Factor-Augmented Vector Autoregression with narrative identification. An application to monetary policy in the US *

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Abstract

I extend the Bayesian Factor-Augmented Vector Autoregressive model (FAVAR) to incorporate an identification scheme based on an external instrument approach. A Gibbs sampling algorithm is provided to estimate the posterior distributions of the model parameters. I use this novel modeling framework to estimate the effects of a monetary policy shock in the United States, and I compare the obtained results with those from a smaller-scale model and different identifying instruments. Results confirm that a tightening monetary policy shock has contractionary effects on the real and financial sides of the economy. Furthermore, the paper suggests that taking into account a large information set helps to mitigate price and real economic puzzles as well as discrepancies across estimates obtained using different monetary policy instruments.

JEL Classification: C32, C38, E52

Keywords: information sufficiency, factor-augmented VARs, instrumental variables, monetary policy, structural VARs

*I would like to thank Haroon Mumtaz for his exceptional and inspiring supervision. The paper benefited from comments by Christiane Baumeister, Andrea Carriero, Emmanuel Guerre, Fabio Parla, Alessia Paccagnini, Mirela Miescu, Michelle Piffer, Ivan Petrella, Gerhard Rünstler, Marek Jarocinski, Massimo Ferrari and participants at the Economics Seminars (Central Bank of Ireland, 2021), at the 34th Annual Irish Economic Association Conference (Trinity College Dublin, 2021), at internal ECB seminars (July and November 2021) and at Workshop in Macroeconometrics (King's Business School, November 2021).

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‡The views expressed in this paper are those of the author and do not reflect the views of the European Central Bank or the ESCB. Any remaining errors are my own.

Non-technical summary

Understanding the impact of a change in monetary policy on an economy of interest is a key question for policymakers and researchers. To answer this question, economists can rely on an increasingly large number of data potentially containing useful information on current and past conditions of the economy of interest. Including a large set of informative data in the analysis is useful for various reasons. First, it can help the economist or the policy maker to better understand the propagation mechanisms of the monetary policy shock across the various sides of the economy considered; second, a more accurate representation of economic concepts that would be otherwise summarised in aggregate figures can be captured in the model; third, large datasets can help in capturing more accurately the driving forces of the underlying economy of interest. However, models which are typically used to study the impact of a certain shock suffer from the so-called "curse of dimensionality" issue, meaning that they do not allow for the inclusion of a large number of variables due to the fast-increasing number of coefficients.

Another issue often faced when studying the effect of an unexpected change in monetary policy is how to identify the shock of interest. Economic systems are complex by nature and hit by many interconnected shocks at the same time. Hence, disentangling the effect of each shock is not a trivial task. Identifying the effect of an unexpected monetary policy change becomes even more challenging if the interest lies in the response of fast-moving variables such as asset prices. Asset prices react swiftly to news concerning monetary policy. It follows that their observed response is particularly sensitive to whether the shock is exogenous to the economy.

This paper proposes a modeling framework to overcome the abovementioned issues. The proposed model can encompass a large set of informative variables by including unobserved factors, which can summarise the information contained in many individual series. This dimensionality-reduction technique is combined with a shock identification

scheme which allows for the identification of the shock of interest without imposing restrictions on the model that can be difficult to justify from an economic point of view. Finally, the model can be easily extended to the case of time variation in parameters, which allows studying how the policy impact differs across different periods.

The model presented in this paper is employed to study the effects of an unexpected increase in the monetary policy rate in the United States over the period 1991-2015, using a large set of macroeconomic and financial variables. It finds that an increase in the policy rate has contractionary effects on both the real and financial side of the economy as well as on prices. Results also suggest that including a large set of information in the model helps draw conclusions that are consistent with economic theory and robust to different instruments used for identification.

1 Introduction and review of the relevant literature

Vector Autoregressive models (VARs) are broadly used to study the effects of a shock on an economy of interest. This is done mainly through the use of impulse response functions, which allow to observe estimated dynamic responses of a variable over time given the occurrence of a certain structural shock. To correctly conclude the impact of a given shock on one or more variables, VARs should hold two key desirable properties: i) a strong model specification that can properly represent the dynamics of the economy of interest, and ii) a credible identification scheme. A VAR that correctly captures the original data-generating process ensures that the reduced form coefficients are unbiased and that the transmission of the shock is correctly traced. At the same time, a credible identification scheme is needed to recover the structural shocks from the reduced form residuals without making assumptions that are economically not meaningful.

A well-known criticism of VARs relates to the small amount of information they can take into account due to the fast-increasing number of parameters. If the model omits information that is relevant to explaining the dynamics of the economy of interest then the estimated dynamic responses will be biased and will potentially lead to wrong economic conclusions. Stock & Watson (2018) suggests overcoming this issue by augmenting VARs with latent factors able to summarise a large amount of information (FAVAR). FAVAR models, originally introduced in the macroeconomic literature by Bernanke *et al.* (2005), represent a popular solution to effectively expand the information set taken into account in VAR models.

In the context of shock identification, a growing strand of literature uses external instruments as a proxy for the shock of interest. To be a valid proxy, an instrument should be correlated only with the structural shock of interest and not with the other shocks. Stock (2008) firstly introduced the idea of using these instruments for identification in SVAR; subsequently, this idea has been used in a growing number of works, most notably in Stock & Watson (2012) Mertens & Ravn (2013) and Gertler & Karadi (2015) (Proxy SVARs). Since these seminal papers, the external instrument approach has become very popular in the literature because, contrarily to widely used recursive identification schemes, it allows to map reduced form residuals into structural shocks without relying on restrictions which tend to be hard to defend.

The external instrument approach has been applied in several works to study the effects of monetary (Caldara & Herbst (2016), which proposes a Bayesian framework for the Proxy SVAR; Jarociński & Karadi (2020)) or fiscal (Mertens & Ravn (2014)) policies. More recently, Plagborg-Møller & Wolf (2021) and Paul (2020) have proposed alternative strategies to use narrative instruments to identify structural shocks in VARs. In the former case (SVAR-IV), the authors include the instrument as an extra endogenous variable ordered first and use recursive identification to recover the structural shocks. In the latter case (VARX), the instrument enters the model as an exogenous variable [1]. These two methodologies differ from those previously proposed in the way in which the proxy enters into the model. While in Mertens & Ravn (2014), Stock & Watson (2012)

[1]Bagliano & Favero (1999) has similarly used a VARX but did not show that this approach consistently identifies the true (relative) impulse responses, as Paul (2020) does.

and in Caldara & Herbst (2016) the proxy enters the model through an extra equation (two-steps approach), in Paul (2020) and Plagborg-Møller & Wolf (2021) it is added directly to the reduced-form specification (one-step approach). It is worth noting that, as mentioned in Paul (2020), the SVAR-IV coincides with the VARX if the instrument is uncorrelated with the regressors in the VAR, including lags of the instrument itself.

A growing number of works have developed instruments to proxy a monetary policy shock in the US, which will be the shock of interest in this paper. A prominent example is the highly-cited narrative measure by Romer & Romer (2004). After the Great Financial Crisis, monetary policy has become more and more of a multi-dimensional phenomenon. The Fed can rely on various instruments to affect the economy, both conventional and unconventional, such as changes in the target policy rate, large-scale asset programs, and forward guidance (Jarociński & Karadi (2020) and Jarociński (2021)). The literature on this topic presents instruments that characterize the Fed policy with one compounded instrument as well as instruments that distinguish between these policy tools. A non-exhaustive list of works that propose instruments to proxy a monetary policy shock in the U.S. is Gürkaynak *et al.* (2005), Bu *et al.* (2021) Gertler & Karadi (2015) and Miranda-Agrippino & Ricco (2021).

This paper aims to contribute to the literature which offers solutions to encompass a large set of information in VAR models while relying on a narrative identification scheme. It does so by proposing an algorithm that incorporates the exogenous variable identification approach à la Paul (2020) within the modeling framework of a FAVAR. The algorithm presented here can be seen as a valid alternative to the Proxy FAVAR models already existing in the literature (Miescu & Mumtaz (2019); Bruns (2021); Kerssenfischer (2019)). I employ the FAVAR with the exogenous instrument to revisit the transmission of monetary policy in the U.S. Moreover, to study the importance of including a large set of information in the model, I compare the results obtained with a FAVAR with an exogenous variable with those obtained with the smaller-scale VARX by Paul

(2020). Furthermore, I estimate the FAVAR with exogenous proxy using different identifying instruments, to understand whether using a data-rich model can mitigate the discrepancies in the impulse responses observed in the literature. The remainder of the paper is the following: section 2 presents the modeling framework; section 3 describes the Gibbs sampling algorithm used to estimate the model; section 4 presents the Monte Carlo experiment; section 5 shows the empirical applications and section 6 concludes.

2 A FAVAR model with exogenous variable

In this section, I present the FAVAR model with an identification scheme through an exogenous proxy; section 2.1 presents the FAVAR model as originally proposed in Bernanke *et al.* (2005), while in section 2.2 I show how this is extended to allow for identification through an exogenous instrument. More details on the model's state-space representation can be found in Appendix A.

