Money Allocation, Unemployment and Monetary Policy*

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Abstract

Firms and consumers both hold significant amounts of money, and the firm share changes over time and is negatively correlated with inflation. Whereas existing studies of monetary policy and unemployment only consider consumer money, we build a quantitative framework of money allocation between consumers and firms. The quantitative results show that incorporating firm money greatly amplifies the effect of monetary policy on unemployment, and that an increase in inflation reduces the firm money share. The positive spillover effect from consumer money to firm money proves quantitatively important in accounting for changes in firm money.

Key words: firm money, money allocation, unemployment, monetary policy

JEL classification numbers: D83, E24, E41, E52, J64

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1 Introduction

Many fundamental questions in economics involve the allocation of scarce resources, for example, the allocation of income to consumption and investment, the allocation of time to work and leisure, and so on. A special resource in the human society is money. Although economists have long recognized the use of money by consumers (Baumol, 1952) and firms (Brunner and Meltzer, 1967), no study has seriously considered how the economy allocates the money between these two parties.

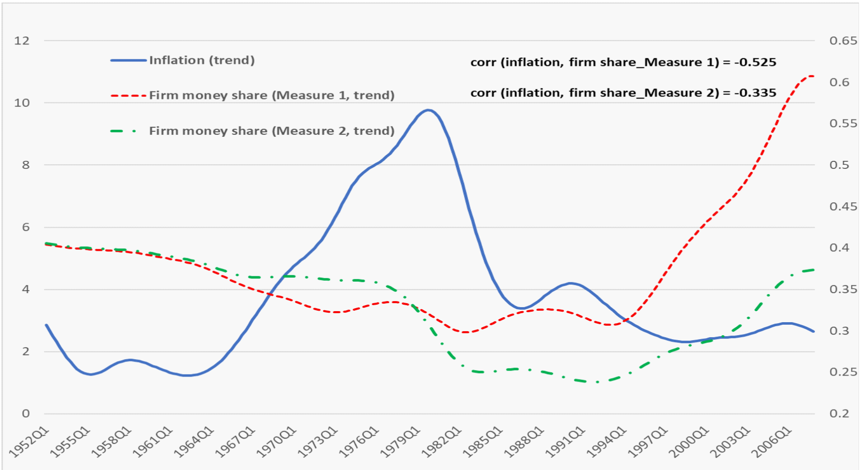

Figure 1: Firm Money Shares and Inflation, 1952–2007

NOTE: The money holdings for firms and consumers in Measure 1 are measured using checkable deposits and currency held by nonfinancial firms, and households, respectively. In Measure 2, the money holdings further include money market mutual funds and commercial papers. The firm money share is the ratio of firm money to the sum of these two holdings. Inflation is seasonally adjusted and measured using the consumer price index (CPI). The times series of the two measures of the firm money share and inflation are the trend part of the raw data. The data on money holdings are from the Federal Reserve and the data on the CPI are from the U.S. Bureau of Labor Statistics.

We look at two measures of money holdings by consumers and firms in the United States over the period from 1952 to 2007. Both the consumers and firms hold significant amounts of money. Figure 1 displays inflation (solid line) and two measures of the firm money share (dashed lines) over the same period, where the firm money share is measured as the ratio of firm money to the sum of these two money holdings. Two messages are delivered: The share of firm money changes substantially over time. Moreover, a negative relationship exists between the firm money share and inflation in the long run.[1] Although

[1] Inflation and the share of firm money are still negatively correlated after 2008, although with smaller correlation coefficients. We stop at 2007 to avoid the financial crisis and unconventional monetary policies, both of which are beyond the scope of this paper.

Figure 1 raises many interesting issues, as a starting point, we focus on the following questions: In the study of money and monetary policy, is it important to incorporate money demand by both consumers and firms? Are there any important macroeconomic and policy implications related to the allocation of money between these two types of agents? Is monetary policy responsible for the movement of the share of firm money in the long run? Existing studies, concerning the frictions that make money essential as well as the cost of carrying money, cannot answer these questions because they study either money demand by consumers or firms. This paper aims to address the above questions in a general equilibrium model featuring both types of money demand.

Our model combines the theory of money demand by consumers in the spirit of Kiyotaki and Wright (1993) where consumers purchase consumption goods from firms, and the theory of money demand by firms where firms purchase capital goods to create job vacancies in the labor market in the spirit of Mortensen and Pissarides (1994).[2] Consumption goods and capital goods are assumed to be traded in frictional markets with limited commitment, which not only makes money essential for transactions, but also leads to endogenous money allocation over two different purposes: consumption and investment. To make the model tractable, we assume that the consumers (or households) are the owners of the firms. Thus, such a money allocation can be interpreted as the allocation between consumers and firms (as equity).

Incorporating both consumer money and firm money allows us to provide some interesting and important insights regarding the implications of monetary policy for unemployment and the firm money share. First, we find that the two types of money demand both imply a positive effect of long-run inflation on unemployment. Therefore, modeling two money demands would *amplify* the policy effects compared to only one. Second, consumer (real) money is found to be *complementary* to firm money. Intuitively, consumers use money to purchase goods from firms. Less consumer money results in less profit for firms from the trade of goods, which lowers the incentive of firms to create job vacancies in the labor market. Due to this spillover effect, the firm's demand for money used to purchase capital declines. Third, monetary policy can influence the *allocation* of money between consumers and firms because the two types of money demand can respond to

[2] Stockman (1981) also assumes that money is used to buy capital goods. An alternative way to model money demand by firms is to assume the working capital requirement for wage payment before production. However, this method involves an underlying limited commitment problem between firms and workers, which is largely mitigated in our work due to the long-term employment relationships.

changes in inflation at different magnitudes.[3]

Compared with studies on monetary policy and unemployment that only consider consumer money, this paper illustrates richer transmission mechanisms of monetary policy caused by considering firm money. In our model, monetary policy directly and indirectly affects the money demand by firms. The direct effect operates through the opportunity cost of carrying money, as noted in the literature on consumer money. An increase in inflation dampens the money demand by firms as it does for the money demand by consumers. The indirect effect operates via the endogenous positive spillover effect from consumer money to firm money, which further imposes a downward pressure on firm money and makes it more responsive to increases in inflation. The direct and indirect effects jointly contribute to the amplified effect of monetary policy on unemployment and the endogenous relationship between inflation and the firm money share.

Calibrated to the key features of various markets in the United States, our model proves successful in several ways. First, with only the changes in monetary policy (inflation or the nominal interest rate), the calibrated model is able to reproduce the overall pattern of the firm money share and unemployment observed from 1952 to 2007. Particularly, the model predicts a negative correlation between inflation and the firm money share, consistent with that observed in Figure 1. Second, the calibrated model is able to generate the observed movement in unemployment when the model economy moves from a low-inflation episode (1996 to 2006) to a high-inflation episode (1980 to 1986). In the baseline calibration where we target the average unemployment in the low-inflation episode, the predicted unemployment increases by 2.85 percentage points in response to the increase in inflation observed between these two episodes, which is very close to its empirical counterpart, at 2.73 percentage points. Lastly, these results remain robust in a variety of calibration strategies, regardless of how money holdings are measured.

Introducing firm money into the analysis proves quantitatively important to understanding the influence of monetary policy on unemployment. In a decomposition exercise, we find that in the face of the same change in inflation, shutting down the firm money channel (i.e., only consumers face the change in inflation) reduces the policy effect by 1.5 percentage points, which suggests that the firm money channel accounts for about 53 percent of the overall movement in unemployment. This result delivers an important

[3] Incorporating firm money also implies that hyperinflation can always eliminate monetary equilibrium, whereas that has not always been the case in previous models with only consumer money.

implication: it is important to take into account the role played by firm money in evaluating the influence of monetary policy on unemployment. Otherwise, the results might be *understated*.

This decomposition exercise also illustrates the quantitative importance of the complementarity between consumer money and firm money. When we remove the firm money channel, we only eliminate the direct effect of inflation on firm money, whereas the indirect effect, the spillover effect from consumer money to firm money, remains. Indeed, the simulation results indicate that such a spillover effect is quantitatively sizable. When consumer money decreases, firms also decrease their money holdings, accounting for 70 percent of the reduction in firm money holdings in baseline simulations in which both channels are present. This result suggests that considering the complementarity between consumer money and firm money, which has received scarce attention in the literature on firm money, might be important for advancing our understanding of the determination of firm money.

In addition, we conduct a counterfactual exercise to demonstrate the key role played by endogenous money allocation adjustments. In baseline simulations, when the average inflation rises from its value in the low-inflation episode to that in the high-inflation episode, the firm money share declines significantly from 31 percent to 13 percent. In the model, this allocation adjustment reflects the direct effect of inflation on the two types of money demand and the indirect effect on firm money via the spillover effect of consumer money. To examine the importance of the endogenous adjustment of money allocation, in the counterfactual exercise, we fix the money allocation at its before-change level. The simulation results show that when 31 percent of the overall money holdings are allocated to the firm, the resulting unemployment increases by only 0.12 percentage points in reaction to the same change in inflation. The reasons for this result are twofold. First, fixing the money allocation eliminates the effect of the monetary policy on firm money. Moreover, imposing a fixed share of firm money violates households' optimal money allocation rule, weakening the effect of monetary policy on consumer money. More money is allocated to fund investments in job vacancies than what would be for the optimal allocation rule. These two factors work together and greatly undo the influence of monetary policy on unemployment.

This paper belongs to the recent advances in monetary economics that apply the dynamic general equilibrium approach to liquidity. As surveyed in Lagos, Rocheteau, and Wright (2017) and Rocheteau and Nosal (2017), the key feature of this strand of literature lies in its concern regarding market frictions that make money essential. Our paper introduces the money allocation between consumers and firms into this strand of literature. Such a consideration allows us to address many interesting and important questions that cannot be examined in the existing studies that either focus on money holdings by consumers (e.g., Shi, 1999; Andolfatto, 2010; Aruoba, Waller, and Wright, 2011; Berentsen, Menzio, and Wright, 2011; Zhang and Huangfu, 2018), or firms (e.g., Chiu, Meh, and Wright, 2017; Rocheteau, Wright, and Zhang, 2018; Wright, Xiao, and Zhu, 2018).

This work is closely related to several recent studies on the long-run effects of monetary policy on unemployment. Berentsen, Menzio, and Wright (2011) pioneer the study along this line of research. Many subsequent studies explore the role of firm ownership as private liquidity (Rocheteau and Rodriguez-Lopez, 2014), the capital pledgeability (Gu, Jiang, and Wang, 2015), the open market operations as in Williamson (2012) (Dong and Xiao, 2018), and the extensive and intensive margins of capital accumulation (Gomis-Porqueras, Huangfu, and Sun, 2020). While these studies focus on money demand by consumers, we stress the importance and richer implications induced by incorporating both consumer money and firm money in the analysis of monetary policy and unemployment.

