Endogenous Production Networks and Non-Linear Monetary Transmission

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CREi, UPF & BSE

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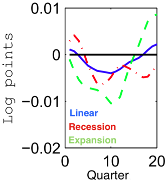
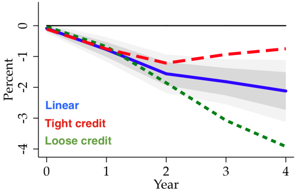
Motivation: non-linear monetary transmission to GDP

Recession vs Expansion



Tenreyro and Thwaites (2016)

Tight vs Loose credit



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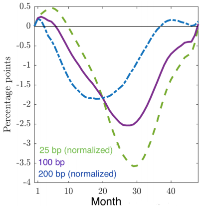
Large vs Small shocks



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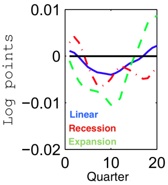

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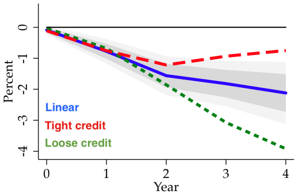

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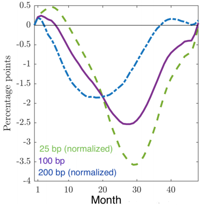
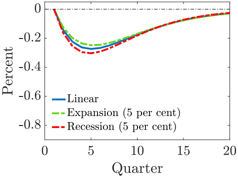
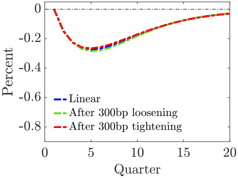
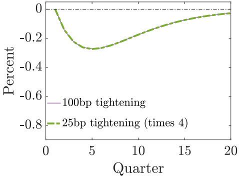


Ascari and Haber (2021)

- Tightening in a fully non-linear medium-scale New Keynesian model:







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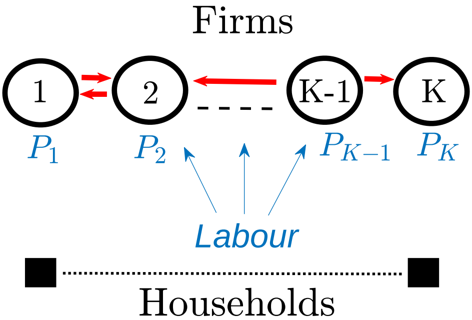
3. Novel model-free empirical evidence on network responses to shocks

A TWO-PERIOD SETTING

Overview



Firms: production and choice of suppliers

- K sectors, continuum of firms Φ_k in each sector

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- *Roundabout Production (for firm j in sector k)*:

$$Y_k(j) = \psi(S,\Omega)\mathcal{A}_k(S_k)N_k(j)^{1-\sum_{r\in S_k}\omega_{kr}}\prod_{r\in S_k} Z_{kr}(j)^{\omega_{kr}}, \quad \forall k, \forall j \in \Phi_k$$

where $S_k \subseteq \{1, 2, ..., K\}$ is sector k's choice of suppliers, $\mathcal{A}_k(.)$ is the technology mapping, $\omega_{kr} = [\Omega]_{kr}$ are input-output weights, $N_k(j)$ is labor, $Z_{kr}(j)$ is intermediates

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- *Marginal Cost (conditional on supplier choice)*:

$$MC_k = \frac{1}{\mathcal{A}_k(S_k)} W^{1-\sum_{r\in S_{kt}}\omega_{kr}}\prod_{r\in S_k} P_r^{\omega_{kr}}, \qquad \forall k, \forall j \in \Phi_k$$

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- *Optimal Network*:

$$S_k^* \in \arg\min_{S_k} MC_k(S,P), \qquad \forall k$$

  where $S = [S_1, S_2, ..., S_K]'$ and $P = [P_1, P_2, ..., P_K]'$

Pricing, Households and Monetary Policy

- *Optimal reset price*:

$$\overline{P}_k = (1 + \mu_k) MC_k, \quad (1 + \mu_k) = (1 + \tau_k) \frac{\theta}{\theta - 1}$$

where τ_k is tax, θ is within-sector elasticity of substitution

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- *Money supply rule*: $\mathcal{M} = \mathcal{M}_0 \exp(\varepsilon^m)$

Pricing Equilibrium

BASELINE $(\varepsilon^m = 0)$

Consider variations in the baseline pair $(\mathcal{A}, \mathcal{M}_0)$

Baseline: a two-sector example

- Two sectors: $\omega_{kk} = 0, \quad \tau_k = -\frac{1}{\theta}, \quad \theta \to 1^+, \quad P_{k,0} = 1, \quad \forall k = 1, 2$

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	Sector 1	Sector 2
$a(.)$	$a_1(\varnothing) = 1, \quad a_1(\{2\}) = \bar{a}$	$a_2(\varnothing) = 1, \quad a_2(\{1\}) = \bar{a}$
Ω	$\omega_{12} = \omega_{c1} = 0.5$	$\omega_{21} = \omega_{c1} = 0.5$
α	$\alpha_1 = 0$	$\alpha_2 = 0.5$

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- Marginal costs: $mc_{k,0} = -a_k(S_{k,0}) + m_0 + \mathbf{1}_{-k \in S_{k,0}} \frac{1}{2}(p_{-k,0} - m_0)$

