Score Disclosure*

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May 26, 2022

Abstract

We study verifiable disclosure by a monopolist when the product has multiple quality attributes. We identify an equilibrium in which the firm discloses a *score*—the average of the qualities—without revealing any further information. While full unraveling is still an equilibrium, it is dominated by the score equilibrium in terms of ex ante as well as ex post profits. Moreover, it is "defeated" by the score equilibrium.

Keywords: Monopoly, quality uncertainty, verifiable information disclosure, multi-dimensional types.
JEL classification: D82, D83, L12, L15.

1 Introduction

Most products in today's world have multiple vertical attributes (durability, safety, acceleration, age, etc.). And in many cases, consumers' purchase decisions are guided by some overall quality score rather than a thorough inspection of all the attributes. For example, while contemplating which laptop to buy, many consumers check out overall review scores on CNet.com without inspecting technical features in details. Many people select over various car or health insurance plans based on their Defacto.com "Star Ratings," which provide a coarse expert summary of their quality attributes and comprehensiveness. Moviegoers typically check out IMDb.com before selecting which movie to watch. Similarly, university applications in the US are heavily influenced by the rankings

*We thank Simon Anderson, Heski Bar-Isaac, Maarten Janssen, Bruno Jullien, Marco Pagnozzi, Régis Renault, Alex Smolin and participants of various seminars and conferences for valuable comments. Alexander Tonis provided excellent research assistance. Levent Celik acknowledges financial support from the Czech Science Foundation under grant number 18-26746S. All errors are our own.

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published by US News. From a behavioral point of view, looking at a score is certainly simpler and less costly than checking out all relevant individual attributes. In this paper, we show that even in the absence of any behavioral motives or any cost differences revealing only an overall quality score rather than fully disclosing each quality attribute can be an equilibrium outcome of a verifiable disclosure game.

Our analysis applies to situations where there is information asymmetry between sellers and buyers regarding product attributes. We assume that consumers do not know all product attributes well as it is generally costly or simply too complicated to gather/learn all details. We also assume that preferences for attributes vary across attributes as well as across consumers. Sellers, on the other hand, are typically a lot better informed about the attributes of their products, and we assume here for simplicity that the seller is informed perfectly. However, she lacks consumer-level information on how much each consumer values each attribute. We then ask how much information the seller discloses about each attribute in a verifiable disclosure game à la Grossman and Hart (1980), Grossman (1981) and Milgrom (1981). That is, the seller can provide vague information but cannot lie.

While in a single-attribute environment all consumers rank any pair of quality realizations in the same way, this is no longer the case with multiple attributes. Two individuals may differ substantially by how much they value each attribute, and as such, may differ substantially in terms of how they rank two particular quality vectors. And this creates a "horizontal" aspect in a multi-attribute environment.

For the main intuition for our results, compare two products with the same average quality $\frac{1}{2}$. One product has first quality equal to 1 and the second one equal to 0, $(q_1, q_2) = (1, 0)$, whereas the second product has both qualities equal to $\frac{1}{2}$, $(q_1, q_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$. The valuation of the buyer is $\theta_1 q_1 + \theta_2 q_2$, where θ_is are independent uniform random variables on $[0, 1]$. In the former case, the seller deals with unidimensional heterogeneity of consumers—only θ_1 matters—and sets price $\frac{1}{2}$ (production costs are zero) at which half of the consumers buy the product. The valuations for the second product have the triangular distribution on $[0, 1]$. By setting price at $\frac{1}{2}$ the seller gets the same profits because half of the consumers buy as before. However, this price is not optimal any longer and the seller can do strictly better.[1] More generally, when θ_i comes from a symmetric log-concave distribution, the valuations for a more balanced good are more concentrated around the mean implying the rotation of the demand around the mean counterclockwise. Optimal prices are (weakly) below the mean and hence, the demand expands for a more

[1]The optimal price is $\sim$0.41 and the profits are $\sim 0.27 > \frac{1}{4}$, the profits under unidimensional heterogeneity. See Example in the Appendix.

balanced good (this effect was absent in the example above). Adjusting the price provides an additional positive effect on profits.

The intuition above shows that consumers prefer balanced goods. By disclosing only the "score", i.e., the average quality—we call this *score disclosure*—the seller effectively creates a balanced good since consumers care about the expected quality in each dimension, and the seller's profits are then higher than under full disclosure. We show that score disclosure constitutes a perfect Bayesian equilibrium (PBE) whereby the seller pools together all (q_1, q_2) for which $\frac{q_1+q_2}{2}$ is the same, and sends a common message m for them. As higher scores are strictly better other things being equal, each different level of $\frac{q_1+q_2}{2}$ is fully revealed in equilibrium. However, while the buyer infers $\frac{q_1+q_2}{2} = m$ upon observing message m, she cannot tell apart q_1 and q_2 beyond that. In other words, the initial quality dimensions (q_1, q_2) are replaced by the score—the vertical dimension—and the balance—the horizontal dimension. There is full unraveling along the vertical dimension and full pooling in the horizontal dimension.

Full disclosure is also a PBE in our model (supported by extreme off-equilibrium beliefs) but it results in lower expected profits than the score disclosure. We show more generally that any symmetric disclosure—that is, when any equilibrium message leads to equal expected qualities in the two dimensions—yields the same expected profit which is higher than under any asymmetric disclosure including full disclosure. However, some symmetric disclosures—such as no disclosure—cannot be maintained as a PBE in the ex post disclosure game where the firm makes its disclosure decision after learning its type. Thus, there is a natural reason for why we might often observe score disclosure in multi-attribute environments.[2,3]

Although we develop our model and present its results in the context of a standard product market, the main elements of the analysis can be applied to many other markets. For instance, one may look at financial markets where managers make disclosures about their assets that have multiple attributes. As Goldstein and Yang (2015) discuss, "...cash flows depend on the demand for firms' products and the technology they develop, on firms' idiosyncratic developments and the way they are affected by the macroeconomy or the industry, and on the success of firms' operations in traditional lines of business and in new speculative lines of business." One may also adapt the analysis to markets where

[2]There are other—non-linear—score disclosure equilibria. They result in the same expected profits as mentioned above but have higher informational requirements as we discuss in Section 3.2.

[3]Also, score disclosure equilibrium dominates the full disclosure equilibrium in the sense of the (un)defeated equilibrium refinement introduced by Mailath, Okuno-Fujiwara and Postlewaite (1993).

there is no price or the price is regulated. For instance, an incumbent party trying to get approval on a set of public projects that differ in terms of various quality aspects needs to think carefully about how to present its proposal. Similarly, the problem of a university to promote itself to potential students can be addressed in a similar framework.

Relation to prior literature A large literature has analyzed quality disclosure when the product has a single vertical attribute. In their seminal papers, Grossman and Hart (1980), Grossman (1981) and Milgrom (1981) show the famous unraveling result assuming a credible and costless way of revealing the quality. The primary driver of this celebrated result is that if quality is ever withheld, then it has to be the lowest quality, because otherwise the seller would have disclosed it.[4]

In a Hotelling framework, Sun (2011) and Celik (2014) show that equilibria typically involve partial revelation when disclosure pertains to the horizontal attribute. Koessler and Renault (2012) study a more general model that allows for both horizontal and vertical differentiation, and find that a fully-revealing equilibrium always exists if product and consumer types are independently distributed. Our setup fits this description, so in line with Koessler and Renault (2012), full disclosure is an equilibrium in our model. They also provide a sufficient condition under which another equilibrium exists—but it is not satisfied in our model.[5] Anderson and Renault (2013) also consider both modes of differentiation, but they model the horizontal aspect as a random utility term, so the seller has no better information about it than the buyer.

