A Tale of Two Debts: Student Loans, Credit Card and Young American Borrowers*

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Abstract

We document that young Americans with student loans are more likely to borrow on credit cards, and their credit card debt tends to be larger and more expensive. Furthermore, while credit card default increases with both types of debt, taking on more credit card loans does not necessarily lead to higher student loan default. We propose a theory that captures the institutional differences of the two credit markets, risk-based pricing of loans and differential default consequences, and demonstrate that these differences help explain the observed behavior. Different from the existing theories of unsecured credit but consistent with the data, our theory delivers a credit card interest rate that is less sensitive to the credit card loan amount, once we allow for student loan borrowing and default to impact on credit card interest rates. We then calibrate the model to the data and quantify the contribution of loan pricing and default consequences to default incentives for the two types of debt across different borrowers. We show that, in such an economy, forgiveness of student loans, modeled via income-driven repayments, induces important redistributional effects but has a net negligible aggregate effect. However, the policy delivers large welfare effects across the board when the economy faces high income risk and tight credit markets.

JEL Codes: D91; I22; G19;

Keywords: Default, Bankruptcy, Student Loans, Credit Cards, Loan Forgiveness

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1 Introduction

Student loans and credit card debt are two major financial liabilities of young Americans. About seventy percent of individuals who enroll in college take out student loans.[1] Over sixty percent of borrowers with student loans also use credit cards.[2] Over the last decade, both student loan and credit card debt have been increasing steadily with student loan balances increasing much faster than credit card debt balances and surpassing them in 2010.[3] Despite similar monthly financial burdens associated with the two types of loans, young U.S. households default at a higher frequency on student debt than on credit card debt. As such, while default rates for both types of loans have been trending up, the increase in student loan default rates has been faster than that for credit card debt default rates.[4] This may be somewhat puzzling given that interest rates on credit card loans are higher on average than student loan interest rates and that credit card debt can be discharged under personal bankruptcy (Chapter 7, "Liquidation"), whereas student loans cannot be discharged except in extreme circumstances under Chapter 13, "Reorganization".[5]

To understand these borrowing and default patterns, we first turn to individual level micro data. Using the Survey of Consumer Finances data, we document key facts about the interaction between the two types of credit and default behavior for young U.S. individuals with student loans. First, student debt is positively correlated with participation in the credit card market and with credit card debt levels. Second, credit card interest rates increase in both student and credit card debt. Lastly, default on credit card debt increases in the size of student loan debt and in credit card debt. In contrast, default on student debt presents a hump-shaped profile in levels of credit card debt while increasing in student loan levels.

We propose a quantitative theory of unsecured credit and default of young U.S. agents with student and credit card debt that accounts for key institutional differences of the two markets: loan pricing and consequences associated with default. Our model features infinitely lived individuals who differ in their student loan balances and income levels. They face uncertainty in income

[1] "Quick Facts About Student Debt," the Institute for College Access and Success, March 2014. https://ticas.org/sites/default/files/pub_files/Debt_Facts_and_Sources.pdf.

[2] Survey of Consumer Finances 2004.

[3] Federal Reserve Board, G19, and the FRBNY Consumer Credit Panel/Equifax.

[4] In the first quarter of 2018, the 90+ days delinquency rate for student loans was nearly 12 percent and far exceeds the delinquency rate for credit card debt. See "agent Debt and Credit Report (Q1 2018)," Federal Reserve Bank of New York.

[5] As a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work. According to Education Credit Management Corporation, which serviced loans for twenty-five lending agencies and the United States Department of Education; in 2008 it was reported that of 72,000 loans in bankruptcy proceedings, only 276 debtors attempted discharge, and by November 2009 of the 134 resolutions, 29 resulted in total or partial discharge.

and make consumption, savings, and payment decisions. As standard in the literature, credit card interest rates are endogenously determined and depend on the likelihood that borrowers may default on their credit card debt (as in Chatterjee et al. (2007)). A key addition to this line of work is that in our model, credit card default risk is determined in equilibrium not only by the size of the credit card loan but also by the size of student debt and default on student loans. As in practice, the interest rate in the student loan market does not account for the risk that some borrowers may default. Lastly, we model consequences of defaulting on student loans and credit card debt to mimic the current environment as follows. For student loans, they include a wage garnishment while for credit card debt, they consist of exclusion from borrowing for several years. Importantly, credit card debt can be discharged in bankruptcy, whereas student loans cannot be discharged. We characterize borrowing and default behavior and demonstrate that these differences in contractual arrangements and the interactions between the two types of credit are key in explaining observed borrowing and default behavior.

The first contribution of the paper is that we demonstrate, both theoretically and quantitatively, that default risk and thus interest rates on credit card loans nontrivially depend on borrowing and default on student loans. Our first main theoretical result shows that credit card interest rates increase with both student and credit card debt as well as in the default status on student loans. As a corollary, our quantitative analysis delivers that, in line with the data, credit card interest rates are less sensitive to credit card loans and to default risk in credit card markets compared to predictions of standard theories of unsecured credit.[6]

Our second contribution is to demonstrate that differences in market arrangements successfully explain observed default patterns inducing a higher incentive to default on student loans than on credit card loans, all else equal. Our second main theoretical result shows that a borrower with high enough student loan debt and credit card debt will choose to default in the student loan market rather than in the credit card market. Quantitatively, while default on student debt increases in the amount of the student loan, default on student loans is hump-shaped in credit card debt. Individuals with high credit card debt levels are typically individuals with low risk, on average, who face better terms on their credit card accounts, whereas individuals with low credit card debt levels are individuals with high risk, on average, who face worse terms on their credit card accounts, an equilibrium result. Individuals are incentivised to default as they accumulate more (and more expensive) credit card debt. However, borrowers may also want to use their credit card debt to repay student loans. We characterize this relationship and show how this trade-off crucially depends on the pricing rules and default consequences in the two credit markets and debt

[6] Using administrative data, Dempsey and Ionescu (2023) document that the slope of interest rate spreads on credit card loans with respect to default probability is much smaller than such standard models predict.

portfolio allocations.

On a methodological level, our paper makes advancements to theories of unsecured credit with default along two dimensions. First, we show how the existence and uniqueness of the agent's problem is achieved in a general equilibrium setup with two endogenous default decisions and characterize such decisions in terms of agent characteristics and market arrangements. The challenge in this respect relates to the non-trivial market clearing conditions, which include a menu of loan prices and importantly, the interaction between the two types of credit. Second, we characterize the trade-offs that differences in default rules between the two types of debt induce and characterize the role they play in shifting default incentives.

Lastly, we explore the policy implications of our model and study the impact of different student loan repayments. Specifically we consider income driven repayment plans that allow for student loan forgiveness. We find that the policy induces significant redistributional effects with poor borrowers with large levels of student loans benefiting the most and middle earners losing the most from this policy. Middle earners choose to default the most under the standard repayment scheme. The net aggregate welfare effect, however, is negligible. Nevertheless, the policy can induce substantial benefits across the entire borrower population in an economy calibrated to feature tight credit markets and high income risk. The large welfare effects are primarily driven by the adverse impact that student loan borrowing and default have on credit card default risk. Put simply, loan forgiveness significantly lowers credit card default risk, a benefit that is highest in an economy with high income and credit risk.

Our findings suggest that partial student loan forgiveness is important in tough times when individuals face stringent terms on their credit card accounts and high income risk. This is particularly important in the recent decade when, due to significant increases in college costs, students borrow more than ever in both student loan and credit card markets. Importantly, our research reveals the importance of accounting for the interactions between student loan and credit card markets when studying borrowing and default behavior in unsecured credit and related policies.

Related literature The novelty of our work lies in its study of two credit markets with distinct financial arrangements, the student loan market and the credit card market. In doing so, our paper relates to two broad strands of the existing literature.

The first strand focuses solely on credit card debt default and personal bankruptcy. This literature includes Athreya et al. (2009), Chatterjee et al. (2007), Chatterjee et al. (2022), Li and Sarte (2006), and Livshits et al. (2007). The first two studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7. Chatterjee et al. (2022) provide a theory that explores the importance of credit scores for consumer credit in an environment with limited information and show how dynamic reputation can incentivize debt repayment. Li and Sarte (2006) model both Chapter 7 and Chapter 13 bankruptcy filing upon credit default. Livshits et al. (2007) quantitatively compare liquidation in the U.S. to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

The second strand related to our paper is student loan research with important recent contributions studying the role of student debt for household decisions, including financial investments, human capital accumulation, labor supply, or investment in housing.[7] Within this line of work, our paper is most closely related to work that focused on student loan default and related policies and their impact on young borrowers (See, for example, Ionescu (2011), Lochner and Monge (2011), Ionescu and Simpson (2016), and Lochner et al. (2021)). Ionescu (2011) studies the welfare consequences of allowing for (partial) discharge of student loans in personal bankruptcy. Lochner and Monge (2011) develop a human capital model with government student loan programs and private lending under limited commitment to explain the strong correlation between ability and schooling. Ionescu and Simpson (2016) also study the interactions between the government student loan market and the private student loan market, and their implications for higher education policies. Lochner et al. (2021) study how parental support impacts student loan borrowing and payment decisions.

Our paper bridges these two lines of work by studying both student loan and credit card debt borrowing and default. On a methodological level, our paper builds on Chatterjee et al. (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In Chatterjee et al. (2007), individual probabilities of credit card debt default are linked only to the size of the credit card loan. We take a step further and allow credit card debt default probabilities to also depend on student loan debt and its default status. As a result, credit card interest rates responds to changes in default incentives induced by different default arrangements in the two markets. We demonstrate that this is key in our economy to deliver a relatively less sensitive pricing schedule to credit card default risk, result consistent with empirical findings in Dempsey and Ionescu (2023) and in stark contrast to results implied by standard theories of unsecured credit. In this direction, our paper relates to recent work that has examined the role of pricing and market features to explain key trends in credit card markets. For example, Herkenhoff and Raveendranathan (2019), while still focusing on credit card borrowing and default, depart from the standard pricing paradigm via long-term credit arrangements and yield pricing relationships that are relatively flat with respect to default risk, consistent with our results.

[7]For an extensive literature review on student loans research see Athreya et al. (2022).

Lastly, in the respect that we study the interaction of two financial markets, our paper is closely related to Mitmann (2016), who develops a general equilibrium model of housing and unsecured debt. As in our paper, Mitmann (2016) allows for default in both mortgages and student loans and analyzes the effects of bankruptcy and foreclosure policies. Apart from modeling different types of credit, our research is different from Mitmann (2016)'s work in several ways. First, our paper focuses on the interplay between two types of unsecured credit that feature dischargeability versus non-dischargeability of loans and pricing rules that incorporate or not the likelihood that some borrowers may default. As such, the tradeoffs we uncover in equilibrium and their implications for borrowing and default are quite different. Second, we focus our policy analysis on forgiveness and repayment arrangements specific to young borrowers with student debt.[8]

The rest of the paper is organized as follows. In section 2, we document the key facts regarding the interaction between borrowing, pricing and default in the student loan and credit card markets. In section 3, we develop our theoretical framework and present the analytical results. Section 4 provides our quantitative analysis where we map our theory to the data, discuss the quantitative predictions and the policy implications of our theory. Section 5 concludes.

2 Empirical Analysis

2.1 Data Source

Our main data come from the Survey of Consumer Finances (SCF). The SCF is normally a triennial cross-sectional survey of U.S. families. The survey data include information on households' balance sheets, pensions, income, and other demographic characteristics of families. The survey also gathers other financial information such as interest rates borrowers pay on their main credit card.[9] For this paper, we will use the full data of SCF from 2001 to 2019. Unless otherwise specified, we define young households as those headed by a person between the age of 22 and 40. The choice of age 22 is to ensure that the individual has a reasonable chance to have finished college. The choice of ending age 40 is to ensure that the individual doesn't have college-age children yet so that we can attribute the student loans to his own education instead of his children's.

[8] In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers, which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

[9] The data are available at https://www.federalreserve.gov/econres/scfindex.htm.

2.2 Empirical Results: Debt Holdings and Interest Rates Charged

Young Americans Are More Likely to Hold Student Loans and Credit Card Debt.

We start by examining various debt holdings by households over the life cycle using SCF 2001 to 2019. For the purpose of this paper, we focus on student loans, credit card debt, and residential mortgages collateralized by primary residences.[10] Figure A1 presents the binned scatter plots of the fraction of households who owe these debts as well as the share of the various debt in value by age, where we controlled for year fixed effects. As can be seen, young households, particularly those under the age of 35, are much more likely to owe student loans and credit card debt, but much less likely to owe residential mortgages than their older counterparts. In terms of the value share of the total debt, for young households, student loans far exceed credit card debt and, for the most part, mortgages as well, though, not surprisingly, mortgages catch up rather quickly as households age.

Young Americans with Student Loans Are More Likely to Hold Credit Card Debt and Charged Higher Credit Card Rates

We now turn to study young households between the age of 22 and 40 who have had some college education,[11] positive student loans, and positive income, but do not own homes. The requirement of positive wage and some college education is to ensure that the individuals are out of school and working. The exclusion of homeowners is to better focus on the interaction of the two forms unsecured debt, student loan and credit card debt. The interest rate reported by the SCF is the interest rate borrowers pay on the card with the largest balance. We classify an individual as filed for bankruptcy if he filed within the last year. For student loan payment status, for surveys between 2001 and 2013, a person is delinquent on his student loan payments if he isn't making student loan payments, student loans are not deferred or in grace period. Surveys in 2016 and 2019 collected more information on student loans than earlier years, which allow us to add additional criteria, that is, the person is not making student loan payment for financial reasons; the student loans are for self education; the person is not currently enrolled in school; and the person is not in any income-based payment plan. The thus constructed sample contains 6,873 observations; 58 percent of them are of age 30 or younger, and 86 percent of them are of age 35 or younger.

Table 1a presents additional summary statistics. In our sample, nearly half have college degree,

[10] We do not study auto loans in this paper as autos are often complements to work, i.e., people need cars for commuting. Not surprisingly, young people are also more likely to owe auto loans.

[11] We exclude those with doctoral or professional degrees from our study as these group people behave very differently from those who borrowed for undergraduate or mater degrees.

Table 1: Summary Statistics of Young American Households

variable	Mean	Median	S.d.
Age	29.25	28.00	5.05
College graduate	0.49	0.00	0.50
Male	0.61	1.00	0.49
Married	0.41	0.00	0.49
Number of kids	0.68	0.00	1.08
Income (2019, $000)	54.97	45.24	49.94
Student loan (2019, $000)	35.40	22.00	43.85
Fraction with credit card balance (%)	0.57	1.00	0.50
Credit Card Debt (2019, $000)	2.80	0.30	5.62
Interest paid on credit card	15.40	15.49	6.45
Filed for bankruptcy within 1 year	0.020	0	0.14
Defaulted on student loans	0.045	0	0.26

Note. This table presents summary statistics of households head by individuals between 22 and 40, with student loans but don't own primary homes. All statistics are weighted. Total number of observattions=6,873. Data source: SCF 2001-2019.

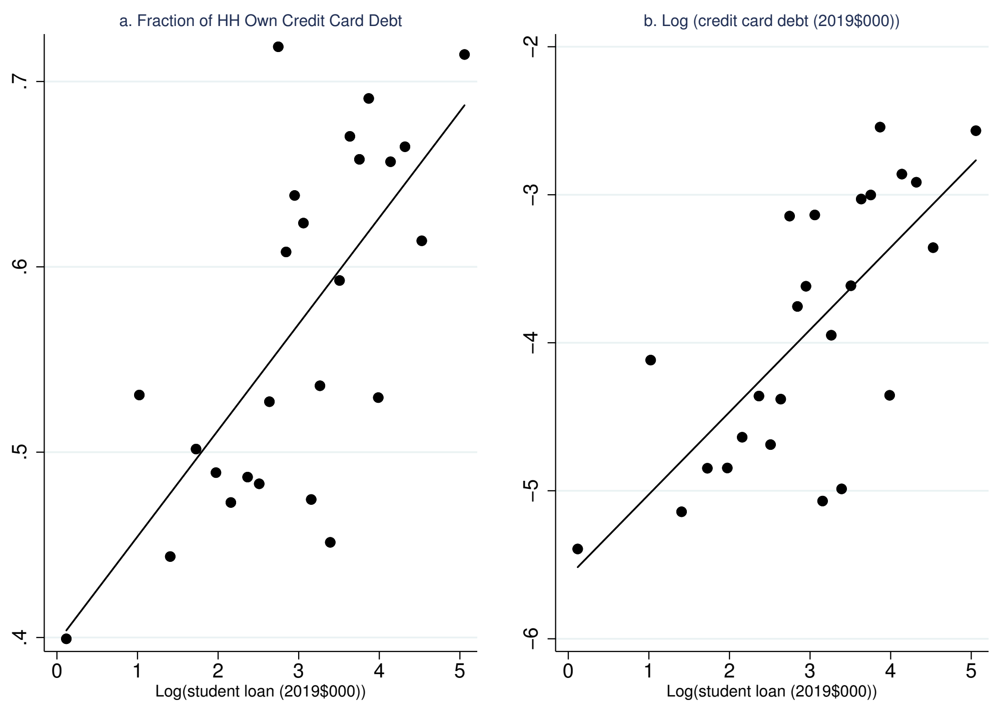
and 60 percent are male. Notably, on average, student loan amounts to over half of the annual income. The median student loan is also less than half of the median income. However, it is important to point out that student loans are positively correlated with income with a correlation coefficient of 0.06. Turning to credit card debt, nearly 60 percent of these young households with student loans also have credit card balance. The credit card balance, however, is unevenly distributed with a mean of $28,000 and a median of $300, both in 2019 dollars. Figure 1a further illustrates this positive relationship between student loan debt and credit card loans with panel a demonstrating that individuals with more student loans are also more likely to owe credit card debt and panel b showing that individuals with more student loans also owe more credit card debt.