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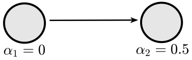
- Optimal network choice over marginal costs:

	$S_2 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(m_0 - 1, m_0 - 1)$	$(m_0 - 1, -\bar{a} + m_0 - \frac{1}{2})$
$S_1 = \{2\}$	$(-\bar{a} + \frac{3}{4}m_0 - \frac{1}{4}, m_0 - 1)$	$(-\frac{10}{7}\bar{a} + \frac{5}{7}m_0, -\frac{12}{7}\bar{a} + \frac{6}{7}m_0)$

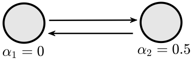
Recession vs Expansion (varying $\overline{a}$)

Recession: $\overline{a} = 0$

Normal: $\overline{a} = 0.65$



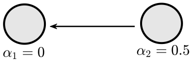
Expansion: $\overline{a} = 0.8$



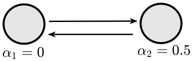
Tight vs Loose initial money (varying m_0)

Tight money: $m_0 = 0$

Normal money: $m_0 = 4$



Loose money: $m_0 = 8$



Baseline: density of the network

Lemma (Baseline supplier choices)

Suppose the marginal cost is quasi-submodular in $(S_k, \mathcal{A}_k(S_k)), \forall k$. *Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$ *such that either* $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 = \underline{\mathcal{M}}_0$ *or* $\overline{\mathcal{A}} = \underline{\mathcal{A}}, \overline{\mathcal{M}}_0 \geq \underline{\mathcal{M}}_0$, *then:*

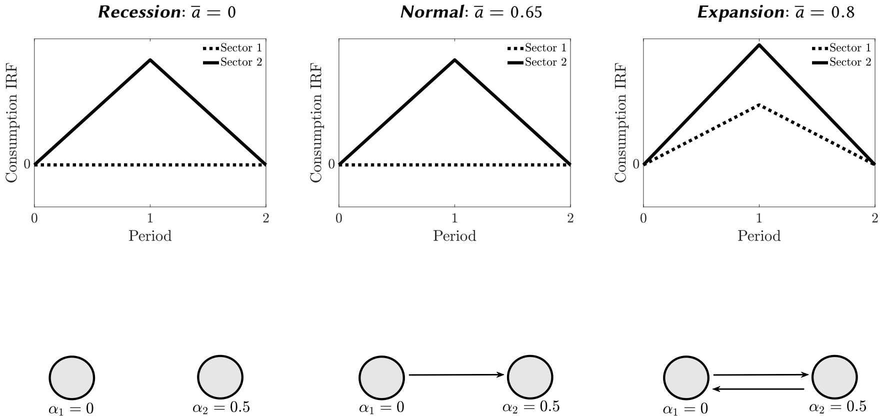
$$S_k(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) \supseteq S_k(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$$

for all $k = 1, 2, ..., K$.

MONETARY SHOCKS

Small Monetary Shocks

IRFs to a small monetary expansion across the cycle $\overline{a}$



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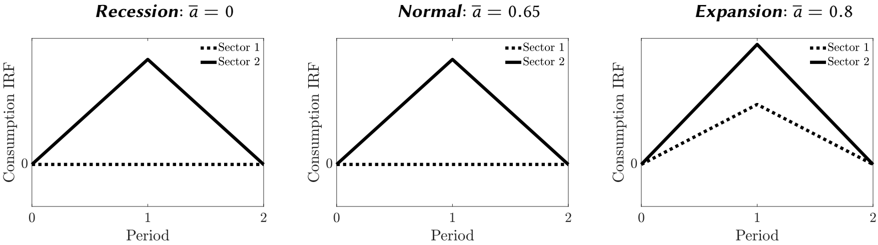
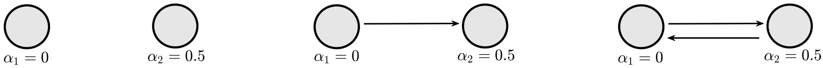


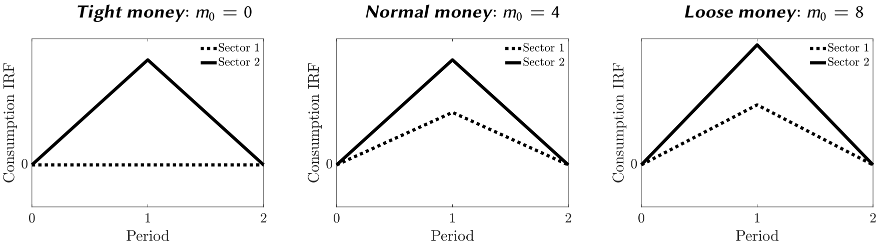


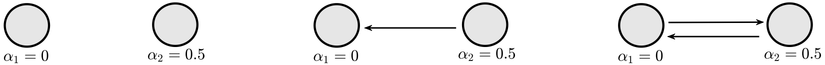
Proposition (Cycle dependence)

For two otherwise identical baselines with $\overline{\mathcal{A}} \geq \underline{\mathcal{A}}$*, following a monetary shock* ε^m *that is small with respect to both baselines:* $\quad c_k(\varepsilon^m; \overline{\mathcal{A}}) \geq c_k(\varepsilon^m; \underline{\mathcal{A}}), \quad \forall k.$

