Implementation with Missing Data[*]

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Abstract

We analyze implementation in environments where planners observe societies' incomplete choice data and are partially informed about how to associate individuals' preferences with states of the economy, on which desired collective goals depend. We identify necessary conditions and show that they are sufficient in economic environments. Further, we establish that more information enriches implementation opportunities. To demonstrate practicality, we examine the implementability of an efficiency notion suited to this setting.

Keywords: Implementation; Incomplete Choice Data; Reliable Nash Equilibrium; Reliable Pareto Optimality; Privacy Preservation.

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1 Introduction

Even though increased data processing opportunities facilitate efficient monitoring and storage of individuals' revealed preferences, in many economic environments of interest, planners charged with the implementation of collective goals do not know all the choice data of individuals. However, planners' knowledge about individuals' choices is essential in designing mechanisms to decentralize desired goals. So how should planners tackle the design of mechanisms when they do not know all the choice data of individuals?

We consider an environment with incomplete information where the incomplete public choice data serves as a correlation device. A planner (she) is responsible for implementing a goal that depends on the states of the economy that she does not observe, while each individual (he) observes the realized state of the economy along with his own preferences. Further, the planner and the individuals have access to public choice data that consists of observations of some of the individuals' choices from some subsets of alternatives at some states of the economy. The novel feature of our model is that the incompleteness of the public choice data induces a missing data problem where the planner and the individuals are not fully informed about how to associate the states of the economy with the underlying payoff relevant characteristics of the society.[1] The incomplete public choice data enables the planner and the individuals to infer the set of preference profiles (payoff states) compatible with their observations. Their inferences are critical in forming foresight about strategic behavior in non-cooperative mechanisms.

To illustrate our setting, consider the situation where a newly appointed CEO (the planner) is responsible for running a firm. She needs to choose one of the following policies (alternatives) depending on the state of the firm that she does not observe: *expansion*, *prudence*, or *contraction*. For simplicity, assume that the firm is composed of two departments: finance and marketing. The incumbent chiefs of finance (CFO) and marketing (CMO) observe the firm's state, be it *strong*, *normal*, or *weak*, as well as their own preferences that are contingent on the firm's state. The CEO should choose *expansion* only when the firm's

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[1] When the planner and the individuals are fully informed, they can associate each state of the economy with individuals' 'true' rankings as in Maskin (1999, circulated in 1977). This is the standard case analyzed in the literature. For more, see Maskin and Sjöström (2002), Palfrey (2002), and Serrano (2004).

state is *strong*. Otherwise, she should go for *prudence*. The CEO seeks to implement this goal by extracting the CFO's and the CMO's information via a non-cooperative mechanism, in which how the CFO and the CMO play depends on their preferences. All three C-level executives have access to partial information about the CFO's and the CMO's preferences from past data on accounting records and meeting minutes, and we assume that their recent choices are in line with their current preferences.[2] In particular, last quarter, the firm's state was *normal*, and the CFO strictly preferred *prudence* to *contraction* and the CMO strictly preferred *prudence* to *expansion*, while there is no further information about their preferences in that state. Hence, how the CFO ranks *expansion* compared to *contraction* and *prudence*, and how the CMO evaluates *contraction* compared to *expansion* and *prudence* is not a part of the public choice data at the firm's *normal* state. In other states of the firm, similar instances of incompleteness of the public choice data about the CFO's and the CMO's preferences may arise.[3]

In our environment, the planner and the individuals are unsure about the 'true' preference profile associated with the realized state of the economy. However, the planner needs to consider individuals' behavior in every possible ranking profile compatible with the incomplete public choice data to make reliable strategic predictions and ensure outcomes consonant with the desired goal. Besides, the individuals do not have incentives to find out others' true preferences whenever they correlate their behavior only on the public choice data. These lead us to the notion of *reliable Nash equilibrium* (RNE): Given a state of the economy and ranking profiles compatible with the incomplete public choice data, an action profile is an RNE of a mechanism at this state of the economy if the prescribed behavior is a Nash equilibrium (NE) at every compatible ranking profile.

A profile of RNE taken across the states of the economy is identical to an ex-post correlated equilibrium (an ex-post equilibrium using the states of the economy as a correlation device, abbreviated by ECE), in which each individual's behavior depends only on the public choice data. In other words, a profile of RNE is equivalent to a public ex-post correlated

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[2] We assume that the public information on individuals' past choice data reflect their current preferences truthfully.

[3] In Section 2, we formalize this example and show that the CEO can implement the desired goal using only the incomplete public choice data.

equilibrium, where each individual's strategy depends only on the state of the economy but not on their own private payoff type (ranking). As a result, the RNE provides the following robustness properties: (*i*) It employs no probabilistic information, no belief updating, and no common prior assumption as it is belief-free, and the equilibrium behavior features the ex-post no-regret property.[4] (*ii*) The RNE, unlike the ECE, refrains from using individuals' private information and relies only on the public choice data. In furtherance, in the Bayesian framework, we observe that any RNE profile is equivalent to a public Bayes correlated equilibrium (a Bayesian equilibrium using the states of the economy as a correlation device, where individuals' behavior depends only on the public choice data). Thus, the RNE preserves these robustness properties in the Bayesian framework as it is a Bayes correlated equilibrium no matter what individuals' private information is.

Full implementation of a collective goal in RNE entails the existence of a mechanism such that at every state of the economy, the set of the RNE of that mechanism equals the set of desired alternatives at that state. Thus, implementation in RNE sustains the robustness properties mentioned above. Therefore, the outcomes of mechanisms implementing the collective goal in RNE are verifiable using only the public information, and vindications based on individuals' private information are not needed. In other words, such mechanisms preserve privacy.[5] Furthermore, we show that individuals' private information is not essential as far as full implementation in ECE is concerned: If a mechanism implements a desired goal in ECE, then the same mechanism implements this goal in RNE (Proposition 3). The reverse of this statement does not hold. We also demonstrate that in our setup, implementation in Bayes correlated equilibrium shares many similarities with implementation in ECE.

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[4] The RNE shares these robustness properties with the ex-post equilibrium (Bergemann and Morris, 2005, 2008, 2009, 2011). A recent related study (on robust implementation in rationalizable strategies) is ? (?). On the other hand, the ex-post no-regret property requires that individuals do not seek to change their behavior even when informed about others' payoff types.

[5] There is a natural connection between privacy and mechanism design (Pai and Roth, 2013; Chen et al., 2016). Nissim et al. (2012) consider mechanism design with *privacy-aware individuals*, "agents [who] also assign non-positive utility to the leakage of information about their private types through the public outcome of the mechanism." Preservation of privacy may even be enforced by law, e.g., the California Consumer Privacy Act and the General Data Protection Regulation of the European Union. In situations where the planner serves as a public officer, e.g., a court-appointed trustee, implementation in RNE provides accountability vindicated by public data.

Even though full implementation in RNE provides useful properties, 'bad' ECE outcomes (not aligned with the desired goal) may still arise in corresponding mechanisms if individuals use their private information. One can dispense with such instances by using only the public choice data: Demanding that the desired goal be consonant with the NE for all states of the economy and all compatible payoff states prevents such bad equilibria since every ECE induces an NE at every payoff state compatible with the given state of the economy. As a result, we obtain the notion of safe implementability in RNE by additionally insisting on every alternative sustained in NE for some ranking profile compatible with a state of the economy to be among the desired alternatives for that state.[6]

Given the planner's inference drawn from the incomplete public choice data, we provide necessary as well as sufficient conditions for a goal to be (safely) implementable in RNE.

Our *necessity* result (Theorem 1) establishes that if a collective goal is implementable (safely implementable) in RNE, then there exists a profile of sets reliable-consistent (safe-consistent, resp.), with the desired goal. Further, our necessity analysis implies that additional information about individuals' choice data enriches implementation opportunities (Theorem 2). Our findings justify the conclusion that the less informed the planner is, the more restricted and invariant the implementable goal becomes. In fact, if there is a state of the economy at which the planner is completely ignorant of the underlying rankings, then every social choice function (SCF) must be constant whenever it is either implementable or safely implementable in RNE. We also obtain *sufficiency* results in economic environments using these consistency concepts (Theorem 3).

To demonstrate the practicality of implementation in RNE, we analyze the notion of reliable Pareto optimality. This efficiency notion constitutes a collective goal that maps states of the economy to alternatives that are (weakly) Pareto optimal at every ranking profile compatible with the given state of the economy. We show that reliable Pareto optimality is implementable in RNE whenever the public choice data induces an economic environment with at least three individuals and for every state of the economy, there is an alternative that is Pareto optimal at every compatible preference profile.

To the best of our knowledge, ours is the first paper that explicitly considers imple-

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[6] The construction of safe implementation is in line with that of secure implementation (Saijo et al., 2007).

mentation with missing data. A related article is Eliaz (2002) that analyzes the implementation problem when some of the players are faulty—in the sense that they can act arbitrarily—while their identity and exact number are not known.[7]. Indeed, one can view a faulty player as an individual about whom there is no public choice data. Yet, our setup differs from Eliaz's as the identity of a player without any public choice data is common knowledge in our setting.

The current paper considers the rational domain for purposes of clarity. Indeed, we can extend our results to a behavioral setup with slight modifications.[8]

The organization of the paper is as follows: Section 2 presents the preliminaries. Section 3 contains our necessity as well as sufficiency results, and Section 4 discusses implementability of suitable notions of efficiency. In Section 5, we consider individuals' behavior using their private information. Section 6 concludes. The relation between our necessary conditions and Maskin monotonicity is relegated to Appendix A.

2 Preliminaries

Let X be a set of *alternatives*, $\mathcal{X}$ the set of all non-empty subsets of X. $N = \{1, ..., n\}$ denotes a *society* with a finite set of individuals where $n \geq 2$.

The set of all states of the world, Ω, is in one-to-one correspondence with all the *admissible payoff-relevant characteristics* of the environment. We assume there is *distributed knowledge* with regard to the state of the world, i.e., $\Omega := \times_{i \in N} \Omega_i$ where Ω_i denotes the set of *payoff types* of individual i. Indeed, the *preferences* of individual $i \in N$ at a *payoff state* $\omega \in \Omega$ is captured by a complete and transitive binary relation $R_i^\omega \subseteq X \times X$. Given a set of alternatives $S \in \mathcal{X}$, *the choice of individual i at payoff state ω from S* is $C_i^\omega(S) := \{x \in S \mid x \, R_i^\omega \, y, \text{ for all } y \in S\}$. Given $i \in N$, $\omega \in \Omega$, and $x \in X$, $L_i^\omega(x) := \{y \in X \mid x \, R_i^\omega \, y\}$ denotes the *lower contour set of i at ω of x*.

We let Θ be the set of *states of the economy*. A *social choice correspondence* (SCC) defined on the states of the economy is $f : \Theta \to \mathcal{X}$, a non-empty valued correspondence

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[7] See also Altun et al. (2023).

[8] Korpela (2012) and de Clippel (2014) provide necessary as well as sufficient conditions for behavioral implementation. Two recent related papers are Hayashi et al. (2020) and Barlo and Dalkıran (2022b).

mapping Θ into X. Given $\theta \in \Theta$, $f(\theta)$ consists of the alternatives the planner desires to sustain at θ, and are referred to as *f-optimal* alternatives at θ. We wish to emphasize that the SCC does not depend on the payoff states.

