A theory of front-line management

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Abstract

Front-line managers are essential to the success of large organizations, yet they must often find a way to incentivize their workers without altering their wage. We propose a model of the interaction between a front-line manager and a worker in a large organization, and show that different "managerial styles" arise endogenously as a function of the information structure and the players' relative patience. In addition to improving the understanding of the interaction between workers and their immediate managers, the theory alludes to novel connections between an organization's personnel policies and the output that front-line managers induce from their workers.

Keywords: Front-line management, Periodic budget, Structure of the firm, Asymmetric discounting.
JEL Classifications: D21, D82, D86, L22.

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1 Introduction

A significant restructuring of large organizations was observed toward the end of the 20^{th} century. While there was a decline in mid-level management, front-line managers became increasingly important and began receiving more discretion.[1] Different levels of managers in hierarchical organizations have different responsibilities and tools at their disposal to incentivize workers. While front-line managers have the expertise and position to monitor their subordinates and effectively adapt their tasks to changing operational needs, their ability to incentivize workers is limited. They are rarely involved in the design of labor contracts and typically cannot adjust wages in response to fluctuations in work.

If labor contracts were complete (in the sense that they could address every possible contingency), the role of the front-line manager would mainly consist of monitoring activities. However, such contracts would be extremely complex and unrealistic.[2] Workers may sometimes need to stay beyond their usual work hours to prepare urgent presentations or comply with an unexpected demand from a major client, deal with unexpected breakdowns, or take on temporary additional duties when a coworker is on sick leave. More generally, while it is hard to fully anticipate all possible events requiring special effort on the part of the worker, it is widely accepted that such occasions are a significant part of the dynamics in many organizations.[3]

In this respect, the front-line manager fulfills a fundamental role within a hierarchical organization beyond the supervisory function discussed in, for example, Williamson (1967), Calvo and Wellisz (1978,1979), Tirole (1986), and Laffont (1988). Namely, if given sufficient discretion, a front-line manager can be an effective means to *complement* the incomplete labor contract of the worker. That is, rather than specifying a long and detailed list of if-then circumstances within the worker's labor contract, organizations can authorize front-line managers to use their discretion to take opportune managerial decisions and provide additional incentives when needed.

[1]See, e.g., Cameron, Freeman and Mishra (1991) and Thomas and Dunkerley (1999).

[2]Indeed, Williamson (1975) and Hart (1995) argue that incomplete contracting is necessary for a cogent theory of the firm.

[3]This has been noted, for example, by Williamson (1967, p. 125) who writes that "the firm is required to adapt to circumstances which are predictable in the sense that although they occur with stochastic regularity, precise advance knowledge of them is unavailable. ... Coordination in these circumstances is thus essential."

Although monthly wages are typically beyond the control of front-line managers, they can often influence other important aspects of a worker's job satisfaction through their choice of managerial practices and the allocation of perks under their control. Some of the key channels by which this can be done are as follows. First, they can improve work atmosphere and allow their subordinates to attain a better work-life balance by permitting remote work or flexible working hours, reducing the monitoring frequency, and exempting them from mundane tasks.[4] Second, front-line managers can enhance their subordinates' prospects in the firm by providing them with training programs or increasing their visibility in the organization, such as by allowing them to participate in meetings with important clients or higher-up managers.[5] Third, front-line managers can convey to their subordinates that the organization values their contribution, cares about their well-being, and will support their socioemotional needs.[6]

We argue that these incentivization tools have a substantially different structure compared to the traditional quasilinear model of monetary incentives. In particular, the per-period incentive budget is typically *small* (e.g., allowing work from home for only one day may not be enough compensation for substantial effort), and also such incentives are usually *perishable* (e.g., the possibility to allow work from home on a given day is wasted if not utilized on that day).

We propose a tractable model to study the interaction between the front-line manager (she) and a worker (he) with the features described above and obtain a number of qualitative results regarding the dynamics at the bottom of the organizational hierarchy. Our analysis also offers insights into the broader organizational level (beyond the interaction between the front-line manager and the worker) and suggests potential links between the organizational promotion and layoff policies and front-line managerial practices (which affect the organization's productivity).

In our model, the (front-line) manager can offer only small and perishable per-period compensation, which she uses to incentivize worker's "extra" effort that

[4]According to a recent survey by Gallup (2022) the second most important factor, after wages, in deciding whether to accept a new job is "work-life balance and personal well-being," and the third most important factor is "the ability to do what they do best."

[5]The importance of career concerns to employees is well established (e.g., Holmström, 1999), and the effect of organizational visibility on these concerns is pointed out by Holmström (2017).

[6]The importance of such behavior is the basis of Organizational Support Theory, which also highlights the importance of front-line managers in shaping workers' perceptions of organizational support. See Rhoades and Eisenberger (2002) for a review of this theory.

is needed on random occasions. The interpretation is that, occasionally, there are peaks at work or events that require special attention and effort, beyond the regular work requirements. We refer to such events as "opportunities." In some cases, the manager observes the arrival of opportunities (e.g., when she receives an urgent task from her superiors), while in other cases, opportunities may be observed privately by the worker (e.g., creative ideas) who then decides whether or not to conceal them from the manager.

As the manager's per-period compensation budget is small, there is structural asynchronicity between work and compensation in our model: the manager is unable to *instantaneously* compensate the worker for his effort. This asynchronicity, in turn, gives rise to a dynamic spillover between opportunities since new opportunities may arrive while the worker is still receiving compensation for his previous effort. Thus, the front-line manager faces a rather complex dynamic problem where the optimal way to resolve tradeoffs depends on the players' relative patience. While most papers focus on the case where the players discount the future using the same discount factor, there is no compelling reason to assume that, within their specific interaction, the worker and the manager consider the future in the same way. For example, a manager who plans to lead the team for only a short while might be impatient (or a short-termist) compared to a worker who expects to stay in the same position for a very long time. In general, each party can be more or less patient than the other and, to an extent, this can be endogenously determined by the organizational promotion and layoff policies.

We characterize the optimal contracts for any pair of the players' discount rates, both for the case where the arrival of opportunities is observable and for the case where opportunities can be concealed by the worker. The optimal contracts generate distinct qualitative features for different specifications of the model that naturally correspond to various "managerial styles." For example, what degree of flexibility do the workers enjoy at work and what affects this flexibility? How do the worker's workload and perks change over time? Do managers generate artificial ranks within their teams or do they maintain equal status among similar workers? If certain perks are granted, are they permanent or temporary?

We begin by considering the case in which the arrival of opportunities is observed by the manager. When the manager is *patient* relative to the worker, the manager treats workers identically, regardless of their past work or tenure. When an opportunity arrives, the manager requires the worker's effort and promises all perks within her discretion for a given amount of time. These promises, however, are "conditional" and do not accumulate: each promise will be nullified upon the arrival of the next opportunity. It is therefore a question not of *how much* but rather *whether* some work was done in the recent past.

When the manager is *impatient*, on the other hand, the interaction features completely different dynamics. There are three distinct phases. Initially, the worker is a *junior* and expected to exert maximal effort on every opportunity that arrives, without enjoying any perks. At some point, the worker moves up to an *intermediate* status, in which he still needs to work whenever an opportunity arrives, but now he also enjoys all the perks at the manager's discretion. Finally, he reaches a *senior* status, in which he enjoys the maximal level of perks without exerting any effort (beyond his unmodeled baseline duties). The transition times between the different phases are fixed (up to the arrival of the first opportunity) and do not depend on the actual amount of effort the worker has exerted over time. Hence, the manager adopts a "*tenure-based seniority system*" – a substantially different managerial style from that of the patient manager.

Note that the above contracts (both for the patient and for the impatient manager) feature very little correlation between work and compensation. In particular, the arrival of an opportunity is typically "bad news" for the worker. Hence, such contracts cannot be used to incentivize opportunities that the worker can conceal. For the worker to reveal that such an opportunity has arrived, the promise of future compensation must increase by at least the cost of the required effort. Thus, the optimal contracts for the case of *concealable* opportunities will have the *perfect bookkeeping* property: on every path of play, the discounted value (using the worker's discount rate) of granted perks is equal to the discounted cost of the effort exerted.

As in the case of observable opportunities, the relative patience of the players affects the managerial style. However, the transition between the different managerial styles is more nuanced in the case of concealable opportunities. A seniority system still becomes more likely as the manager becomes more impatient. As a consequence of the perfect bookkeeping property, however, the seniority system in this case is *performance-based* rather than *tenure-based*. The *junior* and *senior* statuses are similar under both observable and concealable opportunities: junior workers exert effort without receiving perks, whereas senior workers enjoy all the perks without exerting any effort when opportunities arrive. The *intermediate* status, however, has a different structure: when opportunities are concealable, the worker enjoys a *partial* level of perks as a guaranteed baseline, and whenever he exerts effort he enjoys an immediate *temporary* increase in perks.[7] The seniority system is performance-based in that, in contrast to the tenure-based seniority system, promotion is related directly to the realized amount of work over time.

Another substantial difference is that if opportunities are concealable, the seniority system arises only if the manager is *sufficiently* impatient relative to the worker. If the manager is only slightly impatient (or if she is patient), then the worker will enjoy perks immediately whenever he exerts effort. Due to the perfect bookkeeping property, these promises of future compensation accumulate. The slightly impatient manager will continue to incentivize maximum effort until she completely runs out of incentives. Thus, in this case, the worker will eventually stop exerting effort on new opportunities. On the other hand, it is suboptimal for a manager who is more patient than the worker to increase the promises without limit. As a result, the required effort on new opportunities will always be positive, but it may change nonmonotonically over time.

Table 1 summarizes the qualitative dynamics of incentive provision for varying information structures and relative patience.

Opportunities	***Manager***		
	Patient	**Slightly Impatient**	**Very Impatient**
Observable	Conditional promises (Finite)	Tenure-based seniority system	
Concealable	Accumulating promises (Finite)	Accumulating promises (Infinite)	Performance-based seniority system

Table 1: Incentive Dynamics.

Based on the above analysis, we can take the perspective of the organization where the manager–worker interaction takes place and draw broader observations regarding the impact of the endogenous managerial style at the bottom of the hierarchical ladder. Note that the managerial style changes significantly with the identity of the more patient player when opportunities are observable (unlike in the con-

[7]This intermediate status exists so long as the manager is not extremely impatient.

cealable opportunities case). Assuming that both the front-line manager and the worker are less patient than the organization (arguably, a natural assumption for large organizations), we show that when the manager becomes slightly less patient than the worker, the organization's payoff from the manager–worker interaction drops discontinuously. That is, a discontinuity occurs at a point typically assumed in the literature – where the players use the same discount factor. To illustrate that this drop is significant, we show that, perhaps surprisingly, when the manager is slightly impatient relative to the worker, the organization is sometimes better off under the *concealable* rather than the *observable* opportunities case.

This observation regarding incentive provision in hierarchical organizations is interesting in its own right as it suggests that the organization may benefit from increasing information frictions at the bottom of the organizational hierarchy. Thus, an organization might prefer to hire professional managers who lack relevant technical expertise, even if they do not have exceptional managerial skills.

In addition, our analysis establishes an unambiguous effect of uncertainty in the opportunity arrival process on the manager's ability to complement the worker's contract. Namely, as opportunities become *lumpier* – bigger but rarer such that their expected flow value does not change – the manager's ability to extract value decreases in all specifications of the model, except for the case where opportunities are observable and the players share the same discount factor. This result showcases once again the nongeneric nature of the assumption that the players use the same discount factor and how misleading this assumption can be.

