Which Side are You On?

Interest Groups and Relational Contracts

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Abstract

This paper studies *quid-pro-quo* dynamic agreements between an interest group and two political parties. Political parties repeatedly compete for office. Before each election, the interest group decides which party to support. When in power, parties choose the rent they transfer to the interest group to buy its support. Yet, binding agreements are not possible, so agreements must be self-enforcing. When political fluctuations are mild, the interest group favors an opportunistic agreement in which it always supports the current incumbent. As political fluctuations increase, the interest group prefers an exclusive agreement in which it supports a single party even when it is in opposition. An interest group with more inefficient rents is also less likely to favor an opportunistic agreement. The model offers a novel explanation for why studies on the impact of campaign contributions on policymaking find mixed evidence. Besides, my results shed new light on existing empirical findings by showing that interest groups' long-term loyalty does not necessarily imply an ideological alignment, and interest groups' opportunism can be a sign of high-quality institutions. Lastly, I study the impact of crisis periods, weak political parties, and the interest group's entry costs on the interest-group-optimal agreement.

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1 Introduction

Many interest groups try to influence policymaking through *quid-pro-quo* relationships with political parties. However, not all interest groups behave the same as they face the most basic fact of politics: power changes hands in a frequent and not fully predictable way. In particular, not all interest groups remain loyal to a political party after an electoral defeat; for example, firms from heavily regulated sectors often support the party currently in power (Fouirnaies and Hall, 2014). Other interest groups maintain long-term relationships with a single party, as was the case of the monopolistic conglomerates (*zaibatsu*) of Imperial Japan. Similarly, Buonanno et al. (2016) show that the Italian Mafia supported Silvio Berlusconi's Forza Italia between 1994 and 2013, even during the eight years it was out of power. Among the Political Action Committees (PACs) in the US House of Representatives, heterogeneity is the norm: whereas 45 percent of them concentrate at least 75 percent of their contributions on a party, the remaining share tends to support the majority party's incumbents (Chamond and Kaplan, 2013).[1] Two questions naturally arise: How does an interest group optimally allocate its electoral support across parties over time? Why do different interests groups follow different strategies?

This paper explores *quid-pro-quo* agreements between political parties and special interests in a setting where power changes hands stochastically and binding contracts are not possible. This modeling strategy captures the idea that these agreements are seldom public and lack third-party enforcement. Consequently, they are *relational contracts*, that is, they require relational incentives—the credible threat of a cooperation break-up—to be sustained. I show that two contracts are preferred by the interest group depending on its characteristics and the institutional setting. These contracts resonate with reality: either the interest group exchanges favors with each period's incumbent, whichever it is; or it sustains a long-term relationship with a single political party. In

[1] This heterogeneity of relationships appears also when countries choose their allegiances in another country's domestic politics. Ivan Krastev observed: "Poland decided [...] it is not going to be involved in the American domestic politics. So Kaczyński probably feels closer ideologically to Trump, but he is never going to go into traps invented to take Biden [...] Here, Orbán made a choice, which is kind of very unexpected for a small European country. He decided to enter the domestic politics of the United States and totally to bet on Trump and the Republican Party". (Financial Times, Rachman Review, 29th September 2022)

addition to tackling the rationale for different *quid-pro-quo* relationships, my model explains why some political parties provide costly favors without receiving any electoral support in return.

I consider a parsimonious dynamic model of relational incentives between two political parties and an interest group (for concreteness, a firm). Parties compete for office in repeated elections which the firm can influence. In each period, the elected party decides whether to transfer a rent to the firm. The rent in each period is non-negative and bounded-above by a *transactional constraint*. Whereby, there is a limit on how much rent can be transferred in each period, which broadly captures the level of discretion that the government enjoys. Electoral outcomes are stochastic. After the rent is transferred, the firm can support one of the parties, increasing its probability of winning the coming election. Besides, the incumbent may have an incumbency advantage, which exogenously produces persistence in power. Notably, the firm cannot guarantee reelection, which implies that a level of *residual uncertainty* in elections always exists. The firm has no political preferences and solely values rent extraction. Rent extraction bears an *efficiency loss* that both parties suffer, regardless of which currently holds power. This feature of the model reflects the fact that rents are frequently extracted from a common pool of goods enjoyed by the whole of society (e.g., clean air or market competition). Binding agreements are not possible, hence a contract is simply an equilibrium of the game.[2]

I characterize the firm's best contract and focus on the evolution of the relationship between the firm and the parties as power changes hands. The analysis reveals that two relational contacts have particular relevance: *(i)* the *Exclusive contract*, in which the firm always supports the same party, irrespective of whether it currently holds power. In return, the party transfers a large amount of rents. *(ii)* The *Opportunistic contract*, in which the firm's support goes in each period to the current incumbent, whichever it is, and both parties transfer the highest possible rent. An intuitive approach suggests advantages for each kind of relationship. On the one hand, there are gains from long-term cooperation with a single party due to the fact that parties are involved in a zero-sum competition for office. On the other hand, a party out of power has little to offer, so exogenous political turnover can impel the firm to cooperate with each period incumbent, that is, with both

[2] As Dal Bó (2003) emphasizes, the interest groups' literature largely ignores these credibility problems. "Grossman and Helpman (1994), e.g., analyze a two-period model in which lobbies choose political contributions in the first period and the government sets policy in the second. Lobbies pay if the government delivers, although after it delivers there are no incentives for the lobbies to pay."

parties.[3]

The core insight behind this paper's results is that if the amount of transferable rents per period is not too limited, cooperation with both parties is sub-optimal for the firm. Competition for office is zero-sum and improving one party's electoral prospects implies worsening its adversary's. Thus, the interest group can obtain more rents by sustaining an exclusive relationship with one political party. However, when the amount of transferable rents is severely limited, the firm cannot extract the full value of a long-term relationship with a single party. In that case, the firm prefers to gain access to whichever party holds power and the firm's best contract is the *Opportunistic contract*.

This brings us to a central comparative static derived in the paper: as the firm faces greater political fluctuations, it is more likely to prefer an exclusive relationship. The *Exclusive contract* with the initial incumbent is the unique firm's best contract if electoral uncertainty is sufficiently large. On the contrary, if electoral uncertainty is small, the firm's best contract is the *Opportunistic contract*. For intermediate levels of electoral uncertainty, the interest group's best contract combines features of both contracts: the initial incumbent receives electoral support when it holds power and (with some probability) also when it does not. Importantly, this result is independent of whether the increase in political fluctuations is due to a reduction in incumbency advantage or in the firm's electoral influence. The intuition behind this result is twofold. First, incumbency loses value as the incumbent's persistence in power becomes more uncertain. As a result, the value of a long-term relationship with a single party decreases, implying that the transactional constraint is less likely to bind in the *Exclusive contract*. Second, incumbents know their hold on power will likely last for a shorter period in a more uncertain environment. Thus, they assign a greater value to the firm's "loyalty" after an electoral defeat, decreasing their willingness to pay for an opportunistic relationship.

My second central comparative static is that, as rents transferred to a firm become more inefficient, the firm is more likely to prefer an exclusive relationship with the initial incumbent. The intuition is again twofold. First, firms that require more costly concessions actually receive

[3] The importance of incumbency is well conveyed in De Feo and De Luca (2017) discussion on why in postwar Italy many mafiosi preferred to support the Christian Democrats, instead of the Sicilian separatist movement: "Supporting the incumbent party guaranteed several advantages to the mafia, which could directly access important leading figures at the government level to defend its economic interests (e.g., the allocation of public procurement contracts in the areas of its activities) and lobby for softer legislation on mafia-related crimes, the protection of mafia members at different levels in judicial trials, and lower investment in mafia-controlling activities".

less of them, and as a result, the transactional constraint is less likely to bind in the *Exclusive contract*. Second, as the transfer of rents becomes more costly, parties are willing to extract fewer rents in payment for an opportunistic firm, and hence the firm prefers to switch to a more exclusive relationship. Moreover, although rent extraction (weakly) decreases as it becomes more inefficient, parties' welfare is differently affected in each contract region. In the region where the firm optimal strategy is opportunism, the increase in rent's inefficiency is entirely translated to the parties, and rent extraction is unchanged. In the region where the firm's optimal contract is exclusive, rents decrease as they become more inefficient, and parties' welfare is unaffected.[4]

Some interesting implications for empirical research arise from these results. One enduring puzzle has been the mixed empirical evidence for the claim that campaign contributions are rewarded with favorable policies (see Ansolabehere et al., 2003; Fowler et al., 2020). The papers analyzing this question typically associate a *quid pro quo* agreement with a positive relationship between the policies enacted by the party holding power in period t and the electoral support it received in period $(t-1)$. More formally, the time-t rent transferred by the incumbent party is regressed with respect to the interest group's time-$(t-1)$ electoral support to that same party. However, suppose that players follow an *Opportunistic contract*. Then, the interest group supports at a certain period the present officeholder, who may coincide or not with the next period's one. In that case, this empirical exercise would not reflect exchanges of favors but rather the incumbent's electoral persistence.

The second implication of my results relates to the widespread association of exclusive relationships with ideological alignments (e.g., Ferguson and Voth 2008; Acemoglu et al. 2013). Without disregarding the role played by ideology, I show that non-ideological groups can also prefer exclusive agreements. This result suggests a novel and complementary interpretation for different papers that document how certain groups of voters exhibit long-term loyalties to a party that favored them in the past. For example, in their study of the 1950 Italian Land Reform, Caprettini et al. (2022) associate the lasting electoral support to the Christian Democrats of the voters allotted with land in the reform with a change in their ideology and socioeconomic status. Nonetheless, favors

[4] These results can be tested on public procurement data, particularly the presence of different *quid pro quo* agreements. A public contest allows us to estimate the loss in public utility from political favors by comparing the characteristics of the politically connected bidder to ones of the leading bidder. I thank Ferenc Szucs for this suggestion.

to voters—especially if those favors involve relevant transfers of wealth, like in land reforms—can produce long-term loyalties as part of a non-written contract between the voters and the party.

This paper also points out the potential link between opportunistic interest groups and high institutional quality. The restrictions to the incumbent's ability to transfer rents are hard to observe and measure. The disguised mechanisms at the disposal of political actors are often unknown and highly dependent on the particularities of each economic sector.[5] Besides, the supervision of officeholders may come from different branches of the state, like the judiciary, independent agencies, or rival political parties. My results suggest that we can learn about the aggregated quality of these institutions by observing the interest groups' behavior across periods. In particular, it follows from this paper's core insight that opportunistic interest groups in a specific economic sector and country indicate effective restrictions on the incumbent's discretion to transfer rents.

To explore an additional implication of this paper's core insight, I consider the following extension of the baseline model. If a shock temporarily increases the amount of rents that can be transferred (e.g., because of a crisis or a war), the firm can become exclusive towards that period incumbent. Thus, a temporal shock modifies permanently the balance of electoral power.[6]

Following the empirical evidence on special interest politics, I introduce in the baseline model an *entry cost* the first time the interest group affects an election. This cost includes the effort of building a political network or establishing an electoral machine. I show that the interest group's best contract prescribes no active participation on-path for a range of parameters. However, this contract includes positive rent extraction made by both political parties under the off-path threat of active participation, which means that these are deterrence rents. This insight can speak to the well-establish Tullock paradox, as expenditures made by interest groups may be close to zero and, nonetheless, translate into important rent payments by the parties. Interestingly, rents depend on the entry cost even when there is no on-path participation, as the off-path threats' credibility depends on the entry cost.

To gain insight into the importance of long-lived parties, I compare the baseline model with a *weak party system.* The results above rely on the assumption that parties are long-lived organizations

[5]Disguised mechanisms are those that transfer resources to special interests but may be justifiable on other, more palatable, grounds (Tullock, 1983). Coate and Morris (1995) showed that such mechanisms can be used in equilibrium, even when they are Pareto inferior to direct transfers.

[6] This result resonates with the empirical findings of papers studying the New Deal expenditure (Kantor et al., 2013) and the effect of land reforms (Caprettini et al., 2022; Carillo et al., 2022).

resilient to electoral defeat. This feature is not standard in a principal-agent setting, where the principal has the prerogative to fire the agent forever. In a weak party system, parties' decisions are taken by leaders who, once electorally defeated, can no longer run for office and are indifferent to their party's future thereafter. Intuitively, the firm's best contract under weak parties is opportunistic. However, leaders' unwillingness to keep long-term relationships does not necessarily imply that a weak party system is less extractive. Leaders face replacement if defeated, so they have higher stakes in each election than a party and, thus, an incentive to pay more for reelection. As a result, the comparison between both party systems is non-monotonic with respect to governments' transactional constraint. When the amount of transferable rents is sternly restricted, the firm behaves opportunistically in both party systems, and rent extraction is identical. Making rents more transferable leads, first, to the weak party system becoming more extractive due to leaders' higher stakes. But if constraints are eased further, the firm can extract the full value of a long-term relationship, and the long-lived party system becomes more extractive.

The assumption that the rents' burden is evenly distributed between the incumbent and its challenger may not apply to certain cases. In particular, it is natural to assume that the incumbent has the ability to shift the rents' burden towards its adversary's constituency (e.g., if rents spoil local public goods). If rents feature a higher cost for the opposition than for the incumbent, the findings of the baseline model are qualitatively unchanged, and importantly, the firm extracts a greater rent.

I also study the opposite case, in which rents feature a higher cost for the incumbent than for the opposition. This assumption is appropriate if rent extraction has a substantial opportunity cost for the incumbent. For instance, because time and funds that could be devoted to the incumbent's preferred projects are distorted. Under this assumption, the firm does not prefer to concentrate rent extraction on the initial incumbent. Hence, although the firm's behavior is identical to the one in the baseline model for a significant range of parameters, its optimal rent scheme is qualitatively different. Even in the *Exclusive contract*, both parties extract rents on-path: whereas one party exchanges electoral support for rent extraction, the other party transfers rents simply to avoid the off-path punishment. This result shows how, even in the simple framework proposed here, *quid pro quo* relationships can be hard to unpack based only on observable (on-path) behavior.

The rest of the paper is organized as follows. The next section reviews the related literature.

In Sections 3, 4, 5, 6 and 7 the baseline model is presented, followed by a discussion of its main insights and implications. Sections 8 and 9 consider extensions of the baseline model. Section 10 studies the asymmetric distribution of rents' burden. The last section concludes. The appendix contains the proofs of all the results presented in the text.

2 Literature review

This paper is related to a growing literature that studies policy dynamics when today's policy affects tomorrow's allocation of political power (e.g., Padró i Miquel, 2007; Acemoglu and Robinson, 2008; Bai and Lagunoff, 2011). However, my approach differs from these papers in that it focuses on self-enforcing agreements via non-Markov strategies.

More closely related to this work are the models of relational incentives applied to political economy questions (e.g. Dixit et al., 2000; Acemoglu et al., 2008, 2011a; Yared, 2010; and Padró i Miquel and Yared, 2012). The most important difference between my approach and the previous literature is that I assume elections can be affected by agents' actions, but only stochastically. Previously, Acemoglu et al. (2008) and Yared (2010) considered an electoral accountability environment where a representative voter could reelect or oust from power the current incumbent with certainty. Alternatively, Dixit et al. (2000) and Acemoglu et al. (2011a) study a setting where the allocation of political power fluctuates stochastically, but exogenously. This paper also contributes to a question that has already receive some attention in the political economy literature: how can a leader's supporters make their leader, once in power, abide by her promises? Or, similarly, when are leader's promises before attaining power credible? Myerson (2008) emphasizes the importance of communication between the leader's supporters. This paper contributes to this question by studying the role of relational incentives.

This paper also contributes to the extensive research on special interest politics (e.g., Coate and Morris, 1995; Grossman and Helpmann, 2001). Chamond and Kaplan (2013) study an electoral competition model where an interest group can condition campaign contributions on the favors of both parties, allowing threats to play a relevant role. However, because of the static nature of their model, they consider binding contracts where the credibility of threats is not discussed. In contrast, this paper presents a dynamic setting with self-enforcing contracts, which allows to investigates which threats are credible, and to study the relation between persistence in power and campaign contributions.

Various empirical papers have studied the capture of the political system by special interests. The literature on politically connected firms, initiated by Fisman (2001), has accumulated evidence on firms' benefits from political connections in various countries (e.g., Faccio, 2006; Ferguson and Voth, 2008). The influence of organized crime on the political system has been thoroughly studied in the case of the Italian Mafia (Buonanno et al., 2016; De Feo and De Luca, 2017) or the Colombian paramilitaries (Acemoglu et al., 2013), among others. These papers, however, do not study the dynamics of these relationships as power changes hands.

3 Model

Time is discrete and infinite, indexed by $t \geq 0$. There are two political parties, ℓ and r, and one interest group, for concreteness, a firm, f. All players discount their payoffs by the common discount factor β and are all risk neutral. Every action is publicly observable. Political parties like being in power and dislike transferring rents. The firm has no political preferences and solely values rent extraction.