In what follows, we model the planners' and individuals' knowledge of the association of the states of the economy, Θ, with the payoff states, Ω. The *identification function* $\pi^* : \Theta \to \Omega$ captures the 'true' association of states of the economy with the underlying payoff states, where $\pi^*(\theta) \in \Omega$ is the payoff state and $\pi_i^*(\theta) \in \Omega_i$ individual i's payoff type associated with the realized state of the economy $\theta \in \Theta$.

We assume that each individual i knows his own realized payoff type, $\pi_i^*(\theta)$. Meanwhile, the planner does not observe the payoff state and the state of the economy. Further, she does not necessarily know how to associate the states of the economy with the underlying payoff states. Notwithstanding, the planner and the agents publicly observe choice data that contains partial information about individuals' preferences contingent on the states of the economy. As a result, a *missing data* problem emerges.

The access to the incomplete choice data enables the planner and the agents to make deductions about the payoff states that are compatible with the given states of the economy. We model the resulting inferred knowledge via a non-empty-valued *inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, where for any state of the economy $\theta \in \Theta$, $\mathcal{K}(\theta) \subset \Omega$ identifies the non-empty set of payoff states that are compatible with the public choice data at θ. The ***compatibility with the public choice data*** at a state of the economy $\theta \in \Theta$ demands that if the publicly observable choice of any individual $i \in N$ at θ from a set that includes both alternatives x and y contains x, then it is publicly known that $x \in C_i^{\omega}(\{x, y\})$ (alternatively, $xR_i^{\omega}y$) for all $\omega \in \mathcal{K}(\theta)$. For any $\theta \in \Theta$, we note that $\mathcal{K}(\theta)$ equals $\times_{i\in N}\mathcal{K}_i(\theta)$, for some non-empty collection of sets $\{\mathcal{K}_i(\theta)\}_{i\in N}$ with $\mathcal{K}_i(\theta) \subset \Omega_i$ for all $i \in N$. We require that for all $\theta \in \Theta$, $\pi^*(\theta) \in \mathcal{K}(\theta)$. The situation in which $\mathcal{K}(\theta) = \{\pi^*(\theta)\}$ for all $\theta \in \Theta$ corresponds to the standard case analyzed in the literature, the case with an *informed planner*.[9] On the other hand, at the state of the economy θ, individual i observing the incomplete public choice data and his own payoff type $\pi_i^*(\theta)$, infers that the set of payoff states that are compatible with

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[9] An implementation problem with $\mathcal{K}(\theta) = \Omega$ for some $\theta \in \Theta$ corresponds to a situation where the public choice data does not provide any information about individuals' preferences at the state of the economy θ. So, at θ, for any individual, the planner and the other agents consider any preference relation possible.

the public choice data at θ equals $\{(\pi_i^*(\theta), \omega_{-i}) \mid \omega_{-i} \in \mathcal{K}_{-i}(\theta)\}$, where $\mathcal{K}_{-i}(\theta) := \times_{j \neq i} \mathcal{K}_j(\theta)$.

The following formalizes the example we discuss in the introduction and helps us exemplify our formulation:

Example 1. Consider a new CEO (the planner) who is tasked to run a firm and needs to choose one of the following policies without observing the state of the firm: *expansion*, *prudence*, or *contraction*. The incumbent chiefs of finance (CFO) and marketing (CMO) observe the state of the firm, be it *strong (S)*, *normal (N)*, or *weak (W)*, along with their own preferences (payoff types).

We denote firm's policies, alternatives, by $X = \{c, e, p\}$, where c denotes the contractionary, e the expansionary, and p the prudent policy. Moreover, $\Theta = \{S, N, W\}$ captures the states of the firm. For simplicity, we assume that the payoff states, Ω, are all strict ranking profiles of $\{c, e, p\}$; therefore, there are $(6 \times 6)^3$ possible payoff states.

From past data on accounting records and meeting minutes, all three C-level executives have access to partial information about the CFO's and the CMO's true preferences. In particular, last quarter, the firm's state was *strong*; the CFO strictly preferred e to p and p to c, and the CMO strictly preferred e to p, while there is no further information pertaining to this state. Summarizing the rest of the incomplete data on the CFO's and the CMO's preferences recorded in recent quarters featuring the corresponding states delivers Table 1.

	Strong		*Normal*		*Weak*	
	CFO	*CMO*	*CFO*	*CMO*	*CFO*	*CMO*
$\{c, e, p\}$	$\{e\}$				$\{p\}$	$\{p\}$
$\{c, e\}$						
$\{c, p\}$	$\{p\}$		$\{p\}$			
$\{e, p\}$		$\{e\}$		$\{p\}$		

Table 1: Example 1 – An example with missing data.

We assume that the C-level executives' recent choices are compatible with their current preferences. For example, at the *normal* state of the firm, it is common knowledge that the CFO chooses p from $\{c, p\}$ but how he ranks e relative to these policies is not known. As a result, it is public information that at firm's *normal* state, the CFO's ranking compatible with his possible payoff types has to be one in $\{epc, pec, pce\}$, where xyz

denotes the strict preference order with x is strictly preferred to y, y to z, and x, y, z are distinct elements in $\{c, e, p\}$. Similar considerations for the CMO establish the following: $\mathcal{K}(N) = \{\{epc, pce, pec\} \times \{cpe, pce, pec\}\} \subset \Omega$. By repeating these arguments, we obtain $\mathcal{K}(S) = \{\{epc\} \times \{cep, ecp, epc\}\}$, and $\mathcal{K}(W) = \{\{pce, pec\} \times \{pce, pec\}\}$. The true association $\pi^* : \Theta \to \Omega$ is a selection from $\mathcal{K} : \Theta \twoheadrightarrow \Omega$. Thus, at S, the CEO knows the CFO's ranking, R^{ω}_{CFO}, equals epc while the CMO's, R^{ω}_{CMO}, must be in $\{epc, ecp, cep\}$, for all $\omega \in \mathcal{K}(S)$. On the other hand, at N, when the CFO knows his type equals $\pi^*_{CFO}(N) = pec$, then, thanks to the incomplete public choice data, he infers that the set of payoff states compatible with firm's *normal* state N equals $\{\{pec\} \times \{pec, pce, cpe\}\}$.

The CEO adopts an implementation approach and decentralizes her policy decision by extracting the relevant information from the CFO and the CMO. In particular, the objective that the CEO wishes to implement calls for e if the state of the firm is *strong* and p otherwise. This goal is desirable as, for every state of the firm, it involves a policy choice that is *efficient* at all compatible payoff states. □

For clarity, we gather the information structure of our model in the following:

Assumption 1. *The information and knowledge requirements of our model are as follows:*

- (i) *the planner knows N, X, Ω, Θ, and $f : \Theta \to \mathcal{X}$; and*
- (ii) *each individual i knows N, X, Ω, Θ, $f : \Theta \to \mathcal{X}$, the realized state of the economy $\theta \in \Theta$, and his realized payoff state $\pi^*_i(\theta) \in \Omega_i$; and*
- (iii) *items (i), (ii), and $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ characterizing the payoff states compatible with the public choice data are common knowledge among the individuals and the planner.*

In our setup, the only source for making inferences about others' payoff types is the public choice data. That is, at any given state of the economy θ, each individual has private information about his own payoff type but does not have any information about the others' payoff types except for the inferences compatible with the public choice data.[10]

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[10] For example, the compatibility with the public choice data prevents the following inference: Let $N = \{1, 2\}$, $X = \{x, y\}$, and consider θ such that $\mathcal{K}(\theta) = \{(xy, yx), (yx, xy)\}$ where ab denotes the strict preference order with a is strictly preferred to b, and a, b are distinct elements in $\{x, y\}$. If this inference were possible, then individual 1, knowing that his ranking at θ is given by xy (yx), would infer that individual 2's ranking equals yx (xy, resp.).

A mechanism $\mu = (M, g)$ assigns each individual i a non-empty *message space* M_i and specifies an *outcome function* $g : M \to X$ where $M = \times_{j \in N} M_j$. Given μ and $m_{-i} \in M_{-i} := \times_{j \neq i} M_j$, the *opportunity set* of individual i pertaining to others' message profile m_{-i} in mechanism μ is $O_i^{\mu}(m_{-i}) := \{g(m_i, m_{-i}) \mid m_i \in M_i\}$.

We aim to seek implementation of a given SCC based on only the *public information* available in the economy. That is why the subsequent construction refrains from relying on individuals' assessments about others' types as well as their own types.[11]

The NE of a mechanism is defined as follows: Given $\mu = (M, g)$, a message profile $m^* \in M$ is a ***Nash equilibrium*** of μ at payoff state $\omega \in \Omega$ if $g(m^*) \in \bigcap_{i \in N} C_i^{\omega}(O_i^{\mu}(m^*_{-i}))$. In our setting, however, the realized payoff state $\pi^*(\theta)$ is not necessarily common knowledge among the agents. Thus, here, the notion of NE is not plausible. Nevertheless, in what follows, we show that given a state of the economy, the planner can *rely on* a message profile that constitutes an NE for all payoff states that are compatible with the public choice data at that state of the economy. This brings about the following notion of equilibrium:

Definition 1. *Given a mechanism $\mu = (M, g)$, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the message profile $m^* \in M$ is a* ***reliable Nash equilibrium*** *(RNE) of μ at state of the economy $\theta \in \Theta$ if for all $i \in N$, $g(m^*) \in C_i^{\omega}(O_i^{\mu}(m^*_{-i}))$ for all $\omega \in \mathcal{K}(\theta)$.*

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ (that is induced by the public choice data), the message profile $m^* \in M$ is an RNE of $\mu = (M, g)$ at state of the economy $\theta \in \Theta$, whenever the following hold: for every individual i, m^* leads to an alternative which is chosen by i at payoff state ω from his opportunity set that results from others' actions m^*_{-i} at every ω that is compatible with the inference correspondence.

The notion of RNE is similar in spirit to the concept of ex-post equilibrium that employs the public choice data as a correlation device: The ex-post correlated equilibrium (ECE) induces an NE at every payoff state compatible with the public choice data, while each individual's strategy depends on the public choice data as well as his own private information. However, given a state of the economy, in an RNE, the actions of every individual depend only on the public choice data and not on that player's payoff type. Thereupon, a profile

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[11] In Section 5, we analyze situations in which individuals correlate their behavior using their private information.

of RNE taken across the states of the economy is equivalent to an ECE that uses correlated behavior based only on the public choice data, a notion of equilibrium that we designate as public ex-post correlated equilibrium (PECE). We formalize these in Section 5.

Consequently, the RNE shares the following *robustness properties* with ex-post correlated equilibrium: It employs no probabilistic information, no belief updating, and no common prior assumption as it is belief-free; the equilibrium behavior features the ex-post no-regret property. Moreover, the RNE, unlike the ECE, relies only on public information.

