Choosing Employment Protection: the role of On-the-Job Search and Ability Learning

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Abstract

Why would agents insert employment protection in labour contracts? Within a Diamond-Mortensen-Pissarides search and matching model with on-the-job search and heterogeneous match-productivity, I show that firms and workers choose employment protection to improve their joint welfare by reducing workers' search intensity. I use this model, augmented with Bayesian learning about a worker's unobserved ability, to explain the coexistence of fixed-term contracts and open-ended contracts in continental Europe, as well as key facts about the distribution of fixed-term contracts in the workforce: (i) their high incidence among young workers; (ii) their correlation with low wages; and (iii) their persistence in a worker's career. I calibrate the model using Italian administrative data, and I perform welfare comparisons between different Employment Protection Legislations. I show that the endogenous nature of the contractual choice plays a key role in the welfare gains of heterogeneous agents.

JEL codes: J41, J48, J65, E24, K31

Keywords: Employment protection, fixed-term contracts, on-the-job search, heterogeneous agents

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1 Introduction

A labour contract often entails some protection for the worker in case of involuntary separation. In continental Europe, institutional constraints play a central role in the regulation of these contractual aspects. These constraints divide labour contracts into two broad categories: *Open-Ended Contracts* (OECs) and *Fixed-Term Contracts* (FTCs). The former are the traditional labour contracts which generally involve relatively high separation costs. The latter are flexible and less protected contracts, where the two parties already fix an expiration date at the beginning of the working relationship. The economic literature has extensively studied the effects of the introduction of FTCs in Europe, but it has paid little attention to the agents' choice between the two types of contracts. In this regard, while it is quite intuitive to understand the joint advantage of opting for a FTC, that of avoiding the separation costs associated with the OEC, it is less obvious why workers and firms should sign a contract associated with employment protection. Nevertheless, the OEC is still the most common labour contract among the workforce, suggesting that it provides some benefits for both agents.

The first contribution of this paper is to show that, in presence of on-the-job search and heterogeneity in match productivity, agents could optimally choose employment protection to reduce an otherwise excessive search of the worker while employed, hence increasing the joint surplus to share. In the vast majority of the literature with FTCs, the contractual choice is modelled by postulating an exogenous probability of receiving each contract or by assuming an exogenous difference in the duration or the productivity of the two contracts.[1] Instead, in my model, the choice of the type of contract becomes endogenous to the characteristics of workers and matches. Indeed, it is well-documented that FTCs are unevenly distributed among workers and jobs[2], suggesting that their characteristics play an important role in the choice of the type of labour contract. Three facts in particular clearly emerge from an anal-

[1]See among others Postel-Vinay and Robin (2002), Boeri and Garibaldi (2007), Faccini (2014).

[2]A complete characterization can be found in Booth et al. (2002), which focuses on the UK, but a similar characterization is also present in Portugal and Varejão (2009) focusing on Portugal.

ysis of the distribution of FTCs: (i) FTCs are disproportionately widespread among young workers; (ii) they are correlated with low-wage positions; and (iii) they are persistent, that is, a worker employed with a FTC has a higher probability than his peers of being employed with the same type of contract years later. The second contribution of the paper is then to explain these facts by introducing in the model an additional dimension of heterogeneity, namely ex-ante unknown workers' ability. Specifically, agents discover the worker-specific ability over time, through the observation of period-by-period production, which in turn acts as a signal of the underlying worker ability. Experience then naturally emerges as a third dimension of heterogeneity, as it reduces the uncertainty about the worker-specific ability. The third contribution of this paper is to perform welfare comparisons, analyzing costs and benefits of alternative policy interventions for different types of agents. The heterogeneity of welfare effects gives rise to policy trade-offs, as it was highlighted in previous papers that examine the support for specific labour market policies.[3] However, I perform these comparisons in an environment where agents also optimize over the type of labour contract. This is particularly important in presence of heterogeneity, given that the type of contract is not independent of jobs' and workers' characteristics. Indeed, I show that different labour market policies with the same objective of a reduction in the share of FTCs, such as a tax on FTCs or a firing cost cut, can have opposite effects on different types of workers. Moreover, these outcomes are due to the endogenous contractual choices of the agents, that create persistence in the type of contract a worker receives. As a result, compared to a scenario in which the contract type is exogenously given, the burden of different policies is more concentrated on particular segments of the workforce.

The mechanism underlying the contractual choice is illustrated in a simplified version of the full model that will be developed. The simpler model is a discrete-time search and matching Diamond-Mortensen-Pissarides model with on-the-job search and heterogeneity in the match-specific productivity. When agents meet, they observe the match-specific produc-

[3] Cahuc and Postel-Vinay (2002) and Boeri et al. (2012), among others

tivity draw and bargain over the wage, which will be re-bargained at every period. Then, the worker chooses how intensively to search for another job while employed, balancing search costs and benefits in terms of the probability to receive a new offer. Importantly, search intensity cannot be bargained with the firm as the worker keeps the "right-to-manage" his on-the-job search activity. In this setting, the worker chooses an excessively high search intensity from the joint firm-worker perspective, since he is not internalizing the damage to the firm in case he actually quits. Indeed, in this event, the firm will lose its part of the match surplus and will have to open a new vacancy. In other words, the worker's private marginal benefit from searching on-the-job is generally higher than the joint marginal benefit of the firm-worker pair.

One can show that in this environment an optimal labour contract involves a backload of the wage, that is, a commitment to future higher wages against lower wages today. This raises the continuation value of the worker and hence reduces incentives to search. I then show that employment protection, in terms of either firing costs or severance payments, provides the agents with an instrument to commit to this outcome. In fact, via the introduction of separation costs, which credibly grant a higher wage to the worker in the next periods, employment protection shifts worker's utility from the present into the future. This result is in line with the literature on hidden on-the-job search. In a similar environment, Lentz (2014) shows that the optimal labour contract would involve an increasing wage path.[4] This solution requires the two parties to commit to a given wage profile, without subsequent bargaining. This assumption is clearly not realistic with a FTC, as both parties know that continuation beyond the original contract duration, whether with a renewal or with a transformation, will necessarily involve agreement to a new contract and thus a new bargaining. An OEC solves this commitment problem, as employment protection raises future wages by reducing the future outside option of the firm, even in an environment with period-by-period bargaining.

The full model adds to on-the-job search with variable intensity and match-specific pro-

[4]See also Stevens (2004).

ductivity, the additional element of ex-ante unknown worker-specific ability, which is learned over time. Further, to focus on the European setting, it restricts the choice of labour contracts to FTCs and OECs. Specifically, a FTC can be terminated at the end of every period with no additional cost, while an OEC features employment protection in the form of firing costs that the firm has to pay in case of endogenous termination of the match. In making this choice, agents maximize the surplus, balancing the benefits of FTCs in terms of lower firing costs, with the benefits of OECs in terms of lower on-the-job search intensity.

I calibrate the structural model using Italian microdata contained in two rich administrative datasets: (i) "Mercurio", from "Veneto Lavoro", collecting all the job histories of workers of Veneto, one of the largest Italian regions; and (ii) a sample of social security records from INPS, the Italian social security agency. These datasets contain the entire working careers of millions of employed workers, covering several decades. However, I restrict my analysis to the years after 2000, given that FTCs were introduced in Italy in 1997. I also use the Italian Labour Force Survey ("Rilevazione Continua delle Forze di Lavoro", RCFL), a cross-sectional dataset collected every quarter, which contains information related to on-the-job search intensities and wages.

I estimate the parameters of the model performing a Monte Carlo Markov Chain estimation. The large number of observations and the detailed sequence of labour contracts contained in the datasets allow me to estimate the key parameters of the model, using as targets several aggregate moments of the Italian labour market. More specifically, I use: (i) the unemployment rate; (ii) the share of FTCs; (iii) the job-finding, separation, quitting and contract transformation rates; and (iv) selected moments of the wage distribution. The calibrated model can reproduce the (untargeted) three facts mentioned above: FTCs are mostly used by inexperienced and low-skilled workers, and in low-pay jobs. In addition, the model can explain the emergence of a dual labour market, in which some workers experience long sequences of fixed-term contracts and unemployment, while others are able to keep their job position for a long period of time.

Finally, I analyze the effects of some counterfactual scenarios that resemble real policy interventions: (i) a 25% cut in the firing costs; (ii) a lump-sum tax on all FTCs, equivalent to 1% of the average wage and rebated back to all workers.[5]

The richness of the model allows evaluating policies along several dimensions. First, the effects of these two policies are indeed highly heterogeneous across workers. In particular, while both these reforms aim to reduce the share of fixed-term contracts, they have opposite effects on the workers' welfare among low and high expected ability workers: a lump-sum FTC tax is effectively a low-ability tax, reducing the wage of low-ability workers and their job-finding probability; conversely, a firing costs cut reduces the welfare of the "insiders" with an OEC, while it helps low-ability workers to enter the labour market. Secondly, the fact that the contract is chosen endogenously is crucial to assess these heterogeneous effects correctly. For instance, if the type of the contract were to be assigned randomly, a policy such as a lump-sum FTC-tax would fall randomly on the labour force, with a negative impact only on current holders of FTCs. Instead, the active choices of the contract types made by agents, transform this policy in a tax on low-ability workers. Finally, the general equilibrium effects of these policies play an important role in determining the final labour market outcomes, the final share of FTCs and the economy's overall efficiency.

2 Related literature and Contribution

As already anticipated in the introduction, the previous literature that introduced the choice of the contract generally assumed that an exogenous fraction of workers receives an OEC: see for example Cahuc and Postel-Vinay (2002) in which to the standard search and matching model, the authors added the assumption of an exogenous fraction p of the new matches as fixed-term contracts.

Faccini (2014) uses a similar model in which agents are forced to transform the contract into an OECs with some exogenous probability. This paper highlights an important advan-

[5] As I will describe in Section 7, similar reforms were enacted in Spain, Italy and France.

tage of FTCs: the possibility to test the productivity of the match. Indeed, there is ex-ante unknown match-specific productivity that is revealed with a certain probability at the end of every period. My work shares with this paper the idea of productivity as an experience good that is discovered over time. However, in my model, the worker's ability is the unknown variable that the agents will learn over time. This fact has important consequences in the final distribution of FTCs among the population, as it will be clear later.

This representation of the choice of the contract would be consistent with the idea that the presence of permanent contracts is entirely due to imposed institutional constraints. Indeed, it is certainly true that labour market regulation set limits to the use of Fixed-Term Contracts.[6] I will capture these restrictions with an exogenous "transformation" shock, which forces agents to transform a FTC into an OEC with a certain probability. However, there is evidence that the choice in favour of employment protection is not fully due to legislation. Indeed, the OEC is commonly employed even at the beginning of a working relationship, when possible institutional constraints are less binding. Moreover, we can observe that some forms of employment protection are common even in countries in which they are not mandatory. For instance, in the USA severance payments are often included in labour contracts, as part of workers' benefits.[7]

Other papers took a reduced-form approach to the choice between OECs and FTCs, assuming some exogenous differences between contracts. For instance, Caggese and Cuñat (2008) assumes that an OEC assures higher productivity to the match. Instead, Garibaldi (2006) and Cao et al. (2010) assume that FTCs have a lower expected duration. In particular, Cao et al. (2010) assumes that only workers with a FTC can search on-the-job. This could be thought of as a shortcut for this work's main mechanism, allowing for a tractable model. However, in this way, we do not consider the endogenous nature of the searching decision,

[6] In Italy, they varied over time, but they generally imposed a cap on the ratio between FTCs and OECs in the same firm (with several exceptions), they limit the number of renewals and the overall maximum duration.

[7] Unfortunately, precise data about the extension of this type of agreements are lacking. Using available surveys, Parsons (2017) shows that in 2000 23% of the American labour force had access to severance pay in case of separations, a percentage that rose to 34% among workers in medium-large firms.

which could have important consequences in aggregate terms and for the type of workers that receive the FTC. Indeed, these papers abstract from heterogeneity in workers' ability, limiting the persistence in the type of the contract a worker receives.

One paper focused on the explanation of the coexistence of OECs and FTCs is Cahuc et al. (2016). The authors noticed that in reality, FTCs are highly expensive to terminate until the expiration rate. With the additional assumption of a small cost of writing a contract, they can explain the spread of FTCs among the jobs with a limited expected duration, while OECs becomes optimal for jobs with an expected duration long enough. This framework is undoubtedly suited to explain the use of FTCs among very short jobs with a predetermined duration. Instead, my model can explain the use of FTCs even when there is no heterogeneity in the exogenous expected duration of the match. Indeed, in labour surveys[8] a considerable share (between 25 and 33%) of workers claim to be employed with a FTC because they are in a trial period, supporting the idea that the FTCs can be used as a stepping stone towards the OECs or more generally to a long-lasting employment relationship.

Another relevant explanation for the coexistence of FTCs and OECs is described in Crechet (2018), where the author assumes differences in the risk-aversion between employer and employee to explain the surplus gain that an OEC can create by providing some insurance to the worker. This is in line with a more general rationalization for the use of employment protection, described in Pissarides (2010). In my model, I offer an alternative explanation for the use of employment protection, which works even when there is no difference in the agents' risk-aversion. Moreover, my model could explain the counter-intuitive finding in Lalé (2019), that in the presence of risk-averse agents and Nash-bargaining, severance payments actually reduce agents' welfare by inducing an increasing wage path over time, that runs counter to having a smooth consumption path. The increasing wage path following the introduction of employment protection is present in my model, but it is a desirable feature for the agents, since it reduces the otherwise excessively high on-the-jobs search intensity.

[8]For example, the Italian Labour Survey, RIL provided by ISTAT.

This work contributes to this literature, explaining both the coexistence of FTCs and OECs and the peculiar distribution of FTCs in the workforce, adding three relevant heterogeneity dimensions: match-specific productivity, worker-specific ability and experience. These heterogeneity dimensions allow me to explain the three facts I mentioned in the introduction: the spread of FTCs among young workers, the high correlation of FTCs with low wages and the persistence in the type of contract. Regarding the latter, there is in the literature a shared concern regarding FTCs when they become a dead-end for workers, see for example Ichino et al. (2008) and Gagliarducci (2005). However, in these works, they generally found evidence that temporary jobs are stepping stones to reach a permanent position and not a never-ending trap for the worker, meaning that they are still a better option than unemployment. In this work, I investigate these aspects in the evaluation of labour market policies that reduce the share of FTCs, taking into account the possible negative effects on "outsides", meaning unemployed or people employed with FTCs.

My work also contributes to the vast literature focused on the European dual labour market. This strand of literature started right after the liberalization of the temporary labour contracts in Spain, the first country that largely allowed for this kind of contracts. A review can be found in Dolado et al. (2002). This literature has always focused on the aggregate impacts of these labour market reforms on unemployment, productivity and labour market flows. One main finding of these works is that an increase in the share of temporary contracts leads to higher volatility in the labour market. García-Serrano and Jimeno (1999) uses a pooled cross-section data from 17 sectors in 17 Spanish regions to estimate that an increase in the percentage of fixed-term contracts leads to an increase in overall labour mobility. However, FTCs could increase the on-the-job search of workers, counteracting the beneficial effects on the average unemployment duration, with an increase in job-to-job transitions. This mechanism is captured in my model by the presence of endogenous on-the-job search and the same mechanism was present in Boeri (1999). In the latter, the author focused on a specific case of temporary jobs, activated after separation in some European countries, finding that the increased job-to-job transitions of these workers crowd out the job-finding probability of the unemployed. The long-term effect of this increased volatility on the unemployment rate and the employment level is ambiguous, but in the short-term, the simple introduction of fixed-term contracts seems to generate a *honey moon effect*, as shown in Boeri and Garibaldi (2007). This work also provides evidence that, in the long-run, the increase in fixed-term contracts could lead to a reduction in productivity. Moreover, the combination of widespread use of fixed-term contracts and high firing costs on permanent contracts leads to an amplification of the unemployment fluctuations along the business cycle, as shown in Bentolila et al. (2012). There is instead some evidence of less training and investment in human capital for workers with temporary contracts (as documented in Booth et al. (2002), Lucidi and Kleinknecht (2009) and Albert et al. (2005)). However, there is also some evidence of a boost in productivity, at least in the short term, deriving from the use of the same temporary contracts through a reduction in the absenteeism rate (Ichino and Riphahn (2005) and Engellandt and Riphahn (2005)). Overall, it seems that FTCs increase productivity in the short-term, but they could potentially lower long-term productivity.

Finally, I briefly summarize the empirical literature on the relationship between contract type and worker and firm characteristics. Portugal and Varejão (2009) shows that human capital is an important determinant for the contractual choice, both on worker's and firm's side. They find that highly-skilled vacancies are more likely to be filled with a permanent contract, given that the hiring process is harder, that the screening process is faster and that firing a skilled-worker, on which the firm has invested, is relatively more expensive. For these reasons, firms that employed highly-skilled workers are more likely to have OEC with them. They also provide evidence of the FTC as a screening tool, given the correlation between the destruction of permanent positions and the creation of new fixed-term vacancies. This is in line with the observed correlation between OEC and high wages. On the workers' side, they provide evidence, consistent with other works, that young, female and low-educated workers are more likely to be employed by fixed-term contracts. In Booth et al. (2002), the authors give a similar picture in describing the characteristics of workers employed through temporary contracts, with a striking difference that more educated men seem to be more likely employed with these kinds of contracts.

Finally, the recent Italian labour market reform has not yet been studied extensively, but some results can be found in Sestito and Viviano (2016). Using the same administrative source of my work, the authors use a difference-in-difference estimation that gives us the impact of the reduction in firing costs and the tax-incentive for OEC on the number of permanent contracts signed in the first half of 2015. They find that almost half of the total amount of the new contracts can be attributed to the two reforms. Relevant for my work, they found that most of the effects on the share of FTCs signed are due to the tax-incentive, while the cut in the firing costs has a more limited role. This is in line with the results in my counterfactual simulations, where I show that a tax-incentive for OECs is more effective in raising their share. However, this type of policy is at risk of harming low-income workers.

3 Descriptive Evidence

As I have already anticipated, the sources of information of this section are mainly three: the Italian Labour Survey (Rilevazione sulle Forze di Lavoro), elaborated by the Italian Statistical Agency (ISTAT), a sample of working histories from the social security data collected by INPS and the dataset Mercurio, collecting all working histories of workers in Veneto, from the regional office Veneto Lavoro.

Using these datasets, I provide some evidence about the diffusion of the two types of labour contracts and on-the-job search in Italy. In particular, I will document the three important facts about the FTC distribution that I will use as a validation of the model in the empirical section.

3.1 FTCs and OECs among workers and firms

I derive a characterization of the labour force employed with the two possible types of labour contracts from the quarterly Labour Survey of ISTAT in 2013.[9] The population of reference of the survey are the legal workers excluding self-employed, which in Italy are around 25 % of the total legal workers.

The detailed description is presented in Appendix B, where I reported the share of FTCs in different sectors, profession and regions. Moreover, I run a probit estimation on all observable, in order to have more robust evidence of the characteristics of the jobs and workers employed with a FTC. Here I summarize the main points for the theoretical and empirical analysis.

FTCs and OECs are both widely spread in the Italian labour market, across all sectors, jobs and workers categories. However, as I already anticipated, there are prominent differences in the share of FTCs in different segments of the labour market. The first fact that I

will use as a validation of the empirical model is that FTCs are much more diffused among the youngest cohorts, where they reach the majority of the total workers in 2013, while FTCs covered 13.3% of the whole working population.

Moreover, firm tenure is highly correlated with the probability of having an OECs. Indeed, among the matches created in 1 year or less, the FTC share raises to 55.6%. On the contrary, gender and education do not have a strong relationship with the use of FTCs,

The second fact that I will explain with my model is that FTCs are generally associated with lower wages, even considering the observable characteristics of the workers (Booth et al. (2002)). It is hard to claim that there is a causal link in this relationship since many unobservable characteristics could bias the results, and there are clear problems of self-selection of both workers and job positions. Indeed, in a lab experiment presented in Berton et al. (2015), workers ask for a higher salary in order to accept contracts with a lower expected

[9] I described the dataset in appendix A. I use the 2013 data to show the situation before the labour market reforms of the followings years that had some effects on the choice of the contract, as shown in Sestito and Viviano (2016). However, the distribution of the contracts in the population is similar even today.

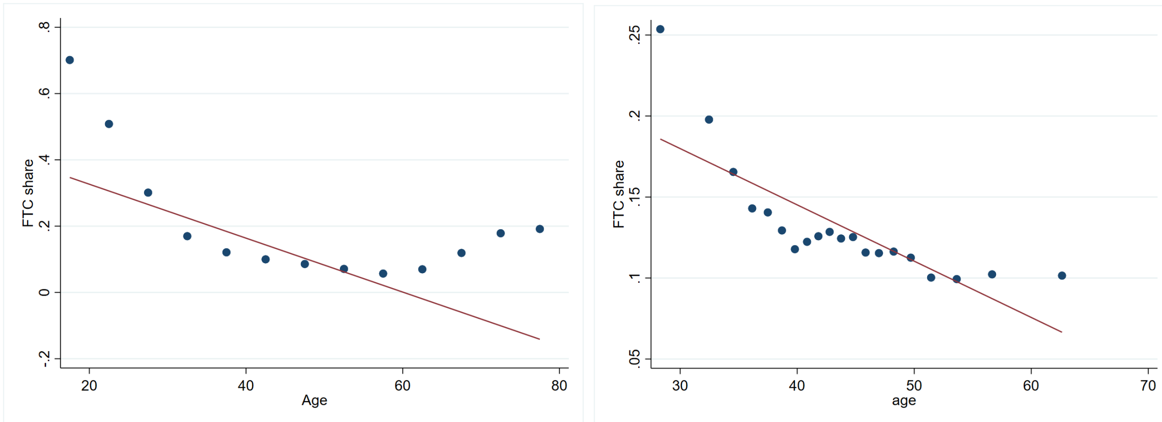


Figure 1: FTC and age. Source: ISTAT RCFL, 2013. These scatter plots pictures FTC share among employees in different age classes. The left plot shows the unconditional data, while the right plots the same data after conditioning for sex, income, educational level, foreigner dummy, years of tenure, firm size, sector (2 digits), occupation (3 digits), part-time dummy, geographical area.

duration, in particular, they asked for compensation to accept FTCs with a duration of fewer than three years. However, there are no conclusive studies on the possible "wage premium" earned by workers employed through less costly fixed-term jobs to the best of my knowledge.

This high correlation between FTCs and income can be observed in figure 2. FTC shares are higher among low-income jobs, even controlling for other characteristics.

In term of sectors, FTCs are most common in agriculture, hotel and restaurants, where seasonal jobs are the standard. In agriculture, FTCs cover more than half of the overall working force. In some sectors, generally skill-intense, they are less common: for instance, they reach just a share of 3.7% in finance.

If we analyzed the job occupations instead, we could see that the highest percentage of FTC is present in unskilled jobs, while the lowest in managerial positions.

Controlling for the other observables, it appears that the highest share of OECs is present in the army and in jobs that require technical specialization, while this is not the case for professional jobs and scientific research, where FTC are quite diffuse.

