Sorting between Real and Financial Constraints: Macroeconomic Implications*

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Abstract

A simple model predicts that financial intermediaries penalize firms with a greater fixed adjustment cost. We show the model prediction is consistent with the observed sorting patterns between the real and financial frictions using a unique data set covering the universe of Portuguese firms. Then, we incorporate the different cost structures and financial frictions into the heterogeneous-firm general equilibrium model to capture the observed sorting pattern. Using the model, we analyze how the recently strengthened sorting pattern affects capital misallocation and aggregate shock sensitivity.

Keywords: Financial frictions, Adjustment cost, Firm dynamics, Misallocation.

JEL Codes: D53, E44, D21

* All errors are our own.

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1 Introduction

A substantial amount of research has been focused on how firm's characteristics influence the borrowing limits the firm faces. Kiyotaki & Moore (1997) highlight how the amount of capital owned by the firm can be used as collateral to determined the total amount a firm can borrow. With the increase in intangible assets, some literature has also focused on how the different types of assets have distinct collateral values, which ends up affecting the financial conditions of the firm. More recently, Lian & Ma (2021) show that the borrowing limits are more dependent on the firm's cash flow than on the collateral the firm can provide. In this paper we show that the firm's cost structure also influences its borrowing limits.

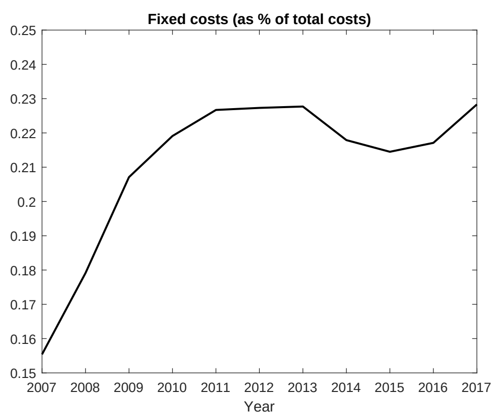


Figure 1: Fixed costs, as % of total costs, evolution over time.

In Figure 1 we can see that fixed costs, as a percentage of total costs, in the Portuguese economy has been increasing over the last decade.[1] This phenomenon is not unique to the Portuguese economy, as De Ridder (2019)

[1] We follow De Ridder (2019) to calculate the share of fixed costs empirically. We use the micro data for the universe of Portuguese firms to find the average for the Portuguese economy.

finds similar patterns in the US and in France.

This paper provides evidence on the sorting pattern between the firms' cost structure and financial frictions, how both affect the firms' investment decisions and end up amplifying recessions during periods of turmoil.

We start by illustrating in a simple setting how the interaction between the real and financial frictions affects both the intensive and extensive margin of investment. In a setting where firms choose investment to maximize profits and face both convex and fixed adjustment costs of capital, as well as a risk-premium on loans, we illustrate two mechanisms: First, firms with a higher fixed costs face higher risk-premiums. As fixed costs increase, conditional on a firm being active, the optimal investment of the firm does not change. As such, when active, the firms will become riskier, as the investment profitability is reduced, and banks charge a higher risk-premium to lend to these firms. On the opposite side, when convex adjustment costs increase, firms' optimal investment is reduced. As such, the probability of default of these firms is not as strongly impacted, which contributes to a smaller increase in the risk-premium.

Second, the type of firms impacted by the interaction between both frictions is different. While the firms more affected by an increase in fixed adjustment costs are the large productive firms, that despite not paying a spread on the loan, due to decreasing returns to scale will not find it optimal to invest, the small productive firms that were facing positive spreads are the ones more affected by convex adjustment costs. In this later case, if the convex adjustment cost increases, the profitability of the investment is greatly reduced and firms will not find it optimal to contract a loan with positive risk-premium and will not invest at all.

Next, using a unique data set covering both the balance sheet and credit situation of the universe of Portuguese firms, we take the predictions of the simple model to the data. We follow Ferreira et al. (2021) in identifying financially constraint firms. First, we validate an important model assumption, which is that fixed and variable costs are negatively correlated, and that firms are either high fixed cost type or high variable cost type.

Next, we find that a larger share of fixed costs as a percentage of total costs is positively and strongly correlated with the the percentage of constrained firms. This means that, firms that have a cost structure more dependent on fixed costs are on average more financially constrained. We equally find that firms with a lumpier investment profile are on average more constrained, in line with model predictions.

Lastly, we assess if the correlation between productivity and constrained firms depends on the share of fixed costs. Similar to model predictions, we find that for low fixed costs firms, the firms more affected by financial frictions are the ones at the bottom of the TFP distribution. For firms with high fixed costs, we find that firms in the top half of the TFP distribution are equally affected by financial frictions, in line with model predictions.

Then, making use of a heterogeneous-firm general equilibrium model, which includes the theoretical mechanisms from the simple model, we test how the sorting between real and financial frictions matter in the aggregate. The model generates a sorting pattern comparable to the empirical one, validating our structural approach.

2 Simple Theory

In this section we present a simple firm and bank model. The objectives are twofold: show 1) the interaction between real and financial frictions; 2) the firms with higher fixed costs are more penalized in terms of spreads they pay.

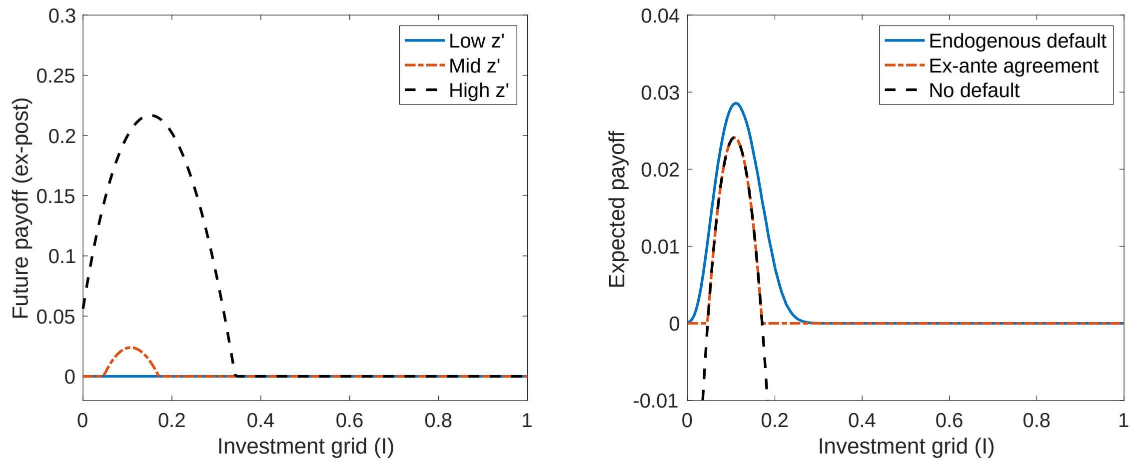
2.1 Firm's investment problem

We consider a firm-level investment problem as follows:

$$I^*(x;Q) = \arg\max_I \ \frac{1}{R}\mathbb{E}_z \max\left\{ \overbrace{z'(k(1-\delta)+I)^\alpha}^{\text{Operating profit}} + \overbrace{(1-\delta)(k(1-\delta)+I)}^{\text{Liquidation (continuation) value}} \underbrace{- \left(I + \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k + \xi \right) Q}_{\text{Debt payment}}, \ \underbrace{0}_{\text{Default payoff}} \right\}$$

where the individual state is defined as $x := (z, k)$, with z being the productivity of the firm and k the amount of capital it starts the period with. The firm then chooses investment I, subject to a debt price Q, to maximize its value given by the operating profits plus the liquidation value of its capital minus the debt repayment. If the firm invests, it needs to pay a fixed adjustment cost ξ and a convex adjustment cost $\frac{\mu}{2}\left(\frac{I}{k}\right)^2 k$. For simplicity, we assume the firm is fully relying on debt financing for the investment, so total debt contracted is equal to the investment value plus the convex and fixed adjustment costs.