Given mechanism μ and state of the economy $\theta \in \Theta$, we let $RNE^{\mu}(\theta) := \{g(m^*) \in X \mid m^* \text{ is an RNE of } \mu \text{ at } \theta\}$ and define implementation in RNE as follows:

Definition 2. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is* ***implementable in reliable Nash equilibrium*** *if there is a mechanism $\mu = (M, g)$ such that $RNE^{\mu}(\theta) = f(\theta)$ for all $\theta \in \Theta$.*

Implementation of an SCC f in RNE by mechanism μ demands that for all states of the economy $\theta \in \Theta$, the f-optimal alternatives at θ *equal* the set of alternatives sustained by an RNE of μ at θ. Hence, it is 'robust' in the sense that for all $\theta \in \Theta$, any $x \in f(\theta)$ is sustained by a message profile $m^x \in M$ that is an NE of μ in every possible payoff state $\omega \in \mathcal{K}(\theta)$. Also, for any $\theta \in \Theta$, any alternative sustained by an RNE of μ at θ must be f-optimal at θ.

On the other hand, implementation in RNE (alternatively, PECE) does not rule out 'bad' ECE resulting in outcomes not aligned with the given SCC (see the example used in the proof of Proposition 3). Therefore, the emergence of equilibrium outcomes sustained by individuals using their private information and resulting in alternatives that are not compatible with the desired goal is a legitimate concern.

One way to deal with such instances is to go for double implementation in RNE and ECE. However, this obliges the planner to consider individuals' private information. As every ECE of a mechanism induces an NE at every payoff state that is compatible with the given state of the economy, we attain another way to deal with the dismissal of bad ECE by using only the public choice data: Ruling out bad NE ensures the elimination of unwanted ECE. That is why we propose the following notion of implementation, which demands that for any message profile $m^* \in M$ and state of the economy $\theta \in \Theta$, m^* being an NE of μ at a payoff state compatible with θ implies $g(m^*) \in f(\theta)$:[12]

Definition 3. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, we say that an SCC $f : \Theta \to \mathcal{X}$ is **safely implementable in reliable Nash equilibrium** by a mechanism $\mu = (M, g)$ if*

(*i*) *$f(\theta) \subset RNE^{\mu}(\theta)$ for all $\theta \in \Theta$; and*

(*ii*) *if $m^* \in M$ and $\theta \in \Theta$ are such that $g(m^*) \in \bigcap_{i \in N} C_i^{\omega}(O_i^{\mu}(m_{-i}^*))$ for some $\omega \in \mathcal{K}(\theta)$, then $g(m^*) \in f(\theta)$.*

It is clear that a planner with inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ infers that if a mechanism μ safely implements an SCC f in RNE, then μ implements f in RNE. Meanwhile, the reverse of this observation does not hold as portrayed in Example 1.

Example 1 (continued). Recall that the SCC under consideration is given by $f(S) = \{e\}$ while $f(\theta) = \{p\}$ for all $\theta \neq S$.[13] We consider two mechanisms μ and μ' both with $M_{CFO} = \{U, M, D\}$ and $M_{CMO} = \{L, M, R\}$ where the outcome functions g associated with μ and g' with μ' are as given in Table 2. The only difference between μ' and μ is: $g(D, R) = c$ while $g'(D, R) = p$.

CMO

		L	M	R
CFO	U	p	e	c
	M	e	p	p
	D	c	p	c

Outcome function of mechanism μ

CMO

		L	M	R
CFO	U	p	e	c
	M	e	p	p
	D	c	p	p

Outcome function of mechanism μ'

Table 2: The mechanisms used in Example 1.

Below, we show that the planner (the CEO) with inference correspondence derived from the incomplete public choice data provided in Table 1 infers that μ safely implements f in RNE, while μ' implements f in RNE but does not do so safely.

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[12] The above justifications for RNE, implementation in RNE, and safe implementation in RNE requires to consider individuals' correlated behavior based also on their private information. For practicality, we postpone the formal treatment of correlation under private information to Section 5.

[13] This SCC is desirable as for every θ, every $x \in f(\theta)$ is Pareto optimal for all ω in $\mathcal{K}(\theta)$.

The detailed workout involving μ is summarized in Table 3 where each RNE corresponding to a state of the economy appears circled. It shows that μ safely implements f in RNE. For example, consider the state of the firm N and recall that $\mathcal{K}(N) = \{\{epc, pce, pec\} \times \{cpe, pce, pec\}\}$. Thus, the planner cannot deduce the best response of the CFO when the CMO chooses L or M because she does not know how the CFO ranks e versus p. But, the planner infers that the CFO's best response is M when the CMO chooses R, as the CFO strictly prefers p to c in all of the (payoff-relevant) contingencies that the planner knows may happen. Similarly, the planner deduces that the CMO's best reply to the CFO choosing M must be either M or R, while she cannot deduce his best replies to the CFO choosing L or R as she does not know how the CMO ranks c versus p. But this suffices to establish that (M, R) is an RNE at N and hence (i) of safe implementation in RNE (Definition 3) holds at N as $g(M, R) = p$ and $f(N) = \{p\}$. To show (ii) of Definition 3 at N, we establish that there is no NE at any $\omega \in \mathcal{K}(N)$ providing an alternative other than p. To that regard, notice that $g(M, L) = e$ and $e \notin C^{\omega}_{CMO}(O^{\mu}_{CMO}(M))$ for all $\omega \in \mathcal{K}(N)$ (because in all such states the CMO strictly prefers p to e) where the CMO's opportunity set for the CFO's message M is $O^{\mu}_{CMO}(M) = \{e, p\}$. Now, $g(D, L) = c \notin C^{\omega}_{CFO}(O^{\mu}_{CFO}(L))$ for all $\omega \in \mathcal{K}(N)$ (because in all such states the CFO strictly prefers p to c) where $O^{\mu}_{CFO}(L) = \{c, e, p\}$. Next, $g(U, M) = e \notin C^{\omega}_{CMO}(O^{\mu}_{CMO}(U))$ for all $\omega \in \mathcal{K}(N)$ (because in all such states the CMO strictly prefers p to e) where $O^{\mu}_{CMO}(U) = \{c, e, p\}$. Similarly, $g(U, R) = g(D, R) = c \notin C^{\omega}_{CFO}(O^{\mu}_{CFO}(R))$ for all $\omega \in \mathcal{K}(N)$ (because in all such states the CFO strictly prefers p to c) where $O^{\mu}_{CFO}(R) = \{c, p\}$. For states of the firm other than N, repeating similar steps establishes that μ safely implements f in RNE.

Similarly, one can verify that μ' implements f in RNE. Indeed, the only difference between the detailed workouts with mechanism μ and μ' involves (D, R). As the corresponding opportunity sets and the RNE do not change, it suffices to consider (D, R) only at S since $g(D, R) = p$ and $f(\theta) = \{p\}$ for all $\theta \neq S$. Now, the planner with inference correspondence $\mathcal{K}$ observes that μ' implements f in RNE but does not do so safely because of this change: (D, R) is not an RNE as $g(D, R) = p \notin C^{\omega}_{CMO}(\{c, p\})$ when $\omega \in \{epc\} \times \{ecp, cep\} \subset \mathcal{K}(S) = \{\{epc\} \times \{cep, ecp, epc\}\}$. But when $\hat{\omega} = (epc, epc) \in \mathcal{K}(S)$, (D, R) is an NE at $\hat{\omega}$ as we have $g(D, R) = p \in C^{\hat{\omega}}_{CFO}(\{c, p\}) \cap C^{\hat{\omega}}_{CMO}(\{c, p\})$ but $p \notin f(S) = \{e\}$.

State of the economy: *Strong*	State of the economy: *Normal*	State of the economy: *Weak*
$f(S) = \{e\}$	$f(N) = \{p\}$	$f(W) = \{p\}$
$\mathcal{K}(S) : \{epc\} \times \{epc, ecp, cep\}$	$\mathcal{K}(N) : \{epc, pec, pce\} \times \{pec, pce, cpe\}$	$\mathcal{K}(W) : \{pec, pce\} \times \{pec, pce\}$
$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & \textcircled{e} & p & p \\ D & c & p & c \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & p & e & c \\ M & e & p & \textcircled{p} \\ D & c & p & c \end{array}$	$\begin{array}{c\|ccc} & L & M & R \\ \hline U & \textcircled{p} & e & c \\ M & e & p & p \\ D & c & p & c \end{array}$
RNE: (M, L) Outcomes: $\{e\}$	RNE: (M, R) Outcomes: $\{p\}$	RNE: (U, L) Outcomes: $\{p\}$

S : (M, L) is an RNE because $g(M, L) = e \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, L) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(U, L) = p \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, L) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, M) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(M, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, M) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(D, M) = p \notin C^{\omega}_{CFO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(U, R) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(S)$,
(M, R) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(M, R) = p \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(S)$,
(D, R) is not an NE at any $\omega \in \mathcal{K}(S)$ as $g(D, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(S)$.

N : (M, R) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, p\}) \cap C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(M, L) is not an NE at any $\omega \in \mathcal{K}(N)$ as $g(M, L) = e \notin C^{\omega}_{CMO}(\{e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(D, L) is not an NE at any $\omega \in \mathcal{K}(N)$ as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, M) is not an NE at any $\omega \in \mathcal{K}(N)$ as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(N)$,
(U, R) is not an NE at any $\omega \in \mathcal{K}(N)$ as $g(U, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(N)$,
(D, R) is not an NE at any $\omega \in \mathcal{K}(N)$ as $g(D, R) = c \notin C^{\omega}_{CFO}(\{c, p\})$ for all $\omega \in \mathcal{K}(N)$.

W : (U, L) is an RNE because $g(M, R) = p \in C^{\omega}_{CFO}(\{c, e, p\}) \cap C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(M, L) is not an NE at any $\omega \in \mathcal{K}(W)$ as $g(M, L) = e \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(D, L) is not an NE at any $\omega \in \mathcal{K}(W)$ as $g(D, L) = c \notin C^{\omega}_{CFO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, M) is not an NE at any $\omega \in \mathcal{K}(W)$ as $g(U, M) = e \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(U, R) is not an NE at any $\omega \in \mathcal{K}(W)$ as $g(U, R) = c \notin C^{\omega}_{CMO}(\{c, e, p\})$ for all $\omega \in \mathcal{K}(W)$,
(D, R) is not an NE at any $\omega \in \mathcal{K}(W)$ as $g(D, R) = c \notin C^{\omega}_{CMO}(\{c, p\})$ for all $\omega \in \mathcal{K}(W)$.

Table 3: Safe implementation of the SCC f via mechanism μ of Table 2.

3 Necessity and Sufficiency

The following conditions are crucial in our necessity and sufficiency results:

Definition 4. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$, a profile of sets* $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ *is* **reliably-consistent** *with f if*

(*i*) *for all $\theta \in \Theta$ and all $x \in f(\theta)$, $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$; and*

(*ii*) *$x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ implies that there is $j \in N$ and $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \notin C_j^{\tilde{\omega}}(S_j(x,\theta))$.*

Moreover, a profile of sets $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ *is* **safely-consistent** *with f if* (*i*) *and the following hold:*

(*iii*) *$x \in f(\theta)$ and $x \notin f(\tilde{\theta})$ implies that for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ there is $j \in N$ with $x \notin C_j^{\tilde{\omega}}(S_j(x,\theta))$.*

It is clear that if a profile of sets is safely-consistent with f, then it is reliably-consistent with f. The reverse of this observation does not hold.