More

IRFs to a small monetary expansion across initial m_0





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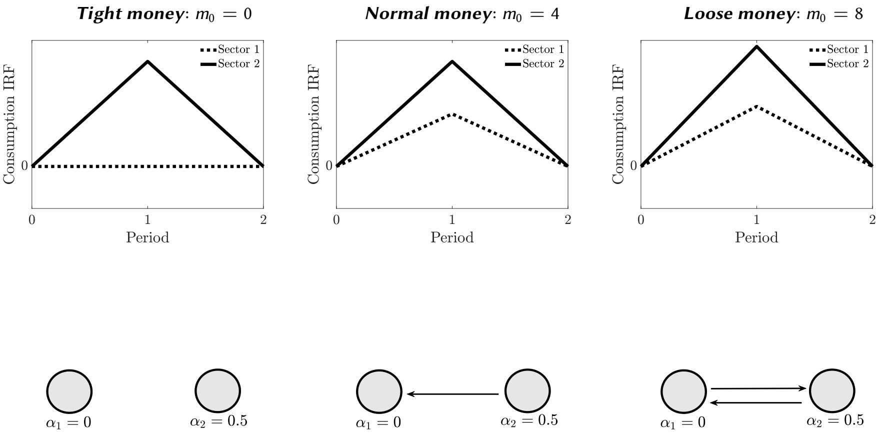


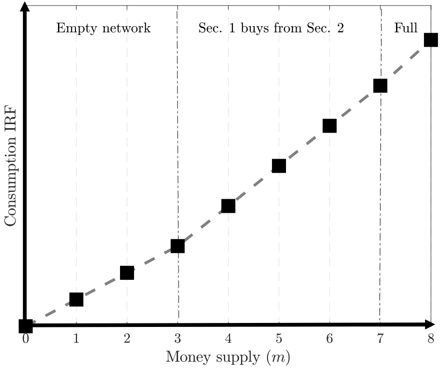
Proposition (Path dependence)

For two otherwise identical baselines with $\overline{\mathcal{M}_0} \geq \underline{\mathcal{M}_0}$, following a monetary shock ε^m that is small with respect to both baselines: $\quad c_k(\varepsilon^m; \overline{\mathcal{M}_0}) \geq c_k(\varepsilon^m; \underline{\mathcal{M}_0}), \quad \forall k.$

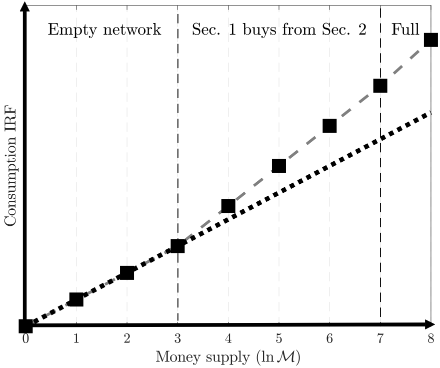
More

Large Monetary Shocks

Large monetary expansions



Large monetary expansions



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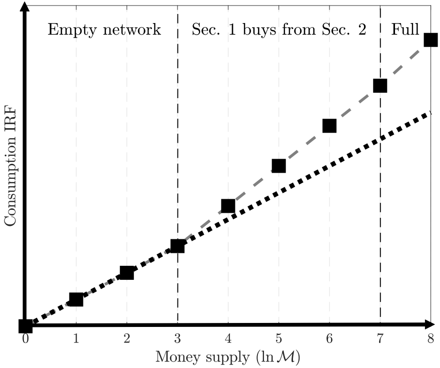
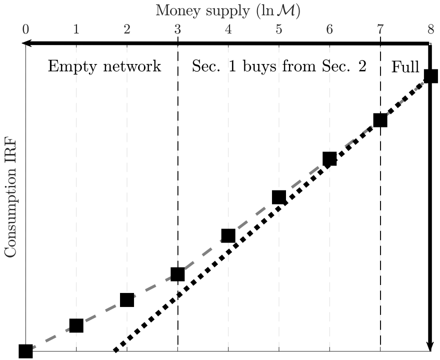


Proposition (Size dependence)

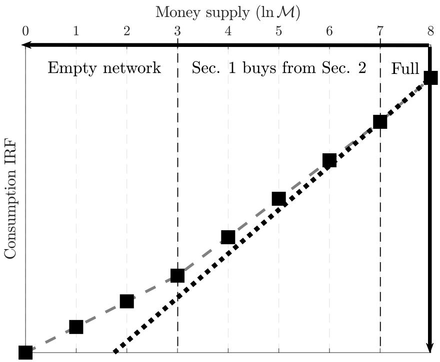
Starting from network S_0, a large monetary expansion E^+ has a more than proportional effect on GDP than a small monetary expansion ε^+: $C(E^+)/C(\varepsilon^+) \geq C(E^+; S_0)/C(\varepsilon^+; S_0)$.

