Verifiability and effective persuasion*

Noémie Cabau† Ming Li‡

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Abstract

We investigate the relationship between the verifiability of statements and effective persuasion when there are multiple experts. We extend the model of Wolinsky [2002] and show that it is possible for experts to fully reveal information to the decision maker when the verifiability goes in the direction of the experts' common bias. Our finding complements Wolinsky's negative result on effective persuasion. We also characterize equilibria that experts prefer to both fully revealing equilibria and uninformative equilibria.

KEYWORDS: Verifiability, Persuasion, Asymmetric equilibria.

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†QSMS Research Group, Budapest University of Technology and Economics, Hungary. Email: cabau.noemie@gtk.bme.hu

‡Concordia University, CIRANO, and CIREQ, 1455 Boulevard de Maisonneuve Ouest, Department of Economics, Concordia University, Montréal, Québec, Canada H3G 1M8. Email: ming.li@concordia.ca.

1 Introduction

In a rapidly changing world, people need to have adequate information to make decisions. During the Covid-19 pandemic, the public's lack of understanding about the severity of the pandemic itself, effectiveness and side effects of the vaccines, and validity of different treatment methods has had large negative effects on efforts to control the pandemic. A significant portion of the US population continue to believe that vaccines are not effective, despite countless news reports and statements from epidemiologists and public health officials that ascertain their effectiveness.[1] On climate change, even though there is a clear scientific consensus, a significant proportion of the population still are skeptical about its impact. A recent Pew Research Center report finds that 39% of US respondents believe that global climate change either will not harm them at all or not too much personally at some point in their lifetime.Furthermore, this number goes up to 72% for respondents who are ideologically conservative.

While there are a variety of factors that contribute to the persistence of arguably misguided beliefs,[2] there is no denying the important role played by the inability of experts to convey their information to the public and the latter's inability to absorb the information provided by the experts. In this paper, we demonstrate in a stylized setup that the nature of information matters in effective persuasion by experts. In particular, what type of verifiable statements experts can make is an important factor that influences whether the experts' information can be utilized by the decision maker.

It is well understood in the economics literature that verifiability has important implications in settings of information transmission. The classic works by Milgrom [1981] and Crawford and Sobel [1982] offer a stark demonstration. Both of them study an environment in which an informed party (sender/expert) communicates information to the uninformed party (receiver/decision maker). However, the na-

[1] According to a poll released by Leger Marketing in January 2022, more than a year after the vaccines first became available, 17% of Canadians and 33% of Americans continued to believe that the vaccines at not effective at "Ensuring that if a person gets COVID-19, they will not get sick enough to be admitted to hospital or suffer major complications of the disease."

[2] One such factor is that people are stuck in "echo chambers," in that they only hear information from people of similar political leanings and cultural backgrounds.

ture of information is different in their models. Milgrom [1981] studies verifiable information, where the expert's set of statements is limited by the information he holds.[3] He is allowed to hide information but not to make factually incorrect statements.[4] In contrast, Crawford and Sobel [1982] study cheap talk, where the expert is unconstrained in what he can say.[5]

Studies on verifiable information identify many instances in which full revelation occurs, utilizing the powerful "unravelling argument" – if it is common knowledge that the sender is perfectly informed and information is verifiable, an informed party will disclose all relevant information as failure to do so induces the uninformed party to believe that the withheld information is bad news from the perspective of the informed party. In contrast, studies on cheap talk have mainly derived results that information revelation is possible only in instances where the two parties' incentives are partially aligned. Comparison of these two extremes gives an indication that verifiability of information tends to favour information revelation.

However, between these two extremes, there are economic and political environments in which partial verifiability of information is common. One such example is a job candidate revealing his qualifications to a potential employer. The job candidate can always credibly reveal that he has a qualification, but he cannot credibly reveal that he does *not* have a qualification. Another example is that a drug company can credibly reveal that its drug has a side effect, but it cannot credibly reveal that it does *not* have a side effect.

It is only through revelation of information that the decision maker can be persuaded. We use "effective persuasion" to refer to the outcome in which experts reveal information that influences the decision-maker's action. In this paper, we study what type of verifiability is conducive to effective persuasion.

Inspired by Wolinsky [2002], we consider a multi-expert model where information is partially verifiable. Each expert observes a piece of binary information that is useful to a decision maker and the decision maker cares about the sum of their information. For each expert, only statements that assert one of the two states is verifiable. The experts' interests are partially aligned with the decision maker, however

[3] We use "he" for experts and "she" for decision makers, to facilitate exposition.

[4] See also Grossman [1981], Milgrom and Roberts [1986], Farrell [1985], and Okuno-Fujiwara et al. [1990].

[5] See also Green and Stokey [2007].

the experts have a bias vis-à-vis the decision maker. Wolinsky [2002] shows that effective persuasion is impossible when experts' and the decision maker's preferences are sufficiently misaligned and each expert can only verify information that is opposite to his bias. As a complement to Wolinsky's findings, we show in our paper, however, if the verifiability goes towards the direction of the experts' bias, it is possible for the experts to fully reveal information to the decision maker. Put together, Wolinsky's and our findings demonstrate the right type of verifiability is one that is in the direction of an expert's bias. We also characterize equilibria that dominate both the fully-revealing equilibria and the uninformative equilibria.

There have been extensive studies about communication of verifiable information by the expert to the decision maker. However, researchers have mainly focused on information transmission between one sender and one receiver, with the exception of Wolinsky [2002] and Bhattacharya and Mukherjee [2013], among others.

In related work, where the expert is imperfectly informed, Shin [1994] finds that the strategy by which the informed party tells the "whole truth" (i.e., reveals the partition of the state space he observes) is never part of an equilibrium. The reason for the failure of the unravelling argument in Shin's settings is that the expert may be uninformed, given which the expert may proclaim to be uninformed to avoid revelation of information. Koessler [2003] shows that if a claim of ignorance is verifiable, a perfectly revealing equilibrium exists.

Jackson and Tan [2013] study a model where the expert could be uninformed and could also receive a signal that does not match the state of the world. They show the existence of fully-revealing equilibria in the case where the two parties' preferences are sufficiently aligned

Other scholars take on a different approach on modelling the uncertainty faced by the uninformed party regarding what the informed one knows. Dziuda [2011] considers the case where the uninformed party is not certain of the number of arguments or signals observed by the informed party. The latter can disclose credibly each of the arguments he observes, however he cannot prove whether or not he has disclosed all arguments. Dziuda shows, similarly to Shin, that disclosure of all arguments (i.e., full revelation) is never part of an equilibrium. The unravelling arguments fails for similar reasons as in Shin's model, that is, the uninformed party does not know the number of arguments the informed party sees.

2 Model

Our model closely follows Wolinsky [2002]. We introduce some modifications that allow us to investigate the verifiability of different statements.

A decision maker, DM, has to choose an action $a \in \{L, H\}$. Action H is riskier than the status quo of not choosing it, L. The appropriate action to take depends on a state of the world unobserved by DM. However, she can consult a panel of n experts. Let $E = \{1, \ldots, n\}$ be the panel, and let i be a typical member of E. Each expert i receives a private signal $s_i \in \{0, 1\}$ and $s_i = 1$ is relatively more favourable for H than $s_i = 0$. Signals are i.i.d., with $q \in (0, 1)$ being the probability for $s_i = 1$. The state S is the sum of each expert's signals, $\sum_{i \in E} s_i$. We use $s = (s_1, \ldots, s_n)$ to denote a signal profile and S_{-i} the sum of the signals of experts other than i.

Following Wolinsky [2002], the payoff of DM is defined by

$$\tilde{V}(S|a) = \begin{cases} V(S) & \text{if } a = H, \\ 0 & \text{if } a = L, \end{cases} \tag{1}$$

and that of experts is identical to each other and defined by

$$\tilde{U}(S|a) = \begin{cases} U(S) & \text{if } a = H, \\ 0 & \text{if } a = L, \end{cases} \tag{2}$$

where both V and U are increasing. Let θ and γ be the lowest state such that $V(S) \geq 0$ and $U(S) \geq 0$, respectively. Their values are different, implying DM and E's preferences are misaligned. Throughout, we suppose that E is more *conservative* than DM: there exist intermediate states where DM prefers H but E prefers L. As an example, a political leader may be more eager to approve a vaccine for emergency use than medical experts. This is without loss of generality as we do not impose restrictions on the default action of DM in the absence of information from E. The following assumption captures the misalignment in preferences and delimits the regions of (dis)agreement.

