Coasian Dynamics in Sequential Search*

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Abstract

Consumer-tracking technology offers new tools for price discrimination in digital markets. We examine the impact of sellers using this technology to adjust prices according to a buyer's prior search length in a competitive search market where buyers differ in their patience. We find a "Coasian equilibrium" wherein sellers reduce prices for buyers with longer search lengths which in turn requires them to reduce prices for buyers with shorter search lengths. There is a Coasian equilibrium that not only yields higher welfare for every buyer than all uniform-pricing equilibria, but is also the unique symmetric equilibrium when some mass of buyers are arbitrarily patient.

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1 Introduction

The shift toward a digital economy has given sellers a greater ability to target buyers with personalized prices and a richer set of consumer data for doing so. One important but largely unexplored dimension for price discrimination online is to adjust prices according to how long a buyer has been searching for the good.

Several aspects of online markets enable this type of price discrimination. For one, the internet features tools like cookies that makes tracking a buyer's past shopping experience easier than it would be offline (Zuiderveen Borgesius and Poort, 2017). At the same time, steady methodological and computational advances have expanded the capacity for sellers to analyze big datasets.[1] Moreover, people are increasingly browsing the internet using mobile phones (rather than computers), where the browsers' default is to allow tracking.[2]

In this paper, we ask whether sellers have an incentive to perform price discrimination based on the length of a buyer's search history. And if so, does this form of price discrimination adversely affect the welfare of buyers? By investigating these questions, we contribute to the broader inquiry exploring online privacy and the effects of large-scale data collection in digital markets.[3] We examine these questions in a competitive market in which buyers sequentially search sellers for a low price. Buyers are distinguished by their *search types* which measure their patience or willingness to continue searching; for example, reflecting their search costs or discount factors.

We find that there are equilibria in which sellers engage in price discrimination based on search duration and that this can actually improve the welfare of all buyers. Our results are driven by two countervailing effects. The first effect could potentially lead to higher overall prices. By cutting the price for buyers who have searched longer, sellers are able to split up buyer types and target the least patient buyers with higher prices. A countervailing effect could, instead, lead to lower overall prices. The promise of lower future prices makes continuing search more desirable. As a result, sellers must also charge lower prices to buyers with shorter search lengths so that they make a purchase and do not continue their search. When considering the net effect, the strategic complementarity inherent to the pricing decisions causes the second effect to dominate and each buyer's welfare increases.

Our result is reminiscent of the celebrated conjecture posed by Ronald Coase regarding

[1] Bourreau and de Streel (2018), Ipsos et al. (2018), Ennis and Lam (2021).

[2] In the EU, 16% of people used a mobile phone to access the internet in 2011 and 71% in 2019 (Eurostat, 2021). Private browsing is the default on neither iPhones' nor Android phones' default browsers in 2022.

[3] See Fuller (2019) for a recent summary of the literature.

a durable good monopolist that sells over time to buyers who vary in their valuation for the good (Coase, 1972). The conjecture posits that the monopolist is compelled to lower current prices because of the pressure it will face at later points in time to charge even lower prices when only the buyers who value the good the least remain in the market. Similarly, sellers in our equilibrium with price discrimination are compelled to lower current prices because of the pressure they face to charge even lower prices when only the buyers who are the least likely to stop searching remain in the market. Our result, thus, demonstrates that price discrimination in digital markets introduces novel leading price discrimination to benefit consumers.

Drawing on this analogy, we call equilibria with a decreasing sequence of prices "Coasian equilibria". In a Coasian equilibrium, buyers with the lowest search types (i.e. most impatient) buy after searching once, buyers with somewhat higher search types buy after searching twice, and so on. We show that only Coasian equilibria exist in addition to uniform-pricing equilibria (Theorems 1 and 2) and that buyers benefit in a Coasian equilibrium as compared to all uniform-pricing equilibria (Theorem 3). Not only is a Coasian equilibrium beneficial to buyers, but such Coasian equilibrium is also the unique equilibrium if the search type distribution admits a mass point of buyers that are infinitely patient (e.g., with zero search costs – so-called "shoppers"; Theorem 4).

We motivate the model with a consumer-search example, but our setup is a general sequential search setting. In particular, the search type is a general ordering of buyers such that continuing to search is less costly for a buyer with a higher search type. The search type embeds the most commonly used ways of measuring cost of search: additive search costs and discounting. Also the instantaneous utility that a buyer gets from transacting immediately is more general than the usual linear utility assumed in consumer search. Our setup also accommodates, for example, log and other concave utility functions. Thus, the model's applications include not only consumer search, but also searching for jobs and financial assets. In these settings, the search length can be inferred from the CV of a job-seeker if the search is for a job and from past transaction records if the search is for a financial asset.

A rich literature has emerged identifying settings where the Coase conjecture holds and where it fails.[4] In this vein, our finding sharply contrasts the result of Board and Pycia (2014) which demonstrates the failure of the Coase conjecture in a search model where

[4] See Nava and Schiraldi (2019) for a discussion of the literature and a general framework for understanding Coasian dynamics for a durable good monopolist.

buyers' tastes vary independently across sellers. The reason behind this discrepancy is that we allow buyers to differ in their propensity to search, i.e., search types, and so equilibria in our model are able to witness deteriorating market conditions from one period to the next. In contrast, by allowing values to vary independently across sellers, the setup in Board and Pycia (2014) introduces stability in sellers' pricing decisions. Our baseline model captures the novel effect of buyer heterogeneity on price dynamics in the simplest possible, yet general, framework, leading to Coasian dynamics in a competitive environment.

