The Rise and Evolution of the Innovative Firm: A Tale of Technology, Market Structure, and Managerial Incentives*

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Abstract

We develop a dynamic general equilibrium model of the transition from an economy with small owner-operated businesses to an economy where part of the control over large-scale organizations is delegated to managers. We model managerial delegation as a principal-agent problem: the owner must offer an incentive contract that makes managers exert effort. The choice of whether and when to delegate managerial tasks, and how many managers to employ is dictated by which organizational form yields the higher rate of return to innovative investments. Owners trade-off the benefit from improved efficiency with the cost of forgoing a share of the gross cash flows as managerial compensation. In equilibrium, the owner-managed organization prevails early on when market size is small; delegation becomes profitable only when market size is sufficiently large to guarantee the viability of the incentive contract. Upon delegation, the economy experiences a productivity growth acceleration fueled by faster innovation. However, the emergence of the managerial class is not hard-wired into the theory: equilibria where firms remain small and owner-managed are possible.

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1 Introduction

A fact of the Second Industrial Revolution from the late nineteenth to the early twentieth century is the emergence of business organizations with large scale of operations and the associated rise of the "managerial class." As emphasized by business historians, technological progress and the expansion of market size drove the transition from the economy with owner-operated small businesses to the economy where some of the control over large-scale organizations is delegated to salaried managers (Chandler, 1962, 1977, 1990). While all developed economies have made such a transition, the timing and the extent to which they have done so varies substantially (Appelbaum and Batt, 2018; Bloom and Van Reenen, 2007, 2010; Chandler, 1977; Freeman and Lazear, 1995; Giorcelli, 2021). In fact, large cross-country differences in market structure in terms of number of firms, firm size and the internal organization of the firm still persist today, more than a century after the Second Industrial Revolution. Understanding the interaction between market structure and the internal organization of the firm is then important to develop a theory of the transition from the economy with small-scale organizations run by owners to the modern economy with large-scale organizations partially, if not fully, run by managers.

In this paper, we develop a theory of this transition. We build a dynamic general equilibrium model in which market size and structure, innovation, and the internal organization of the firm are jointly determined as endogenous outcomes of market forces. We then use the model to investigate, first, what aspects of the economic environment are critical in causing and shaping the transition and, second, the implications of the transition for productivity growth and income distribution dynamics.

The idea driving the theory is that the internal organization of the firm is a choice variable and, like firm entry, employment, and investment, is determined by market conditions and more generally by the economic environment in which firms operate. Managerial services are an essential input to production, marketing and innovation operations. Owners can supply the input on their own or hire the help of salaried managers when profitable. Both forms of organization have access to the same technology, so that the owners' choice is about if and when to delegate, and how many managers to employ.

We model delegation as a principal-agent problem cast as an "effort model": the owner-principal offers a compensation package that pays a fraction of the gross cash flow to the manager-agent in order to elicit optimal effort, that is, the compensation package must satisfy the incentive constraint for the manager to remain in the contractual relationship and exert a level of effort that maximizes the objective function of the owner. The benefit of delegation is improved efficiency in the form of higher product quality and/or higher productivity. The cost is the incentive contract due to the need to provide managers with the right incentives to exert costly effort. The delegation decision is dictated by which organizational structure delivers the higher rate of return to in-house innovative investment, and by no arbitrage, to shareholders.

While managers contribute to the overall efficiency of the firm, they are *not* the source of long-run growth. The gains from delegation are eventually choked off by decreasing returns to managerial services. This is consistent with the view in Penrose (1959) that *finite* managerial services are a natural limit to the growth of the firm. In the tradition of Schumpeterian growth theory, long-run growth comes from knowledge accumulation that manifests itself as sustained increases in aggregate productivity.

The model identifies a novel channel through which the owner's incentives to delegate and to undertake innovative investment reinforce each other. The higher the flow of managerial services due to more managers per firm and more effort per manager, the higher the rate of return to the firm's innovative investment. In turn, the higher the firm's investment, and so the higher the stock of knowledge of the firm, the higher the effect of managerial services on product quality and sales. This positive feedback loop generates strong complementarity between management and innovation.

