Improving output gap estimation – A bottom-up approach

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Abstract

We propose a multivariate unobserved component model to identify potential output and the output gap consistent with the dynamics of the underlying production sectors of the economy and those of inflation and the labor market. Our approach allows us to decompose economic fluctuations and long-term trend growth into its driving factors. Applying our model to the Swiss economy reveals substantial divergence among the considered production sectors. The business cycle and the growth potential of the Swiss economy are most clearly influenced by the sectors that are most dependent on the global economy – manufacturing and financial and other economic services. While activities in trade, transport and hospitality are responsible for the slow decline in trend growth, manufacturing is counteracting this development. A comparison to established estimation approaches shows that our enriched information set can help paint a more comprehensive picture of the business cycle.

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Keywords: business cycle, output gap, potential output, bayesian state-space model, Kalman filter, Gibbs sampling

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1 Introduction

Potential output and its difference to actual observed gross domestic product (GDP)—the output gap—are used to determine the cyclical position of the economy. Potential output measures the level of a sustainable and non-inflationary economy, while the output gap reveals short-term deviations from this level (Hall and Taylor, 1991). A positive output gap indicates that the economy is overheating, while a negative gap signals underutilization of production factors.

In this paper, we present a model that enables the estimation of potential output and the output gap consistent with the cyclical fluctuations and long-term trend dynamics of the sub-sectors of the economy.

For fiscal and monetary policy makers, the output gap serves as a basis for monitoring inflation developments and structural imbalances (Gerlach and Smets, 1999; Coibion and Gorodnichenko, 2015). Generally, most structural models used in macroeconomic forecasting require an estimate of potential output which is key in determining the development of prices and wages (Dupasquier et al., 1999). Central banks rely on a precise estimation of the business cycle to determine possible inflationary and disinflationary pressures. To help maintain a balanced budget, many countries have introduced expenditure caps based on the cyclical position of the economy. For instance, the fiscal surveillance framework of the European Union uses an estimate of the output gap to extract the structural budget balance. To stabilize the business cycle, spending is increased in weak economic phases, while savings are stepped up in boom phases. Moreover, the aftermath of the Financial crisis has revived the debate on secular stagnation and structural changes in potential growth (Summers, 2015; Gordon, 2014), emphasizing the importance of a reliable and informative estimation strategy.

Potential growth and the output gap are unobservable quantities for which a multitude of estimation procedures have been proposed in the literature. The first category of methods comprises univariate filtering techniques which are essentially free of an economic model and decompose output into a permanent and transitory component (Hodrick and Prescott, 1997; Baxter and King, 1999). The next class of methods extends the univariate filters to include other observable variables that are related to the output gap using economic theory. Kuttner (1994) links deviations from potential output to inflation via a Phillips curve relationship and Gerlach and Smets (1999) additionally incorporate the real interest rate through an aggregate demand equation. The model of Blagrave et al. (2015) comprises inflation and labor market developments as well as growth and inflation expectations to inform the output gap. An alternative multivariate approach builds on the identification of temporary versus permanent shocks using structural vector autoregressions (Cochrane, 1994; Dupasquier et al., 1999), following the contribution of Blanchard and Quah (1989). More recently, Jarociński and Lenza (2018) use six indicators of real economic activity alongside a Phillips curve specification to identify the Euro area business cycle. Hasenzagl et al. (2022) propose a semi-structural model that links inflation dynamics and expectations to output, energy prices, and labor market developments. The third class comprises production-function approaches which first decompose output into its production factors—labor, capital, and productivity—and in turn determine their trends and cycles using similar unobserved component models as above (e.g. Havik et al., 2014).

While augmenting univariate filtering techniques with macroeconomic theories has helped to increase the interpretability of output gaps, the current models are silent about the underlying driving factors. Output gaps are usually estimated at the national and supra-regional level, for example for an economic area or a monetary union. By contrast, the business cycles of individual economic sectors within an economic area are rarely considered. Tracking the cycles of individual sectors allows policy actions to be targeted at specific industries, thereby increasing their efficiency and reducing the chance of pro-cyclical outcomes. For instance, the contraction at the outbreak of the COVID-19 pandemic in 2020 varied widely across different industries and was far from being akin to former economic crisis. Industries related to leisure activities such as eating out and vacationing were hit the hardest, while financial and business services experienced a comparably small decline.

A similar argument can be made for estimates of potential output. Decomposing potential growth into contributions by different economic areas can shed light on why recent trend growth rates lag behind those of earlier decades.

We propose a multidimensional state-space model which estimates the aggregate output gap and long-term growth consistent with the dynamics of the various sectors of the economy. This approach enables the decomposition of the business cycle into sector contributions on the one hand, and the separation of the sector cycles into economy-wide and sector-specific contributions on the other hand. Our model connects output to employment and unemployment via Okun's law and captures inflation dynamics via a Phillips curve relationship. The structure of the model is inspired both by Jarociński and Lenza (2018) and Hasenzagl et al. (2022). Its primary innovation is the integration of consistent trends and cycles of sub-sector output and employment. The proposed model profits from a more comprehensive information set and allows for a detailed analysis of the drivers of potential growth and the business cycle.

We apply our model to the Swiss economy and document the anatomy of the various driving forces of the up- and downturns since 1991. Our results suggest that the dynamics of the sectors differ considerably and that their contributions to the aggregate business cycle may vary in size as well as direction. Our potential growth decomposition shows that the sector trade, transport and hospitality is responsible for the slowly decreasing potential growth rate over the past 20 years, with manufacturing counteracting this development. A comparison to established estimation approaches reveals at least three considerable distinctions. Our model points to a stronger underutilization during the 1990s, greater overheating of the economy prior to the Financial Crisis, and a faster recovery afterwards.

This paper is organized as follows. Section 2 details the methodology and Section 3 presents the data. In Section 4, we demonstrate and interpret the application of our model to the Swiss economy. The last section concludes and gives an outlook on possible extensions.

2 Methodology

This section discusses the intuition and structure of our empirical approach. We estimate a multivariate state-space model to extract output gaps for the aggregate economy and its sectors simultaneously. We assume that each sector gap is a linear combination of the common cycle—the output gap—and a sector specific cycle. The sector specific cycles are independent, as are all trend processes. The unemployment and employment gap are each connected to the output gap via Okun's law (Okun, 1963). In the spirit of Stock and Watson (2007); Cogley et al. (2010) and Hasenzagl et al. (2022), among others, we model long-term trend inflation and the resulting inflation gap identifies the trend inflation neutral rate of unemployment (TINRU).

2.1 Econometric model

We use an unobserved component model to estimate the output gap. The observed variables, i.e., aggregate and sector output and employment, the unemployment rate and inflation are each linked to unobserved cycles and trend series.

Let y_t denote log output and y_{it} log output in sector i. We assume log output splits into a trend τ_t and a cycle component g_t—the output gap—i.e.,

$$y_t = \tau_t + g_t \tag{1}$$

with local linear trend

$$\begin{aligned} \tau_t &= \tau_{t-1} + \mu_{t-1} + \varepsilon_{\tau t}, \quad & \varepsilon_{\tau t} &\sim \mathcal{N}\left(0, \sigma_\tau^2\right), \\ \mu_t &= \mu_{t-1} + \varepsilon_{\mu t}, & \varepsilon_{\mu t} &\sim \mathcal{N}\left(0, \sigma_\mu^2\right). \end{aligned} \tag{2}$$

The trend drift μ_t can be interpreted as slowly changing potential growth rate. Shocks to the trend τ_t allow for short-term changes in trend growth. Analogously, we assume that output in each sector i can be separated into trend a τ_{it} and a cycle g_{it}. Each sector cycle is assumed to be linearly connected to the output gap and an idiosyncratic cycle, i.e.,

$$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}.$$

This implies that the sign of the coefficient β_i determines the nature of the correlation among the cycles as

$$\begin{aligned} \operatorname{cov}(g_t, g_{it}) &= \operatorname{cov}(g_t, \beta_i g_t + c_{it}) = \beta_i \operatorname{var}(g_t), \\ \operatorname{cov}(g_{jt}, g_{it}) &= \operatorname{cov}(\beta_j g_t + c_{jt}, \beta_i g_t + c_{it}) = \beta_j \beta_i \operatorname{var}(g_t). \end{aligned}$$

All output sector trends are modeled as in Equation (2) with normal and independent errors

$$\boldsymbol{\varepsilon}_{\tau t} = \left(\varepsilon_{\tau_t}, \varepsilon_{\tau_1 t}, \ldots, \varepsilon_{\tau_n t}\right)', \quad \boldsymbol{\varepsilon}_{\mu t} = \left(\varepsilon_{\mu_t}, \varepsilon_{\mu_1 t}, \ldots, \varepsilon_{\mu_n t}\right)'.$$

To summarize, output in all sectors fluctuates around a longer-term trend whose average growth rate may slowly change over time, driven for instance by technological innovation, globalization, or demographic changes.

We use both labor market and price developments to inform the fluctuations of the business cycle. The output gap is connected to employment as well as unemployment via Okun's law and a Phillips curve relationship links inflation to the unemployment gap and thereby to output. Let e_t denote log employment, u_t the unemployment rate and π_t the inflation rate. We assume

$$\begin{aligned} e_t &= \tau_{et} + \Psi_e(L)\, g_t + c_{et}, \\ u_t &= \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}, \\ \pi_t &= \tau_{\pi t} + \delta\left(u_t - \tau_{ut}\right) + c_{\pi t}, \end{aligned} \tag{3}$$

with $\Psi_{\cdot}(x) = \psi_{\cdot 0} + \ldots + \psi_{\cdot k} x^k$. The slack in the economy affects employment, unemployment and as a result inflation both contemporaneously and with a lag of up to k quarters, capturing labor market frictions (e.g. Hasenzagl et al., 2022). The difference $g_{ut} = u_t - \tau_{ut}$ defines the unemployment gap and τ_{ut} the trend inflation neutral rate of unemployment

(TINRU), i.e., the level of unemployment below which inflation is expected to rise above its trend.[1] In addition, we use Okun's law to help extract the sector output cycles. Since the unemployment rate is usually not available based on economic sectors, we use sector employment e_{it}. We assume

$$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t} \tag{4}$$

for $i = 1, \ldots, m$ with $m \leq n$. The employment trends and the TINRU are each modeled as local linear trends, analogous to Equation (2) with normal and uncorrelated trend and drift innovations

$$\boldsymbol{\varepsilon}_{\tau_e t} = \left(\varepsilon_{\tau^e t}, \varepsilon_{\tau_1^e t}, \ldots, \varepsilon_{\tau_n^e t}\right)', \quad \boldsymbol{\varepsilon}_{\mu_e t} = \left(\varepsilon_{\mu^e t}, \varepsilon_{\mu_1^e t}, \ldots, \varepsilon_{\mu_n^e t}\right)'$$

and $\varepsilon_{\tau_u t}, \varepsilon_{\mu_u t}$, respectively. In the spirit of Stock and Watson (2007); Cogley et al. (2010); Hasenzagl et al. (2022), trend inflation behaves like a random walk without drift, i.e.,

$$\tau_{\pi t} = \tau_{\pi t-1} + \varepsilon_{\tau_\pi t}, \quad \varepsilon_{\tau_\pi t} \sim \mathcal{N}\left(0, \sigma^2_{\tau_\pi}\right).$$

Collecting all trend and drift innovations, we have that

$$\begin{aligned}
\boldsymbol{\varepsilon}_{\tau t} &= \left(\boldsymbol{\varepsilon}'_{\tau t}, \boldsymbol{\varepsilon}'_{\tau_e t}, \varepsilon_{\tau_u t}, \varepsilon_{\tau_\pi t}\right)' \sim \mathcal{N}\left(0, \boldsymbol{\Sigma}_\tau\right), \\
\boldsymbol{\varepsilon}_{\mu t} &= \left(\boldsymbol{\varepsilon}'_{\mu t}, \boldsymbol{\varepsilon}'_{\mu_e t}, \varepsilon_{\mu_u t}\right)' \sim \mathcal{N}\left(0, \boldsymbol{\Sigma}_\mu\right),
\end{aligned}$$

where $\boldsymbol{\Sigma}_\tau$ and $\boldsymbol{\Sigma}_\mu$ are diagonal. The output gap and all idiosyncratic cycles are modeled as stationary autoregressive processes, i.e., for $\mathbf{c}_t = (g_t, c_{1t}, \ldots, c_{nt}, c_{e_t}, c_{e_1 t}, \ldots, c_{e_n t}, c_{ut}, c_{\pi t})'$, we have that

$$\begin{aligned}
&\boldsymbol{\Phi}(L)\,\mathbf{c}_t = \boldsymbol{\varepsilon}_{ct}, \\
&\qquad \boldsymbol{\varepsilon}_{ct} = \left(\varepsilon_{gt}, \varepsilon_{c_1 t}, \ldots, \varepsilon_{c_n t}, \varepsilon_{c^e t}, \varepsilon_{c_1^e t}, \ldots, \varepsilon_{c_n^e t}, \varepsilon_{c_u t}, \varepsilon_{c_\pi t}\right)' \sim \mathcal{N}\left(0, \boldsymbol{\Sigma}_c\right),
\end{aligned}$$

with $\boldsymbol{\Sigma}_c$ diagonal, and the lag polynomial $\boldsymbol{\Phi}(x) = 1 - \boldsymbol{\Phi}_1 x - \ldots - \boldsymbol{\Phi}_p x^p$ with diagonal coefficient matrices $\boldsymbol{\Phi}_j, j = 1 \ldots, p$.