Timing. At the beginning of each period $t \in \mathbb{N}$, the incumbent $\mathcal{I}(t)$ who holds power decides the rent $x(t) \in [0, \tau]$ transferred to the firm in that period, where $\tau > 0$ is a constraint on the transactional ability of each period's incumbent. The transactional constraint captures situations where the incumbent's discretion is limited, e.g., by checks and balances, because inattentive voters punish overt corruption (Ferraz and Finan, 2011; Tullock, 1983), or because politicians have time or agenda limitations.[7] The opposition is inactive. Then the firm decides whether and which party to support in the next election. Formally, it chooses an action $s(t) \in \{L, N, R\}$, where the actions L, N and R denote supporting party ℓ, remaining neutral, and supporting party r, respectively.[8] Finally, an election takes place; the winner holds power in period $t + 1$. I assume that there is a payoff-irrelevant random variable on which players can condition their strategies, which is uniformly

[7] Because rents are a public bad that can be used to punish one party, τ partially resembles the notion of limited liability from an standard principal-agent setting (see Fong and Li, 2017). Nonetheless, τ can also limit the firm's benefits from cooperation, so the two concepts are not entirely comparable.

[8] That the firm backs only one party in each election is supported by Fowler et al. (2020) and Chamon and Kaplan (2013), who show that, in the US congressional elections, it is extremely rare for an interest group to contribute to both parties' campaigns in the same race.

distributed in the interval $[0,1]$ and independent across time. Its realization $\varpi(t)$ is publicly observed after the party chooses the rent extraction. This is a common convenient assumption in the literature on relational contracts, as it convexifies the set of equilibrium payoffs. The timing of the game within a period is depicted in Figure 1. Hereafter I assume, without loss of generality, that the initial incumbent is party ℓ. Throughout, i, x, and s refer to an arbitrary realization of the random variables $\mathcal{I}(t)$, $\mathcal{X}(t)$, and $\mathcal{S}(t)$, respectively.

Election. I denote by $p(i,s)$ party ℓ's probability of wining office in $t+1$ if period t's incumbent is i and the firm's period t action is s. Party r's probability is $1-p(i,s)$. I assume the following functional form for $p(i,s)$:

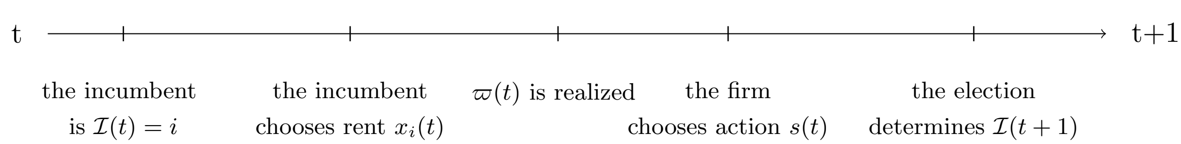
$$p(\ell,s) = \frac{1}{2} + a + m\mathbb{1}_{\{s=L\}} - m\mathbb{1}_{\{s=R\}}$$
$$p(r,s) = \frac{1}{2} - a + m\mathbb{1}_{\{s=L\}} - m\mathbb{1}_{\{s=R\}}$$

The above specification introduces two elements: (i) *incumbency advantage*, $a \geq 0$, which makes reelection exogenously easier for the incumbent; (ii) *firm's action* which increases the reelection probability of the supported party by $m > 0$, a parameter that captures its electoral influence.

Remark 1 *(i)* $\frac{1}{2} + a + m < 1$ *and (ii)* $\frac{1}{2} + a - m > 0$.

Part (i) of Remark 1 implies that the firm is unable to guarantee reelection with probability 1 to an incumbent in case it wants to reward it for keeping its promises. Part (ii) of Remark 1 implies that the firm cannot oust the incumbent from power with probability 1 either, in case it wants to punish it for deviating. As a consequence, elections always have a *residual uncertainty* despite the intervention of the firm.

Figure 1: Timeline



Payoffs. Let $y_j(.)$ be the per-period payoff of player j. The firm's payoff when the incumbent i chooses a level of rent extraction x is:

$$y_f(i,x) = x.$$

Parties suffer from rent extraction and obtain an office benefit $b > 0$ when they hold power. Party ℓ's payoff when it holds power and chooses a rent x_ℓ is

$$y_\ell(\ell, x_\ell) = b - kx_\ell,$$

and when party r holds power and chooses a rent x_r, it is

$$y_\ell(r, x_r) = -kx_r,$$

where parameter $k > 0$ represents how inefficient rent extraction is. Party r's payoff is defined symmetrically. Players maximize their expected discounted stream of payoffs at the current period.

3.1 Discussion of the Model

Political arena. This model departs from the standard principal-agent relationship to capture some distinctive features of the political arena. *(i)* Players are long-lived. Hence, any party will occupy office with certainty at some future period despite the firm's actions. This feature is absent from the literature on relational contracts, which typically assumes that if the principal fires an agent, she does not reappear in any future period. *(ii)* The firm's reward for political parties—electoral support—affects a zero-sum competition between the parties. Thus, improving one party's electoral prospects implies worsening its adversary's. *(iii)* Rent extraction is a public bad for the parties, as both parties suffer from it equally irrespective of which party decided the extraction.

Firm. A firm is characterized by its electoral influence, m, and its social inefficiency, k. A firm's influence is determined by a wide variety of institutional, technological, and social aspects, which go from social prestige to the use of violence against voters. One can imagine an interest group with asymmetric influence—i.e., m_ℓ if $s = L$ and m_r if $s = R$, where $m_\ell \neq m_r$, especially if the group's influence does not take the form of a monetary transfer, but of a public action like the endorsement of candidates. I assume a symmetric influence for simplicity, but the main results do not vary qualitatively if we allow for asymmetric influence.[9] The firm's inefficiency is meant to

[9] Although support can take the form of money donations, many of the most influential interest groups are not the greater donors (Wolton, 2020). In the survey of US legislators and their staff by Fortune magazine in 1999, none the 5 most powerful groups according to the survey—the AARP, the NRA, the NFIB, the American Israel PAC and the AFL-CIO—are a top campaign contributor.

reflect how costly for society is the transfer of rents to the firm.

3.2 Relational contracts

Let $h_t \equiv (i_0, \varpi_0, x_{i_0}, s_0, \ldots, i_{t-1}, \varpi_{t-1}, x_{i_{t-1}}, s_{t-1}, i_t)$ denote a realized history up to date t, including the party that holds power at period t, i_t; and let $\mathcal{H}_t$ be the set of all possible time-t histories. Party j's public strategy function is denoted $x_{j,t}$ and maps any possible time-t history into a rent extraction choice: $x_{j,t} : \mathcal{H}_t \to [0, \tau]$.[10] Similarly, firm's public strategy is denoted s_t and maps any possible time-t history and any incumbent's rent extraction into an electoral support choice: $s_t : \mathcal{H}_t \times [0, \tau] \to \Delta\{L, N, R\}$. Let $\sigma = (x, s)$ denote a profile of public strategies for the three players. Let

$$v_j^\sigma(h_t) \equiv (1-\beta)\mathbb{E}\big[\sum_{s=0}^{\infty} \beta^s y_j(\mathcal{I}(t+s), \mathcal{X}(t+s)) \mid h_t, \sigma\big]$$

denote j's expected discounted payoff from the beginning of period t onwards conditional on all public information at the start of period t and conditional on continuation play $\sigma = (x, s)$. Lastly, let

$$w_j^\sigma(h_t) \equiv \mathbb{E}\big[p(i, \mathcal{S}(t))v_j^\sigma(\mathcal{I}(t+1) = \ell, \mathcal{H}_{t+1}) + (1 - p(i, \mathcal{S}(t)))v_j^\sigma(\mathcal{I}(t+1) = r, \mathcal{H}_{t+1}) \mid h_t, \sigma\big]$$

denote j's expected discounted payoff from period $t+1$ onwards conditional on continuation play $\sigma = (x, s)$ and conditional on all public information at the start of period t (but neither the firm's support choice nor the winner of period t election).[11] Hence, this expression conditions only on the information available after the party extracts the rent, making explicit the uncertainty of future electoral outcomes. Thus,

$$v_j^\sigma(h_t) = (1-\beta)y_j(i(h_t), x(h_t)) + \beta w_j^\sigma(h_t).$$

In what follows, I refer to $v_j^\sigma(.)$ as value and to $w_j^\sigma(.)$ as pre-election value. As shown by Mailath and Samuelson (2006: 192), the techniques from Abreu, Pearce, and Stracchetti (1990) are valid in dynamic games with a finite set of states, as this model. An equilibrium can be represented in a

[10]Since the rent is a continuous variable there is no need to consider mixed strategies for the parties.

[11]Note that as $v_j^{\mathcal{C}}(h_t)$ and $w_j^{\mathcal{C}}(h_t)$ denote on-path payoffs, they are uniquely pinned down by history $h_t \in \mathcal{H}_t$.

recursive fashion.[12]

Definition 1 *A relational contract (or simply, contract) is a Subgame Perfect Nash Equilibrium in which all players use public strategies, i.e., a Perfect Public Equilibrium (hereafter, equilibrium).*

A relational contract describes behavior both on and off the equilibrium path. I am interested in the agent's on-path behavior, so I restrict attention to equilibria in which, after a unilateral deviation, the continuation play corresponds to the worst equilibrium for the deviator (Abreu, 1988). Let $\underline{v}_j(i)$ and $\underline{w}_j(i)$, respectively, denote player j's value and pre-election value in its worst equilibrium at a period in which i holds power.

An strategy profile $\sigma = (x, s)$ is a relational contract—i.e., an equilibrium—if and only if the following set of conditions holds at every history $h_t \in \mathcal{H}_t$: (i) *enforcement constraints:* each party, when it holds office, is willing to extract the prescribed rents,

$$\begin{aligned} (1-\beta)kx_\ell(h_t) &\geq \beta(w_\ell^\sigma(h_t) - \underline{w}_\ell(\ell)), \\ (1-\beta)kx_r(h_t) &\geq \beta(w_r^\sigma(h_t) - \underline{w}_r(r)); \end{aligned} \tag{1}$$

and (ii) the firm is willing to execute its prescribed action,

$$w_f^\sigma(h_t) \geq \underline{w}_f.^{13} \tag{2}$$

Note that the firm's worst contract is to repeatedly play the static equilibrium with zero rents irrespective of the history, hence, $\underline{w}_f = 0$.[14] With a slight abuse of notation, I denote by $\mathcal{C} = (x, s)$ the restriction of the contract to its on-path histories.

As there are many self-enforcing contracts, the question of which to select is relevant. Contrary to Levin (2003) and the subsequent literature, I do not focus on the efficient contract. Because this is a distribution problem between risk-neutral agents, any contract belongs to the Pareto frontier. Instead, this paper focuses on the best contract for the firm, which I understand as the social worst-case scenario. By characterizing the firm's best contract, this paper delves into how interest

[12] In particular, at any public history, the entire public history of the game is subsumed in the continuation value of each player, and associated with these continuation values is a sequence of actions and continuation values.

[13] Note that the only relevant one-shot deviation to consider for an incumbent is to transfer no rent at all.

[14] As this paper focuses on the firm's best contract, results are unchanged if the interest group has commitment power.

groups manage some distinctive features of the political arena, namely the uncertainty of electoral outcomes, and the fact that parties are long-lived (so even if the firm manages to oust a deviator from office, it might stochastically return to it).

Besides, the following class of simple contracts will play an important role in the analysis

Definition 2 *A contract is* (on-path) stationary *if there exists* $x_\ell, x_r \in [0,\tau]$*, and* $s_\ell, s_r \in \Delta\{L,N,R\}$ *such that at every on-path history* $h_t \in \mathcal{H}_t$*,* $x_\ell(h_t) = x_\ell$*,* $x_r(h_t) = x_r$*, and* $s(h_t)$ *is* s_ℓ *(*s_r*) if the current incumbent at history* h_t *is* ℓ *(*r*).*

In other words, under a stationary contract, on-path, all players play an action that depends only on the current incumbent's identity. Thus, an on-path stationary contract is a tuple $\mathcal{C} = (x_\ell, x_r, s_\ell, s_r)$.

For expositional purposes, it is useful to name some contracts after the on-path stationary strategy they prescribe for the firm.

Definition 3 *(i) A contract that prescribes on-path* $s_i = L$ *(*$s_i = R$*) for any* $i \in \{\ell, r\}$ *is referred as a* ℓ-Exclusive contract *(*r-Exclusive contract*), and denoted* $\mathcal{C}^L$ *(*$\mathcal{C}^R$*).*

(ii) A contract that prescribes on-path $s_\ell = L$ *and* $s_r = R$ *is an* Opportunistic contract*, and is denoted* $\mathcal{C}^O$*.*

(iii) A contract in which when ℓ *is in power, the firm deterministically support* ℓ*—i.e.,* $s_\ell = L$*—and when* r *is in power, the firm supports* ℓ *with probability* $\tilde{q} \in (0,1)$ *and* r *with probability* $1-\tilde{q}$*—i.e.,* $s_r = L$ *w.p.* $\tilde{q}$ *and* $s_r = R$ *w.p.* $1-\tilde{q}$*—is called an* ℓ-Biased contract*, which constitutes a class of contracts indexed by* $\tilde{q}$*.*

Thus, in a *ℓ-Exclusive contract*, the firm always supports party ℓ whereas in an *Opportunistic contract*, the firm always support the current incumbent. I denote by v_j^L, v_j^R, v_j^O, and v_j^B player j's value in each of these contracts, respectively.

Definition 4 *For any firm's on-path strategy* s*, the maximum-rent* s*-contract is the contract that maximizes rent extraction among all the contracts with firm's on-path strategy* s *and it is denoted* $\mathcal{C} = (\bar{x}, s)$*.*

4 The Firm's Best Contract

In this section, I characterize the firm's best relational contract. Observe first that given the payoff specification, rents are a public bad for the parties and political competition is zero-sum. Hence, conditional on a level of rent inefficiency k, the sum of playoffs is fixed:

$$y_\ell(i) + y_r(i) + 2ky_f(i) = b.$$

Therefore, to maximize the firm's value is the same as to minimize the sum of the political parties's values.

To determine which contracts can be self-enforcing, the next step is to derive the worst contract for a given political party. To minimize a party's value, the other two players cooperate. Thus, the worst equilibrium for a party is itself a (non-trivial) relational contract. Political parties like being in power and dislike rents, so a party's worst equilibrium should seek to both minimize its probability of being elected and maximize rent extraction. The following lemma characterizes the contract that optimally combines these two goals.

Lemma 1 *Party r's worst contract is a* maximum-rent ℓ-Exclusive contract*. The symmetric description applies for party ℓ.*

Once we have characterized each political party's worst contract, enforcement constraints (1) are well-specified.

The next proposition shows my first main result. Absent transactional constraints, the firm's best contract is such that it gives on path permanent support to the initial incumbent (party ℓ).

Proposition 1 *Suppose there are no transactional constraints, i.e., $\tau \to \infty$. Any firm's best contract is a maximum-rent ℓ-*Exclusive contract.

In any such contract, the value of both parties is minimized among all contracts.

To build some intuition for Proposition 1, it is helpful to compare an *ℓ-Exclusive contract* with an *Opportunistic contract*. Two of the model's features are key for this comparison: rents are a public bad for the parties, and political competition is zero-sum. Naively, one may conjecture that an opportunistic firm benefits from the fact that it incentivizes every period incumbents to extract rents. However, this conjecture is not true since every rent transferred by party r is equally suffered by party ℓ. Thus, every party r's rent expected in the future crowds out party ℓ's willingness to extract rents today by the same (present value) amount. Formally, for any *Opportunistic contract* in which party r extracts some positive rent, there is an alternative *Opportunistic contract* in which party ℓ transfers a rent of equal present value at the initial period instead. This alternative *Opportunistic contract* is an equilibrium if the original contract is so, and both contracts give the same rent extraction.[15] Hence, there always exists a firm's best *Opportunistic contract* such that only party ℓ extracts rents. Recall next that parties are involved in a zero-sum competition for office. Thus, party ℓ is willing to transfer more rents in an equilibrium where the firm grants it permanent electoral support than in an equilibrium where it is opportunistic.

Since binding agreements are impossible, any optimal *Exclusive contract* must be with the initial incumbent, the only party that can make up-front payments in exchange for cooperation. The initial opposition cannot credibly promise to match these up-front payments: it can neither transfer any rent while out of power nor credibly promise to pay these rents as soon as it wins the election because, by then, the effect of the firm's support in previous periods is irreversible. This logic produces an *electoral initial-incumbency advantage*, a form of path-dependence not previously studied in the literature.[16]

Proposition 1 establishes the optimality of an entire class of contracts, which can be constructed in multiple ways. The next remark describes a particularly intuitive maximum-rent *ℓ-Exclusive contract* whose rents are on-path stationary, which helps to bring the Proposition's results to real-world examples.

Remark 2 *Suppose there are no transactional constraints, i.e., $\tau \to \infty$. There exists a* maximum-rent ℓ-Exclusive contract *contract that is stationary, and it takes the following form: only party ℓ*

[15] As a sketch of the proof, consider certain *Opportunistic contract* $\mathcal{C}^O$ in which, without loss of generality, party r extracts a positive rent at certain on-path history $h_t \in \mathcal{H}_t$ reached with positive probability. Then, we can construct an alternative contract identical on-path to $\mathcal{C}^O$ with the only difference that *(i)* party r extracts no rent at on-path history $h_t \in \mathcal{H}_t$ and *(ii)* party ℓ extracts in the initial period a greater rent, equal to its initial period rent in contract $\mathcal{C}^O$ plus the present value of party r's aforementioned rent.