Assumption 1. *For any* $S \in \{1, \ldots, n\}$, $V(S-1) > U(S)$. *Furthermore,* $U(n) > 0 > V(0)$.

An immediate implication of Assumption 1 and the assumption that both U and V are increasing is that $1 \leq \theta < \gamma \leq n$. For states below θ, both DM and the panel prefer L to H, and the converse is true for high states above γ. In contrast, the panel prefers L, and DM prefers H when the state is between θ and $\gamma - 1$. Our setup is slightly more general than that of Wolinsky [2002], given that we do not place restrictions on DM's default action.

Neither E nor DM can commit to a mechanism ex-ante. Instead, after observing their private signal s_i, each expert simultaneously sends DM a report $r_i \in \{0,1\}$, $i = 1,\ldots,n$. Upon receiving the reports, DM makes a decision. Therefore, the experts can report one of the two possible values of the signal, but cannot remain silent. We denote by $r = (r_1,\ldots,r_n)$ a report profile for all experts; we let $R = \sum_{i\in E} r_i$ be the number of 1's the panel reports to DM and $R_{-i} := R - r_i$ its counterpart for experts other than i. A strategy for i, $y_i = Pr(r_i = s_i|s_i) \in [0,1]$, gives the probability that he reports his signal. Let $y = (y_1,\ldots,y_n)$ be a strategy profile for the experts. A strategy for DM, $\rho : \{0,1\}^n \to [0,1]$, gives the probability that she chooses H after seeing a report profile r.

Our equilibrium concept is the Perfect Bayesian Equilibrium.

Definition 1. *An* equilibrium *is a pair* (ρ^*, y^*) *such that,*

1. *For all* $r \in \{0,1\}^n$, $\rho^*(r)$ *maximizes* DM*'s expected payoff given her belief about* s*;*
2. DM*'s belief about* s *must be Bayesian for all* $Pr(r|y^*) > 0$*;*
3. y_i^* *maximizes expert* i*'s expected payoff given* y_{-i}^* *and* ρ^**.*

Said differently, for DM:

$$\rho^*(r) = \begin{cases} 1 & \text{if } \sum_{x\in\{0,\ldots,n\}} Pr(S = x|r, y^*)V(x) > 0 \\ 0 & \text{if } \sum_{x\in\{0,\ldots,n\}} Pr(S = x|r, y^*)V(x) < 0. \end{cases}$$

For expert i, $y_i^* = 1$ only if:

$$\sum_{r_{-i}} Pr(r_{-i}|y_{-i}^*) \sum_{x\in\{0,1\ldots,n\}} Pr\left(S_{-i} = x|r_{-i}, y_{-i}^*\right) U(x + s_i)\left[\rho^*(r_i = s_i, r_{-i}) - \rho^*(r_i \neq s_i, r_{-i})\right] \geq 0.$$

The left-hand side gives i's expected payoff from reporting his signal relative to misreporting it given a strategy profile (ρ^*, y_{-i}^*). Whether the above inequality holds depends on

$$\rho^*(r_i = s_i, r_{-i}) - \rho^*(r_i \neq s_i, r_{-i}),$$

for each report profile by other experts, r_{-i}. If the above expression is nonzero, we say that expert i is *pivotal* at r_{-i}. Note that i is indifferent between all of his strategies if there is no report profile for the other experts that occurs with positive probability and at which i is pivotal.

In the remainder, we investigate information disclosure in equilibrium when certain statements are verifiable. We begin with the benchmark case, where no information is verifiable. This case is analogous to cheap talk. Regardless of their signal, each expert can report or misreport, and DM has no means to assess whether an expert lies about his signal. Then, we consider alternatively the cases where the experts can under-report, i.e., $s_i = 1$ is verifiable, and over-report, i.e., $s_i = 0$ is verifiable.

We shall clarify what we understand by *verifiable statements*. For any $s \in \{0, 1\}$, signal s is *verifiable* if hard evidence exists as of whether the experts have received s; by contrast, signal $s' \neq s$ is *unverifiable* if the experts cannot prove that they did not receive s. Dye [1985] illustrates these notions via the following example. Suppose that the experts are accountants, and DM is the board. Let $s_i = 0$ be the event of finding no errors in the company's accounting, and let $s_i = 1$ be the event of finding an error. If accountant i reports he has found an error (i.e., $r_i = 1$), the board may ask the accountant to hand in proofs, which i can do if he indeed found an error (i.e., $s_i = 1$). Hence, in this example, signal 1 is verifiable. In contrast, i's reporting that he did not find any error (i.e., $r_i = 0$) is unverifiable. Indeed, the board may not have any way to assess whether i did not find any error (i.e., $s_i = 0$) or found one but conceals his finding (i.e., $s_i = 1$). Therefore, in the example, signal 0 is not verifiable.

In what follows, we will use this setup to study when E can effectively persuade DM, namely, influence the decisions made by the latter. In Section 3, we show that this hinges on whether signal 0, the one that is congruent with the expert's bias, is verifiable. In Section 4, we characterize equilibria with effective persuasion and show that, under some mild conditions, the experts prefer some asymmetric and partially revealing equilibria to the fully revealing.

3 When meaningful disclosure is impossible

In this section, we establish a necessary condition on verifiability of certain statements and information disclosure in equilibrium.

First, following Sobel [2013], we say that an equilibrium is *influential* if on the equilibrium path DM chooses the action other than her default action with positive probability. In turn, we say that *meaningful disclosure* is possible if, and only if, an influential equilibrium exists.

We now turn our attention to the case where signal 0 is not verifiable, which includes both the *cheap talk* benchmark and the case where only signal 1 is verifiable [Wolinsky, 2002].

Below, we establish that under cheap talk communication, on the equilibrium path of any influential equilibrium, an expert who received $s_i = 0$ strictly prefers to report his signal. Before introducing the statement, we put an assumption on the experts' strategies. The restriction is without loss of generality, and it simply exempts us from considering situations where a report $r_i = 1$ means in fact $s_i = 0$.

Assumption 2. *For all $i \in E$, for all r_{-i} such that $Pr(r_{-i}|y_{-i}) > 0$, $\rho^*(r_i, r_{-i})$ is weakly increasing in r_i.*

When one of the two signals is verifiable, the above condition is always satisfied since an expert who holds an unverifiable signal has no other choice but to report it. In the case where no signal is verifiable, the assumption implies that on the equilibrium path, an expert is relatively more likely to report a 1 after $s_i = 1$ than after $s_i = 0$.

Lemma 3.1. *Suppose that signal 0 is not verifiable. On the equilibrium path, any expert $i \in E$ who got a signal 0 and that has a positive probability of being pivotal reports his signal with probability 1. Said differently, $y_i^* = 1$ if*

1. *$s_i = 0$,*

2. *$\exists r_{-i}$ s.t. $Pr(r_{-i}|y^*_{-i}) > 0$ and $\rho^*(r_i = 0, r_{-i}) < \rho^*(r_i = 1, r_{-i})$.*

The intuition for the Lemma goes as follows. Consider any strategy profile y_{-i} and report profile r_{-i} for the other experts and any decision rule ρ for DM. If Expert i is not pivotal at r_{-i}, then i is indifferent between all his strategies – hence reporting his signal is indeed a best response. Consider the alternate case where i is pivotal at r_{-i}. By Assumption 2, this means that for i reporting $r_i = 1$ induces DM to choose H with a higher probability than by reporting $r_i = 0$. Therefore, misreporting is a best response for i if, and only if, his expected payoff from doing so is nonnegative (recall that L yields a null payoff):

$$\sum_{x \in \{0,\ldots,n-1\}} Pr(S_{-i} = x | r_{-i}, y_{-i}) U(x) \geq 0.$$

Should DM go with the most optimistic interpretation of i's report, i.e., take $r_i = 1$ at face value and $r_i = 0$ as uninformative about i's signal, DM would consider that the report profile $(r_i = 1, r_{-i})$ is just enough evidence in favour of H. Mathematically,

$$\sum_{x \in \{0,\ldots,n-1\}} Pr(S_{-i} = x | r_{-i}, y_{-i}) V(x+1) \geq 0 \geq \sum_{x \in \{0,\ldots,n-1\}} Pr(S_{-i} = x | r_{-i}, y_{-i}) [qV(x+1) + (1-q)V(x)].$$

Owing to the difference in preferences between the experts and DM, when they are pivotal, the experts strictly prefer L to H. Therefore, the experts have the incentive to report their signal 0 as it tilts the balance towards the decision they prefer.