The ex-post payoff of investment for a shock realization z' is as plotted in panel (a) in figure 2: as expected firms with higher realize productivity will tend to invest more and have higher payoffs. In panel (b) we can see that due to the default option, the ex-ante expected payoff deviates from the non-default payoff: If a bank and a firm can reach an agreement based on z (not z'), the firm's payoff of such a case (orange dash-dotted line) does not deviate from the non-default case (dark dashed line) for the non-default region. However,



(a) Ex-post payoffs

(b) Ex-ante payoffs

Figure 2: Payoffs

due to the default contingent on z', a firm tends to overinvest: the peak in the payoff of the endogenous default (blue solid line) is located right to the peak of the non-default case.

2.2 Loan price setting

Define the expected net benefit for a firm with and without the investment as follows:

$$J^*(x;Q) := \frac{1}{R}\mathbb{E}_z \max\left\{ z'(k(1-\delta)+I^*(x;Q))^\alpha + (1-\delta)(k(1-\delta)+I^*(x;Q)) \right.$$
$$\left. - \left(I^*(x;Q) + \frac{\mu}{2}\left(\frac{I^*(x;Q)}{k}\right)^2 k + \xi \right) Q, 0 \right\}$$
$$J^c(x) := \frac{1}{R}\left[\mathbb{E}_z z'(k(1-\delta))^\alpha + (1-\delta)^2 k\right]$$

where $J^*(x;Q)$ is the value of the firm under the optimal investment policy $I^*(x;Q)$, which is a function of the initial state of the firm x and the loan price

Q, and $J^c(x)$ is the value of a firm if no investment takes place. The equilibrium debt price Q^* is competitively determined in the following equation:

$$\underbrace{I^*(x;Q^*) + \frac{\mu}{2}\left(\frac{I^*(x;Q^*)}{k}\right)^2 k + \xi}_{\text{Bank's total cost}}$$
$$= \underbrace{\frac{1}{R}\mathbb{E}_z\left[\chi(x,z';Q^*) \times \text{Non-default payoff}^B + (1-\chi(x,z';Q^*)) \times \text{Default payoff}^B\right]}_{\text{Bank's discounted expected total benefit}}, \tag{1}$$

where χ is the indicator function of the non-default contingent on the future productivity shock realization z'. That is,

$$\chi(x,z';Q) = \mathbb{I}\Bigg\{ z'(k(1-\delta) + I^*(x;Q))^\alpha + (1-\delta)(k(1-\delta)+I) - \left(I^*(x;Q) + \frac{\mu}{2}\left(\frac{I^*(x;Q)}{k}\right)^2 k + \xi\right) Q \geq 0 \Bigg\}.$$

The default-contingent payoff of the bank is as follows:

$$\text{Non-default payoff}^B = \left(I^*(x;Q^*) + \frac{\mu}{2}\left(\frac{I^*(x;Q^*)}{k}\right)^2 k + \xi\right) Q^*$$
$$\text{Default payoff}^B = z'(k(1-\delta) + I^*(x;Q))^\alpha + (1-\delta)((1-\delta)k + I^*(x;Q)).$$

Equation (1) can be summarized as

$$\text{Total cost}(Q^*) = \frac{1}{R}\text{Expected total Benefit}(Q^*).$$

We rearrange this to get

$$\underbrace{R}_{LHS} = \frac{\text{Expected total benefit}(Q^*)}{\text{Total cost}(Q^*)} = \underbrace{\text{Expected return}(Q^*)}_{RHS}.$$

This implies that the price is competitively determined, implying no arbitrage in the equilibrium. It is worth noting that RHS does not depend on R. Then, the equilibrium price Q^* is determined in equilibrium when the expected return on the loan is equal to the risk-free rate. This is depicted in Figure 3

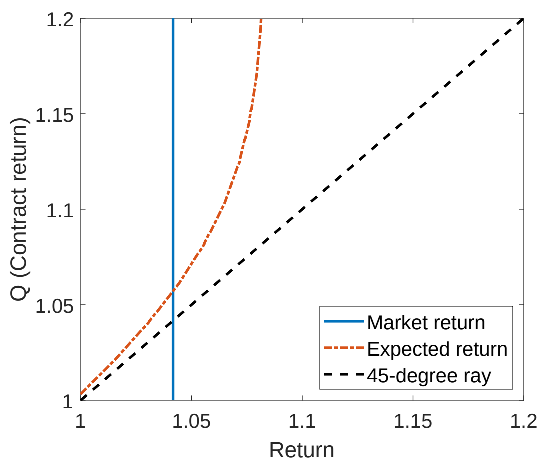


Figure 3: The equilibrium Q determination

2.3 Firm-level implication of the default option

A default option makes a firm that would otherwise never invest, make an investment. For example, as described in Figure 4, a bad firm's investment might be profitable only when the future productivity realization is substantially high (panel (a)). Then, without the default option, the expected payoff is sub-zero for any investment level (panel (b)). So, investment is not a good idea. However, once the default is allowed, the expected payoff soars above zero. So the firm will make an investment.

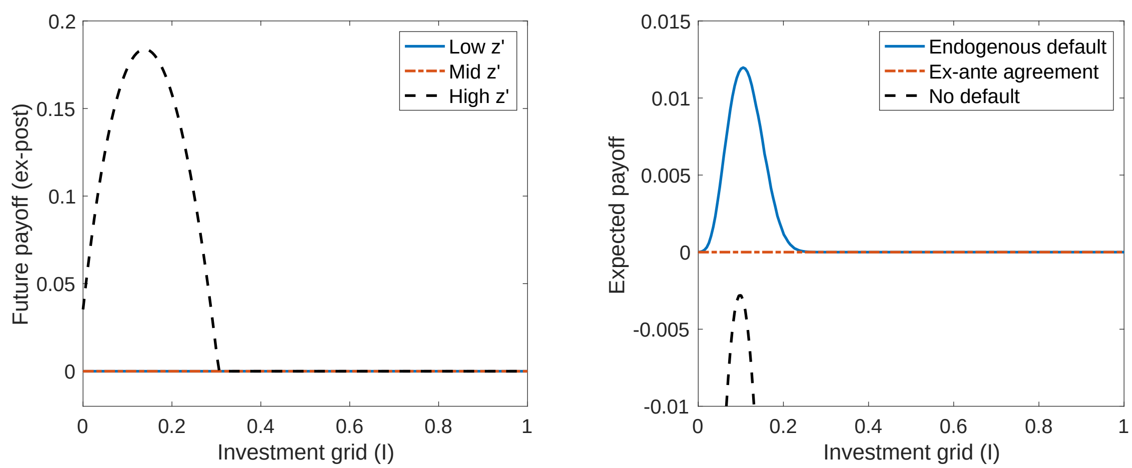
2.4 Endogenous loan decline

If a firm has a bad profile x, a bank might never find it beneficial to give a loan to the firm. Then, Q^* does not exists. This is plotted in Figure 5

