Pay for Performance in Procurement

Juan-José Ganuza[a], Fernando Gómez[b] and Makoto Hanazono[c]

January 2023

Abstract

This paper sheds light on the limitations of "pay-for-performance" in bidding markets with ex post moral hazard. High-powered incentive schemes may lead to worse allocations. We present our analysis in a procurement setting in which potential contractors differ both in their costs and levels of wealth and are protected by limited liability. We show first that competitive mechanisms adversely select undercapitalized firms. Second, more powerful incentive mechanisms induce less desirable allocations: the selected contractor is likely to be at the same time less solvent and less efficient. As a result, low-powered incentives in procurement may be optimal. This is consistent with the fact, that despite the large social welfare losses arising from inadequate performance by contractors (delays, cost overruns, deficient quality of output) in procurement, powerful pay-for-performance mechanisms appear to be rare in reality.

Keywords: Procurement; Incentives; Pay for Performance and Limited Liability

.

Corresponding author: juanjo.ganuza@upf.edu

[a] Universitat Pompeu Fabra and Barcelona School of Economics,[b] Universitat Pompeu Fabra School of Law, [c] Nagoya University.
We are grateful for the comments to Miguel Abito, Francesco Decarolis, Shingo Ishiguro, Steven Tadelis, participants at The 38th Annual Conference of the European Association of Law and Economics (EALE), Simposio de Analisis Economico 2021, Workshop in Organizational Economics at Summer Forum 2021, International Industrial Organization Conference, Asian Meeting of Econometric Society 2022, EARIE 2022, and participants at seminars at Universitat Pompeu Fabra, Kyoto University, Nagoya University, and Waseda University. The usual disclaimers apply. Fernando Gomez acknowledges the financial support of the Spanish Ministry of Science and Innovation under project DER 2017-82673-R. Juan-Jose Ganuza gratefully acknowledges the support of the Barcelona School of Economics Research, the government of Catalonia, and the Spanish Ministry of Science and Innovation through the project PID2020-115044GB-I00. Makoto Hanazono acknowledges the support of Japan Society for the Promotion of Science, grant number 17KK0067.

"One of the main lessons from working on incentive problems for 25 years is, that within firms, high-powered financial incentives can be very dysfunctional...typically, it is best to avoid high-powered incentives and sometimes not use pay-for-performance at all." Bengt Holmstrom, Nobel Prize Lecture (2016).

1 Introduction

Linking payoffs to outcomes (pay-for-performance) provides one of the most powerful economic tools to generate incentives. However, as Holmstrom in his Nobel Prize Lecture (quoted in the opening of the paper) pointed out, it is not difficult in reality to find situations in which pay-for-performance incentive schemes work poorly. Common concerns expressed about pay-for-performance refer to issues of multitasking issues and risk imposed on risk-averse agents. In this paper we explore a different shortcoming affecting pay-for-performance mechanisms: they may lead to worse allocations in settings involving competitive bidding and ex post moral hazard when firms are protected by limited liability.

In this paper we identify a novel trade-off between the provision of incentives and the efficiency of the allocation in bidding markets. The driving force for the inefficiency in the allocation lies in firms with lower capitalization (who will be bankrupt if they fail to perform properly and thus will face lower penalties) exerting lower effort and expecting larger net profits from being awarded the contract. The higher the risk (the larger the incentives), the higher the competitive advantage of less capitalized firms and thus, the worse the allocation will turn out to be.

When all bidding firms are fully solvent, a competitive mechanism leads to the selection of the most efficient firm for the contract or project. This is not true in a more realistic setting, since undercapitalized firms enjoy an advantage in the bidding contest. Incentive mechanisms using penalties (and/or bonuses) affect the actions by the contractor in two ways: first, by directly altering the incentives of the contractor to exert effort; second, by affecting the incentives of firms to bid, and in this way influencing the outcome of the selection process. The higher the incentives (the penalty for mis-performance or the bonus for good performance) the larger the likelihood of a firm with poor capitalization and higher costs of construction to win the selection process.

This trade-off between incentives and efficiency allocation may provide relevant insights into a number of competitive settings, especially in the design and functioning of procurement contracts. In particular, it may explain why the use of pay-for-performance mechanisms in procurement contracts seems to be of modest proportions compared to the size of projects and the repercussions of inadequate performance by contractors. This is a puzzling observation, given the importance of procurement in the functioning of economies, both in developed and developing countries,[1] and the correspondingly large social welfare losses arising from poor performance in procurement contracts (delays, deficient quality of output).[2]

The implementation of a procurement project would typically depend on the terms of the procurement contract, the contractor's effort, and also unforeseen contingencies (weather and other external conditions, shocks to the supply of input and human resources, random catastrophic events) that may arise during the contract's performance. To tackle the contractor's moral hazard problem in such settings, one would expect to find widespread and significant reliance on explicit incentives, given the magnitudes at stake when contractor's output is poor in terms of timing and/or quality. In reality, however, one does not observe high-powered explicit performance incentives in procurement. The most commonly used contractual format appears to be a fixed price contract stipulating price, quality/performance/timing specifications, with some financial penalties if these goals are not met.[3]

Penalties, moreover, seem to be low compared to the size of the negative effects from delays or inferior quality. Lewis and Bajari (2011) discuss this apparent anomaly in real-world procurement contracts, providing an illustrative example regarding highways in California: they estimate that the social cost from delay was $1.75 million per day, while the daily penalty for delay was merely $40,000. Along similar lines, Lewis and Bajari (2014) use data on Minnesota highway construction contracts and show that the penalty system at work provided little incentives to reduce delays, and that switching to a more powerful incentive system would increase welfare with only a small impact on contractors' costs.[4]

We show there may be good efficiency reasons for the reluctance of procurement agencies to resort to explicit powerful incentive schemes (penalties and/or bonuses[5]) to improve the contractors' level of effort. The implications from our analysis may also add another

[1] Bosio et al. (2022) report a figure of 12% of global GDP as the share claimed by government procurement in 2018. They also report significantly larger shares for rich countries such as Switzerland, the Netherlands or Singapore.

[2] Lewis and Bajari (2011, 2014) provide evidence of the large social cost of the delays in highway construction in California and Minnesota. Decarolis et al. (2016) document sizable gains from improving performance in a case study of procurement in the electricity sector.

[3] The financial penalty is typically determined and exacted ex post—when the delay or defective performance is observed by the sponsor—but it may take two forms: first, a retention (retainage) from the contract price that the sponsor withholds until the project output is delivered and evaluated; second, a fine that the sponsor has to claim from the contractor. The use of retainage is explored experimentally by Walker et al. (2020).

[4] Effective penalties are even lower. Penalties are imperfectly enforceable since contractors may dispute them in Courts. This is especially detrimental for incentives when the Court system is inefficient, as it is shown by Coviello et al. (2018) with Italian procurement data.

[5] We will consider also incentive schemes involving positive payments or bonuses to the contractor, although they appear to be extremely rare, despite its prevalence in other settings (employment and managerial relationships). See Bigoni et al. (2014)

explanatory factor to the empirical studies that find poorer outcomes in procurement contracts in less developed countries compared to wealthier ones. This is commonly attributed to poor institutional quality and lower public sector capabilities in the first group of countries. Perhaps also the fact that firms who are candidates for the winning the contract have lower assets may be a contributing factor, given that this is expected to result in lower contractors' effort and worse allocations.

We present our analysis in a procurement setting in which a project with ex post moral hazard has to be allocated among firms that differ in construction costs and assets (capitalization). Final quality of project outcome depends on the contractor's effort level and exogenous shocks. The contractor is protected by limited liability—the contractor cannot pay penalties beyond its total asset level—and incentives to exert effort depend on the incentive (the size of the penalty) and its asset size.

We show three fundamental results. First, competitive mechanisms adversely select undercapitalized firms for undertaking procurement contracts with ex post moral hazard. Second, and counterintuitively, that more powerful incentive mechanisms lead to worse allocations: the winning firm is likely to be less solvent and less efficient in carrying out the project. Increasing the size of the penalty that the sponsor imposes upon the contractor for mis-performance worsens the allocation by granting an additional advantage to firms with less assets. This may result in awarding the contract to a less efficient firm in terms of performance costs. An important implication of this result is that low powered schemes may dominate more powerful incentive mechanisms that would be optimal absent the risk of default by the contractor. Finally, we extend the basic analysis to consider both penalties for failing to provide proper performance and bonuses for achieving the goals set out by the sponsor and show that the allocation does not depend on the format of the explicit incentive (penalty or bonus), but solely on its size.[6]

We also extend our fundamental results to a setting in which the contractor's performance is a continuous variable and show them to hold as well in this more general setup. The tools and techniques developed for addressing this challenging extension of the binary baseline model open the scope of the present paper to apply our results in other competitive environments.

There is an extensive literature dealing with procurement and moral hazard, but previous contributions have analyzed other dimensions of the problem. Seminal papers by Holt (1979, 1980) and McAfee and McMillan (1986) study settings in which contractor's effort reduces construction costs and the principal (sponsor) chooses between fixed-price and cost-sharing

[6]Bigoni et al. (2014) experimentally explore the use of penalties versus bonuses in a procurement setting and provide behavioral arguments for which the sponsors have the preference for penalties. First, penalties allow the sponsor to use exploitative offers taking advantage of contractors' naïveté. Second, loss aversion on the part of sponsors, who are thereby led to offer less generous contracts with penalties than with bonuses.

contracts. These papers mainly analyze the trade-off between the cost of transferring risks to contractors versus providing incentives to reduce construction costs. A second generation of papers, such as McAfee and McMillan (1987) and Laffont and Tirole (1987), characterize the optimal contract under risk neutrality and unlimited liability. We take as given the contract format and the procurement mechanism (the second-price auction), but consider the probability of default and we add the firm's financial condition as another source of firms' private information and heterogeneity. Recently, Chakraborty et al. (2021) characterize the optimal contract in a competitive procurement model with adverse selection and moral hazard, in which the private information refers to the cost of effort, similar to McAfee and McMillan (1986). They find that the "truth-telling" constraint for the less efficient type (not to lie but to shirk ex post) matters and the associated incentive cost increases with competition. This creates a new trade-off between efficiency in the allocation and incentives, so that it may be optimal to limit competition or distort the allocation. We look at the problem from a different perspective and our approach is driven by different forces, obtaining a result along the same lines: more powerful incentives in procurement are likely to lead to less efficient allocations.

Closely related to our work are the papers dealing with a procurement settings with cost uncertainty and limited liability (Waehrer (1995), Zheng (2001), Calveras, Ganuza and Hauk (2004), Engel and Wambach (2006), Board (2007), Chillemi and Mezzetti (2010) and Burguet, Ganuza and Hauk (2012)).What this literature shows is that in the presence of cost uncertainty, the contractors' limited assets pave the way to the likelihood of contractors' default, that limited liability cuts off the potential downside losses of the winning bidder, thus inducing more aggressive bids. This effect is larger for the less solvent bidders, enhancing their chances of winning the contest for the project.[7] This literature, however, does not deal with ex post moral hazard problems in procurement, and does not consider in such a setting the cost and the asset dimensions jointly, a factor that we deem very relevant in the real-world procurement context.

The negative effect of the probability of default over incentives has been analyzed also in the law and economics literature. The so-called judgement proof problem, introduced by Shavell (1986) in a tort environment, is that when injurers are insolvent, first-best behavior in terms of accident prevention cannot generally be attained through liability rules. In a companion paper (Ganuza and Gomez (2020)), we analyze specifically a procurement problem in a tort setting where incentives are given by liability rules, show that the negligence rule dominates strict liability, and argue that negligence standards lower than the first best

[7] Manelli and Vincent (1995) and Lopomo et al. (2022), analyze a similar adverse selection problem and show that standard competitive mechanisms work poorly when low-cost bidders are also linked with lower quality outcomes.

ones may be desirable. The structure of the paper is as follows. In Section 2 we present the basic features of the model. We solve it in Section 3. In Section 4 we show our fundamental result that more powerful pay-for-performance schemes lead to a worse allocation. In Section 5 we analyze the extensions dealing with penalties and bonuses, with negative markups and with a continuous performance variable. Section 6 briefly concludes. All proofs are relegated to the Appendix.

2 Model

We analyze a procurement setting in which a risk-neutral sponsor procures an indivisible project for which he is willing to pay V. We assume V large enough so as to make the possibility of not contracting unattractive for the sponsor. N firms compete for the indivisible contract. Firms differ both in their cost of undertaking the project and in their initial financial status. Let $c_i \geq 0$ and $w_i \geq 0$ denote, respectively, the cost of undertaking the project and the value of the assets of potential contractor $i = 1, 2, ..., N$. Both c_i and w_i are contractor i's private information. The contract is awarded using a second-price auction. Firms submit their bids and the winning firm is the one with the lowest bid, and the price is set equal to the second lowest bid. Ties are resolved using a lottery.

