Reallocation dynamics in production networks with heterogeneous elasticities *

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Abstract

This paper investigates the role of heterogeneous elasticities in the US production networks in shaping the transmission of supply shocks. We develop and estimate a multi-industry general equilibrium model featuring heterogeneous elasticities in input demands. Using quarterly data on 15 US industries, we find that US industries exhibit a relatively large dispersion in input substitution elasticities characterized by higher substitution levels than previously found in the literature. At an industry level, this heterogeneity enriches the propagation patterns of idiosyncratic shocks and amplifies their impact on the supplying industry. At a macro level, this dispersion in elasticities generates greater output volatility, in particular during US recessions, entailing a higher welfare cost of fluctuations.

Keywords: sectoral shocks; propagation; substitution; production network; Bayesian inference.

JEL: E23,E30, E32.

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1 Introduction

In normal times, firms operating in a production network respond to changes in input prices or availability by replacing one input with another in their production process. Recent disruptions in the production network following the Covid-19 epidemic and the Ukrainian war has highlighted the vulnerabilities of supply chains when specific inputs become scarcer. The ability of firms to find a supplier and substitute inputs determine the resilience of supply chains, and shapes in turn the downstream propagation of supply shocks along the production network. This paper argues that elasticities of substitution of inputs are heterogeneous across industries, affecting the propagation patterns of idiosyncratic shocks.

Over the last decade, a literature on production networks has emerged to bring back input-output linkages at the centre of research agenda. Seminal papers, such as Acemoglu et al. [2012], Carvalho and Gabaix [2013] or Baqaee and Farhi [2019], have improved our understanding on the origins of fluctuations and the role of production networks in shaping the propagation of idiosyncratic shocks. With respect to this growing literature, two challenges emerge. The first one questions the empirical foundation of reallocation patterns. The essential contribution of this literature has remained normative, such that the empirical quantification of substitution effects is still an open challenge, in particular to disentangle the sectoral components of aggregate fluctuations.[1] A second challenge for this literature concerns a common but arbitrary assumption that imposes a unique elasticity of input substitution across all industries. If this assumption is widespread for tractability purposes,[2] it considerably limits reallocation possibilities across sectors in the wake of idiosyncratic shocks. In particular, this restriction artificially pushes reallocation dynamics towards complementarity, overstating the transmission of idiosyncratic shocks in the production networks.[3]

Given these challenges, the goal of this paper is to empirically measure the relative dispersion of elasticities of substitution in the production network and assess its quantita-

[1] A notable exception is Baqaee and Farhi [2019] who estimate the intakes of substitution mechanisms in shaping shock transmission and welfare costs, but impose a single elasticity of substitution

[2] A second possible explanation for this assumption is the absence of benchmark value at an industry level for this parameter in the literature.

[3] Atalay [2017] gives an intuition for this mechanism : "When inputs are more complementary a (negative) productivity shock to a supplying industry (e.g., Steel) will lead to larger decreases in output for downstream industries (e.g., Motor Vehicles, Construction, etc...). On the other hand, the output decline in the industry experiencing the productivity shock will be smaller when its output is more complementary to the output of other industries". Conversely, when inputs are substitutes, the mechanism works the other way around.

tive importance in shaping the propagation of idiosyncratic shocks. In presence of sizable changes in the propagation, the goal of the paper is also to investigate the industry and aggregate level effects of dispersed elasticities.

Our approach consists in building a disaggregated version of a real business cycle model for the US economy at a 2-digit level (15 sectors). We next infer the model's structural parameters and idiosyncratic shocks through Bayesian techniques using US quarterly data from 1948 to 2020. This estimation exercise provides a data-consistent measure of elasticities of substitution for the 15 sectors concerned, as well as one additional elasticity for households demand in order to assess for all types of reallocation (inputs and final demand). Based on those estimates, the model is amenable to quantitatively measure the relative importance of heterogeneous elasticities of substitution in shaping the propagation of supply shocks through the production network and its aggregate consequence on fluctuations.

With respect to aggregate business cycle models, the analysis on the transmission of supply shocks gives a central role to the production network's granularity in shaping the propagation mechanism. In this paper, we originally dissect in three complementary forces the contribution of the production network on the propagation mechanism of idiosyncratic shocks. The first force is the *downstream channel* driven by prices: a supply shock modifying the relative price of one industry both propagates to its direct and indirect buyers prices downstream in the production chain. The second force at play is the *input reallocation channel* driven by quantities: a change in one industry price leads other industries to reallocate their budget relatively to the other inputs needed in their production process. Dispersed elasticities of substitution critically affect this channel, and possibly change the way shocks propagate. The last channel is the *final consumption reallocation*: CES consumption preferences allows households to shift their demands across industries following changes in industrial prices. One contribution of this paper is to provide a general mathematical framework for the analysis of such forces, and gauge the empirical importance through the structural inference of the model.

We get four main results from our quantitative investigation. First, we find that intermediate input elasticities of substitution are heterogeneous across industries, with 4 out of 15 elasticities above 1 (while the remaining are below unity). This finding strikingly contrasts with the production networks literature that typically imposes an homogeneous and below unity elasticity of substitution across intermediate inputs for all industries.[4] In particular, we find a relatively higher degree of substitution, while the

[4]See for instance, Atalay [2017], Baqaee and Farhi [2019] or Carvalho et al. [2021b]

latter typically averages out when estimated with common elasticity (e.g. Atalay [2017]). Our second result concerns the micro and macro effects of this relatively more intense substitution across inputs in the production network. In this environment, the impact of idiosyncratic shocks is amplified on the supplying industry, bringing more variability in aggregate fluctuations with respect to models using a common CES parameter across sectors. Third, we find that the downstream effect, i.e. the transmission within the network through prices, quantitatively dominates the propagation of idiosyncratic shocks. However, the contribution of reallocation mechanisms to the propagation is also important in explaining the observed transmission of shocks, and offers a positive support to the normative findings of Baqaee and Farhi [2019]. Our last finding sheds lights on the origin of aggregate fluctuations by revealing that its main source is attributed to sectoral shocks rather than aggregate ones.

Our paper contributes to the literature on the origin and transmission of supply shocks through input-output networks.[5] The analysis of input-output linkages became part of the research agenda of the real business cycle theory through the main contributions of Long and Plosser [1983], Horvath [1998], Dupor [1999] and Horvath [2000]. This literature revisites the initial diversification argument of Lucas [1977] through disaggregated real business models. By validating the diversion of Lucas, the role of idiosyncratic shocks and production network was not part of the real business cycle agenda up to the last decade. Following the contributions of Foerster et al. [2011], Acemoglu et al. [2015], Carvalho and Gabaix [2013], Carvalho et al. [2021a], the interest for the role idiosyncratic shocks in driving fluctuation was renewed, highlighting downstream and upstream propagation patterns. Taking dis-aggregated models to the data was still a challenge up to the contribution of Atalay [2017]. The latter evaluates that sectoral shocks accounts for 83% of US aggregate volatility. Our paper contributes to this literature branch by relaxing the assumption of a unique elasticity of substitution in input demands, and empirically measuring its dispersion via Bayesian methods. Our inference relies on 70 years of US sectoral data, and therefore captures salients features of the production network on a long time period.

Our paper also contributes to the literature analyzing the substitution mechanisms operating within a production network. The substitution channel has first been dissected by Carvalho et al. [2021a] through the lens of nested CES functions in the production

[5]Pioneered by Leontief [1942], this seminal contribution provides the first analysis of sectoral disaggregation of US economy, contrasting for the role of intermediate goods demands shape industrial output. Hulten [1978] provides a conceptual framework on aggregate effects of sectoral shocks that has lead to a flourishing literature on industry-specific shock propagation.

technology. In continuation of this research, Baqaee and Farhi [2018], Baqaee and Farhi [2019], Baqaee and Farhi [2020] and Atalay [2017] assess the importance of reallocation dynamics in shaping the transmission of supply shocks. This literature shows that nonlinearities stemming from CES functions generate large GDP fluctuations.Providing data-grounded quantification of the relative strength of reallocation channels since then has remained an open challenge. With respect to this challenge, our paper provides two contributions. First, we provide a closed-form decomposition of the impact of idiosyncratic shocks on industrial outputs, contrasting input substitution versus final consumption channels in order to take into account all possible types of reallocation within the production network. Second, the inference of our model allows to measure consistently with the data the relative strength of these reallocation channels.

Finally, our paper contributes to the literature bridging sectoral models to the data to estimate elasticities of substitution in consumption and input demands. Regarding final consumption, recent estimates from Herrendorf et al. [2013], Atalay [2017] and Oberfield and Raval [2021] suggests the presence of below 1 elasticity, exhibiting complementarity. On the production side (the elasticity of inputs for producers), Atalay [2017] and Boehm et al. [2019] find values slightly above 0 suggesting that inputs are complement rather than substitutes. However, a common feature of these inferences is the common restriction that all industries exhibit the same elasticity degree in their production functions. Our paper offers new insights by letting the data inform how uniform or dispersed the elasticites are within the production network. To our knowledge, this paper is the first attempt to characterize heterogeneous substitution patterns in multi-sectoral models. Grounded by the inference of these elasticities, we next assess the impact of this heterogeneity in driving the propagation of idiosyncratic shocks at an industrial and aggregate levels.

The rest of the paper is organized as follows. In section 2 provides the multi-sectoral model discusses implications of CES functions in input demands. section 3 describe the estimation method and results. section 4 discusses the output multipliers of idiosyncratic shocks and decomposes these multipliers through three channels to highlight the relevance of substitution in shock transmission. section 5 focuses on the impact of substitution heterogeneity on aggregate fluctuations. section 6 investigate the origins of fluctuations by gauging the relative contributions of aggregate and sectoral shocks. Lastly, section 7 provides additional results stemming from nonlinear effects of the model.

2 The Model

Our model is an extension of the model of Carvalho and Tahbaz-Salehi [2019] characterized by a discrete-time economy that features two types of agents: households and firms. Each agent has a unit mass. Households work, buy and consume from all sectors. Firms use labour from households and inputs from other firms, which creates a production networks. As in recent models of production networks (Atalay [2017],Baqaee and Farhi [2019], Carvalho et al. [2021b]...), the inputs are subject to substituability and the technologies are given by constant elasticity of substitution (CES) functions. The utility function of the household is also CES. The original feature of our economy is that we allow for heterogeneity in substitution parameters, with sectors exhibiting different elasticities of substitution. This particularity has important consequences on firms' reactions to shocks.

2.1 Agents

Consumers are embodied by a representative agent who supplies inelastically one unit of work every period. He seeks to maximize his instantaneous expected utility, which is logarithmic in consumption, the latter being a bundle of final goods from N sectors. The consumption of the agent at period $t \geq 0$ is defined by:

$$C_t = D_t^{\frac{1}{\sigma-1}} \left(\sum_j \phi_j^{\frac{1}{\sigma}} C_{jt}^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where the consumption index C_t is a basket of goods from all sectors, C_{jt} is the consumption from the good produced by sector j (considered as final goods) and D_t is a final demand shock which is unitary on average in the spirit of Carvalho et al. [2021b]. This shock is intended to capture the variations in consumption and sectoral outputs that are due to demand effects and not emerging from supply-side distrubances. The expression of the consumption index is given by a CES technology, where $\sigma > 0$ is the elasticity of substitution across sectors and the $\phi_j \in (0, 1)$ are demand parameters reflecting the importance of industries' goods in the consumer's preferences. The agent is subject to one main budget constraint which writes down in real terms:

$$C_t \leq \sum_j L_{jt} w_t \tag{2}$$

where w_t is the hourly wage in real terms (considered to be the same across sectors) and L_{jt} is the number of hours worked for sector j. The agent supplies inelastically one unit of labour each period, such that $\sum_j L_{jt} = 1, \ \forall t$. These assumptions are standard in this class of models (Carvalho and Tahbaz-Salehi [2019],Acemoglu and Azar [2020],Carvalho et al. [2021a]...) which become analytically tractable. Since there is no deposit services in this economy, the budget constraint can be written:

$$C_t = w_t \tag{3}$$

which is also a measure of the Gross Domestic Product (GDP) of the economy. The total sum of weights of each sectoral consumption share is normalized to 1 :

$$\sum_j \phi_j = 1 \tag{4}$$

Let P_{jt} denote the nominal price of good from sector j at time t, the corresponding price level emerges:

$$P_t = \left(\sum_{j=1}^{N} \phi_j P_{jt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{5}$$

Dividing by P_t on both sides and using p_{jt} as the notation for real prices. we have the following normalization condition:

$$\sum_j \phi_j p_{jt}^{1-\sigma} = 1 \tag{6}$$

Each period, the optimal allocation for each type of sectoral goods is determined by the following optimal control problem:

$$\max_{\{C_{jt}\}} C_t = D_t^{\frac{1}{\sigma-1}} \left(\sum_j \phi_j^{\frac{1}{\sigma}} C_{jt}^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{7}$$

$$s.t. \ : \ C_t = \sum_j p_{jt} C_{jt}$$

First order conditions solving for the previous problem leads to the following demand function for each sector j :

$$C_{jt} = D_t p_{jt}^{-\sigma} \phi_j C_t \tag{8}$$

This condition indicates that the relative consumption of good j in the consumption basket is decreasing with its price p_{jt} and increasing with the preference parameter ϕ_j. It's also a function of the demand shock.

2.2 Firms

Each sector is populated by one firm which operates competitively to produce a homogeneous sectoral good. Firms produce their goods by combining other sectoral goods a well as labor demand. The technology for each sector is Cobb-Douglas in labor input and productivity, but CES for intra-sectoral linkages as follows:

$$Y_{jt} = \xi_j Z_t A_{jt} L_{jt}^{\beta_j} M_{jt}^{1-\beta_j} = \xi_j Z_t A_{jt} L_{jt}^{\beta_j} \left(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}} \right)^{\frac{\sigma_j(1-\beta_j)}{\sigma_j - 1}} \tag{9}$$

with Y_{jt} being the output of sector j at time t, L_{jt} the labour force, M_{jt} the input basket composed of the different M_{jit}, the amount of good from sector i used as input. $\sigma_j > 0$ is the elasticity of substitution across inputs for sector j, Z_t the common state of technology, A_{jt} the sectoral state of technology (which are defined below) and ξ_j a normalization constant defined by :

$$\xi_j = \left(\beta_j^{\beta_j}(1-\beta_j\right)^{\frac{(1-\beta_j)\sigma_j}{\sigma_j - 1}})^{-1}$$

This specification imposes the following condition to reach constant return to scale:

$$\sum_{i=1}^{N} \gamma_{ji} + \beta_j = 1 \qquad for \;\; j = 1, 2...N \tag{10}$$

The resource constraint for good j is given by:

$$Y_{jt} = C_{jt} + \sum_{i=1}^{N} M_{ijt} \tag{11}$$

Therefore, each firm is maximizing its profit stream by choosing at each period its output level and the amount of labour and inputs it wants to use. The underlying program of the firm from sector j writes down at each period $t \geq 0$:

$$\max_{Y_{jt},L_{jt},(M_{ijt})_i} p_{jt}Y_{jt} - w_{jt}L_{jt} - \sum_{i=1}^{N} p_{it}M_{jit} \tag{12}$$

$$s.t:\ Y_{jt} = \xi_j Z_t A_{jt} L_{jt}^{\beta_j} (\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

Solving the program for each sector (as shown in the Appendix) yields the following set of equations for all i,$j = 1, 2...N$:

$$L_{jt} = \beta_j(\frac{p_{jt}}{w_t})Y_{jt} \tag{13}$$

$$M_{jit} = (1-\beta_j)\gamma_{ji}(\frac{p_{jt}}{p_{it}^{\sigma_j}})Y_{jt}(\sum_k \gamma_{jk} p_{kt}^{1-\frac{1}{\sigma_j}})^{-1} \tag{14}$$

Equation 13 ensures that the labour demand from sector j is increasing with the labour intensity parameter β_j, with the price of good j and with the production Y_{jt}, but decreasing with the cost of labour ($i.e$: w_t). Equation 14 shows a relation with a similar pattern, with $(1-\beta_j)\gamma_{ji}$ the intensity parameter of input i for j's technology. However, if the use of good i as an import decreases with the price of that input, it also depends on the overall cost of inputs with the last factor. This relation depends crucially on the value of the elasticity σ_j, and more specifically whether $\sigma_j > 1$ or not.