3 Extensive margin decision

A firm invests only when $J(x; Q) > J^c(x)$. Then there exists a $\overline{Q}$ such that will make the firm indifferent between investing and not:

$$J(x; \overline{Q}) = J^c(x).$$



(a) Ex-post (b) Ex-ante

Figure 4: Payoffs of a bad firm

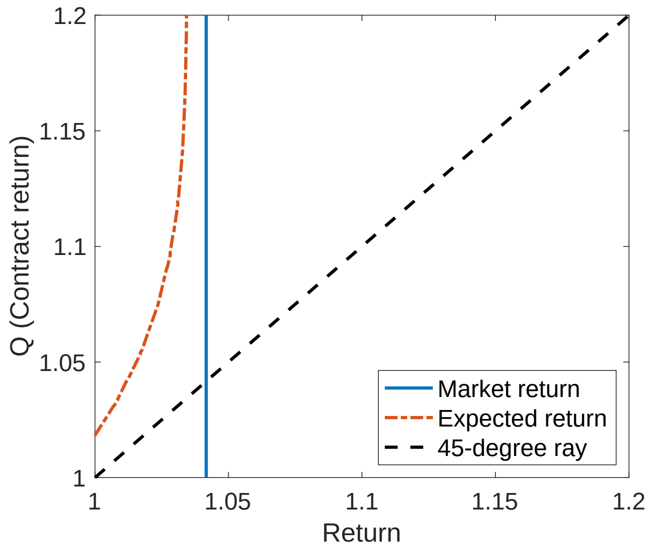


Figure 5: The equilibrium determination: bad firm

Then, for $\forall Q < \overline{Q}$, a firm would invest. More specifically, the $\overline{Q}$ is determined from the following equation:

$$\begin{aligned}
&\mathbb{E}_z \max\Bigg\{ z'(k(1-\delta) + I^*(x;\overline{Q}))^\alpha + (1-\delta)(k(1-\delta) + I^*(x;\overline{Q})) \\
&\qquad\qquad - \left(I^*(x;\overline{Q}) + \frac{\mu}{2}\left(\frac{I^*(x;\overline{Q})}{k}\right)^2 k + \xi \right)\overline{Q}, 0 \Bigg\} \\
&= \mathbb{E}_z z'(k(1-\delta))^\alpha + (1-\delta)^2 k
\end{aligned}$$

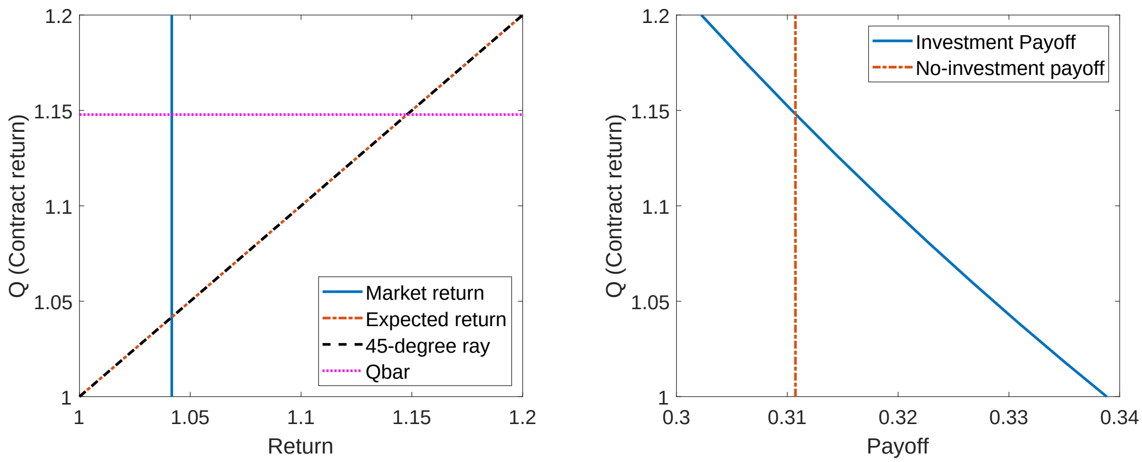
Then, there will be three possible equilibrium outcomes:

(i) If Q^* exists and $Q^* \leq \overline{Q}$, then the firm invests.

(ia) $Q^* = R$: credit spread does not exist.

(ib) $Q^* > R$: credit spread exists.

(ii) If Q^* exists and $Q^* > \overline{Q}$, then the firm does not invest.



(a) Q^* determination

(b) $\overline{Q}$ determination

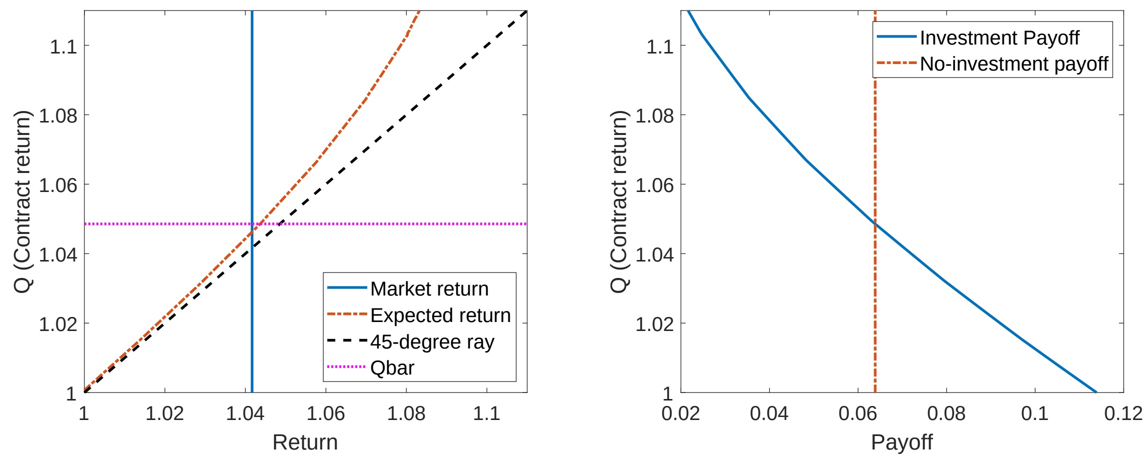
Figure 6: Case (ia): Frictionless outcome

(iii) If Q^* does not exist, the bank will not be willing to lend to the firm and no investment takes place.

Case (i) is the unique case where an investment can be made. Case (ia) is the frictionless case, when the firm invests and pays no spread (i.e. $Q = \frac{1}{R}$), which can be seen in Figure 6. Case (ib) is when the firm still invests but has to pay a risk-premium to get the loan, which can be seen in Figure 7. In cases (ii) and (iii) no investment takes place, but for different reasons: while in the first one the firm does not find it profitable to invest given the interest rate offered by the bank Q^*, in the later case the bank never finds it optimal to lend to the firm at any feasible Q.