Several other papers address information disclosure in the presence of multi-attribute products. Ma and Mak (2014) consider a similar setup to ours, where a monopolist sells a good with two qualities and consumers are heterogeneous in their valuation of one of the qualities. They analyze the properties of full and average quality (similar to our score) disclosure, albeit implemented by a public agency. Also differently from this paper, they consider non-uniform pricing whereby the seller offers a continuum of menus differentiated by price and the amount of each quality component contained in the good. They find that the average quality disclosure generates higher consumer and social surpluses. Martini (2018) assumes convex preferences and allows only full or no disclosure for each type. He obtains that high types on both dimensions pool together while others fully disclose. We instead have linear preferences and introduce heterogeneity

[4]See Dranove and Jin (2010) and Renault (2016) for recent surveys on the subject.

[5]This condition is that the match is "generic" in their terminology. Another difference is that Koessler and Renault (2012) assume a finite type space while we have an infinite one.

of buyers and obtain score disclosure. Other papers take different perspectives. Smolin (2020) considers the seller's optimal screening via informational experiments. However, he assumes that the seller announces its plan before product attributes are realized (as in the Bayesian persuasion literature). Fishman and Hagerty (1990), Shin (1994), Dziuda (2011) and Hoffmann, Inderst and Ottaviani (2020) allow for multiple attributes, but their focus is on partially certifiable disclosure. Finally, Chakraborty and Harbaugh (2010) study disclosure of multiple attributes where the form of communication is cheap talk, and Carroll and Egorov (2019) consider a similar setting where the receiver can verify one of the attributes.[6]

The literature on bundling has provided related results. The main result there is that (pure) bundling dominates separate sales when the marginal cost is low (see, e.g. Fang and Norman, 2006). We obtain that score disclosure dominates full disclosure also when the marginal cost is low. Note, however, the substantial differences in the setup: Bundling concerns selling several goods of known qualities together in a bundle while our interest is on the provision of information about a multi-attribute good, and in the verifiable disclosure setting. In a sense, our setting can be thought of as "informational bundling" of a bundle (since formally, the multi-attribute good can be considered as a bundle) and its connection to the standard physical bundling is an interesting avenue for future research.

The literature on information provision to consumers (e.g., Lewis and Sappington (1994), Anderson and Renault (2006), Johnson and Myatt (2006), Saak (2006), Bar-Isaac, Caruana and Cuñat (2010)) is also related. However, these papers use a random utility setup in which the quality is known while the consumers do not fully know their valuations. Thus, the seller discloses information to make the consumers better understand their valuations.

The rest of the paper is organized as follows. Section 2 introduces the model and shows the relevant benchmarks. Section 3 shows the score disclosure equilibrium and discusses its properties. Section 4 discusses a number of related issues and extensions such as welfare, positive marginal costs and regulated prices. Section 5 concludes. The proofs are contained in the Appendix.

[6] Our analysis and results also relate to the literature on "opaque" products, whereby multi-product sellers keep important characteristics of their products hidden until after purchase. See, for instance, Fay and Xie (2008) and Anderson and Celik (2020).

2 Model

2.1 Setup

There are two players, Seller (S, "she") and Buyer (B, "he"). The seller has a product with two vertical attributes (qualities) $(q_1, q_2) \in [\underline{q}, \bar{q}]^2$. This is her private information, while the buyer only knows that q_1 and q_2 are distributed independently according to some distribution G. The production costs are zero.[7]

The buyer has a unit demand for the product. His utility from buying is $u = \theta_1 q_1 + \theta_2 q_2 - p$, where $(\theta_1, \theta_2) \in [0,1]^2$ is his two-dimensional type and p is the price. Not buying yields zero utility. The buyer knows his type while the seller only knows that both θ_1 and θ_2 are distributed independently according to a symmetric log-concave distribution F on $[0,1]$. The profit of the seller is p in case the sale takes place and 0 otherwise.

The timing is as follows. First, Nature selects values of q_i and $\theta_i, i = 1, 2$, from G and F, respectively. After privately observing (q_1, q_2), the seller sends a verifiable message $m \in M = 2^{[0,1]^2}$ and sets price p.[8,9] "Verifiable" means that the message must be true but it can be imprecise such as $(q_1, q_2) \in \left[0, \frac{1}{2}\right] \times \left[0, \frac{1}{2}\right]$. The buyer privately observes (θ_1, θ_2), then observes message m and price p set by the seller and decides to purchase the product if

$$v(\theta_1, \theta_2, m) = \mathrm{E}[\theta_1 q_1 + \theta_2 q_2 \mid \theta_1, \theta_2, m] \geq p. \tag{1}$$

For a given m, the seller chooses p to maximize her expected profit:

$$\max_p \pi(p, m) = p \Pr[v(\theta_1, \theta_2, m) \geq p \mid m].$$

Finally, the payoffs are realized. All aspects of the game are common knowledge.

We employ the standard perfect Bayesian equilibrium (PBE) as the solution concept for our analysis. A PBE consists of (i) profit maximizing disclosure and pricing decisions by each seller type; (ii) a posterior belief about quality attributes formed via Bayesian updating whenever possible; and (iii) a purchasing decision by each buyer as a best response to pricing and disclosure strategies of each seller type. We also assume that

[7] We discuss the case of positive marginal costs in Section 4.2.

[8] In this environment, any information signaled by the price can be directly revealed by a message at no cost and hence, the price is not used for signaling as is also the case in, e.g., Sun (2011) and Koessler and Renault (2012).

[9] In the end of this section, we introduce sceptical beliefs. For them to be well defined, we restrict messages to be only closed subsets of $[0,1]^2$. Since all the payoffs are continuous in (q_1, q_2), this does not affect any results but only simplifies the notation and the proofs.

the buyer has sceptical beliefs—a standard assumption in the disclosure literature—i.e., after any off-equilibrium message, he thinks that the seller is of the worst possible type compatible with the message.[10]

2.2 Preliminaries

2.2.1 Observable θ_1 and θ_2

Consider a benchmark when the type of the buyer (θ_1, θ_2) is observable. Then, the seller when disclosing the quality (q_1, q_2) can extract the whole consumer surplus by charging $p = \theta_1 q_1 + \theta_2 q_2$. Alternatively, she can disclose a weighted sum, $\theta_1 q_1 + \theta_2 q_2$, and also extract the whole consumer surplus. In other words, the isoprofit curve in the space (q_1, q_2) is linear with the slope $-\frac{\theta_1}{\theta_2}$, and the seller reveals a subset of that line to which her quality belongs—which might be her exact location, the whole line or anything in between. Both full disclosure and disclosing the relevant line with the slope $-\frac{\theta_1}{\theta_2}$ are PBEs under the assumption of sceptical beliefs.

2.2.2 Observable q_1 and q_2

Another benchmark is when both quality attributes are perfectly observable. This also sets the benchmark for later analysis. The seller earns profits

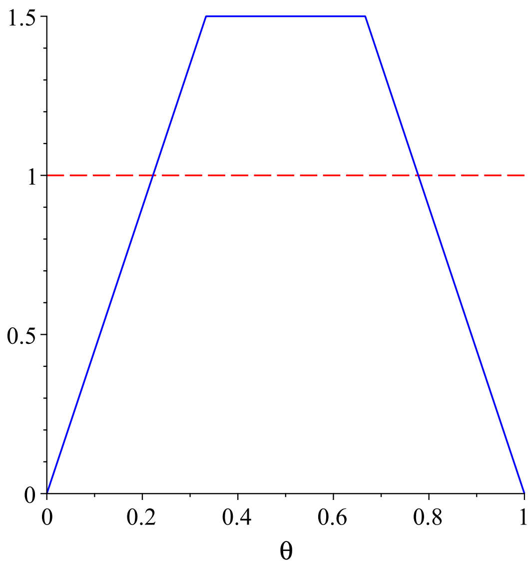
$$\pi_f(q_1, q_2) = \arg\max_p p \Pr[\theta_1 q_1 + \theta_2 q_2 \geq p]. \tag{2}$$

We are interested in the shape of the isoprofit curves. Next Lemma provides the key result that we use throughout the paper.