2.1 The FAVAR model à la Bernanke *et al.* (2005)

The observation equation

Let us suppose to observe a large number of macroeconomic and financial variables containing useful information on the current and past conditions of an economy of interest. I assume that this large number of variables can be summarised by a relatively small number of latent and observable factors, according to the following observation equation:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + v_t \tag{1}$$

$$v_t \sim N(0, R) \tag{2}$$

Where X_t is a Mx1 vector collecting the set of M 'informational' series[2]. The K latent factors are denoted by F_t and Y_t contains N observable variables.

[2]Note that M is "large" and can be potentially larger than the number of periods T

The informational variables are related to the contemporaneous values of the latent factors via the $M \times K$ matrix of factor loading Λ^f and to the contemporaneous values of the observable factors contained in Y_t by Λ^y [3]. Typically, in the literature on monetary policy shocks, Y_t includes only the policy rate. In general, Y_t never includes variables contained in X_t.

Finally, v_t contains the error terms, which are assumed to be zero mean, normally distributed, and with a variance-covariance matrix equal to:

$$VAR(v_{i,t}) = R = \begin{pmatrix} R_1 & 0 & \dots & 0 \\ 0 & \ddots & 0 & 0 \\ \vdots & \dots & R_M & \vdots \\ 0 & \dots & 0 & 0 \end{pmatrix} \tag{3}$$

The transition equation

The unobserved factors are then modeled jointly with the observed variables in a VAR. The idea is that thanks to the inclusion of unobserved factors summarising a large amount of information in a VAR, one can capture that additional information not fully captured by the limited number of variables typically included in the VAR. The dynamics of the unobserved factors F_t and the observed variables in Y_t are assumed to evolve according to the following VAR process:

$$Z_t = c + \sum_{j=1}^{p} B_j Z_{t-j} + u_t \tag{4}$$

$$u_t \sim N(0, Q) \tag{5}$$

Where $Z = \{F_{1,t}, ...F_{K,t}, Y_{1,t}, ...Y_{N,t}\}$.

[3] In the original specification of the FAVAR proposed in Bernanke *et al.* (2005) the elements of Λ^y are non-zero for those variables in X_t that are assumed to react quickly to monetary policy interventions ("fast-moving variables"). Given the different strategies used to identify the structural shock of interest, the model presented here does not include this distinction, and the vector Λ^y is zero regardless of the nature of the variable in X_t.

Equation (4) is a VAR in F_t and Y_t and can be interpreted as a reduced-form model involving both observable and unobservable components. If the true data generating process is given by a FAVAR, omitting the introduction of the unobserved factors in the model will lead to a biased estimate of the VAR coefficients and related impulse responses. The error term u_t is mean zero and with a variance-covariance matrix Q.

2.2 The identification problem and the exogenous variable approach

Equation (4) can be seen as a reduced-form VAR of order p in Z_t. The object of interest here is in understanding the structure of the economy by uncovering the causal relationship among variables in the model; in particular, by exploiting the advantages of the FAVAR, I want to observe the effect of a given shock of interest on the large variables set contained in X_t.

This cannot be done with the reduced-form representation because the error terms u_t are correlated with each other and hence I can not distinguish the impact of the shock on one variable from all the others. In other words, they can not be interpreted as structural shocks. The corresponding structural form of equation (4) is given by:

$$A_0 Z_t = \mu + \sum_{j=1}^{P} \Gamma_j Z_{t-j} + e_t \tag{6}$$

Where A_0 collects the contemporaneous relationships among the endogenous variables. The following relationships between (4) and (6) hold: $c = A_0^{-1}\mu$, $B_j = A_0^{-1}\Gamma_j$ and

$$u_t = Se_t \tag{7}$$

where

$$S = A_0^{-1} \tag{8}$$

and

$$SS' = \Omega \tag{9}$$

where the $\theta \times \theta$ matrix S contains the contemporaneous effect of the structural shocks on the dependent variables, or the impulse response vector of the shock.

Note that the relationship described in (9) produces a system of $\frac{\theta(\theta+1)}{2}$ equations with θ^2 unknown parameters. Since $\theta^2 > \frac{\theta(\theta+1)}{2}$ we are not able to exactly identify this system of equation; to do so, we would need to impose $\frac{\theta(\theta-1)}{2}$ restrictions.
In summary, the econometric problem with identifying the true impulse responses is that the structural shocks are not observed, hence from the data we can not estimate the parameters in the matrix S. In addition, the covariance matrix of the reduced-form innovations does not provide enough identifying restrictions to obtain at least one of the columns in S.
The identification strategy consists of finding a way to estimate the elements in S in an economically meaningful way.
I assume that a proxy for the shock of interest exists and that it satisfies the two following conditions:

$$E(m_t e_{1,t}) = \phi \tag{10}$$

$$E(m_t e_{2,t}) = 0 \tag{11}$$

Where m_t is the proxy for the shock of interest, $e_{1,t}$ is the shock of interest and $e_{2,t}$ contains all the other shocks. Equation (10) captures the so-called relevance condition: the instrument has to be correlated with the shock of interest. The covariance between proxy and shock is $\phi \neq 0$. Equation (11) describes the exogeneity condition, namely that the proxy has to be uncorrelated with all the other shocks; m_t is assumed to have zero mean for simplicity [4].
I include the instrument in the transition equation described in (4), which becomes:

[4] I assume that m_t is not serially correlated.

$$Z_t = \tilde{c} + \sum_{j=1}^{P} \tilde{B}_j \tilde{Z}_{t-j} + Am_t + \tilde{u}_t \quad (12)$$

Where the instrument m_t enters the transition equation as an extra exogenous variable and A contains the contemporaneous relations between the instrument and the endogenous variables. The model in (12) is a FAVAR with an exogenous instrument (henceforth, FAVARX). Hence, the contemporaneous impulse responses become equal to the elements of A which link the instrument and the contemporaneous values of the endogenous variables. It follows that the contemporaneous relative impulse responses[5] are given by:

$$r^*_{ij} = \frac{a_i}{a_j} \quad (13)$$

Where a_i and a_j are elements of A associated to the endogenous variables i and j (with $i \neq j$). The impulse responses for the subsequent periods are obtained by tracing the initial impulse through the model described in (12) via the lagged endogenous variables. It can be shown analytically that this approach to identification delivers consistent estimates of the true relative impulse responses, both contemporaneous and subsequent (see Paul (2020)).

2.3 Comparison between FAVARX and proxy FAVAR

The modeling approach proposed in this paper represents an alternative option to the Bayesian Proxy FAVAR (Miescu & Mumtaz (2019); Bruns (2021)). The Proxy FAVAR and the FAVAR with the exogenous variable share the main advantages of the FAVAR model and the identification through the narrative approach. The main difference between the two approaches lies in the way in which the instrument is used to achieve the

[5]Relative impulse responses normalized the contemporaneous responses of one of the endogenous variables to a given value.

identification. In the case of the Bayesian Proxy FAVAR the instrument is used in a two-step framework: in the first step, the coefficients of the reduced-form innovations are estimated using least squares, while in the second step, the estimated innovations are regressed against the instrument. By contrast, the Bayesian FAVARX is estimated by simply including the instrument as an exogenous variable in the transition equation, as shown in section 2.

Paul (2020) shows that both techniques always deliver the same contemporaneous relative impulse responses, also in small samples, and that if the sample is large enough, the obtained contemporaneous responses converge to the ratio of the true contemporaneous impulse responses. Given that the FAVAR can be seen as a standard VAR where the controls are the latent factors, the same conclusions hold for both models.

Regarding the subsequent values of the relative impulse responses, they can differ across the two methodologies in small samples. In the case of large samples, however, differences can arise only if the instrument is correlated with the other regressors and not because of measurement errors due to the instrument.

More details on the equivalence of the contemporaneous and subsequent relative impulse responses between a Proxy VAR and a VAR with an exogenous variable can be found in Paul (2020).

2.4 Advantages of the methodology

The methodology proposed in this paper is useful to perform structural analysis involving large information and, at the same time, relies on a credible identification strategy. On one hand, the ability to encompass a large amount of information in the model implies many advantages. First, biases stemming from misspecification and information insufficiency can be mitigated. In this context, Stock & Watson (2018) notes that using an external instrument for identification enables, if the instrument meets certain validity requirements, the model to estimate dynamic causal effects without assuming the invertibility of the VAR. While this argument is of extreme relevance for this discussion, it is also worth stressing that in the VARX the subsequent values of the relative impulse response functions (IRFs) are traced using the coefficients of the reduced-form model. Hence, if the econometrician erroneously omits relevant information in the model, this will be absorbed in the reduced-form residuals and the estimated values of the coefficients needed to estimate the IRFs will be distorted. This in turn will result in biased values of subsequent relative impulse responses. A second remark is that the condition for the validity of the instrument mentioned in Stock & Watson (2018) includes a strong lead-lag exogeneity requirement that the instrument is uncorrelated with past and future shocks, which is hard to meet. A second advantage related to the inclusion of a rich information set is the possibility to have a more accurate representation of economic concepts that would be otherwise summarised in aggregate figures can be captured in the model and hence measurement errors can be mitigated. For instance, the concept of "economic activity" can be represented by a larger number of series rather than only industrial production. Third, impulse responses can be observed for a large number of variables, which can help to uncover the transmission of the shock through the economy. On the other hand, the reliance on external information has the important advantage to decrease the number of restrictions required to identify the contemporaneous responses, which can be potentially controversial or hard to defend.