Although it is not surprising that individuals pay higher rates on their credit cards if they owe more credit card debt as shown in Figure 1 panel a, what is interesting is that individuals with more student loans also pay higher interest rates on their credit card debt as demonstrated in Figure 1 panel b. The average interest rate on credit card debt is 15 percent.

2.3 Empirical Results: Loan Performance

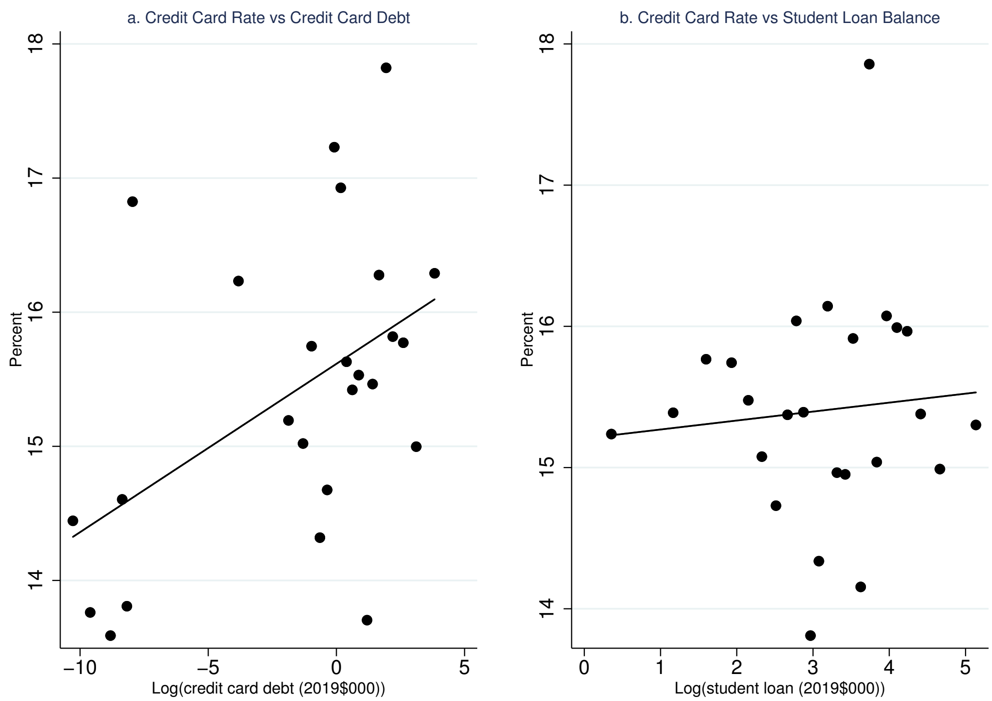
We next examine loan performance by young Americans with student loans of both credit card debt and student loans. According to Table 1a, during our sample period, about 2 percent of the

Figure 1: Credit Card Debt vs Student Loans of Young Households with Student Loans



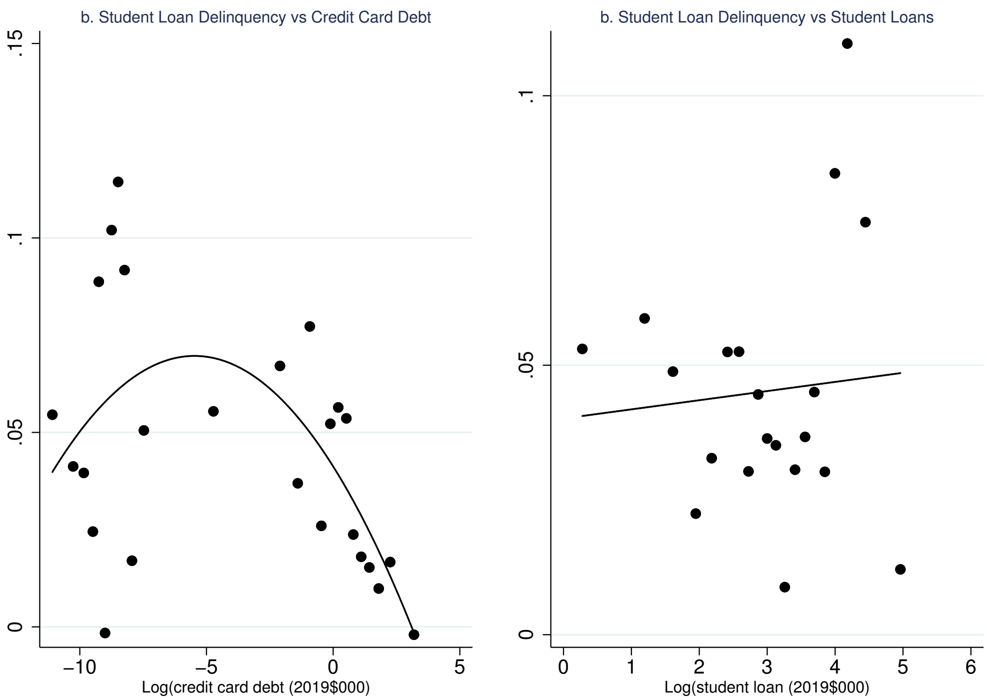
Note: This binned scatter plot depicts credit card debt holdings at the both extensive margin, panel a, and the intensive margin, panel b, with respect to student loans for young American households. Young households are between the age of 22 and 40, with student loans, but don't own homes. Year-fixed effect as well as income is absorbed. Data source: SCF 2007-2019.

Figure 2: Credit Card Interest Rate vs Student Loans of Young Households with Student Loans



Note: This binned scatter plot depicts how credit card interest rates vary with credit card debt balances in panel a and student loans in panel b. Young households are between the age of 22 and 40, with student loans, but don't own homes. Year-fixed effect as well as income is absorbed. Data source: SCF 2007-2019.

Figure 3: Default on Student Loans vs Debt Balances of Young Households with Student Loans

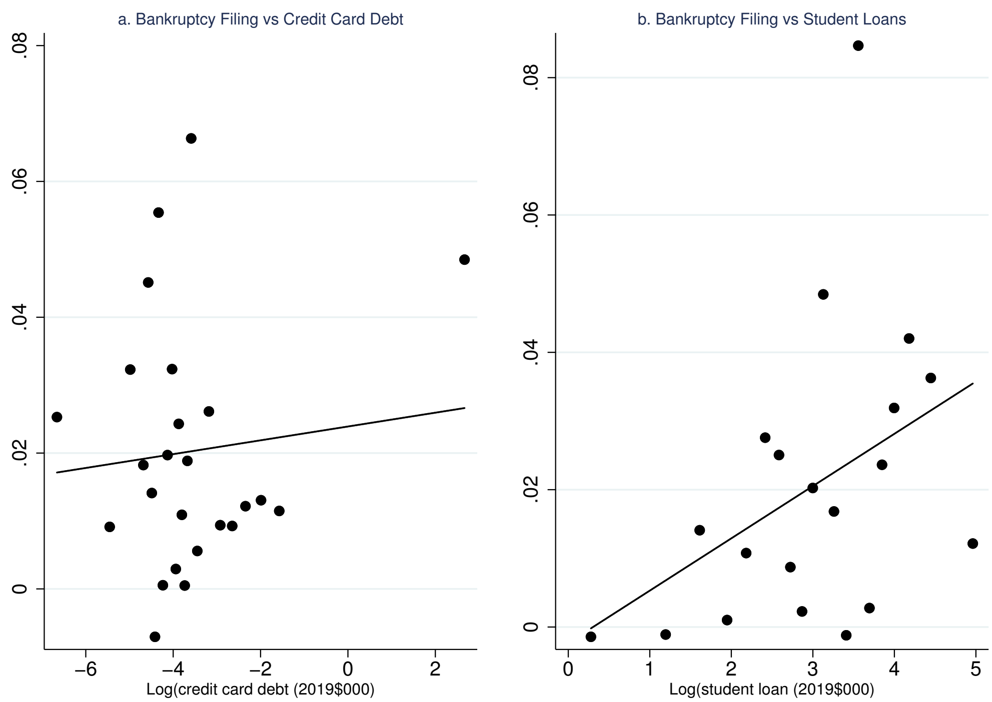


Note: This binned scatter plot depicts how student loan default rates vary with credit card debt balances in panel a and student loans in panel b. Young households are between the age of 22 and 40, with student loans, but don't own homes. Year-fixed effect as well as income and whether the individual finished college are absorbed. Data source: SCF 2001-2019.

households filed for bankruptcy in the survey year and 4.5 percent defaulted on their student loans. The two events are positively correlated with a correlation coefficient of 0.08. Not surprisingly, individuals pay a much higher interest rate on their credit card debt, by about 50 basis points, if they defaulted on their student loans than individuals who were current on their student loan payment.

To explore the relationship between the interaction of loan performance and holdings of different debts, in Figures 3 and 4, we chart how student loan default and bankruptcy filing relate to credit card balance and student loans, respectively. As seen, student loan default rate is hump-shaped with repect to credit card debt, increasing initially with the balance and then decline, while the rate increases monotically with student loan amount. Note that income as well as whether the individual has a college or above degree are controlled for in the construction of the binned scatter chart. By contrast, bankruptcy filing rate increases monotically with credit card debt as well as student loan balances.

Figure 4: Bankruptcy Filing vs Debt Balances of Young Households with Student Loans



Note: This binned scatter plot depicts how bankruptcy filing rates rates vary with credit card debt balances in panel a and student loans in panel b. Young households are between the age of 22 and 40, with student loans, but don't own homes. Year-fixed effect as well as income and whether the individual finished college are absorbed. Data source: SCF 2001-2019.

Summary Our empirical analysis reveals that student debt non trivially affects borrowing in the credit card market at both the extensive and intensive margins. Young adults with student loans are more likely to participate in the credit market and borrow more. Furthermore, student debt matters for pricing of credit card loans with interest rates on credit card loans increasing in student loan balances in addition to the well established increase in credit card debt. Regarding loan performance, we show that bankruptcy filing rate increases monotonically with credit card debt as well as student loan balances. In contrast, student loan default rate is hump-shaped with respect to credit card debt, while it increases monotonically with student loan amount. We next turn to describing a theory that explains these patterns and then use it to examine the importance of contractual arrangements for these patterns and their policy implications.

3 Theoretical Analysis

We develop a tractable infinitely lived agent model where consumers participate in the student loan and credit card markets. Crucial to our analysis, we account for the institutional differences in the two credit markets. We first provide a brief summary of the institutional background. We then proceed with the description of the model and derive the theoretical results. The parsimonious model allows us to characterize analytically agents' decision rules, which in turn help us understand the quantitative version of the model that we develop in the subsequent section to explain the data and to conduct policy analysis.

3.1 Institutional Features

The student loan market and the credit card market differ in many ways, with two important features regarding loan pricing and default consequences.

Interest Rate Determination Once they are out of college, student loan borrowers enter a 10-year repayment plan with fixed payments. The interest rate on student loans is set by the Department of Education and does not incorporate the risk that some borrowers may default. Credit card issuers, by contrast, use consumer repayment and borrowing behavior on all types of loans, including the amount to be borrowed on the credit card itself, to assess the likelihood that a borrower will default and price credit card loans accordingly. The information that credit card issuers use to price their interest rate are summarized by credit scores, either constructed internally or provided by credit bureaus.

Default Consequences Default penalties differ significantly across the student loan market and the credit card market. In particular, defaulting on federal student loans lead to wage garnishment, which can be as high as 15 percent of the defaulter's wage, seizure of federal tax refunds, possible holds on transcripts, and ineligibility for future student loans. More importantly, student loans cannot be discharged under personal bankruptcy except in extreme circumstances under Chapter 13, the "reorganization" bankruptcy. As a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work. Partial dischargeability occurs in less than one percent of the default cases.[12] Once a debtor enters a rehabilitation program for student loans, which typically occurs within a year of default, default status will be erased from the borrowers' records as long as they fulfill the agreed repayment plan. In contrast, credit card debt can be discharged completely under Chapter 7 personal bankruptcy, as well as under Chapter 13 personal bankruptcy. However, bankruptcy filers will have a bankruptcy flag in their credit report and the flag stays on the report for 10 years for a Chapter 7 filing and 7 years for a Chapter 13 filing.

3.2 Preferences and Endowments

The economy is composed of a continuum of infinitely lived agents with unit mass.[13] Agents differ in student loan payment levels, $d \in D = \{d_{\min}, ..., d_{\max}\}$, and income levels, $y \in Y = [y_{\min}, y_{\max}]$. There is a constant probability $(1 - \rho)$ that agents will die at the end of each period. agents that do not survive are replaced by newborns who have not defaulted on student loans $(h = 0)$ or credit cards $(f = 0)$, have zero assets $(b = 0)$, and with labor income and student loan debt drawn independently from the probability measure space $(Y \times D, \mathcal{B}(Y \times D), \psi)$ where $\mathcal{B}(\cdot)$ denotes the Borel sigma algebra and $\psi = \psi_y \times \psi_d$ denotes the joint probability measure. Surviving agents independently draw their labor income at time t from a stochastic process. The amount that the agent needs to pay on her student loan is constant over time.[14] agent characteristics are then defined on the measurable space $(Y \times D, \mathcal{B}(Y \times D))$. The transition function is given by $\Phi(y_{t+1})\delta_{d_t}(d_{t+1})$, where $\Phi(y_t)$ is an i.i.d. process and δ_d is the probability measure supported at d.

[12] According to Education Credit Management Corporation, which serviced loans for twenty-five lending agencies and the United States Department of Education; in 2008 it was reported that of 72,000 loans in bankruptcy proceedings, only 276 debtors attempted discharge, and by November 2009 of the 134 resolutions thus far, 29 resulted in total or partial discharge.

[13] The use of infinitely lived agents is justified by the fact that we focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The use of a continuum of agents is natural, given the size of the credit market.

[14] Federal student loan payments are fixed and computed based on a fixed interest rate and the duration of the loan.

The preferences of the agents are given by:

$$E_0 \sum_{t=0}^{\infty} (\rho\beta)^t U(c_t) \tag{1}$$

where c_t represents the consumption of the agent during period t, $\beta \in (0,1)$ is the discount factor, and $\rho \in (0,1)$ is the survival probability.

Assumption 1. *The utility function $U(\cdot)$ is increasing, concave, and twice differentiable. It also satisfies the Inada condition:* $\lim_{c\to 0^+} U(c) = -\infty$ *and* $\lim_{c\to 0^+} U'(c) = \infty$.

3.3 Market Arrangements

There are several similarities as well as important differences between the credit card market and the student loan market.

3.3.1 The Credit Card Market

The market for privately issued unsecured credit in the U.S. is characterized by a large competitive market in which price-taking lenders issue credit through the purchase of securities backed by repayments from those who borrow. These transactions are intermediated principally by credit card issuers. We model the competitive pricing of default risk that varies with agents characteristics as in Chatterjee et al. (2007).[15] Our model, however, departs from Chatterjee et al. (2007) in several important dimensions: the default risk is based on the borrowing behavior in both markets, the balances in both markets and the payment status in the student loan market. This modeling feature is novel and captures the fact that in practice, the price of the loan depends on past repayment and borrowing behavior in all the markets in which borrowers participate. Unsecured credit card lenders use this behavior (which, in practice, is captured in a credit score) as a signal for agent credit risks and thus their probability of default. They tailor loan prices to individual default risk, not only to individual loan sizes.

An agent can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set $B \subset \mathbb{R}$. The set $B = \{b_{\min}, \ldots, b_{\max}\}$ contains 0 and positive and negative elements. Let N_B be the cardinality of this set. Individuals with $f_t = 1$ (which is a result of defaulting on credit cards in one of the previous periods) are limited in their market participation,

[15] Chatterjee et al. (2007) handle the competitive pricing of default risk by expanding the "asset space" and treating unsecured loans of different sizes for different types of agents (of different characteristics) as distinct financial assets.