In equilibrium, the self-reinforcing feedback between management and investment of incumbent firms interacts with endogenous firm entry. As firms employ more managers who exert more effort, they invest more, which leads to a higher stock market valuation. At the same time, more managers subtract resources from dividend distribution. If the negative effect on distributed dividends dominates that on market valuation, in free-entry equilibrium there is less entry. This in turn implies that firm size is larger, which reinforces the incentives to invest. Following the same logic, a smaller number of managers per firm reduces the incentives to invest, which lowers the firm's stock market valuation even though it raises dividend distribution. If the latter effect dominates, there is more entry and smaller firm size, which further reduces the incentives to invest.

This mechanism can substantially amplify the effects of distortions of firms' entry decisions, such as those due to regulations that increase the cost of setting up a firm, or of government policies that alter the incentives to undertake innovative investments, such as business income taxation and investment tax credits or subsidies. More broadly, *any* policy that alters market structure affects both the organization and the investment of the firm. This feeds back on the number of firms and firm size and causes the final growth effect of the policy to be larger than the initial direct effect.

The complementarity between management and innovation can be sufficiently strong to generate not only a growth acceleration when delegation occurs, but also multiple equilibria. The model admits either one or three steady states. In the latter case, the outer two are saddle-path stable and the one in the middle is unstable. The stable steady states describe two types of equilibria: one in which the economy exhibits low or no growth because firms are small, are operated exclusively by the owner, and invest little if at all; another in which the economy exhibits high growth fueled by innovative investment done by large firms operated with the help of managers. The unstable steady state can be either a "source" or a "focus" with monotonic or spiraling trajectories moving away from it.

We show that along any equilibrium path, the owner-managed form of organization prevails early on when the size of the market is small. The delegation of managerial functions becomes profitable only when market size is large enough to guarantee the viability of the incentive contract. When firms switch to the more complex form of internal organization, the economy experiences a productivity growth acceleration fueled by higher innovative investment. In steady state the economy exhibits sustained productivity growth. Comparing the economy with delegation to the one with no delegation, we find that steady-state growth is faster in the former case.

When multiple steady states occur, we obtain: (i) a growth trap in which the economy is stuck in a low-growth steady state; (ii) indeterminacy in which the economy can fluctuate stochastically driven by self-fulfilling expectations; (iii) history dependence or hysteresis in which initial conditions determine long-run outcomes. This property suggests a possible explanation of why some countries failed to become innovative, knowledge-based economies with large and complex organizations, or why countries with arguably similar fundamentals converged to different growth paths. The mechanism governing these outcomes in our model is novel and provides a rationale for a theory of "convergence clubs,"with the associated bimodal or multi-modal distributions in relative per capita income levels (see Howitt, 2000; Howitt and Mayer-Foulkes, 2005; Quah, 1996*a*,*b*, 1997), based on the interaction of market size, firm-specific innovation, and the internal organization of the firm.

The model has sharp implications for the functional distribution of income, relative pay, and employment dynamics of managers versus production workers. Consistently with one of the best documented empirical regularities in executive compensation, the compensation of the individual manager rises with firm size (Baker, Jensen and Murphy, 1988; Edmans, Gabaix and Landier, 2009; Edmans and Gabaix, 2016; Gabaix and Landier, 2008; Gabaix, Landier and Sauvagnat, 2014; Hall and Liebman, 1998; Murphy, 1999, 2013). Along a path with rising firm size, arguably the empirically relevant case for industrialized economies, the model predicts that managers earn an increasingly higher share of total labor income and provide an increasing share of total employment. These patterns are consistent with trends in managerial compensation and the managerial share of employment in the United States and other major industrialized countries.

The paper is organized as follows. In Section 2 we discuss the related literature. In Sections 3-5 we present the economic environment, the contracting problem, and the general equilibrium of the model. In Sections 6-7 we study the mechanism of multiple equilibria. In Section 8 we examine income distribution dynamics. Section 9 concludes. Appendices A-B provide a discussion of local stability analysis and evidence on managers' income and employment share in the United States.