Large monetary contractions



Large monetary contractions

Next



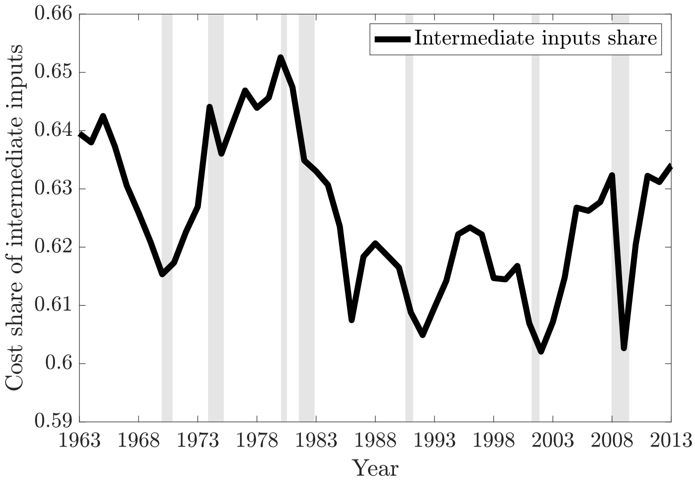
Proposition (Size dependence)

Starting from network S_0, a large monetary contraction E^- has a less than proportional effect on GDP than a small monetary contraction ε^-: $C(E^-)/C(\varepsilon^-) \leq C(E^-;S_0)/C(\varepsilon^-;S_0)$.

EMPIRICAL EVIDENCE

Sectoral Data

Cost share of intermediate inputs (BEA, US)



Cyclical fluctuations in intermediates intensity

- Use BEA annual sectoral accounts (KLEMS) to construct sectoral measures of intermediates intensity between 1987-2017 for 65 sectors (Summary level):

$$\delta_{kt} = \frac{\text{Expenditure on Intermediates}_{kt}}{\text{Expenditure on Intermediates}_{kt} + \text{Compensation of Employees}_{kt}}$$

which exactly matches to $\sum_{r \in S_{kt}} \omega_{kr}, \forall k$, in our theoretical framework

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- Linear local projection:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}$$

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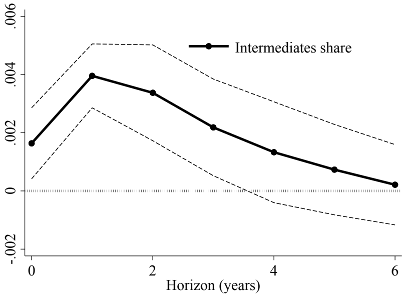
- Non-linear local projection:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H},$$

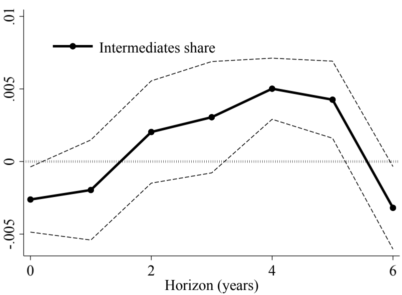
- Use Fernald's TFP shocks and Romer-Romer monetary shocks

Intermediates intensity response: linear local projection

(a) Productivity expansion (+1%)



(b) Monetary easing (-100bp)



Cyclical fluctuations in intermediates intensity

- Use BEA annual sectoral accounts (KLEMS) to construct sectoral measures of intermediates intensity between 1987-2017 for 65 sectors (Summary level):

$$\delta_{kt} = \frac{\text{Expenditure on Intermediates}_{kt}}{\text{Expenditure on Intermediates}_{kt} + \text{Compensation of Employees}_{kt}}$$

  which exactly matches to $\sum_{r \in S_{kt}} \omega_{kr}, \forall k$, in our theoretical framework

- Linear local projection:

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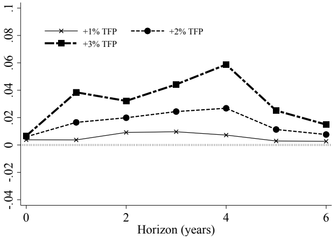
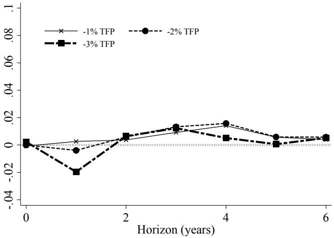
Productivity shocks: non-linear local projection

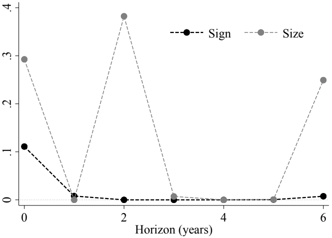
(a) Productivity expansions



(b) Productivity contractions



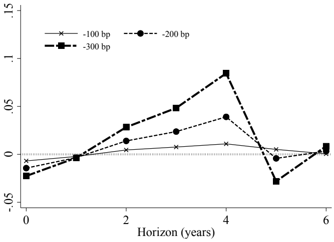
(c) p-values for non-linearities



Model-based

Monetary shocks: non-linear local projection

(a) Monetary expansions



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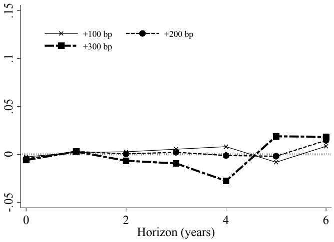
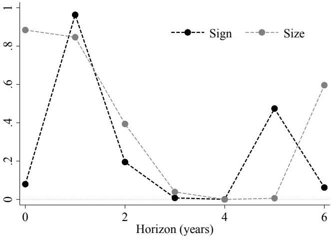