Denote the auction price by P, and the cost and the assets of the winning firm by c and w, respectively. To simplify the presentation, we ignore the subindex i when we refer to the winning firm. As we will focus on the weakly dominant strategy equilibrium of the second price auction, we do not need to specify how firms' types (c_i, w_i) are distributed.

There is a performance/quality measure of the project contract θ (timeliness, quality, etc.) that we initially assume to be binary $\theta \in \{\overline{\theta}, \underline{\theta}\}$. The contractor may invest in effort (for simplicity we assume it to be non-monetary) to improve performance.[8] Let x be the contractor's monetary equivalent of the performance effort, and $p(x)$ be the probability of poor or inadequate performance, where $p(x)$ is decreasing and convex in x.

The sponsor provides incentives to increase effort using a pay-for-performance mechanism $b(\theta)$. We first consider a penalty. In case of bad performance $\underline{\theta}$, the contractor has to pay a monetary sum b, and 0 otherwise. Then, $b(\underline{\theta}) = -b$ and $b(\overline{\theta}) = 0$. Contractors have limited liability, i.e., the losses to contractor i from the penalty cannot be larger than the firm's total wealth, which at that point is $P - c_i + w_i$. Therefore, contractor i will shut down and discontinue the project if performance is of poor quality and the penalty, b, exceeds his total wealth.

We now summarize the timing of the model:

[8]In Appendix B, we show that our main results hold when effort is monetary.

1. Nature chooses the cost c_i and the financial assets w_i of each firm i.

2. A second-price auction takes place.

3. The winning firm receives price P, carries out the project incurring cost c, and exerts the level of effort x. Poor performance takes place or not, according to probability $p(x)$. The contractor pays the penalty if $P-c+w-b>0$. Otherwise, the firm declares bankruptcy and exits.[9]

We want to analyze how this competitive procurement setting works and what are the implications for the design of pay-for-performance mechanisms. For most of this analysis we do not need to specify the sponsor's preferences. However, for some of our results we will assume that the sponsor's preferences are as follows:

$$U_s = \begin{cases} V - P + \theta - b(\theta) & \text{no default} \\ V + \underline{\theta} - c + w - K & \text{otherwise.} \end{cases}$$

This function describes the ex-post sponsor's preferences that are determined by whether or not the winning firm goes bankrupt. In case the winning firm completes the project, the sponsor's welfare depends on the price paid, the kind of performance of the project θ, and the payment/penalty resulting from the incentive mechanism $-b(\theta)$. Bankruptcy may arise when poor performance is observed and the winning firm is unable to pay the penalty. In this case, the sponsor takes the remaining assets of the firm $P-c+w$ and incurs some default cost K.[10]

As a benchmark, we occasionally refer to total welfare, that may depend on the cost of undertaking the project by the winning firm, the project's degree of performance, the cost of effort, and the bankruptcy costs,

$$W = \begin{cases} V + \theta - x - c & \text{non-default} \\ V + \underline{\theta} - x - c - K & \text{otherwise.} \end{cases}$$

3 Solving the Model

We characterize the equilibrium by backwards induction. We start with the performance stage.

[9]In many real-world procurement processes, payments and costs may occur over a period of time. As the firm can borrow funds or defer payments, this should not have an important impact over the firm's bankruptcy decision.

[10]Typically, the bankruptcy of the winning firm will be costly for the sponsor for several reasons: it may give rise to litigation costs and delays and will entail costs to complete the project, either by deploying her own means, or by paying a new contractor to complete the unfinished project.

3.1 Performance Stage

In this stage, the winning bidder faces an effective monetary penalty in case of poor performance. Due to limited liability, her penalty losses, z, are bounded by her available assets, $z = \min\{P - c + w, b\}$. Given z, the winning firm chooses a level of effort $x^*(z)$ which minimizes her expected total cost at the performance stage,

$$x^*(z) \in \arg\min p(x)z + x,$$

where z is the effective penalty faced by the contractor whenever poor performance occurs. Let $\gamma(z)$ be the expected private cost of poor performance for the contractor,

$$\gamma(z) = p(x^*(z))z + x^*(z). \tag{1}$$

Intuitively, $x^*(z)$ and $\gamma(z)$ are increasing in z. For further results, some additional characterization of $\gamma(z)$ is useful. In particular, $\gamma(0) = 0$, $\gamma(z) < b$, $\gamma'(z) = p(x^*(z)) < 1$,[11] and $\gamma''(z) \leq 0$.

3.2 Bidding Stage

We focus on the weakly dominant strategy equilibrium of the second-price auction. In our procurement setting, this implies that firms bid according to their total cost of undertaking the project. Hence, the equilibrium bid of firm i is the minimum price P_i^* for which firm i is willing to accept the project. We start by characterizing the net expected profits of the contractor

$$\pi_N(P, c, w, b) = P - c - \gamma(\min\{P - c + w, b\})$$

As $0 \leq \gamma'(z) \leq 1$, this net expected profits, π_N are increasing in $P - c$ and weakly decreasing in w and b. Then, wealthier firms with higher asset levels have lower net expected profits for a given markup $P - c$.

Then, the equilibrium bid is defined by:

$$E\{\pi_N(P_i^*, c_i, w_i, b)\} = 0, \tag{2}$$

In other words, the equilibrium bid of firm i is the price for which her net expected profits are zero, in case firm i wins the project.

If the firm has no risk of default, the price that makes the net expected profits zero is equal to the total cost, construction cost and performance cost, $P_i^* = c_i + \gamma(b)$. Equivalently,

[11] Notice that $\gamma'(z) = \frac{\partial \gamma}{\partial z} + \frac{\partial \gamma}{\partial x^*}\frac{\partial x^*}{\partial z} = \frac{\partial \gamma}{\partial z} = p(x^*(z))$ since by the envelope theorem $\frac{\partial \gamma}{\partial x^*} = 0$.

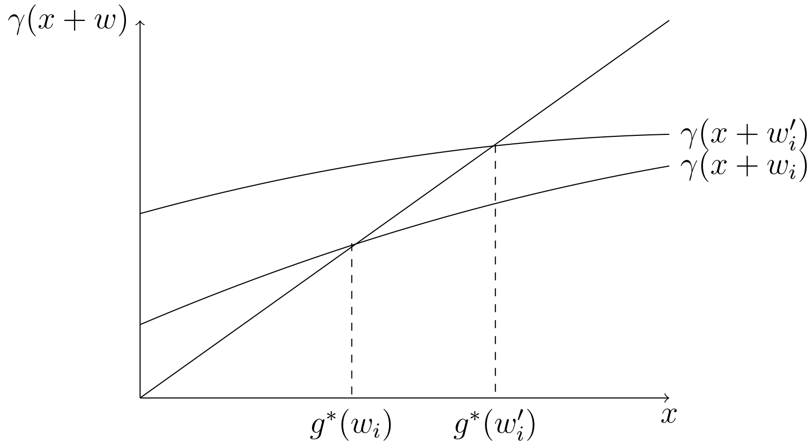

Figure 1 : The equilibrium markup equation $x^* = \gamma(x^* + w)$

if we take the markup between the contract price and the cost as the relevant variable, $P_i^* - c_i$, the non-default markup equals the expected penalty plus the expected costs of effort summarized in $\gamma(b)$.

If the firm defaults in case of poor performance, the price/bid that makes zero net expected profits is given by the expression $P_i^* = c_i + \gamma(P_i^* - c_i + w_i)$. Notice that the price is defined as a fixed point since it also affects the expected total wealth at the performance stage. We rewrite this expression in terms of a default markup function, $g^*(w_i) = P_i^* - c_i$, where $g^*(w_i)$ is implicitly defined by $g^*(w_i) = \gamma(g^*(w_i) + w_i)$.

Lemma 1. *The default markup function, $g^*(w_i)$, is uniquely defined for each w_i. It is increasing in w_i, and $g^*(0) = 0$.*

The proof of Lemma 1 is relegated to the Appendix, albeit the underlying intuition may be grasped from Figure 1: the default markup equation $x^* = \gamma(x^* + w)$ has a unique solution, since $\gamma(0 + w) > 0$ and $0 < \gamma'(x) < 1$ and $\gamma(x^* + w)$ is also increasing in w.

The default markup function is increasing in w, since, intuitively, in case of default, wealthier firms face larger losses. There exists a cut-off level $\widehat{w}(b) = b - \gamma(b)$ such that the default markup function is equal to the no default markup, $g^*(b - \gamma(b)) = \gamma(b)$. If $w = \widehat{w}(b)$ the firm is indifferent between making default or paying the penalty. For $w > \widehat{w}(b)$, firms strictly prefer to pay the penalty and they bid according to the no default markup, $\gamma(b)$. Then, summarizing, the equilibrium bidding markup is

$$g(b, w_i) = \begin{cases} \gamma(b) & \text{if } w_i > \widehat{w}(b) = b - \gamma(b) \\ g^*(w_i) & \text{otherwise.} \end{cases}$$

Then, the equilibrium bid is $P_i^* = c_i + g(b, w_i)$. Notice that firms wealthier than $\widehat{w}(b)$ do not plan to default and that this solvency threshold is increasing in the size of the penalty b.[12] In words, the larger the penalty, the larger the set of firms that may make default.

The next proposition summarizes the characterization of the bidding equilibrium, provides direct comparative statics, and a useful feature of the equilibrium markup function, $g(b, w_i)$.

Proposition 2. *The equilibrium bid in a second-price auction is $P_i^* = c_i + g(b, w_i)$, which is increasing in c_i and weakly increasing in b and w_i. Moreover, $g(b, w_i)$ (and consequently P_i^*) is supermodular in (b, w_i).*

The most important feature of this bidding equilibrium is that the equilibrium bid (probability of winning) is increasing (decreasing) in the level of assets w_i of the firm. In other words, financially weaker contractors are more likely to win the project.

It is important to notice that Proposition 2 is not an artifact of the second-price auction. For example, under perfect information on firms' types, the first-price auction and the second-price auction are revenue equivalent in this setting. In other words, they induce the same prices and allocations.[13] The equivalence between both mechanisms does not extend to private information settings. However, in the Appendix we generalize Proposition 2 by considering general incentive compatible mechanisms in which only the winning firm receives a transfer (as it is the case in the first-price auction). We show that incentive compatibility in this general class of mechanisms requires to adversely selects undercapitalized firms. In words, less capitalized firms face lower penalty expected costs, which gives them an advantage in competitive allocation mechanisms.[14] We can also illustrate Proposition 2 with the following algebraic example.

3.3 Example

Consider that $p(x) = 1 - \sqrt{x}$ and $w \in [0, 1]$. The contractor chooses a level of care $x^*(z)$ which minimizes her expected total cost given the penalty losses z,

$$x^*(z) \in \arg\min(1 - \sqrt{x})z + x. \rightarrow x^*(z) = \frac{z^2}{4} \tag{3}$$

Given $x^*(z)$, the expected total cost of the penalty is

$$\gamma(z) = p(x^*(z))z + x^*(z) = z - \frac{z^2}{4}.$$

[12]This is also implied by $0 < \gamma'(x) < 1$.

[13]A proof of this statement can be found in Appendix C.

[14]The intuition is as follows: In terms of direct mechanisms, a firm can always mimic the behavior of a firm with a higher level of assets and the same cost. Then, incentive compatibility implies that the probability of winning must be non-increasing in the level of assets.

The equilibrium markup if the firm is solvent is $\gamma(b) = b - \frac{b^2}{4}$, otherwise it is given by $g^*(w_i) = \gamma(g^*(w_i) + w_i)$. Then

$$g^*(w_i) = \gamma(g^*(w_i) + w_i) = g^*(w_i) + w_i - \frac{(g^*(w_i) + w_i)^2}{4}$$

This implies that $g^*(w_i) = -w_i + \sqrt{4w_i}$. Notice that $g^*(w_i)$ is increasing, $\frac{g^*(w_i)}{\partial w_i} = -1 + \frac{1}{\sqrt{w_i}} \geq 0$. If $\widehat{w} = b - \gamma(b) = \frac{b^2}{4}$ then $g^*(\widehat{w}) = b - \frac{b^2}{4} = \gamma(b)$ which implies that firms with wealth $w > \widehat{w}$ will not default and will pay the penalty.

The equilibrium bid is

$$P_i^* = \begin{cases} c_i + b - \frac{b^2}{4} & \text{if } w_i > \widehat{w} = b - \gamma(b) = \frac{b^2}{4} \\ c_i - w_i + \sqrt{4w_i} & \text{otherwise.} \end{cases}$$

Figure 2 plots the equilibrium markup $P_i^* - c_i$ and different values of firm wealth, w_i, and shows that undercapitalized firms bid more aggressively.