2 Related Literature

Our paper contributes to the understanding of the macroeconomic origins and effects of the internal organization of the firm. By emphasizing the role of market structure, innovation, and growth, it provides a dynamic general equilibrium model well-suited to study a large set of questions that have been typically addressed in the realm of Organizational Economics and Industrial Organization, abstracting from macroeconomic dynamics. To the best of our knowledge, ours is the first model in which the delegation decision itself, the number of managers per firm, effort per manager, and the growth rate of the economy are jointly determined as endogenous outcomes of a market equilibrium. The model points to an important and so far overlooked interaction between market size, technology, and the internal organization of the firm that can naturally lead to growth accelerations and, possibly, to growth traps.

The model that we build in this paper belongs to the extended class of endogenous growth models featuring both vertical and horizontal innovation with endogenous market structure (see, e.g., Peretto, 1996, 1998, 1999).[1] Two of the authors of this paper have previously examined the effects of corporate governance frictions in the form of managerial resource diversion and empire building using a model of this class (Iacopetta, Minetti

[1]See also Atkeson, Burstein and Chatzikonstantinou (2019) and Etro (2009) for a recent survey of the literature.

and Peretto, 2019; Iacopetta and Peretto, 2021). In this paper we retain the endogenous growth core of the model but modify the environment in two important dimensions.

First, we consider an effort model in which managers optimally choose costly effort that directly improves product quality and thereby firms' sales and profits. That is, managers do not steal or divert resources from better uses, but contribute to the firm's value and are remunerated with an incentive contract. In this sense, we take the "shareholder view" of executive compensation according to which managers are not rent extractors and contracts are chosen to maximize shareholder value (Edmans and Gabaix, 2016). In contrast, Iacopetta, Minetti and Peretto (2019) and Iacopetta and Peretto (2021) follow the "rent extraction view" (Edmans and Gabaix, 2016, p. 1232) and focus on the moral hazard of managers.

Second, we allow the owner of the firm to decide whether, when, and how many managers to employ. Consequently, the delegation decision is endogenous, and the number of managers per firm and effort per manager become two objects determined in general equilibrium alongside other macroeconomic variables. Absent the extensive margin of the number of managers per firm, the model exhibits a unique steady state. In this sense, the model that we present here is new and proposes a qualitatively different theory of the growth of the firm.

The paper builds on the vast literature on innovation-led growth. While this literature has studied several aspects of the growth process, research on the role of managers and the internal organization of the firm has been limited. Notable exceptions are Akcigit, Alp and Peters (2021) and Celik and Tian (2017). Among many differences, arguably the most important one that distinguishes our work from those contributions is the focus on modeling the transition from the economy with small, owner-operated firms to the modern economy with large and complex business organizations. Specifically, we contribute to the debate by identifying a novel channel through which the interaction of market structure and the extensive margin of managerial employment gives raise to a growth acceleration and, possibly, multiple equilibria.

The paper also relates to Dessein and Prat (2019) and Terry (2017) who study how agency problems between the firm's owner and the managers who run it distort investment decisions in intangible assets. Both papers highlight the tension between short-term profit and long-term accumulation induced by agency frictions. While we do not examine the phenomenon of "short-termism," here, as in their work, the incentive problem between managers and owners and its impact on the accumulation of intangibles, knowledge in our case, takes center stage. Acemoglu and Newman (2002) study the effect of labor market conditions on the internal structure of the firm in an efficiency-wage model in which production workers can shirk and the firm employs managers to monitor their effort. While they share our interest in the macroeconomics of the organization of the firm, we address a different question, which centers on the emergence of the managerial class, the innovative investment of incumbent firms, the entry of new firms, and long-run growth.

By emphasizing the role of endogenous firm entry with sunk entry costs, some of the results in this paper relate to a long tradition in industrial organization that views sunk costs as a key determinant of market structure (Sutton, 1991, 2001). Peretto (1996) has incorporated these insights in endogenous growth theory and we view this paper as a continuation of that research agenda. There is also a literature that has studied the broad macroeconomic implications of entry costs for high-frequency movements in output (see, e.g., Alesina et al., 2005; Bertrand and Kramarz, 2002; Bilbiie, Ghironi and Melitz, 2012; Blanchard and Giavazzi, 2003; Cacciatore and Fiori, 2016; Fiori et al., 2012; Hamano and Zanetti, 2017; Nicoletti and Scarpetta, 2003). However, the literature has not explored how such costs affect the internal organization of the firm, thus possibly missing important forces. In our model, for example, as the entry cost parameter becomes arbitrarily large, multiple equilibria vanish. In other words, everything else equal, it is always possible to find a high enough value for the entry cost that rules out multiplicity. To the extent that entry costs capture aspects of the business regulatory environment, and that they can be affected by government policy, e.g., through the stringency and enforcement of national and local regulations, these results can be useful for policy analysis because they highlight the role of the internal organization of the firm as an important transmission channel.