[1] In equilibrium, output equals potential, inflation its long-term trend, and the unemployment rate matches the TINRU. Thus, the TINRU can be thought of as the natural rate of unemployment. In contrast to the non-accelerating inflation rate of unemployment (NAIRU), our unemployment cycle is connected to an inflation gap identifying trend inflation as longer-term inflation expectations.

Some restrictions on the innovation correlations between trends, drifts, and cycles are necessary for identification. The model in Equations (1) and (2) with an autoregressive output gap g_t is identical to the one put forward by Clark (1987). In this model, identification can be achieved by placing restrictions on the innovation covariance structure and by including at least 2 autoregressive lags in the cycle equation (Clark, 1987; Schleicher et al., 2003; Morley et al., 2003; Morley, 2007).[2] We therefore set $p = 2$ and impose that all innovations $\boldsymbol{\varepsilon}_{\tau t}, \boldsymbol{\varepsilon}_{\mu t}$ and $\boldsymbol{\varepsilon}_{ct}$ are mutually independent. This implies that transitional changes in consumption or government expenditures do not affect output trend growth. Similarly, demographic or technological changes are assumed to trigger changes in long-run trend growth but not temporary changes in demand.[3] The same identification restrictions carry over to our full model, as each of our additional observation equations features its own trend and cycle component.

2.2 Aggregation and constraints

To ensure that aggregate outcomes are consistent with sector specific ones, we impose linear constraints on the trend drifts, both for output and employment.[4]

Let Y_{it} and Y_{it}^{nom} denote real and nominal output in sector i and let $P_{it} = 100\, {}^{Y_{it}^{nom}}\!/_{Y_{it}}$ be the corresponding price index. The associate aggregate series are given by Y_t, Y_t^{nom} and P_t. Real aggregate output is defined as the chain-linked volume index, i.e.,

$$Y_t = \sum_{i=1}^{n} \frac{P_{it-1}}{P_{t-1}} Y_{it} = \sum_{i=1}^{n} w_{ti}^{p} Y_{it}$$

[2] To see this, the model can be rearranged into a reduced form for which there exists an equivalent ARIMA representation which is just identified for $p = 2$ (Hamilton, 1994; Morley et al., 2003; Oh et al., 2006). Intuitively, increasing the autoregressive order p increases the number of non-zero autocovariance terms used to estimate the variances.

[3] Even though the presence of correlation between permanent and transitory shocks cannot be ruled out, it will likely be negligible. See, for instance, Clark (1987), Morley et al. (2003), Morley (2007), and Oh et al. (2006) for an analysis and discussion of unobserved component models with correlated trend, drift, and cycle innovations.

[4] This is only relevant if the sectoral series included in the model are exhaustive, i.e., they add up to aggregate output and employment, respectively.

which implies that

$$\begin{aligned}\frac{Y_t}{Y_{t-1}} &= \sum_{i=1}^{n} \frac{P_{it-1}}{P_{t-1}Y_{t-1}} Y_{it-1} = \sum_{i=1}^{n} \frac{P_{it-1}Y_{it-1}}{P_{t-1}Y_{t-1}} \frac{Y_{it}}{Y_{it-1}} \\ &= \sum_{i=1}^{n} \frac{Y_{it-1}^{nom}}{Y_{t-1}^{nom}} \left(\frac{Y_{it}}{Y_{it-1}} \right) = \sum_{i=1}^{n} w_{it}^{nom} \left(\frac{Y_{it}}{Y_{it-1}} \right).\end{aligned}$$

and where w_{ti}^{p} denotes relative previous period prices and w_{it}^{nom} nominal output weights at $t-1$. Since $\sum_{i=1}^{n} w_{it}^{nom} = 1$ for all $t \in \mathbb{Z}$, it holds that

$$\frac{Y_t}{Y_{t-1}} - 1 = \sum_{i=1}^{n} w_{it}^{nom} \left(\frac{Y_{it}}{Y_{it-1}} - 1 \right).$$

The growth rate of output can thus be represented as a weighed average of the growth rates of individual sectors, i.e.,

$$\Delta y_t = \sum_{i=1}^{n} w_{it}^{nom} \Delta y_{it},$$

where $y_t = \ln Y_t$.[5] Consequently, for potential growth, we impose

$$\mu_t = \sum_{i=1}^{n} w_{it}^{nom} \mu_{it} \tag{5}$$

by adding an identity series to the observation equation (Doran, 1992).

For aggregate employment E_t, we have that $E_t = \sum_{i=1}^{n} E_{it}$, where E_{it} represents the number of persons employed in sector i. Analogously to above, we can deduce

$$\begin{aligned}\Delta e_t &= \sum_{i=1}^{n} w_{it}^{e} \Delta e_{it}, \\ \mu_{et} &= \sum_{i=1}^{n} w_{it}^{e} \mu_{e_i t}, \end{aligned} \tag{6}$$

where $e_t = \ln E_t$ and $w_{ti}^{e} = {}^{E_{it-1}}/_{E_{t-1}}$ denotes the share of employment in sector i at point in time $t-1$.

[5]When the data is compiled using the Annual Overlap method, this holds with equality for annual values. For quarterly quantities there can be small deviation, but we assume that these are zero for the trend component we are interested in.

2.3 State-space representation

We stack all observation variables and their cycles, trends, and trend drifts in corresponding order, i.e.,

$$
\begin{aligned}
\underset{\ell\times 1}{\mathbf{y_t}} &= \left(y_t, \mathbf{y}_t^{1:n\prime}, e_t, \mathbf{e}_t^{1:m\prime}, u_t, \pi_t\right)' = (y_1, y_{1t}, \ldots, y_{nt}, e_t, e_{1t}, \ldots, e_{mt}, u_t, \pi_t)' \\
\underset{\ell\times 1}{\mathbf{c}_t} &= \left(g_t, \mathbf{c}_{yt}', c_{et}, \mathbf{c}_{et}', c_{ut}, c_{\pi t}\right)' = (g_t, c_{1t}, \ldots, c_{nt}, c_{et}, c_{e_1 t}, \ldots, c_{e_m t}, c_{ut}, c_{\pi t})' \\
\underset{\ell\times 1}{\boldsymbol{\tau}_t} &= \left(\tau_t, \boldsymbol{\tau}_{yt}', \tau_{et}, \boldsymbol{\tau}_{et}', \tau_{ut}, \tau_{\pi t}\right)' = (\tau_t, \tau_{1t}, \ldots, \tau_{nt}, \tau_{et}, \tau_{e_1 t}, \ldots, \tau_{e_n t}, \tau_{ut}, \tau_{\pi t})', \\
\underset{\ell-1\times 1}{\boldsymbol{\mu}_t} &= \left(\mu_t, \boldsymbol{\mu}_{yt}', \mu_{et}, \boldsymbol{\mu}_{et}', \mu_{ut}\right)' = (\mu_t, \mu_{1t}, \ldots, \mu_{nt}, \mu_{et}, \mu_{e_1 t}, \ldots, \mu_{e_m t}, \mu_{ut})',
\end{aligned}
$$

where $\ell = n + m + 4$ is the number of observables. The state vector can be defined by

$$
\underset{5\ell-1\times 1}{\boldsymbol{\alpha}_t} = \left(\mathbf{c}_t', \mathbf{c}_{t-1}', \mathbf{c}_{t-2}', \boldsymbol{\tau}_t', \boldsymbol{\mu}_t'\right)',
$$

and the measurement and state equations are given by

$$
\begin{aligned}
\underset{\ell\times 1}{\mathbf{y}_t} &= \underset{\ell\times 5\ell-1}{\mathbf{Z}_t} \underset{5\ell-1\times 1}{\boldsymbol{\alpha}_t}, \\
\underset{5\ell-1\times 1}{\boldsymbol{\alpha}_t} &= \underset{5\ell-1\times 5\ell-1}{\mathbf{T}_t} \underset{5\ell-1\times 1}{\boldsymbol{\alpha}_{t-1}} + \underset{5\ell-1\times 3\ell-1}{\mathbf{R}_t} \underset{3\ell-1\times 1}{\varepsilon_t}, \qquad \underset{3\ell-1\times 1}{\varepsilon_t} \sim \mathcal{N}(0, \underset{3\ell-1\times 3\ell-1}{\mathbf{Q}_t}).
\end{aligned}
\tag{7}
$$

For ease of readibility, we split the system matrices into three blocks, one concerning the vector of contemporaneous and lagged cycles $\left(\mathbf{c}_t', \mathbf{c}_{t-1}', \mathbf{c}_{t-2}'\right)'$, one the trends $\boldsymbol{\tau}_t$ and a final block for the drifts $\boldsymbol{\mu}_t$. The structure of the blocks is indicated via vertical and horizontal lines. The system matrices of the state-space model in Equation (7) are given by

$$
\underset{\ell\times 5\ell-1}{\mathbf{Z}_t} = \left[\begin{array}{ccc|c|c} \underset{\ell\times\ell}{\mathbf{Z}_t^0} & \underset{\ell\times\ell}{\mathbf{Z}_t^1} & \underset{\ell\times\ell}{\mathbf{Z}_t^2} & \mathbf{I}_\ell & \mathbf{0} \end{array}\right], \qquad
\underset{5\ell-1\times 5\ell-1}{\mathbf{T_t}} = \left[\begin{array}{ccc|c|c}
\boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 & \mathbf{0} & & \\
\mathbf{I}_\ell & \mathbf{0} & \mathbf{0} & & \\
\mathbf{0} & \mathbf{I}_\ell & \mathbf{0} & & \\
\hline
 & & & \mathbf{I}_\ell & \begin{matrix}\mathbf{I}_{\ell-1} \\ \mathbf{0}\end{matrix} \\
\hline
 & & & & \mathbf{I}_{\ell-1}
\end{array}\right],
$$

$$
\underset{5\ell-1\times 3\ell-1}{\mathbf{R}_t} = \left[\begin{array}{ccc} \mathbf{I}_\ell & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I}_\ell & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{I}_{\ell-1} \end{array}\right], \qquad \underset{3\ell-1\times 3\ell-1}{\mathbf{Q}_t} = \left[\begin{array}{ccc} \mathbf{\Sigma_c} & & \\ \hline & \mathbf{\Sigma}_\tau & \\ \hline & & \mathbf{\Sigma}_\mu \end{array}\right].
$$

The state matrix $\mathbf{T}_t$ contains the autoregressive coefficients $\mathbf{\Phi}_j, j \in \{1,2\}$ of the cycles, defines the trend processes as integrated or regular random walks and additionally includes a number of identities for the lagged cycles. The matrix $\mathbf{R}_t$ connects each state equation to its corresponding innovation term (or none) and the variance-covariance matrix $\mathbf{Q}_t$ contains all cycle, trend and drift variances on its diagonal. The submatrices in $\mathbf{Z}_t$ are defined by

$$
\underset{\ell\times\ell}{\mathbf{Z}_t^0} = \begin{bmatrix} 0 & & & & & \\ \boldsymbol{\beta} & \ddots & & & & \\ 0 & \mathbf{0} & \ddots & & & \\ \vdots & & \ddots & \ddots & & \\ \vdots & & & \mathbf{0} & \ddots & \\ 0 & \cdots & \cdots & \mathbf{0} & \delta & 0 \end{bmatrix} + \mathbf{I}_\ell + \underset{\ell\times\ell}{\tilde{\mathbf{Z}}_t^0}
$$

$$
\underset{\ell\times\ell}{\tilde{\mathbf{Z}}_t^j} = \begin{bmatrix} 0 & & & & & \\ \mathbf{0} & \ddots & & & & \\ \psi_{ej} & \mathbf{0} & \ddots & & & \\ \boldsymbol{\beta}\circ\boldsymbol{\psi}_{e^{1:m}j} & \mathbf{\Psi}_{e^{1:m}j} & \mathbf{0} & \ddots & & \\ \psi_{uj} & \mathbf{0} & \cdots & \mathbf{0} & \ddots & \\ \delta\psi_{uj} & \mathbf{0} & \cdots & \mathbf{0} & 0 & 0 \end{bmatrix}
$$

and $\mathbf{Z}_t^j = \tilde{\mathbf{Z}}_t^j$ for $j \in \{1,2\}$, where $\boldsymbol{\psi}_{e^{1:m}j} = (\psi_{e_1j}, \ldots, \psi_{e_mj})$ and $\mathbf{\Psi}_{e^{1:m}j} = \operatorname{diag}(\boldsymbol{\psi}_{e^{1:m}j})$ for $j \in \{0,1,2\}$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)'$.

The auxiliary matrices $\tilde{\mathbf{Z}}_t^j, j \in \{0,1,2\}$ contain the (lagged) loading of employment ψ_{ej}, sector employment $\boldsymbol{\beta}\circ\boldsymbol{\psi}_{e^{1:m}j}$, unemployment ψ_{uj}, and inflation $\delta\psi_{uj}$ on the business cycle and in the case of sector employment on the sector output cycles ($\mathbf{\Psi}_{e^{1:m}j}$). In addition, the matrix $\mathbf{Z}_t^0$ links each observable with its contemporaneous cycle and it contains the loading coefficients of sector output on the output gap $\boldsymbol{\beta}$ and the one of inflation on the output gap δ.