Lemma 1 *Take a product with attributes (q_1, q_2). The seller's profits (2) from the product with attributes $(\alpha q_1 + (1-\alpha) q_2, \alpha q_2 + (1-\alpha) q_1)$, where $\alpha \in \left[0, \frac{1}{2}\right]$, strictly increase in α.*

In words, consider the straight line joining (q_1, q_2) and (q_2, q_1). The profits are the highest at $\alpha = \frac{1}{2}$, that is, at $(\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2})$ and then monotonically decrease moving away from the diagonal. Due to the symmetry, the seller earns the same profits for (q_1, q_2) and (q_2, q_1), and hence the interval for α can be restricted to $\left[0, \frac{1}{2}\right]$.

[10] Okuno-Fujiwara, Postlewaite and Suzumura (1990) show that the equilibrium involves sceptical beliefs and full disclosure, but their model is unidimensional. In our model with two dimensions, there are multiple equilibria and hence, we require sceptical beliefs only for off-equilibrium messages.



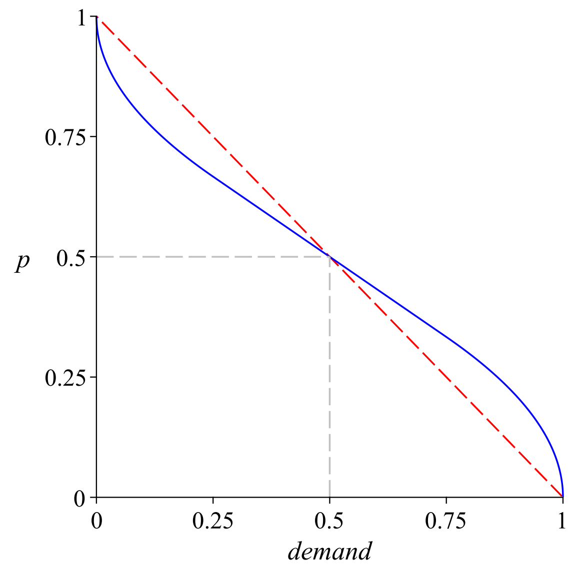


Figure 1: Comparing the distributions of $\theta_1 q_1 + \theta_2 q_2$ (dashed red line) and $\theta_1(\alpha q_1 + (1-\alpha)q_2) + \theta_2(\alpha q_2 + (1-\alpha)q_1)$ (solid blue line) when θ_i is distributed as uniform on $[0, 1]$, $(q_1, q_2) = (1, 0)$ and $\alpha = \frac{2}{3}$. *Left:* PDFs. *Right:* Demands, i.e., 1-CDF with inverted axes.

The intuition for Lemma 1 is the following. Holding $q_1 + q_2$ constant, as the two attributes become more balanced, the seller is able to extract a higher surplus from consumers. This follows from the fact that, as the two attributes become more balanced, the distribution of the valuations gets more concentrated in the middle. This causes the demand curve facing the seller to rotate counterclockwise expanding demand at lower prices. The optimal price is below the average valuation, a well-known feature of the monopoly pricing, and hence, the demand becomes higher at that price. As a result, the seller's profits are higher. See Figure 1 for an illustration.

For a more formal explanation, introduce the *peakedness order* (Birnbaum, 1948): For two symmetric random variables X and Y, X is more peaked than Y in the peakedness order if $|Y - E[Y]|$ first-order stochastically dominates $|X - E[X]|$. It is equivalent to the PDF of X crossing the PDF of Y once on each side of the mean (first from below and then from above) and to CDF of X crossing the CDF of Y once and from below, see Figure 1. When the random variables are also iid and logconcave, their combination is more peaked than any of them (Proschan, 1965).

Lemma 1 implies that the isoprofit curves are convex near the diagonal and that each isoprofit curve lies below the straight line connecting its extreme points, see Figure 2 for an illustration.

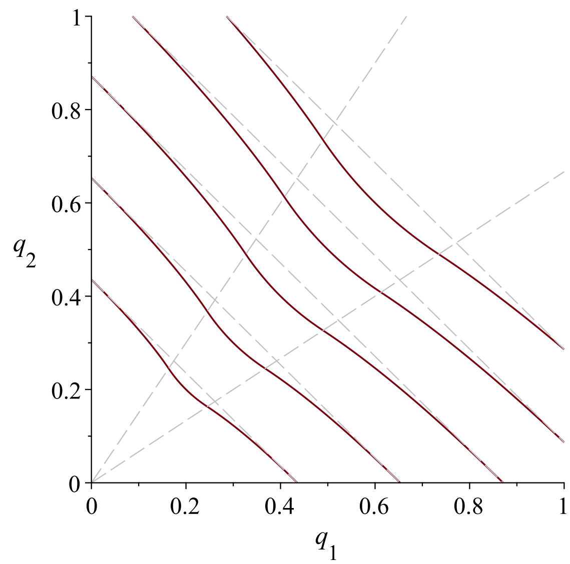


Figure 2: Isoprofit curves when θ_i is distributed as uniform on $[0, 1]$. The rays separate regions where isoprofit curves are convex or concave (see Appendix for details).

3 Score disclosure

We now turn to the main section of the paper. Call the average quality, $\frac{q_1+q_2}{2}$, the *score*. We show in Section 3.1 that there exists an equilibrium in which the seller discloses the score. In Section 3.2 we discuss other equilibria, show that all symmetric disclosure rules are payoff equivalent from an ex-ante perspective and argue why the score equilibrium might be preferred. Section 3.3 shows that the score disclosure equilibrium "defeats" the full disclosure equilibrium in terms of the equilibrium refinement of Mailath, Okuno-Fujiwara and Postlewaite (1993), providing yet another argument in its favor.

3.1 Main result

Next proposition is the main result of the paper.

Proposition 1 *There exists an equilibrium in which the seller discloses only the score.*

The intuition for Proposition 1 is the following. First, it is clear that in the equilibrium the seller cannot inflate her score since the standard unraveling argument applies. Hence, the seller chooses between different messages all leading to the same (and true) average quality. Second, since the valuation of the buyer is linear in the two attributes, his decision whether to buy depends only on the expected qualities. Then, the results of

Section 2.2.2 and, in particular, Lemma 1 apply. Thus, the highest profits are reached when the expected qualities are equal which is achieved by disclosing the score.

Proposition 1 is rather surprising. Indeed, the differentiation along each attribute is vertical and hence, one might think that the unraveling result of Grossman and Hart (1980), Grossman (1981) and Milgrom (1981) should apply and full disclosure should be the only equilibrium. Yet, a *combination* of attributes also has a horizontal dimension and despite the buyer's valuation being linear in the attributes, he is averse to unbalanced goods. Hence, the seller fully pools on this horizontal dimension by only revealing the score.[11]

In contrast to the existing literature on quality disclosure, which predicts full unraveling as the unique equilibrium, score equilibrium in a multi-attribute environment is associated with an intermediate level of information disclosure. This result also differs from the information provision literature (e.g. Lewis and Sappington, 1994; Johnson and Myatt, 2006) which predicts that either full or no information provision is optimal.

3.2 Other equilibria and why score equilibrium is natural

The model presented in Section 2 allows other PBEs. A trivial example is full disclosure of each attribute.[12] If consumers associate any deviation m from full disclosure with the product configuration involving the maximal $|q_1 - q_2|$ given m, then the seller will be forced to disclose the attributes fully. Take the following example. Suppose $(q_1, q_2) = (0.6, 0.2)$. If, while consumers anticipate full disclosure, the seller discloses that $\frac{q_1+q_2}{2} = 0.4$, then consumers might believe that $(q_1, q_2) = (0.8, 0)$ because PBE does not impose any restrictions on beliefs following an off-equilibrium message. The seller will then earn lower profits with $(q_1, q_2) = (0.8, 0)$ than $(q_1, q_2) = (0.6, 0.2)$. Therefore, she would not opt for such a deviation. This reasoning applies to any message other than full disclosure, and for any realization of qualities.[13]

There are many other PBEs. For instance, it is an equilibrium that the seller discloses only the isoprofit curve to which the qualities belong, that is, that $(q_1, q_2) \in \{(q_1, q_2) : \pi(q_1, q_2) = \bar{\pi}\}$. Similarly, any curve in the (q_1, q_2) space which is strictly decreasing, symmetric about the diagonal and is contained in the area between the linear

[11]The results easily extend to a higher number of quality attributes. In fact, the seller has even higher incentives to opt for score than full disclosure as the number of attributes increases.

