Random Double Auction: A Robust Bilateral Trading Mechanism

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Abstract

I construct a novel *random double auction* as a robust bilateral trading mechanism for a profit-maximizing intermediary who facilitates trade between a buyer and a seller. It works as follows. The intermediary publicly commits to charging a *fixed commission fee* and *randomly* drawing a *spread* from a uniform distribution. Then the buyer submits a bid price and the seller submits an ask price simultaneously. If the difference between the bid price and the ask price is greater than the realized spread, then the asset is transacted at the *midpoint price*, and each pays the intermediary half of the fixed commission fee. Otherwise, no trade takes place, and no one pays or receives anything. I show that the random double auction is a *dominant-strategy mechanism*, always gives a *positive* worst-case expected profit, and *maximizes* the worst-case expected profit across all dominant-strategy mechanisms.

Keywords: Random spread, double auction, robust mechanism design, bilateral trade, profit maximization, information design, correlated private values, dominant-strategy mechanisms.

JEL Codes: C72, D44, D82, D83.

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1 Introduction

1.1 Background and Motivation

At every moment, a huge amount of trades are facilitated by intermediaries charging fees for their intermediary services in matching buyers with sellers. For example, stocks are sold through a trading platform that typically gets compensation by means of commissions; cars are sold through an automobile dealer who charges dealer fees; many bonds, commodities and derivatives are sold in the over-the-counter market (OTC) where a market maker earns profits through the bid-ask spread.

There are many situations in which the uncertainty about the value of the asset being traded is large, e.g., a newly public stock, and Tesla's new model. Intermediaries may then know little of the concerned parties' willingness to trade and only have an overall estimate about it. Given the large uncertainty towards the two-sided market, it is natural for the intermediary to seek for a trading mechanism that *guarantees* a good profit. How should a profit-maximizing intermediary design trading rules in such situations? Would the intermediary still be able to guarantee a positive profit and thus have strict incentives to offer intermediary services?

To answer these questions, I study the design of profit-maximizing trading mechanisms for the *two-sided* market when the intermediary has *limited* knowledge about the value distribution of the buyer and the seller. Specifically, I assume that the intermediary knows only the *ex-ante gain from trade*[1], denoted by GFT, but does not know the joint distribution of the traders' private values[2]. A joint distribution consistent with the known ex-ante gain from trade is referred to as a *feasible value distribution*. The intermediary considers the class of all *dominant-strategy mechanisms*[3]. Dominant-strategy mechanisms are attractive because the intermediary can predict trading behavior without making assumptions about the traders' beliefs. The intermediary evaluates a mechanism's performance by the expected profit under the dominant-strategy equilibrium in the worst case across all feasible value distributions, referred to as the *profit guarantee*, and seeks a mechanism that maximizes the profit guarantee across all dominant-strategy mechanisms, referred to as a *maxmin trading mechanism*.

Let me comment briefly on the maxmin modeling approach. At a high level, the maxmin

[1]The ex-ante gain from trade is defined to be $E[\max\{\text{Buyer's value} - \text{Seller's value}, 0\}]$, where the expectation is taken with respect to the joint distribution of the traders' private values.

[2]That is, the intermediary knows neither the marginal distributions nor the correlation structure except for the ex-ant gain from trade, which is a summary statistics of the joint distribution.

[3]A trading mechanism is a dominant-strategy mechanism if each trader has a strategy that is optimal and yields a non-negative ex-post payoff, regardless of the other trader's strategy.

modeling approach addresses an important issue of the classic mechanism design theory, in which the designer is assumed to know the agents' information structure and maximize some objective under her known information structure, e.g., Myerson (1981), Myerson and Satterthwaite (1983) and Crémer and McLean (1985, 1988). Although the classic theory is beautiful and influential, the optimal mechanism is sensitive to the detailed assumptions about the information structure. In contrast, the maxmin modeling approach leads to an answer that depends *less* on the details about the information structure.

Several motivations can be offered for the assumption about the intermediary's limited knowledge. First, the ex-ante gain from trade is a simple summary statistics, whereas the joint distribution is a high-dimensional object. Therefore, it is relatively easy to estimate the ex-ante gain from trade, while obtaining an accurate estimate of the whole joint distribution often requires unrealistically many data about the traders' joint value profiles. In addition, the knowledge of the ex-ante gain from trade is arguably the minimal amount of information under which, as I will show, one obtains a non-trivial answer. Therefore, this model can be viewed as a natural *benchmark*. More importantly, this assumption leads to the discovery of a *novel* trading mechanism with *appealing* properties along with new economic insights.

1.2 Results

The main contribution is the construction of a novel ***random double auction*** as a robust bilateral trading mechanism. It works as follows.

Step 0: ***Fixed commission fee***. The intermediary publicly commits to charging a fixed commission fee $r \in (0,1)$[4], where 1 is the normalized maximum value for each trader.

Step 1: ***Uniformly random spread***. The intermediary publicly commits to randomly drawing a spread s uniformly on $[r,1]$. Then a random spread is drawn whose realization is not observed by either the buyer or the seller. The buyer and the seller both know r and the uniform distribution on $[r,1]$ from which the random spread is drawn.

Step 2: ***Midpoint transaction price***. The buyer submits a bid price b, and the seller submits an ask price a, simultaneously. If the difference between the bid price and the ask price is greater than the realized spread, or $b-a>s$, then the seller sells the asset to the buyer at the midpoint price $\frac{b+a}{2}$, and each pays the intermediary half of the fixed commission fee $\frac{r}{2}$. Otherwise, no trade takes place, and no one pays or receives anything.

Under this mechanism, the uniformly random spread determines whether the transaction is successful for a bid-ask pair. That is, trade takes place randomly. Conditional on trading, the mechanism reduces to a *double auction*, as the transaction price is the midpoint of the

[4] The optimal fixed commission fee r is determined by the known ex-ante gain from trade, details of which are given when deriving the profit guarantee of the random double auction.

bid price and the ask price; in addition, the intermediary earns r as a fixed total commission from both parties. Although both traders have to pay half of the fixed commission fee to the intermediary conditional on trading, this mechanism is ex-post individually rational: Each trader's ex-post payoff is always non-negative by being honest, regardless of the other trader's submission. This is because the lower bound of the random spread is the fixed commission fee.

The random double auction is a *novel* trading mechanism that combines three features: A double auction, a fixed commission fee, and a random spread. Indeed, the first two features are *familiar* in the real world. First, a double auction is widely used in stock exchanges as well as in dark pools[5], e.g., the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE) use a double auction to determine the opening prices; block-trading dark pools such as Liquidnet or POSIT typically match orders at the midpoint of the prevailing bid-ask prices (Duffie and Zhu, 2017). Second, brokerage firms often adopt the fixed-commission practice, e.g., Interactive Brokers offers fixed-commission plans for many financial assets[6]; E*TRADE charges a fixed commission per contract for futures contracts[7]. The main novelty of the random double auction comes from the third feature— a random spread[8]. Importantly, the random spread both *disciplines* the traders for cheating and *hedges* against uncertainty towards the traders' information structure. I next illustrate the key properties of the random double auction along with elaborating the *dual* role played by the random spread.

Strategy-proofness. The random double auction is strategy-proof (Proposition 1), i.e., it is a dominant strategy for the buyer (resp, the seller) to submit a bid price (resp, an ask price) equal to his private value. This is a priori surprising, as conditional on trading, the mechanism reduces to a double auction, and a double auction per se is not strategy-proof (Chatterjee and Samuelson, 1983). This is because, the buyer (resp, the seller) has an incentive to submit a bid price (resp, an ask price) lower (resp, higher) than his true value to lower (resp, raise) the transaction price. A random spread makes it costly for the traders to

[5]A dark pool is a privately organized financial forum or exchange for trading securities that are not accessible by the investing public. Dark pools came about primarily to facilitate block trading involving a huge number of securities.

[6]Interactive Brokers is a brokerage firm. From its official website (interactivebrokers.com), it offers a fixed-commission plan that charges \$0.005 per share for stocks in US; it also offers a fixed-commission plan that charges \$ 0.065 per contract for NANOS Options on CBOE.

[7]E*TRADE is also a brokerage firm. From its official website (us.etrade.com), it charges \$1.5 per contract for futures contracts.

[8]The spread s in the random double auction is closely related to but different from the "bid-ask spread", also called "market-maker spread", which refers to the difference between the price at which a market-maker is willing to buy an asset and the price at which she is willing to sell the asset. Similar to the spread s, the bid-ask spread determines whether a trade takes place given a bid-ask pair. The bid-ask spread is an important source of profit for a market maker when she facilitates a trade successfully. In contrast, the spread s only determines whether a trade takes place, but does not affect the profit conditional on trading.

cheat. This is because, with a random spread, if the buyer (resp, the seller) submits a lower bid price (resp, a higher ask price), then the trade will take place with a lower probability, which limits the buyer's (resp, the seller's) payoff from deviating to a lower bid price (resp, a higher ask price). Remarkably, a judiciously chosen random spread — uniformly random spread — eliminates the traders' incentive to cheat and makes the mechanism strategy-proof. To see this, note that the buyer's ex-post payoff from submitting a bid price b when his true value is v_B and the seller submits an ask price a (assuming trade takes place with a positive probability) is

$$\frac{b-a-r}{1-r} \cdot \left(v_B - \frac{b+a+r}{2}\right), \tag{1}$$

where the first term is the trading probability and the second term is the ex-post payoff of the buyer conditional on trading. Note that (1) is a quadratic function in the bid price b. It is straightforward to show that $b = v_B$ maximizes his ex-post payoff regardless of the seller's submitted ask price a. Similarly, truth-telling maximizes the ex-post payoff for the seller regardless of the buyer's submitted bid price b.

Positive profit guarantee. The profit guarantee of the random double auction is always positive (Proposition 2). This is in sharp contrast to any deterministic dominant-strategy mechanism: As I will show in Theorem 5, the profit guarantee of any deterministic dominant-strategy mechanism is *zero* if the known ex-ante gain from trade is weakly below one half.

To derive the profit guarantee of a random double auction with a general fixed commission fee, I first show that the ex-post profit earned from an arbitrary value profile (v_B, v_S) is $\max\left\{\frac{v_B - v_S - r}{1-r} \cdot r, 0\right\}$. To see this, note that the profit collected from a bid-ask pair (b, a) if $b - a > r$ is

$$\frac{b-a-r}{1-r} \cdot r, \tag{2}$$

where the first term is the trading probability and the second term is the profit conditional on trading. Importantly, (2) is linear in the difference between the bid and the ask, as uniformly random spread translates into a linear trading probability, and the profit conditional on trading is the fixed commission fee. If $b - a \leq r$, then the trade will not take place and the profit is trivially zero. Recall that the bid price (resp, the ask price) is equal to the true value of the buyer (resp, the seller) because the mechanism is strategy-proof. Next, I show that a lower bound on the expected profit is $\max\left\{\frac{GFT-r}{1-r} \cdot r, 0\right\}$. To see this, note that the expected profit [9]

$$E\left[\max\left\{\frac{\max\{v_B-v_S,0\}-r}{1-r} \cdot r, 0\right\}\right] \geq \max\left\{E\left[\frac{\max\{v_B-v_S,0\}-r}{1-r} \cdot r\right], 0\right\} = \max\left\{\frac{GFT-r}{1-r} \cdot r, 0\right\},$$

[9]Observe that $\max\{v_B - v_S, 0\} = v_B - v_S$ when $v_B - v_S > r$.

where the inequality follows from Jensen's inequality, and the equality follows from the linearity of the ex-post profit when it is positive. Finally, I show that the lower bound is tight, i.e., the profit guarantee is $\max\left\{\frac{GFT-r}{1-r}\cdot r, 0\right\}$. To see this, note that a degenerate distribution— a point mass on the value profile $(GFT, 0)$— hits the lower bound. Indeed, a random double auction with any positive fixed commission fee below the ex-ante gain from trade has a positive profit guarantee. A high fixed commission fee translates into a high profit conditional on trading, but also leads to a low trading probability. Optimal fixed commission fee $r = 1 - \sqrt{1 - GFT}$ balances these two effects, resulting in the profit guarantee of $\left(1 - \sqrt{1 - GFT}\right)^2$.

Furthermore, the random double auction exhibits a good hedging property: The intermediary is *indifferent* to any feasible value distribution whose support is contained in the set of value profiles where the difference between values is higher than the fixed commission fee, which renders the random double auction a good candidate for a maxmin trading mechanism. This property holds because the ex-post profit from any value profile in the support of an aforementioned feasible value distribution is linear. Indeed, any aforementioned feasible value distribution minimizes the expected profit under the random double auction.

Optimal profit guarantee. The random double auction gives the optimal profit guarantee across all dominant-strategy mechanisms (Theorem 1). To show this, I construct a feasible value distribution, and show that $\left(1 - \sqrt{1 - GFT}\right)^2$ is the tight upper bound on the expected profit across all dominant-strategy mechanisms against the constructed value distribution. In addition, this upper bound is hit by the random double auction.

The constructed value distribution is a *symmetric triangular value distribution* that can be described as follows. The support is a symmetric triangular subset in the set of joint values, which is the same as the trading region[10] of the random double auction. The marginal distribution for the buyer is a combination of a uniform distribution on $(r, 1)$ and an atom on 1, while for the seller is a combination of a uniform distribution on $(0, 1-r)$ and an atom on 0. The conditional distribution is some truncated generalized Pareto distribution with an atom on 1 (resp, 0) for the buyer (resp, the seller).

There are many different ways to model the intermediary's limited knowledge about the value distribution, and the results can be extended to several other models of the limited knowledge. For the model where the intermediary knows only the difference between the expectations of the traders' values, I show that the random double auction remains a maxmin trading mechanism. For the model where the intermediary knows only the expectations of the traders' values, I show that the random double auction remains a maxmin

[10] I refer to the set of value profiles in which trade takes place with a positive probability as the trading region.

trading mechanism for the symmetric[11] informational environment. For the asymmetric informational environment, I show that a generalized random double auction is a maxmin trading mechanism. It generalizes the random double auction in that it approximates the random double auction as the asymmetric informational environment approximates the symmetric one.

Randomized trading is a salient property of the random double auction. This requires the intermediary to have *full commitment power*, which is a standard assumption in the mechanism design literature (e.g., Myerson (1981)). However, in practice, it is hard for the traders to check whether the randomization is done according to the specified trading rule. The traders then may not trust the specified randomization. This motivates the search for a trading mechanism that maximizes the profit guarantee across all deterministic dominant-strategy mechanisms. Such a trading mechanism is referred to as a *maxmin deterministic trading mechanism*. I characterize the class of maxmin deterministic trading mechanisms for any informational environment with a non-trivial profit guarantee (Theorem 5). Examples of maxmin deterministic trading mechanisms include a *linear trading mechanism*, in which trade takes place with probability one if and only if the difference between the bid price and the ask price exceeds a threshold, and a *double posted-price trading mechanism*, in which trade takes place with probability one if and only if the bid price exceeds a threshold and the ask price falls short of a threshold.

In addition, I extend my result to a more general model in which the intermediary can hold the asset. That is, the sum of the traders' allocations is only required to be weakly less than 1. I show that the random double auction remains a maxmin trading mechanism (Theorem 6). Finally, I apply my result to an information design problem in which a financial regulator can choose a probability distribution of the value profile of the buyer and the seller to maximize their welfare. The intermediary, after observing the choice of the distribution but not the realized joint values, designs a profit-maximizing trading mechanism across all dominant-strategy mechanisms. I show that the symmetric triangular value distribution is a solution to this financial regulator's information design problem (Theorem 7).

The remainder of the introduction discusses the related literature. Section 2 presents the model. Section 3 characterizes the main results. Section 4 characterizes the results for other models of limited knowledge. Section 5 characterizes the class of maxmin deterministic trading mechanisms. Section 6 extends and discusses the main results. Preliminary analysis and omitted proofs are in the Appendix.

[11] Roughly speaking, the (a)symmetric information environment is one where the two-sided markets have (non-)identical willingness to trade on average.