$b_{t+1} \geq 0$.[16]

A purchase of a discount bond in period t with a non-negative face value b_{t+1} means that the agent has entered into a contract where it will receive $b_{t+1} \geq 0$ units of the consumption good in period $t+1$. The purchase of a discount bond with a negative face value b_{t+1} means that the agent receives $q_{d_t,h_t,b_{t+1}}(-b_{t+1})$ units of the period-t consumption good and promises to deliver, conditional on not declaring bankruptcy, $-b_{t+1} > 0$ units of the consumption good in period $t+1$; if it declares bankruptcy, the agent delivers nothing. The total number of credit indexes is $N_B \times N_D \times N_H$. Let the entire set of $N_B \times N_D \times N_H$ prices in period t be denoted by the vector $q_t \in \mathbb{R}^{N_B \times N_D \times N_H}$. We restrict q_t to lie in a compact set $Q \equiv [0, q_{\max}]^{N_B \times N_D \times N_H}$ where $0 < q_{\max} < 1$.

3.3.2 The Student Loan Market

Student loans represent a different form of unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share the features of a competitive market.[17] Unlike credit cards, interest rate on student loans, r_g, is set by the government and does not reflect the risk of default in the student loan market.[18] Second, taking out student loans is a decision made during college years. Once agents are out of college, they need to repay their loans in equal amount over a determined period of time. We model college-loan-bound agents that are out of school and need to repay d per period.[19] Third, defaulters cannot discharge their debt. Instead, a wage garnishment is imposed.

We define the state space of credit characteristics of the agents by $\mathcal{S} = B \times F \times H$ to represent the asset position, the credit card, and student loan default flags. Let $N_{\mathcal{S}} = N_B \times 2 \times 2$ be the cardinality of this set.

3.4 Decision Problems

The timing of events in any period is: (i) idiosyncratic income shocks are drawn for survivors and newborns and student loan debt is drawn for newborns; (ii) agents choose to default/repay

[16] Note that agents are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2002).

[17] There exists a private student loan market that is a hybrid between government loans and credit cards, featuring characteristics of both markets. However, this new market is small, representing less than 20 percent of total student loan balance, according to the Institute for College Access & Success. Concerns about the national default rates are specific to student loans in the government program, because default rates for pure private loans are much smaller (for details see Ionescu and Simpson 2010). We focus on Federal student loans in the current study.

[18] Interest rates on Federal student loans are set in statute after the Higher Education Reconciliation Act of 2005 was passed.

[19] While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.

on both credit card and student loans, make borrowing/savings and consumption decisions, and default flags for the next period are determined. We focus on steady state equilibria.

3.4.1 Credit Cards

Bankruptcy for credit cards in the model resembles Chapter 7 "liquidation" bankruptcy. Consider a agent that starts the period with credit card debt b_t, what happens is as follows:

1. If the agent files for bankruptcy, $\lambda_b = 1$, then his credit card debt is discharged.
2. The agent cannot save during the period when default occurs. This is a simple way to model that U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.
3. The agent begins the next period with a record of default on credit cards. Let $f_t \in F = \{0, 1\}$ denote the default flag for a agent in period t, where $f_t = 1$ indicates in period t a record of default and $f_t = 0$ denotes the absence of such a record.
4. A agent who starts the period with a default flag cannot borrow and the default flag can be erased with a probability p_f.
5. A agent who starts the period with $f_t = 0$ is allowed to borrow and save.

This formulation captures the idea that there is restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, precisely the type of borrowers who are more constrained in their capability to repay their loans.

Finally, we assume that defaulters on credit cards are not completely in autarky. In U.S. consumer credit markets, agents retain a storage technology after bankruptcy, namely, the ability to save. We assume that without loss of generality, defaulters cannot borrow. In practice, borrowers who have defaulted in the past several years are still able to obtain limited credit but at much worse terms.

3.4.2 Student Loans

As in practice, default on student loans in the model at period t (denoted by $\lambda_d = 1$) triggers the following consequences:

1. There is no debt repayment in period t. However, the student loan debt is not discharged. The defaulter must repay the amount owed for payment in period $t + 1$.

2. The defaulter is not allowed to borrow or save in period t, which is in line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is restricted.

3. A fraction γ of the defaulter's wages is garnished starting in period $t+1$. Once the defaulter rehabilitates his student loan, the wage garnishment stops.

4. The agent begins the next period with a record of default on student loans. Let $h_t \in H = \{0, 1\}$ denote the default flag for a agent in period t, where $h_t = 1$ indicates a record of default and $h_t = 0$ denotes the absence of such a record.

5. A agent that begins period t with a record of default must pay the debt owed in period t, d_t. The default flag is erased with probability p_h.

6. There are no consequences on credit card market participation during the periods after a default on student loan occurs. However, there are consequences on the pricing of credit card loans from defaulting on student loans. As discussed earlier, this assumption is justified by the fact that, in practice, student loan default is reported to credit bureaus and so creditors can observe the default status immediately after default occurs. However, immediate repayment and rehabilitation of the defaulted loan will result in the removal of the default status reported by the loan holder to the national credit bureaus. In practice, the majority of defaulters enter repayment plans. Therefore, they are still able to access the credit card market albeit on worse terms.

3.4.3 Agents

We characterize the agents' decision problem recursively where a period t variable x_t is denoted by x and its period $t+1$ value by x'. Each period, given their student loan debt, d, current income, y, and beginning-of-period assets, b, agents choose consumption, c, and asset holdings for the next period, b'. In addition, they must decide whether to repay or default on their student loans, $\lambda_d \in \{0, 1\}$ and credit card debt, $\lambda_b \in \{0, 1\}$.

The agent's current budget correspondence, $B_{b,f,h}(d, y; q)$, depends on the exogenously given income, y, student loan debt, d, beginning of period asset position, b, credit card default record, f, student loan default record, h, and the prices in the credit card market, q. It consists of elements of the form $(c, b', h', f', \lambda_d, \lambda_b) \in (0, \infty) \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$ such that

$$c + q_{d,h,b'}b' \leq y(1 - g) - \tau + b(1 - \lambda_b) - d(1 - \lambda_d),$$

where g denotes the wage garnishment rate and τ denotes the lump-sum tax.

Let $v(d, y; q)(b, f, h)$ or $v_{b,f,h}(d, y; q)$ denote the expected lifetime utility of a agent that starts with student loan debt d, earnings y, asset b, credit card default record f, and student loan default record h, and prices q. Then v is in the set $\mathcal{V}$ of all continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_S}$. The agent's optimization problem can be described in terms of an operator $(Tv)(d, y; q)(b, f, h)$ which yields the maximum lifetime utility achievable if the agent's future lifetime utility is assessed according to a given function $v(d, y; q)(b, f, h)$. Let τ_d and τ_b denote utility costs that the agent incurs in case of default in the student loan market and in the credit card market, respectively.[20]

Definition 1. For $v \in \mathcal{V}$, let $(Tv)(d, y; q)(b, f, h)$ be defined as follows:

1. For $h = 0$ and $f = 0$ (good records in both markets)

$$(Tv)(d, y; q)(b, f, h) \quad = \max_{(c,b',h',f',\lambda_d,\lambda_b)\in B_{b,f,h}(d,y;q)} U(c) - \tau_b\lambda_b + \beta\rho \int v_{b',f',h'}(d, y'; q)\Phi(dy')$$

2. For $h = 0$ and $f = 1$ (good record in the student loan market but bad record in the credit card market; $\lambda_b = 0$ and $f' = 1$ with probability $1 - p_f$ and $f' = 0$ with probability p_f)

$$\begin{aligned}(Tv)(d, y; q)(b, f, h) \quad &= \max{}_{B_{b,f,h}(d,y;q)} \quad \Big\{U(c) - \tau_d\lambda_d + (1 - p_f)\beta\rho \int v_{b',1,h'}(d, y'; q)\Phi(dy') \\ &\quad + p_f\beta\rho \int v_{b',0,h'}(d, y'; q)\Phi(dy')\Big\}.\end{aligned}$$

3. For $h = 1$ and $f = 0$ (bad record in the student loan market but good record in the credit card market; $\lambda_d = 0$ and $h' = 1$ with probability $1 - p_h$ and $h' = 0$ with probability p_h)

$$\begin{aligned}(Tv)(d, y; q)(b, f, h) \quad &= \quad \max\Big\{\max_{B_{b,f,h}(d,y;q)} \Big\{U(c) - \tau_b\lambda_b + (1 - p_h)\beta\rho \int v_{b',f',1}(d, y'; q)\Phi(dy') \\ &\quad + p_h\beta\rho \int v_{b',f',0}(d, y'; q)\Phi(dy')\Big\}, \\ &\quad U(y) - \tau_b + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy')\Big\}.\end{aligned}$$

[20]Consistent with modeling of consumer default in the literature, these utility costs are meant to capture the stigma following default as well as the attorney and collection fees associated with default.

4. For $h = 1$ and $f = 1$ (bad record in both markets)

$$\begin{aligned}
(Tv)(d, y; q)(b, f, h) \quad = \quad & \max\Bigg\{ \max_{B_{b,f,h}(d,y;q)} \Bigg\{ U(c) + (1 - p_f)(1 - p_h)\beta\rho \int v_{b',1,1}(d, y'; q)\Phi(dy') \\
& +(1 - p_f)p_h\beta\rho \int v_{b',1,0}(d, y'; q)\Phi(dy') \\
& +p_f(1 - p_h)\beta\rho \int v_{b',0,1}(d, y'; q)\Phi(dy') \\
& p_f p_h\beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') \Bigg\}, \\
& U(y) + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy') \Bigg\}.
\end{aligned}$$

The first part of this definition says that an agent with good student loan and credit card default records may choose to default on either type of loan, on both or on none of them. For all these cases to be feasible, we need to have that the budget sets conditional on not defaulting on student loans or on credit card debt are non-empty. In the case that at least one of these sets is empty, then the attached option is automatically not available. In the case that both default and no default options deliver the same utility, the agent may choose either. Finally, recall that in the case that the agent chooses to repay her student loans or her credit card debt, she may also choose borrowing and savings, and in the case that she decides to default on either of these loans there is no choice on assets position.

The second part of the definition says that if the agent has a good student loan default record and a default flag on credit cards, he will only have the choice to default/repay on student loans since he does not have any credit card debt. Recall that as long as the agent carries the default flag in the credit card market, he cannot borrow.

The last two parts represent cases for an agent with a bad student loan default record. In these last cases, defaulting on student loans is not an option. In part three, the agent has the choice to default on his credit card loan. As before, this is only an option if the associated budget set is non-empty. In the case that all of these sets are empty, then default involuntarily occurs. We assume that when involuntarily default happens it will occur on both markets (this is captured in the second term of the maximization problem).[21]

In part four, however, there is no choice to default given that $f = 1$ and $h = 1$. Thus, the agent simply solves a consumption/savings decision if the budget set conditional on not defaulting on either loan is non-empty. Otherwise, we assume that default involuntarily occurs. In this case,

[21]This assumption is made such that default is not biased towards one of the two markets.

this happens only in the student loan market since there is no credit card debt.

Note that involuntary default happens when borrowers with very low income realizations and high indebtedness have no choice but default. Under these circumstances we assume that the agent may discharge her student loan and there is no wage garnishment. This feature captures the fact that in practice, a small proportion of agents partially discharge their student loan debt.

Assumption 2. *We assume that it yields higher utility to consume y_{min} today and start with zero assets ($b = 0$), and a bad credit card record ($f = 1$) and student loan default record ($h = 1$) with garnished wages (i.e. the worst utility with a feasible action) than it does to consume zero today and start next period with maximum savings, b_{max}, and a good credit card record ($f = 0$) and student loan default record ($h = 0$) (i.e. the best utility with an unfeasible action).*

3.4.4 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate $r \geq 0$. The intermediary takes prices as given, and chooses loans $\xi_{d_t,h_t,b_{t+1}}$ for all type (d_t, h_t, b_{t+1}) contracts for each t to maximize the present discounted value of current and future cash flows $\sum_{t=0}^{\infty}(1+r)^{-t}\pi_t$, given that $\xi_{d_{-1},h_{-1},b_0} = 0$. The period t cash flow is given by

$$\pi_t = \rho \sum_{d_{t-1},h_{t-1}} \sum_{b_t \in B} (1 - p^b_{d_{t-1},h_{t-1},b_t})\xi_{d_{t-1},h_{t-1},b_t}(-b_t) - \sum_{d_t,h_t} \sum_{b_{t+1} \in B} \xi_{d_t,h_t,b_{t+1}}(-b_{t+1})q_{d_t,h_t,b_{t+1}} \qquad (2)$$

where $p^b_{d_t,h_t,b_{t+1}}$ is the probability that a contract of type (d_t, h_t, b_{t+1}) where $b_{t+1} < 0$ experiences default; if $b_{t+1} > 0$, automatically $p^b_{d_t,h_t,b_{t+1}} = 0$. These calculations take into account the survival probability ρ.

If a solution to the financial intermediary's problem exists, then optimization implies $q_{d_t,h_t,b_{t+1}} \leq \frac{\rho}{(1+r)}(1 - p^b_{d_t,h_t,b_{t+1}})$ if $b_{t+1} < 0$ and $q_{d_t,h_t,b_{t+1}} \geq \frac{\rho}{(1+r)}$ if $b_{t+1} \geq 0$. These conditions hold with equality for any optimal nonzerop $\xi_{d_t,h_t,b_{t+1}}$.

3.4.5 Government

The government in the economy operates the student loan program. The cost to the government is the total amount of college loans plus the interest rate subsidized in college. We denote this cost by L.[22] We compute the per period payment on student loans, d as the coupon payment of

[22]The government pays for the interest accumulated during college for subsidized loans but does not pay interest for unsubsidized loans. For simplicity and ease of comparability, we assume that all student loans were subsidized. Lucas and Moore (2007) find that there is little difference between subsidized and unsubsidized Stafford loans.

a student loan with its face value equal to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate, r_g. In practice, this represents the government interest rate on student loans. When no default occurs, the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

However, due to defaults, the government's budget constraint may not hold. In this case the government revenue from an agent in state b with credit card default status f, income y and student loan debt d is given by $(1-p_d^d)d$ where p_d^d is the probability that a contract of type d is defaulted on. The government chooses lump-sum taxes, τ, to balance the budget,

$$\int d\psi_d(dd) = \int (1-p_d^d) d\psi_d(dd) + \int \tau d\mu.$$

We turn now to the definition of equilibrium and characterize the equilibrium in the economy.

3.5 Steady-state Equilibrium

In this subsection we define a steady state equilibrium, prove its existence, and characterize the properties of the price schedule for individuals with different default risks.

Definition 2. A steady-state competitive equilibrium is a set of non-negative price vector $q^* = (q^*_{d,h,b'})$, non-negative credit card loan default frequency vector $p^{b*} = (p^{b*}_{d,h,b'})$, a non-negative student loan default frequency p^*_d, taxes τ^*, a vector of non-trivial credit card loan measure $\xi^* = (\xi_{d,h,b'*})$, decision rules $b'^*(y,d,f,b,h,q^*)$, $\lambda^*_b(y,d,f,b,h,q^*)$, $\lambda^*_d(y,d,f,b,h,q^*)$, $c^*(y,d,f,b,h,q^*)$, and a probability measure μ^* such that:

1. $b'^*(y,d,f,b,h,q)$, $\lambda^*_b(y,d,f,b,h,q)$, $\lambda^*_d(y,d,f,b,h,q)$, and $c^*(y,d,f,b,h,q)$ solve the agent's optimization problem;

2. τ^* solves the government's budget constraint;

3. $p^{d*}_d = \int \lambda^*_d(y,d,f,b,h)d\mu^*(dy,d,df,db,dh)$ (government consistency);

4. ξ^* solves the intermediary's optimization problem;

5. $p^{b*}_{d,h,b'} = \int \lambda^*_b(y',d,0,b',h'^*)\Phi(dy')H^*(h,dh')$ for $b' < 0$ and $p^{b*}_{d,h,b'} = 0$ for $b' \geq 0$ (intermediary consistency);

6. $\xi^*_{d,h,b'} = \int \mathbf{1}_{\{b'^*(y,d,f,b,h,q^*)=b'\}}\mu^*(dy,d,df,db,h)$ (market clearing conditions (for each type (d,h,b'));

7. $\mu^* = \mu_{q^*}$where $\mu_{q^*} = \Gamma_{q^*}\mu_{q^*}$ (μ^* is an invariant probability measure).

In the next subsection, we first prove the existence and uniqueness of the agent's problem and the existence of the invariant distribution. Then we characterize the default decisions in terms of agent characteristics and market arrangements. Last, we prove the existence of cross-market effects and characterize how financial arrangements in one market affect default behavior in the other market. All proofs are provided in the Appendix.