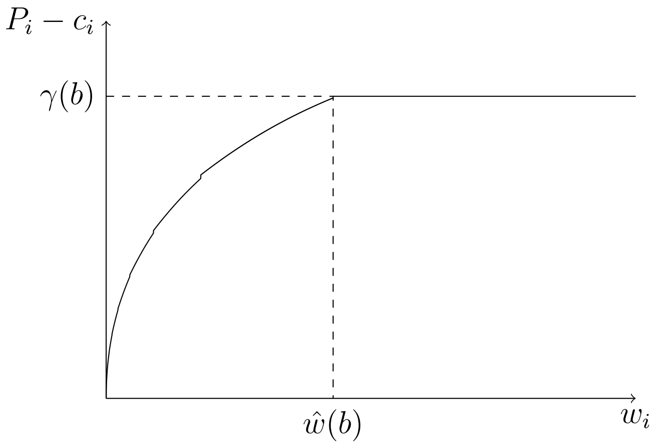

Figure 2 : The bidding equilibrium for b

4 Increasing incentives, worsening performance

The effects of increasing penalties on the outcome of the procurement process are counterintuitive. The following proposition presents one of our core results: more powerful incentive contracts may lead to worse allocations.

Proposition 3. *Let (w, c) be the type of the winner under penalty b. Under any higher penalty, $b' > b$, the winner (w', c') will be weakly less solvent and have weakly higher costs, i.e, $w' \leq w$ and $c' \geq c$.*

Figure 3 shows the bidding equilibrium when the penalty increases from b to b', and illustrates the intuition of Proposition 3

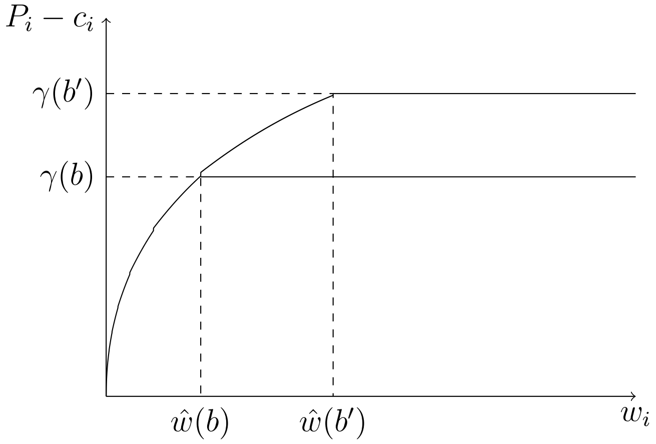

Figure 3 : The bidding equilibrium when the penalty increases from b to b',

If we increase the penalty from b to b', bidders weakly increase their bids. Proposition 2 states that $g(b, w_i)$ and the equilibrium bid are supermodular in (b, w_i). which means that wealthier bidders will increase their bids more than bidders with lower assets. This implies that the winner under b, either will remain the winner under b', or will be replaced by a firm with lower assets (since firms with lower assets than the original winner are now more competitive in relative terms). The second part of Proposition 3 is less intuitive at first blush: the winning firm is not only weakly less solvent, but it may also have higher costs. This is because if one firm wins the project under b', but loses under $b < b'$, the firm must have higher costs than the winner under b, otherwise it would also have won under b. In sum, a higher penalty may have the effect of inducing the selection of both a less solvent and a less efficient winner.

4.1 The optimality of low powered incentive contracts.

Proposition 3 shows that more powerful incentive contracts may lead to awarding the contract to a less efficient firm. In this subsection, we complement this result by showing that increasing incentives may lead to worse overall performance and, consequently, that a low powered incentive contract may be optimal in this setting. In order to do so, we analyze a particular case of the parametric example of Section 3.3 by considering two bidders with types (w_1, c_1) and (w_2, c_2) equal to $(1, 0)$ and $(0, c)$ respectively. We assume that the first bidder always pays the penalty $(b < 1)$ in case of poor performance, has zero marginal cost and she bids her opportunity cost for facing the penalty, $\gamma(b) = b - \frac{b^2}{4}$. The opportunity cost of the second bidder is just her cost c because she is fully insolvent, and she does not plan to pay the penalty in the case of poor performance. As $\gamma(b)$ is increasing, there exists a cut-off $\bar{b} = 2(1 - \sqrt{1-c})$ such that $\gamma(\bar{b}) = c$. Then, if $b \leq \bar{b}$ the first bidder wins at price c. In case of $b > \bar{b}$ the second bidder wins at price $b - \frac{b^2}{4}$. Then, this numerical example illustrates Proposition 3: if we increase the penalty from $b < \bar{b}$ to $b' > \bar{b}$, the winner changes, and the new winner under b' is less solvent and has higher costs.

Now we move to analyze equilibrium effort levels and the performance of the project. If $b < \bar{b}$, the first bidder wins, and will exert the level of effort $x_1^*(b) = \frac{b^2}{4}$. If $b > \bar{b}$ the second firm wins, and it will exert a level of effort $x_2^*(b) = \frac{(b - \frac{b^2}{4} - c)^2}{4}$. Notice that $x_1^*(\bar{b}) > x_2^*(\bar{b}) = 0$. Then, by increasing the penalty from $b < \bar{b}$ to $b' > \bar{b}$, the exerted level of effort may decrease. This highlights an unobserved additional cost of increasing incentives: the choice of effort (and performance) may be poorer under more powerful incentives schemes. Corollary 4 summarizes this insight.

Corollary 4. *Effort exerted by the winning bidder (and expected performance) is non monotonic in the penalty b.*

The next figure illustrates Corollary 4 for a specific value of the cost $c = .64$ which leads to $\bar{b} = 0, 8$.

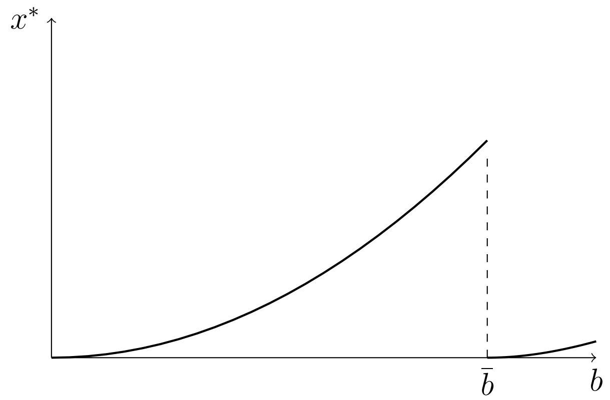

Figure 4 : Effort exerted in Equilibrium.

When the penalty is low, the winning bidder is fully solvent. In this case, a higher penalty translates into higher exerted effort. This happens until the penalty reaches $\bar{b}$, when firm 2 becomes the winning firm and the exerted effort drops to zero. However, for penalties higher than $\bar{b}$ the effort exerted by the winning firm (firm 2) depends on wealth at the performance stage, which is increasing in the procurement price. Then, as the procurement price weakly increases in the penalty, exerted effort may also increase. Along this dimension, as we will show in Figure 4, a higher price may not be necessarily bad news for the sponsor, since it translates into higher effort and performance and a lower probability of default.

As we have discussed above, there are important private and social costs associated with bankruptcy. Then, it is important to assess how incentives (penalties in our binary case) affect the likelihood of bankruptcy of the competitively selected supplier. We will see that the non-monotonicity of exerted effort with respect to the penalty (incentives) extends to the probability of bankruptcy. The probability of default depends on the identity of the winner, as well as on the equilibrium effort level. As long as $b < \bar{b}$, the first firm wins and there will be no bankruptcy. When $b > \bar{b}$ the second firm wins and bankruptcy arises, whenever there is poor performance (since penalty is lower than the current wealth of the firm $b > \gamma(b) - c$). However, conditioning of the second firm being the winner, the larger the penalty, the higher the price and the lower the probability of default.

Corollary 5. *The probability of bankruptcy is non monotonic in the penalty.*

The next figure illustrates Corollary 4 for an specific value of the cost $c = .64$ which leads to $\bar{b} = 0, 8$.

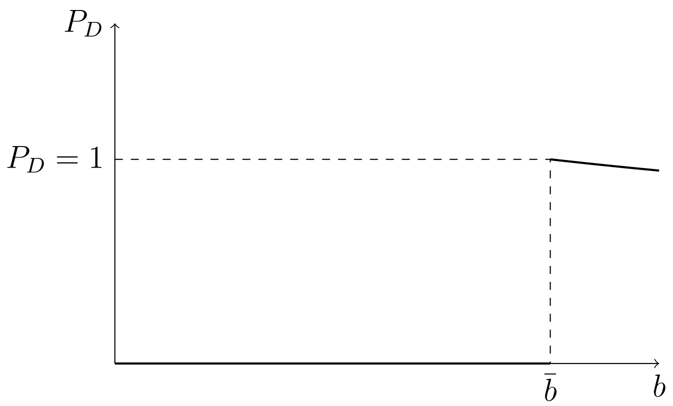

Figure 5 : The probability of bankruptcy in equilibrium

Bankruptcy arises when there is poor performance of the project and the firm is unable to pay the entire amount of the penalty. Then, in our example, when the penalty is low, $b < \bar{b}$ the winner is a fully solvent firm and the probability of default is zero. For larger penalties, $b > \bar{b}$, firm 1 is replaced by firm 2, who has no assets and goes bankrupt in the case of poor performance. Then, for this range of parameters ($b > \bar{b}$), the higher the penalty, the higher the price and the wealth of the winning firm, which translates into higher effort and lower probability of poor performance and bankruptcy.

All these results convey the general message that increasing incentives (penalty) may have adverse effects on performance. As a consequence of such effect, we can make the statement that when there is a default risk it may be optimal for the sponsor to set a penalty lower than the penalty that provides incentives to make the efficient level of effort without a default risk, $x^* \in \arg\max\{(1-p(x))\overline{\theta} + p(x)\underline{\theta} - x\}$

The first-best effort level without default risk is obtained by equalizing the performance variable and the monetary incentives, $b(\theta) = \theta$. Consider that in our binary setting, $\overline{\theta} = 0$, and $\underline{\theta} < 0$. The first-best level effort can be achieved with a penalty $b^E = b(\underline{\theta}) = \underline{\theta} < 0$ that we denote as the efficient penalty. However, when there is a risk of default by the winning firm, this efficient penalty may be far from optimal.

Proposition 6. *Under default risk affecting the winning firm, the efficient penalty may be dominated by lower powered incentives schemes (lower penalties).*

As a simple proof of the previous statement, we focus on our example and start with the simple observation that b^E may be higher or lower than $\overline{b}$. Consider the specification of the sponsor preferences we introduced in the description of the model. If $b^E < \overline{b}$, the optimal penalty in our simple example would be trivially b^E since there is no conflict between allocation and incentives. If $b^E > \overline{b}$, this conflict arises, and indeed, $\overline{b}$ dominates b^E along several dimensions: i) better allocation, $W_{\overline{b}} = (1, 0)$ and $W_{b^E} = (0, c)$, ii) lower price $\gamma(\overline{b}) = c < \gamma(b^E)$, and iii) lower probability of default $P_D(\overline{b}) = 0 < P_D(b^E) = 1 - \frac{(\gamma(b^E)-c)}{2}$. Given that, if the default cost K is sufficiently large, a penalty $\overline{b}$ lower than the efficient one b^E may be optimal.

In general, however, it is difficult to characterize the optimal penalty in this multidimensional setting. In the previous example, we have assumed that the solvent firm has a cost advantage. If both firms have the same costs, the insolvent firm would win under any amount of the penalty and defaults whenever there is poor performance. In such a case, it may be optimal to increase the penalty even beyond b^E in order to increase the price, the wealth of the contractor and the incentives to exert effort. However, in such a situation it may not be optimal to auction the project in the first place.[15] The auction is not selecting the best contractors and the reduction in contractor's rents produced by the auction may be counterproductive in terms of performance. In other words, competition may not help and other mechanisms (such as lotteries or negotiations) may be superior to auctions.[16]

[15]The suboptimality of auctions has been discussed in the literature that analyzes procurement settings with cost uncertainty and limited liability, see for example Burguet, Ganuza and Hauk (2012).Manelli and Vincent (1995) and Lopomo et al. (2022), analyze a similar adverse selection problem and show that standard competitive mechanisms work poorly when low-cost bidders are also linked with lower quality outcomes.

[16]This may partially explain why private firms often use negotiations instead of auctions. Bajari, McMil-

5 Extensions

5.1 Sticks and Carrots

In this extension, we do not impose any constraint over the pay-for-performance mechanisms beside $b(\underline{\theta}) < b(\bar{\theta})$. Consider the possibility of using penalties and positive payments or bonuses, for example, $b(\underline{\theta}) < 0$ and $b(\bar{\theta}) > 0$. We will show that transfers and allocations generated by the pay-for-performance mechanisms do not depend on the particular penalties and bonuses used but only on the power of the incentives scheme $b(\bar{\theta}) - b(\underline{\theta})$. In order to do that, we will show that this more general setting is in fact equivalent to the penalty setting solved in the baseline model.