2.3 Shocks

The states of technology and the demand shifter are assumed to follow classical stochastic process. For sector j and for the whole economy, the processes write down respectively for sectoral, aggregate supply and demand shocks:

$$\log(A_{jt}) = \rho_j \log(A_{j(t-1)}) + \epsilon_{jt} \tag{15}$$

$$\log(Z_t) = \rho_Z \log(Z_{t-1}) + \epsilon_{Zt} \tag{16}$$

$$\log(D_t) = \rho_D \log(D_{t-1}) + \epsilon_{Dt} \tag{17}$$

Where ρ_j denotes the persistence of sectoral technology state, ρ_Z denotes the persistence of common technology state, ρ_D denotes the persistence of the demand shocks and ϵ_{jt}, ϵ_{Zt} and ϵ_{Dt} are zero-mean random variable, normally distributed with respective standard error υ_j, Υ_Z and Υ_D :

$$\epsilon_{jt} \overset{i.i.d}{\sim} \mathcal{N}(0, \upsilon_j^2)$$
$$\epsilon_{Zt} \overset{i.i.d}{\sim} \mathcal{N}(0, \Upsilon_Z^2)$$
$$\epsilon_{Dt} \overset{i.i.d}{\sim} \mathcal{N}(0, \Upsilon_D^2)$$

While the study focuses on the mechanisms from the supply-side, we seek to identify all demand fluctuations in a common demand shock to avoid misidentifying and overestimating our supply shocks, hence the demand shifters ϵ_{Dt}.

2.4 CES function mechanism

To illustrate the functioning if a CES function, consider the CES in Equation 1 (but same conclusion apply for other CES in the paper). Parameter σ determines whether factors behave as good substitutes or not. The CES function actually nests different specification based on the value for σ that worths a discussion. When σ is high, factors are are relatively better substitutes, i.e. a loss in one factor is mechanically compensated by an increase in the others. For $\sigma \to +\infty$, the goods become perfect substitutes such that a decrease in one good is compensated by a proportional increase in another one, while the production function is linear. For $\sigma = 1$, the form of the function is Cobb-Douglas. Finally, for $\sigma \to 0$, the technology becomes Leontieff ($f(x, y) = \min(x, y)$) such that there is no substituability at all: an increase in one specific factor is irrelevant as long as the amount of other factors don't change.

More specifically, when $\sigma > 1$. we say that goods are "gross substitutes": a decrease in the price of a factor relatively to others will increase the share of the budget allocated to this good. In contrast, when $\sigma < 1$, we say that goods are "gross complements": there is limited possibilities for substitution and a increase in the price of a factor relatively to others will lead to an increase in the share of the budget allocated to this good. When $\sigma = 1$, the shares of budget allocated are independent of prices.

2.5 Further notations

The set of factor shares γ_{ji} for $i, j = 1, ...N$ are stacked in a $N \times N$ matrix $\mathbf{\Gamma}$ in the same way as in the Leontief [1942] model of sectoral demands, such that for all i, j, $\mathbf{\Gamma}_{ji} = \gamma_{ji}$. This matrix represents the direct requirements of input for each sector. We also define the Leontieff inverse matrix L as follows:

$$\boldsymbol{L} = \sum_{k=0}^{\infty} \mathbf{\Gamma}^k = (\mathbf{I} - \mathbf{\Gamma})^{-1} \tag{18}$$

This matrix $\boldsymbol{L} = (l_{ji})_{j,i}$ is key in this paper and measures the importance of industries as direct and indirect input supplier to other industries. Finally, we introduce the Domar weights of a sector (or an industry). Domar weights are defined as the share of the sales of a sector on total GDP. Thus, they can be written as:

$$\lambda_{jt} = \frac{p_{jt}Y_{jt}}{GDP_t} = \frac{p_{jt}Y_{jt}}{w_t} \tag{19}$$

We also define wage-relative (or more simply, relative) prices:

$$\hat{p_j} = log(\frac{p_j}{w}) \tag{20}$$

These notations will help us to alleviate the formulae of the paper. We also stack all the shocks in a $1 \times (N+2)$ vector $\epsilon = ((\epsilon_j)_j, \epsilon_Z, \epsilon_D)$ and the elasticities in a $1 \times (N+1)$ vector $\boldsymbol{\sigma} = ((\sigma_j)_j, \sigma)$. A summary of the equation system and the corresponding steady-state closed-form solutions are given in Appendix.

3 Estimation

In this section, we estimate the structural parameters of the model using Bayesian methods. In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. We solve the model using a linear approximation to the model's policy function, and employ the Kalman filter to form the likelihood function and compute the sequence of errors. For a presentation of the method, we refer to the canonical papers of An and Schorfheide [2007] and Smets and Wouters [2007]. This method allows to infer the sectoral substitution parameters which are the most likely to have generated the data.

The following sub-sections discuss the data transformation, calibration, the priors and the posteriors.

3.1 Data

The model is estimated with Bayesian methods on U.S. quarterly data over the sample time period 1948Q2 to 2020Q4, using seasonally adjusted gross output data by industries (as given by the Bureau of Economic Analysis) and the Personal Consumption Expenditure (PCEC index as given by the FRED database). For output data, the original data is a panel of 15 industries spanning 74 years. The name of industries are given in Table 1. We next convert this sample into a quarterly basis through the Chow and Lin [1971] methodology (with flow disaggregation and weighted least squares method) to obtain 291 observations per industry.[6]

AGR :	Agriculture, forestry, fishing and hunting
MIN :	Mining
UTI :	Utilities
CON :	Construction
MAN :	Manufacturing
WHO :	Wholesale trade
RET :	Retail trade
TRA :	Transportation and warehousing
INF :	Information
FIN :	Finance, insurance, real estate, rental and leasing
BUS :	Professional and business services
EDU :	Educational services, health care, and social assistance
ART :	Arts. entertainment, recreation, accommodation and food services
OTH :	Other services, except government
GOV :	Government

Table 1: Description of the 15 industries

Concerning the transformation of series, the aim is to map non-stationary data to a stationary model (namely, industry gross output and consumption). Following Smets and Wouters [2007], data exhibiting a trend or unit root are rendered stationary in two steps. We first divide the sample by the working-age population. Second, data are taken in logs and we apply a first-difference filter to obtain growth rates. Real variables are

[6]The high frequency indicator used for the time disaggregation is the BEA's gross domestic product.

deflated by the implicit GDP deflator price index. The measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth of sector AGR} \\ \text{Real Per Capita Output Growth of sector MIN} \\ \dots \\ \text{Real Per Capita Output Growth of sector GOV} \\ \text{Real Per Capita Consumption Growth} \end{bmatrix} = \begin{bmatrix} \Delta \log \left(Y_{\text{AGR},t}\right) \\ \Delta \log \left(Y_{\text{MIN},t}\right) \\ \dots \\ \Delta \log \left(Y_{\text{GOV},t}\right) \\ \Delta \log \left(C_t\right) \end{bmatrix}. \tag{21}$$

3.2 Calibration and prior distributions

The calibrated parameters are reported in two separate tables Table 8 and Table 9. The calibration strategy of the parameters related to factor shares $(\gamma_{ij})_{i,j}$, reported in Table 8, follows Foerster et al. [2011], Atalay [2017] by using Input-Output tables given by the BEA. More precisely, we use the Commodity-by-industry defined as "Total inputs by commodity directly required in order to produce one dollar of industry output". We use the tables between 2001 and 2020 and we compute the mean in order to calibrate our model. Unlike Foerster et al. [2011], Atalay [2017] who base their calibration on one reference period, we compute the factor shares on an average value observed between 2001 and 2020.[7]

We next turn to the parameters for labour intensity $(\beta_j)_j$ in the technology of firms, and final consumption share per type of sectoral goods $(\phi_j)_j$. Our calibration strategy is inspired by Atalay [2017] as labour intensity parameters $(\beta_j)_j$ are computed using data from the BEA between 2000 and 2020, and calculated as the ratio of compensation of labor on total gross output for our 15 NAICS industry classification. For the consumption expenditure share $(\phi_j)_j$, for each industry we compute the ratios by using the sales in 2002 to the following industry codes: F010 (Personal consumption expenditures). F02R (Residential private fixed investment) and F040 (Exports) for industries other than government and F100 (Government consumption expenditures and gross investment) for the government. One can easily verify that our values reported in Table 9 are very similar to the ones in Atalay [2017].

For the remaining set of parameters and shocks that are not calibrated, we employ Bayesian methods. Table 2 summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. Let us first discuss the prior for

[7]Note that this period allows us to get homogeneous data classification since the BEA changes its methodology on a regular basis

structural disturbances. The prior information on the persistence of the Markov processes ρ_j are inspired from Carvalho et al. [2021b] characterized by a Beta distribution of mean 0.7 and standard deviation of 0.10. Note that the standard deviation is slightly lower to reduce the likelihood of having unit-roots process. Regarding sectoral volatilities υ_j, we impose an inverse gamma distribution with prior mean 0.05 and standard deviation 0.1. The aggregate shock is assumed to be less volatile with same distribution form, but with mean 0.02 and volatility 0.1.

Regarding CES elasticities in the input-demand function, one goal of the paper is to assess quantitatively their relative heterogeneity. To let the data be informative about these key parameter, we impose a very diffuse prior distribution with prior mean 0.9 and standard deviation 2. The latter implies a positive support for elasticities, with posterior value that can either be close to 0 up to 10. Note that such an estimation of sectoral elasticities has never been done so far, and constitutes a first step towards assessing heterogeneity in these parameters across industries.

3.3 Posterior distributions

In addition to prior distributions, Table 2 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from five parallel Markov chain Monte Carlo chains of 100,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor, so as to match an average acceptance rate close to 25-30 percent per chain.

The results of the posterior distributions for each estimated parameter are listed in Table 2. We first discuss the elasticity of substitution. The posterior distribution indicates that most of the sectors at the 15-industry level exhibits relatively low elasticities of substitution ($\sigma_j < 1$), meaning that their inputs are gross complements, as discussed in subsection 2.4. This indicates that US production network exhibits limited possibilities for substitution such that a factor price increase (relative to other inputs) mechanically leads to an increase in the share of the budget allocated to this good. The dispersion across these elasticities is high, as the highest estimated elasticity is 14 times bigger than the lowest one, suggesting that heterogeneity in inputs elasticities is strongly motivated. In particular, the Government sector exhibit a high elasticity of substitution, with $\sigma_{GOV} = 2.98$. The Agricultural industry, the Mining industry and the Arts sector exhibit mean elasticities that are above 1 but the value for the agriculture is really close to 1. However, looking at the 90% confidence interval of the AGR and ART sectors, we can't

Parameter		Prior distribution Type	Mean	SD	Posterior distribution Mean	[5%;95%]
Input Elasticities	σ_{AGR}	$\mathcal{IG}$	0.9	2	1.11	[0.76 , 1.45]
	σ_{MIN}	$\mathcal{IG}$	0.9	2	1.60	[1.19 , 1.99]
	σ_{UTI}	$\mathcal{IG}$	0.9	2	0.31	[0.19 , 0.42]
	σ_{CON}	$\mathcal{IG}$	0.9	2	0.29	[0.19 , 0.38]
	σ_{MAN}	$\mathcal{IG}$	0.9	2	0.21	[0.16 , 0.26]
	σ_{WHO}	$\mathcal{IG}$	0.9	2	0.39	[0.22 , 0.56]
	σ_{RET}	$\mathcal{IG}$	0.9	2	0.76	[0.25 , 1.34]
	σ_{TRA}	$\mathcal{IG}$	0.9	2	0.33	[0.20 , 0.46]
	σ_{INF}	$\mathcal{IG}$	0.9	2	0.27	[0.18 , 0.35]
	σ_{FIN}	$\mathcal{IG}$	0.9	2	0.26	[0.19 , 0.32]
	σ_{BUS}	$\mathcal{IG}$	0.9	2	0.26	[0.18 , 0.33]
	σ_{EDU}	$\mathcal{IG}$	0.9	2	0.72	[0.24 , 1.23]
	σ_{ART}	$\mathcal{IG}$	0.9	2	1.73	[0.44 , 2.84]
	σ_{OTH}	$\mathcal{IG}$	0.9	2	0.63	[0.23 , 1.08]
	σ_{GOV}	$\mathcal{IG}$	0.9	2	2.98	[2.00 , 3.93]
Sectoral shock AR	ρ_{AGR}	$\mathcal{B}$	0.75	0.1	0.978	[0.966 , 0.991]
	ρ_{MIN}	$\mathcal{B}$	0.75	0.1	0.980	[0.970 , 0.991]
	ρ_{UTI}	$\mathcal{B}$	0.75	0.1	0.982	[0.973 , 0.992]
	ρ_{CON}	$\mathcal{B}$	0.75	0.1	0.982	[0.973 , 0.992]
	ρ_{MAN}	$\mathcal{B}$	0.75	0.1	0.986	[0.978 , 0.994]
	ρ_{WHO}	$\mathcal{B}$	0.75	0.1	0.973	[0.958 , 0.986]
	ρ_{RET}	$\mathcal{B}$	0.75	0.1	0.986	[0.978 , 0.995]
	ρ_{TRA}	$\mathcal{B}$	0.75	0.1	0.987	[0.980 , 0.994]
	ρ_{INF}	$\mathcal{B}$	0.75	0.1	0.977	[0.965 , 0.989]
	ρ_{FIN}	$\mathcal{B}$	0.75	0.1	0.970	[0.954 , 0.986]
	ρ_{BUS}	$\mathcal{B}$	0.75	0.1	0.969	[0.955 , 0.985]
	ρ_{EDU}	$\mathcal{B}$	0.75	0.1	0.980	[0.969 , 0.991]
	ρ_{ART}	$\mathcal{B}$	0.75	0.1	0.991	[0.986 , 0.996]
	ρ_{OTH}	$\mathcal{B}$	0.75	0.1	0.987	[0.980 , 0.994]
	ρ_{GOV}	$\mathcal{B}$	0.75	0.1	0.979	[0.968 , 0.990]
Sectoral shock std	υ_{AGR}	$\mathcal{IG}$	0.05	0.1	0.0209	[0.0162 , 0.0255]
	υ_{MIN}	$\mathcal{IG}$	0.05	0.1	0.0303	[0.0261 , 0.0343]
	υ_{UTI}	$\mathcal{IG}$	0.05	0.1	0.0159	[0.0141 , 0.0176]
	υ_{CON}	$\mathcal{IG}$	0.05	0.1	0.0113	[0.0102 , 0.0123]
	υ_{MAN}	$\mathcal{IG}$	0.05	0.1	0.0064	[0.0059 , 0.0070]
	υ_{WHO}	$\mathcal{IG}$	0.05	0.1	0.0109	[0.0100 , 0.0118]
	υ_{RET}	$\mathcal{IG}$	0.05	0.1	0.0111	[0.0098 , 0.0124]
	υ_{TRA}	$\mathcal{IG}$	0.05	0.1	0.0103	[0.0091 , 0.0115]
	υ_{INF}	$\mathcal{IG}$	0.05	0.1	0.0063	[0.0055 , 0.0071]
	υ_{FIN}	$\mathcal{IG}$	0.05	0.1	0.0048	[0.0044 , 0.0053]
	υ_{BUS}	$\mathcal{IG}$	0.05	0.1	0.0089	[0.0081 , 0.0096]
	υ_{EDU}	$\mathcal{IG}$	0.05	0.1	0.0139	[0.0123 , 0.0154]
	υ_{ART}	$\mathcal{IG}$	0.05	0.1	0.0117	[0.0105 , 0.0130]
	υ_{OTH}	$\mathcal{IG}$	0.05	0.1	0.0079	[0.0073 , 0.0086]
	υ_{GOV}	$\mathcal{IG}$	0.05	0.1	0.0109	[0.0096 , 0.0121]
Macroeconomic parameters	σ	$\mathcal{IG}$	0.7	1	0.46	[0.40, 0.52]
	ρ_Z	$\mathcal{B}$	0.75	0.05	0.826	[0.762 , 0.889]
	ρ_D	$\mathcal{B}$	0.75	0.05	0.974	[0.966 , 0.982]
	υ_Z	$\mathcal{IG}$	0.02	0.1	0.0018	[0.0016 , 0.0020]
	υ_D	$\mathcal{IG}$	0.02	0.1	0.0038	[0.0035 , 0.0040]

Notes: $\mathcal{B}$ denotes the Beta and $\mathcal{IG}$ the Inverse Gamma (type 1) distribution.

Table 2: Results of posterior estimation for sectoral parameters

reject the possibility that they also have production technology such that the inputs are gross complements. All other sectors elasticities are interestingly low, with 8 out of 15 with their 90% confidence interval upper bound below unity. These results are consistent with the findings of Atalay [2017], highlighting the low substituability of inputs at a low level of disaggregation in the economy.