3.1 Sorting between real and financial frictions:

To understand the interaction between real and financial frictions we first do some comparative statics of the effects of increasing either the fixed adjustment



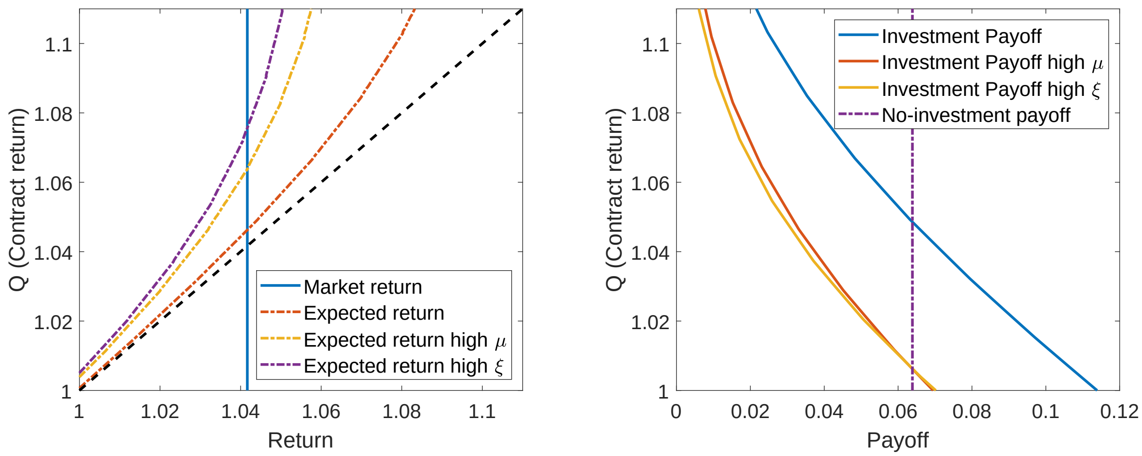
(a) Q^* determination

(b) $\overline{Q}$ determination

Figure 7: Case (ib) with the credit spread

cost ξ or the convex adjustment cost parameter μ on the equilibrium loan pricing. We take the situation in Figure 3 and increase ξ and μ one at a time.

It is important to guarantee that the increase in both parameters is comparable and the increases in the risk-premium do not come down to one having a stronger impact on investment payoff than the other. As such, we increase both parameters by an amount that guarantees the action and inaction regions are the same for both increases. This can be seen in panel (b) in Figure 8, where the investment payoff for both a high ξ and a high μ intersects the no-investment payoff in the same point. In panel (a) we can observe the effect of both increases on the equilibrium interest rate Q. Both a higher ξ and a higher μ are going to translate into a higher risk-premium, but the quantitative impact is more pronounced for a higher fixed than a higher convex adjustment cost. While a firm that has larger fixed adjustment costs does not adjust the investment on the intensive margin, a firm with higher convex adjustment costs will invest less. As such, the fixed cost firm will be more risky and consequently pay a larger spread.



(a) Q^* determination

(b) $\overline{Q}$ determination

Figure 8: Case (ib) with the credit spread

Despite the comparative statics illustrating the mechanism of why firms with higher fixed adjustment costs are more penalized, the analysis was conditional on both firms making investment. To understand the exact impact on the spread as well as on the decision of the firm to take on the loan, or the bank to lend to the firm, in Figure 3.1 we plot the regions capturing the different decisions, when both capital, productivity, fixed and convex adjustment costs change. In each panel, the x-axis has the initial capital of the firm and the y-axis the productivity. Along the horizontal dimension (from left to right) there is an increase in the convex adjustment cost, and along the horizontal dimension (from top to bottom) there is an increase in the fixed adjustment cost. The green and yellow regions capture inaction areas, i.e. when the firm does not invest, but for different reasons. While in the green area the firm can get a loan but due to the real friction decides not to invest, in the yellow region the firm cannot get a loan. The dark-blue area captures firms that invest and get a loan with a price equal to the risk-free interest rate. The light blue is when the firms invest but pay a spread on their loan.

What is possible to observe from this graph is that an increase in the convex adjustment cost is going to affect mainly highly productive small firms that wanted to invest with a lower μ. These are the firms that would pay a spread, but were willing to do it. As the convex cost raises, it becomes less profitable for these firms to accept the loan with a spread and will decide not to invest.

The opposite happens when the fixed adjustment cost raises. In this case, the more penalized firms are the large productive firms that initially would make an investment. As the fixed cost increases, and due to marginal returns to scale, investment for large firms stops being productive. At the bottom of the distribution there is also an impact with low productive small firms deciding not to invest now. But contrary to the case of an increase in the convex cost, there are still a large fraction of firms willing to invest despite paying a positive risk-premium.

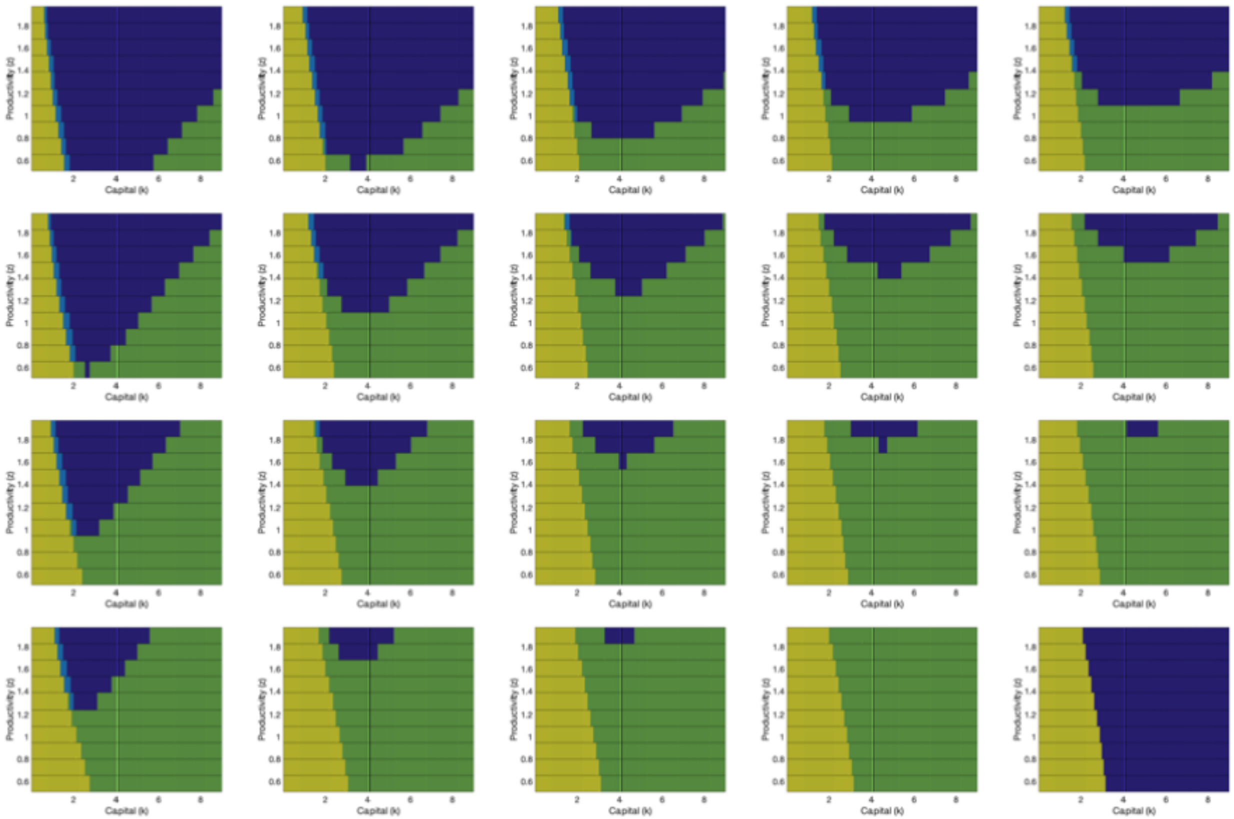


Figure 9: k-z plane: Along the horizontal dimension there is variation on the convex adj. cost. Along the vertical dimension there is variation on fixed adj. cost.