Lastly, this paper contributes to the growing literature on firm cash holdings (Bates et al., 2009). Many studies examine why some firms hold more cash than others from the perspectives of research and development, mergers and acquisitions, and multinational taxation (e.g., Begenau and Palazzo, 2017; Graham and Leary, 2018; Rempel, 2019). Our study complements this literature by bringing consumer money into the analysis. The complementarity relationship found in this paper suggests that it might be insufficient to examine the money holdings of firms in isolation, even if one is only concerned with firm money.

The remainder of the paper proceeds as follows: Section 2 lays out the model featuring endogenous money allocation between consumption and investment. Section 3 characterizes the individual's optimizing problem and the steady-state equilibrium. Section 4 calibrates the model to the data for the United States and quantifies the implications of monetary policy for unemployment and the money allocation in the long run. Counterfactual exercises are conducted to evaluate the contribution of considering consumer money and firm money as well as endogenous money allocation in the transmission of monetary policy. Section 5 concludes.

2 Model

In the economy, there is a continuum of measure one of households, a large number of potential firms and suppliers of capital goods. Time is discrete and continues forever. In each period, there are four stages: First, firms obtain financing from households to buy capital goods. Second, job creation and destruction occur. Third, households purchase goods from firms. Fourth, wages and dividends are paid, and households rebalance their asset portfolios. Stage 1 is in the financial and capital (FC) market. Stages 2 to 4 feature markets in the spirit of Mortensen and Pissarides (MP market; 1994), Kiyotaki and Wright (KW market; 1993), and Arrow-Debreu (AD market), respectively. [4]

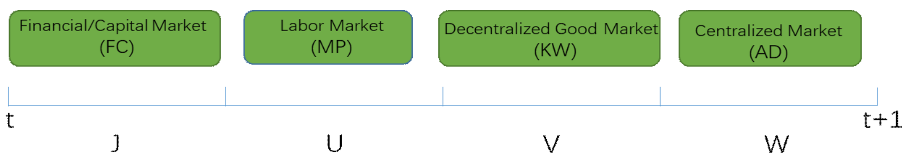

Figure 2: Timeline

In the FC market, firms with business projects first seek financing from households. Funded firms then search for capital suppliers to purchase specialized capital goods that are required for job creation in the MP market. Due to the lack of commitment between firms and capital suppliers, money is needed for the transaction of capital goods. Households choose the number of firms n to fund (equity). The total money allocated for business investment purposes is $n\kappa p_k$, where κ is the quantity of capital goods required for a firm to create a job vacancy, and p_k is the nominal unit price of the specialized capital goods.

Denote B as the number of capital suppliers with free entry into the previous AD market. The meetings between firms and capital suppliers are characterized by a matching

[4] As shown in Lagos and Wright (2005), alternating the KW and AD markets achieves tractability compared to a standard search model.

technology: $M^{FC}(n, B)$, which is assumed to be constant returns to scale (CRS) in both n and B. The matching probabilities for firms and capital suppliers are $\gamma_f = M^{FC}(n, B)/n$ and $\gamma_c = M^{FC}(n, B)/B$, respectively. The firms that fail to acquire the capital goods return the money to the households in the subsequent AD market.

In the MP market, firms with capital goods create job vacancies and search for workers (households). Measure v of firms with vacancies and measure u of workers form matches according to the CRS matching function $M^{MP}(v, u)$. The matching probabilities for firms and workers are $\lambda_f = M^{MP}(v, u)/v$ and $\lambda_h = M^{MP}(v, u)/u$, respectively. Existing matches are subject to an exogenous separation shock that comes at the rate of δ. Upon the exogenous dissolution, the firm loses a fraction $\left(1 - \eta_f\right)$ of its capital stock and can liquidate the rest in the subsequent AD market. The firms that fail to find workers also liquidate their capital in the subsequent AD market.

In the KW market, firms paired with workers in MP produce y units of numeraire goods in KW, and all households want to buy specialized goods regardless of their employment status in MP. Denote by s the number of operating firms. Firms and households meet according to a CRS matching function $M^{KW}(1, s)$. The matching probabilities are $\alpha_h = M^{KW}(1, s)$ and $\alpha_f = M^{KW}(1, s)/s$ for households and firms, respectively. When a match is formed between these two parties, the firm produces q units of the specialized goods for the household at the cost of $c(q)$ units of numeraire goods with $c' > 0$ and $c'' \geq 0$.

In the AD market, one unit of numeraire goods can be transformed into one unit of capital goods, and the process can be reversed. The existing firms pay wages to workers and pay dividends to their shareholders (households). They also purchase capital goods to replenish depreciation, which happens at the rate of δ_k. The households consume numeraire goods, choose the money holdings for the next period, collect debts, and lend to capital suppliers. Each capital supplier borrows κ units of numeraire goods from the households, tries to sell the transformed capital goods to a firm with funds in the next FC market, and repays debts at the real interest rate r in the next AD market.

2.1 Agent Problem

Denote households as h and firms as f. Let e index employment status, where $e = 1$ if an agent is employed, and $e = 0$ otherwise. Denote J_{et}^j, U_{et}^j, V_{et}^j, and W_{et}^j as the value functions for the households and firms in the FC, MP, KW, and AD markets, respectively, which depend on type of the agents $j \in \{h, f\}$, employment status $e \in \{0, 1\}$, and state variables (explained below).

2.1.1 Households

Households of type $e \in \{0, 1\}$ enter the FC market with (real) bond balances b_t, money holdings m_t, and a stock of operating firms s_t. They choose the number of firms to fund n_t. The value function of the households of type e is written by:

$$J^h_{et}(b_t, m_t, s_t) = \max_{n_t} U^h_{et}(b_t, \hat{m}_t, s_t, n_t), \tag{1}$$

$$s.t.\ \hat{m}_t = m_t - n_t \kappa p_{kt}, \tag{2}$$

where p_{kt} is the nominal unit price of capital goods, $n_t \kappa p_{kt}$ is the money balance allocated to fund capital purchases by firms, called firm money, and $\hat{m}_t$ is the money holdings left for consumption purposes, called consumer money.

For employed and unemployed households in the MP market, the respective value functions are expressed as:

$$U^h_{1t}(b_t, \hat{m}_t, s_t, n_t) = \delta V^h_{0t}(b_t, \hat{m}_t, s_{t+1}, n_t, z_t) + (1-\delta) V^h_{1t}(b_t, \hat{m}_t, s_{t+1}, n_t, z_t), \tag{3}$$

$$U^h_{0t}(b_t, \hat{m}_t, s_t, n_t) = \lambda_{ht} V^h_{1t}(b_t, \hat{m}_t, s_{t+1}, n_t, z_t) + (1-\lambda_{ht}) V^h_{0t}(b_t, \hat{m}_t, s_{t+1}, n_t, z_t), \tag{4}$$

where δ is the exogenous job separation rate, and λ_{ht} is the endogenous job finding rate for households. The number of vacancies v_t, stock of operating firms s_t, and number of destroyed firms z_t evolve according to the following equations:

$$v_t = n_t \gamma_{ft}, \tag{5}$$

$$s_{t+1} = v_t \lambda_{ft} + s_t (1-\delta), \text{ and} \tag{6}$$

$$z_t = s_t \delta, \tag{7}$$

where γ_{ft} is the meeting probability of firms in FC, and λ_{ft} is the matching probability of a job vacancy in MP. Equation (5) states that the number of job vacancies equals the number of the funded firms that successfully meet capital suppliers, which occurs with a probability of γ_{ft}.

In the KW market, following Telyukova and Wright (2008), the households are allowed to trade the KW goods using credit with a probability μ, and using cash with the complementary probability $(1-\mu)$. The terms of trade in KW are (d_{ct}, q_{ct}) in credit trade and (d_{mt}, q_{mt}) in money trade, both of which are determined by the Nash bargaining solutions. The value functions for the households of type $e \in \{0, 1\}$ are defined as follows:

$$V_{et}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right) = \begin{array}{l} \alpha_{ht}\mu\left[\upsilon\left(q_{ct}\right) + W_{et}^{h}\left(b_t, \hat{m}_t - d_{ct}/\phi_t, s_{t+1}, n_t, z_t\right)\right] \\ +\alpha_{ht}\left(1-\mu\right)\left[\upsilon\left(q_{mt}\right) + W_{et}^{h}\left(b_t, \hat{m}_t - d_{mt}/\phi_t, s_{t+1}, n_t, z_t\right)\right] \\ +\left(1-\alpha_{ht}\right)W_{et}^{h}\left(b_t, \hat{m}_t, s_{t+1}, n_t, z_t\right), \end{array} \tag{8}$$

where α_{ht} is the matching probability for the households and ϕ_t is the value of money in terms of the numeraire goods.

Following Berentsen, Menzio, and Wright (2011), we require $\hat{m}_t\phi_t \geq d_{mt}$. That is, in money trade, the households can, at most, pay with their money balances. We assume that households can pay their credit debt in the following AD market, so no restriction is imposed on the real debt balance d_{ct} in credit trade. Hence, we interpret the second state variable in the value function $W_{et}^{h}(\cdot)$ as money balances net of credit debt, which can be negative.

In the AD market, the $e-$type households choose how much numeraire goods to consume, x_t, how much nominal money to carry forward to the next period, m_{t+1}, and how much to lend to capital suppliers, b_{t+1}. Their value function is written as follows:

$$W_{et}^{h}\left(b_t, m_t, s_{t+1}, n_t, z_t\right) = \max_{x_t, m_{t+1}, b_{t+1}} \left\{x_t + \beta J_{et+1}^{h}\left(b_{t+1}, m_{t+1}, s_{t+1}\right)\right\}, \tag{9}$$

$$\begin{aligned} s.t.\ x_t + b_{t+1} + \phi_t m_{t+1} &= e w_t + (1-e)\varsigma + \phi_t T_t + b_t(1+r) \\ &\quad + m_t\phi_t + s_{t+1}(R_t - w_t - \delta_k\kappa) + z_t\eta_f\kappa \\ &\quad + n_t\left(1-\gamma_{ft}\right)\kappa p_{kt}\phi_t + n_t\gamma_{ft}\left(1-\lambda_{ft}\right)\left(1-\delta_k\right)\kappa. \end{aligned} \tag{10}$$

In Equation (10) , w_t, ς, T_t, and R_t are real wage, unemployment benefits received while unemployed, (nominal) lump-sum transfers/taxes imposed by the central bank, and the expected periodic revenue of an active firm in real terms, respectively. The last four terms in Equation (10) measure the total real profits (net of capital depreciation) obtained from the stock of operating firms, liquidation of remaining capital goods from exogenously destroyed firms, unspent cash held by the firms that fail to purchase capital goods, and liquidation of undepreciated capital goods held by the firms that fail to form matches in MP. Because the utility function is linear in x_t, households leave the AD market with a degenerate distribution of money, similar to that in Lagos and Wright (2005).[5]

2.1.2 Firms

In the FC market, the number of funded firms, n_t, is determined by households, as shown in Equation (1), rather than the free entry condition, as shown in Mortensen and Pissarides (1994). Funded firms search for capital suppliers to purchase the needed specialized capital goods, which is discussed in detail in Section 2.1.3.[6]

In the MP market, firms with the needed capital open job vacancies. The number of job vacancies is given by Equation (5). The value functions of an active firm (with a worker) and a firm with a vacancy are the following:

$$U_{1t}^f = (1-\delta) V_{1t}^f + \delta \eta_f \kappa. \tag{11}$$

$$U_{0t}^f = \lambda_{ft} V_{1t}^f + (1-\lambda_{ft})(1-\delta_k)\kappa, \tag{12}$$

where λ_{ft} is the matching probability of the firms in the MP market. The last term in Equation (11) represents the liquidation value of an exogenously destroyed firm. Similarly, the last term in Equation (12) measures the value of the capital net of depreciation associated with an unfilled vacancy. Both values are returned to the shareholders (house-

[5] The distribution of the bond balance does not matter for equilibrium. In addition, the distribution of operating firms held by households s_{t+1} is also degenerate because we apply the law of large numbers in the evolution of s_{t+1}, as shown in Equation (6) . That is, it behaves as if all the households hold an index of the stock market.