Our next Proposition establishes that the verifiability of signal 0 is a necessary condition for meaningful disclosure.

Proposition 3.1. *Suppose that signal 0 is not verifiable. In any equilibrium, DM chooses her default action with probability 1.*

To see the reason for this Proposition, observe that by Lemma 3.1, an expert always has the incentive to truthfully reveal his signal 0 when he is pivotal. Now, consider the alternate case where i's signal is $s_i = 1$. If 0 is not verifiable, Expert i can choose to truthfully reveal $s_i = 1$ or misreport it. This is regardless of whether signal 1 is verifiable or not. Consider an instance in which there exists a report profile r_{-i} at which i is pivotal. On the equilibrium path, DM must interpret a report $r_i = 1$ as expert i holding a signal $s_i = 1$ with probability 1. This holds by assumption if signal 1 is verifiable, but it is an equilibrium result under cheap talk since, by Lemma 3.1, an expert who holds a signal 0 and who is pivotal never reports $r_i = 1$.

By the above argument and our current assumption that expert i is pivotal at r_{-i}, we conclude that the report profile $(r_i = 1, r_{-i})$ is just enough favourable evidence for H from DM's viewpoint. Owing to the misalignment in preferences, the same information is not enough evidence for the expert to prefer H. From Expert i's viewpoint

– i.e., when they are pivotal, the experts strictly prefer L to H. Consequently, the experts have the incentive to suppress their signals $s_i = 1$ in equilibrium.

4 Meaningful disclosure and effective persuasion

We have shown above that effective persuasion is only possible if signal 0 is verifiable. In this subsection, we first show that there are equilibria where meaningful information is elicited from the experts. This result follows from our analysis in Section 3, which shows that an expert strictly prefers L to H when he is pivotal, and from the observation that when signal 0 is verifiable, an expert can provide verifiable information in favour of his bias. Then, we investigate how effective persuasion can be. We show that asymmetric strategy profiles allow the experts to persuade DM more effectively than symmetric profiles.

4.1 Meaningful disclosure in equilibrium

The following proposition establishes the existence of a fully revealing equilibrium. An equilibrium is *fully-revealing* if, on the equilibrium path, every expert reports his signal.

Proposition 4.1. *Suppose that $s_i = 0$ is verifiable. Then the full revelation strategy profile (ρ^*, y^*) constitutes an equilibrium, where $y_i^* = 1$ for all $i \in E$ and*

$$\rho^*(r) = \begin{cases} 0 & \text{if } R < \theta \\ 1 & \text{if } R \geq \theta \end{cases}$$

To see that full revelation constitutes an equilibrium, note that in the full revelation strategy profile, each expert is pivotal with a positive probability. As we have shown above, whenever an expert is pivotal, he strictly prefers action L to H. Consequently, the expert's best response is to report $r_i = 0$ when he is pivotal. However, given that 0 is verifiable, this implies that each expert reports his signal $s_i = 0$ truthfully by incentive and reports $s_i = 1$ truthfully by constraint.

The verifiability of statements directly affect meaningful disclosure. We show that meaningful disclosure is possible if, and only if, the signal that is verifiable favours the experts' relative bias. Note that a pivotal expert can persuade DM to follow his recommendation only if he can provide credible evidence supporting his stance. In our setup, the experts are more conservative than DM; thus, $s_i = 0$ goes in the same direction as their relative bias. If the experts were assumed to be more liberal than DM, we would find that meaningful disclosure is conditional on signal $s_i = 1$ being verifiable.

4.2 Asymmetric equilibria

In the previous subsection, we show that information revelation is possible if the experts can disclose verifiable information that favours their bias. As Proposition 3.1 shows, there is an equilibrium in which the experts succeed in persuading DM to take the action they prefer in every state where preferences are aligned.

However, full revelation is not necessarily the best equilibrium for experts. In particular, they may achieve higher payoff in asymmetric equilibria. We say that an equilibrium is *symmetric* if all experts play the same strategy, $y_i = y \in [0,1]$ for all $i \in E$. The experts fully revealing strategies of Proposition 4.1 correspond to the case where $y = 1$.

As shown above, given that 0 is verifiable, misreporting is a best response for an expert only if he is never pivotal on the equilibrium path. If no expert is pivotal, then in equilibrium, DM chooses her default action with probability 1. It follows that all symmetric equilibria other than the fully-revealing are outcome-equivalent, since DM chooses her default action with probability 1.

Thus, in symmetric equilibria, experts either are not influential at all or fully reveal information to DM, which she then uses to implement her preferred decision in each state. In what follows, we show that in asymmetric equilibria experts may persuade DM to take their preferred action when their preferences diverge from that of DM. We first illustrate this possibility with an example.

Example 4.1. Suppose that $n = 3$, $q = 0.35$, $\theta = 1$, and $\gamma = 3$. The payoff from action H is $V(S) = S - 0.995$ for DM and

$$U(S) = \begin{cases} -2 & if\, S \in \{0,1,2\} \\ 0.5 & if\, S = 3. \end{cases}$$

for the experts. Consider the strategy profile $(y_1, y_2, y_3) = (1, 0.7, 0)$ and the decision rule:

$$\rho(r) = \begin{cases} 0 & \textit{if}\, r \in \{(0,0,0),(0,0,1),(0,1,0),(0,1,1)\} \\ 1 & \textit{if}\, r \in \{(1,0,0),(1,1,0),(1,0,1),(1,1,1)\}. \end{cases}$$

It can be verified that (y, ρ) is an equilibrium. Given the experts' behaviour, $Pr(r|y) > 0$ for all r such that $(r_2, r_3) = (1,1)$. Experts 2 and 3 are never pivotal on the equilibrium path. Expert 1 is pivotal at $(r_2, r_3) = (1,1)$, which occurs with positive probability given y_2 and y_3. Consider the cases where the state S is either 1 or 2. In the fully-revealing equilibrium, DM always chooses H. However, in the asymmetric equilibrium above, conditional on $S = 2$, the experts manage to persuade DM to choose L with probability $1/3$, which corresponds to the event $(s_1, s_2, s_3) = (0,1,1)$. Similarly, conditional on the state $S = 1$, they manage to persuade the DM to choose L with probability $2/3$, which corresponds to the the events $(s_1, s_2, s_3) = (0,1,0)$ *and* $(s_1, s_2, s_3) = (0,0,1)$.

Below, we characterize some asymmetric equilibria. We first establish that it is without loss of generality to consider only strategy profiles where the experts either always reveal or always misreport their signal when feasible. From now on, the strategy by which an expert misreports his signal with probability one is referred to as the *babbling strategy*.

Lemma 4.1. *Suppose that $s_i = 0$ is verifiable. If (ρ^*, y^*) is an equilibrium and $y_i^* \in (0,1)$ for some $i \in E$, then the strategy profile (ρ', y') where $y_i' = 0$ and $y_j' = y_j^*$ for all $j \neq i$, and*

$$\rho'(r) = \rho^*(r) \qquad \forall r \in \{0,1\}^n \tag{3}$$

is also an equilibrium. Furthermore, the two equilibria give the experts the same expected payoff.

The Lemma follows from the following observations. Whenever an expert mixes between revealing his signal 0 and misreporting it as 1, the expert is not pivotal, because if he were, he would strictly prefer to reveal it. Thus, by changing his mixed strategy to misreporting with probability 1, it would not affect DM's decisions. But given that DM's decisions are not changed, the expert remains indifferent between revealing his signal and misreporting it and consequently, he finds misreporting with probability 1 to be a best response. In addition, given that DM's decisions are not affected, the other experts' strategies also remain best responses.

Proposition 4.2. *Let t denote any integer in $\{n-(\gamma-\theta),\ldots,n-1\}$. Consider the strategy profile (ρ^t, y^t) where $y_i^t = 1$ for all $i \in \mathcal{T} := \{1,\ldots,t\}$, $y_i^t = 0$ for all $i \in \mathcal{B} := \{t+1,\ldots,n\}$, and:*

$$\rho^t(r) = \begin{cases} 1 & \text{if } \sum_{i\in\mathcal{T}} r_i \geq \theta \\ 0 & \text{otherwise.} \end{cases}$$

The profile (ρ^t, y^t) is an equilibrium if, and only if,

$$\sum_{x\in\{0,\ldots,n-t\}} V(\theta-1+x)\binom{n-t}{x} q^x (1-q)^{n-t-x} \leq 0. \tag{4}$$

From an ex-ante perspective, the probability that the play of (ρ^t, y^t) leads to implementing H in state S is:

$$Pr(H|S, y^t) = \begin{cases} 0 & \text{if } S < \theta \\ \frac{\sum_{z=\theta}^{\min\{S,t\}} \binom{t}{z}\binom{n-t}{S-z}}{\binom{n}{S}} & \text{if } S \in \{\theta,\ldots,\theta-1+(n-t)\} \\ 1 & \text{if } S \geq \theta+(n-t), \end{cases}$$

i.e., the signal profile allocates at least θ among the S signals 1 to the experts in $\mathcal{T}$.

