Self-fulfilling labor wedge fluctuations and unemployment insurance*

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Abstract

We study unemployment insurance within an equilibrium business cycle model with labor search frictions, endogenous participation, and involuntary unemployment. The model yields an endogenous countercyclical labor wedge that may generate self-fulfilling fluctuations. We show that unemployment insurance makes local indeterminacy less pervasive and, therefore, is a powerful automatic stabilizer. In calibrated versions of the model, indeterminacy occurs for replacement rates that are empirically plausible for North America, but not for the replacement rates observed for European countries.

Keywords: labor wedge, indeterminacy, unemployment insurance.
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1 Introduction

Unemployment insurance not only provides timely welfare protection and social insurance to the short-term unemployed but is often also regarded as an important macroeconomic stabilizer. In fact, these two roles of unemployment insurance are often intertwined (Chodorow-Reich and Coglianese, 2019). This is because the stabilizing effect of unemployment insurance is traditionally thought to be most effective in environments featuring incomplete opportunities for private insurance (McKay and Reis, 2016). For example, unemployment insurance may dampen fluctuations in disposable income and thus stabilize the business cycle in environments with nominal rigidities and market incompleteness (Brown, 1955). Similarly, unemployment insurance may redistribute income across individuals that have different marginal propensity to spend and, thus, contribute to aggregate demand stabilization when markets are incomplete (Blinder, 1975). But in this paper we present an additional channel through which unemployment insurance may have both an effective and desirable (welfare improving) stabilization role even in a setting with perfect private insurance markets and, consequently, no motivation for redistribution. Unemployment insurance may help eliminate inefficient self-fulfilling unemployment fluctuations.

To do this, we develop an equilibrium business cycle model with perfect insurance markets, but frictional labor markets. The labor market features transitions across three-states, including employment, unemployment, and non-participation. To obtain an active labor force participation choice, we assume there is an opportunity cost of job search in the form of forgone utility. Conditional on participation, unemployed workers search for jobs in frictional labor markets, and the transition rate from unemployment to employment is assumed to be procyclical: it is easier for unemployed workers to find jobs when output is above its long-run trend. We show that combining an active labor force participation choice with a procyclical job finding rate yields an endogenous and countercyclical labor wedge.[1] The latter is defined as the gap between the marginal rate of substitution of consumption for leisure in the frictionless economy and the marginal product of labor.[2] The countercyclicality of the labor wedge has been shown to be an important feature of observed business cycle fluctuations (Galí et al., 2007; Hall, 2009; Shimer, 2009; Karabarbounis, 2014).

[1]Recent empirical work attributes an important role to the participation margin for labor market transitions, e.g., Elsby et al. (2015) showed that the participation margin accounts for one-third of the cyclical variation in the unemployment rate.

[2]The frictionless economy is obtained by assuming away search frictions in the labor market, and corresponds to the Hansen (1985) and Rogerson (1988) indivisible labor model in which perfectly insured individuals purchase lotteries over employment and non-employment (yielding a two-states labor market without involuntary unemployment). In the frictionless economy's equilibrium, the marginal rate of substitution of consumption for leisure corresponds to the opportunity cost of employment in units of consumption and equals the marginal product of labor.

In our model, the labor wedge is an outcome of search frictions, because it is costly for the stand-in agent to move members into employment. Moreover, because search frictions are more pronounced during downturns, the labor wedge increases in downturns. We show that given the active participation margin, an endogenous countercyclical labor wedge can generate local equilibrium indeterminacy. Specifically, the endogenous participation margin and the procyclical transition rate from unemployment to employment can generate a positive feedback loop leading to self-fulfilling unemployment fluctuations: when output and employment are high the job finding rate increases; this encourages higher labor force participation, in turn, raising employment and output even further. Crucially, the labor wedge is also affected by unemployment insurance and, in particular, the unemployment insurance replacement rate: larger replacement rates lower the volatility of the labor wedge and make it less countercyclical. Because of this, a sufficiently large replacement rate is shown to restore determinacy and, therefore, can be a powerful automatic stabilizer. We consider this to be our main analytical result.

Based on this result, we characterize a necessary condition for local indeterminacy by extending the methodologies of Benhabib and Farmer (1994), Aiyagari (1995) and Wen (2001) to economies with frictional labor markets. This necessary condition places restrictions on the employment elasticity of the labor wedge and, thus, on its countercyclicality. Since the labor wedge cyclicality is affected by the unemployment insurance (UI) replacement rate, the upshot is that by inverting the necessary conditions we are able to obtain a sufficient condition on the size of the UI replacement rate required to restore equilibrium determinacy and, thus, eliminate inefficient fluctuations. Raising the replacement rate sufficiently smoothes the labor wedge and, thus, eliminates belief-driven fluctuations, by making the participation rate less sensitive to changes in the transition rate from unemployment to employment.[3]

We calibrate the model and show numerically that indeterminacy arises for replacement rate policies observed in the United States and Canada (North America), while the equilibrium is locally unique for the policy parameters observed in European countries. Allowing the replacement rate and the (steady state) job finding rate to vary independently, we trace the regions of (in)determinacy of equilibrium across these two dimensions. European characteristics, namely, generous welfare states (high UI replacement rates) and sclerotic labor markets (low job finding rates), fall within the determinacy region, while those of North America, fluid labor markets and less generous policies compared to Europe, fall within the region of indeterminacy. Starting with the replacement rates observed in the United States, holding labor market frictions fixed and raising the replacement rate to

[3]This resonates with the result by Arseneau and Chugh (2012) regarding optimal taxation in models with frictional labor markets and endogenous participation (features that are also important in our model). In their framework, efficiency requires stabilizing the labor wedge over the business cycle.

European levels would be enough to restore the determinacy of equilibrium. This result is consistent with the common empirical finding that raising the replacement rate of unemployment insurance lowers business cycle volatility (see, for example, Di Maggio and Kermani, 2016). Conversely, given a fixed replacement rate, more sclerotic labor markets are more likely to be associated with a unique equilibrium.[4] Crucially, our calibration captures the salient facts distinguishing European and North American labor markets, according to Ljungqvist and Sargent (2008), that displaced workers in Europe suffer smaller earnings losses but also face lower reemployment rates than in North American labor markets.

Our paper contributes to the important literature on automatic stabilizers, understood broadly as features of the tax and transfer system that respond automatically to current conditions in the economy, thereby lowering business cycle volatility. The stabilizing effect of automatic stabilizers, in particular unemployment insurance, is traditionally thought to be potent in environments featuring missing insurance opportunities (Flodén, 2001; Alonso-Ortiz and Rogerson, 2010; Challe and Ragot, 2015; McKay and Reis, 2016). More recently, Kekre (2021) and Gorn and Trigari (2021), in environments with incomplete markets, nominal rigidities and labor market frictions, assess, both theoretically and quantitatively, the stabilising effect of unemployment insurance. Instead, we use the model developed in this paper to study the role of unemployment insurance in a setting with perfect private insurance markets and, consequently, no motivation for redistribution. Still, unemployment insurance is shown to have the potential to eliminate local indeterminacy and, therefore, is a powerful automatic stabilizer that can also have beneficial welfare effects by eliminating inefficient fluctuations driven by extrinsic uncertainty. Although there are other models in which automatic stabilizers reduce business cycle volatility despite the existence of perfect private insurance markets (for example, by lowering the aggregate labor supply elasticity, as in Janiak and Santos Monteiro, 2016), this is in general not efficient. Indeed, it is generally argued that for automatic stabilizers to improve welfare agents must be unable to enter private insurance contracts (McKay and Reis, 2021). In contrast, in the model with search externalities that we propose, unemployment insurance may both lower aggregate volatility and improve welfare in a set-up with perfect private insurance market, by preventing inefficient beliefs-driven business cycle fluctuations.

Finally, our paper is also closely related to the literature on automatic stabilisation in environments where fluctuations in economic activity are possible in the absence of shocks to fundamentals. Schmitt-Grohe and Uribe (1997) show that indeterminacy can arise in the neoclassical growth model

[4]When we conduct these comparative static experiments we are ignoring the possibility that the replacement rate affects the steady state job finding rate. In doing this, we are following the approach in Krusell et al. (2017) in treating labor market frictions as exogenous. However, with an endogenous job creation channel, these two parameters may be codetermined.

if tax rates are determined by a balanced budget with a pre-set level of revenues. Guo and Lansing (1998) argue that progressive income tax schedules can restore saddle path stability and, in related work, Christiano and Harrison (1999) show that indeterminacy can be prevented if the tax rate is an increasing function of aggregate employment. Nakajima (2006) shows that indeterminacy can arise even in the absence of externalities in the neoclassical growth model with asymmetric information and incomplete markets because moral hazard generates a downward-sloping labor supply curve. However, unemployment insurance weakens this mechanism and, thus, can potentially eliminate sunspot fluctuations. In our approach it is the participation choice and the labor market frictions which may generate self-fulfilling unemployment fluctuations, justifying automatic stabilizers in the form of unemployment insurance to eliminate belief-driven fluctuations, even in environments with perfect insurance markets.

The rest of the paper is organized as follows. Section 2 introduces the general equilibrium model. Section 3 looks at the model's dynamics and, in particular, derives necessary conditions for multiple rational expectations equilibrium to arise. Section 4 introduces unemployment insurance. Section 5 considers the calibration to map the model to the data. Finally, in Section 6 we study how labor markets with, in turn, European and North American characteristics compare through the lenses of our model. Section 7 concludes.

2 Model

In our model, labor market adjustments occur only along the extensive margin and there are three possible labor market states: employment, unemployment, and non-participation. Adjustments along the extensive margin are determined by individuals' indivisible choice over labor force participation in frictional labor markets. To overcome the resulting non-convexity, we consider the Hansen (1985) and Rogerson (1988) lottery mechanism, but with individuals purchasing lotteries over labor market participation and, conditional on participation, searching for jobs in frictional labor markets, as in Alvarez and Veracierto (1999, 2012) and Kokonas and Monteiro (2021).

Individuals derive flow utility from consumption, c, and incur a disutility cost from participation in the labor market. The flow utility at each date is given by

$$\mathcal{U}_t = \begin{cases} \ln C_t, & \text{if non-participant} \\ \ln C_t - \xi, & \text{if participant} \end{cases}, \tag{1}$$

where $\xi > 0$ is the opportunity cost of labor market participation due to forgone home production, which is incurred irrespectively of whether the individual is employed or unemployed. As in Hansen

(1985) and Rogerson (1988), individuals purchase lotteries where with probability $\pi \in (0,1)$ they participate in the labor force and, thus, sacrifice utility ξ. But in our model not all labor market participants are employed.

There are frictional labor markets and, conditional on participation, an individual may either be employed or unemployed. An individual who participates in the labor market but begins period t without a job, finds employment with probability $f_t \in (0,1)$. As in Blanchard and Galí (2010), and Michaillat (2012), newly hired workers become productive immediately. Conditional on participation, unemployment is involuntary and any equilibrium with unemployment is not Pareto optimal.

Individuals who fail to find a job are eligible for unemployment insurance, receiving a fraction $\zeta \in [0,1]$ of the total hourly wage rate. Unemployment insurance (UI) is financed with lump-sum taxes $T \geq 0$ levied on every individual. Each job is destroyed with constant probability $\lambda \in (0,1)$. Upon destruction, an individual is allowed to choose between searching for another job or staying out of the labor force. If the existing job is not destroyed, then the individual continues with the existing employment relationship.