As in the previous model, default may only occur if poor performance is observed and $P - c + w + b(\underline{\theta}) < 0$.[17] We rewrite this condition as

$$P - c + w + b(\bar{\theta}) < b(\bar{\theta}) - b(\underline{\theta}) \equiv \tilde{b}.$$

We reformulate the primitives as follows. Let

$$\tilde{c} = c - b(\bar{\theta}).$$

This is the bonus-adjusted cost. Then, the above condition is rewritten as

$$P - \tilde{c} + w < \tilde{b}.$$

This is analogous to the condition in the main model with $\tilde{c}$ and $\tilde{b}$; default occurs if the net penalty, $\tilde{b}$, exceeds the adjusted solvency, $P - \tilde{c} + w$. Using this notation, we can rewrite the payoff of the winning firm as

$$\begin{aligned} & P - c - p(x)\min\{P - c + w, -b(\underline{\theta})\} - x + (1 - p(x))b(\bar{\theta}) \\ = \quad & P - c - p(x)\min\{P - c + w + b(\bar{\theta}), b(\bar{\theta}) - b(\underline{\theta})\} - x + b(\bar{\theta}) \end{aligned}$$

lan, and Tadelis (2009) look at private construction contracts in Northern California and find that 43% of them have been awarded through negotiations with a sole supplier, and only 18% used open competitive tendering. In the same line Decarolis (2014) analyzes empirically a change in Italian public procurement rules, forcing the use of first price auctions for public works. He finds that the increased use of auctions between 2000 and 2006 results in significant discounts in winning bids, but also in significant deterioration of performance from winning bidders. Similarly, Eun (2018) shows with Korean paving contract data that reducing competition by bid screening and excluding abnormally low bids, lead to choose contractors with higher paving costs but that this is compensated by better contractor's performance (lower cost overruns).

[17] To make this point we do not need to assume $b(\bar{\theta}) > 0$. Firms will not default in equilibrium in the high performance state $\bar{\theta}$, since in such a case they will make negative profits and the participation constraint would not be satisfied.

$$= \quad P - \tilde{c} - p(x) \min\{P - \tilde{c} + w, \tilde{b}\} - x.$$

Hence, the use of sticks and carrots amounts to the use of only the penalty $\tilde{b}$ with the bonus-adjusted cost $\tilde{c}$. Let

$$\tilde{z} = \min\{P - \tilde{c} + w, \tilde{b}\}.$$

The optimal effort is then given by $x^*(\tilde{z})$, and the expected cost of effort is expressed by $\gamma(\tilde{z})$. Then, the firms' problem and the bidding equilibrium are equivalent to the ones characterized in the baseline penalty-only model, just replacing c and b by $\tilde{c}$ and $\tilde{b}$. The equilibrium bid of each firm i is then $P^*(\tilde{c}_i, w_i; \tilde{b}) = \tilde{c}_i + g(\tilde{b}, w_i)$ where

$$g(\tilde{b}, w_i) = \begin{cases} \gamma(\tilde{b}) & \text{if } w_i \geq \tilde{b} - \gamma(\tilde{b}) \\ g^*(w_i) & \text{otherwise.} \end{cases}$$

Following the same argument as in Proposition 2, the next corollary obtains immediately:

Corollary 7. *With sticks and carrots, the equilibrium bid in a second-price auction is* $P^*(\tilde{c}_i, w_i; \tilde{b})$*, which is increasing in* $\tilde{c}_i$ *and weakly increasing in* $\tilde{b}$ *and* w_i*, where* $\tilde{c}_i = c_i - b(\bar{\theta})$ *and* $\tilde{b} = b(\bar{\theta}) - b(\underline{\theta})$*.* $g(\tilde{b}, w_i)$ *(and consequently* P_i^**) is supermodular in* $(\tilde{b}, w_i)$*.*

Now we consider two different incentive schemes $b^1(\cdot)$ and $b^2(\cdot)$ for which $\tilde{b} = b^1(\bar{\theta}) - b^1(\underline{\theta}) = b^2(\bar{\theta}) - b^2(\underline{\theta})$. If we analyze the firm's equilibrium bid $P^*(\tilde{c}_i, w_i; \tilde{b}) = \tilde{c}_i + g(\tilde{b}, w_i)$, by construction $g(\tilde{b}, w_i)$ has to be the same for the two incentive schemes. These bids only differ in the adjusted cost part, $\tilde{c}_i = c_i - b^1(\bar{\theta})$ and $\tilde{c}_i = c_i - b^2(\bar{\theta})$. Then, the bids are the same in the both incentive schemes except for a constant shift $b^1(\bar{\theta}) - b^2(\bar{\theta})$. This implies that the winner and the second highest bidder are the same in both schemes, and so are the differences between the two highest bids, and consequently, the rents of the sponsor and the firms are the same. To summarize, we have the following result:

Proposition 8. *Two different incentive schemes* $b^1(\cdot)$ *and* $b^2(\cdot)$ *with the same power of incentives* $\tilde{b} = b^1(\bar{\theta}) - b^1(\underline{\theta}) = b^2(\bar{\theta}) - b^2(\underline{\theta})$ *yield the same equilibrium allocation and distribution of rents.*

In the next subsection, we will illustrate Proposition 8 by computing explicitly the bidding equilibrium for an incentive scheme that includes a bonus, and by obtaining the same outcomes as with just the penalty b for poor performance in our baseline setting.

5.2 "Sticks and Carrots" versus Penalty-Only

In the baseline penalty setup, we have the incentive scheme $b^1(\bar{\theta}) = 0$ and $b^1(\underline{\theta}) = -b$. Remember that the contractor's moral hazard problem in such a case was to minimize

$p(x)z + x$, where $z = \min\{P - c + w, b\}$, are the expected penalty losses. Now, we consider this alternative sticks and carrots scheme with the same power of incentive $b^2(\bar{\theta}) = \bar{b}$ and $b^2(\underline{\theta}) = \underline{b}$ such that $\bar{b} > 0 > \underline{b}$ and $\bar{b} - \underline{b} = b$.

The contractor's incentive problem with sticks and carrots is to minimize

$$-(1 - p(x))\bar{b} + p(x)z_B + x. \tag{4}$$

where $z_B = \min\{P - c + w, -\underline{b}\}$ is the amount of losses incurred by the contractor in the case of poor performance, and $-(1-p(x))\bar{b}$ is the reward the contractor gets for good performance (with a negative sign since the contractor's problem is written as a minimization problem). If we add and subtract $\bar{b}$ from this losses expression z_B, we get

$$z_B = -\bar{b} + \min\{P - c + \bar{b} + w, b\}$$

Let $\tilde{z}_B = \min\{P - \tilde{c} + w, b\}$ be the adjusted losses, that uses the bonus adjusted cost defined above $\tilde{c} = c - \bar{b}$. Then, if we plug $z_B = -\bar{b} + \tilde{z}_B$ into the contractor's objective function, we obtain

$$-\bar{b} + p(x)\tilde{z}_B + x. \tag{5}$$

When we use the adjusted cost $\tilde{c} = c - \bar{b}$, the contractor's incentive problem becomes almost identical to the her problem when the sponsor only uses penalties (they only differ in constant term $\bar{b}$). Then, the contractor chooses optimal effort level functions with the same shape in both cases, $x^*(z)$ (penalties) and $\tilde{x}^*(\tilde{z}_B)$ (sticks and carrots) such that $x^*(z') = \tilde{x}^*(z')$ for all z'.

By the same token, if we plug these optimal efforts in the objective functions, we obtain also two similar expected total cost functions $\gamma(z)$ and $\gamma(\tilde{z}_B) - \bar{b}$, that should be equal to the markup $P - c$ in the equilibrium bidding function. If the firm is solvent, then $z = z_B = b$ and the markup in the penalty case is $\gamma(b)$ and in the sticks and carrots case it is just $\gamma(b) - \bar{b}$. Then, in this latter case, the markup (and the bid) is the same as in the penalty-only case but is shifted down by the amount of the bonus $\bar{b}$.

When the firm may default in the case of penalty-only incentive, we define the markup as a function $p - c = g^*(w_i)$, which was implicitly defined by the equation $g^*(w_i) = \gamma(g^*(w_i) + w_i)$. In the case of sticks and carrots, $\tilde{z}_B = p - \tilde{c} + w_i$, and then the equilibrium markup is $p - c = \gamma(p - \tilde{c} + w_i) - \bar{b}$. This is equivalent to $p - c + \bar{b} = p - \tilde{c} = \gamma(p - \tilde{c} + w_i)$. Then if we take $p - \tilde{c} = \tilde{g}(w_i)$, we can obtain the same implicit equation for the equilibrium $\tilde{g}^*(w_i) = \gamma(\tilde{g}^*(w_i) + w_i)$. As they are characterized by the same equation, they should be the same function $\tilde{g}^*(w_i) = g^*(w_i)$. Finally, $p - \tilde{c} = g^*(w_i)$, using that $\tilde{c} = c - \bar{b}$ we obtain that the equilibrium markup is $p - c = g^*(w_i) - \bar{b}$. As in the no default case, the markup and bidding equilibrium are identical to the penalty-only case but they are shifted down by the size of the bonus.

Given that the default markup function is increasing in w (as in the baseline penalty model) there exists a cut-off level $\widehat{\widehat{w}}(b)$ such that the default markup function is equal to the no default markup, $g^*(\widehat{\widehat{w}}(b)) - \bar{b} = \gamma(b) - \bar{b}$. As the markup equilibrium functions are identical in the "sticks and carrots" case and the penalty-only case except that the amount of the bonus is discounted ex ante, the cut-off level also coincides $\widehat{\widehat{w}}(b) = \widehat{w}(b) = b - \gamma(b)$. Therefore, the equilibrium bidding markup in the "sticks and carrots" case is

$$g(\bar{b}, \underline{b}, w_i) = \begin{cases} \gamma(b) - \bar{b} & \text{if } w_i > \widehat{w}(b) = b - \gamma(b) \\ g^*(w_i) - \bar{b} & \text{otherwise.} \end{cases}$$

Then, the equilibrium bid is $\tilde{P}_i^* = c_i + g(\bar{b}, \underline{b}, w_i) = c_i + g(b, w_i) - \bar{b}$, which is equal to the baseline penalty-only case but with a downward shift in the amount of the bonus $\bar{b}$.

5.3 Negative Markups and Budget Constraints at the Construction Stage.

In the previous analysis we are not taking into account that when using bonuses the net wealth at the construction stage may be negative, i.e., $g(\bar{b}, \underline{b}, w_i) + w < 0$. In this section we consider that firms are constrained to finance ex-ante the project with their own resources and the markup granted by the contract. Then, this is equivalent to imposing a budget constraint at the construction stage. Namely,

$$P - c + w \geq 0,$$

When the bidder follows the bidding strategy as above, this condition is violated if

$$P_i^* - c_i + w_i \quad = \quad g(b, w_i) - \bar{b} + w_i < 0.$$

As $g(b, 0) = 0$, then, for $w_i = 0$ the left hand side is negative. By the previous analysis we know that $g(b, w_i) + w_i$ is increasing and it should be positive for high values of w_i. Then, there is some level of wealth $w(\bar{b}, \underline{b}) > 0$ such that contractors with $w_i \geq w(\bar{b}, \underline{b})$ may bid according to the bidding equilibrium with its own resources. For bidders with $w_i \leq w(\bar{b}, \underline{b})$, each bid is restricted to

$$P_i = c_i - w_i.$$

We will observe that this may destroy the equivalence between bonuses, and more generally "sticks and carrots" schemes, and the exclusive use of penalties. We come back to our numerical example. When the incentive system is penalty b for poor performance, this budget constraint is always satisfied, since markups are non-negative, as the level of wealth is. Consider now a system with the same degree of power in incentives, but based only on a bonus for good performance, $(\bar{b} = b, \underline{b} = 0)$. From the previous section we know that the bidding equilibrium requires to shift down the bidding function downwards in the size of the bonus, that is, in other words, to give a big discount in anticipation of future rents. In such a situation the budget constraint may play an important role. In particular, the equilibrium markup may be constrained by the wealth if $-w_i + \sqrt{4w_i} - b < -w_i$, or equivalently, $w_i < \widehat{w} = \frac{b^2}{4}$. If firms are constrained to finance ex-ante the project with their own wealth and the markup, the equilibrium bid would be

$$P_i^* = \begin{cases} c_i - \frac{b^2}{4} & \text{if } w_i \geq \widehat{w} = \frac{b^2}{4} \\ c_i - w_i & \text{otherwise.} \end{cases}$$

Notice that the equilibrium bidding function is weakly higher and more importantly, weakly decreasing in the level of wealth. The winner would be weakly wealthier, and then the equilibrium effort and project performance is better in this case than when b is purely a penalty.

5.4 Continuous performance

In this section, we extend our baseline model for considering a continuous performance variable, $\theta \in [\underline{\theta}, \overline{\theta}]$. Let x be the contractor's monetary equivalent of the performance effort, with the probability distribution of θ; $F(\theta|x)$, where x orders the distribution in the first-order stochastic dominance sense, so that greater effort increases the probability of obtaining better performance θ, if $x \geq x'$, then $F(\theta|x) \leq F(\theta|x')$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

The sponsor provides incentives to increase effort using a pay for performance mechanism $b(\theta)$, where $b(\theta)$ is increasing and maybe positive or negative. Besides making the performance variable a continuous one, the model remains identical to the binary case. We start with the analysis of the performance stage.