Related Literature

This paper contributes to the literature that has provided various explanations for the prevalence of hierarchical structures in large organizations. Williamson (1967) and Calvo and Wellisz (1978,1979) argue that hierarchies arise due to limitations on the number of employees that a manager can effectively control and monitor.[8] Rosen (1982) suggests that hierarchies enable highly talented senior managers to increase the productivity of their subordinates. Garicano (2000) and Harris and Raviv (2002) propose the idea that hierarchies enable the efficient utilization of ex-

[8] Tirole (1986) and Laffont (1988) show that, in this case, hierarchical structures are susceptible to collusion between the workers and the low-level managers that monitor them. More recently, Letina, Liu and Netzer (2020) show that if low-level managers care about their workers' welfare, but do not collude with them outright, then the firm should induce a contest between the workers.

pert knowledge within the firm, whereas Rajan and Zingales (2001) argue that hierarchies can also prevent employees from stealing a firm's core knowledge. Hart and Moore (2005) show that hierarchies can be an efficient method for allocating resources within the firm. See Mookherjee (2013) for an extensive review of this literature.[9]

Two papers that, like ours, assume that the front-line manager is responsible for determining the responsibilities of her workers are McAfee and McMillan (1995) and Melumad, Mookherjee and Reichelstein (1995). These papers differ from ours in that they consider a static problem in which there is no stochasticity in the tasks that the worker must perform, and the manager can offer monetary incentives to the worker. Moreover, they focus on deriving conditions under which adding an intermediate level between the principal and the worker is beneficial.

Our paper also complements works that study the optimal timing of compensation (e.g., Lazear 1981; Carmichael 1983) and, in particular, those that analyze the mixture between short- and long-term incentives in settings where information changes over time (e.g., Sannikov 2008; Garrett and Pavan 2012, 2015). We contribute to this literature by analyzing the efficient use of limited and perishable incentives in a dynamically changing environment.

Our work also contributes to the recent literature on optimal contracting under different discount factors. Opp and Zhu (2015) study relational contracting in a repeated moral hazard setting, Frankel (2016) studies dynamic delegation, Hoffmann, Inderst and Opp (2021) study a one-shot moral hazard problem in which there is delay in the arrival of information, and Krasikov, Lamba and Schacherer (2020) analyze a canonical adverse selection problem.

Finally, our work is related to the growing literature that studies principal–agent interactions with randomly arriving "projects" under symmetric discounting. Forand and Zápal (2020) and Bird and Frug (2021) consider optimal contracting under symmetric information: Forand and Zápal (2020) study a model with no transfers in which projects of different types arrive randomly over time, whereas Bird and Frug (2021) study a canonical employment model in which the agent's productivity of effort varies over time. Li, Matouschek and Powell (2017), Bird and Frug (2019), and Lipnowski and Ramos (2020) consider transfer-free environments with asymmetric information. More specifically, Li, Matouschek and Powell (2017) derive the

[9] In the broader context of organizational design, Rantakari (2008) studies how the need to coordinate a firm's activities affects the optimal allocation of decision rights within a two-tier hierarchy.

optimal relational contract when the agent has private information on project availability. Bird and Frug (2019) derive the optimal contract under full commitment in a setting where the agent privately observes the stochastic arrival of different types of compensation opportunities. Lipnowski and Ramos (2020) characterize efficient equilibria when the agent has private information on project payoffs.

The paper proceeds as follows. In Section 2 we present the model. In Sections 3 and 4 we analyze the cases where opportunities are observable and concealable, respectively. In Section 5 we present some organizational implications of our analysis and discuss the role of selected modeling assumptions. All proofs are relegated to the Appendix.

2 Model

We consider an infinite-horizon continuous-time interaction between a manager and a worker, in which opportunities arrive stochastically over time according to a Poisson process with an arrival rate $\mu > 0$. The no-effort action, $\alpha = 0$, is always available to the worker. When an opportunity arrives, and only then, in addition to the no-effort action, the worker can exert effort $\alpha \in (0, 1]$.[10] The worker's effort $\alpha \in [0, 1]$ induces a benefit of $\alpha \cdot B$ to the manager and a cost of $\alpha \cdot A$ to the worker, where $B > A > 0$. At each instant, the manager chooses a flow compensation $\varphi \in [0, 1]$. We assume that both the worker's marginal utility from compensation and the manager's marginal cost of compensation are constant and equal to 1.

The players maximize expected discounted payoffs. We denote the worker's discount factor by r_w and focus on the case where there is no fundamental shortage of incentives. That is, we assume that the worker's discounted payoff from setting $\varphi = 1$ indefinitely exceeds his expected discounted cost of full-intensity work, $\alpha = 1$, on all opportunities that arrive, even if one is currently available.[11] Formally,

Assumption 1.

$$A + \frac{\mu A}{r_w} < \frac{1}{r_w}.$$

We denote the manager's discount factor by r_m and refer to the manager as *patient* if $r_m < r_w$ and as *impatient* if $r_m > r_w$.

[10] We discuss the case of storable opportunities in Section 5.2.

[11] Allowing for the opposite inequality would add trivial cases with corner solutions that would not add much of substance but would needlessly impede the exposition.

Our objective is to capture formal as well as informal arrangements between the manager and the worker, and also the asymmetry in their commitment abilities. Our solution concept is the manager's optimal contract (assuming the manager has full commitment power and the worker has none). This approach captures the asymmetry between the manager and the worker more than the relational contracting approach (e.g., MacLeod and Malcomson 1989; Ray 2002; Levin 2003). It is worth emphasizing that due to the upper bound on flow compensation the manager's compensation budget is limited. Hence, the assumption that the manager can fully commit to a contract is weaker than it is in models that allow for (unlimited) monetary compensation and can be justified by standard arguments about her reputational concerns.[12]

Throughout the paper we assume that the worker's effort is perfectly observed by the manager, but we vary our assumptions about whether or not she observes the arrival of opportunities. If the manager does observe the arrival of opportunities ("*observable opportunities*"), then a public history h_t specifies for every $s < t$ whether or not an opportunity was available and the worker's choice of effort.[13] On the other hand, if the manager does not observe the arrival of opportunities ("*concealable opportunities*"), then a public history h_t contains only the worker's effort choices. Given the Poisson arrival of opportunities, any private information that the worker has about the availability of opportunities in the past is irrelevant, and so there is no need to keep track of his private information. Hence, to reduce notation and terminology we refer to a public history as a history under both information structures we consider. We denote the set of all histories of length t by H_t and the set of all finite histories by $H = \cup_{t \in \mathbb{R}_+} H_t$.

At the beginning of the interaction, the manager specifies a *work schedule*

$$\alpha : H \to [0, 1],$$

which assigns a required effort to every history should an opportunity arrive at that history, and she commits to a *compensation policy*

$$\varphi : H \to [0, 1],$$

[12]We discuss relational contracting in Section 5.3.

[13]In this case, whether or not the worker observes the arrival of opportunities is immaterial.

which maps histories into a flow compensation. A pair $\langle \alpha(\cdot), \varphi(\cdot)\rangle$ is referred to as a *contract*. Without loss of generality, we assume that after the manager detects a deviation from the work schedule, she provides no compensation indefinitely.[14]

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot)\rangle$ is *measurable* if, at every history, the worker's continuation utility and the manager's continuation value are well defined. That is, the expectations

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right) ds | h_t\right),$$
$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_m(s-t)}\left(\mu\alpha(h_s)B - \varphi(h_s)\right) ds | h_t\right)$$

exist for every $h_t \in H$.

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot)\rangle$ is *incentive compatible* if it is measurable and, for every $h_t \in H$, it is optimal for the worker to choose $\alpha = \alpha(h_t)$ (conditional on the availability of an opportunity), given the continuation of the contract. Note that if a deviation to a positive effort level is profitable at a given history, then so is a deviation to no effort.[15] Hence, a contract is incentive compatible if and only if the worker (weakly) prefers to follow $\alpha(\cdot)$ than to deviate to $\alpha = 0$ at some history. Since the worker can guarantee himself a payoff of zero by never exerting effort, there is no need to impose an explicit individual rationality constraint. In what follows, we restrict attention to incentive-compatible contracts. We state the relative incentive compatibility constraints formally in Sections 3 and 4.

In our analysis and discussion of the results, we often compare contracts in terms of the timing of effort/compensation. We use the following relations. A work schedule $\alpha(\cdot)$ *postpones* effort relative to $\alpha'(\cdot)$ at a history h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha(h_s) | h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha'(h_s) | h_t\right) \tag{1}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ.[16] Similarly, a compen-

[14]In what follows, the functions $\phi(\cdot)$ and $\alpha(\cdot)$ will be specified only for histories that are on the path of play.

[15]A deviation to a non-zero level of effort is detected by the manager under both information structures, and so following such a deviation the worker will receive a continuation utility of zero. Since exerting effort is costly for the worker, such a deviation provides the worker with a strictly lower discounted payoff than exerting no effort on the current opportunity and then receiving a nonnegative continuation payoff.

[16]The latter requirement implies that a work schedule does not postpone effort relative to itself.

sation policy $\varphi(\cdot)$ *postpones* compensation relative to $\varphi'(\cdot)$ at h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi'(h_s)|h_t\right) \tag{2}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ. Analogous definitions for *expediting* effort and compensation are obtained by reversing the inequalities in (1) and (2). Note that the above definitions use the worker's discount factor.

The manager-discounted marginal cost of providing the worker with a worker-discounted util t units of time from now is $e^{-r_m t}\frac{1}{e^{-r_w t}} = e^{(r_w - r_m)t}$ and, similarly, the manager-discounted marginal benefit from the worker exerting one worker-discounted util of effort t units of time from now is $\frac{B}{A}e^{(r_w-r_m)t}$. Whether these expressions are increasing or decreasing in t is fully pinned down by whether the manager is patient or impatient. The following observation is implied.

Observation 1.

1. *Expediting compensation and postponing effort are both profitable for a patient manager.*

2. *Postponing compensation and expediting effort are both profitable for an impatient manager.*

3 Observable Opportunities

We begin our analysis by studying the case where the manager observes the arrival of opportunities (e.g., assignments allocated to the front-line manager by her superiors). In this case, any deviation by the worker is detected. Since the worker can guarantee himself a continuation payoff of zero at any point in time, the incentive compatibility constraints are given by

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq 0 \quad \forall h_t \in H, \tag{IC_{obs}}$$

where $(h_t, O, \alpha(h_t))$ is the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$.

Patient manager: ***"Have you done anything for me lately?"***

When the manager is patient ($r_m < r_w$), increasing the lag between compensation and effort in a manner that keeps the worker indifferent is detrimental for the manager. It is therefore easy to see that to maximize the gain from the *first* opportunity that arrives, the manager would have to provide the worker with maximal compensation, $\varphi = 1$, immediately after his work, for an interval of time that is just long enough to compensate him for his cost of effort. However, were the manager to do so, it would not be possible for her to provide prompt compensation for any additional opportunities that might arrive within that time interval. Hence, the cheapest form of compensation for the first opportunity reduces the potential gain from further opportunities. Thus, a patient manager faces a complex optimization problem where she endeavors to provide timely compensation for the worker's effort on each of the randomly arriving opportunities. The main result of this section shows that the solution to this problem is simple and qualitatively appealing: under the optimal contract, the worker exerts the same effort α^* on all opportunities, and, along the path of play, he receives a flow compensation of $\varphi = 1$ if an opportunity was available in the last S^* units of time.

This form of compensation can be understood as "conditional promises"; following the worker's effort on a given opportunity, the manager promises a fixed periodic compensation for a given time interval, but this promise is nullified upon the arrival of the next opportunity. The complete nullification of the manager's obligations to the worker upon the arrival of a new opportunity frees incentivization resources precisely when they are needed, and enables the manager to incentivize effort on the currently available opportunity via a new conditional promise.

Assumption 1 implies that there exist incentive-compatible contracts that induce full effort on all opportunities that arrive. However, such contracts need not be optimal for the manager. To see why this is the case, recall that the manager's cost of providing the worker with one worker-discounted util t units of time in the future is $e^{(r_w - r_m)t}$. As the manager's profit from a util worth of effort exerted by the worker is $\frac{B}{A}$, it follows that the *maximal profitable lag* between compensation and work is T^*, where T^* is defined implicitly by

$$e^{T^*(r_w - r_m)} = \frac{B}{A}$$

if $r_m < r_w$, and is given by $T^* = \infty$ if $r_w \leq r_m$. If a conditional promise of length T^* is insufficient to incentivize the worker to exert full effort, then $\alpha^* < 1$; i.e., the manager instructs the worker to forgo part of each opportunity.