To impose constraints on the drifts of sector output and employment as discussed in Section 2.2, $\mathbf{Z}_t$ and $\mathbf{y}_t$ need to be expanded (Doran, 1992). To be precise, we define

$$\hat{\mathbf{y}}_t = (\mathbf{y}_t', 0, 0)', \qquad \underset{\ell+2\times 5\ell-1}{\hat{\mathbf{Z}}_t} = \left[\begin{array}{ccc|c|c} \underset{\ell\times\ell}{\mathbf{Z}_t^0} & \underset{\ell\times\ell}{\mathbf{Z}_t^1} & \underset{\ell\times\ell}{\mathbf{Z}_t^2} & \mathbf{I}_\ell & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \underset{2\times\ell-1}{\mathbf{Z}_t^\mu} \end{array}\right],$$

$$\underset{2\times\ell-1}{\mathbf{Z}_t^\mu} = \begin{bmatrix} -1 & w_{1t}^{nom} & \dots & w_{nt}^{nom} & 0 & & \cdots & & 0 \\ 0 & & \cdots & 0 & -1 & w_{1t}^{e} & \cdots & w_{mt}^{e} & 0 \end{bmatrix}.$$

Note that the weights $w_{it}^{nom}, i = 1, \dots, n$ and $w_{jt}^e, j = 1, \dots m$ as defined in Section 2.2 are time-dependent.

2.4 Estimation

The computational task comprises estimating the unobserved states $\boldsymbol{\alpha}_t$ and the parameters $\boldsymbol{\beta}, \boldsymbol{\psi}_e, \boldsymbol{\psi}_u, \delta, \boldsymbol{\psi}_{e^{1:m}0}, \boldsymbol{\psi}_{e^{1:m}1}, \boldsymbol{\psi}_{e^{1:m}2}, \boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \boldsymbol{\Sigma}_c, \boldsymbol{\Sigma}_\tau, \boldsymbol{\Sigma}_\mu$, where $\boldsymbol{\psi}_e = (\psi_{e0}, \psi_{e1}, \psi_{e2})'$, $\boldsymbol{\psi}_u = (\psi_{u0}, \psi_{u1}, \psi_{u2})'$. Our estimation procedure involves a Gibbs algorithm structured in multiple blocks. The first block draws from the posterior distributions of all trend and trend drift equations. The second block handles all equations involving loading factors and their respective cycles. The third block draws the parameters of the output gap equation. The final block updates the unobserved states conditional on the previously drawn parameters using the simulation smoother of Durbin and Koopman (2012). To compute each posterior we generate 30'000 draws, discard the first 20%, and finally consider each 10th draw to limit possible autocorrelation between draws. See Appendix A.1 for details.

We adopt weakly informative priors in the form of diffuse normal or inverse-gamma priors, see Table 1. To facilitate the estimation of meaningful trends and cycles, we use a smoothing parameter $\lambda = \mathbb{E}[\sigma_c^2]/\mathbb{E}[\sigma_k^2], k \in \{\tau, \mu\}$ defining the ratio between the variance of idiosyncratic and trend innovations. This allows us to mitigate the pile-up problem of Stock and Watson (1998).

Table 1: Prior distributions.

Name	Support	Density	Parameters	
β_i, δ	$\mathbb{R}$	Normal	$\mu = 0,$	$\sigma^2 = 1000$
$(\psi_0, \psi_1, \psi_2)'$	$\mathbb{R}^3$	Normal	$\mu = (0,0,0)',$	$\sigma^2 = 1000\mathbf{I}_3$
$(\phi_1, \phi_2)'$	$\mathbb{R}^2 \times I_{\phi \in S_\phi}$	Normal	$\mu = (0,0)',$	$\sigma^2 = 1000\mathbf{I}_2$
σ_c^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4$
σ_μ^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4\lambda^{-1}$
σ_τ^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4\lambda^{-1}$

Notes: $I_{\phi \in S_\phi}$ denotes the indicator function and S_ϕ the stationary region of an AR(2) process. All indices are suppressed for the sake of readability. The normal distribution is parametrized via mean and variance, the inverse-gamma distribution via degrees of freedom ν and location s with mean $s/\nu-2$. The smoothing constant λ is set to 100.

3 Data

We use quarterly aggregates of gross domestic product for Switzerland according to the production approach. In addition to real GDP, we consider real gross value added before adjustments of five economic sectors. To facilitate the cyclical interpretation of output, we use output series that are adjusted for major international sporting events.[6] Aggregate adjustments are treated as an individual sector to complete the model. All production series are provided by the Swiss State Secretariat for Economic Affairs (SECO). The composition of the sectors is documented in Table 2. For employment, we use full-time equivalents gathered by the Swiss Federal Statistical Office (FSO) as part of the Job Statistic (JOBSTAT). The provided sectoral full-time equivalent series can be aggregated such that the resulting series largely correspond to the production ones specified in Table 2.[7] Finally, we use the unemployment rate based on the definition of the International Labor Organization (ILO) and for inflation we use a measure of core inflation, the year-on-year growth rate of the Consumer Price Index (CPI) excluding oil. Both series are provided by the Swiss FSO. All series are seasonally-adjusted and all output series are

[6] Several international sport organisations are based in Switzerland, including the International Association Football Federation (FIFA), the Union of European Football Associations (UEFA) and the International Olympic Committee (IOC). These associations contribute to output mainly through income from intangible assets such as licenses and patents. However, from a business cycle perspective, the periodicity of their contributions to output disables an economic interpretation. At the same time, output from international sporting events is usually created abroad and therefore only of little relevance to the domestic economy in Switzerland. Excluding output from international sporting events therefore creates a more fitting measure of economic output for business cycle analysis.

[7] For the agricultural sector NOGA 01-03, no employment data is available. However, given the relative size of the sector compared to manufacturing as a whole (less than 1% of output versus 22%), this shortcoming is negligible.

additionally calender-adjusted.

Table 2: Structure of sectors.

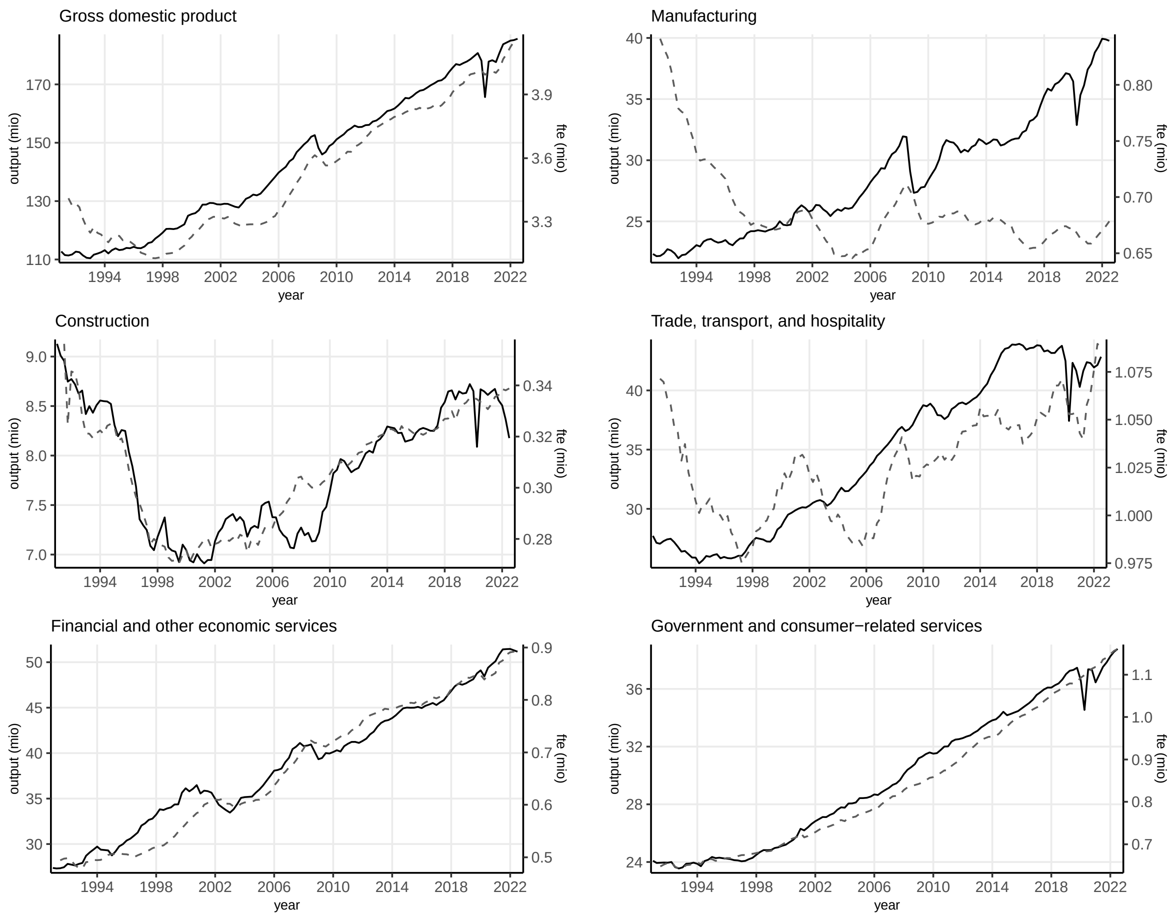
Sector	Sub-sectors	NOGA
Manufacturing	Agriculture, forestry and fishing	01-03
	Mining and quarrying	05-09
	Manufacturing	10-33
	Electricity, gas, steam and air conditioning supply	35
	Water supply, sewerage, waste management and remediation activities	36-39
Construction	Construction	41-43
Trade, transport and hospitality	Trade, repair of motor vehicles and motorcycles	45-57
	Transportation and storage; Information and communication	49-53; 58-63
	Accommodation and food service activities	55-56
Financial and other economic services	Financial service activities	64
	Insurance service activities	65
	Real estate, professional, scientific and technical activities; Administrative and support service activities	68-57; 77-82
Government and consumer-related services	Public administration and defense; compulsory social services	84
	Education	85
	Human health and social work activities	86-88
	Arts, entertainment and recreation	90-93
	Other service activities	94-96
	Activities of housholds as employers and producers for own use	97-98
Adjustments	Taxes on products	
	Subsidies on products	

Notes: The General Classification of Economic Activities (NOGA) provided by the Swiss FSO is derived from the Statistical Classification of Economic Activities in the European Union (NACE). The current NOGA (2008) was enacted in 2008.

Figure 1 shows the development of output (solid lines) and full-time equivalent employment (dashed lines) in Switzerland at the aggregate as well as disaggregate level. The economic development of the various sectors differs, in some cases significantly. For instance, the construction sector experienced a considerable decline in the 1990s, after the bursting of a housing bubble while the overall economic development was decent. Sectors also react differently to crises such as the 2007-2008 financial crisis or the COVID-19 pandemic that began in early 2020. The financial crisis mainly affected manufacturing and financial and other economic services, while consumer-related services did not experience a decline. The coronavirus pandemic initially led to a reduction in output across all sectors, but this decrease was particularly pronounced in trade, transport and hospitality. In general, roughly since the second half of the 1990s, all sectors have been on a steady growth path.

The development of full-time equivalent employment is also heterogeneous across sectors. Approximately until the turn of the century, employment has been decreasing in manufacturing, construction and trade, transport, and hospitality. Most sectors show an upward trend thereafter, while employment in manufacturing has been stagnant. For the aggregate economy and most sectors, there exists a positive correlation between output and employment with the exception of manufacturing which is experiencing elevated levels of productivity growth.

Figure 1: Output and full-time equivalent employment in different sectors.



Notes: The series are depicted from 1990 Q1 until 2022 Q3. The solid lines (left axes) show quarterly output in million 2019 CHF and the dashed lines (right axes) depict full-time equivalent (fte) employment in million.

4 Results

We estimate our model with autoregressive cycles of order $p = 2$ and to capture a potentially lagged reaction of the labor market, we set $k = 2$. We start by elaborating on the role of the prior distributions in identifying the model. The subsequent section discusses the estimated trends and cycles of the Swiss economy at an aggregate level and then moves to the sector contributions of the output and employment gaps. A similar analysis is done for potential growth and employment trend growth. Finally, we compare our measure of the business cycle to alternative estimates.

