The Fiscal Channel of Quantitative Easing*

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VERY PRELIMINARY VERSION

Abstract

Fiscal surpluses/deficits must balance out the losses/gains generated by Quantitative Easing (QE). The general equilibrium effects of QE critically depend on how this fiscal adjustment is made. Following Wallace (1981), it is commonly assumed that only lump-sum taxes are adjusted, making QE irrelevant. We deviate from this premise. When governments also adjust public spending or (distortionary) taxes, QE changes the real allocation of resources. As a result, forward-looking agents adjust their savings-consumption choice, influencing aggregate demand and asset prices. This is the QE's fiscal channel. We show that adjusting spending is optimal in relevant environments, including Wallace's one. Finally, we exploit this channel to show that a targeted QE, such as Green Corporate Bond Programs or the Transmission Policy Instrument, acts as a redistributive and risk-sharing device.

Keywords: QE, Fiscal-Monetary Interactions, Asset Pricing, Optimal Policy.

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1.- Introduction

Central Bank's power to affect asset prices and the broad economy via asset purchases depends on the associated fiscal policy (Wallace (1981), Leeper and Leith (2016), Benigno and Nisticò (2020)). Asset purchases, popularly known as Quantitative Easing (QE), originate gains and losses that enter the consolidated budget constraint of the State and must be offset by movements in other items related to fiscal policy - taxes or spending. From this point of view, asset purchases are always a joint monetary–fiscal operation.[1]

Offsetting QE gains (losses) requires a deficit (surplus). This can be done in many ways, combining tax and spending adjustments. Since Wallace (1981), the literature has focused on a particular way: what Wallace called *unchanged fiscal policy*. The Government would offset QE gains and losses by adjusting only lump-sum taxes affecting precisely the same investors involved in the purchasing program. Spending, deficits, and the distribution of resources would remain the same. Ricardian Equivalence holds, and the Central Bank's operation is irrelevant. In other words, a necessary condition for QE's neutrality is a neutral fiscal reaction.

While this assumption is aimed at analysing the marginal power of the Central Bank, it is somewhat restrictive to grasp the General Equilibrium effects of QE. What if the government uses the extra fiscal space to finance some infrastructure program?[2] What if the government offsets QE losses by raising taxes to non-asset holders? When departing from Wallace's case, QE reallocates real resources (from the private to the public sector, among private agents, among regions, etc.). Rational agents anticipate this reallocation; they adjust their current consumption-savings choices accordingly, affecting aggregate demand and asset prices.[3] This is "the fiscal channel" of QE.

Consider a QE intervention that delivers losses, for instance, due to a sudden increase in reserves' interests. Obeying the intertemporal budget constraint,

[1]This statement assumes full fiscal support. An alternative is the Fed's deferred asset policy: the losses are absorbed against a promise of future gains, but the gains are transferred to the government. This generates an asymmetry we have not exploited. However, if the losses are large enough for long periods, either a fiscal recapitalisation or a loss in controlling interest rates comes. That is why some fiscal support might be needed. We assume that there is always fiscal support, as in the UK.

[2]An example of this behaviour would be the FAST act by the American Congress in 2015.

[3]In this sense, our paper also deviates from Benigno and Nisticò (2020). They allow the Government not to adjust nominal taxes but use an inflation tax. Instead, we will enable the government to implement real adjustments.

the Government must offset them by running a present value surplus. How would it run this surplus? Under Wallace's assumption, the Government would lump-sum tax the private investors that sold the assets to the Central Bank (e.g., pension funds). In this way, although in a different form (a fiscal burden rather than a payoff loss), the loss ends up on the private investors' shoulders. Ultimately, the operation does not affect the expected path of resources accruing to private agents. Prices do not change either. Asset purchases are then irrelevant.

Imagine that, instead, the Government finds it easier to cut spending. If this spending was not really directed towards investors' needs (i.e., it is an imperfect substitute for investors' consumption), they will end up better off; they avoid a loss and are not really affected by the Government's adjustment plan. Anticipating that, rational investors would consume a bit more and save a bit less today. In this case, QE stimulates goods demand and depresses asset prices. All the action is due to a redistribution of resources from the public sector to private investors; private losses are absorbed by cutting unproductive spending.[4]

We formalise these arguments in a stylised two-period Lucas (1978) model extended by a consolidated government that decides on asset purchases and fiscal policy. We restrict ourselves to Central Bank's independence and passive fiscal policy. Thus, given asset purchases financed by short-run risk-free public liabilities, we analyse fiscal policies that adjust to satisfy the intertemporal budget constraint. Fiscal deficits are endogenous to QE, as in Wallace. The innovation lies in exploring different ways of running deficits.