Frictions restrict workers access to employment. However, conditional on employment, the labor market is competitive and wages reflect the marginal product of labor.[5] Goods and capital markets are competitive. There is no aggregate uncertainty (perfect foresight equilibria), but frictional labor markets generate idiosyncratic risk. Competitive firms have (identical) Cobb-Douglas technology which turns labor and capital into output, $Y = F(K,N) = K^{\alpha}N^{1-\alpha}$, $\alpha \in (0,1)$, and (K,N) denote the demand for capital and hours of work. Firms pay wages w to hire workers, r to rent capital, and maximize their flow of profits, $F(K,N) - rK - wN$.

2.1 Stand-in agent's problem

Time is discrete and the horizon is infinite, $t = 0,1,2,\ldots$; the measures of individuals at the end of date $t-1$ in employment, unemployment, or out of the labor force, in turn, N_{t-1}, U_{t-1}, O_{t-1}, are all pre-determined variables. Agents face three salient sources of idiosyncratic risk: the outcome of the lottery over labor force participation, the risk of job loss conditional on employment, and the risk of not finding work conditional on labor force participation. However, as in Andolfatto (1996), despite the random matchings and separations that occur in the labor market inducing different individual employment histories, this heterogeneity does not lead to wealth dispersion because of

[5] The assumption of competitive markets in coexistence with search frictions has a long tradition and follows, for example, Lucas and Prescott (1974), Alvarez and Veracierto (1999, 2012), and Krusell et al. (2008, 2010, 2011).

perfect insurance markets, and all individuals enjoy the same level of consumption no matter their labor force status.[6]

This market structure yields a stand-in agent representation, with life-time utility given by

$$\mathbf{V} = \sum_{t=0}^{\infty} \beta^t \Big(\ln C_t - \xi \Pi_t \Big), \tag{2}$$

with $\beta \in (0,1)$ the discount factor, and where Π_t is the labor force participation at date t, given by

$$\Pi_t = \pi_{e,t} \left(1-\lambda\right) N_{t-1} + \pi_{u,t} U_{t-1} + \pi_{o,t} O_{t-1} + \pi_{\lambda,t} \lambda N_{t-1}, \tag{3}$$

with $\pi_{e,t}$, $\pi_{\lambda,t}$, $\pi_{u,t}$, $\pi_{o,t} \in [0,1]$, the probabilities of labor force participation at date t chosen by individuals that start date t, in turn, matched to a job that survives, a job that is destroyed, in unemployment, and out of the labor force. The stand-in agent must choose state contingent allocations to maximize (2) subject to the budget constraint

$$C_t + I_t = r_t K_t + w_t \Big[N_{t-1}(1-\lambda)\pi_{e,t} + f_t H_t \Big] + b_t (1-f_t) H_t - T_t, \tag{4}$$

where H_t is the measure of individuals searching for jobs at date t, given by

$$H_t = \pi_{u,t} U_{t-1} + \pi_{o,t} O_{t-1} + \pi_{\lambda,t} \lambda N_{t-1}. \tag{5}$$

Thus, the job searchers at date t include all the individuals who were not in employment at date $t-1$ and choose to participate in date t, and also the individuals who were employed at date $t-1$, but who see their job destroyed at date t and choose to remain in the labor force. A proportion $(1-f_t)$ of the job searchers fail to find a job and, thus, earn unemployment benefits, given by $b_t = w_t \zeta$, with ζ the unemployment replacement rate.

Finally, the capital accumulation equation is given by

$$K_{t+1} = I_t + \left(1-\delta\right) K_t, \tag{6}$$

where I_t denotes investment, K_{t+1} is the end of period capital stock holdings and $\delta \in (0,1)$ is capital depreciation.

[6] Appendix A describes how the aggregation result is obtained.

2.2 Equilibrium

The optimality conditions solving the stand-in agent's problem are

$$\xi \leq \frac{f_t w_t + (1 - f_t) b_t}{C_t}, \tag{7}$$

$$\xi \leq \frac{w_t}{C_t}, \tag{8}$$

$$1 = \beta R_{t+1} \left(\frac{C_t}{C_{t+1}} \right), \tag{9}$$

where (7)–(8) are the intratemporal optimality labor supply conditions. Condition (9) is the standard Euler equation with $R_{t+1} = 1 - \delta + r_{t+1}$.

Conditions (7) and (8) determine the participation choices made by individuals. They are written as inequalities because we allow for corner solutions, where some individuals participate with certainty. In the sequel, we restrict attention to equilibria in which some individuals choose not to participate in the labor force. For this to be the case, we must have that (7) holds as equality, meaning that there is an interior solution for the tuple $(\pi_{\lambda,t}, \pi_{u,t}, \pi_{o,t})$. The upshot is that (8) holds as an inequality and, hence, all individuals holding a job choose to participate, $\pi_{e,t} = 1$.

Recalling that $b_t = \zeta_t w_t$, the intratemporal optimality condition (7) can be written as follows

$$w_t = \tau_t \left(\frac{\xi}{1/C_t} \right), \quad \text{with} \quad \tau_t = \frac{1}{f_t + (1 - f_t)\, \zeta}, \quad \text{the labor wedge,} \tag{10}$$

where w_t corresponds to the marginal product of an employed worker, and ξC_t corresponds to the worker's opportunity cost of employment in the frictionless economy. Efficiency requires the marginal product of employment to be equal to the opportunity cost of employment and, therefore, the wedge between these two quantities, $\tau_t \geq 1$, corresponds to the labor wedge. The frictionless economy is achieved when $f_t = 1$ for all t, in which case we obtain the original Hansen (1985) economy, in which there is no involuntary uynemployment and the equilibrium is efficient. Indeed, it is easy to verify that in the frictionless economy $\tau_t = 1$ and, therefore, there is no labor wedge. Thus, the labor wedge in our economy results from labor market search frictions. This is because conditional on participation, inactivity is socially wasteful: in other words, involuntary unemployment is suboptimal in our economy.

The model equilibrium conditions correspond to those of the canonical neoclassical growth model with indivisible labor, augmented with an endogenous labor wedge, as follows

$$1 = \beta R_{t+1} \left(\frac{C_t}{C_{t+1}} \right), \tag{11}$$

$$w_t C_t = \tau_t \xi, \tag{12}$$

$$C_t + K_{t+1} = K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t, \tag{13}$$

$$w_t = (1-\alpha) \left(\frac{K_t}{N_t} \right)^{\alpha}, \tag{14}$$

$$R_t = 1 - \delta + \alpha \left(\frac{N_t}{K_t} \right)^{1-\alpha}, \tag{15}$$

where (11)–(12) are the intertemporal and intratemporal optimality conditions discussed above, condition (13) is the goods' market clearing condition, and (14)–(15) obtain from the firm's optimal decisions. The motion equations for the aggregate labor market variables, are

$$N_t = (1-\lambda) N_{t-1} + H_t f_t, \tag{16}$$

$$U_t = (1 - f_t) H_t, \tag{17}$$

$$\Pi_t = N_t + U_t, \tag{18}$$

where we impose $\pi_{e,t} = 1$ to obtain (16).

Finally, the government budget is given by

$$T_t = b_t U_t, \tag{19}$$

where proceeds from lump-sum taxation finance UI benefits.

The system of equilibrium conditions (11)–(19) fully determines the equilibrium path for a given sequence of job finding rates $\{f_t\}_{t \geq 0}$. In particular, it is possible to define a unique steady state equilibrium up to a choice for the steady state job finding rate, as shown in Appendix B.

3 Search Frictions and Indeterminacy

As is clear from (10), the labor wedge, τ_t, is endogenous and determined by the job finding rate, f_t. Next, we show that if the job finding rate is procyclical (and, thus, the labor wedge is countercyclical), then the equilibrium dynamics around the steady state may feature local indeterminacy, enabling inefficient self-fulfilling fluctuations. We extend the method of Wen (2001) to analyze economies with labor market frictions and determine necessary conditions for local indeterminacy. Our focus is to provide an intuitive explanation for the emergence of inefficient self-fulfilling fluctuations through necessary conditions for local indeterminacy.

3.1 The job finding rate

The job finding rate, which measures the ease of matching job seekers with jobs, has been found to be procyclical in empirical studies. Specifically, during economic expansions, the job finding rate tends to increase, and during recessions, it tends to decrease (Shimer, 2005; Hall, 2005). To align our model with this empirical evidence, we impose the following restriction on the equilibrium sequence of job finding rates, $\{f_t\}_{t\geq 0}$.

Assumption 1 *(procyclical job finding rate)*

$$\widehat{f}_t = \sigma \widehat{Y}_t, \;\; \forall t,$$

where $\sigma > 0$ *denotes the output elasticity of the job finding rate and hat variables,* $\widehat{f}_t$ *and* $\widehat{Y}_t$*, denote log-deviations from the steady state.*

Assumption 1 posits that the job finding rate is procyclical. Crucially, this assumption creates an externality in the model with endogenous participation. The procyclical nature of the job finding rate leads to a positive feedback loop: when output and employment are high, the job finding rate increases, which in turn encourages more individuals to participate in the labor force, further raising employment and output. This externality implies inefficiency in competitive equilibrium and the possibility of sunspot equilibria. Next, we obtain necessary conditions for local indeterminacy and, thus, the emergence of inefficient sunspot equilibria.

3.2 Necessary conditions for local indeterminacy

The stand-in agent consumes a constant fraction $(1-\beta)$ of her permanent income, as follows

$$C_t = (1-\beta)\left[R_t K_t + w_t N_t + b_t U_t - T_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)(w_{t+j}N_{t+j} + b_{t+j}U_{t+j} - T_{t+j})\right]. \quad (20)$$

Thus, indeterminacy in consumption requires indeterminacy in permanent income. Following Wen (2001), we express (20) as a function of the equilibrium path of employment $\{N_t\}_{t=0}^{\infty}$. Once the equilibrium path of employment is determined, then we can also determine the equilibrium path of output, consumption and capital. The task of characterizing the local dynamics of the model around the steady state is, therefore, reduced to the examination of the indeterminacy of the equilibrium path of employment.

Iterating the Euler equation (11) forward, yields

$$\left(\prod_{i=1}^{j} R_{t+j}^{-1}\right) C_{t+j} = \beta^j C_t. \tag{21}$$

In turn, substituting the intratemporal condition (10), the government budget (19), and the iterated Euler equation (21) into (20), yields

$$C_t = R_t K_t \left[\sum_{j=0}^{\infty} \beta^j \left(1 - \xi \tau_{t+j} N_{t+j}\right)\right]^{-1}. \tag{22}$$

Substituting (10) into (22), yields

$$\sum_{j=0}^{\infty} \beta^j \left(1 - \xi \tau_{t+j} N_{t+j}\right) = \frac{\xi \tau_t R_t K_t}{w_t}, \tag{23}$$

where the right-hand side of (23) is the value of wealth in utility terms.[7] Log-linearising (23) around the steady state yields

$$\Gamma \sum_{j=0}^{\infty} \beta^j \left(\widehat{N}_{t+j} + \widehat{\tau}_{t+j}\right) = \widehat{\tau}_t + \widehat{R}_t - \widehat{w}_t + \widehat{K}_t, \tag{24}$$

with $\Gamma = -(1-\beta)\xi\bar{\tau}\bar{N}\left(1 - \xi\bar{\tau}\bar{N}\right)^{-1}$, and where $\bar{X}$ denote the steady state of variable X, and $\widehat{X}$, the log-deviation from steady state. The following two lemmas allow us to express (24) as a function of the sequence $\left\{\widehat{N}_{t+j}\right\}_{j=0}^{\infty}$, and subsequently apply the method of Wen (2001) for deriving necessary conditions for local indeterminacy.