4.1 Prior and posterior distributions

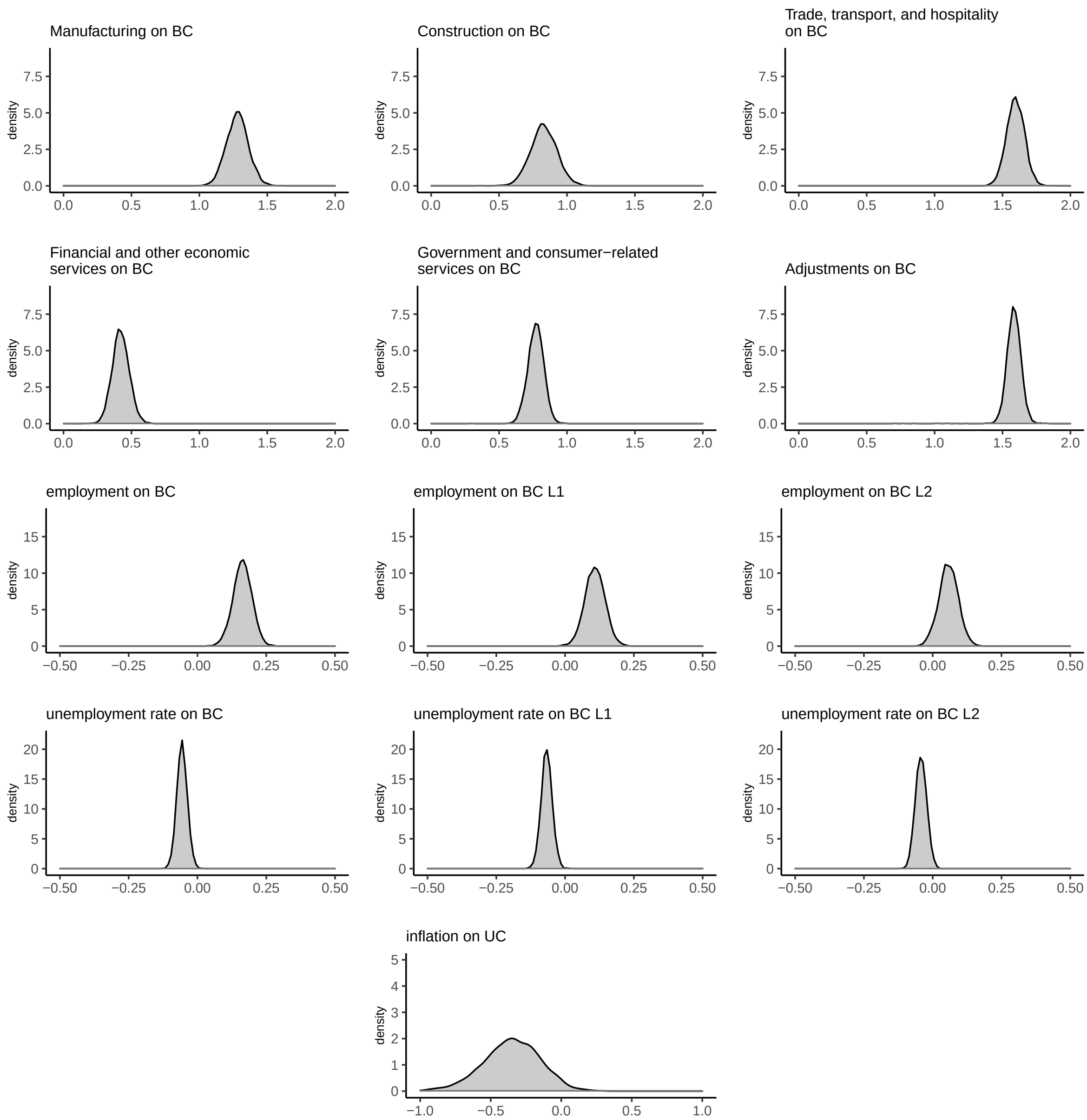
Figure 2 illustrates prior and posterior densities for all parameters that load on the business cycle and the unemployment rate.[8] The posteriors are well peaked indicating that the data is quite informative in estimating the model parameters.

All sector cycles are positively correlated with the output gap, as indicated by the posterior means of the loading factors. As expected, the posterior of the loadings of aggregate employment on the output cycle has positive mean. The contemporaneous effect appears to be the strongest and it wears off with increasing lag order. Accordingly, the posterior distribution of the loadings of the unemployment gap on the output gap and its lags and that of the loading factor of the inflation gap on the unemployment gap has negative mean.

Prior and posterior distributions for the loading factors of sector employment on the respective output series and lags thereof are shown in Figure A.1 and for the cycle, trend, and drift variances in Figure A.2 in Appendix A.2. The posteriors indicate a positive contemporaneous relationship between sector employment and sector output. The model also suggests that employment in manufacturing, trade, transport and hospitality, and financial and other economic services reacts with a lag of up to half a year to changes in output. In contrast, there is no evidence for a lagged reaction of employment in construction and government and consumer-related services.

[8] Table A.1 in Appendix A.3 summarizes the mean, median, first and ninth decile of the posterior distribution of all parameters.

Figure 2: Prior and posterior distributions.



Notes: The prior distributions are specified as in Table 1. BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

4.2 Aggregate trends and cycles in the Swiss economy

Figure 3 plots aggregate output and employment, the unemployment rate and inflation alongside their trend estimates (left column) and corresponding cycles (right column) with 68% highest-posterior density intervals (HPDI). We can clearly recognize the stylized facts of the Swiss business cycle. Despite comparably low trend growth rates, the nineties were marked by a long phase of underutilization, naming it Switzerland's lost decade. The next recession took place after the dot-com bubble. Leading up to Global Financial Crisis in 2007-2008, our model clearly emphasizes an overheating of the economy. Interestingly, the subsequent Great Recession was much less pronounced than the ones before it. During the 2010s, the aggregate Swiss economy was mostly operating close to capacity. The most severe underutilization combined with an unusually swift recovery occured at the outset of the COVID-19 pandemic.

The labor market shares much of the same dynamics. As expected, there is an inverse relation between the output gap and the unemployment gap, while its correlation to the employment gap is evidently positive. Underutilization of economic production factors mostly came hand in hand with elevated levels of unemployment and a negative employment gap and vice versa. At the beginning of the pandemic, however, the labor market response was less pronounced than indicated by the historic relationship to the output gap, suggesting that the massive use of short time working schemes successfully protected the labor market. For instance, the responses of the employment and unemployment gaps in the second quarter of 2020 were roughly 30% below the responses in line with historic declines in capacity utilization, i.e., the ideosycncratic parts of the labor market gaps counteract the effect of the negative shocks to the output gap.[9]

In line with the continuous population growth in Switzerland, particularly since the turn of the millennium, employment alongside its trend have been steadily increasing. The TINRU has experienced a level shift between 2000 and 2007, and has remained relatively stable at rates between 4.5% and 5%. Only lately has the TINRU been decreasing again,

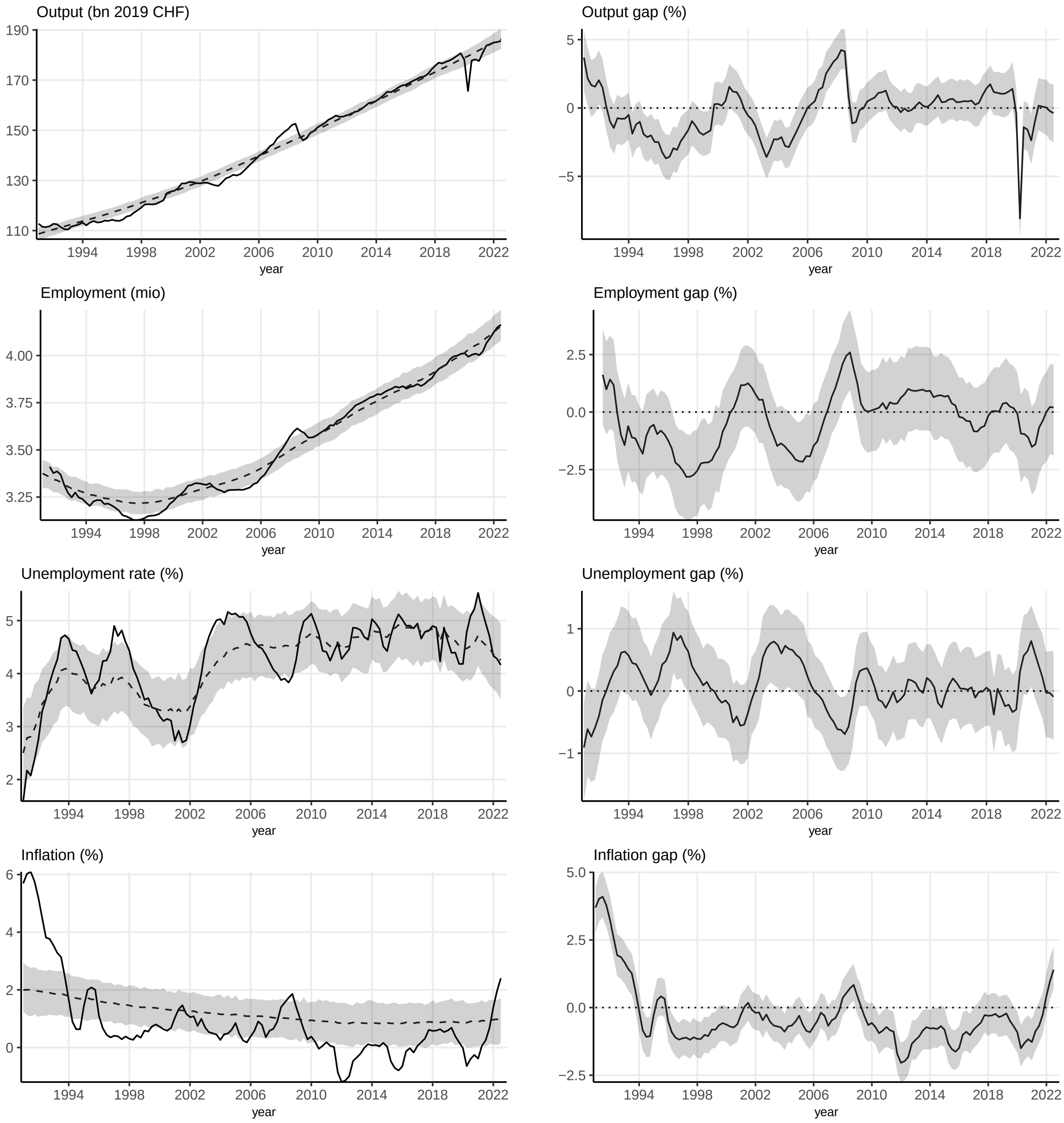
[9]The figures are based on the means of the posterior distributions.

reflecting the current tight situation on the labor market.

The long-term inflation trend has been slowly decreasing from around 2% in 1991 to levels between 0.8% to 0.9% during the European debt crisis. Roughly since 2017, trend inflation started to increase slightly to rates close to 1%. We identify three phases marked by a positive inflation gap: First, the surge in inflation in the early 1990s was triggered by a spillover from the reunification boom in Germany.[10] The second phase occurred before the Great Recession, when the Swiss Franc experienced an unusually long period of weakness and the third phase in 2022, triggered by the war in Ukraine and post COVID-19 pandemic effects. Negative inflation gaps have occurred in times of strong appreciation of the Swiss Franc, e.g., after the SNB scrapped the floor of CHF 1.20 to the Euro in January 2015 and in the course of the demand shortfall during the pandemic.

[10] For more details on this episode, see Rich (1997).

Figure 3: Trends and cycles of output, unemployment and inflation.



Notes: The original data are solid and the trends dashed (left column). The estimated cycles are solid (right column). The shaded areas indicate 68% HPDI.

4.3 A sector perspective on output and employment cycles

The output and employment gaps are decomposed into sectoral contributions in Figure 4. In the lost decade, a broad mix of sectors contributed to the negative output gap. The dot-com recession is mainly attributable to financial services, which enjoyed strong growth before the crisis. The overheating prior to the Financial Crisis is mostly related to manufacturing, trade, transport and hospitality, and financial and other economic services. However, the negative contributions in the aftermath are limited. Even though there was a strong consolidation in the banking sector after the crisis, its negative contribution were somewhat offset by a sizable compensatory expansion in the insurance sector. Moreover, while the manufacturing sector suffered, wholesale trade and especially merchanting activities acted as a stabilizer. A similar picture emerges after the SNB lifted the floor for the euro in Januray 2015: While there were negative contribution from manufacturing and business related sectors, trading activities stabilized the aggregate output gap. When the COVID crisis hit, all sectors were initially affected by the partial lockdown of activities and other containment restrictions. In fact, also personal and government related services contribute to the negative output gap, a sector that normally exhibits no business cycle at all. While the manufacturing sector recovered rapidly, the consumer-oriented services experienced a prolonged period of slowdown.

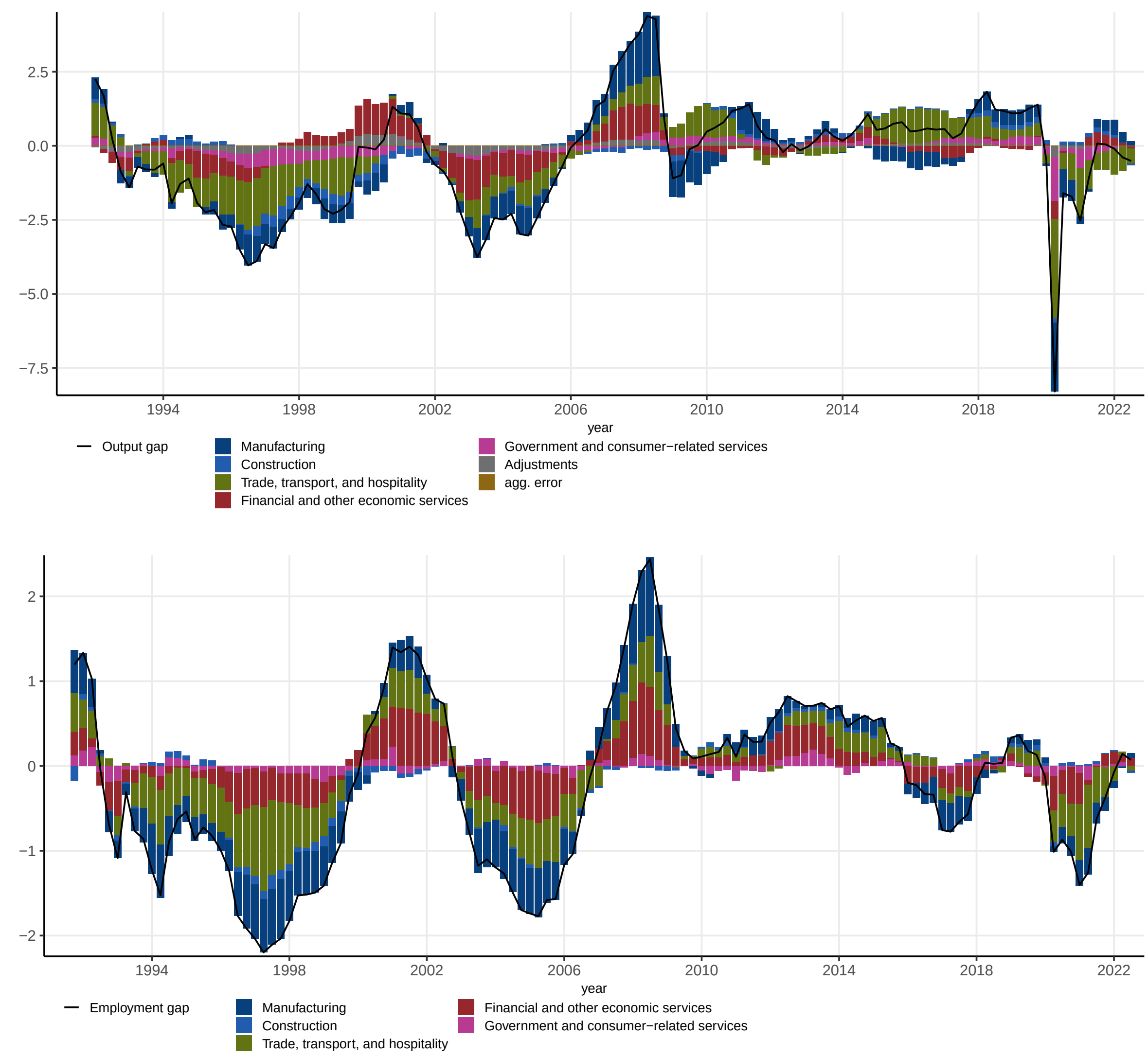
By construction, the picture for employment is similar to the one for value added, but the importance of the underlying sector contributions differs. The three aggregate sectors manufacturing, trade, transport, and hospitality, and financial and other economic services show the largest contributions to the employment gap, whereas those of the remaining sectors, i.e., the construction sector and government and consumer-related services are comparably small. Hence, intuitively, government employment does not react as strongly to the business cycle as other sectors do.[11]