[16] Acemoglu et al. (2011) also features an initial incumbency advantage for an elite political party collaborating with the bureaucracy, which grants the elite persistence in power. However, this initial incumbency advantage appears due to two assumptions absent in my model: a persistent state (the efficiency of bureaucracy) and deterministic elections. The introduction of probabilistic elections in Acemoglu et al. (2011) does not unravel the initial incumbency advantage of the elite party, but turns it into a transitional effect that disappears in the long term. In contrast, in this paper initial incumbency advantage is permanent.

makes positive rent extraction, i.e., $(x_\ell, x_r) = (\hat{x}, 0)$*, where*

$$\hat{x} = \frac{2\beta m}{k(1-\beta)} b. \tag{3}$$

In that contract, party ℓ*'s enforcement constraint (1) binds in every on-path history in which* ℓ *holds power and the firm's value at the initial period is*

$$v_f^L(\ell) = \frac{1 - \beta(1 - p(\ell, L))}{1 - \beta 2a} x_\ell. \tag{4}$$

A symmetric description applies to a stationary maximum-rent r-Exclusive contract

Equation (3) of $\hat{x}$ is key to understand the mechanics of the model. Rent $\hat{x}$ is the highest per-period rent that a firm can receive in exchange for an exclusive relationship. Intuitively, it increases when rent extraction is less inefficient (k), the firm is electorally more influential (m), office benefits (b) are higher, and players are more patient. Any of these parameter changes imply that the party finds cooperation with the firm more valuable relative to the punishment following a deviation.

It may appear puzzling that (3) does not depend on incumbency advantage (a). To see why, we study party ℓ's enforcement constraint (1). When party ℓ transfers a rent equal to $\hat{x}$, its enforcement constraint (1) binds. Hence, when it is elected on-path, party ℓ receives its worst value conditional on holding power, i.e., $v_\ell^L(\ell) = \underline{v}_\ell(\ell)$. So, if party ℓ deviates and is elected, it receives the same value as if it complies with the contract and is elected. Consider now the case in which party ℓ loses the election. If it loses the election on-path, party ℓ receives a better value than its worst value conditional on being in opposition, i.e., $v_\ell^L(r) > \underline{v}_\ell(r)$, as it still receives electoral support and its rival does not transfer any rents. On the contrary, if it loses the election after a deviation, party ℓ receives its worst value conditional on being in opposition—that is, the continuation value of the *r-Exclusive contract.* This implies that the punishment after a deviation consists of *(i)* a decrease of $2m$ in the winning probability of the coming election, and *(ii)* a worse continuation value in the case of defeat. One can rewrite party ℓ's enforcement constraint (1) as:

$$\begin{aligned}(1-\beta)k\hat{x} &= \beta\Big(p(\ell, L)v_\ell^L(\ell) + (1 - p(\ell, L))v_\ell^L(r)\Big) - \beta\Big(p(\ell, R)\underline{v}_\ell(\ell) + (1 - p(\ell, R))\underline{v}_\ell(r)\Big) \\ &= \beta\Big(2m\big(\underline{v}_\ell(\ell) - \underline{v}_\ell(r)\big) + (1 - p(\ell, L))\big(v_\ell^L(r) - \underline{v}_\ell(r)\big)\Big),\end{aligned}$$

where $\underline{v}_\ell(\ell) - \underline{v}_\ell(r) = \frac{1-\beta(1-2m)}{1-\beta 2a} b$ and $v_\ell^L(r) - \underline{v}_\ell(r) = \frac{\beta 2mb + (1-\beta)k\hat{x}}{1-\beta 2a}$. Combining these equations, we obtain rent $\hat{x}$. Observe that the incumbency advantage has two effects. On the one hand, it increases the value of incumbency by increasing the officeholder's persistence in power. On the other hand, it softens the party's punishment by reducing its probability of losing the election after a deviation. These two effects exactly offset each other.

Proposition 1 shows that, despite the uncertain fluctuations of political power, it is beneficial to the firm to promise its support to a single party and keep supporting it even when it loses power. Real-world *quid pro quo* relationships are often sticky. For example, in South Korea, most of the monopolistic conglomerates (*chaebols*) supported the Conservatives—the party in power since the transition to democracy—also after the Liberals attained power in 1993 (Mo and Weingast, 2013). However, Proposition 1 does not reflect the variety of relationships between lobbies and political parties. Opportunistic behavior is commonly observed. In Ghana, a stable two-party system since 1992, most local chiefs endorse the incumbent by influencing voters through persuasion and prestige. In their study of the U.S. House and state legislatures, Fouirnaies and Hall (2014) found a 20-25 percentage-point increase in the share of donations flowing to the new incumbent party—Republican or Democrat. This raises the question: what feature of this stylized model drives the superiority of exclusive relationships?

A key assumption for the optimality of exclusive relationships is the initial incumbent's ability to transfer sufficiently large rents when in power. The following proposition shows how the firm's best relational contract changes with the introduction of transactional constraints.

Proposition 2 *Let*

$$\hat{\tau} \equiv \frac{2\beta m}{k(1-\beta 2(a+m))} b. \tag{5}$$

A firm's best contract is:

*(i) If $\tau \geq \hat{x}$, a maximum-rent ℓ-*Exclusive contract *with $(x_\ell, x_r) = (\hat{x}, 0)$.*

*(ii) If $\tau \in (\hat{\tau}, \hat{x})$, the ℓ-*Biased contract *(see Definition 3) with rents $(x_\ell, x_r) = (\tau, \tilde{x}_r)$, where $\tilde{q} \in (0,1)$ and $\tilde{x}_r > 0$ solve the system of equations given by the parties' binding enforcement*

constraints:

$$(1-\beta)k\tau = \beta(w_\ell^B(\ell;\tilde{x}_r,\tilde{q}) - \underline{w}_\ell(\ell)), \tag{6}$$

$$(1-\beta)k\tilde{x}_r = \beta(w_r^B(r;\tilde{x}_r,\tilde{q}) - \underline{w}_r(r)). \tag{7}$$

(iii) If $\tau \leq \hat{\tau}$, the Opportunistic contract *with rents $(x_\ell, x_r) = (\tau, \tau)$. Moreover, the opportunistic strategy is the only firm's strategy that can support the maximum payments $(x_\ell, x_r) = (\tau, \tau)$ for every $\tau \in [0, \hat{\tau}]$.*

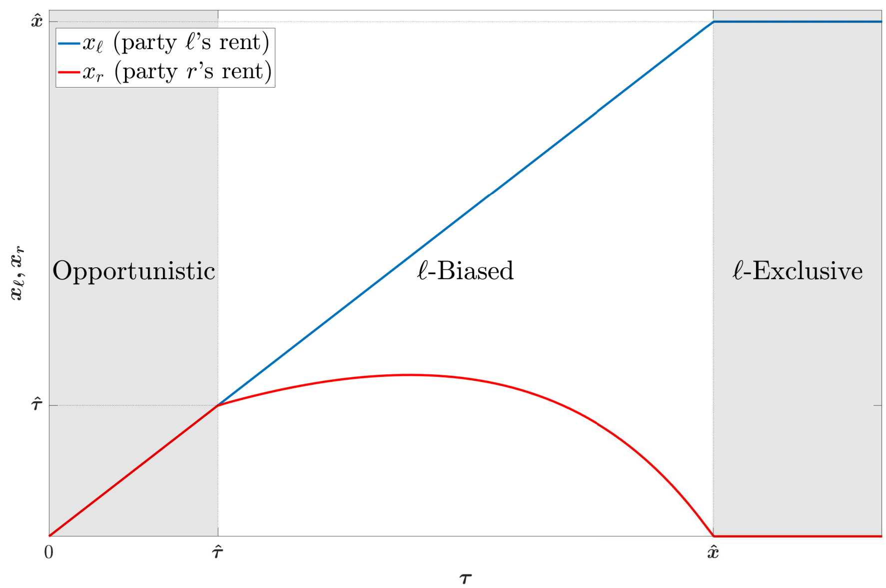


Figure 2: Per-period rent transferred by party ℓ (red) and party r (blue) with respect to the transactional constraint for $m = 0.3$, $\beta = 0.91$, $b = 10$, $a = 0$, and $k = 1$.

The intuition behind Proposition 2 is as follows. When rents are unrestricted, the *ℓ-Exclusive contract* is optimal because long-term relationships with the initial-incumbent are inherently superior, as seen in Proposition 1. When the amount of transferable rents is sufficiently limited, the firm cannot exploit the benefit of an exclusive relationship, so it focuses on having access to rent extraction every period. When the amount of transferable rents is intermediate, the firm can only partially exploit the benefit of an exclusive relationship with the initial incumbent. So roughly speaking, the firm tilts the electoral field in favor of the initial incumbent until its willingness to pay for that advantage hits the transactional constraint and uses the remaining electoral clout to extract some rent when the other party is in power.

When the amount of transferable rents is severely limited, the firm needs to incentivize both parties to transfer rents despite their zero-sum competition, a problem the *Opportunistic contract* solves optimally. To understand why, it is instructive to compare it to a *Neutral contract*, in which the firm is neutral every period. First, an opportunistic firm supports the incumbent in the present and the incumbent's rival when in office in the future, which overall is better than staying neutral because the future is discounted. Moreover, the support for the incumbent's rival is further discounted due to the opportunistic strategy. The "disloyal" behavior of the firm will only materialize after an incumbent's electoral defeat—a more unlikely event with the firm's support on its side. Lastly, the opportunistic strategy increases the value of incumbency, making parties more willing to pay for the firm's reward: reelection. In a *Neutral contract*, a party's gains from holding power are the immediate office benefits plus the extra persistence in power provided by the incumbency advantage. On the contrary, if the firm behaves opportunistically, a party's gains from holding power are the office benefits, the incumbency advantage, and the firm's support. These effects are captured in the denominator of $\hat{\tau}$—i.e., $k(1-\beta 2(a+m))$—which depends on the the incumbent's discount factor and includes $(a+m)$. As a consequence, the *Opportunistic contract* is the unique contract that maximizes rent extraction for every $\tau \in [0, \hat{\tau}]$. Yet, if τ is smaller than $\hat{\tau}$, there exist other contracts that sustain $(x_\ell, x_r) = (\tau, \tau)$, for example a *Neutral contract*.

Equation (5) of threshold $\hat{\tau}$ helps to understand better the mechanics of the model. Threshold $\hat{\tau}$ is the maximum rent transferred in the region where the *Opportunistic contract* with $(x_\ell, x_r) = (\tau, \tau)$ is optimal. When the incumbent transfers a rent equal to $\hat{\tau}$, its enforcement constraint (1) binds. Hence, an elected party receives on-path its worst value conditional on holding power, i.e., $v_\ell^O(\ell) = \underline{v}_\ell(\ell)$. So, if party ℓ deviates and is elected, it receives the same value as if it is elected on path. Consider now the case in which party ℓ is defeated on path. Note that, when party r holds power, on-path actions are the worst possible for party ℓ—i.e., $s_r = R$ and $x_r = \tau$—and when ℓ regains power, it receives $\underline{v}_\ell(\ell)$. So, if party ℓ deviates and is defeated, it receives the same value as if it is defeated on path, i.e., $v_\ell^O(r) = \underline{v}_\ell(r)$. Altogether, this implies that the punishment for a deviation consists only of a change of $2m$ in the coming election probability of success. Thus, one can rewrite party

ℓ's enforcement constraint (1) as:

$$(1-\beta)k\hat{\tau} = \beta\Big(p(\ell, L)v_\ell^O(\ell) + (1-p(\ell,L))v_\ell^O(r)\Big) - \beta\Big(p(\ell,R)\underline{v}_\ell(\ell) + (1-p(\ell,R))\underline{v}_\ell(r)\Big)$$
$$= \beta 2m\big(\underline{v}_\ell(\ell) - \underline{v}_\ell(r)\big),$$

where $\underline{v}_\ell(\ell) - \underline{v}_\ell(r) = \frac{(1-\beta)b}{(1-\beta 2(a+m))}$. Combining the last two equations, we obtain threshold $\hat{\tau}$.

The *ℓ-Biased contract* offers an illustrative example of the role of threats in sustaining relationships. In Proposition 2, as we relax the transactional constraint τ, the firm's strategy becomes closer to a *ℓ-Exclusive contract* in the sense that the probability $1-\tilde{q}$ of supporting party r when it holds power decreases. However, as Figure 2 shows, party r's rent $\tilde{x}_r$ does not monotonically decrease in τ but has a hump-shape. To understand why, observe that as the transactional constraint eases, parties' punishment becomes worse. Thus, for a fixed mixed strategy $\tilde{q}$, the maximum rent each party is willing to pay to sustain the equilibrium increases. This logic counterbalances the effect of the decreasing mixed electoral support. As we relax further the transactional constraint, it becomes non-binding, and the *Exclusive contract* dominates—as stated in Proposition 1.

I conclude this section with three comments. First, the firm's strict preference for different contracts in different institutional settings relies of the existence of *residual uncertainty.* Note that as residual uncertainty tends to zero, the firm becomes indifferent between the three contracts.

Second, these results are qualitatively unchanged if electoral influence is asymmetric. To see this, observe first that the logic behind Proposition 1 does not depend on any particular assumption on the electoral support. Note also that in the *Opportunistic contract* the only relevant aspect is the overall electoral influence of the firm—i.e., $m_\ell + m_r$—as this sum is what determines the firm's maximal effect on an election.

Lastly, given that the game is constant-sum, for any equilibrium and any history, there is no continuation play that gives all players a greater continuation payoff. So the equilibria characterized so far are renegotiation-proof according to standard definitions of renegotiation proofness.[17]

[17] For the case of political games, Acemoglu et al. (2008a) introduce a qualification to this definition of renegotiation-proofness, which is that it should only apply to "all active players". However, despite being relevant in a model with short lived candidates like Acemoglu et al. (2008a), this assumption is less relevant in a model of long-lived parties.

5 Determinants of the Firm's Influence

In this section, I investigate the main drivers of the firm's behavior and its rent extraction. By way of clarification, throughout I refer to the firm's best contract becoming more exclusive in the following sense: thresholds $\hat{\tau}$ and $\hat{x}$ decrease. The proposition below states that a long-term relationship is the firm's preferred agreement when there is great uncertainty about who will be the next holder of power.

Proposition 3 *(i) Rent extraction (weakly) increases in a and m.*

(ii) As a or m increases, the set of firm's best contracts becomes less exclusive.

Part *(i)* of Proposition 3 formalizes the idea that political fluctuations are socially beneficial, independently of whether we focus on the sources of incumbents' stability that are exogenous—the incumbency advantage a—or endogenous—the firm's influence m. Hence, this result speaks not only about the firm's characteristics or the institutional setting but about the political system as a whole. Besides, it unveils a new positive effect of political turnover independent from electoral accountability, as in this model, reelection is unrelated to incumbents' performance.

The intuition behind part *(i)* of Proposition 3 is as follows. Rent extraction decreases because political fluctuations depreciate the party's reward to cooperation: reelection. If being elected implies little persistence in power for the following periods, parties give lower value to office. Now, this logic applies to both opportunistic and exclusive relationships.

The intuition behind part *(ii)* of Proposition 3 is two-fold. First, we must recall that the crucial problem solved by an *Opportunistic contract* is to incentivize both parties to transfer rents despite their zero-sum competition. In a more unstable environment, the moment in which a party will lose power and the firm will shift its electoral clout to support its rival is closer in the future. Effectively, greater political fluctuations imply that incumbents discount less the firm's "disloyalty" after an electoral defeat. Consequently, they are willing to extract lower rents to keep an opportunistic relationship ongoing, which explains why the threshold $\hat{\tau}$ decreases with electoral uncertainty. Second, note that incumbency loses value as the incumbent's persistence in power is less likely. As a result, the value of an exclusive relationship decreases, and hence $\hat{x}$ also decreases, implying that the transactional constraint is less likely to restrict the party's ability to extract the full value of the long-term relationship.

Proposition 4 *(i) Rent extraction (weakly) decreases in k and τ.*

(ii) As k or/and τ increases, the set of firm's best contracts becomes more exclusive.

This proposition has a number of important implications. First and foremost, firms whose rents are more inefficient (higher k) prefer exclusive relationships. Intuitively, if each dollar of rent is more costly, each party is less willing to transfer rents. Hence, the transactions constraints are less likely to bind and the firm does not need to cooperate with both parties. This implication of my model can be tested on public procurement data. Rents are observed if a public contract is awarded to a firm that is not the best bidder. The public utility loss from *quid pro quo* agreements can be estimated by comparing the productivity of the politically connected firm with that of the leading bidder in the contest. Some anecdotal evidence resonates with this result. In Hungary, the foreign-owned firms—often more competitive than their national competitors—have broadly been the more reticent to enter into a long-term relationship with Orban's party, Fidesz.

