Taste for Variety: An Intertemporal Choice Model

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Abstract

Variety-seeking behavior refers to the tendency to alternate between different products to experience diversity or variety in consumption over time. It is a prominent and well-documented driver of individual decision-making and has attracted much attention in the marketing, psychology, and economics literature. In spite of that, the vast majority of intertemporal choice models assume some form of time-separability, implying consumption independence and therefore making them unable to account for variety-seeking behavior. This paper addresses this issue by presenting, studying, and axiomatically characterizing a new discrete choice model of time-risk preferences consistent with variety-seeking behavior. I refer to this model as the history-discounted utility (HDU) model. In the HDU model, consumption independence is relaxed by allowing for a history-dependent utility function. The biological/psychological driver of variety-seeking behavior is a satiation and recovery process in which product enjoyability decreases with consumption and recovers back to its intrinsic level otherwise. The main advantage of the axiomatic characterization presented in this paper is that simple and intuitive axioms allow disentangling the effects from time discounting from history dependence, providing a new framework to axiomatize time-nonseparable preferences. I demonstrate the broad scope of applicability of the HDU model by analyzing two different applications. In the first application, I study a multiproduct monopolist's optimal dynamic pricing strategies in intertemporal discrete choice settings facing variety-seeking consumers. In the second application, I show how the tools provided by the HDU model can help tackle one of the most urgent threats to public health, antibiotic resistance. In particular, I show how the HDU model can be used to design antibiotic treatment plans to fight bacterial infections more effectively while minimizing the threat of developing antibiotic resistance.

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1 Introduction

Variety-seeking behavior refers to the tendency to alternate between different products in order to experience diversity or variety in consumption over time. It is a prominent and well-documented driver of individual decision-making that has attracted much attention in the marketing, psychology, and economics literature.[1] It arises due to the satiation effect individuals experience after consuming a product.[2] The satiation effect increases with consumption frequency (or quantity) and decreases otherwise.

Despite the prominence of variety-seeking behavior in intertemporal choice contexts, neither the leading rational paradigm to study intertemporal choice in economics, the exponentially discounted utility (EDU) model, nor behavioral intertemporal choice models like the quasi-hyperbolic discounting model can accommodate such behavior. These models assume some form of time separability, implying consumption independence. Consumption independence means that the utility generated by current consumption does not depend on past or future consumption streams. For example, it implies that a person's preferences between an Italian and Japanese restaurant tonight do not depend on whether she ate Japanese food last night or expects to have it tomorrow. Therefore, time-separable models are unable to account for variety-seeking behavior.

This paper addresses this issue by presenting, studying, and axiomatically characterizing a new discrete intertemporal choice model of time-risk preferences consistent with variety-seeking behavior. I will refer to this model as the history-discounted utility (HDU) model. To the best of my knowledge, this paper is the first to provide an entirely founded intertemporal choice model consistent with variety-seeking behavior in a discrete choice setting. The HDU model is a simple, intuitive, and flexible model that possesses several desirable properties:

1. It can be estimated empirically with the data typically observed in intertemporal discrete choice settings: the set of available products alongside the chosen product and its price in each period.

[1] See, for example, Ratner et al. (1999), Adamowicz (1994), McAlister (1982), and Ratner et al. (1999) for seminal papers documenting variety-seeking behavior in humans. It has also been reported in animals like in Addessi (2008) and Addessi et al. (2010), highlighting the fundamental nature and ubiquitousness of such behavior. The prominence of such behavior suggests that preferences for variety result from an evolutionary process since it facilitates species' survival in two different but interrelated ways. First, populations that rely on a unique food source are more likely to die of starvation in case of natural occurrences that severely decrease the amount of food availability. Second, populations that rely on a unique food source are less likely to ingest all the nutrients needed to prosper. This evolutionary hypothesis might explain the ubiquitous nature of variety-seeking behavior in humans and non-human animals. See also Zhang (2022) for a recent literature review and Table 1 from Cosguner et al. (2018) for an account of categories of products in which variety-seeking behavior has been empirically demonstrated.

[2] A prominent alternative explanation presented in the psychology literature is that variety-seeking behavior arises due to a psychological need for stimulation. However, both explanations of variety-seeking behavior are entirely consistent with the model I will present in this paper. It just changes the interpretation of the parameters.

2. The HDU model has a wide range of applicability, as I show with two very different but illustrative applications. The first application studies monopolists' optimal dynamic pricing strategies when facing variety-seeking consumers. The second application shows how the HDU model can be used to design optimal antibiotic treatment plans.

3. The model is easily extended to fit different applications, settings, and needs, as I show with three behaviourally founded extensions. In the first extension, I revisit the model by considering agents with limited foresight. In the second extension, I allow desirable goods to become economic bads after a sufficiently long history of past consumption. In the last extension, I consider infinite consumption streams.

The axiomatic characterization of the HDU model allows us to evaluate the logical and mathematical appeal of the model by testing its (falsifiable) axioms and hence assess and interpret its precise predictions. The main advantage of the axiomatic characterization presented in this paper is that simple and intuitive axioms allow disentangling the effects from time discounting from history dependence, providing a new framework to axiomatize time-nonseparable preferences. The key idea of the model is that the biological/psychological driver of variety-seeking behavior is a satiation and recovery process in which product enjoyability decreases with consumption and recovers back to its intrinsic level otherwise.

The HDU model considers a decision maker (DM) with preferences over consumption streams which are sequences of ordered pairs. Each ordered pair consists of a simple lottery over a discrete set of alternatives and a sure monetary amount that the DM chooses at each time point.[3] The DM has additively separable preferences over such ordered pairs. The EDU arises naturally as a particular case of the HDU whenever the DM does not experience any satiation.

Let us describe the HDU model's simplest riskless version to fix ideas. The dynamic nature of the HDU model representation is captured by three key parameters, the time discount rate $\delta \in (0,1)$, the satiation parameter $\lambda \in (0,1]$, and the recovery parameter $\beta \in (0,1]$, and is based on the following simple idea: Every time an alternative is consumed, its utility is discounted by the satiation parameter. Otherwise, its utility might recover up to its intrinsic value at a rate specified by the recovery parameter. On the other hand, monetary amounts do not experience satiation and hence are represented by a static utility function. In sum, a fully forward-looking variety-seeking DM with preferences consistent with the HDU model maximizes the sum of time-discounted and history-discounted utilities where the history-dependent utility representation over the finite set of alternatives follows the aforementioned simple law of motion.

In Section 3, I present the paper's main result, the axiomatic characterization of the HDU model for finite horizon consumption streams. I use a novel axiomatization strategy that

[3]Monetary amounts could be interpreted as prices in some applications, and we can also re-interpret monetary amounts as any other continuous attribute like quality or nutritional levels.

allows isolating the effects of time from pure history dependence. Section 4 presents a simple procedure, based on some results and ideas from Section 3, to uniquely elicitate the parameters and the utility functions of the HDU model with arbitrary precision. This simple procedure can be implemented with easily accessible experimental data. It can also serve as a starting point for developing an empirical strategy to estimate the model with observational data such as consumer supermarket data.

In Section 5, I demonstrate the broad scope of applicability of the HDU model by analyzing two different applications. In the first application, I study a multiproduct monopolist's optimal dynamic pricing strategies in intertemporal discrete choice settings facing variety-seeking consumers with different degrees of foresight. I characterize the monopolist's optimal pricing strategy and show that with such a strategy, the monopolist can impose any consumption path on all consumers, regardless of their degree of foresight. Hence, the monopolist's profit maximization problem is reduced to choosing the optimal consumption path that maximizes total surplus. Therefore, the monopolist can extract all surplus and increase the total surplus in society by inducing myopic consumers and k-period forward-looking agents to behave as a fully-forward agents as patient as the monopolist. This result implies that if the monopolist ignores the variety-seeking nature of the consumer, it will implement a sub-optimal pricing strategy leading to a substantial reduction in profits and total surplus. This application exemplifies how standard economic theory results should be revisited in light of the HDU model whenever variety-seeking behavior occurs. In the second application, I show how the modeling tools provided by the HDU model can help tackle one of the most urgent threats to public health, antibiotic resistance. The HDU model can be used to design antibiotic treatment plans to fight bacterial infections more effectively while minimizing the threat of developing antibiotic resistance. With a simple reinterpretation of the HDU model's main parameters, the HDU model provides an adequate theoretical framework to address this issue.

Finally, in Section 6, I present and axiomatically characterize three extensions of the HDU model. In the first extension, I revisit the HDU model by considering agents with limited foresight. In particular, I characterize the choice behavior of a k-period forward-looking agent. Two special cases of k-period forward-looking agents are worth mentioning. First, the standard fully-forward-looking agent that we have considered up until now corresponds to $k \geq T$ where T is the length of the consumption stream. Second, the myopic agent, which corresponds to $k = 0$, always chooses what he likes the most at each period, ignoring the effects of that choice on future choices. In the second extension, I consider goods that might become bads after a sufficiently long past of consumption history. In the third extension, I illustrate the flexibility of the axiomatization strategy by characterizing the HDU model for infinite consumption plans. This exercise is simplified dramatically since the axiomatization strategy presented in this paper allows disentangling the effects of time from history dependence. Therefore, it reduces to replacing the subset of time discount axioms with appropriate ones to accommodate infinite consumption plans (or any other desired set of time discount axioms like quasi-hyperbolic discounting if we suspect that present bias might interact with variety-seeking). Section 7 concludes this paper.

Related Literature

The study of variety-seeking behavior can be traced back to early experiments in brand loyalty such as McConnell (1968) and Tucker (1964). These studies report switching behavior after controlling for determinant marketing variables such as prices, promotions, or product design.[4] Participants were instructed to make repeated choices among unfamiliar products, and two switching behavior stages were identified. Initially, participants explored all items. Later in the experiment, participants tended to alternate among a subset of preferred products. The first stage is distinguishable from the second because of the information acquisition motive. In the first stage, participants aim to elicit their preferences. In the second stage, once the preferences are known, participants alternate among products to experience variety in consumption over time. The literature refers to such behavior as variety-seeking behavior. This paper fits into the growing literature that deals with switching, as in the second stage, and contributes to the literature in various aspects.

First, numerous papers have studied variety-seeking behavior and have developed models to rationalize such behavior. Seminal contributions in this literature are Dixit and Stiglitz (1977), McAlister (1982), Adamowicz (1994), and Baucells and Sarin (2007, 2010). However, in all these papers, explicitly or implicitly, the satiation and recovery rates are arbitrarily linked, either by definition or by mathematical convenience (in most papers, the satiation and the recovery rates are equal). In principle, however, there is no reason why these two key elements of the dynamic process should be linked. It is easy to imagine products where satiation occurs relatively faster than recovery or vice-versa.[5] Ultimately, whether the satiation and recovery rates are linked in some specific way is an empirical question that should be addressed, and any model attempting to rationalize variety-seeking behavior should not put any artificial restriction on it. This paper fills this gap by providing, to the best of my knowledge, the first entirely founded intertemporal choice model consistent with variety-seeking behavior where the satiation and recovery rates are not linked in any specific way and, as a result, can accommodate richer and more realistic consumption patterns.

[4] As pointed out by Ratner et al. (1999), previous research has identified several factors accounting for switching behavior. The most prominent factors are i) intrapersonal factors, such as the satiation effect individuals experience after consuming a product and/or a psychological need for stimulation, ii) external factors, such as promotions, price changes, or marketing campaigns; and iii) uncertainty about future preferences. However, the only category of factors that can account for variety-seeking behavior in sequential choice contexts among unchanging options is the first one.

[5] For example, a DM might satiate relatively fast from eating oysters and recover its taste back slowly while simultaneously satiate relatively slowly from eating chocolate and recovering faster.

Furthermore, these papers developed quantity-oriented variety-seeking models.[6] Thus, these models are designed to explain how much of the good the variety-seeking DM will consume at each time but are ill-equipped to describe variety-seeking behavior in discrete choice environments such as restaurants, products, or recreational activities choices. The HDU model provides a suitable discrete choice framework to study applications such as the ones presented in this paper.

Second, although some quantity-oriented variety-seeking models have been axiomatically founded like Kaiser and Schwabe (2012), He et al. (2013), Rustichini and Siconolfi (2014), and Baucells and Zhao (2020), the setting presented in this paper requires the development of a completely different technical approach. In addition, this new characterization strategy offers simple and intuitive axioms that easily allow isolating the effects of time from history-dependence. To achieve this, I propose a set of standard axioms that, in conjunction, will enable me to show that for each lottery (alternative) in the choice set, there exists a unique history-dependent compensation that the DM will require to give up that lottery. With this result in hand and an additional simple axiom, I show that there exists a unique stream of appropriate monetary compensations for every consumption stream that makes the DM indifferent. Moreover, this later axiom creates a one-to-one mapping over consumption plans and streams of appropriate monetary compensations, allowing us to rank any pair of consumption plans by ranking the corresponding streams of appropriate monetary compensations. As a result, the central axioms of the HDU model, the satiation and recovery axioms, which capture the history dependence, are entirely independent of the time discount axioms. Therefore, this paper also provides a theoretical framework in which we can independently replace each set of axioms to accommodate different history or time dependencies. For example, if we believe that variety-seeking and present bias interact, we can replace the set of exponential discounting axioms used in this paper with an appropriate set of quasi-hyperbolic discounting axioms.

In sum, this paper presents the first behaviorally founded intertemporal choice model consistent with variety-seeking, where no artificial constraints are put on the satiation and recovery processes and is suitable to analyze discrete choice environments.

2 The History-Discounted Utility Model (HDU model)

In this section, I present the history-discounted utility model (HDU model). The HDU is a history-dependent model of time-risk preferences over intertemporal streams of probability distributions over outcomes. As we will see later, special cases of the HDU model are the EDU model and the expected utility (EU) model. The EDU model states that for any two arbitrary

[6] Except for the Dixit-Stiglitz model, these models can be considered different variants of the inverse models of the intrinsic linear habit formation model prevalent in the macro literature and elegantly axiomatized by Rozen (2010).

sequences of choices over time, $\boldsymbol{x} \equiv (x_0, x_1, \cdots, x_T)$ and $\boldsymbol{y} \equiv (y_0, y_1, \cdots, y_T)$, there exist a discount factor $\delta \in (0,1)$ and a utility function (mapping the set of alternatives into the real numbers) $u : \mathcal{X} \to \mathbb{R}$ such that:

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \text{iff} \quad \sum_{t=0}^{T} \delta^t u(x_t) \geq \sum_{t=0}^{T} \delta^t u(y_t)$$

Importantly, the HDU model generalizes the EDU model by relaxing the assumption of consumption independence over time and hence allowing for history dependence.

2.1 Notation

Let $\mathcal{T} \equiv \{0, 1, 2, \ldots, T\}$, where $T \leq \infty$, denotes *time*. Let $\mathcal{A}$ be a finite set of alternatives with $|\mathcal{A}| = N$. I assume that the alternatives are familiar and frequently consumed by the DM, hence learning place no role in our analysis. Typical elements of $\mathcal{A}$ are a_1, a_2 and a_3. In addition, we denote by $\diamond \in \mathcal{A}$ the *neutral alternative*, that is to say, the option of not consuming anything. Let $\Delta(\mathcal{A})$ be the set of all probability distributions on $\mathcal{A}$, that is the set of all functions $p : \mathcal{A} \to [0,1]$ such that $\sum_{a \in \mathcal{A}} p(a) = 1$. Elements of $\Delta(\mathcal{A})$ are called *lotteries* and are denoted by x, y and z. For any lottery $x \in \Delta(\mathcal{A})$, $p_x(a_i)$ denotes the probability that lottery x assigns to alternative a_i. With a slight abuse of notation, a_i for $i \in \{1, \ldots, N-1\}$ and $\diamond$ will also be used to denote degenerate lotteries that assign probability one to alternative a_i and to the neutral alternative respectively. Hence, $\mathcal{A}$ is also interpreted as the set of all degenerate lotteries. I will interpret the set of real numbers $\mathbb{R}$ as money amounts, typical elements of this set will be denoted by m, m' and m''. Let $\boldsymbol{x} = \big((x_0, m_0), (x_1, m_1), \ldots, (x_T, m_T)\big)$ and $\boldsymbol{y} = \big((y_0, m'_0), (y_1, m'_1), \ldots, (y_T, m'_T)\big)$ both in $\big(\Delta(\mathcal{A}) \times \mathbb{R}\big)^{T+1}$ denote arbitrary *consumption streams*, where $(x_t, m_t) \in \Delta(\mathcal{A}) \times \mathbb{R}$ denotes DM's *choice* at time t, that is, the ordered pair, consisting of a lottery and a monetary amount, that the DM chooses at time t. *Preferences* over consumption streams are denoted by $\succsim$. As usual, $\sim$ and $\succ$ denote the symmetric and the asymmetric part of $\succsim$.

I endow the sets $\Delta(\mathcal{A}) \times \mathbb{R}$ and $\big(\Delta(\mathcal{A}) \times \mathbb{R}\big)^{T+1}$ with the product topology. A *history of length* $t > 0$ is a sequence $\boldsymbol{h}_t \equiv \big((r_0, m_0), (r_1, m_1), \ldots, (r_{t-1}, m_{t-1})\big) \in (\mathcal{A} \times \mathbb{R})^t$ of ordered pairs consisting of the realization of the lottery and the money amount chosen by the DM from time 0 up to time $t-1$. I denote the empty sequence by $\boldsymbol{h}_0 = \emptyset$, i.e., the sequence containing no terms. The *set of all possible histories of length* $t > 0$ is denoted by $H_t = (\mathcal{A} \times \mathbb{R})^t$. The *set of all histories* is $\mathcal{H} = \bigcup_{t=1}^{T} H_t$. In general, I will refer to any distinct histories of the same length as $\boldsymbol{h}_t, \boldsymbol{h}'_t \in \mathcal{H}$. Through the paper, whenever we encounter $\boldsymbol{h}_t$ and $\boldsymbol{h}_{t-1}$, we will assume that $\boldsymbol{h}_t = (\boldsymbol{h}_{t-1}, (r_{t-1}, m_{t-1}))$, that is to say, all elements of the vectors $\boldsymbol{h}_t$ and $\boldsymbol{h}_{t-1}$ are equal up to time $t-2$, but $\boldsymbol{h}_t$ also includes the realization of the choice made at $t-1$.

2.2 Dynamics of the Utility Process

Before I present the full HDU model let me first introduce one of its key elements, the utility process. In order to do so, let's assume for now that the DM is restricted to choose among degenerate lotteries over a finite set of alternatives.[7] The utility process is captured by the sum of a von Neumann-Morgenstern history-dependent utility function $u_{\boldsymbol{h_t}}(x_t)$ and a static utility function for money $v(m_t)$. I will interpret $u_{\boldsymbol{h_t}}(x_t)$ as the utility derived from lottery x_t after a history of past consumption $\boldsymbol{h_t}$. Moreover, $u_{\boldsymbol{h_t}}(x_t)$ is the product of an intrinsic utility function $u_0(x_t)$, which represents the utility derived when the consumer is not satiated at all, and a history discount function $\psi_t(x_t|x_{t-1})$, that discounts intrinsic utility given the history of past consumption as follows:

For any $(x_t, m_t) \in \mathcal{A} \times \mathbb{R}$ and given any history of past consumption $\boldsymbol{h}_t \in H_t$:

$$U_{\boldsymbol{h}_t}(x_t, m_t) = u_{\boldsymbol{h_t}}(x_t) + v(m_t) \tag{1}$$

Moreover,

$$u_{\boldsymbol{h}_t}(x_t) = \psi_t(x_t|x_{t-1})u_0(x_t) \tag{2}$$

where for all $x_t \in \mathcal{A}$

$$\psi_0(x_t|\boldsymbol{h}_0) = 1 \tag{3}$$

and for all $t > 0$

$$\psi_t(x_t|x_{t-1}) = \begin{cases} \lambda_{x_t} \cdot \psi_{t-1}(x_t|x_{t-2}) & \text{if} \quad x_t = x_{t-1} \\ \min\left\{1, \frac{1}{\beta_{x_t}} \cdot \psi_{t-1}(x_t|x_{t-2})\right\} & \text{if} \quad x_t \neq x_{t-1} \end{cases} \tag{4}$$

$$\lambda_{x_t} \in (0, 1] \quad \text{and} \quad \beta_{x_t} \in (0, 1]$$

As mentioned, $u_0(x_t)$ is interpreted as the intrinsic utility of alternative x_t. This is the maximal utility that the agent may enjoy from consuming x_t at any point in time. Thus, it can be seen as the utility that the DM would derive if her taste for x_t has not satiated at all. The parameter λ_{x_t} represents the satiation rate, that is the rate at which the intrinsic utility of the alternative x_t is discounted each time x_t is consumed. Similarly, $\frac{1}{\beta_{x_t}}$ represents the recovery rate, that is the rate at which the intrinsic utility of the alternative x_t recovers from non-consumption. The history discount function $\psi_t(x_t|x_{t-1})$ captures the effects of the satiation-recovery process. Every time an alternative is consumed its utility gets discounted by λ_{x_t}, capturing the satiation effect, and every time an alternative is not consumed its utility recovers at a rate $\frac{1}{\beta_{x_t}}$ capturing the recovery effect. However, notice that the recovery is bounded above by the intrinsic utility and the satiation is bounded below by zero.

[7] I will present the general case in the next subsection with the HDU representation.

The history discount function is individualistic and alternative-specific. That means that each decision maker may have a different history discount function and each alternative in the choice set may be history-discounted differently.

Let me now present a simple example to illustrate this utility process.