Regarding the standard errors of sectoral shocks, there is a high heterogeneity across sectors with mean estimates ranging from 0.48% for Financial services up to 3.03% for the Mining industry. This suggests that the contribution of sectoral productivity shocks on aggregate variable is also heterogeneous. The average sectoral standard errors estimates of productivity shocks is 1.2% which is consistent with estimates of quarterly sectoral TFP shocks from the literature (Carvalho et al. [2021b] for a recent estimate). We also notice that the coefficients of persistence are very high, suggesting that shocks last long over time.

Next, we turn to discussing the inference results concerning common parameters reported in the last rows of Table 2. We first discuss the determination of aggregate demand, stemming here from both the CES for consumption as well as for the exogenous demand shock process. There is little substituability between final goods for consumers at the 15-industry level (we can even assess there is gross complementarity since $\sigma < 1$). This low estimate implies that effects through consumption reallocation won't soar. However, they still play a role in mitigating or amplifying the propagation of shocks, this will be further discussed in the next section. Regarding the exogenous demand shock, its persistence as well as its standard deviation is higher than the aggregate TFP shock, consistently with the findings of the literature such as Smets and Wouters [2007].

Regarding the aggregate production shock, its inferred standard deviation is much smaller than its sectoral counterpart. This result is consistent with the idea that smaller entities of an aggregate economy are more subject to idiosyncratic shocks than aggregate ones. However, the relative bigger standard deviation of sectoral shocks with respect to the aggregate shock does not necessarily imply that its relative contribution is also bigger. Indeed, a common shock affects the whole economy and then spreads through the network such as a sectoral one would, but starting from each industry. Thus, even if the magnitude of the shock is smaller, a common shock of 0.5% is similar as if all the sectors productivity were affected by an idiosyncratic shock of 0.5% at the same time! This comparison of aggregate versus sectoral shocks is deeper assessed in a subsequent section.

3.4 Model comparison

A natural question at this stage is whether our specification of heterogeneous CES technology performs better than the standard specification with uniform CES or Cobb-Douglas. To do so, we perform estimation using these standard specifications: in the first three columns of Table 3, we estimate a model with no or partial substitution (some of the elasticities are unitary, $i.e$: not all functions are CES); in the fourth column, we estimate a model with full substitution but with a common value for the input elasticities (which posterior mode is 0.43); in the last column, we estimate our complete model with CES functions and heterogeneous elasticities across sectors. Using an uninformative prior distribution over models ($i.e$: 20% prior probability for each model), Table 3 shows both the posterior odds ratios and model probabilities taking the standard consumption Cobb-Douglas model as the benchmark.

The posterior odds of the full specification model is 7.36e195 to 1, which is the highest value among all the models we tested. In other words, this statistical test leads us to give credit to CES modelling with heterogeneous elasticities more than other types of specification. Notice that heterogeneity in elasticities is key to fit the data as the main model performs considerably better than the model with a common σ_j value across sectors.

Model type	No substitution ($\sigma = 1$, $\sigma_j = 1$)	Partial substitution ($\sigma \neq 1$, $\sigma_j = 1$)	Partial substitution ($\sigma = 1$, $\sigma_j \neq 1$)	Full substitution ($\sigma \neq 1$, $\sigma_j = 0.43$)	Full substitution ($\sigma \neq 1$, $\sigma_j \neq 1$)
Prior probability	0.2	0.2	0.2	0.2	0.2
Log marginal data density	16240.62	16383.95	16622.51	16649.18	16691.27
Bayes ratio	1	3.45e+62	7.95e+165	4.23e+177	7.36e+195
Posterior model probability	0.00	0.00	0.00	0.00	1.00

Table 3: The comparison of prior and posterior model probabilities with different specifications (with parameters taken at their posterior mode).

4 Substitution dynamics in the propagation of shocks

In this section, we use the estimates of the elasticities (which are key parameters in the model) in order to gauge quantitatively the importance of reallocation in shock transmission. To do so, we first disentangle the different channels through which a shock propagate from a sector to another. This decomposition allows to extract the channels stemming from substitution and compare their effect to the overall shock transmission. Therefore, we use the values from the estimation to compute numerically the contribution of each channel in the transmission. Finally, these computations allow us to assess why reallocation matters in the propagation.

4.1 Dissecting the propagation within the production network

This subsection is dissecting the mechanisms shaping the propagation of sectoral supply shocks along the production network. In particular, the goal is to examine the respective role of substituability in inputs and final goods in driving the transmission of sectoral shocks. These issues are addressed with theoretical considerations on first-order effects of the Taylor expansion, while second order effects are studied in a subsequent section.

Before considering the impact of shocks on output, we first need to get a simple expression for the effect of productivity shocks on wage-relative prices such as in Carvalho and Tahbaz-Salehi [2019]. For clarity purpose, we drop the time subscript (which for each variable are all t). The first theorem on relative log prices is the following:

Theorem 1. *The first-order effect of a productivity shock in sector i on the log relative prices of sector j around the steady state is given by :*

$$\frac{\partial log \frac{p_j}{w}}{\partial \epsilon_i}|_{\epsilon=0} = \frac{\partial \hat{p}_j}{\partial \epsilon_i}|_{\epsilon=0} = -l_{ji} \tag{22}$$

This result is of paramount importance for the rest of the study. It implies that the structure of the production network embodied by $\mathbf{\Gamma}$ is enough to assess to first-order impact of shocks on log relative prices. In particular, it allows us to derive the following decomposition of sectoral shocks through the different channels of propagation as in Carvalho and Tahbaz-Salehi [2019] but with additional detail on final consumption:

Theorem 2. *The first-order effect of a productivity shock in sector i on the log output of sector j around the steady state can be decomposed into three complementary channels as follows:*

$$\frac{\partial log Y_j}{\partial \epsilon_i}|_{\epsilon=0} = TOT_{ji} = DE_{ji} + CR_{ji} + IR_{ji}, \tag{23}$$

where each channel is given by:

$$DE_{ji} = l_{ji}$$

$$CR_{ji} = \frac{1}{\lambda_j}(1-\sigma)\sum_k l_{kj}\phi_k(\sum_r \phi_r l_{ri} - l_{ki})$$

$$IR_{ji} = \frac{1}{\lambda_j}\sum_k l_{kj}\sum_r \gamma_{rj}(1-\sigma_r)\lambda_r(\frac{1}{1-\beta_r}\sum_s \gamma_{rs}l_{si} - l_{ki}).$$

In this expression, DE_{ji} denotes the downstream effect of price fluctuations of input i on the production j, CR_{ji} the effect due to consumption reallocation from the consumer, and IR_{ji} is the effect due to input reallocation from the firms.

The main insight of this theorem is to disentangle and characterize the link between two nodes of the production network. Notice that the expression $\frac{\partial logY_j}{\partial \epsilon_i}|_{\epsilon=0} = TOT_{ji}$ is indeed a multiplier around the steady state. When using a first-order Taylor expansion on the output of the sector j as a function of the shock on sector i, we get that the deviation from the steady-state writes down:

$$Y_j(\epsilon_i) - \overline{Y_j} \approx \frac{\partial logY_j}{\partial \epsilon_i}|_{\epsilon=0} \times \epsilon_i = TOT_{ji} \times \epsilon_i \tag{24}$$

where ϵ_i is the magnitude of the shock in sector i and TOT_{ji} amplifies (or dampens) this shock on sector j. For example, if $\epsilon_i = 3\%$ and $TOT_{ji} = 0.5$, it means that the effect of the shock from sector i on sector j has been halved: the deviation from the steady state output of sector j is only 1.5%. Thus, the $(TOT_{ji})_{j,i}$ are multipliers: they embody the intensity of shock transmission. At this stage of the paper, we'll use this denomination for any element TOT, DE, CR or IR. As shown above, three types of complementary channels make up the overall multiplier.

First, the downstream effect DE_{ji} measures the relative importance of industry j as (direct and indirect) input-supplier to industry i as documented in Carvalho et al. [2021b]. Suppose that industry j is hit by a negative shock that reduces its production and hence increases the price of good j. Such a price increase adversely impacts all the industries that rely on good j as an intermediate input for production, thus creating a direct impact on j's customer industries. But this initial impact will in turn result in further propagation over the production network: the prices of goods produced by industries affected in the first round of propagation will rise, creating an indirect negative effect on their own customer industries, and so on. Formally, the overall effect can be decomposed into direct intra-industry, direct inter-industry and indirect inter-industry effects as follows:

$$l_{ji} = \underbrace{\delta_{j=i}}_{\text{Direct effect on i}} + \underbrace{\gamma_{ji}}_{\text{Effect of i as j's supplier}} + \underbrace{\sum_k \gamma_{jk}\gamma_{ki}}_{\text{Effect of i as supplier of j's suppliers}} + \; ... \tag{25}$$

The second and third channel mechanisms are quite similar. The second channel, CR is the channel through which a sector is affected via final consumption reallocation. When a sector is shocked (either positively or negatively), all the prices in the economy will vary, inducing a trade-off for the consumer between all the products. If the preferences are Cobb-Douglas ($\sigma = 1$), there will be no impact on output through reallocation by the consumers because the shares of the budget allocated to all goods are independent of prices. Thus, there will be additional volatility in a sectoral output due to substitution from the consumer only if $\sigma \neq 1$.

To better understand the mechanism, suppose that goods are complement for the consumer ($\sigma < 1$), and consider CR_{ji}. Take a sector k and suppose that $\sum_r \phi_r l_{ri} > l_{ki}$. It means that $-\sum_r \phi_r l_{ri} < -l_{ki}$, $i.e$ the price variation due to the shock $-l_{ki}$ is bigger than the average variation of the price of the basket of good $-\sum_r \phi_r l_{ri}$ because of the pure downstream channel. However, since there is complementarity, good k becomes more expensive relatively to the average basket and the agent will adapt by increasing its budget share for good k. Thus, since j is a supplier of k in the amount of l_{kj} in total, this reallocation will induce an increase in its production proportional to l_{jk} and to the weight of k in the agent's expenditure ϕ_k due to the increase in the budget share allocated to k by the agent. This is exactly what is captured by $l_{kj}\phi_k(\sum_r \phi_r l_{ri} - l_{ki})$ and is then summed on all sectors k. If there was substituability ($\sigma > 1$), the result would be the opposite: when a sector's relative price increase is lower than the average basket due to downstream effect, this gap will be widened by substitution and thus, sector j will have less inputs to supply him for production.

The same processes are underlying the input reallocation channel. The final consumption weights (ϕ_j) are just replaced by input weights (γ_{rj} and $\frac{\gamma_{rs}}{1-\beta_r}$) and we add the Domar weights λ_r to stress the role of a sector in the overall output. One should notice that if the technology functions are specified as Cobb-Douglas (all elasticities are unity), then the first order multipliers around steady state $\frac{\partial log Y_j}{\partial \epsilon_i}|_{\epsilon=0}$ depend only on the structure of the network: the Input-Output table is sufficient to understand the micro shocks from a sector to another one.

It's noticeable that there might be an effect through the consumption channel on a sector even though it has no weight in final demand ($\phi_j = 0$). Indeed, final goods substitution leads to changes in outputs for sectors where j is a supplier too, and as we described before in the paper, it is sufficient to generate co-movement. The same goes for inputs reallocation: as long as one production function isn't Cobb-Douglas, there might be reallocational impacts due to the mechanisms we explained.

4.2 A numerical quantification

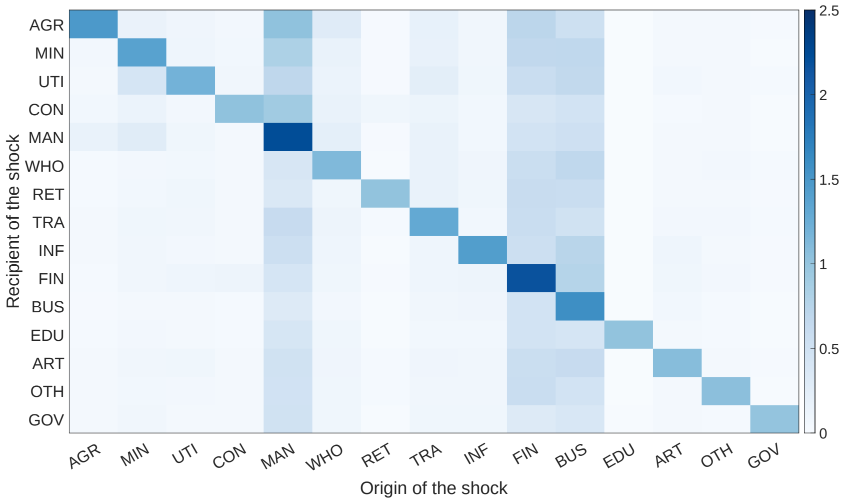
Based on the insights obtained in the previous section, an important question at this stage is quantifying the relative strengths of each force (i.e. downstream, consumption, inputs) in shaping the response of outputs following a realization of a sectoral shock. The estimated model with structural parameters taken at their posterior mean can be used for quantitative purpose to disentangle the relative force discussed in Theorem 2. In this subsection, we decompose the contribution of DE_{ji}, CR_{ji} and IR_{ji} through heat maps for each couple (j, i) of industries in the input-output network. The exact values of the matrices are reported in the Appendix.

The Downstream Effect. First, we quantitatively assess the relative importance of the downstream channel, as the latter plays a major role in the production network. Figure 1 reports the downstream multiplier of a sectoral shock originating from industry in the column, while industry in row reports output response from recipient industries of the shock.[8] The intensity of the downstream multiplier is proportional to the intensity of the blue color.

The first remark on Figure 1 concerns its diagonal elements that are relatively bigger than any other elements both in the same column and row. The relative dominance of the diagonal elements highlights the presence of both column and row wise dominance. Column dominance typically emerges in an economy characterized by a network structure such that for any industry experiencing an idiosyncratic shock, the main recipient of the shock is itself.[9] In contrast, the row dominance emerges in production networks charac-

[8]Note that all the values will be positive since, by definition, $DE_{ji} = l_{ji}$ and the Leontieff inverse matrix is defined as the sum of the powers of the positive matrix $\mathbf{\Gamma}$.

[9]Note that column-wise dominance is specific to the US I-O matrix but isn't true in general for this



Note: For the downstream channel between sectors i, j, the value DE_{ji} indicates the variation of output of sector j when sector i is affected by a sectoral productivity shock of size one. A column i corresponds to the multipliers of shocks from i to other sectors while a line j corresponds to the multipliers of shocks from all sectors to industry j.

Figure 1: Intensity of the downstream multiplier (DE) for the US economy

terized by industrial output that are mostly driven by their own idiosyncratic shock. The fact that diagonal values are bigger than 1 is straightforward since a a specific sector j's output increases relatively more than the size of the initial impulsion.

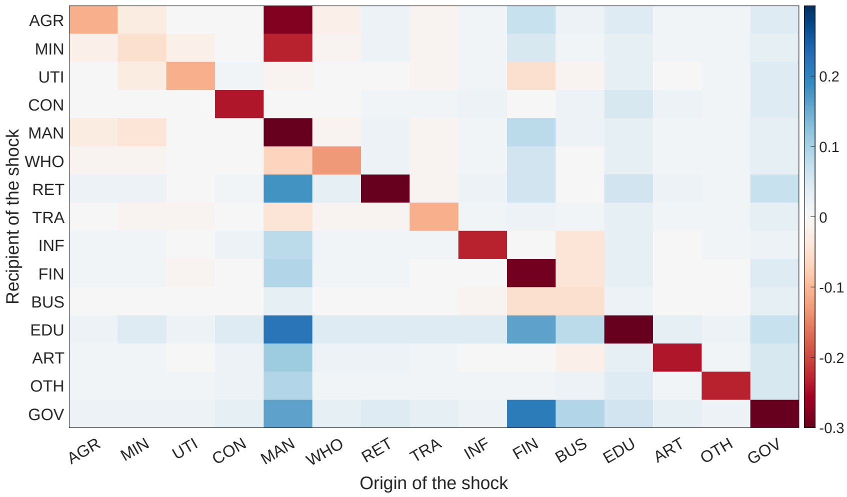
What are idiosyncratic shocks that are the most amplified by the downstream channel? The Manufacturing industry (MAN), the Financial sector (FIN) and the Professional Business and services sector (BUS) are the three sectors exhibiting the highest multiplier: a 1% sectoral productivity shock in one of these sectors yields on average respectively 0.68%, 0.65% and 0.64% of sectoral output (average multiplier of their column). These values strikingly contrast with other sectors, as the next largest average multiplier is 0.22% for Transportation and Warehousing (TRA). The relative importance of these three sectors is explained as they are the largest suppliers within the production network of the US economy.[10] As a result, idiosyncratic shocks in these three sectors are particularly informative, as they trigger large co-movements across sectors via the downstream channel. In contrast, any shock originating from Educational services, health care, and social assistance (EDU) does not translate into sizable fluctuations in other industries,

kind of Leontieff inverse

[10]The relative size of one sector as a supplier is measured by parameter $\gamma_{i,j}$, the latter is calibrated to match the average size within the production network.

as these sectors are negligible suppliers within the production network.