Finally, the paper relates to the literature on "development traps" that emphasizes increasing returns (Matsuyama, 1991), externalities, technological complementarities, endogenous markups (Gali, 1994, 1995), and non-convexities from fixed costs (Ciccone and Matsuyama, 1996), as potential sources of multiple equilibria (see also Azariadis and Stachurski (2005) and Matsuyama (1995) for review articles). Relatedly, Garicano and Rossi-Hansberg (2012) show that in response to improvements in communication technology, organizations in the form of knowledge-based hierarchies may lead to lower growth and even to stagnation. This happens because the return to exploiting available technologies through organizations increases relative to developing new technologies.[2] While

[2]See Garicano and Rossi-Hansberg (2015) for a survey of the literature on knowledge-based hierarchies.

some of the qualitative features of our model resemble those highlighted in this literature, the mechanism of multiplicity in our paper is to the best of our knowledge new.

3 Environment

In this section, we describe the economic environment: preferences, technology, and the set of markets in which agents interact. Time is continuous and lasts forever, indexed by $t \in [0, \infty)$. Variables are continuous functions of time, but to simplify notation we will omit the time argument unless needed for clarity.

3.1 Household

The economy is populated by a representative household with a measure $\Lambda(t) = \Lambda_0 e^{\lambda t}$ of infinitely-lived members. We normalize $\Lambda_0 = 1$ and allow for $\lambda > 0$ so that population grows over time at a constant and exogenous rate. Each household member is endowed with one unit of time which is supplied inelastically as labor services. Household's labor supply is thus equal to $\Lambda(t)$. Similarly, the household is the owner of a fixed factor Ω, e.g., land, whose services too are supplied inelastically.

The household's preferences over per capita consumption are described by

$$U(0) = \int_0^\infty e^{-\rho t} \left[\Lambda(t) \log \left(\frac{C(t)}{\Lambda(t)} \right) \right] dt, \quad \rho > \lambda, \tag{1}$$

where 0 is the arbitrary point in time at which decisions are made, ρ is the intertemporal discount rate, and $C(t)$ is total household consumption.

The household's flow budget constraint is

$$\dot{A} = rA + wL + p_\Omega \Omega - C. \tag{2}$$

In (2), A is household's wealth, which consists of equity shares of firms whose mass N is determined in free-entry equilibrium, L is the mass production workers, and $p_\Omega \Omega$ is income from renting the services of the fixed factor.[3] The rate of return on wealth, r,

[3] For simplicity we separate the household members who are hired as managers from the main household that provides production labor and startup funds. We also assume that managers consume instantly their income flow (see below). We have worked out the model under the alternative assumption that upon being hired managers remain in the household and confer their incomes to all-inclusive household budget and participate in a complete consumption-sharing scheme. The analysis produces the same qualitative in-

the wage rate, w, and the price of the fixed factor, p_Ω, are determined competitively as explained below.

The consumption plan that maximizes (1) subject to (2) satisfies the standard Euler equation

$$r = \rho - \lambda + \frac{\dot{C}}{C}, \tag{3}$$

and the usual boundary conditions.

3.2 Production

The production side of the economy consists of a two-tiered vertical structure. A final sector and an upstream intermediate sector. The latter is where the main corporate-governance action relevant to the paper's research question takes place.

3.2.1 Final Producers

A competitive representative firm produces a final good Y that can be consumed, used to produce intermediate goods, invested in the improvement of the quality of existing intermediate goods, and invested in the creation of new intermediate goods. The final good is the numeraire so its price is $P_Y \equiv 1$.