3.6 Results

Existence and uniqueness of a recursive solution to the agent's problem

Theorem 1. *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

1. *v^* is increasing in y and b.*
2. *Default decreases v^*.*
3. *The optimal policy correspondence implied by Tv^* is compact-valued, upper-hemicontinuous.*
4. *Default is strictly preferable to zero consumption and optimal consumption is always positive.*

Since Tv^* is a compact-valued upper-hemicontinuous correspondence, Theorem 7.6 in **?** (Measurable Selection Theorem) implies that there are measurable policy functions, $c^*(d, y, ; q)(b, f, h)$, $b^*(d, y; q)(b, f, h)$, $\lambda_b^*(d, y; q)(b, f, h)$ and $\lambda_d^*(d, y; q)(b, f, h)$. These measurable functions determine a transition matrix for f and f', namely $F^*_{y,d,b,h,q} : F \times F \to [0, 1]$:

$$F^*_{y,d,b,h,q}(f, f' = 1) = \begin{cases} 1 & \text{if } \lambda_b^* = 1, \\ 1 - p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1, \\ 0 & \text{otherwise;} \end{cases}$$

$$F^*_{y,d,b,h,q}(f, f' = 0) = \begin{cases} 0 & \text{if } \lambda_b^* = 1, \\ p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1, \\ 1 & \text{otherwise.} \end{cases}$$

The policy functions determine a transition matrix for the student loan default record, $H^*_{y,d,b,f,q} : H \times H \to [0,1]$ which gives the student loan record for the next period, $h^{'}$:

$$H^*_{y,d,b,f,q}(h, h^{'} \;\; = \;\; 1) = \begin{cases} 1 & \text{if } \lambda_d^* = 1 \text{ and } h = 0, \\ 1 - p_h & \text{if } h = 1, \\ 0 & \text{otherwise;} \end{cases}$$

$$H^*_{y,d,b,f,q}(h, h^{'} \;\; = \;\; 0) = \begin{cases} 0 & \text{if } \lambda_d^* = 1 \text{ and } h = 0, \\ p_h & \text{if } h = 1, \\ 1 & \text{otherwise.} \end{cases}$$

Existence of invariant distribution

Let $X = Y \times D \times B \times F \times H$ be the space of agent characteristics. In the following we will write $F_q^*(y, d, b, h, f, f') := F^*_{y,d,b,h,q}(f, f')$ and $H_q^*(y, d, b, f, h, h') := H^*_{y,d,b,f,q}(h, h')$. Then the transition function for the surviving agents' state variable $TS_q^* : X \times \mathcal{B}(X) \to [0,1]$ is given by

$$TS_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_f \times Z_h} \mathbf{1}_{\{b^* \in Z_b\}} F_q^*(y, d, b, h, f, df^{'}) H_q^*(y, d, b, f, h, dh^{'}) \Phi(dy^{'}) \delta_d(d'),$$

where $Z = Z_y \times Z_d \times Z_b \times Z_f \times Z_h$ and $\mathbf{1}$ is the indicator function. The agents that die are replaced with newborns. The transition function for the newborn's initial conditions, $TN_q^* : X \times \mathcal{B}(X) \to [0,1]$ is given by

$$TN_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d} \mathbf{1}_{\{(b', h', f') = (0,0,0)\}} \Psi(dy^{'}, dd^{'}).$$

Combining the two transitions, we can define the transition function for the economy, ${T^*}_q : X \times \mathcal{B}(X) \to [0,1]$ by

$${T^*}_q(y, d, b, f, h, Z) = \rho TS_q(y, d, b, f, h, Z) + (1 - \rho) TN_q(y, d, b, f, h, Z).$$

Given the transition function T_q^*, we can describe the evolution of the distribution of agents μ across their state variables (y, d, b, f, h) for any given prices q. Specifically, let $\mathcal{M}(x)$ be the space of probability measures on X. Define the operator $\Gamma_q : \mathcal{M}(x) \to \mathcal{M}(x)$:

$$(\Gamma_q \mu)(Z) = \int T_q^*((y,d,b,f,h),Z)d\mu(y,d,b,f,h).$$

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

3.6.1 Characterization of the Default Decisions

We first determine the set for which default occurs for student loans (including involuntary default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both loans. Let $D^{SL}_{b,f,1}(q)$ be the set for which involuntary default on student loans and partial dischargeability occurs. This set is defined as combinations of earnings, y, and student loan amount, d, for which $B_{b,f,1}(d,y;q) = \emptyset$ in the case $h = 1$. For $h = 0$ let $D^{SL}_{b,f,0}(d;q)$ be the set of earnings for which the value of defaulting on student loans exceeds the value of not defaulting on student loans. Similarly, let $D^{CC}_{b,0,h}(d;q)$ be the set of earnings for which the value of defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case $f = 0$. Finally, let $D^{Both}_{b,0,0}(d;q)$ be the set of earnings for which default on both types of loans occurs with $h = 0$ and $f = 0$. Note that the last two sets are defined only in the case $f = 0$.

Theorem 3 characterizes the sets when default on student loans occurs. Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5 presents the set for which default occurs for both loans.

Theorem 3. *Let $q \in Q$, $b \in B$. If $h = 1$ and the set $D^{SL}_{b,f,1}(q)$ is nonempty, then $D^{SL}_{b,f,1}(q)$ is closed and convex. In particular, the sets $D^{SL}_{b,f,1}(d;q)$ are closed intervals for all d. If $h = 0$ and the set $D^{SL}_{b,f,0}(d;q)$ is nonempty, then $D^{SL}_{b,f,0}(d;q)$ is a closed interval for all d.*

Theorem 4. *Let $q \in Q$, $(b,0,h) \in \mathcal{S}$. If $D^{CC}_{b,0,h}(d;q)$ is nonempty then it is a closed interval for all d.*

Theorem 5. *Let $q \in Q$, $(b,0,0) \in \mathcal{S}$. If the set $D^{Both}_{b,0,0}(d;q)$ is nonempty then it is a closed interval for all d.*

Next, we determine how the set of default on credit card debt varies with the credit card debt, the student loan debt, and the default status on student loans of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result was first demonstrated in Chatterjee et al. (2007).

Theorem 6. *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d;q)$ expand when b decreases.*

In addition, we show two new results: 1) the set of default on credit card loans only shrinks when the student loan amount increases and the set of default on both credit card and student loans expands when the student loan amount increases. These findings imply that individuals with lower levels of student loans are more likely to default only on credit card debt and individuals with higher levels of student loans are more likely to default on both credit card and student loan debt (Theorem 7); and 2) the set of default on credit card loans is larger when $h = 1$ relative to the case in which $h = 0$. In other words, individuals with a default record on student loans are more likely to default on their credit card debt (Theorem 8).

Theorem 7. *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d;q)$ shrink and $D^{Both}_{b,f,h}(d;q)$ expand when d increases.*

Theorem 8. *For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D^{CC}_{b,f,0}(d;q) \subset D^{CC}_{b,f,1}(d;q)$.*

This last set of theorems shows the importance of accounting for borrowing and default behavior in the student loan market when determining the risk of default on credit card debt. These elements will be considered in the decision of the financial intermediary, which we explain next.

3.6.2 Existence and Characterization of Equilibrium

Theorem 9. Existence *A steady-state competitive equilibrium exists.*

In equilibrium, the credit card loan price vector has the property that all possible face-value loans (agent deposits) bear the risk-free rate and negative face-value loans (agent borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability. This probability depends on the loan amount and default status, as well as the size of the credit card debt. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different characteristics in the student loan market is not possible. Each (d, h) market clears in equilibrium and it is not possible for an intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. We turn now to characterizing the equilibrium price schedule.

Theorem 10. Characterization of equilibrium prices *In any steady-state equilibrium, the following is true:*

1. *For any $b^{'} \geq 0$, $q^*_{d,h,b'} = \rho/(1+r)$ for all $d \in D$ and $h \in H$.*

2. *If the grids of D and B are sufficiently fine, and $h = 0$, there are $\underline{d} > 0$ and $\underline{b}' < 0$ such that $q^*_{d,h,b'} = \rho/(1+r)$ for all $d < \underline{d}$ and $b^{'} > \underline{b}'$.*

3. *If the set of income levels for which the agent is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $d_1 < d_2$ implies $q^*_{d_1,h,b'} > q^*_{d_2,h,b'}$ for any $h \in H$ and $b' \in B$.*

4. *If the set of income levels for which the agent is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $q^*_{d,h=1,b'} < q^*_{d,h=0,b'}$ for any $d \in D$ and $b' \in B$.*

Theorem 10 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and on small loan sizes made to individuals with no default record on student loans and small enough levels of student loans (property 2). Property 3 shows that individuals with lower levels of student loans are assigned higher loan prices. The last property shows that individuals with a default record on student loans pay higher prices than individuals with no default record for any loan size, b' and for any amount of student loans they owe, d.

3.6.3 The Interplay Between the Two Markets

We have established that the default probability on credit card loans increases in the amount of student loans. In this section we demonstrate that a borrower with high enough loans will prefer defaulting on his student loans rather than on his credit card debt. Theorem 11 shows that we can find a combination of credit card debt and student loan debt which induces a borrower to default. Furthermore, if the amounts owed to student loans and credit card accounts are higher than the two values in this combination, then the borrower will choose to default on student loans rather than on credit card debt.

Theorem 11. *If the grid of D is fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on student loans.*

The intuition behind this result is that with high enough debt levels, the agent finds it optimal to default in order to preserve his consumption. In the case that the student loan amount and credit card debt are large, defaulting on student loans is optimal since the option of defaulting on credit card debt triggers limited market participation. Defaulting on credit card debt is too costly compared with the benefit of discharging one's debt. The agent therefore delays his repayments on student loans at the expense of having wages garnished in the future. But this penalty is less severe compared to being excluded from borrowing for several periods. These are precisely the types of borrowers who most need the credit card market to help them smooth out consumption.

To conclude, our theory demonstrates that differences in bankruptcy rules and default consequences between the student loans and credit card debt, play an important role in shifting default incentives. In the next section we will first bring our theory to the data and then quantify the role each of these two types of credit played in the increase in student loan default rates in recent years in an extension of our theoretical model.

4 Quantitative analysis

We first calibrate our model, test the model implications for default and borrowing behavior and then conduct counterfactual experiment to isolate the role of differences in default consequences and risk pricing for the two types of loans. We further use our model to study alternative policies for student loan repayments.

4.1 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of student loan debt and income; 3) parameters specific to student loan markets such as default consequences and interest rates on student loans; and 4) parameters specific to credit card markets. we set some parameters to values that are standard in the literature, calibrate some parameters directly to data, and then jointly estimate the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Our model is representative for college-educated individuals who are out of college and have student loans. We calibrate the model to 2004 and use the Survey of Consumer Finances in 2004 for moments in the distribution of income, student loan, and credit card debt.[23] The sample consists of young agents (aged 22-40 years old) with at least some college education, student loan debt, credit card debt, but do not own homes. The age group is specifically chosen to include college dropouts and recent graduates. All individuals are out of college and in the labor force.

[23] We use 2004 as the base year in our calibration so as to remove effects on default behavior driven by the financial crisis and important policy changes in both credit card and student loan markets, such as the Bankruptcy Reform Act in 2005 and the Credit Act in 2010 or policies that introduced a large variety of income driven repayment plans (IDR): IBR in 2009, PAYE in 2012, and REPAYE in 2015. Enrollment in IDR plans was relatively low until the introduction of PAYE 2012, but increased sharply in recent years (Conkling and Gibbs (2019)). This approach allows us to better isolate the effects on default induced by trade-offs in the two credit markets.

All numbers in the paper are provided in 2004 dollars.

The model period is one year and the coefficient of risk aversion chosen ($\sigma = 2$) is standard in the macro literature. So is the calibration of the discount factor ($\beta = 0.96$). We set the interest rate on student loans $r_g = 0.068$ as the most representative rate for student loans.[24] The annual risk-free rate is set equal to $r_f = 0.04$, which is the average return on capital reported by McGrattan and Prescott (2000). Table 1 presents the basic parameters of the model. We set the transaction cost in the credit card market to 0.053 following Evans and Schmalensee (1999). We estimate the survival probability $\rho = 0.975$ to match average years of life to 40.[25] The probabilities to keep default flags in the two markets are set to $1 - p_f = 0.9$ for credit card debt and $1 - p_h = 0.5$ for student loan debt to match average years of punishments, ten for the credit card market and two for the student loan market. The first is consistent with estimates in the literature (see Chatterjee et al. (2007) and Livshits et al. (2007)) and the fact that bankruptcy flag stays on a filer's credit report for 10 years. The second is consistent with regulations from the DoE. Specifically, it takes one period before borrowers restructure and reorganize and another period before completing loan rehabilitation. Borrowers must make 10 consecutive payments to rehabilitate. We assume that the default flag is immediately removed after rehabilitation. We estimate the wage garnishment (γ) and the utility loss from defaulting on credit card loans (τ_p) to match the two year cohort default rate for student loans of 5.2 percent during 2004-2006 (see Figure 2 in section 2.2) and the credit card debt to income ratio in our sample from SCF.[26]

We use the joint distribution of student loan debt and income for young agents as delivered by the SCF 2004. The mean of income is \$49,016 and the standard deviation \$47,397. The mean of the amount of student loan debt owed per year is \$3,237 and the standard deviation \$5,240. And the correlation between the two is 0.3. We assume a log normal distribution with parameters $(\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (0.3316, 0.3342, 0.019, 0.0186, 0.3)$ on $[0, 1] \times [0, 0.12]$.[27]

[24] The interest rate for Federal student loans was set to 6.8 percent in 2006 prior to the Great Recession and it remained to this level for unsubsidized loans. The rate further decreased for new undergraduate subsidized loans after July 1, 2008. Before 2006 the rate was variable, ranging from 2.4 to 8.25 percent, so our parameter falls in the middle of these estimates. For details see **?**.

[25] Since our agents are 27 years old, this calibration matches a lifetime expectancy of 67 years old (from wenli: isn't this a bit low?).

[26] Our estimate is in line with the data where the garnishment can be anywhere from 0 to 15 percent. Also, as in practice, wage garnishments do not apply if income levels are below a minimum threshold below which the borrower experiences financial hardship.

[27] We normalize \$147,810=1. This represents the maximum level of income which is equal to mean of income plus 3 times the standard deviation of income.

Table 2: Parameter Values

Parameter	Name	Value	Target/Source
σ	Coef of risk aversion	2.00	standard
β	Discount factor	0.96	standard
r_g	Interest on student loans	0.068	Dept. of Education
r_f	Risk-free rate	0.04	Avg rate 2004-2007 (FRB-G19)
ϕ	Transaction cost	0.053	Evans and Schmalensee (1999)
P_f	Prob to keep CC default flag	0.9	Avg years of punishment=10
P_h	Prob to keep SL default flag	0.5	Avg years of punishment=2
ρ	Survival probability	0.975	Avg years of life=40
γ	Wage garnishment if SL default	0.022	Default rate on SL =5.2%)
τ_p	Utility loss from CC default	4.5	Default rate on CC =1.5%

4.2 Model versus data aggregate moments

The model does a good job of matching debt to income ratios in the two markets for borrowers in the SCF 2004. It delivers a credit card debt to income ratio of 0.072 The data counterparts is 0.076. By calibration, the student debt to income ratio is in line with the data, 0.067 (both income and student debt are exogenous in our model). Similarly, the model predicts default rates in the two markets consistent with the data, both of them being targeted in the calibration.

The model is consistent overall with borrowing behavior in the credit card market; the model predicts that that the fraction in debt is 10% (the data (counterpart is 18%).[28]

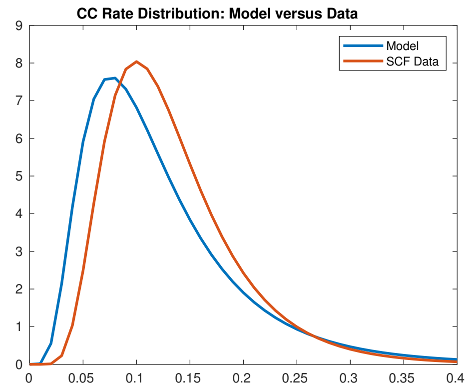
Table 3: Data versus model

	Data	Model
Student loan default rate (targeted)	5.2%	5.1%
Credit card default rate (targeted)	1.5%	1.5%
Credit card interest rate	13.7%	12.6%
Fraction in debt	18%	10%
Credit card debt-to-income ratio	0.076	0.072
Per period college debt-to-income ratio (exog)	0.067	0.067

In terms of credit card pricing, the model replicates the distribution of credit card interest rate quite well, as evident in Figure 5. The model delivers an average credit card interest rate of 12.6

[28]This measure is computed to provide the data counterpart of our model (e.g. net riskfree financial assets in household portfolios aside from student debt) as follows: We use total unsecured debt (but excluding student loans) minus financial assets, defined as the sum of checking and savings accounts, money market deposit accounts, money market mutual funds, value of certificates of deposit, and the value of savings and bonds.