As before, the contractor's losses are bounded by her wealth. If $P - c + w + b(\theta) < 0$, then the winning firm goes bankrupt and loses $P - c + w$. As $b(\theta)$ is increasing, it may exist a cut-off $\widehat{\theta}$ such that $b(\widehat{\theta}) = -(P - c + w)$. Given this definition, if $\theta < \widehat{\theta}$, the winning firm goes bankrupt. Taking into account $b(\theta)$ and her current level of wealth, the winning firm chooses a level of care x_C minimizing her loss function at the performance stage[18]

[18]For consistency with the penalty baseline model, we write the optimization problem of the contractor as a minimization problem of her expected loss at the performance stage. However, depending on the incentive scheme $b(\theta)$, the loss function can be negative (the contractor will have positive profits at the performance stage).

$$x_C^*(\widehat{\theta}) \in \arg\min \int_{\underline{\theta}}^{\widehat{\theta}} (P-c+w)f(\theta|x)d\theta - \int_{\widehat{\theta}}^{\overline{\theta}} b(\theta)f(\theta|x)d\theta + x$$

Let $x_C^*(\underline{\theta})$ be the effort undertaken by the fully solvent firms, i.e $\widehat{\theta} = \underline{\theta}$. We will show that as in the baseline case, the contractor's effort is lower as the probability of default increases.

Lemma 9. *$x_C^*(\widehat{\theta})$ is decreasing in $\widehat{\theta}$.*

Let $\gamma_C(\widehat{\theta})$ be the expected loss of the contractor at the performance stage in the continuous case:

$$\int_{\underline{\theta}}^{\widehat{\theta}} (P-c+w)f(\theta|x_C^*(\widehat{\theta}))d\theta - \int_{\widehat{\theta}}^{\overline{\theta}} b(\theta)f(\theta|x_C^*(\widehat{\theta}))d\theta + x_C^*(\widehat{\theta})$$

As in the baseline case, this loss function is decreasing in the probability of default and then, it is indirectly increasing in the current wealth of the contractor at the performance stage.

Lemma 10. *$\gamma_C(\widehat{\theta})$ is decreasing in $\widehat{\theta}$ and then, increasing in $P-c$ and w.*

The net expected profits of the contractor are

$$\pi_N(P, c, w, b(\theta)) = P - c - \gamma_C(\widehat{\theta}(P-c+w))$$

Now we turn to the analysis of the bidding stage. As in the initial section, we focus on the weakly dominant strategy equilibrium of the second-price auction. Therefore, the equilibrium bid of each firm i is the minimum price P_i^* for which firm i is willing to accept the project, defined by:

$$E\{\pi_N(P_i^*, c_i, w_i, b(\theta))\} = 0, \tag{6}$$

If the firm has no risk of default, the price that makes the net expected profits zero is $P_i^* = c_i + \gamma_C(\underline{\theta})$. Then, the equilibrium markup, $P_i^* - c_i$, is equal to the expected loss in the performance stage $\gamma_C(\underline{\theta})$ (notice that γ_C that can be positive or negative, depending on $b(\theta)$).

If a firm may default for a poor performance realization, the price that makes the net expected profits equal to zero is given by the expression $P_i^* = c_i + \gamma_C(\widehat{\theta}(P_i^* - c_i + w_i))$. Let $g_C^*(w_i)$ be the markup in such a case, where $g_C^*(w_i)$ is implicitly defined by $g_C^*(w) = \gamma_C(\widehat{\theta}(g_C^*(w) + w))$.

Lemma 11. *$g_C^*(w)$ is uniquely defined for each w, it is increasing in w, and $g_C^*(0) = -b(\overline{\theta})$.*

Similarly to the binary penalty case, the default markup function is increasing in w since in the case of default, wealthier firms have larger losses (or lower expected profits if γ_C is negative). There exists a cut-off level $\widehat{w}(b(\theta)) = -b(\underline{\theta}) - \gamma_C(\underline{\theta})$ such that the default markup function is equal to no default markup, $g^*_C(-b(\underline{\theta}) - \gamma_C(\underline{\theta})) = \gamma_C(\underline{\theta})$.[19] Then, the equilibrium bidding markup is

$$g_C(b(\theta), w) = \begin{cases} \gamma_C(\underline{\theta}) & \text{if } w_i \geq -b(\underline{\theta}) - \gamma_C(\underline{\theta}) \\ g^*_C(w_i) & \text{otherwise.} \end{cases}$$

Then, the equilibrium bid is $P^*_i = c_i + g_C(b(\theta), w_i)$. Proposition 12 summarizes the characterization of the bidding equilibrium, and provides direct comparative statics.

Proposition 12. *The equilibrium bid in a second-price auction is $P^*_i = c_i + g_C(b, w_i)$, which is increasing in c_i and weakly increasing in w_i.*

Proposition 12 expresses the same message for the continuous case as Proposition 2 in the penalty binary baseline model: competitive mechanisms select undercapitalized firms.

We can also show that increasing incentives at the performance stage by means of a more powerful pay-for-performance scheme may lead to awarding the contract to a less efficient and less solvent firm, and to worse overall performance. Consequently, a low powered incentive contract may also be optimal in the continuous case.

Take an arbitrary pay for performance mechanism $b(\theta)$, where $b(\theta)$ is increasing for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Now, we consider a more powerful pay-for-performance scheme, $\tilde{b}(\theta) = (1 + \alpha)(b(\theta) - b(\underline{\theta})) + b(\underline{\theta})$, with $\alpha > 0$. Notice that $\tilde{b}(\theta)$ is an upward rotation of $b(\theta)$, and it is a more powerful pay-for-performance scheme because fully solvent firms undertake higher effort under $\tilde{b}(\theta)$ than under $b(\theta)$. Let $\tilde{x}^*_C(\underline{\theta})$ be the optimal effort exerted by fully solvent firms under $\tilde{b}(\theta)$. Then, we can show that $\tilde{x}^*_C(\underline{\theta}) \geq x^*_C(\underline{\theta})$. This is because the difference between the loss functions under $\tilde{b}(\theta)$ and $b(\theta)$ of fully solvent contractors at the performance stage

$$\tilde{\gamma}_C(\underline{\theta}) - \gamma_C(\underline{\theta}) = -\int_{\underline{\theta}}^{\overline{\theta}} \alpha[b(\theta) - b(\underline{\theta})] f(\theta|x) d\theta \leq 0$$

is increasing (as since $b(\theta)$ is increasing and x order the distribution $F(\theta|x)$ in the first-order stochastic dominance sense). In other words, these loss functions have decreasing differences and applying the results of Milgrom and Shannon (1994), we can conclude that $\tilde{x}^*_C(\underline{\theta}) \geq x^*_C(\underline{\theta})$.

[19]For checking this, assume that $g^*_C(\widehat{w}) = \gamma_C(\underline{\theta})$. Then, if we replace $g^*_C(\widehat{w})$ by $\gamma_C(\underline{\theta})$ in the equation $g^*_C(\widehat{w}) = \gamma_C(\widehat{\theta}(g^*_C(\widehat{w}) + \widehat{w}))$ we obtain $\gamma_C(\underline{\theta}) = \gamma_C(\widehat{\theta}(\gamma_C(\underline{\theta}) + \widehat{w}))$. For this equality be true, it has to be that $\widehat{w}$ is such that $\underline{\theta} = \widehat{\theta}(\gamma_C(\underline{\theta}) + \widehat{w}) = b^{-1}(-(\gamma_C(\underline{\theta}) + \widehat{w})$ that implies that $\widehat{w} = -b(\underline{\theta}) - \gamma_C(\underline{\theta})$.

The next step is to prove that when comparing the allocation under $\tilde{b}(\theta)$ and $b(\theta)$, we can also state that increasing incentives may lead to worse allocations. As $\tilde{b}(\theta) > b(\theta)$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$, the expected losses at the performance stage for the fully solvent firms are lower for this more powerful incentive scheme, $\tilde{\gamma}_C(\underline{\theta}) < \gamma_C(\underline{\theta})$. This implies that the default cut-off level is higher when higher incentives are in place, $\widehat{w}(\tilde{b}(\theta)) = -b(\underline{\theta}) - \tilde{\gamma}_C(\underline{\theta}) > \widehat{w}(b(\theta)) = -b(\underline{\theta}) - \gamma_C(\underline{\theta})$. Then, it is possible that a contractor with a level of wealth, $w \in [\widehat{w}(b(\theta)), \widehat{w}(\tilde{b}(\theta))]$ is fully solvent under $b(\theta)$ but not under $\tilde{b}(\theta)$. In such a case, this contractor would have under $\tilde{b}(\theta)$ a competitive advantage over wealthier fully solvent contractors, and it may be the winning firm under $\tilde{b}(\theta)$ even if it has higher costs and it would not have been the winning firm under $b(\theta)$.

Despite the fact that a more powerful incentive scheme leads to higher effort from fully solvent contractors, $\tilde{x}_C^*(\underline{\theta}) \geq x_C^*(\underline{\theta})$, it may also induce lower exerted effort from a winning bidder and thus, poorer performance. Firstly, because, as we show above, under a more powerful incentive scheme, a solvent winner may be replaced by a contractor who has a risk of default and may have lower incentives to exert effort. But even if the winner does not change, it may have less incentives to exert effort under a more powerful incentive scheme since it may default for a larger set of performance realizations.

Lemma 13. *Consider a contractor with a level of wealth such that it defaults for $\theta < \widehat{\theta}$ under $b(\theta)$ and for $\theta < \widehat{\widehat{\theta}}$ under $\tilde{b}(\theta)$ respectively. Then, $\widehat{\theta} < \widehat{\widehat{\theta}}$.*

The intuition for this result is as follows. Under a more powerful incentive scheme, contractors will obtain higher earnings in case of high performance realizations. As in the bidding equilibrium, contractors make zero profits with their bids. This would imply higher losses in the poor performance realization case, which leads contractors to default for a larger set of performance realizations. This result implies that the effort exerted by the winning bidder is not necessarily higher under a more powerful incentive scheme, as we state in Corollary 4 in the binary case.

Corollary 14. *Effort exerted by the winning bidder (and expected performance) is non-monotonic in the power of the incentive scheme.*

The intuition for the lack of monotonicity may be easily grasped by analyzing how the payoffs (and incentives) of the winning bidders depend on the performance realizations under $b(\theta)$ and under $\tilde{b}(\theta)$. If $\theta < \widehat{\theta}$, the firm makes default and loses its wealth under $b(\theta)$ and under $\tilde{b}(\theta)$. If $\theta \in [\widehat{\theta}, \widehat{\widehat{\theta}}]$, then the firm makes default under $\tilde{b}(\theta)$ but its payoff increases on the performance realization under $b(\theta)$. Finally, if $\theta > \widehat{\widehat{\theta}}$ firm does not make default, payoffs are increasing in the performance realization but at a higher rate under $\tilde{b}(\theta)$ than under $b(\theta)$. Then, these pay-off functions do not satisfy a single-crossing condition and the effort exerted under $b(\theta)$ can be higher or lower than the effort exerted under $\tilde{b}(\theta)$. We have shown above that the effort exerted by fully solvent firms is greater under the more powerful incentive scheme. However, we can think of examples where firms that may default may exert lower effort under a more powerful incentive scheme. Take as, in Lemma 13, a contractor that may default under $b(\theta)$ if $\theta < \widehat{\theta}$, and under $\tilde{b}(\theta)$ if $\theta < \widehat{\widehat{\theta}}$, where $\widehat{\theta} < \widehat{\widehat{\theta}}$.

Consider that $\theta \in [\underline{\theta}, \overline{\theta}] = [0,1]$ and there exist only two types of effort, $\underline{x} = c$ leading to a uniform performance distribution function over the interval $[0,1]$ and $\overline{x} = c + \epsilon$ that leads to a three steps distribution function, zero density over the interval $[0, \widehat{\theta}]$, a constant density equal to $\frac{\widehat{\theta}}{\widehat{\widehat{\theta}} - \widehat{\theta}}$ over the interval $[\widehat{\theta}, \widehat{\widehat{\theta}}]$ and a density of 1 over the interval $[\widehat{\widehat{\theta}}, 1]$. The effect of high effort is just to transfer all the measure on $[0, \widehat{\theta}]$ to the interval $[\widehat{\theta}, \widehat{\widehat{\theta}}]$, and then it generates a distribution that dominates the one generated by low effort according to FOSD. It is clear that the contractor will choose high effort, $\overline{x} = c + \epsilon$, under $b(\theta)$, since $b'(\theta) > 0$ on $[\widehat{\theta}, \widehat{\widehat{\theta}}]$ but low effort, $\underline{x} = c$, under $\tilde{b}(\theta)$ since the improvement of the performance distribution will have no effect on their profit (given that it will default if the performance realization is below $\widehat{\widehat{\theta}}$).