1.3 Related Literature

This paper is related to the classic mechanism design literature. Myerson and Satterthwaite (1983) (henceforth MS) study the design of optimal bilateral trading mechanisms assuming the intermediary knows the distribution of the traders' private values and that these values are independently distributed. In contrast, the intermediary in my paper knows only the ex-ante gain from trade, but does not know the joint distribution of the traders' values. Importantly, I permit correlation between values. The intermediary in MS maximizes expected profit, whereas the intermediary in my paper maximizes the worst-case expected profit. The optimal trading mechanism in MS is deterministic, provided that some regularity conditions hold, whereas the maxmin trading mechanism in my paper involves randomized trade. Moreover, the optimal trading mechanism in MS is in general complicated. Under their mechanism, the trade takes place if and only if the buyer's virtual value is greater than the seller's one. These virtual values, however, depend on the fine details of the value distributions, and are non-linear functions of the traders' values in general[12]. In contrast, the maxmin trading mechanism in my paper is simple. Under the random double auction, the trade takes place if and only if the difference between the traders' values is greater than a uniformly random spread.

This paper contributes to the literature on robust mechanism design. One of the main differences is that I focus on a two-sided market, whereas most of the literature focuses on a one-sided market.

Carrasco et al. (2018) study the design of profit-maximizing selling mechanisms when a seller faced with a single buyer only knows the first n moments of the buyer's value distribution (n can be any positive integer), and solve the problem in which the seller only knows the expectation of the buyer's value as a special case. Indeed, their problem in the special case is equivalent to the intermediary's problem when she knows the ex-ante gain from trade and the seller's value is commonly known to be zero. This is because the ex-ante gain from trade is the same as the expectation of the buyer's value if the seller's value is zero. In contrast, my paper studies the intermediary's problem when she knows only the ex-ante gain from trade. Importantly, there is two-sided private information in my paper. This adds complications to the analysis in two ways. First, the mechanism in my paper has to respect the seller's incentive constraint, in addition to the buyer's one. Second, the intermediary is faced with a stronger "adversary" in my paper: The adversary can carefully choose the correlation structure between the traders' values to minimize the expected profit, in addition to choosing the distribution of the buyer's value. Indeed, the worst value distribution in my

[12] Except for a special circumstance in which both traders' value are uniformly distributed.

paper has a rather intricate correlation structure exhibiting a particular positive correlation.

Zhang (2022) considers a model of one-sided auction design in which the designer (the auctioneer) knows the marginal distribution of each bidder's value but does not know the correlation structure. He finds that the second-price auction with the uniformly random reserve price is a maxmin auction across all dominant-strategy mechanisms under certain regularity conditions for the two-bidder case. In contrast, this paper studies a model of two-sided bilateral trade. In addition, the designer (the intermediary) in this paper knows less: She does not know the marginal distribution of each trader's value, in addition to not knowing the correlation structure between the traders' values. Methodologically, both papers construct worst value distributions to proceed the analysis. However, the construction of the worst value distribution is more involved in this paper: It requires me to solve a partial integral equation in addition to ordinary differential equations.

There are other papers seeking robustness to value distributions in a one-side market, e.g., Auster (2018), Bergemann and Schlag (2011), Carroll (2017), Che and Zhong (2021). A separate strand of papers focuses on the case in which the designer does not have reliable information about the agents' hierarchies of beliefs about each other while assuming the knowledge of the payoff environment, e.g., Bergemann and Morris (2005), Chung and Ely (2007), Chen and Li (2018), Bergemann et al. (2016, 2017, 2019), Du (2018), Brooks and Du (2021), Libgober and Mu (2021), Yamashita and Zhu (2018).

This paper contributes to the double auction literature. Chatterjee and Samuelson (1983) analyze the simplest and most well-known double auction mechanism: If the bid price is higher than the ask price, then trade takes place, and the transaction price is the midpoint price; otherwise no trade takes place, and no one pays or receives anything. This mechanism has an undesirable property: Both traders have incentives to cheat under this mechanism. McAfee (1992) shows how to make the double auction mechanism strategy-proof when there are many buyers and sellers. However, McAfee's mechanism reduces to "no trade" when there are only one buyer and one seller. McAfee achieves strategy-proofness by making the price paid by any trader invariant to that trader's report conditional on trading.[13] In contrast, under the random double auction, a trader's report can still affect the price paid (midpoint price) conditional on trading. I achieve the strategy-proofness by introducing a random spread, which lowers the trading probability if the buyer (resp, the seller) underbids (resp, overbids) his value.

[13]Under McAfee's mechanism, the only way a trader can affect the price is by eliminating himself from trading.

2 Model

2.1 Trading Environment

I consider an environment where an asset is traded between two risk-neutral traders through an intermediary. One of the traders is the seller (S), who holds the asset initially, while the other one is the buyer (B), who does not hold the asset initially. I denote by $I = \{S, B\}$ the set of the traders and $i \in I$ is a trader. Each trader i has private information about his value for the asset, which is modeled as a random variable v_i. I denote by V_i the set of possible values of trader i. Throughout, I assume $V_S = V_B$. I assume that V_i is bounded. As a normalization, I assume that $V_i = [0, 1]$. The set of possible value profiles is denoted by $V = [0, 1]^2$ with a typical value profile v. v_B and v_S may be correlated in an arbitrary way. I denote by π the joint distribution of the value profile. In addition, there is no technical assumption on π. That is, π can be continuous, discrete, or any mixtures. The set of all joint distributions on V is denoted by ΔV.

2.2 Knowledge

The intermediary only knows the ex-ant gain from trade GFT, but does not know the joint distribution π. Formally, I denote by

$$\Pi(GFT) = \left\{ \pi \in \Delta V : \int \max\{v_B - v_S, 0\} d\pi(v) = GFT \right\}$$

the collection of joint distributions that are consistent with the known ex-ante gain from trade. I refer to any $\pi \in \Pi(GFT)$ as a *feasible value distribution*. I assume $GFT \in (0, 1)$ to rule out uninteresting cases.

2.3 Dominant-strategy Mechanisms

The intermediary seeks a dominant-strategy mechanism. The revelation principle holds, and it is without loss of generality to restrict attention to direct trading mechanisms. A direct trading mechanism (q, t_B, t_S) consists of a trading rule $q : V \to [0, 1]$, a payment rule $t_B : V \to \mathbb{R}$ and a transfer rule $t_S : V \to \mathbb{R}$.[14] The buyer submits a bid price b and the seller submits an ask price a simultaneously to the intermediary. Upon receiving the bid-ask pair (b, a), the buyer obtains the asset with probability $q(b, a)$ and pays $t_B(b, a)$ to the

[14] q is the probability that the buyer obtains the asset when the asset is indivisible. I allow randomization, which will play a crucial role in my analysis. q can be interpreted as the trading quantity when the asset is divisible.

intermediary, while the seller holds the good with the remaining probability $1 - q(b, a)$ and receives $t_S(b, a)$ from the intermediary. With slight abuse of notation, I sometimes use the true value profile $v = (v_B, v_S)$ to represent the submitted bid-ask pair because each trader truthfully reports his value in the dominant-strategy equilibrium.

A direct trading mechanism (q, t_B, t_S) is a dominant-strategy mechanism if

$$v_B q(v) - t_B(v) \geq v_B q(v'_B, v_S) - t_B(v'_B, v_S), \quad \forall v \in V, v'_B \in V_B; \tag{$DSIC_B$}$$

$$v_B q(v) - t_B(v) \geq 0, \quad \forall v \in V; \tag{$EPIR_B$}$$

$$v_S(1 - q(v)) + t_S(v) \geq v_S(1 - q(v_B, v'_S)) + t_S(v_B, v'_S), \quad \forall v \in V, v'_S \in V_S; \tag{$DSIC_S$}$$

$$v_S(1 - q(v)) + t_S(v) \geq v_S, \quad \forall v \in V. \tag{$EPIR_S$}$$

The set of all dominant-strategy mechanisms is denoted by $\mathcal{D}$.

2.4 Objective

I am interested in the intermediary's *expected profit* in the dominant-strategy equilibrium in which each trader truthfully reports his value of the asset. The expected profit of a dominant-strategy mechanism (q, t_B, t_S) under the joint distribution π is $U((q, t_B, t_S), \pi) = \int_{v \in V} t(v) d\pi(v)$ where $t(v) = t_B(v) - t_S(v)$, referred to as the *ex-post profit*. The intermediary evaluates a trading mechanism by its worst-case expected profit over all feasible value distributions. Formally, the intermediary evaluates a trading mechanism (q, t_B, t_S) by its *profit guarantee* $PG((q, t_B, t_S))$, defined as

$$\inf_{\pi \in \Pi(GFT)} U((q, t_B, t_S), \pi). \tag{PG}$$

The intermediary aims to find a trading mechanism (q^*, t^*_B, t^*_S), referred to as a *maxmin trading mechanism*, that maximizes the profit guarantee. Formally, the intermediary solves

$$\sup_{(q, t_B, t_S) \in \mathcal{D}} PG((q, t_B, t_S)). \tag{MTM}$$

3 Main Results

Recall the random double auction: Given a submitted bid-ask pair (b, a), if $b - a > s$ in which s is a random spread drawn from the uniform distribution on $[r, 1]$ where $r = 1 - \sqrt{1 - GFT} \in (0, 1)$ is the fixed commission fee, then trade takes place at the midpoint price $p = \frac{b+a}{2}$, and each pays the intermediary $\frac{r}{2}$; otherwise, trade does not take place, and no one pays or receives anything.

It is straightforward to show that the random double auction can also be expressed as follows. If $b - a > r$,

$$q^*(b,a) = \frac{1}{1-r} \cdot (b - a - r)\,,$$

$$t_B^*(b,a) = \frac{1}{2(1-r)} \cdot \left[b^2 - (a+r)^2\right]\,,$$

$$t_S^*(b,a) = \frac{1}{2(1-r)} \cdot \left[(b-r)^2 - a^2\right]\,.$$

If $b - a \leq r$,

$$q^*(b,a) = t_B^*(b,a) = t_S^*(b,a) = 0.$$

The trading rule is a *linear* function; the payment rule and the transfer rule are both *quadratic* functions. In addition, this mechanism satisfies the standard weak budget balance property (as in Myerson and Satterthwaite (1983)), i.e., the intermediary never subsidizes the market.

3.1 Strategy-proofness

Proposition 1 (Strategy-proofness)**.** *The random double auction is strategy-proof.*

The proof has been given in the introduction. The key idea is to use a random spread to decrease the traders' incentive to deviate in the double auction.

Remark 1 (Dropping the risk-neutral assumption)**.** This idea does not rely on the assumption that the traders are risk-neutral. Suppose that the traders' von Neumann-Morgenstern utility function is $u(x) = x^\alpha$ where $\alpha > 0$ and $\alpha \neq 1$. Note that the traders are risk-averse (resp, risk-loving) if $\alpha < 1$ (resp, $\alpha > 1$). Now I modify the random spread distribution so that the cumulative distribution function of the random spread s is $\left(\frac{s-r}{1-r}\right)^\alpha$ on the same support $[r, 1]$, then the random double auction is again strategy-proof. To see this, note that the non-risk-neutral buyer's ex-post utility from submitting a bid price b when his true value is v_B and the seller submits an ask price a (assuming trade takes place with a positive probability) becomes

$$\left(\frac{b-a-r}{1-r}\right)^\alpha \cdot \left(v_B - \frac{b+a+r}{2}\right)^\alpha\,,$$

where the first term is the trading probability given the modified random spread distribution and the second term is the ex-post utility of the buyer conditional on trading. It is straightforward that $b = v_B$ maximizes his ex-post utility regardless of the seller's submitted ask price a, as a monotonic transformation preserves the optimal solution. Similarly, truthful-telling maximizes the ex-post utility for the seller regardless of the buyer's submitted bid price b.

3.2 Positive Profit Guarantee

Proposition 2 (Positive profit guarantee)**.** *The random double auction has a positive profit guarantee for any non-trivial informational environment. The amount of the profit guarantee is* $\left(1-\sqrt{1-GFT}\right)^2$.

The proof has been given in the introduction. In its essentials, the ex-post profit under the random double auction is a convex function in the ex-post gain from trade. Therefore, a point mass on the value profile $(GFT, 0)$ minimizes the expected profit across feasible value distributions.

Remark 2 (Positive welfare guarantee)**.** In terms of the traders' welfare, how does the random double auction perform? Define the *ex-post welfare* for a value profile (v_B, v_S) as the sum of the traders' ex-post payoffs, or $q(v)(v_B - v_S) - (t_B(v) - t_S(v))$. The *expected welfare* and the *welfare guarantee* can then be similarly defined. I will show below that the random double auction has a positive welfare guarantee.

To derive the welfare guarantee of the random double auction, I first show that the ex-post welfare given an arbitrary value profile (v_B, v_S) is $\frac{(v_B-v_S-r)^2}{1-r}\mathbb{1}_{v_B-v_S>r}$. To see this, note that the welfare from a bid-ask pair (b, a) if $b - a > r$ is

$$\frac{b-a-r}{1-r} \cdot (v_B - v_S - r)\,,$$

where the first term is the trading probability and the second term is the realized welfare conditional on trading. If $b - a \leq r$, then the trade will not take place and the realized welfare is trivially zero. Recall that the bid price (resp, the ask price) is equal to the true value of the buyer (resp, the seller) because the mechanism is strategy-proof. Next, I show that a lower bound on the expected welfare is $\frac{(GFT-r)^2}{1-r}$. To see this, note that the expected welfare

$$
\begin{aligned}
E\left[\frac{(v_B - v_S - r)^2}{1-r}\mathbb{1}_{v_B - v_S > r}\right] &= E\left[\frac{((v_B - v_S - r)\mathbb{1}_{v_B - v_S > r})^2}{1-r}\right] \\
&\geq \frac{\left(E\left[(v_B - v_S - r)\mathbb{1}_{v_B - v_S > r}\right]\right)^2}{1-r} \\
&= \frac{\left(E\left[\max\{v_B - v_S - r, 0\}\right]\right)^2}{1-r} \\
&\geq \frac{\left(E\left[\max\{v_B - v_S, 0\} - r\right]\right)^2}{1-r} \\
&= \frac{(GFT - r)^2}{1-r},
\end{aligned}
$$

where the first line follows from $\mathbb{1}_{v_B - v_S > r} = \mathbb{1}^2_{v_B - v_S > r}$, the second line follows from Jensen's inequality, the third line follows from $(v_B - v_S - r)\mathbb{1}_{v_B - v_S > r} = \max\{v_B - v_S - r, 0\}$, the fourth line follows from $\max\{v_B - v_S - r, 0\} \geq \max\{v_B - v_S, 0\} - r$, and the last line follows from the definition of GFT. Finally, I show that the lower bound is tight, i.e., the gain from trade guarantee is $\frac{(GFT-r)^2}{1-r}$. To see this, note that a degenerate distribution— a point mass on the value profile $(GFT, 0)$— hits the lower bound. Clearly, any fixed commission fee below the difference between the ex-ante gain from trade leads to a positive welfare guarantee. Raising fixed commission fee leads to both a low welfare conditional on trading and a low trading probability. Therefore, optimal fixed commission fee (for welfare) is zero, resulting in the welfare guarantee of GFT^2.

3.3 Optimal Profit Guarantee

In this section, I will show that the random double auction is a maxmin trading mechanism (Theorem 1) by constructing a feasible value distribution, referred to as a *worst value distribution*, and showing that $\left(1 - \sqrt{1 - GFT}\right)^2$ is the tight upper bound on expected profit across all dominant-strategy mechanisms against the worst value distribution. In addition, the random double auction is an optimal mechanism against the worst value distribution. Essentially, the random double auction and the worst value distribution form a "saddle point": The random double auction maximizes the expected profit given the worst value distribution, and the worst value distribution minimizes the expected profit under the random double auction. The properties of a saddle point imply that the random double auction is maxmin optimal. More details about the saddle point approach are given in Appendix A.1. Subsection 3.3.1 gives details about the construction of the worst value distribution.