Figure 5: Credit card interest rate



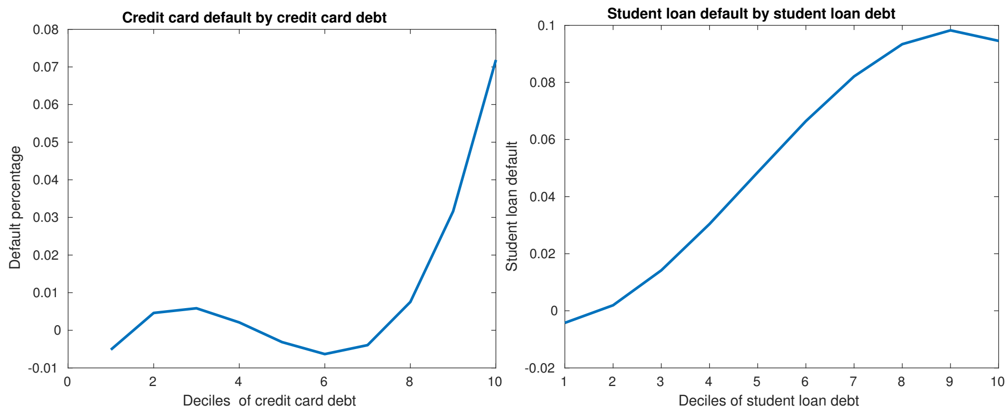
percent. The data counterpart is 13.7 percent. The interest rate in the model is slightly lower compared to the credit card rate in the data since the interest rate in the model represents the effective rate at which borrowers pay, whereas in the data borrowers pay the high rate only in the case that they roll over their debt. The model is in line with borrowing behavior in the credit card market, with The credit card debt to income ratio is a bit lower than in the data. Taxes to cover student loan defaulters in the economy are insignificant (3.615e-004 percent of income, on average). This is because of two factors: these loans are not dischargeable, they are rather delayed; the cost associated with the wage garnishment of 2.2% delivered by the model estimation seems sufficient to cover fees associated with delay costs.

4.3 Benchmark results: understanding default

We study the model's predictions for borrowing and default behavior and evaluate its consistency with the data. We first look at the importance of financial debt burden for default, with focus on the relationship between the two types of debt, then analyze implications for credit card pricing and finally for default across the income distribution.

By calibration, individuals default at higher rates on their student debt than on their credit card debt, on the average. Beyond averages, we examine default behavior across groups of borrowers with different levels of debt in each of the two credit markets. As illustrated in the left panel of Figure 6, consistent with the data presented in Section 3, our model predicts that credit card default increases with the amount of credit card borrowed, from no default for individuals who borrow little to about 7 percent for individuals with debt levels in the top decile (the data counterparts are 0 and 6 percent, respectively). As shown in the right panel of the Figure, student loan default also increases with the level of student debt for most of the range of student debt balances and then flattens out at the top of the student debt distribution.

Figure 6: Default for the two types of credit

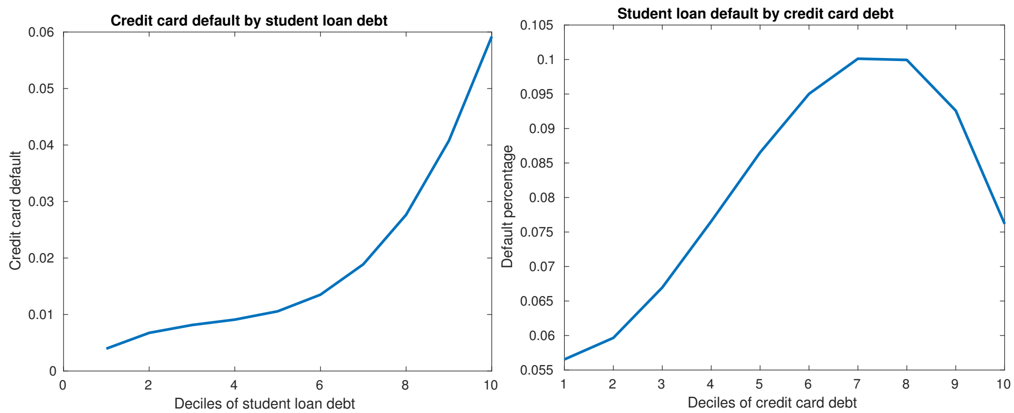


4.3.1 The relationship between credit card borrowing and student debt: implications for default

Regarding the relationship between the two types of debt and their implications for default behavior, recall that data findings reveal that default in credit cards increases in student debt whereas default on student loans is hump-shaped in credit card debt. Our model delivers replicates and explains this behavior. First, as shown in the left panel in Figure 7, default on credit card debt increases in the amount of student loan owed. In line with the data, this pattern confirms that individuals with larger amounts of student loan debt represent a higher risk for the credit card market. In our model, this result is directly implied by the fact that, all else equal, the extra financial burden associated with student debt naturally increases individuals' incentives to default.

Second, individuals with credit card debt default at higher rates on their student loans (8.8 percent) relative to individuals with no credit card debt (4.9 percent) and conditional on having credit card debt, the model delivers a hump-shaped profile for student loan default in credit card debt as the Right panel in Figure 7 shows. As in the data, default rates vary quite significantly across individuals with different levels of credit card debt, ranging from about 5 percent default rate for individuals in the top decile of credit card debt to 10 percent default rate for individuals in the fifth decile of credit card debt. The intuition behind this finding is that individuals with high credit card debt levels are typically individuals with low risk, on average, who face better terms on their credit card accounts, whereas individuals with low credit card debt levels are individuals with high risk, on average, who face worse terms on their credit card accounts, an equilibrium result. Indeed, our model delivers large differences in loan terms on credit card accounts across individuals with different levels of credit card debt (details discussed in the next section). In addition, absent any equilibrium effects, individuals are incentivised to default as they accumulate credit card debt.

Figure 7: The relationship between the two types of debt and default



However, borrowers may also want to use their credit card debt to repay student loans. This trade-off crucially depends on the institutional environments in the two credit markets and the different net effect across the distribution of debt allocation portfolios. As a result, the incentives to default or repay for one type of debt vs. another significantly vary across groups of borrowers, depending on their portfolio positions. We further study these trade-offs and their implications for default incentives in Section 5.4.

4.3.2 Implications of student debt and default for credit card pricing

As shown before, our model predicts that default rates on credit card debt increase with *both* types of loans. In addition, we find that defaulters on student loans have a higher likelihood of default on credit card debt relative to non-defaulters in the student loan market. There are two main reasons behind this result: first, previous defaulters on student loans do not have the option to default on their student loans in the current period, so if they must default, they do so in the credit card market; and second, in addition to being required to repay their student loans, individuals with a default record on student loans also have part of their earnings garnished.

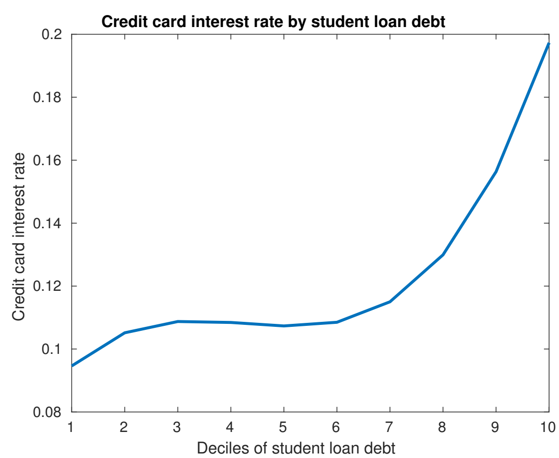
Consistent with our theoretical results on the individual probability of default for credit cards, our quantitative analysis delivers a pricing scheme of credit card loans that varies greatly with individual default risk as proxied by the size of the loan in the credit card market, the amount owed in the student loan market, and the default status in the student loan market.[29] Individuals with a default flag on student loans, $h = 1$ face an interest rate of 12.9 percent, whereas individuals with no default flag, face an interest rate of only 11.5 percent. Furthermore, as shown in Figure 8

[29]These findings represent the quantitative counterpart of our theoretical results in Theorem 10 that shows that the interest rate on credit card debt increases in the amount of each type of loan and it is higher for individuals with a default flag on student loans.

and consistent with our data findings, the interest rate on credit card varies greatly with the amount owed in the student loan market, from about 9 percent for individuals within the bottom decile of student debt to 20 percent for those in the top decide of student debt. Both the amount of student debt and the default status on student loans represent quantitatively important components of credit card loan pricing.

These findings reveal the significance of accounting for risks in all major credit markets in which individuals actively participate when studying default and related policies in theories of quantitative unsecured credit default with endogenous pricing. In this absence of these interactions, such models may miss on accurately accounting for default risks. This is particularly important when thinking about borrowing and default incentives for loans that represent an important part of individuals' portfolios, such as student debt for young agents, like in our economy, or mortgages for individuals later in the life-cycle. Including this margin allows one not only to uncover insights about default incentives in the cross-section (for example, across income levels, as we demonstrate in the next section), but also to accurately capture implications of credit policies (as discussed in Section 5.5).

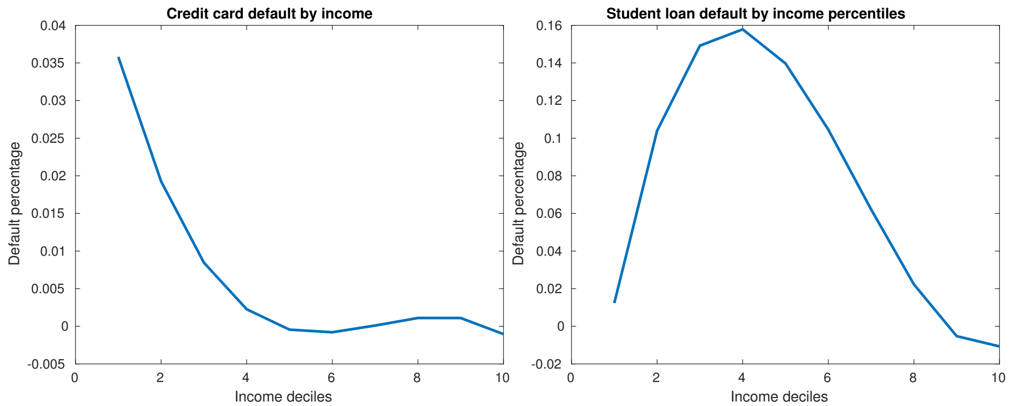
Figure 8: Credit card interest rate



4.3.3 Default across income groups

We now turn to the implications of our model for default behavior across different income groups. As the left panel of Figure 9shows, our model predicts that the likelihood of default on credit card debt decreases with income, result consistent with the empirical literature. However, for the most part, theories of unsecured default have had a hard time capturing this pattern. The intuition is that, in standard models of unsecured credit, agents with relatively low income levels stand to lose more from defaulting on their credit card debt relative to individuals with high income levels, for

Figure 9: Default rates by income



whom the penalties associated with default are relatively less costly. In contrast, in our model, individuals also possess other types of loans, which feature different default consequences or pricing rules. The resulting trade-offs play a key role in delivering the declining profile of the credit card default by income.

As shown in the right panel of 9, default on student loans is hump-shaped in income levels and the differences in the likelihood of default is large across income groups: individuals with medium levels of income experience default rates of about 16 percent, whereas individuals with low or high levels of income (the bottom and the top decile of income, respectively) have default rates around 1 percent, on average. The fact that individuals with high income levels have lower default rates on student loan debt is not surprising: this group of borrowers are not financially constrained and the wage garnishment punishment is too costly for them to warrant default on their student loans. Interestingly, for individuals with low levels of income, incentives to default on student loans are small, and are not amplified by credit card debt. Quite the opposite, poor individuals with large levels of student loans seem to primarily use credit card debt to lower default on student loans. The wage garnishment penalty for them is small. In contrast, for individuals with medium levels of income, we find that having credit card debt amplifies default on student loans quite significantly.

Given that the trade-offs induced by the arrangements in the two markets affect individuals across income groups quite differently, we conjecture that changes in terms in either the credit card or the student loan market will impact default behavior quite differently across income groups as well. In particular, policy proposals to allow for partial student debt forgiveness or to expand income contingent repayments on student loans or to eliminate the bankruptcy option on credit cards would affect incentives to repay or default in both credit markets, with important implications for credit pricing and credit risk. We analyze such policies in Section 6.5.

4.4 The importance of institutional environments in the two credit markets

In this section we analyze the trade-off induced by the institutional environments in the two credit markets and their implications for default. Recall that there are two main differences in credit arrangements between credit card and student loan contracts: 1. consequences to defaulting on the two types of credit and 2. credit pricing rules. Do these trade-offs distort default incentives towards one type of debt? For whom? To answer these questions, we proceed in two steps: We first examine in more detail the predictions of our baseline model for borrowers holding both types of credit across the distribution of debt (the quantitative counterpart of our main result in Theorem 11). Second, we run two counterfactual experiments to quantify the impact of each of these two channels for default incentives.

Our model reveals that, conditional on having low levels of student loan debt, individuals with low levels of credit card debt do not default on their credit card debt, but rather default on their student loans (if they must default). The benefit of discharging their credit card debt upon default is too small compared to the large cost of being excluded from borrowing. At the same time, the penalties associated with default in the student loan market are not contingent on their credit card debt. Similarly, conditional on having high levels of student loan debt, individuals with high levels of credit card debt have a higher likelihood of defaulting on their credit card debt. Furthermore, the gap between default rates by student loan amounts is higher for individuals with low levels of credit card debt relative to individuals with high levels of credit card debt.

These findings confirm our conjecture that while both types of debt increase incentives to default in both credit markets, some individuals may substitute credit card debt for student loan debt, in particular individuals with high levels of student loans. But these individuals represent a high risk for the credit card market and likely receive worse terms on their credit card accounts (e.g. higher interest rates). More expensive credit card debt together with the need to access the credit card market increases incentives to default on student loans. We further examine which individuals are likely to use the credit card market to pay off student loan debt and which ones are likely to default on their student loans at even higher rates because of more (and expensive) credit.

We determine combinations of student loans and credit card debt levels such that above these levels of debt in the two markets, the incentives to default on student loans increase rapidly and no one strictly prefers to default on their credit card debt. This is the quantitative counterpart of our main theoretical result (Theorem 11), which showed that there exists a combination of student loans ($d_1 \in D$) and credit card debt ($b_1 \in B$) such that above this threshold d_1 individuals may prefer default on their student loans. We determine such (d_1, b_1) combinations for the calibrated economy and show that under these thresholds d_1 and b_1, students may be able to use the credit card market to pay off their student loan debt. Details of our quantitative analysis are provided in the Appendix. In brief, our findings confirm that borrowing in one of the two markets affects default incentives in the other credit market in a nontrivial way and interestingly enough, these effects are asymmetric. Student loan debt increases the likelihood of defaulting on credit card, regardless of the student loan amount owed. In contrast, borrowing in the credit card market either amplifies the incentive to default on student loans or helps borrowers reduce their default on student loans. On the one hand, participating in the credit card market and at worse terms pushes borrowers towards more default on their student loans. On the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their student loan debt. The dominance of one of the two factors crucially depends on the portfolio allocation across the two types of debt and the institutional environments in the two credit markets.

We now turn to the second step of our analysis to answer the key question of how much of this default behavior can be explained by differences in the institutional environment: default consequences versus differences in credit pricing. In order to disentangle the effects of the two channels, we run the following two counterfactual experiments: 1. We eliminate the differences in the consequences to default in the two markets in allowing for dischargeability of student loans and exclusion from the credit card market for 10 periods in the case default on student loans occurs. 2. We eliminate the wedge in interest rates between the two types of debt and set the interest rate on credit card debt equal to the fixed rate on student loans.

To be completed.

4.5 Policy implications

4.5.1 Income driven repayment and partial forgiveness for student loans

We use the insights provided by our model economy to asses the effectiveness of student loan policies. Specifically, we quantify the role played by income driven repayments (IDR) for alleviating student loan default and its implications for credit card risk. We analyze the quantitative implications of the IDR by introducing it as the only repayment option in the economy (neither standard repayment or default are available) and preserving budget neutrality via lump-sum transfers.[30]

[30] Having everyone pay under IDR overestimates welfare whereas the absence of default underestimates it. We also abstract from the fact that the policy encourages as many as 5.8 million borrowers with both federally guaranteed student loans and direct loans to move their guaranteed loans into the Direct Loan program. These "split borrowers" have to make loan payments to two different entities. Moving these loans into the Direct Loan program will save the government money, as it will collect all the interest from these loans. This secondary effect of the policy lowers cost on tax payers. Its omission also underestimates welfare.