Lemma 1 *The log-deviation of the labor wedge from the steady state is given by*

$$\widehat{\tau}_t = -\bar{\tau}\bar{f}\left(1 - \zeta\right)\widehat{f}_t, \ \ \forall t. \tag{25}$$

[7] Wen (2001) applies this methodology to the model of Schmitt-Grohe and Uribe (1997), where the labor wedge follows from distortionary labor income taxes and, thus, is exogenous. However, in our set-up the labor wedge is an outcome of labor market frictions and is endogenous. Thus, we generalize the method of Wen (2001) for deriving necessary conditions for local indeterminacy in an economy with endogenous labor wedge fluctuations. More generally, (23) characterises equilibrium of an RBC economy with indivisible labor and time-varying labor income taxes financed with lump-sum subsidies.

Proof. Follows immediately from the log-linearisation of the labor wedge (10) ■

Given (25) and Assumption 1, we can define the employment elasticity of the labor wedge, which is given by

$$\varepsilon_\tau \equiv \frac{\partial \widehat{\tau}_t}{\partial \widehat{N}_t} = -\bar{\tau}\bar{f}\,(1-\zeta)\,(1-\alpha)\,\sigma, \tag{26}$$

where we made use of the log-linear production function, $\widehat{Y} = \alpha\widehat{K}_t + (1-\alpha)\,\widehat{N}_t$. Thus, the labor wedge reacts countercyclicaly to output. Subsequently, (25) implies that the left-hand side of (24) depends on $\sum_{j=0}^{\infty}\beta^j\hat{f}_{t+j}$, which, in turn, after taking into account Assumption 1 and $\widehat{Y} = \alpha\widehat{K}_t + (1-\alpha)\,\widehat{N}_t$, is determined by $\sum_{j=0}^{\infty}\beta^j\hat{K}_{t+j}$ and $\sum_{j=0}^{\infty}\beta^j\widehat{N}_{t+j}$, as follows.

Lemma 2 *The infinite sum* $\sum_{j=0}^{\infty}\beta^j\widehat{\tau}_{t+j}$ *reduces to*

$$\sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t+j} = \omega\widehat{K}_{t-1} + \chi\widehat{N}_{t-1} + \varphi\sum_{j=0}^{\infty}\beta^j\widehat{N}_{t+j}, \tag{27}$$

where ω, χ, φ *are functions of structural parameters and* ε_τ *and* $\varphi < 0$.

Proof. See Appendix D ■

Substituting (25) and (27) into (24) yields

$$\sum_{j=0}^{\infty}\upsilon_j\widehat{N}_{t+j} = \psi_0\widehat{K}_t + \psi_1\widehat{K}_{t-1} + \psi_2\widehat{N}_{t-1}, \tag{28}$$

where the right-hand side depends on predetermined variables at date t, and υ_j and ψ are elasticity parameters.[8] This lead us to the following equilibrium characterisation.

Proposition 1 *If the equilibrium transition path for* N_t *around the steady state follows a stationary AR(1) process, then the equilibrium is locally unique. If the equilibrium path is not unique, then it must follow a stationary AR(2) process around the steady state.*

Proof. The proof follows similar ideas from Wen (2001). We discuss the key points.

First, if $\widehat{N}_t$ is AR(1) then the equilibrium is unique. Let

$$\widehat{N}_{t+1} = \rho\cdot\widehat{N}_t,\ \ \rho\in(0,1).$$

Substituting it into (28) and iterating forward, we obtain that $\widehat{N}_t$ is pinned down by the right hand

[8] See Appendix E for detail derivations of these parameters.

side predetermined variables at date t. The equilibrium is unique.

Under indeterminacy, $\widehat{N}_t$ must follow an autoregressive process with an order higher than one. Consider an AR(2):

$$\widehat{N}_{t+2} = \rho_1 \widehat{N}_{t+1} + \rho_2 \widehat{N}_t, \ \ \rho_1 + \rho_2 < 1. \tag{29}$$

Under repeated iteration, (28) implies

$$\hat{\rho}_1 \widehat{N}_{t+1} + \hat{\rho}_2 \widehat{N}_t = \psi_0 \widehat{K}_t + \psi_1 \widehat{K}_{t-1} + \psi_2 \widehat{N}_{t-1},$$

where $\hat{\rho}_1, \hat{\rho}_2$ are functions of the two stable roots. Given the predetermined variables, $\widehat{N}_t$ is indeterminate unless $\widehat{N}_{t+1}$ is known. Since $\widehat{N}_{t+1}$ cannot be solved by backward iteration using (29), it is unknown at time t. The equilibrium is not unique.

Finally, $\widehat{N}_t$ cannot follow an autoregressive process of a higher order than two, because the state space of the model has the dimension of at most two, including one state variable and one co-state variable ■

Next, through a series of steps, we characterise necessary conditions for local indeterminacy.

Proposition 2 *The employment elasticity of the left-hand side of (23) is negative if and only if*

$$-\varepsilon_\tau < \bar{\varepsilon} \equiv \frac{\alpha\beta\left(\bar{Y}/\bar{K}\right) - \alpha\beta\delta}{\alpha\beta\left(\bar{Y}/\bar{K}\right)}, \tag{30}$$

where $\bar{Y}/\bar{K}$ is the output-capital ratio in steady state.

Proof. Log-linearising the left hand side of (23) yields

$$\Gamma \sum_{j=0}^{\infty} \beta^j \left(\widehat{N}_{t+j} + \widehat{\tau}_{t+j} \right), \tag{31}$$

substituting (25) into (31) and ignoring predetermined variables, yields

$$\Gamma\left(1+\varphi\right) \sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}.$$

with $\Gamma = -(1-\beta)\xi\bar{\tau}\bar{N}\left(1-\xi\bar{\tau}\bar{N}\right)^{-1} < 0$ because intratemporal optimality and labor market clearing imply

$$\xi\bar{\tau}\bar{N} = \frac{\bar{w}\bar{N}}{\bar{C}} = \frac{1-\alpha}{\bar{C}/\bar{Y}} = \frac{1/\beta - 1 + \delta - \alpha\delta - \alpha(1/\beta - 1)}{1/\beta - 1 + \delta - \alpha\delta} < 1,$$

where $\bar{C}/\bar{Y}$ is the steady state consumption-output ratio (see Appendix B). Thus, $\Gamma(1+\varphi)$ is negative if and only if $\varphi > -1$. In Appendix F we show that $\varphi > -1$ is true if and only if (30) holds.

To complete the argument we need to show that

$$\text{sign}\left\{\sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}\right\} = \text{sign}\left\{\widehat{N}_t\right\}. \tag{32}$$

The proof is identical to the proof of Proposition 2 in Wen (2001). The key point is as follows. The equilibrium employment path $\{\widehat{N}_t\}_{t=0}^{\infty}$ is converging with an autoregressive order of at most two (see proposition 1). Solving the second order system and then substituting into $\sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}$ satisfies (32) ■

The emergence of belief driven fluctuations is a result of a positive feedback loop between current actions and expectations about the behaviour of current and future aggregate variables. The result of Proposition 2 shows how current actions reinforce future expectations, providing the first building block towards the construction of necessary conditions for local indeterminacy. Specifically, using the intratemporal condition (10) and rearranging appropriately, the left hand side of (23) reduces to

$$\frac{1}{1-\beta} - \sum_{j=0}^{\infty} \beta^j \left(\frac{1}{C_{t+j}} w_{t+j} N_{t+j}\right), \tag{33}$$

which is a function of the present discounted value of labor income in utility terms. In that respect, condition (30) is necessary and sufficient for the (current) employment elasticity of the present value of labor income in utility terms to be positive. Hence, expectations regarding the future path of aggregate variables and permanent income are reinforced by current actions. To close the positive feedback loop and, hence, complete the construction of necessary conditions for local indeterminacy, we need to show how given expectations influence current actions.

Before stating the necessary condition formally, we prove two useful results.

Proposition 3 *If the labor wedge is constant, then the equilibrium is locally unique.*

Proof. If $\sigma = 0$, then the labor wedge is constant. Then, it suffices to show that given the steady state level of $\bar{K}$ there is no N_t other than steady state $\bar{N}$ that satisfies (23). We prove this by contradiction. Suppose there is $N_t > \bar{N}$ such that (23) is satisfied, then

$$\sum_{j=0}^{\infty} \beta^j \left(1 - \xi \bar{\tau} N_{t+j}\right) = \frac{\xi \bar{\tau} R\left(\bar{K}, N_t\right) \bar{K}}{w\left(\bar{K}, N_t\right)}. \tag{34}$$

By Proposition 2, expression (30) is satisfied and the left hand side of (34) satisfies:

$$\sum_{j=0}^{\infty} \beta^j \left(1 - \xi \bar{\tau} N_{t+j}\right) < \sum_{j=0}^{\infty} \beta^j \left(1 - \xi \bar{\tau} \bar{N}\right).$$

Next, the gross interest rate R is increasing in N_t and the real wage is decreasing in N_t. The right hand side of (23) satisfies:

$$\frac{\xi \bar{\tau} R\left(\bar{K}, N_t\right) \bar{K}}{w\left(\bar{K}, N_t\right)} > \frac{\xi \bar{\tau} R\left(\bar{K}, \bar{N}\right) \bar{K}}{w\left(\bar{K}, \bar{N}\right)},$$

which implies that

$$\sum_{j=0}^{\infty} \beta^j \left(1 - \xi \bar{\tau} N_{t+j}\right) < \sum_{j=0}^{\infty} \beta^j \left(1 - \xi \bar{\tau} \bar{N}\right) = \frac{\xi \bar{\tau} R\left(\bar{K}, \bar{N}\right) \bar{K}}{w\left(\bar{K}, \bar{N}\right)} < \frac{\xi \bar{\tau} R\left(\bar{K}, N_t\right) \bar{K}}{w\left(\bar{K}, N_t\right)}.$$

This is a contradiction ■

Proposition 4 *A necessary and sufficient condition for indeterminacy is that the employment elasticities on both sides of Equation (23) are the same.*

Proof. Starting from an equilibrium path, a small deviation of N away from this path will not violate Eq. (23) if and only if the changes on both sides of (23) brought about by the deviation in N exactly offset each other. In such a case, N is indeterminate given the state K. This is the same as saying that the labor elasticities on both sides of (23) are the same ■

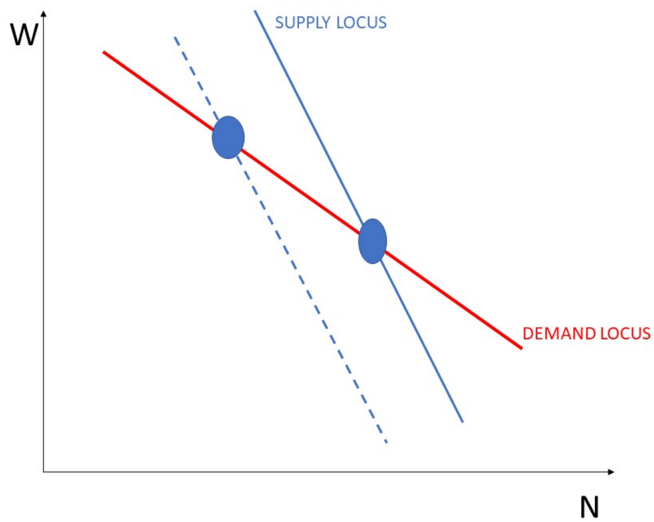
Proposition 5 *(Necessary condition) Suppose* $\alpha < \bar{\alpha} \equiv 1/\left[1 + \delta\left(\alpha\beta\left(\bar{Y}/\bar{K}\right)(1-\delta)\right)^{-1}\right]$. *A necessary condition for local indeterminacy is*

$$-\varepsilon_\tau \in \left(\underline{\varepsilon}, \bar{\varepsilon}\right), \tag{35}$$

where $\underline{\varepsilon} = \varepsilon_R - \varepsilon_w$, $\varepsilon_w < 0$ *is the employment elasticity of real wages,* $\varepsilon_R > 0$ *is the employment elasticity of gross interest and* $\alpha < \bar{\alpha}$ *ensures that* $\bar{\varepsilon} > \underline{\varepsilon}$.