Example 1. *Given $\mathcal{A} = \{a_1, a_2, \diamond\}$, $u_0(a_1) = 10$, $u_0(a_2) = 6$, $u_0(\diamond) = 0$, $\lambda_{a_1} = \lambda_{a_2} = \lambda = 0.8$, $\beta_{a_1} = \beta_{a_2} = \beta = 0.8$, and $\boldsymbol{h}_4 = (a_1, a_1, \diamond, a_2)$, we can compute $u_{\boldsymbol{h}_4}(a_1)$ and $u_{\boldsymbol{h}_4}(a_2)$ recursively as follows.*

Let's first compute $u_{\boldsymbol{h}_0}(a_1)$, given that at the beginning of time there is not satiation, equations (2) and (3) imply that $u_{\boldsymbol{h}_0}(a_1) = \psi_0(x_t|\boldsymbol{h}_0)u_0(a_1) = 1 \cdot u_0(a_1) = 10$. Next, since a_1 is chosen at time $t = 0$, equations (2) and (4) imply that $u_{\boldsymbol{h}_1}(a_1) = \lambda u_0(a_1) = 8$. This means that at time $t = 1$ given that a_1 was chosen in the previous period ($t = 0$) the utility of a_1 is discounted by the factor λ, which is precisely the satiation rate. Similarly at $t = 2$, given that a_1 was chosen again at time $t = 1$, the utility of a_1 is discounted once more and we get, $u_{\boldsymbol{h}_2}(a_1) = \lambda^2 u_0(a_1) = 6.4$. Let's now move to $t = 3$, notice that given that $\diamond$ was chosen at the previous period, the utility of a_1 will now recover, the speed of recovery is precisely $\frac{1}{\beta}$ which is the recovery rate. Hence, according to equations (2) and (4) we get that $u_{\boldsymbol{h}_3}(a_1) = \min\{1, \frac{\lambda^2}{\beta}\}u_0(a_1) = 0.8u_0(a_1) = 8$. Finally, we can now compute $u_{\boldsymbol{h}_4}(a_1)$, notice that a_1 was not chosen at the previous period either, hence the utility of a_1 will recover again. Given that $\lambda = \beta = 0.8$, equations (2) and (4) imply that $u_{\boldsymbol{h}_4}(a_1) = \min\left\{1, \frac{1}{\beta}\min\{1, \frac{\lambda^2}{\beta}\}\right\} u_0(a_1) = \min\{1, \frac{\lambda^2}{\beta^2}\}u_0(a_1) = 10$.[8] Following the same logic we can compute $u_{\boldsymbol{h}_4}(a_2)$, the results are reported in the following table:

t	c_t	$\boldsymbol{h}_t$	$u_{\boldsymbol{h}_t}(a_1)$	$u_{\boldsymbol{h}_t}(a_2)$
0	a_1	$\boldsymbol{h_0} = \emptyset$	$u_{\boldsymbol{h}_0}(a_1) = u_0(a_1) = 10$	$u_{\boldsymbol{h}_0}(a_2) = u_0(a_2) = 6$
1	a_1	$\boldsymbol{h_1} = (a_1)$	$u_{\boldsymbol{h}_1}(a_1) = \lambda u_0(a_1) = 8$	$u_{\boldsymbol{h}_1}(a_2) = u_0(a_2) = 6$
2	$\diamond$	$\boldsymbol{h_2} = (a_1, a_1)$	$u_{\boldsymbol{h}_2}(a_1) = \lambda^2 u_0(a_1) = 6.4$	$u_{\boldsymbol{h}_2}(a_2) = u_0(a_2) = 6$
3	a_2	$\boldsymbol{h_3} = (a_1, a_1, \diamond)$	$u_{\boldsymbol{h}_3}(a_1) = \min\{1, \frac{\lambda^2}{\beta}\}u_0(a_1) = 8$	$u_{\boldsymbol{h}_3}(a_2) = u_0(a_2) = 6$
4		$\boldsymbol{h_4} = (a_1, a_1, \diamond, a_2)$	$u_{\boldsymbol{h}_4}(a_1) = \min\{1, \frac{\lambda^2}{\beta^2}\}u_0(a_1) = 10$	$u_{\boldsymbol{h}_4}(a_2) = \lambda u_0(a_2) = 4.8$

This example has illustrated how the history-dependent utility of each alternative evolve endogenously depending on the history of past consumption. Moreover, it has shown how the satiation-recovery process governs the dynamics of this history-dependent utility process.

2.3 The HDU Model Representation

For the sake of simplicity, in the previous section I have illustrated the utility process when the DM is restricted to choose among degenerate lotteries. Now, I will present the general model in

[8] It is important to notice that even though it is not the case in this example, the min operator is there to prevent the recovery process to exceed the intrinsic value.

which the DM can choose any lottery in the simplex. We say that a decision maker's time-risk preferences are consistent with the HDU model if they admit the following representation:

Definition 2.1 (HDU Representation)**.** *Time-risk preferences are consistent with the history-discounted utility model if they can be represented by: an intrinsic utility function* $u_0 : \Delta(\mathcal{A}) \to \mathbb{R}_+$*, a continuous and increasing utility function for money* $v : \mathbb{R} \to \mathbb{R}$*, a time discount parameter* $\delta \in (0,1)$*, an alternative-specific satiation parameter* $\lambda_{a_i} \in (0,1]$*, and an alternative-specific recovery parameter* $\beta_{a_i} \in (0,1]$ *such that:*

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t \Big[\hat{u}_{\boldsymbol{h}_t}(x_t) + v(m_t)\Big] \geq \sum_{t=0}^{T} \delta^t \Big[\hat{u}_{\boldsymbol{h}_t}(y_t) + v(m'_t)\Big]$$

where for all $z_t \in \Delta(\mathcal{A})$ *and all* $t > 0$*, if* $a_i \in \mathcal{A}$ *was the realization of the lottery chosen by the decision maker at* $t-1$*, that is* $r_{t-1} = a_i$*, then:*

$$\underbrace{\sum_{i=1}^{N} p_{z_t}(a_i) u_{\boldsymbol{h}_t}(a_i)}_{\hat{u}_{\boldsymbol{h}_t}(z_t)} = \underbrace{p_{z_t}(a_i)(\lambda_{a_i} - 1) u_{\boldsymbol{h}_{t-1}}(a_i)}_{\textit{Satiation} \leq 0} +$$

$$\underbrace{\sum_{a_j \in \mathcal{A} \setminus \{a_i\}} p(a_j) \Bigg[\min\Big\{u_0(a_j), \frac{u_{\boldsymbol{h}_{t-1}}(a_j)}{\beta_{a_j}}\Big\} - u_{\boldsymbol{h}_{t-1}}(a_j)\Bigg]}_{\textit{Recovery} \geq 0} + \underbrace{\sum_{i=1}^{N} p_{z_t}(a_i) u_{\boldsymbol{h}_{t-1}}(a_i)}_{\hat{u}_{\boldsymbol{h}_{t-1}}(z_t)}$$

From the above definition, we can clearly see that our DM is fully forward-looking since she maximizes the sum of time-discounted and history-discounted utilities and hence, fully internalizing the satiation and recovery effects. Moreover, for any lottery z_t, its history-dependent utility $\hat{u}_{\boldsymbol{h}_t}(z_t)$ follows a very simple law of motion: $\hat{u}_{\boldsymbol{h}_t}(z_t)$ is equal to its previous value, $\hat{u}_{\boldsymbol{h}_{t-1}}(z_t)$, minus the (potential) negative effect due to satiation and plus the (potential) positive effect due to the recovery process. Only the utility of the alternative $a_i \in \text{supp}(z_t)$ that is the realization of the previous period lottery will experience the satiation effect, all the other alternatives in the support will experience the recovery process.[9]

Finally, notice that if we we restrict our DM to choose only among degenerate lotteries the previous representation can be rewritten more simple as follows:

Definition 2.2 (HDU Representation for Degenerate Lotteries)**.** *If the decision maker can only choose from the set of degenerate lotteries* $\mathcal{A}$*, we can rewrite the previous representation more simply as follows, for any* $\boldsymbol{x}, \boldsymbol{y} \in (\mathcal{A} \times \mathbb{R})^{T+1}$*:*

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t \Big[\psi_t(x_t|x_{t-1}) u_0(x_t) + v(m_t)\Big] \geq \sum_{t=0}^{T} \delta^t \Big[\psi_t(y_t|y_{t-1}) u_0(y_t) + v(m'_t)\Big]$$

[9] The support of a simple lottery $z_t \in \Delta(\mathcal{A})$ is defined as: $\text{supp}(z_t) \equiv \{a_i \in \mathcal{A} \mid p_{z_t}(a_i) > 0\}$.

where for all $z_t \in \mathcal{A}$, *and for all* $t > 0$*:*

$$\psi_t(z_t|z_{t-1}) = \begin{cases} \lambda_{z_t} \cdot \psi_{t-1}(z_t|z_{t-2}) & \textit{if } z_t = z_{t-1} \\ \min\left\{1, \frac{1}{\beta_{z_t}} \cdot \psi_{t-1}(z_t|z_{t-2})\right\} & \textit{if } z_t \neq z_{t-1} \end{cases}$$

$$\psi_0(z_t|\boldsymbol{h}_0) = 1, \quad \lambda_{x_t} \in (0,1], \quad \textit{and} \quad \beta_{x_t} \in (0,1]$$

In words, the HDU model allow us to rank any pair of consumption streams by the sum of their time-discounted and history-discounted utilities. Moreover, whenever the DM is restricted to choose among degenerate lotteries, the utility process that captures the history-dependence in definition 2.2 is precisely the one I presented in the previous section. Observe also that from definition 2.1, 2.2 and the recursive nature of the history discounting, it quickly became evident that in direct analogy to first-order markov chains, we could label the history discount function as "Markovian" type function because although the history discount function depends on the whole history of past consumption, it does so only via the previous period consumption.

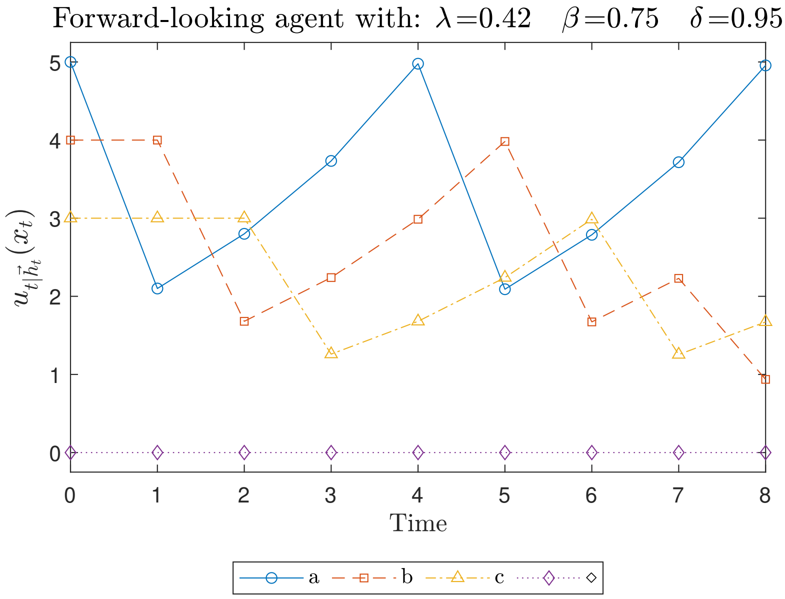
Notice also that the EDU model can be seen as a particular case of the HDU model in which the history discount function equals one for all $t \in \mathcal{T}$, for all $\boldsymbol{h_t} \in \mathcal{H}$ and, for all $z_t \in \Delta(\mathcal{A})$. Therefore, the EDU model is nested within the HDU model. Moreover, one implication of the EDU model in this setting is that the decision maker always chooses the **same** most preferred option in the choice set. However, the HDU model can rationalize sequences of alternating choices over time since the preference ordering changes endogenously depending on the history of past consumption. Therefore, the HDU model is consistent with variety-seeking behavior.

I next present another simple example in which the DM must choose a sequence of degenerate lotteries to illustrate: (i) how a fully forward-looking DM internalizes the satiation-recovery process and (ii) the dynamics of the HDU model.

Example 2. *Let* $\mathcal{A} = \{a, b, c, \diamond\}$, $u_0(a) = 5$, $u_0(b) = 4$, $u_0(c) = 3$, $u_0(\diamond) = 0$, $\lambda_a = \lambda_b = \lambda_c = \lambda = 0.42$, $\beta_a = \beta_b = \beta_c = \beta = 0.75$, $\delta = 0.95$ *and* $T = 8$. *Given this setting the DM must choose the consumption stream that maximizes the sum of time-discounted and history-discounted utilities as prescribed in definition 2.2. The solution to this maximization exercise is* $\boldsymbol{c}^* = (a, b, c, \diamond, a, b, c, b, a)$. *Let me now intuitively explain why is* $\boldsymbol{c}^*$ *the optimal consumption stream. As it can be seen in the following figure, at* $t = 0$ *the DM must choose alternative* a *because there is no gain in choosing a less preferred option since alternative* a *is already at its intrinsic (highest) level and hence, there are no potential gains from recovery. By the same logic, at* $t = 1$ *and* $t = 2$ *the DM must choose* b *and* c *respectively since both are maximal at each point in time and their utilities were at their intrinsic level and hence, there are no potential gains from recovery. At time* $t = 3$ *however, the DM is patient enough to internalize the effects of satiation and recovery and optimally chooses* $\diamond$. *Choosing diamond allows the utilities of the rest of the alternatives to recover and in periods 4, 5 and 6 the DM optimally repeats the same*

a, b, c pattern shown in periods 0,1,2.[10] *At $t = 7$ however, we encounter what we could refer to an end-life effect, the DM does not choose $\diamond$ as we would have expected since the end of her life is too close that the benefits of not choosing anything in order to profit from the recovery process do not outweigh the cost of giving up choosing an alternative that provides positive utility. In fact, at $t = 7$ the DM chooses the current second most preferred alternative, which is b in order to profit from the recovery process that a will experience at $t = 8$ when a will be finally chosen.*

Figure 1: Dynamics of the HDU Model **(Example 2)**.



From this example we can learn some important implications of the HDU model. First, the HDU model can rationalize alternating sequence of choices and hence, variety seeking behavior. However, not all sequences of choices can be rationalized by the HDU model, for example we have learned that it will never be rational not to choose a maximal alternative that it is already at its intrinsic value. Second, the willingness to pay for each alternative is endogenously evolving and it depends on the history of past consumption. For example, the willingness to pay for alternative a at t=0 is much higher than what will be at t=1 due to the satiation process. Third, if the satiation rate is fast enough in comparison to the recovery rate the DM would be willing to endogenously expand the choice set. For example, at t=3, the DM would have been happier choosing any alternative not in the original choice set (even if she has never tried before), as long as she expects that alternative to provide positive utility, instead of choosing $\diamond$.

[10]Given this parameter specification of the HDU model, if $T = \infty$ we would have observed this $(a, b, c, \diamond, a, b, c, \diamond, \dots)$ pattern repeating forever.

The DM has chosen $\diamond$ because is patient enough and anticipates that if it just keeps alternating between a, b and c the utilities of these alternatives will converge to zero in the long run.

In order to fully understand the implications of the HDU model on choice behavior and better understand which consumption streams are consistent with it, I next provide a full axiomatic characterization of the HDU model.

3 Axiomatic Characterization of the HDU Model

In the following axiomatic exercise, I use a novel methodological strategy in order to isolate the effects of time from history dependence in a simple way. I start by laying out three standard axioms in the intertemporal choice literature, namely that the binary relation $\succsim$ on the consumption streams space $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ is a continuous and monotonic weak order. Given these axioms I then induce a history-dependent preference relation $\succsim_{\boldsymbol{h}_t}$ defined on $\Delta(\mathcal{A}) \times \mathbb{R}$. The binary relation $\succsim_{\boldsymbol{h}_t}$ represents DM's preferences after a history of past consumption $\boldsymbol{h}_t$. Since $\succsim_{\boldsymbol{h}_t}$ is induced from $\succsim$, imposing axioms on $\succsim_{\boldsymbol{h}_t}$ is effectively the same as imposing axioms on $\succsim$ defined on a very specific subset of consumption streams.

I then proceed to impose two sets of axioms on this history-dependent preference relation. The first set of axioms, the static axioms, consisting of the Boundedness, Separability and Independence axioms are mainly technical in nature and impose the desired necessary structure to $\succsim_{\boldsymbol{h}_t}$. With this structure, I am able to show that there exist a unique history-dependent monetary compensation $c_{\boldsymbol{h}_t}(x, m) \in \mathbb{R}_+$, such that $(\diamond, m + c_{\boldsymbol{h}_t}(x, m)) \sim_{\boldsymbol{h}_t} (x, m)$. In other words, $c_{\boldsymbol{h}_t}(x, m)$ is the monetary amount that compensates the DM exactly enough to make her indifferent between getting the lottery x and m units of money or getting the degenerate lottery $\diamond$ and $m + c_{\boldsymbol{h}_t}(x, m)$ units of money instead.

The second set of axioms, the dynamic axioms, are the two fundamental axioms of this characterization. Those axioms are the Satiation and Recovery axioms that basically impose some structure on how $\succsim_{\boldsymbol{h}_{t-1}}$ and $\succsim_{\boldsymbol{h}_t}$ are related through the consumption occurred in $t-1$. The Satiation axiom states that if alternative a_i was consumed at $t-1$, then at time t, the DM will require a smaller (or equal) compensation for not getting the degenerate lottery a_i. This axiom captures the fact that, due to satiation, after consuming an alternative the DM might value the alternative less. Similarly, the Recovery axiom states that if alternative a_i was not consumed at $t-1$, then at time t, the DM requires a greater (or equal) compensation for not getting the degenerate lottery a_i. This axiom allows the valuation of an alternative to recover after a period of non-consumption.

Then I turn my attention back to the consumption stream space $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ and I proceed to formally define for any consumption stream $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ its associated stream of appropriate monetary compensations $\Diamond(\boldsymbol{x}) \in (\{\diamond\} \times \mathbb{R})^{T+1}$. $\Diamond(\boldsymbol{x})$ is the stream of monetary compensations that compensates the DM just enough to make her indifferent, at each point in time, between getting the lottery prescribed by $\boldsymbol{x}$, or getting the degenerate lottery $\diamond$ instead.

Finally, I impose the last two axioms of this characterization, the time preference axioms. The first is the Indifference axiom which is a fundamental axiom of this exercise. The Indifference axiom simply states that the DM is always indifferent between any consumption stream $\boldsymbol{x}$ and its associated stream of appropriate monetary compensations $\Diamond(\boldsymbol{x})$. The contribution of this axiom to this exercise is twofold. First, it allows us to rank any pair of consumption plans $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ by using the ranking between $\Diamond(\boldsymbol{x})$ and $\Diamond(\boldsymbol{y})$. Moreover, since there is no history-dependence in the space $(\{\diamond\} \times \mathbb{R})^{T+1}$ this effectively allows us to separate the effects of time from pure history dependence. Therefore, this axiom can be considered a weaker version of a time separability axiom appropriate for history-dependent preferences. Second, notice also that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$. Therefore, this axiom also allows me to make use of all the structure and results derived from standard time-discount axiomatizations. At this point we can easily make use of the results of any standard time preference representations, including exponential, hyperbolic, and quasi-hyperbolic discounting. In this paper, in order to be as close as possible to the most widely used model of intertemporal choice, the EDU model, I impose the Exponential Discounting axiom on the preference relation $\succsim^*$ defined on $\mathbb{R}^{T+1}$, which is the last axiom of this characterization.

3.1 Basic Axioms

I start by laying out three very basic axioms in the intertemporal choice literature.

Axiom 1 *(Weak Order):* The binary relation $\succsim$ on $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ is:

i) *Complete*: for all $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, either $\boldsymbol{x} \succsim \boldsymbol{y}$ or $\boldsymbol{y} \succsim \boldsymbol{x}$.

ii) *Transitive*: for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, if $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$, then $\boldsymbol{x} \succsim \boldsymbol{z}$.

Axiom 2 *(Continuity):* For all $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, the following sets are closed:

$$B(\boldsymbol{x}) = \{\boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1} : \boldsymbol{y} \succsim \boldsymbol{x}\}$$

$$W(\boldsymbol{x}) = \{\boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1} : \boldsymbol{x} \succsim \boldsymbol{y}\}$$

Axiom 3 *(Money Monotonicity):* For all $\boldsymbol{x} = \big((x_0, m_0), \dots, (x_t, m_t), \dots, (x_T, m_T)\big) \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ and all $t \in \mathcal{T}$,

$$\big((x_0, m_0), \dots, (x_t, m_t), \dots, (x_T, m_T)\big) \succ \big((x_0, m_0), \dots, (x_t, m_t'), \dots, (x_T, m_T)\big)$$

if and only if $m_t > m'_t$.