[6] Suppose, in addition to the capital that is needed to set up job vacancies, firms also use variable capital, k, to produce goods (e.g., $y = F(k)$). Assume that the variable capital is obtained in CM. Let k^* satisfy $1 + r = F^{'}(k^*) + (1 + \delta_k)$. Under the condition $c(q^*) < F(k^*)$, which guarantees that $c(q) < y$ is not binding as assumed in Berentsen, Menzio, and Wright (2011), the marginal product of the variable capital is not affected by monetary policy. Therefore, the choice of variable capital and total capital is independent of monetary policy, which is the same as the result obtained in the current setup.

holds), as shown in Equation (10).

In the KW market, an active firm produces y units of numeraire goods. When a firm and a household meet, the firm produces q units of specialized goods for the household at the cost of $c(q)$ units of numeraire goods. The value function of an active firm is expressed as follows:

$$\begin{aligned} V_{1t}^{f} &= (1-\alpha_{ft}) W_{1t}^{f}(0, y_t) + \alpha_{ft}\mu W_{1t}^{f}\left[\frac{d_{ct}}{\phi_t}, y_t - c(q_{ct})\right] \\ &\quad + \alpha_{ft}(1-\mu) W_{1t}^{f}\left[\frac{d_{mt}}{\phi_t}, y_t - c(q_{mt})\right], \end{aligned} \tag{13}$$

where α_{ft} is the matching probability of the firm in the KW market, d_{it}/ϕ_t, where $i = c, m$, is the nominal sales in the KW market, and $y_t - c(q_{it})$ is the unsold numeraire goods.

In the AD market, an active firm with nominal sales $\tilde{m}_t$ and unsold numeraire goods $\tilde{y}_t$ has the following value function

$$W_{1t}^{f}(\tilde{m}_t, \tilde{y}_t) = \tilde{m}_t \phi_t + \tilde{y}_t - w_t - \delta_k \kappa + \beta J_{1t+1}^{f}. \tag{14}$$

The first four terms present the profits net of capital depreciation paid to the shareholders. Combining Equations (13) and (14) gives the expected real revenue of an active firm:

$$\begin{aligned} R_t &= (1-\alpha_{ft}) y_t + \alpha_{ft}\mu [y_t - c(q_{ct}) + d_{ct}] \\ &\quad + \alpha_{ft}(1-\mu)[y_t - c(q_{mt}) + d_{mt}]. \end{aligned} \tag{15}$$

Last, active firms enter the next MP market (in period $t+1$) with the following value function:

$$J_{1t+1}^{f} = U_{1t+1}^{f}. \tag{16}$$

2.1.3 Capital Suppliers

Capital suppliers are assumed to be risk neutral. In period t, a capital supplier borrows κ units of numeraire goods from households in the AD market, sells the transformed capital goods in period $t+1$ in the FC market with a probability of γ_{ct}, and repays the household with the real interest rate r in the subsequent AD market. We impose the free entry condition for the capital suppliers, which implies the zero-profit condition for every period t:

$$\gamma_{ct}\kappa p_{kt}\phi_t + (1-\gamma_{ct})(1-\delta_k)\kappa = (1+r)\kappa. \tag{17}$$

Equation (17) equates the expected benefits from borrowing κ units of numeraire goods (left-hand side; LHS) to its costs (right-hand side; RHS), which implicitly determines the supply of capital. The second term on the LHS of Equation (17) suggests that the capital suppliers bear the cost of depreciation of the unsold capital goods. Equation (17) also means that debtors (households) are risk neutral in the AD market.

3 Equilibrium

This section solves the general equilibrium of the model and focuses on the steady state. Agents take prices as given in AD and bargain over the terms of trade in the FC, MP, and KW markets. The strategy we adopt is to solve the equilibrium in each market and to depict these results in (u, q_m) space to determine the general equilibrium. We start with KW and AD, then FC, and end with MP.

3.1 Consumption Goods Market Equilibrium

When firms and households meet in KW, the terms of trade (q_i, d_i) for $i \in \{c, m\}$ are determined by the generalized Nash bargaining solution with $\theta \in [0,1]$ being the bargaining power for firms as follows:

$$\max_{q_i,d_i} [\upsilon(q_i) - d_i]^{1-\theta} [d_i - c(q_i)]^{\theta}, \tag{18}$$

$$s.t.\ q_i \leq y_i, \text{ and } d_m \leq \hat{m}\phi, \text{ for } i \in \{c, m\}. \tag{19}$$

The first term is the surplus of the households, and the second term is the surplus of the firms, using the linearity of $W_1^f(\cdot)$ and $W_e^h(\cdot)$. Let q^* be the solution to $\upsilon'(q^*) = c'(q^*)$. The Nash bargaining solution in KW is standard. First, in the money trade, as established by Lagos and Wright (2005), the solution to Equation (18) involves $d_m = \hat{m}\phi = g(q_m)$, where

$$g(q) \equiv \frac{(1-\theta)\upsilon'(q)c(q) + \theta\upsilon(q)c'(q)}{(1-\theta)\upsilon'(q) + \theta c'(q)}.$$

Since $g'(q) > 0$, it follows that bringing more money allows a household to get more KW goods but nonlinearly. Second, in the credit trade, one has $q_c = q^*$ and $d_c = g(q^*)$.

In the AD market, it is standard to simplify the value function by eliminating the linear term x using the budget constraint. The resulting value function is linear in d_i, as noted above. Using the above Nash bargaining solution, we rewrite the choice of next-period money balance m' for the households in AD as follows:

$$\max_{m'}\{-m'\phi + \beta\alpha_h (1-\mu)\, \upsilon\left[g^{-1}(m'\phi')\right] + \beta\left[1-\alpha_h(1-\mu)\right] m'\phi'\},$$

where ϕ and ϕ' are the values of money in the current and next periods. The solution satisfies:

$$\phi = \beta\{\alpha_h (1-\mu)\frac{\upsilon'(q_m)}{g'(q_m)} + [1-\alpha_h(1-\mu)]\}\phi'. \tag{20}$$

The above Euler equation shows that households balance the cost of carrying one unit of money (LHS) with the expected marginal benefit obtained in the money trades in the KW market (RHS). Note that $\phi/\phi' = 1+\pi$ and $(1+\pi)/\beta = 1+i$, where π is the steady-state money growth rate (or inflation), and i is the nominal interest rate. Using $\alpha_h = M^{KW}(1, 1-u)/1$, we have the following Euler equation for money demand, the so-called LW curve:

$$\textbf{LW curve}: \qquad \frac{i}{M^{KW}(1,1-u)} = (1-\mu)\left[\frac{\upsilon'(q_m)}{g'(q_m)} - 1\right]. \tag{21}$$

It is different from that in Lagos and Wright (2005) as the matching probability α_h is now endogenous. Moreover, it is different from that in Berentsen, Menzio, and Wright (2011) because it has an additional term $(1-\mu)$ on the RHS due to introducing credit in KW. Note that the RHS is the liquidity premium of money in KW. Simple conditions exist under which $\upsilon'(q_m)/g'(q_m)$ is guaranteed to be monotonic (Wright, 2010); therefore, there exists a unique $q_m > 0$, solving Equation (21), with the property $\partial q_m/\partial u < 0$. Intuitively, a higher u (i.e., less active firms in KW) lowers the trading probability of consumers, which reduces the demand for money by the household, and, thus, reduces the quantity of KW goods in the money trade q_m.

For a given u, the demand for money is affected by other factors. For example, a higher i reduces the demand for money by households because of the higher opportunity cost of holding money, thus decreasing q_m. In addition, a higher μ lowers the probability of using money in KW trade, and a larger θ makes KW goods more expensive for households. Both effects reduce the demand for money by the household, which translates into a reduced q_m. The property of the LW curve is similar to that in Berentsen, Menzio, and Wright (2011) and is summarized in the following proposition.

Proposition 1 *Let q^* solve $v'(q^*) = c'(q^*)$, and q_0 solve $v'(q_0) = g'(q_0)$. For all $i > 0$, the LW curve slopes downward in (u, q_m) space, with $u = 0$ implying $q_m \in (0, q^*)$, and $u = 1$ implying $q_m = 0$. The LW curve shifts downward with i, μ, and θ. As $i \longrightarrow 0$, $q_m \longrightarrow q_0$ for all $u < 1$, and $q_0 = q^*$ if and only if $\theta = 0$.*

3.2 Financial and Capital Market Equilibrium

In FC, the real capital price $p_k\phi$ is determined by the following generalized Nash bargaining problem with σ being the bargaining power of capital suppliers:

$$\max_{p_k} \left[\kappa p_k \phi - (1-\delta_k)\kappa\right]^{\sigma} \left[U_0^f - \kappa p_k \phi\right]^{1-\sigma}.$$

The above equation uses the capital suppliers' zero-profit condition (17) and the fact that the value of a firm with a vacancy is linear in the real money balances in CM.[7] Solving the standard Nash bargaining problem results in the following:

$$\kappa p_k \phi = \sigma U_0^f + (1-\sigma)(1-\delta_k)\kappa. \tag{22}$$

Equation (22) shows that the real price of κ units of capital goods is a weighted average of the value of the firm with a vacancy, U_0^f and the value of the undepreciated capital goods in CM, $(1-\delta_k)\kappa$.[8] Clearly, the real capital price $\kappa p_k \phi$ increases with U_0^f, as it raises the total surplus of a trade.