Consumption Reallocation. This demand-side channel typically emerges from the CES preferences of the representative consumer who substitutes varieties when the relative price of goods change following idiosyncratic shocks. Figure 2 reports the consumption substitution channel multipliers to grasp the impact of this mechanism in sectoral shock transmission.



Note: For the consumption reallocation channel between sectors i, j, the value CR_{ji} indicates the variation of output of sector j when sector i is affected by a sectoral productivity shock of size one. A column i corresponds to the multipliers of shocks from i to other sectors while a line j corresponds to the multipliers of shocks from all sectors to industry j.

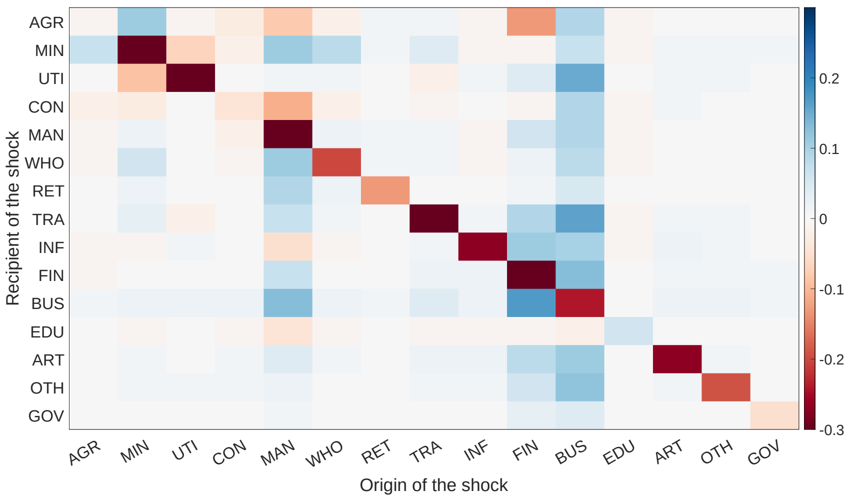
Figure 2: Intensity of the consumption reallocation channel (CR) for the US economy

Despite exhibiting relatively lower values than for the downstream multipliers, the consumption reallocation channel is however quantitatively important. There are two main comments to take on this channel. First, because goods are complement (characterized by estimated substitution elasticity $\sigma < 1$) in the preference of households, a positive sectoral shock doesn't lead to an increase in the consumption spending for that sector specifically, as complementarity forces consumers to increase their demands for other types of sectoral products as well. Complementarity mitigates the propagation of idiosyncratic shocks as diagonal elements are negative, which in turn limits the mechanism of downstream propagation.[11]

[11] To illustrate the relative importance of the consumption reallocation channel, consider the value in Educational, Health and Social services (EDU) sector with $CR_{\text{EDU,EDU}} = -0.47$ (diagonal element in

The second main comment on this channel concerns the heterogeneity in terms of consumption reallocation channels between industries (ie, on non-diagonal values reported in Figure 2). This implies that demand effects can either amplify or mitigate the sectoral response of output. As an example, a shock in the Manufacturing industry (MAN) amplifies the co-movement for some industries via consumption reallocation such as Educational, Health and Social services (EDU) or Retail Trade (RET), but also dampens the response of others such as Mining (MIN) or Agriculture (AGR).

Input Reallocation. The last term that drives the propagation, as discussed in Theorem (2), is the input reallocation channel, that is reported in Figure 3 based on the estimated model. Through this channel, idiosyncratic shocks propagate when sectors substitute their inputs due to price changes.



Note: For the input reallocation channel between sectors i, j, the value IR_{ij} indicates the variation of output of sector j when sector i is affected by a sectoral productivity shock of size one. A column j corresponds to the multipliers of shocks from i to other sectors while a line j corresponds to the multipliers of shocks from all sectors to industry j.

Figure 3: Intensity of the input reallocation channel (IR) for the US economy

On average, the multipliers are larger than for reallocation from consumers in absolute terms, with values ranging from -0.60 (for $IR_{\text{FIN,FIN}}$) to 0.16 (for $IR_{\text{UTI,BUS}}$). Except for the Education, Health and Social services sector (EDU), all the diagonal values are

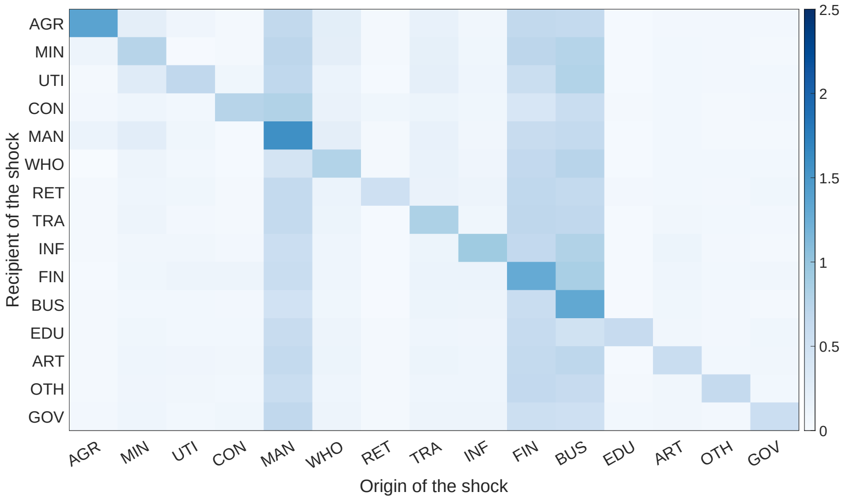
EDU-EDU in Figure 2), a 1% productivity shock in this sector yields to an increase in output by 0.53% (abstracting from any other effects), thus mitigating the sectoral fluctuation through the demand side of the economy.

negative: input reallocation also mitigates one sector's own shock. This result is due to the fact that most elasticities are below 1. Financial services (FIN) and Professional Business and services (BUS) are the two industries whose shocks might affect the economy through input reallocation the most, alongside with Manufacturing (MAN). These results are consistent with the impacts we analysed with the downstream channel.

As underlined by Atalay [2017], two opposite mechanisms occur when inputs are complements. On one side, complementarity generates co-movement with other sectors relying on the shocked industry as a supplier, leading to amplified decreases in output for these industries. On the other side, the shocked industry will suffer lighter output losses due to the limited substitution possibilities from the other sectors. In our case, it seems that the input reallocation channel is overall dampening shock transmission, hinting that the latter effect is prevailing on the other one. This is confirmed with the study of the total multiplier.

The total multiplier. By summing these three complementary channels, namely $DE_{ij} + CR_{ij} + IR_{ij}$, we assess how an idiosyncratic shock generates a sectoral response of output as reported in Figure 4. This multiplier exhibits two interesting features. First, the multipliers are all positive which highlights that an idiosyncratic shock originating from any sector always triggers a positive response of output in other sectors. Such features in the production network actually validates the Lucas [1977]'s assertion that "output movements across broadly defined sectors move together". Therefore, an idiosyncratic shift in productivity spreads along the supply chain always in the same direction. For example, despite being partially buffered by substitution forces, an adverse sectoral shock diminishes the output of all industries. Nevertheless, the impact is more intense for the sector which is primarily affected since the total matrix is diagonal dominant such as for the downstream channel.

This matrix gives interesting insights on shock transmission in the US economy. Firstly, we notice that input-output linkages matter a lot in the propagation of sectoral shocks, even at a low 2-digit level of disaggregation. Without the network structure, this matrix would be the identity matrix. With the network specification, the transmission of shock makes the analysis way more complex. For example, $TOT_{\text{AGR,AGR}}$ and $TOT_{\text{MAN,MAN}}$ are respectively 1.37 and 1.58, suggesting that shocks in these sectors are amplified by 37% and 58% due to the complementary channels considered before. When a sectoral shock hits the Financial sector (FIN), the Business services sector (BUS) will be



Note: For sectors i, j, the value TOT_{ij} indicates the total variation of output of sector j when sector i is affected by a sectoral productivity shock of size one. A column j corresponds to the multipliers of shocks from i to other sectors while a line j corresponds to the multipliers of shocks from all sectors to industry j.

Figure 4: Intensity of the total multiplier (TOT) for the US economy

impacted by a corresponding variation of its output which is almost of the same magnitude as the initial shock ($TOT_{\text{BUS,FIN}} = 0.85$). These propagation patterns exhibit intense deepenings of the idiosyncratic productivity shocks. On the other side, most diagonal values are lower than 1 ($TOT_{\text{MIN,MIN}} = 0.76$, $TOT_{\text{UTI,UTI}} = 0.67$ or $TOT_{\text{GOV,GOV}} = 0.56$), showing that substitution effects can dampen idiosyncratic shocks consequently. We can also assess that the sectors whose shocks propagate the most are the sectors already identified above as the largest suppliers of the network (see the columns of MAN, FIN and BUS in Figure 4).

4.3 How much does substitution matter ?

Does substitution matter ? The decomposition of the first order multipliers we built tends to show that reallocation is of primary importance for shock transmission as substitution mechanisms could constitute a huge part of this radiation through the network. To give a quantitative insight of this role, we compute in Figure 5 the heat-map of multiplier errors from pure downstream values to total values. In other words, we map our grid with the relative difference between the $(DE_{ji})_{j,i}$ and the $(TOT_{ji})_{j,i}$:

$$ERROR_{ji} = \frac{|DE_{ji} - TOT_{ji}|}{TOT_{ji}} \tag{26}$$

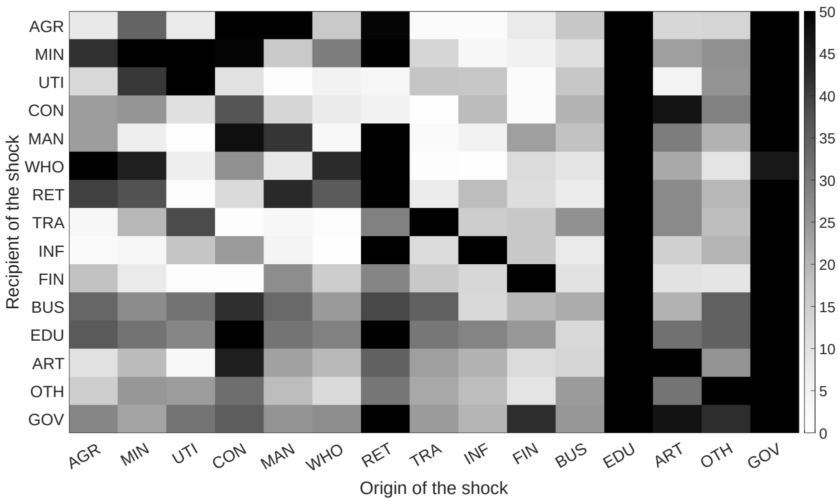


Figure 5: Heatmap of relative difference between Downstream and Total multipliers

Figure 5 shows that most for most couples of sectors (i, j), the pure downstream first-order shock is a poor approximation of the real effect. For 132 couples out of 225 possible (more than one half of the grid), the error is at least 20% and for 48 couples, the error is greater than 50%. For 13 sectors out of 15, the relative error term of their own idiosyncratic shocks (diagonal values) are greater than 30%. Yet, we know from the results above that the diagonal multipliers are among the highest (see Figure 4). This map suggests that reducing first-order multipliers to their pure Downstream counterpart is highly misleading for propagation schemes. In particular, Cobb-Douglas functions constitute a very specific case of production or preferences, which captures poorly the mechanisms of shock transmission. These important intakes of reallocation motivates the study of heterogeneous production technologies with different CES elasticities : if substitution matters, heterogeneity in substitution might matter as well.

5 Heterogeneous elasticities and the business cycles

A unique feature of our model is the heterogeneity in elasticities of substitution within the production network. By estimating industry-specific elasticities, the model allows for a large range of patterns from the input reallocation channel. The literature typically considers a highly restricted version with respect to our case in which all the substitution elasticities are the same across the production network (see recent works from Atalay [2017],Carvalho et al. [2021a], Carvalho et al. [2021b]...). In this section, we investigate how the dispersion in the elasticities estimates affects the sectoral transmission of shocks. More importantly, we show that the restricted models crucially underestimate the volatility of output, especially in times of booms and busts.

5.1 How dispersed are the substitution coefficients in input demands?

At this stage, a natural question would be whether specifying sectoral elasticities is really relevant in shock transmission. As detailed in the subsection 3.4, we ran an additional estimation for a unique elasticity of substitution across industries which yielded an estimate of 0.43 as a common value. However, as highlighted in Table 3, the full-fledged specification (last column) exhibits a Bayes ratio of 1 compared to this restricted model, implying that it performs considerably better in terms of likelihood. This observation gives a first hint at the relevance for heterogeneity of CES elasticities. In Figure 6, we plotted the estimates of the posterior means for each sector, as well as a red dotted line which corresponds to the value when estimating a common elasticity (0.43 as above).

While some values are close to the red line, 4 elasticities out of 15 would be very poorly approximated by a common value of 0.43 (namely for the following sectors: AGR, MIN, ART, GOV). We also notice that 0.43 is outside the 90% confidence interval of estimates from the Retail sector (RET) and the Educational, Health and Social services (EDU). Overall, the estimation results reveal high heterogeneity across sectors with estimates ranging from 0.21 to 2.98.

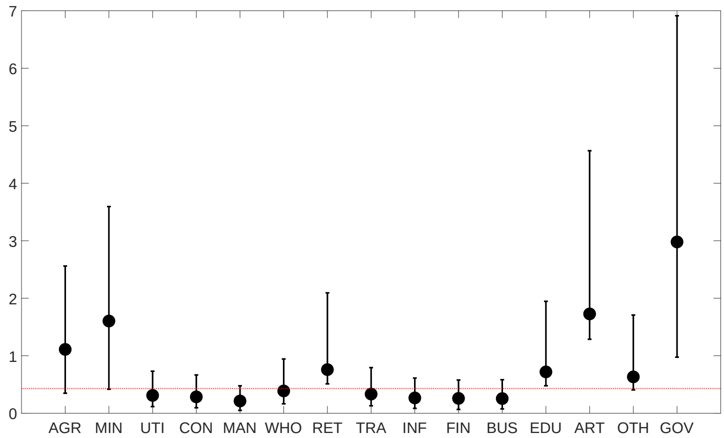


Figure 6: Dispersion of sectoral elasticities (with the 90% confidence interval)

5.2 Implications of elasticity dispersion for shock propagation

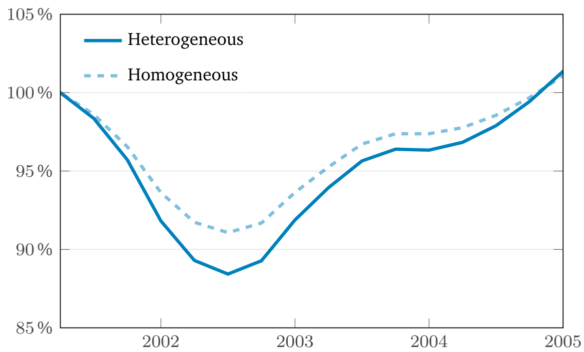
With regards to reallocation dynamics, this heterogeneity affects the intakes of the input reallocation channel. To confirm this hypothesis, we report in Appendix the input reallocation multiplier matrix computed when using the estimates of the homogeneous elasticity of substitution. In other words, we compute $(IR_{ji})_{j,i}$ such as in Figure 3 using $\sigma_j = 0.43$ for all sectors j. We find that this estimation significantly increases the magnitude of the input reallocation channel.

For example, the diagonal values are much higher (in absolute terms) in the homogeneous case than with our full-fledged model. $IR_{\text{AGR,AGR}}$ shifts from -0.01 (which indicates there is almost no shock transmission through this channel) to -0.9 , which indicates a vast dampening of the shock through this channel. Meanwhile, $IR_{\text{BUS,MAN}}$ increased from 0.13 to 0.29. What these results suggest is that relaxing the uniqueness of the elasticity of substitution has major implications on shock transmission. Indeed, we detailed in subsection 4.2 how complementarity was overall mitigating shock propagation in the US specific case. Hence, using a common value of 0.43 forces the reallocation mechanisms towards complementarity and overestimates sectoral shock dampenings.