Axioms 1 and 2 are standard axioms in the intertemporal choice literature and do not need additional explanation. Axiom 3 is also quite standard and it just means that more money is better than less. Given these three axioms we can now induce a history-dependent preference relation $\succsim_{\boldsymbol{h}_t}$ as follows:

Definition 1: We define DM's *preferences*, given a history of past consumption $\boldsymbol{h}_t$, by:

$$(x_t, m_t) \succsim_{\boldsymbol{h}_t} (y_t, m'_t)$$

whenever for any $m''_{t+i} \in \mathbb{R}$, $i \in \{1, \cdots, T-t\}$

$$\boldsymbol{x} = (\boldsymbol{h}_t, (x_t, m_t), (\diamond, m''_{t+1}), \ldots, (\diamond, m''_T)) \succsim (\boldsymbol{h}_t, (y_t, m'_t), (\diamond, m''_{t+1}), \ldots, (\diamond, m''_T)) = \boldsymbol{y}$$

This definition states that from the point of view of the DM, given a history of past consumption $\boldsymbol{h}_t$, time t ordered pair (x_t, m_t) is at least as good as time t ordered pair (y_t, m'_t), if there exist two consumption streams $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, such that $\boldsymbol{x} \succsim \boldsymbol{y}$ and with the following characteristics:

- $\boldsymbol{x}$ and $\boldsymbol{y}$ share a common history $\boldsymbol{h}_t$ of realizations from time 0 up to time $t-1$.
- (x_t, m_t) and (y_t, m'_t) are the choices prescribed, at time t, by $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively.
- $\boldsymbol{x}$ and $\boldsymbol{y}$ prescribe a common sequence of future choices from time $t+1$ up to time T, in which, in every period of time the degenerate lottery $\diamond$ is always chosen in conjunction with a common, but arbitrary and potentially time-varying, amounts of money $m''_i \in \mathbb{R}$ for all $i \in \{t+1, \cdots, T\}$.

Notice that $\succsim_{\boldsymbol{h}_t}$ is a well defined object since A1 in conjunction with A3 prevent any possible inconsistencies. That is to say, if the conditions in the above definition hold for a sequence of monetary amounts $\{m''_{t+i}\}_{i=1}^{T-t}$, then they will also hold for any other sequence $\{m'''_{t+i}\}_{i=1}^{T-t}$. Notice also that in this definition it is embedded that DM's preferences over outcomes can potentially depend on the history of past consumption. In particular, given two distinct histories of past choices $\boldsymbol{h}, \boldsymbol{h'} \in \mathcal{H}$, it is possible for the same DM to rank $(x_t, m_t) \succ_{\boldsymbol{h}} (y_t, m'_t)$ but $(y_t, m'_t) \succ_{\boldsymbol{h'}} (x_t, m_t)$. Given this definition it is almost direct to see that $\succsim_{\boldsymbol{h}_t}$ is a continuous and monotonic weak order (see Lemma A1, A2 and A3 in the appendix). Moreover, the following useful and intuitive lemma will allow me to easily define some objects of interest later on.

Lemma 1 *(Money Solvability):* Suppose Axioms A1, A2, and A3 are satisfied, if $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ and $(y, m') \succsim_{\boldsymbol{h}_t} (x, m'')$, then there exists a unique m^*, with $m'' \leq m^* \leq m$, such that $(x, m^*) \sim_{\boldsymbol{h}_t} (y, m')$.[11]

Money solvability establishes that, given any history of past consumption $\boldsymbol{h}_t \in \mathcal{H}$, whenever an ordered pair (x, m) is preferred to a given ordered pair (y, m') for certain amounts of money but not for smaller amounts, then there exists a unique intermediate amount of money m^*, such that (x, m^*) is exactly indifferent to (y, m').

3.2 Static Axioms

I now proceed to impose the first set of axioms on the induced history-dependent binary relation $\succsim_{\boldsymbol{h}_t}$. This first set of axioms are technical in nature and are meant to impose the desired necessary structure to $\succsim_{\boldsymbol{h}_t}$.

Axiom 4 *(Boundedness):* For all $\boldsymbol{h}_t \in \mathcal{H}$, and for all $(x, m) \in \Delta(\mathcal{A}) \times \mathbb{R}$:

i) *Bounded below*: If $x \neq \diamond$, then $(x, m) \succ_{\boldsymbol{h}_t} (\diamond, m)$.

ii) *Bounded above*: There exists $c \in \mathbb{R}_{++}$, such that $(\diamond, m + c) \succ_{\boldsymbol{h}_t} (x, m)$.

Axiom 4 implies that no matter the history of past consumption any lottery will be preferred to the degenerate lottery that assigns probability one to $\diamond$. This means that no matter the history of past consumption, all lotteries remain goods and never become bads.[12] The second part of axiom 4 implies that there is always a monetary compensation that makes the DM better off by giving up the consumption of any lottery. Moreover, I show in the following lemma that there exist a unique compensation that makes the DM exactly indifferent.

Lemma 2: If axioms A1-A4 are satisfied, then for all $\boldsymbol{h}_t \in \mathcal{H}$, and for all $(x, m) \in \Delta(\mathcal{A}) \times \mathbb{R}$, there exist a unique compensation $c_{\boldsymbol{h}_t}(x, m) \in \mathbb{R}_+$, such that $(\diamond, m + c_{\boldsymbol{h}_t}(x, m)) \sim_{\boldsymbol{h}_t} (x, m)$. Moreover, $c_{\boldsymbol{h}_t}(\diamond, m) = 0$ for all $\boldsymbol{h}_t \in \mathcal{H}$, and for all $m \in \mathbb{R}$.

Lemma 2 ensures that, after any history of past consumption, there exists a unique amount of money that compensates the DM just enough to make her indifferent to get the lottery or alternatively, receiving an appropriate monetary compensation for not getting it. Notice that in the same way that expected utility theory provides commensurability between outcomes and probability, Lemma 2 establishes commensurability between lotteries and money. Notice also that the choice of notation makes clear that for any history of past consumption, and any choice (x, m), the appropriate compensation $c_{\boldsymbol{h}_t}(x, m)$ depends (obviously) on the lottery

[11]All the proofs are contained in Appendix.

[12]In one of the extensions of the HDU model that I will present later this assumption is easily relaxed and a good might become a bad depending of the history of past consumption.

being considered x, but also might depend on the amount of money m, and the history of past consumption $\boldsymbol{h}_t$.

Axiom 5 *(Separability)*:

i) *Coordinate Independence*: For all $\boldsymbol{h}_t \in \mathcal{H}$, $(x, m) \succsim_{\boldsymbol{h}_t} (y, m)$, if and only if, $(x, m') \succsim_{\boldsymbol{h}_t} (y, m')$.

ii) *Thomsen Condition:* For all $\boldsymbol{h}_t \in \mathcal{H}$, if $(x, m) \sim_{\boldsymbol{h}_t} (y, m')$ and $(y, m'') \sim_{\boldsymbol{h}_t} (z, m)$, then $(x, m'') \sim_{\boldsymbol{h}_t} (z, m')$.

Note that coordinate independence for money, $(x, m) \succsim_{\boldsymbol{h}_t} (x, m')$ if and only if $(y, m) \succsim_{\boldsymbol{h}_t} (y, m')$, is already implied by money monotonicity. Given the topological properties of the choice space $\Delta(\mathcal{A}) \times \mathbb{R}$, Coordinate independence and Thomsen condition, in conjunction with the previous axioms, will suffice to ensure the existence of an additive separable representation of $\succsim_{\boldsymbol{h}_t}$.

Axiom 6 *(Independence)*: For all $\boldsymbol{h}_t \in \mathcal{H}$, and for all $(x, m), (y, m) \in \Delta(\mathcal{A}) \times \mathbb{R}$, $z \in \Delta(\mathcal{A})$, and $\theta \in (0, 1]$:

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m) \Leftrightarrow \big(\theta x + (1-\theta)z, m\big) \succsim_{\boldsymbol{h}_t} \big(\theta y + (1-\theta)z, m\big)$$

Axiom 6 is just the standard expected utility independence axiom but imposed only to the first coordinate of the ordered pair, that is the lotteries coordinate.

3.3 Dynamic Axioms

Now, I present the two key axioms of this axiomatization, the satiation and recovery axioms. In order to state the following two axioms in a simpler way, we will make use of the notation introduced in Lemma 2. Recall that $c_{\boldsymbol{h}_t}(x, m) \in \mathbb{R}_+$ is the unique compensation such that $(\diamond, m + c_{\boldsymbol{h}_t}(x, m)) \sim_{\boldsymbol{h}_t} (x, m)$. Moreover, through this paper, whenever we encounter $\boldsymbol{h}_t$ and $\boldsymbol{h}_{t-1}$, we will assume that $\boldsymbol{h}_t = (\boldsymbol{h}_{t-1}, (r_{t-1}, m_{t-1}))$, that is, all elements of the vectors $\boldsymbol{h}_t$ and $\boldsymbol{h}_{t-1}$ are equal up to time $t-2$ but $\boldsymbol{h}_t$ also includes the realization of the lottery chosen at $t-1$. Therefore, we will interpret $c_{\boldsymbol{h}_{t-1}}(x, m)$ and $c_{\boldsymbol{h}_t}(x, m)$ as the appropriate monetary compensations that the DM requires for giving up the lottery x after history $\boldsymbol{h}_{t-1}$ and $\boldsymbol{h}_t$ respectively, taking into account that both histories agree up to $t-2$ but $\boldsymbol{h}_t$ also includes the realization of the lottery chosen at $t-1$. Similarly, $c_{\boldsymbol{h}_0}(x, m)$ is the compensation that the DM requires when there is no history of past consumption.

Axiom 7 *(Satiation):* For every $t, t' \in \mathcal{T}$ and every $(a_i, m) \in \mathcal{A} \times \mathbb{R}$:

i) If $(r_{t-1}, m_{t-1}) = (a_i, m)$, then $\big(\diamond, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$.

ii) If $(r_{t-1}, m_{t-1}) = (r_{t'-1}, m_{t'-1}) = (a_i, m)$ and $\big(ka_i+(1-k)\diamond, m\big) \sim_{\boldsymbol{h}_{t-1}} \big(\diamond, m+c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big)$ for $k \in [0, 1]$, then $\big(ka_i + (1-k)\diamond, m\big) \sim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_{t'}}(a_i, m)\big)$.

Part i) of the Satiation axiom states that if a_i was consumed at $t-1$, then at time t, the DM will require a smaller (or equal) compensation for not getting the degenerate lottery a_i. This captures the key ingredient of the model, the satiation process, and hence it justifies the name the axiom. Part ii) of the Satiation axiom basically states that if at two different moments t and t' the realization of the lottery was the same and the corresponding unique compensations were proportional (in terms of k), then they remain proportional after the consumption of the realization of the lottery. This implies that the satiation process can be described by a constant satiation rate. However, notice that this axiom does not preclude alternative-specific satiation rates.

Axiom 8 *(Recovery):* For every $t, t' \in \mathcal{T}$ and every $(a_i, m) \in \mathcal{A} \times \mathbb{R}$:

i) If $(r_{t-1}, m_{t-1}) \neq (a_i, m)$, then $\big(\diamond, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big)$.

ii) If $(r_{t-1}, m_{t-1}) \neq (a_i, m)$, $(r_{t'-1}, m_{t'-1}) \neq (a_i, m)$, $\big(\diamond, m+c_{\boldsymbol{h}_0}(a_i, m)\big) \succ_{\boldsymbol{h}_t} \big(\diamond, m+c_{\boldsymbol{h}_t}(a_i, m)\big)$ and $\big(ka_i + (1-k)\diamond, m\big) \sim_{\boldsymbol{h}_{t-1}} \big(\diamond, m + c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big)$ for $k \in (0, 1]$, then $\big(ka_i + (1-k)\diamond, m\big) \sim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_{t'}}(a_i, m)\big)$.

Part i) of the Recovery axiom states that if a_i was **not** consumed at $t-1$, then at time t, the DM requires a greater (or equal) compensation for not getting the degenerate lottery a_i. This captures another key ingredient of the model, the recovery process. In the same way the Satiation axiom captures the fact that after consuming an alternative the DM might value the alternative less, the Recovery axiom allows the valuation of an alternative to recover after a period of non-consumption. Notice also that part i) implies that this recovery process has an upper bound. In words, it implies that the valuation of any degenerate lottery a_i will never exceed the valuation of that lottery when there was no history of past consumption. We refer to this maximal valuation of a lottery as its intrinsic valuation. Part ii) of the Recovery axiom states that whenever at two different moments t and t' there is recovery (but not full recovery) and the corresponding unique compensations were proportional (in terms of k), then they remain proportional after the recovery process. Likewise, this implies that the recovery process can be described by a constant recovery rate. Again, notice that this axiom does not preclude alternative-specific recovery rates.

3.4 Time Preference Axioms

Before I introduce the last two axioms of this exercise, the time preferences axioms, let me formally define for any consumption stream $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ its associated stream of appropriate monetary compensations $\Diamond(\boldsymbol{x}) \in (\{\diamond\} \times \mathbb{R})^{T+1}$.

Definition 2: For any sequence of choices $\boldsymbol{x} = \big((x_0, m_0), (x_1, m_1), \ldots, (x_T, m_T)\big) \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ define $\Diamond(\boldsymbol{x})$ as,

$$\Diamond(\boldsymbol{x}) \equiv \Big(\big(\diamond, m_0 + c_{\boldsymbol{h}_0}(x_0, m_0)\big), \big(\diamond, m_1 + c_{\boldsymbol{h}_1}(x_1, m_1)\big), \ldots, \big(\diamond, m_T + c_{\boldsymbol{h}_T}(x_T, m_T)\big)\Big)$$

where $\boldsymbol{h}_t$ is the history generated by $\boldsymbol{x}$ and $c_{\boldsymbol{h}_t}(x_t, m_t)$ are the unique compensations such that $(x_t, m_t) \sim_{\boldsymbol{h}_t} (\diamond, m_t + c_{\boldsymbol{h}_t}(x_t, m_t))$, for every $t \in \mathcal{T}$.

Notice that, as a consequence of Lemma 2, for any given $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, the vector $\Diamond(\boldsymbol{x}) \in (\{\diamond\} \times \mathbb{R})^{T+1}$ is uniquely defined.

I am now prepared to state the following fundamental axiom:

Axiom 9 *(Indifference)*: For any consumption plan $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$, $\boldsymbol{x} \sim \Diamond(\boldsymbol{x})$.

In words, this axiom says that given any consumption stream $\boldsymbol{x}$, if a DM is indifferent at each point in time between getting the lottery prescribed by $\boldsymbol{x}$, or getting an appropriate monetary compensation for getting the degenerate lottery $\diamond$ instead, then she must be indifferent between the consumption plan specified by $\boldsymbol{x}$ and the appropriate stream of monetary compensations specified by $\Diamond(\boldsymbol{x})$. The Indifference axiom extends the commensurability established in Lemma 2, between lotteries and money in the choice space, to the consumption streams space. This axiom is very similar in spirit to "Monotonicity in Prizes" axiom from Anscombe and Aumann (1963), and even more similar to the the "Substitutibility" axiom from Luce and Raiffa (1957).

Moreover, this axiom creates a one to one mapping over consumption plans and streams of appropriate monetary compensations hence, allowing us to rank any pair of consumption plans $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ by using the ranking between $\Diamond(\boldsymbol{x})$ and $\Diamond(\boldsymbol{y})$. Since there is no history-dependence in the space $(\{\diamond\} \times \mathbb{R})^{T+1}$ this allow us to separate the effects of time from pure history dependence (Axiom 7 and 8). Therefore, this axiom can also be considered a weaker version of a time separability axiom appropriate for history-dependent preferences.

Finally, notice also that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$. Therefore, it suffices to impose the standard time-discount axioms on the preference relation $\succsim^*$ defined on $\mathbb{R}^{T+1}$, to get the desired time preference representation, either exponential, quasi-hyperbolic, or any other. In order to be as close as possible to the standard model of intertemporal choice, the EDU model, we present next the last axiom needed to ensure the existence of an exponential discounting representation of $\succsim^*$ and hence of $\succsim$.[13]

Axiom 10 *(Exponential Discounting)*:

i) *(Separability):* All $E \subseteq \mathcal{T}$ are separable.

ii) *(Impatience):* For all $a, b \in \mathbb{R}$ if $a \succ^* b$, then for all $\boldsymbol{x} \in \mathbb{R}^{T+1}$, $(a, b, x_2, x_3, \ldots, x_T) \succ^* (b, a, x_2, x_3, \ldots, x_T)$.

iii) *(Stationarity):* For all $d \in \mathbb{R}$ and $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^{T+1}$ we have $(d, x_0, \cdots, x_{T-1}) \succsim^* (d, y_0, \cdots, y_{T-1})$, if and only if, $(x_0, \cdots, x_{T-1}, d) \succsim^* (y_0, \cdots, y_{T-1}, d)$.

Part iii) of the previous axiom, which is an appropriate finite horizon version of the Stationarity postulated in Koopmans (1960), is the key condition to ensure the existence of an additive separable representation of $\succsim^*$. It amounts to say that if we take two sequences that are shortened by one period it does not matter whether we place d at the beginning or at the end.

3.5 Representation Theorem

We are now ready to present the main result of this paper, the HDU model representation theorem.

Theorem 1. *A binary relation $\succsim$ on $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ satisfies Axioms (1-10) if and only if it has a HDU representation given in definition (2.1).*

Furthermore, if the decision maker can only choose from the set of degenerate lotteries $\mathcal{A}$, we obtain the following corollary:

Corollary 1.1. *A binary relation $\succsim$ on $(\mathcal{A} \times \mathbb{R})^{T+1}$ satisfies Axioms (1-10) if and only if it has a HDU representation given in definition (2.2).*

4 Eliciting the Preference Parameters

In this section I investigate how to elicit the preferences parameters of the model, that is how to elicit $v(\cdot)$, δ, and $u_0(a_i)$, λ_{a_i}, β_{a_i} for all $a_i \in \mathcal{A}$. To that purpose, I present a choice-based elicitation procedure that allows me to elicit the aforementioned key parameters in a very simple way. The methodology I propose is based on the axiomatic characterization presented in the previous section.

[13] See the appendix for additional definitions and results from the exponential discounting literature.

The strategy is the following, recall that due to the Separability axiom, DM's preferences for money are independent of her preferences over the alternatives, therefore $v(\cdot)$ and $u_0(a_i)$ can be elicited independently. I will start by making use of the standard gamble approach and linear interpolation to non-parametrically elicit $v(\cdot)$. Then, with $v(\cdot)$ known, I show how we can easily recover the rest of the parameters. The main advantages of this approach is twofold. First, by making use of the results of the axiomatic characterization, this strategy is able to isolate the effect of time from history dependence and hence, it allows us to independently elicit the time preference parameter δ and the history-dependent parameters λ_{a_i} and β_{a_i}. Second, as lemma 3 will show, $v(\cdot)$ can be elicited with arbitrary precision, and since all other parameters are identified once $v(\cdot)$ is known, the same is true for the rest of the parameters.

4.1 Eliciting $v(\cdot)$

Without loss of generality I will elicit $v(\cdot)$ defined over a bounded domain. In particular I will assume that $v : [0, M] \to \mathbb{R}$ where M can be arbitrarily large and context dependent.[14] I start by creating a partition $\mathcal{P}$ of $[0, M]$ in K equally-spaced subintervals $[m_i, m_{i+1}]$ such that $0 = m_0 < m_1 < m_2 < \cdots < m_K = M$. By expected utility theory we know that for any m_i with $i \in \{1, \ldots, K-1\}$ there exist a unique $\alpha(m_i)$ such that:

$$m_i \sim \alpha(m_i)M + \big(1 - \alpha(m_i)\big)0$$

By using the expected utility representation this means

$$v(m_i) = \alpha(m_i)v(M) + \big(1 - \alpha(m_i)\big)v(0)$$

As it is standard in this approach, I normalize the utility of the best outcome to 1 and the utility of the worse outcome to 0, hence substituting in the above equation $v(M) = 1$ and $v(0) = 0$ we find that

$$v(m_i) = \alpha(m_i) \tag{5}$$

We can use the staircase method to find such $\alpha(m_i)$ and linear interpolation to fill the gaps between $v(m_i)$ and $v(m_{i+1})$.[15] With this procedure we end up with a non-parametric elicitation

[14]By context dependent I mean that M might depend on the set of alternatives $\mathcal{A}$. For example if $\mathcal{A}$ is a set of different types of candies, M can be relatively small but if $\mathcal{A}$ is a set of luxury cars, M must be much larger.

[15]The staircase method consists of a sequence of choices designed to find a preference indifference. In this specific context, in the first choice situation the DM must chose between a sure payment of m_i units of money or a lottery that assigns probability α to M and probability $1 - \alpha$ to 0. If the sure payment is chosen, in the following choice situation α is increased. Otherwise, in the following choice situation α is decreased. This procedure repeats until an indifference with arbitrary precision is found. An example of such a method can be found in Falk, Becker, Dohmen, Enke, Huffman, and Sunde (2017). Linear interpolation consists of using the line segment joining $v(m_i)$ and $v(m_{i+1})$ as an approximation of v in the interval $[m_i, m_{i+1}]$.

of $v : [0, M] \to \mathbb{R}$. Notice that the finer the partition the higher the precision with which we elicit $v(\cdot)$. The following lemma formalizes this idea and implies that using this method we can approximate $v(\cdot)$ with arbitrary precision by using an increasingly finer partition.

Lemma 3 *(Total Interpolation error):* An upper bound for the total interpolation error when using linear interpolation and K equally-spaced subintervals is:

$$S_K = \sum_{i=0}^{K-1} \frac{(m_{i+1} - m_i)(v(m_{i+1}) - v(m_i))}{2}$$

Furthermore, $S_{K+1} < S_K$ and $\lim_{K\to\infty} S_K = 0$.

This lemma provides an upper bound for the interpolation error and shows that it decreases the finer the partition is. Therefore, it implies that the finer the partition the lowest the interpolation error, and hence that v can be estimated with arbitrary precision.