For a more general result of the adverse effect of incentives on the allocation, in the line of Proposition 3, we need to introduce an auxiliary lemma stating that the markup equilibrium function has increasing differences between incentives and wealth:

Lemma 15. *If the probability of default, $F(\widehat{\theta}|x_C^*)$, weakly increases as the incentive scheme becomes more powerful for all w, the equilibrium markups have increasing differences: for $w > w'$*

$$g_C(\tilde{b}(\theta), w) - g_C(b(\theta), w) > g_C(\tilde{b}(\theta), w') - g_C(b(\theta), w').$$

If we compare $b(\theta)$ and $\tilde{b}(\theta)$, the condition that $F(\widehat{\theta}|x_C^*)$ weakly increases is trivially satisfied if $w > \widehat{w}(\tilde{b}(\theta))$ since it is zero in both cases. For $w \in [\widehat{w}(b(\theta)), \widehat{w}(\tilde{b}(\theta))]$, it also holds generally, since the probability of default is zero for $b(\theta)$ and positive for $\tilde{b}(\theta)$. For $w < \widehat{w}(b(\theta))$, as $\widehat{\theta} < \widehat{\widehat{\theta}}$ if the effort is lower under $\tilde{b}(\theta)$, the condition automatically holds. Otherwise it may hold if the effect on the default probability of a higher performance threshold under $\tilde{b}(\theta)$ compensates the also higher performance effort.

Finally, we can state in the continuous case that more powerful schemes may lead the selection of both a less solvent and a less efficient winner.

Proposition 16. *Suppose that $F(\widehat{\theta}|x_C^*)$ weakly increases as the incentive scheme becomes more powerful. Let (w, c) be the type of the winner under $b(\theta)$, and $(\tilde{w}, \tilde{c})$ under a more powerful incentive scheme $\tilde{b}(\theta)$. Then, the winner $(\tilde{w}, \tilde{c})$ will be weakly less solvent and have weakly higher costs, i.e, $\tilde{w} \leq w$, and $\tilde{c} \geq c$.*

Lemma 15 states that wealthier winners decrease (weakly) less of their bids when there is upward rotation of the incentive scheme (which gives bidders more expected rents). As in the binary case, this implies that the winner under $b(\theta)$, either will remain the winner under $\tilde{b}(\theta)$ or will be replaced by a firm with lower assets, since wealthier firms are now less competitive in relative terms. Following the same argument as in Proposition 3, the winning firm is not only weakly less solvent, but it may also have higher costs. The new winner under $\tilde{b}(\theta)$ must have higher costs than the winner under $b(\theta)$ since otherwise it would also have won under $b(\theta)$.

It is important to notice that we have focused on a particular way to increase the incentives, namely a linear upward rotation of the initial incentive scheme, $\tilde{b}(\theta) = (1+\alpha)(b(\theta) - b(\underline{\theta})) + b(\underline{\theta})$. However, our results are more general in two dimensions. Firstly, following similar arguments than in the subsection 5.1 and 5.2, it can be shown that rents and allocation generated by $\tilde{b}(\theta)$ are invariant with respect to a parallel shift of $\tilde{b}(\theta)$ (adding or subtracting a constant). Then, in particular our results holds for a pure rotation of the incentive scheme, or when the more powerful incentive scheme implies a reduction in performance pay for all states. Secondly, our results also hold when α depend on θ, as long as $\alpha(\theta)' \geq 0$ for all θ.

6 Conclusions

The trade-off between providing incentives and achieving efficient allocations affects bidding markets generally. Perhaps most ostensibly, procurement seems to be particularly vulnerable to this. If not for reasons other than sheer magnitude, procurement is of key importance for economies of all kinds and sizes. Understanding what are the effects of rules and practices governing procurement processes is likely to prove relevant for ensuring value from the enormous amounts of public money invested in the provision of infrastructure, goods and services for citizens. Eventually, better understanding may lead to improvements in the design of current arrangements and institutions at play in this sector. Our analysis, we believe, sheds light on widespread features of procurement contracts. In particular, we explain why selection mechanisms based upon competitive bidding may favor undercapitalized firms for undertaking projects and how, when moral hazard is present in the execution of the contract, negatively affect the quality of outcomes. More importantly, we show that more powerful incentives may be counterproductive, leading to worse allocations (chosen contractors will have higher costs and lower assets than their rivals) and poorer performance of the contract. This constitutes an approach to explain the puzzling observation that, despite the important negative effects from inadequate performance in procurement projects, sponsors seem to deploy only mild penalties on non-performing contractors. One way to interpret our results is that the ability of sponsors to use powerful pay-for-performance mechanism is limited by the distribution of assets of potential contractors. In a number of recent empirical studies of procurement, examining the role of granting discretion to public officials in charge of organizing the procurement process and making decisions, it has been found that granting enhanced ability to public buyers results in selected contractors being financially larger and less risky, more experienced as partners of that buyer, and leading to better procurement auctions. This is the case in Coviello et al. (2018) and Coviello et al. (2022). In this spirit, in a recent study linked to the World Bank (Bosio et al. (2022)), analyzing procurement laws, practices and outcomes across a large dataset of countries, a clear negative relationship between timely outcomes in procurement and income level of countries is identified. In that paper, the driving force of the model and the empirical predictions is that in lower income countries, the capabilities of public sector sponsors are also more limited. Discretion, thus is largely exploited in these countries by public officials to choose less efficient contractors who offer benefits to the awarding individuals, whereas in high income countries with solid public sector capabilities, discretion is fruitfully used to improve contractor's selection and, with it, procurement outcomes (in line with Coviello et al. (2018), Coviello et al. (2022), or Decarolis et al. (2020)). In light of our findings, one could add a different explanation for the observed pattern. In lower income countries, the financial standing of the pool of potential contractors is worse than in higher income ones. Therefore, in the former set of countries, the ability of sponsors to use more ambitious incentives mechanisms is also more limited than in the latter. Surely, looking at one side of the interaction (the sponsor, its capabilities, institutional constraints and framework) is important for understanding procurement outcomes, but keeping an eye on the other side (the pool of contracting candidates) is, we believe, also relevant. In the end, the outcomes will result from the incentives faced by both sides in the interaction.

Acknowledgement: We are grateful for the comments to Miguel Abito, Francesco Decarolis, Shingo Ishiguro, Steven Tadelis, participants at The 38th Annual Conference of the European Association of Law and Economics (EALE), Simposio de Analisis Economico 2021, Workshop in Organizational Economics at Summer Forum 2021, International Industrial Organization Conference, Asian Meeting of Econometric Society 2022, EARIE 2022, and participants at seminars at Universitat Pompeu Fabra, Hitotsubashi University, Kyoto University, Nagoya University, and Waseda University. The usual disclaimers apply. Fernando Gomez acknowledges the financial support of the Spanish Ministry of Science and Innovation under project DER 2017-82673-R. Juan-Jose Ganuza gratefully acknowledges the support of the Barcelona School of Economics Research, the government of Catalonia, and the Spanish Ministry of Science and Innovation through the project PID2020-115044GB-I00. Makoto Hanazono acknowledges the support of Japan Society for the Promotion of

Science, grant number 17KK0067.

References

[1] Bajari, P., S. Houghton, and S. Tadelis (2014) "Bidding for Incomplete Contracts: An Empirical Analysis of Adaptation Costs" *The American Economic Review*, 104, 4, 1288-1319.

[2] Bajari, P., R. McMillan, and S. Tadelis (2009) "Auctions Versus Negotiations in Procurement: An Empirical Analysis" *Journal of Law, Economics, and Organization*, 25(2), 372–399.

[3] Bigoni, M, G. Spagnolo and P. Valbonesi (2014) "Sticks and Carrots in Procurement: An Experimental Exploration". *The B.E. Journal of Economic Analysis Policy* 14(3), 893–936.

[4] Board, S. (2007) "Bidding into the Red. A model of Post-Auction Bankruptcy" *Journal of Finance* LXII (6) 2695-2723.

[5] Bosio, E., Djankov, S., Glaeser, E and A. Shleifer. (2022) "Public Procurement in Law and Practice" *American Economic Review*, 112, 4, 1091-1117.

[6] Burguet, R., Ganuza, J. and Hauk, E. (2012) "Limited Liability and Mechanism Design in Procurement" *Games and Economic Behavior*, 76, 1, 15-25.

[7] Calveras, A., Ganuza, J. and Hauk, E. (2004). "Wild Bids. Gambling for Resurrection in Procurement Contracts" *Journal of Regulatory Economics*, 26(1), 41-68.

[8] Chakraborty, I, F. Khalil and J. Lawarree (2020). "Competitive Procurement with Ex Post Moral Hazard1" *RAND Journal of Economics* 52, 1, 179â206.

[9] Chillemi, O. and Mezzetti, C. (2011) "Procurement under default risk: auctions or lotteries?" Marco Fanno-Working Paper 131.

[10] Coviello, D., A. Guglielmo and G. Spagnolo (2018), "The Effect of Discretion on Procurement Performance", *Management Science*, 64(2): 715-738.

[11] Coviello, D., A. Guglielmo, C. Lotti, and G. Spagnolo (2022), "Procurement with Manipulation" mimeo.

[12] Coviello, D., Moretti, L., Spagnolo, G. and Valbonesi, P. (2018), "Court Efficiency and Procurement Performance". *Scandinavian Journal of Economics*, 120: 826-858.

[13] Decarolis, F. (2014) ”Awarding Price, Contract Performance and Bids Screening: Evidence from Procurement Auctions”, *American Economic Journal: Applied Economics*, 6(1), 2014.

[14] Decarolis, F. (2018) “Comparing Procurement Auctions,” *International Economic Review*, 59, 2, 391-419.

[15] Decarolis, F., R. Fisman, P. Pinotti, and S. Vannutelli (2020), “Rules, Discretion, and Corruption in Procurement: Evidence from Italian Government Contracting” mimeo.

[16] Decarolis, F., R. Pacini, and G. Spagnolo (2016), “Past Performance and Procurement Outcomes” NBER Working Paper No. 22814.

[17] Engel, A.R. and Wambach. A. (2006) “Public Procurement under limited liability” *Revista di politica economica* 96(1/2): 13-40.

[18] Eun, D-J. (2018) “Reducing ex-post adjustment by bid screening ex-ante: Evidence from procurement auctions for paving work” *Mimeo.*

[19] Ganuza, J. and Gomez, F. (2020) “Procurement and Accidents. The Liability Curse” *Journal of Legal Studies,* 49, 381-429.

[20] Holt, C. (1979) “Uncertainty and the Bidding for Incentive Contracts” *American Economic Review,* 69, 697-705.

[21] Holt, C. (1980) “Competitive Bidding for Contracts under Alternative Auction Procedures” *Journal of Political Economy,* 88, 433-445.

[22] Laffont, J-J. and J. Tirole. (1987) “Auctioning Incentive Contracts ” *Journal of Political Economy,* 95, 921-937.

[23] Lewis, G. and P. Bajari (2011) “Procurement Contracting With Time Incentives: Theory and Evidence ” *The Quarterly Journal of Economics,* 126, 1173-1211.

[24] Lewis, G. and P. Bajari (2014) “Moral hazard, incentive contracts, and risk: evidence from procurement” *Review of Economic Studies,* 81, 1201-1228.

[25] Lopomo, G, Persico, N and A. Villa (2022) “Optimal Procurement With Quality Concerns” *Federal Reserve Bank of Chicago Working Paper 2022-21.*

[26] Manelli, A. and Vincent, D. (1995) “Optimal procurement mechanisms” *Econometrica*, 591-620.

[27] McAfee, R. P., and McMillan, J. (1986) "Bidding for Contracts: A Principal-Agent Analysis" *RAND Journal of Economics*, 17, 326-338.

[28] McAfee, R. P., and McMillan, J. (1987) "Competition for Agency Contracts" *RAND Journal of Economics*, 18, 296-307.

[29] Milgrom, P and C. Shannon. (1994) "Monotone Comparative Statics" *Econometrica*, 62, 1,157-180.

[30] Shavell, S. (1986) "The Judgement-Proof Problem." *International Review of Law and Economics*, 6, 45-58.

[31] Waehrer, K. (1995). "A Model of Auction Contracts with Liquidated Damages" *Journal of Economic Theory* 67, 531-555.

[32] Walker, M. J., E. Katok and J. Shachat (2020): "Trust and Trustworthiness in Procurement Contracts with Retainage" ESI Working Paper 20-34.