When comparing the output and employment gaps, it is particularly noteworthy that although the impact of the appreciation of the Swiss Franc in 2015 on output was limited, it is very well reflected in the employment gap. The slowdown in output, which was driven by manufacturing and financial and other economic services was counteracted by an expansion in merchanting activities. However, this expansion was not transmitted to the labor market, resulting in a negative employment gap.

The breakdown of the individual output and employment gaps in Figure 5 reveals

[11]See Figure 5 for a decomposition of sector employment cycles into contributions by the business cycle and ideosynchratic employment cycles.

Figure 4: Output and employment gap decomposition.



Notes: Contributions to the output and employment gap are in %.

that despite some commonalities, there is also plenty of room for idiosyncrasies among the sectors. The left panel contains the cycle decomposition for output and the right panel that of employment. Light areas represent sector-specific output cycle contributions and dark areas those of idiosyncratic output and employment cycles, respectively.

For manufacturing, large parts of the output cycles can be explained by the aggregate business cycle which to a considerable degree reflects the state of the global economy. Moreover, the idiosyncratic component of the manufacturing gap seems to broadly exhibit the behavior of the external value of the Swiss Franc. In contrast, the Swiss output gap plays a minor role in explaining employment imbalances in manufacturing. For output in the construction sectors, idiosyncratic factors are predominant, while the opposite is true for employment, which is due to the fact that the corresponding trend is characterized by an erratic development. The impact of the business cycle on both cycles in the sector trade, transport, and hospitality clearly outweigh that of ideosyncratic effects. Again, the expansion of merchanting activities starting in 2015 is apparent in output but not employment. The output cycle of financial and other economic services appears to be less sensitive than its employment cycle. Compared to most other sector and in contrast to the output cycle, the employment gap contains periods of strong under- and overutilization, indicating the sectors' ability to cushion economic fluctuations. Government and consumer-related services are the least sensitive to domestic and foreign events, as the sector's output and employment cycles exhibit the least volatility.

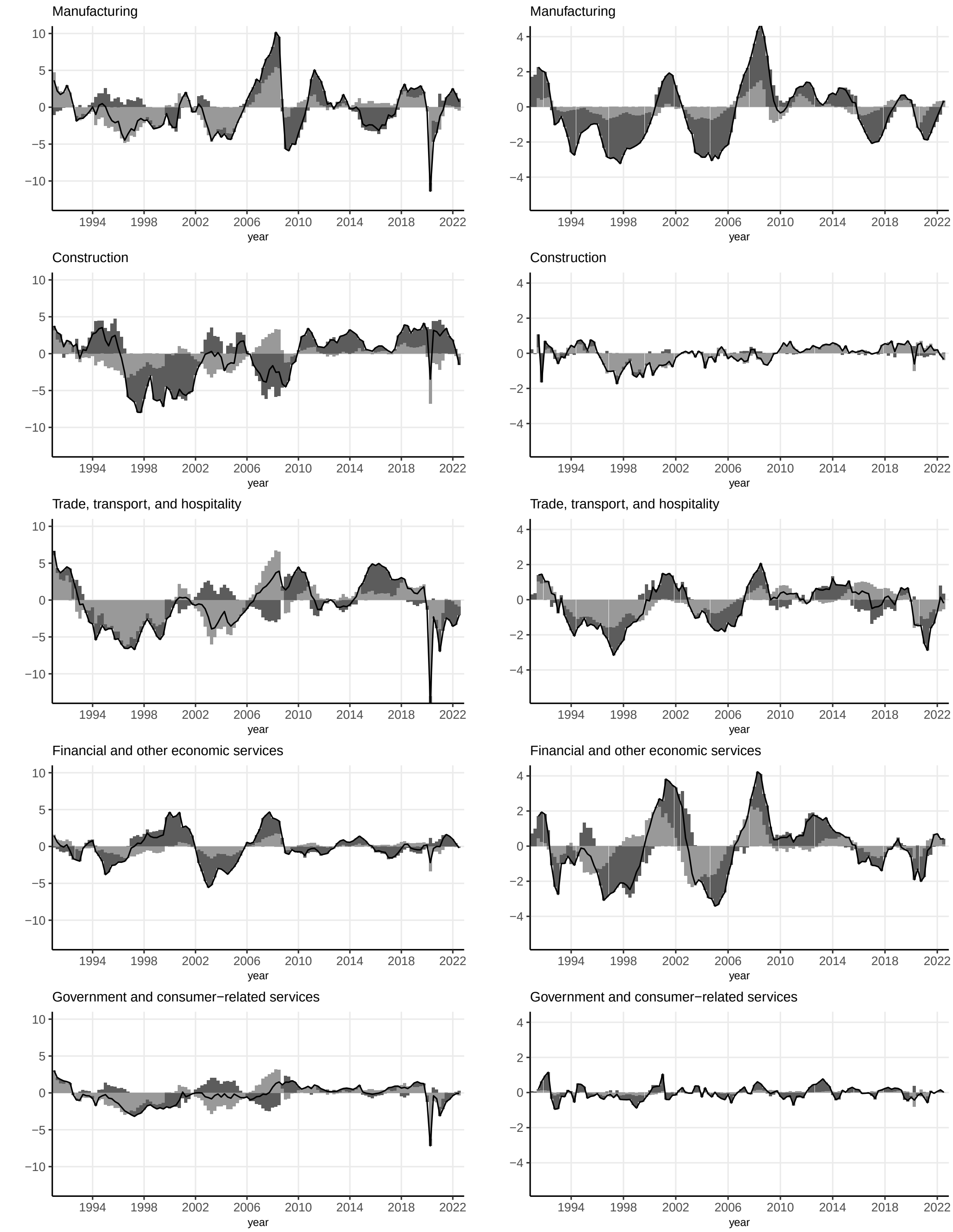
Table 3 presents the correlations between output and employment cycles and the unemployment and inflation gap. The direction and magnitude of the correlation figures confirm our previous results. As expected, the unemployment gap is negatively correlated with the output gap, all sector cycles, and the inflation gap. Interestingly, the output gap shows the lowest correlation to the construction cycle. This can be attributed to the fact that in times of low international demand, emigration to Switzerland increases, which in turn increases the demand for housing.

Table 3: Cycle correlations.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(sector) empl.	weights
(1) Gross domestic product	1.00									0.73	1.00
(2) Manufacturing	0.80	1.00								0.59	0.22
(3) Construction	0.22	0.22	1.00							0.90	0.05
(4) Trade, transport, and hospitality	0.82	0.42	0.29	1.00						0.73	0.25
(5) Financial and other economic services	0.62	0.48	-0.33	0.24	1.00					0.51	0.26
(6) Government and consumer-related services	0.72	0.49	0.49	0.82	0.01	1.00				0.44	0.19
(7) Adjustments	0.76	0.46	-0.09	0.58	0.70	0.38	1.00				0.03
(8) Unemployment rate	-0.75	-0.62	-0.06	-0.61	-0.49	-0.55	-0.56	1.00			
(9) Inflation	0.34	0.29	0.13	0.34	0.06	0.41	0.01	-0.44	1.00		

Notes: Correlation coefficients between the output gap, sector output cycles, the unemployment gap, the inflation gap and (sector) employment cycles. The weights reflect average nominal output weights over the sample period.

Figure 5: Sector output and employment cycle decomposition.

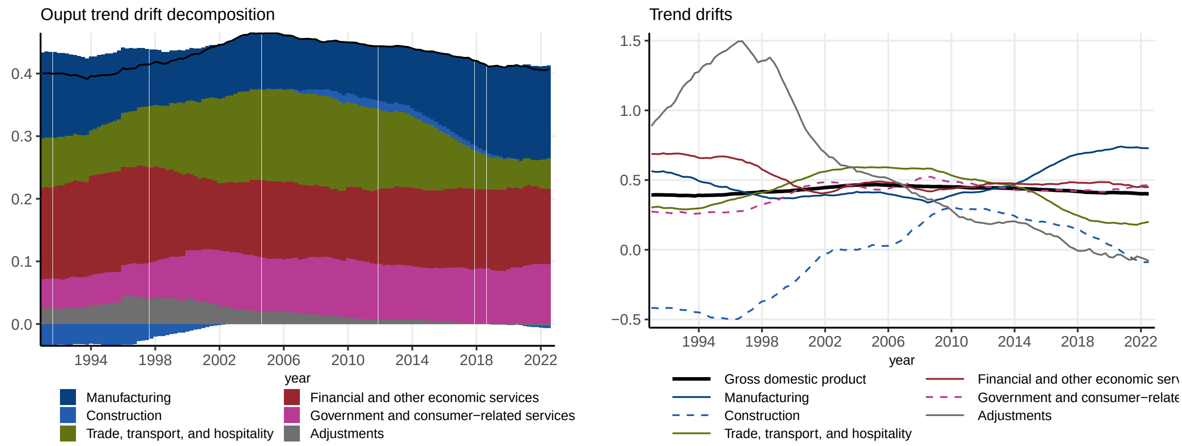


Notes: Sector output (left column) and employment gaps (right column) in %. Light shaded areas represent sector-specific output cycle contributions and dark areas those of ideosynchartic output and employment cycles, respectively.

4.4 A sector perspective on potential growth and trend employment

Figure 6 shows the decomposition of potential growth (left panel) and the sectoral trend growth rates (right panel).[12] Quarterly potential output growth has increased from 0.39% in 1991 to peak levels of 0.46% around 2005 and in turn decreased again to roughly 0.40% in 2022. More recently, the decline has been driven by the sector trade, transport and hospitality, while the increasing contribution of manufacturing has counteracted this development. In 2019, the manufacturing sector surpassed a quarterly trend growth rate of 0.7%, making it the sector with the highest growth potential. This reflects the sector's changing composition, with the highly productive pharmaceutical sector becoming increasingly important.

Figure 6: Long-term trend output growth.



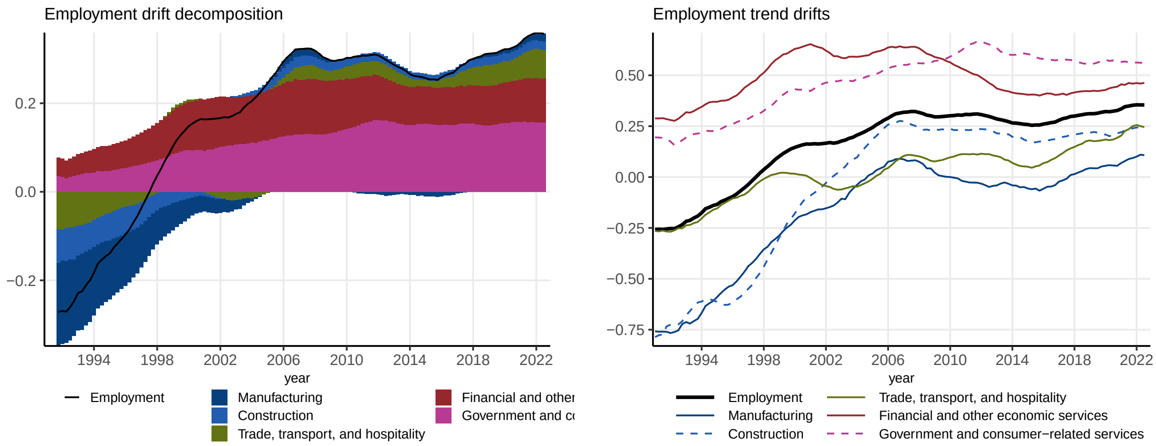
Notes: Contributions and quarterly growth rates are in %.

Looking at the contributions to trend employment growth (Figure 7), the ongoing shift toward a service-based economy is visible. Trend employment growth in manufacturing and construction is no longer as negative as it was in the 1990s, but is now stagnating. This development also reflects the shift to higher productivity activities in these sectors. In contrast, trend employment growth is driven by the service sectors, particularly labor-intensive sectors such as health care and education. Again, employ-

[12]Sector trends alongside credible sets can be found in Figure A.3 in Appendix A.2.

ment trends reflect developments in the international economy, the exchange rate, and emigration to Switzerland.