4.2 Eliciting δ

The simplest way to elicit δ is to pick an arbitrary and small enough $m \in [0, M]$ for which there exists a unique $m' \in [0, M]$ such that

$$\big((\diamond, m), (\diamond, 0), \ldots, (\diamond, 0)\big) \sim \big((\diamond, 0), (\diamond, m'), (\diamond, 0), \ldots, (\diamond, 0)\big)$$

By using the HDU representation this translates to,

$$u_{\boldsymbol{h}_0}(\diamond) + v(m) + \sum_{t=1}^{T} \delta^t \big[u_{\boldsymbol{h}_t}(\diamond) + v(0)\big] = u_{\boldsymbol{h}_0}(\diamond) + v(0) + \delta\big[u_{\boldsymbol{h}_1}(\diamond) + v(m')\big] + \sum_{t=2}^{T} \delta^t \big[u_{\boldsymbol{h}_t}(\diamond) + v(0)\big]$$

Notice that as shown in step 1 of the proof of the representation theorem, $u_{\boldsymbol{h}_t}(\diamond) = 0$ for all $\boldsymbol{h}_t \in \mathcal{H}$. Also by the previous normalization of $v(0) = 0$, the previous equation simplifies to $v(m) = \delta v(m')$, hence

$$\delta = \frac{v(m)}{v(m')} \tag{6}$$

In order to find such a m' we can also use the staircase method. Notice that with this method, the precision with which we elicit δ depends on the precision of the estimation of v.

4.3 Eliciting $u_0(a_i)$

Again we can use the staircase method to find $c_{\boldsymbol{h}_0}(a_i, m) \geq 0$ such that

$$\big((a_i, m), (\diamond, m), \ldots, (\diamond, m)\big) \sim \big((\diamond, m + c_{\boldsymbol{h}_0}(a_i, m)), (\diamond, m), \ldots, (\diamond, m)\big)$$

Using the HDU representation and the fact that $u_{\boldsymbol{h}_t}(\diamond) = 0$ for all $\boldsymbol{h}_t \in \mathcal{H}$ we get that

$$u_0(a_i) = u_{\boldsymbol{h}_0}(a_i) = v(m + c_{\boldsymbol{h}_0}(a_i, m)) - v(m) \tag{7}$$

4.4 Eliciting λ_{a_i}

Similarly to the previous subsection we find $c_{\boldsymbol{h}_1}(a_i, m) \geq 0$ such that

$$\big((a_i, m), (a_i, m), (\diamond, m) \ldots, (\diamond, m)\big) \sim \big((a_i, m), (\diamond, m + c_{\boldsymbol{h}_1}(a_i, m)), (\diamond, m), \ldots, (\diamond, m)\big)$$

and we arrive to the conclusion that

$$u_{\boldsymbol{h}_1}(a_i) = v(m + c_{\boldsymbol{h}_1}(a_i, m)) - v(m)$$

As shown in step 3 of the representation theorem,

$$\lambda_{a_i} = \frac{u_{\boldsymbol{h}_1}(a_i)}{u_{\boldsymbol{h}_0}(a_i)}$$

Using this relationship and the previous results we finally get

$$\lambda_{a_i} = \frac{v(m + c_{\boldsymbol{h}_1}(a_i, m)) - v(m)}{v(m + c_{\boldsymbol{h}_0}(a_i, m)) - v(m)} \tag{8}$$

4.5 Eliciting β_{a_i}

Suppose that for a particular history $\boldsymbol{h}_t$, $\big(\diamond, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \succ_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$ and $(r_{t-1}, m_{t-1}) \neq (a_i, m)$, then as shown in in step 4 of the representation theorem,

$$\frac{1}{\beta_{a_i}} = \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)}$$

then similarly to the previous subsection we elicit β_{a_i} as follows

$$\beta_{a_i} = \frac{v(m + c_{\boldsymbol{h}_{t-1}}(a_i, m)) - v(m)}{v(m + c_{\boldsymbol{h}_t}(a_i, m)) - v(m)} \tag{9}$$

Equations (5) to (9) fully determine the parameters of the model.

5 Applications of the HDU Model

This section aims to demonstrate the broad scope of applicability of the HDU model. I consider two very different but illustrative applications. The first application lies in the realm of theoretical industrial organization. Specifically, I study monopolist's optimal dynamic pricing strategies in intertemporal discrete choice settings facing variety-seeking consumers. This application exemplifies how standard economic theory results should be revisited in light of the HDU model whenever variety-seeking behavior occurs.

In the second application, I show how the modeling tools provided by the HDU model can help tackle one of the most urgent threats to public health, the antibiotic resistance threat. In particular, I show how the results presented in Sections 3 and 4 might be used to design antibiotic treatment plans, to fight bacterial infections more effectively while minimizing the threat of developing antibiotic resistance. Finding such treatment plans is crucial for society since the implacable advance of antibiotic resistance makes treating common infectious diseases increasingly complicated and sometimes even impossible. Moreover, antibiotic resistance has been predicted to cause 10 million deaths per year by 2050 and to have a cumulative cost of 100 trillion USD (O'Neill (2016)).

5.1 IO application: Monopolist's Optimal Dynamic Pricing Strategies Facing Variety-seeking Consumers

Optimal pricing is one of the most fundamental questions any profit-maximizing firm should address. Static pricing strategies that ignore the repeated interaction nature of most customer-seller relationships are often inefficient. In contrast, dynamic pricing strategies have proven effective tools to increase revenue in such environments (see Sweeting (2012)). Moreover, in recent years, dynamic pricing strategies have been widely adopted due to i) increased availability of demand data, ii) the introduction of new technologies that allow sellers to change prices quickly, and iii) the availability of decision-theoretic models that allow firms to analyze demand data better, and set up educated dynamic pricing policies (Elmaghraby and Keskinocak (2003)).

However, most of the dynamic pricing literature does not account for variety-seeking behavior so far. Variety-seeking behavior has been documented to be an essential factor in understanding consumer demand. Variety-seeking behavior has been empirically confirmed in a wide array of product categories. Those categories include hedonistic consumption products like soft drinks, beers, songs, ice creams, fruits, candy, hotels, restaurants, and holiday activities but also utilitarian consumption products like toothpaste, headache remedies, paper towels, or shampoo[16].

In the HDU model, variety-seeking behavior arises primarily due to the satiation effect that consumers experience after consuming a product. This preference-based explanation to variety-seeking behavior implies that current consumption choices affect future choices. Therefore, it is vital to analyze the implications of variety-seeking behavior for the pricing strategies of firms within a dynamic framework.

[16]See Table 1 from Cosguner et al. (2018) for an account of categories of products in which variety-seeking behavior has been empirically demonstrated.

5.1.1 Optimal Pricing Strategy

First, I analyze monopolist's optimal dynamic pricing behavior in intertemporal discrete choice settings facing variety-seeking consumers. With this framework in mind, I will show that instead of taking the traditional but more cumbersome route of defining consumer's dynamic strategic demand and then finding monopolist's profit-maximizing pricing strategy, we can take a different approach circumventing the challenging task of defining consumer's dynamic strategic demand.

Consider a game $\mathcal{G}$ in which a standard profit-maximizing monopolist and a variety-seeking consumer whose preferences are consistent with the HDU model meet in the market for infinitely many periods, each consisting of two phases. At the beginning of a given period t, the monopolist will list the vector of prices for the N_m available commodities. Then, the consumer will purchase her preferred alternative within the set of products offered by the monopolist $\mathcal{A}_{N_m}$, at its respective price.[17] The following figure shows the timeline of the game.

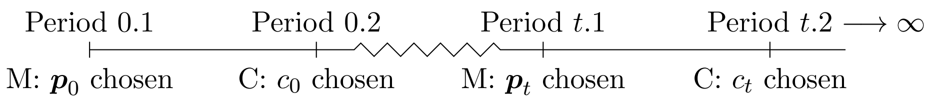


Figure 2: Timeline of $\mathcal{G}$.

I assume that consumer's preferences are consistent with the HDU model representation given in Definition 2.2, the satiation and recovery parameters are the same for all alternatives, and the (dis)utility of prices enter linearly. Consumer's budget for a given period m_t will exceed the intrinsic valuation of the most preferred alternative, so all alternatives are affordable at any time. Furthermore, the consumer cannot make intertemporal transfers of money. Hence, any money not spent in a given period cannot be used in the following period(s). Finally, I assume there are no restrictions on the prices that the monopolist can set on each period and that the marginal cost of producing any units of the available N_m varieties is zero.

Let me now formulate the payoff function for the monopolist in game $\mathcal{G}$. Let $\boldsymbol{p}_t \in \mathbb{R}_+^{N_m}$ be the vector of prices for the N_m alternatives at period t and let $c_t \in \mathcal{A}_{N_m}$ be the consumer's chosen alternative at period t. The monopolist's payoff (profit) function therefore takes the following form:

$$\pi_m \equiv \sum_{t=0}^{\infty} \sum_{a_j \in \mathcal{A}_{N_m}} \delta_m^t p_t^{a_j} \mathbb{1}_{(c_t = a_j)} \tag{10}$$

where δ_m denotes the monopolist's time-discounting factor. Now, consider a particular consumption stream $\boldsymbol{c} = (c_1, c_2, \ldots)$ and the following strategy $\sigma_m(\boldsymbol{c})$ for the monopolist: at phase

[17] Formally, I assume that the consumer has well-defined variety-seeking preferences, consistent with the HDU model, over the set $(\mathcal{A}_{N_m} \times \mathbb{R})^{\infty}$.

1 of each period t, set price vector $\boldsymbol{p}_t$ such that

$$\begin{aligned} p_t^{a_i} &= u_{\boldsymbol{h_t}}(a_i) + \epsilon \text{ for all } a_i \neq c_t \\ p_t^{c_t} &= u_{\boldsymbol{h_t}}(c_t) - \epsilon \text{ if } c_t \neq \diamond, \end{aligned} \tag{11}$$

for an arbitrarily small $\epsilon > 0$.

Result 1. *Suppose the monopolist follows strategy $\sigma_m(\boldsymbol{c})$ for some consumption stream $\boldsymbol{c}$. Then, choosing c_t at period t for all $t \in \mathbb{N}$ is a best response for the consumer. Moreover, $\sigma_m(\boldsymbol{c})$ is the profit-maximizing (cheapest) way to induce $\boldsymbol{c}$.* [18]

Result 1 has two significant implications. Firstly, if the monopolist follows strategy $\sigma_m(\boldsymbol{c})$, it can induce any given consumption path $\boldsymbol{c}$ for any consumer, regardless of her time-discounting parameter δ_c. On the other hand, the monopolist can extract (almost) all surplus at every point in time. This result follows from the fact that the monopolist may alter prices freely every period. Thus, the consumer understands that there is no gain in choosing an alternative different to c_t in any given period. In other words, she knows that, by purchasing c_t, she will gain a surplus of ϵ in period t. In turn, by deviating, she will get an instantaneous payoff of at most zero. Moreover, this deviation does not imply any greater gain in the future than she would get if they follow their original strategy. This is so because the monopolist will react by modifying prices, absorbing all potential gains from the consumer's deviation.

Thus, the monopolist's payoff associated to $\sigma_m(\boldsymbol{c})$ for some consumption stream $\boldsymbol{c}$ can be written as follows:

$$\pi_m(\boldsymbol{c}) = \sum_{t=0}^{\infty} \delta_m^t (u_{\boldsymbol{h_t}}(c_t) - \epsilon). \tag{12}$$

Hence, the monopolist will optimally choose the consumption stream $\boldsymbol{c^*} \in (\mathcal{A}_{N_m})^\infty$ that maximizes $\pi_m(\cdot)$ from (12). It is trivial to see that this is equivalent to solving the following problem:

$$\max_{\boldsymbol{c} \in (\mathcal{A}_{N_m})^\infty} \sum_{t=0}^{\infty} \delta_m^t u_{\boldsymbol{h_t}}(c_t). \tag{13}$$

Result 2. *The monopolist's problem is equivalent to that of a fully forward-looking consumer with utility parameters $(\lambda, \beta, \delta_m)$ who chooses her preferred consumption stream over the set of alternatives $\mathcal{A}_{N_m}$.*

Result 2 has several implications that I must highlight. First of all, it tells us that the monopolist should optimally mimic the behavior of a fictional fully forward-looking consumer

[18] This result, as well as all the results that are derived from this one, hold regardless of consumer's degree of foresight. In particular, they are true for fully forward-looking, k-periods forward-looking, and completely myopic agents.

with parameters $(\lambda, \beta, \delta_m)$. That is, the monopolist should first solve the utility maximization problem of a hypothetical consumer that has the same satiation and recovery parameters as the consumer he is facing but with monopolist's time discount parameter and then use strategy $\sigma_m(\boldsymbol{c}^*)$ to induce such a profit-maximizing plan. This is a consequence of the fact that strategy $\sigma_m(\boldsymbol{c})$ allows the monopolist to extract all surplus generated by consumption stream $\boldsymbol{c}$. Hence, he must choose the stream that maximizes total surplus. This is done by internalizing the effect that satiation and recovery cause on consumer's utility, i.e., by maximizing the discounted sum of history-dependent utilities. Moreover, the resulting allocation is Pareto-efficient.[19]

5.1.2 Optimal Number of Varieties

Finally, I present an extended version of the game that will be referred to as $\mathcal{G}^e$, in which we introduce a new period that precedes our original game. In this initial period, the monopolist endogenously determines the optimal number of varieties he should produce, given the fixed cost of producing a variety.

I assume that the consumer has well-defined variety-seeking preferences, consistent with the HDU model, over the set $(\mathcal{A}_N \times \mathbb{R})^\infty$, where $\mathcal{A}_N = \{a_1, a_2, \ldots, a_{N-1}, \diamond\}$. For simplicity, consider the intrinsic ranking of preferences over the alternatives is such that $a_1 \succsim_0 a_2 \succsim_0 \cdots \succsim_0 a_{N-1} \succ_0 \diamond$, and the satiation and recovery parameters are the same for all alternatives. The monopolist's problem is, therefore, to choose the number of varieties $N_m \leq N$ to produce.[20] Just like before, I also assume that, once N_m is fixed, the monopolist can freely produce any number of units of each of the N_m varieties. However, he must pay a cost for offering a different variety. Let $\zeta(N_m)$ be the cost of producing N_m varieties for the monopolist and assume $\zeta(\cdot)$ is a strictly increasing function.

In this setting, I show how to find the optimal number of varieties the monopolist should produce facing a variety-seeking consumer with any combination of satiation, recovery, and time preferences parameters.

Let $\boldsymbol{c}^*_{\boldsymbol{N_m}} \equiv \underset{\boldsymbol{c} \in (\mathcal{A}_{N_m})^\infty}{\arg\max} \sum_{t=0}^{\infty} \delta_m^t u_{\boldsymbol{h_t}}(c_t | \, N_m)$ and let $V^*(N_m) = U(\boldsymbol{c}^*_{\boldsymbol{N_m}}) = \sum_{t=0}^{\infty} \delta_m^t u_{\boldsymbol{h_t}}(c_t^* | \, N_m)$. Note that $V^*(N_m)$ denotes the monopolist's (maximum) revenues associated to offering N_m

[19] A more rigorous treatment and more results can be found in the companion paper (Puig-Pomés and Sanchez-Moscona 2021). In the aforementioned companion paper, among other results, we show how key properties of the HDU model can be used to find the consumer's optimal consumption path given her satiation, recovery, and time discount parameters. Moreover, given Results 1 and 2 presented in this section, how to find the solution to monopolist's maximization problem.

[20] We can also allow the monopolist to choose $N_m > N$, that is to allow the monopolist to introduce new varieties in the market. In this case, the monopolist must form beliefs about the consumer's intrinsic valuation of each new variety and their associated satiation and recovery parameters. Notice that whenever a variety-seeking consumer chooses a new variety, she will start experimenting for some time until she finds out her intrinsic valuation of this variety and its associated satiation and recovery parameters. After that window of experimentation, she will behave as predicted by the HDU model.

varieties. Hence, in Period 0, the monopolist must choose the number of varieties N_m^* that yields maximum profits.

Result 3. *In equilibrium, the monopolist chooses the number of varieties N_m^* in Period 0 that satisfies*

$$N_m^* = \underset{N_m \in \{1,\ldots,N\}}{\arg\max}\ V^*(N_m) - \zeta(N_m). \tag{14}$$

Thus, in the subgame-perfect equilibrium of $\mathcal{G}^e$, the monopolist chooses N_m^* according to (14) in period 0, and then both players behave according to the strategies discussed in $\mathcal{G}$. Result 3 completely characterizes how the monopolist should optimally choose the number of varieties to produce. Notice that the more impatience the monopolist is, the faster the recovery compared to the satiation parameter, and the higher the fixed cost of producing a new variety, the fewer varieties the monopolist should optimally produce.

5.2 Health Application: Describing the Optimal Sequence of Antibiotics to Maximize Effectiveness and Avoid Resistance

Infections by drug-resistance pathogens constitute a significant threat to society today and will be even more so in the decades to come. Antibiotic resistance has been predicted to cause 10 million deaths per year by 2050 and to have a cumulative cost of 100 trillion USD (O'Neill (2016)). Antibiotic resistance can arise naturally but has also been linked to overuse and misuse of antibiotics (Ventola (2015)). Therefore, it is crucial to optimally design effective treatment plans that minimize the possibility of treatment resistance. One such plan is the so-called *alternating-drug therapy* in which a set of antibiotics are sequentially alternated each round of treatment. This strategy has been proven effective in order to reduce the possibility of treatment resistance while avoiding the toxicity associated with more traditional *combination-drug therapy*, in which a cocktail of antibiotics is administered at each treatment round. In Kim et al. (2014), the authors show that alternating-drug therapy slows the rate of increase in resistance compared with single-drug treatments. Moreover, in another study by Fuentes-Hernandez et al. (2015), it is shown that by using alternating-drug therapy, the elimination of the bacterial infection can be achieved at antibiotic dosages so low that the equivalent two-drug combination treatments are ineffective. Hence, the critical question we should address is: Which alternating sequence of antibiotics should be prescribed to a patient to achieve bacterium clearance while minimizing antibiotic resistance?

In this subsection, I will show how we can exploit the modeling tools provided by the HDU model, in particular the results presented in Sections 3 and 4, to answer this question. Recall that the key elements of the HDU model are: the set of intrinsic utilities $\{u_0(a_1), u_0(a_2), \cdots, u_0(a_{N-1}), u_0(\diamond)\}$, the utility function for money $v : \mathbb{R} \to \mathbb{R}$, the time discount rate δ, the satiation rate λ, and the recovery rate $\frac{1}{\beta}$. In order to apply the HDU model to shed light on this important public health problem, we should reinterpret some of those key elements. In this setting, I interpret $\mathcal{A}$ as the set of available (appropriate) antibiotics to treat the bacterial infection. The intrinsic utility $u_0(a_i)$ is the pretreatment measure of bacteria's sensitivity (susceptibility) to antibiotic a_i. $v(p_t^{a_i})$ as the dis-utility generated by paying the price $p_t^{a_i}$ for antibiotic a_i. λ_{a_i} as the *resistance* rate of antibiotic a_i, the rate at which sensitivity of bacteria to antibiotic a_i decreases. In accordance, I will interpret $\frac{1}{\beta_{a_i}}$ as the recovery rate, the rate at which sensitivity of bacteria to antibiotic a_i is regained. It turns out that the answer to our question of interest is the solution to the following maximization problem:

$$\max_{\{x_t\}_0^T} \sum_{t=0}^{T} \delta^t \left[\psi_t(x_t|x_{t-1}) u_0(x_t) - v(p_t^{x_t}) \right]$$

where for all $z_t \in \mathcal{A}$, and for all $t > 0$

$$\psi_t(z_t|z_{t-1}) = \begin{cases} \lambda_{z_t} \cdot \psi_{t-1}(z_t|z_{t-2}) & \text{if } z_t = z_{t-1} \\ \min\left\{1, \frac{1}{\beta_{z_t}} \cdot \psi_{t-1}(z_t|z_{t-2})\right\} & \text{if } z_t \neq z_{t-1} \end{cases}$$

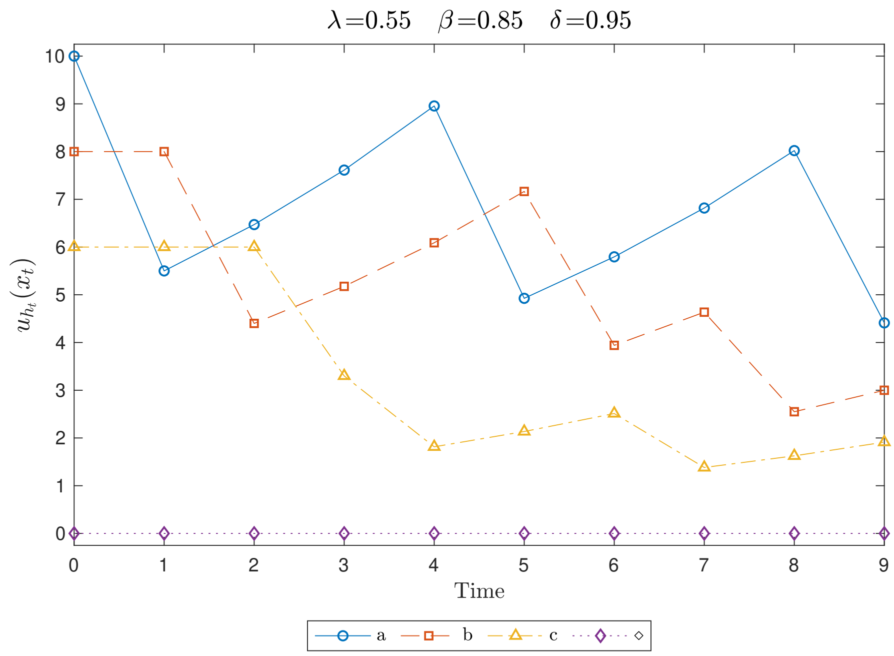
$$\psi_0(z_t|\boldsymbol{h}_0) = 1, \quad \lambda_{x_t} \in (0,1], \quad \text{and} \quad \beta_{x_t} \in (0,1]$$

which is precisely the same problem our fully forward-looking consumer would solve. The following example illustrates this point:

Example 3. *Let* $\mathcal{A} = \{a, b, c, \diamond\}$, $u_0(a) = 10$, $u_0(b) = 8$, $u_0(c) = 6$, $u_0(\diamond) = 0$, $\lambda_a = \lambda_b = \lambda_c = \lambda = 0.55$, $\beta_a = \beta_b = \beta_c = \beta = 0.85$, $\delta = 0.95$, $T = 10$ *and* $p_t^{a_i} = 0$ *for all* $a_i \in \mathcal{A}$ *. Given this parameter specification, the solution to the above maximization problem is the following treatment plan* $\boldsymbol{x}^* = (a, b, c, c, a, b, c, b, a, a)$*. Figure 3 shows the optimal sequence of antibiotic prescription* $\boldsymbol{x}^*$*.*

As the previous example shows, once the model's parameters have been identified, the optimal prescription plan of antibiotics that maximizes efficacy while minimizing antibiotic resistance arises naturally as the solution to a simple maximization problem. Therefore, it is crucial to identify the parameters of the model correctly. Now, building on the results of Section 4, I explain how a medical researcher can identify the model's parameters in this specific application. Note that in this application, the medical researcher acts as a social planner, and hence the utility function for money v and the time discount parameter δ can be elicited in the same way described in section 4. By using quantitative antibiotic susceptibility testing methods (AST), the set of pretreatment measures of the sensitivity (susceptibility) of bacteria to each antibiotic $\{u_0(a_1), u_0(a_2), \cdots, u_0(a_{N-1}), u_0(\diamond)\}$ can be estimated. AST methods are usually used to determine the most effective antibiotic treatment for a bacterial infection and determine if a particular strain of bacteria is becoming drug-resistant (Heller

Figure 3: Optimal treatment plan.



and Spence (2019)). Quantitative AST methods estimate the minimal concentration of a drug that inhibits visible growth of bacteria, this concentration is usually referred to as *minimum inhibitory concentration* (MIC). Antibiotics with lower MIC values are more effective than those with higher values. Thus, for each a_i, we can set $u_0(a_i)$ equal to the negative of its MIC value. Moreover, those methods can also be used to elicit the resistance λ_{a_i} and recovery $\frac{1}{\beta_{a_i}}$ rate to antibiotic a_i. To elicit λ_{a_i}, we perform AST before and after the exposure of a particular strain of bacteria to antibiotic a_i. The resistance rate λ_{a_i} is then simply equal to the ratio between the post-exposure measure of susceptibility $u_{\boldsymbol{h}_t}(a_i)$ and the pre-exposure measure $u_{\boldsymbol{h}_{t-1}}(a_i)$, $\lambda_{a_i} = \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)}$.[21] In a similar fashion, once the bacteria has been exposed to antibiotic a_i, the recovery rate $\frac{1}{\beta_{a_i}}$ can be elicit as the ratio between the susceptibility measure after a period of non-exposure and the susceptibility measure right after the exposure to antibiotic a_i, $\frac{1}{\beta_{a_i}} = \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)}$.[22]

[21]Without loss of generality in this expression it has been implicitly assumed that the exposure to antibiotic a_i occurred at $t-1$.

[22]Again without loss of generality it has been implicitly assumed that the period of exposure to antibiotic a_i was $t-2$ and the period of non-exposure was $t-1$. Also notice that to ensure a unique identification of $\frac{1}{\beta_{a_i}}$, it must be the case that $u_{\boldsymbol{h}_0}(a_i) > u_{\boldsymbol{h}_t}(a_i)$.

6 Extension of the HDU Model

6.1 Different Degrees of Foresight

Up until now, the HDU model has been presented and axiomatically characterized for a standard, fully forward-looking agent. Fully forward-looking agents display infinite foresight, meaning that they anticipate the impact of their current choice on all future decisions. Hence, they choose a consumption path to maximize the sum of time-discounted and history-discounted utilities. However, in some applications, it might be worthwhile to revisit the HDU model by considering agents with limited foresight. Consider a DM choosing restaurants to dine in each Saturday for the rest of his life. Due to the high cognitive cost of such an evaluation, the DM could use the following effort-reduction heuristic: He might recursively choose the sequence of restaurants that maximize the sum of time-discounted and history-discounted utilities of today's choice and the subsequent k periods. Agents using such a heuristic will be referred to as k-periods forward-looking agents. Two special cases of k-periods forward-looking agents are worth mentioning. First, our standard fully-forward looking agent corresponds to $k \geq T$. Second, the myopic agent, which corresponds to $k = 0$ and always chooses what he likes the most at each period, totally ignoring the effects of that choice on future choices.

From now on, in order to avoid any confusion $t = 0$ will represent the present moment. Let's consider the same exact framework as in Section 3 and that Axioms 1 and 2 hold. The following condition called *Future Insensitivity* allow us to generalize the HDU model in order to accommodate the choice behavior of any given k-periods forward-looking agent, including as special cases, the fully forward-looking and the myopic agents.

Axiom I *(Future Insensitivity):* For any $t \in \mathcal{T}$ such that $k < t \leq T$, for all $\boldsymbol{x} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ and any $(x_t, m_t), (x'_t, m'_t) \in \Delta(\mathcal{A}) \times \mathbb{R}$,

$$\big((x_0, m_0), \ldots, (x_k, m_k), \ldots, (x_t, m_t), \ldots (x_T, m_T)\big) \sim \big((x_0, m_0), \ldots, (x_k, m_k), \ldots, (x'_t, m'_t), \ldots, (x_T, m_T)\big)$$

This condition states that the DM is insensitive to consumption allocated further away than k periods from the present ($t = 0$). Hence, the DM will sequentially re-optimize for the subsequent k periods, given that he cannot change his past decisions. With this condition, I obtain the following results:

Theorem 2. *A binary relation $\succsim$ on $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ satisfies Axioms (1-10), and Axiom I if and only if it has an HDU representation given in definition (2.1) where the DM sequentially re-optimize for the subsequent k periods completely ignoring the rest of the future.*

Furthermore, if the decision maker can only choose from the set of degenerate lotteries $\mathcal{A}$, we obtain the following corollary:

Corollary 2.1. *A binary relation $\succsim$ on $(\mathcal{A} \times \mathbb{R})^{T+1}$ satisfies Axioms (1-10), and Axiom I if and only if it has an HDU representation given in definition (2.2) where the DM sequentially re-optimize for the subsequent k periods completely ignoring the rest of the future..*

6.2 Goods Becoming Bads

So far, I have assumed that no matter the history of past consumption, any lottery would always be preferred to the degenerate lottery that assigns probability one to the neutral alternative $\diamond$. This effectively means that regardless of past consumption history, no good will ever become a bad.[23] However, in some applications, it might be interesting to consider goods that might become bads after a sufficiently long history of repeated consumption. For example, think about a long sequence of repeated consumption, let us say of chocolate. It might be the case that the DM might suffer a stomachache if she eats chocolate once again. Hence, in such a case, the DM would prefer to abstain from consumption.

Notice that given the multiplicative structure of the HDU representation, all utilities are greater or equal than zero. Hence, we will use the following intuitive definition of a bad: a bad is an alternative that provides a lower utility than not consuming anything (choosing the neutral alternative). In what follows, I will present an extension of the HDU model in which goods might become bads.[24] I will spell out the (minor) changes to the axiomatic characterization presented in Section 3 needed to accommodate this extension. I will also discuss what those changes mean for the interpretation of the HDU axioms. Finally, I will show how this modified set of axioms leads to a revised representation of the HDU model.

I start by extending the original choice set by adding a new alternative $\rho \in \mathcal{A}^* \equiv \mathcal{A} \cup \{\rho\}$, ρ will be interpreted as the worse possible bad from DM's point of view. Again, with a slight abuse of notation ρ will also be used to denote the degenerate lottery that assigns probability one to the this new alternative ρ. Consequently, the definitions of $\Delta(\mathcal{A})$, $\Delta(\mathcal{A}) \times \mathbb{R}$, $\big(\Delta(\mathcal{A}) \times \mathbb{R}\big)^{T+1}$, and $\mathcal{H}$ are appropriately adjusted to this extended domain, and denoted respectively by $\Delta(\mathcal{A}^*)$, $\Delta(\mathcal{A}^*) \times \mathbb{R}$, $\big(\Delta(\mathcal{A}^*) \times \mathbb{R}\big)^{T+1}$, and $\mathcal{H}^*$. To fix ideas, ρ can be thought of as some sort of non-lethal poison hence, from now on, I will frequently refer to it as poison.

6.2.1 Revised Axioms

Axioms 1-3 remain the same as in section 3, except they now apply to the extended spaces. I impose the following modified version of Axiom 4:[25]

Axiom 4* *(Boundedness):* For all $\boldsymbol{h}_t \in \mathcal{H}^*$, and for all $(x, m) \in \Delta(\mathcal{A}^*) \times \mathbb{R}$:

[23] This is an implication that follows directly from Axiom 4.
[24] I thank Debraj Ray for suggesting me this extension.
[25] In red, I will highlight the changes to the axioms in comparison to those presented in section 3.

i) *Bounded below*: If $x \neq \rho$, then $(x, m) \succ_{\boldsymbol{h}_t} (\rho, m)$.

ii) *Bounded above*: There exists $c \in \mathbb{R}_{++}$, such that $(\rho, m + c) \succ_{\boldsymbol{h}_t} (x, m)$.

In a similar spirit to the original Axiom 4, part i) of Axiom 4* states that no matter past consumption history, any lottery x will always be preferred to the degenerate lottery that assigns probability one to poison. In particular, notice that this implies that $\diamond$ will always be preferred to ρ, which justifies our interpretation of ρ as a bad. More specifically, it implies that regardless of past consumption history, ρ is always the worst bad in the choice set. Part ii) states that for any history of past consumption, there exist a positive amount of money c, that will induce the DM to prefer ρ instead of her original lottery x but receiving this extra amount of money c.[26]

Following Axiom 4*, I state a new Lemma 2 that changes our previous definition of appropriate compensations. In a nutshell, in this extension of the HDU model, we compensate the DM for consuming poison instead of her original lottery (and not for consuming the neutral alternative as was the case in the original formulation of the HDU model).

Lemma 2*: If A1-A3 and A4* are satisfied, then for all $\boldsymbol{h}_t \in \mathcal{H}^*$, and for all $(x, m) \in \Delta(\mathcal{A}^*) \times \mathbb{R}$, there exist a unique compensation $c_{\boldsymbol{h}_t}(x, m) \in \mathbb{R}_+$, such that $(\rho, m + c_{\boldsymbol{h}_t}(x, m)) \sim_{\boldsymbol{h}_t} (x, m)$. Moreover, $c_{\boldsymbol{h}_t}(\diamond, m) > 0$ and $c_{\boldsymbol{h}_t}(\rho, m) = 0$, for all $\boldsymbol{h}_t \in \mathcal{H}^*$, and for all $m \in \mathbb{R}$.

This lemma implies that there exists a unique compensation that makes the DM exactly indifferent between any pair (x, m) and $(\rho, m + c_{\boldsymbol{h}_t}(x, m))$. From now on, I will refer to $c_{\boldsymbol{h}_t}(x, m)$ as the appropriate compensation. The main difference concerning the original Lemma 2 is that the proper compensations are now with respect to poison instead of the neutral alternative. Moreover, notice that in this extension, the neutral alternative serves two purposes. Firstly, as in the original formulation of the HDU model, it allows the consumer to choose not to consume anything in the choice set. Secondly, in this extension, the neutral alternative also serves as a benchmark. After a given history of past consumption, any alternative with lower utility than the neutral alternative is considered a bad.

Again, Axioms 5 and 6 remain unchanged (keeping in mind that they now apply to the extended spaces). While Axiom 7 needs to be reformulated as follows:

Axiom 7* *(Satiation):* For every $t, t' \in \mathcal{T}$, and every $(a_i, m) \in \mathcal{A}^* \times \mathbb{R}$:

i) If $(r_{t-1}, m_{t-1}) = (a_i, m)$, $a_i \neq \diamond$, then $\big(\rho, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$. If $(r_{t-1}, m_{t-1}) = (\diamond, m)$, then $\big(\rho, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big) \sim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$.

ii) If $(r_{t-1}, m_{t-1}) = (r_{t'-1}, m_{t'-1}) = (a_i, m)$ and $\big(ka_i + (1 - k)\rho, m\big) \sim_{\boldsymbol{h}_{t-1}} \big(\rho, m + c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big)$ for $k \in [0, 1]$, then $\big(ka_i + (1 - k)\rho, m\big) \sim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_{t'}}(a_i, m)\big)$.

[26] It is essential to recall that ρ is thought of as a non-lethal poison. If it was lethal, it might well be the case that no amount of money will induce the DM to prefer ρ instead of her original lottery x.

Likewise as in Axiom 7, the first condition in part i) alongside with money monotonicity, implies that after an additional consumption of the alternative a_i (at $t-1$), the appropriate monetary compensation $c_{\boldsymbol{h}_t}(a_i, m)$ will be lower or equal to $c_{\boldsymbol{h}_{t-1}}(a_i, m)$, capturing the satiation effect. However, the main difference concerning Axiom 7 is that now ρ has the same role that $\diamond$ played in the original formulation. Moreover, the new second statement of i) explicitly asserts that $\diamond$ does not experience satiation. Finally, as in Axiom 7, part ii) implies that the satiation rate is constant.

As probably already anticipated, in the following revised version of Axiom 8, ρ plays the role that $\diamond$ had in the original formulation. Furthermore, since the neutral alternative does not experience satiation, it will not experience recovery either, and hence no further changes are required.

Axiom 8* *(Recovery):* For every $t, t' \in \mathcal{T}$, and every $(a_i, m) \in \mathcal{A}^* \times \mathbb{R}$:

i) If $(r_{t-1}, m_{t-1}) \neq (a_i, m)$, then $\big(\rho, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_t}(a_i, m)\big) \succsim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big)$.

ii) If $(r_{t-1}, m_{t-1}) \neq (a_i, m)$, $(r_{t'-1}, m_{t'-1}) \neq (a_i, m)$, $\big(\rho, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \succ_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$ and $\big(ka_i + (1-k)\rho, m\big) \sim_{\boldsymbol{h}_{t-1}} \big(\rho, m + c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big)$ for $k \in (0, 1]$, then $\big(ka_i + (1-k)\rho, m\big) \sim_{\boldsymbol{h}_t} \big(\rho, m + c_{\boldsymbol{h}_{t'}}(a_i, m)\big)$.

The implications of Axiom 8* are mainly analogous to those of Axiom 8. In brief, money monotonicity alongside part i) implies that after a period of non-consumption, the appropriate compensation would be higher or equal than the one demanded before that resting period. Part ii) implies that whenever the recovery is partial, the recovery rate is constant.

Finally, Axioms 9 and 10 remain unchanged with the caveat that, as previously explained, DM's appropriate compensations are now for consuming ρ instead of her original lottery (not for consuming $\diamond$ as it was the case in the original characterization). With this new set of axioms, I obtain the following results.

6.2.2 Revised Representation Theorem

Theorem 3. *A binary relation $\succsim$ on $(\Delta(\mathcal{A}^*) \times \mathbb{R})^{T+1}$ satisfies A1-A3, A4*, A5, A6, A7*, A8*, A9, and A10 if and only if it has a HDU representation given in definition (2.1) where for all $t \in \mathcal{T}$, $u_{\boldsymbol{h}_{t+1}}(\diamond) = u_{\boldsymbol{h}_t}(\diamond) = u(\diamond) > 0$, $\lambda_\diamond = 1$, and $u_{\boldsymbol{h}_{t+1}}(\rho) = u_{\boldsymbol{h}_t}(\rho) = u(\rho) = 0$.*

Furthermore, if the decision-maker can only choose from the set of degenerate lotteries $\mathcal{A}^*$, we obtain the following corollary:

Corollary 3.1. *A binary relation $\succsim$ on $(\mathcal{A}^* \times \mathbb{R})^{T+1}$ satisfies A1-A3, A4*, A5, A6, A7*, A8*, A9, and A10 if and only if it has a HDU representation given in definition (2.2) where for all $t \in \mathcal{T}$, $u_{\boldsymbol{h}_{t+1}}(\diamond) = u_{\boldsymbol{h}_t}(\diamond) = u(\diamond) > 0$, $\lambda_\diamond = 1$, and $u_{\boldsymbol{h}_{t+1}}(\rho) = u_{\boldsymbol{h}_t}(\rho) = u(\rho) = 0$.*

Notice that in the original formulation of the HDU model, the only reason a fully forward-looking DM might choose not to consume anything at a given point in time is exclusively an intertemporal optimization reason.[27] However, in this extension of the HDU model, I have added a new reason for choosing $\diamond$. This new reason is precisely the primary motivation of this extension. I allow a good to become a bad. Specifically, I allow its history-dependent utility to fall below the threshold of the neutral alternative's utility.[28]

I close this section by presenting a simple example that illustrates how this new extension of the HDU model can accommodate that, for some histories of past consumption, some goods become bads.

Example 4. *Given $\mathcal{A} = \{a, \diamond, \rho\}$, $u_0(a) = 10$, $u_0(\diamond) = 4$, $u_0(\rho) = 0$, $\lambda_a = \beta_a = 0.5$. Consider also the following two different histories of past consumption, $\boldsymbol{h}_2 = (a, a)$, and $\tilde{\boldsymbol{h}}_2 = (a, \diamond)$. Simple calculations show that $u_{\boldsymbol{h}_2}(a) = 2.5 < 4 = u_0(\diamond)$, hence after $\boldsymbol{h}_2$, a becomes a bad. However, $u_{\tilde{\boldsymbol{h}}_2}(a) = 10 > 4 = u_0(\diamond)$, hence after $\tilde{\boldsymbol{h}}_2$, a remains a good.*

6.3 Axiomatic characterization of the HDU model when $T = \infty$

In the following lines, I will revise the axiomatic characterization of the HDU model, presented in Section 3, to accommodate infinite horizon consumption plans. The purpose of this exercise is twofold. First, it serves an obvious completeness purpose. The HDU model should also be able to represent variety-seeking preferences over infinite horizon consumption plans. Secondly and more importantly, this exercise will demonstrate another desirable consequence of the axiomatization strategy presented in Section 3. As previously discussed, one of the main methodological advantages of the axiomatization strategy is that it completely separates the effects of time preferences from the effects of pure history dependence. Therefore, time preferences axioms are entirely independent of the rest of the axioms. This axiomatization's feature allows us to easily replace the set of time preferences axioms, which characterizes finite horizon exponential discounting, with any other set that characterizes a different time discounting, e.g., hyperbolic, quasi-hyperbolic, or infinite horizon exponential discounting.[29]

The original axiomatization of infinite horizon exponential discounting is due to Koopmans (1960). Koopmans' axiomatization is probably among the most appealing and intuitive characterizations in the intertemporal choice literature. However, his results suffer from several problems and inaccuracies (Bleichrodt et al. (2008)). One problem is that he assumes the existence

[27] The DM optimally internalizes the satiation and recovery process and hence, might choose $\diamond$ at some periods, provided that this strategy maximizes her overall discounted utility.

[28] Therefore, within this new framework, even a myopic DM might choose $\diamond$ at some periods. Recall that in the original formulation of the HDU model, a myopic DM would never choose $\diamond$.

[29] Recall that as seen in Section 4, another desirable consequence of the axiomatization strategy is that the time preference parameter could be independently elicited from the satiation and recovery parameters.

of a utility function (from the set of infinite horizon consumption streams to the reals) representing the preference relationship that possesses strong continuity properties. Another problem is that you either have to assume bounded utility or restrict the domain of consumption streams into consideration to those for which discounted utility (DU) is well-defined (Bleichrodt et al. (2008) and Strzalecki (2017)). However, knowing in advance for which consumption streams DU is finite and well-defined is complex, and finding conditions stated entirely in terms of observables (preferences) that characterize that subset of consumption streams is even more challenging. Bleichrodt et al. (2008) present a simplification and generalization of Koopman's axiomatization of DU that deals nicely and elegantly with both problems while avoiding complex topological considerations inherent to infinite-dimensional spaces. Therefore, from now on, I will follow their approach.

Preliminaries

Let $\mathcal{T} \equiv \{0, 1, 2, \dots\}$ be the set of time periods. Consumption streams are elements of $\mathbb{R}^{\mathcal{T}}$. For any $c \in \mathbb{R}$, and $\boldsymbol{x} \in \mathbb{R}^{\mathcal{T}}$, let $c\boldsymbol{x} = (c, x_0, x_1, \dots)$. Similarly, for any $c, d \in \mathbb{R}$, and $\boldsymbol{x} \in \mathbb{R}^{\mathcal{T}}$, let $cd\boldsymbol{x} = (c, d, x_0, x_1, \dots)$. An ultimately constant stream is a stream of the form $x_T c = (x_0, x_1, \dots, x_T, c, c, \dots)$ for some $T \in \mathcal{T}$, and some $c \in \mathbb{R}$. For any T, let $X_T \equiv \{x_T c \mid \boldsymbol{x} \in \mathbb{R}^{\mathcal{T}}, c \in \mathbb{R}\}$ be the set of all ultimately constant streams. Finally, let $\succsim^*$ be a weak order defined on a subset $\mathcal{F}$ of $\mathbb{R}^{\mathcal{T}}$ that contains all ultimately constant streams.