[12]This is in line with Koessler and Renault (2012), who show that a fully revealing equilibrium always exists if product and consumer types are independently distributed.

[13]As we show in Section 3.3, PBE with these extreme pessimistic off-equilibrium beliefs is not immune to the refinement of undefeated equilibrium.

score line and the full-information isoprofit curve can also be supported as an equilibrium score function.

More generally, consider a piece-wise continuously differentiable score function $s : [0,1]^2 \to [\underline{m}, \overline{m}]$ which is strictly increasing in q_1 and q_2. Whenever the buyer observes a message $m \in [\underline{m}, \overline{m}]$, she infers that $(q_1, q_2) \in s^{-1}(m)$. Let $\bar{q}_1(m)$ and $\bar{q}_2(m)$ denote the buyer's expectations of q_1 and q_2 upon observing message m:

$$\bar{q}_1(m) = \mathrm{E}[q_1 | s(q_1, q_2) = m],$$
$$\bar{q}_2(m) = \mathrm{E}[q_2 | s(q_1, q_2) = m],$$

and the profits are then

$$\pi(m) = \pi_f(\bar{q}_1(m), \bar{q}_2(m)).$$

That is, the seller's profits are equal to what she would achieve if she fully disclosed $(q_1, q_2) = (\bar{q}_1(m), \bar{q}_2(m))$.

We say that the disclosure is *symmetric* if $\bar{q}_1(m) = \bar{q}_2(m)$ for any message m sent in the equilibrium and otherwise it is asymmetric. We then have the following result.

Proposition 2 *(i) For any symmetric disclosure the expected seller's profit is the same.*

(ii) For any asymmetric disclosure the expected seller's profit is lower than under symmetric disclosure.

The intuition for part (i) is the following. The profit function (2) is homogeneous of degree 1 in qualities (q_1, q_2). Indeed, when qualities double, doubling the price keeps the probability of sale unchanged and hence, doubles the profits. This implies that when (expected) qualities are equal, the profits are linear in them. Hence, by the law of iterated expectations any symmetric disclosure—the expected qualities are equal after any message—yields the same expected profits. The proof actually does not require that the disclosure arises in some PBE and hence, it also covers the case when the seller has commitment power. This implies, in particular, that no disclosure—since it is symmetric—yields the same profits. "Binary" disclosure, that is, disclosing if the average quality is above a certain level, also gives the same profits.

Part (ii) is based on developing further the preference for balancedness as shown in Lemma 1. It shows that replacing any (q_1, q_2) by the score $(\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2})$ increases the profits while the homogeneity of degree 1 mentioned above implies that the expectation can be taken outside, $\pi_f(\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2}) = \frac{1}{2}(\pi_f(q_1, q_1) + \pi_f(q_2, q_2))$. In other words, the profit function (2) is supermodular in (q_1, q_2). By definition, for any asymmetric disclosure, there will be at least one message leading to unequal qualities which—after integrating over all equilibrium messages—leads to lower profits than under symmetric disclosure. A direct implication is of course that full disclosure results in lower profits than score disclosure, and, from part (i), the seller cannot get more than with the score disclosure.

As we mentioned above, any curve in the (q_1, q_2) space which is strictly decreasing, symmetric about the diagonal and is contained in the area between the linear score line and the isoprofit curve can be supported as an equilibrium score function. Because of symmetry, by Proposition 2, part (i), it will result in the same expected profits. Clearly, the isoprofit curve is determined by the distribution of buyer preferences (θ_1, θ_2). Similarly, whether a curve is in between the linear score line and the isoprofit curve also depends on the distribution of buyer preferences. The linear score disclosure is, however, an equilibrium for any (symmetric) preference distribution. Also, any non-linear score disclosure equilibrium is sensitive to the buyer's beliefs about G, the quality distribution. In this sense, the linear score is perhaps the most natural—robust—equilibrium.

3.3 Undefeated equilibrium refinement

It is well understood for games of costless verifiable information disclosure that equilibrium refinements such as intuitive criterion or universal divinity have no bite in selecting equilibria. We instead turn attention to the "undefeated equilibrium" refinement introduced by Mailath, Okuno-Fujiwara and Postlewaite (1993).[14] In contrast to intuitive criterion or universal divinity, undefeated equilibrium selection does not depend on forward induction reasoning. As discussed in Mailath, Okuno-Fujiwara and Postlewaite (1993), it provides a foundation against the selection of the least-costly separating equilibrium in signaling games, which is uniquely selected by criteria based on forward induction reasoning. In our model, it implies, as we show below, that the score disclosure equilibrium is undefeated, and moreover it "defeats" the full disclosure equilibrium.[15]

According to the undefeated equilibrium refinement equilibrium, an equilibrium is defeated if it fails the following test. Consider a proposed equilibrium and take a message that is off the equilibrium path. If there is an alternative equilibrium in which this message is on the equilibrium path for a non-empty set of types and these types obtain a higher

[14] Recently, undefeated equilibrium selection was used in Gill and Sgroi (2012), Celik (2014), Perez-Richet (2014) and Lauermann and Wolinsky (2016).

[15] Score disclosure would also be selected over full disclosure by Pareto-based refinements which rule out equilibria that are payoff dominated for all sender types (strictly for some) by another equilibrium; see Rhodes and Wilson (2018) for an example.

payoff (strictly higher for at least one type) in the alternative equilibrium, then the test requires that the beliefs in the former equilibrium follow Bayes' rule for this set of types. We present the result in the next proposition.

Proposition 3 *(i) Full disclosure equilibrium is defeated by score disclosure equilibrium.*
(ii) Score disclosure equilibrium is undefeated.

The proof and a formal definition of undefeated equilibrium are provided in the Appendix. The intuition is the following. For part (i), consider message $\frac{q_1+q_2}{2} = m$ for some $m \in (0, 0.5)$. It is an equilibrium message in the score disclosure equilibrium but not in the full disclosure equilibrium. Type $(q_1, q_2) = (m, m)$ earns equal profits in both equilibria while all other (q_1, q_2) with the same score m earn strictly higher profits in the score disclosure equilibrium. Therefore, the definition of the undefeated equilibrium requires the off-equilibrium beliefs in the full disclosure equilibrium to be consistent with the equilibrium beliefs in the score disclosure equilibrium. Consider type $(2m, 0)$. Since it is the least balanced among those that send message $\frac{q_1+q_2}{2} = m$, it will get strictly higher profit when the buyer assigns a positive probability to more balanced types. It will then deviate from full disclosure and thus the score disclosure equilibrium defeats the full disclosure one.

Proof of part (ii) follows from Lemma 1, which implies that full-information isoprofit curves are convex near the diagonal and that each isoprofit curve lies below the 135-degree line connecting its extreme points. Take any alternative equilibrium with messages that differ from score disclosure. Within the set of types that send a particular message, the type with the highest score lies on a higher 135-degree line than the expected type implied by the message. Then, this highest score type earns a strictly higher profit in the score disclosure equilibrium than in the alternative equilibrium.

Hence, not only is the full disclosure equilibrium payoff dominated by the score equilibrium (Proposition 2), but it is also defeated by the criterion of undefeated equilibrium. Therefore, full disclosure in the presence of multiple quality attributes has little appeal in terms of its equilibrium properties, and there are natural reasons to focus on other equilibria. As we argued above, score disclosure stands out in terms of its simplicity and robustness.