Proof. A necessary condition for local indeterminacy requires that the employment elasticities

Figure 1: Equilibrium employment-wage loci



of the left and right hand side of (23) have the same sign. Given Proposition 2, the sign of the right hand side of (23) has a negative sign as long as $-\varepsilon_\tau > \varepsilon_R - \varepsilon_w \equiv \underline{\varepsilon}$, which, in turn, reduces to $-\varepsilon_\tau > \alpha\beta\left(\bar{Y}/\bar{K}\right) + \alpha\beta(1-\delta)$. Setting $\underline{\varepsilon} = \bar{\varepsilon}$ we derive $\bar{\alpha}$, which is a function only of $\{\beta, \delta\}$, and for $\alpha < \bar{\alpha}$ (35) is a well-defined interval ■

The restriction on primitive parameters that ensures (35) is a well-defined interval is satisfied by common calibrations in the literature and, in that sense, it is a weak restriction.[9]

The necessary condition (35) consists of two bounds. Satisfaction of the upper bound $\bar{\varepsilon}$, as discussed after Proposition 2, ensures that current actions reinforce future expectations, while satisfaction of the lower bound $\underline{\varepsilon}$ ensures that current actions positively covary with initial held beliefs, which, in turn, can be interpreted in terms of the restriction it places on the slopes of the equilibrium employment-wage loci derived from the demand and supply side of the model, which, for simplicity, we refer to as the labor demand and supply schedules. Specifically, aggregate labor demand is downward sloping ($\varepsilon_w < 0$) and aggregate labor supply is downward sloping as well ($\varepsilon_\tau < 0$). The lower bound of the necessary condition for local indeterminacy requires that aggregate labor supply is steeper than aggregate labor demand, that is, $-\varepsilon_\tau > -\varepsilon_w + \varepsilon_R > -\varepsilon_w$.[10]

[9] In our calibration $\bar{\alpha} \approx 0.61$ and common parameterisations of the capital income share fall below this threshold.

[10] The mechanism in Bennett and Farmer (2000) and Nakajima (2006) rely on downward sloping labor supplies to generate self-fulfilling fluctuations. The former approach augments the one-sector neoclassical growth model with non-separable preferences between consumption and leisure and increasing returns in production. Indeterminacy requires that the labor supply curve is downward sloping and steeper than the (standard) downward-sloping labor

Figure 1 summarises the previous discussion. For example, expecting a recession in the future shifts the labor supply schedule to the left (following a drop in current consumption given an expectation of lower permanent income), which, in turn, is consistent with lower current labor force participation and employment, confirming the initial held beliefs (satisfaction of the lower bound). Furthermore, satisfaction of the upper bound in (35) ensures that the drop in current employment reduces permanent income further (satisfaction of the upper bound), which, in turn, reduces current consumption and employment even further, closing the positive feedback loop between actions and expectations.

4 Unemployment Insurance

The main result of the paper characterizes sufficient conditions regarding optimal replacement rates that restore saddle path stability. The key insight is that by inverting the necessary condition for local indeterminacy we obtain sufficient conditions for saddle path stability.[11]

More formally, we establish the following result.

Proposition 6 *It is always possible to design a UI policy to guarantee a unique equilibrium path around the steady state. In particular, there exists replacement rates* $\zeta \in \left[\bar{\zeta}, 1\right]$*, with*

$$\bar{\zeta} = \frac{\sigma(1-\alpha) - \underline{\varepsilon}}{\sigma(1-\alpha) + \left(1/\bar{f} - 1\right)\underline{\varepsilon}}, \tag{36}$$

for which there is a unique saddle path stable equilibrium.

Proof. Follows immediately by setting $-\varepsilon_\tau = \sigma(1-\alpha)\bar{f}\bar{\tau}(1-\zeta) = \underline{\varepsilon}$, and using the expression of the labor wedge $\bar{\tau} = 1/\left[\bar{f} + (1-\bar{f})\zeta\right]$ at steady state, we construct the bound $\bar{\zeta}$. Then, for all $\zeta \in \left[\bar{\zeta}, 1\right]$ the (lower bound of the) necessary condition is violated and there is a unique path converging to the steady state. ■

demand curve. While Nakajima (2006) considers a variation of the neoclassical growth model with unobservable effort, efficiency wages and limited unemployment insurance for unemployed workers. The labor supply is replaced by the relevant no-shirking condition (NSC). Under partial income insurance, the NSC is downward sloping and steeper than the downward sloping labor demand, and indeterminacy obtains.

[11] It is important to note that for common calibrations of the parameters $\{\alpha, \beta, \delta\}$ found in the literature, there always exists a stable equilibrium around the steady state for any $-\varepsilon_\tau \geq 0$ (see Appendix C). The issue is then whether the equilibrium path is unique (saddle path stability) or whether there exist multiple paths converging to the steady state (local indeterminacy).

To inspect the mechanisms further, consider the intratemporal optimality (10) rearranged as follows:

$$\xi = \frac{w_t f_t}{C_t} + \frac{(1 - f_t)\zeta w_t}{C_t}, \tag{37}$$

which equates the exogenous cost of participation ξ to the expected return that the household receives from participating and searching for a job, for those starting from non-employment or with a job that is destroyed. Beliefs about the state of the economy affect current labor market conditions via externalities through the job finding rate f. If $\zeta = 1$, then the job finding rate cancels from the right hand side of (38) and there is no feedback loop between current actions and expectations. The result of Proposition 6 states that if replacement rates exceed bound $\widetilde{\zeta}$, then the positive feedback loop between current actions and expectations is weakened substantially and inefficient belief-driven fluctuations are eliminated.[12]

Note that replacement rates consistent with Proposition 6 violate the lower bound of the necessary conditions for local indeterminacy. What about violations of the upper bound of the necessary condition that might happen for sufficiently low values of the replacement rate? Setting $\zeta = 0$ in (37) yields $\xi = (w_t f_t)/C_t$, and beliefs about the state of the economy still affect current labor market conditions via externalities through the job finding rate f. In that case, the determinacy properties of the steady state can not be analysed without precise information regarding the strength of the externalities through f, captured by the parameter σ. Following a similar argument with Proposition 6, violation of the upper bound $\bar{\varepsilon}$ requires replacements rates $\zeta < \underline{\zeta}$, with

$$\underline{\zeta} = \frac{\sigma(1 - \alpha) - \bar{\varepsilon}}{\sigma(1 - \alpha) + \left(1/\bar{f} - 1\right)\bar{\varepsilon}}, \tag{38}$$

and the sign of $\underline{\zeta}$ depends non-trivially on the strength of externalities summarised by the parameter σ. For example, if $\underline{\zeta} < 0$, then low replacement rate policies do not violate the necessary condition for local indeterminacy and the bound (38) becomes irrelevant.

However, the result of Proposition 6 can be interpreted in terms of the "sufficient statistic" approach which is discussed in the literature on optimal unemployment insurance provision. According to that interpretation, generous welfare systems suffice to eliminate inefficient fluctuations without needing to have information regarding the structural parameters of the economy. For example, $\zeta = 1$ always exceeds $\widetilde{\zeta}$ and local uniqueness is restored.

[12] If $\bar{\zeta} < 0$, then the lower bound of the necessary condition is violated for any $\zeta \in [0, 1]$ and Proposition 6 is trivial. The interesting case arise when $\bar{\zeta} \in (0, 1)$ and Proposition 6 imposes non-trivial restrictions on replacement rate policies (as will be the case in our numerical examples below).

5 Calibration

We calibrate the parameters of the baseline economy to match selected secular features of the US economy. All the calibrated parameters and the targets determining their calibration are reported in Table 1. A period in our model corresponds to 3 months but the effective replacement ratio earned on average by the typical unemployed worker will be contingent on the cross-sectional distribution of unemployment duration. Thus, in what follows, we calibrate two different models that assume a typical duration of unemployment of 6 and 12 months in the computation of the US average replacement rate. This is done to demonstrate the robustness of our results to different specifications.

The calibration of $\{\alpha, \beta, \delta\}$ is common in the literature. Let us turn to the calibration of $\{\bar{f}, \lambda\}$ (the Ins & Outs). The labor supply side of the model yields a matrix of steady state transition probabilities across the three states of employment, unemployment, and non-participation, as follows:

$$\begin{array}{lll} \bar{\phi}_{ee} = (1-\lambda) + \bar{\pi}_{\lambda}\lambda\,\bar{f}, & \bar{\phi}_{ue} = \bar{\pi}_{u}\,\bar{f}, & \bar{\phi}_{oe} = \bar{\pi}_{o}\,\bar{f}, \\ \bar{\phi}_{eu} = \bar{\pi}_{\lambda}\lambda\left(1-\bar{f}\right), & \bar{\phi}_{uu} = \bar{\pi}_{u}\left(1-\bar{f}\right), & \bar{\phi}_{ou} = \bar{\pi}_{o}\left(1-\bar{f}\right), \\ \bar{\phi}_{eo} = \lambda\left(1-\bar{\pi}_{\lambda}\right), & \bar{\phi}_{uo} = 1-\bar{\pi}_{u}, & \bar{\phi}_{oo} = 1-\bar{\pi}_{o}, \end{array} \tag{39}$$

where $\bar{\phi}_{s,s'}$ denotes the steady state transition probability from labor market state s to s'. For example, $\bar{\phi}_{ee}$ is the probability that an individual who starts the period employed in a given job remains at the same job at the end of the period with probability $(1-\lambda)$, or transits to another job within the same period with probability $\bar{\pi}_{\lambda}\lambda\,\bar{f}$. To set the average job finding rate, $\bar{f}$, we use monthly CPS gross flows data aggregated to the quarterly frequency (based on calculations by Gomes, 2015), and using system (39) obtain $\bar{f} = \bar{\phi}_{ue}/\left(1-\bar{\phi}_{uo}\right) = 0.738$. Similarly, we set the average job separation rate, λ, to match the CPS gross flows, using the relationship $\lambda = \bar{\phi}_{eu}/\left(1-\bar{f}\right) + \bar{\phi}_{eo} = 0.190$. Although we do not use it as a target, this calibration yields an unemployment rate in steady state equal to

$$\bar{u} = \frac{\lambda(1-\bar{f})}{\lambda(1-\bar{f}) + \bar{f}} = 6.32\%,$$

which is very close to the historical average unemployment rate in the US.