The following is our necessity theorem:

Theorem 1. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$,*

(*i*) *if f is implementable in RNE, then there is a profile of sets that is reliably-consistent with f; and*

(*ii*) *if f is safely implementable in RNE, then there is a profile of sets that is safely-consistent with f.*

Proof of Theorem 1. For both (*i*) and (*ii*) of the theorem, denoting the corresponding mechanism $\mu = (M, g)$, we observe that for any $\theta \in \Theta$ and any $x \in f(\theta)$, there is $m^x \in M$ such that $g(m^x) = x$ and $g(m^x) \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^x))$. For all $i \in N$, all $\theta \in \Theta$, and all $x \in f(\theta)$, define $S_i(x,\theta) := O_i^{\mu}(m_{-i}^x)$. Then, for all $\theta \in \Theta$ and all $x \in f(\theta)$, $g(m^x) \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(O_i^{\mu}(m_{-i}^x))$ implies $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$, i.e., (*i*) of Definition 4.

For (ii) of Definition 4, if $x \in f(\theta)$, $x \notin f(\tilde{\theta})$, and $x \in \bigcap_{i\in N,\ \tilde{\omega}\in\mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(S_i(x,\theta))$ (which equals $\bigcap_{i\in N,\ \tilde{\omega}\in\mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^{x})))$, then $m^x \in M$ is an RNE at $\tilde{\theta}$. By Definition 2, $x \in f(\tilde{\theta})$, a contradiction.

Similarly, for (iii) of Definition 4, if $x \in f(\theta)$, $x \notin f(\tilde{\theta})$, and there is $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \in \bigcap_{i\in N} C_i^{\tilde{\omega}}(S_i(x,\theta))$, which equals $\bigcap_{i\in N} C_i^{\tilde{\omega}}(O_i^{\mu}(m_{-i}^{x}))$. Ergo, by ($ii$) of Definition 3, $g(m^x) = x \in f(\tilde{\theta})$, a contradiction. ■

Theorem 1 tells that implementability (safe implementability) of an SCC f in RNE implies the existence of a profile of sets reliably-consistent (safely-consistent, resp.) with f. Reliable-consistency and safe-consistency are related to monotonicity of Maskin (1999) and consistency of de Clippel (2014). We provide formal details of their relations with Maskin monotonicity in Appendix A.

Our next result establishes additional implications of consistency:

Theorem 2. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$, if there exists a profile of sets that is*

(i) *reliably-consistent with f and $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$ for some $\theta, \tilde{\theta} \in \Theta$, then $f(\theta) \subset f(\tilde{\theta})$;*

(ii) *safely-consistent with f and $\mathcal{K}(\theta) \cap \mathcal{K}(\tilde{\theta}) \neq \emptyset$ for some $\theta, \tilde{\theta} \in \Theta$, then $f(\theta) = f(\tilde{\theta})$.*

Proof of Theorem 2. For (i) of the theorem, suppose that the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ is such that there exists a profile of sets $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ reliably-consistent with f and $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$ for some $\theta, \tilde{\theta} \in \Theta$. Then, by ($i$) of reliable-consistency, $x \in f(\theta)$ implies $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$. As $\mathcal{K}(\tilde{\theta}) \subset \mathcal{K}(\theta)$, we observe $x \in \bigcap_{i\in N,\ \tilde{\omega}\in\mathcal{K}(\tilde{\theta})} C_i^{\tilde{\omega}}(S_i(x,\theta))$. Thus, $x \notin f(\tilde{\theta})$ produces a contradiction to (ii) of reliable-consistency. Therefore, $x \in f(\tilde{\theta})$.

To establish (ii) of the theorem, suppose that the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ is such that there exists a profile of sets $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ safely-consistent with f and there is $\omega^* \in \mathcal{K}(\theta) \cap \mathcal{K}(\tilde{\theta})$ for some $\theta, \tilde{\theta} \in \Theta$. Then, by ($i$) of safe-consistency, $x \in f(\theta)$ implies $x \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^{\omega}(S_i(x,\theta))$ and hence $x \in \bigcap_{i\in N} C_i^{\omega^*}(S_i(x,\theta))$. But, $x \notin f(\tilde{\theta})$ implies (by (ii) of safe-consistency) that for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$, $x \notin \bigcap_{i\in N} C_i^{\tilde{\omega}}(S_i(x,\theta))$ which implies (on account of $\omega^* \in \mathcal{K}(\tilde{\theta})$) $x \notin \bigcap_{i\in N} C_i^{\omega^*}(S_i(x,\theta))$, a contradiction. Hence, $x \in f(\tilde{\theta})$. As θ and $\tilde{\theta}$ can be interchanged, we obtain $f(\theta) = f(\tilde{\theta})$. ■

Theorem 2 establishes that information enriches implementation opportunities: Both versions of consistency propel the SCC to display less variation across states of the economy. Part (i) says that if the planner has more information at a state of the economy in comparison to another, then she is able to implement more alternatives in the former in RNE. Moreover, (ii) of the theorem makes a sharp observation: An SCC that is safely implementable in RNE cannot vary across two states of the economy whenever the planner's information is not mutually exclusive across these states. Indeed, sharper implications emerge when there is a state of the economy, at which the planner is completely ignorant of the society's underlying payoff-relevant characteristics: Suppose there is $\tilde{\theta} \in \Theta$ with $\mathcal{K}(\tilde{\theta}) = \Omega$. Then, (i) any SCC that is implementable in RNE must be such that $f(\tilde{\theta}) \subset \bigcap_{\theta\in\Theta} f(\theta)$; and (ii) any SCC that is safely implementable in RNE must be constant. Therefore, if, in addition, f is singleton-valued (i.e., f is an SCF), the existence of $\tilde{\theta} \in \Theta$ with $\mathcal{K}(\tilde{\theta}) = \Omega$ implies $f(\theta) = f(\theta')$ for all $\theta, \theta' \in \Theta$ whenever f is either implementable in RNE or safely implementable in RNE.

We employ the following economic environment assumptions in our sufficiency result:[14]

Definition 5. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the environment is*

(i) ***economic*** *if for all $x \in X$ and for all $\theta \in \Theta$, there exist $i, j \in N$ with $i \neq j$, $\omega \in \mathcal{K}(\theta)$, and $y^i, y^j \in X$ such that $y^i P_i^{\omega} x$ and $y^j P_j^{\omega} x$; and*

(ii) ***strictly economic*** *if for all $x \in X$, all $\theta \in \Theta$, and all $\omega \in \mathcal{K}(\theta)$, there exist $i, j \in N$ with $i \neq j$ and $y^i, y^j \in X$ such that $y^i P_i^{\omega} x$ and $y^j P_j^{\omega} x$.*

The environment being economic (strictly economic) requires that there is a mild form of disagreement in the society: For every state of the economy, it is not possible for all or $n-1$ individuals to agree on their best alternatives in some payoff state (all payoff states, resp.) compatible with that state of the economy.

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[14] Our economic environment assumptions are in line with those in Jackson (1991), Bergemann and Morris (2008), Kartik and Tercieux (2012), and Barlo and Dalkıran (2022a).

The following is our sufficiency theorem:

Theorem 3. *Let* $\#N \geq 3$*. Given an inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *and an SCC* $f : \Theta \to \mathcal{X}$*, if there exists a profile of sets that is*

- (i) *reliably-consistent with* f *and the environment is economic, then* f *is implementable in RNE; and*

- (ii) *safely-consistent with* f *and the environment is strictly economic, then* f *is safely implementable in RNE.*

Proof of Theorem 3. Suppose that for the given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and $f : \Theta \to \mathcal{X}$, the environment is economic (strictly economic) and there is a profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ that is reliably-consistent (safely-consistent, resp.) with f.

The proof is constructive and uses the canonical mechanism $\mu = (M, g)$ (as in Maskin (1999), Moore and Repullo (1990), and de Clippel (2014)) defined as follows: $M_i := \Theta \times X \times \mathbb{N}$ where a generic member $m_i = (\theta^{(i)}, x^{(i)}, k^{(i)}) \in M_i$ with $\theta^{(i)} \in \Theta$, $x^{(i)} \in X$, and $k^{(i)} \in \mathbb{N}$. The outcome function is as given in Table 4.

Rule 1 :	$g(m) = x$	if $m_i = (\theta, x, \cdot)$ for all $i \in N$ with $x \in f(\theta)$,
Rule 2 :	$g(m) = \begin{cases} x' & \text{if } x' \in S_j(x, \theta) \\ x & \text{otherwise.} \end{cases}$	if $m_i = (\theta, x, \cdot)$ for all $i \in N \setminus \{j\}$ with $x \in f(\theta)$, and $m_j = (\theta', x', \cdot) \neq (\theta, x, \cdot)$,
Rule 3 :	$g(m) = x^{(i^*)}$ where $i^* = \min\{j \in N : k^{(j)} \geq \max_{i' \in N} k^{(i')}\}$	otherwise.

Table 4: The outcome function of the mechanism.

We show that μ implements f in RNE via Claims 1 and 2, and μ safely-implements f in RNE via Claims 1 and 3:

Claim 1. *For all* $\theta \in \Theta$ *and all* $x \in f(\theta)$*, let* $m^x \in M$ *be such that* $m^x_i = (\theta, x, 1)$*. Then,* m^x *is an RNE of* μ *at* θ *with* $g(m^x) = x$*.*

Proof. Rule 1 applies and $g(m^x) = x$. The individual deviations can only result in Rules 1 and 2. So, $O_i^\mu(m_{-i}^x) = S_i(x,\theta)$ for all i. By (*i*) of reliable-consistency (safe-consistency, resp.) for all $\theta \in \Theta$ and all $x \in f(\theta)$, $x = g(m^x) \in \bigcap_{i\in N,\ \omega\in\mathcal{K}(\theta)} C_i^\omega(S_i(x,\theta))$. Ergo, m^x is an RNE of μ at θ. ■

Claim 2. *If $m^* \in M$ is an RNE of μ at some $\theta \in \Theta$, the environment is economic, and* **S** *is reliably-consistent, then $g(m^*) \in f(\theta)$.*

Proof. First, we establish that there cannot be an RNE of μ under Rule 2 or 3. Consider an RNE $\bar{m} \in M$ at $\theta \in \Theta$ under either Rule 2 or Rule 3 in order to obtain a contradiction. In both of the cases, $O_j^\mu(\bar{m}_{-j}) = X$ for at least $n-1$ individuals. Then, for at least $n-1$ individuals j, $g(\bar{m}) \in \bigcap_{\omega\in\mathcal{K}(\theta)} C_j^\omega(X)$. This is not possible due to the economic environment assumption.