There are currently four versions of student loan repayment plans based on income, all of which assume loan payments as a percentage of discretionary income and have similar eligibility criteria.[31] Without loss of generality, we account for one such repayment plan and model our experiment closer to income contingent repayment plan (ICR) which provides more flexibility in eligibility criteria. As under the program, we allow borrowers who earn less than 150 percent of the poverty line to have a loan payment of zero and those who have an income higher than this threshold to pay 20 percent of discretionary income.[32] Any remaining debt after 25 years of repayment is forgiven, including both principal and interest.

Overall, we find that the introduction of the ICR induces a small decrease in welfare, on average. On the one hand, the policy induces a high dishargeability rate for student debt relative to the baseline economy and therefore higher taxes are collected when the ICR is introduced. This induces a decline in welfare. On the other hand, the ICR completely eliminates the risk in the credit card market. As a result, credit card debt is less expensive in the economy. More people are borrowing in the credit card market to smooth out consumption and at lower rates. This effect induces an increase in welfare.

These opposite effects vary greatly across borrowers, depending on how much borrowers get to discharge on their student debt and how heavily they rely on the credit card market. Consequently, the welfare effects of the ICR policy vary greatly across groups of borrowers, with poor borrowers with high levels of student loans benefiting the most. This group is most likely to discharge their loans after 25 years of repayment under the ICR. Furthermore, this is the group most likely to use credit cards to smooth out and enjoy the benefits of lower priced credit card loans. In contrast, individuals with relatively low levels of student debt and medium-high income levels lose the most from the ICR implementation given that they likely pay the student loan amount in full and do not rely on credit card markets as much. At the same time, they incur higher taxes to cover for loses associated with the additional discharged debt.

Across groups of student debt, welfare changes are monotonous in student loan levels with individuals in the bottom quartile losing the most. Across income groups, effects are more nuanced, with middle earners (quartiles 2 and 3 of income) losing the most from the ICR policy. Middle earners repay most of their student loans under the ICR without discharging; at the same time they do not benefit from paying their loans faster (as opposed to individuals with high levels of

[31]The four plans are income-contingent, income-sensitive, income based repayment, and pay as you go and they were introduced in stages in the past two decades with IBR in 2009, PAYE in 2012, and REPAYE in 2015. Enrollment in IDR plans was relatively low until the introduction of PAYE 2012, but increased sharply in recent years.

[32]This threshold is $14,148 (in 2004 constant dollars) for a single borrower. We use the value for a single borrower given that our model is representative for U.S. agents aged 20-30 years old.

income) and they pay higher taxes. They no longer have the option to delay their repayment via default either. Recall that middle earners default the most under standard 10-year repayment.

5 Conclusion

We developed a quantitative theory of unsecured credit and default of young U.S. agents with student and credit card debt that accounts for key differences in bankruptcy rules and default consequences as well as risk pricing between the two credit markets. Our economy captures well the observed borrowing and default behavior. It delivers the positive correlation between student debt and credit card debt levels. Importantly, we demonstrate, both theoretically and quantitatively, that credit card default risk and thus pricing of credit card loans increase in both student and credit card debt as well as in the default status on student loans. Consistent with the data and unlike predictions of standard theories of unsecured credit, our model delivers that credit card interest rates are less sensitive to credit card loans.

We further show that the two key differences in market arrangements that we consider successfully explain the observed default pattern. We demonstrate that a borrower with high enough levels of both student loan debt and credit card debt always chooses to default in the student loan market rather than in the credit card market, all else equal. Quantitatively, while default on credit card debt increases in both types of loans, student debt increases in the amount of the student loan, but it is hump-shaped in credit card debt. We characterize this relationship and show how it depends on the pricing rules and default consequences in the two credit markets.

We explore the policy implications of our model and study the impact of income driven repayment plans that allow for student loan forgiveness. In our baseline economy, we find that this policy induces significant redistributional effects, but on aggregate, welfare effects are negligible. In contrast, in an economy calibrated to feature tight credit markets and high income risk, this policy induces substantial benefits across the entire borrower distribution, with these large effects primarily driven by the impact that student loan borrowing and default have on credit card default risk.

Our findings suggest that partial forgiveness may be important in the context of economic conditions when individuals face relatively stringent terms on their credit card accounts and not so great job outcomes. This is particularly important in the recent decade when, due to significant increases in college costs, students borrow more than ever in both student loan and credit card markets. Importantly, our research reveals the importance of accounting for the interactions between student loan and credit card markets when studying borrowing and default behavior in unsecured credit and related policies.

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A Appendix

A1.1 Proofs of theorems

A1.1.1 Proofs of Theorems 1 and 2

Let $c_{\min} = y_{\min}(1-\gamma)$ and $c_{\max} = y_{\max} + b_{\max} - b_{\min}$. Then, if c is the consumption in any of the cases in the definition of T, we have that $U(c_{min}) \leq U(c) \leq U(c_{\max})$ and that $c_{\min}$ is a feasible consumption. Recall that $\mathcal{S} = B \times F \times H$ is a finite set and let $N_{\mathcal{S}}$ be the cardinality of $\mathcal{S}$.

Definition A1. Define $\mathcal{V}$ to be the set of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_{\mathcal{S}}}$ such that

1. For all $(b, f, h) \in \mathcal{S}$ and $(d, y, q) \in D \times Y \times Q$
$$\frac{U(c_{\min})}{1-\beta\rho} \leq v(d, y, q)(b, f, h) \leq \frac{U(c_{\max})}{1-\beta\rho}. \tag{3}$$
2. v is increasing in b and y.
3. v is decreasing in f: $v(d, y, q)(b, 0, h) \geq v(d, y, q)(b, 1, h)$ for all d, y, q, b, h.

Let $\big(C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}), \|\cdot\|\big)$ denote the space of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_{\mathcal{S}}}$ endowed with the supremum norm

$$\|v\| = \max_{(d,y,q)} \|v(d, y, q)\|,$$

where the norm of a vector $w = (w(b, f, h)) \in \mathbb{R}^{N_{\mathcal{S}}}$ is

$$\|w\| = \max_{(b,f,h)\in S} |w(b, f, h)|.$$

Then $\mathcal{V}$ is a subset of $C(D\times Y\times Q; \mathbb{R}^{N_{\mathcal{S}}})$. Define also $C(D\times Y\times Q\times\mathcal{S})$ to be the set of continuous real valued functions $v : D \times Y \times Q \times \mathcal{S} \to \mathbb{R}$ with the norm

$$\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.$$

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

Lemma A1. *The map* $V : C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}) \to C(D \times Y \times Q \times \mathcal{S})$ *defined by*

$$V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)$$

is a surjective isomorphism.

Proof. We prove first that if $v \in C(D\times Y\times Q;\mathbb{R}^{N_S})$ then $V(v)$ is continuous. Let $(d_n, y_n, q_n, b_n, f_n, h_n)_{n\in\mathbb{N}}$ be a sequence that converges to (d, y, q, b, f, h) and let $\varepsilon > 0$. Since $\mathcal{S}$ is a finite set it follows that there is some $N_1 \geq 1$ such that $b_n = b$, $f_n = f$, and $h_n = h$ for all $n \geq N_1$. Since v is continuous then there is $N_2 \geq 1$ such that if $n \geq N_2$ then

$$\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.$$

Thus $|v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon$ for all $n \geq N := \max\{N_1, N_2\}$. Therefore

$$|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N$$

and $V(v)$ is continuous. It is clear from the definition of the norms that $\|V(v)\| = \|v\|$ for all $v \in C(D \times Y \times Q;\mathbb{R}^{N_S})$. Thus V is an isomorphism. Finally, if $w \in C(D \times Y \times Q \times \mathcal{S})$ then one can define $v \in C(D \times Y \times Q;\mathbb{R}^{N_S})$ by

$$v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).$$

Then $T(v) = w$ and T is surjective. $\square$

In the following we are going to tacitly view $\mathcal{V}$ either as a subset of $C(D \times Y \times Q;\mathbb{R}^{N_S})$ or as a subset of $C(D \times Y \times Q \times \mathcal{S})$ via $V(\mathcal{V})$. For example, we are going to prove in the following lemma that $(\mathcal{V}, \|\cdot\|)$ is a complete metric space by showing that it('s image under V) is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$, which is a complete metric space.

Lemma A2. $(\mathcal{V}, \|\cdot\|)$ *is a complete metric space.*

Proof. We are going to show that $\mathcal{V}$ is a closed subspace of $C(D\times Y\times Q\times\mathcal{S})$. Notice first that $\mathcal{V}$ is nonempty because any constant function that satisfies (3) is in $\mathcal{V}$. Let now $\{v_n\}_{n\in\mathbb{N}}$ be a sequence of functions in $\mathcal{V}$ that converge to a function v. Then, since $C(D\times Y\times Q\times\mathcal{S})$ is complete, it follows that v is continuous. Since inequalities are preserved by taking limits it follows immediately that v satisfies the conditions of Definition A1, because each v_n satisfies those conditions. Therefore $v \in \mathcal{V}$ and, thus, $(\mathcal{V}, \|\cdot\|)$ is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$ and, hence, a complete metric space. $\square$

Lemma A3. *The operator T defined on $C(D \times Y \times Q;\mathbb{R}^{N_S})$ maps $\mathcal{V}$ into $\mathcal{V}$ and its restriction to $\mathcal{V}$ is a contraction with factor $\beta\rho$.*

Proof. We will show first that if $v \in \mathcal{V}$ then $Tv \in \mathcal{V}$. Since $v \in \mathcal{V}$ we have that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq v(d,y',q)(b',f',h') \leq \frac{U(c_{\max})}{1-\beta\gamma}$$

for all $(d,y',q) \in D \times Y \times Q$ and $(b',f',h') \in \mathcal{S}$. Integrating with respect to y' we obtain that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq \int v_{(b',f',h')}(d,y';q)\Phi(dy') \leq \frac{U(c_{\max})}{1-\beta\rho},$$

because $\int \Phi(dy') = 1$. Since $U(c_{\min}) \leq U(c) \leq U(c_{\max})$ for all c appearing in the definition of T, it follows that

$$U(c) + \beta\rho \int v_{(b',f',h')}(d,y';q)\Phi(dy') \leq U(c_{\max}) + \frac{\beta\rho U(c_{\max})}{1-\beta\rho} = \frac{U(c_{\max})}{1-\beta\rho},$$

and, similarly

$$\frac{U(c_{\min})}{1-\beta\rho} \leq U(c) + \beta\rho \int v_{(b',f',h')}(d,y';q)\Phi(dy').$$

Thus the condition (3) of Definition A1 is satisfied. To prove that Tv is increasing in b and y and decreasing in f, note that the sets $B_{b,f,h}(d,y,;q)$ are increasing with respect to b and y, and decreasing with respect to f. These facts coupled with the same properties for v (which are preserved by the integration with respect to y') imply that Tv satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since B, F, H and D are finite spaces, it suffices to show that Tv is continuous with respect to y and q. Since Q is compact and v is uniformly continuous with respect to q, it follows by a simple $\varepsilon - \delta$ argument that the integral is continuous with respect q. Since $U(\cdot)$ is continuous with respect to c and c is continuous with respect to d and y, it follows that $T(v)$ is continuous.

Finally we prove that T is a contraction with factor $\beta\rho$ by showing that T satisfies Blackwell's conditions. For simplicity, we are going to view $\mathcal{V}$ one more time as a subset of $C(D \times Y \times Q \times \mathcal{S})$. Let $v, w \in \mathcal{V}$ such that $v(d,y,q,b,f,h) \leq w(d,y,q,b,f,h)$ for all $(d,y,q,b,f,h) \in D \times Y \times Q \times \mathcal{S}$. Then

$$\beta\rho \int v_{(b',f',h')}(d,y';q)\Phi(dy') \leq \beta\rho \int w_{(b',f',h')}(d,y';q)\Phi(dy')$$

for all (d,y,q,b',f',h'). This implies that $Tv \leq Tw$. Next, if $v \in \mathcal{V}$ and a is a constant it follows that

$$\beta\rho \int \big(v_{(b',f',h')}(d,y';q) + a\big)\Phi(dy') = \beta\rho \int v_{(b',f',h')}(d,y';q)\Phi(dy') + \beta\rho a.$$

Thus $T(v+a) = Tv + \beta\rho a$. Therefore T is a contraction with factor $\beta\rho$. $\square$

Theorem 1. *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

1. *v^* is increasing in y and b.*
2. *Default decreases v^*.*
3. *The optimal policy correspondence implied by Tv^* is compact-valued, upper hemi-continuous.*
4. *Default is strictly preferable to zero consumption and optimal consumption is always positive.*

Proof. The first two parts follows from Definition A1 and Lemmas A2 and A3. The last part follows from our assumptions on U. So we need only to prove the third part of the theorem. The optimal policy correspondence is

$$\Xi_{(d,y,q,b,f,h)} = \big\{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q) \text{ that attain } v^*_{b,f,h}(d, y, q)\big\}.$$

For simplicity of our notation we will write $x = (d, y, q, b, f, h)$. For a fixed x we need to show that if Ξ_x is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let $\{(c_n, b'_n, h'_n, f'_n, \lambda^n_d, \lambda^n_b)\}_{n\in\mathbb{N}}$ be a sequence in Ξ_x that converges to some

$$(c, b', h', f', \lambda_d, \lambda_b) \in [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}.$$

Since B, F, and $\{0, 1\}$ are finite sets it follows that there is some $N \geq 1$ such that $b'_n = b'$, $h'_n = h'$, $f'_n = f'$, $\lambda^n_d = \lambda_d$, and $\lambda^n_b = \lambda_b$ for all $n \geq N$. Define

$$\phi(c) = U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy').$$

Then ϕ is continuous and, since $\phi(c_n) = v^*_{(b,f,h)}(d, y; q)$ for all $n \geq 1$, we have that

$$\phi(c) = \lim_{n\to\infty} \phi(c_n) = v^*_{(b,f,h)}(d, y; q).$$

Thus $(c, b', h', f', \lambda_d, \lambda_b) \in \Xi_x$ and Ξ_x is a closed and, hence, compact set.

To prove that Ξ is upper hemi-continuous consider $x = (d, y, q, b, f, h) \in D \times Y \times Q \times S$ and let $\{x_n\} \in D \times Y \times Q \times \mathcal{S}$, $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$ be a sequence that converges to x. Since

D, B, F, and H are finite sets it follows that there is $N \geq 1$ such that if $n \geq N$ then $d_n = d$, $b_n = b$, $f_n = f$, and $h_n = h$. Let $z_n = (c_n, b'_n, h'_n, f'_n, \lambda_d^n, \lambda_b^n) \in \Xi_{x_n}$ for all $n \geq N$. We need to find a convergent subsequence of $\{z_n\}$ whose limit point is in Ξ_x. Since B, H, F, and $\{0, 1\}$ are finite sets we can find a subsequence $\{z_{n_k}\}$ such that $b'_{n_k} = b'$, $h'_{n_k} = h'$, $f'_{n_k} = f'$, $\lambda_d^{n_k} = \lambda_d$, $\lambda_b^{n_k} = \lambda_b$ for some $b' \in B$, $h' \in H$, $f' \in F$, $\lambda_d, \lambda_b \in \{0, 1\}$. Since $\{c_{n_k}\} \subset [c_{\min}, c_{\max}]$ which is a compact interval, there must be a convergent subsequence, which we still label c_{n_k} for simplicity. Let $c = \lim_{k\to\infty} c_{n_k}$ and let $z_{n_k} = (c_{n_k}, b', h', f', \lambda_d, \lambda_b)$ for all k. Then $\{z_{n_k}\}$ is a subsequence of $\{z_n\}$ such that

$$\lim_{k\to\infty} z_{n_k} = z := (c, b', h', f', \lambda_d, \lambda_b).$$

Moreover, since

$$\phi(c_{n_k}) = v^*_{b,f,h}(d_{n_k}, y_{n_k}; q_{n_k}) \text{ for all } k$$

and since ϕ and v^* are continuous functions it follows that

$$\phi(c) = \lim_{k\to\infty} \phi(c_{n_k}) = \lim_{k\to\infty} v^*_{b,f,h}(d_{n_k}, y_{n_k}; q_{n_k}) = v^*_{b,f,h}(d, y; q).$$

Thus $z \in \Xi_x$ and Ξ is an upper hemi-continuous correspondence. □

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

Proof. The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in $X \times \mathcal{B}(X)$ and, thus, T^*_q is well defined. We show first that T^*_q satisfies Doeblin's condition. It suffices to prove that TN^*_q satisfies Doeblin's condition (see Exercise 11.4g of Stockey, Lucas, Prescott (1989)). If we let $\varphi(Z) = TN^*_q(y, d, b, f, h, Z)$ for any $(y, d, b, f, h) \in X$ it follows that if $\varepsilon < 1/2$ and $\varphi(Z) < \varepsilon$ then $1 - \varepsilon > 1/2$ and

$$TN^*_q(y, d, b, f, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon$$

for all $(y, d, b, f, h) \in X$. Thus Doeblin's condition is satisfied.