The production technology is

$$Y = \int_0^N X_i^\theta \left(Q_i L_i^\gamma \Omega^{1-\gamma} \right)^{1-\theta} di, \quad 0 < \theta, \alpha, \gamma < 1, \tag{4}$$

where N is the mass of intermediate goods and L_i and Ω are, respectively, services of labor and of the fixed factor. The subscript i under labor says that the technology features full dilution of labor across intermediate goods, reflecting the property that both labor and intermediate goods are rival inputs. The fixed factor, instead, is non-rival across intermediate goods and labor. Quality, Q_i, is the good's ability to raise the productivity of the other factors.

The profit maximization problem of the final producer yields an isoelastic demand curve for intermediate goods, $X_i = (\theta/p_i)^{\frac{1}{1-\theta}} Q_i L_i^\gamma \Omega^{1-\gamma}$, where p_i is the price of good i, and a demand curve for labor, $L_i = \gamma(1-\theta)(p_i X_i/\theta w)$. Final producers pay $\int_0^N p_i X_i di = \theta Y$, $\int_0^N w L_i di = \gamma (1-\theta) Y$, and $p_\Omega \Omega = (1-\gamma)(1-\theta) Y$, to the suppliers of intermediate

sights as the simpler version in the text at the cost of substantially more algebra. The setup with segregated budgets yields analytical results that make the paper's key mechanisms transparent.

goods, labor, and the fixed factor, respectively, so that total compensation to the factors of production equals output.

3.2.2 Intermediate Producers

We begin with presenting the model's primitives abstracting from delegation. Once those are spelled out, we compare the two regimes with and without managerial delegation to obtain our main results.

Technology and management In line with Schumpeterian growth theory, we view firms as organizations that develop and apply specialized knowledge. Specifically, we write quality as

$$Q_i = M_i Z_i^{\alpha} Z^{1-\alpha}. \tag{5}$$

In words, the contribution of good i to factor productivity downstream depends on the technological component, $Z_i^{\alpha} Z^{1-\alpha}$, and the managerial component, M_i. The technological component depends on the knowledge stock of firm i, Z_i, and on the average knowledge of all firms, $Z = \int_0^N \left(Z_j/N\right) dj$.

We draw a sharp distinction between knowledge and management. While firm's knowledge, Z_i, can grow without bound, we model management as an input subject to natural bounds that apply along an extensive and an intensive margin. Formally, we specify managerial services as

$$M_i = \int_0^{m_i} f\left(h_{ij}\right) dj + \mu, \tag{6}$$

where m_i is the mass of managers employed by firm i, $h_{ij} \in [0,1]$ is effort of manager j in firm i, and μ is the owner's managerial input.[4] The natural bound on m_i stems from the finite size of the workforce; the natural bound on h_{ij} stems from well-understood arguments developed by the literature on effort models.

In-house innovation On the production side, the firm's technology requires one unit of final output per unit of intermediate good and a fixed operating cost, ϕZ_i, also in units

[4] Implicit in (6) are two assumptions. (1) The owner of the firm exerts constant effort. This is a simplifying assumption. Extending the model with an owner's effort choice does not alter in any substantive way our conclusions. (2) The owner remains involved in the firm. Allowing the owner to delegate and walk away does not change the analysis in any substantive way. It only changes the minimum level of hired managerial services from zero (see below) to a positive value.

of final output. The firm accumulates knowledge according to the technology $\dot{Z}_i = I_i$, where I_i is in-house investment in units of final output. To characterize this decision, we use the demand schedule for intermediate goods to write the firm's gross cash flow as

$$\Pi_i \equiv (p_i - 1)\, X_i = (p_i - 1) \left(\frac{\theta}{p_i}\right)^{\frac{1}{1-\theta}} M_i Z_i^{\alpha} Z^{1-\alpha} L_i^{\gamma} \Omega^{1-\gamma}. \tag{7}$$

The traditional approach to this class of models operates with $M_i \equiv 1$ and postulates that the firm chooses the time path of its product's price, $p_i\,(t)$, and its investment, $I_i\,(t)$, to maximize the value of the firm

$$V_i\,(0) = \int_0^{\infty} e^{-\int_0^t r(s)ds} \left[\Pi_i(t) - \phi Z_i - I_i\,(t)\right] dt, \tag{8}$$

subject to $\dot{Z}_i = I_i$, taking as given the path of average quality, Z. The characterization of the firm's decisions yields the return to quality innovation as

$$r = \frac{\partial \Pi_i}{\partial Z_i} - \phi. \tag{9}$$

One of the core ingredients of our analysis will be to be more specific about who makes the $p_i\,(t)$ and $I_i\,(t)$ decisions and what objectives he or she pursues.