Overall, we can already confirm that both contractual forms are used in Italy. Also, this choice is partially related firm and sector characteristics, but partially also on the job

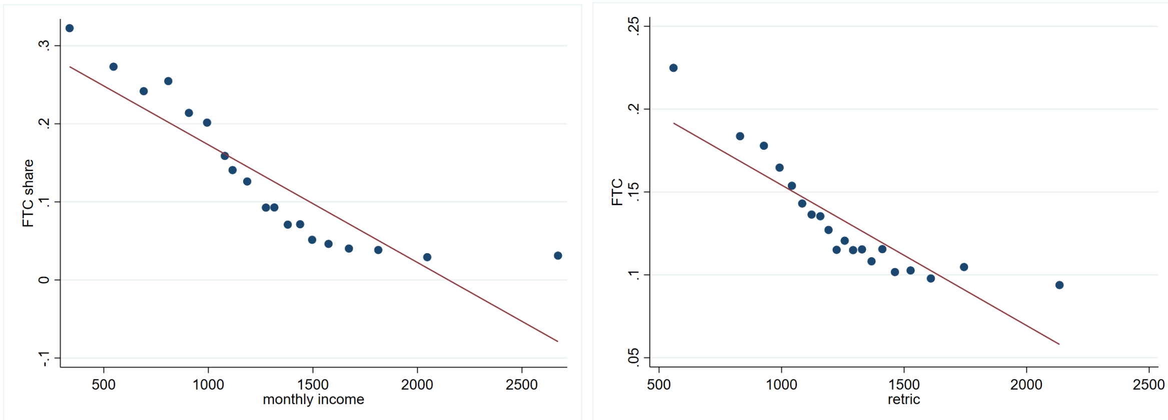


Figure 2: FTC and income. Source: ISTAT RCFL, 2013. These scatter plots pictures FTC share over monthly income. Data are binned in 20 dots. The left plot shows the unconditional data, while the right plots the same data after conditioning for age, age squared, sex, educational level, foreigner dummy, years of tenure, firm size, sector (2 digits), occupation (3 digits), part-time dummy, geographical area.

characteristics and requirements, that translate into workers characteristics.

In the survey, they also asked workers if they were satisfied with the FTCs. The percentage of workers that answered positively is extremely low. More than 95% of workers with a FTC said they accepted it just because they could not find an OEC, not because they prefer it.

3.2 Duration of the Contract

The distribution of the duration of the FTC is reported in figure 3.

The average duration is 12.5 months, but the median is at 9 months. Therefore most of the contracts have a duration of less of one year, but there is a relevant share with a duration exceeding 2 or even 3 years. From 2012 the law set a limit of 3 years for FTCs. However, there were exceptions for staff leasing agencies or if there were specific agreements with the labour unions.

The picture is different if we look at new contracts (figure 5), here the share of FTCs and particularly short temporary contracts is much higher. In this case, I rely on the data coming from the dataset “Mercurio” of Veneto Lavoro, that collects all administrative data

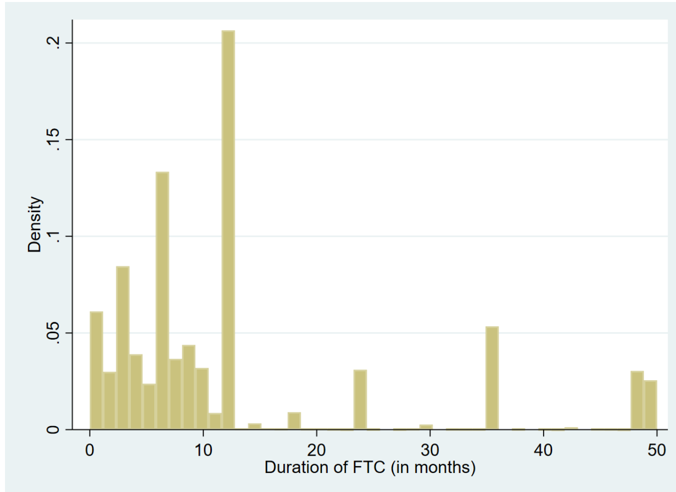


Figure 3: Duration of FTC. Source: ISTAT, 2013, RCFL. Contracts with duration above 50 months are reported at 50.

about hirings and firings in Veneto, a large region in the North-East of Italy.[10]

FTC contracts in Italy are generally longer than 1 month[11]. Moreover, most of the very short contracts are consecutive contracts between the same firm-worker pair. If we consider only new hires (defined as a contract between a new firm-worker pair in 2013), FTCs with a duration shorter then 1 months are less than 20% of the total.

OECs constitute 24.1% of new contracts in Veneto in 2013, and they reach 29% among new hires. This data are consistent with the previous one reporting a high share of FTCs among newly formed matches, and it depicts FTC as a standard practice to start a career in a firm in the Italian labour market.

3.3 Persistence of Type of Contract

The third fact that will use in my model is contract's persistence.

As shown, FTCs are probably related to worker characteristics and occupations. From this, it should not come as a surprise that workers are somehow persistently employed with

[10]The findings are therefore valid only for that specific region since the socio-economic characteristics are different from the rest of Italy. However, there are reasons to believe that the picture in the rest of the country is similar.

[11]Differently from France, where Cahuc et al. (2016) pointed out that temporary contracts shorter than 1 month are more of 60% of all new contracts

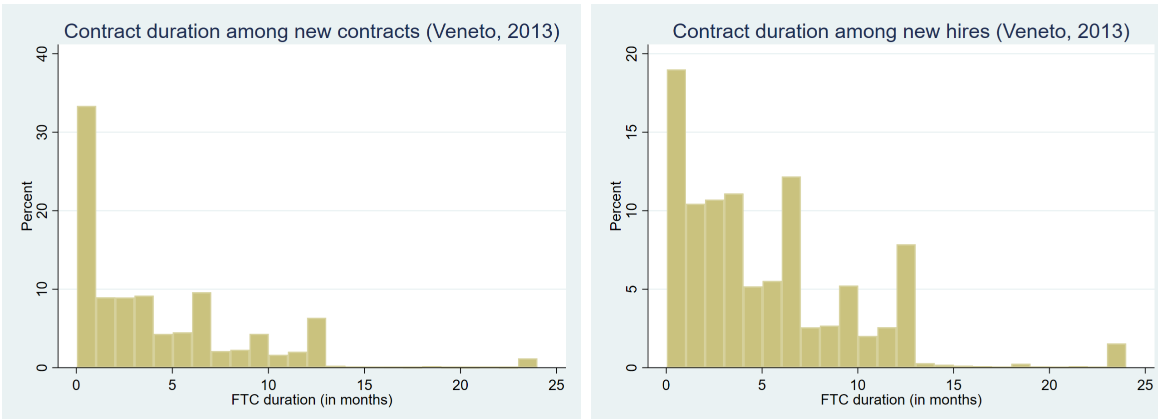


Figure 4: Duration of FTC among new contracts (left) and new hirings (right) Source: Veneto Lavoro, 2013, Mercurio. Population: non-seasonal new contracts in Veneto. Contracts with duration above 24 months are reported at 24.

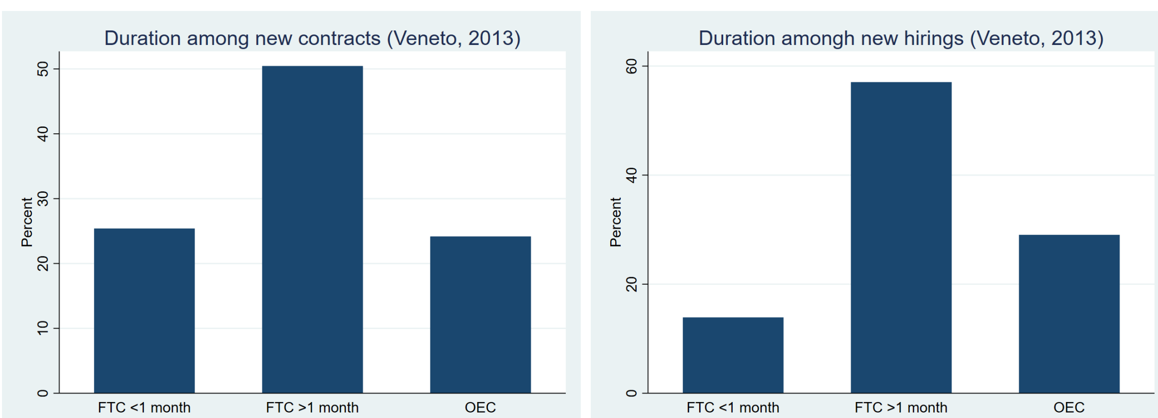


Figure 5: Duration of FTC among new contracts (left) and new hirings (right) Source: Veneto Lavoro, 2013, Mercurio. Population: non-seasonal new labour contracts in Veneto.

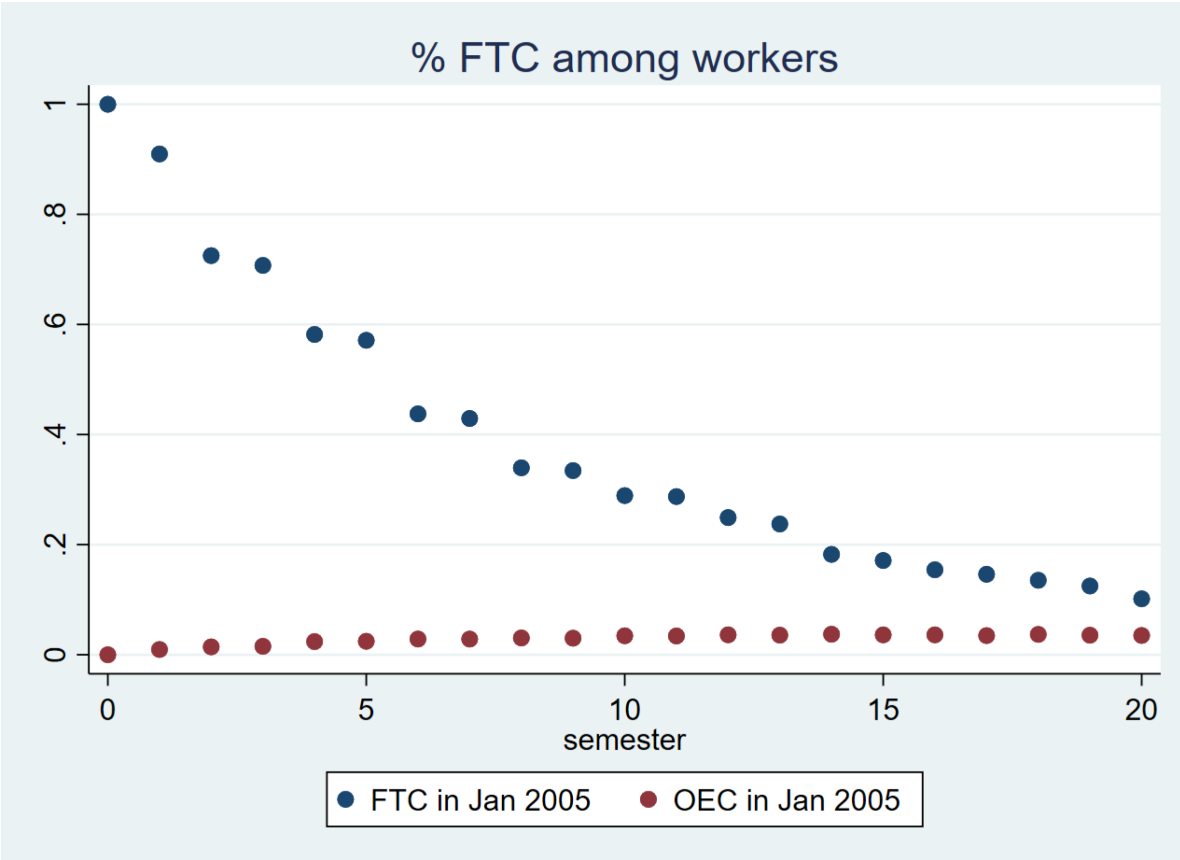


Figure 6: FTCs share among sample of workers starting a job in January 2005, with a FTC (blue) or an OEC (red). INPS data

the same type of labour contract. This is a fact that has been already found and discussed in the literature[12], as mentioned in the previous section.

In figure 6 I show the share of FCTs among a sample of workers that started a job in January 2005, respectively with a FTC or an OEC.[13] Even after 10 years, the share of FTCs among the one that started with a FTC in 2005 was significantly higher.

3.4 On the job search

Job search plays an important role in my model and the Labour Survey provides specific data about on-the-job search by asking workers if they are searching for another job and why

[12] Gagliarducci (2005), for example.

[13] By definition, in one group the initial share of FTCs is 100% and in the other group is 0%, then workers may change labour contract.

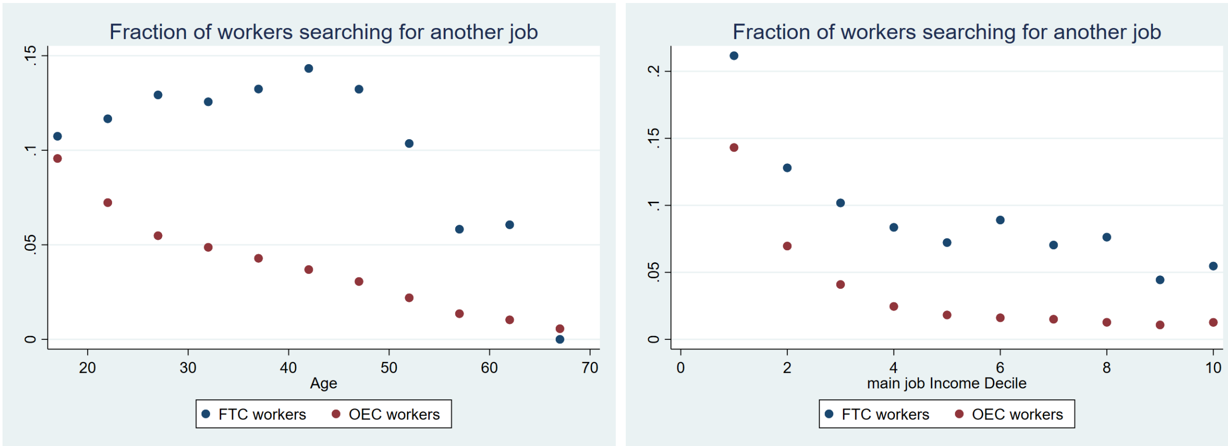


Figure 7: On-the-job searching workers by age (left) and income (right), Italy, 2013. Source: ISTAT

they are doing so. I use this information to provide some evidence that on-the-job search is related to the choice of the contract.

In 2013 the percentage of workforce searching on-the-job was 4.3%. However, it was 12.1% among workers with a FTC and only 3.3% among the others. Among all, other important determinants of the probability of searching are age and income. In figure 7, I show the percentage of searching workers divided by income and age and the difference between FTC and OEC workers is still large.

In appendix B, I report the predictive margins derived from a probit estimation of the probability of performing on-the-job search on all the observables. If we keep all other observable at their mean levels in the populations, an employee with an OEC has a 6.7% probability of being searching on-the-job; however, the probability raises to 11.8% if the employee has a FTC.

Moreover, in the same appendix, I show how the difference in search intensity also translates to a difference in the job-to-job transition rates, using the dataset "Mercurio" of Veneto Lavoro, that collects all working histories of workers from around 2000 till 2018 in Veneto, a region in the North-East of Italy[14].

These results cannot be taken as conclusive evidence that the type of contracts causes

[14]The dataset is described more in details in Appendix A.

different searching intensity in the worker since other variables are possibly driving both the search intensity and the choice of the contract. However, they at least confirm that in the population workers with a FTC search more while on-the-job.

4 Toy Model

In this section, I introduce a variant of the Diamond-Mortensen-Pissarides (DMP) model with on-the-job search to explain the agents' choice about employment protection. I call it the "Toy model" since it will be developed in the next section with the addition of the uncertainty about workers' ability and endogenous firings.

The model is a search and matching model in discrete time. It is characterized by costly searching activity, that can be performed by both employed and unemployed workers, and by heterogeneity in the match-specific productivity x, that is drawn from a uniform distribution $\mathcal{U}(\underline{x}, \bar{x})$ at the beginning of a match. I am assuming that even at $x = \underline{x}$ the match is productive enough to generate a positive surplus. Therefore, the agents will not be willing to separate.

4.1 Search and Matching

There is a continuum of firms and workers, normalized to unity. Workers can be either employed or unemployed. Firms employ just one worker and they post costly vacancies that can be filled by unemployed or searching workers.

The number of matches formed every period is determined by a matching function

$$m(\varsigma, v) = A\varsigma^{\eta}v^{1-\eta}$$

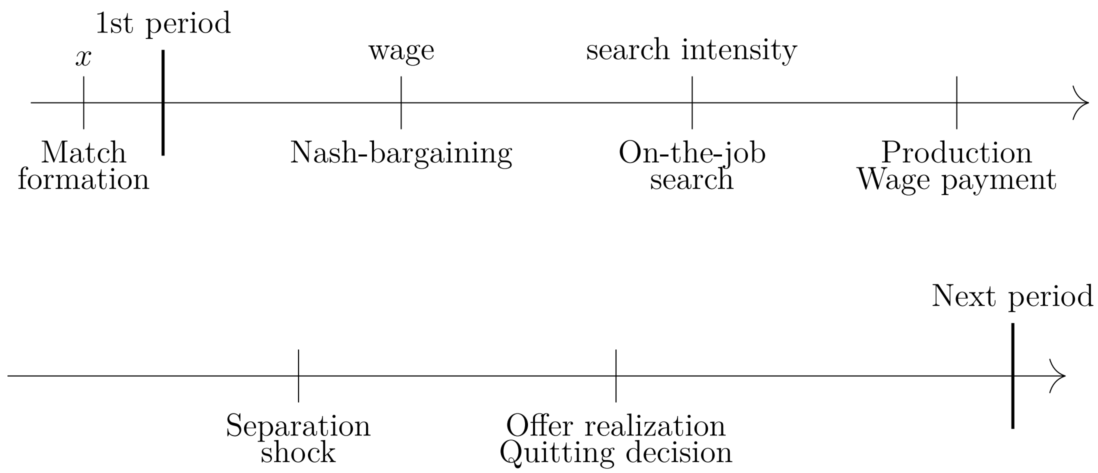
where ς is a search-adjusted unemployment measure. Agents optimize the search intensity as I described in details later on.

Then, I call θ the search-adjusted labour market tightness

$$\theta = \frac{v}{\varsigma}$$

and $p(\theta) = m(1, \theta)$ will refer to the probability of being matched per unit of search.

Timing



From the firm side, the probability of filling a vacancy is q and it is given by:

$$q(\theta) = (1 - \xi)\frac{m(\varsigma, v)}{v} = (1 - \xi)m\left(\frac{1}{\theta}, 1\right)$$

where ξ is the endogenous fraction of matches that are discarded by the workers. This happens because some on-the-job searching workers discard the new matches and they continue their current relationship.

4.2 Timing of Events

Figure 9 summarizes the events' timing that I will describe.

As anticipated, every match has a match-specific component x that it is drawn from a uniform distribution $U(\underline{x}, \bar{x})$ and it is observed when the agents meet.

Agents Nash-bargain over the wage at the beginning of every period, resulting in a split of the match surplus according to their bargaining power γ.[15]

After that, the worker performs his on-the-job searching activity according to a certain search intensity, that affects the subsequent probability of receiving a new offer. In the next

[15] Later, I analyze the implications of this assumption in presence of on-the-job search. In particular, the fact that the Nash-bargain is repeated at every period solves a problem of non-convex bargaining sets raised in Shimer (2006).

section, I will describe this choice in details.

Then, the production realizes and the agents receive their payoffs.

After, a possible separation shock can materialize with a certain exogenous probability λ. In this case, the match is broken and the worker returns to unemployment. However, the worker is still kept until the end of the period, so that he has the possibility to perform a job-to-job transition if he receives a new offer. Indeed, with a certain probability that depends on the worker's previous search intensity, a new offer can materialize at the end of the period. In this case, the worker compares the new offer in hand with the current match and either quit or discard the new offer.

Finally, a new period begins and the agents start from bargaining again over the wage.

4.2.1 Search intensity decision and Quitting

Searching for a job is a costly activity for workers. Nevertheless, they undertake it even while employed. I have already shown some evidence of on-the-job search in Italy; other papers confirmed that an important share of employed workers performs at least some search activity[16]. To capture this empirical evidence, workers spend some effort searching for a job both during unemployment and on-the-job. Following the literature[17] the cost of the worker is a convex function, such as:

$$h_i s^{\nu}$$

with $\nu > 1$ and $i = e, u$ depending on the employment status of the worker. In this way, I am allowing for a difference in the magnitude of the searching costs for employed and unemployed workers. I interpret $s \geq 0$ as a measure of search intensity. Then, $p(\theta)$ will be the probability of finding a job per unit of search intensity s.

Importantly, during the Nash-bargaining, the firm and worker cannot bargain over s,

[16]For example in Faberman et al. (2017), the authors report the result of a relevant survey done in the US on this topic. They show that 20% of employed workers can be classified as "searchers", with 23% of workers looking for a new job in the last 4 weeks and almost 20% actively applying for a vacancy.

[17]See Postel-Vinay and Robin (2004) or Faberman et al. (2017),

since the worker keeps the *right to manage* his search intensity. This natural assumption[18] has important consequences, as we will see. For this reason, search intensity s is chosen by the worker, that compares the benefits and costs of searching. I call s^* the result of this maximization problem.

The worker's benefits are represented by the possible worker's welfare increase coming from a new offer. In computing these benefits, we need to define the bargaining protocol with the old and the new firm. In order to keep the model simple, I make the questionable assumption that workers receive the same offers of unemployed agents. This is in line with some literature that assumes that workers bargain over the wage with the new firm in a subsequent period when they do not have anymore the opportunity to return back to the previous firm.[19]

Given this assumption[20] and using the fact that the agents split the surplus using the Nash-bargaining protocol, the worker decides to perform a job-to-job transition if the surplus of the new match is higher the surplus of the old one, or equivalently if the new x' drawn for the new match is higher than the x of the old match or equivalently.

Therefore, we can already compute the quitting probability of the worker $Q(x)$:

$$Q(x) = \overbrace{p(\theta)s^*(x)}^{\text{new offer prob}} \frac{\bar{x} - x}{\bar{x} - \underline{x}} \tag{1}$$

4.3 Value Functions

We can now analyze the value functions of this Toy model. In all the paper the discount factor is indicated by β, γ refers to the bargaining power of the worker, κ is the per period

[18] It can be rationalized by the fact that the firm cannot monitor the true search intensity performed by the worker.

[19] This assumption is very helpful in simplifying the empirical estimation, given that it allows not to keep track of past wages of the agents. However, it is not qualitatively essential for the results of this section. Indeed, I show in Appendix C that using the sequential auctions protocol, we arrive at similar conclusions.

[20] Another implicit assumption is that workers are not paying any other cost related to the job-to-job transition. In reality, this is hardly the case, leading to the possibility of rejections of more productive matches and an overall reduction in the benefits of on-the-job search.

cost of opening a vacancy, J indicates firm's value function of a filled job, V is the value of a vacancy, W is worker's value function when employed, while U is the value of unemployment. Finally, Z indicates the joint welfare of a match $(W+J)$, while S indicates the joint surplus $(W-U+J-V)$. This surplus is split according to the Nash-bargaining rule. The details of the bargaining in different set-ups are described in Appendix D.

4.3.1 Firms

$$J(x) = x - w(x) + \beta(1-\lambda)\left(1-Q(x)\right)J(x) + \beta(1-\lambda)Q(x)V + \beta\lambda V$$

$$J(x) = \frac{x - w(x) + \beta(1-\lambda)Q(x)V + \beta\lambda V}{1-\beta(1-\lambda)(1-Q(x))} \tag{2}$$

The firm gets the production and it pays the wage, then it gets the discounted continuation value of the filled vacancy if the worker does not separate and he does not quit. Otherwise, it gets the value of the empty vacancy that we now compute.

$$V = -\kappa + q(\theta)\beta\mathbb{E}\left(J(x)\right) + \beta(1-q(\theta))V$$

$$V = \frac{-\kappa + q(\theta)\beta\mathbb{E}\left(J(x)\right)}{1-\beta(1-q(\theta))} \tag{3}$$

where $\mathbb{E}J(x)$ is the expect continuation value from a new match. It is important to notice that this is not simply the expected value of $J(x)$ over the distribution of x, given the presence of workers searching on-the-job and discarding offers.[21]

Using the free-entry condition, I can set $V=0$. In the remaining of the section, I will exploit this in the computations.