Our paper joins an active literature that investigates the various ways price discrimination can emerge in a market with search frictions.[5] This literature has so far mostly ignored price discrimination based on search duration. One exception is a recent working paper by Groh and Preuss (2022) which studies a duopoly version of the Wolinsky (1986) search model. Groh and Preuss (2022) demonstrate that a seller may optimally disclose to its competitor that it was visited by a buyer and the competitor raises the price it charges that buyer later on (so prices increase in search length). A literature also considers price discrimination based on whether a visitor is a first-time or returning customer.[6]

Our paper also relates to recent work studying price discrimination that results from greater access to consumer data in digital markets. Albrecht (2020) demonstrates how, in a market without search frictions in which firms engage in Bertrand competition, better information can increase competition and benefit buyers. Elliott et al. (2021) takes an information design approach to characterize the optimal information structure for competing sellers and also shows that sufficient information can benefit buyers through increased competition.

Our model of price discrimination based on search duration can alternatively be interpreted as introducing dynamic pricing into a sequential search model. From this perspective, we also contribute to the relatively small literature studying truly dynamic search markets, i.e., search markets where market conditions change over time.[7]

We describe the model in Section 2. We tackle equilibrium existence in our model and describe the equilibria in Section 3. The welfare analysis of discriminatory pricing and Coasian dynamics is in Section 4. In Section 5, we show that in a slightly modified model,

[5]See, for example, Fabra and Reguant (2020), Bergemann et al. (2021), Preuss (2021), Ronayne and Taylor (2022), and Mauring (2022).

[6]See, for example, Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003), and Armstrong and Zhou (2016).

[7]Some papers in this literature focus on cyclical equilibria (Janssen and Karamychev, 2002; Mauring, 2020)[8] while others study the sale of goods with an expiration date (Garcia, 2022; Garcia et al., 2022).

a unique Coasian equilibrium exists. Section 6 concludes. The proofs missing from the main text are in the Appendix.

2 Model

A unit mass of sellers each produce an identical good at a constant marginal cost (normalized to zero) and a unit mass of buyers each wish to purchase at most one unit of the good. Buyers search sequentially and randomly for a low price and have free recall. When a buyer visits a seller, the seller first detects how long the buyer has been engaged in search and then offers a price. Our main interpretation is that sellers have access to a tracking technology (e.g., via cookies) and can infer a buyer's shopping history so that p_n^i is the price seller i charges a buyer who has already visited the sites of $n-1$ other sellers.[9] Seller i's strategy thus comprises a sequence of prices $p^i = (p_1^i, p_2^i, \dots)$.

Each buyer receives her first price quote without delay and the payoff for each additional price quote depends on her *search type* $t \in [0,1]$, capturing her degree of patience. Search types are independently and identically distributed according to a differentiable density f. At each step of the search process, the utility obtained by a buyer with search type t from performing m more searches and buying the good at price p is given by the continuously differentiable utility function $U(p,t,m)$. Utility is strictly decreasing in price, strictly increasing in search type when $m \geq 1$, and, for all imperfectly patient types $t < 1$, strictly decreasing in additional search length. Letting $\bar{u}(p) \equiv U(p,t,0)$ be the utility from purchasing the good immediately at price p, we assume that there is a price $v > 0$ satisfying $\bar{u}(v) = 0$.

We solve for symmetric perfect Bayesian equilibria where buyers hold passive beliefs. Passive beliefs mean that if buyers observe a price deviation by a seller, they assume all other sellers continue to follow the equilibrium strategy thereafter.

2.1 Examples

The following are a few examples of possible search types that we later use to demonstrate our results. We mainly focus on the two most commonly used definitions of search types that are used in the sequential search literature.

[9] Alternatively, in a model where all buyers enter the market in the same period, we could take the more traditional "temporal" interpretation so that p_n^i is the price seller i charges in time period n.

Example 1. (Additive search costs)**.** Within the consumer search literature, the classic approach to capture the idea that search is costly in terms of time and effort is to assume that a buyer incurs an additive search cost s to obtain a price quote. Our framework can be seen to embed additive search costs as a special case by relating the search costs and search types by $s = 1 - t$ and expressing utility as

$$U(p, t, m) = \bar{u}(p) - m(1 - t).$$

Example 2. (Exponential discounting)**.** Within the labor literature, the most common way to model costly search is through exponential discounting. For discounting, we define the discount factor δ so that $\delta = t$ and

$$U(p, t, m) = t^m \bar{u}(p).$$

2.2 Adjacent Supermodularity

Our main results hold when the game satisfies a natural strategic complementarity condition we call “adjacent supermodularity". The condition is a weakening of the classic notion of supermodularity.[10] Intuitively, the condition is satisfied if raising other sellers' prices in a manner that diminishes the utility from continued search increases a seller's marginal profit. Formally, letting $\tau : \mathbb{R}^2 \to \mathbb{R}$ denote the function satisfying $\bar{u}(p) = U(p', \tau(p, p'), 1)$, we refer to the game as **adjacent supermodular** if the function $\pi(p, p') \equiv p \cdot F(\tau(p, p'))$ is supermodular; that is, $\frac{\partial^2 \pi(p,p')}{\partial p' \partial p} \geq 0$.