Figure 7: Long-term trend employment growth.



Notes: Contributions and quarterly growth rates are in %.

4.5 Comparison to alternative models

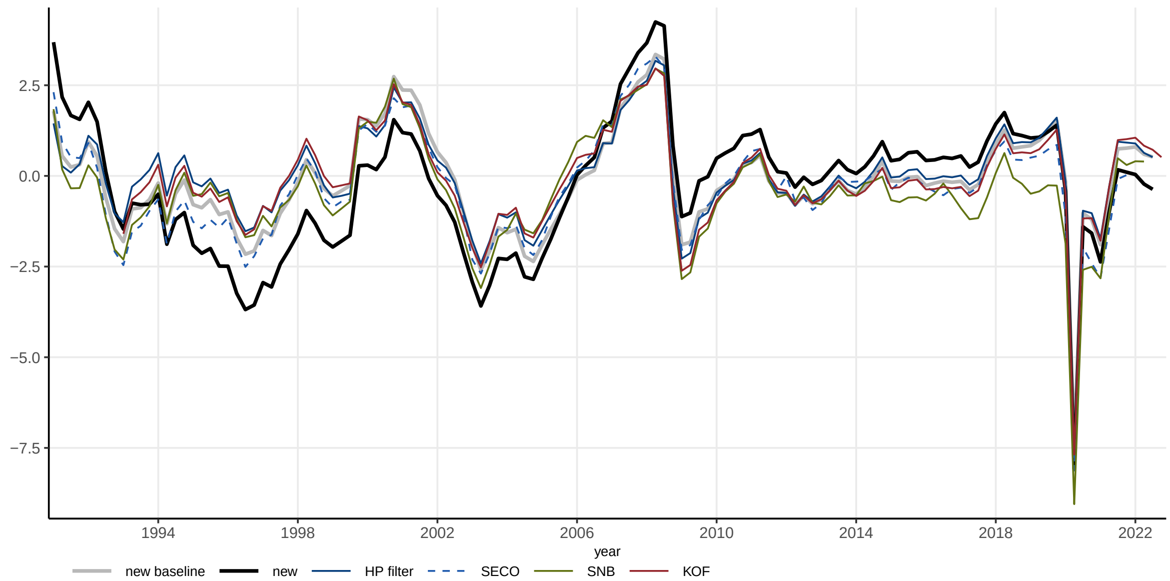
We compare our measure of the business cycles to alternative models in Figure 8. The output gaps published by the Swiss National Bank (SNB), the Swiss State Secretariat of Economic Affairs (SECO) and the KOF Swiss Economic Institute are each based on a production function approach.[13] In addition, we include the HP-filtered output gap, which, in contrast, ignores the economic linkages between output, inflation and the labor market.[14] Finally, we re-estimate our model excluding all sector output and employment equations (baseline).

All estimates of the output gap suggest a similar course of the business cycle in Switzerland. While our baseline model is broadly in line with all remaining models in terms of the level and variability, the model that includes sectoral output and employment shows some divergence. The clearest difference can be observed in two phases. First,

[13]A Cobb-Douglas production function is used to split potential output into three input factors: non-financial capital stock, trend labor input and the trend of total factor productivity. Total factor productivity contains the component of output that cannot be explained by the production factors capital and labor. See e.g. Havik et al. (2014) and Streicher (2022) for details on the methodology.

[14]The HP-filter is a univariate filtering technique including only aggregate output (Hodrick and Prescott, 1997). We set the smoothing constant to 1600 as suggested for quartely data.

Figure 8: Output gap model comparison.



Notes: Output gaps in %. The output gaps from SECO, SNB, and KOF are each based on a production-function approach. The smoothing constant for the HP filter gap is 1600.

our model indicates that the underutilization during the Nineties was more pronounced. Second, the boom leading up to the Financial crisis was even more extreme, with the subsequent recession being less prominent and of shorter duration. As we have seen in Figure 4, the latter fact is mostly attributable to a strong performance in the sector trade, transport, and hospitality.

5 Conclusion

Most conventional methods estimate the output gap consistent with inflation or unemployment dynamics. We go beyond this approach and propose a multivariate state-space model in which potential output and the output gap match the long-term trend growth and cycles of the underlying production sectors. The complementary information on sector output and employment allows for a decomposition of economic fluctuations and long-term developments into its driving factors, thereby providing a more profound estimate. Tracking the economic dynamics of individual sectors, rather than the economy as a whole, can increase the efficiency of fiscal and monetary policy actions and avoid pro-cyclical outcomes.

We use the proposed model to document the dynamics of the Swiss economy, revealing substantial divergence among the considered production sectors. Manufacturing and financial and other economic services are the main drivers of the Swiss business cycle, indicating its dependence on fluctuations of the world economy. The slow decline in growth potential over the past 20 years is mainly due to a slowdown in the sector trade, transport and hospitality, while structural changes in the manufacturing sector toward higher productivity activities have cushioned this development. Our estimate of the business cycle differs from the established methods applied by several national institutions. For instance, our model points to a stronger overheating prior to the Financial Crisis and a faster recovery afterwards. Our output gap decomposition reveals that the latter is caused by an expansion in merchanting activities.

The proposed model is useful for macroeconomic forecasting, as consistent trends and cycles are necessary to inform structural forecasting models. In this context, applying or extending our model to the expenditure side of the economy could be a useful extension. Further research should be devoted to relaxing the covariance assumptions to allow for economic shocks that affect transitory as well as permanent developments. An additional avenue for future work is the isolation of sub-sector dynamics that best contribute to predicting inflation.

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A Appendix

A.1 Estimation Algorithm

To estimate our model parameters and unobserved states, we adopt a Gibbs sampling procedure involving simulation smoothing based on Durbin and Koopman (2012) and related articles.

The parameter set $\Theta = \{\theta_j\}_j$ contains the subsets

$$
\begin{aligned}
\theta_\tau &= \sigma_\tau^2, & \theta_\mu &= \sigma_\mu^2, & \theta_g &= \left\{\phi_1, \phi_2, \sigma_c^2\right\}, \\
\theta_{\tau_i} &= \sigma_{\tau_i}^2, & \theta_{\mu_i} &= \sigma_{\mu_i}^2, & \theta_{c_i} &= \left\{\beta_i, \phi_{i1}, \phi_{i2}, \sigma_{ic}^2\right\}, \\
\theta_{e\tau} &= \sigma_{\tau_e}^2, & \theta_{e\mu} &= \sigma_{\mu_e}^2, & \theta_{c^e} &= \left\{\psi_{e0}, \psi_{e1}, \psi_{e2}, \phi_{e1}, \phi_{e2}, \sigma_{ec}^2\right\}, \\
\theta_{e_i\tau} &= \sigma_{\tau_i^e}^2, & \theta_{e_i\mu} &= \sigma_{\mu_i^e}^2, & \theta_{c_i^e} &= \left\{\psi_{e_i0}, \psi_{e_i1}, \psi_{e_i2}, \phi_{e_i1}, \phi_{e_i2}, \sigma_{e_ic}^2\right\}, \\
\theta_{u\tau} &= \sigma_{\tau_u}^2, & \theta_{u\mu} &= \sigma_{\mu_u}^2, & \theta_{c_u} &= \left\{\psi_u, \phi_{u1}, \phi_{u2}, \sigma_{uc}^2\right\}, \\
\theta_{\pi\tau} &= \sigma_{\tau_\pi}^2, & & & \theta_{c_\pi} &= \left\{\delta, \phi_{\pi1}, \phi_{\pi2}, \sigma_{\pi c}^2\right\},
\end{aligned}
$$

where all trend parameters are listed in the left column, all drift coefficients in the middle column, and all cycle and loading parameters in the right column. Assuming a block independence structure, we have that

$$p(\boldsymbol{\theta}) = \prod_{\theta_j \in \Theta} p(\theta_j),$$

where $\boldsymbol{\theta}$ stacks all components of Θ. Thus, the distribution of the parameters factorizes into all trend, drift, and cycle and loading components, respectively.

A.1.1 Trends

For the local linear trends, the only two parameters are the trend and drift innovation variances, for which we will drop the subscripts for notational simplicity. We impose

$$\pi\left(\sigma^2\right) = \mathcal{IG}(s_0, \nu_0)$$

as prior distribution. Using standard results, we obtain

$$p\left(\sigma^2\middle|\boldsymbol{\tau}\right) = \prod_{t=3}^{T} p\left(\Delta^2\tau_t\middle|\sigma^2\right) p\left(\sigma^2\right) \propto \mathcal{IG}\left(s_*, \nu_*\right) \tag{8}$$

with

$$\begin{aligned} \nu_* &= \nu_0 + T, \\ s_* &= s_0 + \sum_{t=3}^{n} \left(\Delta^2\tau_t\right)^2. \end{aligned} \tag{9}$$

and where $\boldsymbol{\tau} = \{\tau_t\}_t$. For trend inflation, we assume a random walk without drift, for which the second difference in Equations (8) and (9) is replaced by the first difference.

A.1.2 Cycles and loadings

All of our observation equations are variations of a linear model with autoregressive errors, for which we apply the results of Chib (1993). Let now

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t, \quad \Phi\left(L\right)\varepsilon_t = u_t, \quad u_t \sim \mathcal{N}\left(0, \sigma^2\right),$$

where $\Phi\left(L\right) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\boldsymbol{\beta}$ is a $s \times 1$ vector of coefficients and $\mathbf{x_t}$ is a $s \times 1$ vector of covariates. Define

$$y_t^* = \Phi\left(L\right) y_t \qquad\qquad \mathbf{x}_t^* = \Phi\left(L\right) \mathbf{x}_t$$

for $t = p+1, \ldots, T$ and $\mathbf{y}^* = \{y_t^*\}_t, \mathbf{y} = \{y_t\}_t$ and $\mathbf{X}^* = \{\mathbf{x}^*\}_t$ are of dimension $T - p \times 1$ and $T - p \times s$, respectively.

We assume the prior distribution of the involved parameters factorizes, i.e.,

$$\pi\left(\boldsymbol{\beta}, \sigma^2, \boldsymbol{\phi}\right) = \pi\left(\boldsymbol{\beta}\right)\pi\left(\sigma^2\right)\pi\left(\boldsymbol{\phi}\right)$$

with $\boldsymbol{\phi} = \left(\phi_1, \ldots, \phi_p\right)'$ and for the individual prior distributions,

$$\begin{aligned} \boldsymbol{\beta} &\sim \mathcal{N}_s\left(\boldsymbol{\beta}_0, \mathbf{A}_0^{-1}\right) \\ \sigma^2 &\sim \mathcal{IG}\left(\nu_0/2, \delta_0/2\right) \\ \boldsymbol{\phi} &\sim \mathcal{N}_p\left(\boldsymbol{\phi}_0, \boldsymbol{\Phi}_0^{-1}\right) I_{\phi \in S_\phi} \end{aligned}$$

The posterior distribution of the coefficient vector is given by

$$\boldsymbol{\beta}|\mathbf{y} \sim \mathcal{N}_s\left(\tilde{\boldsymbol{\beta}}_0, \tilde{\mathbf{A}}^{-1}\right),$$
$$\tilde{\mathbf{A}} = \mathbf{A}_0 + \sigma^{-2}\mathbf{X}^{*\prime}\mathbf{X}^*,$$
$$\tilde{\boldsymbol{\beta}} = \tilde{\mathbf{A}}^{-1}\left(\mathbf{A}_0\boldsymbol{\beta}_0 + \sigma^{-2}\mathbf{X}^*\mathbf{y}^*\right),$$

that of the variance by

$$\sigma^2|\mathbf{y}, \boldsymbol{\beta}, \boldsymbol{\phi} \sim \mathcal{IG}\left(\frac{T-p+\nu_0+k}{2}, \frac{\delta_0+d_\beta}{2}\right),$$
$$d_\beta = \left(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}\right)'\left(\mathbf{y}^* - \mathbf{X}^*\boldsymbol{\beta}\right),$$

and the autoregressive coefficient by

$$\boldsymbol{\phi}|\mathbf{y}, \boldsymbol{\beta}, \sigma^2 \sim \mathcal{N}_p\left(\tilde{\phi}, \tilde{\boldsymbol{\Phi}}^{-1}\right) I_\phi$$
$$\tilde{\boldsymbol{\Phi}} = \boldsymbol{\Phi}_0 + \sigma^{-2}\mathbf{E}'\mathbf{E}$$
$$\tilde{\boldsymbol{\phi}} = \tilde{\boldsymbol{\Phi}}^{-1}\left(\boldsymbol{\Phi}_0\boldsymbol{\phi}_0 + \sigma^{-2}\mathbf{E}'\mathbf{E}\right)$$

where $\mathbf{E} = \{\boldsymbol{\varepsilon}_t\}_t, \boldsymbol{\varepsilon}_t = (\varepsilon_{t-1}, \ldots, \varepsilon_{t-p})$ is a $T - p \times p$ matrix (Chib, 1993).

It is straightforward to see that each observation equation is a subgroup of this model. For instance, for the sector cycle equations we have $y_{it} - \tau_{it} = \beta_i g_t + c_{it}$ with $c_{it} = \phi_{i1}c_{it-1} + \phi_{i2}c_{it-2} + \varepsilon_{c_it}$, i.e., the above model is of dimension $s = 1$ and $p = 2$. In the case of the output gap, the covariate and coefficient vectors $\mathbf{x}_t$ and $\boldsymbol{\beta}$ are dropped.