Let me first specify the symmetric triangular value distribution, which is the worst value

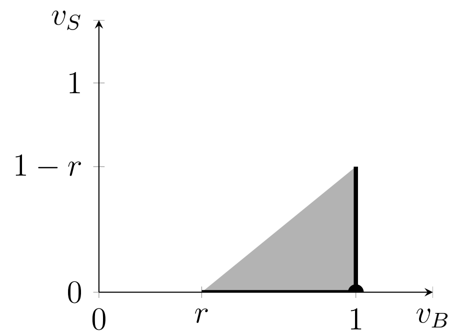

Figure 1: Symmetric Triangular Value Distribution

distribution that I construct. The support is a symmetric triangular subset of joint values $ST := \{v \in V | v_B - v_S > r\}$. The marginal distribution for the buyer is a combination of a uniform distribution on $(r, 1)$ and an atom of size r on 1: $\pi_B^*(v_B) = 1$ for $v_B \in (r, 1)$ and $Pr_B^*(1) = r$. The marginal distribution for the seller is a combination of a uniform distribution on $(0, 1-r)$ and an atom of size r on 0: $\pi_S^*(v_S) = 1$ for $v_S \in (0, 1-r)$ and $Pr_S^*(0) = r$. The conditional distribution for the buyer is a combination of some generalized Pareto distribution on $(v_S + r, 1)$ and an atom on 1: When $v_S \in (0, 1-r)$, $\pi_B^*(v_B|v_S) = \frac{2r^2}{(v_B - v_S)^3}$ for $v_B \in (v_S + r, 1)$ and $Pr_B^*(v_B = 1|v_S) = \frac{r^2}{(1-v_S)^2}$; when $v_S = 0$, $\pi_B^*(v_B|v_S = 0) = \frac{r}{(v_B)^2}$ for $v_B \in (r, 1)$ and $Pr_B^*(v_B = 1|v_S = 0) = r$. The conditional distribution for the seller is a combination of some generalized Pareto distribution on $(0, v_B - r)$ and an atom on 0: When $v_B \in (r, 1)$, $\pi_S^*(v_S|v_B) = \frac{2r^2}{(v_B - v_S)^3}$ for $v_S \in (0, v_B - r)$ and $Pr_S^*(v_S = 0|v_B) = \frac{r^2}{(v_B)^2}$; when $v_B = 1$, $\pi_S^*(v_S|v_B = 1) = \frac{r}{(1-v_S)^2}$ for $v_S \in (0, 1-r)$ and $Pr_S^*(v_S = 0|v_B = 1) = r$.

Equivalently, the symmetric triangular value distribution can be described as a combination of a joint density function on $ST \backslash \{(1, 0)\}$ and an atom of size r^2 on the value profile $(1, 0)$ as follows (See Figure 1).

$$\pi^*(v_B, v_S) = \begin{cases} \frac{2r^2}{(v_B - v_S)^3} & \text{if } v_B - v_S > r,\ v_B \neq 1 \text{ and } v_S \neq 0, \\ \frac{r^2}{(1-v_S)^2} & \text{if } v_B = 1 \text{ and } 0 < v_S < 1-r, \\ \frac{r^2}{(v_B)^2} & \text{if } r < v_B < 1 \text{ and } v_S = 0. \end{cases}$$

$$Pr^*(1, 0) = r^2.$$

To construct the symmetric triangular value distribution, it is useful to define a "virtual value".

Definition 1 (Virtual value)**.** Fix any value distribution π[15], the expected profit of an

[15] For exposition, I assume that π is differentiable everywhere when deriving the virtual values. It can

optimal trading mechanism (q, t_B, t_S) admits a "virtual" representation[16]:

$$E[t(v)] = \int q(v)\phi(v)d\pi(v),$$

where $\phi(v) := (v_B - v_S) - \left(\frac{1-\Pi_B(v_B|v_S)}{\pi_B(v_B|v_S)} + \frac{\Pi_S(v_S|v_B)}{\pi_S(v_S|v_B)}\right)$ is defined to be the "virtual value"[17] of the value profile (v_B, v_S), where the first term is the maximum possible profit the intermediary could have earned if she knew the value profile (v_B, v_S), and the second term is the sum of the traders' information rents, which are pinned down by dominant-strategy incentive compatibility and the binding ex-post participation constraints of zero-value buyer and one-value seller. Here $\pi_B(\cdot|\cdot)$ and $\Pi_B(\cdot|\cdot)$ (resp, $\pi_S(\cdot|\cdot)$ and $\Pi_S(\cdot|\cdot)$) are conditional PDF and conditional CDF for the buyer (resp, the seller).

Using the virtual value, the problem of maximizing the expected profit across all dominant-strategy mechanisms is equivalent to the problem of maximizing the expected virtual value of the value profile in which trade takes places, subject to that the trading rule is *monotone*[18] (a monotonicity constraint associated with dominant-strategy incentive compatibility). This simplifies the problem, as one can now point-wise maximize the objective, ignoring the monotonicity constriant[19]. The symmetric triangular value distribution is constructed by solving a *zero virtual value condition* requiring the virtual value be zero for any value profile in the support except for the highest joint type. The intuition behind this condition is that the intermediary is *indifferent* between trading and no trading for any those value profiles under the random double auction.

Lemma 1. *The symmetric triangular value distribution satisfies a zero virtual value condition for any value profile in the support except for the highest joint type. Formally,*

$$\phi(v) = 0, \quad \forall v \in ST\backslash\{(1,0)\}. \tag{ZVV}$$

Indeed, this condition guarantees that the intermediary is indifferent to any dominant-strategy mechanism in which 1) trade does not take place if the value profile lies outside the support and trade takes place with probability one when the value profile is $(1, 0)$, and 2) ex-post participation constraints are binding for zero-value buyer and one-value seller. In

be easily extended to joint distributions which admits an atom on the value profile $(1, 0)$.

[16]The details are given in Appendix A.2.

[17]This is a straightforward adaptation of the virtual value in Myerson and Satterthwaite (1983) to dominant-strategy mechanisms and the correlated private value environment.

[18]A trading rule q is monotone if q is non-decreasing in v_B and non-increasing in v_S. This is analogous to a monotone allocation rule in the auction design. Details are given in Appendix A.2.

[19]Of course, one need to check that the monotonicity constraint holds in the end.

addition, such a trading mechanism is an optimal trading mechanism given the symmetric triangular value distribution. Using the virtual representation, the optimal expected profit given the symmetric triangular value distribution is

$$Pr^*(1,0) \times 1 = \left(1 - \sqrt{1 - GFT}\right)^2.$$

This is because $(1,0)$ is the only value profile with a positive virtual value, and its virtual value is 1 as it is the highest joint type.

To understand why the symmetric triangular value distribution is a worst value distribution, it is useful to observe that it exhibits a *positive correlation*: If the buyer's value is higher, then the seller's value is more likely to be higher as well. Intuitively, positive correlation levels the maximal gain from trade across value profiles and therefore limits the intermediary's incentive to discriminate across value profiles. Indeed, the symmetric triangular value distribution exhibits "extreme" positive correlation in the following sense: It renders the intermediary *indifferent* across all value profiles in the support but the highest joint type $(1,0)$.

Definition 2 (Positive correlation for bivariate distributions)**.** Let $Z = (X, Y)$ be a bivariate random vector whose distribution is F. I say that Z exhibits positive correlation for D_X and D_Y if $F(X|Y = y)$ *first order stochastically dominates* $F(X|Y = y')$ for any $y > y', y, y' \in D_Y$ and $F(Y|X = x)$ *first order stochastically dominates* $F(Y|X = x')$ for any $x > x', x, x' \in D_X$.

Lemma 2. *The symmetric triangular value distribution exhibits a positive correlation for* $r < v_B < 1$ *and* $0 < v_S < 1 - r$.[20]

Theorem 1. *The random double auction is a maxmin trading mechanism with a profit guarantee of* $\left(1 - \sqrt{1 - GFT}\right)^2$*, and the symmetric triangular value distribution is a worst value distribution.*

Remark 3. It is useful to compare the profit guarantee of the random double auction and the optimal profit across dominant-strategy mechanisms if the value distribution were known to the intermediary. One case could be the following value distribution: The buyer's value follows a uniform distribution on $[GFT, 1]$ and the seller's value follows a uniform distribution on $[0, 1-GFT]$; their values are independent. By a straightforward adaptation of the revenue equivalence theorem, the profit achievable by the optimal dominant-strategy mechanism can

[20]To see this, note that $\Pi_S^*(v_S|v_B) = \frac{r^2}{(v_B - v_S)^2}$ is decreasing w.r.t. v_B for $v_B \in (r, 1)$. When $v_B = 1$, $\Pi_S^*(v_S|v_B = 1) = \frac{r}{1-v_S} \geq \frac{r^2}{(1-v_S)^2}$, so the positive correlation breaks when $v_B = 1$. Similarly, $\Pi_B^*(v_B|v_S) = 1 - \frac{r^2}{(v_B - v_S)^2}$ is decreasing w.r.t. v_S for $v_S \in (0, 1-r)$. When $v_S = 0$, $\Pi_B^*(v_B|v_S = 0) = 1 - \frac{r}{v_B} \leq 1 - \frac{r^2}{(v_B)^2}$, so the positive correlation breaks when $v_S = 0$.

be computed. For example, When $GFT = \frac{3}{4}$, the optimal profit is $\frac{1}{2}$, whereas the profit guarantee of the random double auction is $\frac{1}{4}$, so the ratio between the profit guarantee and the optimal profit is $\frac{1}{2}$. In addition, this ratio is large when GFT is large and converges to 1 as $GFT \to 1$. Another case could be that the value distribution is a point mass on $(GFT, 0)$. Then the optimal profit is GFT. When $GFT = \frac{3}{4}$, the ratio between the profit guarantee and the optimal profit is $\frac{1}{3}$. In addition, this ratio is increasing in GFT and converges to 1 as $GFT \to 1$.

3.3.1 Construction of Symmetric Triangular Value Distribution

In this subsection, I illustrate how I construct a feasible value distribution such that (ZVV) holds. I start from value profiles in which either $v_B = 1$ or $v_S = 0$. Assume that $Pr^*(1,0) = \alpha$. Consider value profiles $(v_B, 0)$ in which $v_B \in (r, 1)$. Let $S^*(v_B, 0) := \int_{(v_B,1)} \pi^*(x,0)dx + Pr^*(1,0)$ for $v_B \in (r,1)$ and $S^*(1,0) := Pr^*(1,0)$. Note that $\pi^*(v_B, 0) = -\frac{\partial S^*(v_B,0)}{\partial v_B}$ for $v_B \in (r,1)$. By (ZVV), I have that for any $(v_B, 0)$ in which $v_B \in (r,1)$,

$$\pi^*(v_B,0)(v_B - 0) - S^*(v_B,0) = 0.$$

Note that this is a simple ordinary differential equation, to which the solution is

$$S^*(v_B,0) = \frac{\alpha}{v_B}, \quad \pi^*(v_B,0) = \frac{\alpha}{v_B^2}, \quad \forall v_B \in (r,1).$$

Then consider value profiles $(1, v_S)$ in which $v_S \in (0, 1-r)$. Similarly, let $S^*(1, v_S) := \int_{(0,v_S)} \pi^*(1,x)dx + Pr^*(1,0)$ for $v_S \in (0, 1-r)$. Note that $\pi^*(1, v_S) = \frac{\partial S^*(1,v_S)}{\partial v_S}$ for $v_S \in (0, 1-r)$. By (ZVV), I have that for any $(1, v_S)$ in which $v_S \in (0, 1-r)$,

$$\pi^*(1,v_S)(1 - v_S) - S^*(1,v_S) = 0.$$

Note that this is also a simple ordinary differential equation, to which the solution is

$$S^*(1,v_S) = \frac{\alpha}{1-v_S}, \quad \pi^*(1,v_S) = \frac{\alpha}{(1-v_S)^2}, \quad \forall v_S \in (0,1-r).$$

Finally consider any value profile (v_B, v_S) in which $v_B - v_S > r$, $v_B \neq 1$ and $v_S \neq 0$. Let $S^*(v_B, v_S) := \int_{(v_B,1)} \pi^*(b, v_S)db + \pi^*(1, v_S)$ if $v_B - v_S > r$, $v_B \neq 1$ and $v_S \neq 0$. Note that $\pi^*(v_B, v_S) = -\frac{\partial S^*(v_B,v_S)}{\partial v_B}$ if $v_B - v_S > r$, $v_B \neq 1$ and $v_S \neq 0$. By (ZVV), I have that if

$v_B - v_S > r$, $v_B \neq 1$ and $v_S \neq 0$,

$$\pi^*(v_B, v_S)(v_B - v_S) - S^*(v_B, v_S) - \int_{(0,v_S)} \pi^*(v_B, s)ds - \pi^*(v_B, 0) = 0. \tag{PIE}$$

Note that (PIE) is a (second order) partial integral equation. It is straightforward to see that $S^*(v_B, v_S)$ is not separable by taking the cross partial derivative. I take the guess-and-verify approach to solve (PIE). I guess that if $v_B - v_S > r$, $v_B \neq 1$ and $v_S \neq 0$,

$$S^*(v_B, v_S) = \frac{\alpha}{(v_B - v_S)^2}.$$

Under this guess, the L.H.S. of (PIE) equals $\frac{2\alpha}{(v_B - v_S)^3}(v_B - v_S) - \frac{\alpha}{(v_B - v_S)^2} - \int_{(0,v_S)} \frac{2\alpha}{(v_B - s)^3} ds - \frac{\alpha}{v_B^2}$, which can be shown to be 0 with simple algebra. Thus, I verified the guess.

To solve for α, I use the requirement that $\pi^*(v)$ is a distribution. Note that the marginal distribution for S is $\pi_S^*(v_S) = S^*(v_S + r, v_S) = \frac{\alpha}{(v_S + r - v_S)^2} = \frac{\alpha}{r^2}$ for $0 < v_S < 1 - r$ and $Pr_S^*(v_S = 0) = S^*(r, 0) = \frac{\alpha}{r}$. Since the integration is 1, I obtain that

$$\frac{\alpha}{r} + \frac{\alpha}{r^2} \cdot (1 - r) = 1.$$

Thus, $\alpha = r^2$.

The final step is to show that the constructed joint distribution is a feasible value distribution. To see this, note that

$$\begin{aligned}
\int \max\{v_B - v_S, 0\}\, d\pi^* &= \int (v_B - v_S)\, d\pi^* \\
&= \left(r \cdot 1 + \int_r^1 v_B dv_B\right) - \left(r \cdot 0 + \int_0^{1-r} v_S dv_S\right) \\
&= GFT,
\end{aligned}$$

where the first line follows from $v_B > v_S$ for any value profile in the support of π^*, the second line uses the marginal distributions of π^*, and the third line uses $r = 1 - \sqrt{1 - GFT}$.

4 Other Models of Limited Knowledge

4.1 Known Difference In Expectations

In this section, I consider a model in which the intermediary only knows the difference between the expectations of the traders' values, denoted by DE, but does not know the joint distribution π. Formally, I denote by

$$\Pi(DE) = \left\{ \pi \in \Delta V : \int (v_B - v_S) d\pi(v) = DE \right\} \tag{KDE}$$

the collection of joint distributions that are consistent with the known difference in expectations. If $DE \leq 0$, then the maxmin profit is zero, as no trading mechanism can generate a positive profit against the point mass on the value profile $(0, -DE)$. Therefore, I focus on *non-trivial* informational environments in which $DE > 0$.

Theorem 2. *Under the model* (KDE)*, The random double auction is a maxmin trading mechanism with a profit guarantee of* $\left(1 - \sqrt{1 - DE}\right)^2$*, and the symmetric triangular value distribution is a worst value distribution.*

Knowing DE is different from knowing GFT. That is, the sets of feasible value distributions are different under these two assumptions. Indeed, for any value distribution in which the seller's value is greater than the buyer's one with a positive probability, GFT is strictly higher than DE. $GFT = DE$ if and only if the seller's value is always weakly lower than the buyer's one. Nonetheless, the results are the same under these two different assumptions. This is because the ex-post profit under the random double auction is convex in either the ex-post gain from trade or the difference between the values[21]. Therefore, any value distribution in which the seller's value is greater than the buyer's one with a positive probability is not a "worst case" for the random double auction under either assumption. In other words, the differences in the sets of feasible value distributions do not matter.