Since there is a continuum of equilibria indexed by each fiscal reaction, a natural question is: what would a rational government do? First, we show that Wallace's *unchanged fiscal policy* is sub-optimal. If lump-sum taxes are available and public spending is unproductive, it is optimal to offset QE gains by lump-sum transfers and QE losses by spending cuts. Thus, the standard irrelevance theorem relies on *non-optimal behaviour*. Moreover, in environments where taxes are distortionary, and spending is productive, the optimal fiscal reaction entails a combination of tax and spending adjustments. For instance, in the event of gains, the government should reduce tax distortions and increase spending, which impacts demand and asset prices as long as agents anticipate it.

Finally, we explore the fiscal channel when there are multiple assets and the

[4]To emphasise QE's redistributive nature, we show that the same allocation achieved by QE with a non-Wallace fiscal reaction can be more easily implemented simply with lump-sum taxes.

Central Bank purchases only one type. The model includes two groups of investors, α and β, each living on an island, consuming only the fruits of their trees but subject to a centralised fiscal system. Let QE consist of buying assets from group α. When the fiscal system is symmetric, QE transfers part of the payoffs and risks of α to β. Consider an expected loss. For agents on island α, QE frees them from part of the loss (i.e., QE implies a transfer of resources). They expect future consumption to decline by less so that today they save a bit less and consume a bit more than otherwise. Consequently, the price of their asset goes down. Investors on island β implicitly subsidise part of the loss on island α. They understand that and, consequently, try to save more to insure against the expected loss in future resources. This represents a boost to the price of their asset. Altogether, the QE program entails asymmetric effects: a transfer of resources to the group/area where the loss occurred; a decrease (increase) in asset prices in the rescued (supporting) area. Thus, QE acts as a risk-sharing device.

The fiscal channel sheds light on two issues: the effects of monetary tightening on a sizeable public balance sheet scenario; the new existing uses of asset purchases by different Central Banks. For instance, both the Bank of England (BoE) and the European Central Bank (ECB) have planned to bias their corporate bond holdings towards "green" firms. The ECB has also announced the Transmission Policy Instrument (TPI), aimed at helping particular countries under some circumstances. These are examples of asset purchases that seek to reallocate real resources.[5]

Related literature. The literature has challenged QE's irrelevance, especially by highlighting the role of financial frictions. For instance, the presence of segmented markets (Vayanos and Vila (2021)); portfolio adjustment costs (Andres et al. (2004) and Harrison (2017)); or collateral constraints (Gertler and Karadi (2011)). Other factors are liquidity-driven propensities to consume (Cui and Sterk (2021)), fiscal crisis Reis (2017), or non-Rational Expectations (Iovino and Sergeyev (2018)). The closest paper to ours is Benigno and Nisticò (2020). They explore a case of active fiscal policy with no tax adjustment to QE losses. In their case, the adjustment goes via the inflation tax. On the contrary, we explore real resource reallocation and the real effects on demand and asset prices it triggers.

[5] These policies deviate from the so-called *market neutrality* approach that has dominated QE so far. We show that these interventions are equivalent to a market-neutral QE with an associated fiscal policy that redistributes the gains towards particular agents (e.g., green firms, pressured countries, etc.).

There are two key dimensions we depart to this recent literature on the interaction between QE interventions and fiscal policy. First, we consider the possibility that the transfer of resources between the government and the private sector is not friction-less. On one side we emphasize the possible distortions imposed by taxation and on the other we do not allow for lump sum transfers and consider the possibility of imperfect substitution between government consumption and private consumption. Second, we have a fundamental reason for taxation and the need of resources by the government: Government expenditures are productive. The government expenditures affects the utility of the household and since it is costly to raise revenues by taxation the flow of resources that QE can potentially generate impact the equilibrium allocation between private and government consumption.

The rest of the paper is structured as follows. Section 3 sets out the model. Section 3 shows an example of non-neutral fiscal reaction. Section 4 analyzes the optimal fiscal reaction in an environment with costly taxation and productive spending. Section 5 introduces heterogeneous private asset-issuers to analyze selective asset purchases programs. Section 6 concludes.

2.- The model

In this section, we present a stylized model that will be used to study the interaction of QE and fiscal policy. The economy is populated by a continuum of measure 1 of identical investors. They last for 2 periods indexed by t = 0, 1, 2. There is a single perishable good in the economy that also act as the numeraire of the economy. There exist two assets: a single risky asset, call it "stock" S, in fixed supply in the form of a contract that delivers D_t goods each period and is marketable at an uncertain price P; a safe asset, "bond" B, that delivers R_t goods with certainty. When the time starts, each investor is endowed with one unit of the stock ($S^i_{-1} = 1$). Payments D_t are exogenous and stochastic. This is the only source of aggregate risk in the economy. Financial markets are competitive but incomplete. A negative amount of stocks is allowed up to some point (specified below). The goods market behaves also competitively. Investors know the stochastic process for risky payments. Besides, homogeneity is common knowledge such that they can use aggregate equilibrium condition to make their optimal choices. Thus, investors hold Rational Expectations.