The zero-profit condition (17) imposed on the capital suppliers determines the supply of capital. Combining Equations (17) and (22) leads to the supplier entry (SE) curve:

$$\textbf{SE curve}: \qquad U_0^f = \frac{(r+\delta_k)}{\sigma \gamma_c(\gamma_f)}\kappa + (1-\delta_k)\kappa, \tag{23}$$

[7] We implicitly assume capital suppliers pool their risk together so that no default occurs by them. Therefore, the difference in the surplus between making a sale or not is $\kappa p_k \phi - (1-\delta_k)\kappa$.

[8] Because the surplus of the two parties is linear in the real capital price, the bargaining solution would be the same if we assume Kalai bargaining.

where we use trading probabilities for firms $\left(\gamma_f\right)$ and capital suppliers (γ_c) to define $\gamma_c = \gamma_c\left(\gamma_f\right)$, with $\gamma_c'\left(\gamma_f\right) \leq 0$ for $\gamma_f < 1$.

The SE curve, ensuring that capital suppliers break even, generally slopes upward in $\left(\gamma_f, U_0^f\right)$ space. Intuitively, a higher U_0^f increases the capital price, which encourages the capital suppliers to enter FC, thus increasing the matching probability for firms γ_f. However, the exact shape of the SE curve can depend on the matching function in FC. Define $\gamma_c^{-1}(1) \geq 0$ as the highest value of γ_f when $\gamma_c = 1$. If $\gamma_c^{-1}(1)$ is positive (e.g., if the matching function is the Cobb-Douglas function), then the SE curve is flat when γ_f is between 0 and $\gamma_c^{-1}(1)$. It becomes a correspondence when $\gamma_f = 1$, as shown in Figure 3. The SE curve is bounded from below at $U_0^f = (r+\delta_k)\kappa/\sigma + (1-\delta_k)\kappa$ because, if U_0^f is lower than this value, then even a matching probability of one $(\gamma_c = 1)$ is insufficient to make the capital suppliers break even.

The demand for capital depends on n, the number of firms financed by households. With some algebra, we can write the FOC of the value function (1) as follows:

$$\gamma_f U_0^f + \left(1-\gamma_f\right)\kappa p_k \phi = \kappa p_k \phi \left[\alpha_h (1-\mu) \frac{\upsilon'(q_m)}{g'(q_m)} + [1-\alpha_h(1-\mu)]\right]. \tag{24}$$

The LHS of the above equation is the expected marginal benefit of investing in one firm. With a probability γ_{ft}, the firm purchases the required capital and creates a job vacancy; otherwise, it returns the unspent cash to the household. The RHS is the expected marginal cost: the foregone marginal benefits from the trade of KW goods. This condition *suggests* the trade-off faced by households in the model economy: more firm money means less money is available for consumption.

Using the Euler equation (20) on the RHS of Equation (24) reduces it to $\kappa p_k \phi (1+i)$. And Equation (24) can be rewritten as:

$$i = \gamma_f \left(\frac{U_0^f}{\kappa p_k \phi} - 1\right), \tag{25}$$

which looks similar to the LW curve (21). The RHS of Equation (25) is the liquidity premium of money in the capital market. Unlike in the work of Berentsen, Menzio, and Wright (2011), Equation (25) shows that the nominal interest rate also directly influences how much money is allocated to fund job vacancies (firm money). The intuition lies in an optimal money allocation rule as stated in Equation (24). In our model, money can be allocated to either finance investment in firms or fund consumption in KW. Equation (24) suggests that these two different money demands must yield the same return. As a result, the nominal interest rate that affects the demand for money in KW trade (consumer money) also influences the demand for money in job creation (firm money).

Combining Equations (22) and (25) results in the liquidity allocation (LA) curve shown below, and Proposition 2 summarizes the main properties of the SE and LA curves.

$$\textbf{LA curve}: \qquad U_0^f = \frac{(\gamma_f + i)(1-\delta_k)}{\gamma_f - i\sigma/(1-\sigma)}\kappa. \tag{26}$$

Proposition 2 *(i) In (γ_f, U_0^f) space, the SE curve is flat when $0 \le \gamma_f \le \gamma_c^{-1}(1)$ and slopes upward when $\gamma_c^{-1}(1) < \gamma_f \le 1$, as shown in Figure 3. (ii) If $i > 0$, then the LA curve approaches infinity from above at $\gamma_f = i\sigma/(1-\sigma)$ and slopes downward in (γ_f, U_0^f) space, as shown in Figure 3. If $i = 0$, then the LA curve becomes a vertical line at $\gamma_f = 0$.*

The steady-state equilibrium in the capital market is a pair (γ_f, U_0^f) that satisfies both the SE curve (23) and LA curve (26). The ratio U_0^f/κ can be considered as a measure of the inefficiency in FC because U_0^f and κ are the social benefit and cost of a job vacancy (i.e., each vacancy requires κ units of numeraire goods as capital). Using Proposition 2, we immediately have the following proposition for the equilibrium in FC.

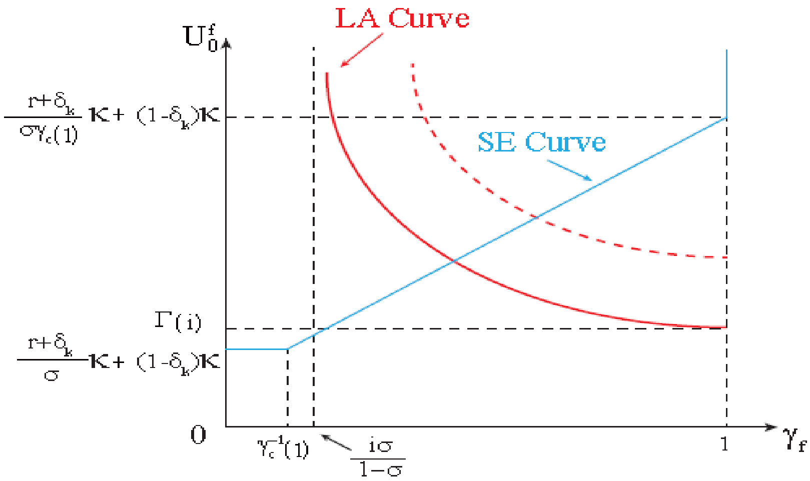

Figure 3: Equilibrium in Financial and Capital (FC) Market

Proposition 3 *A unique equilibrium exists in the capital market. The SE curve shifts upward with r, κ, and δ_k, and downward with σ. The LA curve shifts upward with i, κ, and σ and downward with δ_k.*

Proposition 3 shows the effects of i on the equilibrium outcome $\left(\gamma_f, U_0^f\right)$. As shown in Figure 3, an increase in i shifts the LA curve to the right, and both γ_f and U_0^f tend to increase. Intuitively, an increase in i raises the cost of financing a firm. Therefore, for a given γ_f, the value of U_0^f must be higher to compensate the investors (households), as suggested by the LA curve (26). In equilibrium, γ_f also increases because a higher U_0^f implies a higher capital price as shown in Equation (22). Hence, the capital suppliers require a lower matching probability (γ_c) in FC to break even, which is generally associated with a higher γ_f.

3.3 Labor Market Equilibrium

In the MP market, the real wage w is determined by the Nash bargaining rule with the threat points given by the continuation values, where χ is the bargaining power of the firms. It is routine to solve for the wage as follows:

$$w = \frac{\chi\left[1-\beta\left(1-\delta\right)\right]\varsigma + \left(1-\chi\right)\left[1-\beta\left(1-\delta-\lambda_h\right)\right]\left(R+Q\kappa\right)}{1-\beta\left(1-\delta\right)+\left(1-\chi\right)\beta\lambda_h}, \tag{27}$$

where R is a function of u and q_i for $i \in \{c, m\}$, as shown in Equation (15), $\lambda_h = \left(1-u\right)\delta/u$, and $Q = \beta\delta\eta_f + \beta\left(1-\delta\right)\left(1-\delta_k\right) - 1$ represents an adjusted capital depreciation factor.

The wage equation reduces to the one in Berentsen, Menzio, and Wright (2011) if there is no capital (i.e., $\kappa = 0$). In addition, the magnitude of the wage increases with the liquidation factor η_f. Intuitively, the Nash bargaining rule suggests that the wage is positively correlated with the total surplus of the worker-firm match. A higher η_f indicates a lower capital loss upon an exogenous job separation, which implies a higher total surplus of the operating firm and, consequently, a higher wage. The quantitative importance of η_f is explored in Section 4 in determining the long-run effects of inflation on unemployment.

Combining the Bellman Equations (11)-(16) for the firm, the value of U_0^f can be rewritten as the function of $\left(u, q_m\right)$, which is characterized as the MP (Mortensen Pissarides) curve:

$$\textbf{MP curve}: \quad U_0^f = \lambda_f \frac{R - w - \delta_k \kappa + \beta \delta \eta_f \kappa}{1 - \beta (1 - \delta)} + (1 - \lambda_f)(1 - \delta_k)\kappa, \tag{28}$$

where U_0^f is determined by the FC market equilibrium, R is determined by Equation (15) with $\alpha_f = M^{KW}(1, 1 - u) / (1 - u)$, w is determined by Equation (27), and $\lambda_f = M^{MP}(v(u), u) / v(u)$.[9]

For exposition purposes, we rewrite Equation (28) as follows:

$$\frac{U_0^f}{\kappa} - (1 - \delta_k) = \lambda_f(u) \left[\frac{(R - w)/\kappa - \delta_k + \beta \delta \eta_f}{1 - \beta (1 - \delta)} - (1 - \delta_k) \right].$$

Define the RHS of the above equation as the $e(u, q_m)$ function as follows:

$$e(u, q_m) = \lambda_f(u) \left[\frac{\Pi(u, q_m)/\kappa - \delta_k + \beta \delta \eta_f}{1 - \beta (1 - \delta)} - (1 - \delta_k) \right],$$

where $\Pi(u, q_m) = R - w$, the periodic profit of an active firm. For a given U_0^f, the property of the MP curve is summarized in the following proposition.

Proposition 4 *If $(y - \varsigma)/\kappa > 1 - \beta \left[(1 - \delta)(1 - \delta_k) + \delta \eta_f \right]$, the MP curve slopes downward in (u, q_m) space and shifts upward with U_0^f and downward with θ. If $U_0^f/\kappa < (1 - \delta_k) + e(1, q^*)$, and $e(0, q^*) < (r + \delta_k)/\sigma < e(1, q^*)$, then the MP curve passes through (u_1, q^*), where $u_1 \in (0, 1)$; if $U_0^f/\kappa < (1 - \delta_k) + e(1, 0)$, then it passes through $(u_2, 0)$, where $u_2 < 1$; if $U_0^f/\kappa > (1 - \delta_k) + e(1, 0)$, then it passes through $(1, q_1)$, where $q_1 > 0$.*

See the proof in Appendix 6.[10] □

Proposition 4 shows that a higher U_0^f shifts the MP curve to the right in (u, q_m) space. The reason is that, for a given q_m, an increase in U_0^f requires a higher trading probability for the firm in KW (larger λ_f) as suggested by the MP curve, which is true when the number of active firms in KW becomes lower, or u is higher.