4.2 Known Expectations

In this section, I consider a model in which the intermediary only knows the expectations of the buyer's value and the seller's value respectively, denoted by M_B and M_S, but does not know the joint distribution π. Formally, I denote by

$$\Pi(M_B, M_S) = \left\{ \pi \in \Delta V : \int v_B d\pi(v) = M_B, \int v_S d\pi(v) = M_S \right\} \tag{KE}$$

the collection of joint distributions that are consistent with the known expectations. If $M_B \leq M_S$, then the maxmin profit is zero, as no trading mechanism can generate a positive profit against the point mass on the value profile (M_B, M_S). Therefore, we focus on *non-trivial* informational environments in which $M_B > M_S$.

[21]The ex-post profit $\max\left\{\frac{v_B - v_S - r}{1-r} \cdot r, 0\right\} = \max\left\{\frac{\max\{v_B - v_S, 0\} - r}{1-r} \cdot r, 0\right\}$.

4.2.1 Symmetric Informational Environment: $M_B + M_S = 1$

The higher the seller's value, the lower his willingness to trade. Thus, it is plausible to regard the highest-value seller as the lowest-type seller. When the known expectations sum up to 1, the expectation of the buyer's value and the expectation of the seller's value have the same distance from the lowest-type buyer and the lowest-type seller respectively, i.e., $M_B - 0 = 1 - M_S$. Therefore I refer to this case as the symmetric informational environment. The symmetric informational environment captures situations in which both parties have similar willingness to trade. Likewise, I refer to the case in which $M_B + M_S \neq 1$ as the asymmetric informational environment.

Theorem 3. *Under the model (KE), for the symmetric informational environment, the random double auction is a maxmin trading mechanism with a profit guarantee of* $\left(1 - \sqrt{1 - (M_B - M_S)}\right)^2$*, and the symmetric triangular value distribution is a worst value distribution.*

The derivation of the profit guarantee under the model (KE) is the same as that under the model (KDE). The construction of a worst value distribution is the same. Observe that the symmetric triangular value distribution satisfies $M_B + M_S = 1$, because

$$\begin{aligned}\int (v_B + v_S)\, d\pi^* &= \left(r \cdot 1 + \int_r^1 v_B dv_B\right) + \left(r \cdot 0 + \int_0^{1-r} v_S dv_S\right) \\ &= 1,\end{aligned}$$

where the first line uses the marginal distributions of π^*, and the second line holds for any $r \in (0, 1)$.

Knowing the expectations and knowing the difference in expectations are comparable. Indeed, $\Pi(DE)$ is a larger set: It contains both the symmetric informational environment and the asymmetric ones. For example, if $DE = 0.2$, then it is possible that $M_B = 0.6$ and $M_S = 0.4$ (the symmetric one), and it is possible that $M_B = 0.8$ and $M_S = 0.6$ (an asymmetric one). Therefore, although the random double auction is maxmin optimal under the model (KDE), it is maxmin optimal only for the symmetric informational environment under the model (KE). For the asymmetric one, as I show in the next section, a variation of random double auction does strictly better.

4.2.2 Asymmetric Informational Environment: $M_B + M_S \neq 1$

I extend the analysis to construct a maxmin trading mechanism for the asymmetric informational environment. I will propose a *generalized random double auction* and an *asymmetric triangular value distribution*, and then show that they form a saddle point. The illustration of the result is relegated to Appendix B. This section generalizes the results for the symmetric informational environment, as the generalized random double auction (resp, the asymmetric triangular value distribution) converges to the random double auction (resp, the symmetric triangular value distribution) when the asymmetric informational environment converges to the symmetric informational environment (See Remark 7).

Let (r_1, r_2) in which $r_1 \in (0,1)$, $r_2 \in (0,1)$ and $r_1 + r_2 \neq 1$ be a solution to the following system of equations

$$M_B = \int_{r_1}^{1} \frac{r_1(1-r_2)}{(\frac{1-r_1-r_2}{1-r_1}v_B + \frac{r_1 r_2}{1-r_1})^2} v_B dv_B + r_1 := H_1(r_1, r_2), \tag{KE-B}$$

$$M_S = \int_{0}^{r_2} \frac{r_1(1-r_2)}{(\frac{1-r_1-r_2}{r_2}v_S + r_1)^2} v_S dv_S := H_2(r_1, r_2). \tag{KE-S}$$

Lemma 3. *For the asymmetric informational environment, there exists a solution* $(r_1, r_2) \in (0,1)^2$ *to the system of equations* (KE-B) *and* (KE-S). *In addition,* $r_1 + r_2 \neq 1$.

Let $\gamma := \frac{1-r_2}{r_1}$, $\delta := \frac{2(1-r_1-r_2)}{1-r_1+r_2}$, $\tau := \frac{2r_1 r_2}{1-r_1+r_2}$. The generalized random double auction is described as follows.

Step 0: ***Transformed bid and ask***. The intermediary publicly commits to transforming a bid price b and an ask price a as follows: $b' = \frac{1}{\ln\gamma} \cdot \left[\ln\left(\frac{1-r_1-r_2}{1-r_1}b + \frac{r_1 r_2}{1-r_1}\right)\right], a' = \frac{1}{\ln\gamma} \cdot \left[\ln\left(\frac{1-r_1-r_2}{r_2}a + r_1\right)\right]$. The buyer and the seller both know r_1 and r_2 as well as the transformations.

Step 1: ***Uniformly random spread***. The intermediary publicly commits to randomly drawing a spread s' uniformly on $[0,1]$. Then a random spread is drawn whose realization is not observed by either the buyer or the seller. The buyer and the seller both know the uniform distribution on $[0,1]$ from which the random spread is drawn.

Step 2: ***Exponential transaction price and floating commission fee***. The buyer submits a bid price b, and the seller submits an ask price a, simultaneously. If the difference between the transformed bid price and the transformed ask price is greater than the realized spread, or $b' - a' > s'$, then the seller sells the asset to the buyer at the price $p' = \frac{\gamma^{b'} - \gamma^{a'}}{\delta(\ln\gamma)(b'-a')} - \frac{\tau}{\delta}$, and each pays the intermediary half of the commission fee $\frac{r'}{2} = \frac{\delta p' + \tau}{2}$. Otherwise, no trade takes place, and no one pays or receives anything.

Remark 4. The transaction price p' is no-longer midpoint of the bid price and the ask price. The floating commission fee r', however, has a fixed commission fee component τ, plus an price-adjusted component $\delta p'$, which is linear in the transaction price.

It is straightforward to show that the generalized random double auction can also be expressed as follows. If $r_2 b - (1-r_1)a > r_1 r_2$,

$$q^{**}(b,a) = \frac{1}{\ln\frac{1-r_2}{r_1}} \cdot \left[\ln\left(\frac{1-r_1-r_2}{1-r_1}b + \frac{r_1 r_2}{1-r_1}\right) - \ln\left(\frac{1-r_1-r_2}{r_2}a + r_1\right)\right],$$

$$t_B^{**}(b,a) = -\frac{r_1 r_2}{(1-r_1-r_2)\ln\frac{1-r_2}{r_1}} \cdot \left[\ln\left(\frac{1-r_1-r_2}{1-r_1}b + \frac{r_1 r_2}{1-r_1}\right) - \ln\left(\frac{1-r_1-r_2}{r_2}a + r_1\right)\right] + \frac{1}{\ln\frac{1-r_2}{r_1}} \cdot \left(b - \frac{1-r_1}{r_2}a - r_1\right),$$

$$t_S^{**}(b,a) = -\frac{r_1 r_2}{(1-r_1-r_2)\ln\frac{1-r_2}{r_1}} \cdot \left[\ln\left(\frac{1-r_1-r_2}{1-r_1}b + \frac{r_1 r_2}{1-r_1}\right) - \ln\left(\frac{1-r_1-r_2}{r_2}a + r_1\right)\right] + \frac{1}{\ln\frac{1-r_2}{r_1}} \cdot \left(\frac{r_2}{1-r_1}b - a - \frac{r_1 r_2}{1-r_1}\right).$$

If $r_2 b - (1-r_1)a \leq r_1 r_2$,

$$q^{**}(b,a) = t_B^{**}(b,a) = t_S^{**}(b,a) = 0.$$

Remark 5. The generalized random double auction also satisfies the standard weak budget balance property.

Now let me specify the asymmetric triangular value distribution. The support is an asymmetric triangular subset of joint values $AT := \{v | r_2 v_B - (1-r_1)v_S > r_1 r_2\}$. The marginal distribution for the buyer is a combination of some generalized Pareto distribution on $(r_1, 1)$ and an atom of size r_1 on 1: $\pi_B^{**}(v_B) = \frac{r_1(1-r_2)}{\left(\frac{1-r_1-r_2}{1-r_1}v_B + \frac{r_1 r_2}{1-r_1}\right)^2}$ for $v_B \in (r_1, 1)$ and $Pr_B^{**}(1) = r_1$. The marginal distribution for the buyer is a combination of some generalized Pareto distribution on $(0, r_2)$ and an atom of size $1-r_2$ on 0: $\pi_S^{**}(v_S) = \frac{r_1(1-r_2)}{\left(\frac{1-r_1-r_2}{r_2}v_S + r_1\right)^2}$ for $v_S \in (0, r_2)$ and $Pr_S^{**}(0) = 1-r_2$. The conditional distribution for the buyer is a combination of some generalized Pareto distribution on $\left(r_1 + \frac{1-r_1}{r_2}v_S, 1\right)$ and an atom on 1: When $v_S \in (0, r_2)$, $\pi_B^{**}(v_B|v_S) = \frac{2\left(\frac{1-r_1-r_2}{r_2}v_S + r_1\right)^2}{(v_B - v_S)^3}$ for $v_B \in \left(r_1 + \frac{1-r_1}{r_2}v_S, 1\right)$ and $Pr_B^{**}(v_B = 1|v_S) = \frac{\left(\frac{1-r_1-r_2}{r_2}v_S + r_1\right)^2}{(1-v_S)^2}$; when $v_S = 0$, $\pi_B^{**}(v_B|v_S = 0) = \frac{r_1}{(v_B)^2}$ for $v_B \in (r_1, 1)$ and $Pr_B^{**}(v_B = 1|v_S = 0) = r_1$. The conditional distribution for the seller is a combination of some generalized Pareto distribution on $(0, \frac{r_2(v_B - r_1)}{1-r_1})$ and an atom on 0: When $v_B \in (r_1, 1)$, $\pi_S^{**}(v_S|v_B) = \frac{2\left(\frac{1-r_1-r_2}{1-r_1}v_B + \frac{r_1 r_2}{1-r_1}\right)^2}{(v_B - v_S)^3}$ for $v_S \in \left(0, \frac{r_2(v_B - r_1)}{1-r_1}\right)$ and

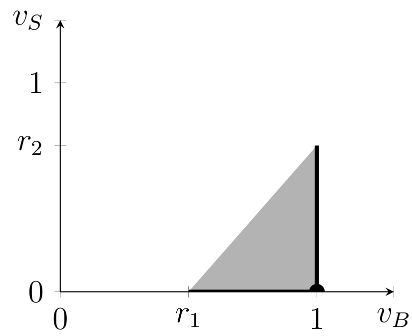

Figure 2: Asymmetric Triangular Value Distribution

$Pr_B^{**}(v_S = 0|v_B) = \frac{\left(\frac{1-r_1-r_2}{1-r_1}v_B + \frac{r_1 r_2}{1-r_1}\right)^2}{(v_B)^2}$; when $v_B = 1$, $\pi_S^{**}(v_S|v_B = 1) = \frac{1-r_2}{(1-v_S)^2}$ for $v_S \in (0, r_2)$ and $Pr_S^{**}(v_S = 0|v_B = 1) = 1 - r_2$.

Equivalently, the asymmetric triangular value distribution can be described as a combination of a joint density function on $AT \backslash \{(1,0)\}$ and an atom of size $r_1(1-r_2)$ on the value profile $(1,0)$ as follows (See Figure 2).

$$\pi^{**}(v_B, v_S) = \begin{cases} \frac{2r_1(1-r_2)}{(v_B - v_S)^3} & \text{if } r_2 v_B - (1-r_1)v_S \geq r_1 r_2,\ v_B \neq 1 \text{ and } v_S \neq 0, \\ \frac{r_1(1-r_2)}{(1-v_S)^2} & \text{if } v_B = 1 \text{ and } 0 < v_S < r_2, \\ \frac{r_1(1-r_2)}{(v_B)^2} & \text{if } r_1 < v_B < 1 \text{ and } v_S = 0. \end{cases}$$

$$Pr^{**}(1,0) = r_1(1-r_2).$$

Lemma 4. *The asymmetric triangular value distribution exhibits a positive correlation for* $r_1 < v_B < 1$ *and* $0 < v_S < r_2$.[22]

Remark 6. If $M_S = 0$, then it is common knowledge that the seller's value $v_S = 0$. Note that $q^{**}(b,0) = \frac{1}{\ln \frac{1-r_2}{r_1}} \cdot \ln\left(\frac{1-r_1-r_2}{r_1(1-r_1)}b + \frac{r_2}{1-r_1}\right)$. If $r_2 = 0$, it is straightforward that $q^{**}(b,0)$ (resp, π^{**}) reduces to the mechanism (resp, the worst-case distribution) found by Carrasco et al. (2018) when the monopolistic seller only knows the expectation of the buyer's value.

Theorem 4. *Under the model* (KE)*, for the asymmetric informational environment, the generalized random double auction is a maxmin trading mechanism with a profit guarantee*

[22] To see this, note that $\Pi_S^{**}(v_S|v_B) = \frac{\left(\frac{1-r_1-r_2}{1-r_1}v_B + \frac{r_1 r_2}{1-r_1}\right)^2}{(v_B - v_S)^2}$ is decreasing w.r.t. v_B for $v_B \in (r_1, 1)$. When $v_B = 1$, $\Pi_S^{**}(v_S|v_B) = \frac{1-r_2}{1-v_S} \geq \frac{(1-r_2)^2}{(1-v_S)^2}$, so the positive correlation breaks when $v_B = 1$. Similarly, $\Pi_B^{**}(v_B|v_S) = 1 - \frac{\left(\frac{1-r_1-r_2}{r_2}v_S + r_1\right)^2}{(v_B - v_S)^2}$ is decreasing w.r.t. v_S for $v_S \in (0, r_2)$. When $v_S = 0$, $\Pi_B^{**}(v_B|v_S = 0) = 1 - \frac{r_1}{v_B} \leq 1 - \frac{(r_1)^2}{(v_B)^2}$, so the positive correlation breaks when $v_S = 0$.

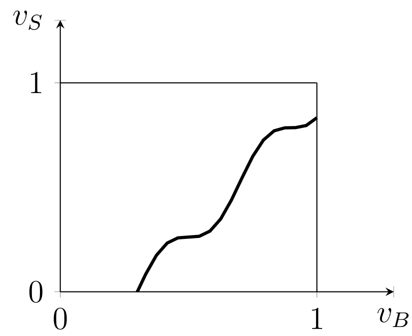

Figure 3: The thick black curve is a trade boundary $\mathcal{B}$ that is non-decreasing.

of $r_1(1-r_2)$*, and the asymmetric triangular value distribution is a worst value distribution.*

Remark 7 (Convergence)**.** If $M_B + M_S \to 1$, it is straightforward to show that there is a solution in which $r_1 + r_2 \to 1$. Then by L'Hôpital's rule, $q^{**} \to q^*$, $p' \to p$, $r' \to r$, $t_B^{**} \to t_B^*$, $t_S^{**} \to t_S^*$. In addition, $\pi^{**} \to \pi^*$.

5 Deterministic Mechanisms

In this section, I restrict attention to the class of deterministic dominant-strategy mechanisms, i.e., the trading rule has an additional property: $q(v)$ [23] is either 0 or 1 for any $v \in V$. I characterize maxmin deterministic trading mechanisms across mechanisms in this class.