Now, consider an RNE at $\theta \in \Theta$ under Rule 1: let m^* be an RNE at θ such that $m_i^* = (\theta', x', \cdot)$ for all $i \in N$. Then, as Rule 1 holds, $g(m^*) = x' \in f(\theta')$. $O_i^\mu(m_{-i}^*) = S_i(x',\theta')$ for all $i \in N$ due to Rules 1 and 2. If $x' \notin f(\theta)$, by (*ii*) of reliable-consistency, there exist $j \in N$ and $\omega \in \mathcal{K}(\theta)$ such that $x \notin C_j^\omega(S_j(x',\theta'))$. This contradicts m^* being an RNE at θ since $O_j^\mu(m_{-j}^*) = S_j(x',\theta')$. Thus, $x' \in f(\theta)$ and hence $g(m^*) \in f(\theta)$ as desired. ■

Claim 3. *If $m^* \in M$ and $\theta \in \Theta$ are such that $g(m^*) \in \bigcap_{i\in N} C_i^\omega(O_i^\mu(m_{-i}^*))$ at some $\omega \in \mathcal{K}(\theta)$, the environment is strictly economic, and* **S** *is safely-consistent, then $g(m^*) \in f(\theta)$.*

Proof. First, we observe that there is no $\hat{m} \in M$ and $\hat{\theta} \in \Theta$ such that either Rules 2 or 3 holds and $g(\hat{m}) \in \bigcap_{i\in N} C_i^{\hat{\omega}}(O_i^\mu(\hat{m}_{-i}))$ at some $\hat{\omega} \in \mathcal{K}(\hat{\theta})$. Because otherwise, $g(\hat{m}) \in C_j^{\hat{\omega}}(X)$ for at least $n-1$ individuals, which contradicts the environment being strictly economic.

To finish the proof, suppose $\hat{m} \in M$ and $\hat{\theta} \in \Theta$ are such that Rule 1 holds and $g(\hat{m}) \in \bigcap_{i\in N} C_i^{\hat{\omega}}(O_i^\mu(\hat{m}_{-i}))$ at some $\hat{\omega} \in \mathcal{K}(\hat{\theta})$. In particular, let $\hat{m}_i = (\theta', x', \cdot)$ for all $i \in N$ with $x' \in f(\theta')$, so $g(\hat{m}) = x'$. Due to Rules 1 and 2, $O_i^\mu(\hat{m}_{-i}) = S_i(x',\theta')$ for all $i \in N$. Then, $x' \notin f(\hat{\theta})$ implies, by (*iii*) of Definition 4, for $\hat{\omega} \in \mathcal{K}(\hat{\theta})$ there is $\hat{j} \in N$ with $x' \notin C_{\hat{j}}^{\hat{\omega}}(O_{\hat{j}}^\mu(\hat{m}_{-\hat{j}}))$, a contradiction to $x' \in \bigcap_{i\in N} C_i^{\hat{\omega}}(O_i^\mu(\hat{m}_{-i}))$ for $\hat{\omega} \in \mathcal{K}(\hat{\theta})$. Thus, $g(\hat{m}) = x' \in f(\hat{\theta})$. ■

This concludes the proof of the theorem. ■

4 Efficiency

Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ characterizing the payoff states compatible with the public choice data, a natural and suitable efficiency notion that comes to mind is ***reliable Pareto optimal SCC***, $RPO : \Theta \twoheadrightarrow X$, defined by $RPO(\theta) := \{x \in X \mid x \in \cap_{\omega \in \mathcal{K}(\theta)} PO(\omega)\}$ where for any $\omega \in \Omega$, $PO(\omega) := \{x \in X \mid \nexists y \in X \text{ such that } yP_i^{\omega}x, \forall i \in N\}$, i.e., the set of (weakly) Pareto optimal alternatives at ω. On the other hand, *reliable efficiency*, which parallels the efficiency of de Clippel (2014), turns out to be also suitable for our environment: Given $\theta \in \Theta$, an alternative $x \in X$ is ***reliably efficient*** at θ if there exists a profile of sets of alternatives $\mathbf{L}_x^{\theta} := (L_{i,x}^{\theta})_{i \in N}$ such that for all $i \in N$, $x \in L_{i,x}^{\theta}$ and $L_{i,x}^{\theta} \subset L_i^{\omega}(x) = \{y \mid xR_i^{\omega}y\}$ for all $\omega \in \mathcal{K}(\theta)$ with the property that $\cup_{i \in N} L_{i,x}^{\theta} = X$. Let $RE : \Theta \twoheadrightarrow X$ denote the ***reliably efficient SCC***.

Below, we show that these efficiency notions are equivalent in our setup.[15] Moreover, both of these efficiency notions (defined in incomplete information environments) are determined by using only the incomplete public choice data; not individuals' private information about their realized payoff types. When the planner is informed, i.e., $\mathcal{K}(\theta) = \{\pi^*(\theta)\}$ for all $\theta \in \Theta$, then $RE(\theta) = RPO(\theta) = PO(\pi^*(\theta))$ for all $\theta \in \Theta$. Thus, reliable Pareto optimality and reliable efficiency are extensions of Pareto optimality to missing data.

Proposition 1. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, $RE(\theta) = RPO(\theta)$ for all $\theta \in \Theta$.*

Proof of Proposition 1. First, we show that $RE(\theta) \subset RPO(\theta)$: If $x \in RE(\theta)$ for some $\theta \in \Theta$, then there is $(L_{i,x}^{\theta})_{i \in N}$ such that $x \in L_{i,x}^{\theta}$ and $x \in C_i^{\omega}(L_{i,x}^{\theta})$ for all $\omega \in \mathcal{K}(\theta)$ and all $i \in N$, and $\cup_i L_{i,x}^{\theta} = X$. Thus, if $x \notin RPO(\theta)$, then there exists $\tilde{y} \in X$ and $\tilde{\omega} \in \mathcal{K}(\theta)$ such that $\tilde{y}P_i^{\tilde{\omega}}x$ for all $i \in N$. As $\cup_i L_{i,x}^{\theta} = X$, there is $j \in N$ such that $\tilde{y} \in L_{j,x}^{\theta}$. But then, $x \notin C_j^{\tilde{\omega}}(L_{j,x}^{\theta})$, a contradiction.

Next, we handle $RPO(\theta) \subset RE(\theta)$:[16] Let $x \in RPO(\theta)$ for some $\theta \in \Theta$. Then, by the public choice data, it is publicly known that there is no $y \in X$ such that $yP_i^{\omega}x$ for all $\omega \in K(\theta)$

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[15] One can verify that in Example 1, the incomplete public choice data is such that the reliable Pareto optimal alternatives coincide with the reliably efficient alternatives at every state of the economy.

[16] Compatibility with the public choice data plays a critical role when showing that reliable Pareto optimality implies reliable efficiency. With more freedom in terms of correlated observations, this implication does

and all $i \in N$. Therefore, for any $y \in X \setminus \{x\}$, there is $j_y \in N$ such that $x \in C^{\omega}_{j_y}(\{x, y\})$ for all $\omega \in \mathcal{K}(\theta)$. If for any $y, y' \in X \setminus \{x\}$, $j_y = j_{y'}$, then due to rationality, we know that $x \in C^{\omega}_{j_y}(\{x, y, y'\})$ for all $\omega \in \mathcal{K}(\theta)$. Consequently, for any $i \in N$, let

$$L^{\theta}_{i,x} := \begin{cases} \{x\} & \text{if } i \neq j_y \text{ for any } y \in X \setminus \{x\}, \\ \{x\} \bigcup \left(\cup_{\{j_y \in N | y \in X \setminus \{x\} \text{ and } j_y = i\}} \{y\} \right) & \text{otherwise.} \end{cases}$$

Then, $x \in C^{\omega}_{i}(L^{\theta}_{i,x})$ for all $\omega \in K(\theta)$ and all $i \in N$, and $\cup_{i \in N} L^{\theta}_{i,x} = X$. Thus, $x \in RE(\theta)$. ■

The following result presents sufficient conditions for implementing the reliable Pareto optimal SCC in RNE:

Proposition 2. *Let* $\#N \geq 3$. *If an inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ *induces an economic environment in which the reliable Pareto optimal SCC,* $RPO : \Theta \twoheadrightarrow X$, *is nonempty-valued, then RPO is implementable in RNE.*

Proof of Proposition 2. Thanks to Proposition 1, it suffices to show that $RE : \Theta \twoheadrightarrow X$ is implementable in RNE. First, we show that given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, if $RE : \Theta \twoheadrightarrow X$ is nonempty-valued, then the associated profile of sets $\mathbf{L} := (L^{\theta}_{i,x})_{i \in N,\ \theta \in \Theta,\ x \in RE(\theta)}$ is reliably-consistent with RE. Let $i \in N$, $\theta \in \Theta$, $x \in RE(\theta)$. Then, $L^{\theta}_{i,x} \subset \cap_{\omega \in \mathcal{K}(\theta)} L^{\omega}_{i}(x)$ implies (i) of reliable-consistency. For (ii) of reliable-consistency, suppose $x \in RE(\theta)$ and $x \notin RE(\tilde{\theta})$. As $x \in RE(\theta)$, there is $\mathbf{L}_x := (L^{\theta}_{i,x})_{i \in N,\ \theta \in \Theta}$ with $x \in \cap_{\omega \in \mathcal{K}(\theta)} C^{\omega}_{i}(L^{\theta}_{i,x})$ and $\cup_{i \in N} L^{\theta}_{i,x} = X$. But, $x \notin RE(\tilde{\theta})$ implies that there is $j \in N$ and $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \notin C^{\tilde{\omega}}_{j}(L^{\theta}_{j,x})$ because otherwise we obtain a contradiction since $\mathbf{L}_x := (L^{\theta}_{i,x})_{i \in N,\ \theta \in \Theta}$ sustains x as a reliably efficient alternative at $\tilde{\theta}$.

Having established the reliable-consistency of the profile $\mathbf{L} := (L^{\theta}_{i,x})_{i \in N,\ \theta \in \Theta,\ x \in RE(\theta)}$ with RE, the rest of the proof follows from Theorem 3. ■

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not hold: Let $N = \{1, 2\}$, $X = \{x, y, z\}$, $\Theta = \{\theta\}$, and $\mathcal{K}(\theta) = \{(xyz, zyx), (zyx, xyz)\}$ where abc denotes the strict preference order with a is strictly preferred to b, b to c, and a, b, c are distinct elements in $\{x, y, z\}$. Then, $y \in RPO(\theta)$. But, $y \notin RE(\theta)$ because of the following: Any profile of sets sustaining y in reliable efficiency, $\mathbf{L}_y := (L^{\theta}_{i,y})_{i \in N,\ \theta \in \Theta}$, must be such that $L^{\theta}_{i,y} \subset \cap_{\omega \in \mathcal{K}(\theta)} L^{\omega}_{i}(y)$, while $\cap_{\omega \in K(\theta)} L^{\omega}_{i}(y) = \{y\}$, for all $i = 1, 2$; hence, we cannot have $\cup_{i=1,2} L^{\theta}_{i,y} = X$. Thus, $RE(\theta)$ is a strict subset of $RPO(\theta)$ (since $RE(\theta) \subset RPO(\theta)$ follows from the first paragraph of the proof of Proposition 1). The above inference is not possible as in the example in Footnote 10 since x being top-ranked by individual 1 at θ in the public choice data implies that x must be top-ranked in all payoff types of individual 1 compatible with θ.

5 Correlation under Private Information

The main purpose of this paper is to attain implementation by using only the incomplete public choice data. In this section, we analyze situations in which individuals may use their private information as well.

When dealing with a setting under incomplete information (see Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1987), Jackson (1991) for the Bayesian, and Bergemann and Morris (2008) and Barlo and Dalkıran (2022a) for the ex-post setting), the main object of interest becomes state contingent allocations, i.e., social choice functions. As a result, social choice sets that are composed of such functions are used instead of SCCs.