Revised Time Preference Axiom

As previously discussed, one of the main advantages of the characterization strategy presented in Section 3, is that it ensures that the time preferences axiom is completely independent from the rest of the axioms. Therefore, we just need to replace our original Axiom 10 by the following revised formulation.

Axiom 10* *(Exponential Discounting Infinite Horizon)*:

i) *(Stationarity):* For all $c \in \mathbb{R}$ and $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{F}$ we have $c\boldsymbol{x} \succsim^* c\boldsymbol{y}$ if and only if $\boldsymbol{x} \succsim^* \boldsymbol{y}$.

ii) *(Ultimate-continuity):* $\succsim^*$ is continuous on each set X_T.

iii) *(Constant-equivalence):* For all $\boldsymbol{x} \in \mathcal{F}$ there exists $c \in \mathbb{R}$ such that $\boldsymbol{x} \sim^* (c, c, \dots)$.

iv) *(Tail-robustness):* For any $c \in \mathbb{R}$ and any $\boldsymbol{x} \in \mathcal{F}$ if $\boldsymbol{x} \succ^* (c, c, \dots)$ $((c, c, \dots) \succ^* \boldsymbol{x})$ then there exists a t such that $\boldsymbol{x}_T c \succ^* (c, c, \dots)$ $((c, c, \dots) \succ^* \boldsymbol{x}_T c)$ for all $T \geq t$.

Revised Representation Theorem

Theorem 4. *A binary relation $\succsim$ on $(\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$ satisfies Axioms (1-9) and Axiom 10* if and only if it has a HDU representation given in definition (2.1) where $T = \infty$.*

Furthermore, if the decision maker can only choose from the set of degenerate lotteries $\mathcal{A}$, we obtain the following corollary:

Corollary 4.1. *A binary relation $\succsim$ on $(\mathcal{A} \times \mathbb{R})^{T+1}$ satisfies Axioms (1-9) and Axiom 10* if and only if it has a HDU representation given in definition (2.2) where $T = \infty$.*

7 Concluding Remarks

This paper has proposed, and axiomatically characterized a new discrete intertemporal choice model consistent with variety-seeking behavior, the HDU model. The axioms presented in this paper are intuitive, as is the representation obtained.

Moreover, the proposed characterization strategy allows us to easily isolate the effects of time from history dependence. The three extensions presented in this paper illustrate the importance of this property.

Finally, the HDU model has a wide range of applicability, as I show with two illustrative applications. In the firs application, I analyze where I demonstrate how the HDU model can be used to design optimal antibiotic treatment plans.

A Additional Results, Definitions, and Mathematical Proofs

Lemma A1 (*Weak Order*): For all $\boldsymbol{h}_t \in \mathcal{H}$, the binary relation $\succsim_{\boldsymbol{h}_t}$ on $\Delta(\mathcal{A}) \times \mathbb{R}$ is :

i) *Complete*: for all $(x, m), (y, m') \in \Delta(\mathcal{A}) \times \mathbb{R}$, either $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ or $(y, m') \succsim_{\boldsymbol{h}_t} (x, m)$.

ii) *Transitive*: for all $(x, m), (y, m'), (z, m'') \in \Delta(\mathcal{A}) \times \mathbb{R}$, if $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ and $(y, m') \succsim_{\boldsymbol{h}_t} (z, m'')$, then $(x, m) \succsim_{\boldsymbol{h}_t} (z, m'')$.

Proof of Lemma A1: Let $\boldsymbol{x} = (\boldsymbol{h}_t, (x, m), (\diamond, 0), \cdots, (\diamond, 0))$, $\boldsymbol{y} = (\boldsymbol{h}_t, (y, m'), (\diamond, 0), \cdots, (\diamond, 0))$ and $\boldsymbol{z} = (\boldsymbol{h}_t, (z, m''), (\diamond, 0), \cdots, (\diamond, 0))$, be consumption streams for an arbitrary $\boldsymbol{h}_t \in \mathcal{H}$.

i) Completeness: By Axiom 1 we know that either $\boldsymbol{x} \succsim \boldsymbol{y}$ or $\boldsymbol{y} \succsim \boldsymbol{x}$, hence by the definition of $\succsim_{\boldsymbol{h}_t}$, it follows that either $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ or $(y, m') \succsim_{\boldsymbol{h}_t} (x, m)$

ii) Transitivity: By Axiom 1 we know that if $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$, then $\boldsymbol{x} \succsim \boldsymbol{z}$, again by the definition of $\succsim_{\boldsymbol{h}_t}$, it follows that if $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ and $(y, m') \succsim_{\boldsymbol{h}_t} (z, m'')$, then $(x, m) \succsim_{\boldsymbol{h}_t} (z, m'')$.

□

Lemma A2 *(Continuity):* For all $\boldsymbol{h}_t \in \mathcal{H}$ and for all $(x, m) \in \Delta(\mathcal{A}) \times \mathbb{R}$, the following sets are closed:

$$B\big((x, m)\big) = \{(y, m') \in \Delta(\mathcal{A}) \times \mathbb{R} : (y, m') \succsim_{\boldsymbol{h}_t} (x, m)\}$$

$$W\big((x, m)\big) = \{(y, m') \in \Delta(\mathcal{A}) \times \mathbb{R} : (x, m) \succsim_{\boldsymbol{h}_t} (y, m')\}$$

Proof of Lemma A2: Let $\{\boldsymbol{x}_k\}_{k=1}^{\infty} = \{(\boldsymbol{h}_t, (x_k, m_k), (\diamond, 0), \cdots, (\diamond, 0))\}_{k=1}^{\infty}$ and $\{\boldsymbol{y}_k\}_{k=1}^{\infty} = \{(\boldsymbol{h}_t, (y_k, m'_k), (\diamond, 0), \cdots, (\diamond, 0))\}_{k=1}^{\infty}$ be a pair of arbitrary vector sequences such that $\boldsymbol{x}_k \succsim \boldsymbol{y}_k$ for all $k \in \mathbb{N}$, and $(x_k, m_k) \to (x, m)$, $(y_k, m'_k) \to (y, m')$ as $k \to \infty$. Then, by the definition of $\succsim_{\boldsymbol{h}_t}$, it follows that $(x_k, m_k) \succsim_{\boldsymbol{h}_t} (y_k, m'_k)$ for all $k \in \mathbb{N}$. Moreover, $\boldsymbol{x}_k \to (\boldsymbol{h}_t, (x, m), (\diamond, 0), \cdots, (\diamond, 0))$ and $\boldsymbol{y}_k \to (\boldsymbol{h}_t, (y, m'), (\diamond, 0), \cdots, (\diamond, 0))$ as $k \to \infty$. Hence, by Axiom 2, we must have $(\boldsymbol{h}_t, (x, m), (\diamond, 0), \cdots, (\diamond, 0)) \succsim (\boldsymbol{h}_t, (y, m'), (\diamond, 0), \cdots, (\diamond, 0))$, which implies $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$. This shows that the set $W\big((x, m)\big)$ is closed. The proof that $B\big((x, m)\big)$ is also closed is analogous and hence omitted. □

Lemma A3 *(Money Monotonicity):* For all $\boldsymbol{h}_t \in \mathcal{H}$, $(x, m) \succ_{\boldsymbol{h}_t} (x, m')$ if and only if $m > m'$.

Proof of Lemma A3: A3 trivially implies:

$$\big(\boldsymbol{h}_t, (x, m), (\diamond, m''_{t+1}), \cdots, (\diamond, m''_T)\big) \succ \big(\boldsymbol{h}_t, (x, m'), (\diamond, m''_{t+1}), \cdots, (\diamond, m''_T)\big)$$

if and only if $m > m'$. Hence, it follows from definition 1 that, $(x, m) \succ_{\boldsymbol{h}_t} (x, m')$ if and only if $m > m'$. $\square$

Proof of Lemma 1: By lemma 2 we know that $\succsim_{\boldsymbol{h}_t}$ on $\Delta(\mathcal{A}) \times \mathbb{R}$ is continuous, and suppose that $(x, m) \succsim_{\boldsymbol{h}_t} (y, m')$ and $(y, m') \succsim_{\boldsymbol{h}_t} (x, m'')$. We want to show that there exist a unique m^*, with $m'' \leq m^* \leq m$, such that $(x, m^*) \sim_{h^t} (y, m')$. Let $B = \{m''' \in \mathbb{R} : (x, m''') \succsim_{h^t} (y, m')\} \subset \mathbb{R}$, and define $m^* \equiv \inf(B)$. By assumption $m \in B$, so we are not taking the infimum of an empty set and hence, m^* is well defined. Since m^* is the infimum of B there exist a sequence $\{m_k\}_{k=1}^{\infty} \in B$ such that $m_k \to m^*$ as $k \to \infty$ and $(x, m_k) \succsim_{\boldsymbol{h}_t} (y, m')$. By continuity, we get that $(x, m^*) \succsim_{\boldsymbol{h}_t} (y, m')$. Since m^* is the greatest lower bound of B, for any $m \in B$ such that $m < m^*$ completeness will imply that $(y, m') \succ_{\boldsymbol{h}_t} (x, m)$. Let $\{m_k\}_{k=1}^{\infty} \equiv m^* - \frac{1}{k}$, clearly, $m_k < m^*$ for all $k \in \mathbb{N}$, hence $(y, m') \succsim_{\boldsymbol{h}_t} (x, m_k)$ but $m_k \to m^*$ as $k \to \infty$, therefore by continuity we get that $(y, m') \succsim_{\boldsymbol{h}_t} (x, m^*)$. Since we have shown that $(x, m^*) \succsim_{\boldsymbol{h}_t} (y, m')$ and $(y, m') \succsim_{h^t} (x, m^*)$ we must conclude that $(x, m^*) \sim_{\boldsymbol{h}_t} (y, m')$. To prove uniqueness, suppose there exists another m^{**} such that $(x, m^{**}) \sim_{\boldsymbol{h}_t} (y, m')$, then by money monotonicity we have that $m^{**} = m^*$. $\square$

Proof of Lemma 2: Suppose $x \neq \diamond$, then by transitivity and axiom 4 (Boundedness), we know that there exist $c \in \mathbb{R}_{++}$ such that $(\diamond, m + c) \succ_{\boldsymbol{h}_t} (x, m) \succ_{\boldsymbol{h}_t} (\diamond, m)$. Therefore, by lemma 3 (Money Solvability), we know that there exists $c_{\boldsymbol{h}_t}(x, m) \in \mathbb{R}$, with $0 < c_{\boldsymbol{h}_t}(x, m) < c$, such that $(x, m) \sim_{\boldsymbol{h}_t} (\diamond, m + c_{\boldsymbol{h}_t}(x, m))$. Finally suppose that $x = \diamond$, applying part ii) of axiom S4 and reflexivity we have, $(\diamond, m + c) \succ_{\boldsymbol{h}_t} (\diamond, m) \sim_{\boldsymbol{h}_t} (\diamond, m)$. Again, by lemma 3 we know that there exists $c_{\boldsymbol{h}_t}(\diamond, m) \in \mathbb{R}$ with $0 \leq c_{\boldsymbol{h}_t}(\diamond, m) < c$ such that $(\diamond, m) \sim_{\boldsymbol{h}_t} (\diamond, m + c_{\boldsymbol{h}_t}(\diamond, m))$. But then, by S3 (Money Monotonicity), we must conclude that $c_{\boldsymbol{h}_t}(\diamond, m) = 0$ for all $\boldsymbol{h}_t \in \mathcal{H}$, and for all $m \in \mathbb{R}$. $\square$

Lemma A4 *(Weak Order*):* The binary relation $\succsim^*$ on $\mathbb{R}^{T+1}$ is *complete* and *transitive.*

Proof of Lemma A4: It is implied by A1-A8 and the fact that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$. $\square$

Lemma A5 *(Continuity*):* $\succsim^*$ is continuous in the product topology on $\mathbb{R}^{T+1}$.

Proof of Lemma A5: It is implied by A1-A8 and the fact that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$. $\square$

Definition A1: For any set $E \subseteq \mathcal{T}$ and any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^{T+1}$ define $\boldsymbol{x}_E \boldsymbol{y} \in \mathbb{R}^{T+1}$ as follows

$$(\boldsymbol{x}_E \boldsymbol{y})_t = \begin{cases} x_t & \text{if } t \in E \\ y_t & \text{if } t \notin E \end{cases}$$

If $E = \{t\}$ is a singleton, then we abuse notation slightly and write $\boldsymbol{x}_t \boldsymbol{y}$ instead of $\boldsymbol{x}_{\{t\}} \boldsymbol{y}$.

Example: Suppose $\boldsymbol{x} = (x_0, x_1, \cdots, x_T)$, $\boldsymbol{y} = (y_0, y_1, \ldots, y_T)$ both in $\mathbb{R}^{T+1}$, and assume that $E = \{t \in \mathcal{T} | \text{ t is odd}\}$ then, $\boldsymbol{x}_E\boldsymbol{y} = (y_0, x_1, y_2, x_3, \ldots)$.

Definition A2 *(Null index)*: A time index $t \in \mathcal{T}$ is *null* if and only if for any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbb{R}^{T+1}$, $\boldsymbol{x}_t\boldsymbol{z} \sim^* \boldsymbol{y}_t\boldsymbol{z}$.

In words we say that an index is *null* if no matter what we give the DM on that index, she does not care. The next lemma establish that our DM cares about the outcomes that she will receive at all points in time. Sometimes non-null indexes are also called essential indexes.

Lemma A6 *(Sensitivity):* Each of the indexes $t \in \mathcal{T}$ is non-null.

Proof of Lemma A6: It is implied by A1-A8 and the fact that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$. □

Definition A3 *(Separable Set)*: A set $E \subseteq \mathcal{T}$ is *separable*, if and only if, for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{z'} \in \mathbb{R}^{T+1}$, $\boldsymbol{x}_E\boldsymbol{z} \succsim^* \boldsymbol{y}_E\boldsymbol{z}$, if and only if, $\boldsymbol{x}_E\boldsymbol{z'} \succsim^* \boldsymbol{y}_E\boldsymbol{z'}$.

When $E = \{t\}$ in the above definition, we refer to it as *Separable index.* In general, given the previous axioms, *singleton separability* (all indexes $t \in \mathcal{T}$ are separable) suffices to ensure the existence of an ordinally separable representation of $\succsim^*$, however its too weak to ensure the existence of an additive separable representation. To that purpose, we need assume not only that all singletons $\{t\}$ are separable, but all subsets of $\mathcal{T}$ are separable.

Proof of Lemma 3:

For any $m \in [m_i, m_{i+1}]$ let's define:

$$\tilde{v}_{i+1}(m) = v(m_i) - \frac{v(m_{i+1}) - v(m_i)}{m_{i+1} - m_i} m_i + \frac{v(m_{i+1}) - v(m_i)}{m_{i+1} - m_i} m$$

By construction, $\tilde{v}_{i+1}(m_i) = v(m_i)$ and $\tilde{v}_{i+1}(m_{i+1}) = v(m_{i+1})$.

The interpolation error in the interval $[m_i, m_{i+1}]$ is:

$$IE_{i+1} = \int_{m_i}^{m_{i+1}} |\tilde{v}_{i+1}(m) - v(m)| dm$$

The total interpolation error for the K equally-spaced partition of $[0, M]$ is therefore:

$$TIE_K = \sum_{i=0}^{K-1} \int_{m_i}^{m_{i+1}} |\tilde{v}_{i+1}(m) - v(m)| dm$$

But since $v(\cdot)$ is increasing and continuous and $\tilde{v}_{i+1}(\cdot)$ is linear we have that

$$IE_{i+1} = \int_{m_i}^{m_{i+1}} |\tilde{v}_{i+1}(m) - v(m)|dm \leq \frac{(m_{i+1} - m_i)(v(m_{i+1}) - v(m_i))}{2}$$

Hence,

$$TIE_K = \sum_{i=0}^{K-1} \int_{m_i}^{m_{i+1}} |\tilde{v}_{i+1}(m) - v(m)|dm \leq \sum_{i=0}^{K-1} \frac{(m_{i+1} - m_i)(v(m_{i+1}) - v(m_i))}{2} = S_K$$

Thus, S_K is an upper bound for TIE_K. Notice also that $S_{K+1} < S_K$ and hence $TIE_{K+1} < TIE_K$ which means that the finer the partition the lowest the interpolation error. □

B Proof of the Representation Theorems

Proof of Theorems 1 and Corollaries 1.1:

Step 1: We show that there exist an additive separable representation of the binary relation $\succsim_{\boldsymbol{h}_t}$ on $\Delta(\mathcal{A}) \times \mathbb{R}$:

I make use of the following theorem due to Wakker (1988), which is already a refinement of the main result Debreu (1959).

Theorem 4.4 (Wakker 1988): Let $\mathcal{C}_1$, $\mathcal{C}_2$ be two connected topological spaces; let $\mathcal{C}_1 \times \mathcal{C}_2$ be endowed with the product topology. Let $\succsim$ be a binary relation on $\mathcal{C}_1 \times \mathcal{C}_2$, with both coordinates essential. Then the following three statements are equivalent:

i) There exists a continuous additive representation for $\succsim$.

ii) The binary relation $\succsim$ is a continuous coordinate independent weak order that satisfies the Thomsen condition.

iii) The binary relation $\succsim$ is a continuous weak order that satisfies triple cancellation.

Given A1-A5 and the topological properties of $\Delta(\mathcal{A}) \times \mathbb{R}$, $\succsim_{\boldsymbol{h}_t}$ satisfies all the premises of the above theorem. In particular, notice that the sets $\Delta(\mathcal{A})$ and $\mathbb{R}$ are both connected. The set $\Delta(\mathcal{A}) \times \mathbb{R}$ is endowed with the product topology. Boundedness and Money Monotonicity ensure that both coordinates are essential. Lemma 1, Lemma 2, Money Monotonicity and Separability ensure that the binary relation $\succsim_{\boldsymbol{h}_t}$ is a continuous coordinate indepedent weak order that satisfies the Thomsen condition, hence we must conclude that for all $(x, m), (y, m') \in \Delta(\mathcal{A}) \times \mathbb{R}$ and for all $\boldsymbol{h}_t \in \mathcal{H}$:

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(x) + v(m) \geq u_{\boldsymbol{h}_t}(y) + v(m')$$

Further notice that $u_{\boldsymbol{h}_t}(\diamond) = 0$, for all $\boldsymbol{h}_t \in \mathcal{H}$. To see this suppose by way of contradiction that $u_{\boldsymbol{h}_t}(\diamond) > 0$, recall that Boundedness imply that if $x \neq \diamond$, then $(x, m) \succ_{\boldsymbol{h}_t} (\diamond, m)$ which implies, in particular, that $u_{\boldsymbol{h}_t}(a_i) > u_{\boldsymbol{h}_t}(\diamond)$ for all $\boldsymbol{h}_t \in \mathcal{H}$ and for all $a_i \in \mathcal{A}$. But Satiation ensures that there always exists a combination of a satiation rate and a history of repeated consumption of a_i such that $u_{\boldsymbol{h}_t}(\diamond) > u_{\boldsymbol{h}_t}(a_i) > 0$ which contradicts Boundedness. Similarly, we can show that $u_{\boldsymbol{h}_t}(\diamond) < 0$ is also not possible.