We illustrate adjacent supermodularity in our running examples.

Example 1.a. (Additive search costs and linear utility)**.** Let $U(p, t, m) = v - p - m(1 - t)$ so that $\tau(p, p') = 1 - p + p'$. The game is adjacent supermodular if

$$\frac{\partial^2 \pi(p, p')}{\partial p' \partial p} = f(\tau) - p f'(\tau) \geq 0,$$

where, for brevity, we denote $\tau := \tau(p, p')$. This condition holds clearly, e.g., for uniform $F(t)$ and all concave $F(t)$.

Example 2.a. (Exponential discounting and linear utility)**.** Let $U(p, t, m) = t^m(v - p)$ so that $\tau(p, p') = \frac{v-p}{v-p'}$. The game is adjacent supermodular if

$$\frac{\partial^2 \pi(p, p')}{\partial p' \partial p} = (v - p')^{-2}[f(\tau)(v - 2p) - p f'(\tau)\tau] \geq 0.$$

[10]See Vives (2005) for an in depth discussion of supermodularity.

Sufficient conditions for this condition to hold are that $f'(t) < 0$ and $-f'(t)t \geq f(t)$. For example, these hold for the family of gamma distributions normalised to $(0, 1]$, $F(t) = \frac{G(t;k,\theta)}{G(1;k,\theta)}$ where $G(t; k, \theta)$ is the standard gamma distribution with scale and shape parameters k and θ respectively, if $k < 1$ and $\theta < 1$. The condition can also be satisfied for other concave $F(t)$ and holds whenever the types are uniformly distributed as long as v is sufficiently large.

Example 2.b. (Exponential discounting and log utility)**.** Let $U(p, t, m) = t^m \ln(w - p)$ where, for brevity, we let $w := 1 + v$. Then $\tau(p, p') = [\ln(w - p)][\ln(w - p')]^{-1}$. The game is adjacent supermodular if

$$\frac{\partial^2 \pi(p, p')}{\partial p' \partial p} = (w - p')^{-1}[\ln(w - p')]^{-2} \left\{ f(\tau) \ln(w - p) - \frac{p}{w - p} [f'(\tau)\tau + f(\tau)] \right\} \geq 0.$$

Sufficient conditions for this condition to hold are that $f'(t) < 0$ and $-f'(t)t \geq f(t)$, which are the same as the sufficient conditions in Example 2.a. Thus, the condition holds for, e.g., the gamma distribution normalised to $(0, 1]$ if its scale and shape parameters are below one and can be satisfied for other concave $F(t)$.

Example 2.c. (Exponential discounting and root utility)**.** Let $U(p, t, m) = t^m (v - p)^{1/k}$ for some $k > 1$ and assume t^k distributed uniformly on $[0, 1]$. Then the results in examples 1.a. and 2.a. on uniform distributions go through directly.

Adjacent supermodularity is often satisfied if the buyers' search type distribution is concave. This is intuitive as a concave F implies relatively more mass of buyers with low search types. If there are many such buyers, then firms have incentives to compete for them, which implies that the firms' prices are strategic complements.

3 Equilibrium existence

Classic results on supermodular games ensure that a pure-strategy equilibrium exists in a supermodular game (Vives, 2005). We show here that the only equilibria in our model are uniform-pricing, Coasian, or are payoff-equivalent to one of these two.[11] A *uniform-pricing equilibrium* is one in which sellers perform no price discrimination, charging each buyer the same price regardless of their prior search length. In a Coasian equilibrium, defined below, the prices decrease in the buyers' search length for all search lengths such that some buyers are still searching.

[11]Notice that for every equilibrium in which all buyers halt their search within n steps of search, if all firms raise their prices off the equilibrium path, then the strategy profile remains an equilibrium.

Lemma 1. *A uniform-pricing equilibrium exists.*

It is worth noting that there may be multiple uniform pricing equilibria. We know from Stahl (1996) that, if the search type is an additive search cost and F is logconcave, then a uniform-pricing equilibrium exists for all prices $p \in [1/f(1), v]$. Thus, the uniform-pricing equilibria may or may not be unique, depending on the nature of the search types and their distribution.

In all uniform-pricing equilibria with active search, buyers buy at the first seller that they visit. This is because searching beyond the first seller brings no extra benefit if buyers expect to face the same price at all the other sellers as at the first one. We will use this fact to derive the welfare comparison between the uniform-pricing and other equilibria of the model.

Now we turn to the more interesting equilibria and show that the only equilibria with active price discrimination that exist are such that prices decrease in the buyers' search length. We make the following definitions.

Definition 1 (Discriminatory equilibria)**.** We say that an equilibrium is a *discriminatory equilibrium* if it is not a uniform-pricing equilibrium. That is, along the equilibrium path in a discriminatory equilibrium, some buyers pay a different price than others.

Definition 2 (Coasian equilibria)**.** We say that an equilibrium is a *(strict) Coasian equilibrium* if it is a discriminatory equilibrium where the prices paid along the equilibrium path weakly (strictly) decrease in the buyers' active search length and strictly for at least some search lengths.