3.3 Entry

To start operations a new firm must sink βZ units of final output. Because of the sunk setup cost, for the new firm is never profitable to supply an existing good in Bertrand competition with the incumbent monopolist, rather it introduces a new good that expands product variety. The new firm enters at the average quality level.[5]

Free entry requires $V_i = \beta Z$. Taking logs and time derivatives of the free-entry condition and of the valuation equation (8) yields the return to entry as

$$r = \frac{\Pi_i - \phi Z_i - I_i}{\beta Z} + \frac{\dot{Z}}{Z}, \tag{10}$$

where the first term on the right-hand side is the dividend/price ratio and the second term is capital gains.

[5]This simplifying assumption preserves symmetry of equilibrium at all times.

4 The Organization of the Firm

In this section, we describe the owner's delegation decision and the manager's incentive to exert effort.

4.1 No-Delegation Benchmark: Owner-Manager

In this subsection we develop our benchmark case of no delegation. Households finance the foundation of intermediate firms covering the entry cost. To make things concrete, we say that households assign the task of setting up and running the firm to a household member, to whom we refer to as the "owner," who solves the same problem as in the standard approach that abstracts from principal-agent considerations.

Specifically, the owner operates at the baseline level of management services, so that $M_i = \mu \in (0, 1)$. This setup yields the same expressions for the rates of return to quality and variety innovation that we obtain in the traditional case, see (9)-(10). The term that captures the specific role of management is the marginal gross profit, $\partial \Pi_i / \partial Z_i = \alpha \Pi_i / Z_i$, which once substituted into (9), after rearranging terms, yields the return to in-house quality innovation under no delegation as

$$r = \alpha \frac{\Pi_i}{Z_i} - \phi. \tag{11}$$

According to this formulation, the owner can perform his or her functions without leaving final production. Thus, one feature of the model is that the entrepreneurial input is distinct from labor in the sense that assigning one household member to setting up and running a firm does not remove that member from the labor supply. This assumption is essentially the one used in general equilibrium theory when we postulate that "firms" are set up and run by risk-neutral agents who maximize shareholder value. Such agents never show up explicitly in the model accounting precisely because they do not exert effort, do not earn compensation, do not consume, and are not removed from the labor force. We deviate from this characterization when we introduce delegation because we want our managers to exert effort, earn compensation, and consume.[6]

[6] Moreover, removing the owner-manager from the workforce does not change the important properties of the model with delegation since it simply adds the time endowment of the owner-manager to the total subtracted from the population size to determine employment in the final sector. That is, we would subtract $\int_0^N (m_i + 1)\, di$ instead of $\int_0^N m_i di$.

4.2 Delegation: Owner-Principal and Manager-Agent

We leave the dynamic investment decision in the hands of the owner and postulate that when he or she employs managers he or she delegates control solely over middle- and lower-level management functions. On the contracting side, since exerting effort is costly, the owner has to provide the right incentives through appropriate compensation. We restrict our attention to a simple contract in which the manager's compensation is proportional to the gross cash flow of the firm (revenues minus variables costs). We then compare the solution for the owner-manager with that for the owner-principal and determine if and when the owner wants to employ managers. Delegation allows the owner-principal to concentrate on higher-level strategic concerns (i.e., formulating the growth strategy of the firm) while managers that do not have such concerns concentrate on lower-level operational functions—setting the price, overseeing production and marketing, and so on—more productively than if they had them. One way to think about delegation in our setup, therefore, is that it achieves a more efficient division of labor within the firm.[7]