We consider a State that participates in the economy by determining monetary and fiscal variables. In particular, it is in charge of public spending G_t; costly taxes T_t with an associated tax cost function $H : T \to \mathbb{R}$ with $0 < H'(T) < 1$;[6] risk-free government debt B_t; the short real interest rate R_t; purchases of risky assets Q_t. The economy starts without debt. Thus, the State budget constraints read as:

$$G_0 + QP = T_0 + \frac{B}{R} \tag{1}$$

$$G_1 + B = T_1 + D_1 Q \tag{2}$$

These constraints can be collapsed into this intertemporal constraint

$$\underbrace{Q\left(P - \frac{D_1}{R}\right)}_{\text{QE losses}} = \underbrace{T_0 + \frac{T_1}{R} - G_0 - \frac{G_1}{R}}_{\text{Primary Surplus}} \tag{3}$$

that points out that QE losses must be offset by the present value of the primary surplus. Note this model implies there is fiscal support in the sense of Del Negro and Sims (2015) and fiscal policy is passive since it adjusts surpluses given the actions of the Central Bank.

Private investors have to decide how much to save and which vehicles to use for that end to obtain the maximum possible welfare. We assume welfare depends on current consumption and a convex combination of utility derived from future consumption and public spending, with y denoting the weight attached to the utility derived from consumption. u and v are concave functions, twice differentiable. Their optimisation can be written as

$$\max_{\{C_0^i, C_1^i, S^i, B^i\}} U = \mathbb{E}_0\{u(C_0^i) + \delta[yu(C_1^i) + (1-y)v(G)]\} \tag{4}$$

s.t.

$$C_0^i + PS^i + \frac{B^i}{R} + T_0 + H(T_0) = (P + D_0)S_{-1}^i \tag{5}$$

$$C_1^i + T_1 + H(T_1) = D_1 S^i + B^i \tag{6}$$

$$0 \leq S^i \leq \bar{S}$$

Competitive Equilibrium. Given $S_{-1}^i = 1$, a Competitive Equilibrium is an asset price P, allocations $\{C_0^i, C_1^i, S^i, B^i\}$ and policies $\{G_0, G_1, T_0, T_1, B, R, Q\}$ that

[6]This is a reduced form for distortionary taxes that simplifies some computations. Bohn (1992) shown its equivalence with labor income taxes.

satisfy:

1. Investor's Euler Equations for stocks and bonds

$$P = \mathbb{E}_0 \left[\delta y \frac{u'(C_1^i)}{u'(C_0^i)} D_1 \right] \tag{7}$$

$$\frac{1}{R} = \mathbb{E}_0 \left[\delta y \frac{u'(C_1^i)}{u'(C_0^i)} \right] \tag{8}$$

2. Investor's budget constraints (expression (5) and (6)).

3. The State's intertemporal budget constraint (3).

4. Assets market clearing conditions

$$\int_0^1 S^i di + Q = 1; \quad \int_0^1 B^i di = B \tag{9}$$

There are 12 endogenous variables and 7 optimality conditions. It follows that economic policy needs to target 5 variables out of $\{G_0, G_1, T_0, T_1, B, R, Q\}$. In the next subsection we model an example of QE.

3.- An example of an endogenous fiscal reaction

In this section we formalize the arguments by expanding the model. First, we explore the effects of QE highlighting the fiscal policy conditions that makes it relevant. Then, we study the demand dynamics that drives QE price effects, showing that stock demand inelasticity is key.

We start by defining a specific economic policy. QE is represented as a purchase of $\hat{Q}$ risky private assets by issuing public risk-free bonds. Public bonds are not used for other purposes. The real risk-free rate is left to be determined by the market. In the first period, there is neither spending nor taxes. In period 1, public spending will be a variable $\hat{G}$ to be determined and taxes will balance the budget. Altogether,

$$\{G_0, G_1, T_0, T_1, B, R, Q\} = \{0, \hat{G}, 0, \hat{G} + Q(PR^m - D_1), \hat{Q}PR^m, R^m, \hat{Q}\}$$