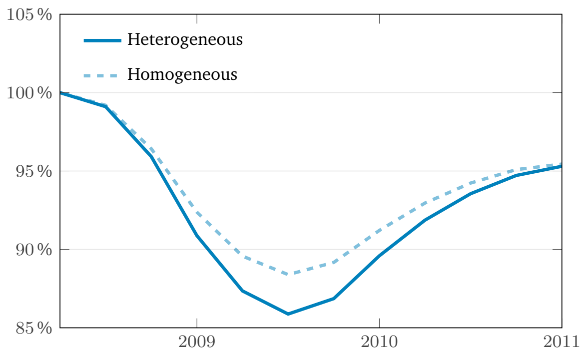
What are the implications of such an overestimation of the input reallocation channel ? As a direct consequence, if this channel intakes are inflated, the volatility of the outputs are underestimated as shocks are over-dampened. Hence, business cycles will be underlooked. The next subsection gives insights on the consequences of this overestimation with counterfactual business cycles analysis.

5.3 A counterfactual analysis of recent crisis (Dotcom, Subprimes, Covid-19...)

To further assess the consequences of homogeneous CES modelling with contrast to our full-fledged model, we compare the aggregate output predictions in both cases during major recessive shocks. In particular, we compare the evolution of aggregate output on a quarterly basis in both specification during three crisis : the Dotcom crisis, the Subprimes crisis and the Covid crisis. To do so, we simulate the evolution of the aggregate output using the sequence of shock identified in our full-fledged model, taking as starting point the level of output before the economic downturn. Graphs for the Dotcom and Subprime crisis are reported in Figure 7 while the graph for the Covid-19 crisis is reported in Appendix.



(a) Dotcom crisis



(b) Subprime crisis

Figure 7: Counterfactual output variations in Homogeneous elasticity and Heterogeneous elasticities model (the homogeneous case corresponds to $\sigma_j = 0.43$)

The intuitions given in subsection 5.2 turn out to be confirmed as the simulated curves for the homogeneous case (dashed line) exhibit significantly smaller decreases of output than for the heterogeneous model. For the Dotcom crisis (Panel (a)), we observe that the aggregate output plummets down to 88.4% of its starting level in 2002Q3 with the unrestricted model while it stays above 91% for the restricted case. Similarly, the output levels during the Subprime crisis (Panel (b)) respectively went down to 85.9% and 88.4%.[12]. As explained above, the model with common elasticity overestimates the

[12]The same observations apply for the Covid-19 crisis in Appendix

impacts of the input reallocation channel which in turn induces a lower volatility of output due to shock dampenings.

6 Aggregate versus Sectoral Shocks

6.1 Aggregate shocks

While we've been focusing on sectoral productivity shocks so far, our specification allows for common productivity shocks to occur as well. These disturbances might be seen as the evolution of the common state of technology of our economy. We can easily obtain an analog theorem on the effects on log relative prices and outputs at first order such as in Theorem (1) and (2) :

Theorem 3. *The first-order effect of a common productivity shock on the log relative prices of sector j around the steady state is given by :*

$$\frac{\partial log\frac{p_j}{w}}{\partial \tilde{\epsilon}}|_{\epsilon=0} = \frac{\partial \hat{p}_j}{\partial \tilde{\epsilon}}|_{\epsilon=0} = -\sum_i l_{ji}$$

Consequently, the first-order effect of a common productivity shock on the log output of sector j around the steady state is given by :

$$\frac{\partial log Y_j}{\partial \tilde{\epsilon}}|_{\epsilon=0} = \sum_i \frac{\partial log Y_j}{\partial \epsilon_i}|_{\epsilon=0} = \sum_i TOT_{ji}$$

As we notice, the common multiplier of a sector is just computed as the sum of the coefficients on the corresponding line in the total multiplier matrix TOT. The fact that all the impacts are just added across all sectors is intuitive since a common shock can be interpreted as the combination of shocks from all industries.[13] Naturally, this means that a common shock impact is always greater than a sectoral one of the same magnitude. The values of the common multipliers are summarized in Table 4:

Sector	AGR	MIN	UTI	CON	MAN	WHO	RET	TRA	INF	FIN	BUS	EDU	ART	OTH	GOV
$\frac{\partial log Y_j}{\partial \tilde{\epsilon}}\|_{\epsilon=0}$	4.49	3.94	3.90	3.48	4.08	3.42	3.48	3.60	3.73	3.78	3.16	3.22	3.49	3.34	3.24

Table 4: Common shock multipliers

[13] However, this result holds only at first-order.

These aggregate disturbances multipliers due to propagation are significant: a 1% shock in common factor productivity will translate into variations from 3.16% of output for Business services (BUS) up to 4.49% for the Agricultural sector (AGR). The standard error of common shocks being around 0.18%, it corresponds to average variations of output from 0.57% to 0.81%.

6.2 The origins of fluctuations

A question that has been largely tackled in the literature concerning sectoral heterogeneity is the decomposition of the aggregate volatility and the understanding of what stems from idiosyncratic variability and what originates from common factors. Three main characteristics shape this decomposition of the aggregate variance : the propagation patterns through the network (which variables are the multipliers), the magnitude of the shocks (embodied by the standard errors of the different shocks) and the persistence of the shocks (stacked in the auto-correlation coefficients ρ). Exploiting these parameters, we can draw a portrait of the contribution of each component in the volatility of the whole production network.

To compare the aggregate and sectoral intakes on fluctuations, we decompose the variance of the sectoral outputs, simulating one shock at a time, that is for a sector k and for a shock $\epsilon_j, j \in \{1, ..., N, Z, D\}$:

$$Var(logY_k|\epsilon_i = 0, \forall i \neq j)$$

Thus, to grasp the role of sectoral shocks on sectoral (or aggregate) fluctuations, we can simply compute the ratio of the sum of the variances stemming from these sectoral shocks on the total sum of the variances, that is to say:

$$\frac{\sum_j Var(logY_k|\epsilon_i = 0, \forall i \neq j)}{\sum_j Var(logY_k|\epsilon_i = 0, \forall i \neq j) + Var(logY_k|\epsilon_i = 0, \forall i \neq Z) + Var(logY_k|\epsilon_i = 0, \forall i \neq D)} \tag{27}$$

The Table 5 summarizes this ratio for each sector. We immediately observe that these results are consistent with the recent literature on the micro origins of aggregate fluctuations. Indeed, on average across sectors, common shocks (supply and demand) are only responsible for 9.3% of the output volatility, with a minimum of 2.73% for the Mining industry (MIN) and a maximum of 13.42% for the Financial services (FIN). We can notice that the industries of goods (AGR, MIN, CON etc.) seem to be more dependent on sectoral shocks than industries of services (BUS, FIN, ART etc.) do. At the aggregate level (Y), notice that common supply shocks are only responsible for 11.68% of the total output volatility, a result which is even smaller than the findings of Atalay [2017] which amout to 17%.

Y	Y_{AGR}	Y_{MIN}	Y_{UTI}	Y_{CON}	Y_{MAN}	Y_{WHO}	Y_{RET}	Y_{TRA}	Y_{INF}	Y_{FIN}	Y_{BUS}	Y_{EDU}	Y_{ART}	Y_{OTH}	Y_{GOV}
88.32	93.72	97.27	93.81	90.05	92.35	87.89	88.72	90.5	86.77	86.58	88.66	89.02	91.66	86.91	87.21

Table 5: Ratio of sectoral volatility due to sectoral shocks

Accordingly, idiosyncratic shocks seem to be the main sources of aggregate fluctuations in the US business cycles since 1948.[14] Taking into account the network structure of the economy leads to complex amplification effects which drive the aggregate fluctuations. The explanation given by the estimates is the following: while the multipliers for aggregate shocks are greater than sectoral ones (see Table 4), the magnitude of these common disturbances are way smaller than idiosyncratic ones. To give a hint on the effect of sectoral shocks on aggregate quantities, Figure 8 plots the fluctuations of US GDP around its trend between 1948 an 2020 in two cases:

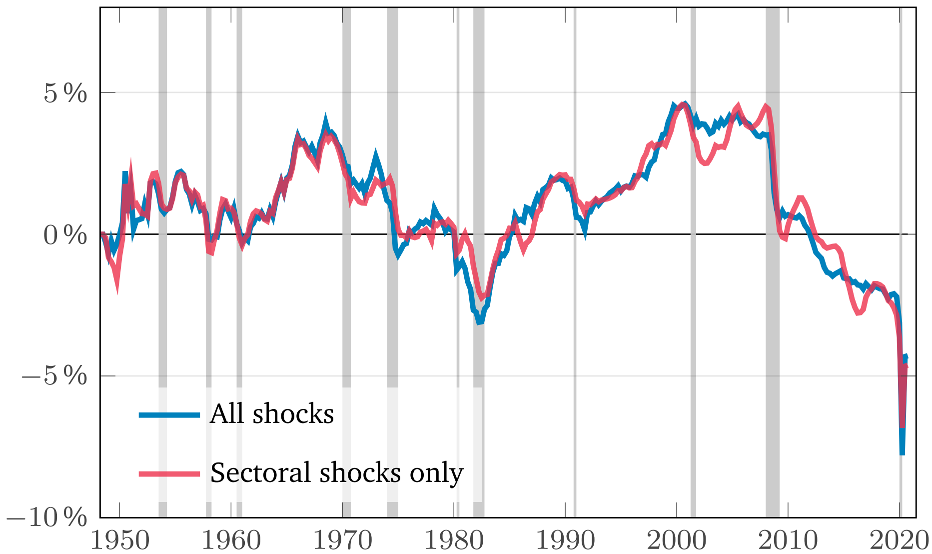


Figure 8: GDP variations around trend between 1948 and 2020 with all/sectoral shocks

[14] These results cast some doubts on the "large law of numbers"-type of argument from Lucas [1977] stating that disaggregating the economy at a granular level is irrelevant since micro shocks should wash out on average.

The blue line represents the observed evolution of GDP in the data (with all shocks taken into account). The red line shows the fluctuations with a simulated economy where the aggregate shocks have been removed. Both curves exhibit very similar patterns as the red line (with sectoral shocks only) does a great job at replicating the observed data, confirming that sectoral shocks are the primary drivers of aggregate volatility.

7 Nonlinear considerations of production networks

This section investigates nonlinear effects of the production network in entailing the welfare cost of fluctuations as well as affecting the propagation of idiosyncratic shocks.

7.1 Welfare cost of sectoral business cycles

A common concern in the business cycles literature is the quantity of consumption that the households would be willing to renounce for, in order to avoid uncertainty for their consumption path. This question goes as far back as Lucas [1987], who defined this quantity as the "Welfare cost of business cycles" (hereafter $\mathcal{W}$c), the difference between the expected utility of consumption and the utility of consumption at the steady state:

$$\mathcal{W} = \mathbb{E}(u(C(\epsilon))) - u(C(\mathbb{E}(\epsilon))) \tag{28}$$

This gap originates from the concave nature of utility and production functions (as function of shocks). Lucas originally found with a baseline model that the cost of fluctuations should be very small (estimates around 0.1% of total consumption). However, Lucas assumptions to get to this result have been largely challenged and new results have shown that the real value might be substantially higher.

Among others, Baqaee and Farhi [2019] have enriched the original computations of Lucas by using non-linear production functions in their model and adding sectoral fluctuations, which is also the case in our specification. As shown by Baqaee and Farhi [2019], adding non-linearities such as substitution possibilities can induce much more consequent values for welfare costs, especially when the degree of non-linearity is increased. We report in Table 6 the computed welfare costs in consumption points (*i.e.* % points of total consumption) for the different specifications such as in Table 3. We compute the derivations using a CRRA utility function[15] and with different coefficient of relative aversion. For each specification, the column on the left ($\mathcal{W}$) displays the welfare cost while the column on the right (Contrib) indicates the contribution of common shocks to the welfare cost.[16]

	No substitution ($\sigma = 1,\, \sigma_j = 1$)		Partial substitution ($\sigma \neq 1,\, \sigma_j = 1$)		Partial substitution ($\sigma = 1,\, \sigma_j \neq 1$)		Full substitution ($\sigma \neq 1,\, \sigma_j = 0.43$)		Full substitution ($\sigma \neq 1,\, \sigma_j \neq 1$)	
	$\mathcal{W}$	Contrib	$\mathcal{W}$	Contrib	$\mathcal{W}$	Contrib	$\mathcal{W}$	Contrib	$\mathcal{W}$	Contrib
$\gamma = 1$	0.002	*(0%)*	0.08	*(0%)*	0.26	*(1%)*	0.21	*(-%)*	0.34	*(0%)*
$\gamma = 1.5$	0.026	*(8%)*	0.11	*(3%)*	0.33	*(1%)*	0.37	*(-%)*	0.41	*(1%)*
$\gamma = 2$	0.05	*(12%)*	0.15	*(4%)*	0.39	*(2%)*	0.53	*(-%)*	0.48	*(2%)*
$\gamma = 4$	0.15	*(12%)*	0.28	*(6%)*	0.66	*(4%)*	1.17	*(-%)*	0.77	*(3%)*

Table 6: Welfare costs of business cycles (in consumption points)

In the full-fledged model, the welfare cost of business cycles neighbours half of a consumption point for realistic calibration of γ, with a value of 0.41% of consumption (for $\gamma = 1.5$). In other words, a household earning 50 000$ annually would be willing to pay 205$ per year to avoid uncertainty in its consumption. These figures seem realistic and broadly in accordance with other estimates in the literature such as in Barlevy [2004] or Baqaee and Farhi [2019]. They are also much higher than the original computations of Lucas. Notice that relaxing Cobb-Douglas assumptions and allowing for substitution increases the welfare costs: it stems from second-order terms which are non-zero when adding non-linearities in the model. For example, the welfare costs are multiplied by 10 when switching from "No substitution" to "Full substitution with heterogeneity" when $\gamma = 2$. This table gives a quantitative assessment of the theoretical contribution of Baqaee and Farhi [2019] and highlights the major implication of substitution mechanisms on households.

Despite increasing with the coefficient of relative risk aversion, the intakes of common disturbances (in Contrib columns) are way smaller than the sectoral shocks, with contributions often below 5%. This observation suggests once again that idiosyncratic shocks are primary drivers of macroeconomic variables fluctuations.

We give here an intuition on the reason why the contribution of common shocks is increasing with γ. Consider the aggregate shocks, for example the aggregate supply shock.

[15] In other words: $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$

[16] The contribution is computed as the ratio of welfare cost when setting all sectoral shocks equal to 0 to the regular welfare cost

The initial impact of the common TFP shock is small compared to sectoral shocks as discussed in section 6. Thus, when the degree of concavity of the utility function is low (small γ), the intakes of the common shocks are low because the biggest shocks drive the variance of the utility. However, when γ increases, u becomes more concave and the variations of utility due to shocks are being squeezed. This gives more relative weight to small shocks as deviations from the mean become small even for large shocks.[17]

7.2 Second-order expansion

So far, this study has only focused on first-order effects, using the first order Taylor expansion. However, the accuracy of this proxy will depend on the size of the shocks ϵ. Indeed, as ϵ increases, the terms of higher order in the Taylor approximation will get bigger and bigger as they are strictly convex. Thus, when the magnitude of the shocks increases too much, the first-order approximation might become gross. To assess the accuracy of the first-order approximation, we can use a slightly more developed Taylor formula (at second order):

$$log(Y_j(\epsilon)) \approx log(Y_j(0)) + \nabla(logY_j)(0)^t * \epsilon + \frac{1}{2}\epsilon^t * D^2(logY_j)(0) * \epsilon \tag{29}$$

In this expression, the additional term $\frac{1}{2}\epsilon^t * D^2(logY_j)(0) * \epsilon$ is likely to have an important weight in the overall variation of log output when ϵ increases. We used simulation to compute the impulse response functions of the sectors' own shock (IRFs) at first-order and second-order. We compare the IRFs by deriving the relative difference between both terms:

$$|\frac{IRF_{1st\ order}(\upsilon_i) - IRF_{2nd\ order}(\upsilon_i)}{IRF_{2nd\ order}(\upsilon_i)}| \tag{30}$$

The values are summarized in Table 7. What emerges from the results is that the 1st order development seems to be an excellent proxy for the size of shocks estimated. Indeed, the relative difference between first order and second order IRFs for "standard error"-wide shocks is, on average, 0.24%. Notice that the biggest error only amounts to 1.15%, for the Utilities sector. What we can conclude from these results is that the first order proxy is (at least) almost as accurate as the second order one. This gives credit to the specification of our study so far.

[17] Notice that for linear utility and consumption, the contribution of each shock would be strictly proportional to the size of the shock

Sector	AGR	MIN	UTI	CON	MAN	WHO	RET	TRA	INF	FIN	BUS	EDU	ART	OTH	GOV
Error	0.30	0.91	1.15	0.33	0.31	0.07	0.07	0.13	0.05	0.15	0.01	0.07	0.00	0.02	0.01

Table 7: Difference between 1st and 2nd order IRFs in relative terms (in %)

8 Conclusion

This paper casts and estimates a general equilibrium model of a multi-industry economy with heterogeneous elasticities of substitution in order to explain and quantify the transmission mechanisms inherent to input-output linkages. Based on the inference of parameters with Bayesian techniques, we show that sectors exhibit highly heterogeneous elasticities of substitution, which contrast with the usual uniformity imposed to these elasticities in the literature. We find that this heterogeneity enriches the propagation patterns of idiosyncratic shocks and amplifies their impact on the supplying industry.