Step 2 (Expected Utility Representation): We are going to show that $u_{\boldsymbol{h}_t}(x)$ has an expected utility representation, $u_{\boldsymbol{h}_t}(x) = \sum_{i=1}^{N} p_x(a_i) u_{\boldsymbol{h}_t}(a_i)$. To this end we start by inducing a preference relation in the first coordinate using Coordinate Independence:

$$x \succsim^1_{\boldsymbol{h}_t} y \quad \Leftrightarrow \quad (x, m) \succsim_{\boldsymbol{h}_t} (y, m)$$

It is trivial to see that given the definition of $\succsim^1_{\boldsymbol{h}_t}$, Lemma 1, Lemma 2 and Independence ensure that $\succsim^1_{\boldsymbol{h}_t}$ is a continuous weak order that satisfies the standard independence axiom from expected utility theory. Therefore, by standard results of expected utility theory we must conclude that for all $x, y \in \Delta(\mathcal{A})$ and for all $\boldsymbol{h}_t \in \mathcal{H}$:

$$x \succsim^1_{\boldsymbol{h}_t} y \quad \Leftrightarrow \quad \sum_{i=1}^{N} p_x(a_i) u_{\boldsymbol{h}_t}(a_i) \geq \sum_{i=1}^{N} p_y(a_i) u_{\boldsymbol{h}_t}(a_i)$$

or equivalently,

$$x \succsim^1_{\boldsymbol{h}_t} y \quad \Leftrightarrow \quad \alpha^x_{\boldsymbol{h}_t} \geq \alpha^y_{\boldsymbol{h}_t} \tag{15}$$

where $\alpha^x_{\boldsymbol{h}_t}$ and $\alpha^y_{\boldsymbol{h}_t}$ are the unique numbers such that:

$$x \sim^1_{\boldsymbol{h}_t} \alpha^x_{\boldsymbol{h}_t} b_{\boldsymbol{h}_t} + (1 - \alpha^x_{\boldsymbol{h}_t})\diamond$$

$$y \sim^1_{\boldsymbol{h}_t} \alpha^y_{\boldsymbol{h}_t} b_{\boldsymbol{h}_t} + (1 - \alpha^y_{\boldsymbol{h}_t})\diamond$$

and $b_{\boldsymbol{h}_t}$ and $\diamond$ are the degenerate lotteries that assign probability one to the best and the worse alternative in $\mathcal{A}$ given history $\boldsymbol{h}_t$. Now recall that from step 1 we got that for all $(x, m), (y, m) \in \Delta(\mathcal{A}) \times \mathbb{R}$ and for all $\boldsymbol{h}_t \in \mathcal{H}$:

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(x) + v(m) \geq u_{\boldsymbol{h}_t}(y) + v(m)$$

which by the definition of $\succsim^1_{\boldsymbol{h}_t}$ it implies that

$$x \succsim^1_{\boldsymbol{h}_t} y \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(x) \geq u_{\boldsymbol{h}_t}(y)$$

Now we can define:

$$0 < \max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\} < \infty$$

We have used Boundedness to bound it away from zero, and the following facts: (i) $\Delta(\mathcal{A})$ is compact (ii) and from Wakker 4.4 we know that $u_{\boldsymbol{h}_t}()$ is continuous, so by the extreme value theorem we know the maximum exists. Therefore, we have that:

$$x \succsim^1_{\boldsymbol{h}_t} y \quad \Leftrightarrow \quad \frac{u_{\boldsymbol{h}_t}(x)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} \geq \frac{u_{\boldsymbol{h}_t}(y)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} \tag{16}$$

Equations (1) and (2) give us two expected utility representations of $\succsim^1_{\boldsymbol{h}_t}$ hence by the uniqueness result of expected utility representation we know that there exist $\eta > 0$ and $\gamma \in \mathbb{R}$, such that

$$\alpha^x_{\boldsymbol{h}_t} = \eta \frac{u_{\boldsymbol{h}_t}(x)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} + \gamma$$

but by construction, we have that $\alpha^{\diamond}_{\boldsymbol{h}_t} = 0$ and $\frac{u_{\boldsymbol{h}_t}(\diamond)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} = 0$ which implies that $\gamma = 0$. Moreover, $\alpha^{best}_{\boldsymbol{h}_t} = 1$ and $\frac{u_{\boldsymbol{h}_t}(best)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} = 1$ which implies that $\eta = 1$. In conclusion,

$$\frac{u_{\boldsymbol{h}_t}(x)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} = \alpha^x_{\boldsymbol{h}_t} \tag{17}$$

Again by step 1 we have that for all $(x, m), (y, m') \in \Delta(\mathcal{A}) \times \mathbb{R}$ and for all $\boldsymbol{h}_t \in \mathcal{H}$:

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(x) + v(m) \geq u_{\boldsymbol{h}_t}(y) + v(m')$$

Also,

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad \frac{u_{\boldsymbol{h}_t}(x) + v(m)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} \geq \frac{u_{\boldsymbol{h}_t}(y) + v(m')}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}}$$

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad \frac{u_{\boldsymbol{h}_t}(x)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} + \hat{v}(m) \geq \frac{u_{\boldsymbol{h}_t}(y)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}} + \hat{v}(m')$$

where $\hat{v}(m) = \frac{v(m)}{\max\{u_{\boldsymbol{h}_t}(z)|z \in \Delta(\mathcal{A})\}}$. Thus, by equation (3):

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad \alpha^x_{\boldsymbol{h}_t} + \hat{v}(m) \geq \alpha^y_{\boldsymbol{h}_t} + \hat{v}(m')$$

or equivalently,

$$(x, m) \succsim_{\boldsymbol{h}_t} (y, m') \quad \Leftrightarrow \quad \sum_{i=1}^{N} p_x(a_i) u_{\boldsymbol{h}_t}(a_i) + \hat{v}(m) \geq \sum_{i=1}^{N} p_y(a_i) u_{\boldsymbol{h}_t}(a_i) + \hat{v}(m')$$

This shows that $u_{\boldsymbol{h}_t}(x)$ has an expected utility representation.

Step 3 (Satiation): We show that if $(r_{t-1}, m_{t-1}) = (a_i, m)$, then $u_{\boldsymbol{h}_t}(a_i) = \lambda_{a_i} u_{\boldsymbol{h}_{t-1}}(a_i)$, where $\lambda_{a_i} \in (0, 1]$:

By Lemma 5 and step 1 we know that:

$$(a_i, m) \sim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big) \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(a_i) + v(m) = v\big(m + c_{\boldsymbol{h}_t}(a_i, m)\big)$$

and

$$(a_i, m) \sim_{\boldsymbol{h}_{t-1}} \big(\diamond, m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big) \quad \Leftrightarrow \quad u_{\boldsymbol{h}_{t-1}}(a_i) + v(m) = v\big(m + c_{\boldsymbol{h}_{t-1}}(a_i, m)\big)$$

Part i) of Satiation implies that, $v\big(c_{\boldsymbol{h}_t}(a_i, m)\big) \leq v\big(c_{\boldsymbol{h}_{t-1}}(a_i, m)\big)$ therefore,

$$u_{\boldsymbol{h}_t}(a_i) + v(m) \leq u_{\boldsymbol{h}_{t-1}}(a_i) + v(m) \quad \Leftrightarrow \quad u_{\boldsymbol{h}_t}(a_i) \leq u_{\boldsymbol{h}_{t-1}}(a_i)$$

Boundedness implies that $0 < u_{\boldsymbol{h}_t}(x) \leq u_{\boldsymbol{h}_{t-1}}(x) < \infty$, hence

$$0 < \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)} \leq 1$$

Now we are going to show that Part ii) of the satiation axiom implies that for all t, t' for which the premises of the axiom hold we have:

$$0 < \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)} = \frac{u_{\boldsymbol{h}'_t}(a_i)}{u_{\boldsymbol{h}_{t'-1}}(a_i)} = \lambda_{a_i} \leq 1$$

First notice that as a result of the independence axiom we have that for any $\kappa \in (0, 1]$

$$u_{\boldsymbol{h}_{t-1}}\big(\kappa a_i + (1-\kappa) \diamond\big) = \kappa u_{\boldsymbol{h}_{t-1}}(a_i)$$

hence by part ii) of Satiation:

$$v\big(m + c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big) = \kappa u_{\boldsymbol{h}_{t-1}}(a_i) + v(m) \tag{18}$$

Also

$$v\big(m + c_{\boldsymbol{h}_{t'-1}}(a_i, m)\big) = u_{\boldsymbol{h}_{t'-1}}(a_i) + v(m) \tag{19}$$

From equation (4) and (5) we must conclude that

$$u_{\boldsymbol{h}_{t'-1}}(a_i) = \kappa u_{\boldsymbol{h}_{t-1}}(a_i)$$

By similar arguments we find that

$$u_{\boldsymbol{h}_{t'}}(a_i) = \kappa u_{\boldsymbol{h}_t}(a_i)$$

Hence, we have that

$$0 < \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)} = \frac{\kappa u_{\boldsymbol{h}_t}(a_i)}{\kappa u_{\boldsymbol{h}_{t-1}}(a_i)} = \frac{u_{\boldsymbol{h}'_t}(a_i)}{u_{\boldsymbol{h}_{t'-1}}(a_i)} = \lambda_{a_i} \leq 1$$

From which we get the desired result,

$$u_{\boldsymbol{h}_t}(a_i) = \lambda_{a_i} u_{\boldsymbol{h}_{t-1}}(a_i) \qquad \textit{where} \qquad \lambda \in (0, 1]$$

Step 4 (Recovery): Similar to satiation.

By Recovery part i) and following similar steps as we did in step 3 we get that:

$$1 \leq \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)} < \infty$$

Suppose now that $\big(\diamond, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \succ_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$, by similar arguments as in step 3 we arrive to the conclusion that:

$$1 \leq \frac{u_{\boldsymbol{h}_t}(a_i)}{u_{\boldsymbol{h}_{t-1}}(a_i)} = \frac{\kappa u_{\boldsymbol{h}_t}(a_i)}{\kappa u_{\boldsymbol{h}_{t-1}}(a_i)} = \frac{u_{\boldsymbol{h}'_t}(a_i)}{u_{\boldsymbol{h}_{t'-1}}(a_i)} = \frac{1}{\beta_{a_i}} < \infty$$

Now suppose that $\big(\diamond, m + c_{\boldsymbol{h}_0}(a_i, m)\big) \sim_{\boldsymbol{h}_t} \big(\diamond, m + c_{\boldsymbol{h}_t}(a_i, m)\big)$, then by step 1 and Lemma 5

$$v\big(m + c_{\boldsymbol{h}_0}(a_i, m)\big) = v\big(m + c_{\boldsymbol{h}_t}(a_i, m)\big) \Leftrightarrow u_{\boldsymbol{h}_0}(a_i) + v(m) = u_{\boldsymbol{h}_t}(a_i) + v(m) \Leftrightarrow u_{\boldsymbol{h}_0}(a_i) = u_{\boldsymbol{h}_t}(a_i)$$

Therefore, taking into account both cases we must conclude that:

$$u_{\boldsymbol{h}_t}(a_i) = \min\left\{u_{\boldsymbol{h}_0}(a_i), \frac{u_{\boldsymbol{h}_{t-1}(a_i)}}{\beta_{a_i}}\right\}$$

Step 5: We show that the binary relation $\succsim^*$ on $\mathbb{R}^{T+1}$ has an exponential discounting representation:

To that purpose we make use of the following theorem:

Theorem 4.6 (Strzalecki 2017): A weak order $\succsim$ on $\mathbb{R}^{T+1}$ satisfies Continuity, Sensitivity, Separablility and Stationarity, if and only if there exists a unique number $\delta > 0$, and a function $v : \mathbb{R} \to \mathbb{R}$ nonconstant, continuous, and unique up to a positive affine transformation such that for any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^{T+1}$:

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t v(x_t) \geq \sum_{t=0}^{T} \delta^t v(y_t)$$

Notice that given A1-A9 and the topological properties of $\mathbb{R}^{T+1}$, $\succsim^*$ satisfies all the premises of the above theorem, hence we must conclude that for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^{T+1}$:

$$\boldsymbol{x} \succsim^* \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t v(x_t) \geq \sum_{t=0}^{T} \delta^t v(y_t)$$

Moreover, since Impatience hold, if $a \succ^* b$, then

$$(a, b, x_2, x_3, \ldots, x_T) \succ^* (b, a, x_2, x_3, \ldots, x_T)$$

From the representation it follows that:

$$v(a) + \delta v(b) > v(b) + \delta v(a)$$

Or equivalently,

$$(1 - \delta)(v(a) - v(b)) > 0$$

Since $v(a) > v(b)$ if follows that $\delta < 1$. Furthermore, from the previous theorem we also know that $\delta > 0$, thus we must conclude that $\delta \in (0, 1)$.

Step 6: We show that for any $\Diamond(\boldsymbol{x}), \Diamond(\boldsymbol{y}) \in (\{\diamond\} \times \mathbb{R})^{T+1}$

$$\Diamond(\boldsymbol{x}) \succsim \Diamond(\boldsymbol{y}) \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t v\big(m_t + c_{\boldsymbol{h}_t}(x_t, m_t)\big) \geq \sum_{t=0}^{T} \delta^t v\big(m'_t + c_{\boldsymbol{h}_t}(y_t, m'_t)\big)$$

This follows immediately from the result in Step 5 and the fact that the space $\{\diamond\} \times \mathbb{R}$ is isometric, and hence also homeomorphic, to $\mathbb{R}$.

Step 7: We show that for any $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$:

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \Diamond(\boldsymbol{x}) \succsim \Diamond(\boldsymbol{y})$$

This follows trivially from the Indifference axiom and the transitivity of $\succsim$.

Step 8: Finally, notice that for all $t \in \mathcal{T}$, $u_{\boldsymbol{h}_t}(x_t) + v(m_t) = v\big(m_t + c_{\boldsymbol{h}_t}(x_t, m_t)\big)$, hence we must conclude that for any $\boldsymbol{x}, \boldsymbol{y} \in (\Delta(\mathcal{A}) \times \mathbb{R})^{T+1}$:

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t \Big[u_{\boldsymbol{h}_t}(x_t) + v(m_t)\Big] \geq \sum_{t=0}^{T} \delta^t \Big[u_{\boldsymbol{h}_t}(y_t) + v(m'_t)\Big]$$

where for all $z_t \in \Delta(\mathcal{A})$ and all $t > 0$ if $a_i \in \mathcal{A}$ was the realization of the lottery chosen by the decision maker at $t - 1$, that is $r_{t-1} = a_i$, then:

$$\underbrace{\sum_{i=1}^{N} p_{z_t}(a_i) u_{\boldsymbol{h}_t}(a_i)}_{u_{\boldsymbol{h}_t}(z_t)} = \underbrace{p_{z_t}(a_i)(\lambda_{a_i} - 1) u_{\boldsymbol{h}_{t-1}}(a_i)}_{\text{Satiation} \leq 0} +$$

$$\underbrace{\sum_{a_j \in \mathcal{A} - \{a_i\}} p(a_j) \left[\min \left\{ u_0(a_j), \frac{u_{\boldsymbol{h}_{t-1}}(a_j)}{\beta_{a_j}} \right\} - u_{\boldsymbol{h}_{t-1}}(a_j) \right]}_{\text{Recovery} \geq 0} + \underbrace{\sum_{i=1}^{N} p_{z_t}(a_i) u_{\boldsymbol{h}_{t-1}}(a_i)}_{u_{\boldsymbol{h}_{t-1}}(z_t)}$$

Furthermore, if the decision maker can only choose from the set of degenerate lotteries $\mathcal{A}$, we can rewrite the previous result more simply as follows, for any $\boldsymbol{x}, \boldsymbol{y} \in (\mathcal{A} \times \mathbb{R})^{T+1}$:

$$\boldsymbol{x} \succsim \boldsymbol{y} \quad \Leftrightarrow \quad \sum_{t=0}^{T} \delta^t \Big[\psi_t(x_t | x_{t-1}) u_0(x_t) + v(m_t) \Big] \geq \sum_{t=0}^{T} \delta^t \Big[\psi_t(y_t | y_{t-1}) u_0(y_t) + v(m'_t) \Big]$$

where for all $z_t \in \mathcal{A}$

$$\psi_0(z_t | \boldsymbol{h}_0) = 1$$

and for all $t > 0$

$$\psi_t(z_t | z_{t-1}) = \begin{cases} \lambda_{z_t} \cdot \psi_{t-1}(z_t | z_{t-2}) & \text{if } z_t = z_{t-1} \\ \min \left\{ 1, \frac{1}{\beta_{z_t}} \cdot \psi_{t-1}(z_t | z_{t-2}) \right\} & \text{if } z_t \neq z_{t-1} \end{cases}$$

The result for the set of degenerate lotteries follow immediately from the previous steps. From this result it follows trivially the result for the set all probability distributions by using the expected utility representation of $u_{\boldsymbol{h}_t}$. □

Proof of Theorems 2, 3, 4 and Corollaries 2.1, 3.1, 4.1:

These proofs follow the same steps that the proof of Theorem 2.1 and Corollary 2.1, hence are omitted. □

C Rationalizable Sequences

At this point we might be wondering which types of sequences this model can accommodate. Despite the fact that in the next section I will provide a complete axiomatic characterization of the HDU model that will allow us to fully discriminate between sequences that are consistent with our set of axioms and those who are not (and hence, cannot be rationalized by the model), in this section I will describe the types of sequences that can be rationalized by the HDU model in its simplest specification: a myopic DM facing alternative-independent satiation and recovery rates. To do so, we will identify some fundamental properties of the sequences generated by this specification of the HDU model.

The purpose of this exercise is twofold. First, it will help the reader to easily grasp the intuition behind the dynamics of the model. Second, it will clearly illustrate the fact that despite the flexibility of the HDU model, only certain types of sequences can be explained by it, therefore, the model is also easily refutable by just cheeking some basic properties of the sequence of choices made by the DM.

Keeping the intrinsic ranking fixed we will examine all possible combinations of the satiation and recovery rates in the admissible range of the parameters, $\lambda \in (0, 1]$ and $\beta \in (0, 1]$. To further clarify the dynamics of the model we will rewrite $\beta \in (0, 1]$ as $\beta = \lambda^n$ where $n \in [0, \infty)$. Moreover, without lost of generality, we will assume that the choice set is $\mathcal{X} = \{a, b, c, d\}$ whose elements are evaluated initially, by the DM, according to the following intrinsic ranking: $a \succ_{\boldsymbol{h_0}} b \succ_{\boldsymbol{h_0}} c \succ_{\boldsymbol{h_0}} d$, represented by $u_0(a) = 5 > u_0(b) = 4 > u_0(c) = 3 > u_0(d) = 2$. All results that follow are general and do not depend on the specific intrinsic ranking taken into consideration nor on the cardinality of the choice set[30]. Even though the ranking of alternatives will evolve endogenously, we will keep referring to a as the best alternative, b as the second best and so on.

Case 1 (No satiation): $\lambda = 1$, $\beta \in (0, 1]$. In this trivial case, there is no satiation thus DM's preferences are static and only the best alternative in the choice set is chosen. Therefore, only the following sequence can be rationalized:

$$\boldsymbol{x}_t = (a, a, a, \cdots)$$

For a graphical illustration see figure Figure 4 in the appendix.

Case 2 (Satiation rate = Recovery rate): $\lambda = \beta \in (0, 1)$. With this parameter configuration only a certain type of sequences can be generated: The best alternative in the choice set is chosen N times, with $N = \left\lfloor ln\left(\frac{u_0(b)}{u_0(a)}\right) * \frac{1}{ln(\lambda)} + 1 \right\rfloor < \infty$,[31] until the second best alternative is chosen for the first time, from that moment on, the DM alternates between both alternatives. The other alternatives in the choice set are never chosen. As an example of this type of sequences we set $\lambda = \beta = 0.9$ and we generate the following sequence:

$$\boldsymbol{x}_t = (\underbrace{a, a, a}_{N=3}, b, a, b, a, \cdots)$$

For a graphical illustration see figure Figure 5 in the appendix.

Case 3 (Satiation rate < Recovery rate): $\lambda \in (0, 1)$, $\beta = \lambda^n$, $n \in (1, \infty)$. Three prototypical cases arise:

[30]Some deviations from the patterns presented here might arise if the DM is ever indifferent between two or more top alternatives. In this case recall that the DM chooses randomly between top indifferent alternatives.

[31]We denote by $\lfloor x \rfloor$ the floor function, $\lfloor x \rfloor = max\{m \in \mathbb{Z} \mid m \leq x\}$.

(i) If $n \in \mathbb{N} \cap (1, \infty)$ and $n < N = \left\lfloor ln\left(\frac{u_0(b)}{u_0(a)}\right) * \frac{1}{ln(\lambda)} + 1 \right\rfloor$: The best alternative in the choice set is chosen N times until the second best alternative is chosen for the first time, from that moment on, the DM repeatedly chooses the best alternative n times followed by the second best alternative once. Again, the other alternatives in the choice set are never chosen. For example, with $\lambda = 0.96$, $\beta = \lambda^3$ ($n = 3 < N = 6$), we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a, a, a, a}_{N=6}, b, \underbrace{a, a, a}_{n=3}, b, \underbrace{a, a, a}_{n=3}, \cdots)$$

See figure Figure 6 in the appendix.

(ii) If $n \in (1, \infty)$ and $n \geq N = \left\lfloor ln\left(\frac{u_0(b)}{u_0(a)}\right) * \frac{1}{ln(\lambda)} + 1 \right\rfloor$: The DM repeatedly chooses the best alternative N times followed by the second best alternative once. As an example, we set $\lambda = 0.96$, $\beta = \lambda^8$ ($n = 8 > N = 6$) and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a, a, a, a}_{N=6}, b, \underbrace{a, a, a, a, a, a}_{N=6}, b, \cdots)$$

See figure Figure 7 in the appendix.

(iii) If $n \in \mathbb{N}^{\complement} \cap (1, \infty)$ and $n < N = \left\lfloor ln\left(\frac{u_0(b)}{u_0(a)}\right) * \frac{1}{ln(\lambda)} + 1 \right\rfloor$: Again the DM repeatedly chooses the best alternative N times followed by the second alternative once. Then, the DM keeps alternating forever between choosing the first alternative either $\lfloor n \rfloor$ or $\lfloor n+1 \rfloor$ times followed by the second best alternative once. The intuition behind this result is simple, this case is just similar to (i) but up to an integer problem. For example, if we set $\lambda = 0.9$, $\beta = \lambda^{2.5}$, ($\lfloor n \rfloor = 2 < n = 2.5 < \lfloor n+1 \rfloor = 3 \leq N = 3$) we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a}_{N=3}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a, a}_{\lfloor n+1 \rfloor = 3}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a, a}_{\lfloor n+1 \rfloor = 3}, b, \cdots)$$

See figure Figure 8 in the appendix.

As a second example, if $\lambda = 0.9$, $\beta = \lambda^{2.3}$, ($\lfloor n \rfloor = 2 < n = 2.3 < \lfloor n+1 \rfloor = 3 \leq N = 3$) we obtain the following sequence with a richer pattern:

$$\boldsymbol{x}_t = (\underbrace{a, a, a}_{N=3}, b, \underbrace{\underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}}_{\texttt{2 times}}, b, \underbrace{a, a, a}_{\lfloor n+1 \rfloor = 3}, b, \underbrace{\underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}}_{\texttt{3 times}}, b, \underbrace{a, a, a}_{\lfloor n+1 \rfloor = 3}, b, \underbrace{\underbrace{a, a}_{\lfloor n \rfloor = 2}, b, \underbrace{a, a}_{\lfloor n \rfloor = 2}}_{\texttt{2 times}}, b, \cdots)$$

See figure Figure 9 in the appendix.