To simplify the exposition, assume $H(T) = 0$ and $y = 1$; we relax these assumptions when dealing with optimal policy. Let $u(C) = log(C)$ and $D_1 = aD_0\varepsilon$

with $\varepsilon \sim log\mathcal{N}(1, \sigma^2)$. For this example, consider the following **spending reaction function**: $\hat{G} = (\hat{Q} - g)D_1$. Spending is assumed to be a proportion of aggregate output and the share depends on $g \leq \hat{Q}$.[7]

The share g determines the response of spending (and taxes) to QE gains. Under Wallace's requirements, $g = \hat{Q}$ so that $\hat{G}$ is insensitive to Q (just equal to zero, as before QE). Away from this corner (i.e., $g < \hat{Q}$), $\hat{G}$ reacts to QE. In general, there is a continuum of Competitive Equilibria indexed by g.[8] This continuum of options is a shortcut for the variety of reasons that precludes extreme movements in taxes (e.g., distortionary effects); $\hat{G} = G(Q)$ is microfounded in the next section. In this setup, the following result holds:

Result 1: QE relevance depends on the way QE losses are offset by fiscal policy, which is entirely determined by g. Two cases can be distinguished:

a. QE irrelevance. If costless lump-sum taxes offset QE losses, QE does not affect allocations or prices, that is, if $g = \hat{Q}$ then $X \perp\!\!\!\perp \hat{Q}$, where X stands for $\{C_1, \hat{G}, P, R^m\}$.

b. QE relevance. If QE losses are offset by a combination of costless lump-sum taxes and spending, QE does affect allocations and prices, that is, if $g < Q$ then $X = X(\hat{Q}, \cdot)$. Moreover, if $0 < g < \hat{Q}$, QE produces a general asset price inflation.

The result can be derived as follows. Given the economic policy, market clearing determines the following equilibrium conditions for investors' controls.[9] First, investors experience a portfolio rebalancing: their holdings of the risky assets are reduced in exchange of an equivalent endowment of safe assets: $S^* = 1 - \hat{Q}$ and $B^* = R^m \hat{Q} P$. Then, whereas in period 0 the agent consumes the whole endowment, future equilibrium consumption is affected by QE: $\{C_t^*\}_{t=0}^1 = \{D_0, D_1(1 - \hat{Q} + g)\}$.

[7]For now, we assume perfect foresight on g. See Appendix A for a version with a stochastic g. In this case, QE increases future consumption risk, giving rise to precautionary savings.

[8]We assume this lack of full tax adjustment to QE takes place. However, exactly the same logic follows if instead of a real lack of adjustment, it is just perceived. In this sense, this formalization also encompasses the subjective tax expectations approach of Iovino and Sergeyev (2018) once we allow for subjective expectations.

[9]The superindex i has been dropped for convenience.

To complete the competitive equilibrium, asset prices must be determined. The key element affected by QE is the Stochastic Discount Factor (SDF)

$$\mathbb{E}_0\left[\delta\frac{C_0}{C_1}\right] = \mathbb{E}_0\left[\delta\frac{D_0}{D_1(1-\hat{Q}+g)}\right] = \frac{\delta}{a(1-\hat{Q}+g)} \tag{10}$$

The SDF is increasing in $\hat{Q}$. This is because future consumption is decreasing in $\hat{Q}$ and lower consumption growth is associated with lower discount rates as more weight is attached to the future to ensure smooth consumption. Then, the risk-free rate R^m is given by the inverse of the SDF and the stock price by

$$P = \mathbb{E}_0\left[\delta\frac{D_0}{D_1(1-\hat{Q}+g)}D_1\right] = \frac{aD_0}{R^m} \tag{11}$$

When $g = \hat{Q}$, $\hat{Q}$ has no effect on allocations or prices. The Wallace's irrelevance holds: investors get exactly the same resources, although from a different source (from the government rather than from their assets), and then the stochastic discount factor is unchanged. On the contrary, on any other point satisfying $g < Q$, QE bears consequences: the State increases its liabilities but gets a fraction of the output; investors reduce their exposure to aggregate shocks but loose part of the output; the stock price goes up and the risk-free rates down.

