeneous firm model are qualitatively the same as the homogeneous firm model. Moreover, as implied from subsection 6.2, the long-run skill premium and growth effects in the two models are quantitatively almost the same.[24] Introducing firm heterogeneity does not change our qualitative results around the symmetric old BGP.

[21] In deriving Eq. (66), we use the fact that $\lambda^i_{jk} = \kappa^i_{jk}(\varphi^i_{jk})^{-\theta}/\sum_l \kappa^i_{jl}(\varphi^i_{jl})^{-\theta} = [(\sigma-1)/\mu]\kappa^i_{jk}(\varphi^i_{jk})^{-\theta}/\kappa^i_j$, which is obtained by rewriting $\lambda^i_{jk} = E^i_{jk}/\sum_l E^i_{jl}$ using $e^i_{jk0}(\varphi) = \sigma(\rho+\gamma)v^i_{jk0}(\varphi)$, $\int_{\varphi^i_{jk}}^{\infty} v^i_{jk0}(\varphi)[g(\varphi)/(1-G(\varphi^i_{jk}))]d\varphi = (\theta/\mu)q^Y_{j0}\kappa^i_{jk}$, and Eq. (55).

[22] The expressions for λ^S_{jj} and λ^L_{jj} are derived from Eqs. (82), (83), and (98).

[23] Ghironi and Melitz (2005) calibrate $\theta = 3.4$ to match the standard deviation of log U.S. plant sales of 1.67 and $\sigma = 3.8$ estimated by Bernard et al. (2003). Bernard et al. (2007) also use the same values of σ and θ in their static two-country, two-sector, two-factor Melitz model.

[24] They are quantitatively not exactly the same, however, as shown in the $\widetilde{\omega}_1, \widetilde{\omega}_2$, and $\widetilde{\gamma}$ rows of Tables 1 and 3. This is because we are not ignoring the second-order effects in the exact hat algebra.

Fig. 4 and the corresponding Table 4 show the results around a factual BGP. Comparing Fig. 4 with Fig. 2, we see that the long-run effects of country- and sector-specific trade cost changes for the heterogeneous firm model are qualitatively the same as the homogeneous firm model.

An interesting result that is common to all four cases is that trade liberalization in country j's S-augmenting machine sector decreases its skill premium and long-run welfare. How can we avoid such losses? For example, in the first and third columns of Fig. 4, or in the first and fifth columns of Table 4, we see that the losses in country j's skill premium and long-run welfare from a 10% decrease in τ_{kj}^{S} are outweighed by their gains from a 10% decrease in τ_{jk}^{S}. This implies that coordinated 10% decreases in τ_{21}^{S} and τ_{12}^{S} can increase the skill premium and long-run welfare of both countries. This is because machine sector S is more closed than machine sector L in both countries around a factual BGP, so encouraging trade in machine sector S improves world resource allocation. Also, the result that such coordinated trade liberalization increases countries' skill premiums is consistent with the literature on trade liberalization and skill premium in static quantitative trade models (e.g., Epifani and Gancia, 2008; Parro, 2013; Burstein and Vogel, 2017; Cravino and Sotelo, 2019).

7 Concluding remarks

The most important policy implication of this paper is that trade liberalization is good for growth, even if it occurs in only one country and only one sector. This provides a stronger rationale for trade liberalization as a growth-enhancing policy. When it comes to welfare, however, it matters in which country and sector trade is liberalized. Even so, this paper identifies what are the possible sources of losses from country- and sector-specific trade liberalization, and suggests how to coordinate international trade liberalization that ensures gains for all countries.

There are some directions for future research. First, while expressing trade liberalization in terms of iceberg trade costs, we do not consider the revenue effect of import tariffs. Although the optimal tariff problem has been attracting attention in the age of the U.S.-China trade war, it is underexplored in a dynamic multi-sector context. It will be valuable to introduce revenue-generating import tariffs and to characterize the optimal tariff structure. Second, while our two-country setting is acceptable as a minimal theoretical assumption, it prevents us from analyzing trade issues involving more than two countries. It will be interesting to extend our model to include three or more countries and to examine the long-run growth, skill premium, and welfare effects of regional trade agreements, economic sanctions, and so on.

References

[1] Acemoglu, D., 2002. Directed technical change. Review of Economic Studies 69, 781–809.

[2] Acemoglu, D., 2003. Patterns of skill premia. Review of Economic Studies 70, 199–230.

[3] Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton.

[4] Acemoglu, D., Gancia, G., Zilibotti, F., 2015. Offshoring and directed technical change. American Economic Journal: Macroeconomics 7, 84–122.

[5] Akcigit, U., Ates, S. T., Impullitti, G., 2021. Innovation and trade policy in a globalized world. unpublished.

[6] Arkolakis, C., Costinot, A., Rodriguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94–130.

[7] Baldwin, R. E., Forslid, R., 1999. Incremental trade policy and endogenous growth: a q-theory approach. Journal of Economic Dynamics and Control 23, 797–822.

[8] Baldwin, R. E., Robert-Nicoud, F., 2008. Trade and growth with heterogeneous firms. Journal of International Economics 74, 21–34.

[9] Bernard, A. B., Eaton, J., Jensen, J. B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93, 1268–1290.

[10] Bernard, A. B., Redding, S. J., Schott, P. K., 2007. Comparative advantage and heterogeneous firms. Review of Economic Studies 74, 31–66.

[11] Burstein, A., Vogel, J., 2017. International trade, technology, and the skill premium. Journal of Political Economy 125, 1356–1412.

[12] Chu, A. C., Cozzi, G., Furukawa, Y., 2015. Effects of economic development in China on skill-biased technical change in the US. Review of Economic Dynamics 18, 227–242.

[13] Cravino, J., Sotelo, S., 2019. Trade-induced structural change and the skill premium. American Economic Journal: Macroeconomics 11, 289–326.

[14] Dekle, R., Eaton, J., Kortum, S., 2008. Global rebalancing with gravity: measuring the burden of adjustment. IMF Staff Papers 55, 511–540.

[15] Dinopoulos, E., Unel, B., 2011. Quality heterogeneity and global economic growth. European Economic Review 55, 595–612.

[16] Egger, P. H., Larch, M., Nigai, S., Yotov, Y. V., 2021. Trade costs in the global economy: measurement, aggregation and decomposition. WTO Staff Working Paper ERSD-2021-2.

[17] Epifani, P., Gancia, G., 2008. The skill bias of world trade. Economic Journal 118, 927–960.

[18] Estevadeordal, A., Taylor, A. M., 2013. Is the Washington Consensus dead? growth, openness, and the Great Liberalization, 1970s-2000s. Review of Economics and Statistics 95, 1669–1690.

[19] Fukuda, K., 2019. Effects of trade liberalization on growth and welfare through basic and applied researches. Journal of Macroeconomics 62, 103058.

[20] Gancia, G., Bonfiglioli, A., 2008. North–South trade and directed technical change. Journal of International Economics 76, 276–295.

[21] Ghironi, F., Melitz, M. J., 2005. International trade and macroeconomic dynamics with heterogeneous firms. Quarterly Journal of Economics 120, 865–915.

[22] Impullitti, G., Licandro, O., 2018. Trade, firm selection and innovation: the competition channel. Economic Journal 128, 189–229.

[23] Irwin, D. A., 2019. Does trade reform promote economic growth? a review of recent evidence. PIIE Working Paper 19–9.

[24] Jones, R. W., 1965. The structure of simple general equilibrium models. Journal of Political Economy 73, 557–572.

[25] Krugman, P., 1980. Scale economies, product differentiation, and the pattern of trade. American Economic Review 70, 950–959.

[26] Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71, 1695–1725.

[27] Naito, T., 2017. Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries. Journal of International Economics 109, 167–173.

[28] Naito, T., 2019. A lab-equipment model of growth with heterogeneous firms and asymmetric countries. Economics Letters 181, 33–36.

[29] Naito, T., 2021. Can the optimal tariff be zero for a growing large country? International Economic Review 62, 1237–1280.

[30] Ourens, G., 2016. Trade and growth with heterogeneous firms revisited. Journal of International Economics 100, 194–202.

[31] Parro, F., 2013. Capital-skill complementarity and the skill premium in a quantitative model of trade. American Economic Journal: Macroeconomics 5, 72–117.

[32] Perla, J., Tonetti, C.,Waugh, M. E., 2021. Equilibrium technology diffusion, trade, and growth. American Economic Review 111, 73–128.

[33] Rivera-Batiz, L. A., Romer, P. M., 1991a. Economic integration and endogenous growth. Quarterly Journal of Economics 106, 531–555.

[34] Rivera-Batiz, L. A., Romer, P. M., 1991b. International trade with endogenous technological change. European Economic Review 35, 971–1004.

[35] Rodriguez, F., Rodrik, D., 2000. Trade policy and economic growth: a skeptic's guide to the cross-national evidence. NBER Macroeconomics Annual 15, 261–325.

[36] Sampson, T., 2016. Dynamic selection: an idea flows theory of entry, trade, and growth. Quarterly Journal of Economics 131, 315–380.

[37] Wacziarg, R., Welch, K. H., 2008. Trade liberalization and growth: new evidence. World Bank Economic Review 22, 187–231.

[38] WTO, 2021. WTO trade cost index: evolution, incidence and determinants, background note. http://tradecosts.wto.org/docs/Trade_Cost_Index_Background_Note_24-03-2021.pdf, last accessed March 19, 2022.