Theorem 1 (All discriminatory equilibria that exist are Coasian)**.** *The only discriminatory equilibria that exist are Coasian.*

For the basic idea behind this result; naturally, for any buyers to be willing to prolong their search in a discriminatory equilibrium, it must be that some price quotes later in the sequence lie below the first price buyers face. Furthermore, if a price is seen with positive probability and it is at least as large as any price earlier in the sequence, then it must deliver no sales and thus a seller can increase profit by reducing this price to some level that ensures a chance of a sale. As a result, prices on the equilibrium path are strictly decreasing in search length.

Finally, we show that Coasian equilibria exist if our game is adjacent supermodular.

Theorem 2 (Existence of Coasian equilibria)**.** *Coasian equilibria exist if the game is adjacent supermodular.*

The result follows from first translating our game into a "related game", identifying that this related game is supermodular and hence admits equilibria, and then showing that these equilibria correspond to the Coasian equilibria of our original game. Notably, there are many Coasian equilibria. In particular, there are Coasian equilibria in which the maximal search length is n for every $n \in \mathbb{N}$ and also equilibria in which there is no maximal search length.

4 Buyers' welfare is higher in Coasian than uniform-pricing equilibria

We now ask how buyers fare in Coasian equilibria. Relative to uniform pricing, in the Coasian equilibria sellers can price discriminate by charging higher prices to buyers with lower search types. At the same time, the cascading sequence of future prices makes continuing search more desirable to buyers, applying downward pressure on earlier prices. When considering the net effect, we obtain the following welfare comparison. In the following result, we say that an equilibrium *delivers buyers higher utility* if it yields a weakly higher payoff for all buyers and a strictly higher for some.

Theorem 3. *If the game is adjacent supermodular, then for every uniform pricing equilibrium there is a Coasian equilibrium delivering buyers higher utility.*

Given the strategic complementarity provided by the adjacent supermodularity condition, starting from any uniform pricing equilibrium, we can construct a corresponding Coasian equilibrium in which all buyers are weakly better off, strictly so for all buyers who obtain at least two price quotes. As supermodular games admit a "smallest" equilibrium, there likewise exists a Coasian equilibrium with the lowest prices, implying the following corollary.

Corollary 1. *If the game is adjacent supermodular, then there is a Coasian equilibrium delivering buyers higher utility than every uniform pricing equilibrium.*

4.1 Example of a Coasian equilibrium

To make the equilibrium construction and the potential magnitude of the welfare gains more concrete, let us return to the environment with additive search costs and quasilinear utility environment (Example 1.a.). While Lemma 1 guarantees the existence of a uniform pricing equilibrium, Stahl (1996) explicitly characterizes the set of equilibria for this environment under the usual assumption that the search type distribution is logconcave.

Proposition 1 (Stahl (1996))**.** *In the environment with an additive search cost and quasi-linear utility, if the search type distribution is logconcave, then a strategy profile is a symmetric equilibrium under uniform pricing if and only if all sellers charge a price p satisfying* $\min\{1/f(1), v\} \leq p \leq v$.

In this environment, we can sharpen the welfare conclusion of Theorem 3 to say that every Coasian equilibrium leads to higher buyer surplus than every uniform pricing equilibrium. What is important for this result is that the optimal price to charge a buyer of search length n is not only increasing in the price other sellers charge a buyer of search length $n + 1$, but it naturally increases at a rate less than one. As a result, the declining sequence of prices in a Coasian equilibrium must all be lower than any price sustaining a uniform pricing equilibrium.

Proposition 2. *In the environment with an additive search cost and quasilinear utility, if the search type distribution is logconcave, then every Coasian equilibrium delivers buyers higher utility than every uniform pricing equilibrium.*

To illustrate these results, suppose the search cost s is uniformly distributed between zero and one and that $v \geq 1$. In a strict Coasian equilibrium wherein a positive fraction of buyers purchase the good at the nth search for each $n \in \mathbb{N}$, we can partition the set $(0, 1) = \cup_{n=1}^{\infty}[s_n, s_{n-1})$ so that a buyer with search cost $s \in [s_n, s_{n-1})$ buys after n searches. The buyer with search cost $s = s_n$ is made indifferent between buying after the nth search and continuing once more, implying $p_n - p_{n+1} = s_n$. Thus, using the recursive definition of the prices, we have $p_n = \sum_{k=0}^{\infty} s_{n+k}$ for all n.

Using this recurrence and equation (5) which characterizes optimal prices, we get that in the Coasian equilibrium we must have

$$\sum_{k=0}^{\infty} s_{n+k} = s_{n-1} - s_n \tag{1}$$

$$\implies s_n = s_{n-1} - 2s_n + s_{n+1}. \tag{2}$$

Figure 1: Coasian equilibrium partitioning of the search costs $\cup_{n=1}^{\infty}[s_n, s_{n-1}]$ for linear utility and additive search costs s with $s \sim U[0,1]$.

which simplifies to $s_n = \frac{s_{n-1}+s_{n+1}}{3}$ with the boundary condition $s_0 = 1$. We solve the recurrence by solving its characteristic equation, getting $s_n = \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^n$. Figure 1 illustrates the implied partitioning of the search costs. Figure 1 demonstrates that the fraction of buyers who buy is decreasing in their search length: the intervals become shorter as we move from the end of the search costs' support (i.e., right of the interval) towards the start (i.e., leftwards). This feature of the partitioning relies on the fact that the search costs are uniformly distributed.