We now look more closely into the properties of the equilibrium. There are two panels of experts: the truthful panel, $\mathcal{T}$ and the babbling panel, $\mathcal{B}$. Certain report profiles fully reveal to the DM which action is optimal. For instance, DM is certain that L is optimal if at least $n-\theta+1$ experts report 0, as even if all the remaining experts are truthfully reporting 1 the state S is still below DM's threshold, θ. In contrast, DM is certain that H is optimal when at least θ experts in the truthful panel $\mathcal{T}$ report 1. Consider the remaining report profiles, which contain fewer than θ reports of 1 among the experts in $\mathcal{T}$ and no more than $n-\theta$ reports of 0. When such report profiles occur, ρ^t dictates that DM follow the recommendation of the truthful panel. In particular, when at least $n-\gamma+1$ experts in $\mathcal{T}$ report 0, ρ^t prescribes DM to choose L.

To see that the experts are playing best responses, observe that the decision rule relies entirely on the submissions by the truthful panel and therefore none of the babbling experts are pivotal. Given that babbling experts always report 1, DM is playing a best response if, and only if, she prefers L to H when the truthful panel hands in $\theta-1$ reports of 1, or (4) is satisfied. The interpretation of the condition is that if DM learns that the state is between $\theta - 1$ and $\theta - 1 + (n - t)$, her default action is L, which coincides with the optimal action for the experts. It is worth noting that (4) does not imply a particular ex ante default action. In Example 4.1, DM's default action is H, and the strategy profile ρ (as defined in the example) and $y' = (1,0,0)$ constitutes an equilibrium. If instead $q = 0.3$, the strategy profile remains an equilibrium, but DM's default action changes to L.

We now demonstrate that the experts are better off in any of the asymmetric equilibria of Proposition 4.2 than in the fully-revealing equilibrium of Proposition 4.1. First, note that the equilibrium outcome is the same when the state lies in the agreement region (i.e., below θ and above γ). However, in an asymmetric equilibrium, DM chooses the action the experts prefer, L, for some signal profiles in the disagreement region. In contrast, DM prefers the fully-revealing equilibrium to any other equilibrium. Note that there are at most $\theta - \gamma$ babbling experts in an equilibrium in Proposition 4.2. There may exist other asymmetric equilibria, where at least $\gamma - \theta + 1$ experts babble, especially when (4) is not satisfied. However the welfare comparison with the fully-revealing equilibrium will be less clear cut. It will depend on the properties of DM's utility function around the threshold θ.

4.3 Mixed panel of experts

A natural extension of the model is to consider a panel consisting of both conservative and liberal experts. The liberal experts are relatively more eager to see H implemented than DM (i.e., $U_L(S) > V(S+1)$ for all $S \in \{0,\dots,n-1\}$, so that $\gamma_L < \theta$) while conservatives are relatively less eager.

Our conclusion regarding information disclosure under cheap talk remains unchanged: no influential equilibrium exists. Indeed, on the equilibrium path, a liberal expert who is pivotal strictly prefers H to L, thus strictly prefers to report 1, regardless of his signal. Similarly, in the analogous case, a conservative expert strictly prefers to report 0, regardless of his signal. Hence, in equilibrium, experts' messages are uninformative and DM implements her default action with probability 1.

Similar to our earlier result, when one of the two signals is verifiable, there exist parameters values for which an influential equilibrium exists. Statements (ii) and (iii) in the next proposition feature a sufficient condition.

Proposition 4.3. *Let C denote the number of conservatives experts, and L denote the number of liberal experts. (i) If no signal is verifiable, then no equilibrium is influential. (ii) Suppose that $s_i = 0$ is verifiable. If:*

$$\sum_{k=0}^{L} \binom{L}{k} q^k (1-q)^{L-k} V(k) \leq 0 \leq \sum_{k=0}^{L} \binom{L}{k} q^k (1-q)^{L-k} V(C+k),$$

then an influential equilibrium exists, where all conservative experts reveal their signals and all liberals babble. (iii) Suppose that $s_i = 1$ is verifiable. If:

$$\sum_{k=0}^{C} \binom{C}{k} q^k (1-q)^{C-k} V(k) \leq 0 \leq \sum_{k=0}^{C} \binom{C}{k} q^k (1-q)^{C-k} V(L+k),$$

then an influential equilibrium exists, where all liberal experts reveal their signals and all conservatives babble.

The proof is omitted. Here, we provide the intuition for the Proposition. Consider the case where $s_i = 1$ is verifiable. Consider the strategy profile stated in the Proposition in which each liberal reveals his signal and each conservative babbles. To see that it is an equilibrium, note that each expert is playing a best response, regardless of DM's strategy. Indeed, truthful reporting of $s_i = 1$ is always a best response for a liberal, whilst misreporting of $s_i = 1$ is always a best response for a conservative.

On the equilibrium path, $r_i = 0$ for all conservatives and $r_i = s_i$ for all liberals. For an equilibrium to be influential, DM must prefer H to L when all liberals report a 1, and she must prefer L to H when they all report 0. Mathematically, this corresponds to the series of inequalities in the proposition. On the left side is DM's expected payoff from H after receiving the report profile where all conservatives say 0 and all liberals say 1. This report profile is on the equilibrium path, and it corresponds to the report profile most favorable to H. The inequality ensures that DM prefers H to L after receiving such reports. On the right side is DM's expected payoff from H after receiving reports 0 from all experts. This report profile is also on path, and it is the report profile least favorable to H. The inequality implies that DM prefers L after receiving bad news.

5 Ex-ante vs equilibrium incentives

In our setup, experts cannot jointly commit to an information disclosure strategy. They would be weakly better off if they could commit, given that they have the same preferences. In an insightful study, McLennan [1998] shows that when voters have the same preferences the ability to commit does not matter in a voting environment. McLennan's result does not apply to the experts-decision maker environment. The difference is that in a voting environment, the mapping from votes/reports to outcomes is preset and independent of the voting/reporting strategies of the voters/experts. However, in an experts-decision maker environment, DM must play a best response to the experts' strategies so the same voting/report profile may lead to different outcomes depending on the voting/reporting strategies of the experts. In our setup, if *DM* committed to a preset threshold, then McLennan's result would continue to apply.

Now we demonstrate this formally. Continue to assume that signal 0 is verifiable. Suppose the experts can now commit to a disclosure strategy profile y. DM then selects ρ to best respond to y. Thus, the ex-ante expected payoff of each expert is:

$$\begin{aligned}\pi(y) &= \sum_{s\in\{0,1\}^n} Pr(s)U(S)Pr(H|s,y)\\ &= \sum_{s\in\{0,1\}^n} Pr(s)U(S)\sum_{r\in\{0,1\}^n} Pr(r|s,y)\rho(r)\end{aligned}$$

where $Pr(S)$ is the probability of state S. A disclosure strategy profile y^* is *optimal* if it statisfies

$$y^* \in \arg\max_{y\in[0,1]^n} \pi(y).$$

We now use Example 4.1 to show that the best disclosure strategy profile sustainable *in equilibrium* may not be optimal in the space of *all* disclosure strategy profiles. All results discussed are formally proved in Appendix F. Let $\mathscr{Y}^*$ be the set of equilibrium strategy profiles. We show that in Example 4.1, the strategy profile $y = (1, 0.7, 0)$ satisifes

$$y \in \arg\max_{\tilde{y}\in\mathscr{Y}^*} \pi(\tilde{y}),$$

but

$$y \notin \arg\max_{\tilde{y}\in[0,1]^n} \pi(\tilde{y}).$$

In particular, we show that y is worse for the experts than $\hat{y} = (0.7, 0.7, 0)$. Note that in y, expert 1 always reveals the truth, but in $\hat{y}$ he misreports his signal with a positive probability. DM's best response to $\hat{y}$ is:

$$\hat{\rho}(r) = \begin{cases} 0 & if\, r \in \{(0,0,0),(0,0,1),(0,1,0),(1,0,0),(0,1,1),(1,0,1)\} \\ 1 & if\, r \in \{(1,1,0),(1,1,1)\}. \end{cases}$$

Notice that $(\hat{\rho}, \hat{y})$ is not an equilibrium. Indeed, Expert 1 is pivotal at $(r_2, r_3) = (1,1)$, which occurs with a positive probability given $\hat{y}$. From Section 3, Expert 1 strictly prefers L to H so strictly prefers to reveal signal 0, however $\hat{y}_1 < 1$.We show in Appendix that $\pi(\hat{y}) > \pi(y)$. Table 1 demonstrates the reason behind this result.