Appendix A. Derivations of key equations

Derivation of Eq. (3)

The time differentiated form of country j's asset market-clearing condition is given by $\dot{A}_j = q_j^Y \sum_i \kappa_j^i \dot{n}_j^i + r_j A_j - \sum_i \sum_k n_j^i \pi_{jk}^i$ (see footnote 6). Using Eq. (1), $\pi_{jk}^i = e_{jk}^i/\sigma, A_j = q_j^Y \sum_i n_j^i \kappa_j^i$, and $E_{jk}^i = n_j^i e_{jk}^i$, it is rewritten as $\dot{A}_j = A_j(\gamma_j + r_j) - (1/\sigma)\sum_i \sum_k E_{jk}^i$. Moreover, since $\sum_i \sum_k E_{jk}^i = (1 - 1/\sigma) q_j^Y Y_j$ from $q_j^Y Y_j = q_j^S D_j^S + q_j^L D_j^L, P_j^i X_j^i = (1 - 1/\sigma) q_j^i Y_j^i, Y_j^i = D_j^i$, and $\sum_i \sum_k E_{jk}^i = \sum_i P_j^i X_j^i$, we obtain:

$$\dot{A}_j = A_j(\gamma_j + r_j) - (1/\sigma)(1 - 1/\sigma) q_j^Y Y_j.$$

Multiplying country j's final good market-clearing condition $Y_j = C_j + \sum_i \kappa_j^i \dot{n}_j^i + \sum_i \sum_k n_j^i y_{jk}^i$ by its price q_j^Y, and using Eq. (1), $E_j = q_j^Y C_j, \pi_{jk}^i = e_{jk}^i - q_j^Y y_{jk}^i = e_{jk}^i/\sigma, A_j = q_j^Y \sum_i n_j^i \kappa_j^i, E_{jk}^i = n_j^i e_{jk}^i$, and $\sum_i \sum_k E_{jk}^i = (1 - 1/\sigma) q_j^Y Y_j$, we obtain $q_j^Y Y_j = E_j + \gamma_j A_j + (1 - 1/\sigma)^2 q_j^Y Y_j$, which is solved for $q_j^Y Y_j$ as:

$$q_j^Y Y_j = \{1/[1 - (1 - 1/\sigma)^2]\}(E_j + \gamma_j A_j). \tag{68}$$

Substituting Eq. (68) into the above expression for $\dot{A}_j$, dividing it by A_j, and noting that $1 - (1 - 1/\sigma)^2 - (1/\sigma)(1 - 1/\sigma) = 1/\sigma$, give:

$$\dot{A}_j/A_j = r_j + \{(1/\sigma)/[1 - (1 - 1/\sigma)^2]\}\gamma_j - \{(1/\sigma)(1 - 1/\sigma)/[1 - (1 - 1/\sigma)^2]\} Z_j; Z_j \equiv E_j/A_j,$$

where the transformed variable Z_j is interpreted as country j's average propensity to consume out of asset. Using this and country j's Euler equation $\dot{E}_j/E_j = r_j - \rho$, the growth rate of Z_j is given by:

$$\dot{Z}_j/Z_j = \{(1/\sigma)(1 - 1/\sigma)/[1 - (1 - 1/\sigma)^2]\} Z_j - \rho - \{(1/\sigma)/[1 - (1 - 1/\sigma)^2]\}\gamma_j. \tag{69}$$

Multiplying country j's two factor market-clearing conditions by their wages, summing them up, and using Eq. (68), $q_j^Y Y_j = q_j^S D_j^S + q_j^L D_j^L, w_j^L L_j^D = (1/\sigma) q_j^L Y_j^L, w_j^S S_j^D = (1/\sigma) q_j^S Y_j^S, A_j = q_j^Y \sum_i n_j^i \kappa_j^i$, and $Y_j^i = D_j^i$, we obtain:

$$\begin{aligned} & w_j^L L_j(s_j \omega_j + 1) = \{(1/\sigma)/[1 - (1 - 1/\sigma)^2]\} p_j^e \kappa_j^L (\kappa_j \nu_j + 1)(Z_j + \gamma_j) \\ & \quad \Leftrightarrow \gamma_j = \{[1 - (1 - 1/\sigma)^2]/(1/\sigma)\}[(s_j \omega_j + 1)/(\kappa_j \nu_j + 1)](L_j/\kappa_j^L)/(p_j^e/w_j^L) - Z_j; \\ & \quad s_j \equiv S_j/L_j, \omega_j \equiv w_j^S/w_j^L, \kappa_j \equiv \kappa_j^S/\kappa_j^L, \nu_j \equiv n_j^S/n_j^L, p_j^e \equiv n_j^L q_j^Y. \end{aligned} \tag{70}$$

Constancy of $\dot{Z}_j/Z_j$ and γ_j in Eq. (69) requires that Z_j is constant on a BGP. Then, substituting Eq. (70) and $\dot{Z}_j/Z_j = 0$ into Eq. (69), Z_j^* is solved as:

$$Z_j^* = \rho + [(s_j \omega_j^* + 1)/(\kappa_j \nu_j^* + 1)](L_j/\kappa_j^L)/(p_j^{e*}/w_j^{L*}). \tag{71}$$

Substituting Eq. (71) back into Eq. (70), and noting that $[1 - (1 - 1/\sigma)^2]/(1/\sigma) - 1 = 1 - 1/\sigma$, we obtain Eq. (3).

Derivation of Eq. (6)

From $P_j^i X_j^i = (1 - 1/\sigma) q_j^i Y_j^i, w_j^L L_j^D = (1/\sigma) q_j^L Y_j^L, w_j^S S_j^D = (1/\sigma) q_j^S Y_j^S$, and country j's two factor market-clearing conditions $L_j = L_j^D, S_j = S_j^D$, we obtain:

$$P_j^L X_j^L = (\sigma - 1) w_j^L L_j, \tag{72}$$
$$P_j^S X_j^S = (\sigma - 1) w_j^S S_j. \tag{73}$$

Using Eqs. (5), (72), (73), and $\sum_l E_{lj}^i = P_j^i X_j^i$, E_{kj}^L and E_{kj}^S are expressed as, respectively:

$$E_{kj}^L = (\sigma - 1)\zeta_{kj}^L w_j^L L_j, \tag{74}$$
$$E_{kj}^S = (\sigma - 1)\zeta_{kj}^S w_j^S S_j. \tag{75}$$

Substituting Eqs. (74) and (75) into $E_{12}^S + E_{12}^L = E_{21}^S + E_{21}^L$, we obtain Eq. (6).

Derivation of Eq. (13)

Taking the ratio of the free entry condition $\sum_k v_{jk}^i = q_j^Y \kappa_j^i$ for the two machine sectors gives:

$$\kappa_j = \textstyle\sum_k v_{jk}^S / \sum_k v_{jk}^L.$$

The next thing to consider is how to express $v_{jk0}^i = \int_0^\infty \pi_{jkt}^i \exp(-\int_0^t r_{js} ds) dt$ on a BGP for $t \geq 0$. Multiplying Eq. (71) by $A_j^* = p_j^{e*}\kappa_j^L(\kappa_j \nu_j^* + 1)$ gives:

$$E_j^* = \rho p_j^{e*} \kappa_j^L (\kappa_j \nu_j^* + 1) + w_j^{L*} L_j (s_j \omega_j^* + 1). \tag{76}$$

Since $w_j^{L*}, \omega_j^*, \nu_j^*$, and p_j^{e*} are constant on a BGP, E_j^* is also constant from Eq. (76). This and country j's Euler equation $\dot{E}_j/E_j = r_j - \rho$ imply that country j's interest rate is fixed at ρ on a BGP:

$$r_j^* = \rho. \tag{77}$$

The profit $\pi_{jkt}^i = [\tau_{jk}^i q_{jt}^Y/(1-1/\sigma)]^{1-\sigma}(P_{kt}^i)^\sigma X_{kt}^i/\sigma = [\tau_{jk}^i/(1-1/\sigma)]^{1-\sigma}(P_{kt}^i/q_{jt}^Y)^{\sigma-1} P_{kt}^i X_{kt}^i/\sigma$ is rewritten, using Eqs. (2), (7), (72), and (73), as:

$$\begin{aligned}
\pi_{jkt}^L &= [\tau_{jk}^L/(1-1/\sigma)]^{1-\sigma}\{[m_k^{L*}/(1-1/\sigma)](q_k^Y/q_j^Y)^*\}^{\sigma-1}(1-1/\sigma) w_k^{L*} L_k (n_{k0}^L \exp(\gamma^* t))^{-1}, \\
\pi_{jkt}^S &= [\tau_{jk}^S/(1-1/\sigma)]^{1-\sigma}\{[m_k^{S*}/(1-1/\sigma)](q_k^Y/q_j^Y)^*\}^{\sigma-1}(1-1/\sigma) w_k^{S*} S_k (n_{k0}^S \exp(\gamma^* t))^{-1} \\
\Rightarrow \pi_{jkt}^i &= \pi_{jk0}^i \exp(-\gamma^* t).
\end{aligned} \tag{78}$$

Using Eqs. (77), (78), and $\pi_{jk0}^i = e_{jk0}^i/\sigma$, $v_{jk0}^i = \int_0^\infty \pi_{jkt}^i \exp(-\int_0^t r_{js} ds) dt$ is rewritten as:

$$v_{jk0}^i = \pi_{jk0}^i/(\rho + \gamma^*) = e_{jk0}^i/[\sigma(\rho + \gamma^*)]. \tag{79}$$

Substituting Eq. (79) into $\kappa_j = \sum_k v_{jk0}^S / \sum_k v_{jk0}^L$, and using Eqs. (74), (75), and $E_{jk0}^i = n_{j0}^i e_{jk0}^i$, we obtain Eq. (13).