(c) p-values for non-linearities



▶▶ Model-based ▶▶ Firm-level

Conclusion

- Develop a sticky-price New Keynesian model with endogenous input-output linkages across sectors, which delivers empirically realistic cyclical variation in production networks
- Results rationalize observed non-linearities associated with monetary transmission: cycle dependence, path dependence and size dependence
- Novel empirical evidence in support of the mechanism

Conclusion

- Develop a sticky-price New Keynesian model with endogenous input-output linkages across sectors, which delivers empirically realistic cyclical variation in production networks
- Results rationalize observed non-linearities associated with monetary transmission: cycle dependence, path dependence and size dependence
- Novel empirical evidence in support of the mechanism
- Future work to develop and enhance the research agenda
  - Formation of input-output linkages across countries, with implications for monetary policy
  - Impact of uncertainty on linkage formation under forward-looking behaviour
  - Misallocation and inefficient production networks: cross-country differences
  - Government policies to address inefficient networks in a decentralized equilibrium

APPENDIX

Firms: pricing under nominal rigidities

- *Profit maximization*:

$$\max_{P_k^*(j)} \Pi_k(j) = [P_k^*(j) Y_k(j) - (1+\tau_k) MC_k Y_k(j)] \quad \text{s.t.} \quad Y_k(j) \quad = \left(\frac{P_k(j)}{P_k} \right)^{-\theta} Y_k$$

- *Optimal reset price*:

$$\overline{P}_k = (1+\mu_k) MC_k, \qquad (1+\mu_k) = (1+\tau_k) \frac{\theta}{\theta - 1}, \qquad \forall k, \forall j \in \Phi_k$$

- *Calvo lotteries (probability of non-adjustment α_k)*:

$$P_k = \left[\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k) \left\{ \frac{1+\mu_k}{\mathcal{A}_k(S_k)} W \prod_{r \in S_k} \left(\frac{P_r}{W} \right)^{\omega_{kr}} \right\}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \forall k$$

Back

Equilibrium

- *Flow Utility*: $\mathcal{U} = \log C - N, \quad C \equiv \prod_{k=1}^{K} C_k^{\omega_{ck}}.$

- *Cash-in-Advance Constraint*: $P^c C = \mathcal{M}$

- *Money supply rule*: $\mathcal{M} = \mathcal{M}_0 \exp(\varepsilon^m)$

- *Equilibrium fixed point problem*:

$$P_k = \left[\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k) \left\{ \min_{S_k} \frac{1+\mu_k}{\mathcal{A}_k(S_k)} \mathcal{M} \prod_{r \in S_k} \left(\frac{P_r}{\mathcal{M}} \right)^{\omega_{kr}} \right\}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \forall k$$

Proposition (Equilibrium)

Equilibrium in my economy: (i) exists; (ii) sectoral prices and final consumptions are unique; (iii) supplier choices and remaining quantities are generically unique.

Back

Small shock $\varepsilon^m \neq 0$ across baselines

Proposition

Let $c_k(\mathcal{A}, \mathcal{M}_0) \equiv \ln C_k(\mathcal{A}, \mathcal{M}) - \ln C_k(\mathcal{A}, \mathcal{M}_0), \forall k$. *Consider any two baseline pairs* $(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)$, $(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0)$, *and* $\varepsilon^m > 0$ *which is small, and* $P_{k,0} = (1 + \mu_k)g(\mathcal{M}_0)MC_k(\mathcal{A}, \mathcal{M}_0)$*:*

$$\mathbb{c}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathbb{c}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0) = \left[\mathcal{L}(\overline{\mathcal{A}}, \overline{\mathcal{M}}_0) - \mathcal{L}(\underline{\mathcal{A}}, \underline{\mathcal{M}}_0)\right] \mathcal{E}^m$$

where $\mathbb{c} = [c_1, c_2, ..., c_K]'$, $\mathcal{E}^m = [\varepsilon^m, \varepsilon^m, ..., \varepsilon^m]'$ *and* $\mathcal{L}$ *is a Leontief inverse given by:*

$$\mathcal{L}(\mathcal{A}, \mathcal{M}_0) = [I - (I - A)\Gamma(\mathcal{M}_0)\Omega(\mathcal{A}, \mathcal{M}_0)]^{-1} [I - (I - A)\Gamma(\mathcal{M}_0)]$$

where $A = diag(\alpha_1, ..., \alpha_K)$, $\Gamma(\mathcal{M}_0) = diag(\gamma_1(\mathcal{M}_0), ..., \gamma_K(\mathcal{M}_0))$, $\gamma_k = \frac{1}{\alpha_k(g(\mathcal{M}_0))^{1-\theta}+1-\alpha_k}$ *and* $[\Omega(\mathcal{A}, \mathcal{M}_0)]_{kr} = \omega_{kr}$ *if* $r \in S_k$ *and 0 otherwise.*

Back

INFINITE-HORIZON MODEL

Pricing in the infinite-horizon model

- Assume "finite-horizon" Calvo (1983) pricing

Assumption (Nominal rigidities)

There exists a finite, deterministic cut-off time period $T > 1$*, such that for* $1 \leq t \leq (T-1)$ *each firm has a sector-specific probability of price adjustment* $\alpha_k \in (0,1)$ *and prices are fully flexible for* $t \geq T$