To formally characterize the optimal contract, let $\sigma_{-1}(h_t)$ denote the supremum of opportunity arrival times along the history h_t. In all subsequent results, equalities and uniqueness statements should be interpreted as holding almost surely.

Proposition 1. *Assume that the manager observes the arrival of opportunities. If $r_m \leq r_w$, then there exist $\alpha^* \in (0, 1]$ and $S^* \in (0, T^*]$ such that*

$$\alpha(h_t) = \alpha^* \quad ; \quad \varphi(h_t) = \begin{cases} 1 & \text{if } t - \sigma_{-1}(h_t) \leq S^* \\ 0 & \text{if } t - \sigma_{-1}(h_t) > S^* \end{cases}$$

is an optimal contract. Moreover, this is the unique optimal contract if $r_m < r_w$.

When $r_w = r_m$ there are multiple optimal contracts. Intuitively, in this case, postponing compensation does not alter the manager's value or violate any of the worker's incentive compatibility constraints. Thus, any contract that results from postponing compensation relative to the optimal contract characterized in Proposition 1 is also optimal. Assumption 1 implies that if $r_w = r_m$, then under the optimal contract described in Proposition 1, $\alpha^* = 1$ and $S^* < \infty$. Hence, postponing compensation is feasible.

Impatient manager: *"Tenure-based seniority"*

By Observation 1, postponing compensation and expediting effort (according to the worker's discount factor) are both profitable for an impatient manager. The first observation underlying the characterization in this section is that neither postponing compensation nor expediting effort violates incentive compatibility. A direct implication of this is that *within* each history, the worker will exert full effort on opportunities that arrive before some (history-dependent) date and enjoy compensation from some other date onward. However, it turns out that the manager is able to postpone compensation and expedite effort *across* histories as well.

We now provide a more detailed intuition for why an impatient manager uses a tenure-based seniority system in which the clock begins at the arrival time of the first opportunity. Assume for a moment that an opportunity is available at the beginning of interaction. Clearly, it is optimal for the impatient manager (and feasible by Assumption 1) to incentivize maximal effort on that opportunity. Consider an arbitrary incentive-compatible contract $\mathcal{C}$ that does so, and denote by X and Y the worker's future discounted expected effort and compensation, respectively. Note that incentive compatibility of $\mathcal{C}$ implies that $Y \geq X + A$. Define τ_X^O implicitly by

$$\int_0^{\tau_X^O} \mu A e^{-r_w t} dt = X,$$

and similarly define τ_Y^C by

$$\int_{\tau_Y^C}^{\infty} e^{-r_w t} dt = Y.$$

Note that, after working on the first opportunity, the worker is indifferent between the continuation of $\mathcal{C}$ and the modified continuation contract where: i) he is required to exert full effort on all opportunities that arrive before time τ_X^O and no effort afterwards, and ii) he will receive no compensation until τ_Y^C and maximal compensation thereafter. Moreover, the modified continuation contract is incentive compatible due to the Poisson arrival of opportunities and the fact that $Y \geq X + A$.

Since this modification of the contract expedites effort and postpones compensation relative to $\mathcal{C}$, it is profitable for the manager. Hence, if the interaction begins with an available opportunity, the optimal contract must have a threshold structure as described above.

The relation between the two thresholds under the optimal contract, τ^O and τ^C, can be identified by two simple optimality conditions. First, because compensation is postponed and effort is expedited, the only relevant incentive compatibility constraint is the one at $t = 0$ (the arrival time of the first opportunity). As compensation is costly to the manager, in optimum, this constraint will be binding

$$A + \int_0^{\tau^O} \mu A e^{-r_w t} dt = \int_{\tau^C}^{\infty} e^{-r_w t} dt. \tag{3}$$

Second, note that $\tau^O > \tau^C$ as otherwise the manager can require the worker to exert effort at τ^O and provide compensation at the (weakly) later time of τ^C. This deviation is profitable for an impatient manager. Indeed, in optimum, $\tau^O - \tau^C$ is such that the net surplus generated by effort is offset by the manager's relative impatience over $\tau^O - \tau^C$ units of time,

$$\frac{B}{A} = e^{(r_m - r_w)(\tau^O - \tau^C)}. \tag{4}$$

Returning to the original interaction (which does not begin with an opportunity), it is straightforward that providing compensation to the worker before he exerts effort for the first time is suboptimal. Hence, in the optimal contract, "the clock is set to zero" at the arrival of the first opportunity. To characterize the optimal contract formally, let $\sigma_1(h_t)$ denote the infimum of the arrival times of opportunities along the history h_t.

Proposition 2. *Assume that the manager observes the arrival of opportunities. If $r_m > r_w$, then the unique optimal contract is*

$$\alpha(h_t) = \begin{cases} 1 \text{ if } t \leq \sigma_1(h_t) + \tau^O \\ 0 \text{ if } t > \sigma_1(h_t) + \tau^O \end{cases} \quad \text{and} \quad \varphi(h_t) = \begin{cases} 0 \text{ if } t \leq \sigma_1(h_t) + \tau^C \\ 1 \text{ if } t > \sigma_1(h_t) + \tau^C \end{cases},$$

where τ^C, τ^O are the unique solution to (3) *and* (4).

Proposition 2 suggests that tenure-based seniority systems arise naturally if the front-line manager is impatient. Under the contract characterized in the proposition, initially, the worker exerts effort but does not receive compensation; at some point ($\sigma_1 + \tau^C$), he begins to receive compensation but still has to work whenever an opportunity arrives; and finally (at $\sigma_1 + \tau^O$), he receives compensation without being required to exert further effort. Furthermore, as the manager becomes more impatient relative to the worker, seniority is attained more quickly and the worker's expected effort decreases. To see this, note that the optimality condition (4) can be written as

$$\frac{B}{A} e^{(r_w - r_m)\tau^O} = e^{(r_w - r_m)\tau^C}, \tag{5}$$

and since $\tau^O > \tau^C$, the derivative, with respect to r_m, of the LHS of (5) is less than that of the RHS. As (3) implies that τ^C is decreasing in τ^O, it follows that increasing r_m will lead to a decrease in τ^O (and an increase in τ^C).

Propositions 1 and 2 are visualized in Figure 1 for a typical sequence of opportunity arrivals. The upper panel depicts the worker's effort (in red) and compensation (in blue) when the manager is patient, and the lower panel depicts the same outcomes when the manager is impatient.

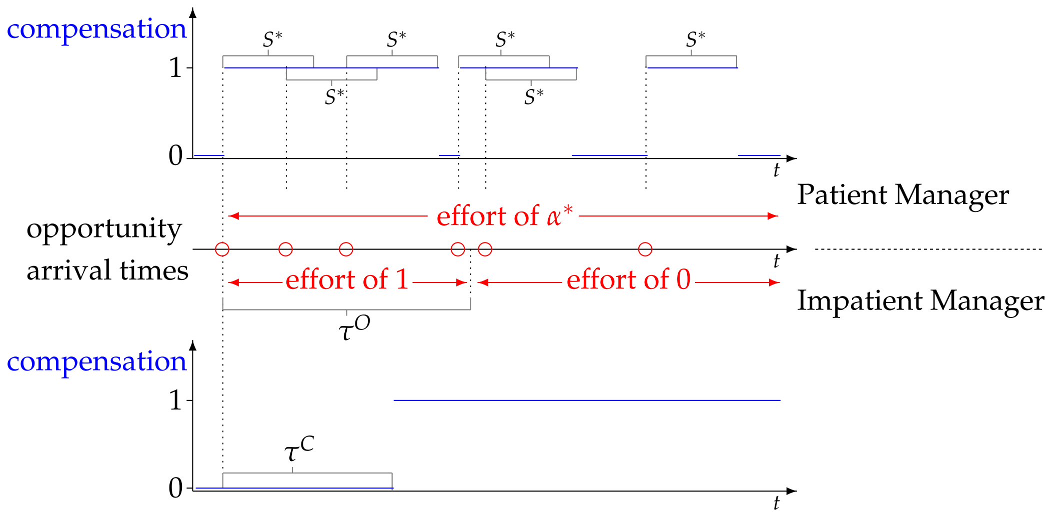

Figure 1: Qualitative dynamics of effort and compensation for patient and impatient managers for a representative sequence of opportunity arrivals.

The effect of lumpiness

The defining feature of the problem faced by the front-line manager is that she uses frequent but low-value compensation to incentivize the worker to exert effort on rare but large opportunities. Thus, it is natural to compare settings where the frequency and magnitude of opportunities vary, while keeping the total value of expected opportunities constant. Formally, we say that the opportunities represented by (A_1, B_1, μ_1) are *lumpier* than those represented by (A_0, B_0, μ_0) if there is $\lambda > 1$ such that $A_1 = \lambda A_0, B_1 = \lambda B_0$, and $\mu_1 = \frac{\mu_0}{\lambda}$.

When $r_w = r_m$, it turns out that making opportunities lumpier does not impact the manager's value so long as Assumption 1 continues to hold. This follows from three observations. First, since $r_w = r_m$, Assumption 1 implies that under the contract characterized in Proposition 1 the worker exerts full effort on all opportunities. Second, the worker's expected utility from that contract is zero. Finally, if $r_w = r_m$, then the timing of compensation does not affect the manager's cost of providing compensation. However, even though common discounting is the standard assumption in the literature, the irrelevance of the degree of lumpiness in this case is a knife-edge result. In general, lumpiness is detrimental to the manager due to the asynchronicity of compensation and effort.

Proposition 3. *Assume that opportunities are observable and that $r_m \neq r_w$. The manager's value strictly decreases when opportunities become lumpier.*

4 Concealable Opportunities

In the optimal contracts derived in the previous section, the arrival of opportunities typically leads to an immediate decrease in the worker's continuation utility. In settings where the arrival of an opportunity is observed only by the worker, such contracts provide incentives for the worker to conceal opportunities from the manager. To provide incentives for the worker to reveal when opportunities become available, the arrival of opportunities must never be "bad news" for the worker. Hence, the incentive compatibility constraints if opportunities are concealable are

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right) ds | (h_t, O, \alpha(h_t))\right) \geq$$

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right) ds | (h_t, N)\right) \quad \forall h_t \in H, \qquad (IC_{conc})$$

where (h_t, N) denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity does not arrive; and, as before, $(h_t, O, \alpha(h_t))$ denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$ on that opportunity.

The need to make the arrival of opportunities not bad news for the worker suggests that when opportunities are concealable, the manager must keep track of both the worker's compensation and his effort. This, in turn, suggests that the optimal contract should depend on the worker's continuation utility. It is well known in the dynamic contracting literature that if the environment is stationary, then the agent's continuation utility can be used as a state variable for deriving the optimal contract (see Spear and Srivastava 1987 and Thomas and Worrall 1990). The argument behind this useful result relies on the property that the continuation payoffs of an efficient contract must always lie on the constrained Pareto frontier. This, in turn, follows from two simple observations: first, if the agent receives a continuation utility u via an inefficient continuation contract, then the principal can increase her value by replacing that continuation contract with a different one that provides the agent with u utils; second, since the agent is indifferent between the original and modified continuations of the contract, this change has no impact on earlier incentive compatibility constraints. Notice that these observations do not depend on the assumption that the players share the same discount factor and, thus, they are valid in our model where the players use different discount factors.

We denote, respectively, by $\alpha(u)$, $\varphi(u)$, and $V(u)$ the Markovian work schedule, the Markovian compensation policy, and the manager's value as a function of the worker's continuation utility. Notice that $u \in [0, \frac{1}{r_w}]$ as, from any point in time onward, the worker can guarantee himself a nonnegative payoff by exerting no effort, and his payoff from receiving the maximal compensation indefinitely is $\int_0^\infty 1 \cdot e^{-r_w t} dt = \frac{1}{r_w}$. The Markovain value function satisfies two properties that help derive the optimal contract:

Lemma 1. *$V(u)$ is strictly decreasing and weakly concave.*

Lemma 1 has two important consequences. First, it directly implies that the worker's expected utility from an optimal contract is zero. Second, it implies that if opportunities are concealable, then the incentive compatibility constraint at every history is binding regardless of the relative patience of the players.