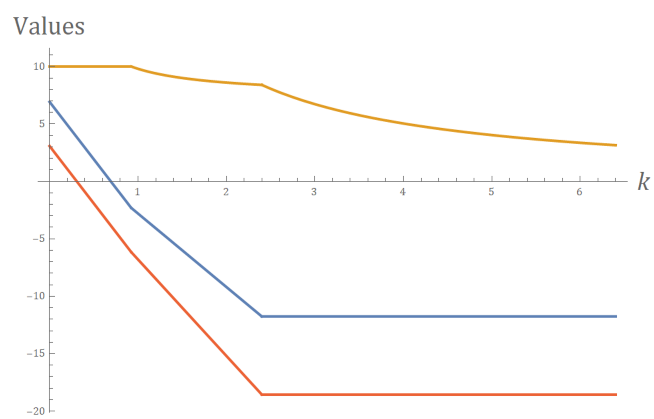


Figure 3: Values of the firm (yellow), party ℓ (blue), and party r (red) with respect to k for $m = 0.3$, $\beta = 0.8$, $b = 10$, $a = 0$.

Proposition 4 states that rent extraction decreases as the rents' inefficiency increases. Nonetheless, how is social welfare (i.e., the sum of parties' welfare) affected? Figure 3 shows that the answer to this question differs in each optimal contract region. In the region where the *Opportunistic contract* is optimal, parties' enforcement constraints do not bind. Thus, an increase in k leaves rent extraction is unaffected and translates into a social welfare loss. On the contrary, in the region where the ℓ-*Exclusive* contract is optimal, rent extraction absorbs an increase in k and social welfare is unchanged.

Lastly, it is interesting to consider the results for m and τ together. Both parameters can be thought to capture the quality of the democratic institutions, specifically whether institutions make it harder for parties and interest groups to sustain the sort of *qui-pro-quo* agreements that this paper studies. Parameter τ measures the extent to which the government can favor certain groups via patronage and parameter m measures the extent to which such groups can reward a party via an electoral advantage. Interestingly, as Figure 4 shows, an increase in institutional quality can make the firm either more or less opportunistic, depending on the dimension in which the change occurs. This result is important if, although unspecified in the model, voters are assumed to attach value to the existence of a balance of power. Then, my model suggests that reforms constraining officeholders' discretion are superior to those that target campaign contributions or other forms of electoral influence.

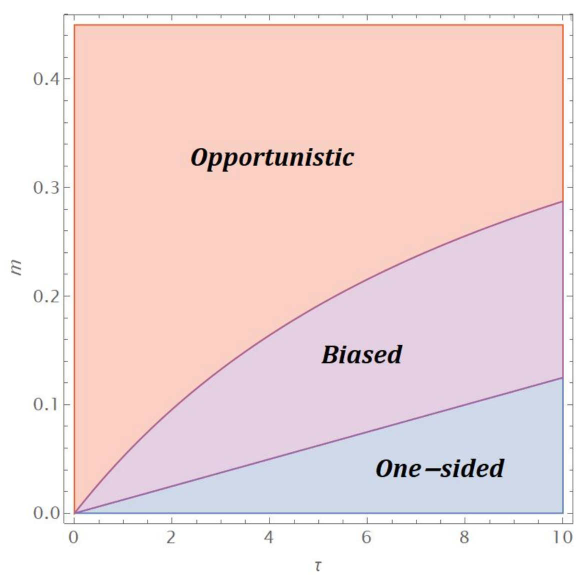


Figure 4: Firm's best contract as a function of the transactional constraint and the influence parameter for $\beta = 0.8$, $b = 10$, $a = 0.05$, $\pi = 1$ and $k = 1$.

6 Implications for Empirical Research

Some interesting implications for empirical research arise from the contracts considered in Proposition 2. First, *Opportunistic contracts* are particularly problematic for the studies which try to show whether interest groups' campaign contributions produce favorable policies in return. Many of these papers regress the rents transferred by the time-t incumbent, $x(t)$, on the interest group's electoral support for the time-t incumbent in the election of period $t-1$, $s(t-1)$, and a number of independent variables:

$$x(t) = \gamma_0 + \gamma_1 s(t-1) + \gamma_2 o(t) + \varepsilon_t, \tag{8}$$

where $o(t)$ is a vector of independent observables and $\varepsilon(t)$ is a vector of unobservables. Different papers measure the rents $x(t)$ differently, e.g., as changes in the firm's market valuation or as roll-call votes in a bill favored by the group. These studies claim that if interest groups' contributions produce policy favors in return, we should see a positive and statistically significant γ_1 coefficient. However, Ansolabehere et al. (2003) review thirty six papers with this type of regression and report that "in three out of four instances, campaign contributions had no statistically significant effects on legislation or had the 'wrong' sign". More recently, Fowler et al. (2020) use a regression discontinuity design exploiting both close congressional, gubernatorial, and state legislative elections and within-campaign changes in market beliefs about US Senate races and conclude that "corporate campaign contributions do not buy significant political favors—at least not on average." Consider now that we run regression (8) in an environment where players follow an *Opportunistic contract*. Then, the interest group's choice of $s(t-1)$ would be to support period $t-1$ officeholder, who may coincide or not with the next period's one. Nonetheless, the time-t incumbent would transfer positive rents to the interest group regardless of $s(t-1)$ because the party understands that its behavior is part of a non-written contract by which the interest group supports the current incumbent at each election. Hence, coefficient γ_1 would not reflect exchanges between the interest group and the parties but rather the level of incumbency advantage.

This paper adds to the discussion on special interest politics by showing how different *quid pro quo* relations serve the interest group's goal of maximizing rents in the presence of power fluctuations. Given that little research has addressed the dynamics of special interest politics, a first step would be to measure the presence of different *quid pro quo* relations, identifying the groups more likely to engage in each. Fouirnaies and Hall (2014, 2017) have delved into the question of opportunistic behavior. This model's prediction that opportunism is optimal when only small transactions are feasible resonates with their finding that firms always supporting incumbents belong to industries under heavy regulation—a group needing constant but relatively minor favors.

Exclusive relationships are typically associated with ideological preferences (Ferguson and Voth 2008; Acemoglu et al. 2013). However, this paper's results suggest that non-ideological groups can also prefer them. One way to test whether an exclusive relationship reflects an ideological alignment or the desire for rent maximization is to look for cases in which an interest group established a relationship when their need for favors arose first despite the lack of ideological alignment with the incumbent at the time. The appearance of private TV in Italy offers an illustrative example. In the 1980s, the need for regulation of the private TV market was palatable. To position himself in an advantageous position, the businessman Silvio Berlusconi sought connections with the political system. Despite the long-lasting dominance of Christian Democrats in Italian politics, he established a close friendship with the current Prime Minister, the Socialist Bettino Craxi. Berlusconi and Craxi's collaboration survived the latter's loss of the premiership and many of his corruption scandals (Ginsborg, 2003).

Note that the main difficulty in testing the existence of exclusive relationships with the initial incumbent is identifying an interest group with an exogenous starting date (an important qualification, as some interest groups appear because they feel neglected by the current government). Technological changes that bring the appearance of new firms constitute exogenous starting dates. Besides, the appearance of new technologies, like the TV, often creates a need for further regulation, increasing the incumbent's discretion during the initial period, thus reinforcing the interest group's drive towards the initial incumbent, according to my analysis.

7 The Long-lasting Effect of Crises

Disturbances like economic recessions, wars, pandemics, or social agitation can force governments to increase public expenditure or pass major pieces of legislation. In these junctures, the constraints under which incumbents operate in ordinary years are relaxed because of the magnitude of the concurring changes. In this section, I build on previous insights to show that a crisis can have a long-lasting effect on the balance of electoral power.

Consider the previous model with the only difference that there exists one crisis period, denoted by $\breve{t}$, in which there is a higher transactional constraint than in an ordinary period, i.e., $\breve{\tau} > \tau$. For simplicity, I assume that $\breve{\tau} \leq \hat{x}$. A crisis is a shock that occurs only once, the players anticipate its possibility, and it is publicly observed at the beginning of the period. In each pre-crisis period, there is a probability $\lambda > 0$ of a crisis, independently of the current incumbent. Let $\chi(t) \in \{0, n, 1\}$, that establishes whether at period t a crisis has not happened yet, 0, has happened before, 1, or it is currently occurring, n.

Lemma 2 *(i) If $\chi = 0$, a party ℓ's worst contract prescribes, if $\chi = 0$, $s_i = R$ for $i \in \{\ell, r\}$ and rents $(x_\ell, x_r) = (0, \tau)$; and from period $\chi = n$ onwards, the firm's best contract.*

*(ii) If $\chi \in \{n, 1\}$, a party ℓ's worst contract is the maximum-rent r-*Exclusive contract.

Let i and o denote period $\breve{t}$ incumbent and opposition, respectively. For expositional purposes, let $\tau < \bar{\tau}$, where

$$\bar{\tau} \equiv \frac{2\beta m\big((1-\beta(1-\lambda)2a)b + (1-\beta 2a)k\lambda\breve{\tau}\big)}{(1-\beta 2a)k(1-\beta(1-\lambda)2a) - 2m\beta k(1-\beta 2a - \lambda(1-\beta 4a))}.$$ [18]

Proposition 5 *A firm's best contract prescribes:*

(i) if $\chi = 0$, the firm's strategy of an Opportunistic contract *with rents $\{x_\ell(0), x_r(0)\} = \{\tau, \tau\}$;*

(ii) if $\chi = n$, the incumbent i extracts a rent $x_i(n) = \breve{\tau}$ and it is supported after;

*(iii) if $\chi = 1$, the firm's strategy of a i-*Biased contract *with rents $\{x_i(1), x_o(1)\} = \{\tau, \tilde{x}_o\}$, where $\tilde{q} \in (0, 1)$ and $\tilde{x}_o > 0$ solve the system of equations:*

$$(1-\beta)k\breve{\tau} = \beta(w_i^B(i, n; \tilde{x}_o, \tilde{q}) - \underline{w}_i(i, n)), \tag{9}$$

$$(1-\beta)k\tilde{x}_o = \beta(w_o^B(o, 1; \tilde{x}_o, \tilde{q}) - \underline{w}_o(o, 1)). \tag{10}$$

Proposition 5 shows that crises have a long-lasting effect on the balance of electoral power. By increasing the government's discretion, crises allow the party in power to 'rewrite' previous agreements and shift them in its favor. This finding resonates with empirical evidence. Kantor et al. (2013) find that the New Deal spending contributed to the persistence of the Democratic party majorities in the mid-20th century. Several papers document the appearance of similar electoral "loyalties" in

[18] To assume that $\tau < \bar{\tau}$ allows us to focus on the region where the contract is more insightful, i.e., the region where if $\chi = 0$ the firm's best contract is opportunistic.

Italy arising from one-time events, in this case, the redistribution of land both by the Christian Democrats in the 1950 Land Reform (Caprettini et al. 2022) and by the Mussolini regime before (Carillo et al. 2022). Also, the American Civil War and the increase in public expenditure it brought along produced a realignment of corporate interests towards the Republican Party, which helped to sediment Republican electoral dominance during the Gilded Age.

8 Weak Political Parties

I have so far assumed political parties that survive electoral defeats. However, often policy decisions are not taken by long-lived parties but by party leaders who are replaced after losing an election, and thus have a shorter time horizon. For the purpose of this section, I take a *weak political party* to be an organization with no ability to constrain its members' actions; in particular, one unable to discipline its short-sighted leaders to internalize the party's long-term objectives. The baseline model corresponds to the case of perfect party discipline, a strong party system. This section studies the effect of weak parties on the dynamics of the relational contracts and their welfare implications.

Consider the following game. Suppose each party j chooses randomly a leader for the election from an infinite pool of mass 1 of identical leaders, denoted Z^j, who are citizens before they are appointed to leadership. After an electoral victory, the leader chooses the rent and keeps her position until she is electorally defeated, then she leaves the political arena and becomes a mere citizen again. The new party leader is drawn randomly from the pool of candidates.

The stage payoff of the long-lived party is unchanged from the baseline model. Because parties are weak, the leader does not internalize her party's interests beyond her replacement. As long as she keeps her leadership position, she has the same stage payoff as its party; but once she becomes a citizen, she only suffers the rents' burden. The stage payoff of leader $z_\ell \in Z_\ell$ when i is the current incumbent is:

$$v_{z_\ell}(i) = \begin{cases} b - kx_{z_\ell} \text{ if } i = z_\ell \\ -kx_i \text{ if } i \neq z_\ell. \end{cases}$$

The following proposition shows that, intuitively, under a weak party system the firm's best contract is opportunistic and its rent scheme has two regions: one in which rents equal the transactional constraint and other in which the transactional constraint is irrelevant.

Proposition 6 *The firm's best contract is the maximum-rent* Opportunistic contract*, which prescribes the same positive rent extraction for any leader, i.e.,* $x_z = \min\{\tau, \hat{x}^O\}$ *for any* $z \in Z^\ell \cup Z^r$*, where*

$$\hat{x}^O = \frac{2\beta m}{k(1-\beta p(\ell, L))} b \tag{11}$$

and it satisfies that incumbent's enforcement constraint binds at every period in which she holds power.

The intuition behind this result is straightforward. If the leader never comes back when voted out of office, she does not care about whom the firm will support in the future. Hence, it is in the firms' interest to support whoever is in power and extract rents from them at the moment at which they value electoral support.

Although there is no a systematic study of strong and weak political parties in the context of special interests, some evidence scattered across the literature supports Proposition 6. Della Vigna et al. (2016) studied connections between media firms and the Italian prime minister Silvio Berlusconi during 1993-2009, a period in which the Italian party system had imploded and individual candidates were more relevant than party organizations. Della Vigna et al. compare a short-sighted and a forward-looking measure of Bersluconi's power and show that political exchanges in that context were short-term and driven by incumbency. Hickey (2014) shows that in Canada—a Parliamentary system with strong parties—personal connections in lobbying have less importance than in the U.S.

8.1 Firm's Influence Under Different Party Systems

Once we have characterized the firm's best contract under both party systems, a natural question arises: do firm's gain from a political system with weak political parties? In addition, long-lived parties are a characteristic feature of *quid pro quo* exchanges in politics. The literature on relational contracts, mainly inspired by labor relationships, typically assumes that if an agent is fired, she will not reappear in any future period. In that sense, comparing both party systems allows us to understand better the implications of the long-liveness assumption.

Note that undisciplined party leaders face higher stakes than long-lived parties in each election: they can lose both office and party leadership.[19] As a consequence, they value electoral support

[19]That electoral stakes are smaller with long-lived parties is shown, for an accountability model, by Ferejohn (1986);

more than long-lived parties, and one might, thus, conjecture that rent extraction is greater under a weak party system. However, a party's expected value of collaborating with the firm can be higher than for a replaceable candidate, as the long-lived party values electoral support also after it is ousted from power. In a nutshell, a weak party system implies higher stakes for the incumbent in each election but no long-term relationships. As Figure 5 shows, no party system is invariably more extractive than the other and, moreover, the difference between the two systems' rent extraction is non-monotonic. The next proposition formalizes this result.

Proposition 7 *Consider $\hat{\tau}$ and $\hat{x}$ from Proposition 2. There exists a $\hat{\hat{\tau}} \in (\hat{x}^O, \hat{x})$ such that in the firm's best contract:*

(i) If $\tau \leq \hat{\tau}$, the weak and the strong party system extract the same rents,

(ii) If $\tau \in (\hat{\tau}, \hat{\hat{\tau}})$, the weak party system extracts greater rents than the strong party system,

(iii) If $\tau > \hat{x}^O$, the strong party system extracts greater rents than the weak party system.

The intuition of Proposition 7 is as follows. If the amount of rents that can be transferred is sternly limited, the firm's best contract is identical in both party systems. Contracts in each party system gradually differ as transactional constraints ease. A region exists—i.e., $\tau \in (\hat{\tau}, \hat{\hat{\tau}})$—where a firm is strictly better-off in a weak party system by taking advantage of leaders' higher electoral stakes. As Figure shows, the firm can make undisciplined party leaders extract the maximum feasible rent in cases where it can no longer do it with long-lived parties. Lastly, if incumbents' discipline is sufficiently unconstrained, the gains from a long-term relationship outweigh any extra electoral motivation of party leaders. Thus, the firm prefers to cooperate with long-lived parties. As a last remark, note that as *residual uncertainty* tends to zero, the firm becomes indifferent between the two party systems, i.e., formally, if $a + m \to \frac{1}{2}$, then $\hat{x}^O \to \hat{x}$.

for democratization, by Przeworski (1991) and Fearon (2011); and in the presence of malfunctioning accountability, like in patronage politics, by Delgado-Vega (2022). This setting differs from these papers in that the electoralist policy is not a prerogative of the incumbent, as the firm can give its electoral support to the opposition or the incumbent.

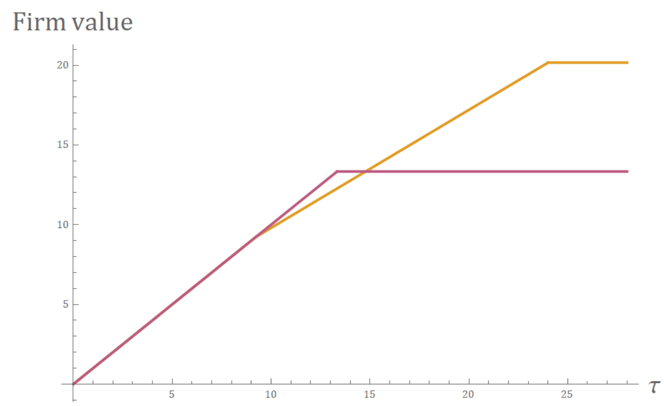


Figure 5: Firm' value under a strong (yellow) and a weak (purple) party system with respect to τ for $m = 0.3$, $\beta = 0.8$, $b = 10$, $a = 0.05$.