Table 1: Expected gain from $\hat{y} = \sum_{s\in\{0,1\}^n} Pr(s) \times \Delta\pi(y|s) = 0.10595$.

Signal profile	$Pr(H\|s,.)$		$\pi(\hat{y}\|s) - \pi(y\|s)$
	$\hat{y}$	y	
$s = (0,0,0)$	0.3^2	0	-0.18
$s = (0,0,1)$	0.3^2	0	-0.18
$s = (0,1,0)$	0.3	0	-0.6
$s = (1,0,0)$	0.3	1	$+1.4$
$s = (0,1,1)$	0.3	0	-0.6
$s = (1,0,1)$	0.3	1	$+1.4$
$s = (1,1,0)$	1	1	0
$s = (1,1,1)$	1	1	0

Consider Expert 1's choice. He expects Experts 2 and 3 to play $(y_2, y_3) = (0.7, 0)$. Expert 1 contemplates the choice between $y_1 = 1$ and $\hat{y}_1 = 0.7$ before he receives his signal. Note that misreporting $s_1 = 0$ entails the following trade-off for Expert 1. On the one hand, it will make DM more likely to take action H in signal profiles where $s_1 = 0$, which has a negative effect on experts' payoffs (see yellow rows in Table 1); on the other hand, it will make DM less likely to take action H in certain signal profiles where $s_1 = 1$ (see grey rows in Table 1). The experts' payoffs are such that the second effect dominates the first. However, if Expert 1 contemplates the choice between full revelation and misreporting after receiving his signal, only the first effect is present. As shown in our previous discussion, Expert 1 does not want to misreport signal $s_1 = 0$ when he is pivotal.

6 Conclusion

In this paper, we address how the verifiability of information affects the feasibility of effective persuasion. We study an environment in which a panel of experts advise a decision maker on a binary decision – between a risky action and the status quo of not undertaking it. The experts are imperfectly informed: Each observes a binary signal, and the state of the world is given by the sum of their signals. The experts' views on the desirability of the risky action differ from the decision maker's in certain states. Consequently, the experts may have the incentive to distort their information, as long as it is feasible to do so. It is not feasible for the expert to claim his signal is of certain realization if that realization is *verifiable*.

We focus on a setup in which the experts and the decision maker's preferences are sufficiently misaligned. Our main finding is that effective persuasion is possible if and only if the signal realization that favours the expert's bias is verifiable. Wolinsky [2002] observes that when an expert is pivotal with a positive probability, he strictly prefers to report his information to be the realization that that tilts the decision to his preferred one. If that realization is not verifiable, such misreporting is made feasible, which renders revelation of information impossible. In contrast, if that realization *is* verifiable, such profitable misreporting is precluded. Truthful revelation becomes an equilibrium, since the only opportunity to misreport and influence the decision maker is towards a decision that experts dislike and therefore will not be utilized. We also show there may exist equilibria where experts are better off compared with truthful revelation.

Appendices

A

PROOF OF LEMMA 3.1: First, observe that if signal 1 is verifiable, when Expert i's signal is $s_i = 0$, he has no choice but to report it truthfully.

Now, we consider the case where neither signal is verifiable. Consider an equilibrium (ρ, y). Take any expert $i \in E$; assume that $s_i = 0$. We argue that i must report it truthfully if he is pivotal, that is $r_i = 0$.

If there does not exists any r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$ and i is pivotal at r_{-i}, then any value of $y_i \in [0,1]$ constitutes a best response for i. Suppose instead that there exists r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$ and i is pivotal at r_{-i}. Recall that if i is pivotal at r_{-i} and Assumption 2 is satisfied, then:

$$\rho(r_i = 1, r_{-i}) > \rho(r_i = 0, r_{-i}).$$

Let $Pr_{DM}(S = x|r, y)$ be DM's posterior belief that the state is x after receiving report profile r and given y. Since r_{-i} is on the equilibrium path, the previous inequality is equivalent to the following ones

$$\sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y)V(x) \leq 0 \leq \sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 1, r_{-i}, y)V(x),$$

with at least one of the two inequalities strict. Let $\mathbb{E}_S[U(S)|r_i = z, r_{-i}, y]$ be i's expected payoff when i reports $r_i = z$, for $z \in \{0,1\}$, the other experts report r_{-i}, and the experts play the strategy profile y. Let $Pr(S_{-i} = x|r_{-i}, y_{-i})$ be i's belief given y_{-i} and r_{-i} that x is the number of signals 1 other experts have received. Given $s_i = 0$, we have

$$\begin{aligned}
\mathbb{E}_S[U(S)|r_i = 1, r_{-i}, y] &= \sum_{x\in\{0,\dots,n-1\}} Pr(S_{-i} = x|r_{-i}, y_{-i})U(x) \\
&= \sum_{x\in\{0,\dots,n-1\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y_{-i}, \tilde{y}_i = 1)U(x) \\
&< \sum_{x\in\{0,\dots,n-1\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y_{-i}, \tilde{y}_i = 1)V(x) \\
&\leq \sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y_{-i}, y_i)V(x) \leq 0.
\end{aligned}$$

In the above chain of (in)equalities, the second equality owes to the observation that conditional on i always reporting truthfully, or $\tilde{y}_i = 1$, on the equilibrium path,

DM infers that $s_i = 0$ with certainty after observing $r_i = 0$. Therefore, conditional on i playing $\tilde{y}_i = 1$, i and DM share the same posterior belief after $(r_i = 0, r_{-i})$. The strict inequality owes to Assumption 1 about the difference between the experts' and DM's preferences, i.e., $U(x+1) < V(x)$ for all $x \in \{0,\ldots,n-1\}$. The last weak inequality owes to the assumption that i is pivotal at r_{-i}. The proof of the first weak inequality is as follows. If i plays y_i, y_i is a best response for i, then on the equilibrium path,

$$\begin{aligned}Pr_{DM}(S = x|r_i = 0, r_{-i}, y) =& Pr_{DM}(s_i = 0|r_i = 0, y_i) Pr_{DM}(S_{-i} = x|r_{-i}, y_{-i}) \\ &+ Pr_{DM}(s_i = 1|r_i = 0, y_i) Pr_{DM}(S_{-i} = x-1|r_{-i}, y_{-i}).\end{aligned}$$

For conciseness, let us set $\omega(x) := Pr_{DM}(S_{-i} = x|r_{-i}, y_{-i})$ and let:

$$\lambda_i := Pr_{DM}(s_i = 0|r_i = 0, y_i) = \frac{(1-q)y_i}{(1-q)y_i + q(1-y_i)}.$$

Earlier, we showed that if $\tilde{y}_i = 1$ and $\tilde{y}_i$ is a best response for expert i, then on the equilibrium path,

$$Pr_{DM}(S = x|r_i = 0, r_{-i}, y_{-i}, \tilde{y}_i) = \omega(x).$$

An expression for $\omega(x)$ is:

$$\omega(x) = \begin{cases} \lambda_i \omega(0) & if\, x = 0 \\ \lambda_i \omega(x) + (1-\lambda_i)\omega(x-1) & if\, x \in \{1,\ldots,n-1\} \\ (1-\lambda_i)\omega(n-1) & if\, x = n. \end{cases}$$

Therefore,

$$\begin{aligned}
&\sum_{x\in\{0,\ldots,n-1\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y_{-i}, \tilde{y}_i) V(x) - \sum_{x\in\{0,\ldots,n\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y) V(x) \\
=& \sum_{x\in\{0,\ldots,n-1\}} \omega(x) V(x) - \lambda_i \omega(0) V(0) - \sum_{x\in\{1,\ldots,n-1\}} [\lambda_i \omega(x) + (1-\lambda_i)\omega(x-1)] V(x) \\
&- (1-\lambda_i)\omega(n-1) V(n) \\
=& (1-\lambda_i) \sum_{x\in\{0,\ldots,n-1\}} \omega(x) V(x) - (1-\lambda_i) \sum_{x\in\{1,\ldots,n\}} \omega(x-1) V(x) \\
=& -(1-\lambda_i) \sum_{x\in\{0,\ldots,n-1\}} \omega(x)[V(x+1) - V(x)] \le 0,
\end{aligned}$$

where the last inequality owes to the assumption that V is weakly increasing in x, for all $x \in \{0,\ldots,n-1\}$.

