Learning from Strategic Sources*

Kailin Chen†

February 10, 2023

Click here for the latest version

Abstract

This paper studies learning from multiple informed agents where each agent has a small piece of information about the unknown state of the world in the form of a noisy signal and sends a message to the principal, who then makes a decision that is not constrained by predetermined rules. In contrast to the existing literature, I model the conflict of interest between the principal and the agents more generally and consider the case where the preferences of the principal and the agents are misaligned in some realized states. I show that if the conflict of interest between the principal and the agents is moderate, there is a discontinuity: when the number of agents is large enough, adding even a tiny probability of misaligned states leads to complete unraveling in which the agents ignore their signals, in contrast to the almost complete revealing that is predicted by the existing literature. Furthermore, I demonstrate that no matter how small the conflict of interest between the principal and the agents is, the information contained in each agent's message must vanish as the number of agents grows large. Finally, no matter how many agents there are, the total amount of information that is transmitted is limited, and the principal always fails to fully learn the unknown state.

Keywords: Learning, cheap talk, voting, information aggregation, information transmission, Chernoff's information

*I would like to thank my advisors Stephan Lauermann, Sven Rady, Johannes Hörner, and Benny Moldovanu for their continual guidance. I am also grateful for comments and suggestions from Yeon-Koo Che, Krishna Dasaratha, Francesc Dilme, Mehmet Ekmekci, Marina Halac, Ryota Iijima, Deniz Kattwinkel, Sophie Kreutzkamp, Axel Niemeyer, Harry Pei, Justus Preusser, Fedor Sandomirskiy, Roland Strauß, Kai Hao Yang, and seminar participants at BGSE Micro Workshop (Bonn) and Yale Micro Theory Lunch.

†University of Bonn, kchen@uni-bonn.de

Contents

1 Introduction

This paper studies learning from multiple informed agents where each agent has a small piece of information about the unknown state of the world in the form of a noisy signal and sends a message to the principal, who then makes a decision that is not constrained by predetermined rules. This framework applies to scenarios that include non-binding shareholder voting, public protests, and survey polls. This paper also contributes to the literature on communication with cheap talk initiated by Crawford and Sobel (1982), and analyzes the case where there are multiple imperfectly informed senders (agents).

If the principal and the agents share the same preferences, then the agents report their signals truthfully and the principal can fully learn the unknown state of the world as the number of agents grows large. However, if the principal and the agents do not share the same preferences, that is, their interests conflict, then the agents might misrepresent information in their messages, as shown by Wolinsky (2002), Morgan and Stocken (2008), Levit and Malenko (2011), Battaglini (2017), and Ekmekci and Lauermann (2022) among others. Several of these studies show that if the conflict of interest between the principal and the agents is below a certain threshold, then as the number of agents grows large, the agents report their signals almost truthfully and the principal can still fully learn the unknown state, and if the conflict is above the threshold, the agents' messages become completely uninformative for any number of agents. However, the results in all of these cases depend on the critical assumption that the preferences of the principal and the agents are aligned if they have complete information about the realized state.

In many situations, the preferences of the principal and the agents might not be fully aligned even if they have complete information about the realized state. Consider the example of non-binding shareholder voting studied by Levit and Malenko (2011), in which the shareholders receive dispersed information concerning the unknown payoff of a proposal to the firm and decide whether to vote in favor of it, while the manager observes the outcome of the vote and ultimately forms his own decision. Both the manager and the shareholders care about the payoff of the proposal and agree on the same decision if the realized payoff is at the extremes of either very high or very low. However, if the manager receives additional private benefits from the proposal, then when the realized payoff is moderate, the preferences of the principal and the agents are more likely to be misaligned: in this case, only the manager might prefer the proposal due to his additional payoff. Similarly, in public protests studied by Battaglini (2017), the citizens receive dispersed information concerning the effect of reform and decide whether to participate in a rally, while the policymaker decides whether to implement the reform after observing the citizens' activities. The preferences of the politician and the citizens are aligned if the reform is dramatically better or worse than the *status-quo*, but are misaligned for less significant changes, where the policymaker's private interests or ideologies may play a larger role. A similar situation also arises in the example of survey polls.

In this paper, I model the conflict of interest between the principal and the agents more generally and consider the case where the preferences of the principal and the agents are misaligned in some realized states. I show that in the framework of the existing literature, if the conflict between the principal and the agents is moderate, there is a discontinuity: when the number of agents is large enough, adding even a tiny probability of misaligned states leads to complete unraveling in which the agents ignore their signals and no information is transmitted. This result stands in contrast to the predicted outcome in the existing literature, in which the agents report almost truthfully. In addition, I demonstrate that no matter how small the conflict of interest between the principal and the agents is, the information contained in each agent's message must vanish as the number of agents grows large. Finally, and more surprisingly, no matter how many agents there are, the total amount of information that is transmitted is limited, and the principal always fails to fully learn the unknown state.

More specifically, I develop a model based on Levit and Malenko (2011) and Battaglini (2017) (henceforth, LMB). Both of these papers analyze a model with one principal and N agents. The principal must decide between policy A and policy B. Both the principal and the agents find A optimal in the high state and B optimal in the low state, that is, their preferences are fully aligned when the state is known. All the agents have the same preferences, while the principal is biased toward A in each state. Therefore, when the realized state is uncertain, they have different "thresholds of acceptance": the principal already prefers A at a relatively low probability of the high state, while the agents prefer it only at a higher probability. In the table below, I provide an example in which the principal receives an additional payoff of 2 from A. In this example, the principal prefers A if the probability that the realized state is high exceeds 3/8, while the agents prefer A in cases where this probability exceeds 5/8. For the information structure, both the principal and the agents share the same prior belief about the unknown state. Each agent has a small piece of private information about the realized state in the form of a noisy signal. She can then choose whether to approve A. The principal, in turn, observes the total number of approvals and then chooses a policy that is most in line with his interests.

Table 1: Payoffs from A

	High	Low
Principal	3+2	-5+2
Agents	3	-5

Table 2: Payoffs from B

	High	Low
Principal	0	0
Agents	0	0

LMB apply this type of model to non-binding shareholder voting and public protests, in which there are usually a large number of agents (shareholders or citizens). LMB show that information transmission is all-or-nothing. If the conflict of interest between the principal and the agents is below a certain threshold, then as N grows large, the agents report their signals almost truthfully. That is, they approve A with a probability approaching 1 when they receive signals favoring A, and they reject A with a probability approaching 1 when they receive signals opposing A. Hence, the principal can fully learn the unknown state, and the information dispersed among the agents is effectively aggregated. However, if the conflict is above the threshold, then complete unraveling happens, in which the agents ignore their signals, and in this case, no information is transmitted.

In what follows, I consider a further possibility, which can be exemplified with a relatively simple scenario. Let us add a middle state to LMB's framework.[1] This middle state is a misaligned state in which the principal prefers A while the agents prefer B. I illustrate the preferences of the principal and the agents in the table below. For the information structure, each agent's signal is ordered by the "monotone likelihood ratio property" (MLRP), which states that, as the realization of the signal increases, it becomes increasingly likely that the state is higher.

Table 3: Payoffs from A

	High	Middle	Low
Principal	3+2	-1+2	-5+2
Agents	3	-1	-5

Table 4: Payoffs from B

	High	Middle	Low
Principal	0	0	0
Agents	0	0	0

I show that when the conflict of interest in LMB's framework is moderate and below the threshold provided by LMB, there is a discontinuity in the results: when N is large enough, adding the middle state with even a tiny probability leads to complete unraveling in

[1]The key difference between this paper and LMB is whether the preferences of the principal and agents are always fully aligned if they know the state, instead of whether there are two or three states.

which the agents ignore their signals and no information is transmitted. This result stands in contrast to LMB's prediction, in which the agents report almost truthfully.

I demonstrate that when the conflict of interest in LMB's framework is sufficiently small, information is still transmitted from the agents to the principal if the middle state is also sufficiently unlikely. However, as the number of agents grows large, the information contained in an agent's message vanishes, that is, the agents reject A with a probability approaching 1 even when they receive the signal that favors A the most. Furthermore, the expected number of total approvals for A in each state is always smaller than a finite number that is independent of N. Similarly, the principal chooses A when the total number of approvals exceeds a cut-off number, and this cut-off is also always smaller than a finite number that is independent of N. Hence, the principal must follow either the unanimity rule under which he chooses A if at least one agent approves A or rules that are similar to the unanimity rule. Finally, I show that no matter how large N is, the total amount of information that is transmitted is limited and the principal always fails to fully learn the unknown state. With a strictly positive probability, the principal chooses the wrong policy in both the high state and the low state, even though the preferences of the principal and the agents are fully aligned in both states.

Another important finding by Battaglini (2017) is that communication among the agents facilitates information transmission and aggregation, benefiting both the principal and the agents. Battaglini thus highlights the value of social media for the effectiveness of petitions and public protests since social media allow citizens to share information. In contrast, by further considering the case in which the agents fully communicate with each other, I show that communication among the agents might impede information transmission and hurt both the principal and the agents. In this case, as N approaches infinity, the agents learn the state, and information is effectively aggregated. However, I find that in some situations, the principal ignores messages from the agents if they fully communicate with each other, while if they cannot communicate, information transmission is restored. A key intuition is that we can interpret the failure of information aggregation as intentional vagueness that mitigates the conflict of interest between the sender (agents) and the receiver (principal), as discussed in the cheap-talk literature initiated by Crawford and Sobel (1982).

In this paper's basic model, the agents can either approve A or reject it, that is, they can only send binary messages. However, I also extend the model to the case where the set of available messages for the agents is not restricted to being binary, which allows the framework of this paper to capture some natural features of applications, for example, the possibilities of abstaining in non-binding shareholder voting, staying neutral in public protests, and sending medium scores in survey polls. In this case, the principal's decision rule becomes multi-dimensional rather than a cut-off in the total number of approvals, which complicates the analysis. I provide a novel and tractable way to analyze this case by taking inspiration from Chernoff's (1952) fundamental connection between simple statistical hypothesis tests and large deviation theory. I show that all of the results presented above are robust in a natural class of equilibria in which the agents follow monotonic strategies.

It is also interesting to see how much information the principal can elicit if he can *ex-ante* commit to a decision rule. In LMB's framework, when N is large, the principal can approach his first-best outcome by committing to a voting mechanism with any qualified majority rule in which he chooses A if the ratio of approvals exceeds a certain cut-off. However, I show that in the present paper's framework, the principal cannot rely on any qualified majority rule since according to the Condorcet jury theorem, they all lead to the first-best outcome for the agents. However, the principal can approach his first-best outcome by randomizing between two qualified majority rules, that is, between two cut-offs in the ratio of approvals.

The rest of this paper proceeds as follows: Section 2 describes the model and characterizes the equilibrium. Section 3 presents the main result that learning is always incomplete no matter how many agents there are. Section 4 discusses information transmission from the agents to the principal. Section 5 analyzes the case where the set of available messages for the agents is not restricted to being binary. Section 6 studies the situation in which the principal can *ex-ante* commit to a decision rule. Section 7 surveys the related literature, and Section 8 concludes the paper. Most of the proofs are sketched in the main text, with the details relegated to the appendix.

2 Model

2.1 Basic Setting

There is one principal (he) and N agents (she). The principal has to decide between two policies, A and B. When he chooses B, the payoffs for all players are normalized to 0. When he chooses A, the payoffs for all players depend on an unknown state of the world $\theta \in \Theta$, with $\Theta = \{\theta_1, \ldots, \theta_n\} \subset \mathbb{R}$ and $\theta_1 < \ldots < \theta_n$.[2] In state θ, the principal receives the

[2]In this paper, we mostly focus on the case where $n = 3$.

payoff $V_{pc}(\theta)$ by choosing A, while the agents all have the same preference and receive the payoff $V_{ag}(\theta)$.

Both the principal and the agents receive higher payoffs from A when the state is higher, that is, both $V_{pc}(\theta)$ and $V_{ag}(\theta)$ strictly increase with θ. There are thresholds $\hat{\theta}_{pc}, \hat{\theta}_{ag} \in \Theta$ such that for each $j \in \{pc, ag\}$:

$$V_j(\theta) > 0 \text{ if } \theta \geq \hat{\theta}_j,$$
$$V_j(\theta) < 0 \text{ if } \theta < \hat{\theta}_j.$$

The principal prefers A more than the agents in every state, that is,[3]

$$V_{pc}(\theta) \geq V_{ag}(\theta), \ \forall \theta \in \Theta, \tag{1}$$

$$\hat{\theta}_{pc} < \hat{\theta}_{ag}. \tag{2}$$

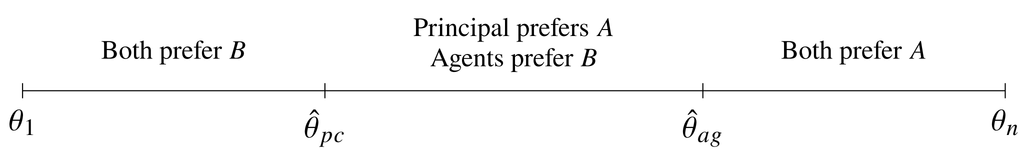

Figure 1

The Preferences of the principal and the agents when the realized state is known. Their preferences are not aligned when $\theta \in [\hat{\theta}_{pc}, \hat{\theta}_{ag})$.

For the information structure, the principal and the agents share a common prior belief $q^0 = (q^0_1, \ldots, q^0_n) \in \Delta^n$ about the unknown state, with $q^0_j > 0$ for each $j \in \{1, \ldots, n\}$. Conditional on the state $\theta \in \Theta$, each agent $i \in \{1, \ldots, N\}$ receives a private, independent signal $s^i \in \{\ell, h\}$, that is, a low or a high signal, with

$$\rho_j = \mathbb{P}[s^i = h | \theta_j], \ \forall \theta_j \in \Theta,$$

$$0 < \rho_1 < \ldots < \rho_n < 1. \tag{3}$$

Hence, the agents are more likely to receive signal h when the state is higher.

After observing the private signal, each agent chooses whether to approve A. The

[3] The condition (2) that the principal and the agents have different cut-offs for states is critical for results, while the condition (1) is for the better exposition. If (1) is violated, the main results of this paper (Theorems 1 and 2) still hold and the other results except for Lemma 1 can also be easily extended.

principal observes the total number of approvals $T \in \{0, ..., N\}$ and chooses the policy that maximizes his expected payoff.

2.2 Three-State Scenario

For simplicity, this paper focuses on the case where $\Theta = \{\theta_1, \theta_2, \theta_3\}$.[4] Both the principal and the agents prefer A to B in state θ_3 and prefer B to A in state θ_1:

$$V_{pc}(\theta_3) > 0 \text{ and } V_{ag}(\theta_3) > 0,$$

$$V_{pc}(\theta_1) < 0 \text{ and } V_{ag}(\theta_1) < 0,$$

while only the principal prefers A in state θ_2:

$$V_{pc}(\theta_2) > 0 \text{ and } V_{ag}(\theta_2) < 0, \tag{4}$$

that is, we have $\hat{\theta}_{pc} = \theta_2$ and $\hat{\theta}_{ag} = \theta_3$. The preferences of the principal and the agents are not aligned in state θ_2, which is a ***misaligned state***. The preferences of the principal and the agents are illustrated by the simplex of belief $q = (q_1, q_2, q_3) \in \Delta^3$ in Figure 2.

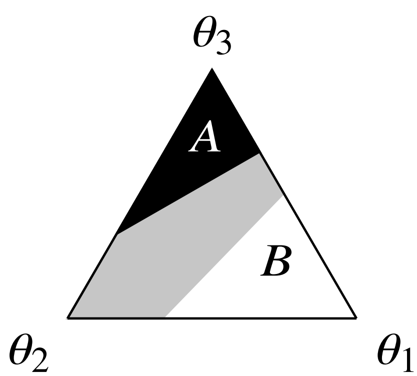

Figure 2

Preferences when the realized state is uncertain.
The corner θ_i for $i \in \{1, 2, 3\}$ corresponds to the belief q with $q_i = 1$. The segment $\theta_i\theta_j$ corresponds to the set of beliefs $\{q | q_i + q_j = 1\}$. Both the principal and the agents prefer A when they all hold a belief q in the black area, while both prefer B in the white area. In the shaded area, only the principal prefers A.

[4]This setting allows us to incorporate and compare the results with the existing literature. The main results of this paper (Theorems 1 and 2) hold for the general setting in Section 2.1. The other results can also be easily extended.

If we assume that $q_2^0 = 0$ and ignore the misaligned state θ_2, then the preferences of the principal and the agents are aligned when the realized state is known and misaligned when the realized state is uncertain. The conflict of interest is generated by ***different payoff intensities in state*** θ_1 ***and*** θ_3 between the principal and the agents. From (1):

$$V_{pc}(\theta_3) \geq V_{ag}(\theta_3) > 0 > V_{pc}(\theta_1) \geq V_{ag}(\theta_1).$$

Therefore,[5]

$$-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)} \leq -\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}. \tag{5}$$

As illustrated in Figure 3,[6] the principal and the agents have different thresholds of acceptance: for each belief $q = (q_1, q_3) \in \Delta^2$, the principal prefers A to B if $\frac{q_3}{q_1} > -\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$, while the agents prefer A to B if $\frac{q_3}{q_1} > -\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$.

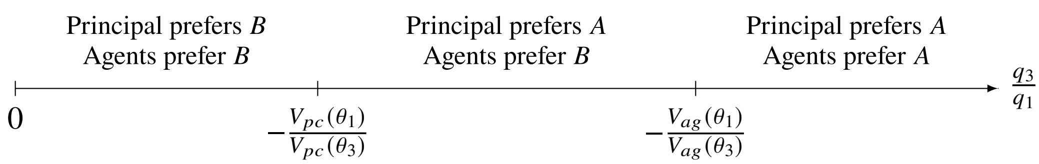

Figure 3

Different thresholds of acceptance when $q_2 = 0$.

2.3 Strategy and Equilibrium

We examine the symmetric *Bayesian Nash equilibrium*, in which all the agents use the same strategy $\mathbf{x} = (x_\ell, x_h) \in [0,1]^2$. Each agent $i \in \{1, ..., N\}$ approves A with probabilities x_ℓ and x_h respectively, when $s^i = \ell$ and $s^i = h$.

We consider equilibria in which the agents who receive signal h are more likely to approve A than the agents who receive signal ℓ, that is, $x_\ell \leq x_h$.[7] Note that there always exists a babbling equilibrium in which $x_\ell = x_h$, that is, the agents ignore their signals. In this equilibrium, the principal finds the total number of approvals uninformative and makes

[5] All results hold if (5) is valid but (1) is violated. When (2) is valid, the condition (5) is equivalent to the argument that under each belief, if the agents prefer A, the principal does so. If (5) is violated, the main results of this paper (Theorems 1 and 2) still hold and the other results except for Lemma 1 can also be easily extended.