9 Entry Cost

This section considers an extension of the baseline game in which the firm faces an entry cost the first time it affects an election. This cost includes the effort of building a political network, gathering funds, hiring lobbying specialists, or establishing an electoral machine. Kerr et al. (2014) document the existence of barriers of entry in lobbying in the US, which are indicative of an entry cost.

Consider a game that differs from Γ only in that, if the firm affects the election outcome for the first time, it incurs in a participation cost. Formally, at the beginning of each period t, if N has been played at all previous periods, either the firm incurs in a cost $\zeta > 0$ and then $s(h_t) \in \Delta\{L, N, R\}$ for h_t onwards, or otherwise $s(h_t) = N$.[20] For simplicity, I assume no transactional constraints, i.e., $\tau \to \infty$.

Definition 5 *A contract that prescribes on-path $s_i = N$ for both $i \in \{\ell, r\}$ is referred as a* Deterred contract*, and it is denoted by $\mathcal{C}^D = (x^D, s^D)$.*

As before, to characterize the firm's best contract it is necessary first to describe the worst equilibrium for a political party. If the firm has participated before on-path, Lemma 1 characterizes a party's worst equilibrium. However, in the Deterred contract, the off-path worst punishment needs to imply participation, and because participation is costly, some off-path threats are not credible given the participation costs.

[20] If the timing is modified such that the cost can be incurred after the incumbent chooses the rent, the main results of the section are not affected, although they are more cumbersome in expositional terms.

Lemma 3 *Let*

$$\bar{\zeta} \equiv \frac{m\beta(2-\beta(1+2(a-m)))}{k(1-\beta)^2(1-\beta 2a)}b.$$

The maximum-rent Deterred contract *is:*

(i) if $\zeta \leq \bar{\zeta}$, such that $(x_\ell, x_r) = (x^D, x^D)$ and

$$(1-\beta)kx^D = \beta(w_\ell^D(\ell; x_D) - \underline{w}_\ell(\ell)),$$

where $\underline{w}_\ell(\ell)$ is such that the firm participates and it is given by a maximum-rent Exclusive contract *with the winner of the first election—i.e.,*

$$\underline{w}_\ell(\ell) = p(\ell, N)v_\ell^L(\ell) + (1-p(\ell, N))v_\ell^R(r) \tag{12}$$

such that $v_\ell^L(\ell)$ ($v_\ell^R(r)$) is given by a maximum-rent ℓ-Exclusive contract *(maximum-rent* r-Exclusive contract*).*

(ii) $\bar{\zeta} < \zeta$, the firm does not participate neither on-path nor off-path and there is no rent extraction.

Lemma 3 shows how off-path threats depend on the entry cost when there has been no on-path participation before. In particular, if $\zeta > \bar{\zeta}$, the firm cannot credibility promise to punish a deviating party because there is no contract that produces sufficient rents to compensate for the entry cost. The following proposition characterizes the firm's best contract.

Proposition 8 *Let*

$$\underline{\zeta} \equiv \frac{m\beta(2-\beta(1-2m))}{k(1-\beta)(1-\beta 2a)}b.$$

where $\underline{\zeta} < \bar{\zeta}$. A firm's best contract is

(i) If $\zeta \leq \underline{\zeta}$, the maximum-rent ℓ-Exclusive contract*.*

(ii) If $\zeta \in [\underline{\zeta}, \bar{\zeta}]$, the maximum-rent Deterred contract*.*

(iii) If $\bar{\zeta} < \zeta$, the firm does not participate neither on-path nor off-path and there is no rent extraction.

In a nutshell, Proposition 8 establishes three regions depending of the entry cost. For a sufficiently low entry cost, the firm participates on-path and off-path. For an intermediate entry cost, the firm does not participate on-path but threatens to participate off-path. For a sufficiently high entry cost, there is no rent extraction because the firm cannot credibility threaten to participate if a deviation occurs.

The Tullock paradox. Tullock (1989) noticed that there are lower lobbying expenditures in the US than one should expect from standard rent-seeking models. Several papers have offered different explanations for this fact. Polborn (2006) argues that because lobbies choose strategically the moment in which they should 'attack' certain piece of legislation they need moderate levels of expenditure. Bombardini and Trebbi (2011) consider that interest groups supply both monetary contributions and votes and the puzzle may be due to the omission of these votes from the calculation of the rate of return. Ansolabehere et al. (2003) argue, among other possible explanations, that, when some legislation helps an entire sector, PACs contributions are subjected to collective action problems.

Proposition 8 offers a parsimonious explanation to the Tullock paradox: rent extraction is not made only in return to interest groups' active support, but also in return to its inaction. Hence, a fraction of rents are *deterrence rents*.

10 The Distribution of Rents' Burden

In this section, I slightly modify our baseline setup to discuss the implications of an asymmetric distribution of the cost of rents between incumbent and opposition. I introduce a parameter $\pi \in [0, 1]$, which I refer as the *distributive parameter*, that enters in parties' payoffs in the following way. Party ℓ's stage payoff when it holds power is

$$y_\ell(\ell) = b - k2(1-\pi)x_\ell,$$

and when party r holds power, it is

$$y_\ell(r) = -k2\pi x_r.$$

Hence, $\pi = \frac{1}{2}$ corresponds to the baseline model, the case in which rents impact equally the party in power and the opposition.

If $\pi > \frac{1}{2}$, rent extraction features a higher cost for the opposition than for the incumbent. It is an appropriate assumption in the case of locally provided public goods. For example, if the firm is competing in local markets in which a part of the demand is serviced by the public sector—like education, health, or transport—rents accrue to the firm when the incumbent mismanages public provision, a practice that it can choose to do more in localities inhabited by its adversary's constituency. Similarly, some regulated aspects—like pollution or telecommunications—or public investments—like infrastructure—have a strong local component.

Let

$$\bar{\beta}(\pi) \equiv \frac{2(1-\pi)}{1+2(2\pi-1)(m-a)},$$

where if $\pi = \frac{1}{2}$, $\bar{\beta}(\pi) = 1$, and if $\pi > \frac{1}{2}$, $\bar{\beta}(\pi) < 1$.

Proposition 9 *Suppose $\pi > \frac{1}{2}$ and $\beta < \bar{\beta}(\pi)$. Propositions 1 and 2 hold. Moreover, absent transactional constraints, Remark 2 characterizes the unique firm's best contract that is (on-path) stationary.*

Besides, maximal rent extraction increases with π and, if there are no transactional constraints and $\beta \geq \bar{\beta}(\pi)$, it tends to infinity.

Proposition 9 establishes, first, that the baseline model results generalize to cases where extraction costs are higher for the opposition than for the incumbent. This result follows because the main insight of Proposition 1—that the firm benefits from concentrating rent extraction on one party, namely, the initial incumbent—also generalizes.

Moreover, the higher π, the greater the rents accruing to the firm in equilibrium. Two effects coincide to produce this result. Firstly, consider party ℓ's worst contract, the *r-Exclusive contract*, and fix its rent $\bar{x}_r^R$. As π rises, party ℓ's punishment through the *r-Exclusive contract* becomes worse because its adversary rents become more harmful. As a consequence, the rent $\bar{x}_\ell^L$ that makes party ℓ's enforcement constraint (1) bind also rises. Secondly, note that rents $\bar{x}_r^R$ and $\bar{x}_\ell^L$ are jointly determined, because the maximum rent that the firm can extract from one party depends on the threat of supporting the other party and extracting from it the maximum rent. As a result, ceteris paribus, a greater $\bar{x}_r^R$ allows the firm to extract a greater $\bar{x}_\ell^L$. This complementarity through off-path threats makes rents tend to infinity for a sufficiently high discount factor.

If $\pi < \frac{1}{2}$, rent extraction features a higher cost for the incumbent than for the opposition. This is not the most natural parametrization, as in most cases the incumbent enjoys sufficient discretion to partially target the rents' cost towards its adversary's constituency. However, this assumption is appropriate if rent extraction has a substantial opportunity cost for the incumbent—for instance, because it requires effort or time to enact the corresponding legislation or because funds that could be devoted to the incumbent's preferred projects have to be deflected to pay the firm. For simplicity, I analyze the setting under the assumption of no transactional constraints and no incumbency advantage.[21]

The following lemma characterizes the scheme that gives maximal rents in a *Exclusive contract*.

Lemma 4 *Suppose $\pi < \frac{1}{2}$ and $\tau \to \infty$. The maximum-rent ℓ-*Exclusive contract *prescribes that party ℓ makes positive rent extraction every period until it is ousted from power for the first time and party r makes positive rent extraction every period it holds power thereafter. Rents satisfy the following:*

(i) Every period in which one party extracts a positive rent it extracts the same rent, which I denote by $(\bar{x}_\ell^{L'}, \bar{x}_r^{L'})$.

(ii) Rents $\bar{x}_\ell^{L'}$ and $\bar{x}_r^{L'}$ are such that the incumbent's enforcement constraint 1 binds every period in which it extracts a positive rent.

*An analogous description applies to the maximum-rent r-*Exclusive contract.

Let

$$\bar{\bar{\beta}}(\pi) \equiv \frac{4}{2 + (1+2m)\pi + \sqrt{4 + (1+2m)\pi(8m - 3(1+2m)\pi)}}.$$

such that if $\pi = 0$ or $\pi = \frac{1}{2}$, $\bar{\bar{\beta}}(\pi) = 1$ and if $\pi \in (0, \frac{1}{2})$, $\bar{\bar{\beta}}(\pi) < 1$ and U-shaped with respect to π.

Proposition 10 *Suppose $\pi < \frac{1}{2}$, and $\beta < \bar{\bar{\beta}}(\pi)$. The maximum-rent ℓ-*Exclusive contract *is a firm's best contract.*

The value of either parties is the lowest possible among all contracts.

[21] Except for the closed form of $\bar{\bar{\beta}}(\pi)$, every other result in the section holds for a positive incumbency advantage.

Therefore, we see that for a sufficiently low discount factor the firm's electoral behavior is the same as in Proposition 1, although the scheme of rent extraction differs. Importantly, the firm does not gain from concentrating rent extraction on the initial incumbent. Note that, keeping the quantity of rents and the electoral strategy fixed, if $\pi < \frac{1}{2}$, a party is better-off if the rents are extracted by its adversary. In the initial periods, the initial incumbent does the rent extraction by itself because it is the only party with the ability to make front-load payments; but the initial opposition does the rent extraction from the moment it attains office for the first time onwards. In this contract—as soon as there has been one change of government—electoral support and rent extraction do not coincide in the same party. This result has interesting derivatives into empirical research. Our first inclination when studying *quid pro quo* agreements is to look for direct exchanges between a party and a firm. Proposition 10 offers a more nuanced picture: a firm and its favored party may use another party as an agent in their exchange, shifting to it the burden of rent extraction in most of periods and disciplining it through off-path threats.

The bound $\bar{\bar{\beta}}(\pi)$ on the discount factor excludes some values of β close to 1 for which either the equilibrium rents tend to infinity, or the firm behaves in a non-stationary way during the initial periods supporting the initial opposition until it wins office for the first time and thereafter following the *ℓ-Exclusive contract*. This non-stationary behavior of the firm appears because, if β is close to 1, equilibrium rents are sufficiently high, and thus the initial opposition is worse-off holding office than being out of power. Because this is unrealistic, I exclude this case from the main discussion and provide a bound for the discount factor under which the *ℓ-Exclusive contract* is the firm's optimal.

Proposition 11 *Suppose $\pi \in (0, \frac{1}{2})$, and $\beta < \bar{\bar{\beta}}(\pi)$, then maximal rent extraction can evolve non-monotonically with respect to the level of commonality π.*

The intuition for this result follows directly from the fact that rent extraction is done by the two parties. Consider the initial incumbent's rent $\bar{x}_\ell^{L'}$ extracted in the initial period, which makes its enforcement constraint bind:

$$(1 - \beta)(1 - \pi)k\bar{x}_\ell^{L'} = \beta(w_\ell^{L'}(h_0; \bar{x}_\ell^{L'}, \bar{x}_r^{L'}) - \underline{w}_\ell(\ell)),$$

where party ℓ's punishment is given by the maximum-rent *r-Exclusive contract*, and thus its worst pre-election value involves rents extracted by both parties, i.e., $\underline{w}_\ell(\ell) = w_\ell^{R'}(h_0; \bar{x}_r^{R'}, \bar{x}_\ell^{R'})$. Now, keeping the rents fixed, we can see that the effect of an increase in the distributive parameter π is two-fold. On one hand, party ℓ's punishment becomes worse because its adversary rents become more harmful, which leads party ℓ to extract a higher rent—an effect already present if $\pi \geq \frac{1}{2}$ (Proposition 9). On the other hand, party ℓ's on-path pre-election value worsens, as it includes rents extracted by party r that now are more harmful, what leads party ℓ to extract a lower rent. The possibility of a non-monotonicity follows from these two effects having opposite signs.

11 Conclusions

This paper models the agreements between political parties and interest groups as relational contracts. This modeling choice captures that these agreements lack third-party enforcement; thus, they need to be self-sustained by the credible threat of a break-up of cooperation. In the model, two political parties repeatedly compete for office in a setting where electoral outcomes are uncertain. In each period, the party in power decides the rent transferred to the interest group. After that, the interest group chooses to support one of the parties, increasing its probability of being in power next period. Rents bear a cost for both parties, and a transactional constraint—a measure of the governments' discretion—limits the rents that can be transferred each period. At the center of our model are the fluctuations of political power, which changes hands over time.

The most relevant insight relates the interest group's best contract with electoral uncertainty. As the firm faces sufficiently great political fluctuations, the *Exclusive contract* with the initial incumbent is the unique firm's best contract. On the contrary, in a stable political arena, the firm's best contract is the *Opportunistic contract.* Intuitively, in an unstable political arena, incumbents know their hold on power to be transitory and thus, give more relevance to the firm's "loyalty" after an electoral defeat, which leads to the superior performance of long-term relationships. Besides, this paper shows that the interest group's best strategy in the presence of exogenous power fluctuations depends on the interest group's characteristics and the institutional setting. These result, and more generally the centrality of incumbency in my analysis, opens interesting directions for the reinterpretation of empirical evidence, for example, in public procurement studies.

This paper is an invitation for future research on special interest politics that puts credibility and dynamics on center stage. A first step in this direction would be the study of settings with multiple organized interests. Interest groups that belong to the same industry may benefit from each other's rents—e.g., if rents favor the industry as a whole—or may suffer from them—e.g., if rents give an advantage to one group over the other. Similarly, interest groups may have conflicting political preferences but be unaffected by each other's rents. Furthermore, given the centrality of the initial period when incumbents' discretion is high, it would be interesting to investigate the dynamics generated when different interest groups start their activity in different periods.

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Appendix

A Proofs

Following Section 3.2, I hereafter refer to a contract as an *equilibrium contract* if it satisfies Definition 3.

Notation: To describe the set of on-path histories of a given contract, denoted $\mathcal{H}$, note that any on-path history $h \in \mathcal{H}$ is characterized by the calendar time $T(h)$; the history of electoral outcomes until $T(h)$, that is, the number $N(h)$ of government changes and the times $G_1(h), \ldots, G_{N(h)}(h)$ in which these changes occurred; the sequence of realizations of the randomization device until $T(h)$, $\varpi_0, \ldots, \varpi_{T(h)}$. So the set of feasible histories is

$$\mathcal{H} \equiv \{(t, n, g_1, \ldots, g_n, \varpi_1, \ldots, \varpi_t) \in \mathbb{N}^{n+2} : 0 < g_1 < \ldots < g_n \leq t \text{ and } n \leq g_n\}.$$

For any $t < t'$ and any pair of histories $h = (n, t, g_1, \ldots, g_n, \varpi_0, \ldots, \varpi_t) \in \mathcal{H}$ and $h' = (n', t', g'_1, \ldots, g'_{n'}, \varpi'_0, \ldots, \varpi'_{t'}) \in \mathcal{H}$, we say that h' follows h if $n \leq n'$,

$$0 < g_1 = g'_1 < g_2 = g'_2 < \ldots < g_n = g'_n \leq g'_{n'},$$

and

$$\varpi_0 = \varpi'_0, \ldots, \varpi_t = \varpi'_t.$$

In that case, let $Pr(h'|s, h)$ denote the probability that h' occurs conditional on h and the firm's on-path strategy s. Also, let $\Omega(s; h, h') \equiv \beta^{t'-t} Pr(h'|s, h)$.

Let $\mathcal{H}(N)$ denote the set of histories $h \in \mathcal{H}$ such that $N(h) = N$, that is, the set of histories in which there are N number of governments changes. Lastly, let $\mathcal{H}^*(N)$ denote the set of histories $h \in \mathcal{H}(N)$ such that $T(h) = G_N(h)$, that is, the set of histories in which there are N number of governments changes and the last government change occurs in the last period.