Finally, as previously pointed out, the one-step procedure allows for an easy extension to the case of time-varying parameters.

3 Estimation

The parameters of the observation and transition equations are estimated with a Gibbs sampling algorithm, while the factors are retrieved using a Kalman filter and Carter and Kohn algorithm (1994).

To start the algorithm, let us imagine observing the latent factors F_t. Given the factors, the observation equation in (1) can be seen as M linear regressions of the form $X_t = \Lambda^f F_t + \Lambda^y Y_t + v_t$ where the elements in Λ^f, Λ^y and R can be drawn from their conditional distributions. Similarly, given the factors, the transition equation is a VAR model where the elements in B, u, and Q can be drawn from their conditional distributions. Given a draw for Λ^f, Λ^y, R, B, u and Q the model is cast into a state-space form as in equations (1) and (12) and the Carter and Kohn algorithm can be used to draw F_t from its conditional distribution. A more detailed description of the steps for the estimation is shown in Appendix B.

4 Monte Carlo Experiment

To test the performance of the algorithm presented above, I run the latter by using artificial data obtained from a FAVAR as a data-generating process. In particular, artificial data is generated from a factor model with 5 unobserved factors and 1 observed factor, described by the following equations:

$$X_{i,t} = b_i F_t + \Gamma_i Y_t + v_t \tag{14}$$

$$F_t = c + BF_{t-1} + Am_t + u_t \tag{15}$$

$$\epsilon t = A_0^{-1} u_t \tag{16}$$

Where the B parameters are calibrated to estimate a VAR with 1 lag and containing 5 factors extracted from the FRED-MD database and the 1-year government bond rate for the US. I generate 500 data sets and discard the first 100 to remove the influence of initial conditions. The estimation of the model with simulated data involves 5000 Gibbs iterations with a burn-in of 1000 iterations. I then compared the responses obtained by estimating the model with artificial data against the true impulse responses. Figure 1 shows the impulse responses of selected variables to a 100 basis point shock on the interest rate at horizons 0 to 40. The paths of the responses along the horizon taken into account are very similar, which suggests that the algorithm performs well at tracking the true responses.

5 Empirical application

In the following subsections, I use the framework described in section 2 to study the effect and transmission of a monetary policy shock in the U.S. Moreover, I compare the results obtained with the FAVAR with exogenous variable to those obtained with a smaller-scale model but with the same identification approach, to shed light on the role played by the inclusion of larger information set on the estimated impulse responses. Furthermore, I compare the estimated impulse responses obtained using different instruments available in the literature to proxy a monetary shock to observe whether the use of a large set of data can help mitigate some of the discrepancies across estimated impulse responses observed in the literature.

5.1 Data and the choice of the proxy for a monetary policy shock

The set of informational series used to extract the factors is sourced from the FRED-MD monthly dataset by St.Louis Fed[6] and it contains 128 macroeconomic and financial series at a monthly frequency. This dataset is augmented with four extra series, i.e. the Excess Bond Premia and credit spread by Gilchrist & Zakrajšek (2012), the 30-year mortgage rate, and Shiller's real dividends. The series taken from FRED-MD is transformed following McCracken & Ng (2015) [7], the Excess Bond Premia, credit spread, and mortgage rate enter into levels while the real dividends are transformed into log-difference. I use the one-year government bond rate as the relevant monetary policy indicator, as in Gertler & Karadi (2015). The latter suggests that using a rate with a longer maturity than the Federal Fund Rate allows considering shocks to forward guidance in the overall measure of policy shocks. Regarding the choice of the instrument

[6] Data set available at $research.stlouisfed.orgeconmccrackefred - databases$. The vintage used is 2020-06

[7] see $https : s3.amazonaws.comfiles.fred.stlouisfed.orgfred - mdAppendix_Tables_Update.pdf$. The series transformed into second difference or log second difference have been transformed into first difference and log difference respectively.

to proxy a monetary policy shock, several options are available in the literature. I compare three instruments by looking at measures of their association to the structural shock of interest, as shown in Table 1. [8]

Reliability measure and F-statistics of the monetary policy instruments

Instrument	MPI	$FF4_{GK}$	MP1
F-statistics	12.48 [6.96 14.92]	39.77 [18.65 48.67]	23.4 [10.83 30.5]
Reliability	0.21 [0.15 0.23]	0.41 [0.29 0.43]	0.56 [0.37 0.57]

Table 1: Relevance of monetary policy instruments. The first line shows the F-statistics of the first-stage regression of the reduced-form innovations of the proxy SVAR on the instrument. The second line shows the reliability measure proposed by Mertens & Ravn (2013). Monetary Policy Instrument (MPI) by Miranda-Agrippino & Ricco (2021); high-frequency instrument (FF4) by Gertler & Karadi (2015); monetary surprise (MP1) by Gürkaynak *et al.* (2005). VAR specification includes industrial production, unemployment rate, consumer price index, commodity price index, excess bond premium, and one-year rate. Sample: 1978M1 - 2019M12.

The Monetary Surprise (MP1) by Gürkaynak *et al.* (2005) displays the highest level of reliability across the instruments taken into account. However, the Monetary Policy Instrument (MPI) proposed by Miranda-Agrippino & Ricco (2021) hhas the advantage to account for the presence of information friction. This is achieved by constructing the instrument by projecting the market-based monetary surprises on their lags and the Central Bank's information set, as summarised by the Greenbook forecast. For this reason, I will use the MPI (plotted in Figure 2) for the benchmark specification. The MPI spans from January 1991 to December 2015, which will be the sample used for the benchmark estimates.

5.2 Model specification and the choice of the number of factors

The benchmark model includes 10 factors and the one-year rate with 13 lags, as is common in the literature. The choice of the number of factors is found to be a key element in the overall model specification and for the reliability of the estimated impulse responses. Jushan Bai (2002) provides a criterion to determine the number of factors present in

[8]The structural shocks have been extracted by using the Bayesian Proxy SVAR à la Caldara & Herbst (2016). The reason for this is that the Bayesian VAR with exogenous instrument does not allow to recover the series of the structural shock of interest as the impact matrix is only partially identified and hence can not be inverted.

the data. However, as noted in Bernanke *et al.* (2005), this does not necessarily address the question of how many factors should be included in the VAR. To have a better understanding of the optimal number of factors to include in the FAVAR, I perform the "structuralness" test for the shock of interest as proposed by Forni & Gambetti (2014). The structuralness test aims to check whether a model contains sufficient information to correctly retrieve the structural shock of interest. The test is based upon the idea that, under suitable conditions, if a shock recovered from a certain specification is orthogonal to the lags of all the variables included in the model, then it is a linear combination of the structural shocks. Based on the latter statement, the test suggests checking for information sufficiency by looking at the orthogonality of the structural shock of interest with respect to the lags of principal components. In practice, orthogonality is tested by regressing the structural shock onto the past values of a number of principal components [9]. I implement the test following a similar procedure as in Forni & Gambetti (2014). First, a model with a low number of principal components is considered and the structural shock of interest is initially extracted from this model specification and regressed against all the other principal components taken once at a time. If orthogonality is rejected, then the past values of the principal components considered are likely to contain information useful to predict the shock, hence these will be included in the model. The model augmented with the principal components which are found to have explanatory power for the shock of interest will be used in the subsequent step and the procedure will be repeated until no further principal components are found to be informative. Following this procedure, the test suggests including 10 principal components (1 to 4, 10, 12, 16, and 18 to 20). This specification will be used in the benchmark exercise shown in the following section. The results of the test are displayed in Table 2.

[9] It is worth noting that, also in this case, the monetary policy shock has been extracted by using the frequentist Proxy FAVAR à la Mertens & Ravn (2013), as for the computation of the reliability measure. Moreover, the principal components are used as proxy for the factors

	Specification	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12	P=13	P=14	P=15	P=16	P17	P=18	P=19	P=20
1-4	ω_t^4	0.60	0.35	0.61	0.95	0.70	0.52	0.32	0.81	0.96	0.28	0.71	0.00	0.20	0.10	0.29	0.68
16	ω_t^5	0.88	0.58	0.32	0.88	0.95	0.46	0.71	0.72	0.84	0.53	0.80	-	0.47	0.12	0.22	0.13
18,20	ω_t^6	0.48	0.71	0.41	0.47	0.45	0.10	0.53	0.18	0.77	0.96	0.87	-	0.25	-	0.28	-
10	ω_t^7	0.67	0.78	0.46	0.84	0.39	-	0.75	0.40	0.24	0.83	0.86	-	0.80	-	0.12	-
19	ω_t^8	0.52	0.75	0.74	0.21	0.62	-	0.49	0.04	0.65	0.75	0.97	-	0.72	-	-	-
12	ω_t^9	0.52	0.56	0.49	0.44	0.53	-	0.89	-	0.34	0.80	0.18	-	0.95	-	-	-

Table 2: p-values of the orthogonality test. F-test for the estimated monetary policy shock. The rows correspond to different specifications ω_t^i where i is the number of principal components included. P refers to the number of principal components used in the test. Structural shocks are extracted from a proxy FAVAR with 13 lags, the proxy for the monetary policy shock is the MPI by Miranda-Agrippino & Ricco (2021) and the base rate is given by the one-year rate.