[9] Note that the steady-state condition for unemployment $(1 - u)\delta = M^{MP}(v, u)$ is used to implicitly define $v = v(u)$.

[10] The first condition in Proposition 4 requires that the lower bound of the total periodic surplus of a worker-firm pair is larger than zero.

3.4 General Equilibrium

Given the above analysis of the individual markets, we can determine the steady-state equilibrium using a system of equations, namely the LW curve (21) , SE curve (23) , LA curve (26), and MP curve (28) . The equilibrium can be solved as follows. First, we use the SE curve (23) and LA curve (26) to determine the equilibrium value of $\left(U_0^f, \gamma_f\right)$, as shown in Figure 3. Next, using the equilibrium U_0^f and the fact that the matching probabilities α_f, λ_f, λ_h, R, and w are all functions of u and q_m, we solve the equilibrium (u, q_m) using the intersection of the LW curve (21) and MP curve (28) , as shown in Figure 4. Last, all other variables easily follow with the solution to (u, q_m) .

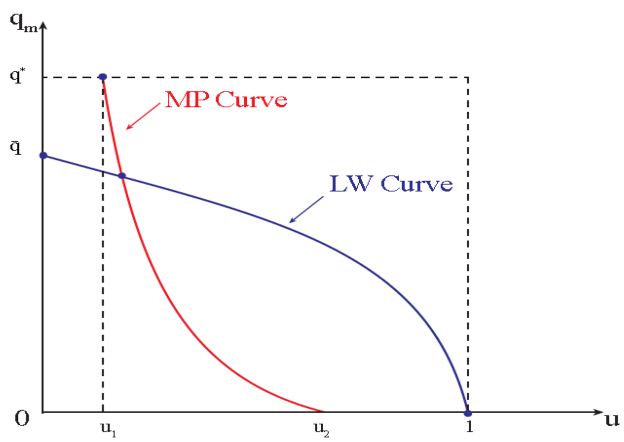

Figure 4: General Equilibrium

Figure 4 illustrates that multiple equilibria could exist. In that case, following Berentsen, Menzio, and Wright (2011), we focus on the equilibrium with the lowest unemployment rate in the quantitative analysis. The following proposition summarizes the conditions for the existence of monetary equilibrium.

Proposition 5 *Assume* $(y - \varsigma)/\kappa > 1 - \beta\left[(1-\delta)(1-\delta_k) + \delta\eta_f\right]$ *and* $e(0, q^*) < (r + \delta_k)/\sigma < e(1, q^*)$*. (i) If* $U_0^f/\kappa < (1-\delta_k) + e(1, 0)$*, then at least one monetary equilibrium exists. (ii) If* $U_0^f/\kappa > (1-\delta_k) + e(1, 0)$*, then no monetary equilibrium exists or an even number of monetary equilibria exist. (iii) If* $U_0^f/\kappa > (1-\delta_k) + e(1, q^*)$*, then no monetary equilibrium exists.*

With the conditions $(y - \varsigma)/\kappa > 1 - \beta\left[(1-\delta)(1-\delta_k) + \delta\eta_f\right]$ and $e(0, q^*) < (r + \delta_k)/\sigma < e(1, q^*)$, the condition in (i) of the above proposition guarantees that the MP curve passes through $(u_2, 0)$, where $u_2 < 1$. Hence, the MP curve and the LW curve can always have at least one interception, as shown in Figure 4. The condition in (iii) ensures that the two curves never cross.

Proposition 6 *In the monetary equilibrium with the lowest u, a higher i increases u and decreases q_m and the number of funded firms n.*

Proof: In the equilibrium with the lowest u, the MP curve crosses the LW curve from above. By Proposition 3, an increase in i shifts the LA curve up, which leads to a larger U_0^f as long as i is not too small. By Proposition 4, an increase in U_0^f shifts the MP curve up. Meanwhile, by Proposition 1, a higher i shifts the LW curve downward. Therefore, an increase in i raises u, while decreasing q_m.

For the last result for n, using the steady-state condition for $u : M^{MP}(v(u), u) = (1-u)\delta$, v decreases with u. Therefore, an increase in i leads to a lower v. In addition, by Proposition 3, a rise in i raises γ_f. By Equation (5), it follows that n decreases with i.□

Proposition 6 delivers rich monetary policy implications for equilibrium unemployment and money allocation. First, Proposition 6 illustrates that a change in i (or inflation) affects equilibrium u through the shifts in both the LW and MP curves. If we interpret the shift of the LW and MP curves as the consumer money and the firm money channels, Proposition 6 demonstrates that both channels generate a positive effect of i on u. This result suggests that firm money plays a key role in evaluating the overall effect of monetary policy on u. More specifically, incorporating firm money amplifies the effect of monetary policy on u, compared to the models that only consider consumer money, such as in the work by Berentsen, Menzio, and Wright (2011) and many subsequent studies.

Second, Proposition 6 reveals that a spillover effect exists from consumer money to firm money, the so-called complementarity channel of monetary policy. When a higher i reduces consumer money (a downward shift in LW), it causes a movement along the MP curve. Intuitively, less consumer money reduces q_m. As a result, the real profit received by the firms in KW declines, which damages job creation and decreases the demand for money by firms. The complementarity channel captures the indirect effect of i on firm money.

Third, Proposition 6 shows that monetary policy can influence the allocation of money between consumers and firms. Regarding firm money as an example, as discussed above, it responds to the increase in i both directly (the shift in MP) and indirectly (the movement along MP induced by the shift in LW). If the two types of money demand respond to the increase in i by different magnitudes, then the monetary policy may explain the movement in the money allocation between these two types of agents.

Lastly, incorporating firm money has a novel implication for hyperinflation. Note that in Berentsen, Menzio, and Wright (2011), hyperinflation could not eliminate all monetary equilibrium. If the MP curve passes through $(u_2, 0)$ where $u_2 < 1$, then monetary equilibrium always exists no matter how high i is because, in their work, monetary policy only affects consumers (i.e., only shift in the LW curve). In our model, a high enough inflation will always eliminate monetary equilibrium because i also directly affects firms by increasing U_0^f in the FC market. If i or U_0^f is high enough, then part (iii) in Proposition 5 implies the following result.

Remark 7 *There is no monetary equilibrium with a high enough i.*

4 Numerical Analysis

This section analyzes the quantitative implications of the results obtained in Section 3. Particularly, we focus on two tasks. First, we examine whether monetary policy can account for the long-run pattern of firm money share as shown in Figure 1. Second, we quantify how monetary policy affects unemployment through various channels, as discussed in Section 3.

Our numerical analysis adopts the following specifications. The matching function in FC takes the following form:

$$M^{FC}(n, B) = nB\left(n^{\zeta^{FC}} + B^{\zeta^{FC}}\right)^{-1/\zeta^{FC}}.$$

Following the labor search literature, the matching function is assumed to be Cobb–Douglas:

$$M^{MP}(u, v) = \mu^{MP}\left(u^{1-\zeta^{MP}} v^{\zeta^{MP}}\right).$$

The matching function in KW is chosen to be the same as the one in Berentsen, Menzio, and Wright (2011):

$$M^{KW}(1,s) = \frac{s}{1+s}.$$

Similarly, the flow utility function and cost function in KW take the forms as follows:

$$u(q) = \frac{A(q)^{1-a}}{1-a}.$$

$$c(q) = (q)^{\varphi}.$$

4.1 Parameterization

Our calibration targets aim to replicate the main rates and flows in the capital, labor, and goods markets in the United States. The model period is set to be one quarter.

Table 1 Calibration targets

Variables		Target Descriptions	Target Values
Discount factor	β	Annual real interest rate	0.048
Productivity in a formed match	y	Normalization	1
UI benefits	ς	Statutory UI replacement ratio in Shimer (2005)	0.4
Elasticity parameter in MP matching func.	ζ^{MP}	Estimated value in Shimer (2005)	0.28
Firm's bargaining power in MP	χ	Hosios' rule	0.28
Curvature parameter in $c(q)$	φ	Normalization in BMW (2011)	1
Separation rate	δ	Quarterly average of monthly rate in Shimer (2005)	0.033
Probability of using a credit card	μ	Prob. of using a credit card in Aruoba *et al.*(2011)	0.15
Fra. of capital returned in destroyed matches	η_f	Random pick	0
Scale parameter in matching func. in MP	μ^{MP}	Average unemployment from 1952 to 2007	0.06
Capital supplier's bargaining power in FC	σ	Labor market tightness in Hall (2005)	0.539
Capital required for a job vacancy	κ	K/Y ratio in Aruoba *et al.* (2011)	2.34
Firm's bargaining power in KW	θ	Markup ratio in Faig and Jerez (2005)	0.30
Scale parameter in utility func.	A	Real demand for money in Aruoba *et al.* (2011)	0.186
Curvature parameter in utility func.	a	Elasticity of money demand w.r.t. i in BMW (2011)	-0.556
Parameter in matching func. in FC	ζ^{FC}	Average firm money share from 1952 to 2007	0.38/0.33

NOTE: These are the targets that our calibration of the model aims to reproduce. These targets correspond to empirical moments in the United States. Each one of the first nine targets determines one parameter. The rest collectively determine the remaining parameters.