To calibrate the output-elasticity of the job finding rate, σ, we target the conditional volatility of unemployment and, specifically, the Okun coefficient corresponding to the correlation coefficient between unemployment and the output gap. The Okun's relationship is derived by log-linearising

Table 1: Calibrated Parameters (time unit of model: quarterly)

Parameter	Value	Target (and source)
α	0.2830	capital's income share of 28.3% (Gomme and Rupert, 2007);
δ	0.0140	annual investment/capital ratio of 7.6% (Cooley and Prescott, 1995);
β	0.9920	annual rate of return on capital of 5.16% (Gomme et al., 2011);
$\bar{f}$	0.7380	quarterly gross worker flows (based on Gomes, 2015, calculations on CPS data);
λ	0.1900	quarterly gross worker flows (based on Gomes, 2015, calculations on CPS data);
σ	1.4180	Okun's Law coefficient of -0.5 (Ball et al., 2013);
$\zeta^{6\mathrm{m}}$	0.4371	6-month unemployment duration average net replacement (OECD statistics);
$\zeta^{12\mathrm{m}}$	0.2500	12-month unemployment duration average net replacement (OECD statistics);
$\xi^{6\mathrm{m}}$	1.1288	civilian labor participation rate of 71.0% (BEA) (6-month duration economy);
$\xi^{12\mathrm{m}}$	1.0640	civilian labor participation rate of 71.0% (BEA) (12-month duration economy).

the system of equations describing the labor market aggregate dynamics.[13] This yields the following

$$\widehat{u}_t = \underbrace{\frac{1-\bar{u}}{\bar{u}}\left[\left(\frac{1}{1-\alpha}-\lambda\sigma\right)\cdot\frac{1}{\lambda+\bar{f}(1-\lambda)}-\frac{1}{1-\alpha}\right]}_{\text{Okun's coefficient}\simeq -0.5}\cdot\widehat{Y}_t+(\cdots), \tag{40}$$

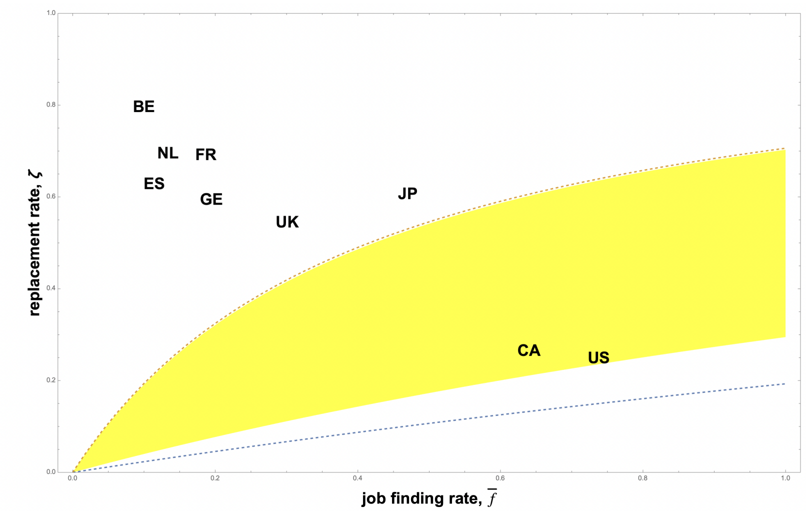
where the term $(\cdots)$ includes predetermined variables. Given the inflow and outflow rates from unemployment, the average unemployment rate and the capital's income share, and targeting an Okun's coefficient of -0.5, yields $\sigma = 1.4180$. The target for the Okun's coefficient is taken from Ball et al. (2013) and is a robust feature of the data which holds for both annual and quarterly periodicities, and for various methods of measuring short-run movements in output and unemployment.

Next, to calibrate ζ we make use of OECD statistics on the US unemployment insurance net replacement rates, available between 2001-2021, for single households with no children previous earnings at the 67% of the average wage, and unemployment duration of, in turn, 6 and 12 months. Computing the 6-month unemployment duration average net replacement rate in unemployment yields $\zeta^{6\mathrm{m}} = 43.71\%$, while computing the 12-month unemployment duration average net replacement rates yields $\zeta^{12\mathrm{m}} = 25\%$.

We set ξ to match the historical average participation rate (16 to 64 year) of 71.0%. The steady state labor market conditions (see Appendix B) yield an employment rate of $\bar{N} = \left(\bar{\Pi}\,\bar{f}\right)/(\lambda + (1-\lambda)\bar{f}) =$

[13] Detailed derivations are presented in Appendix G.

Figure 2: 12-month unemployment duration average replacement rates



0.66. Subsequently, ξ is determined from intratemporal optimality at steady state, as follows

$$\xi^{6\mathrm{m}} = \frac{(1-\alpha)/\bar{N}}{\bar{\tau}^{6\mathrm{m}}} \frac{\bar{Y}/\bar{K}}{\bar{Y}/\bar{K}-\delta} = 1.12884 \quad \text{and} \quad \xi^{12\mathrm{m}} = \frac{(1-\alpha)/\bar{N}}{\bar{\tau}^{12\mathrm{m}}} \frac{\bar{Y}/\bar{K}}{\bar{Y}/\bar{K}-\delta} = 1.064,$$

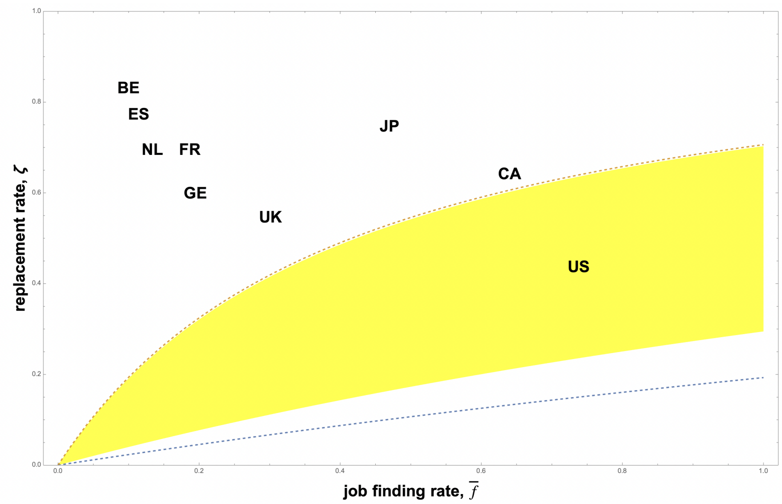
where $\bar{\tau}^{6\mathrm{m}}$ and $\bar{\tau}^{12\mathrm{m}}$ reflect the differences in the computation of the UI average net replacement rate.

6 Replacement rates and sclerotic markets

We discuss the implications of our baseline parametrisation for the local stability properties of the steady state for the US economy. We fix all parameters consistent with the values in Table 1 and let the replacement rate ζ and job finding rate $\bar{f}$ to vary between zero and one, allowing us to trace the regions of local multiplicity and local uniqueness. We plot in the same graph the positions of other major industrialised countries: Germany (GE), the United Kingdom (UK), the Netherlands (NL), Spain (ES), France (FR), Belgium (BE), Canada (CA) and Japan (JP). We compute the average net replacement in unemployment using the same methodology as in Section 5, and to compute $\bar{f}$, we rely on the estimates of the average monthly job-finding rate across OECD countries provided by Hobijn and Sahin (2009), which we convert to quarterly rates to be consistent with the previous analysis.[14]

[14]For example, the monthly finding rate for Germany is 6.98%. The probability that a worker does not find a job in a quarter is $(1-0.0698)^3 = 0.805$, and the probability of finding a job is $1-0.805 = 0.195$.

Figure 3: 6-month unemployment duration average replacement rates



The yellow shaded region in Figure 2 designates the set of points under which satisfaction of the necessary condition imply violation of the Blanchard-Kahn conditions, and there exist multiple paths converging to the steady state. While any point outside the yellow shaded region is consistent with saddle path stability. The region of multiplicity is within the bounds specified by the necessary condition for local indeterminacy. The upper bound is given by $\bar{\zeta}$ computed in Proposition 6, and generous welfare states ($\zeta > \bar{\zeta}$) eliminate inefficient fluctuations. The lower bound in the figure reveals that low replacements rates ($\zeta < \underline{\zeta}$) violate the upper bound of the necessary condition.

Figure 2 uncovers significant differences between North America and Europe, when looking at the generosity of the welfare state and labor market conditions. Europe is characterised by generous welfare states and sclerotic labor markets (job are hard to find), while North America (US and CA) is characterised by less generous welfare states and fluid labor markets. The contribution of the paper is to show what these differences imply about the local determinacy of the equilibrium. European characteristics are consistent with local uniqueness while North America characteristics are consistent with the emergence of inefficient, belief-driven fluctuations. This characterization of two different labor market environments in North America and Europe is evocative of the analysis by Ljungqvist and Sargent (2008), who show that since the 1980s European workers, compared to their North American counterparts, face lower hazard rates of gaining employment and increased unemployment duration, whilst at the same time suffering a lower drop in earnings conditional on job loss due to more generous unemployment compensation.[15]

[15]The job-finding rate estimates for Europe are also consistent with the calibration in Blanchard and Galí (2010), who set the quarterly job-finding rate at 25% to approximate the sclerotic continental European labor markets.

To check the robustness of our results, we perform the same experiment computing the average replacement rate assuming a duration of unemployment of six months. Figure 3 summarises the results. The local stability properties of the model calibrated to European or US characteristics remain consistent with Figure 2. However, the results flip when we calibrate the model to the Canadian characteristics, with average replacement rate rising significantly from 26% to 64% and the economy is located at the boundary above threshold $\bar{\zeta}$.

Both figures confirm the result of Proposition 6, namely, irrespective of labor market conditions, generous replacement rate policies always suffice to eliminate inefficient fluctuations. However, both figures reveal another result, namely, as labor markets become more sclerotic ($\bar{f} \to 0$), keeping replacement rare policy fixed, the economy will exit the multiplicity (yellow shaded area) region and be located above threshold $\bar{\zeta}$ consistent with saddle path stability. To understand the intuition behind this result, we revisit the intratemporal condition once more:

$$\xi = \frac{w_t f_t}{C_t} + \frac{(1 - f_t)\zeta w_t}{C_t},$$

and as $\bar{f} \to 0$, the replacement income gets a higher weight in the expected return of search in the right hand side, weakening the impact of initial held expectations on current actions or, equivalently, violating the lower bound $\underline{\varepsilon}$ of the necessary condition.

7 Conclusion

This papers embeds frictions in the labor market and an endogenous participation choice in an otherwise standard neoclassical growth model and shows that the interaction of these features can generate self-fulfilling fluctuations. Unemployment insurance is shown to make indeterminacy less likely and is a powerful automatic stabiliser. Calibrated versions of the model suggest that this source of instability, and hence, the need to use unemployment insurance as an automatic stabiliser, is not just a theoretical possibility but occurs for empirically plausible values of the parameters. The results give support to generous transfer policies, similar to Europe, that reduce business cycle volatility and restore local uniqueness.

Appendix

A Decentralized Equilibrium and the Lottery Mechanism

The purpose of this section is to derive the stand-in agent representation (2)–(4), obtained by combining an exogenous randomisation mechanism with lotteries over labor force participation. This exogenous randomisation is analogous to the "musical chairs" assumption in Andolfatto (1996), and allows us to specify the problem of the stand-in agent in recursive form. In Kokonas and Monteiro (2021) we show that this hybrid model with musical chairs and lotteries is equivalent in terms of allocations to an economy with Arrow-Debreu markets, where individuals trade according to the realisation of public signals (sunspots) prior and after the realisation of idiosyncratic shocks, along the lines of Prescott and Townsend (1984) and Kehoe et al. (2002).

The economy is populated by a continuum of individuals of unit measure. There is a distinction between job and worker flows in the model. Individuals are distributed across three states or "islands": the employment "island"; the unemployment "island"; and the leisure "island". The mass of individuals on each "island" at the start of date t is, respectively, N_{t-1}, U_{t-1} and O_{t-1}, where N_{t-1} denotes the mass of employed individuals at the end of $t-1$, U_{t-1} denotes the mass of unemployed individuals at the end of $t-1$, and O_{t-1} denotes the mass of non-participants at the end of $t-1$. At the beginning of date t, individuals are allocated randomly across islands, $\imath \in \mathcal{L} \in \{e, u, o\}$. Specifically, individuals assigned to the employment island ($\imath = e$) observe the realisation of the idiosyncratic shock $\kappa \in \{e_d, e_{nd}\}$, where $\kappa = e_d$ denotes job destruction (d) with probability λ, and $\kappa = e_{nd}$ denotes no job destruction (nd) with $1-\lambda$; subsequently they buy lotteries over labor force participation, and engage (or not) in search activity. Individuals assigned to the unemployment or leisure island ($\imath \in \{u, o\}$) buy lotteries over labor force participation and then engage (or not) in search activity.