5.3 Transmission of a monetary policy shock in the U.S.

Figure 3 shows the results for a one-standard-deviation monetary policy shock normalized to give an increase of 100 basis points to the one-year government bond rate. Overall, I find that a tighter monetary policy surprise has contractionary effects on both the demand and supply side of the economy, as well as on prices. In terms of financial variables, financing conditions tighten, leading to a decline in credit and asset prices.

As it can be seen from the top-row panels in Figure 3, industrial production declines across the time horizon observed. The decline in the overall figure reflects a decline in both durable and nondurable goods, with the latter reacting more strongly. Consistently, sales contract across various market segments (in the figure it can be seen the path for manufacturing and trade sales). As firms contract their production and sales decline, business inventories also contract as well as new orders. The response of the ratio of business inventories over sales in the manufacturing market remains marginally positive over the whole horizon, signaling that sales contracts are stronger than inventories. As a result, capacity utilization in the economy declined. A drop on impact in housing starts signals lower levels of investments in the housing market.

Looking at the panels in the third and fourth rows, it can be seen as the labor market is also negatively affected. The unemployment rate increases and the number of employed people go down. The impact on employment is uneven, with employees in manufacturing being more impacted than employees in the public sector. The average number of hours worked also goes up, suggesting that firms are cutting their costs for personnel. The average earnings diminish as expected in a labor market where employees lose their bargaining power due to the higher unemployment rate. Real personal income declines persistently over the horizon as a consequence of lower earnings. Consistently, personal consumption decreases.

Panels in the fifth and sixth rows show the reaction of various measures of prices. As the economy contracts and production and sales decline, prices also adapt by declining, as one would expect, with some adjusting faster than others. Strong uncertainty surrounds the estimated responses of prices, but they are consistent with what theory suggests and overall are not suggestive of the presence of a price puzzle.

Finally, the bottom-row panel shows the responses of credit and financial market variables. Both short-term and long-term costs of credit increase, as suggested by the increase in the short-term rate and the excess bond premium respectively. As a consequence, both consumer and mortgage financing decline with the latter leading to declining property investments (housing starts). It is worth noting that both tightening financing conditions and weakened demand factors negatively weigh on lending dynamics, which might explain the magnitude of the observed decrease. Consistently, asset prices also decline, as reflected in the response of the stock market and house prices, and the increase in the trade-weighted dollar exchange rate. Declining asset prices are, in turn, consistent with a deterioration in households and firms' wealth levels, which corroborates the insights suggested by the responses of real economic variables in the top panels.

5.4 The role of information sufficiency (I): comparison between FAVAR with exogenous instrument and VARX

In this section, I study the importance of taking into account large information sets in the estimation of the dynamic responses to a shock of interest. I do so by comparing the results obtained with the small-scale VARX as proposed in Paul (2020) and the FAVARX. To observe solely the effect played by the information contained, I keep fixed the instrument, the data sample, the number of lags, and the priors across the two models. As shown in Figure 4, the two models deliver standard responses for macroeconomic and financial variables. A contractionary monetary policy shock leads to a decline in asset prices and dividend yields as well as a decrease in industrial production. However, the response of consumer prices in the case of the small-scale VARX is positive on impact and along the horizon considered. The price response estimated with the data-rich models is very similar on impact, but it declines within the first three months and stays negative throughout the estimation horizon, which seems more consistent with standard economic theory. One possible interpretation of these results is that data-rich models, by ensuring a correct retrievement of the original data-generating process and hence of the reduced-form coefficients, can help avoid misleading or counter-intuitive estimations of dynamic responses of the endogenous variables to the structural shock one wants to study (see also section 2.4). As a further illustration of this, Figure 5 shows the responses of CPI and industrial production estimated with a FAVAR model including a lower number of factors than those suggested by the structuralness test, and how these responses change as relevant factors are added to the model. The more relevant factors are added, the more puzzles are solved.

5.5 The role of information sufficiency (II): using different identifying instruments in a data-rich model

While in the previous subsection I explored the importance of having a data-rich model for the estimation of dynamic responses, here I consider this aspect in conjunction with the choice of the instrument used. As documented in Coibion (2012) and Ramey (2016), dynamic responses to monetary policy shocks can vary depending on the information contained in the instrument used as well as on the sample and the model specification. In this section, I explore the performance of different instruments available in the literature when used within a data-rich model and how these compare when used within a standard VAR, to shed light on the potential role of data-rich models in mitigating the discrepancies observed across the dynamic responses estimated when using different instruments available in the literature. Figure 6 shows the responses obtained using three different instruments, both in a FAVARX (left-hand side panels) and an in a VARX (right-hand side panels) model.

Overall, the responses obtained with the three instruments are in line with what the economic theory would suggest. However, the responses obtained with the FAVAR model seem to lead to fewer contradictions. For example, responses of CPI estimated with the VAR suggest the presence of a positive long-run impact on consumer prices if I identify the shock with one instrument, as opposed to what is observed in the case of the other two proxies. These discrepancies vanish in the responses obtained with the FAVAR, which point to a negative long-run impact regardless of the instrument that is employed to proxy the monetary policy shock. Similarly, the response of house price shows opposite paths depending on the instrument used for identification.

6 Conclusions

The main goal of this paper is to propose an algorithm to integrate an identification scheme based on an exogenous instrument approach within a FAVAR model. The FAVARX can be seen as a valid alternative to the Proxy FAVAR model existent in the literature. The main advantage of the proposed methodology with respect to the Proxy FAVAR lies in its computational simplicity and the easy extension to time variation in parameters. The FAVAR with exogenous variable is employed to revisit the transmission of a monetary policy shock in the U.S. The obtained dynamic responses seem consistent with the economic theory, as they suggest that a tightening monetary policy has contractionary effects on real economic variables, causes a decline in various measures of prices, and it tightens financial conditions. Furthermore, the proposed modeling framework is used to study the importance of including sufficient information in the model. Results suggest that including rich information sets plays an important role in mitigating price and real economic puzzles in the estimated impulse responses as well as the discrepancies among the impulse responses obtained with different monetary policy instruments.

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Appendix

A State-space form representation of FAVAR with exogenous variable

Let's rewrite the observation equation in (1) as:

$$
\underbrace{\begin{pmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_{M,t} \\ y_t \end{pmatrix}}_{X_{i,t}} = \underbrace{\begin{pmatrix} \lambda^f_{11} & \lambda^f_{12} & \dots & \lambda^f_{1K} & \lambda^y_1 & 0 & \dots & 0 \\ \lambda^f_{2,1} & \lambda^f_{22} & \dots & \lambda^f_{2K} & \lambda^y_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ \lambda^f_{M,1} & \lambda^f_{M2} & \dots & \lambda^f_{MK} & \lambda^y_M & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} F_{1,t} \\ \vdots \\ F_{Kt} \\ y_t \\ \vdots \\ F_{K,t-p} \\ \vdots \\ y_{t-p} \end{pmatrix}}_{\beta_t} + \underbrace{\begin{pmatrix} v_{1,t} \\ v_{2,t} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ v_{M,t} \\ 0 \end{pmatrix}}_{v_t} \quad (17)
$$

The left-hand side of the equation contains the panel of informational variables X_i, t and the vector of observable variables Y_t (hence, $X_{i,t} = \{X_{i,t}, Y_t\}$). Note that the set of observable variables in Y_t is assumed to include only one variable as this is typically the case in the literature on monetary shocks. The K latent factors are denoted by $F_{i,t}$ with $i = 1...K$ and they summarise the information in $X_{i,t}$.

The structure of this observation equation implies that the informational variables are related to the contemporaneous values of the latent factors via the MxK matrix of the factor loading, given by the elements λ^f_{ij} in matrix H (with $i = 1...M$ and $j = 1...K$); moreover, these informational series are related to the contemporaneous values of the observable variables contained in Y_t by the elements λ^y_i (where, again, $i = 1...M$) in the H matrix.