Prices are given by $p_n = \frac{2\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^n}{\sqrt{5}-1}$. Figure 2 illustrates prices in the Coasian equilibrium (red dots) and in the uniform-pricing equilibrium with the lowest price (black squares). The figure illustrates the general result in Theorem 3: buyers are better off in the Coasian equilibrium than in the uniform-pricing equilibria because all prices are lower in the Coasian equilibrium than in the uniform-pricing equilibrium with the lowest price.

Figure 2 shows that, as a function of the length of search, prices are convex. The reason behind the convexity is that, at each length of search n, buyers with the search cost $s = s_n$ are just indifferent between stopping and searching once more, i.e., $p_n - p_{n+1} = s_n$. Since only buyers with search costs lower than s_n are willing to search longer than n sellers, the next price difference, $p_{n+1} - p_{n+2}$, must be smaller than s_n. In other words, $p_n - p_{n+1} > p_{n+1} - p_{n+2}$ all along the sequence of the equilibrium prices. Intuitively, a buyer with a lower search cost is more willing to continue searching than a buyer with a higher search cost. Thus, to induce some buyers to buy, sellers must decrease future prices at a slower pace when faced with buyers with a lower average search costs (i.e., later along the search length) as opposed to a higher average search costs (i.e., earlier along the search length).

Figure 2 also demonstrates that buyers can get a substantial proportion of the total surplus in Coasian equilibria compared to the uniform-pricing equilibrium. If $v = 1$, buyers get zero surplus in the unique uniform-pricing equilibrium. Conversely, since the first price in the depicted Coasian equilibrium is $p_1 \approx 0.618$, each buyer gets at least 32% of the surplus in the Coasian equilibrium.

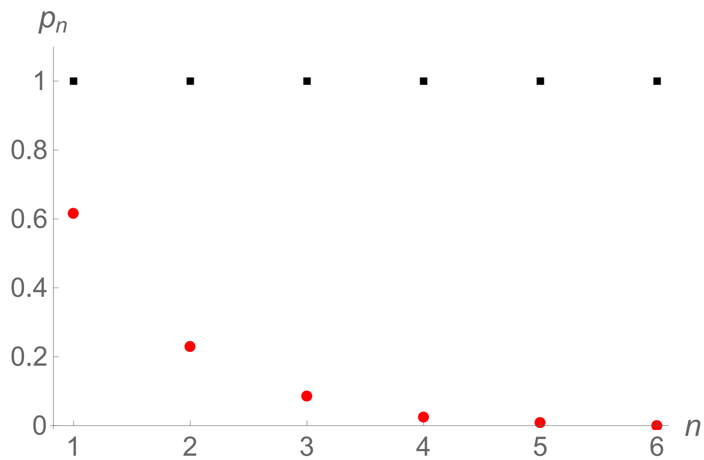

Figure 2: Price charged for a given search duration n in the Coasian equilibrium (red dots) and the uniform-pricing equilibrium with the lowest price (black squares) for $s \sim U[0, 1]$.

5 The Unique Equilibrium with Shoppers is Coasian

In the previous sections, we showed that both the Coasian and uniform-pricing equilibria coexist when sellers have the ability to engage in price discrimination. The key structural difference between the two is that a Coasian equilibrium inovlves a fraction of buyers obtaining multiple price quotes while a uniform pricing equilibrium requires buyers to almost surely accept the first price quote they see.

However, the uniform pricing equilibrium necessarily excludes a mass of buyers who derive enjoyment from shopping and become fully informed about prices before deciding which good to buy. Stahl (1989) and Stahl (1996) show that when a mass of buyers have zero search cost and always break indifference by continuing to search (called "shoppers"), then when there are finitely many sellers, the equilibrium involves price dispersion. As the number of sellers grows large, the equilibrium price distribution converges to a unit mass on the smallest price in the interval described in Proposition 1. In other words, under uniform pricing, introducing shoppers leads prices to be more competitive when there are few sellers, but the effect vanishes as the number of sellers grows large.

We find that when sellers can perform discriminatory pricing, the presence of shoppers ensures that uniform pricing can no longer be sustained as an equilibrium. Specifically, a shopper is a buyer who is perfectly patient and follows the tie-breaking rule of continuing to search when indifferent between continuing and stopping. One additional technical assumption of the game is that if a buyer receives infinitely many price quotes, then they have the option to pay the infimum of these prices. This ensures that a buyer's best reply is always well-defined.

Theorem 4. *When buyers comprise a positive fraction of both shoppers and non-shoppers, then a Coasian equilibrium exists and is the unique symmetric equilibrium.*

For the intuition, observe that if all sellers charge a constant sequence of positive prices, all nonshoppers will buy the good at the first seller they visit, just as before. Therefore, when a seller detects that a buyer has already performed prior searches elsewhere, it knows that the buyer is a shopper and the seller can increase its profit by deviating to a lower price. Thus, the presence of shoppers leads sellers to cut prices later in the sequence, drawing patient nonshoppers to prolong their search, producing Coasian dynamics.