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- Two-period model mimicked by $T = 2$
- Conditional on the path of supplier choices, sector-level solutions can be obtained by backward induction

Optimal networks in the infinite-horizon model

- Firms can re-optimize their supplier choices in every time period

Optimal networks in the infinite-horizon model

- Firms can re-optimize their supplier choices in every time period

- Consider productivity mapping from Acemoglu and Azar (2020), augmented with an aggregate productivity term

Assumption (Productivity mapping)

For every sector $k = 1, 2, ..., K$ *the productivity mapping* $\mathcal{A}_{kt}(S_{kt})$ *takes the following form:*

$$\mathcal{A}_{kt}(S_{kt}) = \mathcal{Z}_t B_0 \prod_{r \in S_{kt}} B_{kr},$$

where $\mathcal{Z}_t$ *is agg. productivity which follows an AR(1) process in logs:* $\ln \mathcal{Z}_t = \rho_z \ln \mathcal{Z}_{t-1} + \zeta_t$, *and* B_0, $\{B_{kr}\}_{kr}$ *are parameters.*

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- Delivers marginal cost cost function in logs: $-z_t - b_0 + w_t + \sum_{r \in S_{kt}} [\omega_{kr}(p_{rt} - w_t) - b_{kr}]$

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- Delivers marginal cost cost function in logs: $-z_t - b_0 + w_t + \sum_{r \in S_{kt}} [\omega_{kr}(p_{rt} - w_t) - b_{kr}]$
- Simple rule for choosing supplier: sector k should be from sector r if and only if:

$$\omega_{kr}(p_{rt} - w_t) < b_{kr}$$

Numerical Algorithm: NK model with endogenous networks

- Start from a guess for sectoral prices, supplier choices and allocations; let $\mathcal{X}_t^-$, $\mathcal{X}_t$ and $\mathcal{X}_t^+$ be, respectively, the full set of past, present and future variables at t

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  (iv) Repeat (i)-(iii) until convergence within the time period;

  (v) If $t > 1$, decrease t by one and go back to (i). Otherwise, compare $\{\{P_{kt}^0\}_{k=1}^K\}_{t=1}^{T-1}$ with $\{\{P_{kt}\}_{k=1}^K\}_{t=1}^{T-1}$; if they are equal, stop the algorithm; if they are not equal, set $P_{kt} = P_{kt}^0$, $\forall k$, $1 \leq t \leq T - 1$, set $t = T - 1$ and return to (i).

Calibration

- Need to make an assumption about the path of money supply:

Assumption (Money supply)

For a given initial money supply $\mathcal{M}_0$, the money supply in $t \geq 1$ takes the following form:

$$\Delta \ln \mathcal{M}_t = \rho_m \Delta \ln \mathcal{M}_{t-1} + \varepsilon_t^m.$$

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  - Aggregate parameters: $\beta = 0.99$, $\theta = 6$, $\rho_a = 0.86$, $\rho_m = 0.80$, $T = 50$ and $\mathcal{Z}_0 = 1$

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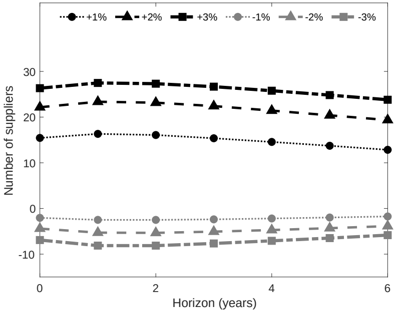
  - Aggregate parameters: $\beta = 0.99$, $\theta = 6$, $\rho_a = 0.86$, $\rho_m = 0.80$, $T = 50$ and $\mathcal{Z}_0 = 1$
  - Sector-specific Calvo parameters (α_k): one minus frequency of price adjustment from Pasten et al. (2020)
  - Sector-specific taxes (τ_k): match sectoral markups from De Loecker et al. (2020)
  - Input-output shares (ω_{kr}): take observed shares from the 2007 BEA Input-Output tables, impute unobserved ones following Acemoglu and Azar (2020)
  - Productivity mapping parameters (B_0, B_{kr}): estimated to ensure the steady-state of my model under $\mathcal{M}_t = \mathcal{Z}_t = 1, \forall t$, simultaneously matches observed input-output linkages an real GDP in 2007

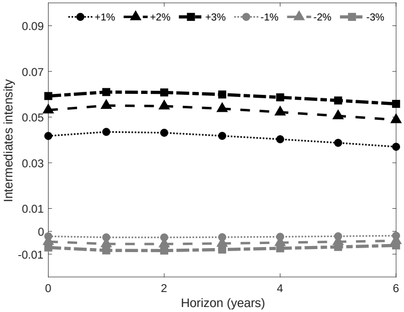
Baselines with Different Productivity Paths and Money Supplies

Baselines with different aggregate productivity paths

(a) Average number of suppliers



(b) Average intermediates intensity



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Baselines with different initial money supplies