[21] In particular, low values of x will be discarded by workers working in a better match, leading to the necessity to open a new vacancy for the searching firm. I will formally define and numerically compute this value later in the full model.

4.3.2 Workers

$$
\begin{aligned}
W(x) =& w(x) - \overbrace{h_e(s^*)^\nu}^{\text{effort cost}} + \beta(1-\lambda)(1-Q(x))W(x) + \beta(1-\lambda)Q(x)\bar{W}^N(x) + \\
&+ \beta\lambda(1-ps^*)U + \beta\lambda ps^*\bar{W}^N(\underline{x})
\end{aligned} \tag{4}
$$

The worker receives the wage and he pays the cost of the on-the-job search, then he gets the continuation value if he does not separate and he does not quit. If he quits, he receives $W^N(x)$ that is the expected value of the new offer *given* that he accepted it[22]. If he separates, he returns to unemployment if he has not received any new offer. Instead, if he has received a new offer, he accepts it for sure. For this reason, I indicated this continuation value as $\bar{W}^N(\underline{x})$, since it is as if the worker is situated in the worst possible match, accepting any other new offer. Similarly, I use this notation for the unemployment value.

$$
U = b - h_u(s_U^*)^\nu + \beta ps_U^*\bar{W}^N(\underline{x}) + \beta(1-ps_U^*)U \tag{5}
$$

The unemployed worker spends costly effort searching for a job, and he gets a flow payment b, that represents government unemployment benefits. Then with a probability depending on s_U^*, the search intensity, he gets the expected continuation value of a new job. Otherwise, he remains in unemployment.

4.4 Optimal Search Intensity

The worker decides the optimal amount of search intensity equating marginal costs and marginal benefits. The marginal costs are increasing in s, given the convexity assumption:

$$
MC = \nu hs^{\nu-1}
$$

From the value function of the worker, we can recover the marginal benefits of s

[22] Hence, it is higher than the simple average over all possible x, given that the lowest values are discarded.

$$MB = \beta p \left[(1-\lambda) \left(\frac{\bar{x}-x}{\bar{x}-\underline{x}} \right) \left(\bar{W}^N(x) - W(x) \right) + \lambda(\bar{W}^N(\underline{x}) - U) \right] \tag{6}$$

They are constant with respect to s, then it is easy to compute the jointly optimal search intensity s^J:

$$s^* = \left(\frac{MB}{\nu h} \right)^{\frac{1}{\nu-1}}$$

4.4.1 A benchmark: jointly optimal solution

As we mentioned, the worker keeps the *right to manage* the search intensity. However, suppose that the agents could bilaterally bargain search intensity at the moment in which they also fix the wage. The two agents could set the search intensity in order to maximize the joint welfare $Z(x) = W(x) + J(x)$, then they would still use the wage as a transfer to split the surplus according to the Nash rule.

From the value functions, we can recover the joint welfare (using the free entry condition).

$$\begin{aligned} Z(x) =& x - h(s^*)^\nu + \beta(1-\lambda)Q(x)\bar{W}^N(x) + \\ &+ \beta(1-\lambda)(1-Q(x))Z(x) + \beta\lambda(1-ps^*)U + \beta\lambda p s^* \bar{W}^N(\underline{x}) \end{aligned}$$

$$Z(x) = \frac{x - h(s^*)^\nu + \beta(1-\lambda)Q(x)\bar{W}^N(x) + \beta\lambda(1-ps^*)U + \beta\lambda p s^* \bar{W}^N(\underline{x})}{1-\beta(1-\lambda)(1-Q(x))} \tag{7}$$

Then the joint marginal benefits of searching are the following:

$$JMB = \beta p \left[(1-\lambda) \left(\frac{\bar{x}-x}{\bar{x}-\underline{x}} \right) \left(\bar{W}^N(x) - W(x) - J(x) \right) + \lambda(\bar{W}^N(\underline{x}) - U) \right] \tag{8}$$

It can be noticed that

$$JMB = MB - \beta p(1-\lambda) \left(\frac{\bar{x}-x}{\bar{x}-\underline{x}} \right) J(x)$$

The marginal costs have not changed, hence the jointly optimal search intensity $s^J = \left(\frac{JMB}{\nu h}\right)^{\frac{1}{\nu-1}}$ and this implies

$$s^J \leq s^*$$

In particular, if $J(x) > 0$, then the search intensity of chosen by the worker is higher than the jointly optimal search intensity.

It is important to notice that both agents would benefit (at least weakly) if they could jointly set the search intensity. The repeated Nash-bargaining assumption assures this.[23] The assumption of the repeated bargain is important also to avoid the problem raised in Shimer (2006) about the non-convexity of the bargaining set in the presence of on-the-job search. Indeed, in my case, the bargaining does not involve future wages, so it does not change the marginal benefits of searching.

4.5 Optimal Labour Contract in the presence of on-the-job search

The result that, in the presence of on-the-job search, the worker could exert an excess of search effort compared to the jointly optimal one[24] is not new to the literature. It has been highlighted in Lentz (2014), that analyzes the optimal labour contract in the presence of hidden on-the-job search and Stevens (2004). In both cases, they claimed that the immediate optimal solution of the problem is for the worker to "buy" his job, with an upfront payment that guarantees him all the future surpluses of the match. In the absence of this possibility, the optimal contract is backloaded: the firm should commit to an increasing utility path for the worker.

Even in my case, it is straightforward to see that if $J(x) = 0$, the optimal joint search intensity s^J coincides with the choice of the worker s^*. Therefore, if the firm could credibly commit, the optimal contract would promise all the joint welfare to the worker in the

[23] In Appendix C, I show that this is not the case if we assume that the firm fixes the wage at the beginning of a match and it has all the bargaining power. Then, the possibility to jointly decide s still increases the joint welfare, but it could reduce the worker's welfare.

[24] The jointly optimal search intensity is different from the socially optimal one since we are not considering search frictions and most importantly the welfare benefits of the new firms matched with the worker.

subsequent periods, while using an upfront payment to split the surplus according to the Nash-bargaining rule.

Formally, I call $W'(x)$ and $J'(x)$ the continuation values, respectively of worker and firm, that the agents set at the beginning of the match when they first bargain, if they could commit not to change them in any subsequent period. Then, the optimal contract would state

$$W'(x) = x - h(s^*)^{\nu} + \beta(1-\lambda)Q(x)\bar{W}^N(x) + \beta(1-\lambda)(1-Q(x))W'(x) + \beta\lambda(1-ps^*)U + \beta\lambda ps^*\bar{W}^N(\underline{x})$$

$$J'(x) = 0$$

The worker receives a wage equal to the entire production at every period. Only in the first period, he receives $w(x)$[25] such that

$$(1-\gamma)(W(x) - U) = \gamma(J(x) - V)$$

where $W(x)$ and $J(x)$ have the same initial expression, but $W'(x)$ and $J'(x)$ as continuation values in the match.

4.5.1 The role of Employment Protection

Suppose that the firm is not able to commit not to re-bargain again in the future.[26] Then, employment protection could be chosen by agents in order to obtain effectively the same wage schedule previously described, but through a standard Nash-bargaining procedure.

In practice, when agents meet for the first time, they set the employment protection in order to maximize the surplus of the match and they contemporaneously bargain over the

[25] It is allowed to be negative so that the worker actually pays the firm.

[26] This assumption can be justified by the acknowledgement that an external judge must sanction a contract violation. This option is generally costly in the first place, and specifically for labour contracts, a violation can also be substantially hard to prove, given the multiplicity of reasons that could justify a contract modification or interruption.

wage.

In this way, they can still use the wage to obtain the usual surplus split. Notice that the outside option of the firm is still the empty vacancy since it has not signed any contract with the worker.

$$(1-\gamma)(W(x)-U)=\gamma(J(x)-V)$$

Here I model employment protection as a pure waste (fc) paid by the firm only in the case of a voluntary separation initiated by the firm.[27] At the end of the section, I will show that the same results carry over if we model the firing costs as severance payment from the firm to the worker.

Proposition

The optimal level of fc will be the following:

$$fc(x)=\frac{x-w^{*}(x)}{\gamma[1-\beta(1-\lambda)(1-Q(x))]} \tag{9}$$

where $w^{*}(x)$ is the wage that would have split the surplus according to the Nash-Bargaining rule in the absence of the firings costs, but still keeping the search intensity at the optimal level s^{J}.

The presence of the firing costs determines a rise in the wage of the worker starting from the second period of the match. This is due to the lower outside option of the firm, that incorporates the firing costs: the Nash-Bargaining from the second period onward split the surplus such that

$$(1-\gamma)(W'(x)-U)=\gamma(J'(x)-V+fc)$$

If fc takes the value stated in equation 9, then $J'(x)=0$ and $W'(x)-U)=Z(x)$ as in the optimal contract with commitment.

[27]There is no cost for quitting, nor in the case of the exogenous separation.

Proof

Notice that absent any firing cost, $w^*(x)$ is such that

$$J(x) - V = (1-\gamma)S(x) = (1-\gamma)(Z(x) - U - V)$$

recalling the value of $J(x)$ from equation 2, and setting $V = 0$ from the free entry condition, we get

$$\frac{x - w^*(x)}{1 - \beta(1-\lambda)(1-Q(x))} = (1-\gamma)(Z(x) - U) \tag{10}$$

If we introduce the firing costs, keeping constant the search intensity s^J, then the value $Z(x) = Z(x')$ does not change, since the firing costs are never actually paid. However, they affect the welfare of the firm and the worker, since the wage must adapt to guarantee the same share $1-\gamma$ of the surplus to the firm.

$$\frac{x - w(x)}{1 - \beta(1-\lambda)(1-Q(x))} - (-fc) = (1-\gamma)(Z(x) - U - (-fc))$$

$$\frac{x - w(x)}{1 - \beta(1-\lambda)(1-Q(x))} = (1-\gamma)(Z'(x) - U) - \gamma fc \tag{11}$$

Subtracting equation 12 from equation 10, we get

$$\frac{w(x) - w^*(x)}{1 - \beta(1-\lambda)(1-Q(x))} = \gamma fc$$

From this, we can conclude that if to conclude that if fc is set to $\frac{x - w^*(x)}{\gamma(1-\beta(1-\lambda)(1-Q(x)))}$, then the wage would increase by $x - w^*(x)$ reducing the firm's profits exactly to zero.

We can notice that the optimal fc are increasing in the firm's profit so in the match-specific productivity x. This result is in line with the fact that labour contracts with stronger employment protection are more diffuse among high-paid jobs.

It is important to highlight the fact that in this toy model firing costs are just a threat to the firm, used to ensure credibility to the wage schedule. If endogenous firings were to be added to the picture, then a trade-off would emerge for the choice of employment protection: the gains from the reduction in on-the-job search and the costs in case of voluntary separation. The full-model will address this issue.

4.5.2 Employment Protection as Severance Payment

If we model employment protection not as pure waste, but a compulsory transfer from the firm to the worker in case of a separation initiated by the firm, then we have to modify the result of the Nash-Bargaining. The reason is that in this case, fc not only decreases the outside option of the firm, but it increases the outside option of the worker.

$$\frac{x - w(x)}{1 - \beta(1-\lambda)(1-Q(x))} - (V - fc) = (1-\gamma)(Z(x) - (U + fc) - (-fc)) \tag{12}$$

Rearranging and using the free-entry condition:

$$\frac{x - w(x)}{1 - \beta(1-\lambda)(1-Q(x))} = (1-\gamma)(Z(x) - U) + fc$$

Therefore we can obtain the optimal labour contract again by setting

$$fc = \frac{w(x) - w^*(x)}{1 - \beta(1-\lambda)(1-Q(x))}$$

The severance payment can be set at a lower level than before, as they affect both the outside option of the agents.

5 Full Model

The full model has the same features of the Toy model with two additional elements. Firstly, workers have a subjective ability that is unknown, so that they are ex-ante homogeneous, but they reveal their type over time through a Bayesian updating process. Secondly, the presence of institutional constraints on the type of labour contracts available: agents can choose only between Fixed-Term Contracts and Open-Ended Contracts.

FTCs involve no firing costs, while OECs are associated with a firing cost fc to be paid by the firm in the form of pure waste, only in case of voluntary separation. Moreover, I assume that for legal limitations, a FTC has a probability φ to experience a "transformation shock", in which case it has to be transformed into a permanent contract or terminated.[28] Importantly, once an OEC is signed, it cannot be reverted back to a FTC in subsequent periods by the same firm. This indeed is generally forbidden and rarely observed in data.

5.1 Search and Matching

There is a continuum of firms and workers, normalized to unity. Workers have an exogenous probability of dying every period called λ_d, and they are immediately replaced by the same amount of newborn unemployed workers. Workers can be either employed (with one of the two kind contracts) or unemployed. Firms employ just one worker and they post costly vacancies that can be filled by searching workers.

As in the Toy model, we have a Cobb-Douglas matching function

$$m(\varsigma, v) = A\varsigma^{\eta}v^{1-\eta}$$

where ς is a search-adjusted unemployment measure. Similarly we define q,θ and p as in section 1.4.1. However, in this case, the match is allowed to have a negative expected surplus. In this case, the match is broken before starting, the worker returns to the unemployment pool and no costs are paid.

[28]This assumption capture legal limitations on FTC present in Italy. For instance, since 2012, FTC has a maximum duration of 3 years, including renewals, recently reduced to 2 years.

5.2 Production, Timing and Bayesian Updating

Every worker i has inner productivity α_i that is unobservable and it is drawn from a known continuous and smooth distribution with cdf $F(\alpha)$. In addition, every match i, j has a match-specific component $x_{i,j}$ that it is drawn from a known distribution $G(x)$ and it is observed when the agents meet.

The output of the match is the sum of these components and a noisy term.[29]. Therefore, it can be used as a noisy signal of the worker's ability. At every period t the production is :

$$y_{i,j,t} = \alpha_i + x_{i,j} + \varepsilon_{i,j,t}$$

where $\varepsilon_{i,j,t}$ is white noise, so it is an i.i.d. shock with mean 0.[30]

All the agents[31] observe the realized production at every period and they perform a Bayesian updating of the distribution of the worker's productivity α_i and therefore they update also the expected production of the future. In particular, I call ϕ the set of moments about the ability distribution of the worker that is relevant for the agents. ϕ will be the moments at the beginning of every period, while ϕ' will be the updated moments after the production realization.

5.2.1 Example

Even if the theoretical section does not assume a specific distribution, it is worth describing the Bayesian Updating procedure I will assume in the empirical session.

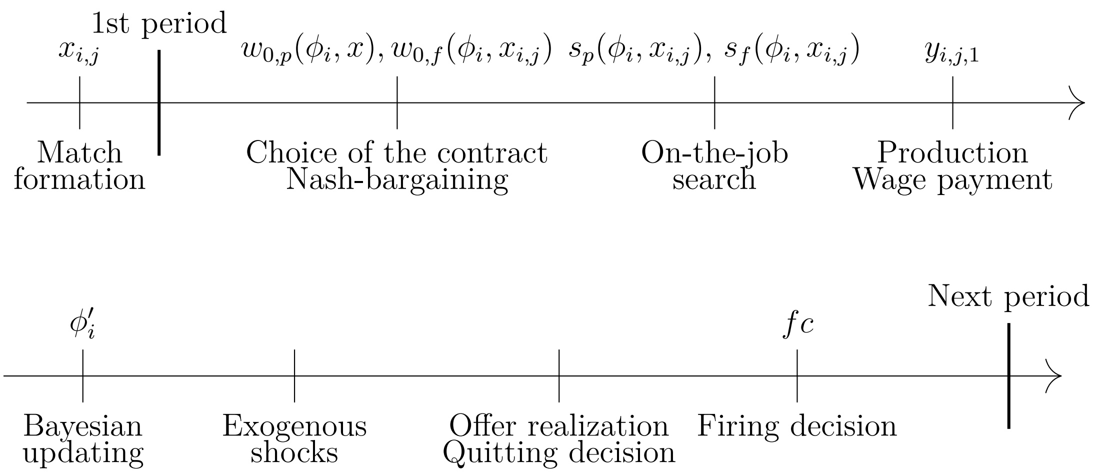
In that section, I assume that the prior distribution for the worker-specific productivity α is a *normal distribution* with parameters $\phi_0 = (\mu_0, \sigma_0^2)$ and the error term is normally

[29]It would be interesting to study a generalization of this assumption, with possible complementarities between worker's ability and match productivity. This would affect the search intensity of different type of workers, possibly leading to a larger distance between s^* and s^J of good workers.

[30]This assumption of a pure transitory shock is used mainly for tractability. Ideally, it would be interesting to allow for some form of persistence and check the robustness of the results. Following the example of Postel-Vinay and Turon (2010), I could introduce a probability of experiencing a shock at every period.

[31]Not only the agents in the match but also all other employers.

Timing



distributed as well, with parameters $(0, \sigma_\varepsilon^2)$.

At every realization of the production y_t, the agents will update their prior beliefs about productivity.

Given the choices about the distributions, the posterior distribution of α will be a normal with the following parameters ϕ_i':

$$\mu_i' = \frac{\sigma_\varepsilon^2 \mu_i + \sigma_i^2 (y_{i,j,t} - x_{i,j})}{\sigma_\varepsilon^2 + \sigma_i^2}$$

$$(\sigma_i^2)' = \frac{\sigma_\varepsilon^2 \sigma_i^2}{\sigma_\varepsilon^2 + \sigma_i^2}$$

The parameter σ_i^2 converges to zero as the number of observed productions growths to infinity: the agents have a better knowledge of the true productivity over time. Consequently, the last observed production is less and less informative as the match continues, as we can observe from the fact that $y_{i,j,t}$ "weights" less in the updating of μ_i as σ_i^2 declines.

5.2.2 Timing

In figure 11 I represent the orders of the events in a period.

As in the Toy model, when a match is formed there is the draw of the productivity parameter $x_{i,j}$. Then, at the beginning of every period agents sign a new contract and they contemporaneously bargain over the wage, that must be the same for every realized production in that specific period.[32]. The wage is Nash-bargained and it is a function of the contractual choices of the agents, the productivity parameter and the beliefs on the moments of the α_i distribution. Both present and past chosen contracts will matter for the wage, as we will see later.

Subsequently, the worker has to choose the optimal amount of search intensity. As in the Toy model, he keeps the right-to-manage this variable.

Then, the production realizes and the agents perform the Bayesian updating described.

After, two possible exogenous shocks can materialize: the first one is a transformation shock, with probability φ, that forces a transformation of a FTC into a OEC or a separation. The second one is the exogenous separation shock that happens with an exogenous probability λ. In this case, the match is broken, regardless of the type of contract. In both cases, the worker will still be kept until the end of the period, so that he has the possibility of performing a job-to-job transition.

The process of offer realization and quitting decision is the same as in the Toy model.

After this, the firm itself has the possibility to terminate the match if the surplus of the match is negative. I call this *firing* of the worker, but it should be noticed that given the repeated bargaining assumption, it is a jointly optimal decision.

In this last case of an endogenous separation, the firm pays a firing cost fc only if she signed an OEC contract. That fc is a pure waste.[33] This fc can be thought as the legal and bureaucratic costs related to the firing procedure. If the contract is a FTC instead, fc is standardized to zero.

Finally, a new period begins and the two agents start from re-bargaining again over the wage and the contract.

[32]There are several possible justifications for the absence of state-contingent wages in the short term, for example, a cost of writing contracts or incentives related to the efficiency wages theory.

[33]In the Toy model, I showed that this assumption is not fundamental. Similarly, in the full-model, a severance payment has similar advantages in terms of lower search intensity and it is still costly to the agents, given the uncertainty over the worker's ability. A OEC imposes inefficient retention of matches with a negative surplus that would have been jointly terminated, absent any severance pay.

5.3 Value Function

I keep the following notation through all the rest of this paper: for clarity, I am not using the individual or match-specific indexes. Subscripts indicate the previous and the actual contract between agents, for example in $W_{f,p}$ the f refers to a previous FTC, while p indicates a present OEC. The past contract is important as well since it determines the outside option of the firm. New matches do not have a previous contract; however, I will still indicate them as if they had a previous FTC contract. Indeed, since FTC does not involve firing costs, a new match is in this sense equivalent to a re-bargained contract after a period of a fixed-term contract.

For wages, we will have $w_{i,j}(\phi, x)$ as the results of a Nash-Bargaining of a worker with a prior distribution of the productivity determined by ϕ and the match productivity draw x. The indexes i, j will be p or f following the rules just stated.

It is also convenient to indicate with β the discount factor incorporating the risk of dying. Formally:

$$\beta = (1 - \lambda_d)\beta^*$$

where β^* is the true discount factor and $(1 - \lambda_d)$ is the surviving probability.

5.3.1 Firms

Differently from the Toy model, firms are matched with workers that are heterogeneous in term of expected ability and previous employment condition. The probability of being matched with a worker of one specific type depends on their searching intensity and their distribution in the population.

In particular, conditioned of being matched with someone, the probability of finding an unemployed worker with an ability distribution characterized by ϕ is the following:

$$q_u(\phi) = \frac{s_u^*(\phi)u(\phi)}{\int_{-\infty}^{\infty}\left[s_u(\phi)u(\phi) + \int_{-\infty}^{\infty}\left(s_p^*(\phi,x)e_p(\phi,x) + s_f^*(\phi,x)e_f(\phi,x)\right)dG(x)\right]dF(\phi)} \tag{13}$$

where I indicate as $u(\phi)$ the measure of unemployed characterized by the moments ϕ and similarly, I call $e_p(\phi, x)$ and $e_f(\phi, x)$ the measure of employed workers in a match with productivity x, characterized by moments ϕ and with a OEC or a FTC respectively.

In the same way, we can get the probability of being matched with a worker characterized by the moments of the ability distribution ϕ and employed in a match with a specific x.

The value of the filled vacancy depends crucially on all this information. For this reason, I introduce here the following average filled vacancy values:

$$J_u(\phi) = \int_{-\infty}^{\infty} \max_{i\in\{f,p\}}(J_{f,i}(\phi,x); V)dG(x)$$

$$J_f(\phi,x) = \int_{-\infty}^{\hat{x}_f} VdG(x') + \int_{\hat{x}_f}^{\infty} \max_{i\in\{f,p\}}(J_{f,i}(\phi,x'))dG(x')$$

$$J_p(\phi,x) = \int_{-\infty}^{\hat{x}_p} VdG(x') + \int_{\hat{x}_p}^{\infty} \max_{i\in\{f,p\}}(J_{f,i}(\phi,x'))dG(x')$$

The first value $J_u(\phi)$ is the average value of a match with an unemployed worker with ability ϕ. J_p and J_f are the average values of a match with a permanent or temporary contract respectively, characterized by ϕ and x. In these cases, $\hat{x}_i$ represents the level of the productivity that is necessary to convince the worker to change job, as we will see later in details.