4 Discussion and extensions

In this section we discuss various issues and extensions of the main model. Section 4.1 discusses the effect of the score disclosure on the optimal price, consumer surplus and welfare. Section 4.2 considers the case of positive marginal costs. In Section 4.3 the price is fixed because, for example, it is regulated.

4.1 Social welfare

As we showed above, score disclosure always increases profits relative to the full disclosure for any given q_1 and q_2 and hence, expected profits also increase. The comparison of the consumer surplus and social welfare is more nuanced. Consider first the effect on the consumer surplus. Under score disclosure consumers get less information and hence, on average make worse choices. For a fixed price, therefore, the consumer surplus is always lower than under full disclosure. When the seller can adjust the price, the price can change in any direction. Indeed, while the demand increases under score disclosure in the relevant price range as in Figure 1(right), the change in the elasticity of the demand is ambiguous. It can be easily seen that the demand cannot become more (or less) elastic everywhere. Hence, in some cases the price increases and in others it decreases.

If the price increases then consumers are hurt not only by less information under score disclosure but also by a higher price. If, instead, it decreases then the effect on the consumer surplus is ambiguous. The effect of the score disclosure on welfare then combines the always positive effect on the profits and the ambiguous effect on the consumer surplus and hence, is ambiguous too. Figure 3 provides an illustration. In the right panel the optimal price decreases with the balancedness of the good and, hence score disclosure decreases the price. While consumer surplus is also lower under score disclosure (not in the figure), the positive effect of profits dominates and score disclosure unambiguously increases welfare. Expected welfare then increases for any distribution of q_i. In the left panel the effect on the price and welfare (and also on the consumer surplus, not in the figure) depends on the initial balancedness of the good. The effect on the expected welfare then depends on the distribution of q_i. For a uniform distribution of q_i score disclosure increases expected welfare relative to full disclosure.

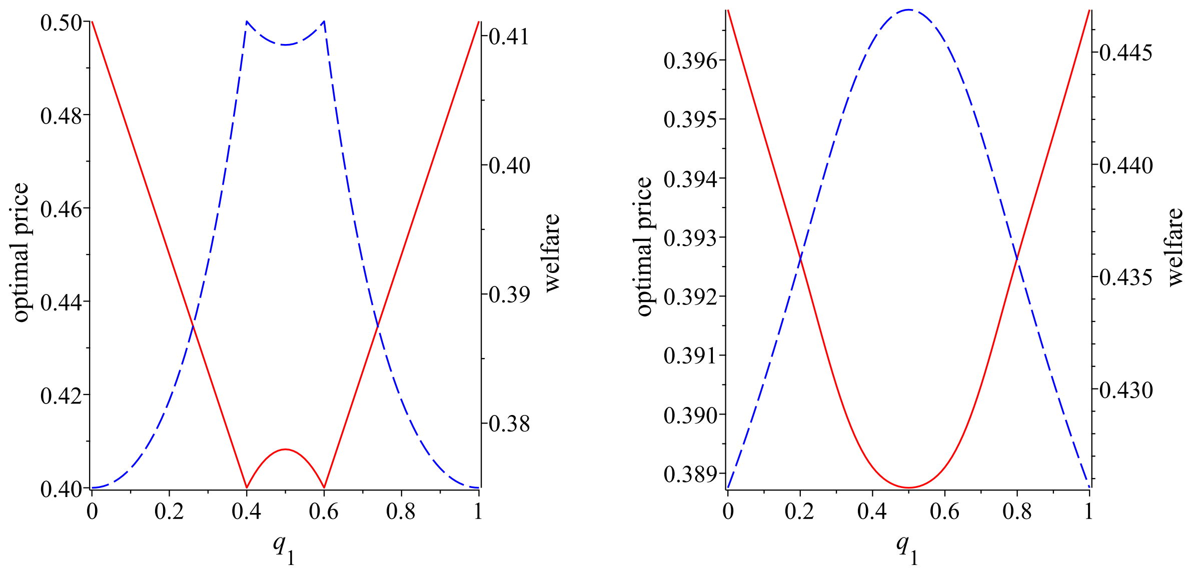


Figure 3: The optimal price (left axis, solid red line) and welfare (right axis, dashed blue line) as a function of q_1 when $q_1 + q_2 = 1$. Score disclosure corresponds to $q_1 = \frac{1}{2}$. *Left:* θ_i is distributed as uniform on $[0, 1]$. *Right:* θ_i is distributed with pdf $f(\theta) = 12 \min\{\theta^2, (1-\theta)^2\}$ on $[0, 1]$.

4.2 Positive marginal costs

In this section we discuss the case of constant positive marginal costs $c > 0$. The next Lemma provides a generalization of Lemma 1.

Lemma 2 *Take a good with attributes (q_1, q_2) and let $\pi(\alpha)$ denote seller's profits from the good with attributes $(\alpha q_1 + (1-\alpha)q_2, \alpha q_2 + (1-\alpha)q_1)$, where $\alpha \in \left[0, \frac{1}{2}\right]$.*

(i) $\pi(\alpha)$ is quasi-convex in α.

(ii) If $c = 0$, $\pi(\alpha)$ strictly increases in α.

(iii) If $c \geq \frac{q_1+q_2}{2}$, $\pi(\alpha)$ strictly decreases in α.

Part (ii) of Lemma 2 is just a repetition of Lemma 1 for the sake of completeness. Part (iii) presents the opposite case: When the price is above the mean valuation $\frac{q_1+q_2}{2}$, the rotation of the demand counter-clockwise makes it lower at this price (see Figure 1, right) and the profits decrease. In both cases, the condition on the marginal costs is sufficient for the respective result but not necessary—a weaker condition can be found for a given distribution of consumer tastes. In the uniform example used in Figures 1 and 2 these conditions can be weakened to $c \leq \frac{1}{2}\min(q_1, q_2)$ in part (ii) and to $c \geq \frac{1}{4}(q_1 + q_2)$ in part (iii).[16]

[16] For a general distribution, the condition $c = 0$ in part (ii) cannot be relaxed because the distribution

Part (i) of Lemma 2 presents the general case. Profits are non-monotonic in α if the price shifts from being below to above the mean valuation of $\frac{q_1+q_2}{2}$ or vice versa. Crucially, if the price is below for some α, then it will stay below for any higher α since it induces a higher demand below the mean and a lower demand above the mean. Hence, the only possibility is that the price is above $\frac{q_1+q_2}{2}$ for low values of α and then switches to be below for higher values of α. This results in a U-shaped $\pi(\alpha)$ which is quasi-convex.[17]

Let us now turn to the equilibria. The full disclosure equilibrium of course always exists. For small costs—or equivalently, high quality—the profits increase in the balancedness of the good by Lemma 2, part (ii) (see fn. 16) and our main results hold, in particular, Proposition 2, part (ii).[18] In words, the score disclosure equilibrium exists and dominates the full disclosure equilibrium in terms of expected payoffs, and also defeats it, as in Proposition 3. For intermediate levels of costs and quality, the seller fully reveals very unbalanced quality realizations and reveals only the score for relatively balanced ones. For high costs or low quality, the score disclosure equilibrium does not exist. The three regions may coexist in which case the score disclosure equilibrium becomes a "partial" score disclosure equilibrium combining regions of full disclosure, score disclosure and a region where the seller fully reveals very unbalanced quality realizations and reveals only the score for relatively balanced ones. Figure 4 provides an illustration.

Note that the regions of full or score disclosure are reversed, in a sense, relative to the standard results on partial disclosure due to, say, certification costs (Grossman and Hart, 1980; Jovanovic, 1982). There, the high types are willing to pay them and separate while low types pool. It is also different in this sense from Johnson and Myatt (2006). They show that maximum provision of information—analogue to our full disclosure—occurs in the niche markets, that is, when costs are high enough. On the other hand, in the mass markets—when the costs are low enough—minimum provision of information is optimal.