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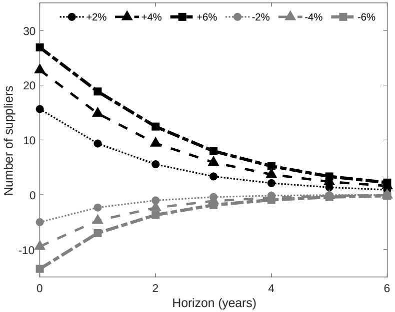

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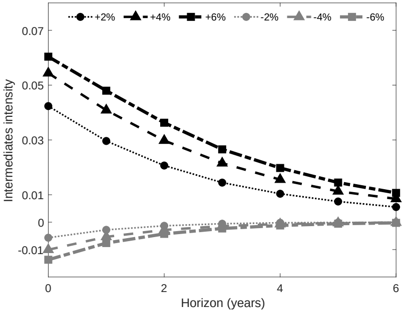

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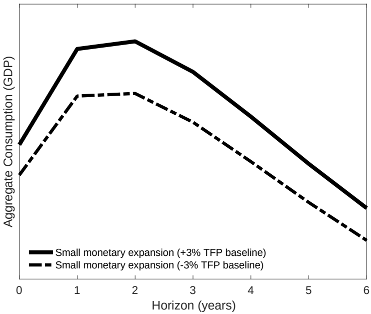
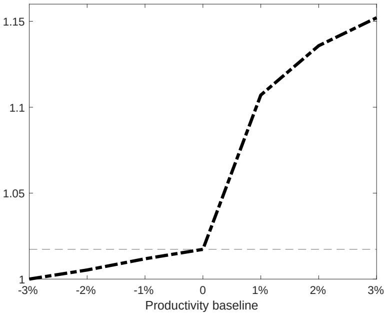
Small Monetary Shocks

Small monetary expansions across productivity baselines

(a) IRFs of GDP under expansion and recession

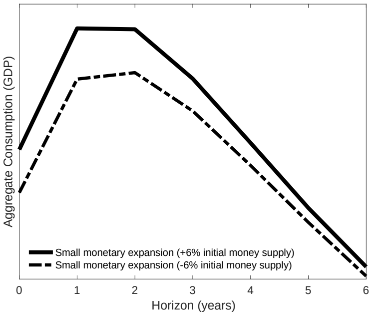


(b) Peaks of IRFs across productivities

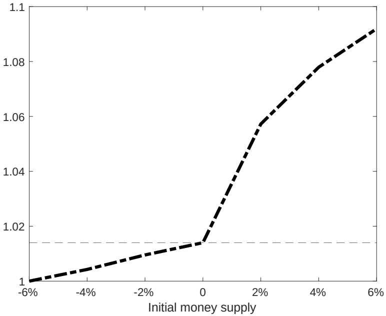


Small monetary expansions across initial money supply

(a) IRFs of GDP under tight and loose money



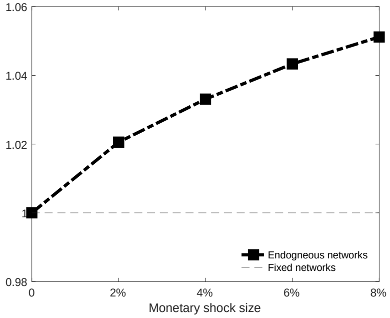
(b) Peaks of IRFs across money supplies



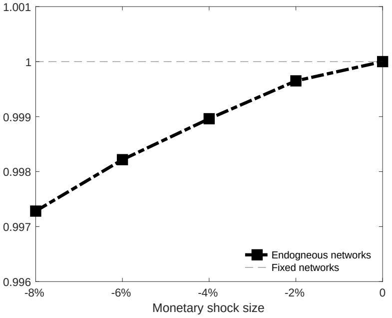
Large Monetary Shocks

Large monetary expansions and contractions

(a) Large monetary expansions



(b) Large monetary contractions



Firm-level Data

Cyclical fluctuations in the number of suppliers

- Measure the number of suppliers at firm level, using data on "in-degree" computed by Atalay et al. (2011) for US publicly listed firms available in Compustat

- Linear local projection:

$$indeg_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}$$

- Non-linear local projection:

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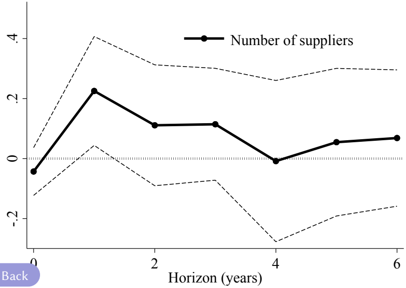
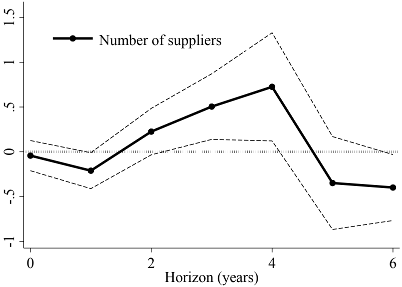
- Use Fernald's TFP shocks and Romer-Romer monetary shocks

Number of suppliers response: linear local projection

(a) Productivity expansion (+1%)



(b) Monetary easing (-100bp)