6 Conclusion

This paper demonstrates that Coasian dynamics can arise in a competitive frictional market. If buyers differ in their patience and sellers can price discriminate based on search length, then buyers benefit in a Coasian equilibrium as compared to uniform pricing. In the Coasian equilibrium, the least patient buyers optimally stop searching first and pay the highest prices. Buyers who are somewhat more patient stop searching next and pay a somewhat lower price, and so on. However, the ability of sellers to thus price discriminate translates into lower profits than under uniform pricing. This is because all buyers benefit from the expectation of future lower prices so all discriminatory prices are below the uniform price. In other words, sellers are trapped in a version of Coasian dynamics in the Coasian equilibrium.

The Coasian equilibrium has an intuitive structure: people more prone to searching in fact search longer and end up paying lower prices. No buyer has an incentive to pretend to have searched less than he really has as prices decrease in search length. In other words, if buyers could reset their search length by deleting cookies, they would have no incentive to do so. The Coasian equilibrium is also the unique symmetric equilibrium in an extended model where some buyers enjoy shopping. Our model, thus, demonstrates the force of Coasian dynamics in frictional markets, in contrast to Board and Pycia (2014).

Our model's results contribute to the broader inquiry that explores online privacy and the effects of large-scale data collection in digital markets. In particular, we show that novel opportunities that sellers have in digital markets, such as using advanced tracking technologies, may backfire for them in equilibrium. Thus, revenue managers may have an incentive to use the information they have about buyers sparingly despite sellers' ever-improving abilities to collect and analyse consumer data.[12]

The paper can be extended in several dimensions. First, allowing for downward-sloping demand would even more starkly demonstrate the extent to which buyers benefit in the Coasian equilibrium as compared to uniform pricing. Second, buyers could search for match values in addition to low prices. If the match values are independent across sellers, the force of Coasian dynamics would still lead to decreasing prices that the Coasian equilibrium features. Although the qualitative effect of this extension on prices is clear, the exact welfare results may be of interest. Finally, price discrimination based on the extent of search more generally may be of both empirical relevance and theoretical interest. Search length, depth, breadth and intensity are all indications of buyers' willingness to search, investigate, compare, and buy products, and, thus, offer other potential bases for price discrimination.

A Appendix

Proof of Lemma 1. We argue that a uniform-pricing equilibrium where $p = v$ always exists. In such a proposed equilibrium, all buyers buy at the first seller they search. Searching further than the first seller brings no benefits to a buyer because all sellers set the same prices in the proposed equilibrium. Thus, each seller expects to serve mass one of buyers. Suppose that all sellers other than i price at $p = v$ and i contemplates a deviation. If i deviates to a higher price $p' > v$, no buyer that visits i first buys from it. Also, no buyer reaches i after visiting another seller j first because all of those buyers buy at j. Thus, deviating to $p' > v$ is not profitable for i. Suppose, instead, that i deviates to some $p' < v$. This is not profitable either because, first, $p' < v$ generates less revenue than $p = v$. Second, $p' < v$ attracts no additional demand because, again, all buyer that visit another seller j first also buy at j. Thus, setting $p^i = v$ is a best response of seller i to others' setting $p = v$. In sum, a uniform-pricing equilibrium where $p = v$ always exists. □

Proof of Theorem 1. Suppose otherwise: that an equilibrium exists where prices strictly increase in the search length for some search lengths, i.e., that $p_m > p_l$ for some $m > l$. We argue that a seller wants to deviate from these prices. The only scenario we must consider is that some buyers continue searching m sellers or beyond. Continuing searching exactly

[12] Bourreau and de Streel (2018), Ipsos et al. (2018), Ennis and Lam (2021).

until the mth seller cannot be optimal for any buyer because the buyer could have paid a lower price by stopping earlier and also engaged in less of the costly search. Thus, it must be that all buyers that search at least m sellers continue beyond the mth seller. But then a seller has a profitable deviation from p_m to a lower price that induces at least some buyers to buy after m searches. This is because, by shortening all buyers' expected search length by one search, the seller can capture as its revenue some of the search costs that the buyers save. The exact details of doing this depend on which buyers are supposed to stop after searching at least m times, but it is always possible. □

Proof of Theorem 2. The proof is in three steps. In Step 1, we transform our game into a related game. In Step 2, we show that this related game is supermodular if we consider strict Coasian equilibria. The existence of a strict Coasian equilibrium in the related game follows from standard results on supermodular games. In Step 3, we show that our original game is adjacent supermodular if we consider strict Coasian equilibria and the related game is supermodular. Thus, the strict Coasian equilibria of the related game remain equilibria in our original game if the latter satisfies adjacent supermodularity.

Step 1: Let us first modify the game we consider. Instead of assuming that each seller faces buyers of all search lengths, assume for now that each seller faces only buyers of a particular single search length. For concreteness, refer to the seller that can sell to buyers of search length n the n-seller. We show that this so-called related game is supermodular if we consider Coasian equilibria.

Step 2: In this related game, consider a hypothetical equilibrium where prices strictly decrease in buyers' search length and either converge to zero as the search length converges to infinity or jump to a price above v after some finite search length N, i.e., a strict Coasian equilibrium. We derive the conditions under which a strict Coasian equilibrium is consistent with supermodularity.

To simplify notation, denote a buyer of type t's utility from performing one more search and buying the good at the price of p by $u(p, t) \equiv U(p, t, 1)$. Suppose that a buyer t prefers to buy in the nth step of search at price p_n rather than continue one step more. This implies that $\bar{u}(p_n) \geq u(p_{n+1}, t)$ and thus, for all $t' < t$, $\bar{u}(p_n) > u(p_{n+1}, t')$: also buyers with a lower search type prefer to buy latest in the nth step of search.