[6] Note that when we assume $q_2^0 = 0$ and ignore the misaligned state θ_2, we suppress Figure 2 to its $\theta_3\theta_1$ segment, which we convert to Figure 3.

[7] For equilibria with $x_\ell \geq x_h$, we relabel approving A as rejecting A, and the following analyses still hold.

a decision based only on his prior belief. There is complete unraveling, and no information is transmitted from the agents to the principal.

We now consider the case where $x_\ell < x_h$. In this case, the principal forms his posterior belief based on his prior belief and the total number of approvals T. The posterior likelihood ratios[8]

$$\frac{\mathbb{P}[T; N|\theta_3]}{\mathbb{P}[T; N|\theta_1]} \text{ and } \frac{\mathbb{P}[T; N|\theta_2]}{\mathbb{P}[T; N|\theta_1]}$$

strictly increase with T since the agents are more likely to receive higher signals and hence are more likely to approve A when the realized state is higher, that is,

$$\rho_1 x_h + (1-\rho_1)x_\ell < \rho_2 x_h + (1-\rho_2)x_\ell < \rho_3 x_h + (1-\rho_3)x_\ell,$$

by (3) and $x_\ell < x_h$. Thus, the principal's posterior belief that the realized state is θ_1 strictly decreases with T.

Hence, a pure strategy for the principal is a cut-off $\hat{T} \in \{0, ..., N+1\}$ such that he chooses A if and only if $T \geq \hat{T}$. A mixed strategy for the principal allows him to randomize when he observes $\hat{T}$ approvals. For simplicity, we assume in the main text that the principal always chooses B when he is indifferent and hence that he always uses pure strategies. All results remain valid when the principal can use a mixed strategy, as shown in the appendix.[9]

We focus on the ***informative equilibrium*** in which the agents use an ***informative strategy*** $\mathbf{x}$ with $x_\ell < x_h$, that is, the agents make decisions according to their private information, and the principal uses a ***responsive strategy*** $\hat{T}$ with $\hat{T} \in \{1, ..., N\}$, that is, the principal makes decisions according to the number of approvals.

2.4 Characterization of Informative Equilibria

Best Response of the Agents: Consider a strategy profile in which the agents choose an informative strategy $\mathbf{x}$ and the principal chooses a responsive strategy $\hat{T}$. An agent is *pivotal* if the principal receives $\hat{T}-1$ approvals from the other $N-1$ agents. When deciding whether

[8] $\mathbb{P}[T; N|\theta_i] = \binom{N}{T}\underbrace{[\rho_i x_h + (1-\rho_i)x_\ell]}_{\text{Prob of approving } A}{}^T \underbrace{[1-\rho_i x_h - (1-\rho_i)x_\ell]}_{\text{Prob of rejecting } A}{}^{N-T}, \forall i \in \{1, 2, 3\}$.

[9] In particular, we do not rely on the principal's mixed strategies to ensure the existence of informative equilibria. In the appendix, we characterize the equilibria where the principal can use a mixed strategy and show that if there exists an equilibrium in which the principal randomizes at $\hat{T} \in \{0, ..., N-1\}$, then there must exist an equilibrium in which the principal chooses A if and only if $T > \hat{T}$. If there exists an equilibrium in which the principal randomizes at $\hat{T} = N$, then there exists an equilibrium in which the principal chooses A if and only if $T = N$.

to approve A, it is optimal for an agent to condition on the pivotal event since this agent's decision cannot affect the outcome in any other event. The likelihood of being pivotal in state $\theta_i \in \{\theta_1, \theta_2, \theta_3\}$ is

$$\mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}] = \binom{N-1}{\hat{T}-1} [\underbrace{\rho_i x_h + (1-\rho_i)x_\ell}_{\text{Prob of approving } A}]^{\hat{T}-1} [\underbrace{1-\rho_i x_h - (1-\rho_i)x_\ell}_{\text{Prob of rejecting } A}]^{N-\hat{T}}.$$

When this agents receives $s \in \{\ell, h\}$, she approves A only if

$$\sum_{i=1}^{3} \underbrace{q_i^0}_{\text{prior}} \cdot \underbrace{\mathbb{P}[s|\theta_i]}_{signal} \cdot \underbrace{\mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}]}_{\text{being pivotal}} \cdot V_{ag}(\theta_i) \geq 0.$$

Rewriting this as a payoff-weighted likelihood ratio, we have

$$\frac{q_3^0 \cdot \mathbb{P}[s|\theta_3] \cdot \mathbb{P}[piv|\theta_3; \mathbf{x}, \hat{T}] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^{2} q_i^0 \cdot \mathbb{P}[s|\theta_i] \cdot \mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}] \cdot V_{ag}(\theta_i)} \geq 1.$$

Letting $L_{ag}(s; \mathbf{x}, \hat{T})$ denote the left side. This agent chooses $\mathbf{x}$ as the best response if for each $s \in \{\ell, h\}$,

$$\begin{cases} x_s = 1 & \text{when } L_{ag}(s; \mathbf{x}, \hat{T}) > 1, \\ x_s \in [0,1] & \text{when } L_{ag}(s; \mathbf{x}, \hat{T}) = 1, \\ x_s = 0 & \text{when } L_{ag}(s; \mathbf{x}, \hat{T}) < 1. \end{cases} \tag{6}$$

By (3), we have

$$L_{ag}(h; \mathbf{x}, \hat{T}) > L_{ag}(\ell; \mathbf{x}, \hat{T}). \tag{7}$$

By (6) and (7), if $\mathbf{x}$ with $x_\ell < x_h$ is the best response to itself and $\hat{T} \in \{1, ..., N\}$, then it must satisfy the following:

$$\begin{cases} x_h = 1 \text{ if } x_\ell > 0, \\ x_\ell = 0 \text{ if } x_h < 1. \end{cases}$$

<ins>Best Response of the Principal</ins>: Consider the case where the agents choose an informative strategy $\mathbf{x}$. When the principal observes T approvals from N agents, he chooses A only if

$$\sum_{i=1}^{3} \underbrace{q_i^0}_{\text{prior}} \cdot \underbrace{\mathbb{P}[T; N|\theta_i]}_{T \text{ approvals}} \cdot V_{pc}(\theta_i) > 0.$$

Rewriting this as a payoff-weighted likelihood ratio, we have

$$\frac{\sum_{i=2}^{3} q_i^0 \cdot \mathbb{P}[T; N|\theta_i] \cdot V_{pc}(\theta_i)}{-q_1^0 \cdot \mathbb{P}[T; N|\theta_1] \cdot V_{ag}(\theta_1)} > 1.$$

Letting $L_{pc}(T; \mathbf{x})$ denote the left side.[10] Note that $L_{pc}(T; \mathbf{x})$ strictly increases with T since

$$\rho_1 x_h + (1 - \rho_1)x_\ell < \rho_2 x_h + (1 - \rho_2)x_\ell < \rho_3 x_h + (1 - \rho_3)x_\ell$$

by (3) and $x_\ell < x_h$, that is, the agents are more likely to approve A when the realized state is higher. The optimal cut-off for the principal is

$$\hat{T} = \min\{T \mid L_{pc}(T; \mathbf{x}) > 1 \text{ and } T \in \{0, \dots, N + 1\}\}. \tag{8}$$

Informative Equilibrium: An ***informative equilibrium*** is characterized by a pair $\{(x_\ell, x_h), \hat{T}\}$ that satisfies (6) and (8), with $x_\ell < x_h$ and $\hat{T} \in \{1, \dots, N\}$.

We can show that in every informative equilibrium, the agents always reject A when they receive signal ℓ:

Lemma 1. *The agents choose $x_\ell = 0$ in every informative equilibrium.*

For a sketch of the proof, consider an informative equilibrium with $x_\ell > 0$. From (6) and (7), we have $x_h = 1$ and hence $x_\ell \in (0, 1)$. Therefore, conditional on being pivotal, the agents always approve A when they receive signal h and are indifferent between A and B when they receive signal ℓ. Note that the principal prefers A more than the agents do. If an agent is indifferent conditional on being pivotal and receiving signal ℓ, then the principal prefers A when this agent is pivotal and receives signal ℓ. Since this agent only rejects A when she receives signal ℓ, the principal prefers A when this agent is pivotal and rejects A, that is, when the principal observes $\hat{T} - 1$ approvals. However, it leads to a contradiction to the optimality of $\hat{T}$ as shown in (8).

3 Information Aggregation

In many situations, including non-binding shareholder voting, public protests, and survey polls, among others, there are usually a large number of agents (shareholders, citizens, and

[10] We suppress $\mathbf{x}$ in $\mathbb{P}[T; N|\theta_i]$ for each $i \in \{1, 2, 3\}$.

interviewees). In this section, we study whether information dispersed among the agents is effectively aggregated and whether the principal fully learns the state as the number of agents grows large:

Definition 1. *A sequence of equilibria* $\{\Gamma_N\}_{N=1}^{\infty}$ ***aggregates information*** *if*

$$\lim_{N\to\infty} \mathbb{P}[A|\theta_1;\Gamma_N] + \mathbb{P}[B|\theta_3;\Gamma_N] = 0.$$

We consider information aggregation with minimal requirements by focusing on stated θ_1 and θ_3 in which the preferences of the principal and the agents are aligned. Note that the failure of information aggregation implies that the principal fails to fully learn the state no matter how many agents there are.

The present paper's framework shares certain qualitative features with elections in which voters decide whether to approve a policy and the total number of approvals matters. As shown by the Condorcet jury theorem (see Ladha, 1992) and its modern versions (Feddersen and Pesendorfer, 1997, 1998, Myerson, 1998, Duggan and Martinelli, 2001), elections effectively aggregate dispersed information among the agents (voters) under any qualified majority rule that depends on the ratio of votes. However, full information aggregation fails under the unanimity rule or rules that are close to it.

The fundamental difference between this paper's framework and elections is that the principal can now choose the policy based on his own decision and is not constrained by predetermined rules. The existing literature extends the idea behind the Condorcet jury theorem and shows that information is still effectively aggregated if the conflict of interest between the principal and the agents is small. However, we show that full information aggregation always fails after adding the misaligned state θ_2.

3.1 Results from the Existing Literature

In this section, we assume that $q_2^0 = 0$ and ignore the misaligned state θ_2. As discussed in Section 2.2, the preferences of the principal and the agents are fully aligned if the realized state is known and misaligned when the state is uncertain. The principal and the agents have different thresholds of acceptance: for each belief $q = (q_1, q_3) \in \Delta^2$, the principal prefers A if $\frac{q_3}{q_1} > -\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$, while the agents prefer A if $\frac{q_3}{q_1} > -\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$. Hence, the ratio of $-\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$ to $-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$ is a natural measure for the conflict of interest between the principal and the agents due to the different payoff intensities in state θ_1 and state θ_3. The existing literature has considered this case and shown that if the ratio $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ is below a certain threshold, information is effectively aggregated.[11]

Proposition 1. *Assume that $q_2^0 = 0$.*

1. If
$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1} \cdot \frac{1-\rho_1}{1-\rho_3}, \tag{9}$$
then there exists a sequence of equilibria that aggregates information.

2. If
$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} > \frac{\rho_3}{\rho_1} \cdot \frac{1-\rho_1}{1-\rho_3}, \tag{10}$$
then only the babbling equilibrium exists for each N.

Figure 4 illustrates the intuition behind Proposition 1. In an informative equilibrium, the agents who receive signal h must (weakly) prefer A conditional on being pivotal. That is, each agent's posterior belief must be higher than $-\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$ conditional on signal h and $\hat{T}-1$ approvals from the other $N-1$ agents. However, the principal optimally chooses the cut-off $\hat{T}$. He must prefer B when he observes $\hat{T}-1$ approvals from N agents. That is, his posterior belief must be less than $-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$ when there are already $\hat{T}-1$ approvals from $N-1$ agents and the pivotal agent rejects A. Therefore, the difference in the thresholds of the posterior likelihood ratio between the agents and the principal depends at most on one signal h and one rejection in every informative equilibrium.

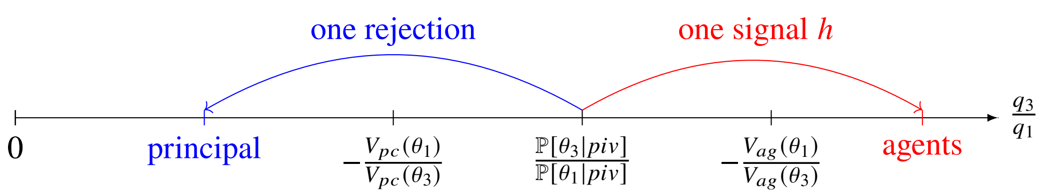

Figure 4

Inference from being pivotal and thresholds of acceptance.

The red line corresponds to the argument that the agents signal h must prefer A conditional on being pivotal. The blue line corresponds to the argument that the principal must prefer B when he observes $\hat{T}-1$ approvals, that is, when the pivotal agent rejects A.

[11] Levit and Malenko (2011) consider the setting with a symmetric information structure. Battaglini (2017) considers the setting in which the number of voters follows a Poisson distribution. Ekmekci and Lauermann (2022) consider the setting with a deterministic population size in their online appendix. We sketch their proof.

Note that if the agents report their signals truthfully, that is, if the agents approve A when they receive signal h and reject A when they receive signal ℓ, the decrease in the posterior likelihood ratio due to one rejection is maximized. Hence, we can replace "one rejection" in Figure 4 with "one signal ℓ" and argue that a necessary condition for the existence of informative equilibria is that the difference in the thresholds of the posterior likelihood ratio depends on at most one signal h and one signal ℓ, and thus we derive (9).

The inequality (9) is indeed a necessary condition for the existence of the informative equilibrium in which the agents report truthfully, that is, it is a necessary condition for the existence of $\hat{T}$ such that

$$\frac{\mathbb{P}[\hat{T};N|\theta_3]}{\mathbb{P}[\hat{T};N|\theta_1]} \geq -\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)},$$
$$\frac{\mathbb{P}[\hat{T}-1;N|\theta_3]}{\mathbb{P}[\hat{T}-1;N|\theta_1]} \leq -\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)},$$

when $x_h = 1$ and $x_\ell = 0$. However, it is not a sufficient condition due to the requirement that $\hat{T}$ must be an integer. We show that as N grows large, the effect of this integer requirement vanishes, and there exists an informative equilibrium in which the agents report almost truthfully, with $x_h \approx 1$ and $x_\ell = 0$:

Lemma 2. *Assume that $q_2^0 = 0$. If $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}\frac{1-\rho_1}{1-\rho_3}$, then for each ϵ, there exists an N_ϵ such that for each $N > N_\epsilon$, there exists an informative equilibrium in which the agents choose $x_h = 1-\epsilon$ and $x_\ell = 0$ (almost truthtelling).*

Therefore, when the conflict generated by the different payoff intensities is below the threshold, the principal can fully learn the unknown state by the law of large numbers as $N \to \infty$, and information is effectively aggregated.

3.2 Failure of Information Aggregation

We consider the setting with the misaligned state θ_2 by assuming that $q_2^0 > 0$ in the rest of this paper. In this setting, the preferences of the principal and the agents might be misaligned even if they know the realized state.

As reviewed in Section 3.1, the existing literature provides the condition under which informative equilibria exist and shows that if this condition is satisfied, there exists a sequence of informative equilibria that aggregates information with the agents reporting their signals almost truthfully as $N \to \infty$. We now show that when $q_2^0 > 0$, full information aggregation always fails even if informative equilibria exist. The principal therefore fails to fully learn the realized state even if he receives a large number of informative messages.

Theorem 1. *No sequence of equilibria aggregates information. That is, there exists a constant $c > 0$[12] such that for each N and each equilibrium Γ with N agents,*

$$\mathbb{P}[A|\theta_1; \Gamma] + \mathbb{P}[B|\theta_3; \Gamma] > c.$$

We illustrate the intuition behind Theorem 1 through two steps.

Step 1. Vanishing Information

We first argue that when $q_2^0 > 0$, the information contained in an agent's message must vanish as $N \to \infty$ in every sequence of informative equilibria, which differs sharply from Lemma 2. We also show how quickly the information vanishes, that is, the rate of convergence for $x_h \to 0$.

Proposition 2. *For each $\epsilon > 0$, there exists N'_ϵ such that when $N > N'_\epsilon$, the agents choose $x_h < \epsilon$ in every informative equilibrium. Furthermore, there exists $T_0 > 0$ such that for each N and each informative equilibrium with N agents,*

$$N \cdot x_h < T_0.$$

To understand the intuition, fix an arbitrary $x \in (0, 1])$ and suppose that the agents behave according to $x_h = x$ and $x_\ell = 0$ for each $N \in \mathbb{N}^+$. The expected number of approvals in each state θ_i is $N \cdot \rho_i x$ for $i \in \{1, 2, 3\}$. Figure 5 illustrates the distributions of the total number of approvals T when N is large.[13] We can see that when N is large, (i) the principal chooses $\hat{T}$ such that $N\rho_1 x < \hat{T} < N\rho_2 x$ since he prefers A when the realized state is θ_2 or θ_3, and (ii) as shown in Figure 5,

$$\frac{\mathbb{P}[\hat{T}; N|\theta_3]}{\mathbb{P}[\hat{T}; N|\theta_2]} \approx 0,$$

and hence

$$\frac{\mathbb{P}[piv|\theta_3]}{\mathbb{P}[piv|\theta_2]} = \frac{\mathbb{P}[\hat{T} - 1; N - 1|\theta_3]}{\mathbb{P}[\hat{T} - 1; N - 1|\theta_2]} \approx \frac{\mathbb{P}[\hat{T}; N|\theta_3]}{\mathbb{P}[\hat{T}; N|\theta_2]} \approx 0.$$

[12]The value of c depends on other parameters, as do N_ϵ, T_0, $\hat{T}^*$, N^*, M_1, and δ, introduced later in this section.

[13]Note that we can approximate these distributions by normal distributions.

Each agent believes that the realized state is very unlikely to be θ_3 conditional on being pivotal. She rejects A even when she receives signal h. Hence, she does not choose $x_h = x$ as a best response.