A.1.3 Algorithm

The algorithm is structured in four blocks: The first three blocks sample the parameter vector $\boldsymbol{\theta}^k$ conditional on the states $\boldsymbol{\alpha}^{k-1}$ and the last block samples $\boldsymbol{\alpha}^k$ conditional on $\boldsymbol{\theta}^k$. More precisely, the first block deals with all trend equations in separate Gibbs steps. In the second block, the parameters of the equations involving loading factors and autoregressive cycles are drawn in another Gibbs step. The third block is an additional Gibbs step to draw the parameters of the output gap equation. The final block applies simulation smoothing as suggested by Durbin and Koopman (2012) conditional on the previously drawn parameters.

Initialization: We use the prior means to initialize all parameters $\boldsymbol{\theta}^0$ and apply the Kalman filter and smoother based on those parameters to initialize the states $\boldsymbol{\alpha}^0$.

Recursion: For $k = 1, \ldots, K$:

1. Trends (Gibbs steps): Draw all trend variances $\sigma^{2^k} | \tau^{k-1}$.

2. Sector output, aggregate and sector employment, unemployment and inflation (Gibbs steps): For each equation, draw autoregressive coefficients, loading coefficients, and cycle variances, i.e.,

$$\boldsymbol{\phi}^k \mid \sigma^{2^{k-1}}, \boldsymbol{\alpha}^{k-1}$$
$$\boldsymbol{\beta}^k \mid \boldsymbol{\phi}^k, \sigma^{2^{k-1}}, \boldsymbol{\alpha}^{k-1}$$
$$\sigma^{2^k} \mid \boldsymbol{\beta}^k, \boldsymbol{\phi}^k, \boldsymbol{\alpha}^{k-1}$$

   sequentially in this order as detailed in Section A.1.1. If the characteristic polynomial $\Phi^k(x)$ has roots inside the unit circle, redraw $\boldsymbol{\phi}^k$.

3. Output gap (Gibbs step): Draw autoregressive coefficients and cycle variance, i.e.,

$$\boldsymbol{\phi}^k \mid \sigma^{2^{k-1}}, \boldsymbol{\alpha}^{k-1}$$
$$\sigma^{2^k} \mid \boldsymbol{\phi}^k, \boldsymbol{\alpha}^{k-1}$$

   sequentially in this order as detailed in Section A.1.1, conditional on the trend τ^{k-1} and cycle g^{k-1}. If the characteristic polynomial $\Phi^k(x)$ has roots inside the unit circle, redraw $\boldsymbol{\phi}^k$.

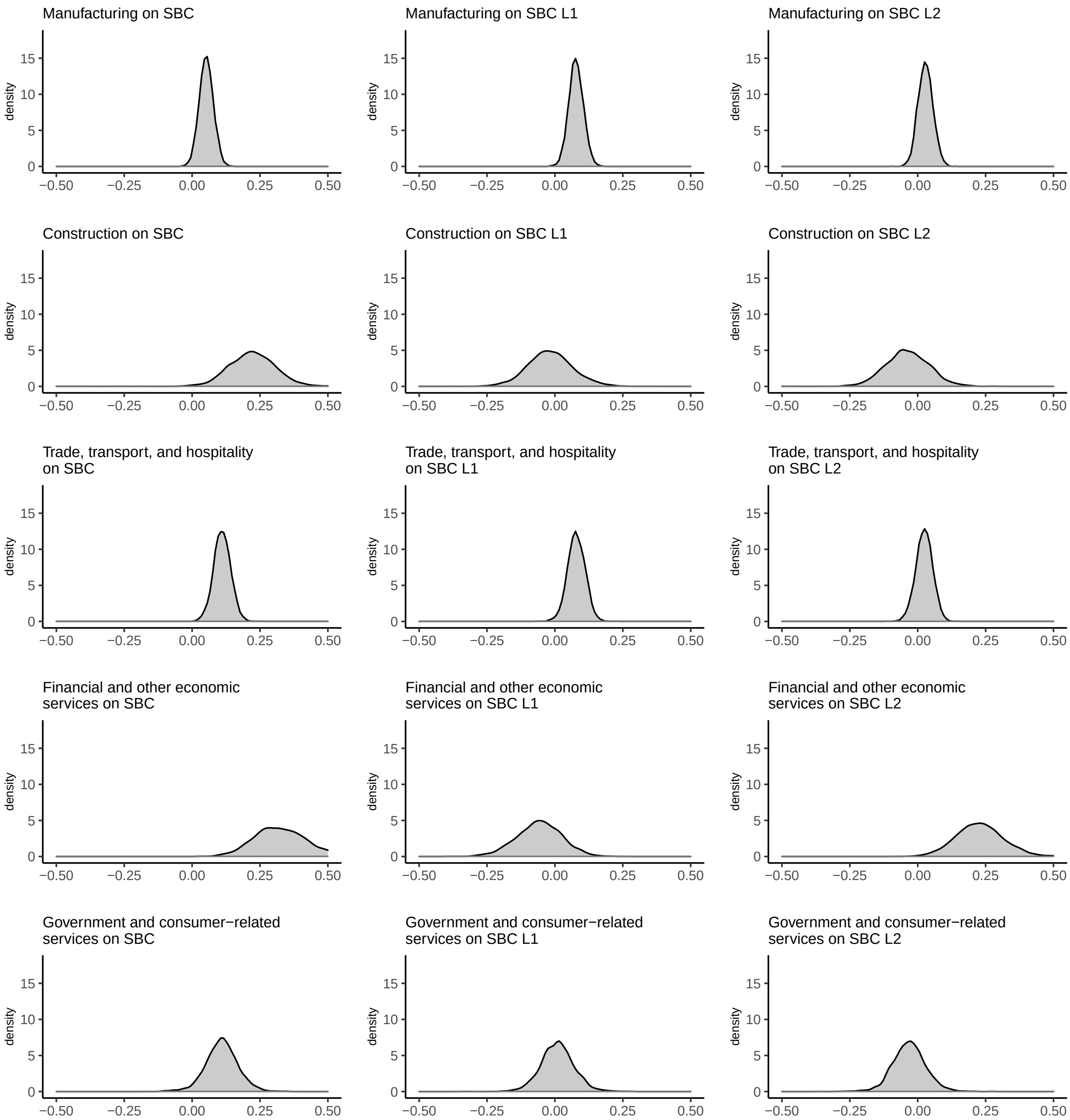
4. States: Apply the simulation smoothing recursion (Durbin and Koopman, 2012) to sample the unobserved states conditional on the parameters

$$\boldsymbol{\alpha}^k \mid \boldsymbol{\theta}^k.$$

Discard the first K_b draws of $\left\{\boldsymbol{\theta}^k\right\}_k$ and $\{\boldsymbol{\alpha}\}_k$ and finally select each 10th draw from the remaining sample.

A.2 Figures

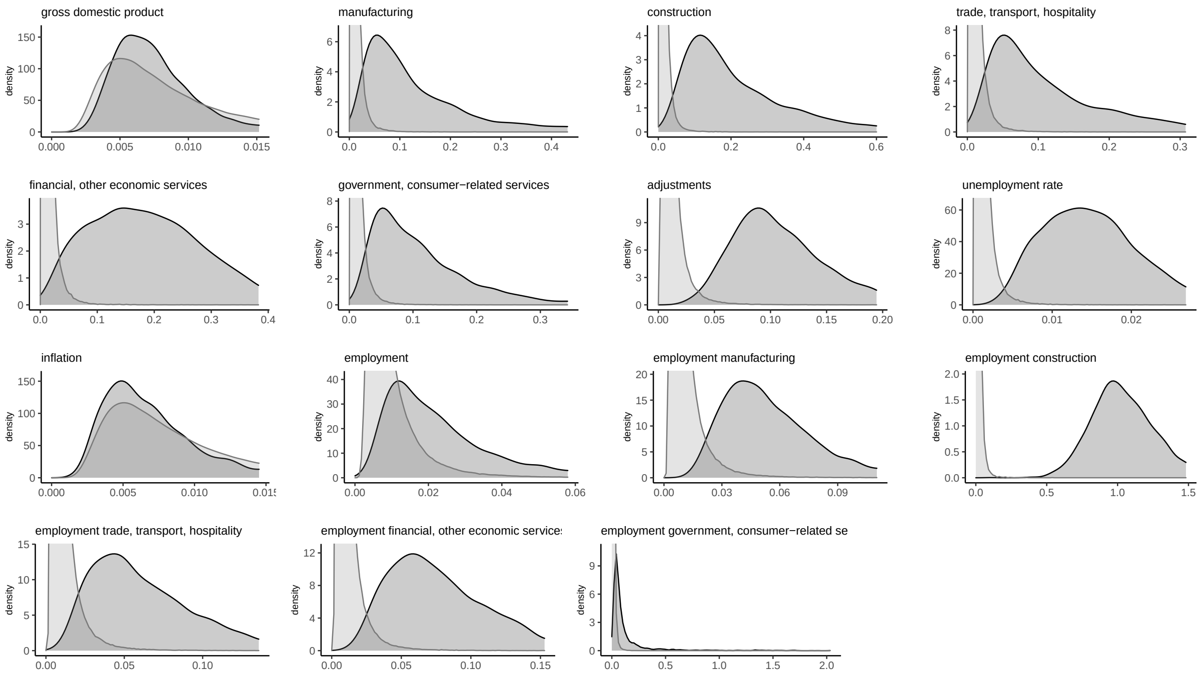
Figure A.1: Prior and posterior distributions of employment loadings.



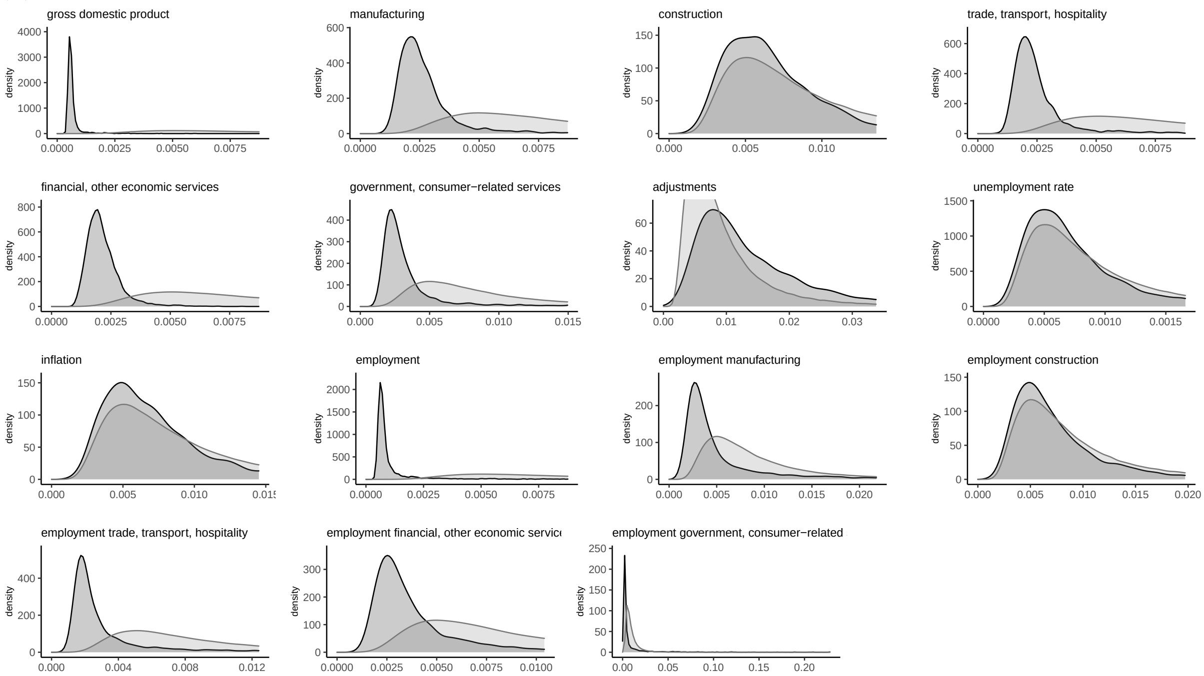
Notes: The prior distributions are specified as in Table 1. BC denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Figure A.2: Prior and posterior distributions of trend, drift, and cycle variances.