We can therefore partition the set $(0, 1) = \cup_{n=1}^{\infty} (t_{n-1}, t_n]$ so that a buyer with search type $t \in (t_{n-1}, t_n]$ buys from the n-seller after n searches. The buyer with search type $t = t_n$ is indifferent between buying after the nth search and continuing once more, implying $\bar{u}(p_n) = u(p_{n+1}, t_n)$, where p_{n+1} is the price that a buyer expects to pay when stopping after $n+1$ searches. Thus, $\tau(p_n, p_{n+1}) = t_n$. Since u strictly increases in t, we have that $\bar{u}(p) \geq u(y,t)$ is equivalent to $t \leq \tau(p,y)$. As $u_1 < 0$ and $u_2 > 0$, we have $\tau_1 < 0$ and $\tau_2 > 0$.

When the n-seller considers which price p to charge, it knows that the buyers with the lowest search type have already bought and exited the market. The distribution of search types among buyers who remain in the market is $[F(t) - F(t_{n-1})]/[1 - F(t_{n-1})]$ for all search types above t_{n-1} and zero otherwise. The total amount of buyers left in the market is $1 - F(t_{n-1})$. Raising the price p has the effect of lowering the marginal buyer type who is willing to make a purchase at the n-seller. In a neighborhood of the equilibrium price, the marginal type's outside option is to continue searching once more and make a purchase. Thus, near the equilibrium price, the n-seller makes a sale to all buyers for whom $\bar{u}(p) \geq u(p_{n+1}, t)$, or $t \leq \tau(p, p_{n+1})$, yielding profit

$$\pi_n(p) = p\left[F\left(\tau(p, p_{n+1})\right) - F(t_{n-1})\right]. \tag{3}$$

The profit of n-seller, $\pi_n(p)$, only depends on other sellers' prices via the price p_{n+1} and, via t_{n-1}, on p_{n-1}. From n-seller's perspective, it cannot affect the price that buyers with search length $n-1$ expect to pay in the next step of search via the choice of p because n-seller's choice takes place after buyers of search length $n-1$ have decided whether to continue searching or not. In other words, the expected price p_n that appears in t_{n-1} is not directly affected by n-seller's choice p (but in equilibrium, of course, $p = p_n$).

Differentiating $\pi_n(p)$, we obtain the first order condition for n-seller's optimal price:

$$F\left(\tau(p, p_{n+1})\right) - F(t_{n-1}) + pf\left(\tau(p, p_{n+1})\right)\frac{\partial \tau(p, p_{n+1})}{\partial p} = 0. \tag{4}$$

Setting $p = p_n$, the condition becomes $F\left(t_n\right) - F(t_{n-1}) + p_n f(t_n)\frac{\partial t_n}{\partial p_n} = 0$ so that the strict Coasian equilibrium prices must satisfy

$$-p_n\frac{\partial t_n}{\partial p_n} = \frac{F\left(t_n\right) - F(t_{n-1})}{f(t_n)}. \tag{5}$$

Let the FOC be denoted by $H = 0$ where

$$H := F\left(\tau(p, p_{n+1})\right) - F(t_{n-1}) + pf(\tau(p, p_{n+1}))\frac{\partial \tau(p, p_{n+1})}{\partial p}.$$

Thus, to show that the related game is supermodular, we need to derive conditions under which $\frac{\partial H}{\partial p_{n-1}} \geq 0$ and $\frac{\partial H}{\partial p_{n+1}} \geq 0$.

We get that

$$\frac{\partial H}{\partial p_{n-1}} = -\frac{\partial t_{n-1}}{\partial p_{n-1}} f(t_{n-1}), \tag{6}$$

and

$$\frac{\partial H}{\partial p_{n+1}} = \frac{\partial \tau(p, p_{n+1})}{\partial p_{n+1}} \left[f(\tau) + p f'(\tau) \frac{\partial \tau(p, p_{n+1})}{\partial p} \right] + p f(\tau) \frac{\partial^2 \tau(p, p_{n+1})}{\partial p \partial p_{n+1}}, \tag{7}$$

where $\tau := \tau(p, p_{n+1})$ for brevity. Since $\frac{\partial t_{n-1}}{\partial p_{n-1}} \propto \tau_1 < 0$, the condition $\frac{\partial H}{\partial p_{n-1}} \geq 0$ is always satisfied. Thus, the necessary and sufficient condition for the related game to be smooth supermodular is that $\frac{\partial H}{\partial p_{n+1}} \geq 0$. This condition is satisfied by adjacent supermodularity. Thus, the related game is supermodular. We know from classic results on supermodular games that supermodularity is a sufficeint condition for a PSE, here, a Coasian pure-strategy equilibrium, to exist (Vives, 2005).