The model is calibrated in two stages. In the first stage, the nine top parameters in Table 1 are determined independently from the rest. The discount factor (β) is set to correspond to the average annual real interest rate of 4.8 percent. The average productivity (y) is normalized to be one. The unemployment insurance (UI) benefits (ς) is set to be the statutory UI replacement ratio, 0.4, as used in Shimer (2005). The elasticity parameter in the matching function in MP $\left(\zeta^{MP}\right)$ is determined to be 0.28, which is the same as the value estimated in Shimer (2005). Using Hosios' rule, this suggests that the firm's bargaining power in MP (χ) is 0.28. The curvature parameter (φ) in the cost function is normalized to be one as in Berentsen, Menzio, and Wright (2011). The exogenous separation rate (δ) is set to be 0.033, which is the quarterly average of the monthly separation rate estimated in Shimer (2005). The probability of using a credit card in KW (μ) is set to be 0.15, the same as the choice in Aruoba *et al.* (2011).[11] The fraction of capital returned in a destroyed match $\left(\eta_f\right)$ is set to be zero.[12]

In the second stage of our calibration, we jointly pick the remaining seven parameters of the model to match the seven targets in the bottom panel in Table 1. Particularly, the scale parameter in the matching function $\left(\mu^{MP}\right)$ in MP is determined to match the average unemployment over the period from 1952 to 2007, at about 6 percent. The bargaining power of the capital suppliers (σ) in FC is chosen to match the labor-market tightness (vacancy-unemployment ratio) used in Hall (2005), which is 0.539. In our model, each operating firm employs one worker and κ units of capital; therefore, $\kappa = K/L$. With output (per worker) being normalized to one, we have $\kappa = \frac{K/L}{Y/L} = \frac{K}{Y}$. Hence, the units of capital required in a job creation process (κ) is determined to matched the observed capital-output ratio, 2.34, as documented in Aruoba *et al.* (2011). The firm's bargaining power in KW (θ) is picked to target the markup ratio, 0.3, as calculated in Faig and Jerez (2005). The markup ratio in the model is measured as follows:

$$markup = 100\left[\mu\frac{g\left(q^c\right)}{c'\left(q^c\right)q^c} + (1-\mu)\frac{g\left(q^m\right)}{c'\left(q^m\right)q^m} - 1\right].$$

The scale parameter in the utility function (A) is chosen to match the real demand for money, 0.186, as documented in Aruoba *et al.* (2011). In our model, the real demand for money is measured as follows:

$$\frac{M}{pY} = \frac{g\left(q^m\right) + nkp_k\phi}{(1-u)R + rb}.$$

[11] Klee (2008), using supermarket scanner data, finds that shoppers in the United States use credit cards in 12 percent of the total transactions. Cooley and Hansen (1991), using earlier consumer data, come up with a measure of 16 percent. Aruoba *et al.* (2011) choose a value in between, and we follow their choice of 15 percent.

[12] For a robustness check, we set $\eta_f = 1$ and find that the main results remain.

The real money holdings in the model contain two parts: one to fund the business projects, $nkp_k\phi$ and the other to finance the cash purchase of KW goods, $g(q^m)$. The curvature parameter in the utility function (a) is determined to match the elasticity of the money demand with respect to the nominal interest rate at -0.556, the same as Berentsen, Menzio, and Wright (2011). Lastly, the parameter in the matching function (ζ^{FC}) in FC is chosen to match the average share of firm money observed over the period from 1952 to 2007, as shown in Figure 1. The firm money share is measured as the ratio of firm money to the sum of firm money and consumer money. In the calibration, the firm money share takes two values. One is 0.38, using data on checkable deposits and currency held by firms and households, called Measure 1, and the other one is 0.33, using the extra data on money market funds and commercial paper held by these two types of agents, called Measure 2.[13]

Table 2 Calibration results, 1952-2007

	Baseline	
	Measure 1: 0.38	Measure 2: 0.33
μ^{MP}	0.5982	0.6446
σ	0.4302	0.4307
ζ^{FC}	10.9206	10.2917
θ	0.0881	0.0880
κ	2.3632	2.3534
A	0.2424	0.2624
a	0.7540	0.7568

NOTE: With these parameters, the model replicates the moments in Table 1. Columns 1 and 2 display the calibration results in which the targets for firm money share take the values of 0.38 and 0.33, respectively.

The calibrated model can successfully replicate all targets listed in Table 1. Table 2 reports the parameter values obtained in the second stage of the calibration. The values in the two columns, 0.38 and 0.33, correspond to the calibrations in which the predicted firm money shares target their respective empirical counterparts, as described above.

[13] The data for household money also contain the checkable deposits and currency held by non-profit organizations, which accounts for a small fraction, but cannot be separated from the money holdings of households.

4.2 Long–Run Behavior of Predicted Unemployment and Firm Money Share

Berentsen, Menzio, and Wright (2011) successfully generate the observed long-run positive relationship between the nominal interest rate (or inflation) and unemployment. This subsection shows that our model, featuring two types of money demands, nicely preserves the long-run property for unemployment, as in Berentsen, Menzio, and Wright (2011). Moreover, our model is able to generate the observed long-run behavior of the firm money share, which is absent in their study.

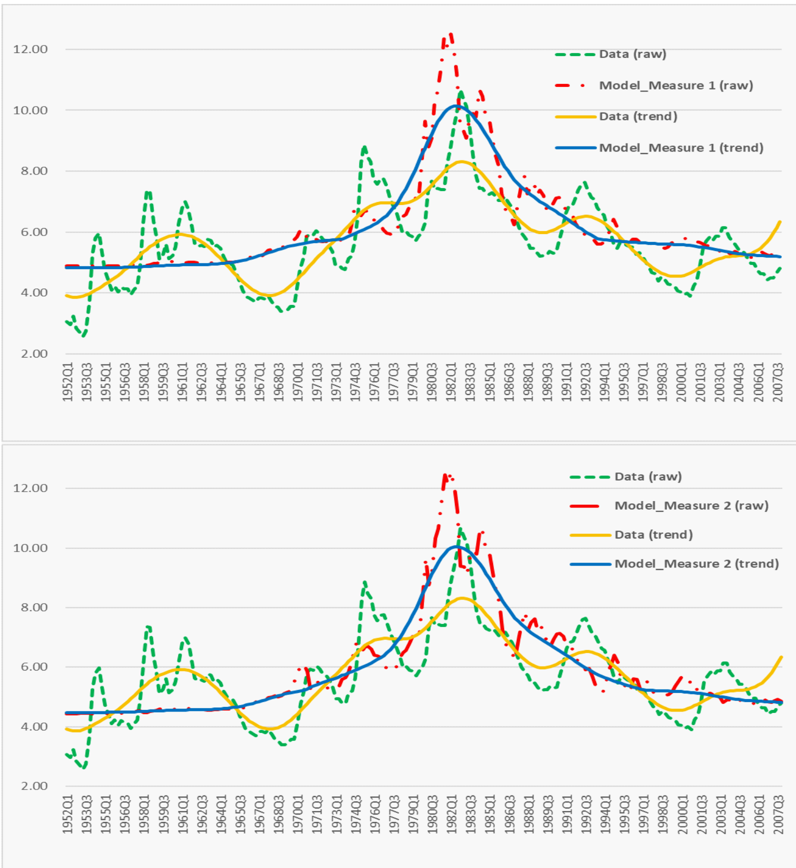

Figure 5 Counterfactual Unemployment, 1952–2007

NOTE: The figure compares the model predicted unemployment in a variety of cases with the empirical counterparts. The solid lines correspond to the simulation results, where the trend part of the time series of the nominal interest rate is imposed on the calibration models. The dashed line correspond to the simulation results, where the time series of the actual nominal interest rate is imposed on the calibration models.

We impose the trend part of the observed time series of the nominal interest rate on the calibrated model to simulate the equilibrium path for unemployment and the firm money share over the period from 1952 to 2007. Figure 5 plots the model predicted unemployment versus its actual time series over the sample period. The two solid lines in Figure 5 illustrate that the predicted unemployment and its actual time series display a clear co-movement. The overall trend of actual unemployment can be captured well by counterfactual unemployment. For robustness purposes, we redo the simulation by feeding the actual nominal interest rate into the model as in Berentsen, Menzio, and Wright (2011). The corresponding model predicted unemployment and its actual trend time series are reported in Figure 5 in dashed lines, and the contrast delivers a similar message.[14] With all these results, we conclude that the calibrated model can account for the overall pattern in unemployment solely by monetary policy, although there still remains much of the movement in the data to be explained by other factors.

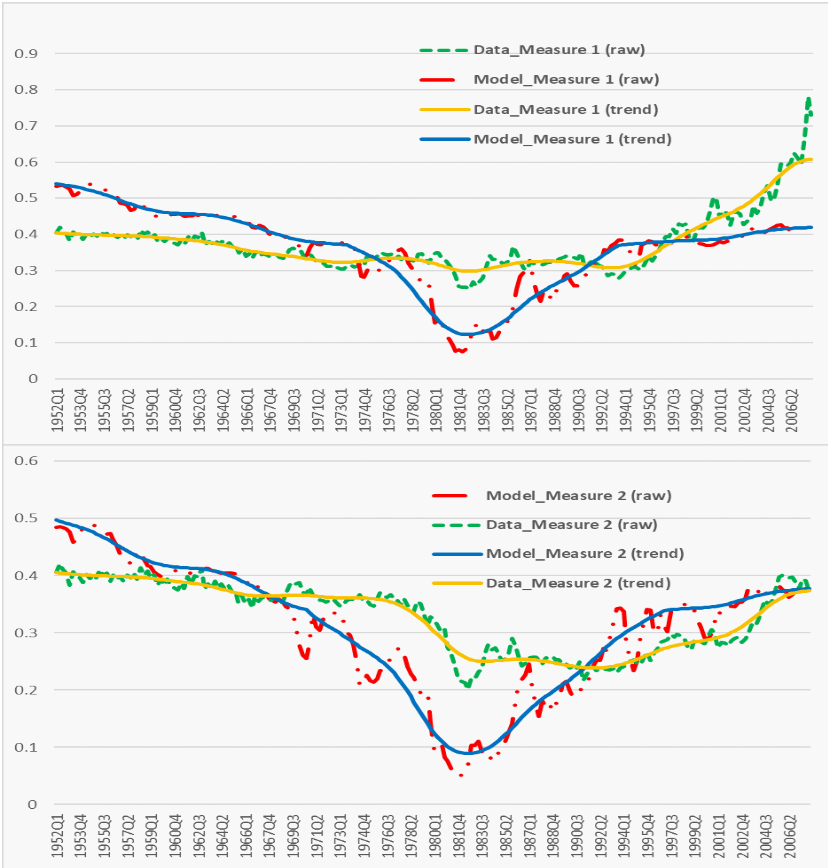

Figure 6: Counterfactual Firm Money Share, 1952–2007

NOTE: The figure compares the model predicted firm money shares in a variety of cases with the empirical counterparts. The solid lines correspond to the simulation results, where the trend part of the time series of the nominal interest rate is imposed on the calibration models. The dashed line correspond to the simulation results, where the time series of the actual nominal interest rate is imposed on the calibration models.

[14]The scatter plot of actual and counterfactual unemployment versus the observed nominal interest rate displays a clear positive relationship between these two variables.

With the confidence in the quantitative performance of the model provided by the results for unemployment, Figure 6 plots the model prediction for firm money share versus the actual time series. It demonstrates that, with the average firm money share used as a target in the calibration, our model is able to generate the overall pattern of firm money share regardless of which measure of money holdings is used in the model calibration. Particularly, when we use Measure 2 with the consideration of the money market funds and commercial papers, the model's fit improves, especially over the period after the 1990s, which squares well with the fact that, since the late 1980s, consumers and big firms have started to hold money market funds and commercial papers as an alternative method of holding liquidity. Admittedly, the movements of the predicted firm money share are stronger than those observed in the data, which might be explained by other factors that are missing in this model.

Table 3 reports the correlation between inflation and the model predicted firm money share in the two baseline models. Over the entire period from 1952 to 2007, the sign of the model predicted correlation is in line with that for the data in all cases. In particular, the magnitude over the first half of the sample period is close to its empirical counterpart. These findings suggest that, prior to the early 1980s, the monetary policy change might serve as the primary factor that drives how money holdings are distributed between firms and consumers. Later, other factors that are missing from the model might play a more important role in explaining the movements in the firm money share.