Corollary 1. *Assume that opportunities are concealable. Under an optimal contract:*

1. *The worker's expected utility is zero.*
2. *All the incentive compatibility constraints are binding.*

Corollary 1 reveals a property of the managerial style of a front-line manager that does not observe the arrival of opportunities: she engages in *perfect bookkeeping* wherein, path by path, compensation and effort discounted according to the worker's discount rate are equal. This is in contrast to the low correlation between work and compensation when opportunities are observable.

When opportunities are concealable, it is convenient to describe the optimal contract via its Markovian representation. By Corollary 1, the worker's continuation utility at the beginning of the interaction is zero and after he exerts effort $\alpha(u)$ his continuation utility increases by $\alpha(u)A$. Moreover, the drift in the worker's continuation utility while no opportunities arrive is

$$du = r_w u - \varphi(u). \tag{6}$$

Hence, the optimal contract is characterized by the solution of the HJB equation:

$$\sup_{\varphi(u),\alpha(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ +\mu\left(\alpha(u)B + V\left(u + \alpha(u)A\right) - V(u)\right)\} = 0, \quad (HJB)$$

subject to (6), where $V'(u)$ exists almost everywhere since $V(\cdot)$ is concave (Lemma 1). The following is the main result of this section.

Proposition 4. *If opportunities are concealable, then the optimal contract is generically unique. Moreover, there exist thresholds $u^C, u^O \in [0, \frac{1}{r_w}]$, such that the optimal contract is given by*

$$\alpha(u) = \min\{1, \frac{u^O - u}{A}\} \quad ; \quad \varphi(u) = \begin{cases} 1 & \text{if } u > u^C \\ r_w u & \text{if } u = u^C \\ 0 & \text{if } u < u^C \end{cases}.$$

The threshold u^O dictates the dynamics of work. The threshold value $u^O = \frac{1}{r_w}$ corresponds to a work schedule in which the manager instructs the worker to fully exploit every opportunity that arrives until all of her compensation budget is exhausted. For lower values of u^O, the manager will sometimes forgo opportunities even though her compensation budget is not exhausted.

The threshold u^C dictates the dynamics of compensation. In particular, it captures the degree of back/front-loading of compensation. So long as the worker's continuation utility is below u^C, compensation is deferred to the future. Setting the compensation threshold at the maximal possible value, $u^C = \frac{1}{r_w}$, corresponds to full back-loading: when the worker's continuation utility reaches that level, it is necessary to set $\varphi = 1$ indefinitely. At the other extreme, the compensation threshold $u^C = 0$ corresponds to full front-loading because, in this case, the manager provides the maximal compensation whenever the worker's promised continuation utility is positive.

For $u^C \in (0, \frac{1}{r_w})$, the compensation dynamics consists of two phases. In the beginning, the *back-loading phase* takes place. So long as $u < u^C$, the worker exerts effort and accumulates promises of future compensation but does not receive any compensation. When his continuation utility attains (or exceeds) u^C, the *front-loading phase* begins. In this phase, the worker receives a permanent base compensation of $r_w u^C$ – which can be thought of as compensation for effort exerted in the back-loading phase – and a temporary additional compensation of $1 - r_w u^C$ whenever $u > u^C$. Observe that if $u = u^C$, then the base compensation of $r_w u^C$ maintains the worker's continuation utility constant at that level. Hence, once the worker's continuation utility reaches u^C, it never drops below this level again, which, in turn, establishes that the optimal arrangement of compensation begins with the back-loading of compensation and transitions to the front-loading of compensation.

The effect of relative patience

The relative patience of the players determines the values of the thresholds u^C and u^O. For the next result denote by $u(T^*) = \int_0^{T^*} e^{-r_w t} dt$ the maximal compensation that can be promised without exceeding the maximal profitable lag between effort and compensation.

Proposition 5. *Assume that opportunities are concealable. If $r_m < r_w$, then the thresholds of an optimal contract defined in Proposition 4 satisfy $u^O \in (0, u(T^*))$ and $u^C = 0$.*

Proposition 5 implies that compensation is provided in the form of "accumulating promises": following the worker's effort on a given opportunity, the manager promises the maximal compensation for a given time interval; to compensate the worker for additional effort, the length of the existing promise is extended. Hence, after the worker has performed a lot of work in a short time interval, requiring additional effort necessitates providing compensation in the distant future. The patient manager will clearly avoid promises greater than $u(T^*)$ as those create negative marginal value. To see why $u^O < u(T^*)$, note that increasing the promise all the way to $u(T^*)$ corresponds to a marginal profit of zero. Hence, if u is near enough to $u(T^*)$ it is profitable for the manager to reserve her compensation budget for future opportunities at which time the lag between effort and compensation will be shorter and the marginal profit will be strictly positive. This feature differentiates between the observable and concealable opportunities cases. In the former, the manager requires the worker to exert a constant effort on all opportunities, whereas in the latter case the required effort fluctuates over time.

Next, we consider an impatient manager. The threshold u^C is determined by the manager's benefit from postponing compensation. Postponing compensation impacts the manager in two ways. First, it reduces her discounted cost of providing compensation, and second, it reduces her ability to incentivize future effort. Intuitively, the threshold u^C balances the cost and benefit from postponing compensation. The next result shows that a slightly impatient manager will fully front-load compensation, a moderately impatient manager will create a seniority system that exhibits a combination of front- and back-loading of compensation, and full back-loading occurs only if the manager is extremely impatient.

Proposition 6. *Assume that opportunities are concealable. Fix A, B, μ, and r_w and suppose that $r_m \geq r_w$. The thresholds of an optimal contract defined in Proposition 4 satisfy*

- $u^O = \frac{1}{r_w}$.
- *There exists $r''_m > r'_m > r_w$ such that* $u^C \in \begin{cases} \{0\} & \text{if } r_m < r'_m \\ (0, \frac{1}{r_w} - A] & \text{if } r_m \in (r'_m, r''_m) \\ \{\frac{1}{r_w}\} & \text{if } r_m > r''_m \end{cases}$.

The optimal contract offered by an impatient manager requires the worker to exert effort for as long as it is possible to incentivize him to do so. Hence, due to perfect bookkeeping, the worker will eventually stop exerting effort.

A slightly impatient manager ($r_w < r_m < r'_m$) could reduce the direct cost of compensation by deferring it to the future. However, due to the perishability of her compensation budget, she prefers to absorb the higher direct cost of providing compensation immediately in order to free up future compensation resources.

A sufficiently impatient manager ($r_m > r'_m$) will establish a system of seniority, albeit a more nuanced one than the tenure-based one described in Section 3. If $r_m \in (r'_m, r''_m)$, the manager will institute a seniority system with three levels: juniors that work and don't receive compensation, intermediates that work and receive compensation, and seniors the receive compensation but don't work.[17] However, this seniority system is performance-based in the sense that "promotion" times depend on the entire history. In particular, the worker attains intermediate status when his continuation utility reaches u^C for the first time, and he reaches senior status when his continuation utility reaches the absorbing state of $\frac{1}{r_w}$. Moreover, the worker's compensation while he is of intermediate status oscillates between $r_w w^C$ and 1 according to the random arrival of opportunities.

Propositions 5 and 6 are visualized in Figure 2.

[17] If $r_m > r''_m$, the intermediate status does not exist.

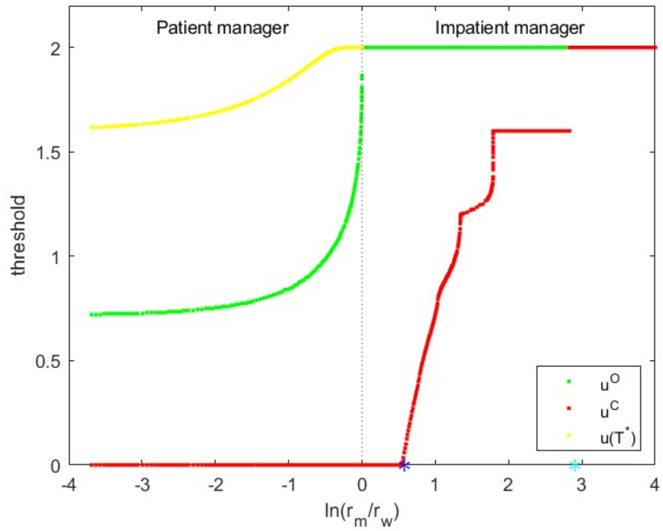

Figure 2: u^O, u^C, and $u(T^*)$ as a function of $ln(\frac{r_m}{r_w})$, for $A = \frac{2}{5}, B = 2, r_w = \frac{1}{2}$, and $\mu = 2$. Note that the middle of the x-axis represents the case where $r_m = r_w$. The left (dark blue) $*$ corresponds to the value of r'_m, and the right (light-blue) $*$ corresponds to the value of r''_m[18]

As can be seen in Figure 2 (and Proposition 6) there is a discontinuity in the optimal compensation threshold at $r_m = r''_m$. This discontinuity, as well as the kinks and abrupt changes in curvature of that threshold, is due to the discrete arrival of opportunities. This effect of discreteness becomes more dominant as the number of opportunities needed to reach the absorbing state of $u = \frac{1}{r_w}$ from u^C becomes small.

The effect of lumpiness

We conclude this section by revisiting the question of how the lumpiness of opportunities impacts the manager's value from the interaction. Intuitively, due to perfect bookkeeping, when opportunities become lumpier, the asynchronicity of compensation and effort increases, which, in turn, reduces the manager's ability to use compensation resources effectively.

Proposition 7. *Assume that opportunities are concealable. The manager's value strictly decreases when opportunities become lumpier.*

[18]The optimal thresholds were derived via Monte Carlo simulations. Note that on the extreme right of the figure the green dots are obscured by the red ones as $u^C = u^O = \frac{1}{r_w}$ when r_m is sufficiently large.

Recall that Proposition 3 shows that when opportunities are observable lumpiness is immaterial if players use the same discount factor. By contrast, under concealable opportunities, lumpiness is detrimental for any pair of discount factors.

5 Discussion

5.1 Organizational Implications

The managerial choices at the bottom of an organization's hierarchy affect the organization at large. Our analysis illustrates that the organization may benefit from information frictions between the front-line managers and their subordinates.

Assume that both the front-line manager and the worker are less patient than the organization. If the manager is more patient than the worker, the intertemporal preferences of the manager and the organization are generally aligned: they both prefer to front-load the worker's compensation and value his future effort. Since, as we showed in the previous sections, information frictions (i.e., the worker's ability to conceal opportunities) impede the effective utilization of the manager's limited compensation budget, such frictions harm the organization.

Consider what happens as the manager becomes less patient. If the arrival of opportunities is observable, the managerial style of the manager changes discontinuously around the focal point where the players use the same discount factor: once the manager becomes even slightly impatient relative to the worker, she fully back-loads compensation and front-loads effort, which is exactly the opposite of the organization's preferences. By contrast, if opportunities are concealable, the managerial style of a slightly patient manager is similar to that of a slightly impatient one. Example 1 demonstrates that if the manager is slightly impatient, then information frictions can be beneficial for the organization; i.e., the alignment of the manager's intertemporal preferences may be more important to the organization than the manager's ability to utilize her compensation budget efficiently.