We have shown above that H yields a strictly negative payoff to expert i when the latter is pivotal at r_{-i} and $Pr(r_{-i}|y_{-i}) > 0$. We thus conclude that:

$$\mathbb{E}_S[U(S)|r_i = 1, r_{-i}, y_{-i}] < \mathbb{E}_S[U(S)|r_i = 0, r_{-i}, y_{-i}],$$

since DM chooses H with a strictly lower probability after $(r_i = 0, r_{-i})$ than after $(r_i = 1, r_{-i})$. Thus, $y_i = 1$; i.e., on the equilibrium path, if i is pivotal at some r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$, then i reports his signal $s_i = 0$.

B

Proof of Proposition 3.1: Consider an equilibrium (ρ, y). We already elicited an expert's behavior on the equilibrium path when the expert holds a signal $s_i = 0$. Suppose that expert i holds a signal $s_i = 1$. By assumption, $s_i = 1$ is not verifiable. Therefore, i can choose any strategy y_i in $[0,1]$. If there does not exist any r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$ and i is pivotal at r_{-i}, then i is indifferent between all of his strategies. Consider the case where there exists r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$ and i is pivotal at r_{-i}. If i is pivotal at r_{-i} and Assumption 2 is satisfied, then this means that:

$$\rho(r_i = 0, r_{-i}) < \rho(r_i = 1, r_{-i}).$$

Since r_{-i} is on the equilibrium path, $\rho(r_i = 0, r_{-i})$ and $\rho(r_i = 1, r_{-i})$ are best responses for DM to $(r_i = 0, r_{-i})$ and $(r_i = 1, r_{-i})$, respectively. In other words, the above inequality is equivalent to the following ones: for $Pr_{DM}(S = x|r, y)$ DM's posterior belief that the state is x after r and given y,

$$\sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y) \le 0 \le \sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 1, r_{-i}, y),$$

and at least one of the two inequalities is strict. Let $\mathbb{E}_S[U(S)|r_i = z, r_{-i}, y]$ and $Pr(S_{-i} = x|r_{-i}, y_{-i})$ be analogously defined as in Appendix A. Observe the following:

$$\begin{aligned}
&\mathbb{E}_S[U(S)|r_i = 1, r_{-i}, y] \\
&= \sum_{x\in\{0,\dots,n-1\}} Pr(S_{-i} = x|r_{-i}, y_{-i})U(x+1) \\
&= \sum_{x\in\{0,\dots,n-1\}} Pr_{DM}(S = x+1|r_i = 1, r_{-i}, y)U(x+1) \\
&< \sum_{x\in\{0,\dots,n-1\}} Pr_{DM}(S = x+1|r_i = 1, r_{-i}, y)V(x) \le \sum_{x\in\{0,\dots,n\}} Pr_{DM}(S = x|r_i = 0, r_{-i}, y)V(x) \le 0
\end{aligned}$$

The second equivalence owes to Lemma 3.1 and its implication that $Pr_{DM}(s_i = 1|r_i = 1) = 1$: On the equilibrium path, after $r_i = 1$, DM infers that $s_i = 1$ since an expert who received a signal 0 never misreports it. The first strict inequality owes to the difference between the experts' and DM's preferences, i.e., $U(x+1) < V(x)$ for all $x \in \{0, \ldots, n-1\}$. The last weak inequality owes to the assumption that i is pivotal at r_{-i}. The proof for the first weak inequality follows that in Appendix A, hence the demonstration is omitted.

We conclude that H yields a strictly negative payoff to expert i when the latter is pivotal at some r_{-i} s.t. $Pr(r_{-i}|y_{-i}) > 0$. Since $\rho(r_i = 0, r_{-i}) < \rho(r_i = 1, r_{-i})$, we have that:

$$\mathbb{E}_S[U(S)|r_i = 1, r_{-i}, y_{-i}] < \mathbb{E}_S[U(S)|r_i = 0, r_{-i}, y_{-i}].$$

A direct implication of the above inequality is that if $s_i = 1$ and if $Pr(r_{-i}|y_{-i}) > 0$ for some r_{-i} at which i is pivotal, then i has a unique best response, $y_i = 0$.

We now show that in any equilibrium, DM chooses her default action with probability 1. By contradiction, suppose there exist two report profiles, $\hat{r}$ and r', s.t. $Pr(\hat{r}|y), Pr(r'|y) > 0$ and $\rho(\hat{r}) > \rho(r') = 0$. Recall that $\rho(r_i, r_{-i})$ is monotone and weakly increasing in r_i for any (r_i, r_{-i}) s.t. $Pr(r_i, r_{-i}|y) > 0$. Also, observe that our result in the above paragraph and Lemma 3.1 imply that the report profile of all 0s, $r^0 = (0, \ldots, 0)$ occurs with positive probability on the equilibrium path (i.e., $Pr(r^0|y) > 0$ if the experts and DM best respond to each other). These observations together with $\rho(\hat{r}) > \rho(r') = 0$ imply that $\rho(r^0) = 0$. In turn, $Pr(r^0|y), Pr(\hat{r}|y) > 0$ imply the following. There exists an expert i and a report profile for the other experts r''_{-i} s.t.: (i) $r_i = 1$, (ii) $r''_{-i} \le \hat{r}_{-i}$ (iii) $Pr(r''_{-i}|y_{-i}) > 0$ and (iv) i is pivotal at r''_{-i}. But (i), (iii) and (iv) contradict the result that $y_i = 0$. Hence, on the equilibrium path, DM chooses her default action with probability 1.

C

Proof of Proposition 4.1: Given y^*, on the equilibrium path, DM's belief after r_i is:

$$Pr(s_i = r_i | r_i, y_i^*) = 1.$$

Furthermore, observe that $Pr(r|y^*) > 0$ for all $r \in \{0,1\}^n$. I.e., every possible report profile occurs with positive probability on the equilibrium path. It thus follows that:

$$\mathbb{E}_S[V(S)|r, y^*] = V(R).$$

By assumption, $V(S) \geq 0$ for all $S \in \{\theta, \ldots, n\}$, and $V(S) < 0$ otherwise. We thus conclude that ρ^* given in Proposition 3.1 is a best response for DM to y^*.

Given ρ^* and y^*_{-i}, $y^*_i = 1$ is a best response for expert i if, and only if, i has no strictly profitable deviation. For all r_{-i} s.t. i is not pivotal at r_{-i}, i is indifferent between $r_i = 0$ and $r_i = 1$ since $\rho^*(r_i = 0, r_{-i}) = \rho^*(r_i = 1, r_{-i})$. Consider any r_{-i} at which i is pivotal, and suppose that $s_i = 0$. If i is pivotal at r_{-i}, given y^*_{-i}, i's belief about the state is:

$$Pr(S = x | r_{-i}, y^*_{-i}) = \begin{cases} 1 & if\, x = \theta - 1 \\ 0 & if\, x \neq \theta - 1. \end{cases}$$

Since $\theta - 1 < \gamma$ by assumption, since i is pivotal at r_{-i}, it follows that i' expected payoff from reporting $r_i = 1$ when $s_i = 0$ is:

$$\rho^*(r_i = 1, r_{-i}) U(\theta - 1) < 0.$$

Expert i's expected payoff from reporting $r_i = 0$ when i is pivotal at r_{-i} is:

$$\rho^*(r_i = 0, r_{-i}) U(\theta - 1) < 0.$$

Since $\rho^*(r_i = 0, r_{-i}) < \rho^*(r_i = 1, r_{-i})$, it follows that i strictly prefers $r_i = 0$ to $r_i = 1$. If i plays $y^*_i = 1$ and given that $s_i = 0$, then $r_i = 0$. If i deviates and reports $r_i = 1$, we know from our preceding conclusion that the deviation is strictly unprofitable. Thus $y^*_i = 1$ is a best response for expert i.