Preliminary steps to the proof of Propositions 1 and 10

Note first that as each period rent is bounded by τ, player's values are bounded. If the transactional constraint is assumed to be sufficiently high so that the incumbent chooses unconstrained at any period, I slightly abuse notation, stating that $\tau \to \infty$; however, I still assume that rents are bounded, i.e., $\tau < \infty$.

Let $\mathfrak{F}(\alpha)$ be the set of contracts such that, at every on-path history $h \in \mathcal{H}$, the present incumbent i's value

$$v_i(i, h) \geq (1 - \beta)b + \beta\alpha, \tag{13}$$

where α is an arbitrarily specified value that satisfies:

$$-k\pi\tau < (1 - \beta)b + \beta\alpha < b, \tag{14}$$

which are bounds satisfied by any feasible party's value. Hereafter, we refer to any contract in $\mathfrak{F}(\alpha)$ as a *compatible* contract. We consider the problem of characterizing the compatible contract that gives the lowest expected pre-election value for an incumbent—i.e., the incumbent's worst compatible contract. To that end, Lemma 5 characterizes the compatible contract that maximizes rent extraction and Lemma 6 relates this contract to the incumbent's worst compatible contract.

Lemma 5 *Let $\tau \to \infty$ and let $\mathcal{C} = (x, s)$ be a compatible contract. There exists another compatible contract $\mathcal{C} = (x', s')$ with the same on-path strategy for the firm, i.e., $s' = s$, and such that x' yields a weakly greater expected discounted rent at history h_0 and further satisfies the following properties:*

(i) if $\pi \in [0, \frac{1}{2}]$, the rents extracted by each party in the first period they hold office, i.e., $(x_\ell(h_0), \{x_r(h)\}_{h \in \mathcal{H}^*(1)})$, are positive and, at any other history, rents are zero;

(ii) if $\pi \in [\frac{1}{2}, 1)$, the rent extracted by the initial incumbent, i.e., $x_\ell(h_0)$, is positive and, at any other history, rents are zero.

In both cases, positive rents are such that (14) binds in the period rent extraction occurs.

Proof of case $\pi \in [0, \frac{1}{2}]$

The structure of the proof is as follows: I start with an arbitrary compatible contract $\mathcal{C} = (x, s)$ and construct through various steps a contract $\mathcal{C}^*$ which satisfies the following properties: (a) $\mathcal{C}^*$

gives a weakly greater level of rent extraction than $\mathcal{C}$, (b) $\mathcal{C}^*$ is compatible, and (c) $\mathcal{C}^*$ satisfies Lemma 5.

Step 1. *We first construct a compatible contract $\mathcal{C}^1$ that gives a weakly greater level of rent extraction than $\mathcal{C}$ and such that the incumbent ℓ's constraint (14) binds at h_0.*

Suppose that contract $\mathcal{C}$ prescribes a rent in history h_0 such that the incumbent's constraint (14) does not bind. Then, consider an alternative contract, denoted $\mathcal{C}^1$, identical on-path to contract $\mathcal{C}$ except that the incumbent extracts a rent in history h_0 such that its constraint binds. Such rent satisfies the incumbent's constraint (14), hence if $\mathcal{C}$ is compatible, so it is $\mathcal{C}^1$. By construction, expected rent extraction at h_0 is greater in contract $\mathcal{C}^1$ than in contract $\mathcal{C}$.

Step 2. *Consider the compatible contract $\mathcal{C}^1 = (x^1, s)$ constructed in Step 1. In this step, we construct a compatible contract $\mathcal{C}^2 = (x^2, s)$ that gives a weakly higher rent extraction and such that for any $h \in \mathcal{H}(0)$, $x^2(h) > 0$ if and only if $h = h_0$.*

Suppose contract $\mathcal{C}^1$ prescribes a positive rent in some history $\hat{h} \in \mathcal{H}(0)$ with $\hat{h} \neq h_0$—i.e., $x^1_\ell(\hat{h}) > 0$. We consider an alternative contract, denoted $\tilde{\mathcal{C}}$, identical on-path to contract $\mathcal{C}^1$ except for (i) contract $\tilde{\mathcal{C}}$ prescribes no rent extraction in history $\hat{h} \in \mathcal{H}(0)$—i.e., $\tilde{x}_\ell(\hat{h}) = 0$—and (ii) in the initial period, contract $\tilde{\mathcal{C}}$ prescribes

$$\tilde{x}_\ell(h_0) = x^1_\ell(h_0) + \Omega(s; h_0, \hat{h}) x^1_\ell(\hat{h}).$$

By construction of $\tilde{x}_\ell(h_0)$, (i) both contracts give the same expected rent extraction and, (ii) because party ℓ's constraint (14) at h_0 is the same in contracts $\mathcal{C}^1$ and $\tilde{\mathcal{C}}$, if $\mathcal{C}^1$ is compatible, so it is $\mathcal{C}^2$. Using the same procedure for any history $h \in \mathcal{H}(0)$ different from h_0, we construct a contract $\mathcal{C}^2 = (x^2, s)$ such that $x^2(h) > 0$ if and only if $h = h_0$ and such that it gives the same expected rent extraction as $\mathcal{C}^1$ at h_0.

Step 3. *Consider the compatible contract $\mathcal{C}^2 = (x^2, s)$ constructed in Step 2. In this step, we construct a compatible contract $\mathcal{C}^3 = (x^3, s)$ that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^2$ and such that for any $h \in \mathcal{H}(1)$, $x^3_r(h) > 0$ if and only if $h \in \mathcal{H}^*(1)$ and $x^3_r(h) > 0$ makes party r's constraint (14) bind.*

Suppose in contract $\mathcal{C}^2 = (x^2, s)$ there exists a history $\hat{h} \in \mathcal{H}^*(1)$ in which party r's constraint (14) does not bind. Then, consider an alternative contract, denoted $\tilde{\mathcal{C}}$, identical on-path to contract

$\mathcal{C}^2$ except for (i) in $\hat{h}$, contract $\tilde{\mathcal{C}}$ prescribes $\tilde{x}_r(\hat{h}) = x_r^2(\hat{h}) + \varepsilon$, where $\varepsilon > 0$, and (ii) in h_0, contract $\tilde{\mathcal{C}}$ prescribes

$$\tilde{x}_\ell(h_0) = x_\ell^2(h_0) - \frac{\pi}{1-\pi}\Omega(s; h_0, \hat{h})\varepsilon.^{22}$$

Note that contract $\tilde{\mathcal{C}}$ implies an increase in total rent extraction of $(1 - \frac{\pi}{1-\pi})\Omega(s; h_0, \hat{h})\varepsilon$ with respect to contract $\mathcal{C}^2$. Further, by construction of $\tilde{x}_\ell(h_0)$, party ℓ's constraint at h_0 is unchanged, and thus, if $\mathcal{C}^2$ is compatible, so it is $\tilde{\mathcal{C}}$. Therefore, by the same logic as the previous paragraph, there exists a contract $\mathcal{C}^3 = (x^3, s)$ that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^2$ and prescribes a rent $x_r^3(h) > 0$ at each history $h \in \mathcal{H}^*(1)$ such that party r's constraint binds, and no rent extraction for any other history $h \in \mathcal{H}(1)$.

Step 4. *Consider the compatible contract $\mathcal{C}^3 = (x^3, s)$ constructed in Step 3. In this step, we construct a compatible contract $\mathcal{C}^4 = (x^4, s)$ that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^3$ and such that $x_\ell^4(h) = 0$ for any history $h \in \mathcal{H}(2)$.*

Suppose there exists a history $\hat{h} \in \mathcal{H}(2)$ in which contract $\mathcal{C}^3$ prescribes $x_\ell^3(\hat{h}) > 0$. Then, consider an alternative contract, denoted $\tilde{\mathcal{C}}$, identical on-path to contract $\mathcal{C}^3$ except for (i) there is no rent extraction at history $\hat{h}$—i.e., $\tilde{x}_\ell(\hat{h}) = 0$—(ii) in any history $\hat{\hat{h}}$ such that history $\hat{\hat{h}}$ follows $\hat{h}$ and $\hat{\hat{h}} \in \mathcal{H}^*(1)$—which I hereafter denote by $\hat{h}_{*(1)}$ for clarity—contract $\tilde{\mathcal{C}}$ prescribes rent

$$\tilde{x}_r(\hat{h}_{*(1)}) = x_r^3(\hat{h}_{*(1)}) + \frac{\pi}{1-\pi}\Omega(s; \hat{h}_{*(1)}, \hat{h})kx_\ell^3(\hat{h}), \tag{15}$$

and (iii) in the initial period, contract $\tilde{\mathcal{C}}$ prescribes

$$\tilde{x}_\ell(h_0) = x_\ell^3(h_0) + \big(1 - \frac{\pi^2}{(1-\pi)^2}\big)\Omega(s; h_0, \hat{h})x_\ell^3(\hat{h}). \tag{16}$$

To see why $\tilde{x}_r(\hat{h}_{*(1)})$ is incentive compatible, note that (see definition of pre-election value in Section 3.2)

$$\beta\tilde{w}_r(\hat{h}_{*(1)}) = \beta w_r^3(\hat{h}_{*(1)}) + (1-\beta)\pi\Omega(s; \hat{h}_{*(1)}, \hat{h})kx_\ell^3(\hat{h}); \tag{17}$$

[22] Note that this new rent is well-defined because, if $\pi > 0$, it cannot be that $\mathcal{C}^2$ prescribes $x_\ell^2(h_0) = 0$. The incumbent's pre-election value in h_0 satisfies that $w_\ell^2(\ell, h_0) > \tilde{w}_\ell(\ell, h_0) \geq \alpha$, and hence $x_\ell^2(h_0) > 0$ is necessary to make constraint (14) bind in h_0.

and party r's enforcement constraint in any history $\hat{h}_{*(1)}$ is

$$(1-\beta)(1-\pi)k\tilde{x}_r(\hat{h}_{*(1)}) = \beta(\tilde{w}_r(\hat{h}_{*(1)}) - \alpha).$$

Thus, substituting (15) and (17), we see that party r's constraint (14) in any history $\hat{h}_{*(1)}$ is the same as in $\mathcal{C}^3$. To see why $\tilde{x}_\ell(h_0)$ is incentive compatible, note that

$$\beta\tilde{w}_\ell(h_0) = \beta w_\ell^3(h_0) + (1-\beta)(1-\pi)\big(1 - \frac{\pi^2}{(1-\pi)^2}\big)k\Omega(s; h_0, \hat{h})x_\ell^3(\hat{h}); \tag{18}$$

and party ℓ's constraint in h_0 is

$$(1-\beta)(1-\pi)k\tilde{x}_\ell(h_0) = \beta(\tilde{w}_\ell(h_0) - \alpha).$$

Thus, substituting (16) and (18), we see that party ℓ's constraint (14) in h_0 is the same as in $\mathcal{C}^3$. Hence, if $\mathcal{C}^3$ is compatible, so it is $\tilde{\mathcal{C}}$. Note that contract $\tilde{\mathcal{C}}$ implies an increase in total rent extraction of $\frac{\pi(1-2\pi)}{(1-\pi)^2}\Omega(s; h_0, \hat{h})x_\ell^3(\hat{h})$ with respect to contract $\mathcal{C}^3$. Using the same procedure for any history $h \in \mathcal{H}(2)$, we construct a contract $\mathcal{C}^4$ that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^3$ and prescribes $x_\ell^4(h) = 0$ for any history $h \in \mathcal{H}(2)$.

Step 5. *Consider the compatible contract $\mathcal{C}^4 = (x^4, s)$ constructed in Step 4. In this step, we construct the compatible contract $\mathcal{C}^*$, which gives a weakly greater expected rent extraction at h_0 and satisfies Lemma 5.*

Using the same reasoning as in Step 1, there exists a compatible contract that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^4$ and such that no rent is extracted in any history $h \in \mathcal{H}(3)$. Lastly, note that our argument in Step 3 extends to any history $h \in \mathcal{H}(4)$. Thus, a contract that maximizes expected rent extraction at h_0 must prescribe no rent extraction in any history $h \in \mathcal{H}(4)$. By applying this two arguments recursively, we construct contract $\mathcal{C}^*$, which gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}^4$, it is compatible, and satisfies Lemma 5. □

Proof of case $\pi \in [\frac{1}{2}, 1)$

I start with an arbitrary compatible contract $\mathcal{C} = (x, s)$ and construct a contract $\mathcal{C}^*$ that (a) gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}$, (b) it is compatible, and (c) satisfies the property (ii) of Lemma 5.

Steps 1 and **2** of the proof for the case $\pi \in [0, \frac{1}{2}]$ follow directly for $\pi \in [\frac{1}{2}, 1)$.

Step 3. *Consider the compatible contract* $\mathcal{C}^2 = (x^2, s)$ *constructed in Step 2. In this step, we construct a compatible contract* $\mathcal{C}^* = (x^*, s)$ *that gives a weakly greater expected rent extraction at* h_0 *than* $\mathcal{C}^2$ *and such that, for any* $h \in \mathcal{H}(n)$ *with* $n \geq 1$, $x^*(h) = 0$.

Suppose that contract $\mathcal{C}$ prescribes for some history $\hat{h} \in \mathcal{H}(1)$ a rent $x_r(\hat{h}) > 0$. Then, consider an alternative contract, denoted $\tilde{\mathcal{C}}$, identical on-path to contract $\mathcal{C}$ except for (i) there is no rent extraction at $\hat{h} \in \mathcal{H}(1)$—i.e., $\tilde{x}_r(\hat{h}) = 0$—(ii) in h_0, it prescribes:

$$\tilde{x}_\ell(h_0) = x_\ell(h_0) + \frac{\pi}{1-\pi}\Omega(s; h_0, \hat{h})x_r(\hat{h}). \tag{19}$$

To see that $\tilde{x}_\ell(h_0)$ is incentive compatible, see that

$$\beta\tilde{w}_\ell(h_0) = \beta w_\ell(h_0) + (1-\beta)\pi\Omega(s; h_0, \hat{h})kx_r(\hat{h}); \tag{20}$$

and party ℓ's constraint in h_0 is

$$(1-\beta)(1-\pi)k\tilde{x}_\ell(h_0) = \beta(\tilde{w}_\ell(h_0) - \alpha).$$

Thus, substituting (19) and (20), we see that party ℓ's constraint in h_0 is the same as in $\mathcal{C}$. Hence, if $\mathcal{C}$ is compatible, so it is $\tilde{\mathcal{C}}$. Note that contract $\tilde{\mathcal{C}}$ implies an increase in total rent extraction of $\frac{2\pi-1}{1-\pi}\Omega(s; h_0, \hat{h})x_r(\hat{h})$ with respect to contract $\mathcal{C}$. By applying recursively this argument, we construct a contract $\mathcal{C}^*$ that gives a weakly greater expected rent extraction at h_0 than $\mathcal{C}$, it is compatible, and satisfies Lemma 5. □

Lemma 6 *Consider any history* $h \in \mathcal{H}$ *with incumbent* i *and opposition* j.

(i) The worst compatible contract for j *from* h *onwards is the firm's best compatible contract from* h *onwards.*

(ii) The worst compatible contract for i *from* h *onwards is such that for any history* h' *that follows* h *and such that* $N(h') = N(h)$, *no rent is extracted and the firm supports the opposition* j,

and from any history $h'' \in \mathcal{H}^*(N(h)+1)$ onwards, the firm's best compatible contract from h'' onwards is played.

Proof:

Without loss of generality, consider h_0 as our history of reference and party r as the opposition party.

Proof of Part (i)

Let $\mathcal{C}^w = (x^w, s^w)$ denote the opposition's worst compatible contract. First, by the same logic as Step 1 of the proof of Lemma 5, in $\mathcal{C}^w$ the incumbent's constraint (14) must bind at h_0—i.e., $v_\ell(\ell, h_0) = (1-\beta)b + \beta\alpha$. Now, since the game is constant sum, one can write the opposition's value as a function of the other two player's values, i.e., $v_r^w(\ell, h_0) = \beta b - \beta\alpha - k v_f^w(\ell, h_0)$. Thus, minimizing $v_r^w(\ell, h_0)$ is equivalent to maximizing $v_f^w(\ell, h_0)$ among all contracts in which the incumbent's constraint (14) binds at h_0. Further, from Lemma 5, note that incumbent's constraint (14) at h_0 also binds in the firm's best compatible contract. Hence, it is not a constraint in the maximization of $v_f^w(\ell, h^0)$. Therefore, to minimize $v_r^w(\ell, h_0)$ is to maximize $v_f^w(\ell, h_0)$. □

Proof of Part (ii)

Step 1. First, note that in party ℓ's worst compatible contract, party ℓ's constraint (14) binds at h_0. So if contract $\tilde{\mathcal{C}}$ with $\tilde{x}_\ell(h_0) \geq 0$ and $\tilde{w}_\ell(h_0)$ is party ℓ's worst contract, it satisfies:

$$(1-\beta)(b - k\tilde{x}_\ell(h_0)) + \beta\tilde{w}_\ell(h_0) = (1-\beta)b + \beta\alpha.$$

and hence, there exists another contract, denoted $\mathcal{C}^w = (x^w, s^w)$, that minimizes party ℓ's value by prescribing $x_\ell^w(h_0) = 0$ and $w_\ell^w(h_0) = \alpha$. Hereafter, I focus on contract $\mathcal{C}^w$, which has

$$w_\ell^w(h_0) \equiv p(\ell, s^w(h_0))v_\ell^w(\ell, h^\ell) + (1 - p(\ell, s^w(h_0)))v_\ell^w(r, h^r). \tag{21}$$

where $T(h^\ell) = T(h^r) = 1$, $h^\ell \in \mathcal{H}(0)$ and $h^r \in \mathcal{H}^*(1)$.