Example 1. *Assume that* $A = 1, B = 1.2, \mu = 0.5, r_m = 0.11, r_w = 0.1$, *and that the organization discounts future payoffs at a rate of* 0.09. *The organization's value is* 0.477 *if the arrival of opportunities is observable, and* 1.031 *if opportunities are concealable.*[19]

[19]The value for the organization in Example 1 is calculated by solving the manager's problem numerically, and evaluating the resulting streams of effort and compensation according to the orga-

A possible interpretation of the example is that, under some circumstances, the organization may prefer to hire managers that lack the technical expertise (to observe the arrival of opportunities), even if they do not possess superior managerial skills.[20]

Our model also highlights a novel aspect of the effectiveness of multidimensional compensation packages. In a hierarchical organization, a significant part of an employee's incentives are often provided via promotion opportunities rather than salary as such compensation may have a lower direct cost for the organization. Our model suggests a potential indirect cost for such compensation through its impact on managerial style. In particular, vertical and lateral promotion (as well as layoff) policies may affect the relative patience within the interaction between a manager and her subordinate. This is particularly relevant if opportunities are observable, since managerial style is discontinuous around the focal point where players use the same discount factor. In fact, this suggest that professional managers will enjoy faster promotions than technical ones.

5.2 Storable Opportunities

The focus of our analysis was on applications in which opportunities are wasted if they are not acted upon immediately. However, our analysis is also relevant for some applications where it is possible to store opportunities. If the manager is impatient, expediting effort is profitable. Hence, storing opportunities is suboptimal for an impatient manager.

By contrast, if the manager is patient, it is profitable for her to reduce the time lag between effort and compensation, even if doing so delays the time at which the worker exerts the amount of worker-discounted effort. Intuitively, by requiring immediate effort (rather than storing the opportunity) the manager is, in essence, using the worker's higher discount factor to discount future benefits. To see this, observe that requiring effort in t units of time in return for a util of compensation at the same time creates a current benefit of $e^{-r_m t}\frac{B}{A}$ for the manager. Whereas, requiring immediate effort in return for a util of compensation in t units of time, generates a current benefit of $e^{-r_w t}\frac{B}{A} < e^{-r_m t}\frac{B}{A}$.

This suggests that if opportunities are perfectly strorable, a patient manager

nization's discount factor.

[20]In fact, it is easy to construct examples in which the front-line manager will generate profit for the organization only if she does not observe the arrival of opportunities.

would slice arriving opportunities into infinitesimal bits, and have the worker exert a constant flow effort in return for immediate compensation. However, as opportunities oftentimes represent random events that require immediate action, they may depreciate or become obsolete over time. Indeed, if stored opportunities depreciate at a rate greater than $r_w - r_m$, then the cost of depreciation outweighs the manager's gain from reducing the time lag between effort and compensation. Hence, our analysis holds also for applications in which storing opportunities is sufficiently costly.

5.3 Relational Contracting

We assumed that the manager can fully commit to a contract to capture the inherent asymmetry between workers and managers. However, our main insights would remain valid were we to use a relational contracting approach instead (i.e., were we to add a restriction that the manager's continuation value must be nonnegative).

If the manager is patient, then, under the optimal contract, she requires effort on all opportunities and refrains from accumulating a large debt to the worker, regardless of whether opportunities are observable or concealable. Hence, in most cases of interest, i.e., when players are not too myopic and the profit from opportunities is not too low, the optimal contracts characterized in Propositions 1 and 5 are also relational contracts. Even if these conditions do not hold, the optimal relational contracts are identical to the optimal contracts, apart from inducing slightly lower effort requirements.

If the manager is impatient, she is unable to use the seniority systems that arise under the optimal contracts characterized in Propositions 2 and 6, as they induce a senior rank in which workers enjoy compensation without exerting effort. Under observable opportunities, the optimal relational contract will still consist of a three-tier tenure-based promotion system, albeit one in which senior workers are required to exert a (constant) level of effort. Under concealable opportunities, the optimal relational contract will generate a two-tier performance-based promotion system: juniors that exert full effort and do not receive compensation, and non-juniors that receive compensation and exert some effort on all opportunities.

References

Bird, Daniel, and Alexander Frug. 2019. "Dynamic nonmonetary incentives." *American Economic Journal: Microeconomics*, 11(4): 111–150.

Bird, Daniel, and Alexander Frug. 2021. "Optimal contracts with randomly arriving tasks." *Economic Journal*, 131: 1905–1918.

Calvo, Guillermo A., and Stanislaw Wellisz. 1978. "Supervision, loss of control, and the optimum size of the firm." *Journal of Political Economy*, 86(5): 943–952.

Calvo, Guillermo A., and Stanislaw Wellisz. 1979. "Hierarchy, ability, and income distribution." *Journal of Political Economy*, 87(5): 991–1010.

Cameron, Kim S., Sarah J. Freeman, and Aneil K. Mishra. 1991. "Best practices in white-collar downsizing: Managing contradictions." *Academy of Management Perspectives*, 5(3): 57–73.

Carmichael, Lorne. 1983. "Firm-specific human capital and promotion ladders." *Bell Journal of Economics*, 14(1): 251–258.

Forand, Jean Guillaume, and Jan Zápal. 2020. "Production priorities in dynamic relationships." *Theoretical Economics*, 15(3): 861–889.

Frankel, Alexander. 2016. "Discounted quotas." *Journal of Economic Theory*, 166: 396–444.

Gallup. 2022. "The top 6 things employees want in their next job." Gallup. https://www.gallup.com/workplace/389807/top-things-employees-next-job.aspx. Accessed on 21^{th} of September 2022.

Garicano, Luis. 2000. "Hierarchies and the organization of knowledge in production." *Journal of Political Economy*, 108(5): 874–904.

Garrett, Daniel F., and Alessandro Pavan. 2012. "Managerial turnover in a changing world." *Journal of Political Economy*, 120(5): 879–925.

Garrett, Daniel F., and Alessandro Pavan. 2015. "Dynamic managerial compensation: A variational approach." *Journal of Economic Theory*, 159: 775–818.

Harris, Milton, and Artur Raviv. 2002. "Organization design." *Management Science*, 48(7): 852–865.

Hart, Oliver. 1995. *Firms, Contracts, and Financial Structure.* Oxford:Oxford University Press.

Hart, Oliver, and John Moore. 2005. "On the design of hierarchies: Coordination versus specialization." *Journal of Political Economy*, 113(4): 675–702.

Hoffmann, Florian, Roman Inderst, and Marcus M. Opp. 2021. "Only time will tell: A theory of deferred compensation." *Review of Economic Studies*, 88(3): 1253–1278.

Holmström, Bengt. 1999. "Managerial incentive problems: A dynamic perspective." *Review of Economic Studies*, 66(1): 169–182.

Holmström, Bengt. 2017. "Pay for performance and beyond." *American Economic Review*, 107(7): 1753–1777.

Krasikov, Ilia, Rohit Lamba, and Thomas Schacherer. 2020. "Implications of unequal discounting in dynamic contracting." *SSRN Electronic Journal*.

Laffont, Jean-Jacques. 1988. "Hidden gaming in hierarchies: Facts and models." *Economic Record*, 64(4): 295–306.

Lazear, Edward P. 1981. "Agency, earnings profiles, productivity, and hours restrictions." *American Economic Review*, 71(4): 606–620.

Letina, Igor, Shuo Liu, and Nick Netzer. 2020. "Delegating performance evaluation." *Theoretical Economics*, 15(2): 477–509.

Levin, Jonathan. 2003. "Relational incentive contracts." *American Economic Review*, 93(3): 835–857.

Li, Jin, Niko Matouschek, and Michael Powell. 2017. "Power dynamics in organizations." *American Economic Journal: Microeconomics*, 9(1): 217–241.

Lipnowski, Elliot, and João Ramos. 2020. "Repeated delegation." *Journal of Economic Theory*, 188: 105040.

MacLeod, W. Bentley, and James M. Malcomson. 1989. "Implicit contracts, incentive compatibility, and involuntary unemployment." *Econometrica*, 57(2): 447–480.

McAfee, R. Preston, and John McMillan. 1995. "Organizational diseconomies of scale." *Journal of Economics & Management Strategy*, 4(3): 399–426.

Melumad, Nahum D., Dilip Mookherjee, and Stefan Reichelstein. 1995. "Hierarchical decentralization of incentive contracts." *RAND Journal of Economics*, 26(4): 654–672.

Mookherjee, Dilip. 2013. "Incentives in hierarchies." In *The Handbook of Organizational Economics.* , ed. Robert Gibbons and Roberts John (pp. 764-798). Princeton University Press.

Opp, Marcus M., and John Y. Zhu. 2015. "Impatience versus incentives." *Econometrica*, 83(4): 1601–1617.

Rajan, Raghuram G., and Luigi Zingales. 2001. "The firm as a dedicated hierarchy: A theory of the origins and growth of firms." *Quarterly Journal of Economics*, 116(3): 805–851.

Rantakari, Heikki. 2008. "Governing adaptation." *Review of Economic Studies*, 75(4): 1257–1285.

Ray, Debraj. 2002. "The time structure of self-enforcing agreements." *Econometrica*, 70(2): 547–582.

Rhoades, Linda, and Robert Eisenberger. 2002. "Perceived organizational support: a review of the literature." *Journal of Applied Psychology*, 87(4): 698–714.

Rosen, Sherwin. 1982. "Authority, control, and the distribution of earnings." *Bell Journal of Economics*, 13(2): 311–323.

Sannikov, Yuliy. 2008. "A continuous-time version of the principal–agent problem." *Review of Economic Studies*, 75(3): 957–984.

Spear, Stephen E., and Sanjay Srivastava. 1987. "On repeated moral hazard with discounting." *Review of Economic Studies*, 54(4): 599–617.

Thomas, Jonathan, and Tim Worrall. 1990. "Income fluctuation and asymmetric information: An example of a repeated principal–agent problem." *Journal of Economic Theory*, 51(2): 367–390.

Thomas, Robyn, and David Dunkerley. 1999. "Careering downwards? Middle managers' experiences in the downsized organization." *British Journal of Management*, 10(2): 157–169.

Tirole, Jean. 1986. "Hierarchies and bureaucracies: On the role of collusion in organizations." *Journal of Law, Economics, and Organization*, 2(2): 181–214.

Williamson, Oliver E. 1967. "Hierarchical control and optimum firm size." *Journal of Political Economy*, 75(2): 123–138.

Williamson, Oliver E. 1975. *Markets and Hierarchies: Analysis and Antitrust Implications.* New York: Free press.

A Appendix: Proofs

Proof of Proposition 1.

Providing the worker with compensation before he exerted effort is suboptimal. Moreover, by the definition of T^*, the worker will not receive compensation if he has not exerted effort in the last T^* units of time. Otherwise, the manager would benefit from reducing compensation at t and decreasing the required effort on the last opportunity along h_t on which effort was exerted. Hence, for the rest of this proof we focus only on contracts under which $\varphi(h_t) = 0$ if $t - \sigma_{-1}(h_t) \geq T^*$.

Assume that an opportunity is currently available and denote by σ the random amount of time that will pass until the arrival of the next opportunity. Denote by

$$\alpha^* \equiv \min\{\frac{1}{A}\mathbb{E}\left(\int_0^{\min\{T^*,\sigma\}} e^{-r_w t} dt\right), 1\}$$

the maximal effort that the worker is willing to exert on an opportunity in exchange for a conditional promise of length T^* (i.e., setting $\varphi = 1$ until either T^* units of time have passed or an opportunity arrives).

The first step in the proof establishes that it is suboptimal for the manager to require effort on a given opportunity that will necessitate providing compensation after the next opportunity arrives.

Lemma A.1. *Under an optimal contract $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$.*

Proof. Assume by way of contradiction that under an optimal contract $\alpha(h_t) > \alpha^*$ for a set of histories with positive measure. Since $\alpha^* = 1$ if $r_w = r_m$, within this lemma we consider only the case where $r_m < r_w$.[21] Denote by ν the set of finite histories (of various lengths) such that for each $h \in \nu$, and every h' that is a (proper) prefix of h, $\alpha(h) > \alpha^*$ and $\alpha(h') \leq \alpha^*$. Note that the contract reaches a history in ν with positive probability and that if $h \in \nu$ and $\tilde{h} \in \nu$, then neither history is a prefix of the other. Thus, it is sufficient to construct a profitable modification of the continuation contract conditional on an opportunity arriving at every $h \in \nu$.