D

Proof of Lemma 4.1: The proof is organized as follows. First, we show that:

$$\rho^*(r) = \rho'(r) \qquad \forall r \in \{0,1\}^n$$

is a best response for DM to y'. Second, we show that (ρ^*, y^*) and (ρ', y') give the same expected payoff to the experts. Third, we show that (ρ', y') is an equilibrium.

Lemma D.1. *If* (ρ^*, y^*) *is an equilibrium, then* $\rho^*(r) = \rho'(r)$ *for all* $r \in \{0,1\}^n$ *is a best response for* DM *to* y'*.*

Proof. Consider all report profiles r s.t. $Pr(r|y^*) = 0$. Since $y'_i = 0 < y^*_i$, then $Pr(r|y') = 0$ as well. Thus,

$$\rho'(r) = \rho^*(r)$$

can be prescribed by a best response to y'. Consider all report profiles r s.t. $Pr(r|y') = 0 < Pr(r|y^*)$. Then, $r_i = 0$ and $Pr(r_{-i}|y^*_{-i}) > 0$. Since $Pr(r|y') = 0$,

$$\rho'(r) = \rho^*(r)$$

can be prescribed by a best response to y'. Next, consider all r s.t. $Pr(r|y') > 0$. Then, $Pr(r|y^*) > 0$ and $r_i = 1$. If (ρ^*, y^*) is an equilibrium, then $y^*_i \in (0,1)$ is a best response for expert i to the profile (ρ^*, y^*_{-i}). We already showed that $y^*_i \in [0,1)$ is a best response only if

$$\rho^*(r_i = 0, r_{-i}) = \rho^*(r_i = 1, r_{-i})$$

for all r_{-i} s.t. $Pr(r_{-i}|y^*_i) > 0$. For any such r_{-i}, recall that $Pr(r_i = 0, r_{-i}|y') = 0$, and we are assuming that:

$$\rho'(r_i = 0, r_{-i}) = \rho^*(r_i = 0, r_{-i}).$$

Since $y'_i < y^*_i$ and ρ' is a best response for DM to y', it follows that:

$$\rho^*(r_i = 1, r_{-i}) \geq \rho'(r_i = 1, r_{-i})$$

for any r_{-i} s.t. $Pr(r_{-i}|y^*_{-i}) > 0$. Combining the three inequalities above, we get that for any r_{-i} s.t. $Pr(r_{-i}|y^*_{-i}) > 0$,

$$\rho'(r_i = 0, r_{-i}) = \rho^*(r_i = 0, r_{-i}) = \rho^*(r_i = 1, r_{-i}) \geq \rho'(r_i = 1, r_{-i})$$

Since, generally, a best response is increasing in r_i (by the verifiability assumption), it follows that the weak inequality above cannot be strict. Therefore,

$$\rho'(r_i = 1, r_{-i}) = \rho^*(r_i = 1, r_{-i}),$$

i.e., $\rho'(r) = \rho^*(r)$ for all r s.t. $Pr(r|y') > 0$ is a best response to y'. $\square$

Lemma D.2. *If* (ρ^*, y^*) *is an equilibrium and* $\rho'(r) = \rho^*(r)$ *for all* $r \in \{0,1\}^n$*, then:*

$$\mathbb{E}_S[\tilde{U}(S); (\rho^*, y^*)] = \mathbb{E}_S[\tilde{U}(S); (\rho', y')]$$

i.e., the profiles (ρ^*, y^*) *and* (ρ', y') *are payoff-equivalent for the experts.*

Proof. The difference in expected payoff between the play of (ρ^*, y^*) and (ρ', y') for any $i \in E$ is: if $s_i = 1$,

$$\begin{aligned}
&\mathbb{E}_S[\tilde{U}(S);(\rho^*, y^*)] - \mathbb{E}_S[\tilde{U}(S);(\rho', y')] \\
=& \sum_{r_{-i}\in\{0,1\}^{n-1}} Pr(r_{-i}|y^*_{-i}) \sum_{x\le R_{-i}} Pr(S_{-i}=x|r_{-i}, y^*_{-i})[\rho^*(r_i=1, r_{-i}) - \rho'(r_i=1, r_{-i})]U(x) \\
=&0
\end{aligned}$$

since $(r_i = 1, r_{-i})$ occurs with positive probability if, and only if, $Pr(r_{-i}|y^*_{-i}) > 0$, and $\rho^*(r_i = 1, r_{-i}) = \rho'(r_i = 1, r_{-i})$ for any r_{-i} s.t. $Pr(r_{-i}|y^*_{-i}) > 0$ by Lemma D.1. If instead $s_i = 0$,

$$\begin{aligned}
\mathbb{E}_S[\tilde{U}(S);(\rho^*, y^*)] - \mathbb{E}_S[\tilde{U}(S);(\rho', y')] =& \sum_{r_{-i}\in\{0,1\}^{n-1}} Pr(r_{-i}|y^*_{-i}) \sum_{x\le R_{-i}} Pr(S_{-i}=x|r_{-i}, y^*_{-i})[y^*_i\rho^*(r_i=0, r_{-i}) \\
&+ (1-y^*_i)\rho^*(r_i=1, r_{-i}) - \rho'(r_i=1, r_{-i})]U(x) \\
=&0
\end{aligned}$$

At the end of the proof of Lemma D.1, we showed that:

$$\rho^*(r_i=0, r_{-i}) = \rho^*(r_i=1, r_{-i}) = \rho'(r_i=1, r_{-i}).$$

The last equality to 0 follows from this result. □

Lemma D.3. *The profile (y', ρ') with $\rho'(r) = \rho^*(r)$ for all $r \in \{0,1\}^n$ is an equilibrium.*

Proof. In Lemma D.1, we showed that ρ' is a best response for DM to y'. It remains to show that y'_j is a best response for expert j to the profile, for all $j \in E$. If $i = j$, we already concluded that:

$$\rho^*(r_i=1, r_{-i}) = \rho^*(r_i=0, r_{-i}) \quad \forall r_{-i} \;\; s.t. \;\; Pr(r_{-i}|y^*_{-i}) > 0.$$

Since $\rho^*(r) = \rho'(r)$ for all r,

$$\rho'(r_i=1, r_{-i}) = \rho'(r_i=0, r_{-i}) \quad \forall r_{-i} \;\; s.t. \;\; Pr(r_{-i}|y'_{-i}) > 0.$$

The above implies that i is indifferent between all of his strategies. Thus $y'_i = 0$ is a best response for expert i to the profile. Next, consider any expert $j \neq i$. Recall that $y'_j = y^*_j$. If $y^*_j = 1$, then j plays a best response to the profile. If $y^*_j < 1$, then j best responds to the profile if, and only if,

$$\rho'(r_j=1, r_{-j}) = \rho'(r_i=0, r_{-j}) \quad \forall r_{-i} \;\; s.t. \;\; Pr(r_{-i}|y'_{-i}) > 0.$$

Since $y'_i = 0$ while $y^*_i > 0$, $Pr(r_{-j}|y'_{-j}) > 0$ implies that $Pr(r_{-j}|y^*_{-j}) > 0$; since (y^*, ρ^*) is an equilibrium, then

$$\rho^*(r_j = 1, r_{-j}) = \rho^*(r_j = 0, r_{-j}) \quad \forall r_{-j} \;\; s.t. \;\; Pr(r_{-j}|y'_{-j}) > 0.$$

The result follows from the fact that $\rho^*(r) = \rho'(r)$ for all $r \in \{0,1\}^n$. $\square$

E

PROOF OF PROPOSITION 4.2: Consider a strategy profile (ρ^t, y^t) as defined in the Proposition. We first show that each expert's strategy is a best response to ρ^t. Since ρ^t only depends on the count of reports 1 in the truthful panel $\mathcal{T}$, no expert in $\mathcal{B}$ is ever pivotal; hence $y^t_i = 0$ is a best response to the profile played by the others, for any $i \in \mathcal{B}$. Observe that any expert $i \in \mathcal{T}$ has a positive probability of being pivotal. Hence, reporting his signal with probability 1 is the unique best response of expert i to the profile played by the others, for all $i \in \mathcal{T}$.