In the original specification of the FAVAR proposed in Bernanke *et al.* (2005) the

elements of the λ_i^y are non-zero for those variables in $X_{i,t}$ that are assumed to react quickly to monetary policy interventions ("fast-moving variables"). The identification through exogenous proxy allows for a non-zero contemporaneous impact on all the endogenous variables, hence the model presented here does not include this distinction, and the vector λ_i^y is zero regardless of the nature of the variable in $X_{i,t}$.
The remaining $(M+N)*p$ elements of the matrix H beyond λ_{ij}^f and λ_i^y are equal to zero; this implies that all the elements in $X_{i,t}$ only depend on the current and not on the lagged values of the factors, contained in the β vector (or the state vector). The reason why the lags of the state variables are still included in the state vector is that we want these lags to enter the VAR that forms the transition equation, described below. It is worth noting that assuming that the elements in $X_{i,t}$ depend only on contemporaneous values of the latent factors is not restrictive as the FAVAR framework allows for dynamic relationships between the informational variables in $X_{i,t}$ and the latent factors. If $X_{i,t}$ is assumed to depend on an arbitrary number of lags of the factor, then we would have a dynamic factor model (Stock & Watson, 1998). Regarding the observable in Y_t, it is simply described as an identity, as indicated by the zeros in the last rows of the H matrix and the 1 placed below the λ_i^y vector.

The unobserved factors are then modeled jointly with the observed variables in the VAR. The idea is that thanks to the inclusion of unobserved factors summarising a large amount of information in the VAR one can capture that additional information not fully captured by the limited number of variables typically included in the VAR. The dynamics of the unobserved factors F_t and the observed variables in Y_t are assumed to evolve according to the following VAR process:

$$
\underbrace{\begin{pmatrix} F_{1,t} \\ \vdots \\ F_{Kt} \\ y_t \\ \vdots \\ F_{1,t-p} \\ \vdots \\ F_{K,t-p} \\ y_{t-p} \end{pmatrix}}_{\beta_t} = \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_\theta \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\mu} + \underbrace{\begin{pmatrix} A_{1,1} & A_{1,2} & \dots & \dots & \dots & A_{1,\theta*p} \\ A_{2,1} & A_{2,2} & \dots & \dots & \dots & A_{2,\theta*p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{\theta,1} & A_{\theta,2} & \dots & \dots & \dots & A_{\theta,\theta*p} \\ I_1 & 0 & \dots & 0 & \dots & 0 \\ 0 & I_2 & \dots & 0 & \dots & 0 \\ 0 & 0 & \ddots & 0 & \dots & 0 \\ 0 & 0 & 0 & I_\theta & \dots & 0 \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} F_{1,t-1} \\ \vdots \\ F_{Kt-1} \\ y_{t-1} \\ \vdots \\ F_{1,t-(p+1)} \\ \vdots \\ F_{K,t-(p+1)} \\ y_{t-(p+1)} \end{pmatrix}}_{\beta_t - 1} + \underbrace{\begin{pmatrix} m_1 * a_1 \\ m_2 * a_2 \\ \vdots \\ m_\theta * a_\theta \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{m_t} + \underbrace{\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{\theta,t} \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{u_t} \tag{18}
$$

Where:

$$
VAR(u_t) = Q = \begin{pmatrix} Q_{1,1} & Q_{1,2} & \dots & Q_{1,\theta} & 0 & \dots & 0 \\ Q_{1,2} & \ddots & \dots & Q_{2,\theta} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \\ Q_{1,\theta} & \dots & \dots & Q_{\theta,\theta} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & \ddots & 0 & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 & 0 & \dots & 0 \end{pmatrix} \tag{19}
$$

where $\theta = K + N$. The zeros result from the fact that the last θ equations in the transition equation describe identities. The Carter and Kohn algorithm will be generalised to take the singularity of the matrix Q into account.

Equation (18) is a VAR in F_t and Y_t, that can be interpreted as a reduced-form model involving both observable and unobservable components, written in first-order companion form. It is worth noting that this model is reduced to a standard form VAR if the coefficients that relate Y_t to past values of F are zero. If those elements are different from zero, then we have a FAVAR. As noted in Bernanke *et al.* (2005), if the true system is given by a FAVAR, omitting the introduction of the unobserved factors in the model will lead to a biased estimate of the VAR coefficients and related impulse responses.

B Estimation via Gibbs sampling algorithm

B.1 Setting priors and starting values

The observation equation: the prior for the factor loadings is normal. Let's define $H = \{\Lambda_i^f, \Lambda_i^y\}$ then $p(H) \sim (H_0, \Sigma_H)$. The diagonal elements of the variance-covariance matrix of the error terms R_i are drawn from an Inverse Gamma $p(R_i) \sim IG(R_{ii0}, V_{R0})$. As starting values, the elements of R are arbitrarily set equal to one and $\Sigma_H = I$.

The transition equation: I introduce a natural conjugate prior for the VAR parameters à la Sims & Zha (1998) as in Bańbura *et al.* (2010), slightly modified to accommodate the presence of the extra exogenous variable.

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\gamma_1\sigma_1 \ldots \gamma_\theta\sigma_\theta)}{\tau} \\ 0_{\theta\times(p-1)\times\theta} \\ \ldots \\ diag(\sigma_1 \ldots \sigma_\theta) \\ \ldots \\ 0_{(1+ex)\times\theta} \end{pmatrix} \tag{20}$$

$$X_{D,1} = \begin{pmatrix} \frac{J_p \otimes diag(\sigma_1 ... \sigma_\theta)}{\tau} 0_{\theta p \times ex} & \\ 0_{\theta \times \theta p + ex} & 0_{\theta p \times (1+ex)} \\ \cdots & \\ 0_{(1+ex) \times \theta} & diag(c_{(1+ex)\times(1+ex)}) \end{pmatrix} \tag{21}$$

Where γ_1 and γ_θ denote the prior mean for the coefficients of the first lag, τ is the tightness of the prior of the VAR coefficients, ex is the number of exogenous variables in the VAR and c is a $(1 + ex) \times (1 + ex)$ matrix for the tightness of the constant and the proxy. Following Alessandri & Mumtaz (2017), the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable. Similarly to Mumtaz & Theophilopoulou (2020), I use the principal components estimates of the factors F_t for this purpose. As is standard for U.S. data, τ is set equal to 0.1. The scaling factors σ_i are set using the standard deviation of the error terms from the AR(1) regressions. The values in c are set equal to 1/1000. Additionally, for those variables that have a unit root, I impose a sum of coefficients prior. This incorporates the belief that coefficients on lags of the dependent variable sum to 1. This prior can be implemented in the transition equation in (4) via the following dummy observations.

$$Y_{D,2} = \left(\frac{diag(\gamma_1 \mu_1 ... \gamma_\theta \mu_\theta)}{\lambda} \right) \tag{22}$$

$$X_{D,2} = \left(\frac{(1_{1\times p}) \otimes diag(\gamma_1 \mu_1 ... \gamma_\theta \mu_\theta)}{\lambda} 0_{\times (1+ex)} \right) \tag{23}$$

Where μ_i denotes the sample means of the endogenous variables. As in Bańbura *et al.* (2010), the tightness of this sum of coefficients prior is set as $\lambda = 10\tau$. The variance-covariance of the error terms is drawn from an Inverse Wishart $p(B_i) \sim IW(B_0, \Sigma_B)$ and $p(r_i) \sim IG(R_{ii0}, V_{R0})$.

Finally, the Kalman filter requires the initial value of the state vector; for this purpose, the principal components extracted from the set of informational variables are used as a first estimate of the factors to set the initial values of the state vector in the Kalman filter.

B.2 Steps of the Gibbs sampling algorithm

Step 1 - Sample the matrix of factor loadings

Conditional on the (starting values) of the factors F and on R_{ii}, sample the factor loading from their conditional posterior. For each variable in the informational set X_t the factor loadings have normal conditional posteriors $H(H_i|F_t, R_{ii}) \sim N(H_i, H_i^*, V_i^*)$, where:

$$H_i^* = (\Sigma_{H_i}^{-1} + \frac{1}{R_{ii}} Z_t' Z_t)^{-1} (\Sigma_{H_i}^{-1} H_{i0} + \frac{1}{R_{ii}} Z_t' X_t) \tag{24}$$

$$V_i^* = (\Sigma_{H_i}^{-1} + \frac{1}{R_{ii}} Z_t' Z_t)^{-1} \tag{25}$$

Where $Z_t = \{F_1, \ldots, F_K\}$ .It is worth noting that in the original FAVAR specification outlined in Bernanke et al. (2005), $Z_t = \{F_1, \ldots, F_K, FEDFUND\}$ if a data series is fast-moving, it has a contemporaneous relationship with the policy rate. On the contrary, for those series considered slow-moving, $Z_t = \{F_1, \ldots, F_K\}$. In the FAVAR with an exogenous variable, this distinction is not needed as the exogenous variable approach allows for the shock to impact all the variables contemporaneously.

Also, note that as the factors and H_i are both estimated, the model is unidentified. Following Bernanke *et al.* (2005), the top $K \times K$ block of b_{ij} is fixed to an identity matrix.

Step 2 - Variance of the errors of the observational equation

Conditional on the factors F_t and the factor loadings $H_i = \{\lambda_{ij}^f, \lambda_i^y\}$, sample the variance of the error terms of the observation equation R_{ii} from the inverse Gamma distribution with scale parameter $(X_{ij}Z_tH_i)'(X_{ij}Z_tH_i) + R_{ii}$ with degrees of freedom $T + V_{R0}$ where T is the length of the estimation sample. Prior degrees of freedom and the prior scale matrix are set to 0 (hence I use information from the data only).