Let us now briefly comment on a setting where θ_1 and θ_2 are distributed independently

of $\theta_i, i = 1, 2$, has its support on $[0, 1]$. If the lower bound is strictly positive, then part (ii) holds for small positive costs.

[17] Johnson and Myatt (2006) present a related result. In their setting, the monopolist provides information or designs the product making the demand more or less disperse—which they model as a rotation of the demand.

[18] Proposition 2, part (i), does not hold because the profit function is not homogeneous of degree 1 and hence, different symmetric disclosures yield different expected profits. Indeed, consider some (q_1, q_2), associated optimal price p and take $\lambda > 1$:

$$\pi_f(\lambda q_1, \lambda q_2) \geq \Pr[\theta_1 \lambda q_1 + \theta_2 \lambda q_2 \geq \lambda p](\lambda p - c) > \Pr[\theta_1 q_1 + \theta_2 q_2 \geq p](\lambda p - \lambda c) = \lambda \pi_f(q_1, q_2),$$

where the first inequality holds because price equal to λp is in general not optimal for $(\lambda q_1, \lambda q_2)$. In other words, $\pi_f(q_1, q_2)$ has increasing returns to scale. This implies that at $q_1 = q_2 = q$, $\pi_f(q, q)$ is convex in q.

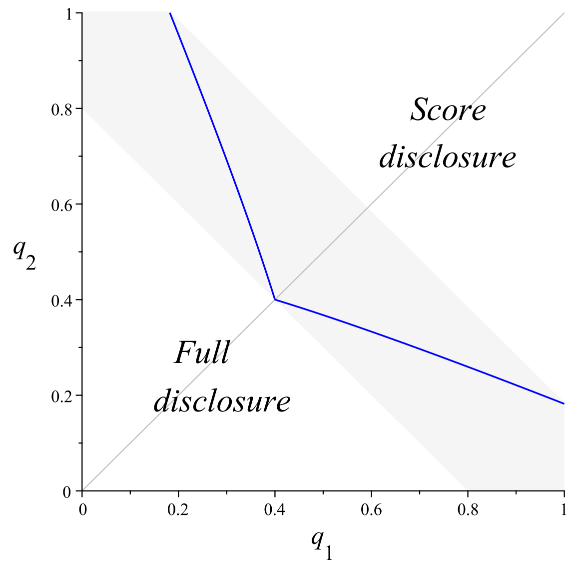


Figure 4: The score disclosure equilibrium for $c = 0.2$ when θ_i is distributed as uniform on $[0, 1]$. In the grey region there is partial score disclosure: For a given score types below the blue line fully disclose while those above it disclose only the score.

according to an asymmetric log-concave distribution. In this case, the demand typically still rotates, albeit the rotation point is generically not in the middle, and it changes with the balancendess of the good α.[19] When the costs are low enough—that is, such that the optimal price is lower than the rotation point for any α—Lemma 2, part (ii), and Proposition 2, part (ii), hold, that is, the score disclosure equilibrium exists and dominates the full disclosure one in terms of ex ante payoff. In the opposite case of the high enough cost the seller fully discloses. In the intermediate case, however, the asymmetry of distribution may yield a profit function which is not quasi-concave in α—depending on how the rotation point changes with α. The equilibrium disclosure then has a more complicated pattern.

4.3 Markets with regulated prices

In several sender-receiver interactions, price is either irrelevant (e.g. elections) or regulated (e.g. university tuition in many European countries). The main results we developed earlier apply to these situations too. Suppose the price is fixed at $\overline{p}$. We know from

[19] We say "typically" because we have not proved the single crossing of cdfs but it holds in all the examples we tried. Diaconis and Perlman (1990) prove it for the Gamma distribution. In general, a more balanced combination dominates the less balanced one in terms of the second-order stochastic dominance (Marshall and Proschan, 1965) but the single crossing of cdfs has not been studied much.

Lemma 1 that score disclosure leads to a higher demand than full disclosure for prices below $\frac{q_1+q_2}{2}$. The seller then strictly prefers score to full disclosure if the fixed price $\overline{p}$ is below the average quality (i.e., her score). Conversely, she prefers full over score disclosure when price is higher than the average quality. Thus, the demand rotation effect we uncovered in Section 2 is still at work.

This result is important for various applications. If, for instance, tuition rates are set sufficiently low by a regulator, then universities will predominantly resort to providing average scores about their qualities rather than more precise information. While students will be worse off under score disclosure because of less information—the tuition is fixed—the positive effect of the demand enlargement may dominate if there are spillovers from the education.

5 Conclusion

In this paper, we study a standard monopoly quality disclosure game when the product has multiple quality attributes. We identify a novel *score* equilibrium where the seller pools together product configurations for which the sum of qualities is the same, and discloses the average quality only. While full unraveling is still an equilibrium, it results in lower profits—both ex ante and ex post—than the score disclosure. We show more generally that any symmetric disclosure rule yields the same expected profit, and this is higher than under any asymmetric disclosure including full disclosure. However, some symmetric disclosure rules—such as no disclosure—cannot be maintained as an equilibrium in the disclosure game. Moreover, the full disclosure equilibrium is defeated by the score disclosure one in the sense of the equilibrium refinement of Mailath, Okuno-Fujiwara and Postlewaite (1993) while the latter one is not defeated by any equilibrium. Hence, the score disclosure equilibrium is very natural in the settings with multi-attribute products.

Appendix

Example with the uniform distribution This example is used to plot isoprofit curves in Figure 2. Suppose that $\theta_i, i = 1, 2$, are distributed as $U[0, 1]$. Suppose q_1 and q_2 are observable as in Section 2.2.2 and assume, without loss of generality, $q_1 \geq q_2$. Then,

$$\Pr(\theta_1 q_1 + \theta_2 q_2 \geq p) = \begin{cases} 1 - \dfrac{p^2}{2q_1 q_2}, & \text{if } p \leq q_2 \\ 1 + \dfrac{q_2}{2q_1} - \dfrac{p}{q_1}, & \text{if } q_2 < p < q_1 \\ \dfrac{(q_1 + q_2 - p)^2}{2q_1 q_2}, & \text{if } p \geq q_1 \end{cases}. \tag{3}$$

Solving the seller's problem (2) obtain the optimal price

$$p(q_1, q_2) = \begin{cases} \dfrac{q_1}{2} + \dfrac{q_2}{4}, & \text{if } q_2 < \dfrac{2}{3} q_1 \\ \sqrt{\dfrac{2q_1 q_2}{3}}, & \text{if } \dfrac{2}{3} q_1 \leq q_2 \leq q_1 \end{cases}. \tag{4}$$

Finally, plug in the optimal price (4) into (2) and allow for $q_2 > q_1$ to get the seller's profits:

$$\pi(q_1, q_2) = \begin{cases} \dfrac{(2q_1 + q_2)^2}{16 q_1}, & \text{if } q_2 < \dfrac{2}{3} q_1 \\ \sqrt{\dfrac{8 q_1 q_2}{27}}, & \text{if } \dfrac{2}{3} q_1 \leq q_2 \leq \dfrac{3}{2} q_1 \\ \dfrac{(q_1 + 2q_2)^2}{16 q_2}, & \text{if } q_2 > \dfrac{3}{2} q_1 \end{cases}. \tag{5}$$

Proof of Lemma 1 For the sake of notation, given some (q_1, q_2) denote the seller's profits $\pi(\alpha)$. We need to show that

$$\pi(\alpha'') > \pi(\alpha')$$

for any $\alpha' < \alpha'' \leq \frac{1}{2}$.