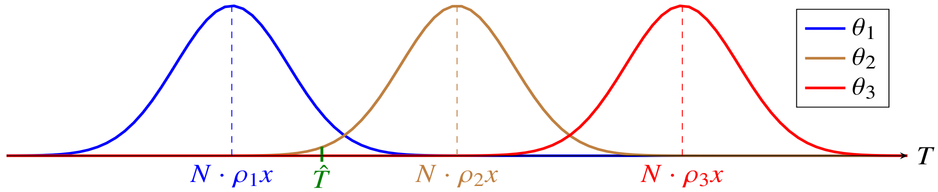

Figure 5

The distribution of the total number of approvals for A in each state when the agents choose $x_h = x \in (0, 1)$ and $x_\ell = 0$. The principal optimally chooses $\hat{T}$.

Each agent makes her decision conditional on her signal and being pivotal. However, the number of approvals that makes an agent pivotal is endogenous, since the principal must be nearly indifferent between A and B when he observes this number. If the agents' messages are informative and the principal receives a large number of messages, then each agent believes that the realized state must be either θ_1 or θ_2 given that the principal is uncertain whether the realized state is θ_1 and indifferent between A and B, as shown in Figure 5. Hence, each agent prefers B regardless of her signal, conditional on being pivotal.

To prevent the inference conditional on being pivotal from overwhelming each agent's private information, the distributions of the total number of approvals in different states must be close to each other, as shown in Figure 6.[14,15] Hence, the information contained in an agent's message must vanish as $N \to \infty$. We then show that the information in an agent's message must vanish at a high speed to make the differences in the mean $N \cdot \rho_i x_h$ of different states finite. Thus, the rate of convergence for $x_h \to 0$ must be comparable to $\frac{1}{N}$.

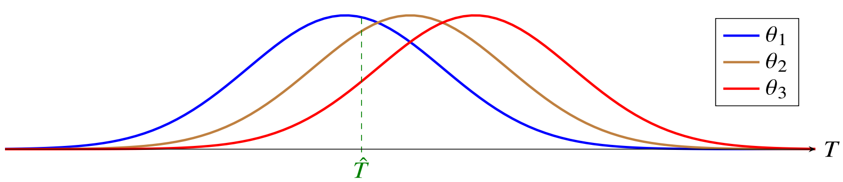

Figure 6

The distributions of the total number of approvals in different states must be close to each other.

[14]When $x_\ell = 0$ and Nx_h is finite, we can approximate the distributions of the total number of approvals by Poisson distributions.

[15]One might directly see the failure of information aggregation from Figure 6. The distribution in state θ_1 must be close to the distribution in state θ_3. Hence, the principal cannot find a $\hat{T}$ to separate them.

For an alternative intuition behind Proposition 2, once again fix an arbitrary $x \in (0, 1])$ and suppose that the agents behave according to $x_h = x$ and $x_\ell = 0$. In this case, the principal and the agents have different preferences, that is, they have different cut-offs for the total number of approvals above which A should be implemented. If the difference in cut-offs is large, the strategy profile with $x_h = x$ and $x_\ell = 0$ cannot be a part of an informative equilibrium. However, the difference in cut-offs increases with x, which measures the information in an agent's message, and the number of agents N. Therefore, the information contained in an agent's message must vanish as $N \to \infty$ in every sequence of informative equilibria. Note that in LMB's setting analyzed in Section 3.1, the difference in cut-offs is constant with respect to N and decreases with x if we ignore the integer requirement for $\hat{T}$.

Step 2. Unanimity Rule

By Proposition 2, the expected number of approvals in each state is always smaller than a finite number that is independent of N. We also show that the principal's cut-off $\hat{T}$ is always smaller than a finite number that is independent of N. Hence, the principal must follow either the unanimity rule ($\hat{T} = 1$) such that he chooses B only if all the agents reject A or rules that are similar to the unanimity rule.

Proposition 3. *There exists $T_0 > 0$ such that for each N and each informative equilibrium with N agents,*

$$\hat{T} < T_0.$$

Note that for each N and each informative equilibrium with N agents, the posterior beliefs about state θ_3 and state θ_1 must have the same magnitude conditional on being pivotal, that is, there exists an $M_1 > 0$ such that

$$\frac{1}{M_1} < \frac{\mathbb{P}[\theta_3|piv]}{\mathbb{P}[\theta_1|piv]} < M_1. \tag{11}$$

If $\frac{\mathbb{P}[\theta_3|piv]}{\mathbb{P}[\theta_1|piv]} \to 0$, then the agents believe that the realized state is either θ_1 or θ_2 conditional on being pivotal, and they reject A when they receive $s = h$. If $\frac{\mathbb{P}[\theta_3|piv]}{\mathbb{P}[\theta_1|piv]} \to \infty$, then the principal believes that the realized state is either θ_2 or θ_3 when he observes $\hat{T} - 1$ approvals, and then chooses A.

In what follows, we show that there is no sequence of equilibria that satisfies

$$\lim_{N\to\infty} \sum_{T=0}^{\hat{T}-1} \mathbb{P}[T; N|\theta_1] = 1 \text{ and } \lim_{N\to\infty} \sum_{T=0}^{\hat{T}-1} \mathbb{P}[T; N|\theta_3] = 0,$$

since (i) $\mathbb{P}[\hat{T}-1; N|\theta_1]$ and $\mathbb{P}[\hat{T}-1; N|\theta_3]$ have the same magnitude as shown in (11), and (ii) $\hat{T}$ is always smaller than a finite number T_0 as shown in Proposition 3. Note that the left term is the probability that the principal chooses B in state θ_1, while the right term is the probability that the principal chooses B in state θ_3. Therefore, no sequence of equilibria aggregates information. Further, we can show that the principal chooses the wrong policy with a strictly positive probability in each state:

Corollary 1. *There exists $\bar{\delta} > 0$ such that for each N and each informative equilibrium with N agents,*

$$\mathbb{P}[A|\theta] > \bar{\delta},\ \forall \theta \in \{\theta_1, \theta_2, \theta_3\},$$
$$\mathbb{P}[B|\theta] > \bar{\delta},\ \forall \theta \in \{\theta_1, \theta_2, \theta_3\}.$$

4 Information Transmission

In this section, we first discuss conditions under which informative equilibria exist. If there exist multiple informative equilibria, we can rank them in both the Blackwell order and the Pareto order. Hence, we can identify the most informative equilibrium that also maximizes the payoffs of the principal and the agents. We then discuss the amount of information transmission by focusing on the most informative equilibrium and show that the amount of information transmission decreases with the conflict of interest between the principal and the agents. Finally, we argue that it might be better to disperse information among the agents instead of letting one agent receive all the information and further argue that communication among the agents might impede information transmission and hurt both the principal and the agents.

4.1 Existence of Informative Equilibria

We say ***information transmission persists*** if there exists N_1 such that for each $N > N_1$, an informative equilibrium exists. We say ***information transmission fails*** if there exists N_2 such that for each $N > N_2$, only the babbling equilibrium exists.

Proposition 1 indicates that when $q_2^0 = 0$, information transmission fails if

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} > \frac{\rho_3}{\rho_1} \cdot \frac{1-\rho_1}{1-\rho_3}.$$

We now provide a new condition when $q_2^0 > 0$:

Proposition 4. *When $q_2^0 > 0$, information transmission fails if*

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} > \frac{\rho_3}{\rho_1}.$$

Therefore, when $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} \in (\frac{\rho_3}{\rho_1}, \frac{\rho_3}{\rho_1}\frac{1-\rho_1}{1-\rho_3})$, there exists a sequence of equilibria that aggregates information if $q_2^0 = 0$ from Proposition 1, while information transmission fails if $q_2^0 > 0$, that is, only the babbling equilibrium exists when N is large enough.

When we assume $q_2^0 = 0$ and ignore the misaligned state θ_2, we show that informative equilibria exist if the difference in the thresholds of the posterior likelihood ratio between the principal and the agents depends at most on one signal h and one rejection. We then let the agents report messages sincerely to maximize the information contained in one rejection and hence in one message.

However, the agents cannot send messages sincerely when $q_2^0 > 0$ and N is large, as discussed before. Otherwise, the agents infer that the state must be either θ_1 or θ_2 conditional on being pivotal, and ignore their signals. Instead, they choose $x_h \approx 0$ and their messages are nearly uninformative according to Proposition 2. Therefore, when N is large, in an informative equilibrium, the agents signal h are indifferent between A and B conditional on being pivotal, that is, the posterior likelihood ratio of state θ_3 to state θ_1 conditional on signal h and $\hat{T} - 1$ approvals from $N - 1$ agents must be higher than $-\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$. However, since the principal prefers B when he observes $\hat{T} - 1$ approvals from N agents, the posterior likelihood ratio of θ_3 to state θ_1 conditional on $\hat{T} - 1$ approvals from N agents must be lower than $-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$. Then since almost no information is contained in an agent's message hence in one rejection, the posterior likelihood ratio conditional on $\hat{T} - 1$ approvals from $N - 1$ agents must also be lower than $-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$. Therefore, the difference in the thresholds of the posterior likelihood ratio $-\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}$ and $-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}$ depends on at most one signal h.

Note that when $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} \in (\frac{\rho_3}{\rho_1}, \frac{\rho_3}{\rho_1}\frac{1-\rho_1}{1-\rho_3})$ and $q_2^0 > 0$, informative equilibria might exist when N is small. In this situation, the agents can choose an x_h away from 0, which increases the information contained in one rejection and hence makes up for a larger difference in the thresholds of the posterior likelihood ratio. Note that both the principal and the agents receive higher expected payoffs from any informative equilibria than from the babbling equilibrium. Hence, the amount of information transmission and the welfare for the principal and the agents[16] are not monotonic with respect to N. The expected payoffs of both the principal and the agents are maximized if the number of agents equals some finite number. In contrast, both the principal and the agents receive a lower expected payoff when we let the number of agents go to infinity. We discuss the effect of the number of agents on the welfare more generally in Section 8.

When $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}$, we argue that information transmission persists if the misaligned state θ_2 is unlikely, that is, the prior q_2^0 is small. However, we cannot freely vary q_2^0 due to the constraint that $q^0 \in \Delta^3$. We replace q^0 with $\lambda = \{\lambda_1, \lambda_2\}$ such that

$$\lambda_1 = \frac{q_1^0}{q_3^0} \text{ and } \lambda_2 = \frac{q_2^0}{q_3^0}.$$

The ratio λ_2 measures the conflict of interest between the principal and the agents concerning the misaligned state θ_2. Both q_1^0 and q_3^0 are smaller while q_2^0 is larger when λ_2 is larger and λ_1 is constant.

Proposition 5. *If* $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}$, *then there exists* $\hat{\lambda}_2 > 0$ *such that*[17]

(i) if $\lambda_2 < \hat{\lambda}_2$, *information transmission persists,*

(ii) if $\lambda_2 > \hat{\lambda}_2$, *information transmission fails.*

In the appendix, we provide sufficient and necessary conditions under which there exists an informative equilibrium with $\hat{T} = 1$, that is, the equilibrium in which the principal chooses the unanimity rule. We show that there exists $\hat{\lambda}_{2,1}$ such that when N is large, an informative equilibrium with $\hat{T} = 1$ exists if $\lambda_2 < \hat{\lambda}_{2,1}$ and only if $\lambda_2 \leq \hat{\lambda}_{2,1}$. We then extend this approach and further derive $\hat{\lambda}_{2,j}$ for each $j \in \mathbb{N}$ corresponding to the informative equilibrium with $\hat{T} = j$ and derive[18]

$$\hat{\lambda}_2 = \sup_{j \in \mathbb{N}} \hat{\lambda}_{2,j}.$$

We can further show that

$$\lim_{j \to \infty} \hat{\lambda}_{2,j} = 0.$$

[16] In Section 4.3, we show that if there exists at least one informative equilibrium, there exists an informative equilibrium that maximizes the amount of information transmission and the welfare of the principal and the agents among all informative equilibria.

[17] The value of $\hat{\lambda}_2$ depends on the value of other parameters except λ_2 and N.

[18] When $\lambda_2 > \hat{\lambda}_2$, we only need to consider informative equilibria with $\hat{T} < T_0$ by Proposition 3 that guarantees that there exists N_2 above which no informative equilibrium exists.

Hence, for each $\lambda_2 > 0$, there exists T^* that is independent of N such that

$$\lambda_2 > \hat{\lambda}_{2,j}, \ \forall j > T^*,$$

which is also indicated by Proposition 3 that $\hat{T}$ in any informative equilibrium is always smaller than a number that is independent of N. The principal must follow the unanimity rule or rules that are close to it, which leads to the failure of information aggregation.

We now investigate how $\hat{\lambda}_2$ changes with other parameters.

Corollary 2. *The threshold $\hat{\lambda}_2$ increases with $V_{ag}(\theta_i)$ and decreases with $V_{pc}(\theta_i)$ for each $i \in \{1, 2, 3\}$.* [19]

From Corollary 2, the threshold $\hat{\lambda}_2$ decreases with $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ when we only vary one term. Note that $\hat{\lambda}_2$ measures the conflict of interest concerning the misaligned state θ_2 while $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ measures the conflict of interest concerning the payoff intensities in state θ_1 and state θ_3. Therefore, information transmission persists if both types of conflict are small, as shown by Figure 7.

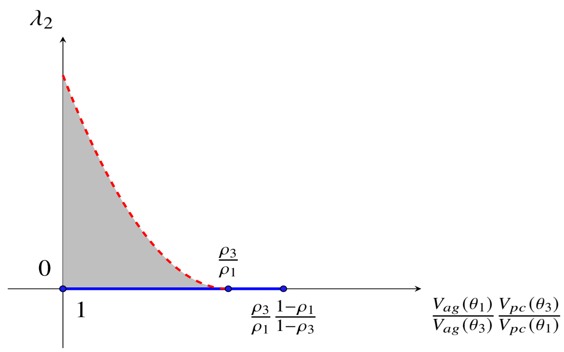

Figure 7

Information transmission and aggregation.

The existing literature considers the case where $\lambda_2 = 0$, and shows that there exists a sequence of equilibria that transmits and aggregates information when $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}\frac{1-\rho_1}{1-\rho_3}$. When $\lambda_2 > 0$, we show that information transmission persists in the shaded area while information aggregation always fails.

Corollary 3. *The threshold $\hat{\lambda}_2$ decreases with ρ_1 while it is not monotonic with ρ_2 and ρ_3.*

Consider the case where ρ_2 decreases. On the one hand, each agent has a higher incentive to approve A conditional on receiving signal h since this signal favors state θ_3

[19] When discussing the comparative statistics in Corollary 2, Corollary 3, and Corollary 4, we always change one parameter and keep the others fixed.

more. On the other hand, each agent has a lower incentive to approve A conditional on being pivotal since the distribution of the total number of approvals in state θ_2 moves closer to the distribution in state θ_1, which decreases this agent's posterior belief of state θ_3 conditional on being pivotal. Thus, a smaller ρ_2 has an ambiguous effect on the agents' incentives to approve A conditional on being pivotal and receiving signal h, and hence has an ambiguous effect on information transmission. We can apply a similar intuition to the case where ρ_3 increases. However, when ρ_1 decreases, the two effects mentioned above move the agents' posterior beliefs of state θ_3 in the same direction. Thus, a smaller ρ_1 increases the agents' incentives to approve A conditional on being pivotal and receiving signal h, and hence contributes to information transmission.

Corollary 3 indicates that the boundary of information transmission, that is, the red dashed line in Figure 7, moves outward when ρ_1 decreases. However, changes in ρ_3 and ρ_2 have ambiguous effects on it.

Corollary 4. *The threshold $\hat{\lambda}_2$ is not monotonic with λ_1.*

In Figure 8, we plot $\hat{\lambda}_{2,j}$ for $j \in \{1, 2, 3\}$ and $\hat{\lambda}_2$ as functions of λ_1. As stated before, an informative equilibrium with $\hat{T} = j$ exists if $\lambda_2 < \hat{\lambda}_{2,j}$ when N is large.

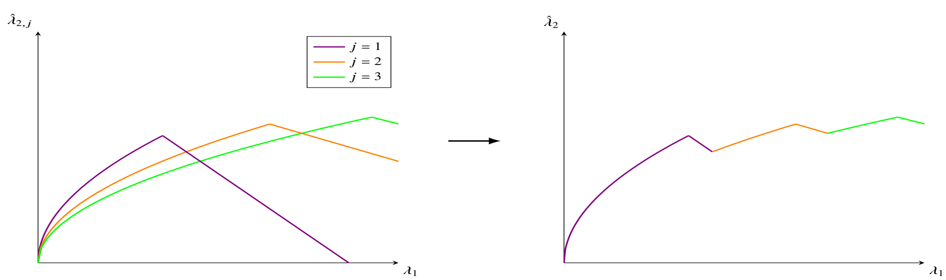

Figure 8
Non-monotonic boundaries.

The threshold $\hat{\lambda}_{2,j}$ is not monotonic with λ_1 for each $j \in \mathbb{N}$. To see the intuition, let us fix an arbitrary $j \in \mathbb{N}^+$ and let the principal always choose $\hat{T} = j$. We then consider the case where λ_1 increases while other parameters are constant. The prior q_2^0 is smaller and hence the conflict of interest between the principal and the agents is smaller, which contributes to information transmission and increases $\hat{\lambda}_{2,j}$. However, the prior q_3^0 also decreases, and hence the state θ_3 is less likely, which decreases the agents' incentives to approve A when they receive signal h, which impedes information transmission and decreases $\hat{\lambda}_{2,j}$.

4.2 Ranking Informative Equilibria

When we fix all parameter values, there cannot exist more than one informative equilibrium in which the principal chooses the same $\hat{T}$ since there exists at most one x_h solving (6). However, there might exist multiple informative equilibria with different $\hat{T}$, as shown by the left panel of Figure 8. We now rank them according to the payoffs of the principal and the agents. Let $U_{pc}(\Gamma)$ and $U_{ag}(\Gamma)$ be the expected payoff of the principal and the expected payoff of the agents respectively for a given equilibrium Γ.

Proposition 6. *Fix all parameter values. If there exist two informative equilibria* $\Gamma_1 = \{x_{h,1}, \hat{T}_1\}$ *and* $\Gamma_2 = \{x_{h,2}, \hat{T}_2\}$ *such that* $\hat{T}_1 < \hat{T}_2$*, then*

$$\begin{aligned} x_{h,1} &\leq x_{h,2}, \\ U_{pc}(\Gamma_1) &\leq U_{pc}(\Gamma_2), \\ U_{ag}(\Gamma_1) &\leq U_{ag}(\Gamma_2). \end{aligned}$$

All inequalities are strict if $x_{h,1} < 1$.