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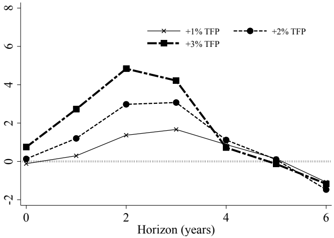
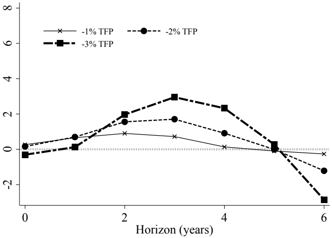
Productivity shocks: non-linear local projection

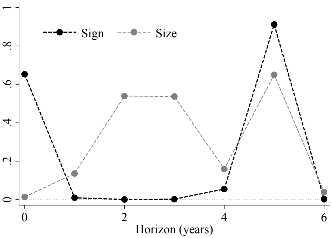
(a) Productivity expansions



(b) Productivity contractions



(c) p-values for non-linearities



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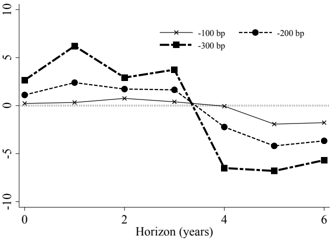
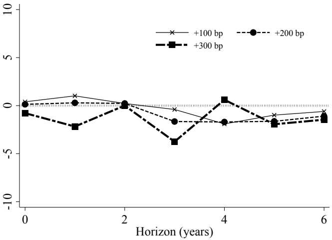
Monetary shocks: non-linear local projection

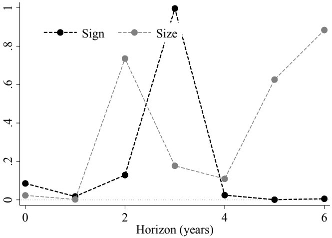
(a) Monetary expansions



(b) Monetary contractions



(c) p-values for non-linearities



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Firm-level Data

Cyclical fluctuations in the number of suppliers

- Measure the number of suppliers at firm level, using data on "in-degree" computed by Atalay et al. (2011) for US publicly listed firms available in Compustat
- Linear local projection:

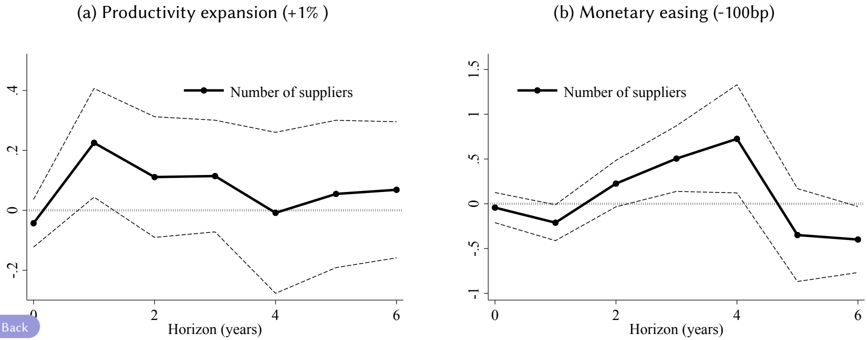
$$indeg_{k,t+H} = \alpha_{k,H} + \beta_H s_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}$$

- Non-linear local projection:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H^{lin} s_t + \beta_H^{sign} s_t \times \mathbf{1}\{s_t > 0\} + \beta_H^{size} s_t \times |s_t| + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H},$$

- Use Fernald's TFP shocks and Romer-Romer monetary shocks

Number of suppliers response: linear local projection



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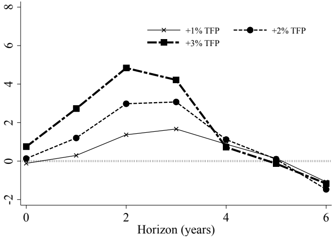
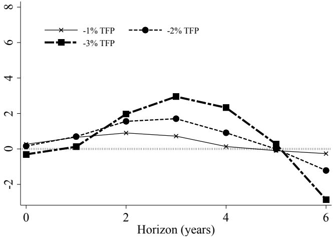
Productivity shocks: non-linear local projection

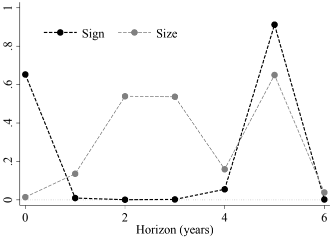
(a) Productivity expansions



(b) Productivity contractions



(c) p-values for non-linearities



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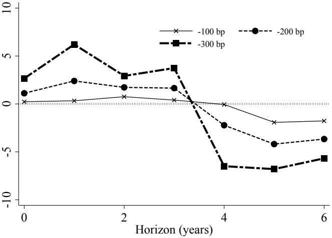
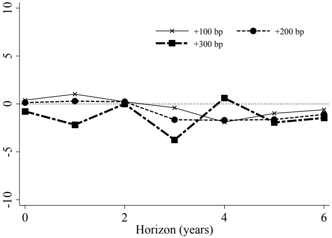
Monetary shocks: non-linear local projection

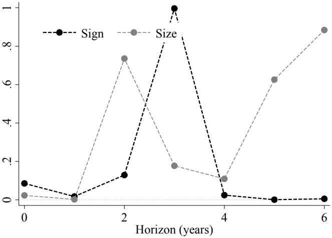
(a) Monetary expansions



(b) Monetary contractions



(c) p-values for non-linearities



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