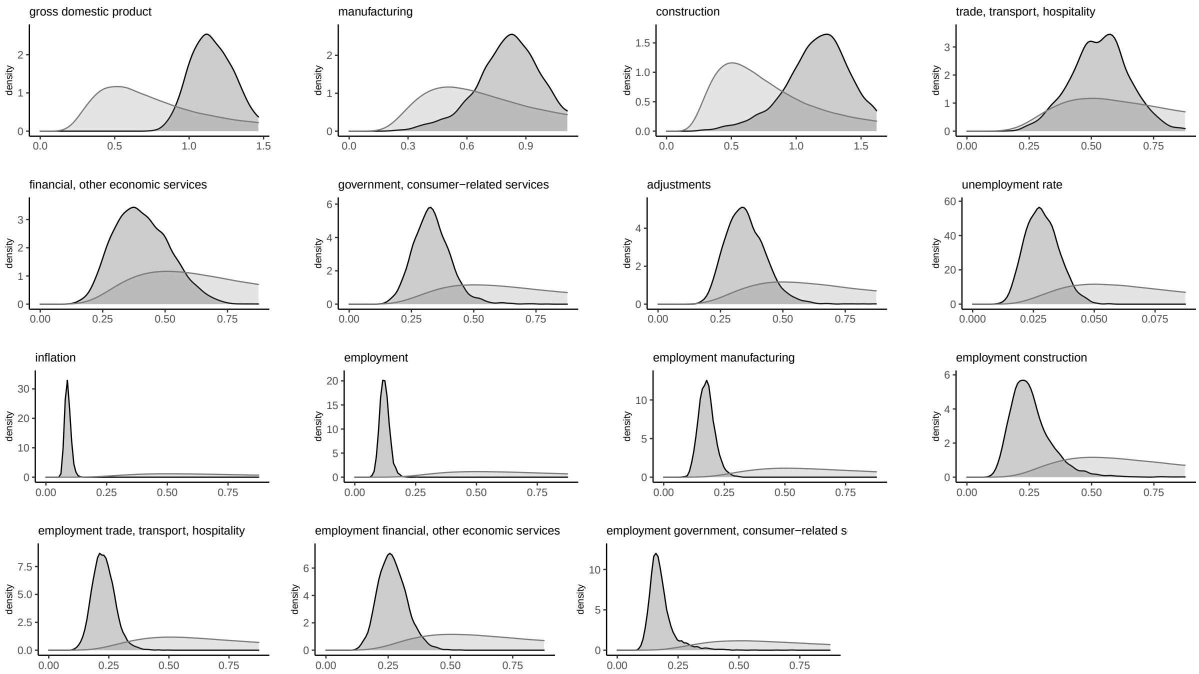
(a) Trend variances



(b) Trend drift variances

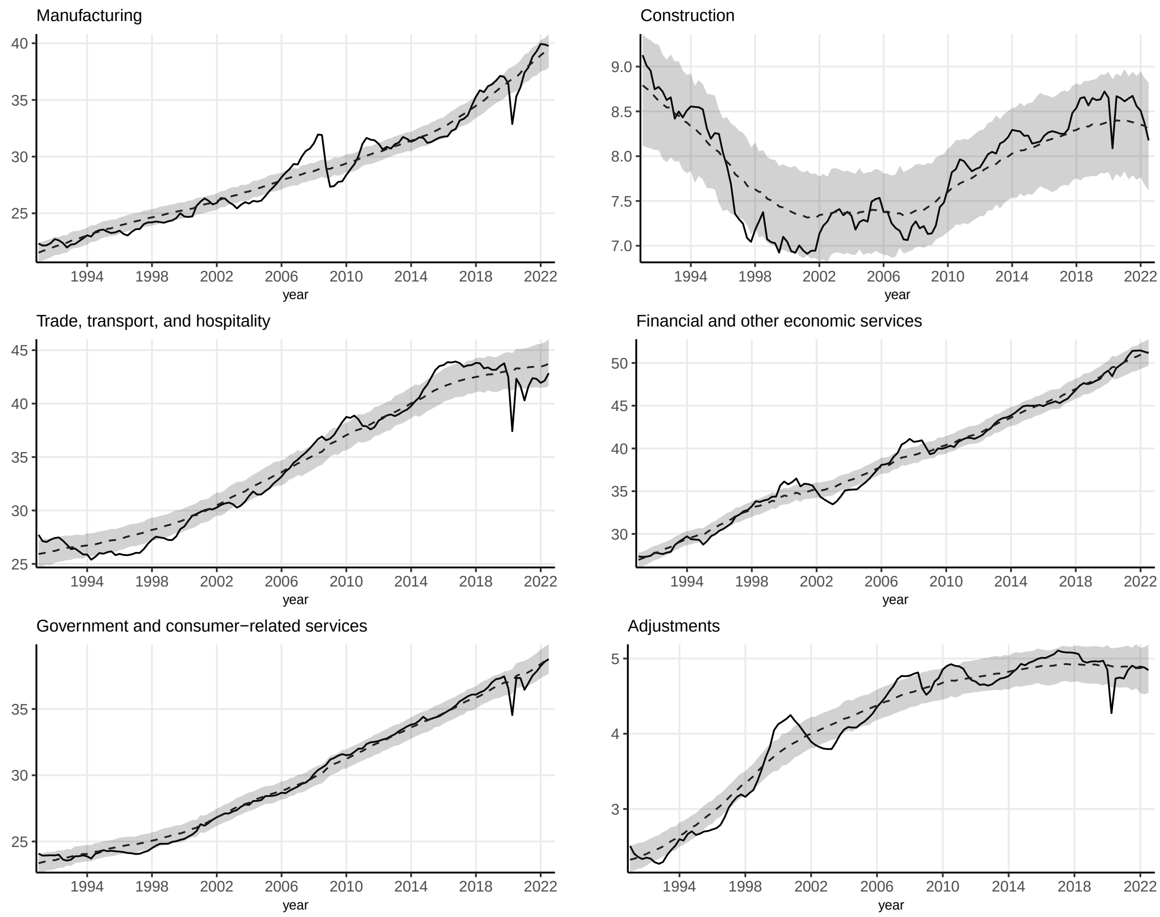


(c) Cycle variances



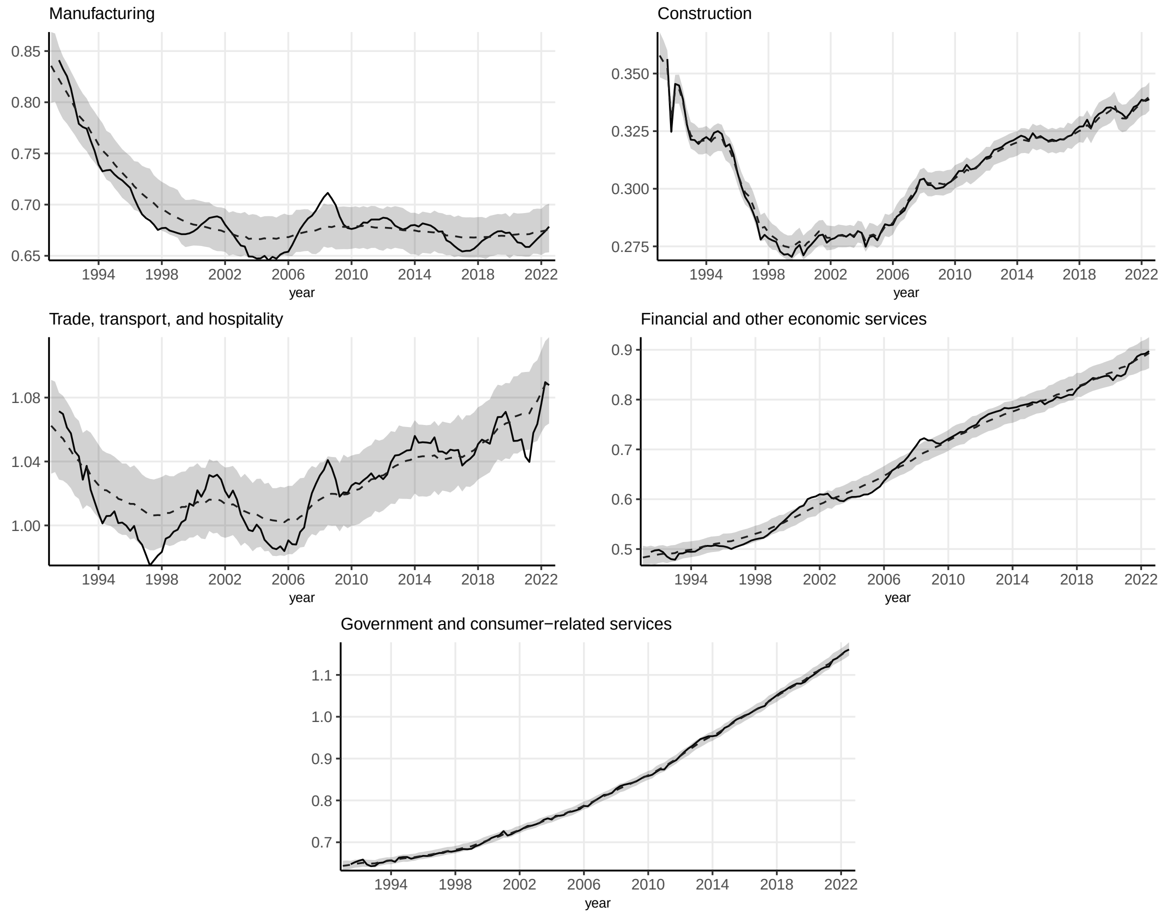
Notes: The prior distributions are specified as in Table 1. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped. For visibility, the x-axis limits are chosen to span zero and the maximum of the 95% quantile of the posterior and the 60% quantile of the prior distribution.

Figure A.3: Sector output trends.



Notes: Ouput in bn CHF. The original data are solid and the trends dashed. The shaded areas indicate 68% HPDI.

Figure A.4: Sector employment trends.



Notes: Employment in mio. The original data are solid and the trends dashed. The shaded areas indicate 68% HPDI.

A.3 Tables

Table A.1: Posterior distributions.

	Cycle variance				Trend variance				Trend drift variance			
	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9
Gross domestic product	1.1762	1.1525	0.9776	1.3784	0.0081	0.0071	0.0043	0.0121	0.0007	0.0006	0.0005	0.0008
Manufacturing	0.8229	0.8277	0.6024	1.0351	0.1503	0.1002	0.0359	0.3191	0.0029	0.0024	0.0016	0.0041
Construction	1.2334	1.1947	0.8442	1.5130	0.2342	0.1771	0.0739	0.4680	0.0070	0.0061	0.0034	0.0113
Trade, transport, and hospitality	0.5423	0.5386	0.3832	0.6842	0.1231	0.0891	0.0334	0.2516	0.0028	0.0022	0.0016	0.0038
Financial and other economic services	0.4088	0.3988	0.2697	0.5583	0.2014	0.1830	0.0637	0.3361	0.0022	0.0020	0.0015	0.0029
Government and consumer-related services	0.3344	0.3277	0.2442	0.4261	0.1391	0.0949	0.0366	0.2461	0.0051	0.0027	0.0018	0.0077
Adjustments	1.1425	0.3455	0.2555	0.4611	0.1109	0.1024	0.0608	0.1718	0.0144	0.0113	0.0056	0.0267
Employment	0.1226	0.1214	0.0987	0.1477	0.0248	0.0193	0.0084	0.0461	0.0019	0.0007	0.0006	0.0034
Empl. Manufacturing	0.1774	0.1756	0.1373	0.2198	0.0566	0.0508	0.0285	0.0927	0.0064	0.0036	0.0021	0.0141
Empl. Construction	0.3214	0.2445	0.1675	0.3743	1.0857	1.0270	0.7708	1.3576	0.0084	0.0064	0.0035	0.0147
Empl. Trade, transport, and hospitality	0.2289	0.2260	0.1733	0.2866	0.0652	0.0559	0.0260	0.1147	0.0038	0.0022	0.0015	0.0079
Empl. Financial and other economic services	0.2689	0.2644	0.1997	0.3434	0.0806	0.0717	0.0361	0.1321	0.0042	0.0032	0.0020	0.0076
Empl. Government and consumer-related services	0.1806	0.1686	0.1317	0.2392	4.3822	0.0639	0.0257	0.5020	0.1103	0.0027	0.0016	0.0625
Unemployment rate	0.0287	0.0282	0.0201	0.0379	0.0153	0.0145	0.0074	0.0238	0.0008	0.0006	0.0003	0.0013
Inflation	0.0901	0.0889	0.0752	0.1064	0.0073	0.0063	0.0034	0.0121				
	Loading on output gap				Loading on output gap, lag 1				Loading on output gap, lag 2			
	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9
Manufacturing	1.2796	1.2808	1.1755	1.3816								
Construction	0.8282	0.8271	0.7056	0.9486								
Trade, transport, and hospitality	1.5956	1.5947	1.5120	1.6771								
Financial and other economic services	0.4210	0.4196	0.3418	0.5005								
Government and consumer-related services	0.7703	0.7720	0.6956	0.8424								
Adjustments	1.5848	1.5841	1.5214	1.6482								
Employment	0.1634	0.1636	0.1201	0.2061	0.1083	0.1083	0.0624	0.1541	0.0574	0.0575	0.0128	0.1008
Unemployment rate	-0.0560	-0.0560	-0.0798	-0.0317	-0.0667	-0.0670	-0.0922	-0.0416	-0.0424	-0.0424	-0.0683	-0.0158
	Loading on sector gap				Loading on sector gap, lag 1				Loading on sector gap, lag 2			
	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9	Mean	Median	D_1	D_9
Empl. Manufacturing	0.0523	0.0521	0.0192	0.0866	0.0780	0.0777	0.0445	0.1125	0.0287	0.0284	-0.0063	0.0638
Empl. Construction	0.2179	0.2208	0.1156	0.3269	-0.0166	-0.0193	-0.1199	0.0889	-0.0357	-0.0399	-0.1396	0.0644
Empl. Trade, transport, and hospitality	0.1106	0.1105	0.0719	0.1506	0.0790	0.0782	0.0391	0.1204	0.0239	0.0244	-0.0160	0.0636
Empl. Financial and other economic services	0.3276	0.3216	0.2070	0.4536	-0.0582	-0.0571	-0.1662	0.0442	0.2285	0.2264	0.1193	0.3410
Empl. Government and consumer-related services	0.1093	0.1095	0.0364	0.1834	0.0090	0.0091	-0.0665	0.0879	-0.0312	-0.0314	-0.1059	0.0449
	Loading on unemployment gap											
	Mean	Median	D_1	D_9								
Inflation	-0.3524	-0.3452	-0.6120	-0.1006								

Notes: Mean, median, and first and ninth decile D_1 and D_9 of the posterior distribution.