When the principal requires a higher $\hat{T}$, the agents approve A with a higher probability and hence increase x_h. The principal observes N messages from the agents that are identically distributed and independent conditional on the state and makes his decision to maximize his expected payoff. When x_h is higher, each message is more Blackwell informative, and hence the joint N messages are also more Blackwell informative. Thus, the principal receives a higher expected payoff from the equilibrium with a higher $\hat{T}$.

For the agents, consider an informative equilibrium $\Gamma = \{x_h, \hat{T}\}$ with $x_h < 1$. Each agent is indifferent between A and B conditional on receiving signal h and being pivotal. Hence, she is indifferent conditional on the event that there are $\hat{T}$ approvals from N agents since the agents randomize when she receives signal h. Therefore, the principal would still choose $\hat{T}$ if he shared the same preference with the agents. Thus, an informative equilibrium with a higher $\hat{T}$ also benefits the agents since the principal chooses $\hat{T}$ under a more Blackwell informative information structure.

As discussed above, in every informative equilibrium, when we fix the strategy of the agents, the principal and the agents agree on the same threshold $\hat{T}$, that is, they share common interests. Therefore, we can rank all informative equilibria in the Blakweell order or the Pareto order, and these two orders coincide with each other.

From Proposition 6, the informative equilibrium with the highest cut-off $\hat{T}_{max}$ maximizes the expected payoffs of the principal and the agents among all informative equilibria. We denote this equilibrium by the ***most informative equilibrium***. Note that the agents also choose the highest x_h in the most informative equilibrium among all informative equilibria. From Proposition 3, the highest cut-off $\hat{T}_{max}$ is always smaller than a number that is independent of N since messages from the agents cannot be too informative. Otherwise, the inference from being pivotal overwhelms each agent's private information.

We can show that for almost all parameter values that satisfy $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}$ and $\lambda_2 < \hat{\lambda}_2$, that is, for almost all parameter values under which information transmission persists, the highest cut-off $\hat{T}_{max}$ is independent of N when N is above some threshold. For the other parameter values that satisfy both conditions above, the highest cut-off $\hat{T}_{max}$ takes a value between two adjacent numbers. In the left panel of Figure 8, we can see that when N is large, the cut-off $\hat{T}_{max} = 1$ if (λ_1, λ_2) is above the orange line and below the purple line while $\hat{T}_{max} = 2$ if (λ_1, λ_2) is above the green line and below the orange line. However, for some points of (λ_1, λ_2) exactly on the orange line, the highest cut-off $\hat{T}_{max}$ might be either 1 or 2 when N is large.

4.3 Amount of Information Transmission

In this section, we discuss the maximal amount of information transmission by focusing on the most informative equilibrium $\Gamma_{max} = \{x_{h,max}, \hat{T}_{max}\}$.

Proposition 7. *In the equilibrium Γ_{max}, the agents' equilibrium strategy $x_{h,max}$ increases with $V_{ag}(\theta_i)$ and decreases with $V_{pc}(\theta_i)$ for each $i \in \{1, 2, 3\}$. Furthermore, it decreases with λ_2.* [20]

As discussed in Section 4.2, the principal receives more information from the agents if the agents choose a higher x_h. By Proposition 7, the agents' equilibrium strategy $x_{h,max}$ decreases with $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ when we only vary one term, and also decreases with λ_2. Hence, the maximal amount of information transmission decreases with both types of conflict between the principal and the agents.

We now compare the maximal amount of information transmission as $N \to \infty$ in the setting with the misaligned state θ_2 with the one in the setting with no misaligned state analyzed by the existing literature. We measure the maximal amount of information

[20] We always change one parameter and keep others including N fixed.

transmission by[21]

$$I = \limsup_{N\to\infty} \frac{V_{pc}^{max} - V_{pc}^{0}}{V_{pc}^{full} - V_{pc}^{0}} \in [0,1],$$

where (i) V_{pc}^{max} is the principal's expected payoff from the most informative equilibrium, (ii) V_{pc}^{0} is the principal's expected payoff from the uninformative babbling equilibrium, and (iii) V_{pc}^{full} is the principal's expected payoff if he can observe the realized state.

Figure 9 illustrates the maximal amount of information transmission regarding the two types of conflict between the principal and the agents, the conflict generated by the different payoff intensities in state θ_1 and θ_3 that is measured by $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$,[22] and the conflict concerning the misaligned state θ_2 that is measured by λ_2.

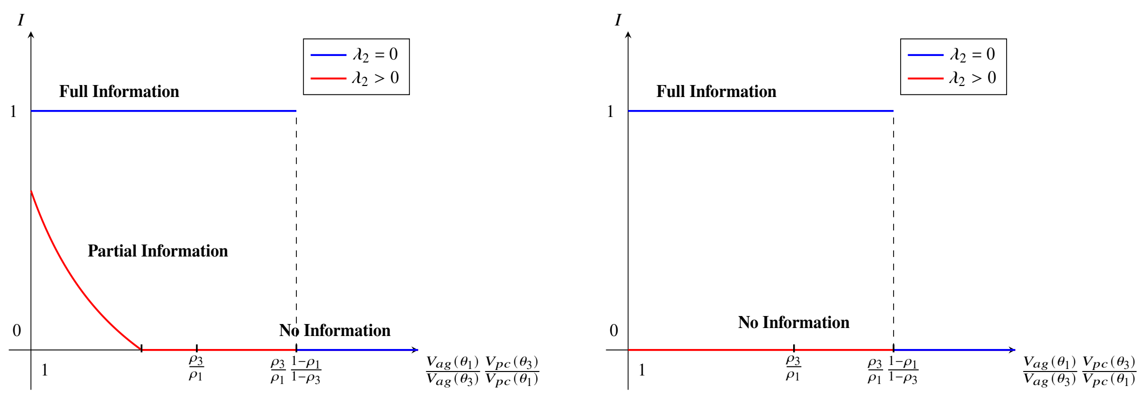

Figure 9

Maximal amount of information transmission.

We plot I as a function of $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ in different cases. The blue line corresponds to the case where $\lambda_2 = 0$. The red line corresponds to the case where $\lambda_2 > 0$. In the left panel, we choose $\lambda_2 < \hat{\lambda}_2$ given that $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} = 1$ for the red line. In the right panel, we chooses $\lambda_2 > \hat{\lambda}_2$ given that $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} = 1$ for the red line.

When $\lambda_2 = 0$, Proposition 1 shows that the principal fully learns the state as $N \to \infty$ if $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ is below the threshold $\frac{\rho_3}{\rho_1}\frac{1-\rho_1}{1-\rho_3}$. Otherwise, information transmission fails and only the babbling equilibrium exists.

This paper analyzes the setting with $\lambda_2 > 0$. In the left panel with a small λ_2, even if

[21] The limit always exists in the situations where $\lambda_2 = 0$.

[22] When varying $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$, we either only change $V_{ag}(\theta_1)$ or only change $V_{ag}(\theta_3)$, while keeping other parameters fixed.

the principal and the agents have the same payoffs in state θ_1 and state θ_3 with

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} = 1,$$

information aggregation fails and the amount of information transmission is limited. As $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ increases, the principal receives less information. When $\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}\frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)}$ is above a threshold that is lower than $\frac{\rho_3}{\rho_1}$, the principal receives no information. In this case, information transmission fails and only the babbling equilibrium exists. Note that the threshold above which information transmission fails decreases with λ_2. In the right panel with a large λ_2, information transmission always fails according to Proposition 5.

4.4 Information Aggregation and Transmission

We claim that the failure of information aggregation might facilitate information transmission and further argue that communication among the agents might impede information transmission and hurt both the principal and the agents.

Consider the case where there is only one agent and this agent receives all N signals. She advises the principal to choose A or not. As $N \to \infty$, this agent is fully informed about the realized state. She advises the principal to choose A in state θ_3 and choose B in state θ_2 and state θ_1. The principal follows this agent's advice of choosing B if he receives a negative expected payoff from choosing A,

$$q_2^0 V_{pc}(\theta_2) + q_1^0 V_{pc}(\theta_1) < 0,$$

that is, if

$$\frac{\lambda_2}{\lambda_1} < -\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_2)}.$$

Proposition 8. *There exists $\bar{\lambda}_1$ such that*[23]

$$\frac{\hat{\lambda}_2}{\lambda_1} > -\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_2)} \text{ iff } \lambda_1 < \bar{\lambda}_1.$$

We plot $\hat{\lambda}_2$ as a function of λ_1 in Figure 10, which illustrates Proposition 8. Consider a pair of (λ_1, λ_2) in the shaded area. If there are N agents and each of them receives a private signal, full information aggregation fails but information transmission persists since

[23]The value of $\bar{\lambda}_1$ depends on the value of other parameters except λ_1 and N.

$\lambda_2 < \hat{\lambda}_2$. If there is only one agent who receives all N signals, as $N \to \infty$, she fully learns the realized state but information transmission fails since

$$\lambda_2 V_{pc}(\theta_2) + \lambda_1 V_{pc}(\theta_1) > 0.$$

The principal chooses A even if the agent advises him to choose B.

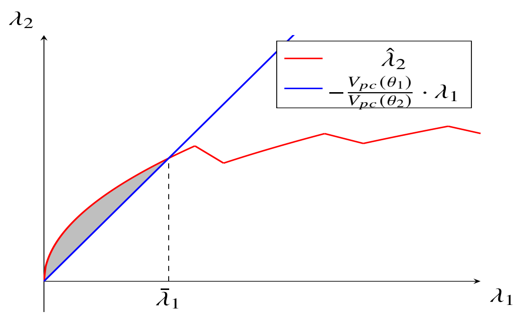

Figure 10

Information transmission with the failure of information aggregation.

The intuition for the argument that the failure of information aggregation might facilitate information transmission goes as follows. Many studies in cheap-talk literature, initiated by Crawford and Sobel (1982) consider a model of information transmission between one sender and one receiver. They show that the sender might make her message intentionally vague since intentional vagueness mitigates the conflict of interest between the sender and the receiver and further facilitates information transmission. Now, we can also interpret the failure of information aggregation as intentional vagueness if we regard all N agents as the sender and the principal as the receiver. Such intentional vagueness disappears in the case where an agent fully learns the state but does not have commitment power.

Furthermore, we can show that for each (λ_1, λ_2) in the shaded area, there always exists an informative equilibrium with $\hat{T} = 1$ when N is large since the unanimity rule aggregates information the least efficiently and hence generates the largest intentional vagueness.

Both the principal and the agents benefit from the failure of information aggregation when (λ_1, λ_2) is in the shaded area since both of them receive higher payoffs from any informative equilibrium than from the babbling equilibrium. Hence, it might be better to disperse the information among the agents instead of letting an agent receive all the signals when this agent cannot commit to generating intentional vagueness.

An important finding of Battaglini (2017) is that communication among the agents facilitates information transmission and aggregation, benefiting both the principal and the agents. He hence highlights the value of social media to the effectiveness of petitions and public protests, since social media allow citizens to share information. In contrast, we show that the communication among the agents might impede information transmission and hurt both the principal and the agents. Note that the case where the agents fully communicate with each other and share their signals is equivalent to the case analyzed above in which there is only one agent and this agent receives all N signals.

5 Beyond the Binary Situation

We now extend the model to the case where neither the signal space nor the message space is binary. Each agent $i \in \{1, ..., N\}$ receives a private signal $s^i \in \{s_1, ..., s_J\}$ with $J \geq 2$. The signals are identically distributed and independent across the agents conditional on the state $\theta \in \{\theta_1, \theta_2, \theta_3\}$. There exists $\alpha > 0$ such that

$$\mathbb{P}[s_j|\theta] > \alpha,\ \forall j \in \{1, ..., J\} \text{ and } \theta \in \{\theta_1, \theta_2, \theta_3\}.$$

That is, an agent cannot exclude any state if she receives a particular signal. We generalize (3) by assuming the strict ***Monotone Likelihood Ratio Property (MLRP)***:

$$\frac{\mathbb{P}[s_j|\theta_3]}{\mathbb{P}[s_j|\theta_2]} \text{ and } \frac{\mathbb{P}[s_j|\theta_2]}{\mathbb{P}[s_j|\theta_1]} \text{ strictly increase with } j. \tag{12}$$

Each agent i can send a message $z^i \in \{z_1, ..., z_K\}$ with $K \geq 2$. The principal observes $T = (T_1, ..., T_K) \in \Delta^K(N)$, that is, the total number of each kind of message, and chooses between A and B.

In the example of non-binding shareholder voting, besides voting in favor of or rejecting the new proposal, the shareholders can also stay neutral and abstain. Similarly, in the example of public protests, the citizens can choose among joining the rally for implementing reform, joining the rally for keeping the *status-quo*, or staying neutral and remaining silent. We can also use this framework to study survey polls in which each interviewee sends a score rating the desirability of a new policy.

We examine symmetric Bayesian Nash equilibrium in which the agents use the same strategy $P = \{p_{j,k}\}_{J\times K}$ such that an agent sends the message z_k with a probability $p_{j,k}$

when she receives the signal s_j. The strategy of the principal is a function

$$\psi : \Delta^K(N) \to [0,1]$$

such that he chooses A with probability $\psi(T)$ when he observes $T = (T_1, ..., T_K)$.

The agents follow a ***monotonic strategy*** if they are more likely to send higher messages when they receive higher signals, that is,[24]

$$p_{j',k} \cdot p_{j,k'} \leq p_{j,k} \cdot p_{j',k'} \text{ for each } j < j' \text{ and } k < k'. \tag{13}$$

The principal follows a ***monotonic strategy*** if he chooses A with a higher probability when an agent switches from a lower message to a higher one, that is, for each $T = (T_1, ..., T_K) \in \Delta^K(N-1)$ and each $m < m'$,

$$\psi(T_1, ..., T_m + 1, ..., T_{m'}, ..., T_K) \leq \psi(T_1, ..., T_m, ..., T_{m'} + 1, ..., T_K).$$

Note that when one side uses a monotonic strategy, it is without loss of generality to let the other side use a monotonic strategy as the best response. We focus on the ***monotonic equilibrium*** in which both the principal and the agents use monotonic strategies.

Monotonic equilibria are reasonable and fit applications well while non-monotonic equilibria are counterintuitive and hard to be implemented. Intuitively, a shareholder should support the new proposal more, a citizen should be more likely to quit the rally for keeping the *status-quo* and join the one for implementing reform, and an interviewee should rate the new policy with a higher score if they are more optimistic about the new proposal, reform, or new policy based on their private information. It is also reasonable that a manager should accept the new proposal with a higher probability if fewer shareholders object to it or more shareholders support it, a politician should implement reform with a higher probability if fewer citizens join in the rally for keeping the *status-quo* or more citizens join the rally for implementing the reform, and an interviewer should choose the new policy with a higher probability if more interviewees rate it with higher scores. There is growling literature studying the monotonic equilibrium in communication games, as discussed in Section 7.

Proposition 9. *For each $\epsilon > 0$, there exists N''_ϵ such that for each $N > N''_\epsilon$, in every monotonic equilibrium except the babbling one, the agents only send z_1* [25] *when they receive*

[24] It is equivalent to $\frac{p_{j',k}}{p_{j,k}} \leq \frac{p_{j',k'}}{p_{j,k'}}$ when both $p_{j,k}$ and $p_{j,k'}$ are positive.

[25] We ignore the degenerate case where agents never send z_1. In this case, just relabel the lowest message

$s \in \{s_1, ..., s_{J-1}\}$ and send z_1 with probability larger than $1-\epsilon$ when they receive s_J, that is,

$$\begin{aligned} p_{j,1} &= 1, \quad \forall\, j \in \{1, ..., J-1\}, \\ p_{J,1} &> 1-\epsilon. \end{aligned}$$

By Proposition 9, when N is large enough, there is no difference among $\{z_2, ..., z_K\}$. The agents send these messages only if $s = s_J$. Therefore, when N is large enough, we return to the basic model with binary signals and binary messages such that

$$\rho_i = \mathbb{P}[s_J|\theta_i], \ \forall i \in \{1, 2, 3\}.$$

Hence, we can easily extend all results in Section 3.2 and Section 4. In particular,

Theorem 2. *No sequence of monotonic equilibria aggregates information. That is, there exists a constant $c > 0$ such that for each N and each monotonic equilibrium Γ with N agents,*

$$\mathbb{P}[A|\theta_1;\Gamma] + \mathbb{P}[B|\theta_3;\Gamma] > c.$$

We now sketch the proof for Proposition 9. For simplicity, consider the case where each agent can send a message $z \in \{z_1, z_2, z_3\}$. Let the agents use a monotonic strategy P. From (12), (13) and some additional regularity assumptions concerning P to avoid degenerate cases, we can show that the distributions of the message from an agent also satisfy strict MLRP,

$$\frac{\mathbb{P}[z_k|\theta_3]}{\mathbb{P}[z_k|\theta_2]} \text{ and } \frac{\mathbb{P}[z_k|\theta_2]}{\mathbb{P}[z_k|\theta_1]} \text{ strictly increase with } k.$$

Denote the distributions of the message from an agent in θ_1, θ_2, and θ_3 by G_1, G_2, and G_3 respectively. We have

$$G_3 \succ G_2 \succ G_1 \tag{14}$$

in the monotone likelihood ratio order.

Consider the ***set of pivotal events***,

$$E_N = \{T = (T_1, T_2, T_3) \in \Delta^3(N-1) \mid \psi(T_1+1, T_2, T_3) \neq \psi(T_1, T_2, T_3+1)\},$$

that is, consider the set of events in which one additional message might change the

that the agents send with a positive probability by z_1.

principal's decision.[26]

Now, let us fix the strategy of the agents and let $N \to \infty$. We can show that

$$\lim_{N\to\infty} \frac{\mathbb{P}[E_N|\theta_3]}{\mathbb{P}[E_N|\theta_2]} = 0 \text{ and } \lim_{N\to\infty} \frac{\mathbb{P}[E_N|\theta_3]}{\mathbb{P}[E_N|\theta_1]} = 0. \tag{15}$$

Intuitively, given that the principal is not sure whether the realized state is θ_1 or not, the realized state must be either θ_1 or θ_2 since the "distance" between distributions G_1 and G_2 is smaller than the one between G_1 and G_3 according to (14). We extend the intuition in Section 3.2 to higher dimensions.