4 Data

We draw on a unique combination of data sets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal Microdata Research Laboratory.

The *Informação Empresarial Simplificada* (IES) Central Balance Sheet Database (CBSD) is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this data set covers the population of virtually all non-financial corporations in Portugal from 2006 onward. We combine this data set with the Central Credit Register (CCR), which contains monthly information on the actual and potential credit above 50 euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal.[2] Actual credit includes loans that are truly taken up, such as mortgages, consumer loans, overdrafts, and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialized into actual credit, such as available credit on credit cards, credit lines, pledges granted by participants, and other credit facilities. We then merge these two databases on the firm level. Moreover, we also add the Monetary Financial Institutions Balance Sheet Database in order to gain information on the balance sheets of banks that extend credit to non-financial institutions. We merge this data set on a firm level using the bank identifier and the share of loans extended by one firm to arrive at our detailed data set.

We restrict the set of firms in this panel data set to those with at least five consecutive observations and to firms that are in business at the time of

[2]Given that the firm balance sheet data is of yearly frequency, we consider the month in which the balance sheet data was reported. Results were robust to shifting and averaging the monthly credit data.

Table 1: Descriptive statistics of Portuguese firms between 2006 and 2017

				Size group median			
Variable	Mean	Median	Std. Dev.	0 - 90th	90th-99th	99-99.5th	>99.5th
Total Assets (€ mio.)	3.48	0.31	92.92	0.26	5.34	45.44	145.00
Turnover (€ mio.)	2.06	0.25	35.74	0.22	3.45	20.73	29.07
Potential credit (€ mio.)	0.21	0.01	4.76	0.00	0.16	1.01	3.12
Effective credit (€ mio.)	0.57	0.05	6.18	0.04	1.22	7.34	13.00
Leverage	0.28	0.20	0.41	0.20	0.24	0.16	0.08
Liquidity ratio	0.13	0.06	0.19	0.06	0.02	0.01	0.01
Age	16	13	12	12	21	23	22
Employees	15	5	140	4	26	98	99
Bank relationships	3	2	2	2	4	4	5

reporting. Furthermore, we only consider privately or publicly held firms and drop micro firms, i.e., those with overall credit amounts of less than 10,000 €. Descriptive statistics for the relevant variables can be found in Table 1.

4.1 Measures of financial constraints

We follow Ferreira et al. (2021), who use the credit information in the data to construct several binary measures indicating whether a firm is financially constrained. Financial constraints are most commonly conceived as a supply side phenomenon. Firms that could potentially obtain credit in perfect credit markets are unable to do so due to asymmetric information considerations on the supply side. For example, a firm that has a profitable investment project that requires external financing cannot realise it as the bank is not satisfied with the creditworthiness of that firm. This may happen either via the price dimension, ie. a risk premium on the interest rate, or on the quantity dimension ie. the credit is denied altogether.

In this paper, we classify constrained firms along the quantity dimension, using the credit information for each firm. Given that credit allowances are changing over time, this provides us with a time-varying and firm-specific measure for being financially constrained. It should be noted, however, that while credit information offers a far more detailed notion of a firm being constrained compared to standard financial ratios such as leverage or liquidity, it is still a proxy.

Measure As outlined above, potential credit summarizes all the irrevocable commitments by credit institutions. Even though this measure enables an understanding of whether firms have drawn down their credit lines and are thus potentially constrained it also encompasses a lot of noise. One problem might be that although firms have exhausted their committed credit line they could still successfully apply for a short- or long-term loan. To account for this, in our baseline definition, we consider a firm to be credit constrained at time t, if it has no potential credit available at time t and neither its short- nor long-term credit (ie. effective credit) is growing:

$$\text{Constrained I} := \mathbf{1}_{\text{Potential credit}_t = 0 \text{ \& } \Delta\text{Effective credit}_t \leq 0}.$$

Fixed cost One important dimension that is not directly reported by the firms are the fixed and variable costs. To estimate the fixed costs we follow De Ridder (2019) and identify fixed costs as

$$\frac{f_{it}}{py_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{py_{it}} \tag{2}$$

where f_{it} is the fixed cost of firm i in year t, py_{it} is revenues and π_{it} the firm's profits. μ_{it} is the firm's markup, which is calculated following De Loecker et al. (2020). For more details please see Appendix B. The idea is that the profit ratio captures the average profits of the firms, while the markup captures the marginal profit. The two only differ in the presence of fixed costs.

4.2 Empirical Results

Using the aforementioned data, we test the implications of the simple model. First, we validate an important assumption we make when testing the implications of the model: that fixed and convex adjustments costs are negatively correlated. This is, that firms do not have both fixed and convex adjustments costs both high or low at the same time. To test this assumption empirically we measure both fixed and variable costs as a share of the firm's turnover. Figure 10 validates our assumption that fixed and variable costs are negatively correlated.

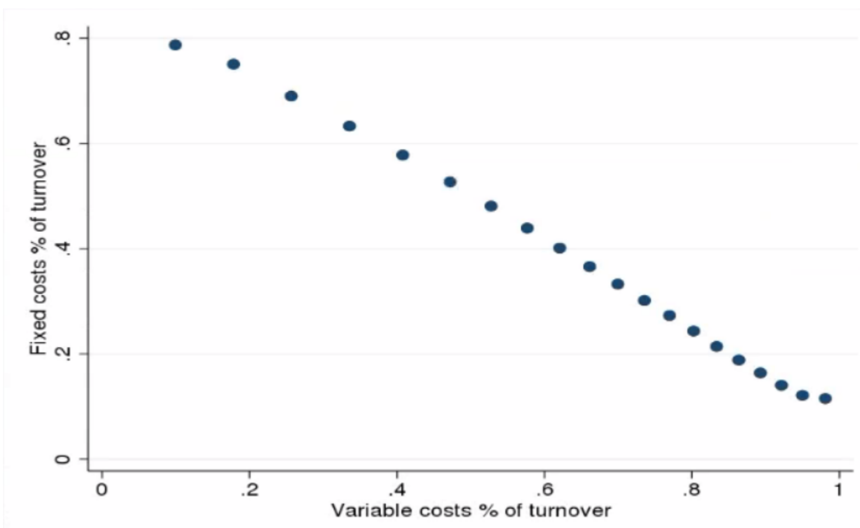


Figure 10: On the y-axis the fixed costs as a % of turnover. On the x-axis variable costs as a % of turnover.

We are now ready to test the predictions of the simple model. Following the predictions from the model, we test if firms with a higher share of fixed costs (as a % of total costs) are more financially constrained. Figure 11 shows that, in line with the simple model predictions, firms with higher fixed costs, as a % of total costs, are on average more financially constraint.