Fix $h_t \in \nu$ and assume that an opportunity is available. A conditional promise of length T^* is not enough to compensate the worker for exerting an effort of more than α^* at h_t. Hence, with positive probability some of the compensation for the effort exerted at h_t must be provided after the arrival of the next opportunity. Formally, there exists a set $\nu_1(h_t)$ of continuation histories of h_t (of various lengths) with positive measure such that for every $h_s \in \nu_1(h_t)$ opportunities do not arrive in (t, s), and the worker's continuation utility conditional on an opportunity arriving at h_s is strictly greater than $A \cdot \alpha(h_s)$.

If there exists $\tilde{\nu} \subset \nu_1(h_t)$ with positive measure (conditional on reaching h_t) such that $\alpha(h_s) < 1$ for every $h_s \in \tilde{\nu}$, then postponing effort (according to the worker's discount factor) from h_t to the histories in $\tilde{\nu}$ is strictly profitable for the (patient) manager (Observation 1) and does not violate incentive compatibility.

If, on the other hand, $\alpha(h_s) = 1$ for almost all $h_s \in \nu_1(h_t)$, then for every such h_s there exists a set of continuation histories (of various lengths) with positive measure (conditional on reaching h_s) $\nu_2(h_s)$ such that for every $h_{s'} \in \nu_2(h_s)$: 1) no opportunities arrive in (s, s'), and 2) the worker's continuation utility if an opportunity arrives at $h_{s'}$ is greater than his continuation utility at h_s by at least $(1 - \alpha^*)A$. To see why such histories exist, recall that $\varphi = 0$ if no opportunity arrived in the last T^* units of time, and so the maximal worker-discounted expected compensation that

[21] If $r_w = r_m$, then $T^* = \infty$, and so it is both profitable and feasible (Assumption 1) to incentivize full effort on all opportunities.

can be provided between two successive opportunities is $\alpha^* A$. Hence, to compensate the worker for exerting an effort of 1 on the opportunity at h_s, at least $(1-\alpha^*)A$ of this compensation is provided after the next opportunity arrives with positive probability.

If there exists $\tilde{\nu} \subset \nu_2(h_s)$ with positive measure (conditional on reaching h_s) such that $\alpha(h_{s'}) < 1$ for every $h_{s'} \in \tilde{\nu}$, then postponing effort from h_s to the histories in $\tilde{\nu}$ does not violate incentive compatibility. Moreover, if effort can be postponed in this manner from a subset of $\nu_1(h_t)$ with positive measure, then doing so increases the manager's value at h_t. Otherwise, for almost all $h_s \in \nu_1(h_t)$, it holds that $\alpha(h_{s'}) = 1$ for almost all $h_{s'} \in \nu_2(h_s)$. In this case, with strictly positive probability the worker's continuation utility at $h_{s'}$ is greater than his continuation utility at h_t by at least $(1-\alpha^*)2A$. Continuing in an iterative manner shows that the manager's value can be increased by postponing effort, as otherwise the worker's continuation utility increases without bound with positive probability (which cannot be the case as it is bounded from above by $\frac{1}{r_w}$). ■

Denote by S^* the length of the conditional promise needed to provide the worker with $\alpha^* A$ worker-discounted utils, i.e.,

$$\mathbb{E}\left(\int_0^{\min\{S^*,\sigma_1\}} e^{-r_w t} dt\right) = \alpha^* A.$$

Note that $S^* \leq T^*$ due to the definition of α^*.

The next part of the proof establishes that, under an optimal contract, the manager will require the maximal effort that can be required without accumulating debts or exceeding the maximal profitable lag.

Lemma A.2. *Under an optimal contract $\alpha(h_t) = \alpha^*$ for almost all $h_t \in H$.*

Proof. By Lemma A.1, $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$. Assume by way of contradiction that under an optimal contract $\alpha(h_t) < \alpha^*$ on a set of histories with positive measure. Let n be the minimal element of $\mathbb{N}$ for which the worker's effort on the n^{th} opportunity to arrive is st rictly less than α^* with positive probability. Let ν denote the set of histories (of various lengths) that end at the arrival of the $n+1-$th opportunity such that the required effort on the n-th opportunity is strictly less than α^*. Finally, for $k \in \{1,\ldots,n\}$, define $NOC(k)$ to be the set of prefixes of histories in ν along which exactly k opportunities have arrived and the $k-$th opportunity arrived at most S^* units of time ago. We say that maximal compensation is provided in $NOC(k)$ if $\varphi(h) = 1$ for almost all $h \in NOC(k)$.

By construction, maximal compensation in $NOC(k)$ for all $k \in \{1, \ldots, n\}$ is sufficient to incentivize effort α^* on the first n opportunities to arrive. Hence, if maximal compensation is provided in $NOC(k)$ for all $k \in \{1, \ldots, n\}$, compensation can be decreased in $NOC(n)$ without violating any of the incentive-compatibility constraints. Otherwise, there exists a maximal $\bar{k} \leq n$ such that maximal compensation is not provided in $NOC(\bar{k})$. If $\bar{k} = n$, an improvement can be reached by increasing the required effort on the n-th opportunity and increasing the compensation within $NOC(n)$, without affecting any of the earlier or later IC constraints. Finally, if $\bar{k} < n$, since the required effort on opportunity $\bar{k}$ is α^*, some compensation for effort on that opportunity must be postponed until after future opportunities have arrived. Hence, the IC constraint at the arrival of the $\bar{k} + 1$ opportunity is not binding with positive probability. Since maximal compensation is provided within $NOC(\bar{k} + 1)$, the principal can reach an improvement by expediting compensation from $NOC(\bar{k} + 1)$ to $NOC(\bar{k})$. ■

To conclude the proof, note that the contract described in the proposition is the only incentive compatible contract under which $\alpha(\cdot) \equiv \alpha^*$ and the worker does not receive compensation if he has not exerted effort in the last S^* units of time. ■

Proof of Proposition 2. When opportunities are observable, the manager's problem can be solved separately for each possible arrival time of the first opportunity. This is because the manager will not provide compensation prior to the first opportunity, and the worker must have a nonnegative continuation utility at all times.

Consider an arbitrary first arrival time σ_1. By Assumption 1 the manager can incentivize the worker to exert maximal effort on the opportunity at σ_1. As the manager is impatient, she will do so in an optimal contract. Moreover, as we established in the main text, the manager will use a threshold structure. Hence, the manager's objective function (conditional on σ_1) is

$$\max_{\tau^O(\sigma_1), \tau^C(\sigma_1)} e^{-r_m \sigma_1} B + \int_{\sigma_1}^{\sigma_1 + \tau^O(\sigma_1)} \mu B e^{-r_m t} dt - \int_{\sigma_1 + \tau^C(\sigma_1)}^{\infty} e^{-r_m t} dt$$

$$\text{s.t. } A + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu A e^{-r_w t} dt = \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_w t} dt. \tag{7}$$

Assumption 1 implies that, in optimum, both $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are interior. To see this, note that the constraint (7) is violated if $\tau^C(\sigma_1) = \infty$ or $\tau^C(\sigma_1) = 0$. Furthermore, setting $\tau^O(\sigma_1) = 0$ implies that $\tau^C(\sigma_1) > \tau^O(\sigma_1)$, and so by slightly increasing $\tau^O(\sigma_1)$ (and decreasing $\tau^C(\sigma_1)$ to maintain incentive compatibility) the worker will exert more effort on opportunities for which he will receive compensation after he has exerted effort. As the manager is impatient, this change is profitable. Finally, setting $\tau^O(\sigma_1) = \infty$ implies that the worker continues exerting effort for an arbitrarily long period of time after he begins receiving compensation. Because the manager is impatient, slightly increasing $\tau^C(\sigma_1)$ (and decreasing $\tau^O(\sigma_1)$ to maintain incentive compatibility) is profitable.

The above discussion implies that the optimal thresholds are given by the FOC of the Lagrangian that corresponds to the above (concave) maximization problem. The first-order conditions with respect to $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are, respectively,

$$\mu B e^{-r_m \tau^O(\sigma_1)} - \gamma(\sigma_1) e^{-r_w \tau^O(\sigma_1)} \mu A = 0$$
$$e^{-r_m \tau^C(\sigma_1)} - \gamma(\sigma_1) e^{-r_w \tau^C(\sigma_1)} = 0,$$

where γ is the Lagrange multiplier. It follows that $\frac{B}{A} = e^{(r_m - r_w)(\tau^O(\sigma_1) - \tau^C(\sigma_1))}$. Hence, $\tau^O(\sigma_1) = \tau^C(\sigma_1) + K$, where $K > 0$ is a constant that does not depend on σ_1. This relation implies that the LHS of (7) is increasing in $\tau^O(\sigma_1)$ while the RHS is decreasing in $\tau^O(\sigma_1)$. Hence, there is a unique solution that does not depend on σ_1. ■

Proof of Proposition 3. We establish this proposition separately for the case where the manager is patient and the case for which she is impatient.

Case 1: patient manager. We start this proof by establishing the comparative statics of α^* with respect to λ. If the worker's expected utility from a conditional promise of length T^* is strictly greater than A, then the worker exerts full effort on all opportunities. Moreover, this will remain the case if opportunities become marginally lumpier. Thus, we focus on the case where the worker's expected utility from such a promise is at most A.

The worker's expected utility from a conditional promise of length T^*, as a function of the parameter λ, is $\frac{1}{r_w+\mu/\lambda}(1 - e^{-T^*(r_w+\mu/\lambda)})$. The marginal increase in the value of such a promise from making opportunities lumpier is $\frac{\mu}{(\mu+r_w)^2}(1 - e^{-T^*(\mu+r_w)}T^*(\mu + r_w))$. Thus, to establish that the α^* is decreasing in λ it is enough to show that making opportunities marginally lumpier has a larger impact on the cost of exerting the required effort than on the value of a conditional promise of length T^*.

Under the assumption that a conditional promise of length T^* does not provide excess compensation, it holds that $A \geq \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}$. Hence, it is sufficient to show that

$$\frac{\mu}{(\mu + r_w)^2}(1 - e^{-T^*(\mu+r_w)}T^*(\mu + r_w)) < \frac{1 - e^{-T^*(\mu+r_w)}}{\mu + r_w}.$$

When $\mu + r_w$ is kept constant, this inequality is harder to satisfy for higher values of μ. Thus, it is sufficient to show that it holds for $r_w = 0$, i.e., to show that

$$1 - e^{-\mu T^*}(1 + \mu T^*) < 1 - e^{-\mu T^*},$$

which holds for any $\mu T^* > 0$. Note that the above calculation does not depend on the value of T^*. Hence, the same calculation shows that when it is possible to induce full effort, S^* increases when opportunities become marginally lumpier.

Next, we show that making opportunities lumpier is detrimental for the manager. If $\alpha^* = 1$, this is an immediate consequence of S^* being increasing in λ. Assume that $\alpha^* < 1$ and let $f(r, \lambda) = \frac{1-e^{-T^*(r+\frac{\mu}{\lambda})}}{r\lambda+\mu}$ denote the $r-$discounted compensation that is provided via a conditional promise of length T^* as a function of λ. Note that the average cost of providing a util of compensation is $\frac{f(r_m,\lambda)}{f(r_w,\lambda)}$.

The cross-derivative of $f(r, \lambda)$ evaluated at $\lambda = 1$ is equal to

$$\frac{\partial^2 f(r, \lambda)}{\partial r \partial \lambda}|_{\lambda=1} = \frac{\mu e^{-T(\mu+r)} \left(T^*(\mu + r)(T^*(\mu + r) + 2) - 2e^{T^*(\mu+r)} + 2\right)}{(\mu + r)^3}.$$

The sign of this cross-derivative is the sign of $x(x + 2) + 2 - 2e^x$, where $x = T^*(r + \mu)$. As this sign is negative, the cross-derivative is negative. As $f(r, \lambda)$ is positive and decreasing in λ, it follows that the average cost of compensating the worker for his effort is increasing in λ (recall that $r_w > r_m$). As the worker's total effort is also decreasing in λ, we can conclude that making opportunities lumpier reduces the manager's value.