Step 2. *In this step, we characterize the sequence of actions following h^ℓ.*

Note that party ℓ's value $v_\ell^w(\ell, h^\ell)$ does not affect player's incentives in any history that follows h^r. Using Step 1 reasoning, there exists a contract $\mathcal{C}^{w,1} = (x^{w,1}, s^{w,1})$ with identical actions as $\mathcal{C}^w$

at h_0 and at any history that follows h^r, and that prescribes $x_\ell^{w,1}(h^\ell) = 0$ and $w_\ell^{w,1}(h^\ell) = w_\ell^w(h_0)$. By construction, (i) $\mathcal{C}^{w,1}$ prescribes a $w_\ell^{w,1}(\ell, h_0) \leq \alpha$, and (ii) if $\mathcal{C}^w$ is compatible, so it is $\mathcal{C}^{w,1}$.

Step 3. *In this step, we characterize the sequence of actions following h^r.*

We can now rearrange (21) to obtain

$$w_\ell^w(h_0) = \frac{1}{1 - \beta p(\ell, s^w(h_0))}(p(\ell, s^w(h_0))(1-\beta)b + (1 - p(\ell, s^w(h_0)))v_\ell^w(r, h^r)),$$

and it follows from $v_\ell^w(r, h^r) < b$ that $w_\ell^w(h_0)$ is minimized for $s^w(h_0) = R$. Therefore, to minimize party ℓ's value contract $\mathcal{C}^w$ must prescribe $s^w(h) = R$ and $x^w(h) = 0$ for any $h \in \mathcal{H}(0)$. Lastly, it follows from Part (i) of the Lemma that party ℓ's value once it becomes the opposition party for the first time—i.e., $v_\ell(r, h^r)$—is minimized by the firm's best compatible contract from history h^r onwards. Therefore, a party's worst compatible pre-election value given an arbitrary α is:

$$\underline{w}(\alpha) = \frac{1}{1 - \beta p(\ell, R)}(p(\ell, R)(1-\beta)b + \beta(1 - p(\ell, R))v_j^*(i; \alpha)), \tag{22}$$

where $v_j^*(i; \alpha)$ denotes the opposition's value in the firm's best compatible contract from that period onwards. □

B For $\pi \in [\frac{1}{2}, 1]$

Lemma 7 *Let $\pi \in [\frac{1}{2}, 1]$ and let $\mathcal{C} = (x, s)$ be a compatible contract that maximizes rent extraction for a given α and satisfies Lemma 5. Then, $s(h) = L$ for any on-path history $h \in \mathcal{H}$.*

Proof:

The proof is as follows: to define the firm's best compatible contract, instead of maximizing among all possible contracts, we can maximize among all contracts that prescribe a rent scheme satisfying Lemma 5—which implies maximizing w.r.t. the firm's on-path strategy.

From Lemma 5, contract $\mathcal{C} = (x, s)$ exists and it prescribes a rent $x_\ell(h_0)$ that makes party ℓ's constraint (14) bind at h_0:

$$(1-\beta)kx_\ell(h_0) = \beta(w_\ell(\ell, h_0) - \alpha), \tag{23}$$

The above equation shows that, among all contracts that satisfy Lemma 5, the contract $\mathcal{C} = (x, s)$ that maximizes $x_\ell(h_0)$ is such that s maximizes $w_\ell(\ell, h_0)$, so it prescribes $s(h) = L$ for any on-path $h \in \mathcal{H}$. Recall that the firm's enforcement constraint is always satisfied, hence the choice of s is unconstrained. $\square$

Proof of Proposition 1:

Together, Lemmas 5 and 7 characterize a firm's best compatible contract from some history h onwards as a function of an arbitrary value α. From Lemma 6 characterizing a party's worst equilibrium contract is equivalent to characterizing the firm's best contract. Hence, we have obtained a party's worst compatible pre-election value from any history h onwards as a function of α, that is, we have obtained an application $\alpha \to \underline{w}(\alpha)$, where $\underline{w}(\alpha)$ is given by (22) where

$$v_j^*(i; \alpha) = \beta \frac{(2(1 - p(\ell, L)) - 2\pi(1 - \beta 2a))b + 2\pi(1 - \beta 2a)\alpha}{2(1 - \beta 2a)(1 - \pi)},$$

which is the value of party r in h_0 in contract $\mathcal{C}^L = (x^L, s^L)$ satisfying Lemmas 5 and 7.

Step 1 shows that $\alpha \to \underline{w}(\alpha)$ has a fixed point. Step 2 gives a closed form solution for this fixed point and thus, characterizes a party's worst equilibrium contract and a firm's best equilibrium contract. Step 3 constructs a firm's best contract which is on-path stationary.

Step 1. *There exists a worst equilibrium contract for a party. That is, we show that there exists an α^* such that $\underline{w}(\alpha^*) = \alpha^*$ and it is a fixed point of the application $\alpha \to \underline{w}(\alpha)$.*

By the Intermediate Value Theorem, this follows from (i) $\underline{w}(\alpha)$ being continuous and linear w.r.t. α, (ii) $\underline{w}(\alpha) > \alpha$ as $\alpha \to -\infty$, and (iii) $\underline{w}(\alpha) < \alpha$ for some α.

Step 2. *We solve the fixed point α^* of the application $\alpha \to \underline{w}(\alpha)$ and derive from it a firm's best equilibrium contract.*

As $\alpha \to \underline{w}(\alpha)$ is a linearly increasing contraction, it has a unique fix point. Evaluated on α^*, the initial incumbent's rent given by (23) becomes:

$$x_\ell(h_0; \alpha^*) = \frac{2\beta m b(1 - \beta(1 - p(\ell, L))}{(1 - \beta)(1 - \beta 2a)k(2(1 - \pi) - \beta(1 + 2(2\pi - 1)(m - a)))};$$

an increasing function of m, a, β, and π, bounded for any $\beta < \bar{\beta}(\pi)$. Therefore, the firm's best equilibrium contract, denoted by $\mathcal{C}^* = (x^*, s^*)$, prescribes (i) $x^*(h_0) = x_\ell(h_0; \alpha^*)$, (ii) $x^*(h) = 0$ for any on-path history $h \neq h_0$, and (iii) $s(h) = L$ for any on-path history $h \in \mathcal{H}$.

Step 3. *Consider the equilibrium contract constructed in Step 2* $\mathcal{C}^* = (x^*, s^*)$ *and let* $\mathcal{C}^L = (\hat{x}, s^L)$ *denote the on-path stationary* ℓ-Exclusive equilibrium contract *with maximum rents as describes in Remark 2. In this step, we show that contract* $\mathcal{C}^L$ *can attain the same initial value for the three players as contract* $\mathcal{C}^*$*.*

First, note that party ℓ's value at h^0 (i) in $\mathcal{C}^*$, is $v^*_\ell(h_0) = -(1-\beta)kx^*_\ell(h_0) + \frac{1-\beta(1-p(\ell,L))}{1-\beta 2a}b$ and (ii) in $\mathcal{C}^L$, is $v^L_\ell(h_0) = \frac{1-\beta(1-p(\ell,L)}{1-\beta 2a}(b - k\hat{x})$. Then, observe that rent $\hat{x}$ satisfies the following equation:

$$\frac{1-\beta(1-p(\ell,L))}{1-\beta 2a}(b - 2(1-\pi)k\hat{x}) = -(1-\beta)2(1-\pi)kx^*_\ell(h^0) + \frac{1-\beta(1-p(\ell,L)}{1-\beta 2a}b,$$

which yields

$$\hat{x} = \frac{4\beta m b}{k(2(1-\pi) - \beta(1 + 2(2\pi-1)(m-a)))}.\square$$

Proof of Proposition 2:

The proof is as follows: steps 1, 2, and 3 prove the first part of the result, steps 4 and 5 prove the second and third part of the result.

Step 1. *Let* x^τ *be the strategy that prescribes the incumbent to always extract the maximal rent, that is,* $x^\tau(h) = \tau$ *for any on-path history* $h \in \mathcal{H}$*. In this step, we show that if there exists a* τ *and a* s *such that* $\mathcal{C} = (x^\tau, s)$ *is a compatible contract, then* $\mathcal{C}^O = (x^\tau, s^O)$ *is also compatible for the same* τ*, where* s^O *is the opportunistic strategy.*

First, note that, given strategy x^τ, any on-path pre-election value of the firm is the same—i.e., $\bar{w}_f = \tau$. Thus, at any $h \in \mathcal{H}$,

$$w_\ell(h) = b - w_r(h) - k\tau. \tag{24}$$

Second, note that $\mathcal{C} = (x^\tau, s)$ is compatible if it satisfies for any on-path history $h \in \mathcal{H}$ the incumbent's constraint (14):

$$(1-\beta)(1-\pi)k\tau \leq \beta(w_i(i,h) - \alpha).$$

Let $w_i^O(i)$ ($w_j^O(i)$) denote the incumbent's (opposition's) pre-election value in $\mathcal{C}^O$. If for some τ, there exists at least one on-path history $\hat{h} \in \mathcal{H}$, such that $\mathcal{C} = (x^\tau, s)$ prescribes a pre-election value $w_i(i, \hat{h}) \leq w_i^O(i)$, $\mathcal{C}^O = (x^\tau, s^O)$ is also compatible for the same τ.

We show now that for any compatible contract $\mathcal{C} = (x^\tau, s)$ there exists at least one history $\hat{h} \in \mathcal{H}$, such that $\mathcal{C}$ prescribes a pre-election value $w_i(i, \hat{h}) \leq w_i^O(i)$.

Suppose that contract $\mathcal{C} = (x^\tau, s)$ is such that, for every $h \in \mathcal{H}$, $w_i(i, h) > w_i^O(i)$, and it is compatible. We build now $\mathcal{C}$ by maximizing $w_\ell(\ell, h_0)$ under the constraint of $w_i(i, h) > w_i^O(i)$ for every on-path history $h \neq h_0$. Consider

$$w_\ell(\ell, h_0) = p(\ell, s(h_0))((1-\beta)(b - k(1-\pi)\tau) + \beta w_\ell(\ell, h_1^\ell)) + (1 - p(\ell, s(h_0)))(-(1-\beta)k\pi\tau + \beta w_\ell(r, h_1^r))$$

where $T(h_1^\ell) = T(h_1^r) = 1$, $h_1^\ell \in \mathcal{H}(0)$ and $h_1^r \in \mathcal{H}^*(1)$. From $w_i^O(i) > w_j^O(i)$ and (24), it follows that $w_\ell(\ell, h^\ell) > w_i^O(i) > w_j^O(i) > w_\ell(r, h^r)$. Thus, $s(h_0) = L$ maximizes $w_\ell(\ell, h_0)$. Applying the same reasoning to any other on-path history $h \in \mathcal{H}(0)$, we have that $s(h) = L$ and

$$w_\ell(\ell, h_0) = \frac{p(\ell, L)(1-\beta)(b - k(1-\pi)\tau) - (1 - p(\ell, L))(1-\beta)k\pi\tau}{1 - \beta p(\ell, L)} + (1 - p(\ell, L)) \sum_{h_t^r \in \mathcal{H}^*(1)} \beta^{t+1} p(\ell, L)^t w_\ell(r, h_t^r),$$

where h_t^r is such that $T(h_t^r) = t$ and $h_t^r \in \mathcal{H}^*(1)$; and where $w_\ell(r, h_t^r) = b - w_r(r, h_t^r) - k\tau$. Besides, consider the incumbent's on-path pre-election value in contract $\mathcal{C}^O = (x^\tau, s^O)$,

$$w_i^O(i) = \frac{p(\ell, L)(1-\beta)(b - k(1-\pi)\tau) + (1 - p(\ell, L))(-(1-\beta)k\pi\tau + \beta(b - w_i^O(i) - k\tau))}{1 - \beta p(\ell, L)}.$$

Therefore, $w_\ell(\ell, h_0) > w_i^O(i)$ implies that $w_i^O(i) > w_r(r, h_t^r)$ for at least for one history $h_t^r \in \mathcal{H}^*(1)$.

Step 2. *In this step, we show that there exists a unique transactional constraint $\hat{\tau}$ such that contract $\mathcal{C}^O = (x^\tau, s^O)$ is an equilibrium and its enforcement constraint (1) binds.*

Let transactional constraint $\hat{\tau}(\alpha)$ be such that the incumbent's constraint (14) in contract $\mathcal{C}^O = (x^\tau, s^O)$ binds, i.e.,

$$(1-\beta)k\hat{\tau}(\alpha) = \beta(w_\ell^O(\ell) - \alpha), \tag{25}$$

which yields

$$\hat{\tau}(\alpha) = -\beta \frac{(1 + 2a + 2(m - 2\beta(a+m)))b + (-2 + 4\beta(a+m))\alpha}{2k(-2 + \beta + 2a\beta + 2\beta m + 2\pi - 2\beta\pi)}.$$

First, we characterize for a given α and its associated transactional constraint $\hat{\tau}(\alpha)$ a party ℓ's worst compatible contract from history h_0 onwards, which I denote by $\mathcal{C}^w = (x^w, s^w)$.

Since $\mathcal{C}^O = (x^\tau, s^O)$ is a firm's best contract with constraint (25) binding, it follows from Lemma 6 that contract $\mathcal{C}^w$ prescribes $x^w(h) = 0$ and $s^w(h) = R$ for any $h \in \mathcal{H}(0)$ and $x^w(h) = x^\tau$ and $s^w(h) = s^O$ for any $h \in \mathcal{H}(n)$ with $n \geq 1$.

Once contract $\mathcal{C}^w$ is characterized, we can obtain party ℓ's worst compatible pre-election value from some history h in which it holds power onwards as a function of the arbitrary value α and when the incumbent's constraint (25) binds, which is (22) where $v_j^*(i, h; \alpha)$ is party ℓ's value in $\mathcal{C}^O = (x^\tau, s^O)$ when r holds power and the transactional constraint is $\hat{\tau}(\alpha)$ is given by (25), i.e.,

$$v_j^*(i, h; \alpha) = v_\ell^O(r) = \frac{\beta(1 - 2(a + m))b - 2k(2\pi + \beta k(1 - 2(a + m + \pi)))\hat{\tau}(\alpha)}{2(1 - 2\beta(a + m))}.$$

The application $\alpha \to \underline{w}(\alpha)$ is linearly increasing and by the same reasoning as in the proof of Proposition 1 one can show that it has a unique fix point, which is

$$\alpha^* = \frac{1 - 2m - \pi + 2(-1 + 2\beta)(a(-1 + \pi) - m\pi)}{-4a\beta(1 - \pi) + 2(1 - \pi - 2\beta m\pi)} b.$$

Evaluated on α^*, the transactional constraint $\hat{\tau}(\alpha)$ becomes:

$$\hat{\tau} \equiv \frac{\beta m}{k(1 - \pi - 2\beta(a(1 - \pi) + m\pi)} b.$$

One can check that $\hat{\tau} < \infty$ for any $\beta \leq \bar{\beta}(\pi)$ and $\hat{\tau} < \bar{x}_\ell^L$, where $\bar{x}_\ell^L$ is defined in Remark 2. Therefore, when the transactional constraint $\tau = \hat{\tau}(\alpha^*)$, party ℓ's worst equilibrium contract from h_0 onwards is $\mathcal{C}^w = (x^w, s^w)$.

Step 3. *Consider the transactional constraint $\hat{\tau}$ defined in Step 2. In this step, we show that for $\tau \leq \hat{\tau}$ contract $\mathcal{C}^O = (x^\tau, s^O)$ is an equilibrium and, otherwise, it is not.*

Consider contract $\mathcal{C}^w = (x^w, s^w)$ from Step 2. An *Opportunistic contract* such that $x^w(h) = x^\tau$ is an equilibrium for some τ if and only if

$$(1 - \beta)k\tau \leq \beta(w_\ell^O(\ell) - w_\ell^w(\ell, h_0)).$$

As $w_\ell^O(\ell) - w_\ell^w(\ell, h_0)$ is monotonically decreasing in τ, this condition holds for any $\tau \leq \hat{\tau}$.

Step 4. *Consider a value α and a τ such that $\tau > \hat{\tau}(\alpha)$. In this step, we characterize the parties' worst compatible contract.*

Consider a *ℓ-Biased contract* denoted by $\mathcal{C}^B = (x^B, s^B)$ and characterized by the pair $(\tilde{x}_r(\alpha), \tilde{q}(\alpha))$ such that it satisfies:

$$\begin{aligned} (1-\beta)k\tau =& \beta(w_\ell^B(\ell; \tilde{x}_r(\alpha), \tilde{q}(\alpha)) - \alpha), \\ (1-\beta)k\tilde{x}_r(\alpha) =& \beta(w_r^B(r; \tilde{x}_r(\alpha), \tilde{q}(\alpha)) - \alpha). \end{aligned} \tag{26}$$

Note that $\mathcal{C}^B$ makes the incumbent's constraint (14) bind always on-path—i.e., $v_i^B(i) = (1-\beta)b + \beta\alpha$ for any $i \in \{\ell, r\}$—, and it is compatible by construction.