In our setup, where individuals' behavior can be correlated on publicly observable economic states, how to formalize suitable social choice sets is not obvious. In general, a *correlated social choice set* (CSCS) is $\Phi := (\Phi_\theta)_{\theta\in\Theta}$ with Φ_θ being a non-empty subset of all functions mapping $\mathcal{K}(\theta)$ to X, $\{\varphi \mid \varphi : \mathcal{K}(\theta) \to X\}$, for all $\theta \in \Theta$. Given the desirable alternatives as specified by an SCC $f : \Theta \to \mathcal{X}$ and a state of the economy $\theta \in \Theta$, a CSCS associated with f at θ, $\Phi_{f,\theta}$, is a non-empty subset of the set of all functions mapping $\mathcal{K}(\theta)$ into $f(\theta)$, i.e., $\{\varphi_\theta \mid \varphi_\theta : \mathcal{K}(\theta) \to f(\theta)\}$. We denote a CSCS associated with f by $\Phi_f := (\Phi_{f,\theta})_{\theta\in\Theta}$.[17, 18]

The notion of reliability inherent in RNE parallels the following: A CSCS associated with a given SCC f, Φ_f, satisfies the *reliability criterion* if for all $\theta \in \Theta$, $\Phi_{f,\theta}$ consists of constant functions mapping $\mathcal{K}(\theta)$ to $f(\theta)$ with the requirement that for all $x \in f(\theta)$ there is a function in $\Phi_{f,\theta}$ that maps $\mathcal{K}(\theta)$ to $\{x\}$. That is, $\Phi_{f,\theta} := \cup_{x\in f(\theta)} \{\bar{\varphi}_{(\theta,x)}\}$ where $\bar{\varphi}_{(\theta,x)} : \mathcal{K}(\theta) \to f(\theta)$ is defined by $\bar{\varphi}_{(\theta,x)}(\omega) = x$ for all $\omega \in \mathcal{K}(\theta)$.

In what follows, given an SCC f, we focus on its associated CSCS that is uniquely

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[17] For example, let $N = \{1,2\}$, $X = \{x,y,z\}$, $\Theta = \{\theta_1,\theta_2\}$ and $\Omega_i = \{\omega_{i1},\omega_{i2},\omega_{i3}\}$ for $i = 1,2$ with $\mathcal{K}(\theta_1) = \{(\omega_{11},\omega_{21}),(\omega_{11},\omega_{22}),(\omega_{12},\omega_{21}),(\omega_{12},\omega_{22})\}$ and $\mathcal{K}(\theta_2) = \{(\omega_{12},\omega_{22}),(\omega_{12},\omega_{23}),(\omega_{13},\omega_{22}),(\omega_{13},\omega_{23})\}$. Given an SCC f with $f(\theta_1) = \{x,y\}$ and $f(\theta_2) = \{z\}$, a CSCS associated with f, Φ_f, could be such that $\Phi_{f,\theta_1} = \{\langle x,x,x,x\rangle, \langle x,x,y,y\rangle, \langle y,y,y,x\rangle\}$ and $\Phi_{f,\theta_2} = \{\langle z,z,z,z\rangle\}$ where $\langle a_1,a_2,a_3,a_4\rangle$ denotes the function on $\mathcal{K}(\theta_1)$ and $\mathcal{K}(\theta_2)$ defined accordingly (e.g., $\langle y,y,y,x\rangle$ denotes the function on $\mathcal{K}(\theta_1)$ which maps the payoff state $(\omega_{12},\omega_{22})$ to x and all the other payoff states in $\mathcal{K}(\theta_1)$ to y).

[18] It is customary in the implementation literature to have the CSCS be exogenously given. In the current context, an innocuous requirement on the CSCS Φ_f that is coherent with our setting involves the restriction that for all $\theta \in \Theta$ and all $x \in f(\theta)$, there exists $\varphi_\theta \in \Phi_{f,\theta}$ such that $x \in \varphi_\theta(\mathcal{K}(\theta))$.

determined by the reliability criterion, and we denote such a CSCS by $\bar{\Phi}_f$.[19]

Given a mechanism $\mu = (M, g)$, individual i's strategies are mappings that are measurable with respect to his information that include the following (as formalized in Assumption 1): (i) the realized state of the economy $\theta \in \Theta$ along with the public choice data implying the inference correspondence $\mathcal{K}_j(\theta)$ for all $j \in N$ (*public information*), and (ii) individual i's realized payoff state $\pi_i^*(\theta) \in \mathcal{K}_i(\theta)$ (*private information*).

Consequently, for each state of the economy $\theta \in \Theta$, individual i's *correlated strategy* at θ is a function $\sigma_{i\theta} : \mathcal{K}_i(\theta) \to M_i$. We let $\Sigma_{i\theta}$ be the set of individual i's correlated strategies at $\theta \in \Theta$. We denote a profile of correlated strategies at θ by $\sigma_\theta := (\sigma_{i\theta})_{i\in N} \in \Sigma_\theta$, where $\Sigma_\theta := \times_{i\in N}\Sigma_{i\theta}$; a correlated strategy profile by $\sigma := (\sigma_{i\theta})_{i\in N,\theta\in\Theta} \in \Sigma := \times_{\theta\in\Theta}\Sigma_\theta$.

When correlation is based only on public information, it is appropriate to work with the following notion of strategies: For each state of the economy $\theta \in \Theta$, individual i's *public correlated strategy* at θ is given by $\varsigma_{i\theta} \in M_i$. We let $\Sigma^P_{i\theta}$ be the set of individual i's public correlated strategies at $\theta \in \Theta$. We denote a profile of public correlated strategies at θ by $\varsigma_\theta := (\varsigma_{i\theta})_{i\in N} \in \Sigma^P_\theta := \times_{i\in N}\Sigma^P_{i\theta}$; a public correlated strategy profile by $\varsigma := (\varsigma_{i\theta})_{i\in N,\theta\in\Theta} \in \Sigma^P := \times_{\theta\in\Theta}\Sigma^P_\theta$.

5.1 Ex-Post Correlated Equilibrium

Definition 6. *Given a mechanism $\mu = (M, g)$, and the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the correlated strategy profile $\sigma^* \equiv (\sigma^*_{i\theta})_{i\in N,\theta\in\Theta} \in \Sigma$ is an* ***ex-post correlated equilibrium*** *(ECE) of μ if for all states of the economy $\theta \in \Theta$, all $i \in N$, and all $\omega_i \in \mathcal{K}_i(\theta)$,*

$$g(\sigma^*_{i\theta}(\omega_i), \sigma^*_{-i\theta}(\omega_{-i})) \in C_i^{(\omega_i,\omega_{-i})}(O_i^\mu(\sigma^*_{-i\theta}(\omega_{-i}))), \ \ \textit{for all } \omega_{-i} \in \mathcal{K}_{-i}(\theta).$$

Moreover, the public correlated strategy profile $\varsigma^* \equiv (\varsigma^*_{i\theta})_{i\in N,\theta\in\Theta} \in \Sigma^P$ is a ***public ex-post correlated equilibrium*** *(PECE) of μ if for all $\theta \in \Theta$, all $i \in N$, and all $\omega_i \in \mathcal{K}_i(\theta)$,*

$$g(\varsigma^*_{i\theta}, \varsigma^*_{-i\theta}) \in C_i^{(\omega_i,\omega_{-i})}(O_i^\mu(\varsigma^*_{-i\theta})), \ \ \textit{for all } \omega_{-i} \in \mathcal{K}_{-i}(\theta).$$

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[19] Using the example of Footnote 17, the CSCS associated with f that satisfies the reliability criterion is uniquely determined as follows: $\bar{\Phi}_{f,\theta_1} = \{\langle x, x, x, x\rangle, \langle y, y, y, y\rangle\}$ and $\bar{\Phi}_{f,\theta_2} = \{\langle z, z, z, z\rangle\}$.

Any RNE behavior profile, $(m_\theta)_{\theta\in\Theta}$, is equivalent to a PECE $\varsigma \in \Sigma^P$ with $\varsigma_{i\theta} = m_{i\theta}$ for all $i \in N$ and all $\theta \in \Theta$. In furtherance, any PECE strategy profile $\varsigma \in \Sigma^P$ induces an ECE $\sigma^\varsigma \in \Sigma$ of μ, where $\sigma^\varsigma_{i\theta}(\omega_i) = \varsigma_{i\theta}$ for all $i \in N$, all $\theta \in \Theta$, and all $\omega_i \in \mathcal{K}_i(\theta)$.

Therefore, the aforementioned robustness property of RNE follows as every RNE profile induces both an ECE and PECE, and hence individuals base their equilibrium behavior only on the public choice data.

The implementation of a given CSCS Φ (that is not necessarily associated with an SCC f) in ECE is obtained as follows: Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, a CSCS Φ is implementable by a mechanism μ in ECE if (i) for all $\theta \in \Theta$ and all $\varphi_\theta \in \Phi_\theta$, there is an ECE $\sigma^* \in \Sigma$ with $g(\sigma^*_\theta(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$; and ($ii$) if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$ there is $\varphi_\theta \in \Phi_\theta$ such that $g(\sigma^*_\theta(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$. As we focus on the CSCS that is associated with the SCC f and satisfies the reliability criterion, $\bar{\Phi}_f$, for all $\theta \in \Theta$ and all $\varphi_\theta \in \bar{\Phi}_{f,\theta}$ it must be that $\varphi_\theta(\omega) = x$ for some $x \in f(\theta)$ for all $\omega \in \mathcal{K}(\theta)$. As a result, given an SCC f, the implementability of the CSCS $\bar{\Phi}_f$ is equivalent to the following definition, which does not employ social choice sets:

Definition 7. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is* ***implementable in ex-post correlated equilibrium*** *by a mechanism $\mu = (M, g)$ if*

(i) *for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is an ECE $\sigma^{(x,\theta)} \in \Sigma$ with $g(\sigma^{(x,\theta)}_{i\theta}(\omega_i), \sigma^{(x,\theta)}_{-i\theta}(\omega_{-i})) = x$ for all $\omega \in \mathcal{K}(\theta)$; and*

(ii) *if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$, there exists $y \in f(\theta)$ such that for all $\omega \in \mathcal{K}(\theta)$, $g(\sigma^*_{i\theta}(\omega_i), \sigma^*_{-i\theta}(\omega_{-i})) = y$.*

We observe that when f is implementable in ECE, individuals use their private information. Demanding that they use only the public information delivers the following:

Definition 8. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is* ***implementable in public ex-post correlated equilibrium*** *by a mechanism $\mu = (M, g)$ if*

(i) *for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is a PECE $\varsigma^{(x,\theta)} \in \Sigma^P$ with $g(\varsigma^{(x,\theta)}_{i\theta}, \varsigma^{(x,\theta)}_{-i\theta}) = x$; and*

(ii) *if $\varsigma^* \in \Sigma^P$ is a PECE of μ, then for all $\theta \in \Theta$, $g(\varsigma^*_{i\theta}, \varsigma^*_{-i\theta}) \in f(\theta)$.*

The following relates implementability in PECE to implementability in RNE:

Remark 1. *Given an inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, *an SCC* $f : \Theta \rightarrow \mathcal{X}$ *is implementable in PECE by a mechanism* μ *if and only if it is implementable in RNE via* μ.