Case 2: Impatient manager. Solving the optimal thresholds for the contract characterized in Proposition 2, τ^O, τ^W, as a function of λ gives

$$\tau^O(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + \lambda r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w},$$
$$\tau^C(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + \lambda r_w))}{r_w}.$$

Recall that the manager's value is

$$\mathbb{E}\left(e^{-r_m\sigma_1}\left(B\lambda + \frac{B\mu(1 - e^{-r_m\tau^O(\lambda)})}{r_m} - \frac{e^{-r_m\tau^C(\lambda)}}{r_m}\right)\right).$$

Plugging in the expressions for the optimal thresholds, differentiating with respect to λ, and evaluating at $\lambda = 1$ shows that making opportunities marginally lumpier changes the manager value by

$$(r_w - r_m)\frac{\mu\left(\frac{B}{A}\right)^{-\frac{r_m}{r_w - r_m}}\left(B\mu\left(\frac{B}{A}\right)^{\frac{r_m}{r_w - r_m}} + 1\right) e^{-r_m\left(\frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w}\right)}}{(\mu + r_w)(\mu + r_m)^2}.$$

This expression is negative as $(r_w - r_m) < 0$ and the ratio is positive. ■

In the analysis that follows we use a technical lemma that states that for every incentive-compatible contract for which $u > 0$, there exists another incentive-compatible contract that implements the same work schedule via a compensation policy that is (pointwise) weakly lower.

Lemma A.3. *Assume that opportunities are concealable. Moreover, assume that under an incentive-compatible contract the continuation contract at h_t, $\langle \alpha(\cdot), \varphi(\cdot) \rangle$, is such that the worker's continuation utility is $u > 0$. There exists $\tilde{u} < u$ such that for every $u' \in (\tilde{u}, u)$ there exists an incentive-compatible contract $\langle \alpha'(\cdot), \varphi'(\cdot) \rangle$ that provides the worker with a continuation value of u', and for which $\varphi'(h_s) \leq \varphi(h_s)$ and $\alpha'(h_s) = \alpha(h_s)$ at every h_s that is a continuation of h_t.*

Proof of Lemma A.3. Consider an incentive-compatible contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ under which the worker's continuation utility is $u > 0$ and normalize the current time to zero. If the worker's expected discounted compensation along the histories in which there are no binding incentive compatibility constraints is positive, then the worker's continuation utility at time zero can be decreased by reducing his compensation along those histories. If this is not the case, then the worker's compensation is almost surely zero prior to a binding incentive compatibility constraint. Hence, concealing all opportunities is a best response for the worker. However, as $\varphi = 0$ before the worker exerts effort, this best response provides a payoff of $0 < u$. ■

Proof of Lemma 1. Let $\langle \hat{\alpha}(\cdot), \hat{\varphi}(\cdot) \rangle$ be an incentive-compatible (continuation) contract under which the worker's expected discounted payoff is $u > 0$. From Lemma A.3 it follows that there exists $\tilde{u} < u$ such that if the worker's continuation utility is in $(\tilde{u}, u)$, then the manager can induce the same work schedule for a lower compensation. Thus, there is an open neighborhood to the left of u for which the manager can obtain a value strictly greater than $V(u)$. The strict monotonicity of $V(\cdot)$ follows from the fact that the choice of u is arbitrary.

Next, we show that $V(u)$ is weakly concave. Let $u_1 < u_2$ such that $u_1, u_2 \in [0, \frac{1}{r_w}]$. One (unnatural) way the manager can deliver a promise of $\frac{u_1+u_2}{2}$ is to fictitiously split all opportunities and compensation in half and create two (perfectly correlated) fictitious worlds, each of which contains half of the compensation flow and half of each opportunity. Observe that scaling all payoffs by $\frac{1}{2}$ multiplies the players' discounted payoffs by half in any contract, and so any optimal contract in the original non-scaled world is also an optimal contract in each fictitious world. The manager can then use the continuation contract that supports $V(u_1)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_1}{2}$ in fictitious world 1, and the continuation contract that supports $V(u_2)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_2}{2}$ in fictitious world 2. Since using these continuation contracts cannot increase the manager's payoff, $V(\frac{1}{2}(u_1 + u_2)) \geq \frac{1}{2}V(u_1) + \frac{1}{2}V(u_2)$, which establishes the concavity of $V(\cdot)$. ■

Proof of Proposition 4. We establish this proposition separately for the case where the manager is weakly patient and the case where she is impatient. In each case, we first derive one part of the optimal contract (the work schedule when the manager is impatient, and the compensation policy when she is impatient), and then use the HJB equation to derive the optimal contract and show that it is, generically, unique.

Case 1: impatient manager ($r_m > r_w$). The first step of the proof is to show that under any optimal contract the work schedule is $\overline{\alpha}(u) = \min\{1, \frac{1/r_w - u}{A}\}$.

Assume by way of contradiction that $\alpha(\hat{u}) < \min\{1, \frac{1/r_w - \hat{u}}{A}\}$ for some $\hat{u} \in [0, \frac{1}{r_w}]$. Suppose that the current state is $\hat{u}$ and that an opportunity is currently available. If the worker's expected discounted future effort is zero, then it is both possible and profitable to increase α and increase the worker's compensation in the future without changing his continuation utility. If, on the other hand, the worker's expected discounted future effort is positive, then the manager can expedite effort (in the non-Markovian representation of the contract) without altering the compensation policy. By Observation 1 it is profitable for the manager to expedite effort, and, since she does so according to the worker's discount factor, it also relaxes all incentive-compatibility constraints.

The above claim enables us to simplify the HJB equation to

$$(HJB_{imp}) \qquad \sup_{\varphi(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) + \mu\left(\overline{\alpha}(u)B + V\left(u + \overline{\alpha}(u)A\right) - V(u)\right)\} = 0.$$

From the FOC of (HJB_{imp}) it follows that $\varphi(u) = 1$ if $V'(u) < -1$ and that $\varphi(u) = 0$ if $V'(u) > -1$. Since $V(\cdot)$ is weakly concave (Lemma 1), there is a (possibly degenerate) interval $I \subset [0, \frac{1}{r_w}]$ over which $V'(u) = -1$. Note that for any $u^\dagger \in I$ the compensation policy given by

$$\varphi_{u^\dagger}(u) = \begin{cases} 1 & \text{if } u > u^\dagger \\ r_w u^\dagger & \text{if } u = u^\dagger \\ 0 & \text{if } u < u^\dagger \end{cases}$$

is an optimal compensation policy.

Next, we show that, generically, I is degenerate. Fix A, B, μ, and r_w, and let $I(r_m)$ denote the interval (or point) for which $V'(\cdot) = 1$ for a manager with discount rate r_m. To establish the generic uniqueness of optimal contracts we will show that if there exist $\tilde{r}_m < \hat{r}_m$ such that both $I(\tilde{r}_m)$ and $I(\hat{r}_m)$ have a positive measure, then these intervals have a disjoint interior. The result then follows from a standard argument about the density of rational numbers.

Assume by way of contradiction that for some $\tilde{r}_m < \hat{r}_m$, the set $I^* \equiv I(\tilde{r}_m) \cap I(\hat{r}_m)$ has a nonempty interior. Select u^* and $\epsilon > 0$ such that $u^*, u^* - \epsilon \in int(I^*)$.

Fix the optimal compensation policy $\varphi_{u^*}(\cdot)$, and let $\Delta\varphi_s = \mathbb{E}(\varphi_s | u_0 = u^* - \epsilon) - \mathbb{E}(\varphi_s | u_0 = u^*)$ and $\Delta\alpha_s = E(\alpha_s | u_0 = u^* - \epsilon) - \mathbb{E}(\alpha_s | u_0 = u^*)$. Since the chosen compensation policy, $\varphi_{u^*}(\cdot)$, is optimal, we have

$$V(u^* - \epsilon) - V(u^*) = \int_0^\infty e^{-r_m s}(\mu B \Delta\alpha_s - \Delta\varphi_s) ds. \tag{8}$$

As path by path u_s is monotone in u_0, and the work schedule and compensation policies are threshold policies, it follows that $\mu B \Delta\alpha_s - \Delta\varphi_s \geq 0$ for all s, with a strict inequality on a set of times with strictly positive measure. Hence, differentiating the RHS of (8) with respect to r_m shows that the RHS of (8) is decreasing in r_m. However, as $V'(u) = -1$ for all $u \in I^*$ it holds that $V(u^* - \epsilon) - V(u^*) = \epsilon$. Hence, (8) can be satisfied for at most one r_m and so the interior of I^* is empty.

It follows that if the interior of $I(r_m)$ is nonempty, then the compensation policies corresponding to elements of $int(I(r_m))$ are suboptimal for any $r'_m \neq r_m$. Thus, we can index every r_m for which the optimal contract is not unique by a rational number from the interior of $I(r_m)$. Hence, the set of manager-discount factors for which the optimal contract is not unique is (at most) countable.

Case 2: weakly patient manager ($r_m \leq r_w$). We begin by showing that the optimal compensation policy is

$$\overline{\varphi}(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u = 0. \end{cases}$$

To do so, we show that if $u(h_t) > 0$ then in an optimal contract $\varphi(h_t) = 1$ in the next dt units of time conditional on no opportunity arriving in that interval. If $u(h_t) = \frac{1}{r_w}$, this is immediate. Assume by way of contradiction that $u(h_t) \in (0, \frac{1}{r_w})$, and that the worker does not receive the maximal compensation with probability 1 in the next dt units of time conditional on no opportunity arriving in that interval. By arguments analogous to those used in the proof of Lemma A.3, it is possible to expedite compensation into the interval $[t, t + dt]$ (conditional on no opportunity arriving) without violating the incentive compatibility constraints in any history that is a continuation of h_t. If $r_m < r_w$, then expediting compensation is profitable (Observation 1). If, on the other hand, $r_m = r_w$, expediting compensation is profitable as it enables the manager to require more effort in the future.

The above claim enables us to simplify the HJB equation to

$$(HJB_p) \quad V(u) = \sup_{\alpha(u)\in[0,\min\{1,\frac{\frac{1}{r_w}-u}{A}\}]} \{-r_m V(u) + V'(u)[r_w u - \overline{\varphi}(u)] - \overline{\varphi}(u) + \mu\left(\alpha(u)B + V\left(u+\alpha(u)A\right) - V(u)\right)\} = 0.$$

The FOC of HJB_p with respect to $\alpha(u)$ is $B + V'\left(u+\alpha(u)A\right)A = 0$. Thus, to show that there is a unique optimal contract, it is sufficient to show that $V(\cdot)$ is strictly concave. To do so, we return to the construction used to establish the weak concavity in Lemma 1 and further the analysis by utilizing the structure of $\bar{c}(\cdot)$.

In the event with strictly positive probably where no opportunity arrives for T units of time, where T solves $u_1 = \frac{1-e^{-r_w T}}{r_w}$, the worker's continuation utility in fictitious world 1 is zero while his continuation utility in fictitious world 2 is strictly positive. At this point, the manager can temporarily merge the two fictitious worlds and expedite compensation in world 2 by using the compensation from world 1. By Observation 1 this modification is profitable for a strictly patient manager and, hence, $V(u)$ is strictly concave if $r_m < r_w$. If $r_w = r_m$, then merging the fictitious worlds increases the discounted effort the worker can be incentivized to exert in the future, which also strictly increases the manager's profit. ■

Proof of Proposition 5.

In Proposition 4 we established that if $r_m \leq r_w$, then $u^C = 0$ and there is a unique optimal contract. Setting $u^O = 0$ implies that the worker never exerts effort, which, in turn, implies that the manager's value is zero; an outcome that is clearly suboptimal. If the manager is patient, the setting $u^O > u(T^*)$ is suboptimal as when the worker exerts effort that increases his continuation utility to above $u(T^*)$, the manager will have promised compensation more than T^* units of time in the future. By the definition of T^*, this reduces the manager's value. Thus, to establish the proposition it remains to show that setting $u^O = u(T^*)$ is also suboptimal.