4.2.1 Decisions: Manager-Agent

The typical manager of firm i chooses his or her own effort, h_i, to maximize

$$U_i^{\text{manager-agent}}(0) = \int_0^{\infty} e^{-\int_0^t r(s)ds} \left[a_i \Pi_i - s(h_i) Z\right] dt, \tag{12}$$

where a_i denotes the contractually stipulated fraction of the firm's gross cash flow that constitutes his or her remuneration. Effort entails a utility cost, $s(h_i)$, where the function $s(h_i)$ is increasing and convex, with $s(0) = 0$, $\lim_{h_i \to 1} s(h_i) = +\infty$ and $\lim_{h_i \to 1^-} s'(h_i) = +\infty$. Since the problem has no dynamic constraint, it yields the well-known pricing rule of a constant markup over the marginal cost, $p_i = 1/\theta$, and a simple intratemporal first-order condition for managerial effort:

$$a_i \frac{\partial \Pi_i}{\partial M_i} \frac{\partial M_i}{\partial h_i} \frac{1}{Z} = s'(h_i). \tag{13}$$

Looking ahead, (13) will play a key role in the optimal contracting problem below, since the owner strategically takes managers' effort decisions as constraints in making his or her decisions about I_i and a_i.

[7]See Edmans and Gabaix (2016) for a discussion of why real-world boards of directors may prefer simple to sophisticated fully "optimal" incentive contracts, even if such contracts are designed to maximize shareholder value. Arrow (1984) makes a similar point in a survey article on principal-agent theory.

To streamline notation, it is useful to define now a new variable which will be used repeatedly throughout the analysis below:

$$\pi_i \equiv (p_i - 1)\left(\frac{\theta}{p_i}\right)^{\frac{1}{1-\theta}}\left(\frac{Z_i}{Z}\right)^{\alpha} L_i^{\gamma}\Omega^{1-\gamma} = \frac{(p_i - 1) X_i}{Z M_i} = \frac{\Pi_i}{Z M_i}. \tag{14}$$

Using (14) and $\frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial h_i}\frac{1}{Z} = \pi_i \frac{\partial M_i}{\partial h_i} = \pi_i f'(h_i)$, we write (13) in a more compact form as

$$a_i \pi_i f'(h_i) = s'(h_i), \tag{15}$$

which gives manager's effort implicitly as $h_i = \tilde{h}(a_i \pi_i)$.

The variable π_i captures the dependence of the managerial incentives to exert effort on the "baseline" profitability of the firm, i.e., profitability as driven by: (i) the Lerner index, the term $p_i - 1$; (ii) the scale of use of the product, which depends on the pricing decision, the term $(\theta/p_i)^{\frac{1}{1-\theta}}$, and the final producer's employment decision, the term L_i^{γ}; (iii) the firm's relative knowledge, the term $(Z_i/Z)^{\alpha}$.

For our purposes, the key property of this variable is that it abstracts from (or nets out) the managerial input, M_i. It thus provides information on how profitability depends on the traditional forces at play in the model, i.e., those identified by the previous literature that abstracts from the organizational aspects that we study here. One of the decisions of the owner is then the determination of the internal organization of the firm in response to baseline profitability. We now turn to this problem.

4.2.2 Decisions: Owner-Principal

The owner maximizes

$$V_i(0) = \int_0^{\infty} e^{-\int_0^t r(s)ds}\left[\Pi_i(t) - \phi Z_i(t) - I_i(t) - \int_0^{m_i(t)} a_{ij}(t)\,\Pi_i(t)\,dj\right] dt, \tag{16}$$

subject to the demand and technology constraints specified above and the behavior of the manager as summarized in (13). Specifically, the owner internalizes $h_{ij} = \tilde{h}\left(a_{ij}\pi_i\right)$. The owner's problem is separable in the individual compensation decision and the employment and investment decisions. It is useful to discuss these components of the model in a few steps.

Incentives and effort The owner's compensation decision is described by the first-order condition for the fraction of the gross cash flow rendered to the manager:

$$\forall j \in [0, m_i], a_{ij}: \quad \frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial h_{ij}}\frac{\partial \tilde{h}\left(a_{ij}\pi_i\right)}{\partial a_{ij}} = \Pi_i + a_{ij}\frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial h_{ij}}\frac{\partial \tilde{h}\left(a_{ij}\pi_i\right)}{\partial a_{ij}}. \tag{17}$$

Since the technology for managerial services (6) treats managers symmetrically and they have identical utility cost of effort, the compensation decision reduces to a symmetric structure where managers supply identical effort and are paid the same amount.