Taking all this into account, the value of an open vacancy is equal to:

$$\begin{aligned} V = -\kappa + \beta^*(1-q(\theta))V+ \\ +\beta^* q(\theta)\frac{\int_{-\infty}^{\infty}\left(s_u u(\phi)J_u(\phi) + \int_{-\infty}^{\infty}\left[s_f e_f(\phi,x)J_f(\phi,x) + s_p e_p(\phi,x)J_p(\phi,x)\right]dG(x)\right)dF(\phi)}{\int_{-\infty}^{\infty}\left[s_u(\phi)u(\phi) + \int_{-\infty}^{\infty}\left(s_p(\phi,x)e_p(\phi,x) + s_f(\phi,x)e_f(\phi,x)\right)dG(x)\right]dF(\phi)} \end{aligned} \tag{14}$$

As usual, using the free entry condition, we can set the value of the open vacancy equal to zero.

$$V = 0$$

Permanent contract

$$\begin{aligned} J_{i,p}(\phi, x) = {} & \mathbb{E}(y) - w_{i,p}(\phi, x) + \beta\lambda V + \beta^{*}\lambda_{d} V + \\ & + \beta(1-\lambda)p(\theta)s_{p}^{*}(\phi, x)\int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} Max(J_{p,p}(\phi', x), V - fc) + (1 - \mathbb{1}_{W})V\right] dF(y) \\ & + \beta(1-\lambda)(1 - p(\theta)s_{p}^{*}(\phi, x))\int_{-\infty}^{+\infty} Max(J_{p,p}^{t+1}(\phi', x), V - fc) dF(y) \end{aligned} \tag{15}$$

where $i \in \{f, p\}$ indicate the previous contract and $\mathbb{1}_{W}$ indicates worker's choice to continue the relationship.

In addition to the Toy model, we have the probability for the worker to die and terminate the match, but more importantly, the uncertainty about the worker's ability. This feature creates the possibility for the worker to be fired. However, the permanent contract implies firing costs in case of an endogenous separation. Therefore agents separate only if the loss for the firm from continuing the match is higher then the firing costs.

Temporary contract

$$\begin{aligned} J_{f,f}(\phi, x) = {} & \mathbb{E}(y) - w_{f,f}(\phi, x) + \beta\lambda V + \beta^{*}\lambda_{d} V + \\ & + \beta(1-\lambda)\Bigg[\varphi\Bigg(p(\theta)s_{f}^{*}(\phi, x)\int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} Max(J_{f,p}(\phi', x), V) + (1 - \mathbb{1}_{W})V\right] dF(y) \\ & + (1 - p(\theta)s_{f}^{*}(\phi, x))\int_{-\infty}^{+\infty} Max\left(J_{f,p}(\phi', x), V\right) dF(y)\Bigg) + \\ & + (1-\varphi)\Bigg(p(\theta)s_{f}^{*}(\phi, x)\int_{-\infty}^{+\infty}\left[\mathbb{1}_{W} Max(\max_{i\in\{p,f\}} J_{f,i}(\phi', x), V) + (1 - \mathbb{1}_{W})V\right] dF(y) \\ & + (1 - p(\theta)s_{f}^{*}(\phi, x))\int_{-\infty}^{+\infty} Max\left(\max_{i\in\{p,f\}} J_{f,i}(\phi', x), V\right) dF(y)\Bigg)\Bigg] \end{aligned} \tag{16}$$

The equation is similar to the previous one, but the firms do not have to pay the firing costs anymore in case of termination. Also, with probability $1-\varphi$ it has the option to continue with another period of FTC (last two lines), while with probability φ the firm is forced to transform the contract or to terminate. Notice that the choice of the worker also depends on this realization of the "transformation shock", since when considering the new offer, the worker already knows if the next period he will be employed with a FTC or an OEC or if he is going to be fired.

5.3.2 Workers

Workers decide the search intensity as in the Toy model. Now, it is a function of both job and worker characteristics, so it should be indicate as $s_i^*(\phi, x)$ and $i \in \{f, p\}$. For notational convenience, I indicate it only as a function of the labour contract.

I refer to x^n to indicate the new productivity draw for a new match. Finally, to simplify further the notation, I indicate as $W(\phi, x) = \max_{i \in \{p,f\}} W_{f,i}(\phi, x)$, that is the worker's welfare maximized by the best labour contract.

Permanent contract

$$\begin{aligned}
W_{i,p}(\phi,x) =& w_{i,p}(\phi,x) - h_e \left(s_p^*\right)^{\nu} + \\
&+\beta\lambda \int_{-\infty}^{+\infty} \left[(1-p(\theta)s_p^*)U(\phi') + p(\theta)s_p^* \int_{-\infty}^{+\infty} Max(W(\phi',x^n), U(\phi')dG(x)\right] dF(y)+ \\
&+\beta(1-\lambda) \int_{-\infty}^{+\infty} \left[p(\theta)s_p^* \int_{-\infty}^{+\infty} Max(W_{p,p}(\phi',x), W(\phi',x^n), U(\phi'))dG(x)+ \right. \\
&\left. +(1-p(\theta)s_p^*)\left(Max(W_{p,p}(\phi',x), U(\phi')\right)\right] dF(y)
\end{aligned} \tag{17}$$

where $i \in \{p, f\}$.[34]

Again, the only important difference with the Toy model is the possibility of firings.

[34]The previous contract just determines the current wage of the worker, as shown in Appendix D.

I recall that the Nash-bargaining assures that any endogenous separation is beneficial to both agents. This explains the presence of the maximization in the continuation values: the decision about the endogenous firing can also be modelled as if the worker were comparing the value of continuing the match with the value of unemployment.

Fixed-term contract

$$\begin{aligned} W_{f,f}(\phi,x) = & w_{f,f}(\phi,x) - h_e\left(s_f^*\right)^\nu + \\ & +\beta\lambda\int_{-\infty}^{+\infty}\left[(1-p(\theta)s_f^*)U(\phi',x) + p(\theta)s_f^*\int_{-\infty}^{+\infty} Max(W(\phi',x^n),U(\phi'))dG(x^n)\right]dF(y)+ \\ & +\beta(1-\lambda)(1-\varphi)\int_{-\infty}^{+\infty}\left[p(\theta)s_f^*\int_{-\infty}^{+\infty}\left[Max(W(\phi',x),W(\phi',x^n),U(\phi'))\right]dG(x^n)+\right. \\ & \left. +\left(1-p(\theta)s_f^*\right)\left(Max(W(\phi',x),U(\phi')\right)\right]dF(y)+ \\ & +\beta(1-\lambda)\varphi\int_{-\infty}^{+\infty}\left[p(\theta)s_f^*\int_{-\infty}^{+\infty}\left[Max(W_{f,p}(\phi',x),W(\phi',x^n),U(\phi'))\right]dG(x^n)+\right. \\ & \left. +\left(1-p(\theta)s_f^*\right)\left(Max(W_{f,p}(\phi',x),U(\phi')\right)\right]dF(y) \end{aligned} \tag{18}$$

FTC that can be renewed or voluntarily transformed into an OEC with probability $1-\varphi$, otherwise in the continuation value the agents will simply have to decide between the endogenous separation and the permanent contract (last two lines of the value function).

Comparing this with the OEC value function, there are differences in wage, different optimal search intensity, different continuation values due to the absence of firing costs and the option to choose for a FTC in the future, that is possible only in this case.

Unemployed

The unemployed worker gains the unemployment benefit, pays the searching costs and then he gets the expected discounted continuation value.

$$U(\phi) = b - h_u\left(s_u^*(\phi)\right)^\nu + \beta p(\theta)s_u^*\int_{-\infty}^{+\infty} Max(W(\phi',x^n),U(\phi))dG(x) + \beta(1-p(\theta)s_u^*)U(\phi)$$

$$U(\phi) = \frac{b - h_u\left(s_u^*(\phi)\right)^{\nu} + \beta p(\theta) s_u^* \int_{-\infty}^{+\infty} Max(W(\phi', x^n), U(\phi)) dG(x)}{1 - \beta(1 - p(\theta) s_u^*)} \tag{19}$$

5.4 Firing Thresholds

The endogenous separation happens when the surplus of the match becomes negative. Given the Nash-bargaining protocol, we can analyze the case as if the firm decides to fire the worker.

This decision has a reservation property: there exists a level of production below which the match is broken and above which the match is continued.

To see this it is sufficient to notice that a separation occurs when $J_{p,p}(\phi', x) < fc$ or $\max\left(J_{f,f}(\phi', x), J_{f,p}(\phi', x)\right) < 0)$ and the firm's value function is monotonically increasing in the expected production. The expected production depends on the updated ability distribution, and this is monotonically increasing in the realized production. Therefore, given our assumptions about the ability distribution, we can conclude that there exists a level of production that acts as a threshold, below which the match is destroyed.

However, we will have different thresholds for every ϕ, x and two different thresholds for each of them, depending on the type of contract.

Given the firing costs, the outside option is lower for a permanent contract compared to a FTC. A firm has to pay fc to terminate an OEC, while the outside option for a FTC is the vacancy $V = 0$. For this reason, the threshold will be higher for the FTC.

Therefore, I call $\bar{y}_p(\phi, x)$ and $\bar{y}_f(\phi, x)$ respectively, the level of production such that:

$$J_{p,p}(\phi', x) = -fc$$

$$\max_{i \in \{p,f\}} J_{f,i}(\phi', x) = 0$$

In Appendix E I show that indeed the threshold for the FTC is at least as high as the one for OEC. [35]

I also need a third threshold, $\bar{y}_{f'}(\phi, x)$ for the case in which the agents are forced to transform a FTC in an OEC. In this case, the continuation surplus of the match is weakly decreased[36].

5.5 Optimal search intensity

The quitting decision of the worker is similar to the Toy model: the worker leaves if his welfare is higher with the new firm rather than with the old one. However, now the welfare depends on the ability of the worker and the type of contract. Given the presence of employment protection, the simple fact that the match-specific draw is higher for the new match does not assure that the worker decides to quit.

To see this, suppose a worker employed with an OEC receives a new offer by a firm with the exact match-specific productivity. However, a FTC in the new match allows increasing the total surplus by an infinitesimal amount. In this case, the worker discards the new offer, since his welfare is higher in the old firm[37].

I call $\hat{x}_i(\phi, x)$ the quitting thresholds, with $i \in \{f, p, f'\}$ where we will have

$$W_{p,p}(\phi', x) = \max_{i \in \{p,f\}} W_{f,i}(\phi', \hat{x}_p)$$

$$\max_{i \in \{p,f\}} W_{f,i}(\phi', x) = \max_{i \in \{p,f\}} W_{f,i}(\phi', \hat{x}_f)$$

$$W_{f,p}(\phi', x) = \max_{i \in \{p,f\}} W_{f,i}(\phi', \hat{x}_{f'})$$

As for the firing threshold, I need this third case f' to describe the situation in which a transformation shock forces the agents to transform the FTC contract in an OEC.

Given the thresholds, we can compute the probability $Q_i(\phi', x)$ that a worker quits for

[35] The proof starts form $J_{p,p}(\phi', x) = J_{f,p}(\phi', x) - \gamma f c$ and uses the fact that $S_{f,p}(\phi', x) - S_{f,f}(\phi', x) < fc$

[36] This comes directly from the fact that they could have chosen to voluntarily transform the labour contract if that was optimal). Therefore we have $\bar{y}_f(\phi, x) \leq \bar{y}_{f'}(\phi, x)$

[37] More precisely, his wage is higher by $\gamma f c$.

every possible updated worker's ability distribution and therefore for every observed production. Given the distribution of the match-specific component, it will be:

$$Q(\phi', x)_i = p(\theta) s_i^*(\phi, x)(1 - G(\hat{x}_i))$$

Even the optimal search intensity $s_i^*(\phi, x)$ depends on the labour contract, and it is obtained as in the Toy model, by equating the marginal benefits of the worker (MB) to the marginal costs $MC = \nu h_e s^{\nu-1}$.

$$\begin{aligned} MB_f =& \beta p(\theta) \Big[\lambda \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (max((W(\phi', x) - U(\phi'), 0))\, dG(x) dF(y) + \\ & +(1-\lambda)(1-\varphi) \int_{-\infty}^{\infty} \int_{\bar{x}_f(\phi,x)}^{\infty} \left(max(W(\phi', x^n) - \max_{i \in \{p,f\}} (W_{f,i}(\phi', x), U(\phi')), 0) \right) dG(x) dF(y) + \\ & + (1-\lambda)\varphi \int_{-\infty}^{\infty} \int_{\bar{x}_{p'}(\phi,x)}^{\infty} (max(W(\phi', x^n) - Max(W_{f,p}(\phi', x), U(\phi')), 0))\, dG(x) dF(y) \Big] \end{aligned}$$

Equalizing it to MC, we get the expression for the optimal amount of search intensity.

Similarly for the OEC

$$\begin{aligned} MB_p =& \beta p(\theta) \Big[\lambda \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (max((W(\phi', x) - U(\phi'), 0))\, dG(x) dF(y) + \\ & + (1-\lambda) \int_{-\infty}^{\infty} \int_{\bar{x}_p(\phi,x)}^{\infty} (max(W(\phi', x^n) - Max(W_{p,p}(\phi', x), U(\phi')), 0))\, dG(x) dF(y) \Big] \end{aligned}$$

The marginal benefits of increasing s are generally higher for the FTC, since in the OEC the employment protection raises the wage of the worker, increasing his continuation value in the match and the probability of staying.[38]

[38] This is not true for all possible value of x and ϕ, since the FTC has the advantage that it provides the possibility to continue with a FTC, possibly increasing the welfare of the worker.

5.6 Surplus

$$
\begin{aligned}
&S_{f,p}(\phi, x) = S_{p,p}(\phi, x) - fc = J_{f,p}(\phi, x) - V + W_{f,p}(\phi, x) - U(\phi) = \\
&= \mathbb{E}(y) - h_e \left(s_p^*\right)^{\nu} + \\
&+ \beta\lambda \int_{-\infty}^{\infty} \left[(1 - p(\theta)s_p^*)(U(\phi')) + p(\theta)s_p^* \int_{-\infty}^{\infty} max(W(\phi', x^n), U(\phi'))dG(x)\right] dF(y) + \\
&+ \beta(1-\lambda) \left[\int_{-\infty}^{\bar{y}_p} \left[-fc(1 - Q_p) + (1 - p(\theta)s_p^*)U(\phi') + p(\theta)s_p^* \int_{-\infty}^{\infty} max(W(\phi', x^n), U(\phi'))dG(x)\right] dF(y) + \right. \\
&+ \left. \int_{\bar{y}_p(\phi,x)}^{\infty} \left[(1 - p(\theta)s_p^*) \left(W_{p,p}(\phi', x) + J_{p,p}(\phi', x)\right) + p(\theta)s_p^* \int_{\hat{x}_p}^{\infty} W(\phi', x^n)dG(x)\right] dF(y)\right] - U(\phi)
\end{aligned}
\tag{20}
$$

Now, we can write the surplus of a match and look at the jointly optimal decision about the labour contract.

The surplus equation is composed of different lines: in the first one we have the productivity minus the searching costs, therefore the immediate component of the surplus. The second line is the continuation value when there is an exogenous separation. The third line is continuation value when the observed production is so low that the worker will be fired. Finally, we have the continuation value when the expected productivity of the worker is high enough to be kept in the match.

$$
\begin{aligned}
&S_{f,f}(\phi,x) = J_{f,f}(\phi,x) - V + W_{f,f}(\phi,x) - U(\phi) = \\
&= \mathbb{E}(y) - h_e\left(s_f^*\right)^\nu + \\
&+ \beta\lambda\int_{-\infty}^{\infty}\left[(1-p(\theta)s_f^*)(U(\phi')) + p(\theta)s_f^*\int_{-\infty}^{\infty} max(W(\phi',x^n),U(\phi'))dG(x)\right]dF(y) + \\
&+ \beta(1-\lambda)(1-\varphi)\left[\int_{-\infty}^{\bar{y}_f}\left[(1-p(\theta)s_f^*)U(\phi') + p(\theta)s_f^*\int_{-\infty}^{\infty} max(W(\phi',x^n),U(\phi'))dG(x)\right]dF(y) + \right.\\
&+ \left.\int_{\bar{y}_p(\phi,x)}^{\infty}\left[(1-p(\theta)s_f^*)\max_{i\in\{p,f\}}\left(W_{f,i}(\phi',x) + J_{f,i}(\phi',x)\right) + p(\theta)s_f^*\int_{\hat{x}_f}^{\infty} W(\phi',x^n)dG(x)\right]dF(y)\right] + \\
&+ \beta(1-\lambda)\varphi\left[\int_{-\infty}^{\bar{y}_{f'}}\left[(1-p(\theta)s_f^*)U(\phi') + p(\theta)s_f^*\int_{-\infty}^{\infty} max(W(\phi',x^n),U(\phi'))dG(x)\right]dF(y) + \right.\\
&+ \left.\int_{\bar{y}_{f'}(\phi,x)}^{\infty}\left[(1-p(\theta)s_f^*)\left(W_{f,p}(\phi',x) + J_{f,p}(\phi',x)\right) + p(\theta)s_f^*\int_{\hat{x}_{f'}}^{\infty} W(\phi',x^n)dG(x)\right]dF(y)\right] - U(\phi)
\end{aligned} \tag{21}
$$

For FTC the expression is similar, but there are no firing costs and we have to separate the two cases in which a transformation shock realizes or not.

5.7 Optimal Contractual decision

We can finally look at the contractual decision made by the agent at the beginning of a period.

In the Toy model, we have already shown the fact that the agents can increase the joint welfare by reducing the searching intensity of the worker. Employment protection is a way to achieve this reduction in search intensity. This fact is true even in the full model.

However, if we take the difference of the two surpluses $S_{f,p}(\phi,x) - S_{f,f}(\phi,x)$, we notice that employment protection comes at a cost in the full model. The costs are composed by the firing costs in case of an endogenous separation, by the inefficient retention of unproductive matches and by the reduction in the continuation surplus.

$$S_{f,p}(\phi, x) - S_{f,f}(\phi, x) = \beta(1-\lambda)(FC + IR + CONT) + \\ + \Delta Effort + \beta \Delta Quit \tag{22}$$

In details

$$FC = -fc\left(\int_{-\infty}^{\bar{y}_p} (1 - Q_p(\phi', x)) dF(y)\right) \leq 0$$

FC represents the expected firing costs actually paid by the firm.

$$IR = \int_{\bar{y}_p}^{\bar{y}_{f'}} (1 - Q_p(\phi', x)) \left(J_{p,p}(\phi', x) + W_{p,p}(\phi', x) - U(\phi')\right) dF(y) \leq 0$$

IR are the costs of the unproductive match kept only for the presence of the employment protection.

$$CONT = (1-\varphi) \int_{\bar{y}_f}^{\infty} \left[W_{p,p}(\phi', x) + J_{p,p}(\phi', x) - \max_{i \in \{f,p\}} \left(J_{p,i}(\phi', x) + W_{p,i}(\phi', x)\right)\right] dF(y) \leq 0$$

Finally, these are the costs link with the absence of choice in the next period, when a FTC can allow selecting the best labour contract again, while the OEC forces the agents to continue with an OEC.

The red components are the ones that we have already encountered in the Toy model: they are due to the change in search intensity depending on the amount of employment protection.

They are composed of the difference in search costs

$$\Delta Effort = h_e(s_f^*(\phi, x)^\nu - s_p^*(\phi, x)^\nu)$$

and the difference in the results of the search effort

$$\Delta Quit = p(s^*_f - s^*_p)\Big(\lambda(U(\phi) - W(\phi', x^n))+$$
$$+(1-\lambda)\int_{-\infty}^{\infty}\int_{\hat{x}}^{\infty}\Big[max(W_{p,p}(\phi', x) + J_{p,p}(\phi', x), -fc) - W(\phi', x^n)\Big]dG(x)dF(y)$$

However, there is an important difference with respect to the Toy model: here, there is no assurance that this red component has a positive effect on the overall surplus. This is the consequence of the fact that now matches can have a negative surplus, on top of the assumption that the agents do not choose employment protection, but it is set exogenously.

An example of this is the case in which for every realized productivity, the continuation surplus is low enough that the match would be terminated (negative continuation surplus), but it is not low enough to lead the firm's profit below the firing costs.

In this situation, the jointly marginal benefits of search intensity are higher than the worker's marginal benefits, since he is not internalizing the negative welfare of the firm in continuing the match. This leads to an inefficiently *low* search intensity of the worker.[39] In these circumstances, these red components are negative.

Nevertheless, if the expected surplus of the match is high enough, the worker will tend to over-search as in the Toy model.[40] This generates a trade-off between the costs of employment protection and the gains in terms of lower search intensity.

It can be noted that employment protection costs are decreasing in the worker's ability and the match-productivity draw, since the higher the expected match surplus, the lower the possibility of endogenous firings. However, they are also decreasing in the quitting probability, since the higher the chance that the worker quits, the lower the disbursement by the firm.

Similarly, the benefits of an OEC are lower for low-quality matches, since the difference

[39] This can happen if the match is discovered unproductive later on when the firing costs limit the possibility to terminate it.

[40] I underline once again the fact that this is not the benevolent social planner point of view, since we are not considering the surplus of the new firm.

between the MB of search intensity and the JMB decreases (eventually becoming negative) as the match surplus decreases.

In the calibration section, I will show how the optimal labour contract varies with x and the expected worker's ability.

5.8 Steady-State Equilibrium

In order to solve for the steady-state equilibrium, I need to compute all the flows that characterize it. Indeed, we have many different possibilities, for every employment status, ϕ, x and type of contract.

Also, the matching function will take into account the presence of on-the-job search.

In particular, it is now possible to give a formal description of the searching-adjusted unemployment:

$$\varsigma = \int_{\phi} \left[s_u^*(\phi)u(\phi) + \int_x \sum_{i=p,f} s_i^*(\phi,x)e_i(\phi,x)d(x) \right] d(\phi)$$

Searching workers will decrease the effective market tightness.

We can now formally compute the parameter ξ that indicates the shares of new job offers that are discarded by agents.