On the other hand, thanks to the reliability criterion, the following holds: if an SCC f is implementable in ECE by a mechanism μ, then μ also implements f in PECE, while the reverse does not hold:

Proposition 3. *Given an inference correspondence* $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, *if an SCC* $f : \Theta \rightarrow \mathcal{X}$ *is implementable in ECE via a mechanism* μ, *then it is implementable in PECE via* μ. *But the reverse does not hold.*

Proof of Proposition 3. Suppose $\mu = (M, g)$ implements an SCC f in ECE. Then, by (i) of Definition 7 for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is an ECE $\sigma^{(x,\theta)} \in \Sigma$ such that $g(\sigma_\theta^{(x,\theta)}(\omega)) = x$ for all $\omega \in \mathcal{K}(\theta)$.

Let $\theta \in \Theta$. As μ implements f in ECE by hypothesis, for all $i \in N$ and $\tilde{\omega}_i \in \mathcal{K}_i(\theta)$, $g(\sigma_{i\theta}^{(x,\theta)}(\tilde{\omega}_i), \sigma_{-i\theta}^{(x,\theta)}(\omega_{-i})) = x \in C_i^{(\tilde{\omega}_i,\omega_{-i})}(O_i^\mu(\sigma_{-i\theta}^{(x,\theta)}(\omega_{-i})))$ for all $\omega_{-i} \in \mathcal{K}_{-i}(\theta)$.

Fix $\omega^* \in \mathcal{K}(\theta)$, and define ς^* by $\varsigma_{i\theta}^* := \sigma_{i\theta}^{(x,\theta)}(\omega_i^*)$ for all $i \in N$. So, for all $i \in N$ and $\tilde{\omega}_i \in \mathcal{K}_i(\theta)$, $g(\sigma_\theta^{(x,\theta)}(\omega^*)) = x = g(\varsigma_\theta^*) \in C_i^{(\tilde{\omega}_i,\omega_{-i})}(O_i^\mu(\sigma_{-i\theta}^{(x,\theta)}(\omega_{-i}))$ for all $\omega_{-i} \in \mathcal{K}_{-i}(\theta)$. Hence, ς^* is a PECE of μ resulting in x; establishing (i) of Definition 8.

Finally, let ς^* be a PECE of μ. Then, σ^* defined naturally via ς^* such that $\sigma_\theta^*(\omega) = \varsigma_\theta^*$ for all $\theta \in \Theta$ and all $\omega \in \mathcal{K}(\theta)$ is an ECE that satisfies the properties demanded by (ii) of Definition 7. Thus, there is $y \in f(\theta)$ such that $g(\sigma_\theta^*(\omega)) = y$ for all $\omega \in \mathcal{K}(\theta)$. This implies $y = g(\varsigma^*) \in f(\theta)$ and hence establishes (ii) of Definition 8.

To see that the reverse does not hold, consider the following example: $N = \{1, 2\}$, $X = \{x, y\}$, $\Theta = \{\theta_1, \theta_2\}$, Ω_i consists of all strict rankings of $\{x, y\}$, $\mathcal{K}_i(\theta_1) = \{xy, yx\}$, and $\mathcal{K}_i(\theta_2) = \{xy\}$ for all $i = 1, 2$ (where xy denotes the situation where x is strictly preferred to y by i at θ), while the SCC at hand is f such that $f(\theta_1) = \{y\}$ and $f(\theta_2) = \{x, y\}$.

Consider the following mechanism: The message sets are $M_1 = M_2 = \{a_1, a_2\}$ and the outcome function $g : M_1 \times M_2 \rightarrow X$ is as given Table 5.

Observe that at θ_1, we have $\mathcal{K}(\theta_1) = \{(xy, xy), (xy, yx), (yx, xy), (yx, yx)\}$. At $\omega = (xy, xy)$ the NE are (a_1, a_1) and (a_2, a_2); at $\omega = (xy, yx)$ the NE are (a_1, a_2) and (a_2, a_2);

		Individual 2	
		a_1	a_2
Individual 1	a_1	x	y
	a_2	y	y

Table 5: The mechanism used in the proof of Proposition 3.

at $\omega = (yx, xy)$ the NE are (a_2, a_1) and (a_2, a_2); and finally, at $\omega = (yx, yx)$ the NE are (a_1, a_2), (a_2, a_1), and (a_2, a_2). Since (a_2, a_2) is the only NE of μ at every $\omega \in \mathcal{K}(\theta_1)$, if ς^* is a PECE of μ, then at θ_1 is $\varsigma^*_{i\theta_1} = a_2$. Moreover, at θ_2, $\mathcal{K}(\theta_2) = \{(xy, xy)\}$, and when $\omega = (xy, xy)$, (a_1, a_1) and (a_2, a_2) constitute the NE of μ. Thus, if ς^* is a PECE of μ, then at θ_2, $\varsigma^*_{\theta_2} \in \{(a_1, a_1), (a_2, a_2)\}$. Ergo, there are two PECEs of μ given by $\varsigma^{(1)}$ and $\varsigma^{(2)}$: $\varsigma^{(1)}$ is defined by $\varsigma^{(1)}_{i\theta_1} = \varsigma^{(1)}_{i\theta_2} = a_2$ for all $i = 1, 2$; $\varsigma^{(2)}$ is defined by $\varsigma^{(2)}_{i\theta_1} = a_2$ and $\varsigma^{(2)}_{i\theta_2} = a_1$ for all $i = 1, 2$. As a result, we observe that $\varsigma^{(1)}$ is a PECE such that $g(\varsigma^{(1)}_{\theta_1}) = g(\varsigma^{(1)}_{\theta_2}) = y$, and $\varsigma^{(2)}$ is a PECE such that $g(\varsigma^{(2)}_{\theta_1}) = y$ and $g(\varsigma^{(2)}_{\theta_2}) = x$. Therefore, μ implements f in PECE as (i) and (ii) of Definition 8 hold.

Notwithstanding, there is an ECE that does not conform to the requirements of (ii) of Definition 7: Let σ^* be defined by $\sigma^*_{i\theta_1}(xy) = \sigma^*_{i\theta_2}(xy) = a_1$ and $\sigma^*_{i\theta_1}(yx) = a_2$ for all $i = 1, 2$. If player 2's payoff type at θ_1 equals xy, then $\sigma^*_{2\theta_1}(xy) = a_1$ and hence $O^\mu_1(a_1) = \{x, y\}$; so, individual 1 chooses a_1 if his payoff type is xy and a_2 if it is yx, as is specified under $\sigma^*_{1\theta_1}$. Similarly, when individual 2's payoff type at θ_1 is fixed to yx, then $\sigma^*_{2\theta_1}(yx) = a_2$ and so $O^\mu_1(a_2) = \{y\}$; so, individual 1 choosing as specified under $\sigma^*_{1\theta_1}$ is a best reply. Further, at θ_2, player 2's payoff type is xy and $\sigma^*_{2\theta_2} = a_1$ so $O^\mu_1(a_1) = \{x, y\}$. As player 1's payoff type at θ_2 equals xy, $\sigma^*_{1\theta_2}(xy) = a_1$ is a best reply. Thanks to symmetry, these establish that σ^* is a 'bad' ECE such that $g(\sigma^*_{\theta_1}(xy, xy)) = x$, while $(xy, xy) \in \mathcal{K}(\theta_1)$ but $x \notin f(\theta_1) = \{y\}$ which violates (ii) of Definition 7. ■

Implementability in PECE does not prevent 'bad' ECE that the planer wishes to dismiss as the example used in the proof of Proposition 3 displays. The dismissal of bad ECE can be attained by the use of secure implementation in PECE and ECE as follows:[20] Given

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[20] Our intuition resembles that of Saijo et al. (2007) that considers "double implementation in Nash equilibrium and in dominant strategies."

$\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is *securely implementable in public ex-post correlated equilibrium* by a mechanism $\mu = (M, g)$ if (i) of Definition 8 and (ii) of Definition 7 hold.

The dismissal of unwanted ECE via secure implementability in PECE compels the planner to consider individuals' private information. As we aim to obtain implementation by using only the public information, we propose an alternative way to eliminate undesirable ECE. To that regard, we employ a variation of secure implementability in PECE: Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, we say that an SCC $f : \Theta \to \mathcal{X}$ is *protectedly implementable in public ex-post correlated equilibrium* by a mechanism $\mu = (M, g)$ if (i) of Definition 8 and the following holds: (ii) if $\sigma^* \in \Sigma$ is an ECE of μ, then for all $\theta \in \Theta$ and all $\omega \in \mathcal{K}(\theta)$, $g(\sigma^*_\theta(\omega)) \in f(\theta)$. Observe that every ECE of a mechanism μ, $\sigma^* \in \Sigma$, induces an NE strategy profile at every $\omega \in \mathcal{K}(\theta)$ for any given $\theta \in \Theta$; i.e., σ^* is such that for all $\theta \in \Theta$, $g(\sigma^*_\theta(\omega)) \in \cap_{i \in N} C^\omega_i(O^\mu_i(\sigma^*_{-i\theta}(\omega_{-i})))$ for all $\omega \in \mathcal{K}(\theta)$. Therefore, dismissing 'bad' NE ensures the elimination of unwanted ECE as specified in (ii) of protected implementability in PECE. This leads us to the following notion of implementation: Given $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is ***safely implementable in public ex-post correlated equilibrium*** by $\mu = (M, g)$ if (i) of Definition 8 and the following hold: (ii) if $m^* \in M$ and $\theta \in \Theta$ are such that $g(m^*) \in \bigcap_{i \in N} C^\omega_i(O^\mu_i(m^*_{-i}))$ for some $\omega \in \mathcal{K}(\theta)$, then $g(m^*) \in f(\theta)$. This notion ensures that implementation is based only on public information and prevents the emergence of undesirable ECE that may arise due to individuals' coordination using their private information. The following summarizes our motivation for safe implementability in RNE:

Remark 2. *Given an inference correspondence $\mathcal{K} : \Theta \to \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is safely implementable in RNE if and only if it is safely implementable in PECE.*

5.2 Bayes Correlated Equilibrium

In order to discuss the Bayesian setup, we consider the following probabilistic setting: For each state of the economy $\theta \in \Theta$, and for each payoff state compatible with θ, $\omega \in \mathcal{K}(\theta)$, individual i's preferences admit a conditional expected utility representation via the expected utility function $u_{i\theta}(\,\cdot \mid \omega_i) : X \to \mathbb{R}$. Moreover, for each state of the economy $\theta \in \Theta$, individual i's *belief* at his payoff type $\omega_i \in \mathcal{K}_i(\theta)$ is given by $p_{i\theta}(\omega_i) \in \Delta(\mathcal{K}_{-i}(\theta))$, where $\Delta(\mathcal{K}_{-i}(\theta))$ denotes the probability simplex on $\mathcal{K}_{-i}(\theta)$. The belief profile is given by $\mathbf{p} := (p_{i\theta}(\omega_i))_{i \in N,\ \theta \in \Theta,\ \omega_i \in \mathcal{K}_i(\theta)}$.