Step 3 - Coefficients of the transition equation

Conditional on the factors F_t and the error covariance matrix Ω, the posterior for the coefficients of the transition equation B is normal and given by $H(B|F_t, \Omega) \sim N(B^*, D^*)$ where:

$$B^* = vec((\bar{X}_t'\bar{X}_t')^{-1}(\bar{X}_t'\bar{Y}_t)) \tag{26}$$

$$D^* = (\Omega \otimes \bar{X}_t'\bar{X}_t')^{-1} \tag{27}$$

Where

$$\bar{X} = \begin{pmatrix} X \\ X_{D,1} \\ X_{D,2} \end{pmatrix} \tag{28}$$

$$\bar{Y} = \begin{pmatrix} Y \\ Y_{D,1} \\ Y_{D,2} \end{pmatrix} \tag{29}$$

Step 4. Error covariance of the transition equation

Conditional on the factors F and the VAR coefficients B, the error covariance Ω has a inverse Wishart posterior with scale matrix $(Y_t - \hat(X)B)'(Y_t - \hat(X)B) + \Omega_0$ and degrees of freedom $T + V_0$.

Step 5 - Backward recursion to obtain the factors

Given H, R, B and Ω the model can be cast into state-space form, and then the factors F_t are sampled via the Carter and Kohn algorithm. In the Carter and Kohn algorithm, I have to take into account the fact that the matrix Q is singular in the FAVAR model. This implies that the recursion has to be generalized slightly to take this singularity into account. This modification implies that I use μ^*, F^*, Q^* and β^*_{t+1}, where μ^*, F^*, Q^* and β^*_{t+1} denote the first jv rows of μ, F, Q and β_{t+1} and jv is the number of factors plus the number of observable variables in the model, which corresponds to the size of the non-zero elements in matrix Q. It is worth noting that while in the FAVAR with a Cholesky identification as originally proposed in Bernanke *et al.* (2005) the μ vector is fixed as it contains the coefficients associated with the constant, in the FAVAR proposed here, the μ also contains the coefficients associated with the exogenous proxy for the structural shock of interest; hence, the μ vector will change in every time period through the Kalman filter and the backward recursion.

Repeat steps 1 to 5 as many times as needed to reach convergence.

C Convergence of the Gibbs sampler

I assess the convergence of the Gibbs sampling algorithm by looking at the efficiency factors for the estimated coefficients of the observation equation. The inefficiency factor is defined as the ratio of the numerical variance of the sample posterior mean to the variance of the sample mean from the hypothetical uncorrelated draw (see Chib, 2001). The ineffciency factors for the estimated coefficients range between 2.7 and 10.8, suggesting good mixing properties of the Gibbs sampler (see Figure 7).

D Comparison between exogenous instrument approach and Cholesky

Figure 8 compares the responses of a number of selected variables obtained with the FAVAR with the exogenous variable against the ones obtained with a standard FAVAR with shock identified via Cholesky decomposition, as originally proposed in Bernanke *et al.* (2005). The two sets of responses look substantially different. In general, responses obtained through the exogenous variable approach are greater in magnitude than the ones obtained when the shock is identified through zero restrictions. Moreover, the latter present a price puzzle in the response of CPI and a real economy puzzle in the responses of personal consumption and unemployment.

The impact on industrial production is substantially stronger in the case of the exogenous variable than with Cholesky, but both lie in negative territory over the whole horizon. Inflation responds positively in the short-term in the case of Cholesky, indicating the presence of a prize puzzle, which disappears with the exogenous variable approach. The impact on consumption diverges in the two cases; it increases in the case of Cholesky, contrarily to what theory would suggest. The unemployment rate reacts with delay in the case of Cholesky, while it increases on impact and over the rest of the sample in the case of the exogenous variable approach. Finally, the impact on the equity market is visibly different. When the shock is identified with zero restrictions, the impact is marginally positive but close to zero, while with FAVAR with exogenous variable it declines sharply in the short-term before rebounding.

E Figures

Figure 1: Monte Carlo Simulation

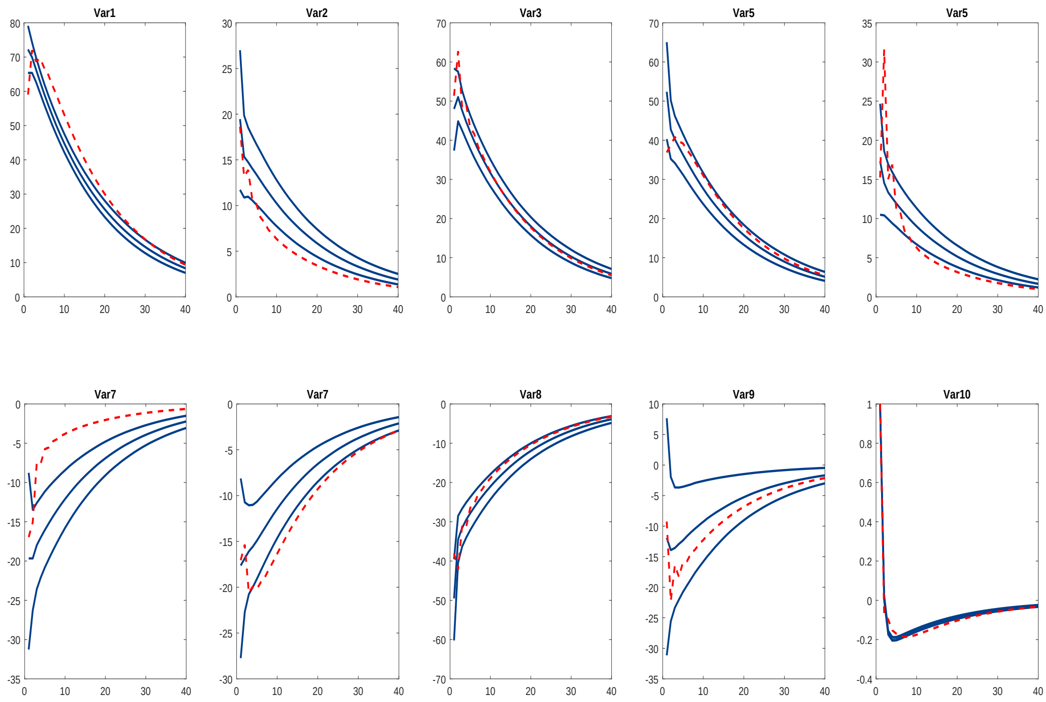

Figure 1: Comparison between impulse responses estimated with a FAVAR with exogenous instrument and the true impulse responses. Dashed lines represent the true IRFs; the blue lines reproduce the median and 68 bands.

Figure 2: Monetary Policy Instrument (MPI)

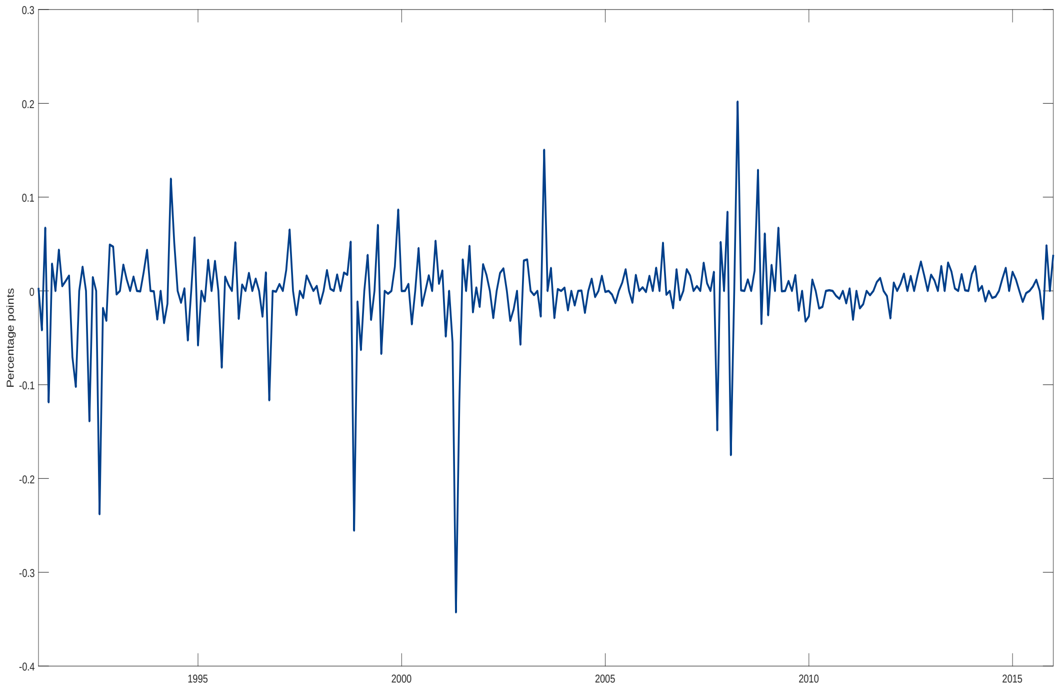

Figure 2: Monetary Policy Instrument by Miranda-Agrippino & Ricco (2021). Sample: 1991M1-2015M12.