Appendices

A Proofs

Proof of Lemma 1. As $g^*(w_i) = \gamma(g^*(w_i) + w_i)$ for proving the result, we have to show that the equation $x^* = \gamma(x^* + w_i)$ has a unique solution. This follows for the features of $\gamma(x)$, that it is an increasing function, it has a slope lower than one, and $\gamma(0 + w_i) \geq 0$. Then, $\gamma(0 + w_i) \geq 0$ and given that $\gamma'(x) \leq 1$ it cross the 45 degree line only once, $x^* = \gamma(x^* + w_i)$. Moreover, $g^*(w_i)$ is increasing in w_i. Let $g^* - \gamma(g^* + w_i) = 0$ by taking the total derivative $(1 - \gamma'(g^* + w_i))dg^* - \gamma'(g^* + w_i))dw_i = 0$. Then, $\frac{dg^*}{dw_i} = \frac{\gamma'(g^*+w_i)}{1-\gamma'(g^*+w_i)} > 0$. Finally, as $\gamma(0) = 0$, the unique solution of $g^*(0) = \gamma(g^*(0))$ is $g^*(0) = 0$. □QED

Proof of Proposition 2. The proof of the first part of the Proposition 2 (P_i^* being increasing in c_i, and weakly increasing in b, and w_i) is immediate from the arguments in the main text. Now, we need to show that $g(b, w_i)$ is supermodular, if $\overline{x}' > \overline{x}$ then $g(b', w) - g(b, w)$ is weakly increasing in w.

$$g(b, w_i) = \begin{cases} \gamma(b) & \text{if } w_i \geq \widehat{w}(b) = b - \gamma(b) \\ g^*(w_i) & \text{otherwise.} \end{cases}$$

As $\gamma(b)$ has a slope lower than 1, $\widehat{w}(b) = b - \gamma(b)$ is increasing. Consider $b' > b$,

$$g(b', w) - g(b, w) = \begin{cases} \gamma(b') - \gamma(b) & \text{if } w \geq \widehat{w}(b'). \\ g^*(w) - \gamma(b) & \text{if } \widehat{w}(b') \geq w \geq \widehat{w}(b) \\ 0 & \text{if } w < \widehat{w}(b) \end{cases}$$

Notice that as $g^*(w)$ is increasing in w, $g^*(\widehat{w}(b)) = \gamma(b)$ and $g^*(\widehat{w}(b')) = \gamma(b')$, then $g(b', w) - g(b, w)$ is a continuous and weakly increasing function of w. This implies that $g(b, w)$ is a supermodular function.□QED

Proof of Proposition 3: Let (w', c') be the wealth and cost of the winning firm under b', and (w, c) under b. Consider contrary to the Proposition 3 that the winner under b' is more solvent than the winner under b, that is, $w' > w$. Given that P^* is increasing in c and w, and that $P(w', c', b') < P(w, c, b')$, we must have $c' < c$. Then, the next two conditions have to be satisfied.

$$P(w', c', b') = c' + g(b', w') < c + g(b', w) = P(w, c, b'),$$

$$P(w', c', b) = c' + g(b, w') > c + g(b, w) = P(w, c, b).$$

They are equivalent to

$$g(b', w') - g(b', w) < c - c'$$

$$g(b, w') - g(b, w) > c - c'.$$

These two conditions imply that

$$g(b, w') - g(b, w) > g(b', w') - g(b', w)$$

or equivalently,

$$g(b', w) - g(b, w) > g(b', w') - g(b, w').$$

This leads to a contradiction with Proposition 1 , since $g(b, w)$ is supermodular, which implies that

$$g(b', w) - g(b, w) \leq g(b', w') - g(b, w')$$

Finally, consider contrary to the Proposition 3, that the winner under b' is more efficient than the winner under b, $c' < c$. The case in which the winner under b' is also more solvent, is discussed above. But, if the winner under b' is more efficient and less solvent, he should have won also under $\overline{x}$, since her bid is lower than the bid of (w, c) for any penalty.□QED

Proof of Corollaries 4 and 5: See the explanations given in the main text as well as the example and figures 3 and 4. □QED

Proof of Proposition 6 : The example provided in the main text and the discussion after Proposition 6 is a simple proof the result. □QED

Proofs of Corollary 7 and Proposition 8: Follow from the arguments of the main text.□QED

Proof of Lemma 9: We can rewrite loss function of the contractor at the performance stage as

$$\gamma_C(x, \widehat{\theta}) = -\int_{\underline{\theta}}^{\widehat{\theta}} b(\widehat{\theta}) f(\theta|x) d\theta - \int_{\widehat{\theta}}^{\overline{\theta}} b(\theta) f(\theta|x) d\theta + x$$

Using the results of Milgrom and Shannon (1994), it is enough to show that $\gamma_C(x, \widehat{\theta})$ has increasing differences. Notice that

$$\frac{\partial \gamma_C(x, \widehat{\theta})}{\partial \widehat{\theta}} = -b'(\widehat{\theta}) F(\widehat{\theta}|x)$$

Given that $b'(\widehat{\theta}) > 0$ and that x orders the distribution in the first-order stochastic dominance sense, $\frac{\partial \gamma_C(x,\widehat{\theta})}{\partial \widehat{\theta}}$ is increasing in x, which concludes the proof. □QED

Proof of Lemma 10: Given $b(\widehat{\theta}) = -(P - c + w)$. we rewrite the loss function $\gamma_C(\widehat{\theta})$ as follows

$$-\int_{\underline{\theta}}^{\widehat{\theta}} b(\widehat{\theta}) f(\theta|x_C^*(\widehat{\theta}))d\theta - \int_{\widehat{\theta}}^{\overline{\theta}} b(\theta) f(\theta|x_C^*(\widehat{\theta}))d\theta + x_C^*(\widehat{\theta})$$

Then by the envelope theorem

$$\frac{d\gamma_C(\widehat{\theta})}{d\widehat{\theta}} = -b'(\widehat{\theta})F(\widehat{\theta}|x^*(\widehat{\theta})) < 0$$

Finally, as $\widehat{\theta} = b^{-1}(-(P - c + w))$, is decreasing in $P - c$ and w, γ_C is increasing in $P - c$ and w. □QED

Proof of Lemma 11: The proof is very similar to Lemma 1 related to the binary penalty case. We start by showing that $g_C^*(0) = -b(\overline{\theta})$. Intuitively, the contractor with zero wealth only can make zero profits when she defaults with probability one. Then, $\widehat{\theta}(g_C^*(0))) = b^{-1}(-g_C^*(0)) = \overline{\theta}$ which implies that $g_C^*(0) = -b(\overline{\theta})$. Alternatively, $\gamma_C(\widehat{\theta}(g_C^*(0))) = \gamma_C(\overline{\theta}) = -b(\overline{\theta})$.

The next step is to show that $g_C^*(w_i) = \gamma_C(\widehat{\theta}(g_C^*(w_i) + w))$ has a unique solution. For the same arguments as Lemma 1, this is because γ_C is increasing in w, it has a slope lower than one ($\frac{\partial \gamma_C}{\partial w} = F(\widehat{\theta}|x_C^*(\widehat{\theta})) < 1$), and $\gamma_C(0 + w_i) > g_C^*(0)$. Finally, we show that $g^*(w_i)$ is increasing in w_i, following also the same arguments in Lemma 1. Let $g_C^* - \gamma_C(g_C^* + w) = 0$ by taking the total derivative $(1 - \gamma_C'(g_C^* + w))dg_C^* - \gamma_C'(g_C^* + w))dw = 0$. Then, $\frac{dg_C^*}{dw} = \frac{\gamma_C'(g_C^*+w)}{1-\gamma_C'(g_C^*+w)} > 0$. This is because, $0 < \gamma_C' < 1$.. □QED

Proof of Lemma 13: We know that the bidding equilibrium mark up has to lead to zero profit ex-ante with the optimal level of effort, we can write this condition for both incentive schemes as follows, for $b(\theta)$:

$$-wF(\widehat{\theta}|x^*) + \int_{\widehat{\theta}}^{\overline{\theta}} (b(\theta) + g^*(w))f(\theta|x^*)d\theta - x^* = 0$$

for $\tilde{b}(\theta)$

$$-wF(\widehat{\tilde{\theta}}|\tilde{x}^*) + \int_{\widehat{\tilde{\theta}}}^{\overline{\theta}} (\tilde{b}(\theta) + \tilde{g}^*(w)))f(\theta|\tilde{x}^*)d\theta - \tilde{x}^* = 0.$$

If we replace in this later equilibrium condition, $\tilde{x}^*$ by x^*, the expected ex-ante pay-off have to be negative since we are introducing a non optimal effort. Then, it has to be true that

$$-wF(\widehat{\theta}|x^*) + \int_{\widehat{\theta}}^{\overline{\theta}} (b(\theta) + g^*(w))f(\theta|x^*)d\theta > -wF(\widehat{\tilde{\theta}}|x^*) + \int_{\widehat{\tilde{\theta}}}^{\overline{\theta}} (\tilde{b}(\theta) + \tilde{g}^*(w))f(\theta|x^*)d\theta$$

Consider, contrary to the Lemma that $\widehat{\theta} \geq \widehat{\tilde{\theta}}$. Then, the previous inequalty can be written in such a case as follows:

$$\int_{\widehat{\tilde{\theta}}}^{\widehat{\theta}} (-w - (\tilde{b}(\theta) + \tilde{g}^*(w)))f(\theta|x^*)d\theta > \int_{\widehat{\theta}}^{\overline{\theta}} ((\tilde{b}(\theta) + \tilde{g}^*(w) - b(\theta) - g^*(w))f(\theta|x^*)d\theta$$

However, the left hand side is negative (because $-w < (\tilde{b}(\theta) + \tilde{g}^*(w))$ for $\theta > \widehat{\tilde{\theta}}$) and the right hand side is positive since (because $(\tilde{b}(\theta) + \tilde{g}^*(w) > b(\theta) + g^*(w)))$. Then, we have a contradiction, which implies that $\widehat{\theta} < \widehat{\tilde{\theta}}$. This concludes the proof. □QED

Proof of Lemma 15: Recall:

$$g_C(b(\theta), w) = \begin{cases} \gamma_C(\underline{\theta}) & \text{if } w \geq \widehat{w}(b(\theta)) = -b(\underline{\theta}) - \gamma_C(\underline{\theta}) \\ g_C^*(w) & \text{otherwise.} \end{cases}$$

and

$$g_C(\tilde{b}(\theta), w) = \begin{cases} \tilde{\gamma}_C(\underline{\theta}) & \text{if } w \geq \widehat{w}(\tilde{b}(\theta)) = -\tilde{b}(\underline{\theta}) - \tilde{\gamma}_C(\underline{\theta}) \\ \tilde{g}_C^*(w) & \text{otherwise.} \end{cases}$$

where $\tilde{\gamma}_C(\theta)$ and $\tilde{g}_C^*(w)$ are accordingly defined with $\tilde{b}(\theta)$.

Since $\tilde{b}(\theta)$ is a rotation of $b(\theta)$, the expected losses of performance stage for fully solvent firms are lower under $\tilde{b}(\theta)$ than under $b(\theta)$, i.e., $\tilde{\gamma}_C(\underline{\theta}) < \gamma_C(\underline{\theta})$. Noting that $\tilde{b}(\underline{\theta}) = b(\underline{\theta})$, we have $\widehat{w}(\tilde{b}(\theta)) > \widehat{w}(b(\theta))$.

Then

$$g_C(\tilde{b}(\theta), w) - g_C(b(\theta), w) = \begin{cases} \tilde{\gamma}_C(\underline{\theta}) - \gamma_C(\underline{\theta}) & \text{if } w \geq \widehat{w}(\tilde{b}(\theta)), \\ \tilde{g}_C^*(w) - \gamma_C(\underline{\theta}) & \text{if } \widehat{w}(\tilde{b}(\theta)) \geq w \geq \widehat{w}(b(\theta)), \\ \tilde{g}_C^*(w) - g_C^*(w) & \text{if } w < \widehat{w}(b(\theta)). \end{cases}$$

Notice that as $\tilde{g}_C^*(w)$ is increasing in w, $g_C^*(\widehat{w}(b(\theta))) = \gamma_C(\underline{\theta})$ and $\tilde{g}_C^*(\widehat{w}(\tilde{b}(\theta))) = \tilde{\gamma}_C(\underline{\theta})$, then $g_C(\tilde{b}(\theta), w) - g_C(b(\theta), w)$ is a continuous and weakly increasing function of w for $w \geq \widehat{w}(b(\theta))$ as seen in the binary penalty case. To complete the proof, we need to show that $\tilde{g}_C^*(w) - g_C^*(w)$ is also weakly increasing in w.