Implementation with lump-sum taxes. The previous distribution of goods between the private and the public sector and asset prices can be replicated by simply using a lump-sum tax $T_1 = \tau D_1$ with $\tau = \hat{Q} - g$. With this tax, goods market clearing becomes $\{C_t^*\}_{t=0}^1 = \{D_0, D_1 - T_1 = D_1(1-\tau) = D_1(1-\hat{Q}+g)\}$. Lump-sum taxes affect the equilibrium stochastic discount factor without affecting asset payoffs, delivering bond and stock prices exactly as (10) and (11). This implementation leaves balance sheets unchanged (no need of public debt issuance, no public ownership of stocks).[10]

3.1.- Behind (ir)relevance

In this section we show the demand dynamics behind the previous proposition. Different demand elasticities to the policy are driving the results. In the case of

[10]If lump-sum taxes are unavailable, the government can seize the same amount of resources via a tax on dividends at a price that does not stimulate asset prices. In this sense, QE would be a way of redistributing capital income without harming capital prices.

full tax pass-through, the private stock demand reacts 1-to-1 to the QE intervention in a way that the reduction in supply due to QE is offset by a proportional reduction in the private demand, leaving prices unchanged. Contrarily, the demand adjustment is smaller when QE net inflows are accommodated via some public spending adjustment. Then, investors wish to protect themselves against lower future consumption by saving a bit more (relative to the case of full tax pass-trough). Hence, the reduction in private demand is not enough to offset QE purchases, so prices go up. Result 2 summarizes this logic.

Result 2: Private consumption and (stock) demand elasticities to a QE intervention depends on the way the QE losses are offset by fiscal policy. There are two scenarios:

a. Stock demand unitary elasticity. When taxes fully absorb QE losses, agents do not change their demand for goods but reduce the stock demand one-to-one with QE, that is, if $g = \hat{Q}$ *then:*

$$\frac{\partial C_t}{\partial Q} = 0; \quad \frac{\partial S_t}{\partial Q} = -1$$

b. Stock demand elasticity lower than 1. When QE losses are accommodated via a combination of tax and spending adjustments, agents adjust their consumption-savings decision, that is, if $g < \hat{Q}$ *then:*

$$\frac{\partial C_0}{\partial \hat{Q}} = -\frac{P + D_0}{1+\delta} < 0; \quad \frac{\partial S}{\partial \hat{Q}} = -\frac{\delta}{1+\delta}\Big(1 + \frac{D_0}{P}\Big) > -1$$

To obtain the result, we derive the demands for goods and stocks. Keeping bond holdings and taxes at their equilibrium value, optimal demands for consumption and risky assets can be derived using investors' optimality conditions. Thus, the demand for C_0 reads as:

$$C_0^d = \frac{D_0 + P}{1+\delta} - \frac{\hat{Q} - g}{1+\delta} \tag{12}$$

This is the familiar expression: optimal consumption as a function of lifetime resources. Plugging it into the budget constraint, we obtain the optimal stock demand:

$$S^d = \frac{1}{P}\left[\frac{\delta}{1+\delta}(D_0 + P)\right] - \frac{(\delta\hat{Q} + g))}{1+\delta} \tag{13}$$

These demands show that the response of agents to a QE intervention depends on the fiscal pass-through. In the particular case of $g = \hat{Q}$, consumption remains unaffected by $\hat{Q}$ and stock demand goes down 1-to-1 with $\hat{Q}$. Away from this corner, agents would opt for a reduction in demand of both consumption and stocks. In other words, when they understand the risk of lower future consumption, they reduce their savings less than in the $g = \hat{Q}$ case to insure against future lower consumption. That triggers a general asset price inflation. Graph 1 illustrates the mechanism. Thus, the fiscal consequences of asset purchases explain the relative inelasticity of asset demands.[11]

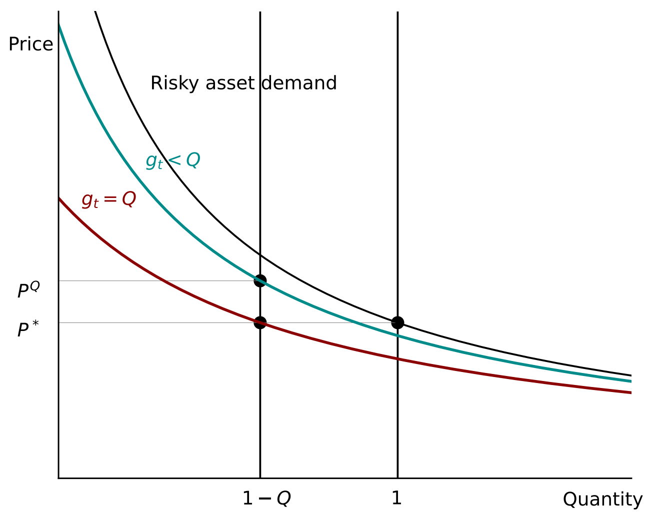

Figure 1: Relative demand inelasticity gives rise to price effects of QE. *Vertical lines plot aggregate supply of the risky asset available to the public. A QE intervention reduce it from 1 to 1-Q. When taxes absorb all QE net inflows ($g_t = Q$, red line), investors reduce their demand to exactly offset QE purchases, leaving prices unchanged. However, when QE is accommodated via a combination of tax and spending adjustments ($g_t < Q$, blue line), agents reduce their stock demand by less (relative inelasticity), giving rise to price effects.*

[11]Relative demand inelasticity also plays a part in other QE analysis; explicitly in Gabaix and Koijen (2021) and implicitly in the segmented market approach (e.g., Vayanos and Vila (2021)) and the portfolio adjustment costs approach (Andres et al. (2004), Harrison (2017))[12]. In this respect, the 'lack of tax adjustment' is comparable to many of the frictions explored in the QE literature.