Definition 3. The *trade boundary* of a given deterministic dominant-strategy mechanism (q, t_B, t_S) is a set of value profiles $\mathcal{B} := \{\bar{v} = (\bar{v}_B, \bar{v}_S) \in V | q(\bar{v}) = 0$[24] and for any small $\epsilon > 0$, $q(\bar{v}_B + \epsilon, \bar{v}_S) = 1$ or $q(\bar{v}_B, \bar{v}_S - \epsilon) = 1\}$.

I observe that the trade boundary of a deterministic dominant-strategy mechanism is non-decreasing (See Figure 3).

Remark 8 (Non-decreasing trade boundary)**.** If $\bar{v} = (\bar{v}_B, \bar{v}_S) \in \mathcal{B}, \bar{v}' = (\bar{v}'_B, \bar{v}'_S) \in \mathcal{B}$ and $\bar{v}_B > \bar{v}'_B$, then $\bar{v}_S \geq \bar{v}'_S$. [25]

The main idea of searching for a maxmin deterministic trading mechanism is as follows. I divide all possible deterministic dominant-strategy mechanisms into four classes according

[23] I define $q(v)$ to be 0 if $v \notin V$.

[24] For exposition, I assume that trade does not take place on the trade boundary. As will be clear, this is to guarantee that a best response for adversarial Nature exists. This assumption does not affect the solution and the value of the problem. Similar assumption is also made in Kos and Messner (2015).

[25] To see this, note that by the definition of the trade boundary, I have that $q(\bar{v}_B, \bar{v}'_S) = 1$ because $\bar{v}' \in \mathcal{B}$ and $\bar{v}_B > \bar{v}'_B$. Then, again by the definition of the trade boundary, I have that $\bar{v}_S \geq \bar{v}'_S$ because $\bar{v} \in \mathcal{B}$.

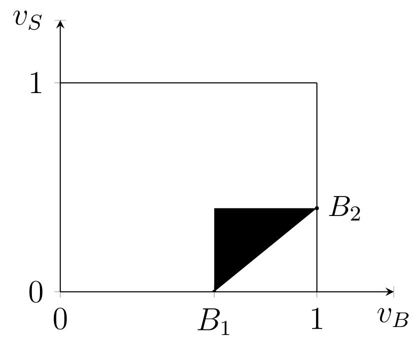

Figure 4: $B_1 = (1-\sqrt{\frac{1-GFT}{2}}, 0)$, $B_2 = (1, \sqrt{\frac{1-GFT}{2}})$. If $GFT > \frac{1}{2}$, then $B_1 \in \mathcal{B}, B_2 \in \mathcal{B}$, and $\mathcal{B}$ lies in the black region for a maxmin deterministic trading mechanism.

to the trade boundary. By *strong duality*[26], I can work on the dual program. I propose a relaxation of the dual program by ignoring a lot of constraints. The merit of doing so is to have a finite-dimensional linear programming problem. Then I derive an upper bound of the value of the relaxation and show that it can be attained by constructing deterministic dominant-strategy mechanisms as well as a feasible value distribution.

Theorem 5. *When $GFT > \frac{1}{2}$, any deterministic dominant-strategy mechanism satisfying the following properties is a maxmin deterministic trading mechanism (See Figure 4):*
(i). $\left(1-\sqrt{\frac{1-GFT}{2}}, 0\right) \in \mathcal{B}$, $\left(1, \sqrt{\frac{1-GFT}{2}}\right) \in \mathcal{B}$.
(ii). $\mathcal{B}$ is above (including) the line $v_B - v_S = \sqrt{\frac{1-GFT}{2}}$.
(iii). The payment rule and the transfer rule are characterized by Lemma 5.
The profit guarantee is $\left(1-\sqrt{2(1-GFT)}\right)^2$. The worst value distribution puts probability masses of $\sqrt{\frac{1-GFT}{2}}$, $\sqrt{\frac{1-GFT}{2}}$ and $1-2\sqrt{\frac{1-GFT}{2}}$ on the value profiles $\left(1-\sqrt{\frac{1-GFT}{2}}, 0\right)$, $\left(1, \sqrt{\frac{1-GFT}{2}}\right)$ and $(1,0)$ respectively.

When $GFT \leq \frac{1}{2}$, the Never Trading Mechanism[27] is a maxmin deterministic trading mechanism with a profit guarantee of 0.

That is, I characterize the class of maxmin deterministic trading mechanisms for any informational environment with a non-trivial profit guarantee (i.e., $GFT > \frac{1}{2}$). The worst value distribution is discrete, and is the same for the mechanisms in this class. Now I provide examples of some maxmin deterministic trading mechanisms.

Linear Trading Mechanism: Trade takes place with probability one if $v_B - v_S > \sqrt{\frac{1-GFT}{2}}$, and conditional on trading, the buyer pays $1-\sqrt{\frac{1-GFT}{2}} + v_S$ and the seller receives

[26] That is, given a dominant-strategy mechanism, the value of the primal minimization problem equals that of its dual maximization problem, details of which are in Appendix D.

[27] Trade never takes place, and no one pays or receives anything.

$v_B - \left(1 - \sqrt{\frac{1-GFT}{2}}\right)$; otherwise, no trade takes place, and no one pays or receives anything.

Double Posted-Price Trading Mechanism: Trade takes place with probability one if $v_B > 1 - \sqrt{\frac{1-GFT}{2}}$ and $v_S < \sqrt{\frac{1-GFT}{2}}$, and conditional on trading, the buyer pays $1 - \sqrt{\frac{1-GFT}{2}}$ and the seller receives $\sqrt{\frac{1-GFT}{2}}$; otherwise, no trade takes place, and no one pays or receives anything.

6 Extension and Discussion

6.1 Can-hold Case

Consider a more general model in which the intermediary can hold the asset. To wit, this only requires that the sum of the buyer's allocation (denoted by q_B) and the seller's allocation (denoted by q_S) do not exceed 1. Recall that this sum is required to be 1 for the main results. Formally, the intermediary seeks a trading mechanism (q_B, q_S, t_B, t_S) such that the following constraints hold:

$$v_B q_B(v) - t_B(v) \geq v_B q_B(v'_B, v_S) - t_B(v'_B, v_S), \quad \forall v \in V, v'_B \in V_B; \tag{$DSIC_B$}$$

$$v_B q_B(v) - t_B(v) \geq 0, \quad \forall v \in V; \tag{$EPIR_B$}$$

$$v_S q_S(v) + t_S(v) \geq v_S q_S(v_B, v'_S) + t_S(v_B, v'_S), \quad \forall v \in V, v'_S \in V_S; \tag{$DSIC'_S$}$$

$$v_S q_S(v) + t_S(v) \geq v_S, \quad \forall v \in V; \tag{$EPIR'_S$}$$

$$q_B(v) + q_S(v) \leq 1, \quad \forall v \in V. \tag{CH}$$

I denote the set of such trading mechanisms as $\mathcal{D}'$[28]. The intermediary's problem is to seek for a trading mechanism that solves

$$\sup_{(q_B, q_S, t_B, t_S) \in \mathcal{D}'} \inf_{\pi \in \Pi(GFT)} \int t(v) d\pi(v). \tag{MTM'}$$

Theorem 6. *The random double auction is a solution to (MTM').*

That is, the solution to the more general problem (MTM') coincides with the solution to the problem (MTM). To see this, first note that the value of (MTM') is weakly higher than the value of (MTM) because $\mathcal{D} \subset \mathcal{D}'$. I will show that the value of (MTM) is weakly higher than the value of (MTM'). Indeed, given the symmetric triangular value distribution,

[28]Note that here the monotonicity constraints are that $q_B(v_B, v_S)$ is non-decreasing w.r.t. v_B for any v_S and $q_S(v_B, v_S)$ is non-decreasing w.r.t. v_S for any v_B.

the random double auction is an optimal mechanism even among this wider class of trading mechanism $\mathcal{D}'$. To show this, first note that a simple adaptation of Lemma 5 yields an analogous virtual representation of the expected profit for this more general model:

$$E[t(v)] = \int_v [q_B(v)\phi_B(v) + q_S(v)\phi_S(v)]d\pi(v) - 1,$$

where $\phi_B(v) = v_B - \frac{1-\Pi_B(v_B|v_S)}{\pi_B(v_B|v_S)}$ and $\phi_S(v) = v_S + \frac{\Pi_S(v_S|v_B)}{\pi_S(v_S|v_B)}$. Here $\phi_B(v)$ (resp, $\phi_S(v)$) is the buyer's (resp, the seller's) virtual value when the value profile is $v = (v_B, v_S)$. Given the symmetric triangular value distribution, $\phi_B = \phi_S > 0$ for any value profile in the support except for the highest joint type (1,0), in which $\phi_B(1,0) > \phi_S(1,0) = 0$; in addition, $\phi_B \leq 0$ and $\phi_S \geq 0$ for any value profile outside the support. Then any trading mechanism in $\mathcal{D}'$ will be optimal if 1) the ex-post participation constraints are binding for zero-value buyer and one-value seller, and 2) $q_B = 0$ and $q_S = 1$ for any value profile outside the support, $q_B + q_S = 1$ for any value profile inside the support and $q_B(1,0) = 1, q_S(1,0) = 0$. It is straightforward to see that the random double auction[29] is such a mechanism and therefore remains optimal to the symmetric triangular value distribution in this general model. Indeed, given the property that the buyer's virtual value is equal to the seller's virtual value for any value profile in the support except for (1,0), the intermediary does not have an incentive to hold the asset in an optimal trading mechanism.

6.2 Information Design Problem

A well-known result in models of private information is that the distribution of agents' private information is a key determinant of their welfare. For example, in the environment of bilateral trade, Myerson and Satterthwaite (1983) consider the independent private value model and show that the two trading parties' welfare is not the full surplus for general distributions and the amount of their welfare depends on their distributions of private values. Indeed, most of the existing models of private information in the environment of bilateral trade assume that the distribution of the two trading parties' private information is exogenous. However, it is conceivable that a *financial regulator*, e.g., the Security and the Exchange Commission (SEC), may optimally design the nature of the private information held by the two trading parties to maximize their welfare, given the fact that their welfare is affected by the distribution of their private information.

In this section, I consider an information design problem of a *financial regulator* whose objective is to maximize the expected welfare. Recall that the expected welfare is defined as

[29] In this more general model, $q_B = q^*$ and $q_S = 1 - q^*$.

the sum of the traders' expected profits. I assume that the financial regulator can carefully design the private information of the traders by choosing a value distribution subject to the constraint that the ex-ante gain from trade is GFT, i.e., $\pi \in \Pi(GFT)$. The intermediary, after observing the choice of the distribution but not the realized joint values, designs a profit-maximizing trading mechanism across dominant-strategy mechanisms. Formally, the financial regulator solves [30]

$$\sup_{\pi \in \Pi(GFT)} \int [q^*(v)(v_B - v_S) - t^*(v)] d\pi(v) \tag{MW}$$

subject to

$$(q^*, t_B^*, t_S^*) \in \arg \sup_{(q, t_B, t_S) \in \mathcal{D}} \int_v t(v) d\pi(v).$$

Theorem 7. *The symmetric triangular value distribution is a solution to* (MW)*.*

That is, the symmetric triangular value distribution is an optimal information structure for the financial regulator's information design problem (MW).

Recall that a symmetric triangular value distribution has the property that the virtual value is zero for any value profile in the support except for the value profile $(1, 0)$. This property has two implications. First, it implies that an *efficient* [31] trading mechanism is a best response for the intermediary. Second, it implies that in a best response, the intermediary does not discriminate across all value profiles in the support but the value profile $(1, 0)$. These two implications render a symmetric triangular value distribution a good candidate as a solution.

Under the symmetric triangular value distribution, the expected welfare is the difference between the ex-ante gain from trade GFT and the expected profit of the intermediary $\left(1 - \sqrt{1 - GFT}\right)^2$. Indeed, the symmetric triangular value distribution minimizes the expected profit of the intermediary. This is because the expected profit under the random double auction is weakly higher than $\left(1 - \sqrt{1 - GFT}\right)^2$ for any feasible value distribution (Recall Proposition 2). Therefore, the symmetric triangular value distribution solves (MW). In addition, the expected welfare is equally shared by the traders: Each trader obtains an expected profit of $\sqrt{1 - GFT}(1 - \sqrt{1 - GFT})$.

The information design problem (MW) is closely related to Condorelli and Szentes

[30] If the intermediary has multiple optimal trading mechanisms, I break ties in favor of the financial regulator by selecting one that maximizes the gain from trade for the traders. This is a standard tie-breaking rule in the information design literature (e.g., Kamenica and Gentzkow (2011), Roesler and Szentes (2017) and Condorelli and Szentes (2020)).

[31] Precisely, trade takes place with probability one for any value profile in the support of the symmetric triangular value distribution, and 0 otherwise.

(2020) who consider a *buyer-optimal* information design problem: The buyer can choose the probability distribution of her valuation for the good to maximize her profit. The seller, after observing the buyer's choice of the distribution but not the realized valuation, designs a revenue-maximizing selling mechanism. The problem (MW) may be interpreted as a *traders-optimal* information design problem. Critically, trade is efficient under the solution in either problem.

Appendix A Preliminaries

A.1 Zero-Sum Game

The intermediary's maxmin optimization problem (MTM) can be interpreted as a two-player sequential *zero-sum game*. The two players are the intermediary and adversarial Nature. The intermediary first chooses a trading mechanism $(q, t_B, t_S) \in \mathcal{D}$. After observing the intermediary's choice of the trading mechanism, adversarial Nature chooses a feasible value distribution $\pi \in \Pi(M_B, M_S)$. The intermediary's payoff is $U((q, t_B, t_S), \pi)$, and adversarial Nature's payoff is $-U((q, t_B, t_S), \pi)$. Instead of solving directly for such a subgame perfect equilibrium, I can solve for a Nash equilibrium $((q^*, t_B^*, t_S^*), \pi^*)$ of the simultaneous-move version of this zero-sum game, which corresponds to a saddle point of the payoff functional U. Formally, for any $(q, t_B, t_S) \in \mathcal{D}$ and any $\pi \in \Pi(M_B, M_S)$,

$$U((q^*, t_B^*, t_S^*), \pi) \geq U((q^*, t_B^*, t_S^*), \pi^*) \geq U((q, t_B, t_S), \pi^*). \tag{SP}$$

Indeed, the first inequality implies that the mechanism (q^*, t_B^*, t_S^*)'s profit guarantee is the expected profit when adversarial Nature chooses the value distribution π^*, and the second inequality implies that no other dominant-strategy mechanism can yield a strictly higher expected profit under the value distribution π^*. Hence, the two inequalities together imply the mechanism (q^*, t_B^*, t_S^*) is a maxmin trading mechanism.

A.2 Revenue Equivalence

When searching an optimal dominant-strategy mechanism given a value distribution, it will be useful to simplify the problem. I will use the following proposition: Its proof is standard but included in Appendix A.2.1 for completeness.

Lemma 5 (Revenue Equivalence)**.** *When searching an optimal dominant-strategy mechanism, it is without loss to restrict attention to trading mechanisms satisfying the following properties:*

(i). $q(v)$ *is non-decreasing in* v_B *and non-increasing in* v_S*.*
(ii). $t_B(v) = v_B q(v) - \int_0^{v_B} q(x, v_S)dx$*.*
(iii). $t_S(v) = v_S q(v) + \int_{v_S}^1 q(v_B, x)dx$*.*
(iv). $t(v) = (v_B - v_S)q(v) - \int_0^{v_B} q(x, v_S)dx - \int_{v_S}^1 q(v_B, x)dx$*.*

That is, the trading rule $q(v)$ is monotone; the payment rule t_B and the transfer rule t_S admit an envelope representation. In addition, the ex-post participation constraints for zero-value buyer and one-value seller are binding. Lemma 5 is standard in the mechanism design literature. The envelope representation of the ex-post profit (property (iv)) implies $E[t(v)] = \int q(v)\phi(v)d\pi(v)$, using integration by parts.