Definition 9. *Given a mechanism $\mu = (M, g)$, the inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, and the belief profile $\mathbf{p}$, the correlated strategy profile $\sigma^* \equiv (\sigma^*_{i\theta})_{i \in N,\ \theta \in \Theta} \in \Sigma$ is a* ***Bayes correlated equilibrium*** *(BCE) of μ if for all $i \in N$, for all $\theta \in \Theta$, and for all $\omega_i \in \mathcal{K}_i(\theta)$,*

$$\textstyle\sum_{\omega_{-i} \in \mathcal{K}_{-i}(\theta)} p_{i\theta}(\omega_{-i}|\omega_i) \ \left[u_{i\theta}\left(g(\sigma^*_{i\theta}(\omega_i), \sigma^*_{-i\theta}(\omega_{-i})) \mid \omega_i\right) - u_{i\theta}\left(g(m_i, \sigma^*_{-i\theta}(\omega_{-i})) \mid \omega_i\right)\right] \geq 0,$$

for all $m_i \in M_i$.

If we were to consider public strategies in the Bayesian framework, a correlated public strategy profile $\varsigma^* \in \Sigma^P$ is a *public Bayes correlated equilibrium* (PBCE) of μ if for all $i \in N$, all $\theta \in \Theta$, and all $\omega_i \in \mathcal{K}_i(\theta)$, $0 \leq \sum_{\omega_{-i} \in \mathcal{K}_{-i}(\theta)} p_{i\theta}(\omega_{-i}|\omega_i) \ \left[u_{i\theta}\left(g(\varsigma^*_\theta)|\omega_i\right) - u_{i\theta}\left(g(m_i, \varsigma^*_{-i\theta})|\omega_i\right)\right] = u_{i\theta}\left(g(\varsigma^*_\theta)|\omega_i\right) - u_{i\theta}\left(g(m_i, \varsigma^*_{-i\theta})|\omega_i\right)$, for all $m_i \in M_i$. Thus, the PBCE is equivalent to the PECE.

Since any RNE profile is equivalent to a PECE, the equivalence of PBCE and PECE delivers further robustness properties for RNE as every RNE profile induces a PBCE and hence a BCE.

The concept of BCE delivers the following full implementation notion: Recall that a CSCS associated with a given SCC $f : \Theta \to \mathcal{X}$ is $\Phi_f := (\Phi_{f,\theta})_{\theta \in \Theta}$ with $\Phi_{f,\theta}$ is a non-empty subset of $\{\varphi_\theta \mid \varphi_\theta : \mathcal{K}(\theta) \to f(\theta)\}$ for all $\theta \in \Theta$.

Definition 10. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, the belief profile* $\mathbf{p}$*, and an an SCC $f : \Theta \to \mathcal{X}$ we say that a CSCS Φ_f associated with f is* ***implementable in Bayes correlated equilibrium*** *by a mechanism $\mu = (M, g)$ if*

(*i*) *for all $\theta \in \Theta$ and all $\varphi_\theta \in \Phi_{f,\theta}$, there exists a BCE $\sigma^{(\varphi_\theta)} \in \Sigma$ with $g(\sigma^{(\varphi_\theta)}_\theta(\omega)) = \varphi_\theta(\omega)$ for all $\omega \in \mathcal{K}(\theta)$; and*

(*ii*) *if $\sigma^* \in \Sigma$ is a BCE of μ, then for all $\theta \in \Theta$, there exists $\varphi \in \Phi_{f,\theta}$ such that $g(\sigma^*_\theta(\omega)) = \varphi(\omega)$ for all $\omega \in \mathcal{K}(\theta)$.*

Recall that $\bar{\Phi}_f$ denotes the unique CSCS that satisfies the reliability criterion associated with the given SCC f (i.e., $\bar{\Phi}_f = (\bar{\Phi}_{f,\theta})_{\theta \in \Theta}$ s.t. $\bar{\Phi}_{f,\theta} := \cup_{x \in f(\theta)} \{\bar{\varphi}_{(\theta,x)}\}$ where $\bar{\varphi}_{(\theta,x)} : \mathcal{K}(\theta) \to$

$f(\theta)$ is defined by $\bar{\varphi}_{(\theta,x)}(\omega) = x$ for all $\omega \in \mathcal{K}(\theta)$). So, when attention is restricted to the implementation of CSCS $\bar{\Phi}_f$ in BCE, (*i*) of Definition 10 becomes for all $\theta \in \Theta$ and all $x \in f(\theta)$, there is a BCE $\sigma^{(x,\theta)} \in \Sigma$ with $g(\sigma_\theta^{(x,\theta)}(\omega)) = x$ for all $\omega \in \mathcal{K}(\theta)$, while (*ii*) of the same definition turns into the following: if $\sigma^* \in \Sigma$ is a BCE of μ, then for all $\theta \in \Theta$, there exists $y \in f(\theta)$ such that $g(\sigma_\theta^*(\omega)) = y$ for all $\omega \in \mathcal{K}(\theta)$.

Consequently, implementation in BCE (of a CSCS satisfying the reliability criterion associated with an SCC f) shares many similarities with implementation in ECE (of the same SCC f). Both involve opportunity sets which the planner needs to identify considering the private information of each individual pertaining to his assessment about others' payoff types.

Meanwhile, with implementation in RNE, the planner achieves her goal without the need for detailed examinations and inferences about individuals' private information.

6 Conclusion

We consider an incomplete information setting in which the planner and the individuals have access to incomplete public choice data. Hence, the planner as well as the individuals are partially informed about how to associate states of the economy, on which the SCC is defined, with payoff states (individuals' underlying preferences). We propose suitable notions of implementation in this setting and identify associated necessary conditions, which we use to establish the following observation: More information induces richer implementation opportunities. We also show that our necessary conditions are sufficient in economic environments. We use these to analyze the implementability of efficiency notions suited to this setting.

As a final remark, we note that our results can be extended to behavioral domains in which individuals' choices are not necessarily derived from preference maximization.

Appendix

A Relation to Maskin Monotonicity

We extend Maskin monotonicity to our environment as follows.

Definition 11. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$, an SCC $f : \Theta \to \mathcal{X}$ is*

- (i) **reliably Maskin monotonic** *if $x \in f(\theta)$ and $L_i^{\omega}(x) \subseteq L_i^{\tilde{\omega}}(x)$ for all $i \in N$, all $\omega \in \mathcal{K}(\theta)$, and all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ implies $x \in f(\tilde{\theta})$.*
- (ii) **safely Maskin monotonic***, if the following holds: if $x \in f(\theta)$ and for some $\omega \in \mathcal{K}(\theta)$ and some $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ we have $L_i^{\omega}(x) \subseteq L_i^{\tilde{\omega}}(x)$ for all $i \in N$, then $x \in f(\tilde{\theta})$.*

The following proposition displays the relationship between our necessary conditions and Maskin monotonicity:

Proposition 4. *Given an inference correspondence $\mathcal{K} : \Theta \twoheadrightarrow \Omega$ and an SCC $f : \Theta \to \mathcal{X}$, there is a profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ that is*

- (i) *reliably-consistent with f if and only if f is reliably Maskin monotonic.*
- (ii) *safely-consistent with f if and only if f is safely Maskin monotonic.*

Proof of Proposition 4. For the necessity of (i), the existence of a reliably-consistent profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ implies that if $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$, there is $j \in N$ and $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $x \notin C_j^{\tilde{\omega}}(S_j(x, \theta))$. Hence, $S_j(x, \theta)$ is not a subset of $L_j^{\tilde{\omega}}(x)$. But, $S_j(x, \theta) \subset L_j^{\omega}(x)$ with $x \in S_j(x, \theta)$ for all $\omega \in \mathcal{K}(\theta)$ due to (i) of reliable-consistency. Thus, f is reliably Maskin monotonic. For sufficiency of (i), define the profile of sets $\mathbf{S} := (S_i(x, \theta))_{i \in N,\ \theta \in \Theta,\ x \in f(\theta)}$ as follows: for any $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$, let $S_i(x, \theta) := \bigcap_{\omega \in \mathcal{K}(\theta)} L_i^{\omega}(x)$. Then, for all $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$, we have $x \in \bigcap_{\omega \in \mathcal{K}(\theta)} C_i^{\omega}(S_i(x, \theta))$; establishing (i) of reliable-consistency. For (ii) of reliable-consistency, suppose $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$. Then, by reliable Maskin monotonicity, there is $j \in N$ and $\omega \in \mathcal{K}(\theta)$ and $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ such that $L_j^{\omega}(x) \not\subset L_j^{\tilde{\omega}}(x)$. As $S_j(x, \theta) = \bigcap_{\omega' \in \mathcal{K}(\theta)} L_j^{\omega'}(x)$, we conclude that $S_j(x, \theta) \not\subset L_j^{\tilde{\omega}}(x)$ implying $x \notin C_j^{\tilde{\omega}}(S_j, \theta)$. Thus, $\mathbf{S}$ is reliably-consistent with f.

The necessity of (*ii*) implies that there is a safely-consistent $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ such that if $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$, then for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$, there is $j_{\tilde{\omega}} \in N$ with $x \notin C^{\tilde{\omega}}_{j_{\tilde{\omega}}}(S_{j_{\tilde{\omega}}}(x,\theta))$. Hence, for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$, $S_{j_{\tilde{\omega}}}(x,\theta)$ is not a subset of $L^{\tilde{\omega}}_{j_{\tilde{\omega}}}(x)$. But, $S_{j_{\tilde{\omega}}}(x,\theta) \subset \bigcap_{\omega\in\mathcal{K}(\theta)} L^{\omega}_{j_{\tilde{\omega}}}(x)$ with $x \in S_{j_{\tilde{\omega}}}(x,\theta)$ due to (*i*) of safe-consistency. Thus, if $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$, then we observe that for all $\omega \in \mathcal{K}(\theta)$ and all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$, there is $j_{\tilde{\omega}} \in N$ such that $L^{\omega}_{j_{\tilde{\omega}}}(x) \not\subset L^{\tilde{\omega}}_{j_{\tilde{\omega}}}(x)$. Therefore, f is safely Maskin monotonic. For sufficiency of (*ii*), define the profile of sets $\mathbf{S} := (S_i(x,\theta))_{i\in N,\ \theta\in\Theta,\ x\in f(\theta)}$ as follows: for any $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$, let $S_i(x,\theta) := \bigcap_{\omega\in\mathcal{K}(\theta)} L^{\omega}_i(x)$. Then, for all $i \in N$, $\theta \in \Theta$, and $x \in f(\theta)$, we have $x \in \bigcap_{\omega\in\mathcal{K}(\theta)} C^{\omega}_i(S_i(x,\theta))$; establishing (*i*) of safe-consistency. For (*ii*) of safe-consistency, suppose $x \in f(\theta)$ and $x \notin f(\tilde{\theta})$. Then, by safe Maskin monotonicity, for all $\omega \in \mathcal{K}(\theta)$ and all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ there is $j \in N$ such that $L^{\omega}_j(x) \not\subset L^{\tilde{\omega}}_j(x)$. As $S_j(x,\theta) = \bigcap_{\omega'\in\mathcal{K}(\theta)} L^{\omega'}_j(x)$, we observe that for all $\tilde{\omega} \in \mathcal{K}(\tilde{\theta})$ there is $j \in N$ such that $S_j(x,\theta) \not\subset L^{\tilde{\omega}}_j(x)$, which implies $x \notin C^{\tilde{\omega}}_j(S_j(x,\theta))$. So, $\mathbf{S}$ is safely-consistent with f. ■

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