4.- Optimal Fiscal Reaction

In this section, we examine the optimal response of the fiscal authority to QE. In the model described in Section 2, the government has to choose a combination of taxes and spending to maximize social welfare given the Central Bank's decision about Q. In other words, the fiscal reaction to QE has to be optimally chosen. The government's problem can be written as

$$\max_{\{T,G\}} U = \mathbb{E}_0\Big\{u(C_0) + \delta\big[yu(C_1) + (1-y)v(G)\big]\Big\} \tag{14}$$

subject to private and public budget constraints (3), (5), (6) and given the QE policy $\{Q, B\} = \{\hat{Q}, \hat{Q}PR\}$. *Asset prices are taken as given.*[13] The optimality conditions boils down to

$$y\mathbb{E}_0[u'(D_1 - X - T - H(T))(1 + H'(T))] = (1-y)\mathbb{E}_0[v'(T + X)] \tag{15}$$

with $X = \hat{Q}P(R^s - R)$ being QE gains. The left hand side is the marginal cost of taxes, related to lower current consumption and higher distortions; the right hand side is the marginal benefit of taxes, those that fund additional productive spending. In this setup, the following result holds.

Result 3: The optimal spending reaction to QE depends on the degree of tax distortions and the productivity of public spending. Different cases can be pinned down:

*a. General case. When taxes are costly (i.e., $H'(T) > 0$) and spending is productive (i.e., $y < 1$ and $v'(G) > 0$), the optimal reaction is to adjust **both** spending and taxes, that is,*

$$0 < \frac{dG^*}{dX} < 1; \quad -1 < \frac{dT^*}{dX} < 0 \tag{16}$$

*b. Lump-sum technology. When lump-sum taxes are available (i.e., $H'(T) = 0$) and spending is productive (i.e., $y < 1$ and $v'(G) > 0$), QE must be offset **only** by adjusting taxes*

$$\frac{dG^*}{dX} = 0; \quad \frac{dT^*}{dX} = -1 \tag{17}$$

*c. Unproductive G. When spending is unproductive (i.e., $y = 1$ or $v'(G) = 0$), QE gains must be offset by **transfers** and QE losses by spending **cuts**, that*

[13] This is an information friction: The government does not know the equilibrium pricing function and take prices as beyond its control.

is, if $X > 0$

$$\frac{dG^*}{dX} = 0; \quad \frac{dT^*}{dX} = -1 \tag{18}$$

and if $X < 0$

$$\frac{dG^*}{dX} = 1; \quad \frac{dT^*}{dX} = 0 \tag{19}$$

Corollary. Wallace unchanged fiscal policy is suboptimal.

To derive this result, we proceed by contradiction. First, the Marginal Rate of Substitution between C and G without QE (i.e., $X = 0$) is given by the ratio of weights, that is,

$$\frac{\mathbb{E}_0[u'(D_1 - T - H(T))(1 + H'(T))]}{\mathbb{E}_0[v'(T)]} = \frac{(1-y)}{y} \tag{20}$$

Now, without loss of generality, consider a QE program $X > 0$. Denote $\bar{T}$ the new tax level. If all the adjustment goes through taxes, the government's budget constraint implies $\bar{T} = T - X$. That means a higher consumption in period 1 since $\bar{C}_1 = D_1 - X - (T - X) - H(T - X)$ and $H' > 0$. $\bar{C}_1 > C_1$ implies a lower marginal utility from consumption by the concavity of u. $\bar{T} < T$ such that $(1 + H'(T - X)) < (1 + H'(T))$. It follows that

$$\frac{(1-y)}{y} = \frac{\mathbb{E}_0[u'(D_1 - T - H(T-X))(1 + H'(T-X))]}{\mathbb{E}_0[v'(T)]} < \frac{\mathbb{E}_0[u'(D_1 - T - H(T))(1 + H'(T))]}{\mathbb{E}_0[v'(T)]} = \frac{(1-y)}{y} \tag{21}$$

which is a contradiction. Hence, *a fiscal reaction entailing a solo tax adjustment cannot, in general, be optimal* (i.e.,$\bar{T} = T - X \neq T^*$).