Derivation of Eq. (14)

From $q_j^Y Y_j = q_j^S D_j^S + q_j^L D_j^L, w_j^L L_j^D = (1/\sigma) q_j^L Y_j^L, w_j^S S_j^D = (1/\sigma) q_j^S Y_j^S, S_j = S_j^D, L_j = L_j^D$, and $Y_j^i = D_j^i$, we obtain:

$$
\begin{aligned}
w_j^L L_j &= (1/\sigma) q_j^L Y_j^L, \\
w_j^S S_j &= (1/\sigma) q_j^S Y_j^S, \\
w_j^S S_j + w_j^L L_j &= (1/\sigma)(q_j^S Y_j^S + q_j^L Y_j^L) = (1/\sigma) q_j^Y Y_j.
\end{aligned}
$$

Using this, $\beta_j = q_j^S D_j^S / (c_j^Y Y_j) = q_j^S Y_j^S / (q_j^Y Y_j)$ is rewritten as:

$$
\beta_j = w_j^S S_j / (w_j^S S_j + w_j^L L_j) = s_j \omega_j / (s_j \omega_j + 1) \Rightarrow 1 - \beta_j = 1/(s_j \omega_j + 1), \beta_j/(1-\beta_j) = s_j \omega_j. \quad (80)
$$

Using $\beta_j = \alpha^\varepsilon (q_j^S)^{1-\varepsilon} / [\alpha^\varepsilon (q_j^S)^{1-\varepsilon} + (1-\alpha)^\varepsilon (q_j^L)^{1-\varepsilon}]$, $s_j \omega_j = \beta_j/(1-\beta_j)$ in Eq. (80) is rewritten as $s_j \omega_j = [\alpha/(1-\alpha)]^\varepsilon (q_j^S / q_j^L)^{1-\varepsilon}$. Substituting Eq. (9) into this, s_j is solved as Eq. (14).

Derivation of Eq. (19)

Substituting $E_{jt} = E_j^*$ and $q_{jt}^Y = p_j^{e*}/n_{jt}^L = q_j^{Y*} \exp(-\gamma^* t); q_j^{Y*} \equiv q_{j0}^Y$ into $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho t) dt = \int_0^\infty (\ln E_{jt} - \ln q_{jt}^Y) \exp(-\rho t) dt$, and applying integration by parts, we obtain:

$$
\rho U_j = \ln E_j^* - \ln q_j^{Y*} + (1/\rho)\gamma^*.
$$

Eqs. (2) and (3) imply that:

$$
\begin{aligned}
\rho + \gamma^* &= (1 - 1/\sigma)[(s_j \omega_j^* + 1)/(\kappa_j \nu_j^* + 1)](L_j/\kappa_j^L)/(p_j^{e*}/w_j^{L*}) \\
\Leftrightarrow p_j^{e*} \kappa_j^L (\kappa_j \nu_j^* + 1) &= (1 - 1/\sigma) w_j^{L*} L_j (s_j \omega_j^* + 1)/(\rho + \gamma^*). \quad (81)
\end{aligned}
$$

Substituting Eq. (81) into Eq. (76) gives:

$$
E_j^* = w_j^{L*} L_j (s_j \omega_j^* + 1)\eta^*; \eta^* \equiv (1 - 1/\sigma)\rho/(\rho + \gamma^*) + 1 > 1.
$$

Using Eq. (17), $w_j^{L*}/q_j^{Y*} = w_j^{L*}/(p_j^{e*}/n_{j0}^L)$ is rewritten as $w_j^{L*}/q_j^{Y*} = n_{j0}^L [m_j^{L*}/(1-1/\sigma)]^{1-\sigma} (L_j/\kappa_j^L)^{-1} R_j^*$. Substituting this into the above expression for E_j^* divided by q_j^{Y*}, we obtain:

$$
\begin{aligned}
E_j^*/q_j^{Y*} &= (w_j^{L*}/q_j^{Y*}) L_j (s_j \omega_j^* + 1)\eta^* = K_j W_j^* \eta^*; \\
K_j &\equiv n_{j0}^L (1 - 1/\sigma)^{\sigma-1} \kappa_j^L, \\
W_j^* &\equiv (m_j^{L*})^{1-\sigma} R_j^* (s_j \omega_j^* + 1).
\end{aligned}
$$

Substituting this into $\rho U_j = \ln E_j^* - \ln q_j^{Y*} + (1/\rho)\gamma^*$, we obtain Eq. (19).

Derivations of Eqs. (21) and (22)

Logarithmically differentiating δ_j in Eq. (13), and using Eq. (8), give:

$$
\begin{aligned}
\widehat{\delta}_j =& \frac{\zeta_{jj}^S}{\zeta_{jj}^S + \zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)}(\sigma-1)\widehat{m}_j^S \\
&+ \frac{\zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)}{\zeta_{jj}^S + \zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)}\left[\frac{-\zeta_{kk}^S}{1-\zeta_{kk}^S}(\sigma-1)\widehat{m}_k^S + \widehat{w}_k^L - \widehat{w}_j^L + \widehat{\omega}_k - \widehat{\omega}_j\right] \\
&- \left\{\frac{\zeta_{jj}^L}{\zeta_{jj}^L + \zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)}(\sigma-1)\widehat{m}_j^L \right. \\
&\left. + \frac{\zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)}{\zeta_{jj}^L + \zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)}\left[\frac{-\zeta_{kk}^L}{1-\zeta_{kk}^L}(\sigma-1)\widehat{m}_k^L + \widehat{w}_k^L - \widehat{w}_j^L\right]\right\}.
\end{aligned}
$$

Noting from Eqs. (74) and (75) that:

$$
\begin{aligned}
E_{jj}^S + E_{jk}^S &= (\sigma-1)\zeta_{jj}^S w_j^S S_j + (\sigma-1)\zeta_{jk}^S w_k^S S_k \\
&= (\sigma-1) w_j^S S_j [\zeta_{jj}^S + \zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)], \\
E_{jj}^L + E_{jk}^L &= (\sigma-1)\zeta_{jj}^L w_j^L L_j + (\sigma-1)\zeta_{jk}^L w_k^L L_k \\
&= (\sigma-1) w_j^L L_j [\zeta_{jj}^L + \zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)], k \neq j,
\end{aligned}
$$

Country j's domestic revenue shares are given by:

$$
\lambda_{jj}^S = \frac{E_{jj}^S}{E_{jj}^S + E_{jk}^S} = \frac{\zeta_{jj}^S}{\zeta_{jj}^S + \zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)}, \tag{82}
$$

$$
\lambda_{jj}^L = \frac{E_{jj}^L}{E_{jj}^L + E_{jk}^L} = \frac{\zeta_{jj}^L}{\zeta_{jj}^L + \zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)}, k \neq j. \tag{83}
$$

At the symmetric BGP, they are simplified to:

$$
\begin{aligned}
\lambda_{jj}^S &= \frac{\zeta_{jj}^S}{\zeta_{jj}^S + \zeta_{jk}^S (L_k/L_j)(w_k^L/w_j^L)(s_k/s_j)(\omega_k/\omega_j)}, k \neq j \\
&= \frac{\zeta_{jj}^S}{\zeta_{jj}^S + \zeta_{kj}^S (L/L)(1/1)(s/s)(\omega/\omega)} \\
&= \zeta_{jj}^S, \\
\lambda_{jj}^L &= \frac{\zeta_{jj}^L}{\zeta_{jj}^L + \zeta_{jk}^L (L_k/L_j)(w_k^L/w_j^L)} \\
&= \frac{\zeta_{jj}^L}{\zeta_{jj}^L + \zeta_{kj}^L (L/L)(1/1)} \\
&= \zeta_{jj}^L.
\end{aligned}
$$

Using Eq. (8) and $m_j^i = (1+\tau^{1-\sigma})^{1/(1-\sigma)}$, $\lambda_{jj}^i = \zeta_{jj}^i$ is calculated as:

$$
\begin{aligned}
\lambda_{jj}^i &= \zeta_{jj}^i \\
&= (m_j^i)^{\sigma-1} \\
&= \frac{1}{1+\underbrace{\tau^{1-\sigma}}_{\in(0,1)}} \\
&\equiv \lambda \in (1/2, 1).
\end{aligned}
$$

Therefore, we obtain:

$$
\begin{aligned}
\lambda_{jj}^i &= \zeta_{jj}^i = 1/(1+\tau^{1-\sigma}) \equiv \lambda \in (1/2,1) \forall i \forall j, \\
\lambda_{jk}^i &= \zeta_{kj}^i = 1-\lambda \in (0,1/2) \forall i \forall j,k, k \neq j.
\end{aligned}
\tag{84}
$$

Then $\widehat{\delta}_j$ is simplified to:

$$
\begin{aligned}
\widehat{\delta}_j &= (\sigma-1)\lambda[(\widehat{m}_j^S - \widehat{m}_j^L) - (\widehat{m}_k^S - \widehat{m}_k^L)] - (1-\lambda)(\widehat{\omega}_j - \widehat{\omega}_k), k \neq j \\
\Rightarrow \widehat{\delta}_k &= (\sigma-1)\lambda[(\widehat{m}_k^S - \widehat{m}_k^L) - (\widehat{m}_j^S - \widehat{m}_j^L)] - (1-\lambda)(\widehat{\omega}_k - \widehat{\omega}_j) \\
&= -\widehat{\delta}_j.
\end{aligned}
\tag{85}
$$

Next, logarithmically differentiating Eq. (16) gives:

$$
\widehat{\omega}_j = (\psi-1)[(1-\sigma)(\widehat{m}_j^S - \widehat{m}_j^L) + \widehat{\delta}_j]. \tag{86}
$$

Eqs. (85) and (86) imply that $\widehat{\omega}_j - \widehat{\omega}_k = (\psi-1)\{(1-\sigma)[(\widehat{m}_j^S - \widehat{m}_j^L) - (\widehat{m}_k^S - \widehat{m}_k^L)] + 2\widehat{\delta}_j\}, k \neq j$. Substituting this into Eq. (85), and solving it for $\widehat{\delta}_j$, we obtain Eq. (21). $A > \lambda + (\psi-1)(1-\lambda)$ follows from $A - [\lambda + (\psi-1)(1-\lambda)] = \psi(1-\lambda) \in (0, \psi/2)$.