Figure 3: Benchmark results - transmission of a monetary policy shock

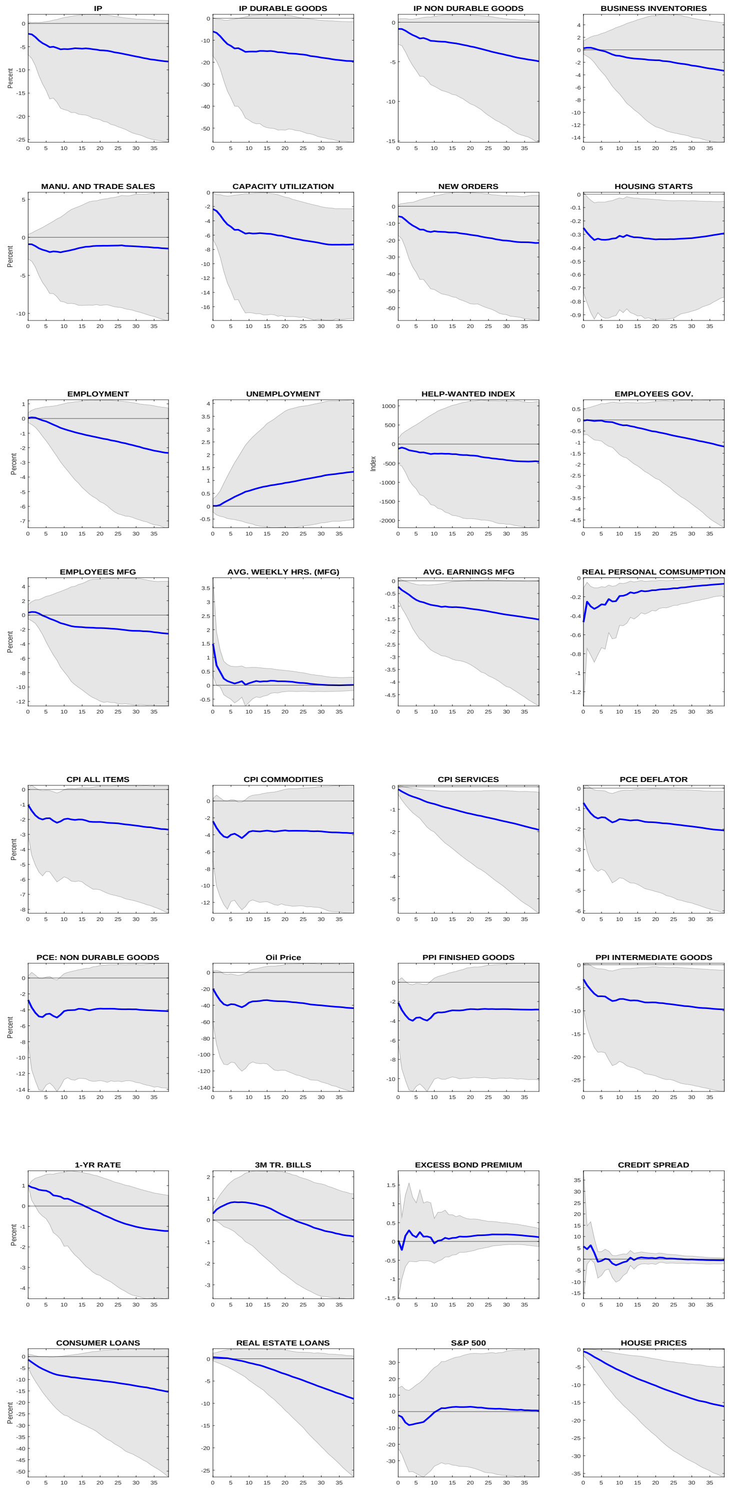

Figure 3: Transmission of a Monetary Policy shock in the US. Impulse responses to a contractionary monetary policy shock, normalized to give an initial increase in the federal funds rate of 100 basis points. Blue solid lines and shaded gray areas correspond to the median responses obtained with FAVAR with exogenous proxy and 68 percent confidence bands. The FAVAR includes 10 factors extracted from the FRED dataset augmented with extra variables and the Federal Fund Rate. The instrument used to proxy the policy shock is the Monetary Policy Instrument (MPI) by Miranda-Agrippino & Ricco (2021). Sample: 1991M1-2015M12.

Figure 4: Small-scale VARX and FAVAR with exogenous proxy

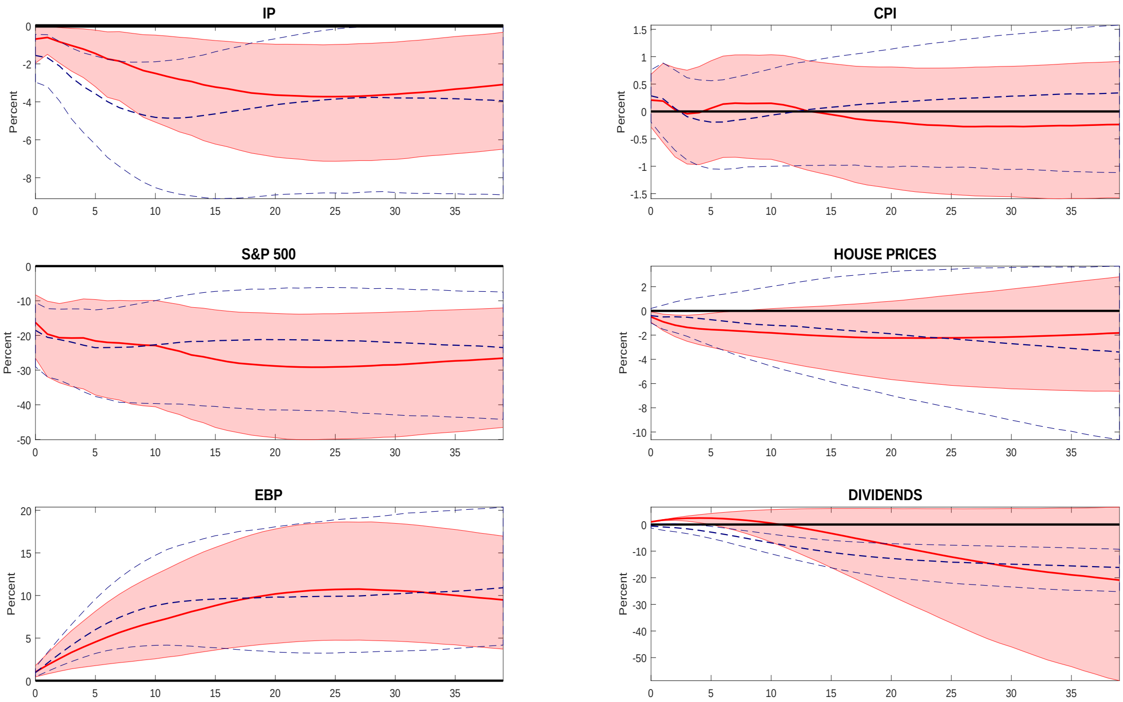

Figure 4: Impulse responses to a contractionary monetary policy shock, normalized to give an initial increase in the one-year government bond rate of 100 basis points. Dashed-lines indicate the median response obtained with the small-scale VARX along with 68 percent confidence intervals. Red solid lines and shaded areas correspond to the responses obtained with FAVAR with exogenous proxy and 68 percent confidence bands. The small-scale VARX includes stock prices, dividends, house prices, excess bond premium, CPI, industrial production, and the rate. The FAVAR includes 13 factors extracted from the FRED dataset augmented with extra variables and the rate. The instrument used to proxy the policy shock is the monetary surprise (MP1) by Gürkaynak *et al.* (2005). Sample: 1988M11-2017M9.

Figure 5: Information sufficiency, price and real activity puzzles

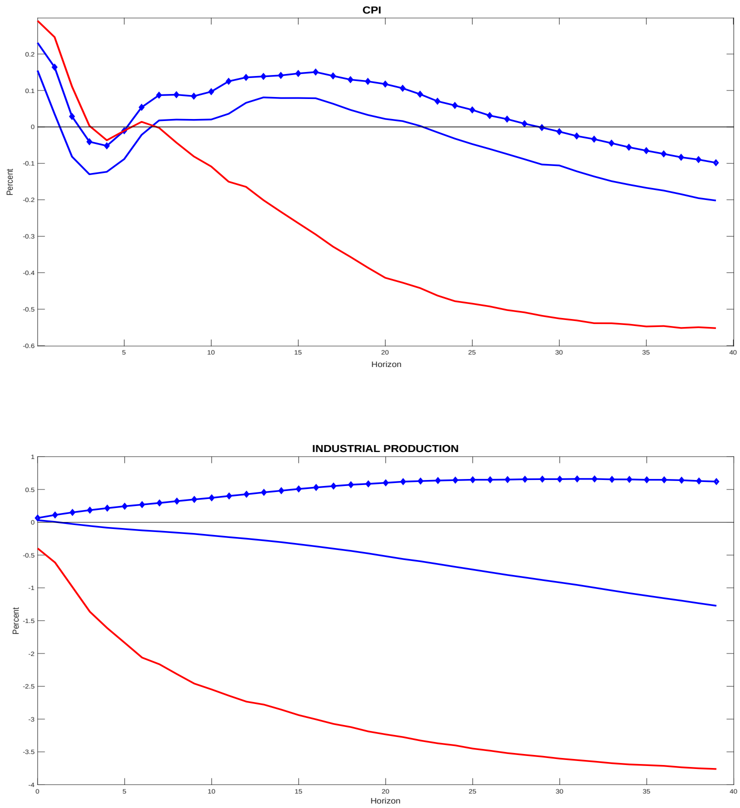

Figure 5: Price and Industrial Production puzzles with different numbers of factors. Impulse responses to a contractionary monetary policy shock, normalized to give an initial increase in the one-year government bond rate of 100 basis points. The instrument used to proxy the Monetary Policy Instrument (MPI) by Gürkaynak *et al.* (2005). Sample: 1988M11-2017M9. Top-panel: CPI responses obtained with different number of factors. Blue line with markers: 8 factors; blue line: 9 factors; red line: 10 factors. Bottom-panel : IP responses obtained with different number of factors. Blue line with markers line: 5 factors; blue line: 6 factors; red line: 7 factors.