Consider now all the adjustment going through G. Then, $\bar{T} = T$. By the concavity of v, $v'(T + X) < v'(T)$ such that

$$\frac{(1-y)}{y} = \frac{\mathbb{E}_0[u'(D_1 - T - H(T) - X)(1 + H'(T))]}{\mathbb{E}_0[v'(T+X)]} > \frac{\mathbb{E}_0[u'(D_1 - T - H(T))(1 + H'(T))]}{\mathbb{E}_0[v'(T)]} = \frac{(1-y)}{y} \tag{22}$$

which is another contradiction. Then, *no tax adjustment cannot be optimal either* (i.e.,$\bar{T} = T \neq T^*$). Hence, the **optimal tax revenue** T^* must lie somewhere in the middle, that is,

$$-1 < \frac{\partial T^*}{\partial X} < 0 \tag{23}$$

Since the government budget constraints $G = T + X$, then $\frac{\partial G^*}{\partial X} = 1 + \frac{\partial T^*}{\partial X}$ and

$$0 < \frac{\partial G^*}{\partial X} < 1 \tag{24}$$

as stated in the result.

Let's focus on some particular cases. If taxes are lump-sum (i.e., $H'(T) = 0$), $\bar{C_1} = C_1$ such that adjusting only taxes is optimal (i.e., $\bar{T} = T - X = T^*$). If spending is unproductive (i.e., $y = 1$), the government would always want to minimize taxes. It follows that if $X > 0$, it would be optimal to transfer the QE gains (i.e., $\frac{dT^*}{dX} = -1$; $\frac{dG^*}{dX} = 0$), but if $X < 0$, the optimal thing would be to cut spending (i.e., $\frac{dT^*}{dX} = 0$; $\frac{dG^*}{dX} = 1$). *A Wallace economy is therefore characterized by lump-sum taxes and unproductive spending.* In that setup, cutting spending is the optimal reaction to QE losses.

An example with perfect foresight. There is no uncertainty about dividends. Assume $u(\cdot) = v(\cdot) = ln(\cdot)$. The tax cost function is $H(T) = \alpha T$. Then, it can be shown that

$$G^* = aD_1 + bX \tag{25}$$

with $a = \frac{1-y}{1+\alpha} > 0$ and $b = 1 - \frac{1+\alpha y}{1+\alpha} > 0$. Accordingly, $T^* = aD_1 + (b-1)X$, which differs from the Wallace assumption that $T = aD_1 - X$.[14]

Figure 2 plots the optimal and Wallace's reaction functions for taxes and spending separately. When dividends are low, a QE program is likely to generate losses. In this event, Wallace's policy is to increase taxes; optimally, though, there would a combination of lower spending and higher taxes. With high dividends and QE gains, Wallace's policy is to transfer back these gains while optimal policy prescribes a combination of lower taxes and higher spending.

5.- QE with heterogeneity

In this section, we analyse the consequences of a QE targeted at a specific group of investors (e.g., particular regions). We extend the previous setup by including two groups of investors, type α and type β, that live on an island, consuming only the fruits of their trees net of taxes/transfers to a central government. QE consists of buying some trees from, say, island α in exchange for public debt. To repay the debt, the government collect taxes from both islands. Hence, only one group sells assets to the government, but both are impacted by their effects through the centralised fiscal system. Thus, QE entails a redistribution between groups: if QE generates gains (losses), the redistribution is from group α to β (β

[14]Note that if spending is unproductive (i.e., $y = 1$) or tax costless (i.e., $\alpha = 0$), $b = 0$, Wallace assumption turns optimal.