We now verify that ρ^t is a best response for DM to y^t. Observe that $Pr(r|y^t) > 0$ if, and only if, $r_i = 1$ for all $i \in \mathcal{B}$. Consider any such report profile r. Denote by $R_{\mathcal{T}} := \sum_{i \in \mathcal{T}} r_i$ the count of reports 1 among the experts in $\mathcal{T}$. DM's expected payoff from H given r and y^t is:

$$\begin{aligned}
\mathbb{E}_S[V(S)|r, y^t] &= \sum_{x \in \{R_{\mathcal{T}}, \ldots, R_{\mathcal{T}} + (n-t)\}} V(x) Pr(S = x | r, y^t) \\
&= \sum_{z \in \{0, \ldots, n-t\}} V(R_{\mathcal{T}} + z) \binom{n-t}{z} q^z (1-q)^{n-t-z}
\end{aligned}$$

since $y^t_i = 1$ for all $i \in \mathcal{T}$ and $y^t_i = 0$ for all $i \in \mathcal{B}$. Thus, after r and given y^t, a best response for DM is:

$$\rho(r) = \begin{cases} 1 & if \mathbb{E}_S[V(S)|r, y^t] > 0 \\ 0 & if \mathbb{E}_S[V(S)|r, y^t] < 0. \end{cases}$$

Recall that by assumption, $V(S) > 0$ for all $S \geq \theta$. It follows that if ρ^t is a best response for DM to y^t, and if r s.t. $R_{\mathcal{T}} \geq \theta$, then $\rho^t(r) = 1$. The mapping ρ^t satisfies this condition. Next, a best response for DM to y^t after r s.t. $\mathcal{R}_{\mathcal{T}} \leq \theta - 1$ is $\rho^t(r) = 0$ only if:

$$\sum_{z \in \{0, \ldots, n-t\}} V(\theta - 1 + z) \binom{n-t}{z} q^z (1-q)^{n-t-z} \leq 0,$$

since V is increasing. If the condition in the Proposition is satisfied, then the mapping ρ^t prescribes L after any such r. We thus conclude that ρ^t is a best response to y^t.

F

We first show that the equilibrium profile (ρ, y) featured in the Example is the profile that the experts weakly prefer to any other equilibrium profile. Said differently,

$$y \in \arg\max_{\tilde{y} \in \mathscr{Y}^*} \pi(\tilde{y})$$

By Lemma 4.1 and since the experts are ex-ante identical, it is without loss that we can restrict attention to the following strategy profiles for the experts:

- the truthful profile, $y^1 = (1,1,1)$,
- the profile where only expert 3 misreports, $y^2 = (1,1,0)$
- the profile featured in the example, where only expert 1 reports correctly, $y^3 = (1,0,0)$,
- the babbling profile, $y^4 = (0,0,0)$.

We established in Proposition 4.1 that the truthful profile can be sustained in an equilibrium, for any possible parameter values. Below, we show that given the parameters of the Example, (1) the babbling profile y^4 cannot be sustained in an equilibrium, (2) the profile y^2 can be sustained in an equilibrium.

Lemma F.1. *Given $n = 3$, $q = 0.35$ and $V(S) = S - 0.995$, the strategy profile y^4 cannot be sustained in an equilibrium; however, the strategy profile y^2 and ρ^2 below form an equilibrium:*

$$\rho^2(r) = \begin{cases} 0 & \textit{if } r \in \{(0,0,0),(0,0,1)\}, \\ 1 & \textit{if } r \in \{(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0),(1,1,1)\}. \end{cases}$$

Proof. We prove the first statement. If the experts play y^4 and $y_i^4 = 0$ is a best response for expert i, then i must not be pivotal at $r_{-i} = (1,1)$. Using Bayes' rule, DM's expected payoff after $r = (1,1,1)$ given y^4 is:

$$3 \times 0.35 - 0.995 > 0.$$

Hence, a best response prescribes DM to choose H after $r = (1,1,1)$. Consider the following report profile $r' = (0,1,1)$. Due to the verifiability assumption, $Pr(s_1 = 0|r_1 = 0) = 1$; since $Pr(r_2 = r_3 = 1|y_2, y_3) > 0$, it follows that DM's expected payoff after r' given y^4 is:

$$2 \times 0.35 - 0.995 < 0.$$

Hence, a best response for DM after r' prescribes L. But then expert 1 is pivotal at $(r_2, r_3) = (1,1)$; hence the expert's strategy $y_1^4 = 0$ is not a best response. Therefore, y^4 cannot be sustained in an equilibrium.

We now show that (ρ^2, y^2) is an equilibrium. It can be verified that expert 3 is not pivotal given ρ^2, for all $r_{-i} \in \{0,1\}^2$. Thus, $y_3^2 = 0$ is a best response for expert 2. Generally, $y_i = 1$ is a best response, for any expert i. Hence experts 1 and 2 are playing a best response. We now verify whether ρ^2 is a best response for DM to y^2. Observe that $Pr(r|y^2) > 0$ if, and only if, r belongs to the following set:

$$\mathscr{R} = \{(0,0,1), (0,1,1), (1,0,1), (1,1,1)\}.$$

Using Bayes' rule, DM's expected payoff from H after any $r \in \mathscr{R}$ is:

$$\begin{cases} 0.35 - 0.995 < 0 & \textit{if } r = (0,0,1) \\ 1.35 - 0.995 > 0 & \textit{if } r \in \{(0,1,1),(1,0,1)\} \\ 2.35 - 0.995 > 0 & \textit{if } r = (1,1,1). \end{cases}$$

Thus ρ^2 is a best response for DM to y^2. □

We now elicit the ex-ante expected payoff from the play of the equilibria (ρ^i, y^i) for all $i \in \{1,2,3\}$. Observe that in any of these equilibria, by Proposition 4.2, DM chooses L with probability 1 when $S = 0$ and she chooses H with probability 1 when $S = 3$.

1. Consider the truthful equilibrium (ρ^1, y^1). DM implements H whenever the state is weakly higher than 1. Thence,

$$\pi(y^1) = -2[3q(1-q)^2 + 3q^2(1-q)] + 0.5q^3.$$

2. Consider the equilibrium (ρ^2, y^2). It can be verified that DM will choose H with probability 1 whenever $S \in \{2,3\}$, and will choose L when $S = 1$ only if $s = (0,0,1)$. Thus,

$$\pi(y^2) = -2[2q(1-q)^2 + 3q^2(1-q)] + 0.5q^3$$

3. We already showed in the example that if (ρ^3, y^3), DM chooses H when $S = 1$ only if $s = (1,0,0)$. Also, DM chooses L when $S = 2$ only if $s = (0,1,1)$. Thus,

$$\pi(y^3) = -2[q(1-q)^2 + 2q^2(1-q)] + 0.5q^3.$$

It follows that $\pi(y^3) > \pi(y^2) > \pi(y^1)$. By Lemma 4.1, this implies that:

$$\max_{\tilde{y}\in\mathscr{Y}^*} \pi(\tilde{y}) = \pi(y^3).$$

Next, we elicit DM's best response to the profile $\hat{y} = (0.7, 0.7, 0)$. The set of reports r s.t. $Pr(r|\hat{y}) > 0$ is:

$$\hat{\mathscr{R}} = \{(0,0,1), (0,1,1), (1,0,1), (1,1,1)\}.$$

Using Bayes' rule, DM's expected payoff from H after any $r \in \mathscr{R}$ is:

$$\begin{cases} 0.35 - 0.995 < 0 & if\, r = (0,0,1) \\ 0.992 - 0.995 < 0 & if\, r \in \{(0,1,1),(1,0,1)\} \\ 1.63 - 0.995 > 0 & if\, r = (1,1,1). \end{cases}$$

Thus $\hat{\rho}$ is a best response for DM to $\hat{y}$. We also conclude that for a given play of $(\hat{\rho}, \hat{y})$, DM will choose H if and only if $r = (1,1,1)$. This event occurs with positive probability given $\hat{y}$; also, observe that $Pr(r = (1,1,1)|s = (0,0,0), \hat{y}) > 0$. Said differently, there is a positive probability that DM chooses H in every state. Observe that $\hat{y}_3 = 1$ implies that expert 3 always reports $r_3 = 1$. Thus, the probability that DM chooses H in state $S \in \{0,1,2,3\}$ equals the probability that experts 2 and 3 both report 1. This gives:

$$\begin{aligned}\pi(\hat{y}) = &\sum_{s\in\{0,1\}^3} U(S)q^S(1-q)^{3-S}\Pi_{i\in E}(1-\hat{y}_i)^{1-s_i} \\ =&q^3 \times 0.5 - 2\left[(1-q)^3 \times (0.3)^2 + q(1-q)^2 \times (0.69) + q^2(1-q) \times (1.6)\right]\end{aligned}$$

It can be directly verified that $\pi(\hat{y}) > \pi(y^3)$.

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