Next, notice that if $\varphi(Z) > 0$ then $TN^*_q(y, d, b, f, h, Z) > 0$ and, thus,

$$T^*_q(y, d, b, f, h, Z) = \rho TS^*_q(y, d, b, f, h, Z) + (1 - \rho)TN^*_q(y, d, b, f, h, Z) > 0.$$

Then Theorem 11.10 of Stockey, Lucas, Prescott (1989) implies the conclusion of the theorem. □

A1.1.2 Proofs of Theorems 3-8

Let $(b, f, h) \in \mathcal{S}$ and $q \in Q$ be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For $y \in Y$, $d \in D$ we define the following maps:

$$\psi_{nodef}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 0) := U(c) + \beta\rho \int v_{b',f',h'}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 0) \in B_{b,f,h}(d, y; q)$;

$$\psi_{sl}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 0) = U(c) + \beta\rho \int v_{b',f',1}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 1, 0) \in B_{b,f,h}(d, y; q)$;

$$\psi_{cc}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 1) = U(c) + \beta\rho \int v_{b',1,h'}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)$; and

$$\psi_{both}(y, d)(c, b', f', h', \lambda_d = 1, \lambda_b = 1) = U(c) + \beta\rho \int v_{0,1,1}(d, y'; q)\Phi(dy')$$

for all $(c, b', f', h', 0, 1) \in B_{b,f,h}(d, y; q)$. Note that these functions are continuous in y and d. Also, these functions depend on b, f, and q. Also, we will write $\omega_{b,f,h}(q, d)$ for the expected utility of an agent that starts next period with (b, f, h, q, d).

Theorem 3. *Let $q \in Q$, $f \in F$, $b \in B(f)$. If $h = 1$ and the set $D^{SL}_{b,f,1}(q)$ is nonempty, then $D^{SL}_{b,f,1}(q)$ is closed and convex. In particular the sets $D^{SL}_{b,f,1}(d; q)$* are closed intervals for all d. If $h = 0$ and *the set $D^{SL}_{b,f,0}(d; q)$ is nonempty,* then $D^{SL}_{b,f,0}(d; q)$ is a closed interval for all d.

Proof. If $h = 1$ then $D^{SL}_{b,f,1}(q)$ is the combinations of earnings y and student loan amount d for which $B_{b,f,1}(d, y; q) = \emptyset$. Then they satisfy the inequality $y(1-\gamma) + b(1-\lambda_b) - d - q_{b',d,h}b' \leq 0$ for all $\lambda_b \in \{0, 1\}$ and $b' \in B$. Thus $D^{SL}_{b,f,1}(q)$ is closed. Moreover, if (y_1, d_1) and (y_2, d_2) are elements in $D^{SL}_{b,f,1}(q)$ then if $(y, d) = t(y_1, d_1) + (1-t)(y_2, d_2)$ with $t \in (0, 1)$ it follows easily that

$$y(1-\gamma) + b(1-\lambda_b) - d - q_{b',d,h}b' \leq 0$$

and, thus, $(y, d) \in D^{SL}_{b,f,1}(q)$. So $D^{SL}_{b,f,1}(q)$ is convex.

Assume now that $h = 0$ and let $d \in D$ be fixed. Let y_1 and y_2 with $y_1 < y_2$ be in $D^{SL}_{b,f,0}(d; q)$.

Therefore

$$\psi_{sl}(y_i,d)(c_i^*,b_i'^*,f_i'^*,h_i'^*,1,0) \geq \max \begin{array}{l}\{\psi_{nodef}(y_i,d)(c,b',f',h',0,0),\\ \psi_{cc}(y_i,d)(c,b',h',0,1),\\ \psi_{both}(y,d)(c,b',h',1,1)\}\end{array} \tag{4}$$

for all $(c,b',f',h',0,0),(c,b',f',h',0,1),(c,b',f',h',1,1) \in B_{b,f,0}(d,y_i;q)$, $i=1,2$. Let $y \in (y_1,y_2)$ and assume, by contradiction, that $y \notin D^{SL}_{b,f,0}(d;q)$. Assume, without loss of generality, that the agent chooses not to default on either market, i.e.

$$\psi_{sl}(y,d)(c,b',f',h',1,0) < \psi_{nodef}(y,d)(c^*,b'^*,f'^*,h'^*,0,0), \tag{5}$$

for all $(c,b',f',h',1,0) \in B_{b,f,0}(d,y;q)$, where $(c^*,b'^*,f'^*,h'^*,0,0) \in B_{b,f,0}(d,y;q)$ is the optimal choice for the maximization problem. Let $\bar{c}_1 = c^* - (y-y_1)$. If $\bar{c}_1 \leq 0$ then $\bar{c}_1 < y_1 + b$ and thus

$$c^* = \bar{c}_1 + (y-y_1) < y_1 + b + (y-y_1) = y+b. \tag{6}$$

If $\bar{c}_1 > 0$ we have that $(\bar{c}_1, b'^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d,y_1;q)$ and, thus,

$$\psi_{sl}(y_1,d)(c_1^*,b_1'^*,f_1'^*,h_i'^*,1,0) \geq \psi_{nodef}(y_1,d)(\bar{c},b'^*,f'^*,h'^*,0,0).$$

Therefore

$$U(y_1+b) + \beta\rho \int v_{b_1'^*,f_1'^*,1}(d,y';q)\Phi(dy') \geq U(\bar{c}_1) + \beta\rho \int v_{b'^*,f'^*,0}(d,y';q)\Phi(dy'), \tag{7}$$

Subtracting (7) from (5) we have that

$$U(y+b) - U(y_1+b) < U(c^*) - U(\bar{c}_1).$$

Since $(y+b)-(y_1+b) = y-y_1 = c^* - \bar{c}_1$ and U is strictly concave it follows that $c^* < y+b$.

Consider now $\bar{c}_2 = c^* + (y_2 - y)$. Then $(\bar{c}_2, b'^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d,y_2;q)$ and thus

$$U(y_2+b) + \beta\rho \int v_{b_2'^*,f_2'^*,1}(d,y';q)\Phi(dy') \geq U(\bar{c}_2) + \beta\rho \int v_{b^*,f^*,0}(d,y';q)\Phi(dy'). \tag{8}$$

Using inequalities (5), and (8) we obtain that

$$U(y_2+b) - U(y+b) > U(\bar{c}_2) - U(c^*).$$

Thus $c^* > y+b$, and we obtain a contradiction with $c^* < y+b$. Therefore $y \in D^{SL}_{b,f,0}(d;q)$ and, thus, $D^{SL}_{b,f,0}(d;q)$ is an interval. It is also a closed set because the maps ψ_{sl}, ψ_{both}, ψ_{cc}, and ψ_{nodef} are continuous with respect to y. Thus, $D^{SL}_{b,f,0}(d;q)$ is a closed interval. $\square$

Theorem 4. *Let $q \in Q$, $(b,f,0) \in \mathcal{S}$.* If $D^{CC}_{b,f,0}(d;q)$ is nonempty then it is a closed interval for all *d.*

Proof. If $b \geq 0$ then $D^{CC}_{b,f,0}(d;q)$ is empty. If $b < 0$ the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it. $\square$

Theorem 5. *Let $q \in Q$, $(b,f,0) \in \mathcal{S}$. If the set* $D^{Both}_{b,f,0}(d;q)$ is nonempty then it i*s a closed interval* for all d.

Proof. If $b \geq 0$ then the set $D^{Both}_{b,f,0}(d;q)$ is empty. For $b < 0$ the proof is similar with the proof of Theorem 3. $\square$

Theorem 6. *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d;q)$ expand when b decreases.*

Proof. Let $b_1 > b_2$. Then

$$\begin{aligned}
\big\{(c,b',f',h',0,1) \in B_{b_1,f,h}(d,y;q)\big\} &= \big\{(c,b',f',h',0,1) \in B_{b_2,f,h}(d,y;q)\big\},\\
\big\{(c,b',f',h',1,1) \in B_{b_1,f,h}(d,y;q)\big\} &= \big\{(c,b',f',h',1,1) \in B_{b_2,f,h}(d,y;q)\big\},\\
\big\{(c,b',f',h',0,0) \in B_{b_1,f,h}(d,y;q)\big\} &\supseteq \big\{(c,b',f',h',0,0) \in B_{b_2,f,h}(d,y;q)\big\},\\
\big\{(c,b',f',h',1,0) \in B_{b_1,f,h}(d,y;q)\big\} &\supseteq \big\{(c,b',f',h',1,0) \in B_{b_2,f,h}(d,y;q)\big\}.
\end{aligned}$$

Thus, if for b_1,

$$\begin{aligned}
\psi_{cc}(y,d)(c^*,b'^*,f'^*,h'^*,0,1) \;\geq \max\;& \big\{\psi_{nodef}(y,d)(c,b',f',h',0,0),\\
&\psi_{sl}(y,d)(c,b',h',1,0),\\
&\psi_{both}(y,d)(c,b',h',1,1)\big\},
\end{aligned}$$

it follows that the same inequality will hold for b_2 as well. Therefore, $D^{CC}_{b_1,f,h}(d;q) \subseteq D^{CC}_{b_2,f,h}(d;q)$. $\square$

Theorem 7. *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d;q)$ shrink and $D^{Both}_{b,f,h}(d;q)$ expand when d increases.*

Proof. Let $d_1 < d_2$. Then

$$\begin{aligned}
\big\{(c,b',f',h',0,1) \in B_{b,f,h}(d_1,y;q)\big\} &\supseteq \big\{(c,b',f',h',0,1) \in B_{b,f,h}(d_2,y;q)\big\},\\
\big\{(c,b',f',h',1,1) \in B_{b,f,h}(d_1,y;q)\big\} &= \big\{(c,b',f',h',1,1) \in B_{b,f,h}(d_2,y;q)\big\},\\
\big\{(c,b',f',h',0,0) \in B_{b,f,h}(d_1,y;q)\big\} &\supseteq \big\{(c,b',f',h',0,0) \in B_{b,f,h}(d_2,y;q)\big\},\\
\big\{(c,b',f',h',1,0) \in B_{b,f,h}(d_1,y;q)\big\} &= \big\{(c,b',f',h',1,0) \in B_{b,f,h}(d_2,y;q)\big\}.
\end{aligned}$$

Thus, if

$$\begin{aligned}
\psi_{both}(y,d_1)(c^*,b'^*,f'^*,h'^*,1,1) \;\geq \max \;& \big\{\psi_{nodef}(y,d_1)(c,b',f',h',0,0),\\
& \psi_{sl}(y,d_1)(c,b',h',1,0),\\
& \psi_{cc}(y,d_1)(c,b',h',0,1)\big\},
\end{aligned}$$

it follows that the same inequality holds for d_2. Therefore, $D^{Both}_{b,f,h}(d_1;q) \subseteq D^{Both}_{b,f,h}(d_2;q)$. On the other hand, if

$$\begin{aligned}
\psi_{cc}(y,d_1)(c^*,b'^*,f'^*,h'^*,0,1) \;\geq \max \;& \big\{\psi_{nodef}(y,d_1)(c,b',f',h',0,0),\\
& \psi_{sl}(y,d_1)(c,b',h',1,0),\\
& \psi_{both}(y,d_1)(c,b',h',1,1)\big\},
\end{aligned}$$

the inequalities can reverse for d_2. Therefore $D^{CC}_{b,f,h}(d_1;q) \supseteq D^{CC}_{b,f,h}(d_2;q)$. □

Theorem 8. *For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D^{CC}_{b,f,0}(d;q) \subset D^{CC}_{b,f,1}(d;q)$.*

Proof. Let $y \in Y$. For $h = 1$ we have that

$$\big\{(c,b',f',h',1,1) \in B_{b,f,1}(d,y;q)\big\} = \emptyset$$

and

$$\big\{(c,b',f',h',1,0) \in B_{b,f,1}(d,y;q)\big\} = \emptyset.$$

Therefore, if for $f = 0$ we have that

$$\begin{aligned} \psi_{cc}(y, d_1)(c^*, b'^*, f'^*, h'^*, 0, 1) \quad \geq \max \quad & \{\psi_{nodef}(y, d_1)(c, b', f', h', 0, 0), \\ & \psi_{sl}(y, d_1)(c, b', h', 1, 0), \\ & \psi_{both}(y, d_1)(c, b', h', 1, 1)\}, \end{aligned}$$

then the same inequalities hold for $f = 1$. $\square$

A1.1.3 Proofs of Theorems 9 and 10

Theorem 9. Existence *A steady-state competitive equilibrium exists.*

We see that once q^* is known, then all the other components of the equilibrium are given by the formulas in Definition 2. We can rewrite part 5 of the Definition as

$$\begin{aligned} q^*_{d,h,b'} &= \frac{\rho}{1+r}(1 - p^b_{d,h,b'}) \\ &= \frac{\rho}{1+r}\left(1 - \int \lambda^*_b(y', d, 0, b', h', q^*)\phi(dy')H^*(h, dh')\right), \end{aligned}$$

where λ^*_b and f'^* are measurable selections guaranteed by Theorem 1, and H^* is the transition matrix provided by Theorem 1. Thus q^* is a fixed point of the map $T : [0, q_{\max}]^{N_D \times N_H \times N_B} \mapsto [0, q_{\max}]^{N_D \times N_H \times N_B}$

$$T(q)(d, h, b') = \frac{\rho}{1+r}\left(1 - \int \lambda^*_b(y', d, 0, b', h', q)\phi(dy')H^*(h, dh')\right). \tag{9}$$

Since $Q := [0, q_{\max}]^{N_D \times N_H \times N_B}$ is a compact convex subset of $\mathbb{R}^{N_D \times N_H \times N_B}$ we can apply the Schauder theorem (Theorem V.19 of **?***) if we prove that the map*

$$q \mapsto \int \lambda^*_b(y', d, 0, b', h', q)\phi(dy')H^*(h, dh')$$

is continuous.

Before starting the proof we remark that the above map is well defined because even though apriori the transition matrix H^* depends on (y, d, b, f, q), in fact, knowing the pair (h, b') completely determines $H^*(h, dh')$ when $b' < 0$. If $b' < 0$ then $f = 0$, $\lambda^*_d = 0$. Thus $H^*(0, 0) = 1$, $H^*(0, 1) = 0$, $H^*(1, 0) = p_h$ and $H^*(1, 1) = 1 - p_h$. Also, if $b' \geq 0$ then $p^b_{d,h,b'} = 0$ by definition.

We begin by showing that the sets of discontinuities of $\lambda^*_b(\cdot, q)$ and $b^*(\cdot, q)$, $q \in Q$, and $\lambda^*_b(x, \cdot)$ and $b^*(x, \cdot)$, $x \in X$, have measure 0. This will follow from the following lemmas. Let us begin by *noticing that the sets of discontinuities of these functions are contained in the sets of indifference.*

We fix $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ and we will suppress the dependence of functions on these variables. That is, we study the behavior with respect to y. Since B, F, H, and D are finite sets this will suffice to prove the continuity of $\lambda_b^*(\cdot, q)$. The first step is to study in more detail the maximization problem on the no default path. Recall that

$$\psi_{nodef}(y,d)(c,b',f',h',0,0) = U(c) + \beta\rho \int v_{b',0,0}(d,y';q)\Phi(dy')$$

for all $(c, b', 0, 0, 0, 0) \in B_{b,f,h}(d, y; q)$. For $y \in Y$ we write $b'(y)$ for the the values of b' that maximize ψ_{nodef}. Recall that b, f, h, d, and q are fixed and that $b'(y)$ can be a correspondence. Since t is a lump sum tax that is paid by every agent in the economy, it does not affect the choices. For simplicity we assume that $t = 0$ in the following.

Lemma A4. *Let $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ be fixed. Then for any $y_0 \in Y$ there is $\varepsilon > 0$ such that the following holds:*

1. *If $b'(y_0)$ is a single valued then b' is constant and single valued on $(y_0 - \varepsilon, y_0 + \varepsilon)$.*
2. *If $b'(y_0)$ is multi-valued then either $b'(y)$ is single valued on $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$ and there is $\bar{b} \in b'(y_0)$ such that $b'(y) = \bar{b}$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$, or $b'(y) = b'(y_0)$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$.*

Proof. If $b'(y_0)$ is single valued, then

$$\begin{gathered} U(y_0 + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0,}(d,y';q)\Phi(dy') \quad > \\ U(y_0 + b - d - q_{d,h,b'} b') + \beta\rho \int v_{b',0,0}(d,y';q)\Phi(dy'), \end{gathered} \tag{10}$$

for all $b' \in B \setminus \{b'(y_0)\}$ (the right hand side is $-\infty$ if $(c, b', 0, 0, 0, 0) \notin B_{b,f,h}(y_0, d; q)$, where, here, $c = y_0 + b - d - q_{d,h,b'} b'$). Then, since $B(f)$ is finite and U is continuous with respect to y, we can find $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$ then

$$\begin{gathered} U(y + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0}(d,y';q)\Phi(dy') \quad > \\ U(y + b - d - q_{d,h,b'} b') + \beta\rho \int v_{b',0,0}(d,y';q)\Phi(dy'), \end{gathered} \tag{11}$$

for all $b \in B(f) \setminus \{b'(y_0)\}$. Thus $b'(y) = b'(y_0)$ for all $|y - y_0| < \varepsilon$.