Finally, substituting Eq. (21) back into Eq. (86), and using $A - [\lambda + (\psi-1)(1-\lambda)] = \psi(1-\lambda)$, we obtain Eq. (22).

Derivation of (24)

Logarithmically differentiating R_j in Eq. (17) gives:

$$
\begin{aligned}
\widehat{R}_j = &\frac{1}{\psi-1} \frac{\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1} (m_j^S/m_j^L)^{(1-\sigma)(\psi-1)} \delta_j^{\psi-1}}{\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1} (m_j^S/m_j^L)^{(1-\sigma)(\psi-1)} \delta_j^{\psi-1} + (1-\alpha)^\varepsilon (L_j/\kappa_j^L)^{\psi-1}} \\
&\times [(1-\sigma)(\psi-1)(\widehat{m}_j^S - \widehat{m}_j^L) + (\psi-1)\widehat{\delta}_j].
\end{aligned}
$$

Substituting Eq. (16) into Eq. (80) gives:

$$
\beta_j = \frac{\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1} (m_j^S/m_j^L)^{(1-\sigma)(\psi-1)} \delta_j^{\psi-1}}{\alpha^\varepsilon (S_j/\kappa_j^S)^{\psi-1} (m_j^S/m_j^L)^{(1-\sigma)(\psi-1)} \delta_j^{\psi-1} + (1-\alpha)^\varepsilon (L_j/\kappa_j^L)^{\psi-1}}.
$$

This implies that:

$$
\widehat{R}_j = \beta_j [(1-\sigma)(\widehat{m}_j^S - \widehat{m}_j^L) + \widehat{\delta}_j].
$$

At the symmetric old BGP, β_j is simplified to:

$$\beta_j = \frac{\alpha^{\varepsilon}(S/\kappa)^{\psi-1}}{\alpha^{\varepsilon}(S/\kappa)^{\psi-1} + (1-\alpha)^{\varepsilon}(L/\kappa)^{\psi-1}} \equiv \beta \forall j.$$

Using Eq. (22) and $\beta_j = \beta$, the above expression for $\widehat{R}_j$ is rewritten as Eq. (24).

Derivations of Eqs. (29), (30), and (31)

From Eqs. (25) to (28), $\widehat{m}_1^S - \widehat{m}_1^L, \widehat{m}_2^S - \widehat{m}_2^L$, and $(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)$ are calculated as:

$$\widehat{m}_1^S - \widehat{m}_1^L = (1-\lambda)[\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L + (\widehat{\nu}_1 - \widehat{\nu}_2)/(\sigma-1)], \tag{87}$$

$$\widehat{m}_2^S - \widehat{m}_2^L = (1-\lambda)[\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L - (\widehat{\nu}_1 - \widehat{\nu}_2)/(\sigma-1)], \tag{88}$$

$$(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L) = (1-\lambda)[(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L) + 2(\widehat{\nu}_1 - \widehat{\nu}_2)/(\sigma-1)]. \tag{89}$$

Using Eq. (23) to calculate the difference $\widehat{\nu}_1 - \widehat{\nu}_2$, and noting that $A(\psi-1) - 2\psi[\lambda + (\psi-1)(1-\lambda)] = -A - \psi(2\lambda-1)$, we obtain:

$$\widehat{\nu}_1 - \widehat{\nu}_2 = (1/A)(\sigma-1)[A + \psi(2\lambda-1)][(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)]. \tag{90}$$

Substituting Eq. (90) into Eq. (89), and noting that $A - 2(1-\lambda)[A + \psi(2\lambda-1)] = (2\lambda-1)^2$, $(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)$ is solved as (29).

Substituting Eq. (29) back into Eq. (90) gives:

$$\widehat{\nu}_1 - \widehat{\nu}_2 = (\sigma-1)[A + \psi(2\lambda-1)][(1-\lambda)/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]. \tag{91}$$

Substituting Eq. (91) back into Eqs. (87) and (88), we obtain (30) and (31).

Derivation of Eq. (37)

Eq. (37) is derived in the following steps: (i) using the relative price of the final good $q_1^Y/q_2^Y = [(L_1/\kappa_1^L)/(L_2/\kappa_2^L)](w_1^L/\chi)(m_1^L/m_2^L)^{\sigma-1}R_2/R_1$ in subsection 3.6, we solve for $\widehat{q_1^Y/q_2^Y} = \widehat{q_1^Y/q_2^Y}(\widehat{w}_1^L, \widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$; (ii) substituting $\widehat{q_1^Y/q_2^Y} = \widehat{q_1^Y/q_2^Y}(\widehat{w}_1^L, \widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$ in step (i) into the logarithmically differentiated form of the balanced trade condition (6), we solve for $\widehat{w}_1^L = \widehat{w}_1^L(\widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$; (iii) substituting $\widehat{w}_1^L = \widehat{w}_1^L(\widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$ in step (ii) back into $\widehat{q_1^Y/q_2^Y} = \widehat{q_1^Y/q_2^Y}(\widehat{w}_1^L, \widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$, and substituting it into the differentiated form of the international balanced growth condition (2), together with country j's revised growth equation (36), we solve for $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$; (iv) substituting $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$ in step (iii) back into $\widehat{q_1^Y/q_2^Y} = \widehat{q_1^Y/q_2^Y}(\widehat{w}_1^L(\widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L), \widehat{\chi}, \widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$, and substituting it into Eq. (36) for $j = 2$, we solve for $d\gamma = d\gamma_2(\widehat{\tau}_{21}^S, \widehat{\tau}_{21}^L, \widehat{\tau}_{12}^S, \widehat{\tau}_{12}^L)$.

Step (i):

Subtracting Eq. (28) from Eq. (27) gives:

$$\widehat{m}_1^L - \widehat{m}_2^L = (1-\lambda)\{\widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L + [2/(\sigma-1)]\widehat{\chi} - 2(\widehat{q}_1^Y - \widehat{q}_2^Y)\}. \tag{92}$$

Substituting Eqs. (33) and (92) into the logarithmically differentiated form of $q_1^Y/q_2^Y = [(L_1/\kappa_1^L)/(L_2/\kappa_2^L)](w_1^L/\chi)(m_1^L/m_2^L)^{\sigma-1}R_2/R_1$, $\widehat{q}_1^Y - \widehat{q}_2^Y$ is solved as:

$$\widehat{q}_1^Y - \widehat{q}_2^Y = (1/B)[\widehat{w}_1^L - (2\lambda-1)\widehat{\chi} + (\sigma-1)(1-\lambda)C]; \tag{93}$$
$$B \equiv 1 + 2(\sigma-1)(1-\lambda) > 1,$$
$$C \equiv \widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L - [\beta/(2\lambda-1)][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)].$$

Step (ii):
Logarithmically differentiating Eq. (6), and using Eq. (8), give:

$$\frac{\zeta_{12}^S s_2\omega_2}{\zeta_{12}^S s_2\omega_2 + \zeta_{12}^L}\left[\frac{-\zeta_{22}^S}{1-\zeta_{22}^S}(\sigma-1)\widehat{m}_2^S + \widehat{\omega}_2\right] + \frac{\zeta_{12}^L}{\zeta_{12}^S s_2\omega_2 + \zeta_{12}^L}\left[\frac{-\zeta_{22}^L}{1-\zeta_{22}^L}(\sigma-1)\widehat{m}_2^L\right]$$
$$= \widehat{w}_1^L + \frac{\zeta_{21}^S s_1\omega_1}{\zeta_{21}^S s_1\omega_1 + \zeta_{21}^L}\left[\frac{-\zeta_{11}^S}{1-\zeta_{11}^S}(\sigma-1)\widehat{m}_1^S + \widehat{\omega}_1\right]$$
$$+ \frac{\zeta_{21}^L}{\zeta_{21}^S s_1\omega_1 + \zeta_{21}^L}\left[\frac{-\zeta_{11}^L}{1-\zeta_{11}^L}(\sigma-1)\widehat{m}_1^L\right].$$

Noting that $\zeta_{kj}^S s_j\omega_j/(\zeta_{kj}^S s_j\omega_j + \zeta_{kj}^L) = s_j\omega_j/(s_j\omega_j+1) = \beta\forall i\forall j,k,k\neq j$ at the symmetric old BGP from Eqs. (24), (80), and (84), and using Eq. (85), the above expression is simplified to:

$$0 = -\widehat{w}_1^L + [\beta/(1-\lambda)]\widehat{\delta}_1 + [\lambda/(1-\lambda)](\sigma-1)(\widehat{m}_1^L - \widehat{m}_2^L).$$