Recall $g_C^*(w)$ is implicitly defined by $g_C^*(w) = \gamma_C(\widehat{\theta}(g_C^*(w)+w))$, and $\widehat{\theta}(g_C^*(w)+w)$ satisfies $b(\widehat{\theta}(g_C^*(w)+w)) = -(g_C^*(w)+w)$. Substituting and rearranging, we have

$$g_C^*(w)(1 - F(\widehat{\theta}|x_C^*(\widehat{\theta}))) = wF(\widehat{\theta}|x_C^*(\widehat{\theta})) - \int_{\widehat{\theta}}^{\overline{\theta}} b(\theta) f(\theta|x_C^*(\widehat{\theta}))d\theta + x_C^*(\widehat{\theta}).$$

where $\widehat{\theta} = \widehat{\theta}(g_C^*(w)+w)$. Differentiating with respect to w, we obtain

$$g_C^{*\prime}(w) = \frac{F(\widehat{\theta}|x_C^*(\widehat{\theta}))}{1 - F(\widehat{\theta}|x_C^*(\widehat{\theta}))}.$$

Under the assumption that the equilibrium probability of default weakly increases as the incentive scheme becomes more powerful, we have

$$\tilde{g}_C^{*\prime}(w) \geq g_C^{*\prime}(w)$$

Integrating this for $w > w'$ and rearranging,

$$\tilde{g}_c^*(w) - g_c^*(w) \geq \tilde{g}_c^*(w') - g_c^*(w').$$

This concludes that the equilibrium markups have increasing differences.□QED

Proof of Proposition 16: As Lemma 15 has stablished the supermodularity of the markup functions, it follows from the same arguments than the proof of Proposition 3. □QED

B Monetary Care.

Let $z = P - c + w$ be now the initial wealth (before the effort decision) at the construction stage, of a firm that plans to make default in case of bad performance. This firm chooses a level effort $x^{M*}(z)$ which minimizes his expected total cost,

$$x^{M*}(z) \in \arg\min p(x)z + (1 - p(x))x,$$

In case of a bad performance the firm looses all its wealth and otherwise, its cost is limited to the exerted effort.[20] Compared to the non monetary effort case, the cost of effort is subsidized in the case of bankruptcy. Let $\gamma^M(z)$ be the expected private cost of being

[20]Notice that the problem remains regular since the second order condition is satisfied, $p(x)''(z-x) - 2p'(x)) \geq 0$.

liable when effort is monetary.

$$\gamma^M(z) = p(x^{M*}(z))z + (1 - p(x^{M*}(z))x^{M*}(z).$$

As in the case of non monetary effort, $x^{M*}(z)$ and $\gamma^M(z)$ are increasing functions, and $\gamma^M(0) = 0$, and $\gamma^{M\prime}(z) < 1$. Compared with the non monetary case, when effort is monetary and firms are not fully solvent, $x^{M*}(z) \geq x^*(z)$ (since care is partially subsidized) and $\gamma^M(z)$ $\leq \gamma(z)$ (since liability costs are lower). However, for the fully solvent firms, the equilibrium level of effort is the same than in the non monetary case, and then also the expected payoffs during the contracting stage.

The equilibrium level of effort will be

$$x_E^M(\overline{x}, w) = \begin{cases} x^*(b) & \text{if } P - c + w > b + x^*(b) \\ x^{M*}(z) & \text{otherwise.} \end{cases}$$

Following the arguments of the main text, we characterize the bidding equilibrium bid of each firm i as the minimum price P_i^* for which firm i is willing to accept the project. As in the non monetary case, if the firm does not plan to make default in the case of bad performance, the price that makes the net expected profits zero is $P_i^{M*} = c_i + \gamma(b)$, and the equilibrium markup in such case is: $P_i^{M*} - c_i = \gamma(b)$. Similarly to the main text, if the firm is not solvent, the price that makes the net expected profits equal to zero is given by the expression $P_i^{M*} = c_i + \gamma^M(P_i^{M*} - c_i + w_i)$ or, in terms of the markup, $g^{M*}(w_i) = \gamma^M(g^{M*}(w_i) + w_i)$, where $g^{M*}(w_i)$ is unique and increasing in w_i for the same arguments than in the non monetary case. It is important to notice that $\gamma^M(z) \leq \gamma(z) \Rightarrow g^{M*}(w_i) \leq g^*(w_i)$

As in the non monetary case, there exists a cut-off level $\widehat{w^M}$, such that the default markup function is equal to non-default markup, $\gamma(b)$. Then for this cut-off $g^{M*}(\widehat{w^M}) = \gamma^M(\gamma(b) + \widehat{w^M}) = \gamma(b)$. For this later equality to be true, it is necessary that $\gamma(b) + \widehat{w^M} = b + x^*(b)$. Then, the cut-off level $\widehat{w^M} = (1 - p(x^*(b)))b$, where $\widehat{w^M}(b)$ is increasing on b. As $\widehat{w}(b) = b - \gamma(b) \leq \widehat{w^M}(b) = b - \gamma(b) + x^*(b)$, more types will be liable under monetary care, but the equilibrium bidding markup is qualitatively the same:

$$g^M(b, w_i) = \begin{cases} \gamma(b) & \text{if } w_i > \widehat{w^M}(b) \\ g^{M*}(w_i) & \text{otherwise.} \end{cases}$$

In particular, $g^M(b, w_i)$ is supermodular in (b, w_i), if $b' > b$ then $g^M(b', w) - g^M(b, w) -$

is weakly increasing in w. Then, the proof is identical to the non-monetary case, if $b' > b$,

$$g^M(b', w) - g^M(b, w) = \begin{cases} \gamma(b') - \gamma(b) & \text{if } w \geq \widehat{w^M}(b'), \\ g^{M*}(w) - \gamma(b) & \text{if } \widehat{w^M}(b') \geq w \geq \widehat{w^M}(b), \\ 0 & \text{if } w < \widehat{w^M}(b). \end{cases}$$

As $g^{M*}(w)$ is increasing in w, and by definition $g^{M*}(\widehat{w^M}(b)) = \gamma(b)$ and $g^{M*}(\widehat{w^M}(b')) = \gamma(b')$, then $g^M(b', w) - g^M(b, w)$ is increasing in w. This implies that $g^M(b, w)$ and the equilibrium bid, $P^{M*} = c + g^M(b, w)$, are supermodular functions, and consequently Proposition 2 and Proposition 3 hold under monetary care.

C Revenue Equivalence between Second-Price Auction and First Price Auction with Perfect Information.

Let $P_1^* < P_2^* < ... < P_i^* < ... < P_n^*$ be the bids of the second-price auction, where we have labeled firms according to their bids, being firm 1 the firm with the lowest bid, and firm n, the firm with the highest bid. Hence, the outcome of the second-price auction will be that firm 1 wins at a price equal to P_2^*. In a first-price auction with perfect information, there exists a Nash equilibrium that generates the same outcome. Consider the following profile of bidding strategies $(P_2^*, P_2^* + \varepsilon, P_3^*, ..., P_i^*, ..., P_n^*)$ where ε is the minimum amount in which a bid can be modified. This is a Nash equilibrium because no one can profit by unilaterally deviating and it generates the same allocation and revenue than the second-price auction: firm 1 wins at a price equal to P_2^*. Before proceeding with the proof, it is important to notice that the bids in the second-price auction are the opportunity costs of the firm from accepting the contract. Meaning that firm i with a price lower than P_i^* would make negative profits in expectation, and with a price higher than P_i^* will make positive profits. Then, if firm 1 deviates and bids more than P_2^*, she looses the contract (50% of probability of loosing in the case of bidding $P_2^* + \varepsilon$) and gets zero instead of a positive payoff. Bidding lower than P_2^* reduces directly her profits. If firm 2 deviates and bids P_2^*, she gets zero profits as with the current bid. Bidding lower than P_2^* generates negative profits for Firm 2. Finally, making a bid higher than $P_2^* + \varepsilon$, does not modify the allocation and consequently, firm 2 also obtains zero profits. For some firm other than firm 1 and 2, if she bids a lower amount, either it does not win yet, or she gets a negative payoff in case she bids lower than the winning bid. If she bids higher, her payoff remains at zero. So this strategy profile is a Nash Equilibrium since no profitable unilateral deviations exists.

D General Allocation Mechanisms.

Proposition 2 in the main text states that a second-price auction mechanism adversely selects undercapitalized firms for the project, since for a given level of private cost in delivering the project, the lower the wealth w, the larger the probability of winning.

This market failure does not depend on the competitive mechanism used for allocating the project, since incentive compatibility leads to adverse selection in this incentive setting in which firms are protected by limited liability.

In order to formalize this claim, in this appendix we consider an arbitrary direct incentive compatible mechanism $\Psi(w, c), P(w, c)$ specifying a probability of winning Ψ_i and a price P_i in the case of winning for each firm i and for every vector of assets and costs (w, c).[21]

We focus on the incentives of agents with the same cost, but different assets, to truthfully reveal their types. Notice that by looking at the case of firms with identical costs and different levels of wealth, we do not restrict the domain of the types we are considering but we want to characterize a minimal necessary condition that must be satisfied for any incentive compatible mechanism. In order to do so, by taking expectations over the other types (w_{-i}, c_{-i}), we can redefine the mechanism according to the individual probability of winning

$$\Psi_i(w_i) = E_{w_{-i}}, c_{-i}\{\Psi(w_i, w_{-i}, c_{-i})\}$$

and a distribution of prices for each w_i, $P_i(w_i)$.

Proposition 17. *In any IC mechanism $(\Psi_i(w_i), P_i(w_i))$, for which the distribution of prices can be ranked in terms of First Order Stochastic Dominance, the probability of winning of an individual firm, $\Psi_i(w_i)$, is monotonically decreasing in w_i, and the expected price in case of winning, $P_i(w_i)$, is monotonically increasing in w_i.*

Proof: The firm indirect utility function is

$$\Pi_i(\widehat{w}, \widehat{c}, w_i, c_i) = \Psi_i(\widehat{w}, \widehat{c})\pi_i(P_i(\widehat{w}, \widehat{c}), w_i)$$

where $\widehat{w}$ and $\widehat{c}$ refer to revealed types while w_i and c_i are real types. We want to analyze the ex-ante incentives to reveal truthfully the level of assets of firms with the same cost but different assets. For each w, $\Psi_i(w_i)$ is a number, and $P_i(w_i)$ is a distribution of prices.

[21] We are assuming that only the winning firm receives a transfer. This is consistent with the motivation of the paper, focused on firms with different wealth levels and protected by limited liability, and moreover it is a common feature of most of the real world procurement processes. Alternatively, we could consider, for example, that all firms pay for participating in the auction and that the money thus raised is added to the price paid to the winning firm, reducing or eliminating the probability of expost bankruptcy. Chillemi and Mezzetti (2011) explores these procurement mechanisms.

Then, we define the ex-ante profit function.

$$\Pi_i(w_i', w_i) = \Psi_i(w_i')E_{P_i(w_i')}\{\pi_i(P, w_i)\}.$$

Where $\pi_i(P, w_i)$ is the profit function defined in the baseline model but to economize in notation we are omitting the dependence of profits with respect to b and c_i. Consider that $w_i' > w_i$. The incentive compatibility constraint on the general mechanism, $\Psi(w, c), P(w, c)$, has to imply the following conditions over the ex-ante incentives to reveal truthfully the level of assets of firms with the same cost but different levels of wealth

$$\Pi_i(w_i, w_i) \geq \Pi_i(w_i', w_i) \tag{7}$$

and

$$\Pi_i(w_i', w_i') \geq \Pi_i(w_i, w_i'). \tag{8}$$

Combining these two equations we obtain

$$\Pi_i(w_i, w_i) - \Pi_i(w_i, w_i') \geq \Pi_i(w_i', w_i) - \Pi_i(w_i', w_i')$$

which is equivalent to

$$\Psi_i(w_i)E_{P_i(w_i)}\{\pi_i(P, w_i) - \pi_i(P, w_i')\} \geq \Psi_i(w_i')E_{P_i(w_i')}\{\pi_i(P, w_i) - \pi_i(P, w_i')\}. \tag{9}$$

From Proposition 3 of the basic model, we know that $\pi_i(P, w_i) - \pi_i(P, w_i')$ is weakly decreasing in P. Now, assume for contradiction that $P_i(w_i') < P_i(w_i)$ (in terms of FOSD) since $\pi_i(P, w_i) - \pi_i(P, w_i')$ is decreasing in P, this implies that

$$E_{P_i(w_i)}\{\pi_i(P, w_i) - \pi_i(P, w_i')\} \leq E_{P_i(w_i')}\{\pi_i(P, w_i) - \pi_i(P, w_i')\}. \tag{10}$$

Given (10), inequality(9) can only be satisfied if $\Psi_i(w_i') \leq \Psi_i(w_i)$. But $P_i(w_i') < P_i(w_i)$ and $\Psi_i(w_i') \leq \Psi_i(w_i)$ violates the incentive compatibility constraint of type w_i' (inequality (8)). Hence $P_i(w_i') > P_i(w_i)$. But then $\Psi_i(w_i') \leq \Psi_i(w_i)$. Otherwise, the incentive compatibility constraint of type w_i would be violated (inequality (7)). This concludes the proof.

Then, financially weaker contractors are selected with higher probability in any incentive compatible mechanism. In our setting, the opportunity cost of undertaking the project is decreasing in the wealth of the contractor and then, any competitive mechanism would adversely select undercapitalized bidders.