Individuals choose state-contingent allocations before knowing in which island they will be allocated initially, and there exist insurance opportunities for every possible randomization contingency. Insurance contracts are provided by competitive firms that make zero profits in equilibrium. We focus on separating equilibria where firms offer different prices to different types and insurance is actuarially fair. The price of an insurance contract is denoted by q and the quantity bought is denoted by y. At the end of each period, spot markets open, where individuals execute contacts, buy consumption and capital, and receive capital and labor income. To simplify presentation, we abstract from time subscripts and formulate individual decisions recursively—subscripts "-1" denote previous period values and superscripts "prime" denote next period values.

The Bellman equation characterising the problem solved by each individual is

$$\mathbf{V}(K,k) = \max_{c,\pi,y,k'} \Big\{ N_{-1}\left[\lambda v_{e_d} + (1-\lambda) v_{e_{nd}}\right] + U_{-1} v_u + O_{-1} v_o \Big\}, \tag{A.1}$$

with

$$v_{e_{nd}} = -\pi_e \xi + \pi_e \left[\ln\left(c_{e_{nd},e}\right) + \beta \mathbf{V}\left(K', k'_{e_{nd},e}\right)\right] + (1-\pi_e)\left[\ln\left(c_{e_{nd},o}\right) + \beta \mathbf{V}\left(K', k'_{e_{nd},o}\right)\right],$$

$$\begin{aligned} v_{e_d} = & -\pi_\lambda \xi + f\pi_\lambda \left[\ln\left(c_{e_d,e}\right) + \beta \mathbf{V}\left(K', k'_{e_d,e}\right)\right] + (1-f)\,\pi_\lambda \left[\ln\left(c_{e_d,u}\right) + \beta \mathbf{V}\left(K', k'_{e_d,u}\right)\right] \\ & + (1-\pi_\lambda)\left[\ln\left(c_{e_d,o}\right) + \beta \mathbf{V}\left(K', k'_{e_d,o}\right)\right], \end{aligned}$$

$$\begin{aligned} v_u = & -\pi_u \xi + f\pi_u \left[\ln\left(c_{u,e}\right) + \beta \mathbf{V}\left(K', k'_{u,e}\right)\right] + (1-f)\,\pi_u \left[\ln\left(c_{u,u}\right) + \beta \mathbf{V}\left(K', k'_{u,u}\right)\right] \\ & + (1-\pi_u)\left[\ln\left(c_{u,o}\right) + \beta \mathbf{V}\left(K', k'_{u,o}\right)\right], \end{aligned}$$

$$\begin{aligned} v_o = & -\pi_o \xi + f\pi_o \left[\ln\left(c_{o,e}\right) + \beta \mathbf{V}\left(K', k'_{o,e}\right)\right] + (1-f)\,\pi_o \left[\ln\left(c_{o,u}\right) + \beta \mathbf{V}\left(K', k'_{o,u}\right)\right] \\ & + (1-\pi_o)\left[\ln\left(c_{o,o}\right) + \beta \mathbf{V}\left(K', k'_{o,o}\right)\right]. \end{aligned}$$

Subscripts denote (personal) labor market states (however, notation with respect to the participation probabilities and the opportunity cost of employment is consistent with the main text). The first subscript denotes the assignment of the randomisation induced by musical chairs, that is, (e_d) denotes pre-existing jobs that destroyed with probability λ, (e_{nd}) denotes pre-existing jobs that survived with probability $1-\lambda$, (u) denotes unemployment and (o) denotes out of labor force (non-participation). The second subscript denotes the labor market state of an individual at the end of the period, following the outcome of the lottery over participation and the search process. To simplify exposition, we denote labor market transitions with the pair $(\tilde{\imath}, \jmath)$, with $\tilde{\imath} \in \{e_d, e_{nd}, u, o\}$ and $\jmath \in \{e, u, o\}$; $\tilde{\imath}$ denotes the assignment induced by the musical chairs' randomisation and the realisation of idiosyncratic shocks and $\jmath$ the assignment induced by the lottery over participation and the outcome of the resulting search process.

The flow budget constraint for each pair $(\tilde{\imath}, \jmath)$ is

$$c_{\tilde{\imath},\jmath} + k'_{\tilde{\imath},\jmath} + \sum_{\tilde{\imath}} \sum_{\jmath} q_{\tilde{\imath},\jmath} y_{\tilde{\imath},\jmath} = Rk + w\underline{h}\mathbb{1}_\jmath + \zeta w \underline{h} \mathbf{1}_\jmath + y_{\tilde{\imath},\jmath} - T, \tag{A.2}$$

where $\mathbb{1}_\jmath$ is an indicator function which equals one if $\jmath = e$ and zero otherwise, $\mathbf{1}_\jmath$ is another indicator function which equals one if $\jmath = u$ and zero otherwise, $\zeta \in [0,1]$ is the replacement rate and T are lump-sum taxes.

Actuarially fair insurance and strict concavity of the instantaneous utility function imply

$$c_{\tilde{\imath},\jmath} = c, \tag{A.3}$$

for all pairs $(\tilde{\imath},\jmath)$. Optimality with respect to k' requires

$$c_{\tilde{\imath},\jmath}^{-1} = \beta \mathbf{V}_{k'}(K', k'_{\tilde{\imath},\jmath}); \tag{A.4}$$

$\mathbf{V}_{k'}(\cdot)$ denotes the derivative of the value function with respect to k'. Combining (A.3) and (A.4), yields

$$k'_{\tilde{\imath},\jmath} = k', \tag{A.5}$$

for all pairs $(\tilde{\imath},\jmath)$; moreover, (A.5) implies that $K = k$.

The individual's consolidated decision reduces to

$$\begin{gathered}
\mathbf{V}(k) = \max_{c,k',\pi} \Big\{ \ln(c) - \xi \left(N_{-1}(1-\lambda)\pi_e - N_{-1}\lambda\pi_\lambda + U_{-1}\pi_u + O_{-1}\pi_o\right) + \beta \mathbf{V}(k') \Big\}, \\
s.t. \\
c + k' = Rk + w\Big[N_{-1}(1-\lambda)\pi_e + \left(N_{-1}\lambda\pi_\lambda + U_{-1}\pi_u + O_{-1}\pi_o\right)\left(f + \zeta(1-f)\right)\Big] - T,
\end{gathered} \tag{A.6}$$

which corresponds to the stand-in agent representation (2)–(4) in the main text.

B Steady State

Let $\underline{X}$ denote the steady state of X. The steady state values of the first block are given by

$$\left(\bar{K}/\bar{Y}\right) = \left[\frac{\alpha}{1/\beta - (1-\delta)}\right], \tag{B.1}$$

$$\left(\bar{C}/\bar{K}\right) = \frac{1/\beta - (1-\delta)}{\alpha} - \delta, \tag{B.2}$$

$$\bar{N} = \frac{1-\alpha}{\bar{\tau}\xi}\frac{\bar{Y}/\bar{K}}{\bar{Y}/\bar{K} - \delta}, \tag{B.3}$$

$$\bar{K} = \bar{N}\left(\bar{K}/\bar{Y}\right)^{1/(1-\alpha)}; \tag{B.4}$$

and the steady state values of the second block are given by

$$(\bar{N}/\bar{H}) = \bar{f}/\lambda, \tag{B.5}$$

$$(\bar{\Pi}/\bar{H}) = (\lambda + \bar{f}(1-\lambda))/\lambda, \tag{B.6}$$

$$(\bar{N}/\bar{\Pi}) = \bar{f}/(\lambda + (1-\lambda)\bar{f}), \tag{B.7}$$

$$\bar{u} = \frac{\lambda(1-\bar{f})}{\lambda(1-\bar{f}) + \bar{f}}. \tag{B.8}$$

C Eigenvalue approach

The log-linearized equilibrium conditions (around the deterministic steady state) of the first block are given by

$$\widehat{w}_t = \widehat{C}_t + \widehat{\tau}_t, \tag{C.1}$$

$$\widehat{\tau}_t = -\bar{\tau}\bar{f}\,(1-\zeta)\,\sigma\widehat{Y}_t, \tag{C.2}$$

$$\widehat{C}_{t+1} - \widehat{C}_t = \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})\left(\widehat{N}_{t+1} - \widehat{K}_{t+1}\right), \tag{C.3}$$

$$\widehat{R}_t = \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})\left(\widehat{N}_t - \widehat{K}_t\right), \tag{C.4}$$

$$\widehat{w}_t = \alpha\left(\widehat{K}_t - \widehat{N}_t\right), \tag{C.5}$$

$$\left(\bar{C}/\bar{K}\right)\widehat{C}_t + \widehat{K}_{t+1} = \left(\bar{Y}/\bar{K}\right)\widehat{y}_t + (1-\delta)\widehat{K}_t, \tag{C.6}$$

$$\widehat{Y}_t = \alpha\widehat{K}_t + (1-\alpha)\widehat{N}_t. \tag{C.7}$$

The model in log-linear form can be written as

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} = \Gamma \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \tag{C.8}$$

with

$$\Gamma_{11} = \frac{\alpha\left(\bar{Y}/\bar{K}\right)}{\alpha - (-\varepsilon_\tau)} + 1 - \delta, \tag{C.9}$$

$$\Gamma_{12} = -\left(\left(\bar{C}/\bar{K}\right) + \frac{(1-\alpha)(\bar{Y}/\bar{K})}{\alpha - (-\varepsilon_\tau)}\right), \tag{C.10}$$

$$\Gamma_{21} = \frac{\alpha\beta\left(\bar{Y}/\bar{K}\right)(-\varepsilon_\tau)\,\Gamma_{11}}{\alpha - (-\varepsilon_\tau) + \alpha\beta\left(\bar{Y}/\bar{K}\right)(1-\alpha)}, \tag{C.11}$$

Table 2: Determinacy/Indeterminacy

necessary condition	$-\varepsilon_\tau$	eigenvalue approach
violated	$\in [0, 0.298694]$	Blanchard-Kahn conditions satisfied
satisfied	$\in (0.298694, 0.302159]$	Blanchard-Kahn conditions satisfied
satisfied	$\in (0.302159, 0.717)$	**indeterminacy (two stable eigenvalues)**
satisfied	$\in (0.717, 0.820436)$	Blanchard-Kahn conditions satisfied
violated	$\in [0.820436, +\infty)$	Blanchard-Kahn conditions satisfied

Note: At $-\varepsilon_\tau = 0.717$ one eigenvalue is on the unit circle.

$$\Gamma_{22} = \frac{\alpha - (-\varepsilon_\tau) + \alpha\beta\left(\bar{Y}/\bar{K}\right)(-\varepsilon_\tau)\Gamma_{12}}{\alpha - (-\varepsilon_\tau) + \alpha\beta\left(\bar{Y}/\bar{K}\right)(1-\alpha)}. \tag{C.12}$$

The eigenvalues of the matrix Γ are

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Gamma_{11} + \Gamma_{22} - \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \\ \Gamma_{11} + \Gamma_{22} + \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \end{bmatrix}. \tag{C.13}$$

Local uniqueness (saddle path stability) requires one stable and one unstable eigenvalue (Blanchard-Kahn conditions); while local indeterminacy requires two stable eigenvalues and no solution requires two unstable eigenvalues.[16] (C.9)-(C.12) are indexed by $\varepsilon_{\mathrm{lw}}$. In turn, setting $\alpha = 0.283$, $\beta = 0.992$, and $\delta = 0.014$ (see Section 5), makes the necessary condition for local indeterminacy equal to

$$-\varepsilon_\tau \in \Big(0.298694, 0.820436\Big). \tag{C.14}$$

Table 2 summarises the result. Specifically, violation of the Blanchard-Kahn conditions occur only when the necessary condition for local indeterminacy is satisfied, while allowing the elasticity parameter to take a wide range of values shows that violations of the necessary condition for local indeterminacy is associated with saddle path stability, and as $-\varepsilon_\tau \to +\infty$, then (C.9)-(C.12) converge to

$$\Gamma_{11} = 1 - \delta, \tag{C.15}$$

$$\Gamma_{12} = -\left(\bar{C}/\bar{K}\right), \tag{C.16}$$

[16] An eigenvalue of a matrix is stable if it has modulus less than one. It is unstable if it has modulus greater than one.

$$\Gamma_{21} = -\alpha\beta \left(\bar{Y}/\bar{K}\right)(1-\delta), \tag{C.17}$$

$$\Gamma_{22} = 1 + \alpha\beta \left(\bar{Y}/\bar{K}\right) \cdot \left(\bar{C}/\bar{K}\right), \tag{C.18}$$

and using the previous parametrisation yields

$$\lambda_1 = 0.955755 < 1 \quad \text{and} \quad \lambda_2 = 1.03164 > 1, \tag{C.19}$$

which is consistent with saddle path stability.