To see more precisely why (15) is true, consider the posterior likelihood ratio for $T \in \Delta^3(N-1)$ and each $i, i' \in \{1,2,3\}$,

$$\begin{aligned}\frac{\mathbb{P}[T|\theta_i]}{\mathbb{P}[T|\theta_{i'}]} &= \prod_{k=1}^{3}\left[\frac{\mathbb{P}[z_k|\theta_i]}{\mathbb{P}[z_k|\theta_{i'}]}\right]^{T_k} = \exp\left\{\sum_{k=1}^{3} T_k \log\frac{\mathbb{P}[z_k|\theta_i]}{\mathbb{P}[z_k|\theta_{i'}]}\right\} \\ &= \exp\left\{(N-1)\cdot\left[KL\left(\gamma(T), G_{i'}\right) - KL\left(\gamma(T), G_i\right)\right]\right\},\end{aligned} \tag{16}$$

where $\gamma(T)$ is the sample frequency with

$$\gamma(T) = (\gamma_1(T), \gamma_2(T), \gamma_3(T)) = \left(\frac{T_1}{N-1}, \frac{T_2}{N-1}, \frac{T_3}{N-1}\right),$$

and $KL(\cdot,\cdot)$ is the Kullback–Leibler divergence (KL divergence) with

$$KL(\gamma, G_i) = \sum_{k=1}^{3} \gamma_k \log\frac{\gamma_k}{\mathbb{P}[z_k|\theta_i]}, \ \forall i \in \{1,2,3\}.$$

It measures how γ (observed frequency) deviates from G_i (mean in state θ_i). The larger $KL(\gamma, G_i)$ is, the more rare that a sample with a frequency γ in state θ_i is.

From (16), as when N is large, instead of focusing on the set of pivotal events E_N, we can work with the ***set of pivotal frequencies***,

$$F = \left\{\gamma \in \Delta^3(1) | KL(\gamma, G_1) = \min\left[KL(\gamma, G_2), KL(\gamma, G_3)\right]\right\}.$$

For each $\gamma \notin F$, we have $T \notin E_N$ for each T with $\gamma(T) = \gamma$ when N is large. For example,

[26]Note the in any monotonic equilibrium except the babbling equilibrium, for each $T = (T_1, T_2, T_3) \in \Delta^3(N-1)$, if $\psi(T_1+1, T_2, T_3) \neq \psi(T_1, T_2+1, T_3)$ or $\psi(T_1, T_2+1, T_3) \neq \psi(T_1, T_2, T_3+1)$, then we must have $\psi(T_1+1, T_2, T_3) \neq \psi(T_1, T_2, T_3+1)$.

consider a $\tilde{\gamma} \notin F$ such that

$$KL(\tilde{\gamma}, G_1) < \min\left[KL(\tilde{\gamma}, G_2), KL(\tilde{\gamma}, G_3)\right].$$

When N is large and the principal observes $\tilde{T}$ from $N-1$ agents such that $\gamma(\tilde{T}) = \tilde{\gamma}$, he must be sure that the state is θ_1 by (16). Hence, one additional message cannot change his decision. We have $\tilde{T} \notin E_N$.

Note that in the binary setting analyzed in Section 3.2, the set F is a singleton, which is not true when we move beyond the binary setting. We provide a way to identify the unique most likely pivotal frequency

$$\gamma^* = \arg\min_{\gamma \in F} KL(\gamma, G_1).$$

Not that we only need to consider the pivotal events with frequencies concentrated around γ^* since the unconditional likelihoods of them dominate the unconditional likelihoods of other pivotal events at an exponential rate as shown in (16). We further show that

$$KL(\gamma^*, G_1) = KL(\gamma^*, G_2) < KL(\gamma^*, G_3). \tag{17}$$

We prove (15) by using (16) and (17).

To find the most likely pivotal frequency, let us consider a type of statistical distance between distribution G_i and $G_{i'}$ for $i \neq i'$, the ***Chernoff Information***:

$$c(G_i, G_{i'}) = \min_{\gamma \in \Delta^3(1)} KL(\gamma, G_i) \;\; s.t. \; KL(\gamma, G_i) = KL(\gamma, G_{i'}).$$

The minimizing problem has a unique minimizer.[27] Denote it by $\gamma_{i,i'}$ or $\gamma_{i',i}$.

It can be show that[28]

$$c(G_1, G_2) < c(G_1, G_3),$$

if (14) is satisfied. We further show that

$$KL(\gamma_{1,2}, G_1) = KL(\gamma_{1,2}, G_2) < KL(\gamma_{1,2}, G_3).$$

[27]Both the function $KL(\gamma, G_i)$ and the set $\left\{\gamma \in \Delta^3(1) \mid KL(\gamma, G_i) = KL(\gamma, G_{i'})\right\}$ are convex.

[28]Frick et al. (2021a) first find this result. There will be a note forthcoming for further discussion.

The key step is to show that

$$G_3 \succ G_2 \succ \gamma_{1,2} \succ G_1$$

in the monotonic likelihood ratio order if we regard $\gamma_{1,2}$ as a signal distribution. Therefore, the frequency $\gamma_{1,2}$ is the most likely pivotal frequency and satisfies (17).

We plot the simplex of distributions and frequencies in Figure 11, which illustrates the reasoning presented above.

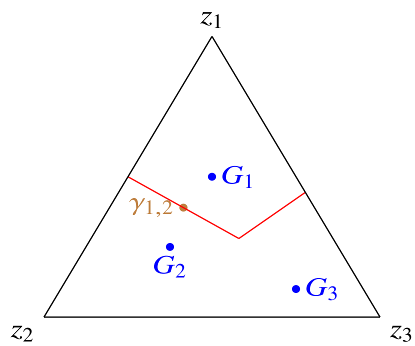

Figure 11
The most likely pivotal frequency.
The corner z_k corresponds to the distribution or the sample frequency that the message is always z_k. The segment $z_k z_{k'}$ corresponds to the set of distributions and sample frequencies that the message is always either z_k or $z_{k'}$. The red line is the set of pivotal frequencies from which the "distance" (KL divergence) to G_1 equals the minimum of distances to G_2 and G_3. Among all pivotal frequencies, the frequency $\gamma_{1,2}$ has the shortest distance to G_1 and hence it is the most likely pivotal frequency.

We show that in every sequence of monotonic equilibria, we must have

$$\lim_{N\to\infty} KL(G_i, G_{i'}) = 0, \quad \forall i, i' \in \{1, 2, 3\},$$

$$\lim_{N\to\infty} c(G_i, G_{i'}) = 0, \quad \forall i, i' \in \{1, 2, 3\},$$

that is, the distributions of messages in different states must be close to each other. Otherwise, the agents realize that the state must be either θ_1 or θ_2 conditional on being pivotal by (15) and ignore their own signals. Hence, the information contained in an agent's message must vanish as $N \to \infty$.

By extending Lemma 1, we show that the agents only send the lowest message z_1 when they receive the lowest signal s_1. Finally, we demonstrate that as N grows large, the agents only send z_1 when they receive $s \in \{s_1, ..., s_{J-1}\}$ and send z_1 with probability near 1 when they receive $s = s_J$ since (i) the strategy of the agents must be a monotonic mapping and satisfy a single crossing condition[29] due to the strict MLRP of signals, and (ii) information contained in an agent's message must vanish.

6 Commitment Case

In the basic model, the principal cannot *ex-ante* commit to a decision rule. Consequently, conditional on being pivotal, each agent learns that the principal must be nearly indifferent between A and B and infers the realized state from such an event. We now consider the case where the principal can design and commit to a decision mechanism. In this section, we show that the principal can approach his first-best outcome as $N \to \infty$ by committing to mechanisms with a simple structure.

With no misaligned state

We first consider the case with no misaligned state, that is, the case where $q_2^0 = 0$. Let us start with direct and anonymous mechanisms that depend only on T, that is, the total number of agents reporting signal h. The principal commits to a ***cut-off mechanism*** if there exists $\hat{T} \in \mathbb{N}$ such that the principal chooses A when $T \geq \hat{T}$ and choose B otherwise.

Note that when the agents can observe all signals together, for each N, there exists a cut-off $\bar{T}_N$ such that the agents prefer A if and only if more than $\bar{T}_N$ of them receive signal h. The principal then commits to a sequence of cut-off mechanisms $\{\hat{T}_N\}_{N=1}^{\infty}$ with $\hat{T}_N = \bar{T}_N$ for each N. It is always incentive compatible for the agents to report truthfully. The principal can approach his first-best outcome as $N \to \infty$.

The principal can also pick any $t \in (0, 1)$ and run an election among the agents following a qualified majority rule with t, in which the agents choose whether to vote for A, and A is chosen if the ratio of votes for it exceeds t. By the Condorcet jury theorem and its modern versions (Feddersen and Pesendorfer, 1997, 1998, Myerson, 1998, Duggan and Martinelli, 2001), as $N \to \infty$, dispersed information among the agents is effectively aggregated and the principal approaches his first-best outcome.

With the misaligned state

We now consider the case with the misaligned state θ_2, that is, with $q_2^0 > 0$. First, the principal cannot approach his first-best outcome by committing to a sequence of cut-off mechanisms. Figure 12 illustrates this argument. When N is large, the principal must

[29] That is, for each $k' > k$, if the agents send message z_k with a positive probability when they receive signal s_j, then they never send message $z_{k'}$ when they receive signals $s_{j'}$ with $j' < j$.

choose $\hat{T}_N \in (N\rho_1, N\rho_2)$ to approach his first-best outcome. However, each agent realizes that the state must be either θ_1 or θ_2 conditional on being pivotal, that is, conditional on the event that from the other $N-1$ agents, $\hat{T}_N - 1$ of them receive signal h. She does not have the incentive to report truthfully when she receives signal h.

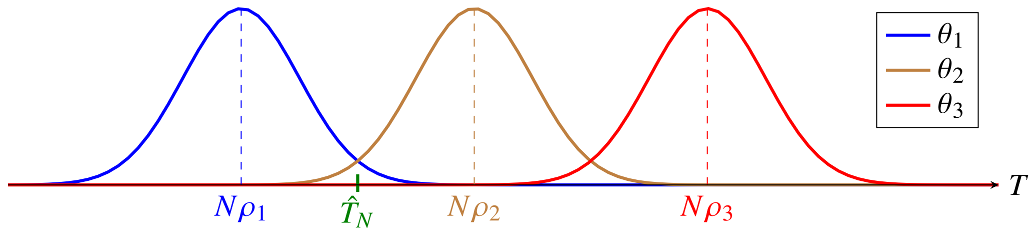

Figure 12
Distributions of the total number of signal h and one cut-off $\hat{T}_N$.
The cut-off mechanism with $\frac{\hat{T}_N}{N} \in (\rho_1, \rho_2)$ is not incentive compatible.

Furthermore, for each $t \in (0,1)$, the principal cannot approach his first-best outcome by committing to an election following a qualified majority rule with t, in which according to the Condorcet jury theorem, information is aggregated as $N \to \infty$ but the agents approach their first-best outcome.

The principal can approach his first-best outcome by mixing two cut-off mechanisms. Consider a mechanism $M = (\mu, \hat{T}^\alpha, \hat{T}^\beta)$ such that the principal commits to choosing the cut-off mechanism $\hat{T}^\alpha$ with probability μ and choosing the cut-off mechanism $\hat{T}^\beta$ with probability $1-\mu$. The agents cannot observe the principal's choice.

Proposition 10. *there exists a sequence of mechanisms $\{M_N\}_{N=1}^\infty$ with $M_N = (\mu_N, \hat{T}_N^\alpha, \hat{T}_N^\beta,)$ for each N such that*

$$\begin{aligned}
\lim_{N\to\infty} Pr(A|\,\theta_3; M_N) &= 1,\\
\lim_{N\to\infty} Pr(A|\,\theta_2; M_N) &= 1,\\
\lim_{N\to\infty} Pr(B|\,\theta_1; M_N) &= 1.
\end{aligned}$$

Note that each agent makes a decision conditional on being pivotal, that is, conditional on the event that her report can change the decision of the principal. In the mechanism $M_N = (\mu_N, \hat{T}_N^\alpha, \hat{T}_N^\beta,)$ with N agents, when the principal chooses $\hat{T}_N^\alpha$, an agent is pivotal if from the other $N-1$ agents, there are $\hat{T}_N^\alpha - 1$ agents who receive signal h, and when the principal chooses $\hat{T}_N^\beta$, an agent is pivotal if from the other $N-1$ agents, there are $\hat{T}_N^\beta - 1$

agents who receive signal h. The principal hence can mix between $\hat{T}_N^\alpha$ and $\hat{T}_N^\beta$ to manipulate the agents' inferences from being pivotal.

As illustrated by Figure 13, the principal chooses $\hat{T}_N^\alpha \in (N\rho_1, N\rho_2)$ for his first-best outcome and chooses $\hat{T}_N^\beta$ close to $N\rho_3$ to fulfill the incentive-compatible constraint. Note that the agents prefer B conditional on being pivotal in the cut-off mechanism with $\hat{T}_N^\alpha$, while they prefer A conditional on being pivotal in the cut-off mechanism with $\hat{T}_N^\beta$. By mixing between $\hat{T}_N^\alpha$ and $\hat{T}_N^\beta$, the principal can make the agents indifferent between A and B conditional on being pivotal. Hence, they have incentives to report their signals truthfully. Furthermore, by choosing $\hat{T}_N^\beta$ close to $N\rho_3$, the principal can choose $\hat{T}_N^\beta$ with a probability approaching 0 as $N \to \infty$. He pays almost no information rent to the agents and approaches his first-best outcome.

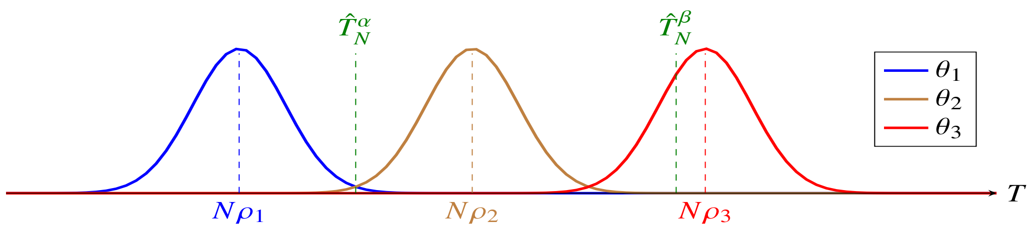

Figure 13

Distributions of the total number of signal h and two cut-offs $\hat{T}_N^\alpha$ and $\hat{T}_N^\beta$.

The principal mixes between two cut-offs and lets $\frac{\hat{T}_N^\beta}{N}$ be close to ρ_3.

Similarly, the principal can approach his first-best outcome by randomizing between two qualified majority rules with different t.

7 Related Literature

This paper is related to the literature on cheap talk with multiple senders. This paper further considers the case where senders (agents) have the same preference. Besides Levit and Malenko (2011) and Battaglini (2017), Wolinsky (2002) analyzes a similar model and also shows that information transmission fails and complete unraveling happens if the conflict of interest between the principal and the agents is large. Ekmekci and Lauermann (2022) follow the setting of Battaglini (2017) but add costly participation, that is, each agent in our basic model needs to pay a cost drawn from a distribution when rejecting A.[30] They show

[30] In this case, approving A is the default choice for the agents.

that information is aggregated even if the conflict of interest is above the threshold given by Levit and Malenko (2011) and Battaglini (2017). However, It is ambiguous whether a similar result holds in our setting with the misaligned state θ_2. Morgan and Stocken (2008) consider a model where the agents have heterogeneous preferences. They show that when the principal and the agents have similar preferences, information is effectively aggregated and the principal fully learns the state when the number of agents grows large.

This paper is also related to the literature on information aggregation in elections (Feddersen and Pesendorfer, 1997, 1998, Myerson, 1998, Duggan and Martinelli, 2001), which demonstrates that information dispersed among voters is effectively aggregated in elections with pre-determined qualified majority rules while information aggregation fails under the unanimity rule. In contrast, we consider the case where the principal cannot *ex-ante* commit to a rule. We show that he optimally follows the unanimity rule or rules close to it, and information aggregation fails. Razin (2003) considers a novel model in which the voters vote between two candidates and the winning candidate chooses the policy based on his own decision. He shows that if both candidates have large conflicts of interest with voters, full information aggregation fails in a special subset of symmetric equilibria under a symmetric setting. We consider a different setting and show that the principal always fails to fully learn the state in all symmetric equilibria, even if the conflict of interest between him and the agents is small.

We further demonstrate that if the principal has the commitment power, he cannot approach his first-best outcome by committing to a qualified majority rule.[31] However, the principal can approach his first-best outcome by committing to randomizing between two qualified majority rules to manipulate the agents' inferences from being pivotal. Gerardi et al. (2009) consider a similar mechanism in which the principal mixes between asking different numbers of agents to manipulate the agents' inferences from being pivotal. Kattwinkel and Winter (2022) characterize the optimal mechanism in the setting of Levit and Malenko (2011) and Battaglini (2017), which is a non-monotonic voting mechanism in which the principal chooses A when the number of agents voting for it is neither too high nor too low. However, the principal cannot rely on this mechanism to approach his first-best outcome in our setting with the misaligned state θ_2.

We take inspiration from the literature on comparisons of statistical experiments. Moscarini and Smith (2002) consider a model in which a decision-maker is uncertain about the state of the world but can draw signals that are identically distributed and in-

[31] However, the voters might obtain information aggregation, as discussed in Section 4.4.

dependent conditional on the state by performing an experiment repeatedly. Frick et al. (2021b) further consider the case of misspecified learning. They both provide rankings over statistical experiments by calculating the expected payoff of the decision-maker when he can perform a large number of experiments. Both rankings depend critically on Chernoff's information introduced in Section 6. Note that when the number of experiments approaches infinity, the speed at which the belief of the decision-maker converges depends crucially on the most likely events in which the principal stays uncertain about the realized state and hence the frequency that we derive when calculating Chernoff's information.

Finally, Section 6 contributes to the growing literature on the monotonic equilibrium in communication games, including Cho and Sobel (1990), Krishna and Morgan (2001), Chen et al. (2008), Ivanov (2010), Gordon et al. (2021), Kolotilin and Li (2021), and Vida et al. (2022). In addition, most of the literature studying the communication game in which each sender receives a noisy signal about the unknown state, including Austen-Smith (1990, 1993), Morgan and Stocken (2008), Hagenbach and Koessler (2010), Galeotti et al. (2013), and Currarini et al. (2020) among others, focuses on binary signals and messages like our basic model, which guarantees the monotonicity of the equilibrium. The proof of Proposition 9 provides a novel and tractable way to analyze the case with multiple signals and messages.

8 Concluding Remarks

This paper analyzes a model of learning from multiple agents. In contrast to the existing literature, this paper considers the situation in which the preferences of the principal and the agents might not be completely aligned even if they fully know the state of the world, and introduce a different way to model the conflict of interest between the principal and the agents. The paper provides new insights regarding information transmission and demonstrates that learning is always incomplete no matter how many agents there are.