Substituting Eqs. (32), (92), and (93) into the above expression, we obtain:

$$0 = -(2\sigma-1)\widehat{w}_1^L + 2\sigma\lambda\widehat{\chi} + (\sigma-1)\{\lambda(\widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L) + [\beta F/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}$$
$$\Leftrightarrow \widehat{w}_1^L = [1/(2\sigma-1)]\{2\sigma\lambda\widehat{\chi} + (\sigma-1)\{\lambda(\widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L) + [\beta F/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}\}; \tag{94}$$
$$F \equiv B[\lambda + (\psi-1)(1-\lambda)] + 2(2\lambda-1)\lambda(\sigma-1)(1-\lambda) > 0.$$

Step (iii):
Substituting Eq. (94) back into Eq. (93) gives:

$$\widehat{q}_1^Y - \widehat{q}_2^Y = [1/(2\sigma-1)]\{\widehat{\chi} + (\sigma-1)\{\widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L + [\beta/(2\lambda-1)^2](J/B)[(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}\}; \tag{95}$$
$$J \equiv F - (2\lambda-1)(2\sigma-1)(1-\lambda).$$

Substituting Eq. (36) into the differentiated form of Eq. (2) gives:

$$0 = \beta[(\widehat{m}_1^S - \widehat{m}_1^L) - (\widehat{m}_2^S - \widehat{m}_2^L)] + \widehat{m}_1^L - \widehat{m}_2^L.$$

Using Eqs. (29), (92), and (95), the above expression is solved for $\widehat{\chi}$ as:

$$\widehat{\chi} = -[(\sigma-1)/(2\sigma B)]\{B(\widehat{\tau}_{21}^L - \widehat{\tau}_{12}^L) + [\beta L/(2\lambda-1)^2][(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}; \tag{96}$$
$$L \equiv (2\sigma-1)BA - 2(\sigma-1)J.$$

Step (iv):
Using Eqs. (95) and (96), Eq. (28) is rewritten as:

$$\widehat{m}_2^L = (1/2)(1-\lambda)\{\widehat{\tau}_{12}^L + \widehat{\tau}_{21}^L + [\beta/(2\lambda-1)^2]A[(\widehat{\tau}_{21}^S - \widehat{\tau}_{21}^L) - (\widehat{\tau}_{12}^S - \widehat{\tau}_{12}^L)]\}. \quad (97)$$

Substituting Eqs. (31) and (97) into Eq. (36) for $j=2$, and noting that $-2[A+\psi(2\lambda-1)](1-\lambda)+A = (2\lambda-1)^2 \Rightarrow -\beta[A+\psi(2\lambda-1)][(1-\lambda)/(2\lambda-1)^2] + (1/2)[\beta/(2\lambda-1)^2]A = \beta/2$, we obtain Eq. (37).

Derivation of the model in relative changes

In the definition of δ_j in Eq. (13), rewriting ζ_{jj}^i and $\zeta_{jk}^i, k \neq j$ using Eq. (8), and replacing all endogenous variables x with their new values $x' = x\widetilde{x}$, give:

$$\begin{aligned}\delta_j\widetilde{\delta}_j = &\{\zeta_{jj}^S(\widetilde{m}_j^S)^{\sigma-1} + (1-\zeta_{kk}^S(\widetilde{m}_k^S)^{\sigma-1})[(w_k^S S_k)/(w_j^S S_j)](\widetilde{w}_k^L/\widetilde{w}_j^L)(\widetilde{\omega}_k/\widetilde{\omega}_j)\}\\ &\div \{\zeta_{jj}^L(\widetilde{m}_j^L)^{\sigma-1} + (1-\zeta_{kk}^L(\widetilde{m}_k^L)^{\sigma-1})[(w_k^L L_k)/(w_j^L L_j)](\widetilde{w}_k^L/\widetilde{w}_j^L)\}.\end{aligned}$$

Eq. (80) implies that:

$$\begin{aligned}&(w_k^S S_k)/(w_j^S S_j) = (\beta_k/\beta_j)(1/y_j),\\ &(w_k^L L_k)/(w_j^L L_j) = [(1-\beta_k)/(1-\beta_j)](1/y_j); y_j \equiv (w_j^S S_j + w_j^L L_j)/(w_k^S S_k + w_k^L L_k), k \neq j. \quad (98)\end{aligned}$$

Using Eq. (98), the above expression for $\widetilde{\delta}_j$ is solved as Eq. (39).

Eqs. (40), (41), and (42) are obtained from Eqs. (16), (15), and (17), respectively, together with the general expression for β_j in Eq. (24).

Using $\widetilde{n}_1^L/\widetilde{n}_2^L = \widetilde{\chi}, \widetilde{n}_1^S/\widetilde{n}_2^S = (\widetilde{\nu}_1/\widetilde{\nu}_2)\widetilde{\chi}$, and $\zeta_{jj}^i = (m_j^i)^{\sigma-1} = 1/[1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}], k \neq j$, the relative change in $m_j^i = [1+(n_k^i/n_j^i)(\tau_{kj}^i q_k^Y/q_j^Y)^{1-\sigma}]^{1/(1-\sigma)}, k \neq j$ in Eq. (7) is calculated as Eqs. (43) to (46).

For Eq. (12), straightforward calculation using Eq. (17) gives Eq. (47).

Using Eq. (8), the relative change form of Eq. (6) is given by:

$$L_2[(1-\zeta_{22}^S(\widetilde{m}_2^S)^{\sigma-1})s_2\omega_2\widetilde{\omega}_2 + 1 - \zeta_{22}^L(\widetilde{m}_2^L)^{\sigma-1}] = w_1^L L_1 \widetilde{w}_1^L[(1-\zeta_{11}^S(\widetilde{m}_1^S)^{\sigma-1})s_1\omega_1\widetilde{\omega}_1 + 1 - \zeta_{11}^L(\widetilde{m}_1^L)^{\sigma-1}].$$

Dividing this by Eq. (6): $L_2[(1-\zeta_{22}^S)s_2\omega_2 + 1 - \zeta_{22}^L] = w_1^L L_1[(1-\zeta_{11}^S)s_1\omega_1 + 1 - \zeta_{11}^L]$, and using Eq. (80), $\widetilde{w}_1^L$ is solved as Eq. (48).

Eq. (49) is the relative change form of Eq. (2). Using Eq. (80), the relative change form of Eq. (18) is given by Eq. (50).

The relative change form of the general expression for β_j in Eq. (24) is given by:

$$\widetilde{\beta}_j = (\widetilde{m}_j^S/\widetilde{m}_j^L)^{(1-\sigma)(\psi-1)}\widetilde{\delta}_j^{\psi-1}/[\beta_j(\widetilde{m}_j^S/\widetilde{m}_j^L)^{(1-\sigma)(\psi-1)}\widetilde{\delta}_j^{\psi-1} + 1 - \beta_j].$$

Using Eq. (40), this is rewritten as Eq. (51). The relative change in ζ_{jj}^i is calculated from Eq. (8) as Eq. (52). Using Eq. (80), the relative change in y_1 in Eq. (98) is calculated as Eq. (53).

The relative changes in $W_j = (m_j^L)^{1-\sigma}R_j(s_j\omega_j + 1)$ and $\eta = (1-1/\sigma)\rho/(\rho+\gamma) + 1$ in Eq. (19) are calculated using Eq. (80) as, respectively:

$$\widetilde{W}_j = (\widetilde{m}_j^L)^{1-\sigma}\widetilde{R}_j(\beta_j\widetilde{\omega}_j + 1 - \beta_j),$$
$$\widetilde{\eta} = [(1-1/\sigma)\rho/(\rho+\gamma)+1]^{-1}[(1-1/\sigma)\rho/(\rho+\gamma\widetilde{\gamma})+1].$$

This implies Eq. (54).

.