A.2.1 Proof of Lemma 5

(i). Dominant-strategy incentive compatibility (DSIC) for a type v_B of B requires that for any v_S and $v_B' \neq v_B$:

$$v_B q(v_B, v_S) - t_B(v_B, v_S) \geq v_B q(v_B', v_S) - t_B(v_B', v_S).$$

DSIC for a type v_B' of B requires that for any v_S and $v_B \neq v_B'$:

$$v_B' q(v_B', v_S) - t_B(v_B', v_S) \geq v_B' q(v_B, v_S) - t_B(v_B, v_S).$$

Adding the two inequalities, I have that:

$$(v_B - v_B')(q(v_B, v_S) - q(v_B', v_S)) \geq 0.$$

It follows that $q(v_B, v_S) \geq q(v_B', v_S)$ whenever $v_B > v_B'$.
Similarly, DSIC for a type v_S of S requires that for any v_B and $v_S' \neq v_S$:

$$v_S(1 - q(v_B, v_S)) + t_B(v_B, v_S) \geq v_S(1 - q(v_B, v_S')) + t_S(v_B, v_S').$$

DSIC for a type v_S' of S requires that for any v_B and $v_S \neq v_S'$:

$$v_S'(1 - q(v_B, v_S')) + t_S(v_B, v_S') \geq v_S'(1 - q(v_B, v_S)) + t_S(v_B, v_S).$$

Adding the two inequalities, I have that:

$$(v_S - v_S')(q(v_B, v_S') - q(v_B, v_S)) \geq 0.$$

It follows that $q(v_B, v_S) \leq q(v_B, v'_S)$ whenever $v_S > v'_S$.

(ii). Fix v_S, and define

$$U_B(v_B) := v_B q(v_B, v_S) - t_B(v_B, v_S).$$

By the first two inequalities in (i), I obtain that

$$(v'_B - v_B)q(v_B, v_S) \leq U_B(v'_B) - U_B(v_B) \leq (v'_B - v_B)q(v'_B, v_S).$$

Therefore $U_B(v_B)$ is Lipschitz, hence absolutely continuous w.r.t. v_B and therefore differentiable w.r.t. v_B almost everywhere. Then applying the envelope theorem to the above inequality at each point of differentiability, I obtain that

$$\frac{dU_B(v_B)}{dv_B} = q(v_B, v_S).$$

Then I have that

$$t_B(v_B, v_S) = v_B q(v_B, v_S) - \int_0^{v_B} q(x, v_S)dx - U_B(0).$$

Note that $U_B(0) \geq 0$ by the ex-post individually rational constraint. If $U_B(0) > 0$, then I can reduce it to 0 so that I can increase the payment from B for any value profile in which the seller's value is v_S. And the profit will be weakly greater. Thus, when searching for an optimal dominant-strategy mechanism, it is without loss of generality to let $U_B(0) = 0$. Then I obtain that $t_B(v_B, v_S) = v_B q(v_B, v_S) - \int_0^{v_B} q(x, v_S)dx$.
(iii). Similarly, fix v_B, and define

$$U_S(v_S) := v_S(1 - q(v_B, v_S)) + t_S(v_B, v_S).$$

By the fourth and fifth inequalities in (i), I obtain that

$$(v'_S - v_S)(1 - q(v_B, v_S)) \leq U_S(v'_S) - U_S(v_S) \leq (v'_S - v_S)(1 - q(v_B, v'_S)).$$

Therefore $U_S(v_S)$ is Lipschitz, hence absolutely continuous w.r.t. v_S and therefore differentiable w.r.t. v_S almost everywhere. Then applying the envelope theorem to the above inequality at each point of differentiability, I obtain that

$$\frac{dU_S(v_S)}{dv_S} = 1 - q(v_B, v_S).$$

Then I have that

$$t_B(v_B, v_S) = U_S(1) - v_S(1 - q(v_B, v_S)) - \int_{v_S}^{1} q(v_B, x)dx.$$

Note that $U_S(1) \geq 1$ by the ex-post individually rational constraint. If $U_S(1) > 1$, then I can reduce it to 1 so that I can decrease the payment to S for any value profile in which the buyer's value is v_B. And the profit will be weakly greater. Thus, when searching for an optimal dominant-strategy mechanism, it is without loss of generality to let $U_S(1) = 1$. Then I obtain that $t_S(v) = 1 - (1 - q(v))v_S - \int_{v_S}^{1}(1 - q(v_B, x)dx = q(v)v_S + \int_{v_S}^{1} q(v_B, x)dx$.
(iv). This is implied by (ii) and (iii).

Appendix B Illustration of Theorem 4

B.1 Construction of Generalized Random Double Auction

Lemma 6. *Given a trading mechanism $(q, t_B, t_S) \in \mathcal{D}$, if π minimizes the expected profit over $\Pi(M_B, M_S)$, then there exist real numbers λ_B, λ_S and μ such that*

$$\lambda_B v_B + \lambda_S v_S + \mu = t(v), \quad \forall v \in supp(\pi). \tag{CS}$$

(CS) is a *complementary slackness condition*, stating that the ex-post profit is a linear function of the true values for any value profile in the support of a worst value distribution. The complementary slackness condition is a result of strong duality. The proof is similar to the one for the main model (See Appendix D for details). The complementary slackness condition is useful in the construction of a maxmin trading mechanism for the asymmetric informational environment.

For the asymmetric informational environment, it is natural to attach different weights to the submitted bid price and the submitted ask price. I thus form an educated guess of the trading region in a maxmin trading mechanism: Trade takes place with positive probability if and only if the difference between a *weighted* bid (true value of the buyer) $r_2 \cdot v_B$ and a (different) *weighted* ask (true value of the seller) $(1 - r_1) \cdot v_S$ exceeds a threshold $r_1 r_2 > 0$, or $r_2 v_B - (1 - r_1)v_S > r_1 r_2$. In addition, again, the support of a worst value distribution coincides with the trading region (including the boundary). Together with *(iv)* of Lemma 5, the complementary slackness condition (CS) can be expressed as follows: For any $(v_B, v_S) \in AT$,

$$\lambda_B^{**} v_B + \lambda_S^{**} v_S + \mu^{**} = (v_B - v_S) \cdot q^{**}(v_B, v_S) - \int_{\frac{1-r_1}{r_2} v_S + r_1}^{v_B} q^{**}(x, v_S) dx - \int_{v_S}^{\frac{r_2}{1-r_1}(v_B - r_1)} q^{**}(v_B, x) dx. \tag{CS-A}$$

Now I solve for the trading rule q^{**}. Similarly, I first take the first order derivatives with respect to v_B and v_S respectively, and I obtain that for any $(v_B, v_S) \in AT$,

$$(v_B - v_S) \cdot \frac{\partial q^{**}(v_B, v_S)}{\partial v_B} - \frac{\partial \int_{v_S}^{\frac{r_2}{1-r_1}(v_B - r_1)} q^{**}(v_B, x) dx}{\partial v_B} = \lambda_B^{**}, \tag{FOC-B}$$

$$(v_B - v_S) \cdot \frac{\partial q^{**}(v_B, v_S)}{\partial v_S} - \frac{\partial \int_{\frac{1-r_1}{r_2} v_S + r_1}^{v_B} q^{**}(x, v_S) dx}{\partial v_S} = \lambda_S^{**}. \tag{FOC-S}$$

Then, I take the cross partial derivative, with some algebra, I obtain that

$$(v_B - v_S) \cdot \frac{\partial q^{**}(v_B, v_S)}{\partial v_B \partial v_S} = 0.$$

Thus, $q^{**}(v_B, v_S)$ is separable, which can be expressed as the sum of two functions f^{**} and g^{**}: For any $(v_B, v_S) \in AT$,

$$q^{**}(v_B, v_S) = f^{**}(v_B) + g^{**}(v_S). \tag{B.1.1}$$

Again, the separable nature is crucial for solving (CS-A). Plugging (B.1.1) into (FOC-B) and (FOC-S), I obtain that for any $(v_B, v_S) \in AT$,

$$\left[\left(1 - \frac{r_2}{1-r_1}\right) v_B + \frac{r_1 r_2}{1-r_1}\right] \cdot (f^{**})'(v_B) - \frac{r_2}{1-r_1} \cdot \left[f^{**}(v_B) + g^{**}\left(\frac{r_2}{1-r_1}(v_B - r_1)\right)\right] = \lambda_B^{**}, \tag{B.1.2}$$

$$\left[\left(\frac{1-r_1}{r_2} - 1\right) v_S + r_1\right] \cdot (g^{**})'(v_S) + \frac{1-r_1}{r_2} \cdot \left[f^{**}\left(\frac{1-r_1}{r_2} v_S + r_1\right) + g^{**}(v_S)\right] = \lambda_S^{**}. \tag{B.1.3}$$

Note that $f^{**}(v_B) + g^{**}\left(\frac{r_2}{1-r_1}(v_B - r_1)\right) = 0$ and that $f^{**}\left(\frac{1-r_1}{r_2} v_S + r_1\right) + g^{**}(v_S) = 0$ because trade does not take place in the boundary of the trading region, i.e., $q^{**}(v_B, v_S) = 0$ for $r_2 v_B - (1 - r_1) v_S = r_1 r_2$. Then it is straightforward to solve for $f^{**}(v_B)$ and $g^{**}(v_S)$, and I obtain that

$$f^{**}(v_B) = \frac{(1-r_1)\lambda_B^{**}}{1-r_1-r_2} \cdot \ln\left[\left(1 - \frac{r_2}{1-r_1}\right) v_B + \frac{r_1 r_2}{1-r_1}\right] + c_B^{**}, \tag{B.1.4}$$

$$g^{**}(v_S) = \frac{r_2 \lambda_S^{**}}{1 - r_1 - r_2} \cdot \ln\left[\left(\frac{1-r_1}{r_2} - 1\right) v_S + r_1\right] + c_S^{**}, \tag{B.1.5}$$

where c_B^{**} and c_S^{**} are some constants. Observe that

$$g^{**}\left(\frac{r_2(v_B - r_1)}{1 - r_1}\right) = \frac{r_2 \lambda_S^{**}}{1 - r_1 - r_2} \cdot \ln\left[\left(1 - \frac{r_2}{1-r_1}\right) v_B + \frac{r_1 r_2}{1 - r_1}\right] + c_S^{**}.$$

Then, again, using that $q^{**}(v_B, v_S) = 0$ for $r_2 v_B - (1 - r_1) v_S = r_1 r_2$, I have that

$$(1 - r_1)\lambda_B^{**} + r_2 \lambda_S^{**} = c_B^{**} + c_S^{**} = 0. \tag{B.1.6}$$

Now plugging (B.1.4),(B.1.5) and (B.1.6) into (B.1.1), I obtain that for any $(v_B, v_S) \in AT$,

$$q^{**}(v_B, v_S) = \frac{(1 - r_1)\lambda_B^{**}}{1 - r_1 - r_2} \cdot \left[\ln\left(\frac{1 - r_1 - r_2}{1 - r_1} v_B + \frac{r_1 r_2}{1 - r_1}\right) - \ln\left(\frac{1 - r_1 - r_2}{r_2} v_S + r_1\right)\right].$$

Likewise, to solve for λ_B^{**}, I let $q^{**}(1, 0)$ be 1 and obtain that $\lambda_B^{**} = \frac{1 - r_1 - r_2}{(1 - r_1)\ln\frac{1 - r_2}{r_1}}$. So far I have obtained the trading rule q^{**}[32]. The payment rule t_B^{**} (resp, the transfer rule t_S^{**}) is then characterized by (*ii*) (resp, (*iii*)) of Lemma 5.

B.2 Construction of Asymmetric Triangular Value Distribution

Similar to the symmetric informational environment, I impose a *zero virtual value condition* on the joint distribution, stating that virtual value is 0 for any value profile in the support except for the highest joint type. Formally,

$$\phi(v) = 0, \quad \forall v \in AT \backslash \{(1, 0)\}. \tag{ZVV-A}$$

The construction procedure for the joint distribution is exactly the same. Therefore I omit it. Note that the marginal distribution no longer has a uniform distribution part since $v_B - v_S$ is no longer a constant on the boundary of the trading region due to different weights for the buyer and the seller. The final step is to make sure that the constructed joint distribution has the known expectations. Given the marginal distributions for the buyer and the seller, I have a system of two equations (KE-B) and (KE-S). Lemma 3 states that a solution exists for the asymmetric informational environment, details of which are given in Appendix C.

[32]Plugging the trading rule q^{**} into (CS-A), it is straightforward that $\mu^{**} = -\frac{r_1(1 - r_1 - r_2)}{(1 - r_1)\ln\frac{1 - r_2}{r_1}}$.

Appendix C Proofs for Section 3

C.1 Proof of Lemma 3

I start from establishing the following four claims regarding some properties of the functions $H_1(r_1, r_2)$ and $H_2(r_1, r_2)$, which play a crucial role in establishing Lemma 3. First, by simple calculation, I have that for $(r_1, r_2) \in (0, 1)^2$,

$$H_1(r_1, r_2) = \frac{r_1(1 - r_2)(1 - r_1)^2}{(1 - r_1 - r_2)^2} \cdot \ln \frac{1 - r_2}{r_1} - \frac{r_1 r_2(1 - r_1)}{1 - r_1 - r_2} + r_1, \tag{C.1.1}$$

$$H_2(r_1, r_2) = \frac{r_1(1 - r_2)r_2^2}{(1 - r_1 - r_2)^2} \cdot \ln \frac{1 - r_2}{r_1} - \frac{r_1 r_2^2}{1 - r_1 - r_2}. \tag{C.1.2}$$

First note that H_1 and H_2 are not well-defined when $0 < r_1 = 1 - r_2 < 1$. Using L'Hôpital's rule, it is straightforward to show that $\lim_{1-r_2 \to r_1} H_1(r_1, r_2) = \frac{1-r_1^2+2r_1}{2}$ and $\lim_{1-r_2 \to r_1} H_2(r_1, r_2) = \frac{(1-r_1)^2}{2}$. I thus define $H_1(r_1, r_2) := \lim_{1-r_2 \to r_1} H_1(r_1, r_2)$ and $H_2(r_1, r_2) := \lim_{1-r_2 \to r_1} H_2(r_1, r_2)$ when $0 < r_1 = 1 - r_2 < 1$. This makes H_1 and H_2 continuous on $(0, 1)^2$. In addition, using L'Hôpital's rule, it is straightforward to show that $\lim_{r_1 \to 0} H_1(r_1, r_2) = 0$ for $r_2 \in (0, 1)$, $\lim_{r_1 \to 1} H_1(r_1, r_2) = 1$ for $r_2 \in (0, 1)$, $\lim_{r_2 \to 0} H_1(r_1, r_2) = r_1 - r_1 \ln r_1$ for $r_1 \in (0, 1)$, $\lim_{r_2 \to 1} H_1(r_1, r_2) = 1$ for $r_1 \in (0, 1)$, $\lim_{r_1 \to 0} H_2(r_1, r_2) = 0$ for $r_2 \in (0, 1)$, $\lim_{r_1 \to 1} H_2(r_1, r_2) = (1 - r_2) \ln (1 - r_2) + r_2$ for $r_2 \in (0, 1)$, $\lim_{r_2 \to 0} H_2(r_1, r_2) = 0$ for $r_1 \in (0, 1)$ and $\lim_{r_2 \to 1} H_1(r_1, r_2) = 1$ for $r_1 \in (0, 1)$. Therefore I define $H_1(r_1, r_2)$ and $H_2(r_1, r_2)$ as follows.