One promising direction for future research is to understand the effect of the number of agents on the welfare of the principal and the agents. We can show that in some situations, the expected payoffs of both the principal and the agents are maximized if the number of agents equals some finite number, while both the principal and the agents receive a lower expected payoff when we let the number of agents go to infinity, whenever we focus on the sequence of informative equilibria that maximize the welfare of the principal and the agents or the sequence of informative equilibria that minimize the welfare. As discussed in Section

4.3, there exist situations in which informative equilibria exist only if the number of agents is below some threshold. Furthermore, even when information transmission persists, we can find situations in which the maximal amount of information transmission is non-monotonic with the number of agents, and more surprisingly, the maximal amount of information transmission, the cut-off chosen by the principal, and the expected payoffs of the principal and the agents, are all maximized when the number of agents equals to a finite number. We expect that such results should hold generally for all parameter values.

Appendices

The appendices proceed as follows:

(i) In Appendix A, we prove Lemma 1, Proposition 2, Proposition 3.

(ii) In Appendix B, we provide sufficient and necessary conditions under which there exists an informative equilibrium with $\hat{T} = 1$, that is, the equilibrium in which the principal chooses the unanimity rule. We then extend this approach for the informative equilibrium with $\hat{T} = i$ for each $i \in \mathbb{N}$.

(iii) In Appendix C, we prove the results in Section 4 based on results in Appendix B.

(iv) In Appendix D, we characterize the equilibria in which the principal uses mixed strategies and demonstrate that it is without loss of generality to focus on the equilibria in which the principal uses pure strategies.

(v) In Appendix E, we construct the mechanisms in which the principal approaches his first-best outcome as $N \to \infty$.

Appendix A

A.1 Proof of Lemma 1

Assume there exists an informative equilibrium with $x_\ell > 0$. From (6) and (7), we have

$x_h = 1$. Hence, we have $x_\ell \in (0, 1)$. From (6),

$$\frac{q_3^0 \cdot (1-\rho_3) \cdot \mathbb{P}[piv|\theta_3; \mathbf{x}, \hat{T}] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q_i^0 \cdot (1-\rho_i) \cdot \mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}] \cdot V_{ag}(\theta_i)} = 1. \tag{18}$$

In state $\theta_i \in \{\theta_1, \theta_2, \theta_3\}$, the probability that one agent rejects A is $(1-\rho_i)(1-x_\ell)$. Hence,

$$\frac{(1-\rho_i) \cdot \mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}]}{(1-\rho_{i'}) \cdot \mathbb{P}[piv|\theta_{i'}; \mathbf{x}, \hat{T}]} = \frac{(1-\rho_i) \cdot \mathbb{P}[T-1; N-1|\theta_i]}{(1-\rho_{i'}) \cdot \mathbb{P}[T-1; N-1|\theta_{i'}]} = \frac{\mathbb{P}[T-1; N-1|\theta_i]}{\mathbb{P}[T-1; N-1|\theta_{i'}]}.$$

Plug it into (18),

$$\frac{q_3^0 \cdot \mathbb{P}[T-1; N|\theta_3] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q_i^0 \cdot \mathbb{P}[T-1; N|\theta_i] \cdot V_{ag}(\theta_i)} = 1.$$

From (1) and (4),

$$\frac{\sum_{i=2}^3 q_i^0 \cdot \mathbb{P}[T-1; N|\theta_i] \cdot V_{pc}(\theta_i)}{-q_1^0 \cdot \mathbb{P}[T-1; N|\theta_1] \cdot V_{pc}(\theta_1)} > 1.$$

That is,

$$L_{pc}(\hat{T}-1; \mathbf{x}) > 1,$$

which contradicts (8).

A.2 Proof of Proposition 2

Consider a strategy profile that the agents choose $x_h \in (0, 1)$ and $x_\ell = 0$, we have

$$\begin{aligned}\frac{\mathbb{P}[T; N|\theta_i]}{\mathbb{P}[T; N|\theta_{i'}]} &= \frac{(\rho_i x_h)^T (1-\rho_i x_h)^{N-T}}{(\rho_{i'} x_h)^T (1-\rho_{i'} x_h)^{N-T}} = \exp\left[T \cdot \log \frac{\rho_i x_h}{\rho_{i'} x_h} + (N-T) \cdot \log \frac{1-\rho_i x_h}{1-\rho_{i'} x_h}\right] \\ &= \exp\left\{N\left[KL(\frac{T}{N}, \rho_{i'} x_h) - KL(\frac{T}{N}, \rho_i x_h)\right]\right\}.\end{aligned} \tag{19}$$

where $KL(\cdot, \cdot)$ is the relative entropy with

$$KL(x, y) = x \log \frac{x}{y} + (1-x) \log \frac{1-x}{1-y}.$$

Fix some arbitrary $x \in (0, 1)$. Consider a sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, \hat{T}_N)\}$ with

$$\lim_{N \to \infty} x_{h,N} = x. \tag{20}$$

We first claim that there exists $\hat{N}_1$ such that when $N > \hat{N}_1$, we must have

$$\hat{T}_N < N \cdot \rho_2 x_{h,N}. \tag{21}$$

Note that if (21) does not hold,

$$\begin{aligned} KL(\frac{\hat{T}_N}{N}, \rho_1 x_{h,N}) - KL(\frac{\hat{T}_N}{N}, \rho_2 x_{h,N}) &= \frac{\hat{T}_N}{N} \log \frac{\rho_2 x_{h,N}}{\rho_1 x_{h,N}} + (1 - \frac{\hat{T}_N}{N}) \log \frac{1 - \rho_2 x_{h,N}}{1 - \rho_1 x_{h,N}} \\ &\geq \rho_2 x_{h,N} \cdot \log \frac{\rho_2 x_{h,N}}{\rho_1 x_{h,N}} + (1 - \rho_2 x_{h,N}) \log \frac{1 - \rho_2 x_{h,N}}{1 - \rho_1 x_{h,N}} \\ &= KL(\rho_2 x_{h,N}, \rho_1 x_{h,N}) \\ &> 0. \end{aligned}$$

The first inequality is from taking the derivative in $\frac{\hat{T}_N}{N}$. The second inequality is a result known as Gibbs' inequality. From (20), we can see that $KL(\rho_2 x_{h,N}, \rho_1 x_{h,N})$ is always larger than a strictly positive number independent of N. Therefore, if (21) does not hold, then

$$KL(\frac{\hat{T}_N}{N}, \rho_1 x_{h,N}) - KL(\frac{\hat{T}_N}{N}, \rho_2 x_{h,N})$$

is always smaller than a positive number independent of N. Hence, if we cannot find an $\hat{N}_1$ such that (21) holds when $N > \hat{N}_1$, then from (19), for each $\hat{M}_1 > 0$, we can find $\bar{N}_1$ such that

$$\frac{\mathbb{P}[\hat{T}_{\bar{N}_1}; \bar{N}_1 | \theta_2]}{\mathbb{P}[\hat{T}_{\bar{N}_1}; \bar{N}_1 | \theta_1]} > \hat{M}_1,$$

and

$$\frac{\mathbb{P}[\hat{T}_{\bar{N}_1} - 1; \bar{N}_1 | \theta_2]}{\mathbb{P}[\hat{T}_{\bar{N}_1} - 1; \bar{N}_1 | \theta_1]} > \hat{M}_1,$$

By choosing $\hat{M}_1$ large enough, we can see that the principal strictly prefers A when he observes $\hat{T}_{\bar{N}_1} - 1$ approvals from $\bar{N}_1$ agents, which contradicts the optimality of $\hat{T}_{\bar{N}_1}$ as shown in (8).

However, if there exists $\hat{N}_1$ such that (21) holds when $N > \hat{N}_1$, we must have

$$\lim_{N \to \infty} \frac{\mathbb{P}[\hat{T}_N; N | \theta_2]}{\mathbb{P}[\hat{T}_N; N | \theta_3]} = \infty. \tag{22}$$

Note that when $\hat{T}_N < N \cdot \rho_2 x_{h,N}$,

$$\begin{aligned} KL(\frac{\hat{T}_N}{N}, \rho_3 x_{h,N}) - KL(\frac{\hat{T}_N}{N}, \rho_2 x_{h,N}) &= \frac{\hat{T}_N}{N} \log \frac{\rho_2 x_{h,N}}{\rho_3 x_{h,N}} + (1 - \frac{\hat{T}_N}{N}) \log \frac{1 - \rho_2 x_{h,N}}{1 - \rho_3 x_{h,N}} \\ &\geq \rho_2 x_{h,N} \cdot \log \frac{\rho_2 x_{h,N}}{\rho_3 x_{h,N}} + (1 - \rho_2 x_{h,N}) \log \frac{1 - \rho_2 x_{h,N}}{1 - \rho_3 x_{h,N}} \\ &= KL(\rho_2 x_{h,N}, \rho_3 x_{h,N}). \\ &> 0. \end{aligned}$$

From (20), we can see that $KL(\rho_2 x_{h,N}, \rho_3 x_{h,N})$ is always larger than a strictly positive number independent of N. We then prove (22) by (19).

From (22),

$$\lim_{N\to\infty} \frac{\mathbb{P}[\hat{T}_N - 1; N|\theta_3]}{\mathbb{P}[\hat{T}_N - 1; N|\theta_2]} = 0,$$

Hence, the agents choose $x_{h,N} = x_{l,N} = 0$ when N is above some threshold by (6), which leads to a contradiction.

Therefore, in every sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, \hat{T}_N)\}$, we must have

$$\lim_{N\to\infty} x_{h,N} = 0. \tag{23}$$

Otherwise, we can construct a subsequence from it and show that x_h converges to a positive number along this sub-sequence, which leads to a contraction as shown above.

We now assume that there exists a sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, \hat{T}_N)\}$ with

$$\lim_{N\to\infty} N \cdot x_{h,N} = \infty. \tag{24}$$

From the proof above, we have

$$KL(\frac{\hat{T}_N}{N}, \rho_1 x_{h,N}) - KL(\frac{\hat{T}_N}{N}, \rho_2 x_{h,N}) \geq KL(\rho_2 x_{h,N}, \rho_1 x_{h,N}), \text{ if } \hat{T}_N \geq N \cdot \rho_2 x_{h,N}, \tag{25}$$

$$KL(\frac{\hat{T}_N}{N}, \rho_3 x_{h,N}) - KL(\frac{\hat{T}_N}{N}, \rho_2 x_{h,N}) \geq KL(\rho_2 x_{h,N}, \rho_3 x_{h,N}),, \text{ if } \hat{T}_N \leq N \cdot \rho_2 x_{h,N}. \tag{26}$$

We can linearize $KL(\rho_2 x_{h,N}, \rho_i x_{h,N}))$ with respect to $x_{h,N}$ when $x_{h,N} \approx 0$ for $i \in \{1, 3\}$,

$$KL(\rho_2 x_{h,N}, \rho_i x_{h,N}) = \kappa_i x_{h,N} + o(x_{h,N}), \ \forall i \in \{1, 3\}, \tag{27}$$

where

$$\kappa_i = \rho_2 \log \frac{\rho_2}{\rho_i} + \rho_i - \rho_2 > 0, \ \forall i \in \{1, 3\}.$$

Similar to the proof before, when (23) and (24) hold, from (19), (25) and (27), when N is above some threshold, we must have (21), which leads to (22) from (19), (26) and (27), and a further contradiction.

Therefore, there exists a finite number T_0 independent to N such that for each N and each informative equilibrium with N agnets,

$$N \cdot x_h < T_0.$$

Otherwise, we can construct a sequence of informative equilibria and construct a subsequence from it, showing that $N \cdot x_{h,N}$ grows without bound along this sub-sequence, which leads to a contradiction as shown above.

A.3 Proof of Proposition 3

Consider a sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, \hat{T}_N)\}$. From Proposition 2,

$$x_{h,N} < \frac{T_0}{N}, \ \forall N \in \mathbb{N}^+.$$

Hence,

$$\frac{\mathbb{P}[T; N|\theta_3]}{\mathbb{P}[T; N|\theta_1]} = \frac{(\rho_3 x_{h,N})^T (1 - \rho_3 x_{h,N})^{N-T}}{(\rho_1 x_{h,N})^T (1 - \rho_1 x_{h,N})^{N-T}} > \left(\frac{\rho_3}{\rho_1}\right)^T \left(\frac{1 - \rho_3 \cdot \frac{T_0}{N}}{1 - \rho_1 \cdot \frac{T_0}{N}}\right)^{N-T} > \left(\frac{\rho_3}{\rho_1}\right)^T \left(\frac{1 - \rho_3 \cdot \frac{T_0}{N}}{1 - \rho_1 \cdot \frac{T_0}{N}}\right)^{N}.$$

Since

$$\lim_{N\to\infty} \left(\frac{1 - \rho_3 \cdot \frac{T_0}{N}}{1 - \rho_1 \cdot \frac{T_0}{N}}\right)^{N} = \exp[(\rho_1 - \rho_3)T_0] > 0.$$

We can find $\gamma > 0$ independent of T and N such that

$$\frac{\mathbb{P}[T; N|\theta_3]}{\mathbb{P}[T; N|\theta_1]} > \left(\frac{\rho_3}{\rho_1}\right)^T \cdot \gamma, \ \forall N \in \mathbb{N}^+ \text{ and } \forall T \in \{0, ..., N\}. \tag{28}$$

Note that $\frac{\mathbb{P}[\hat{T}_N - 1; N|\theta_3]}{\mathbb{P}[\hat{T}_N - 1; N|\theta_1]}$ must be always smaller than a number independent of N for each $N \in \mathbb{N}^+$. Otherwise, the principal chooses A when he observes $\hat{T}_N - 1$ approvals, which contradicts the optimality of $\hat{T}_N$ as shown in (8). From (28), for each $N \in \mathbb{N}^+$, the equilirium cut-off $\hat{T}_N$ is always smaller than a number indepenedent of N.

Appendix B

We provide sufficient and necessary conditions under which there exists an informative equilibrium with $\hat{T} = 1$, that is, the equilibrium in which the principal chooses the unanimity rule. We then extend this approach for the informative equilibrium with $\hat{T} = i$ for each $i \in \mathbb{N}$.

We only consider the case where

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1}.$$

Otherwise, from Proposition 4, information transmission fails when N is large enough.

For each $i \in \mathbb{N}^+$, we say ***information transmission persists with*** $\hat{\mathbf{T}} = \mathbf{i}$ (ITP_i) if there exists $\hat{N}_1$ such that for each $N > \hat{N}_1$, an informative equilibrium with $\hat{T} = i$ exists. We say ***information transmission fails with*** $\hat{\mathbf{T}} = \mathbf{i}$ (ITF_i) if there exists $\hat{N}_2$ such that for each $N > \hat{N}_2$, there does not exist an informative equilibrium with $\hat{T} = i$.

B.1 Unanimity Rule

We now provide the sufficient and necessary conditions for ITP_1 and IFP_1. We consider the case where the principal always chooses $\hat{T} = 1$, that is, he follows the unanimity rule under which B is chosen if all the agents reject it. From Proposition 2, when N is large enough, the agents must choose $x_h < 1$ in each informative equilibrium. Therefore, if ITP_1 holds, then there exists $\hat{N}_1$ and a sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, x_{\ell,N}, \hat{T})\}_{N=\hat{N}_1}^{\infty}$ with

$$x_{h,N} \in (0,1), x_{\ell,N} = 0, \hat{T} = 1,$$

for each $N > \hat{N}_1$. We suppress $x_{h,N}$ to x_N to save notation. If ITP_1 holds, there exists a strictly positive sequence $\{x_N\}_{N=\hat{N}_1}^{\infty}$ satisfying

$$\frac{\rho_3 \cdot (1 - \rho_3 x_N)^{N-1} \cdot V_{ag}(\theta_3)}{-\lambda_2 \cdot \rho_2 \cdot (1 - \rho_2 x_N)^{N-1} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot \rho_1 \cdot (1 - \rho_1 x_N)^{N-1} \cdot V_{ag}(\theta_1)} = 1, \tag{29}$$

$$\frac{\rho_3 \cdot (1-\rho_3 x_N)^{N-1} \cdot V_{pc}(\theta_3) + \lambda_2 \cdot \rho_2 \cdot (1-\rho_2 x_N)^{N-1} \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot \rho_1 \cdot (1-\rho_1 x_N)^{N-1} \cdot V_{pc}(\theta_1)} > 1, \tag{30}$$

$$\frac{(1-\rho_3 x_N)^{N} \cdot V_{pc}(\theta_3) + \lambda_2 \cdot (1-\rho_2 x_N)^{N} \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (1-\rho_1 x_N)^{N} \cdot V_{pc}(\theta_1)} \leq 1. \tag{31}$$

for each $N > \hat{N}_1$. We derive (29) from (6) since the agents are indifferent conditional on receiving signal h and being pivotal. We derive (30) and (31) from (6) since the principal prefers A when one agent approve it and prefers B when all the agents reject it.

Note that we can interpret the agents as voters who vote between A and B under the unanimity rule. An informative voting equilibrium under the unanimity rule exists if there is a positive x_N satisfying (29). We now provide conditions under which there exists $\hat{N}_1$ such that when $N > \hat{N}_1$, there exists an informative equilibrium under the unanimity rule. Note that the left side of (29) decreases with x_N. Furthermore, for each $x \in (0,1)$, if we fix $x_N = x$ for each N and let N go to infinity, the left side approaches 0. Therefore, by the intermediate value theorem, a sufficient and necessary condition under which informative voting equilibria exist when N is large enough is that the value of the left side (29) given that $x_N = 0$ is strictly bigger than 1, by which we have

$$\lambda_2 < -\frac{\rho_3 V_{ag}(\theta_3) + \lambda_1 \rho_1 V_{ag}(\theta_1)}{\rho_2 V_{ag}(\theta_2)}.$$

Denote the right side by $\hat{\lambda}'_{2,1}$. Note that if $\lambda_2 < \hat{\lambda}'_{2,1}$. There exists a unique informative voting equilibrium since (29) admits a unique solution. Therefore, a necessary condition for ITP_1 is $\lambda_2 < \hat{\lambda}'_{2,1}$ while a sufficient condition for ITF_1 is $\lambda_2 \geq \hat{\lambda}'_{2,1}$.