In addition, the paper propose to decompose the role of the production network in three complementary forces through which shocks propagate along the supply chain. Based on our inference exercise, we let the data determine their relative contribution in shaping the transmission of idiosyncratic shocks. We find that reallocation channels (input reallocation and final demand reallocation) account for a large part of shock transmission. This finding stresses out how important are the reallocation dynamics in disaggregated environments, such as theoretically suggested by the literature.

Production networks model are a becoming more and more sophisticated as we better grasp the complexity of input-output linkages. We identify two natural avenues for further research in our understanding of reallocation dynamics. First, we considered in our model that the input-output structure was endogenous. Recent works from Taschereau-Dumouchel [2017] and Acemoglu and Azar [2020] endogenize the choice of supplier, showing that the propagation of sectoral shocks might lead to cascade shutdowns in the production networks. Estimating the elasticity of substitution in a model with endogenous production networks such as theirs could be highly informative in order to understand the interaction between reallocation possibilities and supplier choices. Second, we estimated a model with CES production function and preferences which are typically used in multi-sector model. However, one wants to allow for more complex form such as the non-homothetic CES preferences studied by Comin et al. [2021]. The estimation of such state-of-the-art preferences could be a promising way of understanding transmission mechanisms and multi-sectoral macroeconomics.

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9 Appendix

9.1 Summary of the equations

The equations summarizing our system are thus for all $i, j = 1, 2...N$ and $t \geq 0$:

$$C_t = w_t \tag{31}$$

$$\sum_j \phi_j p_{jt}^{1-\sigma} = 1 \tag{32}$$

$$C_{jt} = D_t p_{jt}^{-\sigma} \phi_j C_t \tag{33}$$

$$L_{jt} = \beta_j (\frac{p_{jt}}{w_t}) Y_{jt} \tag{34}$$

$$M_{jit} = (1 - \beta_j)\gamma_{ji}(\frac{p_{jt}}{p_{it}^{\sigma_j}}) Y_{jt} (\sum_k \gamma_{jk} p_{kt}^{1-\sigma_j})^{-1} \tag{35}$$

$$Y_{jt} = C_{jt} + \sum_{i=1}^{N} M_{ijt} \tag{36}$$

$$Y_{jt} = Z_t A_{jt} \xi_j L_{jt}^{\beta_j} (\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j - 1}} \tag{37}$$

$$\log(A_{jt}) = \rho_j \log(A_{j(t-1)}) + \epsilon_{jt} \tag{38}$$

$$\log(Z_t) = \rho_Z \log(Z_{t-1}) + \epsilon_{Zt} \tag{39}$$

$$\log(D_t) = \rho_D \log(D_{t-1}) + \epsilon_{Dt} \tag{40}$$

9.2 Steady-state

The corresponding steady state system for our set of equations is for all $i, j = 1, 2...N$ (see Appendix for proof):

$$\overline{A_j} = \overline{Z} = \overline{D} = 1 \tag{41}$$

$$\overline{C} = \overline{w} = \overline{p_j} = 1 \tag{42}$$

$$\overline{C_j} = \phi_j \tag{43}$$

$$\overline{Y_j} = \sum_i \phi_i l_{ij} \tag{44}$$

$$\overline{L_j} = \beta_j \overline{Y_j} \tag{45}$$

$$\overline{M_{ji}} = \gamma_{ji} \overline{Y_j} \tag{46}$$

9.3 Proofs

9.3.1 Agents

The FOCs of the consumption basket optimization yields (with η_t the corresponding Lagrange multiplier):

$$(C_{jt}) \ : \qquad -\eta_t p_{jt} + \frac{1}{1-\sigma}((1-\sigma)\phi_j^{\sigma} C_{jt}^{-\sigma}) C_t^{\frac{\sigma}{1-\sigma}} = 0$$

which can be manipulated when putting to power $1-\sigma$:

$$C_t^{-1} \phi_j^{\sigma-1} C_{jt}^{1-\sigma} = \eta_t^{1-\frac{1}{\sigma}} p_{jt}^{1-\frac{1}{\sigma}}$$

Summing on all sectors j and using the aggregate index of C_t we find that:

$$\eta_t = C_t^{-\sigma - \frac{\sigma}{\sigma-1}}$$

Which gives the Equation 8 when reinjecting in the FOC (C_{jt}).

9.3.2 Firms

The Lagrangian of the firm j which wants to maximize its profit writes down (with θ_{jt} the corresponding Lagrange multipliers):

$$\mathrm{L}((\mathrm{Y}_{jt})_t, (L_{jt})_t, (M_{jit})_{i,t}, (\theta_{jt})_t) = p_{jt}Y_{jt} - w_t L_{jt} - \sum_{i=1}^{N} p_{it}M_{jit} - \theta_{jt}(Y_{jt} - Z_t A_{jt}\xi j L_{jt}^{\beta_j}(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}) \tag{47}$$

The FOCS yield the following equations:

$$(Y_{jt}) \ : \qquad p_{jt} = \theta_{jt}$$

$$(L_{jt}) \ : \qquad w_{jt} = \theta_{jt}\beta_j \frac{Y_{jt}}{L_{jt}}$$

$$(M_{jit}) \ : \qquad p_{it} = \theta_{jt}Y_{jt}\frac{\sigma_j}{\sigma_j - 1}(1-\beta_j)(\sum_{k=1}^{N} \gamma_{jk}^{\frac{1}{\sigma_j}} M_{jkt}^{1-\frac{1}{\sigma_j}})\gamma_{ji}^{\frac{1}{\sigma_j}}\frac{\sigma_j - 1}{\sigma_j}M_{jit}^{-\frac{1}{\sigma_j}}$$

$$(\theta_{jt}) \ : \qquad Y_{jt} = Z_t A_{jt}\xi_j L_{jt}^{\beta_j}(\sum_{i=1}^{N} \gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

Plugging the expressions of the Lagrange multipliers θ_{jt} given by the equation (Y_{jt}), we immediately get the equations from subsection 2.2 except the ones for the input demands M_{jit}. Simplifying the condition (M_{jit}) gives:

$$p_{it} = p_{jt}Y_{jt}(1-\beta_j)(\sum_{k=1}^{N} \gamma_{jk}^{\frac{1}{\sigma_j}} M_{jkt}^{1-\frac{1}{\sigma_j}})\gamma_{ji}^{\frac{1}{\sigma_j}} M_{jit}^{-\frac{1}{\sigma_j}} \tag{48}$$

We put this equation to power $1-\sigma_j$ and we sum on all i, giving that:

$$\sum_{k=1}^{N} \gamma_{jk}^{\frac{1}{\sigma_j}} M_{jkt}^{1-\frac{1}{\sigma_j}} = (p_{jt}Y_{jt}(1-\beta_j))^{\frac{1}{\sigma_j}-1}(\sum_k \gamma_{jk}p_{kt}^{1-\sigma_j})^{-\frac{1}{\sigma_j}}$$

When plugging back this expression in Equation 48, we immediately get the expression of input demand.

9.3.3 Determination of the steady states

The normalization conditions make it possible to determine the closed-form solution of the steady states. The steady state of the sectoral and aggregate TFPs and the demand shocks are straightforward, such that :

$$\overline{A_j} = 1, \forall j$$

$$\overline{Z} = \overline{D} = 1$$

Now we want to compute the steady states for the prices and the wage (in real terms). Reinjecting the expressions of M_{ji} and L_j at the steady states given by Equation 34 and Equation 35, we get that:

$$\overline{Y_j} = \xi_j(\beta_j(\frac{\overline{p_j}}{\overline{w}})\overline{Y_j})^{\beta_j}(\sum_{i=1}^{N}\gamma_{ji}^{\frac{1}{\sigma_j}}((1-\beta_j)\gamma_{ji}(\frac{\overline{p_j}}{\overline{p_i}^{\sigma_j}})\overline{Y_j}(\sum_k \gamma_{jk}\overline{p_k}^{1-\sigma_j})^{-1})^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

Using the expression of ξ_j, simplifying the $\overline{Y_j}$ and putting the $\overline{p_j}$ together, we get that this equation reduces to :

$$1 = (\frac{\overline{p_j}}{\overline{w}})^{\beta_j}\overline{p_j}^{1-\beta_j}((\sum_k \gamma_{jk}\overline{p_k}^{1-\sigma_j})^{\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

Finally, we can log-linearize, giving the following equation :

$$\log(\frac{\overline{p_j}}{\overline{w}}) = \frac{1-\beta_j}{(1-\sigma_j)}\log(\frac{1}{1-\beta_j}\sum_k \gamma_{jk}(\frac{\overline{p_k}}{\overline{w}})^{1-\sigma_j})$$

From now on, we can prove that this equation implies for all j:

$$\overline{p_j} = \overline{w}$$

Denote by $\tilde{j}$ the sector such that the real price is maximal across sectors at the steady state and suppose that $\overline{p_{\tilde{j}}} \geq \overline{w}$. We have:

$$\log(\frac{\overline{p_{\tilde{j}}}}{\overline{w}}) = \frac{1-\beta_{\tilde{j}}}{(1-\sigma_{\tilde{j}})}\log(\frac{1}{1-\beta_{\tilde{j}}}\sum_k \gamma_{\tilde{j}k}(\frac{\overline{p_k}}{\overline{w}})^{1-\sigma_{\tilde{j}}})$$

We notice that:

$$\sum_k \gamma_{\tilde{j}k}(\frac{\overline{p_k}}{\overline{w}})^{1-\sigma_{\tilde{j}}} \leq \sum_k \gamma_{\tilde{j}k}(\frac{\overline{p_{\tilde{j}}}}{\overline{w}})^{1-\sigma_{\tilde{j}}} = (\frac{\overline{p_{\tilde{j}}}}{\overline{w}})^{1-\sigma_{\tilde{j}}}\sum_k \gamma_{\tilde{j}k} = (1-\beta_{\tilde{j}})(\frac{\overline{p_{\tilde{j}}}}{\overline{w}})^{1-\sigma_{\tilde{j}}}$$

Thus, for sector $\tilde{j}$, we have:

$$\log(\frac{\overline{p_{\tilde{j}}}}{\overline{w}}) \leq (1-\beta_{\tilde{j}})\log(\frac{\overline{p_{\tilde{j}}}}{\overline{w}})$$

This implies that $\overline{p_{\tilde{j}}} = \overline{w}$ because $(1-\beta_{\tilde{j}}) < 1$ and $\frac{\overline{p_{\tilde{j}}}}{\overline{w}} \geq 1$. Moreover, the inequality is binding if and only if all the prices are equal. Thus, the real prices are all equal to the real wage at equilibrium. If $\overline{p_{\tilde{j}}} < \overline{w}$, then we can use the exact same reasoning with $\tilde{j}$ denoting the smallest real price.

Moreover, we know that:

$$\sum_j \phi_j \overline{p_j}^{1-\sigma} = 1$$

Thus:

$$\overline{w}^{1-\sigma} \sum_j \phi_j = 1$$

$$\overline{w} = 1$$

Furthermore, since $\overline{C} = \overline{w}$, we have :

$$\overline{C} = 1$$

This equation immediately implies that:

$$\overline{C_j} = \phi_j$$

Now we want to get the expression of $(\overline{Y_j})_j$. Remember that we have:

$$\overline{Y_j} = \overline{C_j} + \sum_i \gamma_{ij} \overline{Y_i}$$

which allows us to write in matrix form:

$$(\mathbf{I} - \mathbf{\Gamma})\overline{\mathbf{Y}} = \mathbf{\Phi}$$

$$\overline{\mathbf{Y}} = \boldsymbol{L}\mathbf{\Phi}$$

This gives the closed-form expression for the $(\overline{Y_j})_j$. It's straightforward to get the other equations of the steady states knowing the $(\overline{Y_j})_j$.

9.3.4 Proof of Theorem 1

We begin from the same equation as for the steady state but we use the temporal indices. We suppose that at $t-1$, the economy is at the steady state and thus the variables are defined by the equations in subsection 9.2 :

$$Y_{jt} = \xi_j Z_t A_{jt}(\beta_j(\frac{p_{jt}}{w_t})Y_{jt})^{\beta_j}(\sum_{i=1}^{N}\gamma_{ji}^{\frac{1}{\sigma_j}}((1-\beta_j)\gamma_{ji}(\frac{p_{jt}}{p_{it}^{\sigma_j}})Y_{jt}(\sum_k \gamma_{jk}p_{kt}^{1-\sigma_j})^{-1})^{1-\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

$$1 = Z_t A_{jt}(\frac{p_{jt}}{w_t})^{\beta_j}p_{jt}^{1-\beta_j}((\sum_k \gamma_{jk}p_{kt}^{1-\sigma_j})^{\frac{1}{\sigma_j}})^{\frac{\sigma_j(1-\beta_j)}{\sigma_j-1}}$$

We log-linearize and rearrange the terms to obtain:

$$\log(\frac{p_{jt}}{w_t}) = -\tilde{\epsilon}_t - \epsilon_{jt} + \frac{1-\beta_j}{1-\sigma_j}\log(\frac{1}{1-\beta_j}\sum_k \gamma_{kj}(\frac{p_{kt}}{w_t})^{1-\sigma_j}) \tag{49}$$

Denote $\tilde{j}$ the sector with highest price at time t such as in the proof for the steady state and suppose once again that $\epsilon = 0$ and $p_{\tilde{j}t} \geq w_t$. The equation of normalization of prices ensures that $p_{\tilde{j}} \geq 1$. Thus:

$$\frac{1-\beta_{\tilde{j}}}{1-\sigma_{\tilde{j}}}\log(\frac{1}{1-\beta_{\tilde{j}}}\sum_k \gamma_{\tilde{j}k}(\frac{p_{kt}}{w_t})^{1-\sigma_{\tilde{j}}}) \leq (1-\beta_{\tilde{j}})log(\frac{p_{\tilde{j}t}}{w_t})$$

Remember that the equality only holds if all the prices are all equal across sectors. Plugging this into the log-linearized expression from Equation 49, we get that :

$$\log(\frac{p_{\tilde{j}t}}{w_t}) \leq (1-\beta_{\tilde{j}t})\log(\frac{p_{\tilde{j}t}}{w_t})$$

This can be true if and only if all the prices are equal to the wage in real terms. Once again, if $p_{\tilde{j}t} < w_t$, we use the same reasoning with $\tilde{j}$ being the smallest price. Thus, we get that when the productivity shocks are all zero at time t, prices are all equal to the wage and the normalization condition imposes that for all j:

$$p_{jt} = w_t = 1$$

Now we differentiate the Equation 49 and we evaluate it for $\epsilon = 0$. This leads us to the equality:

$$\frac{\partial log\frac{p_j}{w}}{\partial \epsilon_{it}}|_{\epsilon=0} = -\mathbb{1}_{j=i} + \sum_k \gamma_{jk} \frac{\partial log\frac{p_k}{w}}{\partial \epsilon_{it}}|_{\epsilon=0}$$

Using the notation $\hat{p_{jt}} = log(\frac{p_{jt}}{w_t})$ and $\hat{\boldsymbol{p}}$ the associated vector, we get that:

$$(\mathbf{I} - \mathbf{\Gamma})\frac{\partial \hat{\boldsymbol{p}}}{\partial \epsilon_{it}}|_{\epsilon=0} = -\boldsymbol{e_i}$$

$$\frac{\partial \hat{\boldsymbol{p}}}{\partial \epsilon_{it}}|_{\epsilon=0} = -\boldsymbol{Le_i}$$

which ends the proof of the first theorem.

9.3.5 Proof of Theorem 2

We begin from the resource constraint which implies:

$$\lambda_{jt} = (C_{jt} + \sum_{k=1}^{N} M_{kjt})\frac{p_{jt}}{w_t}$$

$$\frac{\partial \lambda_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = (\frac{\partial C_{jt}\frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} + \sum_{k=1}^{N} \frac{\partial M_{kjt}\frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0})$$

We need to compute a few terms before being able to obtain the expression of the proposition.