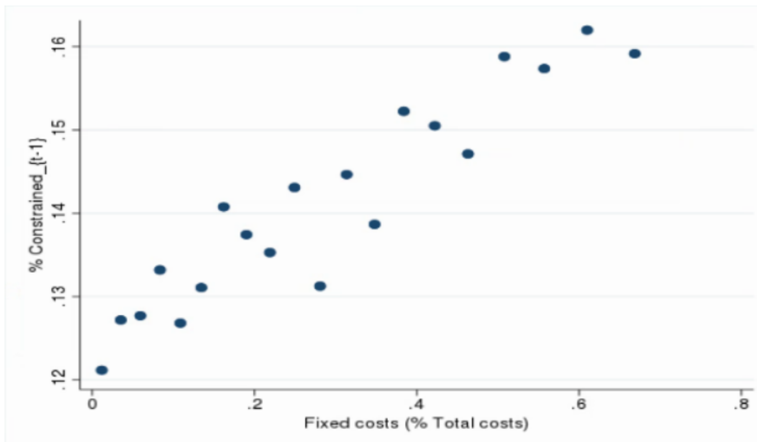


Figure 11: On the y-axis the % of the constrained firm at the end of period $t-1$. On the x-axis fixed costs, as a % of total costs.

Next we check if the firm's investment profile is equally in accordance with the model predictions and if it is correlated with the financial situation of the firm. From the simple model we know that firms with a more lumpy investment will invest less often, as the threshold for a firm to become active is higher. In the data, we follow the literature and classify a firm as active if it has an investment rate above 20% in any given period. On the right panel of Figure 12 it is possible to observe that, indeed, firms that have a lumpier investment profile on average are active for fewer periods. At the same time, these firms, are, on average, more financially constrained, as can be seen on the left panel.

Lastly, we test if the relation between TFP and constrained firms depends on the share of fixed costs, as predicted by the model. We estimate TFP following Ackerberg et al. (2015). In Figure 13 on the right-hand side it is possible to observe the strong and negative correlation between TFP and share of constrained firms, when fixed costs are low, whit the more productive firms being less financially constrained. On the left-hand side, the non-linear relation between TFP and financial constraints for high fixed costs firms is plotted. When fixed costs are high, the firms in the second half of the productivity distribution are more affected by financial constraints, as predicted by the model.

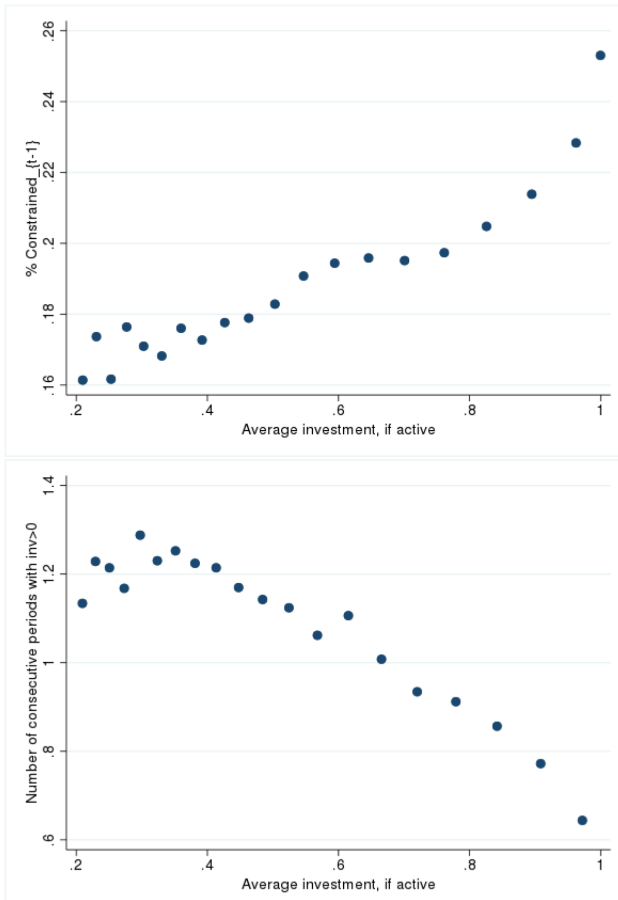


Figure 12: On the x-axis in both figures, the investment rate is conditional on a firm being active. On the left panel, on the y-axis, the % of constrained firms, and on the right panel, the number of consecutive periods a firm is active.

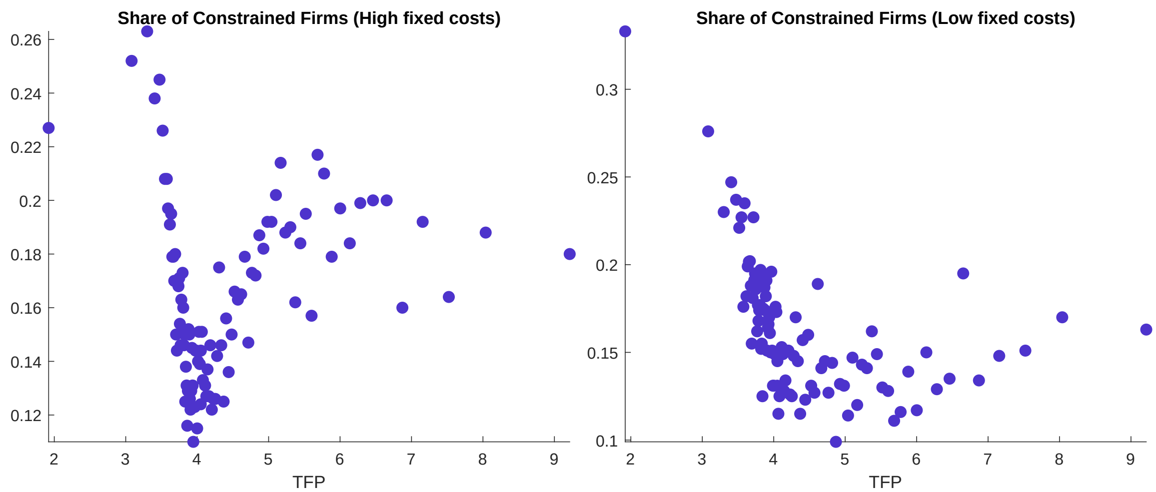


Figure 13: On the x-axis in both figures, the estimated Total Factor Productivity (TFP) at the firm level, following Ackerberg et al. (2015). On the left-hand side the share of constrained firms with high fixed costs (firms in the top quartile of the fixed cost distribution). On the right-hand side the share of constrained firms with low fixed costs (firms in the bottom quartile of the fixed cost distribution).

5 Baseline Model

In this section, we describe the full quantitative baseline model that includes the theoretical mechanism described in the previous section. We consider a heterogeneous-firm general equilibrium model, where a measure of one of the heterogeneous firms belongs to two different types: F (fixed cost) or C (convex adjustment cost) types. Each type is distinguished from the other by the nature of its real friction in capital adjustment: F type firms with greater fixed adjustment costs and C type firms with greater convex adjustment costs. This difference leads to a difference in financial frictions, which we call as the sorting between the real and financial frictions. A detailed explanation of the types follows after the production function. In the model, time is discrete and lasts forever.

5.1 Production Technology

At the beginning of each period, a firm is given the capital stock k_t that is determined from the previous period's investment decision and an idiosyncratic productivity z_t. The firm (manager of the firm) is aware of the distribution Φ_t of the individual states (k_t, z_t) and the aggregate productivity A_t, rationally expecting the dynamics of these aggregate allocations. Each firm operates using the Cobb-Douglas production function, where the inputs are labor and capital stock. The labor demand n_t is determined in the static operating profit maximization problem as follows:

$$\max_{n_t} A_t z_t k_t^{\alpha} n_t^{\gamma} - w_t n_t$$

where w_t is wage at period t. The logged idiosyncratic productivity follows an AR(1) process that is discretized by the Tauchen method:

$$log(z_{t+1}) = \rho_z log(z_t) + \sigma_z \epsilon_{t+1}, \quad \epsilon_{t+1} \sim_{iid} N(0,1).$$

Similarly, the aggregate productivity process also follows an AR(1) process, which we discretize in the quantitative analysis:

$$log(A_{t+1}) = \rho_A log(At) + \sigma_A \epsilon_{t+1}, \quad \epsilon_{t+1} \sim_{iid} N(0,1).$$

5.2 Investment decision and real and financial constraints

Each firm owns the capital stock and makes an investment decision. Each firm is subject to two real frictions: a fixed adjustment cost and a convex adjustment cost. The fixed adjustment cost occurs when the investment is beyond a range Ω that is proportional to the existing capital stock. The Ω is defined as follows:

$$\Omega = [-\nu k, \nu k].$$

A capital adjustment is subject to convex adjustment cost regardless of the size.