	$\widetilde{\tau}_{21}^{S} = 0.9$	$\widetilde{\tau}_{21}^{S} = 1.1$	$\widetilde{\tau}_{21}^{L} = 0.9$	$\widetilde{\tau}_{21}^{L} = 1.1$	$\widetilde{\tau}_{12}^{S} = 0.9$	$\widetilde{\tau}_{12}^{S} = 1.1$	$\widetilde{\tau}_{12}^{L} = 0.9$	$\widetilde{\tau}_{12}^{L} = 1.1$
$\widetilde{\delta}_1$	0.952015	1.03766	1.0504	0.963711	1.05057	0.963632	0.951863	1.03774
$\widetilde{\delta}_2$	1.05057	0.963632	0.951863	1.03774	0.952015	1.03766	1.0504	0.963711
$\widetilde{\omega}_1$	0.996996	1.00205	1.00301	0.997956	1.0253	0.980606	0.975323	1.01978
$\widetilde{\omega}_2$	1.0253	0.980606	0.975323	1.01978	0.996996	1.00205	1.00301	0.997956
$\widetilde{\nu}_1$	0.949154	1.03978	1.05357	0.961741	1.07715	0.944944	0.928373	1.05826
$\widetilde{\nu}_2$	1.07715	0.944944	0.928373	1.05826	0.949154	1.03978	1.05357	0.961741
$\widetilde{R}_1$	0.997855	1.00146	1.00215	0.99854	1.01812	0.986176	0.98242	1.01416
$\widetilde{R}_2$	1.01812	0.986176	0.98242	1.01416	0.997855	1.00146	1.00215	0.99854
$\widetilde{m}_1^S$	0.988942	1.00817	1.00492	0.995992	0.999271	1.00041	0.994416	1.00365
$\widetilde{m}_2^S$	0.999271	1.00041	0.994416	1.00365	0.988942	1.00817	1.00492	0.995992
$\widetilde{m}_1^L$	1.00492	0.995992	0.988942	1.00817	0.994416	1.00365	0.999271	1.00041
$\widetilde{m}_2^L$	0.994416	1.00365	0.999271	1.00041	1.00492	0.995992	0.988942	1.00817
$\widetilde{q_1^Y/q_2^Y}$	0.979089	1.01631	0.979089	1.01631	1.02136	0.983952	1.02136	0.983952
$\widetilde{w}_1^L$	1.01704	0.991211	0.988959	1.01289	0.983248	1.00887	1.01116	0.987279
$\widetilde{\chi}$	1.0915	0.940026	0.961798	1.03437	0.916169	1.0638	1.03972	0.966772
$\widetilde{\beta}_1$	0.998496	1.00102	1.0015	0.998977	1.01249	0.990208	0.987507	1.00979
$\widetilde{\beta}_2$	1.01249	0.990208	0.987507	1.00979	0.998496	1.00102	1.0015	0.998977
$\widetilde{\zeta}_{11}^S$	0.969344	1.02306	1.01384	0.988817	0.997961	1.00115	0.984442	1.01026
$\widetilde{\zeta}_{22}^S$	0.997961	1.00115	0.984442	1.01026	0.969344	1.02306	1.01384	0.988817
$\widetilde{\zeta}_{11}^L$	1.01384	0.988817	0.969344	1.02306	0.984442	1.01026	0.997961	1.00115
$\widetilde{\zeta}_{22}^L$	0.984442	1.01026	0.997961	1.00115	1.01384	0.988817	0.969344	1.02306
$\widetilde{y}_1$	1.00282	1.00194	1.00282	1.00194	0.997185	0.998062	0.997185	0.998062
$\widetilde{\gamma}$	1.02518	0.982164	1.02518	0.982164	1.02518	0.982164	1.02518	0.982164
Δu_1	-0.007546	0.006783	0.044638	-0.032228	0.056057	-0.040827	-0.018265	0.015803
Δu_2	0.056057	-0.040827	-0.018265	0.015803	-0.007546	0.006783	0.044638	-0.032228

Note: $\rho = 0.02, \sigma = 3.8, \psi = 1.7, \beta_1 = \beta_2 = 0.5, \zeta_{11}^S = \zeta_{22}^S = \zeta_{11}^L = \zeta_{22}^L = 0.9, y_1 = 1, \gamma = 0.01.$

Table 1. Long-run effects of relative changes in τ_{jk}^i around a symmetric BGP: homogeneous firms.

	$\widetilde{\tau}_{21}^S = 0.9$	$\widetilde{\tau}_{21}^S = 1.1$	$\widetilde{\tau}_{21}^L = 0.9$	$\widetilde{\tau}_{21}^L = 1.1$	$\widetilde{\tau}_{12}^S = 0.9$	$\widetilde{\tau}_{12}^S = 1.1$	$\widetilde{\tau}_{12}^L = 0.9$	$\widetilde{\tau}_{12}^L = 1.1$
$\widetilde{\delta}_1$	0.971367	1.01948	0.997475	1.00191	1.05833	0.961876	0.923304	1.06782
$\widetilde{\delta}_2$	1.02495	0.983462	1.0052	0.997451	0.956014	1.0287	1.06274	0.944528
$\widetilde{\omega}_1$	0.992356	1.0051	0.971694	1.02365	1.03107	0.978487	0.938219	1.05372
$\widetilde{\omega}_2$	1.0075	0.994804	0.979333	1.01757	0.994555	1.00398	0.991106	1.00752
$\widetilde{\nu}_1$	0.963942	1.02468	0.969241	1.0256	1.09122	0.941183	0.866262	1.12519
$\widetilde{\nu}_2$	1.03265	0.978352	0.98443	1.01497	0.950808	1.03279	1.05329	0.951628
$\widetilde{R}_1$	0.99437	1.00376	0.979202	1.01748	1.02298	0.984181	0.954776	1.03983
$\widetilde{R}_2$	1.00745	0.994855	0.979583	1.01745	0.994609	1.00394	0.991198	1.00746
$\widetilde{m}_1^S$	0.995717	1.00281	1.00094	0.999118	1.00083	0.999335	0.995824	1.00261
$\widetilde{m}_2^S$	1.00043	0.999705	0.998799	1.00096	0.994199	1.00363	1.00359	0.996074
$\widetilde{m}_1^L$	1.00217	0.99851	0.987279	1.01042	0.996203	1.00212	0.991812	1.00588
$\widetilde{m}_2^L$	0.995449	1.00301	0.986383	1.01081	1.00749	0.995549	0.97755	1.02048
$\widetilde{q_1^Y/q_2^Y}$	0.99405	1.00403	0.960794	1.03637	1.00325	0.99724	1.03072	0.974195
$\widetilde{w}_1^L$	0.981062	1.01306	0.950626	1.05117	1.01225	0.985976	0.915003	1.0737
$\widetilde{\chi}$	1.01893	0.987538	0.992322	1.01314	0.950525	1.02732	0.959738	1.02562
$\widetilde{\beta}_1$	0.996285	1.00246	0.986096	1.01131	1.0148	0.98947	0.96911	1.0253
$\widetilde{\beta}_2$	1.00229	0.998402	0.993577	1.00532	0.998326	1.00122	0.997259	1.00229
$\widetilde{\zeta}_{11}^S$	0.988054	1.00788	1.00265	0.997532	1.00234	0.99814	0.988352	1.00732
$\widetilde{\zeta}_{22}^S$	1.00119	0.999175	0.99664	1.00269	0.983843	1.01019	1.01009	0.989047
$\widetilde{\zeta}_{11}^L$	1.00609	0.995835	0.964787	1.02944	0.989405	1.00596	0.977242	1.01656
$\widetilde{\zeta}_{22}^L$	0.987308	1.00844	0.962338	1.03057	1.02111	0.987587	0.938403	1.0584
$\widetilde{y}_1$	0.972131	1.0194	0.950368	1.05119	1.03237	0.972349	0.891335	1.09774
$\widetilde{\gamma}$	1.0035	0.997624	1.03211	0.975125	1.01466	0.989986	1.02169	0.983212
Δu_1	-0.013076	0.008798	0.023976	-0.017968	0.060156	-0.040465	-0.039546	0.037542
Δu_2	0.027975	-0.018939	0.027179	-0.019105	-0.019202	0.011764	0.064652	-0.056556

Note: $\rho = 0.02, \sigma = 3.8, \psi = 1.7, \beta_1 = 0.515948, \beta_2 = 0.693663, \zeta_{11}^S = 0.968899, \zeta_{22}^S = 0.963665, \zeta_{11}^L = 0.826309, \zeta_{22}^L = 0.64393, y_1 = 0.537046, \gamma = 0.0174589.$

Table 2. Long-run effects of relative changes in τ_{jk}^i around a factual BGP: homogeneous firms.

	$\widetilde{\tau}_{21}^S = 0.9$	$\widetilde{\tau}_{21}^S = 1.1$	$\widetilde{\tau}_{21}^L = 0.9$	$\widetilde{\tau}_{21}^L = 1.1$	$\widetilde{\tau}_{12}^S = 0.9$	$\widetilde{\tau}_{12}^S = 1.1$	$\widetilde{\tau}_{12}^L = 0.9$	$\widetilde{\tau}_{12}^L = 1.1$
$\widetilde{\delta}_1$	0.94169	1.04446	1.06192	0.957436	1.0622	0.957316	0.94144	1.04459
$\widetilde{\delta}_2$	1.0622	0.957316	0.94144	1.04459	0.94169	1.04446	1.06192	0.957436
$\widetilde{\omega}_1$	0.996892	1.00199	1.00312	0.998011	1.02569	0.980903	0.974955	1.01947
$\widetilde{\omega}_2$	1.02569	0.980903	0.974955	1.01947	0.996892	1.00199	1.00312	0.998011
$\widetilde{\nu}_1$	0.938763	1.04654	1.06523	0.955532	1.08949	0.939034	0.917861	1.06492
$\widetilde{\nu}_2$	1.08949	0.939034	0.917861	1.06492	0.938763	1.04654	1.06523	0.955532
$\widetilde{R}_1$	0.997781	1.00142	1.00223	0.99858	1.0184	0.986387	0.982159	1.01394
$\widetilde{R}_2$	1.0184	0.986387	0.982159	1.01394	0.997781	1.00142	1.00223	0.99858
$\widetilde{m}_1^S$	0.986912	1.00929	1.00672	0.994745	1.00099	0.999075	0.992403	1.00483
$\widetilde{m}_2^S$	1.00099	0.999075	0.992403	1.00483	0.986912	1.00929	1.00672	0.994745
$\widetilde{m}_1^L$	1.00672	0.994745	0.986912	1.00929	0.992403	1.00483	1.00099	0.999075
$\widetilde{m}_2^L$	0.992403	1.00483	1.00099	0.999075	1.00672	0.994745	0.986912	1.00929
$\widetilde{q_1^Y/q_2^Y}$	0.978748	1.01603	0.978748	1.01603	1.02171	0.98422	1.02171	0.98422
$\widetilde{w}_1^L$	1.0184	0.992037	0.989804	1.01337	0.981936	1.00803	1.0103	0.98681
$\widetilde{\chi}$	1.10546	0.934943	0.952524	1.04198	0.9046	1.06958	1.04984	0.959713
$\widetilde{\beta}_1$	0.998444	1.001	1.00156	0.999004	1.01268	0.990359	0.987318	1.00964
$\widetilde{\beta}_2$	1.01268	0.990359	0.987318	1.00964	0.998444	1.001	1.00156	0.999004
$\widetilde{\zeta}_{11}^S$	0.962474	1.02721	1.01657	0.986887	0.997066	1.00163	0.980901	1.01187
$\widetilde{\zeta}_{22}^S$	0.997066	1.00163	0.980901	1.01187	0.962474	1.02721	1.01657	0.986887
$\widetilde{\zeta}_{11}^L$	1.01657	0.986887	0.962474	1.02721	0.980901	1.01187	0.997066	1.00163
$\widetilde{\zeta}_{22}^L$	0.980901	1.01187	0.997066	1.00163	1.01657	0.986887	0.962474	1.02721
$\widetilde{y}_1$	1.00392	1.0026	1.00392	1.0026	0.996096	0.997408	0.996096	0.997408
$\widetilde{\gamma}$	1.02584	0.982562	1.02584	0.982562	1.02584	0.982562	1.02584	0.982562
Δu_1	-0.012421	0.010378	0.050772	-0.035110	0.062449	-0.043582	-0.023260	0.019322
Δu_2	0.062449	-0.043582	-0.023260	0.019322	-0.012421	0.010378	0.050772	-0.035110