To do so, we need the amount of discarded offers ($Disc$):

$$Disc = \int_{\phi} \Big[s_u^* u(\phi) G(\hat{x}_u) + \int_x s_p^* e_p^*(\phi,x) \left(\lambda G(\hat{x}_u) + (1-\lambda)G(\hat{x}_p)\right) dx +$$

$$+ \int_x s_f^* e_f^*(\phi,x) \Big(\varphi \left(\lambda G(\hat{x}_u) + (1-\lambda)G(\hat{x}_{f'})\right) + (1-\varphi)\left(\lambda G(\hat{x}_u) + (1-\lambda)G(\hat{x}_f)\right) \Big) dx \Big] d\phi$$

The share of searching units performed by workers that will not accept the new offers will be:

$$\xi = \frac{Disc}{\varsigma}$$

5.8.1 Inflows and Outflows

The inflow of workers into unemployment every period is given by both endogenous and exogenous separations:

$$
\begin{aligned}
In_u(\phi) = (1-\lambda_d)\int_y \Bigg(\int_\phi \mathbb{1}_{\phi'=\phi}\Bigg[\int_x \Big[\lambda + (1-\lambda)\mathbb{1}_{y<y_p}(1-p(\theta)s_p^*(1-G(\hat{x_p}))\Big] e_p(\phi,x)dx+ \\
+(1-\varphi)\int_x \Big[\lambda + (1-\lambda)\mathbb{1}_{y<\bar{y}_f}(1-p(\theta)s_f^*(1-G(\hat{x_f}))\Big] e_f(\phi,x)dx+ \\
+\varphi \int_\phi \int_x \Big[\lambda + (1-\lambda)\mathbb{1}_{y<\bar{y}_{f'}}(1-p(\theta)s_f^*(1-G(\hat{x_{f'}}))\Big] e_f(\phi,x)dx\Bigg] d\phi\Bigg) dF(y)
\end{aligned}
$$

The outflows from unemployment are instead given by the match with unemployed:

$$
Out_u(\phi) = (1-\lambda_d)p(\theta)s_u^*(\phi)u(\phi)(1-G(\hat{x}_u))
$$

Then we have inflows to employment, coming by either unemployed and employed workers.

$$
In_{e_i}(\phi,x) = Out + JtJ = (1-\lambda_d)p(\theta)\int_\phi s_u^*(\phi)u(\phi)f(x)\mathbb{1}_{x\geq\hat{x}_u}d\phi + JtJ
$$

where Job-to-Job transitions:

$$
\begin{aligned}
JtJ(\phi,x^n) =&(1-\lambda_d)p(\theta)\int_y\int_\phi \mathbb{1}_{\phi'=\phi}\Bigg(\int_{-\infty}^{+\infty} s_p^* e_p(\phi,x)\Big[\lambda f(x^n)\mathbb{1}_{x^n>\hat{x}_u} + (1-\lambda)\mathbb{1}_{x^n>\hat{x}_p}dx\Big] + \\
&+(1-\varphi)\int_{-\infty}^{+\infty} s_f^* e_f(\phi,x)\Big[\lambda f(x^n)\mathbb{1}_{x^n>\hat{x}_u} + (1-\lambda)\mathbb{1}_{x^n>\hat{x}_f}dx\Big] + \\
&+\varphi\int_{-\infty}^{+\infty} s_f^* e_f(\phi,x)\Big[\lambda f(x^n)\mathbb{1}_{x^n>\hat{x}_u} + (1-\lambda)\mathbb{1}_{x^n>\hat{x}_{f'}}dx\Big]\Bigg) d\phi dF(y)
\end{aligned}
$$

Finally, at every period, a fraction λ_d of the population is created. In the calibration section, I describe which prior distribution ϕ they are endowed with. I indicate them as $n(\phi)$ and they have an immediate probability $p(\theta)s_u(\phi)$ to be employed. Otherwise, they enter in unemployment.

The equilibrium conditions that close the model are then:

$$In_u(\phi) = Out_u(\phi) + \int_\phi (1 - p(\theta)s_u^*(\phi))n(\phi)d\phi$$

For every ϕ, x and t

$$In_{e_i(\phi,x)} = e_i(\phi, x)$$

This last condition comes from the fact that at every period all employed workers change condition, either by separating or by updating ϕ.

5.8.2 Equilibrium conditions

I summarize the equilibrium conditions of a steady-state of the model:

1. Matching function

   $$m(\varsigma, v) = A\varsigma^\eta v^{1-\eta}$$

   whit the appropriate definition of search-adjusted unemployment;

2. From the free entry condition $V = 0$, we get

   $$\frac{\kappa}{\beta q(\theta)} = \bar{J}$$

   where $q(\theta) = (1 - \xi)m\left(\frac{1}{\theta}, 1\right)$ and $\bar{J}$ is the average profit of the firm, as described by equation 1.15.

3. Flows equilibrium

   $$In_u(\phi) = Out_u(\phi)$$

   For every ϕ, x and t

   $$In_{e_i(\phi,x)} = e_i(\phi, x)$$

4. Firms and Workers maximize their value functions;

5. Wages are Nash-bargained

6. Search-effort is chosen such that the FOC conditions hold.

6 Calibration

In this section, I provide a calibration of the model using the dataset I already described: the Labour Force Surveys by ISTAT, the microdata from INPS collecting a sample of Italian working histories from 2000 until 2015 and the dataset "Mercurio" from Veneto Lavoro, collecting all the working histories from the region Veneto.[41]

Some parameters are taken from the literature or externally calibrated, while others are estimated using a Monte Carlo Markov Chain estimation.

6.1 Externally Calibrated Parameters

In my model, one period is equivalent to 6 months.

6.1.1 Discount Factor

I set the discount factor $\beta^* = 0.976$, that is equivalent to an annual discount rate of 5%.

6.1.2 Matching Function and Bargaining Power

I assume that the unemployment coefficient in the matching function is the same estimated in Shimer (2005), $\eta = 0.72$. This is a standard value in the search and matching literature. Then, I set the matching efficiency parameter $\varsigma = 0.5$. It is known that in a standard search and matching model, the vacancy cost and the matching efficiency parameter are only jointly identified, if there is no information on the measure of vacancies, but only on the unemployment level. Therefore, I set the matching efficiency arbitrarily, using the observed unemployment levels to calibrate the correct vacancy cost associated. I set the bargaining

[41] A more detailed description is presented in Appendix A.

power of the worker equal to the elasticity of the matching function to unemployment, $\gamma = 0.72$, that is the standard Hosios condition for optimality.[42]

6.1.3 Unemployment Benefit

The OECD collects the net replacement ratio of the unemployment benefits for different categories of workers across the developed countries. In my work, I use the replacement rate that was in place in Italy in 2013, taking as a benchmark a single worker without children, into unemployment for 6 months and with a previous wage equal to the average wage. Therefore I set the unemployment benefit to 59% of the average wage.

6.1.4 Transitory Shock

Since 2012, in Italy the FTC cannot last more than 3 years, summing up all subsequent FTC between the same employee-employer couple. However, there were several exceptions to this law, for instance, for employment agencies or whenever there were special agreements between the firm and the trade unions.

To capture these institutional constraints, I set the transformation shock probability $\varphi = 0.16$, meaning that on average, a FTC experience a transformation shock after about 6 periods (three years).

6.1.5 Death Probability

I set the probability of workers to drop out from the model equal to the probability from which the workers dropped out from the INPS dataset from 2000 till 2010 and that did not return back in the dataset. This happened with probability $\lambda_d = 3\%$. [43]

[42] Note that in my model with endogenous search intensity, this condition does not assure social optimality of the search and matching process. Nevertheless, it is still useful to use this condition in order to compare the results with other papers in the literature.

[43] I am probably slightly overestimating the dropping out probability since some of these workers could come back in the dataset in the following years. However, this rarely occurred in the past.

This probability included all possible motivations for which an employee could exit the labour force, including the decision to become self-employed or inactive.

I assume that when workers are born, they have an initial draw of the production observed by everyone, based on their true ability and setting $x = 0$. In this way, they enter the labour force with a slightly heterogeneous prior distribution.

6.2 Estimated Parameters

The parameters that remain to estimate using a set of the moments are the following: the average ability of the worker μ_α, the variance in the ability of worker σ^2_α, the variance in the match-specific component σ^2_x, the variance in the white noise in the production σ^2_ε, the parameters for searching costs ν, h_e and h_u, the firing costs fc, the vacancy cost κ, the exogenous separation rate λ.

For the estimation, the MCMC procedure starts from an initial guess of the parameters.

For every iteration, I draw a new set of parameters from a normal distribution centred on the parameters last accepted in the iteration process. Then, I estimate a loss function taking the square differences of some realized moments with respect to the targets from the data. The ratio of the value of this loss function to the one of the previous step determines the probability to accept or reject the new draw of parameters.

The process continues until converging to a distribution of parameters.

In this procedure, all parameters are jointly estimated; however, the chosen moments are thought to target specific parameters in the set. Here I describe all the chosen targets.

6.2.1 Maximum wages percentiles

From the INPS dataset, I considered the period from 2005 until 2015. I considered a worker actually employed in a firm in the 6 months window if he worked at least 13 weeks in the period. As a preliminary step, I regressed the income on the following observables: sex, geographical area, profession (6 categories), time fixed effect (semester of the year). I considered the constant plus the residuals of this regression as the wage of the worker for the period.

I performed this preliminary step in order to eliminate some heterogeneity in the data that are strongly influencing the distribution of wages in the sample, but that I am not capturing in the model. The underlying assumption is that the model can be applied within each category.

Then, for every worker, I compute the maximum wage earned in the dataset and I build the distribution of the maximum wages. As moments, I am using the following percentiles: 20%, 40%, 50%, 60%, 80%. Using the maximum wage avoids one issue of the important role of working experience in the ability of the worker, that is absent in the model.[44]

6.2.2 Standard Deviation in wages for the same worker

From the same data, cleaned as described previously, I take as moments the standard deviation in the wages between firms for the same worker. In practice, I estimate the average wage for the same worker-firm pair. Then I compute the standard deviation of this measure among the life-time of the worker. This measure should provide important information for the estimation of the variance in match-specific productivity.

6.2.3 Separation Rates

The same dataset is useful also to compute the separation rates. I called a separation only a labour contract that is followed by a contract in a firm different from the previous one unless the time between the two contracts is shorter than one month (then it is a Job-to-Job transition). I consider a separation even a contract followed more than 1 year later by another contract with the same worker-firm pair. I compute the separation rates for the two types of labour contracts.

[44] I am considering adding a deterministic growth in the worker's ability to capture the main trend. This should also increase the incentive for the agents to provide employment protection in order to reduce the search intensity.

6.2.4 Job-to-Job transition rates

I have already shown some data about the job-to-job transition rates from the Veneto Lavoro dataset. Here, I compute the job-to-job transition rates inside from the INPS dataset, in order to keep consistency with the other targeted moments. I define a job-to-job transition a new hire that happens less than 1 month after the termination of the previous job. I computed the rates for both types of labour contracts.

6.2.5 Job Finding Rate for Unemployed

I compute the job-finding rate for unemployed in 2013 starting from a subsample of the workers in the INPS dataset that satisfied the following conditions: (i) individuals that worked at some point between 2000 and 2012, (ii) they were not employed in the second semester of 2012 (iii) they worked at a certain point from 2013 to 2016.[45]. Then, I compute the share of these workers that found a job in the first semester of 2013 and I compute the job-finding probability of the first semester. I repeated the same measurement for the second semester, with the corrected conditions (i), (ii) and (iii).

6.2.6 FTC share

I compute the FTC in the INPS dataset in order to have a useful moment for the determination of the firing costs.

6.2.7 FTC transformation rate

From the INPS dataset, I compute the share of FTC that becomes OEC in the next period between the same firm-worker pair.

[45]Therefore, I am not including the workers who are not returning back into the dataset after 2012, since I am classifying them as out of labour force. For this reason, this job-finding probability is probably an upper bound, since some of these individuals were unemployed failing in finding a job.

Name	Parameter	Median	90 confidence interval
Vacancy Cost	κ	0.0558	0.0365–0.1004
Firing Costs	fc	1.189	0.5962–1.759
Average Workers Productivity	μ_α	1.893	1.607– 2.179
Workers Productivity SD	σ_α	1.499	1.017–2.005
Match Productivity SD	σ_x	0.6862	0.5009–0.8780
Production Shock SD	σ_ε	2.742	1.9506–4.109
Searching cost Employed	h_e	4.931	4.513–8.4606
Searching cost Unemployed	h_u	0.2646	0.0838–0.6177
Searching cost Convexity	ν	2.659	2.219–3.130
Exogenous Separation Probability	λ	0.0221	0.0171–0.0288

Table 1: Estimated Parameters. *Median* indicates the median values taken by the parameters in the simulations of the MCMC estimation. *90 confidence interval* gives the interval in which lies 90 percent of the MC simulations.

6.2.8 Unemployment rate

I take the unemployment rate from the ISTAT dataset in 2013.

In the following tables, I reported the estimated parameters and the ability of the model to target the chosen moments. I chose the parameters that minimize the loss function and I report their 90-per cent confidence interval.

Moments	Targets	Model
Unemployment rate	8.65%	13.5%
FTC share	12.9%	12.2%
Separation Rate FTC	9.1%	8.27%
Separation Rate OEC	2.8%	2.0%
Transformation Rate (FTC-OEC)	10.5%	14.4%
Job-to-Job Rate FTC	3.0%	3.4%
Job-to-Job Rate OEC	1.0%	0.81%
Max Wage 20th percentile	1.954	2.423
Max Wage 40th percentile	2.889	3.032
Max Wage 50th percentile	3.04	3.338
Max Wage 60th percentile	3.613	3.649
Max Wage 80th percentile	4.652	4.413
SD Wage across Firms	0.5463	0.3605
Job finding Rate of Unemployed	0.265	0.268

Table 2: Targeted Moments and Model Simulated Moments. Parameters set at the median level in the MCMC estimation

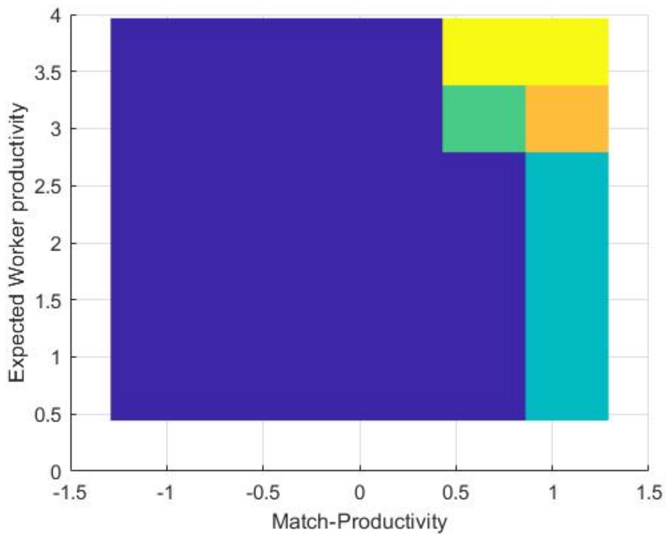


Figure 12: Contractual decision by match-specific productivity draw and expected worker ability. In yellow the agents choose an OEC, in blue a FTC. Other colors are mixed areas due to interpolations between grid points.

6.3 Results

First of all, I describe the optimal contract for new workers just entering the labour market. In figure 13, we can see the choice of the contract for workers with no experience, varying their expected productivity and match-specific draw. Along the horizontal axes, I vary the match-specific productivity, while the expected ability is on the vertical axes. It can be noted that low-ability workers and the low-productivity match draws are the ones receiving the FTC, while the opposite is true for high-ability and high-match draws. Different colours are present because the grid-points are not enough to find the exact discontinuity, where the optimal contract switch from FTC to OEC. To roughly determines what kind of contract is present in a determined region, I rely on linear interpolation.

The results are in line with the observed fact that OEC is correlated with high wages, while FTCs dominate all low-wages regions.

Similarly, I plot the preferred contract for a fixed expected worker ability ($\alpha = 3.158$) and for a fixed match-specific productivity (x=0.28), varying the experience and the match-specific productivity and worker ability respectively.

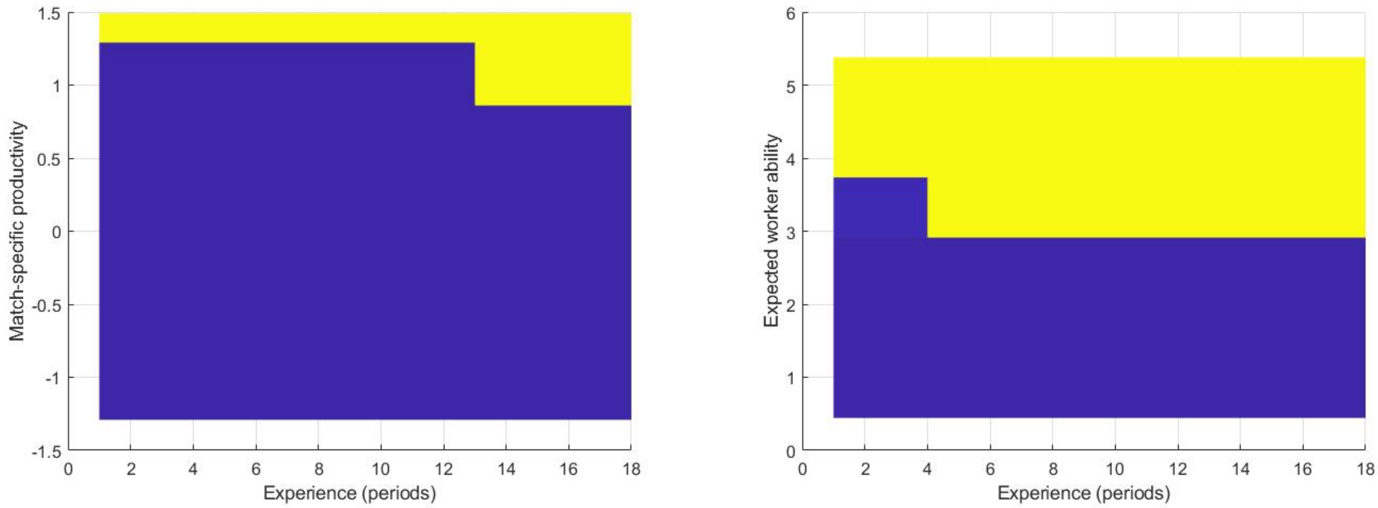


Figure 13: Contractual decision by experience and match-specific productivity (left) or expected worker ability (right). In yellow the agents choose an OEC, in blue a FTC. Other colors are mixed areas due to interpolations between grid points. On the left the match-specific productivity is fixed at 0.28, while on the right the expected worker ability is fixed at 3.158.

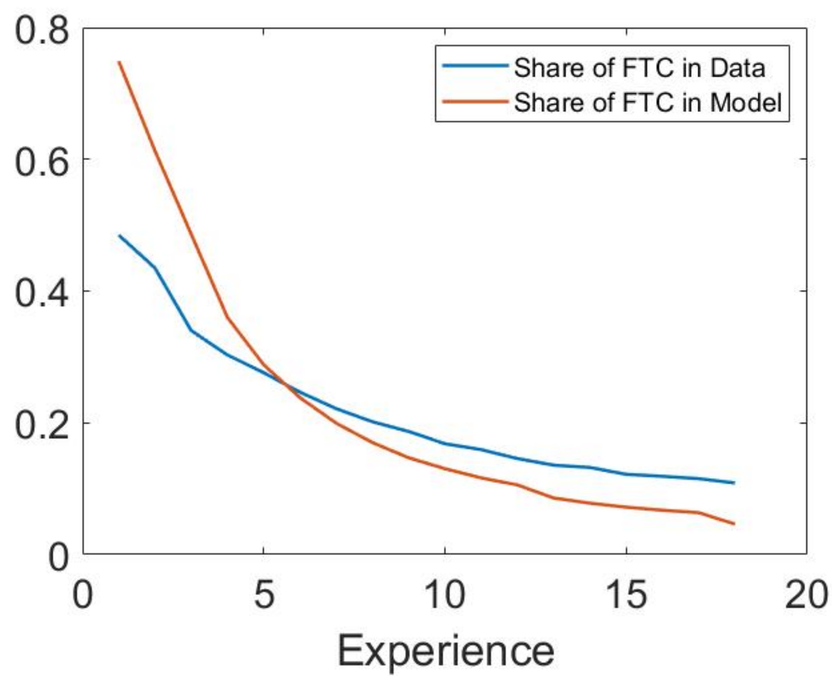


Figure 14: Share of FTC employees among workers with different working experience. Data from INPS Dataset, 2013.

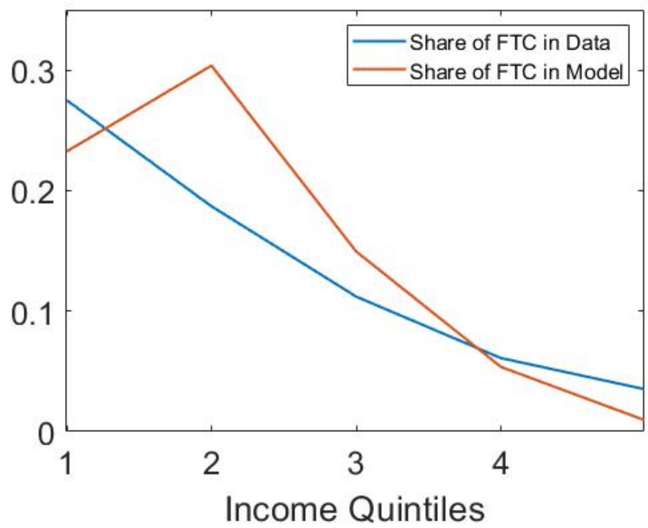


Figure 15: Share of FTC among Wage Quintiles

In figure 14, instead, I report the share of FTC among workers with different working experience. My model can reproduce the observed increase in the share of OEC among more experienced workers.

If we look at the share of FTC per income quintile (figure 15), we see that the share is decreasing in the income, with an exception: the rise in the share in the second quintile. The raise is explained by the large use of FTC when the worker enters the labour market for the first time. Since his ability at the peak of uncertainty, his expected productivity is close to the average of the entire population. This raises the share of FTCs in that section of the wage distribution, given that in my model there is no direct impact of training or learning-by-doing and therefore new-born workers are in expectations as productive as the rest of workers. In the real world, it is reasonable to assume that young workers start with lower productivity that would translate in lower initial wages.

Until now, we have just looked at the cross-section description, but it is interesting to look at some aspects of the working histories of agents. In figure 16, we have the average unemployment duration for workers with 5 years of working experience, according to their expected ability, following them for 10 years after an unemployment episode.

It can be noted that there is a substantial rise in the unemployment duration of low-

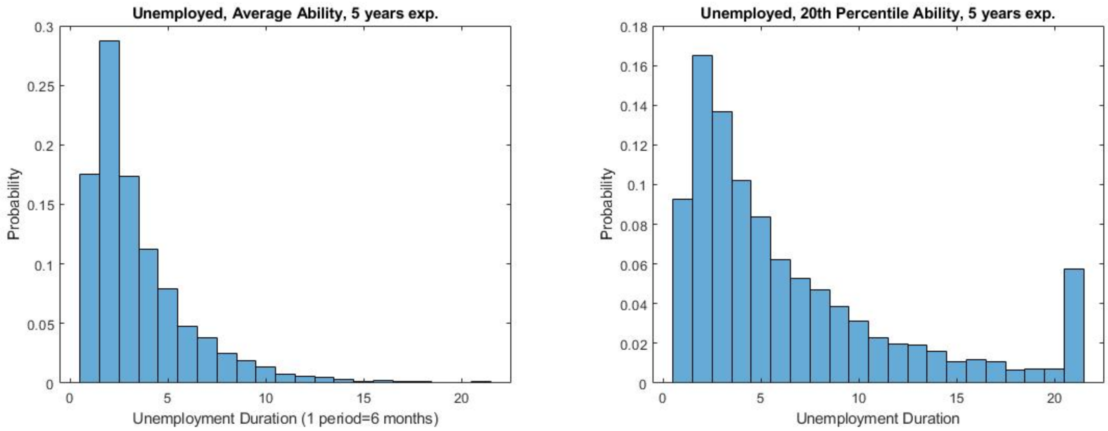


Figure 16: Unemployment Duration for workers with 5 years of Working Experience. Average Expected Productivity on the left and 20th Percentile on the right

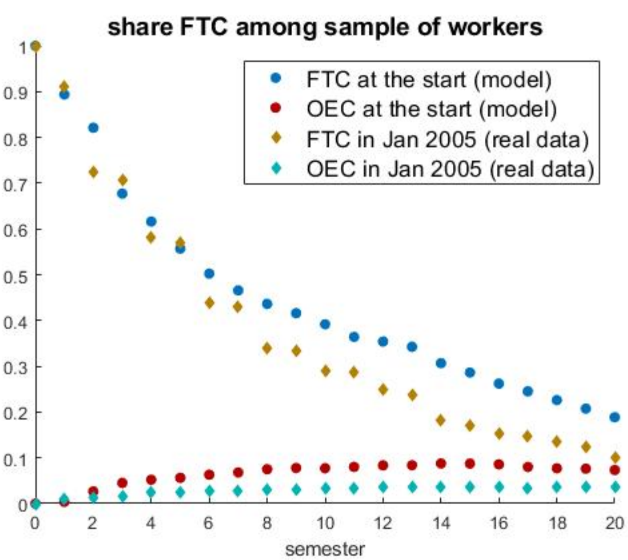


Figure 17: Share of FTC over time among newly employed workers. Circles are simulated working histories, while diamonds are real data.

ability workers. Moreover, in figure 17, I show how the model is able to reproduce the third descriptive fact that I previously documented in figure 6, that described the persistence in the type of contract that we observe in the real data.