$$H_1(r_1, r_2) = \begin{cases} \frac{(1-r_2)r_1(1-r_1)^2}{(1-r_1-r_2)^2} \cdot \ln \frac{1-r_2}{r_1} - \frac{r_1 r_2(1-r_1)}{1-r_1-r_2} + r_1 & \text{if } (r_1, r_2) \in (0, 1)^2 \text{ and } r_1 + r_2 \neq 1, \\ \frac{1-r_1^2+2r_1}{2} & \text{if } \quad 0 < r_1 = 1 - r_2 < 1, \\ 0 & \text{if } r_1 = 0 \text{ and } r_2 \in (0, 1), \\ 1 & \text{if } r_1 = 1 \text{ and } r_2 \in (0, 1), \\ r_1 - r_1 \ln r_1 & \text{if } r_2 = 0 \text{ and } r_1 \in (0, 1), \\ 1 & \text{if } r_2 = 1 \text{ and } r_1 \in (0, 1). \end{cases}$$

$$H_2(r_1, r_2) = \begin{cases} \frac{r_1(1-r_2)r_2^2}{(1-r_1-r_2)^2} \cdot \ln \frac{1-r_2}{r_1} - \frac{r_1 r_2^2}{1-r_1-r_2} & \text{if } (r_1, r_2) \in (0, 1)^2 \text{ and } r_1 + r_2 \neq 1, \\ \frac{(1-r_1)^2}{2} & \text{if } \quad 0 < r_1 = 1 - r_2 < 1, \\ 0 & \text{if } r_1 = 0 \text{ and } r_2 \in (0, 1), \\ (1 - r_2) \ln (1 - r_2) + r_2 & \text{if } r_1 = 1 \text{ and } r_2 \in (0, 1), \\ 0 & \text{if } r_2 = 0 \text{ and } r_1 \in (0, 1), \\ 1 & \text{if } r_2 = 1 \text{ and } r_1 \in (0, 1). \end{cases}$$

Claim 1. *Fix any $r_2 \in [0,1)$, $H_1(r_1,r_2)$ is strictly increasing in r_1. Moreover, for any $r_2 \in (0,1)$, $\lim_{r_1 \to 1-r_2} \frac{\partial H_1(r_1,r_2)}{\partial r_1}$ exists and is positive. In addition, for any $r_2 \in [0,1)$, as $r_1 \to 1$, $H_1(r_1,r_2) \to 1$.*

Proof of Claim 1. When $r_2 = 0$, $H_1(r_1,r_2) = r_1 - r_1 \ln r_1$. This is an strictly increasing function because $\frac{\partial H_1(r_1,r_2)}{\partial r_1} = -\ln r_1 > 0$. In addition, by L'Hôpital's rule, $\lim_{r_1 \to 1} H_1(r_1,r_2) = 1$. Thus, Claim 1 holds when $r_2 = 0$. When $0 < r_2 < 1$, I already have that $\lim_{r_1 \to 1} H_1(r_1,r_2) = 1$. Now taking the first order derivative w.r.t. r_1 to (C.1.1), I obtain that

$$\frac{\partial H_1(r_1,r_2)}{\partial r_1} = \frac{(1-r_1)(1-r_2)}{(1-r_1-r_2)^2} \cdot \left[\left(1 - 3r_1 + \frac{2r_1(1-r_1)}{1-r_1-r_2} \right) \cdot \ln \frac{1-r_2}{r_1} - 2r_2 \right]. \tag{C.1.3}$$

Then to show the first part of Claim 1, it suffices to show that if $(r_1,r_2) \in (0,1)^2$ and $r_1 + r_2 \neq 1$,

$$\left(1 - 3r_1 + \frac{2r_1(1-r_1)}{1-r_1-r_2} \right) \cdot \ln \frac{1-r_2}{r_1} - 2r_2 > 0. \tag{C.1.4}$$

Let $\beta := \frac{1-r_2}{r_1}$, then $\beta \in (0,1) \cup (1,\infty)$. Plugging $r_2 = 1 - \beta r_1$ into (C.1.4), it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$,

$$\left(1 - 3r_1 + \frac{2(1-r_1)}{\beta - 1} \right) \cdot \ln \beta - 2 \cdot (1 - \beta r_1) > 0. \tag{C.1.5}$$

Slightly rewriting (C.1.5), it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$,

$$\frac{\beta+1}{\beta-1} \cdot \ln \beta - 2 + \left(-\frac{3\beta - 1}{\beta - 1} \cdot \ln \beta + 2\beta \right) \cdot r_1 > 0. \tag{C.1.6}$$

Then, it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$, the following two inequalities hold:

$$\frac{\beta+1}{\beta-1} \cdot \ln \beta - 2 > 0, \tag{C.1.7}$$

$$-\frac{3\beta-1}{\beta-1} \cdot \ln \beta + 2\beta > 0. \tag{C.1.8}$$

Now to prove (C.1.7), it suffices to show that $f(\beta) := \ln \beta - \frac{2(\beta-1)}{\beta+1} > 0$ for $\beta \in (1,\infty)$ and $f(\beta) < 0$ for $\beta \in (0,1)$. Taking the first order derivative to $f(\beta)$, I obtain that

$$f'(\beta) = \frac{(\beta-1)^2}{\beta(\beta+1)^2}. \tag{C.1.9}$$

Therefore, $f(\beta)$ is strictly increasing. Note that $f(1) = 0$. Thus, I proved (C.1.7). To prove

(C.1.8), it suffices to show that $h(\beta) := (1-3\beta)\ln\beta + 2\beta(\beta-1) > 0$ for $\beta \in (1,\infty)$ and $h(\beta) < 0$ for $\beta \in (0,1)$. Taking the first order derivative to $h(\beta)$, I obtain that

$$h'(\beta) = 4\beta - 3\ln\beta + \frac{1}{\beta} - 5. \tag{C.1.10}$$

Now taking the second order derivative to $h(\beta)$, I obtain that

$$h''(\beta) = \frac{(4\beta+1)(\beta-1)}{\beta^2}. \tag{C.1.11}$$

Note that $h''(\beta) > 0$ when $\beta > 1$, $h''(\beta) < 0$ when $\beta < 1$ and $h''(1) = 0$. This implies that $h'(\beta)$ is minimized at $\beta = 1$. Note that $h'(1) = 0$. This implies that $h(\beta)$ is strictly increasing. Finally, note that $h(1) = 0$. This implies that (C.1.8) holds.

Using L'Hôpital's rule, I have that $\lim_{r_1 \to 1-r_2} \frac{\partial H_1(r_1,r_2)}{\partial r_1} = \frac{r_2(6-5r_2)}{1-r_2} > 0$ for $r_2 \in (0,1)$. □

Claim 2. *Fix any $r_1 \in (0,1)$, $H_1(r_1,r_2)$ is strictly increasing in r_2. Moreover, for any $r_1 \in (0,1)$, $\lim_{r_2 \to 1-r_1} \frac{\partial H_1(r_1,r_2)}{\partial r_2}$ exists and is positive. In addition, for any $r_1 \in (0,1)$, as $r_2 \to 1$, $H_1(r_1,r_2) \to 1$.*

Proof of Claim 2. When $0 < r_1 < 1$, I already have that $\lim_{r_2 \to 1} H_1(r_1,r_2) = 1$. Now taking the first order derivative w.r.t. r_2 to (C.1.1), with some algebra, I obtain that

$$\frac{\partial H_1(r_1,r_2)}{\partial r_2} = \frac{(1-r_1)^2 r_1}{(1-r_1-r_2)^2} \cdot \left[\left(-1 + \frac{2(1-r_2)}{1-r_1-r_2}\right) \cdot \ln\frac{1-r_2}{r_1} - 2\right]. \tag{C.1.12}$$

Then it suffices to show that if $(r_1,r_2) \in (0,1)^2$ and $r_1 + r_2 \neq 1$,

$$\left(-1 + \frac{2(1-r_2)}{1-r_1-r_2}\right) \cdot \ln\frac{1-r_2}{r_1} - 2 > 0. \tag{C.1.13}$$

Plugging $r_2 = 1 - \beta r_1$ into (C.1.13), it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$,

$$\frac{\beta+1}{\beta-1} \cdot \ln\beta - 2 > 0. \tag{C.1.14}$$

This is exactly (C.1.7) and has been shown in the Proof of Claim 1.

Using L'Hôpital's rule, I have that $\lim_{r_2 \to 1-r_1} \frac{\partial H_1(r_1,r_2)}{\partial r_2} = \frac{(1-r_1)^2}{6r_1} > 0$ for $r_1 \in (0,1)$. □

Claim 3. *Fix any $r_2 \in (0,1)$, $H_2(r_1,r_2)$ is strictly increasing in r_1. Moreover, for $r_2 \in (0,1)$, $\lim_{r_1 \to 1-r_2} \frac{\partial H_2(r_1,r_2)}{\partial r_1}$ exists and is positive.*

Proof of Claim 3. Taking the first order derivative w.r.t. r_1 to (C.1.2), I obtain that

$$\frac{\partial H_2(r_1,r_2)}{\partial r_1} = \frac{(1-r_2)r_2^2}{(1-r_1-r_2)^2} \cdot \left[\left(1+\frac{2r_1}{1-r_1-r_2}\right)\cdot \ln\frac{1-r_2}{r_1} - 2\right]. \tag{C.1.15}$$

Then it suffices to show that if $(r_1, r_2) \in (0,1)^2$ and $r_1 + r_2 \neq 1$,

$$\left(1+\frac{2r_1}{1-r_1-r_2}\right)\cdot \ln\frac{1-r_2}{r_1} - 2 > 0. \tag{C.1.16}$$

Plugging $r_2 = 1 - \beta r_1$ into (C.1.16), it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$, $\frac{\beta+1}{\beta-1}\ln\beta - 2 > 0$, which is exactly (C.1.7) and has been shown in the Proof of Claim 1.

Using L'Hôpital's rule, I have that $\lim_{r_1 \to 1-r_2} \frac{\partial H_2(r_1,r_2)}{\partial r_1} = \frac{(r_2)^2}{6(1-r_2)} > 0$ for $r_2 \in (0,1)$. $\square$

Claim 4. *Fix any $r_1 \in (0,1]$, $H_2(r_1,r_2)$ is strictly increasing in r_2. Moreover, for any $r_1 \in (0,1)$, $\lim_{r_2\to 1-r_1} \frac{\partial H_2(r_1,r_2)}{\partial r_2}$ exists and is positive. In addition, for any $r_1 \in (0,1]$, as $r_2 \to 1$, $H_2(r_1,r_2) \to 1$.*

Proof of Claim 4. When $r_1 = 1$, $H_2(r_1,r_2) = (1-r_2)\cdot\ln(1-r_2) + r_2$. This is an strictly increasing function because $\frac{\partial H_2(r_1,r_2)}{\partial r_2} = -\ln(1-r_2) > 0$. In addition, by L'Hôpital's rule, $\lim_{r_2\to 1} H_2(r_1,r_2) = 1$. Thus, Claim 4 holds when $r_1 = 1$. When $0 < r_1 < 1$, I already have that $\lim_{r_2 \to 1} H_1(r_1,r_2) = 1$. Now taking the first order derivative w.r.t. r_2 to (C.1.2), I obtain that

$$\frac{\partial H_2(r_1,r_2)}{\partial r_2} = \frac{r_1 r_2}{(1-r_1-r_2)^2}\cdot\left[\left(2-3r_2+\frac{2r_2(1-r_2)}{1-r_1-r_2}\right)\cdot\ln\frac{1-r_2}{r_1} - 2\cdot(1-r_1)\right]. \tag{C.1.17}$$

Then to show the first part of Claim 4, it suffices to show that if $(r_1,r_2) \in (0,1)^2$ and $r_1 + r_2 \neq 1$,

$$\left(2-3r_2+\frac{2r_2(1-r_2)}{1-r_1-r_2}\right)\cdot\ln\frac{1-r_2}{r_1} - 2\cdot(1-r_1) > 0. \tag{C.1.18}$$

Plugging $r_2 = 1 - \beta r_1$ into (C.1.18), it suffices to show that for any $\beta \in (0,1)\cup(1,\infty)$,

$$\left(3\beta r_1 - 1 + \frac{2\beta(1-\beta r_1)}{\beta - 1}\right)\cdot \ln\beta - 2\cdot(1-r_1) > 0. \tag{C.1.19}$$

Slightly rewriting (C.1.19), it suffices to show that for any $\beta \in (0,1)\cup(1,\infty)$,

$$\frac{\beta+1}{\beta-1}\cdot\ln\beta - 2 + \left(\frac{\beta^2-3\beta}{\beta-1}\ln\beta + 2\right)\cdot r_1 > 0. \tag{C.1.20}$$

Then, it suffices to show that for any $\beta \in (0,1) \cup (1,\infty)$, the following two inequalities hold:

$$\frac{\beta+1}{\beta-1} \cdot \ln \beta - 2 > 0, \tag{C.1.21}$$

$$\frac{\beta^2 - 3\beta}{\beta - 1} \cdot \ln \beta + 2 > 0. \tag{C.1.22}$$

Note that (C.1.21) is exactly (C.1.7), which has been shown in the Proof of Claim 1. To prove (C.1.22), it suffices to show that $g(\beta) := (\beta^2 - 3\beta)\ln\beta + 2(\beta - 1) > 0$ for $\beta \in (1,\infty)$ and $g(\beta) < 0$ for $\beta \in (0,1)$. Taking the first order derivative to $g(\beta)$, I obtain that

$$g'(\beta) = (2\beta - 3) \cdot \ln \beta + \beta - 1. \tag{C.1.23}$$

Now taking the second order derivative to $g(\beta)$, I obtain that

$$g''(\beta) = 2\ln\beta - \frac{3}{\beta} + 3. \tag{C.1.24}$$

Note that $g''(\beta)$ is strictly increasing and $g''(1) = 0$. This implies that $g'(\beta)$ is minimized at $\beta = 1$. Note that $g'(1) = 0$. This implies that $g(\beta)$ is strictly increasing. Finally, note that $g(1) = 0$. This implies that (C.1.22) holds.

Using L'Hôpital's rule, I have that $\lim_{r_2 \to 1 - r_1} \frac{\partial H_2(r_1,r_2)}{\partial r_2} = \frac{(1-r_1)(5r_1+1)}{6r_1} > 0$ for $r_1 \in (0,1)$. □

I am now ready to prove Lemma 3. Fix any (M_B, M_S) in which $0 < M_S < M_B < 1$ and $M_B + M_S \neq 1$. By Claim 1, 2 and the Intermediate Value Theorem, I have that for any $r_2 \in [0,1)$, there exists a unique $I(r_2) \in (0,1)$ such that $r_1 = I(r_2)$ is a solution to $H_1(r_1, r_2) = M_B$. In addition, $I(r_2)$ is a strictly decreasing function. Moreover, by the Implicit Function Theorem[33], $I(r_2)$ is continuous at each $r_2 \in [0,1)$. By Claim 3, 4 and the Intermediate Value Theorem, I have that for any $r_1 \in (0,1]$, there exists a unique $J(r_1) \in (0,1)$ such that $r_2 = J(r_1)$ is a solution to $H_2(r_1, r_2) = M_S$. In addition, $J(r_1)$ is a strictly decreasing function. Moreover, by the Implicit Function Theorem[34], $J(r_1)$ is continuous at each $r_1 \in (0,1]$. Thus it suffices to prove that there exists $r_2 \in (0,1)$ such that

$$J(I(r_2)) = r_2. \tag{C.1.25}$$

[33]The Implicit Function Theorem applies for any $r_2 \in [0,1)$ because by Claim 1 and 2, $\frac{\partial H_1(I(r_2),r_2)}{\partial r_1} > 0$ and $\frac{\partial H_1(I(r_2),r_2)}{\partial r_2} > 0$ for any $r_2 \in [0,1)$.

[34]The Implicit Function Theorem applies for any $r_1 \in (0,1]$ because by Claim 3 and 4, $\frac{\partial H_2(r_1,J(r_1))}{\partial r_1} > 0$ and $\frac{\partial H_2(r_1,J(r_1))}{\partial r_2} > 0$ for any $r_1 \in (0,1]$.