Table 3 Correlation of counterfactual firm money share with inflation

	Model		**Data**	
	Measure 1	Measure 2	Measure 1	Measure 2
Raw data results				
1952-2007	−0.676	−0.704	−0.416	−0.216
1952-1981	−0.874	−0.914	−0.772	−0.738
1982-2007	−0.855	−0.833	−0.529	0.510
Trend data results				
1952-2007	−0.698	−0.726	−0.460	−0.224
1952-1981	−0.913	−0.943	−0.876	−0.837
1982-2007	−0.828	−0.828	−0.524	−0.437

NOTE: The raw data results correspond to the simulation results, where the actual time series of the nominal interest rate is imposed on the calibration model. The trend data results correspond to the simulation results, where the trend of the nominal interest rate is imposed on the calibration model. The reported results are the correlation between the model predicted firm money share and the observed inflation.

4.3 Policy Effects: A Tale of Two Episodes

This subsection aims to quantify the effect of monetary policy (changes in inflation) on unemployment. Particularly, we explore the importance and richer transmission mechanisms of monetary policy induced by including money demand by both consumers and firms.

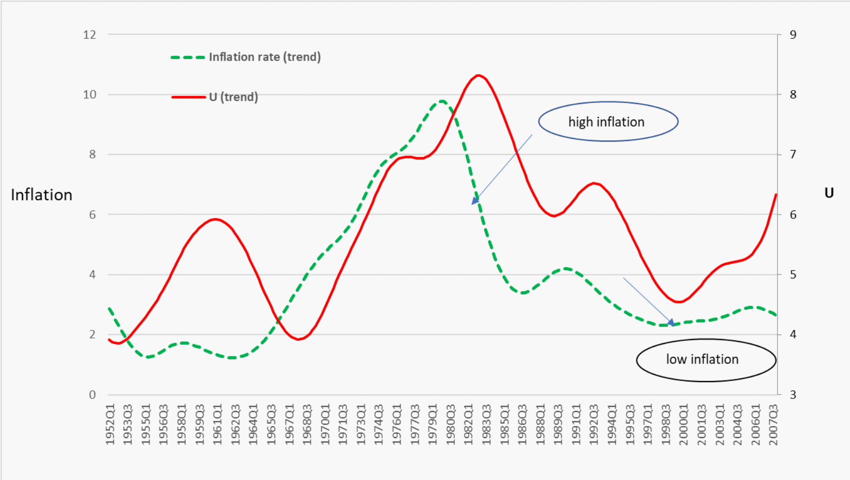

Figure 7: Inflation and Unemployment, 1952–2007

NOTE: The figure plots the observed trend of inflation and trend of unemployment over the period from 1952 to 2007.

To pursue this goal, we focus on two periods: a high-inflation episode and a low-inflation episode. As shown in Figure 7, the high-inflation episode refers to the period from 1980 to 1986, with an average inflation rate of 6.1 percent. The low-inflation episode is from 1996 to 2006, with an average inflation rate of 2.6 percent. The gap in the observed unemployment over these two episodes is 2.73 percentage points, and unemployment is higher in the high-inflation episode.

The strategy we adopt is to feed in the trend part of the actual inflation over the low-inflation episode to *re-calibrate* the model. Particularly, we ask the model to match the same targets as listed in Table 1 except for two deviations. One deviation is the target for average unemployment, which is replaced by the average unemployment over the low-inflation episode, at 5 percent. The other deviation is that we ask the model to match the observed firm money share over the low-inflation episode. The share is 0.48 using Measure 1 and 0.31 using Measure 2.

Table 4 Calibration results, 1996-2006 (low-inflation episode)						
	Baseline		**Benefit**	**Depreciation**	**Curvature**	**Markup**
	Measure 1: 0.48	Measure 2: 0.31				
μ^{MP}	0.6116	0.6773	0.6292	0.6180	0.6898	0.6764
σ	0.4686	0.4411	0.3909	0.4818	0.4492	0.4432
ζ^{FC}	7.7481	9.6753	14.9945	10.2056	9.8530	9.8160
θ	0.1101	0.0601	0.0946	0.0852	0.0612	0.0871
κ	2.3653	2.3606	2.2819	2.3027	2.3268	2.4014
A	0.1984	0.2400	0.2713	0.2141	0.2404	0.2576
a	0.7036	0.8235	0.7538	0.7621	0.8386	0.8084

NOTE: With these parameters, the model replicates the moments in Table 1 except for two targets, namely unemployment and firm money share. Columns 1 and 2 indicate the baseline calibration results, where the targets for firm money share take the values of 0.48 and 0.31, respectively. Columns 3 to 6 consider some alternative calibrations in which the values of the key parameters ς, η_f, φ, and θ are changed. In Column 3, the UI benefits change the value from 0.4 to 0.5. Column 4 reports the results, where no capital losses occur upon exogenous match dissolutions (i.e., η_f changes from 0 to 1). Column 5 corresponds to a change in the curvature of the cost production in KW, from 1 to 1.05. Column 6 corresponds to a change in markup ratio from 0.3 to 0.4.

The calibration results are reported in Table 4. The first two columns report the results for the baseline models where two measures of money holdings are considered. Following Berentsen, Menzio, and Wright (2011), we also consider some alternative calibrations in which the values of the key parameters ς, η_f, φ, and θ are changed, with Measure 2 being used for the money holdings. In the third column, which is termed Benefit, ς is set at 0.5, as used in Berentsen, Menzio, and Wright (2011). In the fourth column, which is termed Depreciation, η_f is set to be 1. In the fifth column, which is termed Curvature, φ is set to be 1.05. In the last column, termed Markup, the target for the markup ratio changes to 0.4.

The parameters $\{\varsigma, \eta_f, \varphi, \theta\}$ influence the effects of inflation on unemployment as follows: An increase in UI benefits (higher ς) makes households choosier about job opportunities and leads to higher unemployment. Similarly, making production in KW more costly (higher φ) reduces the profit of the firm in KW, which hurts job opening, thus resulting in higher unemployment. Reducing capital loss in an exogenously separated match (higher η_f) increases the firm's profit, thus reducing unemployment. Lastly, a rise in the markup ratio of firms generates two opposing effects on unemployment. On one hand, the firm's bargaining power in KW becomes stronger (higher θ) in response to a higher markup ratio. The improved profitability of firms results in higher vacancies in the labor market, thus lowering unemployment. On the other hand, higher real profits of firms in KW require a lower matching probability by the firms in KW (lower λ_f) to keep the value of a vacancy U_0^f unchanged, as suggested by Equation (28). As a result, the firms' incentive for job creation is discouraged, and unemployment gets higher. The intuition discussed above suggests that the predicted influence on unemployment should increase in the cases of Benefits and Curvature, and decrease in the case of Depreciation, while remaining uncertain in the case of Markup. Although these alternative parameter values are arbitrarily set, they are useful for illustrating how the results depend on the parameter values.

With the model successfully calibrated, we increase inflation by 3.50 percentage points, which is the observed increase in the average inflation from the low-inflation episode to the high-inflation episode, and we impose the enlarged time series of inflation on the re-calibrated model to simulate the response of unemployment.

4.3.1 Response of Unemployment

Table 5 reports the predicted response of unemployment. In the baseline calibration (the first line), the increase in inflation raises unemployment by 3.04 and 2.85 percentage points, respectively, depending on which measure of money holding is used, which is close to its empirical counterpart, at 2.73 percentage points.

The results remain robust when some key parameters take alternative values, as shown in the lower panel in Table 5 (the first line). Confirming the intuitions discussed above, compared with the results for Measure 2, the effect on unemployment rates significantly increases in the case of Benefit where workers face higher outside options (higher ς). In contrast, unemployment reacts less pronounced in the case of Depreciation, where capital experiences no losses upon an exogenous match dissolution ($\eta_f = 1$). The effect slightly increases in the cases of Curvature, where production in KW becomes more costly (larger φ), and in Markup, where the markup ratio of the firm gets higher (higher θ). In the case of Markup, the rise in the markup ratio gives rise to two opposing effects on unemployment, and the effect that operates through the matching probability of the firm in KW dominates, which leads to a higher response of unemployment.

Table 5 Response of unemployment					
	Model				**Data**
Baseline results:					
	Measure 1		**Measure 2**		
Model with firm money channel	3.04		2.85		2.73
Model without firm money channel	1.27		1.35		2.73
Robustness check results:					
	Benefit	**Depreciation**	**Curvature**	**Markup**	
Model with firm money channel	3.40	2.63	2.87	2.93	2.73
Model without firm money channel	1.86	1.38	1.50	1.39	2.73

NOTE: Across the two baseline cases (upper panel) and four cases with the alternative values of ς, η_f, φ, and θ (lower panel) the comparison shows significant and sizable increases in unemployment in response to rising inflation. In the lower panel, with Measure 2 used to measure the firm money share, Column 1 reports the result when the UI benefits change the value from 0.4 to 0.5. Column 2 reports the results in which no capital losses occur upon an exogenous match dissolution ($\eta_f = 1$). Column 3 reports the results when the curvature of the cost function increases from 1 to 1.05. In Column 4, the target for markup ratio changes from 0.3 to 0.4. The last column of the table reports the change in unemployment between the two inflation episodes observed in the data.

4.3.2 Importance of Including Firm Money

Compared with the studies that consider only consumer money, monetary policy directly and indirectly affects the money demand by firms in our model. This subsection conducts a decomposition exercise to explore the contribution of including firm money into the analysis of the effect of monetary policy on unemployment.

As discussed in Propositions 2 to 4, the direct effect on firm money operates through the channel of optimal money allocation, which requires equal rates of return from financing investments in firms and funding the consumption in KW. As a result, increasing inflation dampens money demand by firms as it does for consumer money (opportunity cost of holding money). The indirect effect on firm money operates via an endogenous positive spillover effect from consumer money to firm money, which further imposes a downward pressure on firm money and makes it more responsive to the increase in inflation. The direct and indirect effects jointly generate a stronger effect of monetary policy on unemployment as opposed to that in the model with only consumer money. Moreover, these two effects also contribute to the endogenous relationship between inflation and the firm money share.