7 Counterfactual scenarios

In this section, I run some simulation of possible policy interventions that resemble the ones implemented or discussed by governments over the last decades.

7.1 Cut in the firing costs

The first policy change is a cut in the firing costs. A reform that has been generally invoked by international organizations to increase the labour market flexibility, labour mobility and to reduce the duality between FTCs and OECS [46]. This policy was enacted in Spain in 2012 during the sovereign debt crisis when the government imposes a cap on the severance payments. Similar policies were passed in Italy in 2014 (the so-called *Jobs Act*) and more recently in France in 2017. However, in these two cases, the governments mostly tried to reduce the uncertainty associated with firing costs, limiting the role of judges in the decision about unfair dismissals.

I assume that the government implements a large cut of 25% of the firing costs. Then, I let the model reach the new steady-state, and I compare it with the previous steady-state obtained from the calibration. I keep all the other parameters of the model fixed.

7.2 Lump-sum Tax on FTC

The second policy I analyze is a lump-sum tax on all fixed-term contracts, rebated to employed workers. In this way, it could be seen equivalently as a lump-sum subside to open-ended contracts. Similar policies have been enacted by the Italian government in 2014 and

[46] See for example for Economic Co-operation et al. (2013), where they investigate the success of this policy if coupled with an investment in active labour market policies.

Macro Indicator	Initial Steady State	Cut in Firing costs	Change	Tax on FTCs	Change
Unemployment rate	13.5%	13.5%	-0.0%	13.8%	+0.3%
Output	2.746	2.737	-0.35%	2.730	-0.6%
Share of FTCs	12.2%	11.8%	-0.4%	5.1%	-7.1%
Job-find of Unempl.	0.268	0.267	-0.1%	0.261	-0.7%
Average Productivity	3.175	3.165	-0.3%	3.167	-0.2%
Average Wage	3.135	3.123	-0.4%	3.101	-1.1%
Average Welfare	64.86	64.57	-0.5%	64.74	-0.2%
% of unprod. matches	17.9%	12.2%	-5.7%	24.9%	+7.0%

Table 3: Change in main indicators at the steady-state, after a 25% cut in firing costs or a FTC-only lump-sum tax equal to 1% of the average wage. Unproductive matches are defined as matches that are kept only for the presence of firing costs, but that would be terminated otherwise.

subsequently in 2018. In the first case, the government introduces a three-year social security exemption for all newly signed OECs (including transformations from existing FTCs), in 2018 the government increases the social security contribution at every subsequent FTC between the same firm-worker couple. In my exercise, I am not able to reproduce exactly these policies, since they would require to insert the tenure in the model. However, I am able to simulate the effect of a permanent increase in taxation on one type of contract.

I assume that at every period, the government charges all FTC of a lump-sum tax equivalent to 1% of the average wage. The collected sum is then rebated back in equal shares to all workers.

7.3 Counterfactual results and discussion

In the following table, I report the change in the most important economic and labour market indicators following the two policy interventions.

Both the two policy changes lead to a decrease in the share of FTCs since they reduce the upper productivity thresholds above which an OEC is preferred. However, while the tax on FTC has considerable effects in shifting the choice of agents towards the OEC, the cut in the firing cost has a much weaker effect. This is due to the change in the bottom productivity thresholds for firms, that are now accepting both workers with lower ability and lower match-specific draws. These low-productive workers are generally employed with FTCs, therefore while the cut in the firing costs lower the threshold for which an OEC is optimal, it contemporaneously allows for more job-creation of low-quality matches with FTCs. The opposite is true with a FTC-tax. This tax raises the lower threshold for the job-creation since firms have to sustain the tax for FTCs. The region where a FTC is optimal is therefore squeezed from both sides, leading to a considerable reduction in the share of FTCs.

It can be noted that both policies have negative effects on production and average welfare, but similar aggregate results hide strongly different effects on different types of workers. In Appendix F, I collect the figures describing the change in the welfare of different groups of workers. Here, I described the most important effects of these policies.

The firing costs reduction reduces the wages of all workers with an OEC, by raising the firm outside option, so the reform reduces the welfare of all workers already protected by the OEC. Moreover, even considering the symmetric gains of firms, the policy induces a net loss in the surplus for matches that seek employment protection to reduce the excessive worker on-the-job searching intensity.[47] These considerations particularly harms high-ability workers and low-ability workers in a highly productive match with OECs, as highlighted in figure 28. After the reform, there is a reduction in the share of inefficient matches, that are matches with a negative surplus, kept only because of the firing costs. Before the reform, they were around 15% of all matches, while this share is reduced to 10% after the reform. However, this cleansing effect is not enough to compensate the reduction in the average productivity, due

[47] A limit of this counterfactual is the assumption that agents cannot agree to stipulate a private agreement regarding severance payments. It is rarely observed in Italy but could become a possibility if indeed firing costs were to be consistently lowered. This would also allow a comparison with a *laissez-fair* scenario, resembling the US labour market. This interesting possibility is left for future research.

to a lower upfront selectivity of firms. The endogeneity in the contractual choice plays an important role in reducing the magnitude of this cleansing effect. Indeed, the fact that OECs are used for high-quality matches, both in terms of worker-specific ability and match-specific productivity, reduces both the share and the negative burden of these unproductive matches, compared to a scenario in which the contract was exogenously assigned. The policy change has mixed effects on unemployed and workers with FTCs (figures 26 and 27). In these cases, workers with medium-low ability benefit from firms being less selective and having a higher chance of obtaining a longer employment spell. High-productive workers instead pay the cost of a lower surplus, without particular gains. Another remark is that the policy does not considerably increase the job-finding rate of unemployed. This is a consequence of the crowding-out effect of employed workers, that increase the on-the-job search intensity. Before the reform, unemployed constituted 65% of the aggregate search-intensity, then this share decreases to 60%, compensated by employed workers.

The effects of FTC-tax are different. They predominantly hurt low-ability workers, by raising the minimum productivity draw to be accepted with a FTC, leading to longer unemployment spells. Instead, the high ability workers benefit from the tax-rebate and from the higher probability of receiving a permanent contract, even in a context of higher unemployment rate and lower average productivity. Firms use fewer FTCs and tend to transform them earlier, reducing the role of FTCs as a screening device for workers. As a consequence, the share of inefficient matches increases considerably from 15% to 20% of the total, leading to a reduction of the average productivity, even in a context in which firms are more selective at the recruitment stage. It is useful to compare the obtained welfare changes depicted in figure 29, 30 and 31, with the same welfare changes in the case we were to assume that the agents could not choose the type of contract and the only possibility to obtain the OEC were through the exogenous "transformation" shock. This case is illustrated in figures from 32 to 37. It is apparent that the welfare implications of the policies are largely modified by the absence of an endogenous choice. The intuition is the following: the endogenous choice of the contract creates a persistence in the type of contract a worker receives, since it is linked to his expected ability. For this reason, a policy that reduces that for example reduces the firing costs harm predominantly workers that are expecting to be employed with this type of contracts, that are high-ability workers. Even more clear is the fact that a FTC-tax will be paid exclusively by workers that are expecting to experience some periods of temporary contracts. The correlation between the contract type and the worker ability determines the winner and loser in welfare terms of these policies.

8 Further Research and Conclusion

In conclusion, in this work, I developed a theoretical model that is able to explain the choice about the juridical form of a labour contract and in particular that could justify the choice in favour of employment protection. A temporary contract has the advantage that it saves the firing costs in case the overall match-productivity is too low and does not force the firm to keep unproductive matches. However, in the presence of on-the-job search, there is an incentive for the agents to choose for some employment protection. This is due to an excess of on-the-job search performed by the worker from a joint firm-worker perspective since, in his optimal effort choice, he does not take into account the welfare of the firm. The employment protection can credibly shift some utility of the worker from the present to the future and this reduces the searching incentive, increasing the joint surplus. Descriptive evidence confirms the fact that workers with a FTC indeed are more likely searching on-the-job and they perform more job-to-job transitions.

Adding in the model the fact that the ability of workers is discovered over time, the calibrated model reproduces the observation that FTCs are used mainly by young workers, that they correlate with low-wages and that they are persistent, meaning that workers with a FTC tend to be employed with a similar contract in the future.

This reveals the importance of taking into account the endogenous nature of the choice of the contract, that leads to differences in the impact of labour market reforms on different workers. Popular policies to limit the spread of fixed-term contracts, such as a cut in firing costs and a tax on FTCs, can have a different impact among workers. In particular, a reduction in the firing costs hit workers with an OECs and high-ability workers, that experience a reduction in wages and they react by increasing their on-the-job search, crowding-out unemployed workers. Low-ability workers among unemployed and temporary workers can benefit from such a policy. On the contrary, a tax on FTC hurts, particularly low-ability workers, while it increases the welfare of high-ability wor-kers in good matches. Overall, a tax on FTC has a considerable negative impact on the average productivity, by forcing firms to limit to the role of FTCs as a screening device for unproductive matches.

In 2015 the government of Italy decided to modify the firing rules for new OECs, introducing a fixed severance payment to be paid from the firm to the worker, proportional to worker's number of years in the firm and wage. For future research, it would be possible to compare the realized effect of this reform with the predictions of the model, evaluating the welfare effects on different categories of workers.

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Appendix A. Datasets

8.1 INPS Dataset

This dataset contains the annual records of a sample (based on workers) of Italian labour contracts from 1985 until 2016. It is based on the mandatory declarations of employers for social contributions. The dataset has a total of more than 35 millions entries. However, an institutional framework for FTCs was introduced in Italy only in 1997. For this reason, in this paper, I generally use only information from 2000 onward.

On top of the year, the worker and firm identifiers, the dataset give the following information about the labour contracts: type of contact (OEC, FTC, seasonal), annual wage, profession (6 categories), Time (full or part-time), number of days paid, starting and ending date (if in the year), the reason for separation. Concerning workers, the dataset gives information about gender, age, region of residence. About firms, the dataset provides sector (100 sectors) and dimensional class (14 classes).

Mercurio from Veneto Lavoro

This dataset of working histories is collected by the regional agency in Veneto, one region of the North-East of Italy.

The Institution collects all the mandatory communications of working relationships in Veneto. The panel can reconstruct all the working histories of the inhabitants after 2000, even if they migrate outside the region, as long as workers are in Italy and they declare their change of living place.

The dataset gives access to many characteristics of the labour contracts, of workers and firms: age, sex, education, living place, occupation, firm sector. Importantly, it provides information about the starting date, type, duration of the hiring contracts, transformation and also job-destruction exact date and motivations.

However, the dataset has two main drawbacks: it does not provide the wage of the contract and it can miss the share of employed workers that started their job before the dataset was created and they never quit or had a change in their labour contract. For this reason, for this paper, I relied mostly on data from the previously described INPS dataset, using this rich dataset only when other sources are missing.

In terms of numbers, the dataset covers the working outcomes of more than 3 million workers, 8 hundred employer and more than 15 millions of job relationships. The dataset starts before 2000 and is updated until 2018. From this large dataset, I extracted a random sub-sample based on the day of birth of the worker.

Rilevazione Continua delle Forze di Lavoro

This dataset is the quarterly Labour Force Survey organized by the Italian Statistical Office. The survey uses a questionnaire standardized at European level. Annually, approximately 250 thousand households are interviewed, selected as representative of the Italian population. Every household is interviewed 4 times in 15 months: in two consecutive quarters and then for the other 2 quarters after a break of 3 months.

I use this dataset in this paper for the descriptive evidence section and the moments targeted in the calibration. I use the 2013 data in order to avoid the important labour market reforms that the Italian government started in 2014. However, I performed some robustness checks using the years from 2010 to 2014, and the results do not change significantly.

Restricting the data to the year 2013, I have 611 255 observations across 4 quarters, of which 410 750 in the working-age 15-65. As shown in figure 18, among people 15-65 years old, workers are 49.7% per cent. Among these, 24.9% are self-employed. Therefore most of the data used in this paper come from the remaining 153 417 employed workers.

Figure 18: Source: ISTAT RCFL, 2013. Employment Condition among 15-65 years old.

Appendix B. Descriptive Statistics

In figure 19, I report the fraction of the labour force employed with a FTC in the main European countries in 2017. It is apparent that FTCs are now widely used in all continental Europe and if we restrict the attention to people under30, we can see that in Italy and Spain (but also in others like Portugal and Poland)[48] more than 40% of the young labour force is employed with this kind of contracts.

[48]Source Eurostat, 2017

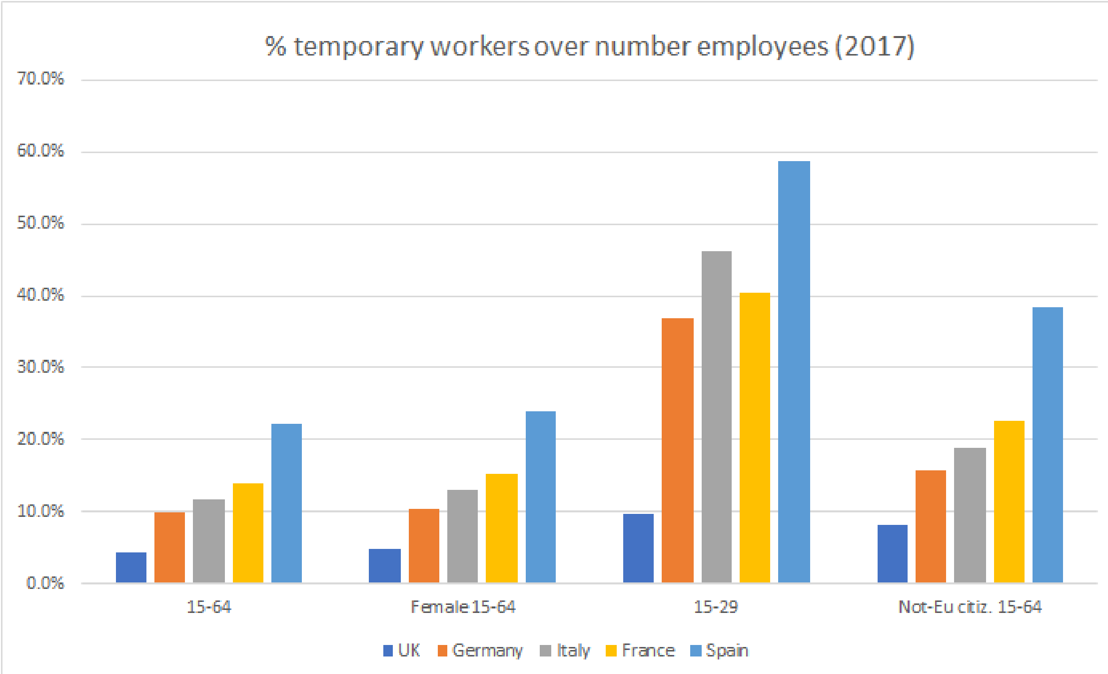

Figure 19: Share of labor force using FTC

Focusing now on Italy, the following tables give some descriptive statistics about FTC diffusion in different sectors, professions and Italian geographical areas. Then I perform a probit estimation of having a FTC contract on all possible observables.

	Managerial and lawmaking	Professional and scientific activities	Highly Technical activity	Administrative
Agriculture	0%	5.0%	11.3%	9.3%
Industry	0.8%	5.7%	5.5%	7.2%
Constructions	2.8%	8.6%	8.9%	8.5%
Commerce	6.7%	8.0%	5.6%	8.8%
Hotels & Restaurants	12.8%	23.8%	18.9%	34.6%
Stock & Transports	3.2%	4.1%	6.7%	5.1%
Inform. & Commun.	3.3%	8.3%	10.28%	9.8%
Finance & Insurance	0%	1.2%	3.3%	5.3%
Real est., firm services	2.7%	16.2%	13.3%	15.0%
Public Administration	13.9%	4.8%	4.1%	6.9%
Education & Health	4.3%	15.3%	7.0%	8.8%
Other services	10.9%	12.8%	20.7%	14.1%
OVERALL	5.7%	12.8%	7.3%	9.2%

	Skilled activity in services	Skilled activity in industries	Transport	Unskilled activity	Army	OVERALL
Agriculture	34.0%	41.0%	46.9%	71.2%		58.6%
Industry	7.9%	10.3%	9.6%	14.4%		8.9%
Constructions	27.3%	16.2%	10.3%	22.2%		14.9%
Commerce	16.1%	13.8%	15.9%	19.4%		13.5%
Hotels & Restaurants	32.6%	25.3%	25.0%	35.8%		32.4%
Stock & Transports	18.8%	10.4%	10.1%	17.2%		9.0%
Inform. & Commun.	17.7%	4.3%	7.1%	15.8%		9.4%
Finance & Insurance	12.5%	0%	0%	0%		3.7%
Real est., firm services	19.2%	24.2%	23.3%	15.3%		15.6%
Public Administration	3.6%	13.9%	13.8%	19.7%	5.1%	6.2%
Education & Health	13.2%	12.6%	13.7%	11.5%		12.0%
Other services	17.2%	19.8%	22.6%	7.1%		13.2%
OVERALL	18.7%	14.4%	11.0%	21.4%	5.1%	13.3%

Table 4: Percentage of FTCs over the total in different sectors and occupations in Italy, 2013

	North-West	North-East	Centre	South	Islands
Agriculture	34.5%	47.0%	48.9%	70.4%	64.2%
Industry	7.2%	9.5%	10.8%	9.4%	10.6%
Constructions	13.9%	12.4%	12.8%	18.6%	19.6%
Commerce	11.7%	14.9%	14.2%	14.2%	12.3%
Hotels & Restaurants	27.4%	38.7%	29.4%	33.9%	31.6%
Stock & Transports	8.0%	10.0%	7.5%	10.7%	9.4%
Inform. & Commun.	9.5%	9.5%	10.2%	7.7%	8.2%
Finance & Insurance	3.1%	5.1%	3.7%	2.9%	2.9%
Real est., firm services	13.7%	16.5%	15.0%	18.9%	16.2%
Public Administration	3.7%	4.8%	2.8%	6.5%	14.6%
Education & Healthcare	11.2%	13.5%	12.2%	11.5%	12.0%
Other services	9.9%	15.1%	11.4%	18.6%	16.1%
OVERALL	10.5%	13.6%	12.6%	16.3%	17.0%

Table 5: Caption

Margins of Probit Estimation: probability of having a FTC

Predictive margins of probit Estimation: probability of performing on-the-job search

Variables	FTC
Age 15-19	0.525***
	(0.0189)
Age 20-24	0.397***
	(0.00609)
Age 25-29	0.253***
	(0.00402)
Age 30-34	0.157***
	(0.00277)
Age 35-39	0.123***
	(0.00214)
Age 40-44	0.105***
	(0.00186)
Age 45-49	0.0926***
	(0.00172)
Age 50-54	0.0787***
	(0.00172)
Age 55-59	0.0619***
	(0.00175)
Age 60-64	0.0639***
	(0.00284)
Age 65-69	0.0827***
	(0.00820)
Male	0.128***
	(0.00122)
Female	0.134***
	(0.00135)
Italian	0.134***
	(0.000872)
Foreigner-European	0.127***
	(0.00380)
Foreigner-Non European	0.104***
	(0.00251)
Primary School or lower	0.152***
	(0.00412)
Middle School	0.122***
	(0.00132)
High-School	0.128***
	(0.00125)
Degree or more	0.164***
	(0.00323)
Full time	0.129***
	(0.000990)
Part time	0.135***
	(0.00193)

Variable	FTC
1st income decile	0.251***
	(0.00428)
2nd income decile	0.204***
	(0.00347)
3rdincome decile	0.175***
	(0.00276)
4th income decile	0.145***
	(0.00287)
5th income decile	0.127***
	(0.00246)
6th income decile	0.105***
	(0.00245)
7th income decile	0.0853***
	(0.00225)
8th income decile	0.0676***
	(0.00204)
9th income decile	0.0545***
	(0.00198)
10th income decile	0.0431***
	(0.00213)
<10 employees	0.127***
	(0.00140)
10-15 employees	0.137***
	(0.00231)
16-19 employees	0.143***
	(0.00327)
20-49 employees	0.141***
	(0.00198)
50-249 employees	0.130***
	(0.00190)
250+ employees	0.116***
	(0.00258)
North-West	0.121***
	(0.00143)
North-East	0.145***
	(0.00165)
Centre	0.127***
	(0.00175)
South	0.129***
	(0.00179)
Islands	0.136***
	(0.00251)
Sectors (2 digits) dummies	YES
Occupations (3 digits) dummies	YES
Observations	146,475

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 6: Source: ISTAT RCFL, 2013. Probit estimation of having a FTC on observables.

Variables	On-the-Job search
OEC	0.06696***
	(0.00476)
FTC	0.11828 ***
	(0.006893)
Male	0.09212 ***
	(0.00575)
Female	0.06434 ***
	(0.00470)
Full time	0.0710 ***
	(0.00502)
Part time	0.10317 ***
	(0.00609)
Educational level dummies	Yes
Firm class size	Yes
Italian dummy	Yes
Sector dummies (2 digits)	Yes
Occupation dummies (3 digits)	Yes
Observations	143,328

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7: Source: ISTAT RCFL, 2013. Probit estimation, predictive margins of performing on-the-job search. Age and income at their means.

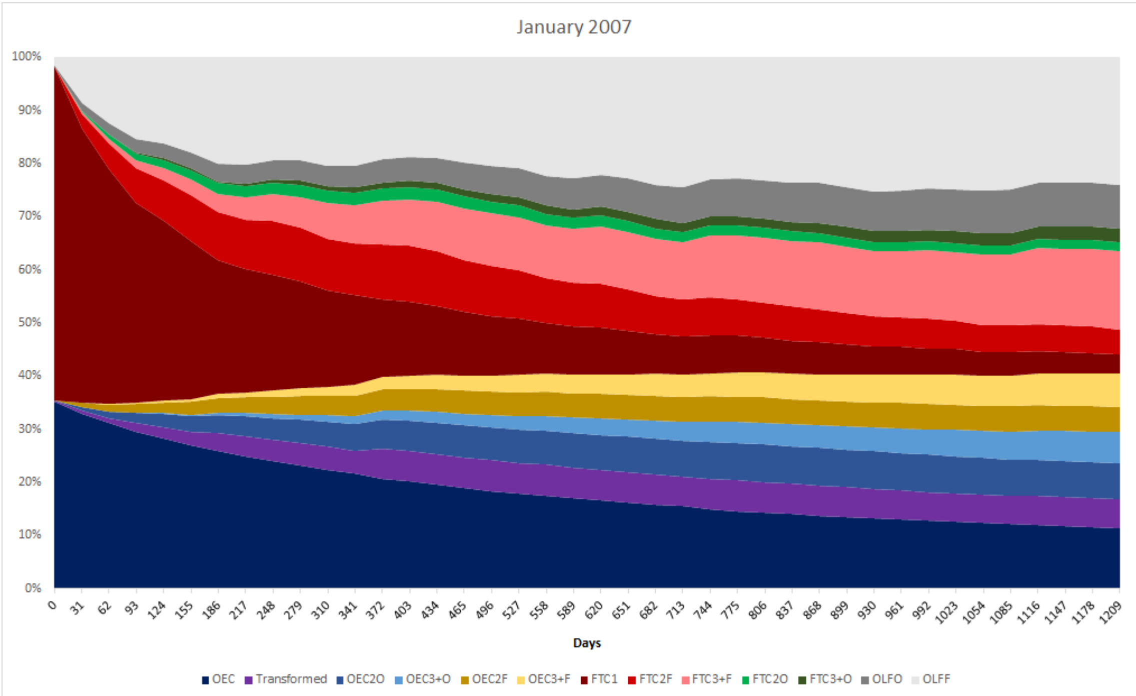

Figure 20: Workers histories, January 2007. Source: Veneto Lavoro

Veneto Lavoro: "Mercurio"

Figures 20 and 21 represent workers' job histories of a random sample of employees (based on the date of birth) hired in Veneto in January 2007 and 2013. I excluded all workers that were employed for less than 120 days in the time-span considered (1209 days). In addition, I excluded workers that spent more than half of their working days in the following sectors: agriculture, tourism and domestic. Indeed, workers in these sectors are subject to special regulations and they are mostly influenced by seasonality.