Now consider the case where $\lambda_2 < \hat{\lambda}'_{2,1}$, there exists $\hat{N}_1$ such that there exists a strictly positive sequence $\{x_N\}_{N=\hat{N}_1}^{\infty}$ satisfying (29) for each $N > \hat{N}_1$. Note that (29) implies (30) since

$$V_{pc}(\theta_2) > 0 > V_{pc}(\theta_2),$$
$$-\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)} \leq -\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}.$$

Assume that there exists $\hat{N}_1$ such that there exists a strictly positive sequence $\{x_N\}_{N=\hat{N}_1}^{\infty}$ satisfying (29) and (31) for each $N > \hat{N}_1$, by which we have ITP_1. By Proposition 2,

$$\lim_{N\to\infty} x_N = 0. \tag{32}$$

Therefore, we can find $\{\epsilon_N\}_{\hat{N}_1}^{\infty}$ with $\lim_{N\to\infty}\epsilon_N = 0$ such that for each $N > \hat{N}_1$

$$\frac{(1-\rho_3 x_N)^{N-1}\cdot V_{pc}(\theta_3) + \lambda_2 \cdot (1-\rho_2 x_N)^{N-1}\cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (1-\rho_1 x_N)^{N-1}\cdot V_{pc}(\theta_1)} < \frac{1}{1-\epsilon_N} \tag{33}$$

by (31) and (32).

Note that we can rewrite (29) as

$$a_1 \left(\frac{1-\rho_1 x_N}{1-\rho_3 x_N}\right)^{N-1} + a_2 \left(\frac{1-\rho_2 x_N}{1-\rho_3 x_N}\right)^{N-1} = 1, \tag{34}$$

where $a_1 > 0$ and $a_2 > 0$ are calculated from (29). We can also rewrite (33) as

$$b_1 \left(\frac{1-\rho_1 x_N}{1-\rho_3 x_N}\right)^{N-1} - b_2 \left(\frac{1-\rho_2 x_N}{1-\rho_3 x_N}\right)^{N-1} > 1-\epsilon_N, \tag{35}$$

where $b_1 > 0$ and $b_2 > 0$ are calculated from (33). By (34) and (35),

$$\left(\frac{1-\rho_1 x_N}{1-\rho_3 x_N}\right)^{N-1} > \frac{b_2 + a_2(1-\epsilon_N)}{a_2 b_1 + a_1 b_2}, \tag{36}$$

$$\left(\frac{1-\rho_2 x_N}{1-\rho_3 x_N}\right)^{N-1} < \frac{b_1 - a_1(1-\epsilon_N)}{a_2 b_1 + a_1 b_2}. \tag{37}$$

Note that for each $t > 0$, if there exists a sequence $\{y_N\}_{N=\hat{N}_1}^{\infty}$ such that

$$\left(\frac{1-\rho_1 y_N}{1-\rho_3 y_N}\right)^{N-1} > t,\ \forall N > \hat{N}_1.$$

then

$$\liminf_{N\to\infty} \left(\frac{1-\rho_2 y_N}{1-\rho_3 y_N}\right)^{N-1} > t^{\frac{\rho_3-\rho_2}{\rho_3-\rho_1}},$$

which is shown by considering the sequence $\{y_N\}_{N=\hat{N}_1}^{\infty}$ such that

$$\left(\frac{1-\rho_1 y_N}{1-\rho_3 y_N}\right)^{N-1} = t,\ \forall N > \hat{N}_1.$$

Therefore, if both (36) and (37) for each x_N when $N > \hat{N}_1$ with $\lim_{N\to\infty}\epsilon_N = 0$, we must have

$$\left(\frac{b_2+a_2}{a_2b_1+a_1b_2}\right)^{\frac{\rho_3-\rho_2}{\rho_3-\rho_1}} \le \frac{b_1-a_1}{a_2b_1+a_1b_2}$$

That is,

$$\lambda_2 \le \lambda_1^{\frac{\rho_3-\rho_2}{\rho_3-\rho_1}} \cdot \frac{\rho_3 u_{13} - \rho_1 v_{13}}{(\rho_3 u_{23} + \rho_2 v_{23})^{\frac{\rho_3-\rho_2}{\rho_3-\rho_1}} (\rho_1 u_{23} v_{13} + \rho_2 u_{13} v_{23})^{\frac{\rho_2-\rho_1}{\rho_3-\rho_1}}}.$$

with

$$u_{ij} = \frac{|V_{pc}(\theta_i)|}{|V_{pc}(\theta_j)|},\ \forall i,j \in \{1,2,3\},$$

$$v_{ij} = \frac{|V_{ag}(\theta_i)|}{|V_{ag}(\theta_j)|},\ \forall i,j \in \{1,2,3\}.$$

Denote the right side by $\hat{\lambda}''_{2,1}$. A nessessary condition for ITP_1 is $\lambda_2 \le \hat{\lambda}''_{2,1}$. Furthermore, if $\lambda_2 > \hat{\lambda}''_{2,1}$, we can find $\hat{N}_2$ such that for each $N > \hat{N}_2$, there does not exist x_N satisfying both (36) and (37). Therefore, a sufficiecnt condition for ITF_1 is $\lambda_2 > \hat{\lambda}''_{2,1}$.

Let

$$\hat{\lambda}_{2,1} = \min\{\hat{\lambda}'_{2,1}, \hat{\lambda}''_{2,1}\}.$$

A nessessary condition for ITP_1 is $\lambda_2 \le \hat{\lambda}_{2,1}$ while a sufficient condition for ITF_1 is $\lambda_2 > \hat{\lambda}_{2,1}$.

We now show that a sufficient condition for ITP_1 is $\lambda_2 < \hat{\lambda}_{2,1}$. Note that we can find sequences $\{y_N\}_{N=1}^{\infty}$, $\{\epsilon_N\}_{N=1}^{\infty}$, and $\{\epsilon'_N\}_{N=1}^{\infty}$ with

$$\lim_{N\to\infty} y_N = \lim_{N\to\infty} \epsilon_N = \lim_{N\to\infty} \epsilon'_N = 0,$$

such that

$$\frac{\rho_3 \cdot (1-\rho_3 y_N)^{N-1} \cdot V_{ag}(\theta_3)}{-\hat{\lambda}''_{2,1} \cdot \rho_2 \cdot (1-\rho_2 y_N)^{N-1} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot \rho_1 \cdot (1-\rho_1 y_N)^{N-1} \cdot V_{ag}(\theta_1)} = 1 + \epsilon_N,$$

$$\frac{(1-\rho_3 y_N)^N \cdot V_{pc}(\theta_3) + \hat{\lambda}''_{2,1} \cdot (1-\rho_2 y_N)^N \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (1-\rho_1 y_N)^N \cdot V_{pc}(\theta_1)} = 1 + \epsilon'_N$$

Hence, for each $\lambda_2 < \hat{\lambda}_{2,1}$, we can find $\hat{N}_1$ such that for each $N > \hat{N}_1$,

$$\frac{\rho_3 \cdot (1-\rho_3 y_N)^{N-1} \cdot V_{ag}(\theta_3)}{\lambda_2 \cdot \rho_2 \cdot (1-\rho_2 y_N)^{N-1} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot \rho_1 \cdot (1-\rho_1 y_N)^{N-1} \cdot V_{ag}(\theta_1)} > 1,$$

$$\frac{(1-\rho_3 y_N)^N \cdot V_{pc}(\theta_3) + \lambda_2 \cdot (1-\rho_2 y_N)^N \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (1-\rho_1 y_N)^N \cdot V_{pc}(\theta_1)} < 1.$$

Note that the left sides of both equations above decrease with y_N. Therefore, for each $N > \hat{N}_1$, if we can find $x_N \leq 1$ satisfying (29), then we have $x_N > y_N$ and (31) is satisfied. In this case, since $\lambda_2 < \hat{\lambda}_{2,1} \leq \hat{\lambda}_{2,1}$ and x_N is the unique solution of (29), we must have $x_N > 0$. Therefore, we construct an informative equilibrium with $\hat{T} = 1$. If we cannot find $x_N \leq 1$ satisfying (29), we have

$$\frac{\rho_3 \cdot (1-\rho_3)^{N-1} \cdot V_{ag}(\theta_3)}{\lambda_2 \cdot \rho_2 \cdot (1-\rho_2)^{N-1} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot \rho_1 \cdot (1-\rho_1)^{N-1} \cdot V_{ag}(\theta_1)} > 1, \tag{38}$$

$$\frac{(1-\rho_3)^N \cdot V_{pc}(\theta_3) + \lambda_2 \cdot (1-\rho_2)^N \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (1-\rho_1)^N \cdot V_{pc}(\theta_1)} < 1. \tag{39}$$

From (38),

$$\frac{\rho_3 \cdot (1-\rho_3)^{N-1} \cdot V_{pc}(\theta_3) + \lambda_2 \cdot \rho_2 \cdot (1-\rho_2)^{N-1} \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot \rho_1 \cdot (1-\rho_1)^{N-1} \cdot V_{pc}(\theta_1)} < 1. \tag{40}$$

From (39),

$$\frac{(1-\rho_3) \cdot (1-\rho_3)^{N-1} \cdot V_{ag}(\theta_3)}{\lambda_2 \cdot (1-\rho_2) \cdot (1-\rho_2)^{N-1} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot (1-\rho_1) \cdot (1-\rho_1)^{N-1} \cdot V_{ag}(\theta_1)} < 1, \tag{41}$$

From (38) and (41), the agents strictly prefer A conditional receiving signal h and being pivotal while strictly prefer B conditional on receiving signal ℓ and being pivotal. Hence, it is optimal for the agents to choose $x_h = 1$ and $x_\ell = 0$. From (39) and (40), it is optimal for the principal to choose $\hat{T} = 1$. Hence, we construct an informative equilibrium. Therefore, for each $\lambda_2 < \hat{\lambda}_{2,1}$, we can find $\hat{N}_1$ such that for each $N > \hat{N}_1$, an informative equilibrium with $\hat{T} = 1$ exists.

In Figure 14, we plot $\hat{\lambda}'_{2,i}$, $\hat{\lambda}''_{2,i}$ and $\hat{\lambda}_{2,i}$ as functions of λ_1.

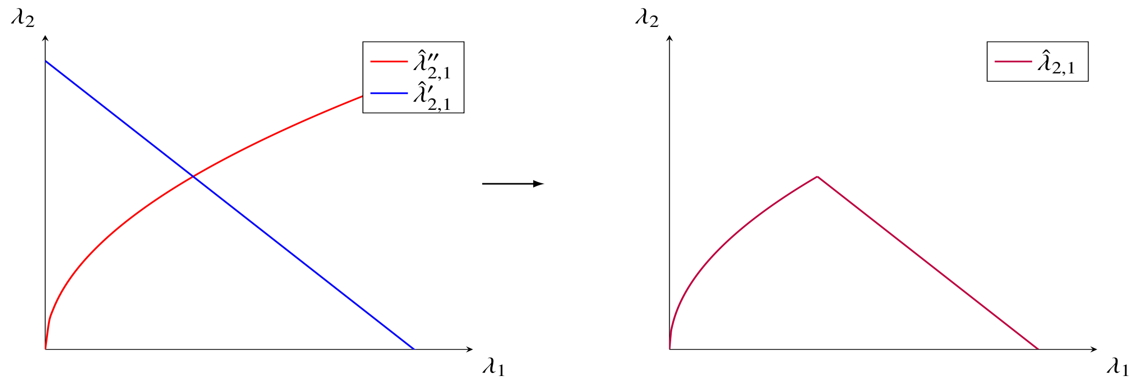

Figure 14

Conditions for ITP_1 and ITF_1.

B.2 General Case

We now discuss the sufficient conditions for ITP_i and IFP_i for each $i > 1$. Fix an arbitrary $i > 1$ and consider the case where the principal always chooses $\hat{T} = i$. From Proposition 2, when N is large enough, the agents must chooses $x_h < 1$ in each informative equilibrium. Therefore, if ITP_i holds, then there exists $\hat{N}_1$ and a sequence of informative equilibrium $\{\Gamma_N = (x_{h,N}, x_{\ell,N}, \hat{T})\}_{N=\hat{N}_1}^{\infty}$ with

$$x_{h,N} \in (0,1), x_{\ell,N} = 0, \hat{T} = i,$$

for each $N > \hat{N}_1$. We suppress $x_{h,N}$ to x_N to save notation. If ITP_i holds, there exists a strictly positive sequence $\{x_N\}_{N=\hat{N}_1}^{\infty}$ satisfying

$$\frac{\rho_3 \cdot (\rho_3 x_N)^{i-1} \cdot (1-\rho_3 x_N)^{N-i} \cdot V_{ag}(\theta_3)}{-\lambda_2 \cdot \rho_2 \cdot (\rho_2 x_N)^{i-1} \cdot (1-\rho_2 x_N)^{N-i} \cdot V_{ag}(\theta_2) - \lambda_1 \cdot \rho_1 \cdot (\rho_1 x_N)^{i-1} \cdot (1-\rho_1 x_N)^{N-i} \cdot V_{ag}(\theta_1)} = 1, \tag{42}$$

$$\frac{\rho_3 \cdot (\rho_3 x_N)^{i} \cdot (1-\rho_3 x_N)^{N-i} \cdot V_{pc}(\theta_3) + \lambda_2 \cdot \rho_2 \cdot (\rho_2 x_N)^{i} \cdot (1-\rho_2 x_N)^{N-i} \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot \rho_1 \cdot (\rho_1 x_N)^{i} \cdot (1-\rho_1 x_N)^{N-i} \cdot V_{pc}(\theta_1)} > 1, \tag{43}$$

$$\frac{(\rho_3 x_N)^{i-1} \cdot (1-\rho_3 x_N)^{N-i+1} \cdot V_{pc}(\theta_3) + \lambda_2 \cdot (\rho_3 x_N)^{i-1} \cdot (1-\rho_2 x_N)^{N-i+1} \cdot V_{pc}(\theta_2)}{-\lambda_1 \cdot (\rho_3 x_N)^{i-1} \cdot (1-\rho_1 x_N)^{N-i+1} \cdot V_{pc}(\theta_1)} \leq 1. \tag{44}$$

Note that we can choose

$$\lambda_1' = \lambda_2 \cdot \left(\frac{\rho_1}{\rho_3}\right)^{i-1},$$

$$\lambda_2' = \lambda_2 \cdot \left(\frac{\rho_2}{\rho_3}\right)^{i-1},$$

and convert (42), (43), and (44) regading λ_1 and λ_2 to (29), (30), and (31) regarding λ_1' and λ_2'. In this way, we can follow the analysis in A.2.1 and find $\hat{\lambda}_{2,i}$ such that ITP_i holds if $\lambda_2 < \hat{\lambda}_{2,i}$ and while ITF_i holds if $\lambda_2 > \hat{\lambda}_{2,i}$. The left panel of Figure 8 illustrateS $\hat{\lambda}_{2,1}$, $\hat{\lambda}_{2,2}$, and $\hat{\lambda}_{2,3}$.

Appendix C

C.1 Proof of Proposition 4

Assume that there exists an informative equilibrium with $x_h \in (0, 1)$ and $x_\ell = 0$, that is, the agents are indifferent conditional on receiving signal h and being pivotal. From (6),

$$\frac{q_3^0 \cdot \rho_3 \cdot \mathbb{P}[T-1; N-1|\theta_3] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q_i^0 \cdot \rho_i \cdot \mathbb{P}[T-1; N-1|\theta_i] \cdot V_{ag}(\theta_i)} = 1.$$

Therefore,

$$\frac{q_3^0 \cdot \rho_3 \cdot \mathbb{P}[T-1; N-1|\theta_3] \cdot V_{ag}(\theta_3)}{-q_1^0 \cdot \rho_1 \cdot \mathbb{P}[T-1; N-1|\theta_1] \cdot V_{ag}(\theta_1)} > 1,$$

$$\frac{q_3^0 \cdot \mathbb{P}[T-1; N-1|\theta_3]}{q_1^0 \cdot \mathbb{P}[T-1; N-1|\theta_1]} > -\frac{\rho_1}{\rho_3}\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)}. \tag{45}$$

Furthermore, since the principal must prefer B when he observes $\hat{T} - 1$ approvals by (8),

$$\frac{\sum_{i=2}^3 q_i^0 \cdot \mathbb{P}[T-1; N|\theta_i] \cdot V_{pc}(\theta_i)}{-q_1^0 \cdot \mathbb{P}[T-1; N|\theta_1] \cdot V_{pc}(\theta_1)} \leq 1.$$

Therefore,

$$\frac{q_3^0 \cdot \mathbb{P}[T-1; N|\theta_3] \cdot V_{pc}(\theta_3)}{-q_1^0 \cdot \mathbb{P}[T-1; N|\theta_1] \cdot V_{pc}(\theta_1)} \leq 1,$$

$$\frac{q_3^0 \cdot (1-\rho_3 x_h) \cdot \mathbb{P}[T-1; N-1|\theta_3] \cdot V_{pc}(\theta_3)}{-q_1^0 \cdot (1-\rho_1 x_h) \cdot \mathbb{P}[T-1; N|\theta_1] \cdot V_{pc}(\theta_1)} \leq 1,$$

$$\frac{\sum_{i=2}^{3} q_i^0 \cdot \mathbb{P}[T-1; N|\theta_i] \cdot V_{pc}(\theta_i)}{-q_1^0 \cdot \mathbb{P}[T-1; N-1|\theta_1] \cdot V_{pc}(\theta_1)} \leq -\frac{1-\rho_1 x_h}{1-\rho_3 x_h} \frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)}. \tag{46}$$

From (45) and (46),

$$-\frac{\rho_1}{\rho_3}\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} < -\frac{1-\rho_1 x_h}{1-\rho_3 x_h}\frac{V_{pc}(\theta_1)}{V_{pc}(\theta_3)},$$

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} < \frac{\rho_3}{\rho_1} \cdot \frac{1-\rho_1 x_h}{1-\rho_3 x_h}. \tag{47}$$

By Proposition 2, when N is large enough, in every informative equilibrium, we must have $x_h \approx 0$. Therefore, if

$$\frac{V_{ag}(\theta_1)}{V_{ag}(\theta_3)} \cdot \frac{V_{pc}(\theta_3)}{V_{pc}(\theta_1)} > \frac{\rho_3}{\rho_1},$$

there does not exist any informative equilibrium when N is large enough, by which we prove Proposition 4.

C.2 Proof of Proposition 5

We choose

$$\hat{\lambda}_2 = \sup_{i \in \mathbb{N}+} \hat{\lambda}_{2,i}.$$

From the analysis in Appendix B, it is direct to see that information transmission persists if $\lambda_2 < \hat{\lambda}_2$.

For each $\lambda_2 > \hat{\lambda}_2$, by Proposition 3, we can find T_0 independent of N such that in any informative equilibrium, the principal chooses $\hat{T} < T_0$. We further have

$$\lambda_2 > \max_{i \in \mathbb{N}+} \hat{\lambda}_{2,i} > \max_{i < T_0} \hat{\lambda}_{2,i}.$$

Hence, we have ITF_i for each $i < T_0$. We then can find $\hat{N}_2$ such that when $N > \hat{N}_2$, only the babbling equilibrium exists since T_0 is a finite number independent to N, by which information transmission fails.