Case 4 (Satiation rate > Recovery rate): $\lambda \in (0, 1)$, $\beta = \lambda^n$, $n \in [0, 1)$.

With this parameter configuration there are two important facts worth to highlight. First, it is the only configuration in which the DM will ever choose the third or the fourth best alternatives. Second, once the DM starts alternating no alternative will be ever chosen twice consecutive.

(i) If $n \in \left[\frac{1}{2}, 1\right)$: With this parameter configuration only the three best alternatives can be chosen.

If $n = \frac{1}{2}$: The best alternative in the choice set is chosen N times until the second best alternative is chosen for the first time. From that moment on, no alternative is ever chosen twice, and the DM alternates between the first two alternatives until the third one is chosen for the first time. Once the third alternative is chosen for the the first time, the same repeating sub-sequence of length three is repeated forever. As an example, we set $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.5$, and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a,}_{N=3} \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, c}, \underbrace{a, b, c,} \underbrace{a, b, c,} \cdots)$$

See figure Figure 10 in the appendix.

If $n \in \left(\frac{1}{2}, 1\right)$: Similar to the previous case but up to an integer problem. Here we are recovering a bit more than needed to keep the clean pattern outlined in the previous case. Because of this excess recovery we can generate richer patterns. As an example, we set $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.6$, and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a,}_{N=3} \underbrace{b, a}, \cdots, \underbrace{b, a}, \underbrace{\underbrace{c, b, a,} \underbrace{b, c, a,} \underbrace{b, a, c,} \underbrace{b, a, b,} \underbrace{c, a, b,} \underbrace{a, c, b,} \underbrace{a, b, c,} \underbrace{a, b, a,}}_{\texttt{Repeating sub-sequence}} \cdots)$$

See figure Figure 11 in the appendix.

(ii) If $n \in \left[\frac{1}{3}, \frac{1}{2}\right)$ The four alternatives of the choice set are chosen in the long run but history-dependent utilities do not tend to zero when $t \to \infty$.

If $n = \frac{1}{3}$: The best alternative in the choice set is chosen N times until the second best alternative is chosen for the first time. From that moment on, no alternative is ever chosen twice, and the DM alternates between the first two alternatives until the third one is chosen for the first time. Once the third alternative is chosen for the the first time, the same repeating sub-sequence of length three is repeated until the fourth best alternative is chosen. Once the fourth alternative is chosen, the same repeating sub-sequence of length four is repeated forever. As an example, we set $\lambda = 0.82$, $\beta = \lambda^n$, $n = 1/3$, and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a}_{N=2}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c, d}, \underbrace{a, b, c, d}, \cdots)$$

See figure Figure 12 in the appendix.

If $n \in \left(\frac{1}{3}, \frac{1}{2}\right)$: Similar to the previous case but up to an integer problem. Here we are recovering a bit more than needed to keep the clean pattern outlined in the previous case. Because of this excess recovery we can generate richer patterns. As an example, we set $\lambda = 0.7$, $\beta = \lambda^n$, $n = 0.4$, and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a,}_{N=1} \underbrace{b, a}, \underbrace{b, a}, \underbrace{c, b, a}, \underbrace{c, b, a}, \underbrace{c, b, a}, \underbrace{c, b, a}, \quad \underbrace{\underbrace{c, b, a}, \underbrace{c, b, a, d},}_{\texttt{Repeating sub-sequence}} \quad \cdots)$$

See figure Figure 13 in the appendix.

(iii) If $n \in \left[0, \frac{1}{3}\right)$ The four alternatives of the choice are chosen in the long run, moreover history-dependent utilities do tend to zero when $t \to \infty$.

The best alternative in the choice set is chosen N times until the second best alternative is chosen for the first time. From that moment on, no alternative is ever chosen twice, and the DM alternates between the first two alternatives until the third one is chosen for the first time. Once the third alternative is chosen for the the first time, the same repeating sub-sequence of length three is repeated until the fourth best alternative is chosen. Once the fourth alternative is chosen, the same repeating sub-sequence of length four is repeated forever. As an example, we set $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.1$, and we obtain:

$$\boldsymbol{x}_t = (\underbrace{a, a, a}_{N=3}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, a}, \underbrace{b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c}, \underbrace{a, b, c, d}, \underbrace{a, b, c, d}, \cdots)$$

See figure Figure 14 in the appendix.

D Figures

Figure 4: $\lambda = 1$, $\beta \in (0, 1]$.

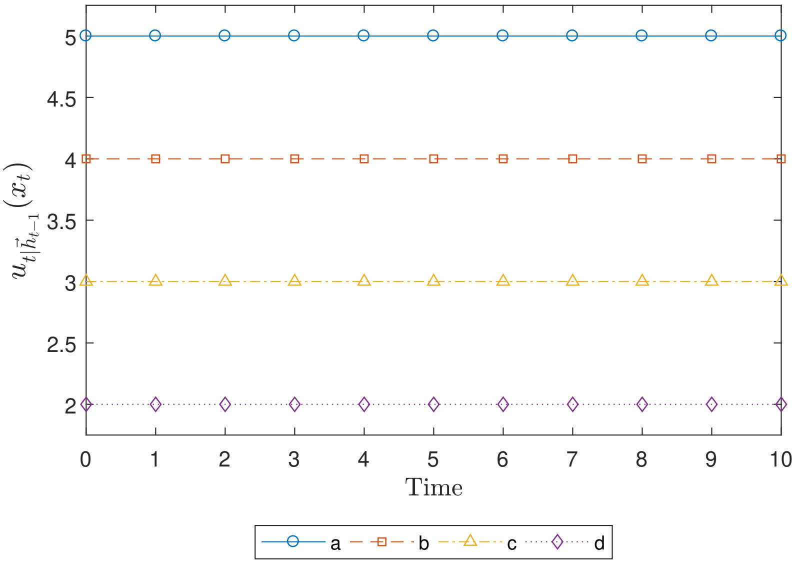


Figure 5: $\lambda = \beta = 0.9$.

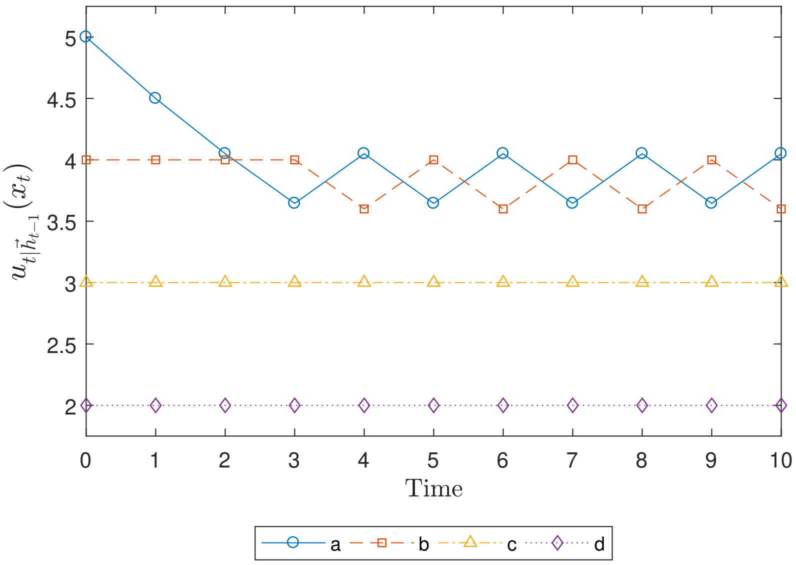


Figure 6: $\lambda = 0.96$, $\beta = \lambda^n$, $n = 3 < N = 6$.

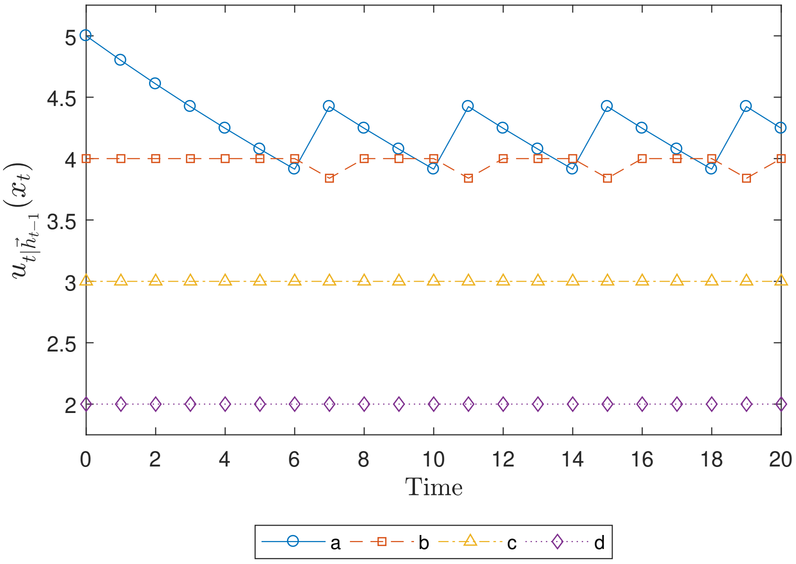


Figure 7: $\lambda = 0.96$, $\beta = \lambda^n$, $n = 8 > N = 6$.

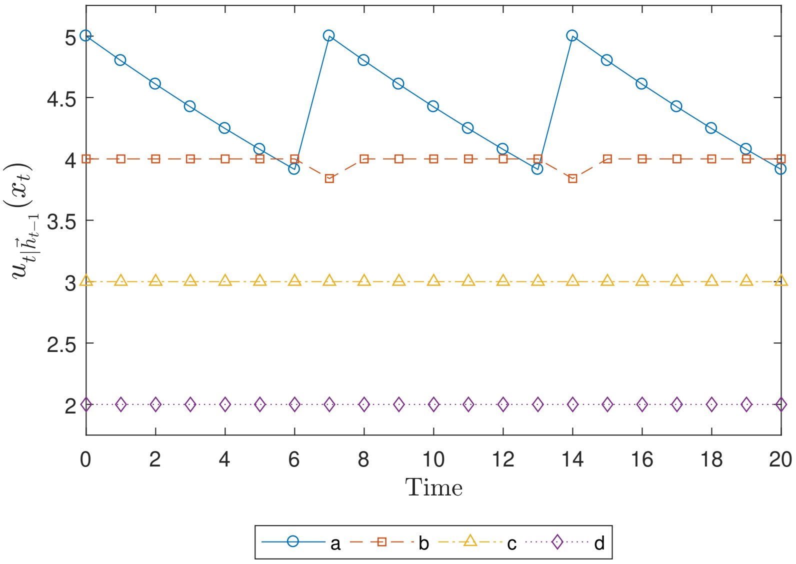


Figure 8: $\lambda = 0.9$, $\beta = \lambda^n$, $n = 2.5 < N = 3$.

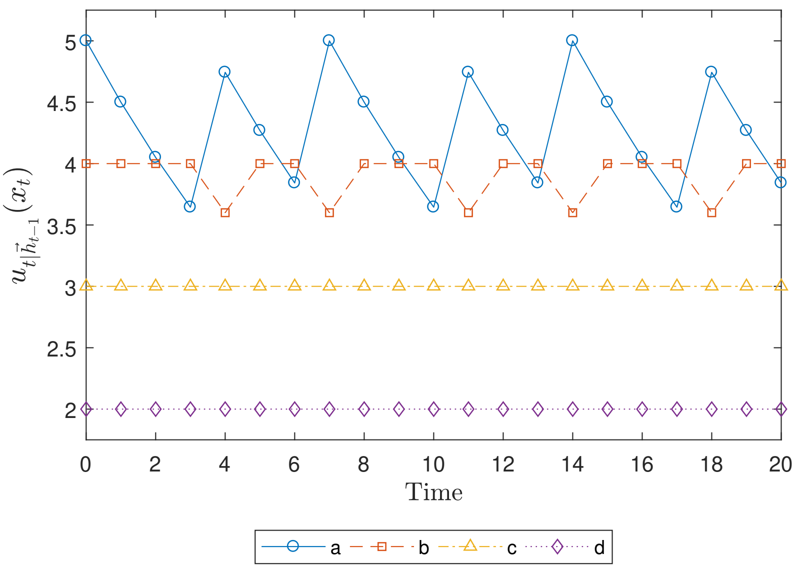


Figure 9: $\lambda = 0.9$, $\beta = \lambda^n$, $n = 2.3 < N = 3$.

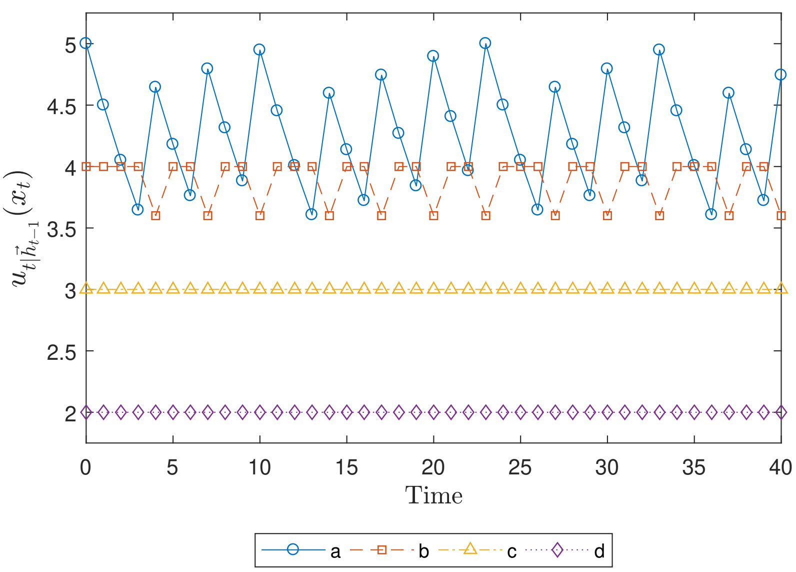


Figure 10: $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.5$.

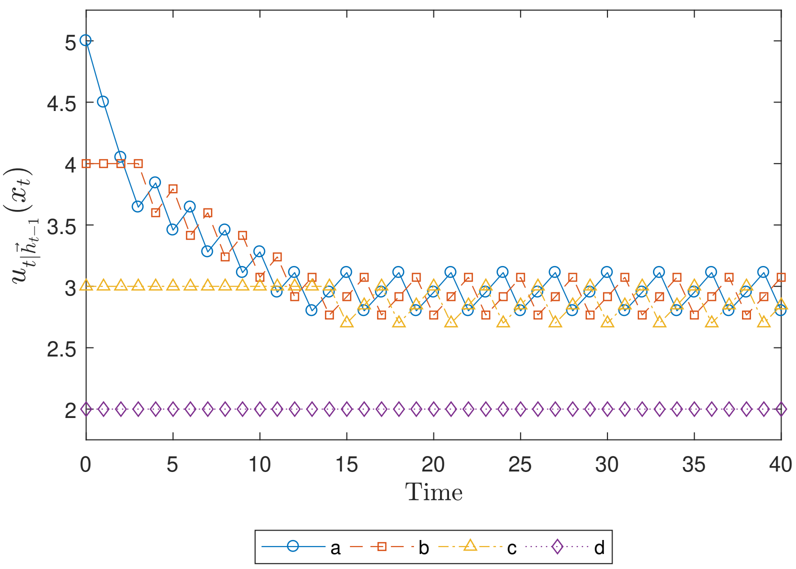


Figure 11: $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.6$.

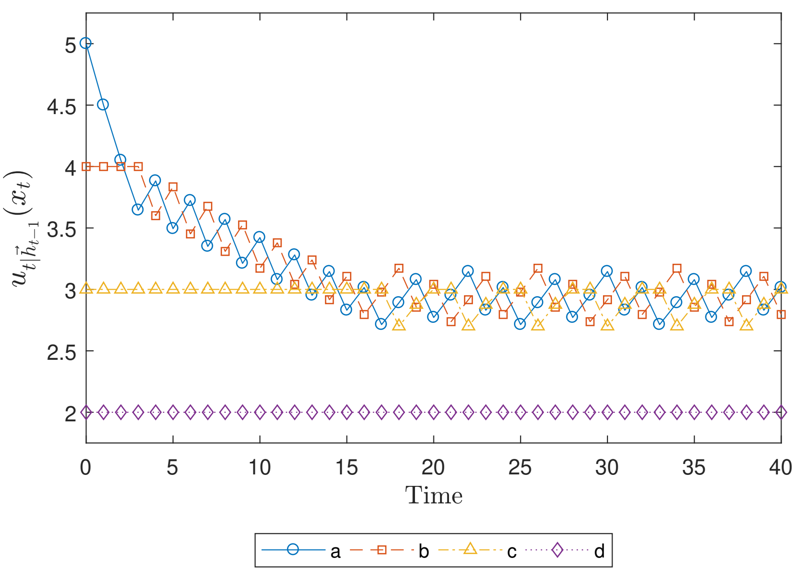


Figure 12: $\lambda = 0.82$, $\beta = \lambda^n$, $n = 1/3$.

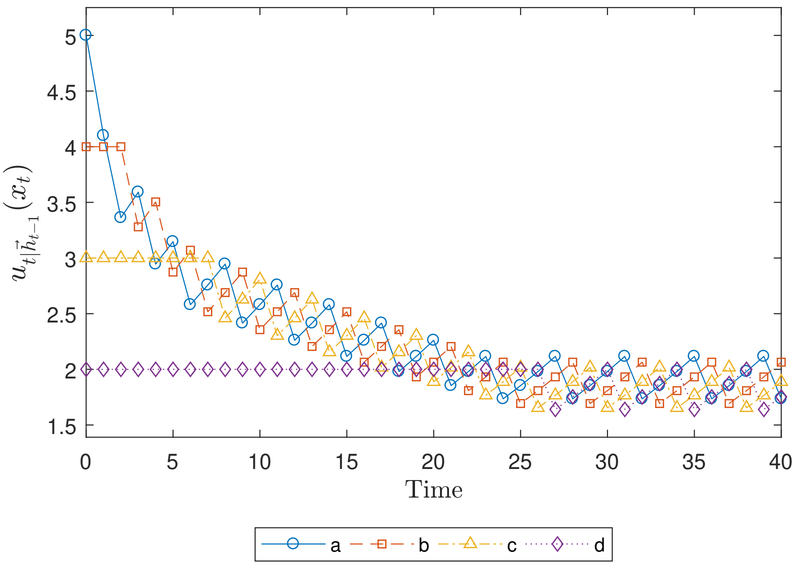


Figure 13: $\lambda = 0.7$, $\beta = \lambda^n$, $n = 0.4$.

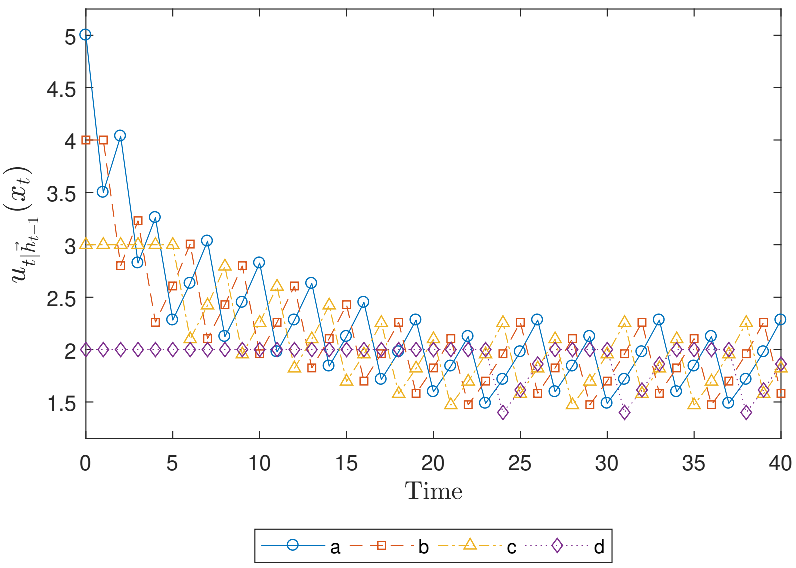
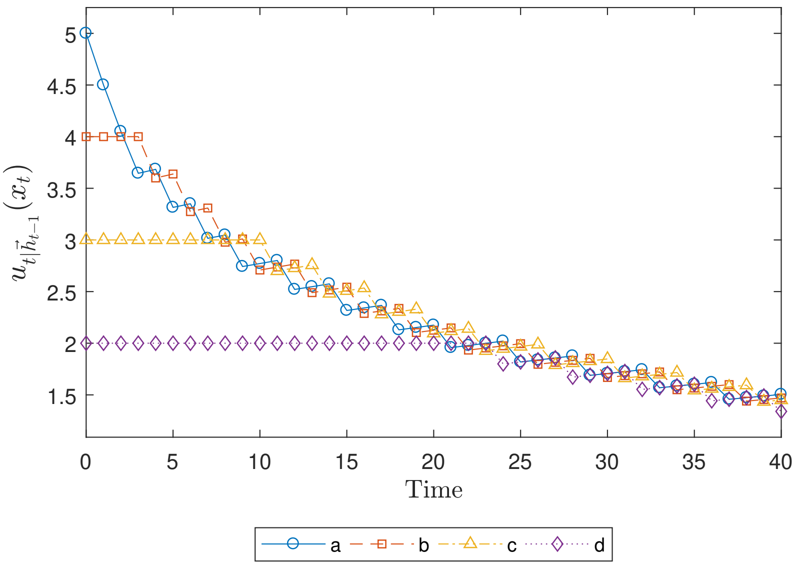


Figure 14: $\lambda = 0.9$, $\beta = \lambda^n$, $n = 0.1$.



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