The colours indicate different labour contacts and different employers. In particular the dark blue and red are the workers that keep a job in the same firm respectively with an OEC and a FTC. Therefore the graph indicate that in 2007 around 65% of the hired workers sign a FTC, while around 35% sign an OEC. Half of this 35% will leave the firm in the following 1200 days, but the percentage of "leavers" is much higher for FTC.

The purple area is composed of workers that had their FTC transformed into a OEC in the same firm. They are a considerable but not extremely high percentage of the labour

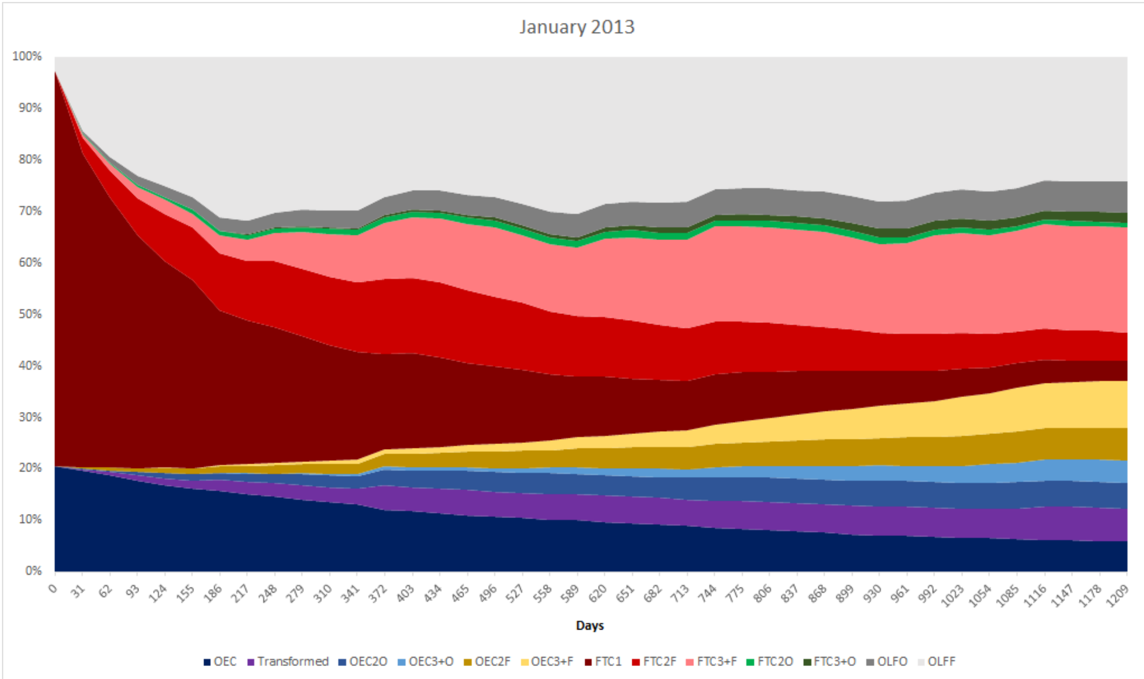

Figure 21: Workers histories, January 2013. Source: Veneto Lavoro

force.

The lighter blue and red are the workers that move to another firm (intermediate colour) or a third or more (the lightest colour). An important share of workers performs these transitions, keeping the same type of contract from one firm to another.

The yellow area instead represents workers moving from an FTC to an OEC but in a second (intermediate colour) or third or more (the lightest colour) firm. It is interesting to notice that an important share of workers can arrive at an OEC from an initial FTC.

Green areas instead indicate the opposite case: workers that leave an OEC for a FTC in another firm. The percentage of workers in this situation is quite small, especially in 2013.

Finally, the grey area is workers that are not employed any more. I cannot distinguish between unemployed, self-employed or out of the labour force. The lighter area indicates workers whose last contract was an FTC, the opposite for the darker one. It is not surprising that there is a large share of workers that use the FTC just for some occasional jobs.

The comparison between the two years can reveal two different situations in the labour market. In 2007 Veneto was in a situation of full employment, entering the Great Recession, while in 2013 Italy and Veneto hit the bottom of the second recession and a slow recovery started from 2014. In addition, in 2015 fiscal incentive for OEC and the labour market reform took place.

Using the same population of workers, figures 22 and 23 report the percentage of workers performing job-to-job transitions every month. I identify a job-to-job transition by two consecutive job-contracts interrupted by an unemployment spell of at most 30 days.

The labour force that is interested in this job-to-job transition is around 2% of the overall labour force every month, and it is higher for FTCs where it reaches the 5% in the peak 1 year after the beginning of a contract.

It is important to notice that we are considering a selected population of workers since they are the ones starting a new contract in January. It is reasonable to believe that workers decrease the search intensity as the match continues. For this reason, job-to-job transitions are probably less common among the entire workforce. However, from these graphs, it seems that the higher on-the-job search that is correlated with FTC also translate in a higher propensity to perform job-to-job transitions.

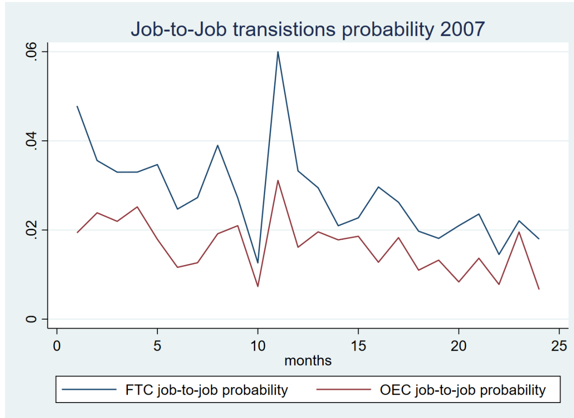

Figure 22: Monthly J2J transitions over Total Employees, for workers starting in January 2007

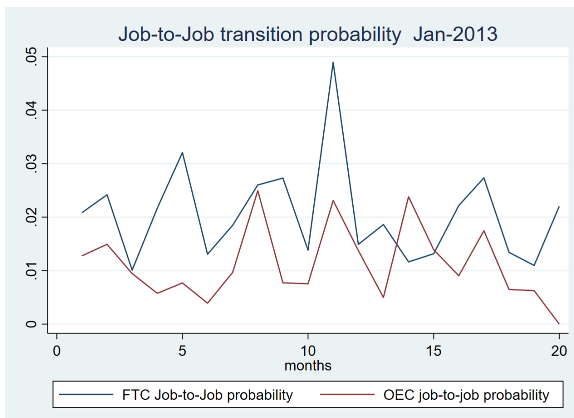

Figure 23: Monthly J2J transitions over Total Employees, for workers starting in January 2013

Going to the other side of the labour market, figures 24 and 25 refer to job positions histories of a sample of firms, opened in January 2007 and 2013. I defined a job position as a sequence of job-contracts in the same firm for the same job-title, signed with the same or ever different workers. Job-titles are divided into 3-digits categories, allowing for quite precise identification.

The sequence of contracts could also partially overlap or with a short break, depending on the duration of the contracts itself. I choose the following “interval ”, that is the number of days between the end of the previous contract and the start of the new one:

$$-63 < interval < max\{15, min\{duration, 360\}\}$$

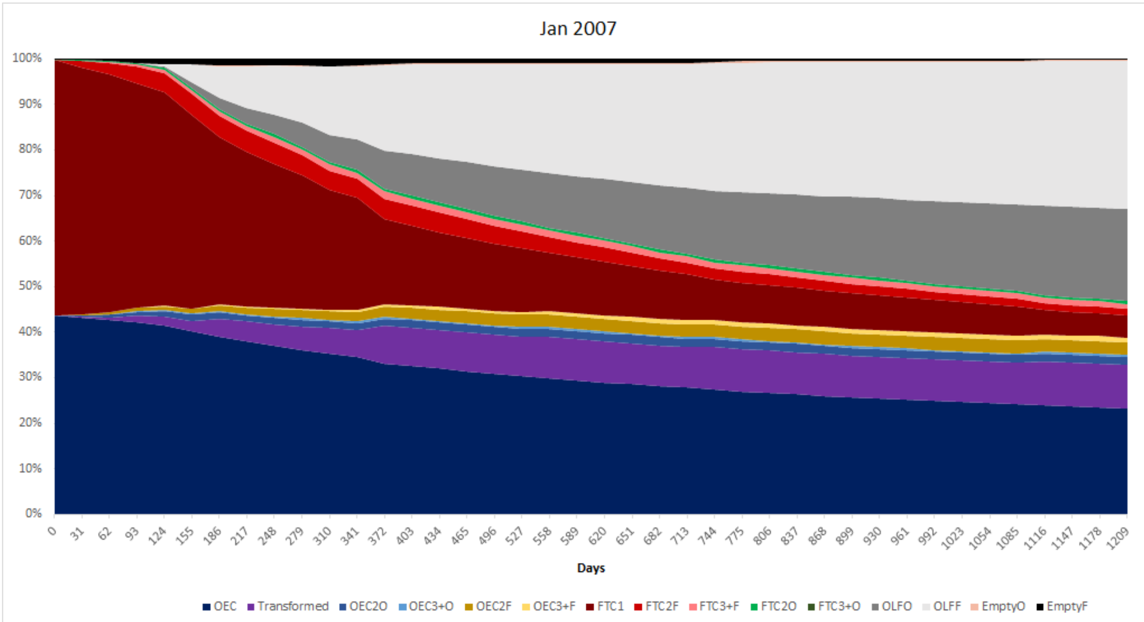

Figure 24: Job positions histories, January 2007. Source: Veneto Lavoro

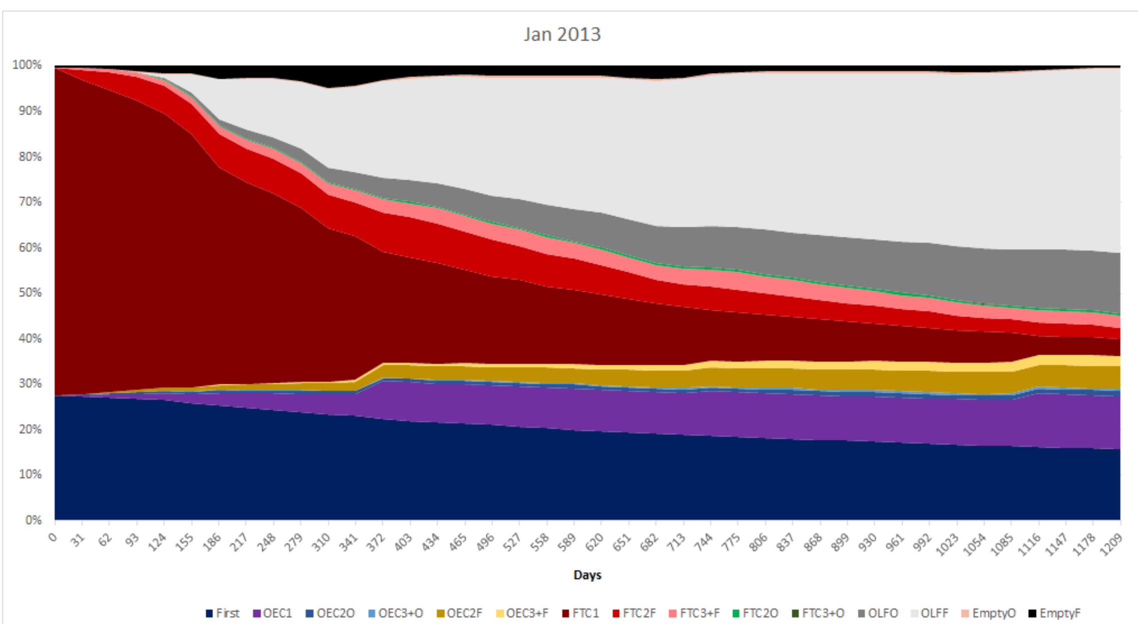

Figure 25: Job positions histories, January 2013. Source: Veneto Lavoro

Where the duration is the length of the previous contract.

The graphs are similar to the previous ones: lighter colours indicate a change in the worker. Red areas indicate FTCs, while the blue and yellow are OECs. Black areas are vacancy waiting to be filled, while grey areas are job-positions destroyed and never re-filled. The graphs show that a large number of job-positions (almost 50%) are expiring within the 1200 days interval. In addition, it can be noticed that a considerable share of job-positions is filled by workers at first “tested” with a FTC. Moreover, job-positions covered with FTC are not usually filled by a sequence of different workers, but they are most likely be covered by an OEC or be destroyed.

APPENDIX C

Toy model with Sequential Auction

In this section, I rewrite the Toy model assuming that the wage is determined not by Nash-bargaining, but by a sequential auction framework as in Postel-Vinay and Robin (2002).

I summarize here the main assumptions of this protocol. Firms have all the bargaining power, and they decide the wage in order to maximize their profit under the participation constraint of the worker. For the moment, I assume wages are fixed at the beginning for the entire duration of the match unless workers receive new offers. In this case, workers use the new offer to make the firms competing à la Bertrand: the new firm make a take-it-or-leave-it offer to the worker, and the old firm can counter the offer-

Formally, I call $W(w, x)$ the value function of a worker in a match of productivity x whose wage is w, similarly for the value function $J(w, x)$.

Suppose the worker is in a match with productivity x and he obtains a new offer from a firm whose productivity is x'. I call $\delta(x, x')$ the minimum wage for which the worker would switch from a match with productivity x to a firm whose productivity is x'.

$$W(\delta(x, x'), x') = W(x, x)$$

Indeed, the old firm can at most offer a wage equal to the match-productivity and the welfare associated with it is the maximum the worker can obtain remaining in the old match.

When a worker obtains a new offer, three different situations can emerge: the workers discard the new offer, the worker remains in the old firm, but he uses the new offer to raise his wage or the worker quits and join the new firm.

The first situation happens if $x > x'$ and $W(w, x) > W(x', x')$ so that even the best offer the new firm can make is no match for the current worker's welfare. The second situation realizes if x is still larger than x', but $W(w, x) < W(x', x')$. In this case, the old firm raises the wage of the worker in order to at least match the best outside offer and keep the worker. Finally, if $x' > x$, then the worker quits and the new firm offers exactly the wage to at least match the best offer of the old firm $\delta(x, x')$[49]

Value Functions

8.1.1 Firms

$$J(w,x) = x - w + \beta(1-\lambda)\left(1 - ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)J(w,x) + \beta(1-\lambda)ps^*\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{J}(x)+$$

$$+\beta(1-\lambda)Q(x)V + \beta\lambda V$$

I called $\hat{x}$ the minimum level of productivity for which the firm has at least to raise the wage of the worker to match the outside offer and I called $\hat{J}$ the average continuation value for the firm in the interval from $\hat{x}$ till x, at which point the worker quits.

$$\hat{J}(x) = \frac{\int_{\hat{x}}^{x} J(\delta(x,x'),x)dx'}{x-\hat{x}}$$

The quitting probability is unchanged with respect to the main model

$$Q(x) = ps^*\left(\frac{\bar{x}-x}{\bar{x}-\underline{x}}\right)$$

Nevertheless, the firm has a generally lower probability of remaining with the same welfare in the subsequent period, because of the possible renegotiation.

It is also important to notice that s^* this time is a function of both x and w.

Using the free-entry condition, I can set directly $V = 0$.

8.1.2 Workers

$$W(w,x) = w - h(s^*)^{\nu} + \beta(1-\lambda)\left(1 - ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)W(w,x) + \beta(1-\lambda)Q(x)\bar{W}^N(x)+$$

[49]This wage can be lower than w, since there are expectations of future wage increases, as shown in details in Postel-Vinay and Robin (2002).

$$+\beta(1-\lambda)ps^*\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{W}(x)+\beta\lambda(1-ps^*)U+\beta\lambda ps^*\bar{W}^N(U)$$

As before, the probability that the workers keeps the same welfare in the subsequent period is reduced by the probability of renegotiation, that happens if the new offer is between $\hat{x}$ and x. I called $\hat{W}$ the average value of this continuation value if the new offer is in that range.

$$\hat{W}(x)=\frac{\int_{\hat{x}}^{x} W(\delta(x,x'),x)dx'}{x-\hat{x}}$$

Then, I called $\bar{W}^N(U)$ the average welfare of a new job coming from unemployment.

8.1.3 Optimal search intensity

As usual, I assume the *right to manage* of the worker regarding the search intensity. Then the marginal benefit of search are:

$$MB=\beta p\left[(1-\lambda)\left(\frac{\bar{x}-x}{\bar{x}-\underline{x}}\right)\left(\bar{W}^N(x)-W(w,x)\right)+\lambda(\bar{W}^N(U)-U)+\right.$$
$$\left.+(1-\lambda)\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\left(\hat{W}(x)-W(w,x)\right)\right]$$

Again, the search intensity is obtained by equating MB with the marginal costs

$$s^*=\left(\frac{MB}{\nu h}\right)^{\frac{1}{\nu-1}}$$

In comparison with equation 6, the marginal benefits have an additional last term in the second line: the worker has an extra-incentive to search in order to increase his continuation value *with the incumbent firm.* This suggests another fact: the firm can actively influence the optimal searching effort by choosing the wage. For this reason, the optimal wage offered by the firm can potentially be higher than the reservation wage of the worker, as I show later.

8.1.4 Optimal joint search intensity

As in the main model, we compute the joint welfare to obtain the jointly optimal searching effort. However, it is important to notice that while in the main model the Nash-bargaining assured that moving from s^* to the optimal joint search intensity is beneficial for both agents, this is not the case in this framework, since the firms can seize the entire surplus. Therefore, moving from s^* to s^J the worker is at most indifferent.

$$JMB = MB - \beta(1-\lambda)p\left[\left(\frac{\bar{x}-x}{\bar{x}-\underline{x}}\right)J(w,x) + \left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\left(J(x)-\hat{J}(w,x)\right)\right]$$

The difference between the joint marginal benefits and the marginal benefits of the worker is now larger because now higher search intensity leads to a higher probability of renegotiation, that is a loss for the firm not internalized by the worker.

From the equation of JMB and the marginal cost we obtain again that

$$s^J \leq s^*$$

Therefore, even in this scenario, the worker performs an excess of on-the-job search compared to what would maximize the joint surplus of the match.

8.1.5 Optimal contract discussion

The optimal contract in this environment is well analyzed in the paper Lentz (2014). What is important in my case is that the perfect solution would be to allow the worker to "buy" his job: his wage would coincide with the production for all the duration of the match in exchange of a payment upfront that would leave him at the reservation utility.

If we restrict the model not to allow negative wages, but we allow the possibility to commit to future wages, the solution exists, it is unique, and it consists of an increasing wage path.

If instead, we force the firm to commit to a flat wage, then we can notice that the firm could potentially decide to raise the wage above the reservation wage, even if this would give a part of the surplus to the worker.

To see this, notice that the maximization problem of the firm would be

$$\max_w J(w,x) \;\; s.t$$

$$w \geq \delta(x,x')$$
$$s^* = \left(\frac{MB}{\nu h}\right)^{\frac{1}{\nu-1}}$$

where x' could be the unemployment benefit if the worker is not employed.

The FOC of $J(w,x)$ with respect to w is a complicated object because it depends on both the derivative of the searching intensity and the change in the probability and expected value of a renegotiation. Indeed, a higher w reduced the search intensity raises the value of $\hat{x}$ and it, therefore, it changes also $\hat{J}(x)$.

I call $\hat{J}'$ the following derivative

$$\hat{J}'(w,x) = \frac{\left(\partial \frac{x-\hat{x}}{\bar{x}-\underline{x}}\hat{J}(x)\right)}{\partial w} < 0$$

This derivative is smaller than zero because a raise in w determines a higher value of $\hat{x}$ and a reduction in the profit flows.

$$FOC = \frac{-\left[1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right] - (x-w)\beta(1-\lambda)p\overbrace{\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\frac{\partial s^*}{\partial w} - \frac{s^*}{\bar{x}-\underline{x}}\frac{\partial \hat{x}}{\partial w}\right]}^{<0}}{\left(1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right)^2} +$$

$$+\beta(1-\lambda)p\left(\frac{\overbrace{\left[\frac{\partial s^*}{\partial w}\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{J}(x) + s^*\frac{\partial \hat{J}(x)}{\partial w}\right]}^{<0}\left(1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right)}{\left(1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right)^2} - \right.$$

$$\left.\frac{\left(s^*\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{J}(x)\right)\beta(1-\lambda)p\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\frac{\partial s^*}{\partial w}-\frac{s^*}{\bar{x}-\underline{x}}\frac{\partial\hat{x}}{\partial w}\right]}{\left(1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right)^2}\right)$$

Overall, the FOC has an ambiguous sign, since by raising the wage the firm loses part of the profit flows, but it increases the expected duration of them, by diminishing the search intensity and the probability of a future re-bargaining.

We can see this more clearly rewriting the numerator as

$$FOC_{num} = -\left(x - w + \beta(1-\lambda)ps^*\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{J}(x)\right)\overbrace{\left[\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\frac{\partial s^*}{\partial w} - \frac{s^*}{\bar{x}-\underline{x}}\frac{\partial\hat{x}}{\partial w}\right]}^{<0}+$$

$$+\overbrace{\left(\beta(1-\lambda)p\left[\frac{\partial s^*}{\partial w}\left(\frac{x-\hat{x}}{\bar{x}-\underline{x}}\right)\hat{J}(x) + s^*\frac{\partial\hat{J}(x)}{\partial w}\right]-1\right)}^{<0}\left[1-\beta(1-\lambda)\left(1-ps^*\left(\frac{\bar{x}-\hat{x}}{\bar{x}-\underline{x}}\right)\right)\right]$$

The first line represents the gains from a higher wage, due to the higher duration, while the second line is the costs. The FOC does not have to hold with equality, since there is a reservation wage for the worker.[50]If at the reservation wage the FOC is negative, then the optimal solution is for the firm to pay the reservation wage and not to try to reduce the search intensity. However, if the FOC holds with equality, the firm pays a higher wage than the reservation wage in an attempt to reduce s^*.

8.1.6 Optimal contract

Finally, relying on the paper of Lentz (2014), we can show that the previous contract with a fixed wage is inefficient and that the firms can improve the joint welfare by offering a contract that is backloaded.

[50]Notice that due to the envelope theorem, it is straightforward to see that the worker always benefits from higher wages.

APPENDIX D. Nash-Bargaining

8.2 Nash-Bargaining without Employment Protection

The wage is set using the standard result of Nash bargaining, where $\gamma \in [0, 1]$ represents the contractual power of the worker.