Consumption. Firstly, we determine the wage multiplier around the steady state $\frac{\partial w_t}{\partial \epsilon_i}|_{\epsilon=0}$. We divide the normalization Equation 6 by $w_t^{1-\sigma}$ and we differentiate around the steady-state giving the equation :

$$\sum_k \phi_k (1-\sigma)\frac{\partial \frac{p_{kt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} = (\sigma - 1)\frac{\partial w_t}{\partial \epsilon_{it}}|_{\epsilon=0}$$

which immediately gives:

$$\frac{\partial w_t}{\partial \epsilon_{it}}|_{\epsilon=0} = \sum_k \phi_k l_{ki} = \lambda_i$$

Note that we come back to the theorem from Hulten [1978]. Then, we use the fact that $C_{jt}\frac{p_{jt}}{w_t} = \phi_j p_{jt}^{1-\sigma}$, giving:

$$\frac{\partial C_{jt}\frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} = \phi_j(1-\sigma)\frac{\partial p_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0}$$

Since we have :

$$\frac{\partial p_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = \frac{\partial \frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} + \frac{\partial w_t}{\partial \epsilon_{it}}|_{\epsilon=0},$$

we get that:

$$\frac{\partial p_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = -l_{ij} + \sum_k \phi_k l_{ik},$$

and thus:

$$\frac{\partial C_{jt}\frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} = \phi_j(1-\sigma)(\sum_k \phi_k l_{ik} - l_{ij}).$$

Inputs. Define ω_{kjt}, such that :

$$\omega_{kjt} = \frac{p_{jt}M_{kjt}}{p_{kt}Y_{kt}}$$

It follows that :

$$\frac{\partial M_{kjt}\frac{p_{jt}}{w_t}}{\partial \epsilon_{it}}|_{\epsilon=0} = \frac{\partial \omega_{kjt}\lambda_{kt}}{\partial \epsilon_{it}}|_{\epsilon=0}$$

We use the exact same proof as in Carvalho and Tahbaz-Salehi [2019] to show that:

$$\sum_k \frac{\partial \omega_{kjt}\lambda_{kt}}{\partial \epsilon_{it}}|_{\epsilon=0} = \sum_k \gamma_{kj}\frac{\partial \lambda_{kt}}{\partial \epsilon_{it}}|_{\epsilon=0} + \sum_k \gamma_{kj}(\sigma_k - 1)\lambda_{kt}(l_{ij} - \frac{1}{1-\beta_k}\sum_s \gamma_{ks}l_{is})$$

Compilation. Thus if we use the following notation:

$$B_{ji} = \phi_j(1-\sigma)(\sum_k \phi_k l_{ik} - l_{ij})$$

$$D_{ji} = \sum_k \gamma_{kj}(1-\sigma_k)\lambda_{kt}(l_{ij} - \frac{1}{1-\beta_k}\sum_s \gamma_{ks}l_{is})$$

Compiling all the information we got so far allows us to write :

$$\frac{\partial \lambda_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} - \sum_k \gamma_{kj} \frac{\partial \lambda_{kt}}{\partial \epsilon_{it}}|_{\epsilon=0} = B_{ji} + D_{ji}$$

Thus, we have:

$$\frac{\partial \lambda_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = \sum_k l_{kj}(B_{ki} + D_{ki})$$

Finally, notice that since $\lambda_{jt} = \frac{p_{jt}Y_{jt}}{w_t}$. we have :

$$\frac{\partial logY_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = \frac{1}{\lambda_j}\frac{\partial \lambda_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} - \frac{\partial \hat{p_{jt}}}{\partial \epsilon_{it}}|_{\epsilon=0}$$

$$\frac{\partial logY_{jt}}{\partial \epsilon_{it}}|_{\epsilon=0} = \underbrace{\frac{1}{\lambda_j}\sum_k l_{jk}B_{ki}}_{Cons_{ji}} + \underbrace{\frac{1}{\lambda_j}\sum_k l_{jk}D_{ki}}_{Inp_{ji}} \underbrace{- l_{ji}}_{Dir_{ji}}$$

9.3.6 Proof of Theorem 3

We use the exact same proof as for Theorem 1 in subsubsection 9.3.4 until we get to the Equation 49:

$$\log(\frac{p_{jt}}{w_t}) = -\tilde{\epsilon_t} - \epsilon_{jt} + \frac{1-\beta_j}{1-\sigma_j}\log(\frac{1}{1-\beta_j}\sum_k \gamma_{kj}(\frac{p_{kt}}{w_t})^{1-\sigma_j}) \qquad (1)$$

Here, we differentiate and evaluate for $\epsilon = 0$:

$$\frac{\partial \hat{p_{jt}}}{\partial \tilde{\epsilon_t}}|_{\epsilon=0} = -\mathbb{1} + \sum_k \gamma_{jk}\frac{\partial \hat{p_{kt}}}{\partial \tilde{\epsilon_t}}|_{\epsilon=0}$$

where $\mathbb{1}$ is a $1 \times N$ vector with all values being 1. We easily get to the analog equation:

$$\frac{\partial \hat{\boldsymbol{p}}}{\partial \tilde{\epsilon_t}}|_{\epsilon=0} = -\boldsymbol{L}\mathbb{1}$$

which gives the first part of the theorem on log prices. The second part is straightforward when replacing the expression of the $\frac{\partial \hat{p_{jt}}}{\partial \tilde{\epsilon}}|_{\epsilon=0}$ with sum on i in the proof.

9.4 Additional Data

9.4.1 Data Calibration

The input-output parameters (γ_{ji}) are summarized in the following table :

Γ	AGR	MIN	UTI	CON	MAN	WHO	RET	TRA	INF	FIN	BUS	EDU	ART	OTH	GOV
AGR	0.324598	0.000743	0	0.002971	0.067805	3.48E-05	0.002729	0.00016	0	1.76E-06	0.001230	0.000225	0.010222	0.000673	0.003492
MIN	0.008136	0.236967	0.270576	0.021419	0.10834	8.68E-05	8.49E-05	0.000463	0.000684	7.33E-06	0.000976	0.000444	0.002250	0.002161	0.023451
UTI	0.022	0.035396	0.132605	0.007703	0.02216	0.023014	0.05811	0.03264	0.010177	0.041124	0.01257	0.020360	0.058883	0.017368	0.022930
CON	0.006762	0.025454	0.031282	0.000369	0.004161	0.002934	0.006512	0.011101	0.005236	0.062358	0.001419	0.001994	0.006142	0.014111	0.064662
MAN	0.293157	0.244634	0.118161	0.518919	0.54764	0.08847	0.075144	0.222182	0.129758	0.027287	0.104006	0.227856	0.174281	0.233906	0.311867
WHO	0.128932	0.048567	0.030769	0.093017	0.089573	0.075681	0.036096	0.04544	0.027703	0.010641	0.020728	0.055388	0.03815	0.041686	0.046214
RET	0.007020	0.002668	0.009308	0.113926	0.005016	0.001178	0.007938	0.023370	0.001061	0.003141	0.002979	0.001364	0.02846	0.033814	0.001037
TRA	0.03854	0.055199	0.119027	0.033661	0.039773	0.113776	0.128658	0.258758	0.026101	0.015740	0.036338	0.023029	0.022710	0.02037	0.051157
INF	0.002707	0.007784	0.015648	0.013225	0.005999	0.035487	0.038585	0.014954	0.333411	0.028273	0.063851	0.031291	0.026006	0.041028	0.076812
FIN	0.141291	0.151305	0.076426	0.065756	0.025898	0.199180	0.271761	0.187082	0.104439	0.527489	0.217406	0.269449	0.214979	0.31995	0.102352
BUS	0.018103	0.183348	0.153682	0.114990	0.072530	0.373144	0.313027	0.129140	0.286061	0.223934	0.455784	0.246474	0.306268	0.189038	0.223970
EDU	0.000464	0	0.001039	9.34E-06	2.41E-05	0.002490	0.007447	0.000323	0.000595	1.70E-05	0.000731	0.040980	0.003199	0.009530	0.020850
ART	0.002873	0.003280	0.01756	0.001591	0.003587	0.013341	0.012935	0.022437	0.055750	0.0315	0.050926	0.045243	0.06466	0.020171	0.018812
OTH	0.004846	0.004586	0.007880	0.01242	0.006110	0.037814	0.026523	0.033199	0.013434	0.020757	0.023260	0.027101	0.028339	0.046636	0.024922
GOV	0.00018	6.22E-05	0.016024	1.55E-05	0.001371	0.033360	0.014438	0.018732	0.005583	0.007723	0.00778	0.008794	0.015424	0.009544	0.007465

Table 8: Calibration values of the Input-Output matrix $\mathbf{\Gamma}$

The parameters values for labor share (β_j) and consumption expenditure (ϕ_j) are summarized in the following table :

Sector	AGR	MIN	UTI	CON	MAN	WHO	RET	TRA	INF	FIN	BUS	EDU	ART	OTH	GOV
β	0.12	0.15	0.15	0.33	0.18	0.31	0.37	0.30	0.21	0.15	0.46	0.50	0.34	0.43	0.52
ϕ	0.006	0.003	0.015	0.057	0.199	0.040	0.071	0.021	0.034	0.172	0.032	0.121	0.049	0.035	0.145

Table 9: Calibration values of the labor share and consumption expenditure

9.4.2 Multiplier matrices

The estimates of the multiplier matrices using the values from Table 2 are the following :

$$
\begin{pmatrix}
1.48 & 0.16 & 0.11 & 0.06 & 1.03 & 0.30 & 0.03 & 0.19 & 0.08 & 0.71 & 0.53 & 0.00 & 0.06 & 0.05 & 0.02 \\
0.07 & 1.38 & 0.11 & 0.08 & 0.83 & 0.17 & 0.02 & 0.18 & 0.09 & 0.67 & 0.68 & 0.00 & 0.06 & 0.04 & 0.02 \\
0.06 & 0.43 & 1.20 & 0.08 & 0.69 & 0.15 & 0.03 & 0.25 & 0.10 & 0.58 & 0.65 & 0.00 & 0.07 & 0.05 & 0.03 \\
0.07 & 0.14 & 0.06 & 1.03 & 0.91 & 0.17 & 0.09 & 0.13 & 0.07 & 0.41 & 0.46 & 0.00 & 0.04 & 0.04 & 0.01 \\
0.17 & 0.28 & 0.09 & 0.05 & 2.23 & 0.23 & 0.02 & 0.17 & 0.07 & 0.46 & 0.52 & 0.00 & 0.05 & 0.04 & 0.02 \\
0.03 & 0.07 & 0.07 & 0.04 & 0.40 & 1.12 & 0.01 & 0.17 & 0.10 & 0.57 & 0.68 & 0.00 & 0.06 & 0.06 & 0.04 \\
0.03 & 0.07 & 0.09 & 0.05 & 0.36 & 0.09 & 1.02 & 0.17 & 0.10 & 0.60 & 0.59 & 0.01 & 0.05 & 0.05 & 0.02 \\
0.05 & 0.10 & 0.09 & 0.05 & 0.61 & 0.12 & 0.03 & 1.31 & 0.08 & 0.58 & 0.50 & 0.00 & 0.06 & 0.06 & 0.03 \\
0.04 & 0.08 & 0.07 & 0.04 & 0.54 & 0.11 & 0.02 & 0.12 & 1.43 & 0.54 & 0.74 & 0.00 & 0.11 & 0.05 & 0.02 \\
0.04 & 0.08 & 0.12 & 0.13 & 0.43 & 0.09 & 0.02 & 0.11 & 0.13 & 2.17 & 0.76 & 0.00 & 0.10 & 0.06 & 0.03 \\
0.03 & 0.05 & 0.05 & 0.03 & 0.32 & 0.07 & 0.01 & 0.08 & 0.10 & 0.47 & 1.59 & 0.00 & 0.07 & 0.04 & 0.01 \\
0.03 & 0.06 & 0.05 & 0.03 & 0.41 & 0.09 & 0.01 & 0.07 & 0.07 & 0.47 & 0.43 & 1.02 & 0.06 & 0.04 & 0.01 \\
0.05 & 0.09 & 0.10 & 0.04 & 0.49 & 0.10 & 0.03 & 0.10 & 0.09 & 0.57 & 0.61 & 0.00 & 1.09 & 0.05 & 0.02 \\
0.04 & 0.07 & 0.06 & 0.05 & 0.48 & 0.10 & 0.03 & 0.09 & 0.09 & 0.59 & 0.46 & 0.01 & 0.05 & 1.06 & 0.02 \\
0.04 & 0.09 & 0.05 & 0.06 & 0.50 & 0.09 & 0.01 & 0.09 & 0.09 & 0.32 & 0.39 & 0.01 & 0.04 & 0.03 & 1.01
\end{pmatrix}
$$

Figure 9: Downstream channel matrix of US economy (DE)

$$
\begin{pmatrix}
-0.11 & -0.03 & 0.00 & 0.00 & -0.28 & -0.02 & 0.02 & -0.01 & 0.01 & 0.07 & 0.02 & 0.04 & 0.01 & 0.01 & 0.04 \\
-0.02 & -0.05 & -0.02 & 0.00 & -0.23 & -0.01 & 0.02 & -0.01 & 0.01 & 0.05 & 0.01 & 0.03 & 0.01 & 0.01 & 0.03 \\
0.00 & -0.03 & -0.11 & 0.01 & -0.01 & 0.00 & 0.00 & -0.01 & 0.01 & -0.05 & -0.01 & 0.03 & 0.00 & 0.01 & 0.04 \\
0.00 & 0.00 & 0.00 & -0.24 & 0.00 & 0.00 & 0.01 & 0.01 & 0.02 & 0.00 & 0.02 & 0.05 & 0.02 & 0.01 & 0.04 \\
-0.03 & -0.04 & 0.00 & 0.00 & -0.32 & -0.01 & 0.02 & -0.01 & 0.01 & 0.08 & 0.02 & 0.03 & 0.01 & 0.01 & 0.03 \\
-0.01 & -0.01 & 0.00 & 0.00 & -0.07 & -0.13 & 0.02 & -0.01 & 0.01 & 0.06 & 0.00 & 0.03 & 0.01 & 0.01 & 0.03 \\
0.02 & 0.02 & 0.00 & 0.01 & 0.18 & 0.03 & -0.37 & -0.01 & 0.02 & 0.06 & 0.00 & 0.06 & 0.02 & 0.01 & 0.07 \\
0.00 & -0.01 & -0.01 & 0.00 & -0.04 & -0.01 & -0.01 & -0.11 & 0.01 & 0.02 & 0.01 & 0.03 & 0.01 & 0.01 & 0.03 \\
0.01 & 0.01 & 0.00 & 0.02 & 0.08 & 0.01 & 0.01 & 0.01 & -0.23 & 0.00 & -0.04 & 0.03 & 0.00 & 0.01 & 0.02 \\
0.01 & 0.01 & -0.01 & 0.00 & 0.09 & 0.01 & 0.01 & 0.00 & 0.00 & -0.29 & -0.04 & 0.03 & 0.00 & 0.00 & 0.04 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.03 & 0.00 & 0.00 & 0.00 & -0.01 & -0.05 & -0.05 & 0.02 & 0.00 & 0.00 & 0.03 \\
0.02 & 0.04 & 0.02 & 0.04 & 0.22 & 0.04 & 0.04 & 0.04 & 0.04 & 0.16 & 0.08 & -0.47 & 0.03 & 0.02 & 0.07 \\
0.01 & 0.01 & 0.00 & 0.02 & 0.11 & 0.02 & 0.02 & 0.01 & 0.00 & 0.00 & -0.02 & 0.03 & -0.24 & 0.01 & 0.05 \\
0.01 & 0.01 & 0.01 & 0.02 & 0.09 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.02 & 0.04 & 0.01 & -0.23 & 0.05 \\
0.02 & 0.02 & 0.02 & 0.03 & 0.16 & 0.03 & 0.04 & 0.03 & 0.02 & 0.21 & 0.09 & 0.06 & 0.03 & 0.02 & -0.40
\end{pmatrix}
$$