Figure 6: Different identifying instruments and information sufficiency

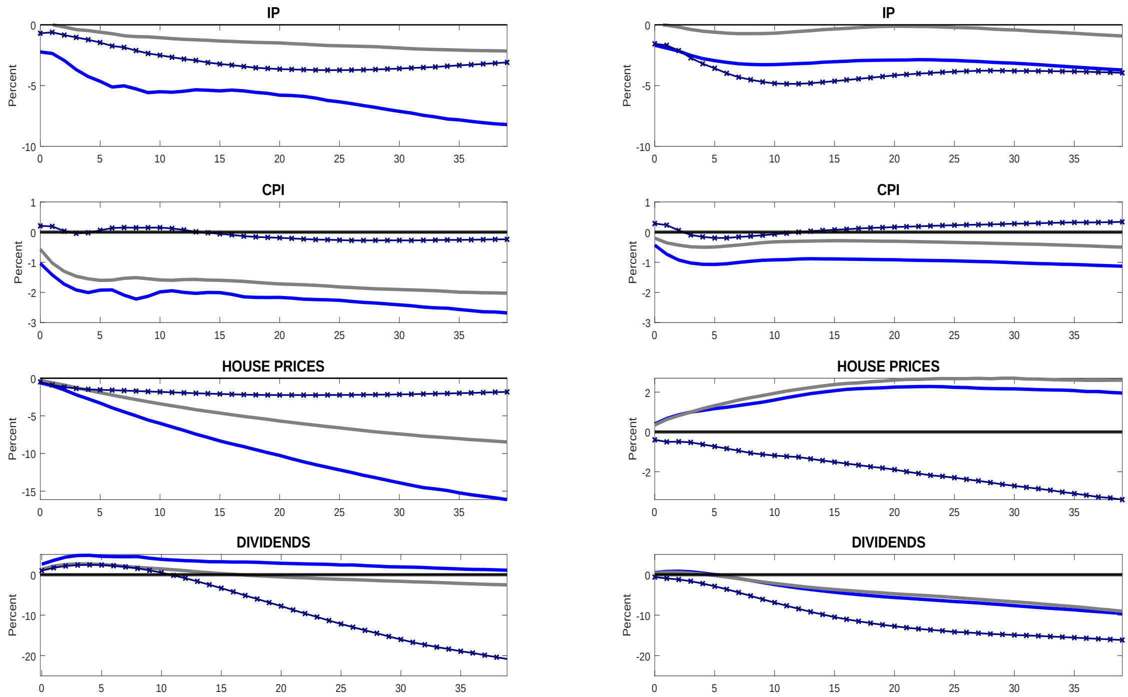

Figure 6: Monetary policy instruments comparisons - Impulse responses to a one-standard-deviation shock normalized to have an increase of 100 basis points in the one-year government bond rate estimated with FAVARX (left-hand side panels) and VARX (right-hand side panels). Solid blue lines indicate impulse responses resulting from a shock identified with the MPI by Miranda-Agrippino & Ricco (2021); blue lines with markers indicate the impulse response resulting from a shock identified using the MP1 by Gürkaynak *et al.* (2005). Gray lines represent responses to a shock using the MPI by FF4 by Gertler & Karadi (2015). Model specifications: FAVAR with MPI and GK include 10 factors as in the benchmark specification; FAVAR with MP1 includes 13 factors; VARX includes stock prices, dividends, house prices, excess bond premium, CPI, industrial production, and the Federal Fund Rate. For each instrument, all the available sample is used.

Figure 7: Inefficiency Factors

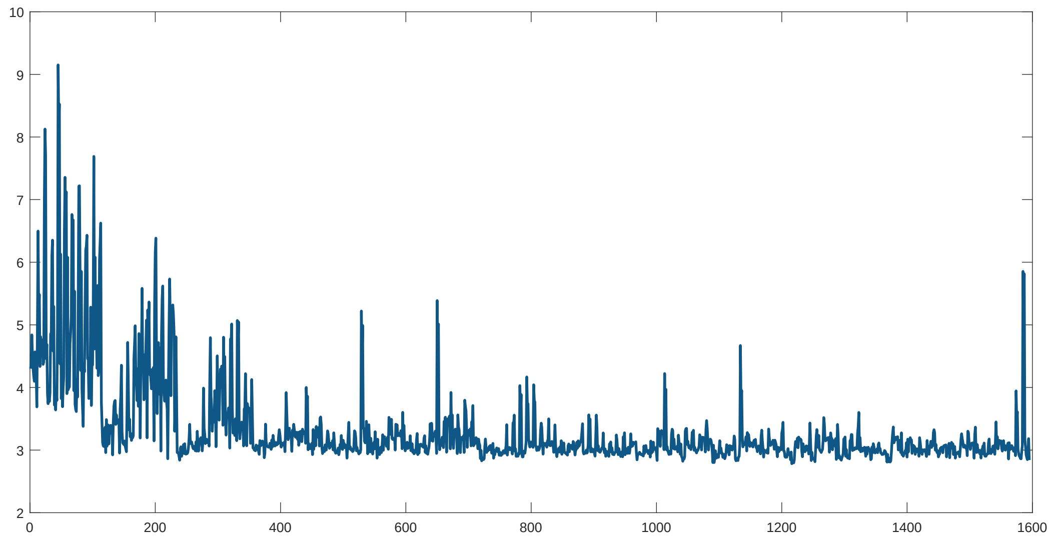

Figure 7: Inefficiency factors for the parameters of the transition equation, obtained from a 100000 iteration of the Gibbs sampler, 50000 burns and retaining every 10th iteration. Lags for the autocorrelation=20.

Figure 8: Comparison between the exogenous instrument approach and Cholesky

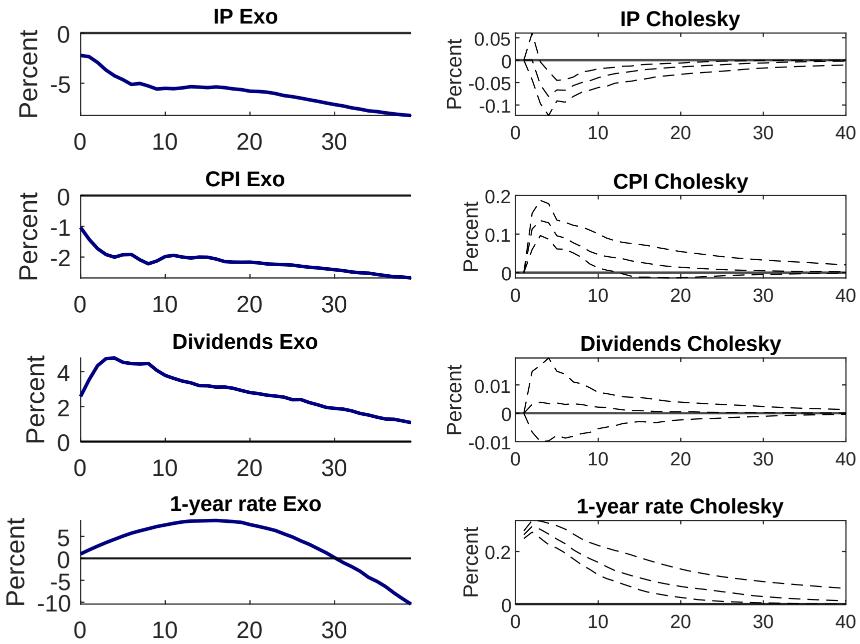

Figure 8: FAVAR including 10 factors plus the one-year rate. Impulse responses to a one-standard-deviation shock normalized to have an increase of 100 basis points in the one-year government bond rate. The gray-dashed lines indicate the impulse response resulting from a shock identified with Cholesky scheme. Blue lines represent responses to a shock with the exogenous variable approach. The instrument used is the MPI narrative instrument by Miranda-Agrippino & Ricco (2021). Sample 1991-2015.