D Lemma 2

Combining Assumption 1 in the main text and (C.7) yields

$$\sum_{j=0}^{\infty} \beta^j \hat{f}_{t+j} = \sigma\alpha \sum_{j=0}^{\infty} \beta^j \widehat{K}_{t+j} + \sigma(1-\alpha) \sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}. \tag{D.1}$$

To complete the proof we need to express $\sum_{j=0}^{\infty} \beta^j \widehat{K}_{t+j}$ as a function of predetermined variables and $\sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}$.

Combining (C.1), (C.2), (C.3), (C.5), (C.7), yields

$$\widehat{K}_{t+1} = \Lambda_1 \widehat{K}_t + \Lambda_2 \widehat{N}_t + \Lambda_3 \widehat{N}_{t+1}, \tag{D.2}$$

with

$$\Lambda_1 = \frac{\alpha\left(1 - \frac{\epsilon_\tau}{1-\alpha}\right)}{\alpha\left(1 - \frac{\epsilon_\tau}{1-\alpha}\right) + \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})}, \tag{D.3}$$

$$\Lambda_2 = -\frac{\alpha + \epsilon_\tau}{\alpha\left(1 - \frac{\epsilon_\tau}{1-\alpha}\right) + \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})}, \tag{D.4}$$

$$\Lambda_3 = \frac{\alpha + \epsilon_\tau + \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})}{\alpha\left(1 - \frac{\epsilon_\tau}{1-\alpha}\right) + \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})}, \tag{D.5}$$

and where

$$\varepsilon_\tau = -\sigma\bar{\tau}\bar{f}\,(1-\zeta)\,(1-\alpha)\,, \tag{D.6}$$

is the employment elasticity of the labor wedge.

Using (D.2) to solve for $\widehat{K}_{t+j}$ for all j backwards yields

$$\begin{aligned}
\widehat{K}_t &= \Lambda_1 \widehat{K}_{t-1} + \Lambda_2 \widehat{N}_{t-1} + \Lambda_3 \widehat{N}_t, \\
\widehat{K}_{t+1} &= \Lambda_1^2 \widehat{K}_{t-1} + \Lambda_1 \Lambda_2 \widehat{N}_{t-1} + (\Lambda_1 \Lambda_3 + \Lambda_2) \widehat{N}_t + \Lambda_3 \widehat{N}_{t+1}, \\
\widehat{K}_{t+2} &= \Lambda_1^3 \widehat{K}_{t-1} + \Lambda_1^2 \Lambda_2 \widehat{N}_{t-1} + \left(\Lambda_1^2 \Lambda_3 + \Lambda_1 \Lambda_2\right) \widehat{N}_t + \left(\Lambda_1^2 \Lambda_3 + \Lambda_2\right) \widehat{N}_{t+1} + \Lambda_3 \widehat{N}_{t+2}, \\
\widehat{K}_{t+3} &= \Lambda_1^4 \widehat{K}_{t-1} + \Lambda_1^3 \Lambda_2 \widehat{N}_{t-1} + \left(\Lambda_1^3 \Lambda_3 + \Lambda_1^2 \Lambda_2\right) \widehat{N}_t + \left(\Lambda_1^2 \Lambda_3 + \Lambda_1 \Lambda_2\right) \widehat{N}_{t+1} + (\Lambda_1 \Lambda_3 + \Lambda_2) \widehat{N}_{t+2} + \Lambda_3 \widehat{N}_{t+3}, \\
\widehat{K}_{t+4} &= \Lambda_1^5 \widehat{K}_{t-1} + \Lambda_1^4 \Lambda_2 \widehat{N}_{t-1} + \left(\Lambda_1^4 \Lambda_3 + \Lambda_1^3 \Lambda_2\right) \widehat{N}_t + \left(\Lambda_1^3 \Lambda_3 + \Lambda_1^2 \Lambda_2\right) \widehat{N}_{t+1} + \left(\Lambda_1^2 \Lambda_3 + \Lambda_1 \Lambda_2\right) \widehat{N}_{t+2} + \\
&\quad (\Lambda_1 \Lambda_3 + \Lambda_2) \widehat{N}_{t+3} + \Lambda_3 \widehat{N}_{t+4}, \\
&\quad \vdots \\
\widehat{K}_{t+j} &= \Lambda_1^{j+1} \widehat{K}_{t-1} + \Lambda_1^{j} \Lambda_2 \widehat{N}_{t-1} + (\Lambda_1 \Lambda_3 + \Lambda_2) \sum_{i=0}^{j-1} \Lambda_1^{j-i-1} \widehat{N}_{t+i} + \Lambda_3 \widehat{N}_{t+j}. \qquad \text{(D.7)}
\end{aligned}$$

Since $\Lambda_1 \in (0,1)$, then substituting (D.7) into $\sum_{j=0}^{\infty} \beta^j \widehat{K}_{t+j}$ yields

$$\begin{aligned}
\sum_{j=0}^{\infty} \beta^j \widehat{K}_{t+j} = {} & \frac{\Lambda_1 \widehat{K}_{t-1}}{1-\beta\Lambda_1} + \frac{\Lambda_2 \widehat{N}_{t-1}}{1-\beta\Lambda_1} + && \text{(D.8)} \\
& \widehat{N}_t \left[\Lambda_3 \left(1 + \beta\Lambda_1 + \beta^2 \Lambda_1^2 + ...\right) + \beta\Lambda_2 \left(1 + \beta\Lambda_1 + \beta^2 \Lambda_1^2 + ...\right)\right] + && \text{(D.9)} \\
& \beta \widehat{N}_{t+1} \left[\Lambda_3 \left(1 + \beta\Lambda_1 + \beta^2 \Lambda_1^2 + ...\right) + \beta\Lambda_2 \left(1 + \beta\Lambda_1 + \beta^2 \Lambda_1^2 + ...\right)\right] + ... && \text{(D.10)} \\
= {} & \frac{\Lambda_1 \widehat{K}_{t-1}}{1-\beta\Lambda_1} + \frac{\Lambda_2 \widehat{N}_{t-1}}{1-\beta\Lambda_1} + \frac{\Lambda_3 + \beta\Lambda_2}{1-\beta\Lambda_1} \sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}, && \text{(D.11)}
\end{aligned}$$

with

$$\frac{\beta\Lambda_2 + \Lambda_3}{1-\beta\Lambda_1} = \frac{(1-\beta)(\alpha+\epsilon_\tau) + \alpha\beta(1-\alpha)\left(\bar{Y}/\bar{K}\right)}{\alpha\left(1 - \frac{\epsilon_\tau}{1-\alpha}\right)(1-\beta) + \alpha\beta(1-\alpha)(\bar{Y}/\bar{K})}.$$

Substituting into (D.1) and rearranging, yields

$$\sum_{j=0}^{\infty} \beta^j \hat{f}_{t+j} = \sigma\alpha \left(\frac{\Lambda_1 \widehat{K}_{t-1}}{1-\beta\Lambda_1} + \frac{\Lambda_2 \widehat{N}_{t-1}}{1-\beta\Lambda_1}\right) + \sigma(1-\alpha)\left(1 + \frac{\alpha}{1-\alpha} \frac{\beta\Lambda_2 + \Lambda_3}{1-\beta\Lambda_1}\right) \sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}, \qquad \text{(D.12)}$$

and finally, substituting into the expression for the labor wedge derived in Lemma 1 in the text, yields

$$\sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t+j} = \varepsilon_\tau \frac{\alpha}{1-\alpha} \left(\frac{\Lambda_1 \widehat{K}_{t-1}}{1-\beta\Lambda_1} + \frac{\Lambda_2 \widehat{N}_{t-1}}{1-\beta\Lambda_1}\right) + \varepsilon_\tau \left(1 + \frac{\alpha}{1-\alpha} \frac{\beta\Lambda_2 + \Lambda_3}{1-\beta\Lambda_1}\right) \sum_{j=0}^{\infty} \beta^j \widehat{N}_{t+j}, \qquad \text{(D.13)}$$

with

$$\omega = \varepsilon_\tau \frac{\alpha}{1-\alpha}\frac{\Lambda_1}{1-\beta\Lambda_1} = \varepsilon_\tau \frac{\alpha}{1-\alpha} \frac{\alpha\left(1-\varepsilon_\tau/(1-\alpha)\right)}{\alpha\left(1-\varepsilon_\tau/(1-\alpha)\right)(1-\beta) + \alpha\beta(1-\alpha)\left(\bar{Y}/\bar{K}\right)}$$

$$\chi = \varepsilon_\tau \frac{\alpha}{1-\alpha}\frac{\Lambda_2}{1-\beta\Lambda_1} = -\varepsilon_\tau \frac{\alpha}{1-\alpha} \frac{\alpha + \varepsilon_\tau}{\alpha\left(1-\varepsilon_\tau/(1-\alpha)\right)(1-\beta) + \alpha\beta(1-\alpha)\left(\bar{Y}/\bar{K}\right)}$$

$$\varphi = \varepsilon_\tau \left(1 + \frac{\alpha}{1-\alpha}\frac{\beta\Lambda_2 + \Lambda_3}{1-\beta\Lambda_1}\right) = \varepsilon_\tau \times \frac{\alpha(1-\beta)/(1-\alpha) + \alpha\beta\left(\bar{Y}/\bar{K}\right)}{\alpha\left(1-\varepsilon_\tau/(1-\alpha)\right)(1-\beta) + \alpha\beta(1-\alpha)\left(\bar{Y}/\bar{K}\right)}$$

E Derivation of elasticity parameters in (28)