Note that $J(I(r_2))$ is a continuous and strictly increasing function for $r_2 \in [0,1)$. Also note that $J(I(0)) \in (0,1)$ because $I(0) \in (0,1)$ and $J(r_1) \in (0,1)$ when $r_1 \in (0,1)$. Now, by the Intermediate Value Theorem, it suffices to show that there exists some $r_2 \in (0,1)$ such that

$$J(I(r_2)) \leq r_2. \tag{C.1.26}$$

Because J is strictly decreasing, it is equivalent to showing that there exists some $r_2 \in (0,1)$ such that

$$I(r_2) \geq J^{-1}(r_2). \tag{C.1.27}$$

By Claim 1, this is equivalent to showing that there exists some $r_2 \in (0,1)$ such that

$$H_1(J^{-1}(r_2), r_2) \leq M_B. \tag{C.1.28}$$

Let $\epsilon := M_B - M_S > 0$. I observe a relationship between the two functions H_1 and H_2 when $(r_1, r_2) \in (0,1)^2$:

$$H_1(r_1, r_2) - H_2(r_1, r_2) = \left(\frac{(1-r_1)^2}{r_2^2} - 1\right) \cdot H_2(r_1, r_2) + r_1 \cdot (2 - r_1). \tag{C.1.29}$$

Note that when $r_2 \to 1$, $J^{-1}(r_2) \to 0$. To see this, suppose not, then by Claim 4, $H_2(J^{-1}(r_2), r_2) \to 1$ when $r_2 \to 1$, a contradiction to $H_2(J^{-1}(r_2), r_2) = M_S < 1$. Then by (C.1.29), as $r_2 \to 1$,

$$\begin{aligned}
H_1(J^{-1}(r_2), r_2) - M_S &= H_1(J^{-1}(r_2), r_2) - H_2(J^{-1}(r_2), r_2) \\
&= \left(\frac{(1 - J^{-1}(r_2))^2}{(r_2)^2} - 1\right) \cdot H_2(J^{-1}(r_2), r_2) + J^{-1}(r_2) \cdot (2 - J^{-1}(r_2)) \\
&= \left(\frac{(1 - J^{-1}(r_2))^2}{(r_2)^2} - 1\right) \cdot M_S + J^{-1}(r_2) \cdot \big(2 - J^{-1}(r_2)\big) \\
&\to \left(\frac{(1-0)^2}{1^2} - 1\right) \cdot M_S + 0 \cdot (2 - 0) \\
&= 0.
\end{aligned}$$

This implies that there exists some $r_2 \in (0,1)$ such that

$$\left|H_1(J^{-1}(r_2), r_2) - M_S\right| \leq \frac{\epsilon}{2}. \tag{C.1.30}$$

Note that (C.1.30) implies (C.1.28) because $H_1(J^{-1}(r_2), r_2) \leq M_S + \frac{\epsilon}{2} < M_S + \epsilon = M_B$.

Finally, suppose that $r_1 + r_2 = 1$ for the solution (r_1, r_2), then $M_B + M_S = H_1(r_1, r_2) +$

$H_2(r_1, r_2) = 1$ by the definition of $H_1(r_1, r_2)$ and $H_2(r_1, r_2)$, a contradiction to the assumption that $M_B + M_S \neq 1$. Therefore, I have that $r_1 + r_2 \neq 1$ for the solution (r_1, r_2).

C.2 Proof of Theorem 4

Step 1: The generalized random double auction maximizes the expected profit under the asymmetric triangular value distribution. To show this, first note that (ZVV-A) holds by construction. In addition, the virtual value is non-positive for any value profile outside the support and positive for the value profile $(1, 0)$. Then any dominant-strategy mechanism in which 1) ex-post participation constraints are binding for zero-value buyer and one-value seller, and 2) trade does not take place when $r_2 v_B - (1 - r_1) v_S < r_1 r_2$ and trade takes place with probability one when $(v_B, v_S) = (1, 0)$ is an optimal trading mechanism. It is straightforward to see that the generalized random double auction is such a mechanism.
Step 2: The asymmetric triangular value distribution minimizes the expected profit under the generalized random double auction. I use the duality theory to show this. Note that the asymmetric triangular value distribution is a feasible value distribution by construction. By the virtual representation, the expected profit (the value of the primal problem) given the random double auction and the symmetric triangular value distribution is $Pr(1, 0) \times (1 - 0) = r_1(1 - r_2)$. Second, the constraints in the dual problem hold for all value profiles. To see this, note that for any value profile $v = (v_B, v_S)$ inside the support (or $v \in AT$), $\lambda_B^{**} v_B + \lambda_S^{**} v_S + \mu^{**} = t^{**}(v)$ by (CS-A). Also, for any value profile $v = (v_B, v_S)$ in which $r_2 v_B - (1 - r_1) v_S = r_1 r_2$, $\lambda_B^{**} v_B + \lambda_S^{**} v_S + \mu^{**} = 0 = t^{**}(v)$ because $\lambda_B^{**} = \frac{1 - r_1 - r_2}{(1 - r_1) \ln \frac{1 - r_2}{r_1}}$, $\lambda_S^{**} = -\frac{1 - r_1 - r_2}{r_2 \ln \frac{1 - r_2}{r_1}}$ and $\mu^{**} = -\frac{r_1(1 - r_1 - r_2)}{(1 - r_1) \ln \frac{1 - r_2}{r_1}}$. Then, for any value profile $v = (v_B, v_S)$ in which $r_2 v_B - (1 - r_1) v_S < r_1 r_2$, $\lambda_B^{**} v_B + \lambda_S^{**} v_S + \mu^{**} < 0 = t^{**}(v)$ because $\lambda_B^{**} > 0$ and $\lambda_S^{**} < 0$. Finally, the value of the dual problem given the constructed dual variables is equal to $r_1(1 - r_2)$ by simple calculation. The details of the constructed dual variables as well as the characterization are given in Appendix B.

Appendix D Proof of Theorem 5

To facilitate the analysis, I first establish a strong duality result. Given a dominant-strategy mechanism (q, t_B, t_S), the primal minimization problem of adversarial Nature is as follows (with dual variables in the bracket):

$$\inf_{\pi} \int t(v) d\pi(v) \tag{P}$$

subject to

$$\int \max\{v_B - v_S, 0\} d\pi(v) = GFT, \quad (\lambda)$$

$$\int d\pi(v) = 1. \quad (\mu)$$

It has the following dual maximization problem:

$$\sup_{\lambda \in \mathcal{R}, \mu \in \mathcal{R}} \lambda GFT + \mu \tag{D}$$

subject to

$$\lambda \max\{v_B - v_S, 0\} + \mu \leq t(v), \quad \forall v \in V.$$

Note that the value of (P) is bounded by 1 as $t(v) \leq 1$. In addition, the joint distribution that puts all probability masses on the value profile $(\frac{1+GFT}{2}, \frac{1-GFT}{2})$ is in the interior of the primal cone. Then, by Theorem 3.12 in Anderson and Nash (1987), strong duality holds. Theorem 5 is established by the following three steps.

Step 1: Narrow down the search to a class of mechanisms.

I divide all deterministic dominant-strategy mechanisms that satisfy the properties stated in Lemma 5 into the following four classes:

Class 1: The trade boundary touches on the value profiles $(c_1, 1)$ and $(0, c_2)$ for some $0 \leq c_1 \leq 1, 0 \leq c_2 \leq 1$.

Class 2: The trade boundary touches on the value profiles $(0, c_1)$ and $(1, c_2)$ for some $0 \leq c_1 \leq 1, 0 \leq c_2 \leq 1$.

Class 3: The trade boundary touches on the value profiles $(c_1, 0)$ and $(c_2, 1)$ for some $0 \leq c_1 \leq 1, 0 \leq c_2 \leq 1$.

Class 4: The trade boundary touches on the value profiles $(c_1, 0)$ and $(1, c_2)$ for some $0 < c_1 \leq 1, 0 \leq c_2 < 1$[35].

By (iv) of Lemma 5, I can show that the ex-post profit from the value profile $(0, 0)$ or $(0, 1)$ will never be positive for any mechanism in *Class 1*, *Class 2* or *Class 3*. To see this, note that for any mechanism in *Class 1*: $t(0,0) = 0 - c_2 = -c_2 \leq 0$, $t(1,0) = (1-0) \cdot 1 - 1 - 1 = -1 < 0$; for any mechanism in *Class 2*: $t(0,0) = 0 - c_1 = -c_1 \leq 0$, $t(1,0) = (1-0) \cdot 1 - 1 - c_2 = -c_2 \leq 0$; for any mechanism in *Class 3*: $t(0,0) = 0$, $t(1,0) = (1-0) \cdot 1 - (1 - c_1) - 1 = -(1 - c_1) \leq 0$. Consider the joint distribution that puts probability masses GFT and $1 - GFT$ on the value profiles (1,0) and (0,0) respectively. It is straightforward to verify that this is a feasible value distribution; moreover, the profit under this joint distribution cannot be positive. Therefore, I can restrict attention to *Class 4* only.

[35] The cases where $c_1 = 0$ are included in *Class 2*, and the cases where $c_2 = 1$ are included in *Class 3*.

Step 2: Identify an upper bound of the profit guarantee.

I propose a relaxation of (D) by ignoring many constraints. Specifically, the only remaining ones are the constraints for the following four value profiles: (0,0), (1,0), $(c_1, 0)$ and $(1, c_2)$. Formally, I have the following relaxed problem:

$$\max_{\lambda \in \mathcal{R}, \mu \in \mathcal{R}} \lambda GFT + \mu \tag{D'}$$

subject to

$$\mu \leq 0, \tag{D.1.1}$$

$$\lambda + \mu \leq c_1 - c_2. \tag{D.1.2}$$

$$\lambda c_1 + \mu \leq 0, \tag{D.1.3}$$

$$\lambda(1 - c_2) + \mu \leq 0. \tag{D.1.4}$$

Note that the value of (D'), denoted by val(D'), is weakly greater than the value of (D). Now I will find an upper bound of the value of (D') across $0 < c_1 \leq 1$ and $0 \leq c_2 < 1$, and show that it is attainable by constructing deterministic dominant-strategy mechanisms and a feasible value distribution.

If $\lambda \leq 0$, then by (D.1.1), val(D')≤ 0 for any $0 < c_1 \leq 1$ and $0 \leq c_2 < 1$. Henceforth, I restrict attention to $\lambda > 0$. If $c_1 \geq GFT$, then $\lambda GFT + \mu \leq \lambda c_1 + \mu \leq 0$, where the first inequality follows from $\lambda > 0$ and the second inequality follows from (D.1.3). If $c_2 \leq 1 - GFT$, then $\lambda GFT + \mu \leq \lambda(1 - c_2) + \mu \leq 0$, where the first inequality follows from $\lambda > 0$ and the second inequality follows from (D.1.4). If $c_1 \leq c_2$, then $\lambda GFT + \mu \leq \lambda + \mu \leq 0$, where the first inequality follows from $\lambda > 0$ and the second inequality follows from (D.1.2). Therefore, I can restrict attention to $1 - GFT < c_2 < c_1 < GFT$, because otherwise the profit guarantee cannot be positive. This also implies that I can restrict attention to informational environments in which $GFT > \frac{1}{2}$, because otherwise the profit guarantee cannot be positive. Now I am left with (D.1.2), (D.1.3) and (D.1.4) as they imply (D.1.1).

If $c_1 \geq 1 - c_2$, then I am left with (D.1.2) and (D.1.3), as (D.1.4) is not binding. It is straightforward that the solution occurs when both (D.1.2) and (D.1.3) are binding, because $c_1 < GFT < 1$. The solution is $\lambda = \frac{c_1 - c_2}{1 - c_1}$, $\mu = -\frac{c_1(c_1 - c_2)}{1 - c_1}$. val(D')$= \frac{(c_1 - c_2)(GFT - c_1)}{1 - c_1} := K(c_1, c_2)$. Now I maximize $K(c_1, c_2)$ subject to the constraints that $1 - GFT < c_2 < c_1 < GFT$ and $c_1 \geq 1 - c_2$. Observe that $c_2 = 1 - c_1$ in the optimal solution as $K(c_1, c_2)$ is decreasing in c_2. Now, with some algebra,

$$K(c_1, 1 - c_1) = 1 + 2(1 - GFT) - 2(1 - c_1) - \frac{1 - GFT}{1 - c_1}.$$

Then it is straightforward that the optimal solution is $c_1 = 1 - \sqrt{\frac{1-GFT}{2}}$, $c_2 = 1 - c_1 = \sqrt{\frac{1-GFT}{2}}$, and the maximized value of $K(c_1, c_2)$ is $\left(1 - \sqrt{2(1-GFT)}\right)^2$.

If $c_1 \leq 1 - c_2$, then I am left with (D.1.2) and (D.1.4), as (D.1.3) is not binding. It is straightforward that the solution occurs when both (D.1.2) and (D.1.4) are binding, because $1 - c_2 < GFT < 1$. The solution is $\lambda = \frac{c_1 - c_2}{c_2}$, $\mu = -\frac{(1-c_2)(c_1-c_2)}{c_2}$. val(D')$=\frac{(c_1-c_2)(GFT-1+c_2)}{c_2} := L(c_1, c_2)$. Now I maximize $L(c_1, c_2)$ subject to the constraints that $1 - GFT < c_2 < c_1 < GFT$ and $c_1 \leq 1 - c_2$. Observe that $c_1 = 1 - c_2$ in the optimal solution as $L(c_1, c_2)$ is increasing in c_1. Now, with some algebra,

$$L(1-c_2, c_2) = 1 + 2(1-GFT) - 2c_2 - \frac{1-GFT}{c_2}.$$

Then it is straightforward that the optimal solution is $c_2 = \sqrt{\frac{1-GFT}{2}}$, $c_1 = 1 - c_2 = 1 - \sqrt{\frac{1-GFT}{2}}$, and the maximized value of $L(c_1, c_2)$ is $\left(1 - \sqrt{2(1-GFT)}\right)^2$.

Step 3: Show that the upper bound is attainable.

The last step is to construct deterministic trading mechanisms whose profit guarantee is $\left(1 - \sqrt{2(1-GFT)}\right)^2$ when $GFT > \frac{1}{2}$. Consider any deterministic trading mechanism satisfying (i), (ii) and (iii) of Theorem 5. Let $\lambda = \frac{1-\sqrt{2(1-GFT)}}{\sqrt{\frac{1-GFT}{2}}}$ and $\mu = -\frac{(1-\sqrt{2(1-GFT)})(1-\sqrt{\frac{1-GFT}{2}})}{\sqrt{\frac{1-GFT}{2}}}$. I will show that they are feasible for the original dual problem (D).

Note that the constraint for the value profile (1,0) holds with equality by construction. Then the constraint holds for any *interior* value profile[36]. Indeed, the constraint is the most stringent for the value profile (1,0) because the trade boundary is non-decreasing. To see this, note that the constraint for any interior value profile (v_B, v_S) is equivalent to that

$$\lambda \max\{v_B - v_S, 0\} + b_1(v_B) - b_2(v_S) + \mu \leq 0, \tag{D.1.5}$$

where $(v_B, b_1(v_B))$ and $(b_2(v_S), v_S)$ are in the trade boundary. Then the L.H.S. of (D.1.5) is maximized at (1,0) because that $\lambda > 0$ and that b_1 as well as b_2 are non-decreasing. For any *exterior* value profile[37], the constraint also holds if (ii) of Theorem 5 holds. To see this, note that given the constructed λ and μ, $\lambda \max\{v_B - v_S, 0\} + \mu = 0$ for the value profiles $\left(1 - \sqrt{\frac{1-GFT}{2}}, 0\right)$ and $\left(1, \sqrt{\frac{1-GFT}{2}}\right)$. Then, by linearity, $\lambda(v_B - v_S) + \mu = 0$ for any value

[36] A value profile in which trade takes place with probability one is referred to as an interior value profile.

[37] A value profile in which trade does not take place is referred to as an exterior value profile. Note that by the definition of the trade boundary, a value profile on the trade boundary is also an exterior value profile.

profile on the line linking $\left(1-\sqrt{\frac{1-GFT}{2}},0\right)$ and $\left(1,\sqrt{\frac{1-GFT}{2}}\right)$. Therefore, if (*ii*) of Theorem 5 holds, the constraint also holds for any exterior value profile because that $\lambda > 0$ and $\mu < 0$. Finally, the value of (D) under the constructed dual variables is $\left(1-\sqrt{2(1-GFT)}\right)^2$ by simple calculation.

Now consider the joint distribution described in Theorem 5. First, it is straightforward to verify that it is a feasible value distribution. Second, given any trading mechanism satisfying (*i*), (*ii*) and (*iii*) of Theorem 5, the value of (P) is $\left(1-\sqrt{2(1-GFT)}\right)^2$ under the joint distribution by simple algebra. This finishes the proof.

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