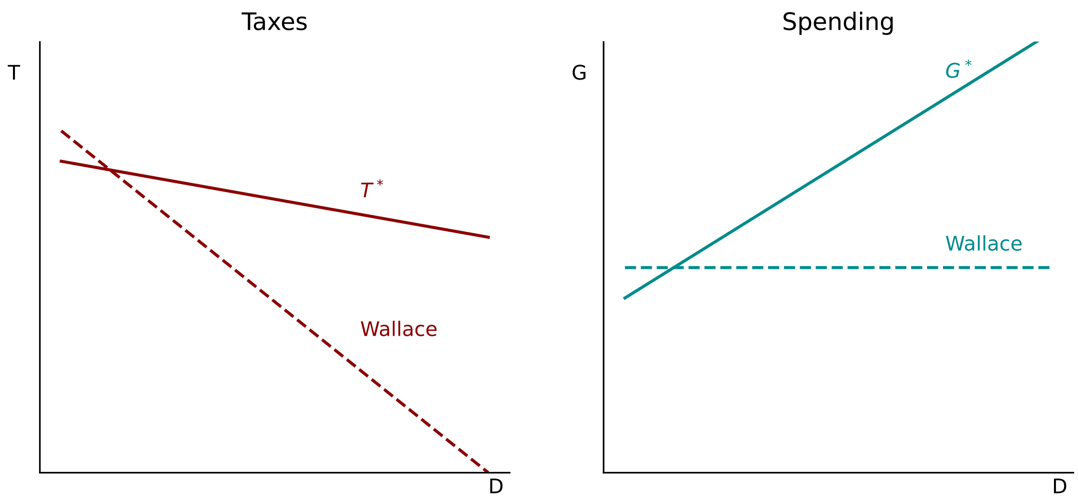

Figure 2: Wallace's vs. Optimal reaction function. *The graph plots taxes (left) and spending (right) as a function of dividends. Dotted lines are Wallace's policies; continuous lines are optimal policies.*

to α). Asset prices would increase (decrease) in island α (β) as a result.[15]

The following example formalizes this point. Consider the economy of section 2 and two groups of agents, α and β. Each group solves an optimization problem exactly as stated in equations (4), (5), and (6). Define the QE policy as the purchase of $\hat{Q}$ of group α's stocks by issuing public debt, that is, $\{Q, B\} = \{\hat{Q}, \hat{Q}P^{\alpha}R^{m}\}$. For simplicity, assume no spending and no tax distortions. Then, the period 1 government's budget constraint reads as

$$B = T^{\alpha} + T^{\beta} + \hat{Q}D_1^{\alpha} \tag{26}$$

Impose a symmetry clause according to which the government has to tax both groups equally (i.e., $T^{\alpha} + T^{\beta} = T$). Hence, in equilibrium, $T = \hat{Q}(P^{\alpha}R^{m} - D_1^{\alpha})/2$. The goods market clearing conditions on each island imply $\{C_0^{\alpha}, C_1^{\alpha}\} = \{D_0^{\alpha}, D_1^{\alpha}(1 - \frac{\hat{Q}}{2}) + \frac{\hat{Q}P^{\alpha}R^{m}}{2}\}$ and $\{C_0^{\beta}, C_1^{\beta}\} = \{D_0^{\beta}, D_1^{\beta} - \frac{\hat{Q}(P^{\alpha}R^{m} - D_1^{\alpha})}{2}\}$. To grasp the effect on asset prices, it suffices to analyze the impact of QE on the equilibrium stochastic discount factor on each island. On island α,

$$\mathbb{E}_0\left[\delta\frac{C_0^{\alpha}}{C_1^{\alpha}}\right] = \mathbb{E}_0\left[\delta\frac{D_0^{\alpha}}{D_1^{\alpha}(1 - \frac{\hat{Q}}{2}) + \frac{\hat{Q}P^{\alpha}R^{m}}{2}}\right] \tag{27}$$

[15]Asset price changes can trigger second-round effects, for instance, through endogenous collateral constraints that counteract the redistributive effects of the 1st round. We leave these effects for future research.

and on island β,

$$\mathbb{E}_0\left[\delta\frac{C_0^\beta}{C_1^\beta}\right] = \mathbb{E}_0\left[\delta\frac{D_0^\beta}{D_1^\beta - \frac{\hat{Q}(P^\alpha R^m - D_1^\alpha)}{2}}\right] \tag{28}$$

It is clear, then, that QE affects the SDF, followed by asset prices on both islands. In what direction? Consider an expected loss (i.e. $\mathbb{E}_0[\hat{Q}(P^\alpha R^m - D_1^\alpha)] > 0$ without loss of generality). For agents on island α, QE frees them from part of the loss (i.e., QE implies a transfer of resources). They expect future consumption to decline by less so that today they save a bit less and consume a bit more today than otherwise. Consequently, the stochastic discount factor goes down and so does the price of their tree P^α. For the same reason, the return on government debt goes up.[16] Investors on island β are implicitly subsidizing part of the loss on island α. They understand that and consequently, try to save more to insure against the expected loss in future resources so that P^β goes up. Altogether, the QE program entails asymmetric effects: a transfer of resources to the group/area where the loss occurred; decrease (increase) in asset prices of the rescued area. **QE acts as a risk-sharing device.**[17]

6.- Multiperiod model

Implications for optimal public debt management. Fiscal propagation of QE.

7.- Empirical validation

To be added

8.- Conclusions

To be added

[16] The effects of QE on bond returns would be the opposite if QE is implemented by buying assets from α with the resources obtained from selling bonds to β.

[17] In this example, the risk of group α is shared with group β. A QE involving purchases of assets from both groups would entail a symmetric risk-sharing.

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