There are two types of firms in the economy: F (fixed cost) type and C (convex adjustment cost) type. Each firm's type follows an exogenous Markov process, represented by the following transition kernel Γ_{type}:

$$\Gamma_{type} = \begin{bmatrix} p_{type} & 1 - p_{type} \\ 1 - p_{type} & p_{type} \end{bmatrix}$$

F-type firms are subject to a greater fixed adjustment cost parameter than C-type firms: $\overline{\xi}_F > \overline{\xi}_C$. For each firm, the fixed cost is assumed to follow an $i.i.d.$ random uniform shock:

$$\xi \sim_{iid} Unif([0, \overline{\xi}_j]), \quad j \in \{F, C\}$$

Therefore, the expected fixed adjustment cost is greater for F-type firms than C-type firms. We assume the fixed cost is a labor overhead cost, so the total cost is computed by combining the wage and the fixed cost, $w\xi$ (Khan & Thomas, 2008). In contrast, F-type firms' convex adjustment cost parameter is smaller than C-type firms' convex adjustment cost parameter: $\mu_F < \mu_C$. The convex adjustment cost follows the conventional form in the literature:

$$\frac{\mu_j}{2} \left(\frac{I}{k} \right)^2 k, \quad j \in \{F, C\}.$$

On top of the two real constraints, firm-level investments are subject to financial constraints. Following Lian & Ma (2021), we assume financial intermediaries impose the cash-flow-based borrowing limits:

$$I \leq \theta_j \pi(k, z; S), \quad j \in \{F, C\},$$

where $\pi(k, z; S)$ is the operating profit of a firm with capital stock k and productivity z in the aggregate state S. Importantly, we assume financial intermediaries discriminate F-type firms due to the illiquid nature of the investment: $\theta_F < \theta_C$. Although we do not explicitly model the intermediary sector in the full baseline model, the exogenously designed sorting between financial and real frictions that captures the intermediaries' incentives helps properly capture the observed patterns in the firm-level investments.

5.3 Household

We close the model by introducing the representative household, who consumes, saves, and supplies labor. The household specification closely follows Khan & Thomas (2008) and Bachmann et al. (2013). Specifically, we assume a log utility and disutility for indivisible labor supply in the following form:

$$log(c) - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}},$$

where c is consumption; L is the labor supply; η is the scaling parameter; χ is the Frisch elasticity of the labor supply. In the current calibration, we assume $\chi \to \infty$, but we plan to adopt the level in the empirically supported range in future work.

The recursive formulation of the household's problem is as follows:

$$\begin{aligned} V(a;S) = \max_{c,a',L} \quad & log(c) - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}}, + \beta \mathbb{E} V(a';S') \\ \text{s.t. } \ & c + \int \Gamma_{A,A'} q(S,S') a'(S') dS' = w(S) l_H + a(S) \\ & G_\Phi(S) = \Phi', \quad \mathbb{P}(A'|A) = \Gamma_{A,A'}, \quad S = \{\Phi, A\} \end{aligned}$$

where a is the state-contingent equity portfolio value; A is the aggregate productivity; Φ is the joint cumulative distribution of the individual state variable; q is the state-contingent price; Γ is the transition kernel of the aggregate productivity; G_Φ is the expected dynamics of the individual state distribution Φ.

From the first order condition with respect to state contingent saving a', we characterize the state price as follows:

$$q(S, S') = \beta \frac{C(S)}{C(S')}.$$

As in Khan & Thomas (2008), we define $P(S) := 1/C(S)$, which we use for normalizing the firm's value function for easier computation.

5.4 Recursive Formulation of the firm's problem

In the analysis of the baseline model economy, we assume a stationary environment, $S = \{A, \Phi\}$, where $A(= 1)$ is the fixed aggregate productivity, and Φ is the stationary distribution of the individual firms. However, in the business cycle analysis, we analyze the recursive competitive equilibrium where the aggregate productivity shocks generate the dynamics in the aggregate state variables.

A firm is given with a type $j \in \{F, C\}$, and there are type-specific fixed cost $\bar{\xi}_j$, convex adjustment cost μ_j and financial constraint θ_j.

$$\begin{aligned}
J(k, z, j; S) = \quad & \pi(k, z; S) + (1 - \delta)k \\
& + \int_0^{\bar{\xi}_j} \max\left\{R^*(k, z; S) - \xi w(S), R^c(k, z, j; S)\right\} dG_\xi(\xi) \\
R^*(k, z, j; S) = & \max_{k' \in \Theta} \; -k' - c(k, k', j) + \mathbb{E}q(S)J(k', z', j'; S) \\
R^c(k, z, j; S) = & \max_{k^c \in \Omega \cap \Theta} \; -k^c - c(k, k^c, j) + \mathbb{E}q(S)J(k^c, z', j'; S)
\end{aligned}$$

The following lines explain the details of each component in the value function.

$$
\begin{aligned}
\text{(Operating profit)} \quad & \pi(z,k;S) := \max_{n_d} zAk^{\alpha}n_d^{\gamma} - w(S)n_d \\
\text{(Convex adjustment cost)} \quad & c(k,k',j) := \left(\mu_j^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k, \\
& j \in \{F,C\} \text{ and } \mu_F < \mu_C \\
\text{(Fixed adjustment cost)} \quad & \xi \sim_{iid} Unif[0,\overline{\xi}_j], \quad j \in \{F,C\} \text{ and } \overline{\xi}_F > \overline{\xi}_C \\
\text{(Real constraint of investment)} \quad & \Omega(k;\nu) := [-k\nu, k\nu] \quad (\nu < \delta) \\
\text{(Financial constraint of investment)} \quad & \Theta(k,z,j;\theta) := (-\infty, \theta_j\pi(k,z) + (1-\delta)k], \\
& j \in \{F,C\} \text{ and } \theta_F < \theta_C \\
\text{(Idiosyncratic productivity)} \quad & z' = G_z(z) \text{ (AR(1) process)} \\
\text{(Firm type transition)} \quad & j' = G_j(j) \text{ (Markov chain)} \\
\text{(Discount factor)} \quad & q(S) = \beta \\
\text{(Aggregate states)} \quad & S = \{A, \Phi\}
\end{aligned}
$$

As a benchmark model to the baseline, we also consider an economy without the type-specific heterogeneity in the frictions. The benchmark is separately calibrated from the baseline model to be properly compared with the baseline with respect to the data patterns in the firm-level investment.