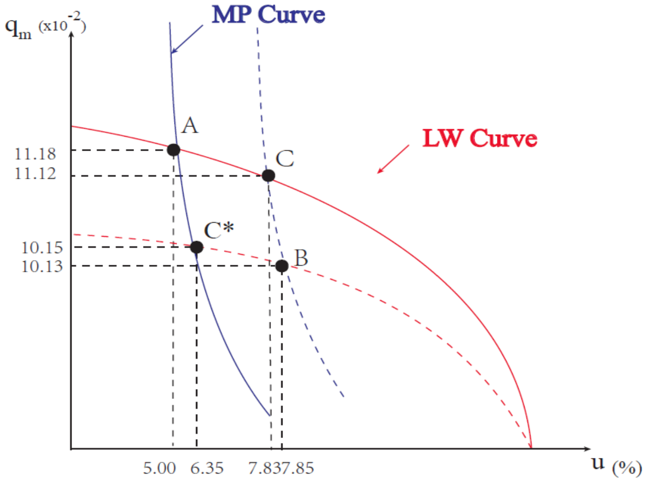

Figure 8: Decomposition of Monetary Policy Effects

NOTE: The solid lines are the LW and MP curves. The dashed lines display the shifts in these curves in response to increasing inflation. Point A is the equilibrium before the change, and Point B is the equilibrium after the change. Point C^* is the equilibrium in which the change in inflation only affects the LW curve.

Figure 8 graphically displays the effect of inflation (or the nominal interest rate) on the two money holdings via the shifts of the MP curve and LW curve. The shift in MP captures the direct effect on firm money, whereas the shift in LW reflects the direct effect on consumer money, which, in return, results in an indirect effect on firm money via the spillover effect. Hence, the strategy used in the decomposition exercise is to shut down the shift of these curves. That is, when we shut down the direct effect of monetary policy on firms (firm money channel), the MP curve remains unchanged, whereas only the LW curve moves. Similarly, when we eliminate the direct effect of monetary policy on consumers (consumer money channel), the LW curve remains unchanged, whereas only the MP curve moves. To shut down the shift in LW, we impose the before-change inflation (or the nominal interest rate) on Equation (21). To eliminate the shift in MP, we impose the before-change inflation (or nominal interest rate) on Equation (25). As suggested by Proposition 4, the MP curve remains when we fix the value of a job vacancy, U_0^f, in FC.

Figure 8 illustrates the decomposition results for the baseline case, where Measure 2 is used for the measure of money holdings. In the baseline case, in the face of increasing inflation, the LW curve shifts downwards, and the MP curve shifts to the right. As a result, the equilibrium moves from A to B, and unemployment increases by 2.85 percentage points (from 5 percent to 7.85 percent). Eliminating the firm money channel (fixed

MP) moves the equilibrium from A to C^* in response to the same change in inflation, and the movement in unemployment substantially decreases by 1.5 percentage points, about 53 percent of the overall effect on unemployment in the baseline case. In sharp contrast, removing the consumer money channel (fixed LW) moves the equilibrium from A to C, while the policy effect on unemployment declines by only 0.02 percentage points. The comparison sheds light on the quantitative importance of firm money, which suggests that it is important to take into account firm money in evaluating the effects of monetary policy on unemployment. Otherwise, the predictions might be significantly *understated.* Our decomposition results are in line with those in Berentsen, Menzio, and Wright (2011). In the absence of firm money demand, their model accounts for around $40 - 60$ percent of changes in unemployment in responses to the observed inflation changes.

The decomposition exercise is also useful to illustrate the quantitative importance of the spillover effect from consumer money to firm money, the complementarity channel emphasized in this work. Eliminating the firm money channel (fixed MP) means that monetary policy only affects firm money through the positive spillover effect of consumer money, the indirect effect of inflation on firm money. Indeed, the simulation results show that the complementarity channel is quantitatively sizable. When the equilibrium moves from A to C^*, consumer money falls, and firms also reduce their money holdings. Quantitatively, the decline in firm money accounts for 70 percent of the overall decline in the baseline case (A to B) in response to the same change in inflation. This result suggests that considering the complementarity between consumer money and firm money might be important for advancing our understanding of the determination of firm money, which has not *yet* received much attention in the literature.

Table 5 summarizes the decomposition results for the other cases in which Measure 1 is used for the measure of money holdings (upper panel, the second line), and some key parameters are calibrated differently (lower panel, the second line). Consistent with the finding in Figure 8, the results demonstrate that, in all cases, removing the firm money channel significantly reduces the response of unemployment to the same change in inflation. Moreover, the remaining response of unemployment reflects the quantitative importance of the complementarity channel.

4.3.3 Importance of the Endogenous Money Allocation Adjustment

As reported in Table 6, an increase in inflation (or the nominal interest rate) moves the money allocation towards the consumption of KW goods (consumer money) and away from investment in firms (firm money). The firm money share decreases sharply from 31 percent to 13 percent. In the model, the adjustment of the money allocation is driven by the direct effect of inflation on two types of money holdings, and the indirect effect on firm money that operates via the complementarity channel, as discussed above. The above-mentioned change in the firm money share suggests that firm money is more responsive to the increment in inflation compared to consumer money in terms of magnitude.

Table 6 Contribution of money allocation adjustment

	Before change	After change	
	Baseline (Measure 2)	w/ adj.	w/o adj.
Total money holdings	0.2130	0.1525	0.1983
Firm money	0.0671	0.0199	0.0619
Firm money share	**31%**	**13%**	**31%**
Vacancies	0.0275	0.0079	0.0253
Unemployment	0.0500	0.0785	0.0512

NOTE: The comparison between with and without money allocation changes shows that the response of unemployment is lower when the firm money share is fixed.

To examine the importance of the endogenous adjustment of the money allocation in generating the amplified effect on unemployment, we conduct a counterfactual exercise. Particularly, we ask the model to allocate the money holdings over consumption and business investment in the same way as its before-change level. Thus, with the same inflation change, the households in this counterfactual exercise still allocate 31 percent of the money holdings to business investment. Table 6 reports the results in the last column. The resulting unemployment drops substantially to 5.12 percent.

The reasons for this result are two-fold. First, imposing the fixed money allocation eliminates the effect of monetary policy on firm money. Meanwhile, imposing a fixed money allocation results in a misallocation of money between consumption and business investment. The optimal allocation rule stated in Equation (24) is violated, and more money is allocated to fund business projects than would occur in an optimal allocation.[15]

[15] We find that the marginal benefits of business investment are smaller than those with the consumption in this counterfactual exercise, indicating the existence of misallocation to business projects.

These two factors work together and greatly undo the effect of inflation on job creation, as shown in Table 6. Therefore, unemployment remains almost unchanged. Of note is that this exercise demonstrates the importance of endogenous money allocation in the transmission of monetary policy.

5 Conclusions

While previous studies of money often focus on consumer money or firm money, this paper builds a quantitative framework incorporating both of them, which makes it possible to study the macroeconomic and policy implications related to money allocation between consumers and firms.

Using this framework, we find that monetary policy indeed can play a significant role in explaining the movement in money allocation. The calibrated model is able to account for the overall pattern of the firm money share observed over the period from 1952 to 2007, and its negative relationship with long-run inflation.

The framework with two types of money demand proves useful in evaluating the effects of monetary policy on unemployment. The policy-related response of unemployment operates through both the consumer money channel and firm money channel. The latter channel, absent in the existing literature, proves quantitatively important. In addition, the endogenous money allocation between consumers and firms is instrumental in understanding monetary policy transmission.

The consideration of both consumer money and firm money also provides interesting insights into the determinants of firm money. The complementary relationship between the two types of money demand found in this work demonstrates that policies, which affect consumer money, would also influence firm money. This result suggests that even if one is only concerned with firm money, it might be useful to consider this general equilibrium effect of consumer money into the analysis.

Our approach to incorporating different uses of money opens new research avenues. Although we focus on the demand for money by firms for investment purposes, other reasons exist regarding why firms need money, such as working capital, research and development, and mergers and acquisitions. Future studies can explore how money is allocated not only between firms and consumers but also between the different uses of liquidity by firms. One of the possible extensions could be to consider how other macroeconomic factors influence the money allocation for different uses.

6 Appendix

Proof of Proposition 4: Using (27), we can write

$$R - w = \frac{\chi \left[1 - \beta \left(1 - \delta\right)\right] \left(R - \varsigma\right) - \left(1 - \chi\right) \left[1 - \beta \left(1 - \delta - \lambda_h\right)\right] Q\kappa}{1 - \beta \left(1 - \delta\right) + \left(1 - \chi\right) \beta \lambda_h},$$

where the expected periodic revenue of firms in the steady state is as follows according to Equation (15) is as follows:

$$R = y + \alpha_f \mu \left[g\left(q^*\right) - c\left(q^*\right)\right] + \alpha_f \left(1 - \mu\right) \left[g\left(q_m\right) - c\left(q_m\right)\right].$$

As is generally true with Nash bargaining, the trade surplus of firms, $g\left(q\right) - c\left(q\right)$, in a meeting equals $\frac{\theta[v(q)-c(q)]}{(1-\theta)v'(q)/c'(q)+\theta}$ and increases in q if $q \leq q^*$. Therefore, R increases in q_m and u because $\alpha_f = M^{KW}\left(1, 1-u\right) / \left(1-u\right)$ increases in u.

The MP curve can be written as follows:

$$U_0^f / \kappa = \left(1 - \delta_k\right) + e\left(u, q_m\right). \tag{29}$$

A sufficient condition for the RHS of Equation (29)or $e\left(u, q_m\right)$ to be increasing in λ_f is the following:

$$y - \varsigma - \left\{\kappa - \beta \left[\left(1 - \delta\right)\left(1 - \delta_k\right) + \delta \eta_f\right] \kappa\right\} > 0,$$

which merely requires that the periodic total surplus of a worker-firm pair is positive. To summarize, an increase of u makes λ_h lower and λ_f higher, so the RHS of Equation (29) increases in u (and q_m). Therefore, the MP curve slopes downward in the $\left(u, q_m\right)$ space.

Using Equation (29) and Proposition 3, anything that affect U_0^f/κ would shift the MP curve. The rest of this proposition involves comparing $U_0^f/\kappa - \left(1 - \delta_k\right)$ and $e\left(0, q^*\right)$, $e\left(1, q^*\right)$, or $e\left(1, 0\right)$. If $e\left(0, q^*\right) < \left(r + \delta_k\right)/\sigma < e\left(1, q^*\right)$, then for the lower bound of U_0^f/κ: $\left(r + \delta_k\right)/\sigma + \left(1 - \delta_k\right)$, we know the MP curve pass a point $\left(u_1, q^*\right)$ in the $\left(u, q_m\right)$ space, where $u_1 \in (0, 1)$. Otherwise, it is impossible for the MP curve to cross the LW curve.

More specifically, if $U_0^f/\kappa < (1-\delta_k) + e(1,q^*)$, then the MP curve passes a point (u_1, q^*) in the (u, q_m) space, where $u_1 \in (0,1)$. if $U_0^f/\kappa < (1-\delta_k) + e(1,0)$, then it passes through $(u_2, 0)$, where $u_2 < 1$; if $U_0^f/\kappa > (1-\delta_k) + e(1,0)$, then it passes through $(1, q_1)$, where $q_1 > 0$.

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