Assume towards a contradiction that under the optimal contract $u^O = u(T^*)$, and let τ be the first time at which the worker's continuation utility reaches u^O. Since $u^C = 0$, an opportunity must be available at τ. If the first opportunity to arrive after τ arrives sufficiently quickly, then the worker will not exert full effort on that opportunity. Choose $s \in (0, T^*)$ such that if the first opportunity to arrive after τ arrives at $\tau + s$, then the worker's effort on that opportunity is strictly less than 1, and denote the probability that the first opportunity after τ arrives in $[\tau + s/2, \tau + s]$ by $p > 0$. For $\epsilon > 0$ define the following (non-Makrovian) modification of the contract at time τ: reduce the worker's required effort at τ by $u(\epsilon) = \left(\int_{T^*-\epsilon}^{T^*} e^{-r_w t}dt\right)/A$ and refrain from promising compensation in $[\tau + T^* - \epsilon, \tau + T^*]$. Then, use the freed compensation in $[\tau + T^* - \epsilon, \tau + T^*]$ to increase the worker's effort on the first opportunity to arrive after τ, if it arrives in $[\tau + s/2, \tau + s]$. If it does not arrive in this interval, never provide compensation in $[\tau + T^* - \epsilon, \tau + T^*]$.

The manager's loss of value at time τ from this deviation is bounded from above by the manager's value from the worker exerting an effort of $u(\epsilon)$ and receiving compensation in $T^* - \epsilon$ units of time. This upper bound is given by

$$\frac{e^{-r_w T^*}\left(e^{r_w \epsilon} - 1\right)\left(\frac{B}{A} - e^{(r_w - r_m)(T^* - \epsilon)}\right)}{r_w} = e^{-r_w T^*}\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right)\epsilon + O\left(\epsilon^2\right) = O\left(\epsilon^2\right),$$

since $\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right) = 0$ by the definition of T^*.

If ϵ is small enough, then the worker will not exert full effort on the first opportunity in $[\tau + s/2, \tau + s]$ after this deviation. In this case, the gain from the additional effort on that opportunity is bounded from below by the product of p and the manager's time τ discounted gain should the the next opportunity arrive at $\tau + s/2$ (Observation 1). This bound is given by

$$p\left(\frac{B\left(e^{r_w \epsilon} - 1\right)e^{r_w(s/2 - T^*) - r_m s/2}}{A r_w} - \frac{e^{-r_m T^*}\left(e^{r_m \epsilon} - 1\right)}{r_m}\right)$$
$$= \epsilon p\left(e^{(r_w - r_m)s/2} - 1\right)\left(\frac{B}{A}\right)^{\frac{r_m}{r_m - r_w}} + O\left(\epsilon^2\right).$$

Since $r_w > r_m$ and $s > 0$, the linear term in the approximation is strictly positive. Hence, for sufficiently small ϵ, the modification is profitable. ■

Proof of Proposition 6. In Proposition 4 we established that $u^O = \frac{1}{r_w}$ if $r_m > r_w$. Furthermore, when $r_m = r_w$ the manager will use the same threshold, to avoid wasting her limited capacity to compensate the worker. To establish the second part of the proposition we begin by showing that $u^C = \frac{1}{r_w}$ is an optimal threshold if and only if $r_m \geq r''_m \equiv r_w + (\frac{B}{A} - 1)\mu$. An upper bound on the manager's marginal net gain from providing the worker with a util at present is attained by the worker exerting a worker-discounted util on the first opportunity to arrive. Note that if $u^C > \frac{1}{r_w} - A$ and $u = u^C$, then this upper bound is attained. The value of this upper bound is given by

$$\int_0^\infty \mu e^{-\mu t} \frac{B}{A} e^{(r_w - r_m)t} dt - 1 = \frac{\mu}{\mu + r_m - r_w} \frac{B}{A} - 1.$$

It is straightforward to show that this expression is positive if and only if $r_m \leq r''_m$. It follows that if $r_m > r''_m$, then providing compensation while $u < \frac{1}{r_w}$ is suboptimal. On the other hand, if $r_m < r''_m$, then it is strictly suboptimal to set $u^C = \frac{1}{r_w}$ since for such discount rates it is profitable to compensate the worker when $u > \frac{1}{r_w} - A$.

Next, we establish that there exists $r'_m > r_w$ for which $u^C = 0$. Let τ denote the random arrival time of the first opportunity on which the worker will not exert full effort under the contract with $u^C = 0$. Note that under any other contract, the worker will not exert full effort (weakly) earlier. It follows that the marginal value from decreasing the worker's continuation utility is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$. If $r_w = r_m$ the timing of compensation does not affect the manager's cost of providing compensation. This, in turn, implies that when $r_w = r_m$ the manager's marginal gain from providing compensation is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$ for all u. By the continuity of payoffs in r_m, it follows that there exists $r'_m > r_w$ such that the manager strictly benefits from full front-loading of compensation if $r_m < r'_m$.

To conclude the proof we use the following lemma that states that u^C is increasing in r_m (proof below) to show that $r'_m < r''_m$

Lemma A.4. *Fix B, C, μ, and r_w and assume that the manager is impatient. If u^C is an optimal threshold for r_m and $\tilde{u}^C$ is an optimal threshold for $\tilde{r}_m > r_m$, then $\tilde{u}^C \geq u^C$.*

To see why this inequality follows from the lemma, observe that when $r_m = r''_m$ there exist histories for which under the optimal contract the worker exerts effort on only the first opportunity, whereas, when $r_m = r'_m$ in every history the worker's discounted cost of effort must equal his discounted compensation, which, by Assumption 1, is greater than the cost of effort on a single opportunity. ■

Proof of Lemma A.4. Consider two contracts, $\mathcal{C}_1$ and $\mathcal{C}_2$, that differ in their compensation threshold, $u_2^C > u_1^C$. Denote by $u_{t,i}$ the worker's continuation utility at time t under contract $\mathcal{C}_i$ and let $\overline{\tau} = \sup\{t : u_{t,2} \leq u_2^C\}$. That is, $\overline{\tau}$ is the latest time at which the worker's continuation utility is lower than u_2^C under $\mathcal{C}_2$. Note that $\overline{\tau}$ is finite (almost surely) by the Borel–Cantelli lemma.

Observe that since $u_2^C > u_1^C$, it holds that $u_{t,1} \leq u_{t,2}$ for all t. This implies that the worker exerts the same effort on every opportunity that arrives before $\overline{\tau}$ under both $\mathcal{C}_1$ and $\mathcal{C}_2$. In addition, it implies that the compensation for effort exerted on those opportunities is postponed under $\mathcal{C}_2$ relative to $\mathcal{C}_1$. Denote by $g(r_m)$ the gain from this postponement as a function of r_m. Moreover, from $\overline{\tau}$ onwards, under $\mathcal{C}_2$ the worker exerts weakly less effort than he does under $\mathcal{C}_1$, and he receives a compensation of $\varphi_t = 1$ at all times. Let $d(r_m)$ denote the difference in the time-zero discounted continuation value from $\overline{\tau}$ onward between $\mathcal{C}_1$ and $\mathcal{C}_2$. The net gain from replacing $\mathcal{C}_1$ with $\mathcal{C}_2$ is $g(r_m) - d(r_m)$. Note that $g(\cdot)$ is increasing and $d(\cdot)$ is decreasing. Hence, whenever $g(r_m) \geq d(r_m)$, we also have $g(r'_m) > d(r'_m)$ for all $r'_m > r_m$, which establishes the monotonicity of u^C. ■

Proof of Proposition 7. To establish this proposition, it is convenient to think of each opportunity as being composed of many "small opportunities." We will show that making opportunities lumpier in the original model is equivalent to a certain change in the correlation structure of these small opportunities.

First, we consider the case where opportunities become lumpier by a rational factor. Assume that opportunities become lumpier by $\frac{N}{M} > 1$, where $N, M \in \mathbb{N}$. We analyze this change by considering an auxiliary representation of the model in which there are $M \times N$ Poisson processes, each with an arrival rate of $\frac{\mu}{N}$, that govern the arrival of the small opportunities. Moreover, we assume that the payoff from exerting full effort on each small opportunity is $(-\frac{A}{M}, \frac{B}{M})$. Both the original and the lumpy versions of the model correspond to appropriately defined correlation structures of the arrival processes in the auxiliary representation.

To map the auxiliary representation to the original model, divide the Poisson processes into N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. To see why this correlation structure represents the original model, note that when a group of opportunities is available the payoff vector from exerting full effort on all opportunities in the group is $M \times (-\frac{C}{M}, \frac{B}{M}) = (-A, B)$, which is exactly the payoff vector from exerting full effort on a single opportunity in the original model. Moreover, the probability that a given group arrives in an (infinitesimal) interval dt is $\frac{\mu}{N}dt$, and since the groups are independent, the probability that some group arrives in that interval is $\sum_{i=1}^{N} \frac{\mu}{N}dt = \mu dt$.

To map the auxiliary representation to the lumpy model, divide the processes into M groups of N processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. For this correlation structure, the payoff from exerting full effort on all opportunities in a group is $N \times (-\frac{A}{M}, \frac{B}{M}) = (-\frac{N}{M}A, \frac{N}{M}B)$ and the probability that some group of opportunities is available in an interval dt is $\frac{\mu}{N/M}dt$.

Next, we construct a sequence of modifications that begins with the lumpy representation and ends with the original one, such that the first two modifications do not impact the manager's value, and the third modification strictly increases it.

Consider the lumpy representation. The first modification utilizes the idea of splitting the interaction into fictitious worlds introduced in Lemma 1. In particular, we create N fictitious worlds, denoted by $(1, \ldots, N)$, that each contain $\frac{1}{N}$ of the flow compensation and M arrival processes, one from each group. We denote the processes in fictitious world n by $(P_1^n, \ldots, P_M^n)$. Note that the arrival processes within each fictitious world are independent of one another, and so each fictitious world is a scaled version of the lumpy representation. Hence, by the argument used in Lemma 1, the sum of the manager's values across all fictitious worlds is equal to her value in the lumpy representation.

The second modification is to the correlation structure of the processes across fictitious worlds. Changing the correlation structure of two arrival processes that are assigned to *different* fictitious worlds does no impact the manager's value in either fictitious world. Hence, so long as the processes within each fictitious world are independent of one another, the correlation across fictitious worlds is immaterial. Thus, we can replace the original correlation structure with the following correlation structure: P_m^n and $P_{m'}^{n'}$ are perfectly correlated if $m - n \overset{(\text{mod} N)}{=} m' - n'$, and independent otherwise. This modification maintains the independence of the processes within each fictitious world. To see this note that for any $n \leq N$ and $m, m' \leq M$, such that $m' \neq m$, the fact that $M < N$ implies that $m - n \overset{(\text{mod} N)}{\neq} m' - n$.

The third modification is to re-merge the fictitious worlds. Note that under the correlation structure created in the second modification, there are N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. Thus, merging these fictitious worlds creates the auxiliary representation of the original model. Regardless of the relative patience, there are instances in which the manager benefits from merging two fictitious worlds: if $r_m < r_w$ this occurs when in fictitious world i the worker's continuation is positive while in fictitious world j it is zero, whereas if $r_m \geq r_w$ this occurs when in fictitious world i an opportunity is (partially) forgone while in fictitious world j the worker's continuation utility is below its maximal level. It follows that the sum of the manager's values across all fictitious worlds is strictly less than her value in the original model.

Finally, consider the case where $\lambda \notin \mathbb{Q}$. The manager's value is continuous in λ as i) the distribution of arrival times is continuous in λ, and ii) if opportunities are made slightly lumpier then the manager can instruct the worker to incur the same *cost* of effort on every opportunity that arrives by using the same compensation policy. As the set of rational numbers is dense, this establishes the proposition. ■