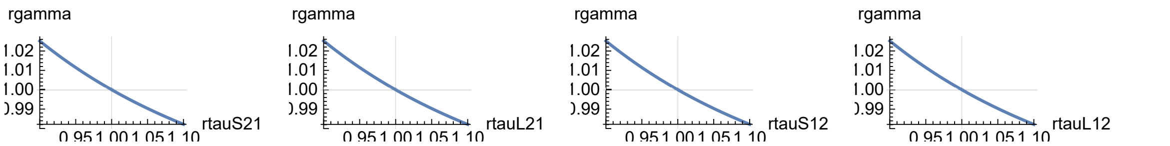
Note: $\rho = 0.02, \sigma = 3.8, \psi = 1.7, \theta = 3.4, \beta_1 = \beta_2 = 0.5, \zeta_{11}^S = \zeta_{22}^S = \zeta_{11}^L = \zeta_{22}^L = 0.9, y_1 = 1, \gamma = 0.01$.

Table 3. Long-run effects of relative changes in τ_{jk}^i around a symmetric BGP: heterogeneous firms.

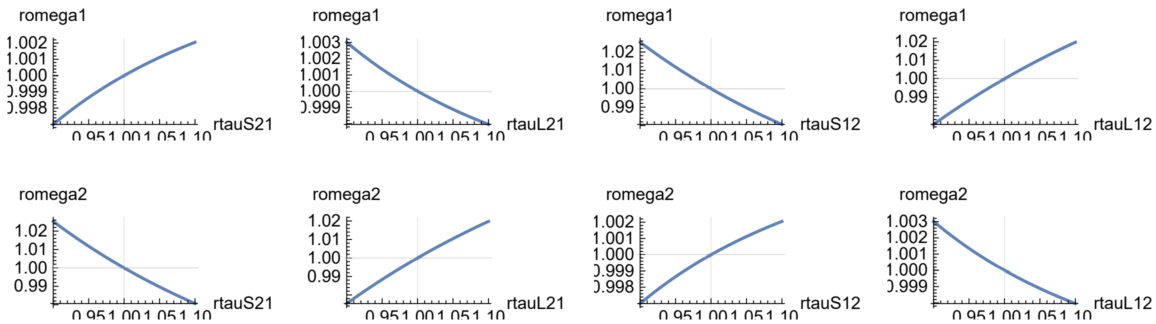
	$\widetilde{\tau}_{21}^S = 0.9$	$\widetilde{\tau}_{21}^S = 1.1$	$\widetilde{\tau}_{21}^L = 0.9$	$\widetilde{\tau}_{21}^L = 1.1$	$\widetilde{\tau}_{12}^S = 0.9$	$\widetilde{\tau}_{12}^S = 1.1$	$\widetilde{\tau}_{12}^L = 0.9$	$\widetilde{\tau}_{12}^L = 1.1$
$\widetilde{\delta}_1$	0.961667	1.02409	0.987377	1.00902	1.0708	0.957256	0.893667	1.08547
$\widetilde{\delta}_2$	1.02971	0.981287	1.00231	1.00131	0.945174	1.03449	1.06728	0.935595
$\widetilde{\omega}_1$	0.990454	1.00586	0.965152	1.02801	1.03023	0.980281	0.929728	1.05644
$\widetilde{\omega}_2$	1.00866	0.994354	0.983149	1.01465	0.995633	1.00315	0.995511	1.00511
$\widetilde{\nu}_1$	0.952488	1.03009	0.952969	1.03728	1.10317	0.93838	0.830868	1.14674
$\widetilde{\nu}_2$	1.03863	0.975747	0.98542	1.01598	0.941047	1.03775	1.06249	0.940379
$\widetilde{R}_1$	0.992972	1.00432	0.974414	1.02071	1.02235	0.985498	0.94861	1.04186
$\widetilde{R}_2$	1.00859	0.994409	0.983343	1.01455	0.995675	1.00312	0.995554	1.00507
$\widetilde{m}_1^S$	0.99458	1.00325	1.00193	0.998437	1.00332	0.997817	0.993059	1.00393
$\widetilde{m}_2^S$	1.00099	0.999353	0.999183	1.00079	0.993098	1.00412	1.00489	0.994577
$\widetilde{m}_1^L$	1.00364	0.997724	0.98844	1.00936	0.994095	1.00326	0.996018	1.00265
$\widetilde{m}_2^L$	0.994942	1.00322	0.989741	1.00777	1.01104	0.993623	0.979535	1.02116
$\widetilde{q_1^Y/q_2^Y}$	0.992372	1.00478	0.955948	1.04153	1.00145	0.998418	1.02482	0.97824
$\widetilde{w}_1^L$	0.958433	1.02655	0.877303	1.1282	0.993011	0.996098	0.826911	1.14425
$\widetilde{\chi}$	1.00519	0.996163	0.922736	1.08145	0.921059	1.04334	0.887323	1.07204
$\widetilde{\beta}_1$	0.995357	1.00283	0.982823	1.01336	1.01441	0.990357	0.964705	1.02655
$\widetilde{\beta}_2$	1.00264	0.998263	0.994777	1.00444	0.998658	1.00096	0.99862	1.00156
$\widetilde{\zeta}_{11}^S$	0.984393	1.00943	1.00149	0.998473	1.00215	0.998326	0.982441	1.00921
$\widetilde{\zeta}_{22}^S$	1.0019	0.998777	0.998181	1.00171	0.980869	1.01153	1.01344	0.985128
$\widetilde{\zeta}_{11}^L$	1.00564	0.996316	0.94772	1.04073	0.98437	1.00823	0.961362	1.02465
$\widetilde{\zeta}_{22}^L$	0.988249	1.00748	0.970318	1.02423	1.03074	0.98274	0.945487	1.06062
$\widetilde{y}_1$	0.948018	1.0337	0.871719	1.13299	1.01156	0.983815	0.79942	1.17341
$\widetilde{\gamma}$	1.00149	0.998932	1.02398	0.981141	1.0135	0.990991	1.01147	0.988999
Δu_1	-0.021052	0.012917	0.006298	-0.005217	0.064184	-0.040623	-0.070005	0.054164
Δu_2	0.029848	-0.019311	0.018123	-0.011090	-0.028092	0.016535	0.058832	-0.058185

Note: $\rho = 0.02, \sigma = 3.8, \psi = 1.7, \theta = 3.4, \beta_1 = 0.515948, \beta_2 = 0.693663, \zeta_{11}^S = 0.968899, \zeta_{22}^S = 0.963665, \zeta_{11}^L = 0.826309, \zeta_{22}^L = 0.64393, y_1 = 0.537046, \gamma = 0.0174589$.

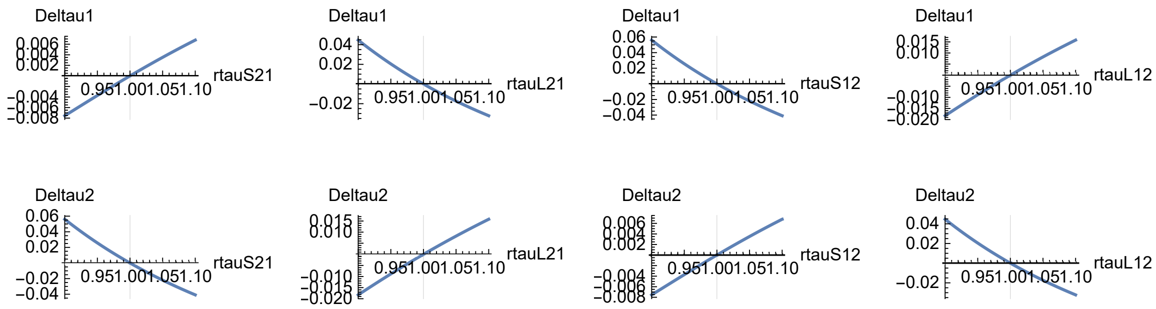
Table 4. Long-run effects of relative changes in τ_{jk}^i around a factual BGP: heterogeneous firms.