Figure 10: Final good substitution channel matrix of US economy (CR)

$$
\begin{pmatrix}
-0.01 & 0.11 & -0.01 & -0.03 & -0.08 & -0.02 & 0.01 & 0.01 & -0.01 & -0.13 & 0.09 & -0.01 & 0.00 & 0.00 & 0.00 \\
0.07 & -0.57 & -0.07 & -0.02 & 0.11 & 0.08 & 0.01 & 0.04 & -0.01 & -0.01 & 0.07 & -0.01 & 0.01 & 0.01 & 0.01 \\
0.00 & -0.09 & -0.42 & 0.00 & 0.01 & 0.01 & 0.00 & -0.02 & 0.01 & 0.04 & 0.15 & 0.00 & 0.01 & 0.01 & 0.00 \\
-0.02 & -0.03 & 0.00 & -0.04 & -0.11 & -0.02 & 0.00 & -0.01 & 0.00 & -0.01 & 0.09 & -0.01 & 0.01 & 0.00 & 0.00 \\
-0.01 & 0.02 & 0.00 & -0.02 & -0.33 & 0.02 & 0.01 & 0.01 & -0.01 & 0.06 & 0.09 & -0.01 & 0.00 & 0.00 & 0.00 \\
-0.01 & 0.06 & 0.00 & -0.01 & 0.11 & -0.20 & 0.01 & 0.01 & -0.01 & 0.02 & 0.08 & -0.01 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.02 & 0.00 & 0.00 & 0.09 & 0.02 & -0.13 & 0.00 & 0.00 & 0.01 & 0.05 & 0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.03 & -0.02 & 0.00 & 0.07 & 0.01 & 0.00 & -0.37 & 0.01 & 0.09 & 0.16 & -0.01 & 0.01 & 0.01 & 0.00 \\
-0.01 & -0.01 & 0.01 & 0.00 & -0.05 & -0.01 & 0.00 & 0.01 & -0.27 & 0.11 & 0.10 & -0.01 & 0.02 & 0.01 & 0.00 \\
-0.01 & 0.00 & 0.00 & 0.00 & 0.07 & 0.00 & 0.00 & 0.02 & 0.02 & -0.60 & 0.13 & 0.00 & 0.01 & 0.01 & 0.01 \\
0.01 & 0.02 & 0.02 & 0.02 & 0.13 & 0.02 & 0.01 & 0.04 & 0.02 & 0.17 & -0.24 & 0.00 & 0.02 & 0.02 & 0.01 \\
0.00 & -0.01 & 0.00 & -0.01 & -0.04 & -0.01 & 0.00 & -0.01 & -0.01 & -0.01 & -0.02 & 0.06 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.01 & 0.00 & 0.01 & 0.04 & 0.01 & 0.00 & 0.02 & 0.02 & 0.08 & 0.11 & 0.00 & -0.27 & 0.01 & 0.00 \\
0.00 & 0.01 & 0.01 & 0.01 & 0.02 & 0.00 & 0.00 & 0.01 & 0.01 & 0.06 & 0.12 & 0.00 & 0.01 & -0.19 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 0.03 & 0.04 & 0.00 & 0.00 & 0.00 & -0.05
\end{pmatrix}
$$

Figure 11: Input substitution channel matrix of US economy (IR)

$$
\begin{pmatrix}
1.37 & 0.24 & 0.10 & 0.04 & 0.66 & 0.25 & 0.05 & 0.18 & 0.08 & 0.66 & 0.64 & 0.03 & 0.06 & 0.06 & 0.06 \\
0.12 & 0.76 & 0.02 & 0.05 & 0.71 & 0.24 & 0.05 & 0.21 & 0.09 & 0.71 & 0.77 & 0.03 & 0.07 & 0.06 & 0.05 \\
0.05 & 0.30 & 0.67 & 0.09 & 0.68 & 0.15 & 0.03 & 0.22 & 0.11 & 0.57 & 0.79 & 0.03 & 0.07 & 0.06 & 0.07 \\
0.06 & 0.11 & 0.07 & 0.76 & 0.80 & 0.16 & 0.09 & 0.13 & 0.09 & 0.40 & 0.58 & 0.04 & 0.07 & 0.05 & 0.06 \\
0.14 & 0.26 & 0.09 & 0.03 & 1.58 & 0.24 & 0.05 & 0.18 & 0.08 & 0.60 & 0.64 & 0.03 & 0.07 & 0.05 & 0.05 \\
0.01 & 0.12 & 0.07 & 0.03 & 0.44 & 0.79 & 0.04 & 0.17 & 0.10 & 0.65 & 0.75 & 0.03 & 0.07 & 0.07 & 0.07 \\
0.05 & 0.11 & 0.09 & 0.05 & 0.64 & 0.14 & 0.52 & 0.16 & 0.12 & 0.68 & 0.64 & 0.06 & 0.07 & 0.06 & 0.09 \\
0.05 & 0.12 & 0.06 & 0.05 & 0.64 & 0.13 & 0.03 & 0.83 & 0.09 & 0.69 & 0.67 & 0.03 & 0.08 & 0.07 & 0.06 \\
0.04 & 0.08 & 0.08 & 0.06 & 0.56 & 0.11 & 0.03 & 0.13 & 0.92 & 0.65 & 0.80 & 0.03 & 0.13 & 0.06 & 0.05 \\
0.03 & 0.09 & 0.12 & 0.12 & 0.58 & 0.11 & 0.03 & 0.14 & 0.14 & 1.28 & 0.85 & 0.03 & 0.11 & 0.07 & 0.08 \\
0.04 & 0.07 & 0.07 & 0.06 & 0.48 & 0.09 & 0.02 & 0.13 & 0.12 & 0.58 & 1.31 & 0.02 & 0.09 & 0.06 & 0.05 \\
0.05 & 0.09 & 0.07 & 0.07 & 0.60 & 0.12 & 0.05 & 0.11 & 0.10 & 0.62 & 0.49 & 0.61 & 0.08 & 0.06 & 0.09 \\
0.05 & 0.11 & 0.10 & 0.08 & 0.64 & 0.13 & 0.05 & 0.13 & 0.11 & 0.64 & 0.70 & 0.03 & 0.58 & 0.07 & 0.08 \\
0.05 & 0.10 & 0.08 & 0.07 & 0.59 & 0.11 & 0.05 & 0.11 & 0.11 & 0.65 & 0.61 & 0.04 & 0.08 & 0.63 & 0.07 \\
0.06 & 0.11 & 0.07 & 0.09 & 0.67 & 0.12 & 0.05 & 0.12 & 0.12 & 0.55 & 0.52 & 0.07 & 0.08 & 0.06 & 0.56
\end{pmatrix}
$$

Figure 12: Total multiplier matrix of US economy (TOT)

9.4.3 Reallocational effects relatively to downstream effects

The error matrix between DE and TOT effects is the following :

$$
\begin{pmatrix}
8.3 & 34.1 & 8.1 & 56.1 & 55.5 & 16.3 & 49.1 & 1.8 & 2.1 & 8.2 & 16.8 & 93.2 & 13.1 & 13.4 & 63.0 \\
41.8 & 81.1 & 518.1 & 49.2 & 16.2 & 29.4 & 58.1 & 13.1 & 3.4 & 6.1 & 11.1 & 95.0 & 23.5 & 25.8 & 64.8 \\
12.8 & 40.9 & 78.6 & 10.1 & 0.6 & 5.6 & 4.0 & 17.0 & 16.7 & 2.1 & 16.7 & 91.0 & 4.9 & 25.6 & 54.4 \\
23.7 & 25.1 & 10.6 & 36.8 & 13.1 & 7.7 & 5.4 & 0.5 & 18.9 & 2.0 & 20.4 & 96.5 & 46.9 & 28.6 & 75.6 \\
23.7 & 7.0 & 0.5 & 47.8 & 41.3 & 3.2 & 58.7 & 2.3 & 5.4 & 23.6 & 17.7 & 94.9 & 29.5 & 20.5 & 65.3 \\
133.7 & 44.4 & 6.7 & 25.9 & 8.6 & 42.7 & 66.1 & 1.0 & 0.5 & 11.9 & 9.8 & 89.0 & 22.0 & 9.7 & 46.1 \\
39.8 & 37.4 & 1.3 & 12.1 & 43.0 & 36.1 & 97.2 & 7.5 & 18.7 & 11.4 & 7.3 & 90.3 & 27.0 & 19.9 & 74.8 \\
3.6 & 19.8 & 38.4 & 1.0 & 3.6 & 1.5 & 28.5 & 57.5 & 15.4 & 16.6 & 25.6 & 93.9 & 27.0 & 18.4 & 52.5 \\
2.2 & 3.5 & 17.1 & 24.2 & 4.5 & 0.6 & 50.1 & 12.0 & 55.1 & 16.5 & 7.9 & 92.4 & 14.5 & 20.2 & 58.1 \\
17.7 & 7.9 & 1.3 & 1.0 & 26.6 & 15.3 & 27.8 & 16.6 & 13.1 & 69.8 & 10.3 & 93.8 & 10.2 & 9.5 & 66.4 \\
33.6 & 26.7 & 31.1 & 42.1 & 33.2 & 24.3 & 38.8 & 34.8 & 12.6 & 19.7 & 21.5 & 90.4 & 20.6 & 34.8 & 70.7 \\
36.0 & 31.2 & 27.7 & 54.0 & 30.9 & 28.7 & 79.6 & 30.4 & 28.1 & 24.5 & 12.8 & 66.5 & 31.5 & 34.6 & 83.1 \\
10.4 & 19.1 & 3.1 & 44.7 & 23.1 & 19.5 & 34.6 & 23.6 & 20.6 & 11.8 & 13.4 & 89.3 & 89.0 & 25.5 & 68.9 \\
15.2 & 24.7 & 23.9 & 32.1 & 18.7 & 12.2 & 30.5 & 22.3 & 18.6 & 9.7 & 24.1 & 82.2 & 30.9 & 66.8 & 74.2 \\
27.7 & 22.8 & 30.9 & 35.4 & 25.5 & 26.5 & 76.2 & 24.1 & 20.0 & 42.2 & 24.7 & 83.4 & 47.2 & 42.2 & 80.0
\end{pmatrix}
$$

Figure 13: Relative error between Downstream and Total multipliers in absolute terms (in %)

$$
\begin{pmatrix}
-0,90 & 0,06 & 0,01 & 0,00 & -0,38 & 0,01 & 0,01 & 0,03 & 0,03 & 0,10 & 0,25 & 0,00 & 0,01 & 0,01 & 0,01 \\
0,04 & -0,75 & -0,10 & -0,01 & -0,23 & 0,07 & 0,01 & 0,03 & 0,03 & 0,10 & 0,20 & 0,00 & 0,02 & 0,02 & 0,01 \\
0,01 & -0,14 & -0,58 & 0,01 & 0,02 & 0,02 & 0,00 & -0,03 & 0,03 & 0,01 & 0,19 & 0,00 & 0,01 & 0,01 & 0,00 \\
0,00 & -0,01 & 0,01 & -0,26 & -0,03 & 0,00 & -0,02 & 0,01 & 0,03 & -0,04 & 0,18 & 0,00 & 0,02 & 0,01 & 0,00 \\
-0,03 & -0,04 & 0,00 & 0,00 & -0,75 & 0,01 & 0,01 & 0,01 & 0,03 & 0,17 & 0,20 & 0,00 & 0,02 & 0,01 & 0,00 \\
0,01 & 0,05 & 0,01 & 0,00 & 0,06 & -0,40 & 0,01 & 0,01 & 0,02 & 0,12 & 0,15 & 0,00 & 0,01 & 0,01 & 0,00 \\
0,01 & 0,02 & 0,00 & -0,02 & 0,09 & 0,02 & -0,14 & -0,01 & 0,01 & 0,02 & 0,06 & 0,00 & 0,00 & 0,00 & 0,00 \\
0,01 & 0,03 & -0,02 & 0,01 & 0,06 & 0,01 & 0,00 & -0,62 & 0,04 & 0,15 & 0,27 & 0,00 & 0,02 & 0,01 & 0,00 \\
0,02 & 0,03 & 0,02 & 0,02 & 0,17 & 0,02 & 0,01 & 0,04 & -0,65 & 0,10 & 0,09 & 0,00 & 0,01 & 0,01 & 0,01 \\
0,01 & 0,02 & 0,00 & -0,01 & 0,19 & 0,03 & 0,00 & 0,03 & 0,02 & -0,77 & 0,11 & 0,00 & 0,00 & 0,00 & 0,01 \\
0,03 & 0,04 & 0,03 & 0,03 & 0,29 & 0,04 & 0,01 & 0,07 & 0,02 & 0,14 & -0,37 & 0,00 & 0,02 & 0,02 & 0,01 \\
0,00 & 0,00 & 0,00 & 0,00 & 0,01 & 0,00 & 0,00 & 0,00 & 0,00 & 0,01 & 0,01 & -0,03 & 0,00 & 0,00 & 0,00 \\
0,01 & 0,02 & 0,00 & 0,02 & 0,13 & 0,02 & 0,00 & 0,03 & 0,01 & 0,03 & 0,10 & 0,00 & -0,32 & 0,01 & 0,00 \\
0,01 & 0,03 & 0,02 & 0,01 & 0,12 & 0,01 & 0,00 & 0,02 & 0,02 & 0,04 & 0,17 & 0,00 & 0,01 & -0,32 & 0,01 \\
0,00 & 0,01 & 0,00 & 0,00 & 0,02 & 0,00 & 0,00 & 0,00 & 0,00 & 0,03 & 0,04 & 0,00 & 0,00 & 0,00 & -0,07
\end{pmatrix}
$$

Figure 14: Input substitution channel matrix of US economy (IR) of the restricted model ($\sigma_j = 0.42, \forall j$

9.5 Graphs

The graphs of priors and posteriors marginal density for the main estimated parameters can be found here:

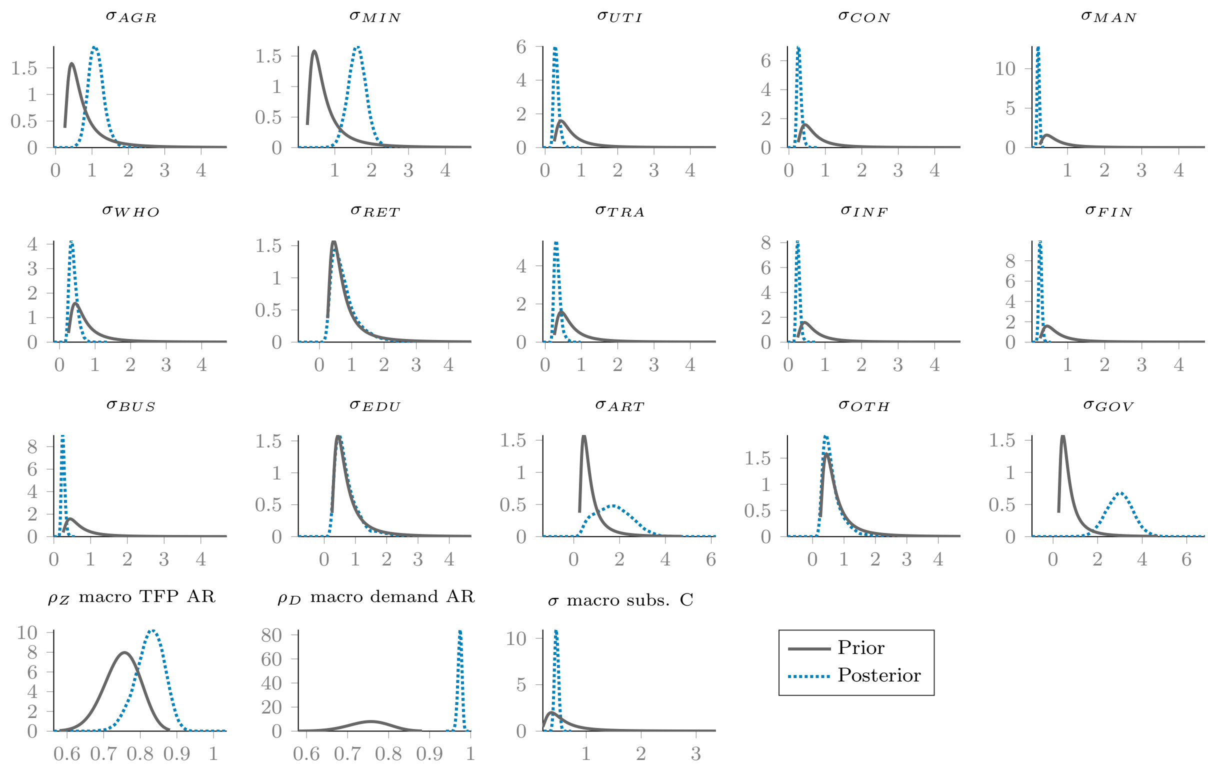

Figure 15: Priors and posteriors distribution of the main estimated parameters

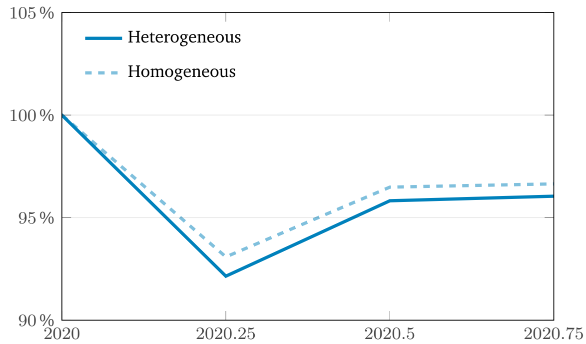

Figure 16: Covid Crisis counterfactuals