Suppose now that $b'(y_0)$ is multi-valued. WLOG, assume that $b'(y_0)$ consists of two elements b'_1 and b'_2 (we can assume this since B is finite). Then

$$\begin{aligned} U(y_0 + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y_0 + b - d - q_{d,h,b'_2} b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') & \end{aligned}$$

and they both satisfy inequality (10) for all $b' \in B \setminus \{b'_1, b'_2\}$. There is $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$, then (11) is satisfied for both b'_1 and b'_2. We need to compare, thus, $U(y + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy')$ and $U(y+b-d-q_{d,h,b'_2} b'_2)+\beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. If $q_{d,h,b'_1} b'_1 = q_{d,h,b'_2} b'_2$, then it follows that $\int v_{b'_1,0,0}(d, y'; q)\Phi(dy') = \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. Therefore

$$\begin{aligned} U(y + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') &= \\ U(y + b - d - q_{d,h,b'_2} b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') & \end{aligned}$$

for all y. Thus $b'(y) = b'(y_0)$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$. Suppose now that $q_{d,h,b'_1} b'_1 < q_{d,h,b'_2} b'_2$. Then

$$s_0 := y_0 + b - d - q_{d,h,b'_1} b'_1 > y_0 + b - d - q_{d,h,b'_2} b'_2 =: t_0.$$

Assume that ε is so that $t_0+\varepsilon < s_0-\varepsilon$. Then, if $|y-y_0| < \varepsilon$ we have that $t_0 < y+b-d-q_{d,h,b'_1} b'_1 =: s_1$, $t_1 := y + b - d - q_{d,h,b'_2} b'_2 < s_0$, and $t_1 < s_1$. Then we have

$$\begin{aligned} U(t_1) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') &= U(t_1) - U(t_0) + U(t_0) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) - U(s_1) \\ &+ U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy'). \end{aligned}$$

Since U is strictly concave, $t_0 < s_0$, $t_0 < s_1$, $t_1 < s_1$, $t_1 < s_0$, and $t_1 - t_0 = s_1 - s_0 = y - y_0$, it follows that $U(t_1) - U(t_0) > U(s_1) - U(s_0)$. Thus

$$U(t_1) + \beta\rho \int v_{b'_2,f',0}(d, y'; q)\Phi(dy') > U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy')$$

and b'_2 is *the only* solution to the maximization problem. Therefore b' is single valued and equals

b_2' on $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$. The case $q_{d,h,b'} b_1' > q_{d,h,b_2'} b_2'$ is similar. $\square$

Lemma A5. *Let $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ be fixed. Suppose that y_1 is a point of indifference between not defaulting and defaulting on student loans. Then, if ε is small enough, either there is no other point y of indifference with $|y - y_1| < \varepsilon$ or all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$ are points of indifference.*

Proof. Let $\varepsilon > 0$ be such that for all $y \in Y$ with $|y - y_1| < \varepsilon$ we have that $b'(y) = b'(y_1) =: b'$. We can find such an ε by Lemma (A4): if $b'(y_1)$ is single-valued, then this is the first part of the lemma; if $b'(y_1)$ is multi-valued, the second part of the lemma implies that we can pick $\bar{b} \in b'(y_1)$ such that $\bar{b} \in b'(y)$ or $b'(y) = \bar{b}$ for all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$. We will consider $b'(y) = \bar{b}$ in both cases (note that this choice does not alter the measurability of b'^*). Assume first that $d \neq q_{d,h,b'} b'$, which implies that $c_1 \neq y_1 + b$, and assume, by contradiction, that y_2 is another point of indifference and the distance between y_1 and y_2 is smaller than ε. Then

$$U(c_1) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_1 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy')$$

and

$$U(c_2) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_2 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy').$$

Therefore $U(c_1) - U(c_2) = U(y_1 + b) - U(y_2 + b)$. However, we have that

$$c_1 - c_2 = y_1 - y_2 = (y_1 + b) - (y_2 + b).$$

This is a contradiction with U being strictly concave. If $d = q_{d,h,b'} b'$ then $c_1 = y_1 + b$, and, hence, $c = y + b$ for all y, then all points y with $|y - y_1| < \varepsilon$ are indifference points. $\square$

The above lemma holds also for for all types of indifference. Thus, since Y is compact, if we fix d and q, there are only a finite number of earning levels that are discontinuity points for λ_d^*, λ_b^*, and b^*.

Lemma A6. *The set of pairs $\{y, d\}$ that are points of discontinuity for λ_d^*, λ_b^*, and b^* has measure 0.*

Proof. Lemma A5 implies that we can change the maps in a Borel way so that for each $d \in D$ the set of $y \in Y$ for which these maps are discontinuous is finite. The conclusion follows now since D is finite. $\square$

Proof. of Theorem 9 Let $\{q_n\}_{n\in\mathbb{N}} \subset Q$ be a sequence that converges to q. We will show that $\lim_{n\to\infty} \lambda_b^*(y,d,f,b,h,q_n) = \lambda_b^*(y,d,f,b,h,q)$ almost everywhere. Since the sequence $\{q_n\}$ is countable, by Lemma A5 we can find a set $E \subset X$ of measure 0 that contains all the points of indifference for the prices q_n, $n \in \mathbb{N}$, and q. Let $(y,d,f,b,h) \in X \setminus E$ be fixed. Since $v_{b,f,h}(d,y;\cdot)$ is continuous and Q is a compact space it follows that $v_{b,f,h}(d,y;\cdot)$ is uniformly continuous. Therefore, since B is finite, there is $\delta > 0$ such that if $\|q' - q''\| < \delta$ and

$$\begin{aligned}\psi_{nodef}(q')(c^*,b^*,f',h',0,0) > \max\Big\{ &\max_{(c,b',f',h',1,0)} \psi_{sl}(q')(c,b',f',h',1,0),\\ &\max_{(c,b',f',h',0,1)} \psi_{cc}(q')(c,b',f',h',0,1),\\ &\max_{(c,b',f',h',1,1)} \psi_{both}(q')(c,b',f',h',1,1)\Big\}\end{aligned}$$

then the same inequality holds for q''. In the inequality above we suppressed the dependence on (y,d,f,b,h) to simplify the notation. Thus, if $\lambda_b^*(y,d,f,b,h,q') = 0$ and $\lambda_d^*(y,d,f,b,h,q') = 0$ then $\lambda_b^*(y,d,f,b,h,q'') = 0$ and $\lambda_d^*(y,d,f,b,h,q'') = 0$. Similar statements hold for all possible combinations of values of λ_b^* and λ_d^*. Therefore, by shrinking δ if necessary, we have that if $\|q' - q''\| < \delta$ then $\lambda_b^*(y,d,f,b,h,q') = \lambda_b^*(y,d,f,b,h,q'')$. This implies that $\lim_{n\to\infty} \lambda_b^*(y,d,f,b,h,q_n) = \lambda_b^*(y,d,f,b,h,q)$ for all $(y,d,f,b,h,q) \in X \setminus E$. Finally, since $|\lambda_b^*(y,d,f,b,h,q)| \leq 1$ and X is a compact space, the Lebesgue's Dominated Convergence Theorem (see, for example, **?**, Theorem 1.34) implies that

$$\lim_{n\to\infty} \int \lambda_b^*(y',d,f',b',h',q_n)\Phi(dy)H(h,dh') = \int \lambda_b^*(y',d,f',b',h',q_n)\Phi(dy)H(h,dh').$$

Thus the map T defined in (9) is continuous and, hence, has a fixed point. □

Theorem 10. *In any steady-state equilibrium the following is true:*

1. *For any $b' \geq 0$, $q^*_{d,h,b'} = \rho/(1+r)$ for all $d \in D$ and $h \in H$.*
2. *If the grids of D and B are sufficiently fine, and $h = 0$ there are $\underline{d} > 0$ and $\underline{b}' < 0$ such that $q^*_{d,h,b'} = \rho/(1+r)$ for all $d < \underline{d}$ and $b' > \underline{b}'$.*
3. *If the set of income levels for which the agent is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $d_1 < d_2$ implies $q^*_{d_1,h,b'} > q^*_{d_2,h,b'}$ for any $h \in H$ and $b' \in B$.*

4. If the set of income levels for which the agent is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $q^*_{d,h=1,b'} > q^*_{d,h=0,b'}$ for any $d \in D$ and $b' \in B$.

Proof. The first part follows from part 5) of the definition of an equilibrium.

For the second part, assume that there are $b_1 < 0$ and $\underline{d} > 0$ such that $y + b_1 - d_1 > 0$ for all $y \in Y$ and consider any agent with $b_1 < b < 0$ and $0 < d < \underline{d}$. In particular the agent must have a clean default flag on the credit card market and on the student loan market. If an agent with debt $b < 0$ defaults only on the credit card market then its utility is

$$u(y-d) - \tau_b + \beta\rho \int u\big(y' - d - q^*_{b'^*(d,y';q)(b,0,0),d,0} b'^*(d,y';q)(b,0,0)\big)\Phi(dy') \\ + (\beta\rho)^2 \int (1-p_f)\omega_{b'^*(d,y';q)(b,0,0),1,0}(q^*,d) + p_f \omega_{b'^*(d,y';q)(b,0,0),0,0}(q^*,d)\Phi(dy').$$

On the other hand, one feasible action of the agent is to not default on any market, pay off the debt and save in the following period $b'^*(d,y';q)(b,0,0)$. The utility from this course of action is

$$u(y+b-d) + \beta\rho \int u\big(y' - d - q^*_{b'^*(d,y';q)(b,0,0),d,0} b'^*(d,y';q)(b,0,0)\big)\Phi(dy') \\ + (\beta\rho)^2 \int \omega_{b'^*(d,y';q)(b,0,0),0,0}(q^*,d)\Phi(dy').$$

Then property 3) of Definition A1 implies that the utility gain by not defaulting is at least

$$u(y+b-d) - u(y-d) + \tau_b.$$

Assuming that the grid of B is sufficiently fine so that we can find $\underline{b} > b_1$ such that the above expression is positive for all $b > \underline{b}$ and $d < \underline{d}$ the conclusion follows. The proof for the case when the agent defaults on both markets is similar.

Assuming that the set of income levels for which the agent is indifferent between defaulting on credit card debt and any other available option, Theorem 7 implies that if $d_1 < d_2$ then $p^{b*}_{d_1,h,b'} \leq p^{b*}_{d_2,h,b'}$ for any $h \in H$ and $b' \in B$. The third part of the theorem follows. One can similarly prove the last part of the theorem. □

A1.1.4 Proof of Theorem 11

Theorem 11. *If the grids of D and B are fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on* *student loans.*

Proof. Suppose that D is fine enough so that we can find $d_1 > 0$ such that given $A > 1$ to be specified below we have that $|u'(y - d_1)| \geq A$ for all $y \in Y$ such that $y > d_1$. Since $q_{\max} < 1$ then we can find $b_1 < 0$ such that $b - q_{\max} b' < 0$ for all $b' \in B$. The utility from defaulting on the credit card for b_1 is

$$u(y - d_1) - \tau_b + \beta\rho\omega_{0,1,0}(q^*, d_1)$$

and the utility from not defaulting on either path is

$$u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h)) + \beta\rho\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1).$$

Using the mean value theorem we can find c' such that $y + b_1 - d_1 - c_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h) < c' < y - d_1$ and

$$u(y-d_1)-u(y+b_1-d_1-q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d, y; q)(b, f, h)) = u'(c')(b_1-q_{b'^*(d,y;q)(b,f,h),d_1,h}b'^*(d, y; q)(b, f, h)).$$

In particular, $|u'(c')| > A$. We chose A such that

$$A(q_{b'} b' - b_1) > \tau_b + \beta\rho(\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1) - \omega_{0,1,0}(q^*, d_1)),$$

for all $b' \in B$. It follows that the utility from defaulting on credit card is higher than the utility of not defaulting at all.

Suppose now that the grids of D and B are fine enough so that we can find d_2 and b'_2 such that $u(y + b'_2) - u(y - d_2) - \tau_d + \tau_b$ is zero or as close to zero as we want. That is, the agent's current utility from defaulting on student loans or credit card are basically the same. Then, if an agent chooses to default on the credit card market today, in the next period her utility will be

$$u(y' - d_2 - q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}) + \beta\rho\big((1 - p_f)\omega_{b^{"*}_{CC},0,1}(d_2, q^*) + p_f\omega_{b^{"*}_{CC},0,0}(d_2, q^*)\big),$$

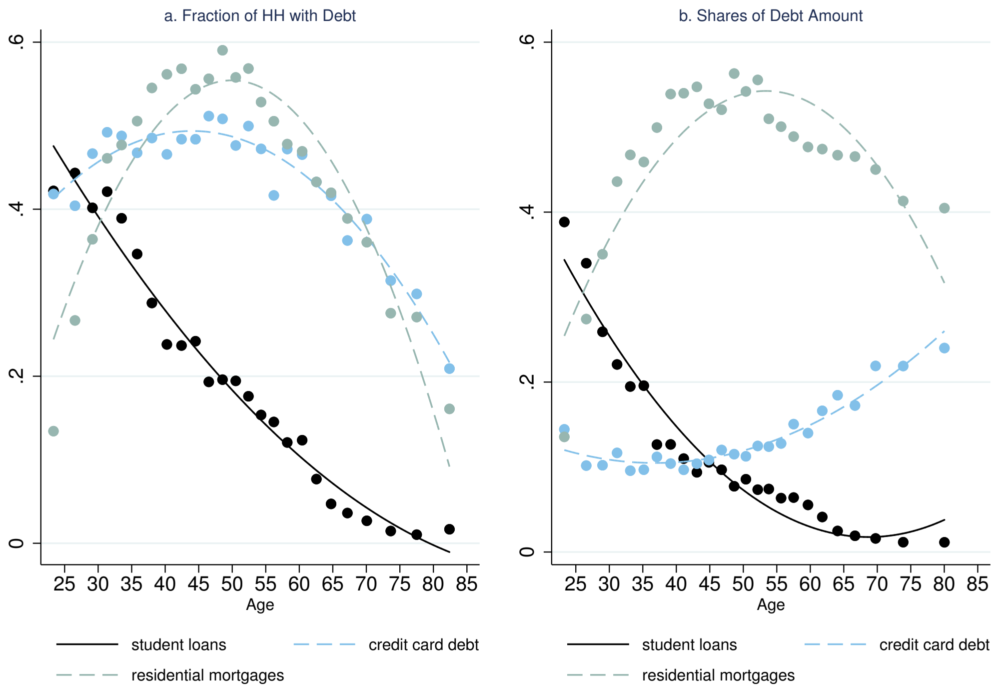
where $b^{"*}_{CC} \geq 0$. If the agent chooses to default on student loans, she can chose to borrow $b^{"}_2 < 0$ such that $y'(1 - \gamma) - d_2 - q_{b^{"}_2} b^{"}_2 > y' - d_2 - q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}$ and $|u'(y'(1 - \gamma) - d_2 - q_{b^{"}_2} b^{"}_2)| > B$, where B is so that

$$\begin{aligned} u'(c')(-\gamma y' - q_{b^{"}_2} b^{"}_2 + q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}) \;\; &\geq \;\; (1 - p_h)\omega_{b^{"}_2,0,1}(d_2, q^*) + p_h\omega_{b^{"}_2,0,0}(d_2, q^*) \\ &- \;\; \big((1 - p_f)\omega_{b^{"*}_{CC},0,1}(d_2, q^*) + p_f\omega_{b^{"*}_{CC},0,0}(d_2, q^*)\big). \end{aligned}$$

Thus, if $b_2 = \min\{b_2', b_2''\}$ it follows that the agent chooses to default on student loans. $\square$

A1.2 Additional Figures and Tables

Figure A1: Household Debt Holdings Over the Life Cycle



Note: This figure provides binned scatter plots of fraction of households with student loans, credit card debt, or home mortgages, separately, in Panel a; and their respect average balance shares in total debt in Panel b. Year-fixed effects are absorbed. The numbers are constructed using weights provided by SCF. Data source: SCF 2001-2019.