Denote the CDF of $\theta_1(\alpha q_1 + (1-\alpha) q_2) + \theta_2(\alpha q_2 + (1-\alpha) q_1)$ by G_α. Then,

$$\pi(\alpha) = \max_p (1 - G_\alpha(p))p.$$

G_α is symmetric around $\frac{q_1+q_2}{2}$ and log-concave. By Lemma 2.1 in Proschan (1965), $G_{\alpha''}$ is strictly more peaked than $G_{\alpha'}$ for any $\alpha' < \alpha'' \leq \frac{1}{2}$. This implies that $G_{\alpha''}(p) < (>) G_{\alpha'}(p)$ for $p < (>) \frac{q_1+q_2}{2}$.

Solving $\max_p (1 - G_\alpha(p))p$ yields $p^*_\alpha \leq \frac{q_1+q_2}{2}$. Indeed, the first-order condition is $p \frac{g_\alpha(p)}{1-G_\alpha(p)} = 1$. Evaluating the left-hand side at the mean, $p = \frac{q_1+q_2}{2} \equiv \mu$, gives $\mu \frac{g(\mu)}{\frac{1}{2}} = 2\mu g(\mu) \geq 1$ since g_α, being symmetric and log-concave, has a peak at μ while $g(\mu) = \frac{1}{2\mu}$

for the uniform distribution on $[0, 2\mu]$. Since the hazard rate $\frac{g_\alpha(p)}{1-G_\alpha(p)}$ is increasing due to log-concavity of g_α, the left-hand side is increasing. Hence, $p^*_\alpha \leq \mu = \frac{q_1+q_2}{2}$. Hence,

$$\pi(\alpha'') > (1 - G_{\alpha''}(p^*_{\alpha'})p^*_{\alpha'} \geq (1 - G_{\alpha'}(p^*_{\alpha'})p^*_{\alpha'} = \pi(\alpha').$$

The first inequality is due to the fact that $p^*_{\alpha'}$ is not optimal under $G_{\alpha''}$. ■

Proof of Proposition 1 Consider the seller with the product (q_1, q_2) and suppose that all other types of the seller disclose the score. If she also discloses the score, then the buyer's valuation (1) is equal to $\mathrm{E}[\theta_1 + \theta_2]\frac{q_1+q_2}{2}$.

Suppose now that the seller deviates. First consider messages which are compatible with different scores. With sceptical beliefs, the buyers assign the lowest possible score compatible with the message. Hence, the seller cannot inflate her score.

Now consider messages with the true score $\frac{q_1+q_2}{2}$. If $\mathrm{E}[q_1 \mid m] = \mathrm{E}[q_2 \mid m](= \frac{q_1+q_2}{2})$, the seller has the same profits as when disclosing the score. Otherwise, if $\mathrm{E}[q_1 \mid m] \neq \mathrm{E}[q_2 \mid m]$, by Lemma 1 she gets lower profits.

Hence, the seller does not have any incentives to deviate from disclosing the score. ■

Proof of Proposition 2 We start with the following lemma.

Lemma A1 *Profit function $\pi_f(q_1, q_2)$ given by (2) is supermodular.*

Proof of Lemma A1 We need to show $\pi_f(q_1, q_2) \leq \frac{\pi_f(q_1,q_1)+\pi_f(q_2,q_2)}{2}$. We show first that $\pi_f(q_1, q_2)$ is homogeneous of degree 1, that is, $\pi_f(\lambda q_1, \lambda q_2) = \lambda \pi_f(q_1, q_2)$:

$$\max_p p \Pr[\theta_1 \lambda q_1 + \theta_2 \lambda q_2 \geq p] = \lambda \max_p p \Pr[\theta_1 q_1 + \theta_2 q_2 \geq p].$$

Rewrite it as

$$\max_p \frac{p}{\lambda} \Pr\left[\theta_1 q_1 + \theta_2 q_2 \geq \frac{p}{\lambda}\right] = \max_p p \Pr[\theta_1 q_1 + \theta_2 q_2 \geq p],$$

which is true (and the optimal price is homogeneous of degree 1 as well).

Then,

$$\pi_f(q_1, q_2) \leq \pi_f\left(\frac{q_1+q_2}{2}, \frac{q_1+q_2}{2}\right) = \frac{\pi_f(q_1, q_1) + \pi_f(q_2, q_2)}{2},$$

where the inequality follows from Lemma 1 and the equality follows from the fact that $\pi_f(q_1, q_2)$ is homogeneous of degree 1, particularly, from the relations $\pi_f(q_1, q_1) = \frac{q_1}{q_1+q_2}\pi_f(q_1 + q_2, q_1 + q_2)$ and $\pi_f(q_2, q_2) = \frac{q_2}{q_1+q_2}\pi_f(q_1 + q_2, q_1 + q_2)$. ■

Consider the seller's ex ante profit under no disclosure. Since the buyer's valuation (1) is linear in q_1 and q_2, the seller's profits under no disclosure are equal to

$$\bar{\pi} = \pi_f(\mathrm{E}[q_1], \mathrm{E}[q_2]) = \pi_f(\mathrm{E}[q_1], \mathrm{E}[q_1]) = k\mathrm{E}[q_1], \tag{6}$$

where $k = \pi_f(1, 1)$. In the second equality in (6) we used the fact that q_1 and q_2 come from the same distribution and in the last equality we used the fact that $\pi_f(q_1, q_2)$ is homogeneous of degree 1 as we showed in the proof of Lemma A1.

Part (i) Consider a score function s and a message m. If $\bar{q}_1(m) = \bar{q}_2(m)$ for any $m \in [\underline{m}, \overline{m}]$, then

$$\pi(\bar{q}_1(m), \bar{q}_2(m)) = \pi(\bar{q}_1(m), \bar{q}_1(m)) = \pi_f(\bar{q}_1(m), \bar{q}_1(m)) = k\bar{q}_1(m),$$

where in the second equality we used the linearity of π_f and in the last inequality we used the homogeneity of degree 1 of π_f. Then, by the law of iterated expectations, $\mathrm{E}[\bar{q}_1(m)] = \mathrm{E}[q_1]$ which completes the proof of part (i).

Part (ii) Now, $\bar{q}_1(m) \neq \bar{q}_2(m)$ for some $m \in [\underline{m}, \overline{m}]$. Then,

$$\pi(\bar{q}_1(m), \bar{q}_2(m)) = \pi_f(\bar{q}_1(m), \bar{q}_2(m)) \leq \frac{\pi_f(\bar{q}_1(m), \bar{q}_1(m)) + \pi_f(\bar{q}_2(m), \bar{q}_2(m))}{2},$$

since π_f is supermodular by Lemma A1. Then,

$$\mathrm{E}[\pi(\bar{q}_1(m), \bar{q}_2(m))] \leq \mathrm{E}\left[\frac{\pi_f(\bar{q}_1(m), \bar{q}_1(m)) + \pi_f(\bar{q}_2(m), \bar{q}_2(m))}{2}\right] = \frac{k\mathrm{E}[q_1] + k\mathrm{E}[q_2]}{2} = \bar{\pi}.$$

■

Proof of Proposition 3 Mailath, Okuno-Fujiwara and Postlewaite (1993) studied signaling games with a uni-dimensional type space. We extend their definition to a two-dimensional type space. Denote $\boldsymbol{q} = (q_1, q_2)$.