Log-linearising (23) around the steady state yields

$$\Gamma \sum_{j=0}^{\infty} \beta^j \left(\widehat{N}_{t+j} + \widehat{\tau}_{t+j}\right) = \widehat{\tau}_t + \widehat{N}_t \left(1 - (1-\alpha)\beta(1-\delta)\right) + \widehat{K}_t \left(1-\alpha\right)\beta(1-\delta) \tag{E.1}$$

with

$$\Gamma = -\frac{(1-\beta)\xi\bar{\tau}\bar{N}}{1-\xi\bar{\tau}\bar{N}};$$

Substituting for $\widehat{\tau}_t$ and $\sum_{j=0}^{\infty} \beta^j \widehat{\tau}_{t+j}$ from Lemmas 1 and 2, and using $\widehat{Y}_t = \alpha\widehat{K}_t + (1-\alpha)\widehat{N}_t$, yields

$$\sum_{j=0}^{\infty} \upsilon_j \hat{N}_{t+j} = \psi_0 \widehat{K}_t \;+\; \psi_1 \widehat{K}_{t-1} \;+\; \psi_2 \widehat{N}_{t-1}, \tag{E.2}$$

with

$$\upsilon_0 = \Gamma(1+\varphi) - \left(\varepsilon_\tau + 1 - \beta(1-\alpha)(1-\delta)\right) \tag{E.3}$$

$$\upsilon_j = \Gamma(1+\varphi) \tag{E.4}$$

$$\psi_0 = \beta(1-\alpha)(1-\delta) + \frac{\alpha}{1-\alpha}\varepsilon_\tau \tag{E.5}$$

$$\psi_1 = -\Gamma\omega \tag{E.6}$$

$$\psi_2 = -\Gamma\chi. \tag{E.7}$$

F Proposition 2

We derive conditions under which $\varphi > -1$, with

$$\varphi = \varepsilon_\tau \times \frac{\alpha(1-\beta)/(1-\alpha) + \alpha\beta\left(\bar{Y}/\bar{K}\right)}{\alpha\left(1-\varepsilon_\tau/(1-\alpha)\right)(1-\beta) + \alpha\beta(1-\alpha)\left(\bar{Y}/\bar{K}\right)}.$$

Specifically,

$$\varphi > -1 \Rightarrow \tag{F.1}$$

$$-\varepsilon_\tau > \frac{\alpha(1-\beta) + (1-\alpha)\alpha\beta\left(\bar{Y}/\bar{K}\right)}{\alpha\beta\left(\bar{Y}/\bar{K}\right)}, \tag{F.2}$$

and using the expression for $\bar{Y}/\bar{K}$ (see Appendix B), the condition reduces to

$$-\varepsilon_\tau > \frac{\alpha\beta\left(\bar{Y}/\bar{K}\right) - \alpha\beta\delta}{\alpha\beta\left(\bar{Y}/\bar{K}\right)}. \tag{F.3}$$

G Okun's Law

The labor market equations for log-linearisation are

$$N_t = (1-\lambda)N_{t-1} + H_t f_t, \tag{G.1}$$

$$\Pi_t = (1-\lambda)N_{t-1} + H_t, \tag{G.2}$$

$$1 - u_t = \frac{N_t}{\Pi_t}. \tag{G.3}$$

Log-linear version:

$$\widehat{N}_t = (1-\lambda)\widehat{N}_{t-1} + \frac{\bar{H}\bar{f}}{\bar{N}}\left(\widehat{f}_t + \widehat{H}_t\right), \tag{G.4}$$

$$\widehat{\Pi}_t = \frac{\bar{N}}{\bar{\Pi}}(1-\lambda)\widehat{N}_{t-1} + \frac{\bar{H}}{\bar{\Pi}}\widehat{H}_t, \tag{G.5}$$

$$\widehat{u}_t = \frac{1-\bar{u}}{\bar{u}}\left(\widehat{\Pi}_t - \widehat{N}_t\right). \tag{G.6}$$

Combining (F.4),(F.5) yields

$$\hat{\Pi}_t = \widehat{N}_{t-1}\left(1 - \frac{1}{\bar{f}}\right)\frac{\bar{N}}{\bar{\Pi}}(1-\lambda) + \frac{\bar{N}}{\bar{\Pi}}\frac{1}{\bar{f}}\widehat{N}_t - \frac{\bar{H}}{\bar{\Pi}}\widehat{f}_t. \tag{G.7}$$

Combining Assumption 1 with expressions (B.6)-(B.7) and (C.7), yields

$$\widehat{\Pi}_t = \widehat{N}_{t-1}\left(1 - \frac{1}{\bar{f}}\right)\frac{\bar{N}}{\bar{\Pi}}(1-\lambda) - \frac{\bar{N}}{\bar{\Pi}}\frac{1}{\bar{f}}\frac{\alpha}{1-\alpha}\widehat{K}_t + \widehat{Y}_t\left(\frac{1}{1-\alpha} - \lambda\sigma\right)\cdot\frac{1}{\lambda + \bar{f}(1-\lambda)}. \tag{G.8}$$

Substituting (F.8) into (F.6) and using (C.7) once more, yields

$$\widehat{u}_t = \underbrace{\frac{1-\bar{u}}{\bar{u}}\left(\left(\frac{1}{1-\alpha} - \lambda\sigma\right)\cdot\frac{1}{\lambda + \bar{f}(1-\lambda)} - \frac{1}{1-\alpha}\right)}_{\text{Okun's coefficient}}\cdot\widehat{Y}_t + (\cdots), \tag{G.9}$$

with $(\cdots)$ including the predetermined variables $\widehat{K}_t$ and $\widehat{N}_{t-1}$.

References

Aiyagari, S. R. (1995). Comments on Farmer and Guo's "the econometrics of indeterminacy: an applied study". Staff Report 196, Federal Reserve Bank of Minneapolis.

Alonso-Ortiz, J. and R. Rogerson (2010). Taxes, transfers and employment in an incomplete markets model. *Journal of Monetary Economics 57*, 949–958.

Alvarez, F. and M. Veracierto (1999). Labor-market policies in an equilibrium search model. *NBER macroeconomics annual 14*, 265–304.

Alvarez, F. and M. Veracierto (2012). Fixed-term employment contracts in an equilibrium search model. *Journal of Economic Theory 147*(5), 1725–1753.

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review 86*(1), 112–132.

Arseneau, D. M. and S. K. Chugh (2012). Tax smoothing in frictional labor markets. *Journal of Political Economy 120*(5), 926–985.

Ball, L. M., D. Leigh, and P. Loungani (2013). Okun's law: Fit at fifty? NBER Working Papers 18668, National Bureau of Economic Research.

Benhabib, J. and R. E. Farmer (1994). Indeterminacy and increasing returns. *Journal of Economic Theory 63*(1), 19–41.

Bennett, R. L. and R. E. Farmer (2000). Indeterminacy with non-separable utility. *Journal of Economic Theory 93*, 118–143.

Blanchard, O. and J. Galí (2010). Labor markets and monetary policy: A New Keynesian model with unemployment. *American Economic Journal: macroeconomics 2*(2), 1–30.

Blinder, A. S. (1975). Distribution effects and the aggregate consumption function. *Journal of Political Economy 83*(3), 447–475.

Brown, E. C. (1955). The static theory of automatic fiscal stabilization. *Journal of Political Economy 63*(5), 427–440.

Challe, E. and X. Ragot (2015). Precautionary Saving Over the Business Cycle. *The Economic Journal 126*, 135–164.

Chodorow-Reich, G. and J. M. Coglianese (2019). Unemployment insurance and macroeconomic stabilization. In *Recession Ready, ed. Heather Boushey, Ryan Nunn, and Jay Shambaugh*, pp. 153–179. Brookings Institution.

Christiano, L. J. and S. G. Harrison (1999). Chaos, sunspots and automatic stabilizers. *Journal of Monetary Economics 44*(1), 3–31.

Cooley, T. F. and E. C. Prescott (1995). Economic growth and business cycles. *Frontiers of business cycle research 1*, 1–38.

Di Maggio, M. and A. Kermani (2016). The importance of unemployment insurance as an automatic stabilizer. Technical report, National Bureau of Economic Research.

Elsby, M. W. L., B. Hobijn, and A. Şahin (2015). On the importance of the participation margin for labor market fluctuations. *Journal of Monetary Economics 72*, 64–82.

Flodén, M. (2001). The effectiveness of government debt and transfers as insurance. *Journal of Monetary Economics 48*, 81–108.

Galí, J., M. Gertler, and J. D. Lopez-Salido (2007). Markups, gaps, and the welfare costs of business fluctuations. *The Review of Economics and Statistics 89*, 44–59.

Gomes, P. (2015). The importance of frequency in estimating labour market transition rates. *IZA Journal of Labor Economics 4*(1), 6.

Gomme, P., B. Ravikumar, and P. Rupert (2011). The return to capital and the business cycle. *Review of Economic Dynamics 14*(2), 262–278.

Gomme, P. and P. Rupert (2007). Theory, measurement and calibration of macroeconomic models. *Journal of Monetary Economics 54*(2), 460–497.

Gorn, A. and A. Trigari (2021). Assessing the (de)stabilizing effects of unemployment benefit extensions. Cepr discussion papers, C.E.P.R. Discussion Papers.

Guo, J.-T. and K. J. Lansing (1998). Indeterminacy and stabilization policy. *Journal of Economic Theory 82*, 481–490.

Hall, R. (2009). Reconciling cyclical movements in the marginal value of time and the marginal product of labor. *Journal of Political Economy 117*, 281–323.

Hall, R. E. (2005). Job loss, job finding, and unemployment in the us economy over the past fifty years. *NBER macroeconomics annual 20*, 101–137.

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics 16*(3), 309–327.

Hobijn, B. and A. Sahin (2009). Job-finding and separation rates in the oecd. *Economics Letters 104*, 107–111.

Janiak, A. and P. Santos Monteiro (2016). Towards a quantitative theory of automatic stabilizers: The role of demographics. *Journal of Monetary Economics 78*, 35–49.

Karabarbounis, L. (2014). The labor wedge: Mrs vs. mpn. *Review of Economic Dynamics 17*, 206–223.

Kehoe, T. J., D. K. Levine, and E. C. Prescott (2002). Lotteries, sunspots, and incentive constraints. *Journal of Economic Theory 107*(1), 39–69.

Kekre, R. (2021). Unemployment insurance in macroeconomic stabilization. Working Paper 29505, National Bureau of Economic Research.

Kokonas, N. and P. S. Monteiro (2021). Aggregation in economies with search frictions. *Journal of Mathematical Economics 96*, .

Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2008). Aggregate implications of indivisible labor, incomplete markets, and labor market frictions. *Journal of Monetary Economics 55*(5), 961–979.

Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2010). Aggregate labor market outcomes: the roles of choice and chance. *Quantitative Economics 1*(1), 97–127.

Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2011, May). A three state model of worker flows in general equilibrium. *Journal of Economic Theory 146*(3), 1107–1133.

Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2017). Gross worker flows over the business cycle. *American Economic Review 107*(11), 3447–76.

Ljungqvist, L. and T. J. Sargent (2008). Two questions about european unemployment. *Econometrica 76*(1), 1–29.

Lucas, R. E. and E. C. Prescott (1974). Equilibrium search and unemployment. *Journal of Economic theory 7*(2), 188–209.

McKay, A. and R. Reis (2016). The role of automatic stabilizers in the US business cycle. *Econometrica 84*(1), 141–194.

McKay, A. and R. Reis (2021). Optimal automatic stabilizers. *The Review of Economic Studies 88*(5), 2375–2406.

Michaillat, P. (2012). Do matching frictions explain unemployment? Not in bad times. *American Economic Review 102*(4), 1721–1750.

Nakajima, T. (2006). Unemployment and indeterminacy. *Journal of Economic Theory 126*, 314–327.

Prescott, E. C. and R. M. Townsend (1984). General competitive analysis in an economy with private information. *International Economic Review 25*(1), 1–20.

Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics 21*(1), 3–16.

Schmitt-Grohe, S. and M. Uribe (1997). Balanced-budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy 105*(5), 976–1000.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review 95*(1), 25–49.

Shimer, R. (2009). Convergence in macroeconomics: The labor wedge. *American Economic Journal: Macroeconomics 1*(1), 280–297.

Wen, Y. (2001). Understanding self-fulfilling rational expectations equilibria in real business cycle models. *Journal of Economic Dynamics and Control 25*(8), 1221–1240.