In the general case without employment protection, the wage maximize the following expression

$$\max_{w} (W - U)^{\gamma} (J - V)^{1-\gamma}$$

This result in the surplus being split according to this rule

$$(1 - \gamma)(W - U) = \gamma(J - V)$$

or in other terms

$$W - U = \gamma S$$

$$J - V = (1 - \gamma)S$$

In the Toy model, all the value functions are dependent on x, therefore even the wage is depending on x. In the full model, in the case of the FTC, the wage depends on both x and the worker prior distribution, characterized by ϕ.

8.3 Nash-Bargaining with Firing Costs as a pure waste

If the two agents are already in a contract with firing costs, the outside option of the firm is to pay the firing costs. Therefore the surplus of the match is

$$S = W - U + J - (V - fc)$$

The wage maximizes

$$\max_{w}(W-U)^{\gamma}(J-V+fc)^{1-\gamma}$$

and it is generally higher than without firing costs, everything else equal.

However, this is not the case when two agents have just met and they are bargaining for a new contract. In this last case, the outside option of the firm is the empty vacancy, since it can terminate the match immediately without any further cost.

To give an example, I show the specific case of the OEC in the full model. We have to separately consider the case in which the two agents have just met (or equivalently, they are transforming a FTC into an OEC) and when they are bargaining after an entire period with an OEC.

In the former case, the wage is the one that maximizes

$$\max_{w_{f,p}(\phi,x)}(W_{f,p}(\phi,x)-U(\phi))^{\gamma}(J_{g,p}(\phi,x)-V)^{1-\gamma}$$

so, the outside option of firm is only the empty vacancy and the resulting wage split the surplus in the usual way:

$$(1-\gamma)(W_{f,p}(\phi,x)-U(\phi))=\gamma(J_{f,p}(\phi,x)-V)$$

In the second case, the outside option becomes the empty vacancy minus the firing costs. Then the wage maximizes

$$\max_{w_{p,p}(\phi,x)}(W_{p,p}(\phi,x)-U(\phi))^{\gamma}(J_{p,p}(\phi,x)-V+fc)^{1-\gamma}$$

and the two shares are

$$J_{p,p}(\phi,x)-V+fc=(1-\gamma)S_{p,p}(\phi,x)=(1-\gamma)(W_{p,p}(\phi,x)-U+J_{p,p}(\phi,x)-V)$$

$$W_{p,p}(\phi, x) - U(\phi) = \gamma S_{p,p}(\phi, x) = \gamma(W_{p,p}(\phi, x) - U + J_{p,p}(\phi, x) - V)$$

Notice that $W_{p,p} + J_{p,p} = W_{f,p} + J_{f,p}$, since the two contracts are identical, but from the initial wage, that is just a transfer between the two agents. From this, we can verify that

$$J_{p,p}(\phi, x) - V + fc = (1 - \gamma)S_{p,p}(\phi, x) = (1 - \gamma)(S_{f,p}(\phi, x) + fc)$$

Therefore

$$J_{p,p}(\phi, x) - V + fc = J_{f,p}(\phi, x) - V + (1 - \gamma)fc$$

$$J_{p,p}(\phi, x) = J_{f,p}(\phi, x) - V - \gamma fc$$

Similarly

$$W_{p,p}(\phi, x) = W_{f,p}(\phi, x) + \gamma fc$$

8.4 Nash-Bargaining with Severance Payment

This case is similar to the previous one, with the difference that also the outside option of the worker is affected by employment protection. Indeed, the firing cost for the firm is transferred to the worker in case of separation. In this case, the wage is determined through this maximization:

$$\max_w (W - U - fc)^\gamma (J - V + fc)^{1-\gamma}$$

and the two shares are

$$J - V + fc = (1 - \gamma)S = (1 - \gamma)(W - U + J - V)$$

$$W - fc - U = \gamma S = \gamma(W - U + J - V)$$

APPENDIX E. Thresholds Proof

8.5 Higher firing threshold for FTC ($\hat{y}_f > \hat{y}_p$)

I start from observing (as shown in Appendix D) that

$$J_{p,p}(\phi', x) = J_{f,p}(\phi', x) - \gamma fc$$

Then if $\max_{i \in \{p,f\}} J_{f,i}(\phi', x) = J_{f,p}(\phi', x)$, it is immediate that at $y = \hat{y}_f$ (implying $J_{f,p}(\phi', x) = 0$), $J_{p,p}(\phi', x) = -\gamma fc > -fc$.

Then, $\hat{y}_p$ must be lower in order to reach the threshold where $J_{p,p}(\phi', x) = -fc$.

If instead $\max_{i \in \{p,f\}} J_{f,i}(\phi', x) = J_{f,f}(\phi', x)$, then consider the highest possible difference between the surplus of the two contracts in favor of FTC. This happens when the contract is going to be terminated by the firm in any case for a too low productivity of the match. In this case the worker is going to choose the exact same amount of search intensity, since in both cases he is going to be fired. The only difference is the presence of the firing costs. This costs are going to be paid with probability $(1 - \lambda)$. Therefore, the maximum difference between the two contracts can be $S_{f,f}(\phi', x) - S_{f,p}(\phi', x) \leq \beta(1 - \lambda)fc < fc$. From this we can claim that $J_{f,f}(\phi', x) - J_{f,p}(\phi', x) \leq (1 - \gamma)\beta(1 - \lambda)fc$. Then we can reach the conclusion that even if $J_{f,f}(\phi', x)$ can be higher than $J_{f,p}(\phi', x)$, at most it can be

$$J_{f,f}(\phi', x) = J_{f,p}(\phi', x) + (1 - \gamma)\beta(1 - \lambda)fc$$

but then we can finally arrive at

$$J_{f,f}(\phi', x) - J_{p,p}(\phi', x) = (1 - \gamma)\beta(1 - \lambda)fc + \gamma fc < fc$$

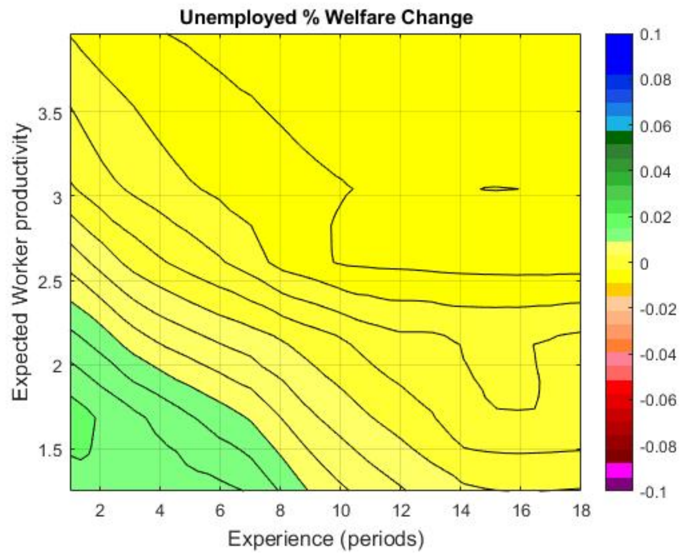

Figure 26: Change in welfare of unemployed workers at the steady state with a cut of 25% of firing costs.

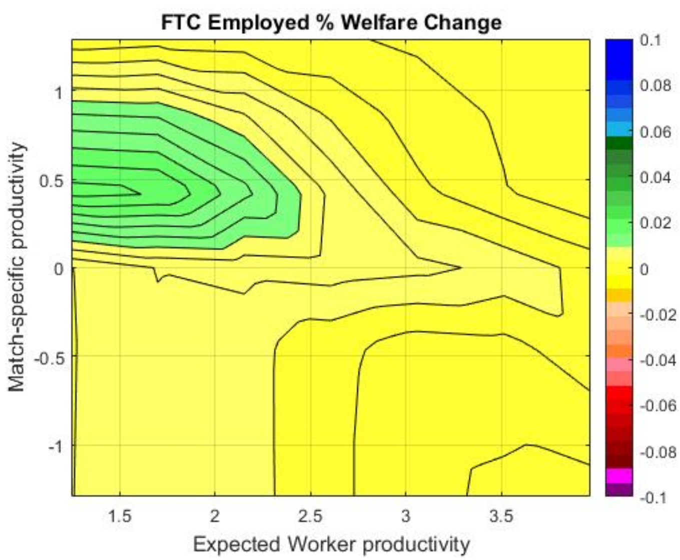

Figure 27: Change in welfare of FTC employed workers at the steady state with a cut of 25% of firing costs.

APPENDIX F. Welfare Comparisons

The first three graphs report the changes in welfare after a **cut in firing costs** for workers, depending on their employment status. The second three graphs report the same changes after a **FTC-tax**.

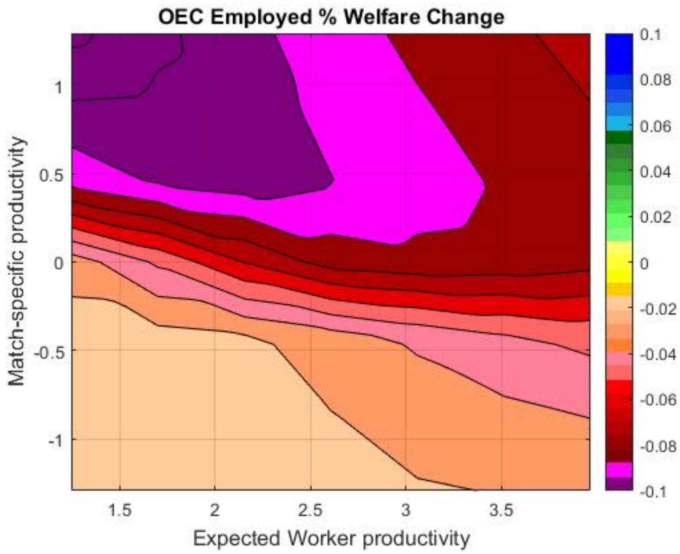

Figure 28: Change in welfare of OEC employed workers at the steady state with a cut of 25% of firing costs.

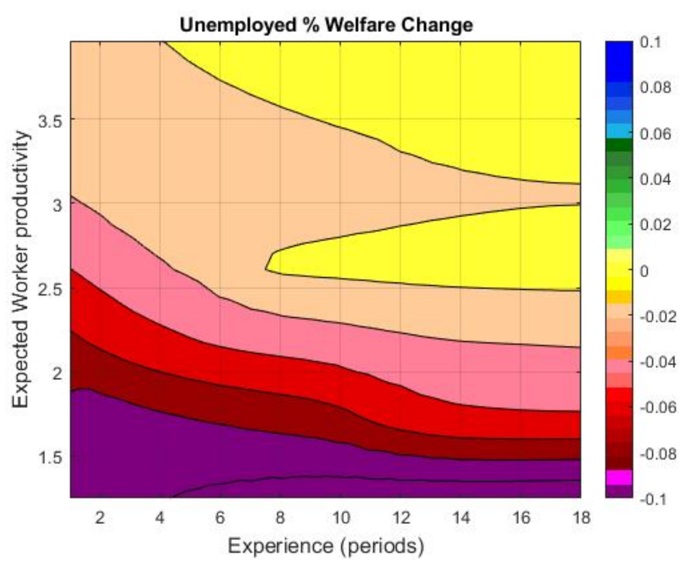

Figure 29: Change in welfare of unemployed workers at the steady state with a lump-sum on FTCs equal 1% of average wage.

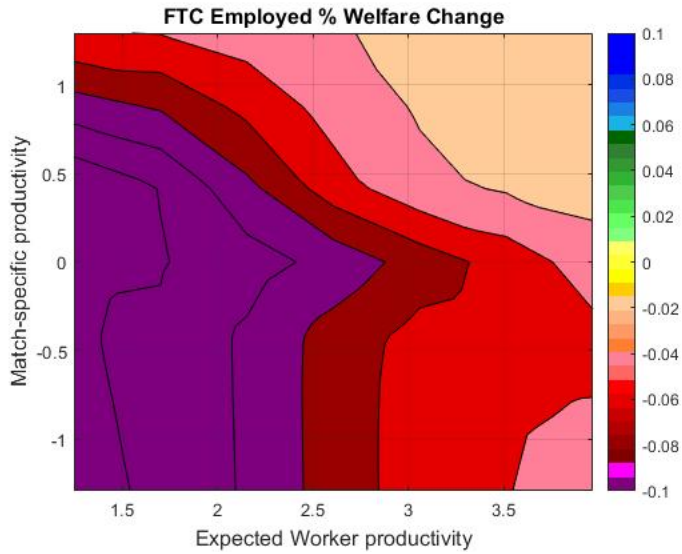

Figure 30: Change in welfare of FTC employed workers at the steady state with a lump-sum on FTCs equal 1% of average wage.

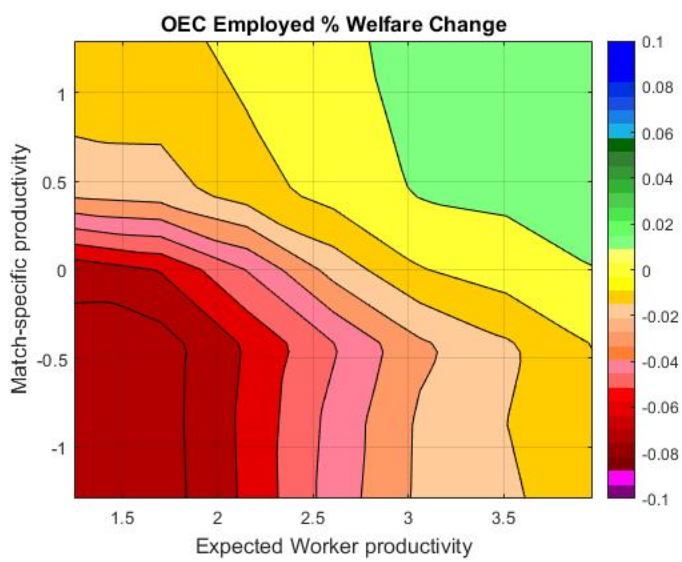

Figure 31: Change in welfare of OEC employed workers at the steady state with a lump-sum on FTCs equal 1% of average wage.

Macro Indicator	Initial Steady State	Cut in Firing costs	Change	Tax on FTCs	Change
Unemployment rate	13.2%	13.0%	-0.2%	13.3%	+0.1%
Output	2.741	2.739	-0.0%	2.730	-0.2%
Share of FTCs	14.5%	14.4%	-0.1%	14.2%	-0.3%
Job-find. of Unempl.	0.263	0.266	+0.3%	0.262	-0.1%
Average Productivity	3.160	3.150	-0.3%	3.155	-0.2%
Average Wage	3.122	3.111	-0.4%	3.101	-0.7%
Average Welfare	64.73	64.50	-0.4%	64.67	-0.1%
% of unprod. matches	23.5%	16.9%	-6.6%	23.9%	+0.4%

Table 8: Change in main indicators at the steady-state, after a 25% cut in firing costs or a FTC-only lump-sum tax equal to 1% of the average wage. Unproductive matches are defined as matches that are kept only for the presence of firing costs, but that would be terminated otherwise.

Absence of Endogenous Choice

In this section the table with the aggregate indicators and the graphs with the welfare changes, after the implementation of policy changes, in an environment in which agents cannot choose the type of contract. More precisely, I am keeping the same structural parameters estimated in the main model. However, I am assuming that the agents cannot choose to sign an OEC. The only way in which the agents can obtain an OEC is through the "transformation" shock. I am assuming that this exogenous shock realizes at the beginning of every period (including the initial match-formation stage). Also, I am calibrating this parameter in order to match the observed share of FTC in Italy.

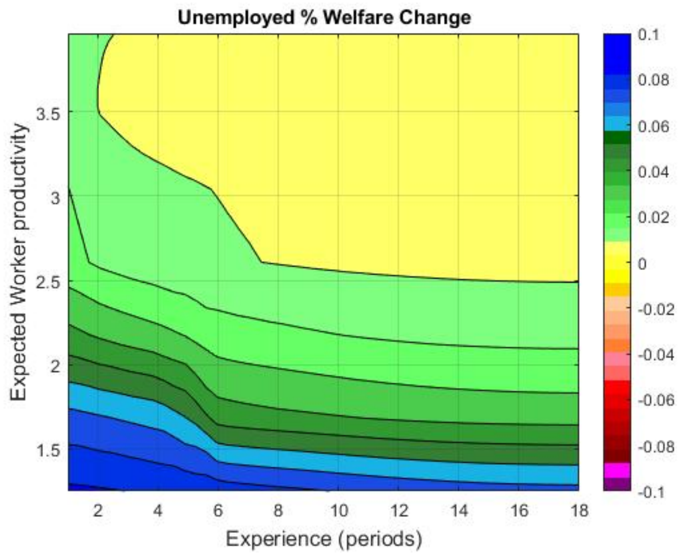

Figure 32: Change in welfare of unemployed workers at the steady state with a cut of 25% of firing costs. Choice of the contract NOT allowed.

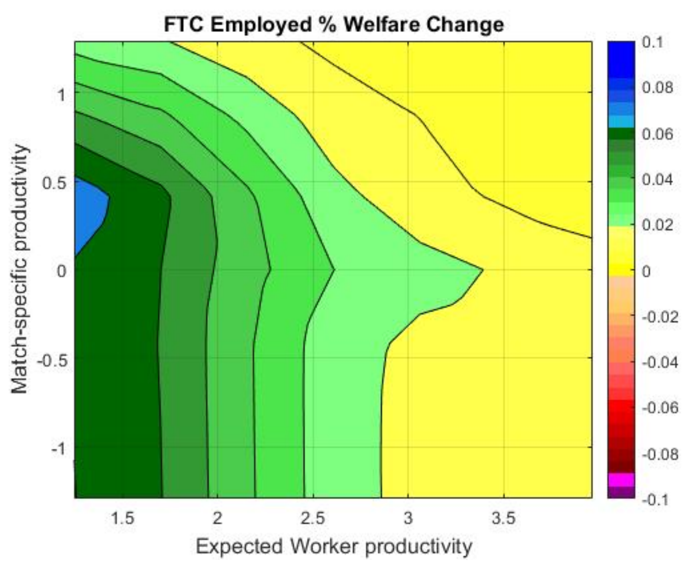

Figure 33: Change in welfare of FTC employed workers at the steady state with a cut of 25% of firing costs. Choice of the contract NOT allowed.

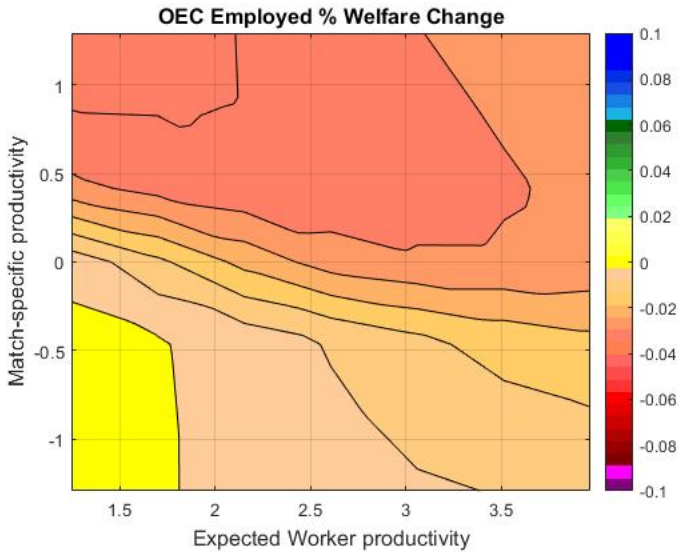

Figure 34: Change in welfare of OEC employed workers at the steady state with a cut of 25% of firing costs. Choice of the contract NOT allowed.

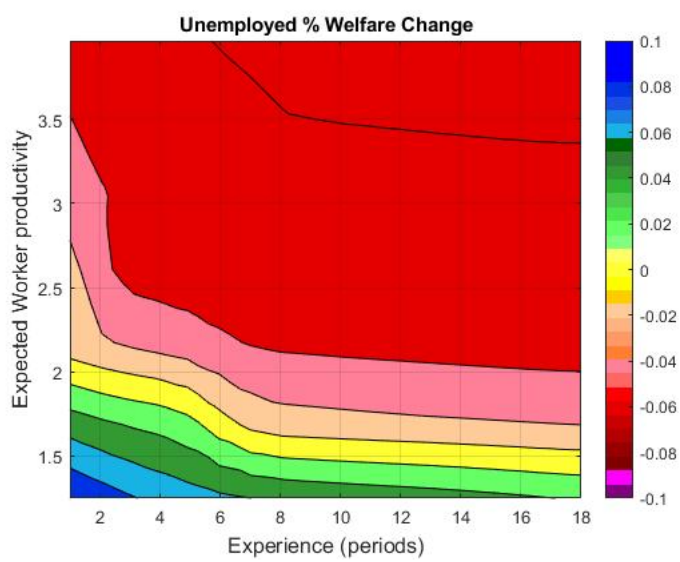

Figure 35: Change in welfare of unemployed workers at the steady state with a lump-sum on FTCs equal 1% of average wage. Choice of the contract NOT allowed.

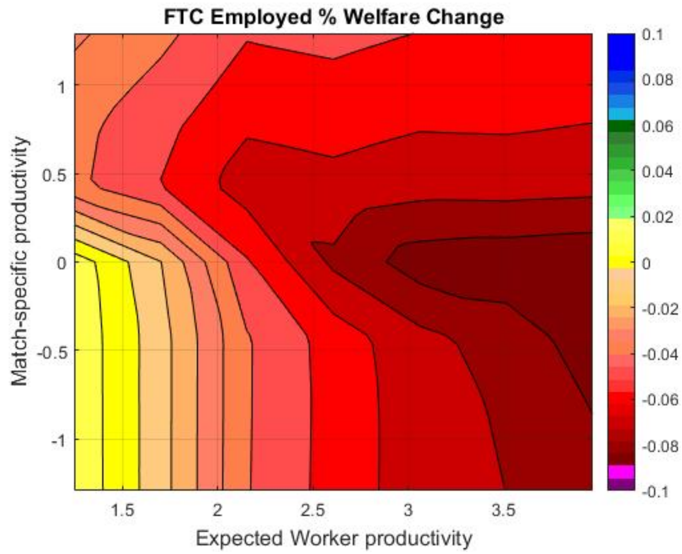

Figure 36: Change in welfare of FTC employed workers at the steady state with a lump-sum on FTCs equal 1% of average wage. Choice of the contract NOT allowed.

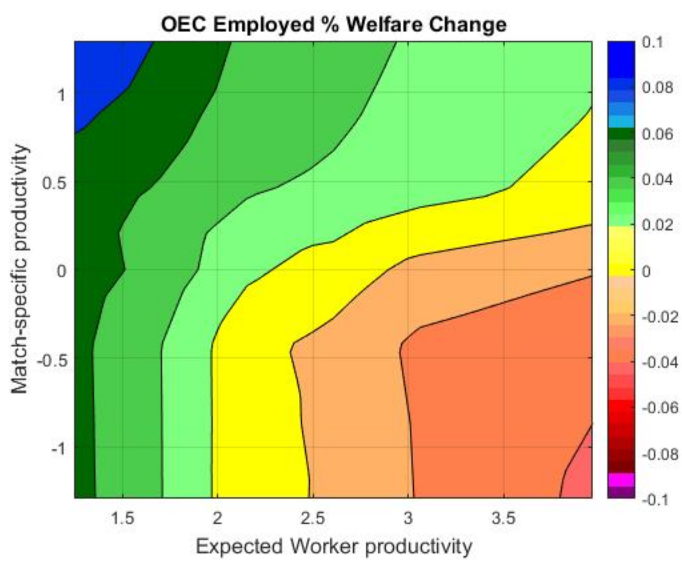

Figure 37: Change in welfare of OEC employed workers at the steady state with a lump-sum on FTCs equal 1% of average wage. Choice of the contract NOT allowed.