(a) relative changes in the balanced growth rate

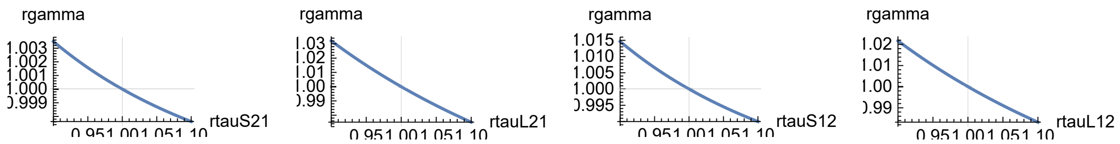


(b) relative changes in countries' skill premiums

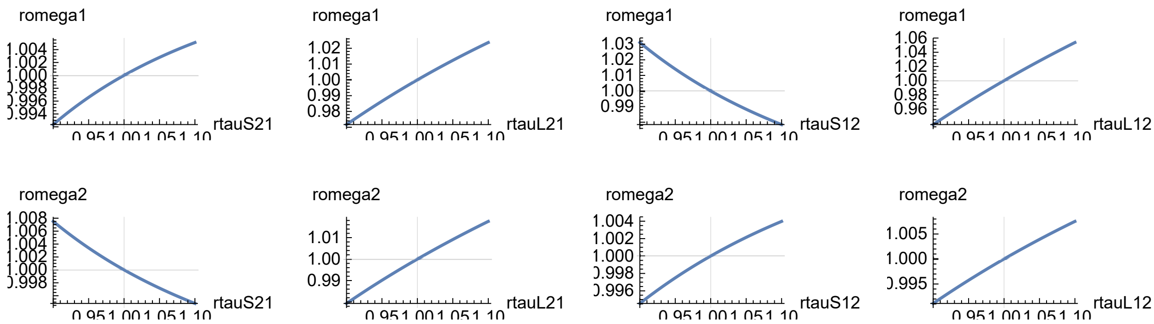


(c) changes in countries' long-run welfare in flow terms

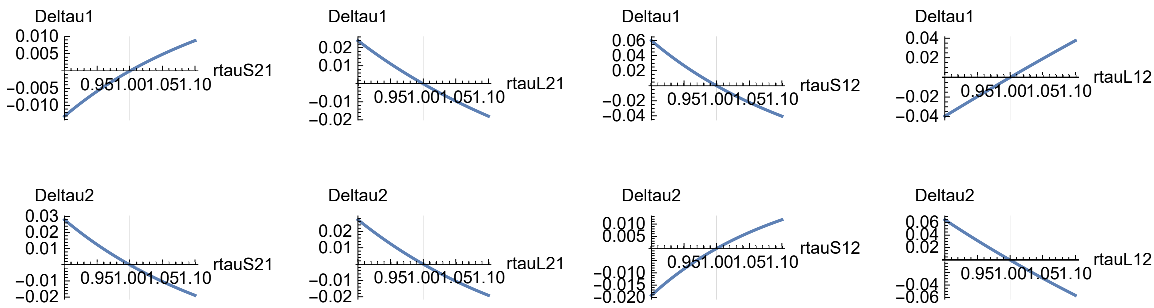
Fig. 1. Long-run effects of relative changes in τ_{jk}^{i} around a symmetric BGP: homogeneous firms.



(a) relative changes in the balanced growth rate

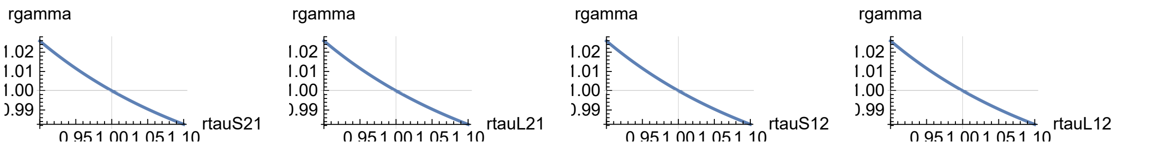


(b) relative changes in countries' skill premiums

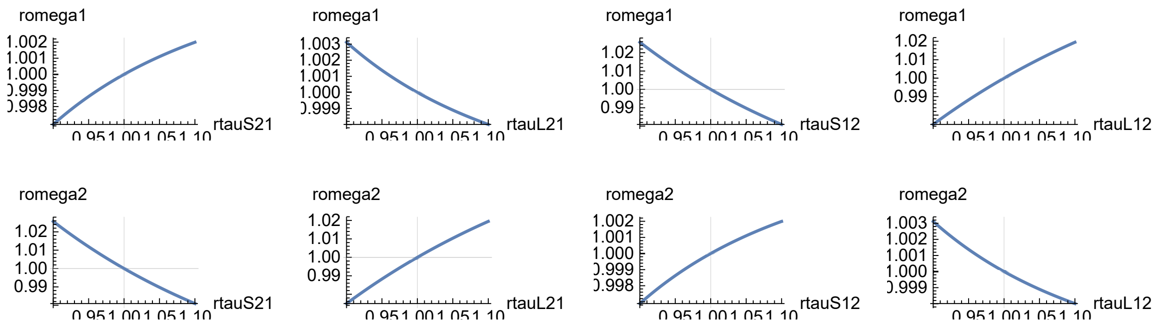


(c) changes in countries' long-run welfare in flow terms

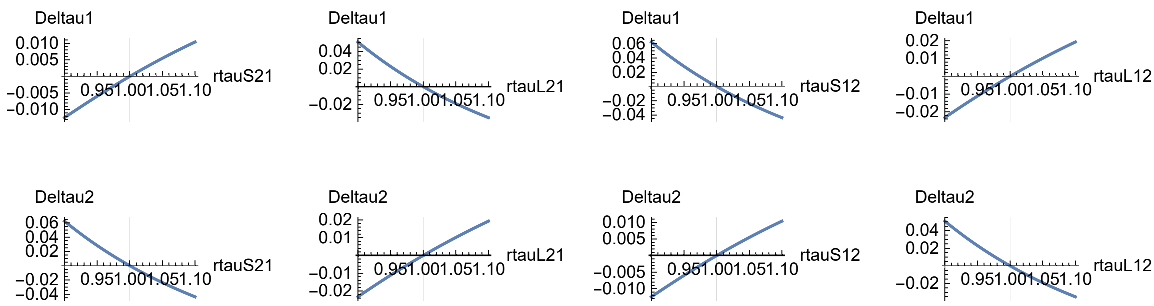
Fig. 2. Long-run effects of relative changes in τ^i_{jk} around a factual BGP: homogeneous firms.



(a) relative changes in the balanced growth rate

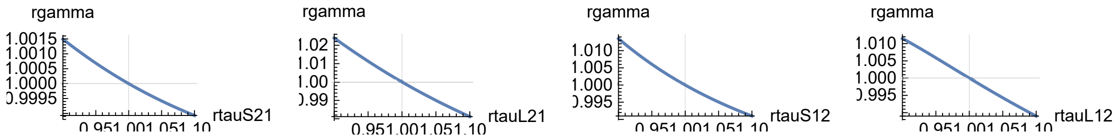


(b) relative changes in countries' skill premiums

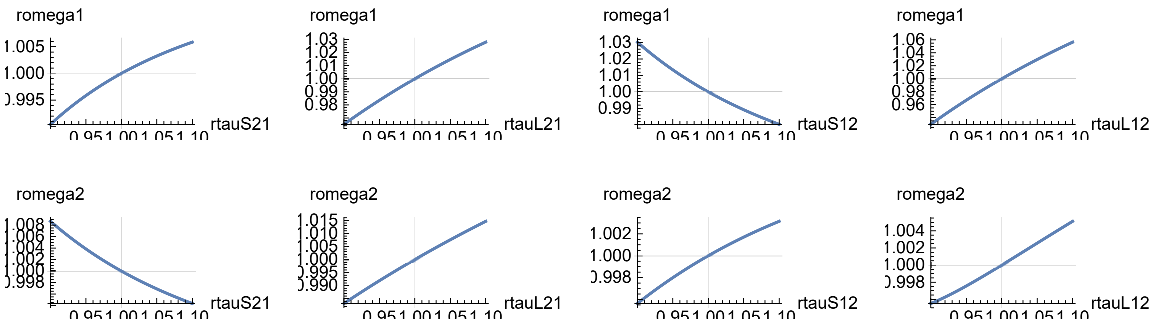


(c) changes in countries' long-run welfare in flow terms

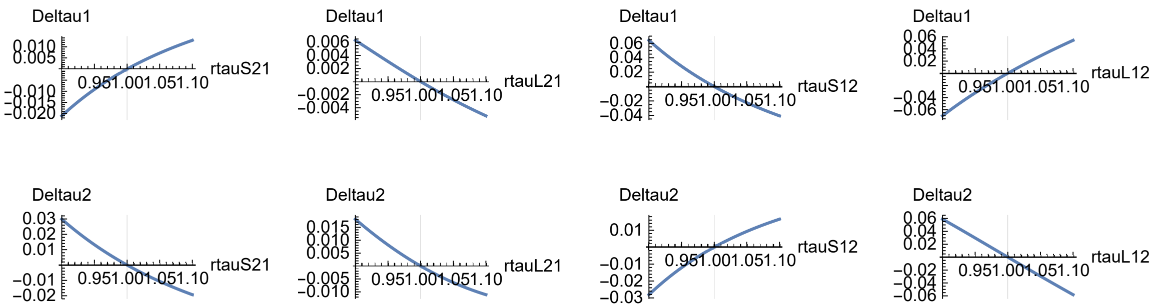
Fig. 3. Long-run effects of relative changes in τ^i_{jk} around a symmetric BGP: heterogeneous firms.



(a) relative changes in the balanced growth rate



(b) relative changes in countries' skill premiums



(c) changes in countries' long-run welfare in flow terms

Fig. 4. Long-run effects of relative changes in τ^i_{jk} around a factual BGP: heterogeneous firms.