First, we show that $\mathcal{C}^B$ is party r's worst compatible contract at h_0.

To that end, we build a contract $\tilde{\mathcal{C}}$ by minimizing party r's value at h_0, which is:

$$\tilde{v}_r(\ell, h_0) = -(1-\beta)\pi k\tilde{x}_\ell(h_0) + \beta(p(\ell, \tilde{s}(h_0))\tilde{v}_r(\ell, h_1^\ell) + (1 - p(\ell, \tilde{s}(h_0)))\tilde{v}_r(r, h_1^r)),$$

where $h_1^\ell \in \mathcal{H}(0)$ and $h_1^r \in \mathcal{H}^*(1)$. By the same reasoning as Step 2 in the Proof of Lemma 6, the actions prescribed at h_0 and h_1^ℓ are the same. Thus, the equation can be rearranged to obtain:

$$\tilde{v}_r(\ell, h_0) = \frac{1}{1 - \beta p(\ell, \tilde{s}(h_0))}(-(1-\beta)k\tilde{x}_\ell(h_0) + \beta(1 - p(\ell, \tilde{s}(h_0)))\tilde{v}_r(r, h_1^r)).$$

Suppose that, given α, $\tilde{x}_\ell(h_0) = \tau$ and $\tilde{v}_r(r, h_1^r) = (1-\beta)b + \beta\alpha$ are together compatible for any $\tilde{s}_0 \in \Delta\{L, N, R\}$, then $\tilde{\mathcal{C}}$ must prescribe both. Lastly, given that $(1-\beta)b + \beta\alpha > -k\tau$,

$$\tilde{v}_r(\ell, h^0) = \frac{1}{1 - \beta p(\ell, \tilde{s}(h_0))}(-(1-\beta)\pi k\tau + \beta(1 - p(\ell, \tilde{s}(h_0)))((1-\beta)b + \beta\alpha))$$

attains its lowest value for $\tilde{s}(h_0) = L$ because the numerator of the derivative with respect to the probability of playing L at h_0 yields: $\beta(1-\beta)(-k\tau - (1-\beta)b - \beta\alpha)$, which is negative.

Note that party r's value at h_0 is the same in contracts $\tilde{\mathcal{C}}$ and $\mathcal{C}^B = (x^B, s^B)$. Hence, contract $\mathcal{C}^B = (x^B, s^B)$ is party r's worst compatible contract (and the firm's best contract) at h_0.

Second, it follows from Lemma 6 that party r's worst compatible contract from a history h in which it holds power, which I denote $\mathcal{C}^w$, prescribes $x^w(h) = 0$ and $s^w(h) = L$ for any $h \in \mathcal{H}(0)$ and

$x^w(h) = x^B(h)$ and $s^w(h) = s^B$ for any $h \in \mathcal{H}(n)$ with $n \geq 1$. The symmetric applies to party ℓ with an *r-Biased contract* symmetric to the *ℓ-Biased contract* we have just described.

Step 5. *Consider contract $\mathcal{C}^w$ build in Step 4. In this step, we obtain the firm's best equilibrium contract for $\tau \in [\hat{\tau}, \hat{x}]$.*

Using the characterization of contract $\mathcal{C}^w$, we have party ℓ's worst compatible pre-election value from some history h in which it holds power onwards as a function of the arbitrary value α and if $\tau > \hat{\tau}(\alpha)$, which is (22) where:

$$v_j^*(i, h; \alpha) = v_r^B(\ell) = \frac{\beta(1 - 2(a + m))(b - 2k(1 - \pi)\tilde{x}_r(\alpha)) - 2k(2 - \beta(1 + 2a + 2(1 - 2\tilde{q}(\alpha))m))\pi\tau}{2(1 - 2\beta(a + m(1 - \tilde{q}(\alpha))))}$$

and $(\tilde{x}_r(\alpha), \tilde{q}(\alpha))$ are given by (26). The application $\alpha \to \underline{w}(\alpha)$ is linearly increasing and by the same reasoning as in the proof of Proposition 1 one can show that it has a unique fix point, which is

$$\alpha^* = \frac{(1 - 2(m - a) - 4\beta a)b - 2k(1 - 2(a - m))\pi}{2(1 - 2\beta a)}.$$

Evaluated on α^* and, for simplicity, assuming $\pi = \frac{1}{2}$, $(\tilde{x}_r(\alpha), \tilde{q}(\alpha))$ become:

$$\tilde{x}_r(\alpha^*) = \frac{(b + k\tau)(2b\beta m - (1 - \beta)k\tau)}{k(b\beta(1 - 2a) - (1 - \beta)k\tau)},$$
$$\tilde{q}(\alpha^*) = -\frac{(1 - \beta)(2b\beta m - k(1 - 2\beta(a + m))\tau)}{2\beta m((1 - 2a)b\beta - (1 - \beta)k\tau)}.$$

One can check that if $\tau = \hat{\tau}$, $(\tilde{q}(\alpha^*), \tilde{x}_r(\alpha^*)) = (0, \hat{\tau})$, if $\tau = \hat{x}$, $(\tilde{q}(\alpha^*), \tilde{x}_r(\alpha^*)) = (1, 0)$, and for any intermediate values of τ, $\tilde{q}(\alpha^*) \in (0, 1)$ and $\tilde{x}_r(\alpha^*) \geq 0$. Further,

$$\frac{\partial \tilde{q}}{\partial \tau} = \frac{(1 - \beta)b(1 - 2\beta a)(1 - 2(a + m))kb}{2m((1 - 2a)b\beta - (1 - \beta)k\tau)^2},$$

is positive. Therefore, if $\tau \in [\hat{\tau}, \hat{x}]$ the firm's best equilibrium contract, denoted by $\mathcal{C}^* = (x^*, s^*)$, is an *$\ell$-Biased contract* with probability $\tilde{q}(\alpha^*)$ and stationary rents $(x_\ell^*, x_r^*) = (\tau, \tilde{x}_r(\alpha^*))$. $\square$

C For $\pi \in [0, \frac{1}{2}]$

Lemma 8 *Let $\pi \in (0, \frac{1}{2}]$ and let $\mathcal{C} = (x, s)$ be a compatible contract that maximizes rent extraction for a given α and satisfies Lemma 5. Then, (i) $s(h) = s(h')$ for any on-path $h, h' \in \mathcal{H}(0)$; and (ii) $s(h) = s(h')$ and $s(h) = L$ for any on-path $h, h' \in \mathcal{H}(n)$, with $n \geq 1$.*

Proof:

From Lemma 5, there exists a compatible contract $\mathcal{C} = (x, s)$ that maximizes rent extraction and such that (i) $x(h) > 0$ if and only if $h = h_0$ or $h \in \mathcal{H}^*(1)$ and (ii) every $x(h) > 0$ makes the incumbent's constraint (14) bind at history h. Then, party r's value in the initial period is:

$$v_r(h_0) = -(1-\beta)\pi k x_\ell(h_0) + \beta w_r(\ell, h_0)$$

and, as party r's constraint binds at any $h \in \mathcal{H}^*(1)$, then $w_r(\ell, h_0) = \Omega(s; h_0, \mathcal{H}^*(1))((1-\beta)b + \beta\alpha)$, and hence,

$$v_r(h_0) = -(1-\beta)\pi k x_\ell(h_0) + \Omega(\sigma; h_0, \mathcal{H}^*(1))((1-\beta)b + \beta\alpha).$$

Hereafter, the firm's strategy at every on-path history $h \in \mathcal{H}(0)$ is denoted s_0, and at every on-path history $h \in \mathcal{H}(n)$ with $n \geq 1$, is denoted $s_{n \geq 1}$.

Step 1. *In this step, we show that $\mathcal{C} = (x, s)$ must be such that $s_n = L$.*

Note, first, that from Lemma 6, contract $\mathcal{C}$ is both the firm's best contract and the opposition's worst contract. Suppose that there exists a history $\hat{h} \in \mathcal{H}(n)$, with $n \geq 1$, in which $\mathcal{C}$ prescribes $s(\hat{h}) \neq L$. Then, consider an alternative contract, denoted $\tilde{\mathcal{C}} = (\tilde{x}, \tilde{s})$, identical on-path to contract $\mathcal{C}$ except for (i) at history $\hat{h}$ it prescribes $\tilde{s}(\hat{h}) = L$. Let Δb denote some positive value defined such that for history $\hat{\hat{h}}$, that follows $\hat{h}$ and satisfies $\hat{\hat{h}} \in \mathcal{H}^*(1)$—which I hereafter denote by $\hat{h}_{*(1)}$ for clarity—party r's pre-election value is

$$\tilde{w}_r(\hat{h}_{*(1)}) = w_r(\hat{h}_{*(1)}) - \Delta b.$$

(ii) In history $\hat{h}_{*(1)}$, contract $\tilde{\mathcal{C}}$ prescribes rent

$$\tilde{x}_r(\hat{h}_{*(1)}) = x_r(\hat{h}_{*(1)}) - \beta \frac{\Delta b}{(1-\pi)k};$$

and (iii) in h_0, contract $\tilde{\mathcal{C}}$ prescribes

$$\tilde{x}_\ell(h_0) = x_\ell(h_0) + (1+\pi)\Omega(\sigma; h_0, \hat{h}_{*(1)})\beta \frac{\Delta b}{(1-\pi)k}.$$

Since $\mathcal{C}$ is compatible and $\tilde{w}_r(h_{*(1)}) = w_r(h_{*(1)}) - \Delta b$, rents $\tilde{x}_\ell(h_0)$ and $\tilde{x}_r(\hat{h}_{*(1)})$ are incentive compatible and thus, $\tilde{\mathcal{C}}$ is compatible. Note that contract $\tilde{\mathcal{C}}$ implies an increase in total rent extraction of $\pi\Omega(\sigma; h_0, \hat{h}_{*(1)})\beta\frac{\Delta b}{(1-\pi)k}$ with respect to contract $\mathcal{C}$.

Step 2. Once s_n has been characterized, it follows that rents $\{x_r(h)\}_{h\in\mathcal{H}^*(1)}$ in contract $\mathcal{C}$ are also characterized: $x_r(h) = x_r(h')$ for any on-path histories $h, h' \in \mathcal{H}^*(1)$ and

$$x_r(h,\alpha) = \frac{\beta((\frac{1}{2}-m)b-\alpha)}{k2(1-\beta)(1-\pi)}. \tag{27}$$

To fully characterize contract $\mathcal{C}$ it only remains to determine s_0. Since any player's value at any history $h \in \mathcal{H}^*(1)$ is the same, there exists a contract $\mathcal{C}^1 = (x^1, s^1)$ that satisfies Lemma 5 and such that s_0^1 is stationary. □

Proof of Proposition 10

We follow the same steps as in the proof of Proposition 9. In Lemmas 5, 6 and 8 we have characterized a compatible contract that gives party ℓ's worst pre-election value from some history h onwards as a function of α, which is (22) where:

$$v_j^*(i,h;\alpha) = -(1-\beta)\pi k\hat{x}_r + \beta\frac{p(r,s_0)((1-\beta)b+\beta\alpha)}{1-\beta(1-p(r,s_0))},$$

and from constraint (14),

$$(1-\beta)(1-\pi)k\hat{x}_r = \beta\left(\frac{(1-p(r,s_0))(1-\beta)b + p(r,s_0)(-(1-\beta)\pi k\hat{x}_\ell + \beta\frac{p(r,R)}{1-\beta 2a}b)}{1-\beta(1-p(r,s_0))} - \alpha\right),$$

and $\hat{x}_\ell$ is given by (27), and where, slightly abusing notation, s_0 denotes the probability of playing L—i.e., $s_0 = 1$ implies $s_0(h) = L$ and $s_0 = 0$ implies $s_0(h) = R$. Lastly, it can be checked by simple algebra that the sign of the derivative of $\underline{w}(\alpha, s_0)$ w.r.t. s_0 is independent of s_0. Hence, $s_0(h)$ can only take only two forms: either $s_0(h) = L$ or $s_0(h) = R$ for any history $h \in \mathcal{H}(0)$.

Step 1. *There exists a worst equilibrium contract for a party. That is, for any $\beta < \bar{\bar{\beta}}$, we show* *that there exists an α^* such that $\underline{w}(\alpha^*; s_0) = \alpha^*(s_0)$ for any $s_0 \in \Delta\{L, R\}$ and it is a fixed point of the application $\alpha \to \underline{w}(\alpha, s_0)$.*

Note, first, that $\underline{w}(\alpha, s_0)$ is a linearly increasing function of α and that $\underline{w}(\alpha, s_0)$ derivative with respect to α and with respect to $\tilde{s}_0$ is positive. Hence, if the $\frac{\partial \underline{w}(\alpha, s_0)}{\partial \alpha} < 1$ for $\tilde{s}_0 = 1$ so it is for any $\tilde{s}_0 \in [0, 1]$. Consider

$$\frac{\partial \underline{w}(\alpha, L)}{\partial \alpha} = \frac{\beta(1+2m)((2-\beta)(1-\pi)\pi + \beta(1-2\pi+2m(1-\pi-\pi^2)))}{(2-\beta(1+2m))^2(1-\pi)^2}.$$

which is monotone with respect to β, and hence there exists a threshold $\bar{\bar{\beta}}$ such that, for any $\beta < \bar{\bar{\beta}}$ and any $s_0 \in \Delta\{L, R\}$, $\alpha \to \underline{w}(\alpha, s_0)$ is a contraction.

Proof of Lemma 3 and Proposition 8:

First, conditional on participation on-path, the firm's best contract follows from Proposition 1. Second, it follows from Proposition 1 that party ℓ's worst pre-election value is given by (12). Third, conditional on not participation on-path, any Deterred contract such that the initial incumbent's constraint binds is a maximum-rent Deterred contract. This follows from Lemma 5 with $\pi = \frac{1}{2}$. Hence, Lemma 3 describes a maximum-rent Deterred contract.

Lastly, threshold $\underline{\zeta}$ is the entry cost such that the firm's values at h_0 in contracts $\mathcal{C}^L = (\bar{x}^L, s^L)$ and $\mathcal{C}^D = (\bar{x}^D, s^D)$ are equal:

$$-(1-\beta)\underline{\zeta} + v_f^L(\ell) - v_f^D(\ell) = 0.$$

Threshold $\bar{\zeta}$ is the entry cost such that the firm's value at h_0 in contract $\mathcal{C}^L = (\bar{x}^L, s^L)$ is zero:

$$-(1-\beta)\bar{\zeta} + v_f^L(\ell) = 0.\square$$

Lemma 9 *The leader z_ℓ's worst contract prescribes, when z_ℓ is the incumbent, $s = R$ and no rent extraction; and when z_ℓ is not the incumbent, it is identical to the firm's best contract.*

Proof of Lemma 9 and Proposition 6:

The structure of the leader's worst contract and the firm's best contract in a weak party system are easily deductible. Note first that any leader extracts the same rent, which follows from the symmetry of the firm's best contract. Let v^{out} and x^{out} denote, respectively, a leader's expected value after an electoral defeat and the other leaders' rent extraction. To characterize the rent scheme, consider leader z's enforcement constraint as $\tau \to \infty$,

$$(1-\beta)k\hat{x}^O = \beta(cv_z^O(z, \hat{x}^O, x^{out}) - \underline{cv}_z(z, x^{out})),$$

where $\underline{cv}_z(z, x^{out})$ is leader z's worst pre-election value. Solving this equation yields

$$\hat{x}^O(x^{out}) = \frac{4\beta m(b + kx^{out})}{k2(1-\beta p(\ell, R)}.$$

Hence, we obtain z's rent as an application $x^{out} \to \hat{x}^O(x^{out})$, which is a contraction as it is linearly increasing with respect to x^{out} with an slope smaller than 1, i.e., $\frac{\partial \hat{x}^O(x^{out})}{\partial x^{out}} = \frac{4\beta m}{2(1-\beta p(\ell,R)} < 1$ for any $\beta < \frac{1}{p(\ell,L)}$. Lastly, we solve the fixed of the application $x^{out} \to \hat{x}^O(x^{out})$, obtaining (11). $\square$

Proof of Proposition 7:

First, we compute both firm's values as $\tau \to \infty$ in the strong party system—i.e., $v_f^S(\ell) = \frac{\beta 2m(1-\beta p(\ell,R))}{k(1-\beta)(1-\beta 2a)}b$—, and the weak party system—i.e., $v_f^W(\ell) = \hat{x}^O$—, obtaining:

$$v_f^S(\ell) - v_f^W(\ell) = \frac{1 - 4a + 4(a^2 - m^2)}{k(1-\beta)(1-\beta 2a)2(1-\beta p(\ell, L))}\beta^3 mb,$$

which is positive, so the firm is better off in a strong party system as $\tau \to \infty$. Second, note that $\hat{x}^O \in (\hat{\tau}, \hat{x})$, which follows from straightforward algebra. Lastly, for $\tau \leq \hat{x}^O$, the firm in a weak party system extracts every period the highest feasible rent. Thus, the firm is strictly better-off in a weak party system if $\tau \in (\hat{\tau}, \hat{x}^O)$ and indifferent between the party systems if $\tau \leq \hat{\tau}$. $\square$