C.3 Rest Results of Section 4

The proofs of Corollary 2, Corollary 3, and Proposition 7 are similar. We only need to show that given that there exists an informative equilibrium, there always exists an informative equilibrium with a higher x_h when there are a lower λ_2, a lower ρ_1, higher $V_{ag}(\theta_i)$, and lower $V_{pc}(\theta_i)$ for each $i \in \{1, 2, 3\}$. We skip the proof here since we already do a similar construction when proving a sufficient condition for ITP_1 is $\lambda_2 < \hat{\lambda}''_{2,1}$ in Appendix B.

The part of Corollary 3 that $\hat{\lambda}_2$ is non-monotonic with ρ_2 and ρ_3 is proved by taking the derivative of $\hat{\lambda}_2$ over ρ_2 and ρ_3. Proposition 6 is direct from Blackwell's informative ranking. Proposition 8 is based on calculation, which can be directly seen by the fact that $\hat{\lambda}''_{2,1}$ is concave in λ_1.

Appendix D

In this section, we characterize the informative equilibrium in which the principal uses a mixed strategy. Consider an equilibrium in which the agents choose an informative strategy $\mathbf{x}$ with $x_\ell < x_h$ and the principal chooses $\hat{T} \in \{1, ..., N-1\}$ and $p \in (0, 1)$ such that he chooses B when $T < \hat{T}$, chooses A with probability p when $T = \hat{T}$, and chooses A when $T > \hat{T}$.[32]

In this case, if one agent is pivotal, with probability p, there are $\hat{T} - 1$ approvals from $N - 1$ agents while with probability $1 - p$, there are $\hat{T}$ approvals from $N - 1$ agents. Hence,

$$\mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}, p] = p \cdot \mathbb{P}[\hat{T} - 1, N - 1|\theta_i; \mathbf{x}, \hat{T}, p] + (1 - p) \cdot \mathbb{P}[\hat{T}, N - 1|\theta_i; \mathbf{x}, \hat{T}, p].$$

We then define

$$L_{ag}(s; \mathbf{x}, \hat{T}, p) = \frac{q_3^0 \cdot \mathbb{P}[s|\theta_3] \cdot \mathbb{P}[piv|\theta_3; \mathbf{x}, \hat{T}, p] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^{2} q_i^0 \cdot \mathbb{P}[s|\theta_i] \cdot \mathbb{P}[piv|\theta_i; \mathbf{x}, \hat{T}, p] \cdot V_{ag}(\theta_i)}.$$

[32] We skip the case where $\hat{T} = 0$ and $p \in (0, 1)$ since if there exists such an equilibrium, there must exist one informative with $\hat{T} = 1$ and $p = 1$. Similarly, we skip the case where $\hat{T} = N$ and $p \in (0, 1)$.

We have

$$\begin{cases} x_s = 1 & \text{when } L_{ag}(s;\mathbf{x},\hat{T},p) > 1, \\ x_s \in [0,1] & \text{when } L_{ag}(s;\mathbf{x},\hat{T},p) = 1, \\ x_s = 0 & \text{when } L_{ag}(s;\mathbf{x},\hat{T},p) < 1. \end{cases} \tag{48}$$

For the principal, he must be indifferent when he observes $\hat{T}$,

$$\frac{\sum_{i=2}^{3} q_i^0 \cdot \mathbb{P}[\hat{T};N|\theta_i] \cdot V_{pc}(\theta_i)}{-q_1^0 \cdot \mathbb{P}[\hat{T};N|\theta_1] \cdot V_{ag}(\theta_1)} = 1. \tag{49}$$

Therefore, an informative equilibrium $\{\mathbf{x},\hat{T},p\}$ with $p \in (0,1)$ is characterized by (48) and (49).

It is direct to verify that Lemma 1, Proposition 2, Proposition 3, and hence Theorem 1 stay valid when we allow the principal to use a mixed strategy. We extend results in Section 4 based on the following lemma.

Lemma 3. *If there exists an informative equilibrium in which the principal chooses A with probability $p \in (0,1)$ when he observes $\hat{T}$ approvals with $\hat{T} \in \{1,\dots,N-1\}$, then there exists an informative equilibrium in which the principal chooses A if and only if $T > \hat{T}$.*

Proof. Consider the case that there exists an informative equilibrium $\{x_h,\hat{T},p\}$ in which the principal chooses A with probability $p \in (0,1)$ when he observes $\hat{T}$ approvals. Since the agents chooses $x_h > 0$, from (48), we have

$$\frac{q_3^0 \cdot \rho_3 \cdot \mathbb{P}[piv|\theta_3;\mathbf{x},\hat{T},p] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^{2} q_i^0 \cdot \rho_i \cdot \mathbb{P}[piv|\theta_i;\mathbf{x},\hat{T},p] \cdot V_{ag}(\theta_i)} \leq 1. \tag{50}$$

Note that since $x_\ell < x_h$, we have

$$\frac{\mathbb{P}[T;N|\theta_3]}{\mathbb{P}[T;N|\theta_1]} \text{ and } \frac{\mathbb{P}[T;N|\theta_2]}{\mathbb{P}[T;N|\theta_1]}$$

both strictly increase with T. Hence, from (50),

$$\frac{q_3^0 \cdot \rho_3 \cdot \mathbb{P}[\hat{T},N-1|\theta_3;\mathbf{x},\hat{T},p] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^{2} q_i^0 \cdot \rho_i \cdot \mathbb{P}[\hat{T},N-1|\theta_i;\mathbf{x},\hat{T},p] \cdot V_{ag}(\theta_i)} > 1. \tag{51}$$

Note that if $x_h = 1$, equations (49) and (51) guarantee that there exists an information equilibrium in which the agents choose the same x_h and the principal chooses A if and only if $T > \hat{T}$. Note that (49) implies that the agents must choose $x_\ell = 0$ conditional on receiving signal ℓ and being pivotal. If $x_h < 1$, we can follow the construction when proving a sufficient condition for ITP_1 is $\lambda_2 < \hat{\lambda}''_{2,1}$ in Appendix B, showing that there must exist $x'_h \in (x_h, 1]$ such that there exists an informative equilibrium in which the agents choose the same x'_h and the principal chooses A if and only if $T > \hat{T}$.

□

From Lemma 3, we can see that we only need to focus on the informative equilibrium in which the principal follows pure strategy when discussing the existence of informative equilibria.

Appendix E

E.1 Proof of Proposition 10

Fix $N \in \mathbb{N}^+$ and conider a mechanism $M_N = (\mu_N, \hat{T}^\alpha_N, \hat{T}^\beta_N,)$. When one agent is pivotal, with probability μ_N, there are $\hat{T}^\alpha_N$ agents receving signal h for $N-1$ agents, and with probability $1-\mu_N$, there are $\hat{T}^\beta_N$ agents receving signal h for $N-1$ agents.

Define

$$\mathbb{P}[piv_\alpha|\theta_i; \hat{T}^\alpha_N] = \binom{N-1}{\hat{T}^\alpha_N - 1}[\rho_i]^{\hat{T}^\alpha_N - 1}[1-\rho_i]^{N-\hat{T}^\alpha_N}, \ \forall i \in \{1,2,3\},$$

$$\mathbb{P}[piv_\beta|\theta_i; \hat{T}^\beta_N] = \binom{N-1}{\hat{T}^\beta_N - 1}[\rho_i]^{\hat{T}^\beta_N - 1}[1-\rho_i]^{N-\hat{T}^\beta_N}, \ \forall i \in \{1,2,3\},$$

$$\mathbb{P}[piv|\theta_i; \mathbf{\hat{T}_N}, \mu_N] = \mu_N \cdot \mathbb{P}[piv_\alpha|\theta_i; \hat{T}^\alpha_N] + (1-\mu_N)\cdot \mathbb{P}[piv_\beta|\theta_i; \hat{T}^\beta_N], \ \forall i \in \{1,2,3\},$$

where $\mathbf{\hat{T}_N} = (\hat{T}^\alpha, \hat{T}^\beta)$. The mechanism M_N must satisfy the incentive compatibility constraints under which the agents report truthfully,

$$\frac{q^0_3 \cdot \rho_3 \cdot \mathbb{P}[piv|\theta_3; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q^0_i \cdot \rho_i \cdot \mathbb{P}[piv|\theta_i; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_i)} \geq 1, \tag{52}$$

$$\frac{q^0_3 \cdot (1-\rho_3) \cdot \mathbb{P}[piv|\theta_3; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q^0_i \cdot (1-\rho_i) \cdot \mathbb{P}[piv|\theta_i; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_i)} \geq 1. \tag{53}$$

From (19),

$$\frac{\mathbb{P}[T;N|\theta_i]}{\mathbb{P}[T;N|\theta_{i'}]} = \exp\left\{N\left[KL(\frac{T}{N},\rho_{i'}) - KL(\frac{T}{N},\rho_i)\right]\right\}. \tag{54}$$

Note that for $i' > i$, if

$$t > \frac{\log\frac{1-\rho_i}{1-\rho_{i'}}}{\log\frac{\rho_i'}{\rho_i} + \log\frac{1-\rho_i}{1-\rho_{i'}}},$$

then

$$KL(t,\rho_{i'}) - KL(t,\rho_i) > 0.$$

While if

$$t < \frac{\log\frac{1-\rho_i}{1-\rho_{i'}}}{\log\frac{\rho_{i'}}{\rho_i} + \log\frac{1-\rho_i}{1-\rho_{i'}}},$$

then

$$KL(t,\rho_{i'}) - KL(t,\rho_i) < 0.$$

Hence, we can choose t_α and t_β such that

$$t_\alpha \in (\rho_1, \frac{\log\frac{1-\rho_1}{1-\rho_2}}{\log\frac{\rho_2}{\rho_1} + \log\frac{1-\rho_1}{1-\rho_2}}), \tag{55}$$

$$t_\beta \in (\frac{\log\frac{1-\rho_2}{1-\rho_3}}{\log\frac{\rho_3}{\rho_2} + \log\frac{1-\rho_2}{1-\rho_3}}, \rho_3), \tag{56}$$

and let $\hat{T}_N^\alpha, \hat{T}_N^\beta$ be the integers closest to Nt_α and Nt_β respectively. We have

$$\lim_{N\to\infty} \frac{\mathbb{P}[piv_\alpha|\theta_i;\hat{T}_N^\alpha]}{\mathbb{P}[piv_\alpha|\theta_1;\hat{T}_N^\alpha]} = 0; \forall i \in \{2,3\}, \tag{57}$$

$$\lim_{N\to\infty} \frac{\mathbb{P}[piv_\beta|\theta_i;\hat{T}_N^\beta]}{\mathbb{P}[piv_\beta|\theta_3;\hat{T}_N^\beta]} = 0; \forall i \in \{1,2\}. \tag{58}$$

By Sterling approximation and (19),

$$\frac{\mathbb{P}[piv_\alpha|\theta_i;\hat{T}_N^\alpha]}{\mathbb{P}[piv_\beta|\theta_j;\hat{T}_N^\beta]} = \exp\left\{N\left[KL(t_\beta,\rho_j) - KL(t_\alpha,\rho_i) + o(1)\right]\right\}, \ \forall i,j \in \{1,2,3\}. \tag{59}$$

Note that for each $i \in \{1, 2, 3\})$, the function $KL(t, \rho_i)$ strictly decreases with t when $t < \rho_i$ and strictly increases with t when $t > \rho_i$. We further have $KL(t, \rho_i) = 0$ if and only if $t = \rho_i$. We further choose t_α and t_β such that

$$KL(t_\alpha, \rho_1) > KL(t_\beta, \rho_3),$$

and choose $\mu_N \in (0, 1)$ such that

$$\frac{\mu_N}{1 - \mu_N} \cdot \frac{\mathbb{P}[piv_\alpha|\theta_1; \hat{T}_N^\alpha]}{\mathbb{P}[piv_\beta|\theta_3; \hat{T}_N^\beta]} \cdot \frac{-V_{ag}(\theta_1)}{V_{ag}(\theta_3)} = 1. \tag{60}$$

From (59), we can see that μ_N must exist when N is large enough and

$$\lim_{N\to\infty} \mu_N = 0. \tag{61}$$

From (58) and (60),

$$\lim_{N\to\infty} \frac{\mu_N}{1 - \mu_N} \cdot \frac{\mathbb{P}[piv_\alpha|\theta_1; \hat{T}_N^\alpha]}{\mathbb{P}[piv_\beta|\theta_1; \hat{T}_N^\beta]} = \infty.$$

Hence,

$$\lim_{N\to\infty} \frac{\mu_N \cdot \mathbb{P}[piv_\alpha|\theta_1; \hat{T}_N^\alpha]}{\mathbb{P}[piv|\theta_1; \mathbf{\hat{T}_N}, \mu_N]} = 1.$$

Similarly,

$$\lim_{N\to\infty} \frac{(1 - \mu_N) \cdot \mathbb{P}[piv_\beta|\theta_3; \hat{T}_N^\beta]}{\mathbb{P}[piv|\theta_3; \mathbf{\hat{T}_N}, \mu_N]} = 1.$$

Furthermore, from (58), (59) and (60),

$$\lim_{N\to\infty} \frac{\mathbb{P}[piv|\theta_2; \mathbf{\hat{T}_N}, \mu_N]}{\mathbb{P}[piv|\theta_1; \mathbf{\hat{T}_N}, \mu_N]} = 0,$$

$$\lim_{N\to\infty} \frac{\mathbb{P}[piv|\theta_2; \mathbf{\hat{T}_N}, \mu_N]}{\mathbb{P}[piv|\theta_3; \mathbf{\hat{T}_N}, \mu_N]} = 0.$$

Therefore, when N is large,

$$\frac{q_3^0 \cdot \mathbb{P}[piv|\theta_3; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_3)}{-\sum_{i=1}^2 q_i^0 \cdot \mathbb{P}[piv|\theta_i; \mathbf{\hat{T}_N}, \mu_N] \cdot V_{ag}(\theta_i)} \approx \frac{\mu_N}{1 - \mu_N} \cdot \frac{\mathbb{P}[piv_\alpha|\theta_1; \hat{T}_N^\alpha]}{\mathbb{P}[piv_\beta|\theta_3; \hat{T}_N^\beta]} \cdot \frac{-V_{ag}(\theta_1)}{V_{ag}(\theta_3)} = 1,$$

by which (52) and (53) are satisfied and we hence construct an incentive-compatible mechanism. By (55), (56), (61), and the law of large numbers, we can see that the principal can approach his first-best outcome as $N \to \infty$.

References

D. Austen-Smith. Information transmission in debate. *American Journal of Political Science*, 34(1):124–152, 1990.

D. Austen-Smith. Interested experts and policy advice: Multiple referrals under open rule. *Games and Economic Behavior*, 5(1):3–43, 1993.

M. Battaglini. Public protests and policy making. *Quarterly Journal of Economics*, 132(1): 485–549, 2017.

Y. Chen, N. Kartik, and J. Sobel. Selecting cheap-talk equilibria. *Econometrica*, 76(1): 117–136, 2008.

H. Chernoff. A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations. *Annals of Mathematical Statistics*, 23(4):493–507, 1952.

I.-K. Cho and J. Sobel. Strategic stability and uniqueness in signaling games. *Journal of Economic Theory*, 50(2):381–413, 1990.

V. P. Crawford and J. Sobel. Strategic information transmission. *Econometrica*, 50(6): 1431–1451, 1982.

S. Currarini, G. Ursino, and A. Chand. Strategic transmission of correlated information. *Economic Journal*, 130(631):2175–2206, 2020.

J. Duggan and C. Martinelli. A Bayesian model of voting in juries. *Games and Economic Behavior*, 37(2):259–294, 2001.

M. Ekmekci and S. Lauermann. Informal elections with dispersed information. Working paper, 2022.

T. Feddersen and W. Pesendorfer. Voting behavior and information aggregation in elections with private information. *Econometrica*, 65(5):1029–1058, 1997.

T. Feddersen and W. Pesendorfer. Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political Science Review*, 92(1):23–35, 1998.

M. Frick, R. Iijima, and Y. Ishii. Belief convergence under misspecified learning: A martingale approach. Working paper, 2021a.

M. Frick, R. Iijima, and Y. Ishii. Welfare comparisons for biased learning. Working paper, 2021b.

A. Galeotti, C. Ghiglino, and F. Squintani. Strategic information transmission networks. *Journal of Economic Theory*, 148(5):1751–1769, 2013.

D. Gerardi, R. McLean, and A. Postlewaite. Aggregation of expert opinions. *Games and Economic Behavior*, 65(2):339–371, 2009.

S. Gordon, N. Kartik, M. P.-y. Lo, W. Olszewski, and J. Sobel. Effective communication in cheap talk games. Working paper, 2021.

J. Hagenbach and F. Koessler. Strategic communication networks. *Review of Economic Studies*, 77(3):1072–1099, 2010.

M. Ivanov. Communication via a strategic mediator. *Journal of Economic Theory*, 145(2): 869–884, 2010.

D. Kattwinkel and A. Winter. Optimal decision mechanisms for juries: Acquitting the guilty. Working paper, 2022.

A. Kolotilin and H. Li. Relational communication. *Theoretical Economics*, 16(4):1391–1430, 2021.

V. Krishna and J. Morgan. A model of expertise. *Quarterly Journal of Economics*, 116(2): 747–775, 2001.

K. K. Ladha. The Condorcet jury theorem, free speech, and correlated votes. *American Journal of Political Science*, 36(3):617–634, 1992.

D. Levit and N. Malenko. Nonbinding voting for shareholder proposals. *Journal of Finance*, 66(5):1579–1614, 2011.

J. Morgan and P. C. Stocken. Information aggregation in polls. *American Economic Review*, 98(3):864–96, 2008.

G. Moscarini and L. Smith. The law of large demand for information. *Econometrica*, 70 (6):2351–2366, 2002.

R. B. Myerson. Extended Poisson games and the Condorcet jury theorem. *Games and Economic Behavior*, 25(1):111–131, 1998.

R. Razin. Signaling and election motivations in a voting model with common values and responsive candidates. *Econometrica*, 71(4):1083–1119, 2003.

P. Vida, T. Honryo, and H. Azacis. Strong forward induction in monotonic multi-sender signaling games. Working paper, 2022.

A. Wolinsky. Eliciting information from multiple experts. *Games and Economic Behavior*, 41(1):141–160, 2002.