6 Quantitative Analysis

6.1 Sorting between the real and financial frictions in the baseline model

In this section, we analyze the firm dynamics of constrained firms in the simulated data in comparison with the observed pattern from the data. The

10,000 firms are simulated using the equilibrium allocations in the baseline model over 100 periods after the initial 100 periods of burn-in periods that are discarded from the sample. Consistent with the data counterpart, we define financially constrained firms as those with a binding financial constraint. We define active firms as firms making a large-scale investment greater than 20% of existing capital stock, interchangeably with lumpy investment, following the literature on firm-level investment.

Figure 14 is the baseline-model counterpart of Figure 12. The right-hand side panel of Figure 14 is the firm-level (*not* observation-level) scatter plot along the two dimensions: one is the intensive margin in the investment when a firm makes a large-scale investment, and the other is a portion (%) of constrained firms at the last period. The figure displays a significant positive sorting pattern between the portion of constrained firms and the average size of the lumpy investments. If a firm is financially constrained in the last period, the firm's lumpy investment plan must have been delayed to the next period due to the lack of funding to implement the investment. Therefore, the more firms are financially constrained in the last period, the more likely the firms are to make lumpy investments in the following period, implementing the delayed plan in the last period. This effect can generate synchronized investment patterns over the business cycle, leading to endogenous fluctuations in aggregate investment (Lee, 2022).

The left-hand side panel of Figure 14 is another firm-level scatter plot along the two dimensions: one is the intensive margin in the investment when a firm makes a large-scale investment, and the other is the number of consecutive periods with a positive investment. When a firm is strongly constrained by the convex adjustment cost, the firm needs to split the size of the investment out of the concern of reducing the marginal cost of investment. This makes

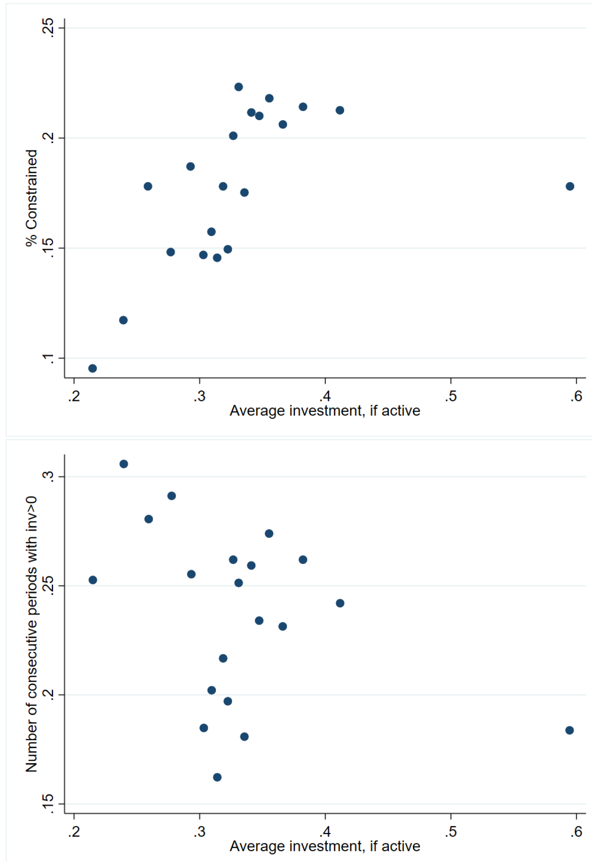


Figure 14: On the x-axis in both figures, the investment rate is conditional on a firm being active. On the left panel, on the y-axis, the % of constrained firms at the last period, and on the right panel, the number of consecutive periods a firm is active.

firms invest more frequently with a smaller size. Therefore, as can be seen from Figure 14, the number of consecutive periods of positive investment is significantly negatively correlated with the average size of the lumpy investment. Importantly all these patterns in the simulated data are consistent with the observed patterns in the data, validating our structural approach at the firm level.

7 Concluding remarks and future plan

The starting point of this paper is the change in firm-level operating cost structure: the importance of fixed cost is rising. By investigating how the real frictions associated with the fixed cost interplay with the other real frictions (convex adjustment cost) and the financial frictions, we (plan to) analyze how the macroeconomic change affects the economy through resource misallocation and aggregate shock propagation.

From a simple theory, we establish that when the fixed adjustment cost meets financial friction, the lumpiness of firm-level investment becomes more severe. On top of that, due to the option value of the liquid investment project in comparison with the illiquid investment project, financial intermediaries tend to prefer lending money to convex-adjustment-cost type firms than fixed-adjustment-cost type firms, which intensifies the lumpiness of the latter type's investment even further. The theory is consistent with the empirical patterns we analyze in the unique Portuguese firm-level data set.

To investigate the macroeconomic impact of the rising fixed cost, we introduce a heterogeneous-firm general equilibrium model, where firms are subject to real and financial frictions with the sorting pattern between the two frictions, as our simple theory predicts. From the simulated data based on the stationary equilibrium, we compare model-implied firm-level investment patterns and the data counterpart, validating our structural approach.

In future work, we plan to sharpen the theoretical points on the financial intermediaries' incentive on lending money to two different types of firms: fixed-cost types and convex-adjustment-cost types. This will also help us build the micro-foundation of the full model better, allowing us a richer analysis of the counterfactual. In particular, we plan to analyze the capital misallocation in the stationary equilibrium that arises due to the sorting pattern between the real and financial frictions. Going a step further, we will measure how much of additional misallocation occurs due to the rising fixed adjustment cost, as we showed as a motivating fact. Then, we will analyze the business cycle implication of the sorting between the two frictions. Due to the strengthened lumpiness in the investment pattern, we expect the sorting leads to a stronger tendency for synchronization of investments across the firms, resulting in substantial amplification of the aggregate investment fluctuations at the cost of households' welfare.

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A Proofs

A.1 Proposition ??

Taking the derivative of the threshold for z with respect to the investment we have

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{1+\mu\frac{I}{k}}{\beta^F\left[\left((1-\delta)k+I\right)^{\alpha}-\left((1-\delta)k\right)^{\alpha}\right]} - \frac{I(b)+\xi+\frac{\mu}{2}\left(\frac{I(b)}{k}\right)^2 k}{\beta^F\alpha\left((1-\delta)k+I\right)^{\alpha-1}}$$

Notice that this can also be rewritten as

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{MC(I)}{TB(I)} - \frac{TC(I)}{MB(I)}$$

Reorganizing we get

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{MB(I)}{TB(I)} - \frac{TC(I)}{MC(I)}$$

Which is smaller than zero as the first term is smaller than 1 (marginal benefit of investment is lower or equal to the total benefit) and the second term is higher than 1 (total cost of investment is higher or equal to the marginal cost). This means that if the financially constrained level of investment $I(b)$ is lower than I^*, the financial constraints will have both an impact on the intensive and extensive margin, as the productivity threshold for active investment will be higher, causing some firms not to invest. The presence of the convex adjustment cost will have a similar effect given that $I^c \leq I^*$.

A.2 Proposition ??

Taking the derivative with respect to p^*, we obtain

$$u(I^*) - u(0) = u'(b)\frac{\partial b}{\partial p^*}$$

Thus,

$$\frac{\partial b}{\partial p^*} = \frac{u(I^*) - u(0)}{u'(b)} > 0$$

Consider a concave u, so $u'' < 0$. Then,

$$\frac{\partial^2 b}{\partial (p^*)^2} = -\frac{u(I^*) - u(0)}{u'(b)^2} u''(b) \frac{\partial b}{\partial p^*} > 0.$$

This implies that a risk-averse bank would convexly reduce the lending amount for firms with a high fixed cost (firms with low p^*).

A.3 Proposition ??

B Markup Estimation