Step 3: Now return to our original game where each seller faces buyers of all active search lengths. We want to show that the strict Coasian equilibrium as constructed in Step 2 remains an equilibrium in our original game. Recall that, in that equilibrium, buyers with a lower search type buy and exit the market earlier and the prices satisfy the FOC (4). Now we must consider a seller's problem when setting a price p to buyers of search length n. A seller i makes a sale to buyers whose search type exceeds t_{n-1}, who visit seller i, and for whom $\bar{u}(p) \geq u(p_{n+1}, t)$, or $t \leq \tau(p, p_{n+1})$, yielding profit

$$\pi_n^i(p) = p \left[F\left(\tau(p, p_{n+1})\right) - F(t_{n-1}) \right]. \tag{8}$$

Note that the price p_n that appears in t_{n-1} is price that buyers of search length $n-1$ expect to pay if they stop at the next search step. This price is determined in equilibrium by all sellers' choices, but is not a direct choice of seller i when it decides on the price p to set to buyers of search length n. The FOC that seller i's optimal choice satisfies is, thus, exactly the same condition as we derived in the related problem, equation (4) or $H = 0$. By assumption, our original game satisfies adjacent supermodularity so $\frac{\partial H}{\partial p_{n+1}} \geq 0$. In other words, the strict Coasian equilibria derived in Step 2 remain equilibria in our original game if the original game satisfies adjacent supermodularity. □

Proof of Theorem 3. Let p^* correspond to the price charged in a uniform pricing equilibrium. Consider the related game from before, except now restrict the price charged by Seller 1 to lie in $[0, p^*]$. From the adjacent supermodularity, there exists a Coasian equilibrium of this game, in particular there exists a smallest equilibrium. As the equilibrium p_1^{**} lies below p^*, Seller 2's best reply p_2^{**} must likewise lie below p^*.

Removing the restriction on Seller 1's price, if the other sellers play according to the Coasian equilibrium identified above, Seller 1 likewise would desire to do so. To see why, let $P_1(p_2)$ denote Seller 1's best replies as a correspondence of Seller 2's price. Because there is a uniform pricing equilibrium at price p^*, we have $p^* \in P_1(p^*)$. From adjacent supermodularity, we also know Seller 1's smallest best reply is monotone increasing in Seller 2's price, hence, $\min P_1(p_2^{**}) \leq \min P_1(p^*) \leq p^*$. But this means that Seller 1 has a best reply in $[0, p^*]$, hence, the Coasian equilibrium with the price restriction continues to be an equilibrium without the price restriction. In this Coasian equilibrium each buyer does weakly better and some strictly better than in the uniform pricing equilibrium. □

Proof of Proposition 2. Notice that if the only uniform pricing equilibrium involves firms charging v in every period, then the conclusion follows immediately because the first price in any Coasian equilibrium is at most v and each subsequent price is strictly less than this. Now suppose there exists an interior uniform pricing equilibrium in which all firms charge $1/f(1) \leq v$. In a Coasian equilibrium, given that all other firms charge p_2 on the second search, a firm's objective is to set $\tilde{p}_1$ to maximize $\tilde{p}_1 \cdot F(1 - \tilde{p}_1 + p_2)$. An interior optimum satisfies

$$p_1^* = \frac{F(1 - p_1^* + p_2)}{f(1 - p_1^* + p_2)}.$$

Denoting $t_1 \equiv 1 - p_1^* + p_2$ we thus have $p_1^* = \frac{F(t_1)}{f(t_1)}$. By the logconcavity of F, we know that $\frac{F(x)}{f(x)}$ is strictly increasing in x. In a Coasian equilibrium, $t_1 < 1$ while in the buyers' preferred uniform pricing equilibrium, all buyer's buy at the price $p = \frac{F(1)}{f(1)}$. Thus, relative to the uniform pricing equilibrium, each buyer obtains a strictly higher payoff from buying at the first seller inspected in the Coasian equilibrium, and obtains a higher payoff still from playing their equilibrium strategy. □

Proof of Theorem 4. To connect this result more closely with Stahl (1996), we prove the claim in the more general setting in which sellers are permitted to set a sequence of mixtures over prices, $\{\mu_n^i\}_{n=1}^{\infty}$.

Firstly, it is clear that the Coasian equilibrium in which a fraction of buyers search every search length constructed in Theorem 2 continues to exist when a fraction of buyers are shoppers. If all other sellers play their Coasian strategy, then a seller cannot obtain positive profit from any shoppers as the infinimum of their observed prices is zero and thus the seller exhibits the same best replies as in the game without shoppers.

Secondly, to show that the Coasian equilibrium is the unique existing symmetric equilibrium, suppose $\{\mu_n\}_{n=1}^{\infty}$ are the price distributions in a symmetric equilibrium. It cannot be that the infimum of the lowest price in the support of the price distributions, $\inf_n \min(\operatorname{supp}(\mu_n))$, is strictly positive. Otherwise, a seller would have an incentive to charge a lower price at some point along the sequence when the fraction of nonshoppers remaining in the market is small and a positive profit could be made from all visiting shoppers. This implies that the expected profit from shoppers is zero and so a seller's optimal pricing only depends on nonshoppers. It also cannot be that the price of zero lies in the support of any price distribution μ_n for any n, otherwise a positive fraction of nonshoppers would perform enough searches for a positive chance to obtain the price in the neighborhood of zero and thus a seller could increase profit by charging a small but positive price. Taken together, since $\min(\operatorname{supp} \mu_n) > 0$ for all n and $\inf_n \min(\operatorname{supp} \mu_n) = 0$, a positive fraction of nonshoppers perform n searches for every n. It cannot be that a seller generates zero sales when being the nth search for any n as there are prices that can obtain positive profit from nonshoppers. But then, following the construction of Theorem 2 provides that the equilibrium is in fact the Coasian equilibrium. □

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