Definition 1 *Denote by $\pi_\sigma(\boldsymbol{q})$ the profit S earns in PBE σ when her quality is $\boldsymbol{q}$. Let the corresponding equilibrium message be $m_\sigma(\boldsymbol{q})$ and the beliefs $\mu_\sigma(\boldsymbol{q} \mid m_\sigma)$. Fix two possible PBEs σ' and σ''. Then, σ' defeats σ'' if there exists message $m \in M$ such that*

(i) No seller type in σ'' sends m, while the set of types in σ' that send m is non-empty, i.e., $\forall \boldsymbol{q} \in [0, 1]^2$, $m_{\sigma''}(\boldsymbol{q}) \neq m$, and $K = \{\boldsymbol{q} \in [0, 1]^2 \mid m_{\sigma'}(\boldsymbol{q}) = m\} \neq \varnothing$;

(ii) All seller types that send m in σ' earn higher profits and at least one type earns strictly higher profits in σ' than σ'', i.e., $\forall \boldsymbol{q} \in K$: $\pi_{\sigma'}(\boldsymbol{q}) \geq \pi_{\sigma''}(\boldsymbol{q})$ and $\exists \boldsymbol{q} \in K : \pi_{\sigma'}(\boldsymbol{q}) >$

$\pi_{\sigma''}(\boldsymbol{q})$;

(iii) Beliefs in σ'' *are inconsistent in the following sense:* $\exists \boldsymbol{q} \in K$ *for which* $\mu_{\sigma''}(\boldsymbol{q} \mid m) \neq \frac{\beta(\boldsymbol{q})g(q_1)g(q_2)}{\int_{\boldsymbol{q'} \in K} \beta(\boldsymbol{q'})dG(q'_1)dG(q'_2)}$ *for any* $\beta : [0,1]^2 \to [0,1]$ *satisfying (1)* $\beta(\boldsymbol{q}) = 1$ *if* $\boldsymbol{q} \in K$ *and* $\pi_{\sigma'}(\boldsymbol{q}) > \pi_{\sigma''}(\boldsymbol{q})$*, (2)* $\beta(\boldsymbol{q}) = 0$ *if* $\boldsymbol{q} \notin K$*.*

We are now in a position to present the proof.

Part (i) Denote the score and full disclosure equilibria by σ' and σ'', respectively. Take message $m \in (\underline{q}, \overline{q})$ sent in σ', i.e., $K\{(q_1, q_2) \mid \frac{q_1+q_2}{2} = m\} \neq \varnothing$. Such a message is not sent in σ'' and hence, part (i) of Definition 1 is satisfied. Using Lemma 1, all the types that send m in σ' earn strictly higher profits in σ' than in σ'' except for the type $(q_1, q_2) = (m, m)$ who earns the same. Hence, part (ii) is satisfied.

Part (iii) of Definition 1 requires that there is a type that sends message m for which beliefs in σ'' do not follow the Bayes' rule. Suppose to the contrary that beliefs in σ'' do follow the Bayes' rule, that is, $E_{\mu_{\sigma''}}[q_1 \mid m] = E_{\mu_{\sigma''}}[q_2 \mid m] = m$. The seller with $q_1 \neq q_2$ then strictly prefers to deviate in σ'' by sending message $m = \frac{q_1+q_2}{2}$, a contradiction. Thus, σ' defeats σ''.

Part (ii) Consider another PBE $\tilde{\sigma}$ and an equilibrium message m there which is off-equilibrium in the score disclosure PBE σ'. Let the set of types that send m in $\tilde{\sigma}$ be denoted by $K = \{\boldsymbol{q} \in [0,1]^2 \mid m_{\tilde{\sigma}}(\boldsymbol{q}) = m\}$ and take type $(\tilde{q}_1, \tilde{q}_2) \in K$ that has the highest score $q_1 + q_2$ within K, i.e., $\tilde{q}_1 + \tilde{q}_2 \geq q_1 + q_2$ for all $(q_1, q_2) \in K$. It follows that $\tilde{q}_1 + \tilde{q}_2 > E_{\tilde{\sigma}}[q_1 + q_2 \mid m] = E_{\tilde{\sigma}}[q_1 \mid m] + E_{\tilde{\sigma}}[q_2 \mid m]$ as long as there are at least two types in K with different scores. By Lemma 1, the profits in $\tilde{\sigma}$ when sending message m, $\pi_f\left(E_{\tilde{\sigma}}[q_1 \mid m], E_{\tilde{\sigma}}[q_2 \mid m]\right)$, are strictly lower than the profits $(\tilde{q}_1, \tilde{q}_2)$ in the score disclosure equilibrium σ', $\pi_f\left(\frac{\tilde{q}_1+\tilde{q}_2}{2}, \frac{\tilde{q}_1+\tilde{q}_2}{2}\right)$. This reasoning applies to all possible PBEs and all off-equilibrium messages. Hence, no other PBE defeats score disclosure equilibrium. ■

Proof of Lemma 2 As in the proof of Lemma 1, denote the CDF of $\theta_1(\alpha q_1 + (1-\alpha) q_2) + \theta_2(\alpha q_2 + (1-\alpha) q_1)$ by G_α and the optimal price by p^*_α. G_α is symmetric around $\frac{q_1+q_2}{2}$ and log-concave. By Lemma 2.1 in Proschan (1965), $G_{\alpha''}$ is strictly more peaked than $G_{\alpha'}$ for any $\alpha' < \alpha'' \leq \frac{1}{2}$. This implies that $G_{\alpha''}(p) < (>) G_{\alpha'}(p)$ for $p < (>) \frac{q_1+q_2}{2}$.

Hence, if $p^*_\alpha < \frac{q_1+q_2}{2}$ for any $\alpha \in \left[0, \frac{1}{2}\right]$, then $\pi(\alpha) = (1 - G_\alpha(p^*_\alpha))(p^*_\alpha - c)$ strictly increases in α. The proof of Lemma 1 shows why this holds for $c = 0$ when p^*_α might be equal to $\frac{q_1+q_2}{2}$. This is part (ii).

If $p^*_\alpha > \frac{q_1+q_2}{2}$ for any $\alpha \in \left[0, \frac{1}{2}\right]$, then the demand shrinks with α and hence, $\pi(\alpha)$ strictly decreases in α. A sufficient condition is $c \geq \frac{q_1+q_2}{2}$ since $p^*_\alpha > c$. This is part (iii).

Finally, suppose that p^*_α is higher or lower than $\frac{q_1+q_2}{2}$ depending on α. Take $\alpha' < \alpha'' \leq \frac{1}{2}$. Let us show that if $p^*_{\alpha'} \leq \frac{q_1+q_2}{2}$, then $p^*_{\alpha''} < \frac{q_1+q_2}{2}$. By contradiction, suppose that $p^*_{\alpha''} \geq \frac{q_1+q_2}{2}$. Then,

$$\begin{aligned}(1 - G_{\alpha''}(p^*_{\alpha''}))(p^*_{\alpha''} - c) &> (1 - G_{\alpha''}(p^*_{\alpha'}))(p^*_{\alpha'} - c)\\ &\geq (1 - G_{\alpha'}(p^*_{\alpha'}))(p^*_{\alpha'} - c)\\ &> (1 - G_{\alpha'}(p^*_{\alpha''}))(p^*_{\alpha''} - c),\end{aligned}$$

where the first inequality holds because $p^*_{\alpha''}$ is optimal under α'', the second inequality holds because $G_{\alpha''}(p^*_{\alpha'}) \leq G_{\alpha'}(p^*_{\alpha'})$ since $p^*_{\alpha'} \leq \frac{q_1+q_2}{2}$, and the third inequality holds because $p^*_{\alpha'}$ is optimal under α'. Hence, $(1 - G_{\alpha''}(p^*_{\alpha''}))(p^*_{\alpha''} - c) > (1 - G_{\alpha'}(p^*_{\alpha''}))(p^*_{\alpha''} - c)$ but $G_{\alpha''}(p^*_{\alpha''}) \geq G_{\alpha'}(p^*_{\alpha''})$ since $p^*_{\alpha''} \geq \frac{q_1+q_2}{2}$, a contradiction. Hence, $p^*_{\alpha''} < \frac{q_1+q_2}{2}$.

Thus, there is $\bar{\alpha}$ such that $p^*_\alpha \gtreqless \frac{q_1+q_2}{2}$ if and only if $\alpha \lesseqgtr \bar{\alpha}$. Then, $\pi(\alpha)$ is U-shaped in α, with a minimum at $\bar{\alpha}$. Since both monotonic and U-shaped functions are quasi-convex, this concludes part (i). $\blacksquare$

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