Using the definition of π_i in (14), and $\frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial h_i}\frac{1}{Z} = \frac{\Pi_i}{ZM_i}\frac{\partial M_i}{\partial h_i}$, so that $\frac{\partial \Pi_i}{\partial M_i} = \frac{\Pi_i}{M_i}$, we obtain $\frac{\Pi_i}{M_i}f'\left(h_i\right) = \Pi_i + a_i\frac{\Pi_i}{M_i}f'\left(h_i\right)\frac{\partial \tilde{h}\left(a_j\pi_i\right)}{\partial a_i}$, which in turn gives $\left(1-a_i\right)\frac{f'\left(h_i\right)}{M_i}\frac{\partial \tilde{h}\left(a_i\pi_i\right)}{\partial a_i} = 1$. It is then useful to express this last relation as

$$\left(1-a_i\right) f'\left(h_i\right) h_i \frac{\partial \tilde{h}\left(a_i\pi_i\right)}{\partial a_i}\frac{a_i}{h_i} = a_i M_i, \tag{18}$$

which describes what the owner is willing to pay to elicit effort, whereas the first-order condition of the manager describes the manager's response to such incentive.

Employment and investment Given the symmetry across managers, the dividend flow accruing to the owner becomes $\left(1-m_i a_i\right)\Pi_i - \phi Z_i - I_i$. The employment and investment decisions are then described by the following first-order conditions:

$$m_i: \quad a_i\Pi_i = \left(1-m_i a_i\right)\frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial m_i}; \tag{19}$$

$$I_i \text{ and } Z_i: \quad 1 = q_i \quad \text{and} \quad \left(1-m_i a_i\right)\left(\frac{\partial \Pi_i}{\partial Z_i} + \frac{\partial \Pi_i}{\partial M_i}\frac{\partial M_i}{\partial Z_i}\right) - \phi = r - \frac{\dot{q}_i}{q_i}. \tag{20}$$

Recalling that $\partial \Pi_i/\partial M_i = \Pi_i/M_i$, the owner's first-order condition with respect to m_i becomes $a_i\Pi_i = \frac{\left(1-m_i a_i\right)\Pi_i}{M_i}\frac{\partial M_i}{\partial m_i}$, which can be further simplified as

$$a_i = \frac{1-m_i a_i}{M_i}\frac{\partial M_i}{\partial m_i}. \tag{21}$$

The investment decision reduces to

$$r = \left(1-m_i a_i\right)\left(\frac{\partial \Pi_i}{\partial Z_i} + \frac{\Pi_i}{M_i}\frac{\partial M_i}{\partial Z_i}\right) - \phi, \tag{22}$$

which describes the firm's rate of return to in-house investment.

4.2.3 Incentive Contract and Delegation

To characterize the behavior of the firm, it is useful to separate the solution for the contract offered to managers and the resulting internal organization of the firm from the solution for the investment plan.

Incentive contract The owner's tradeoffs in setting compensation are described by (18) and (21). Specifically, (21) determines the mass of managers the owner wants to employ given what he or she pays each of them; (18) determines the contractual incentive needed to elicit the typical manager's effort that is optimal for the owner. We can solve the two equations for decision rules of the form $(a(\pi_i), m(\pi_i))$, so that the effort of the manager follows the decision rule $h_i = \tilde{h}(a(\pi_i)\pi_i)$.

To make progress on the existence and characterization of these rules, we specify managerial services as $M_i = \int_0^{m_i} h_{ij}^{\xi} dj + \mu = m_i h_i^{\xi} + \mu$, and the utility cost of effort as $s_i(h_i) = -\varepsilon \log(1 - h_i)$. Given these functional forms, (18) and (21) for the owner, and the manager's first-order condition for effort (15), give a system of three equations in three unknowns, (m_i, a_i, h_i), whose solution fully characterizes the optimal incentive contract:

$$\frac{(1-a_i)\,\xi h_i^{\xi}}{1-\xi+\frac{h_i}{1-h_i}} = a_i m_i h_i^{\xi} + \mu a_i; \tag{23}$$

$$(1-m_i a_i)\, h_i^{\xi} = a_i m_i h_i^{\xi} + \mu a_i; \tag{24}$$

$$h_i^{1-\xi}(1-h_i)^{-1} = \frac{\xi}{\varepsilon} a_i \pi_i. \tag{25}$$

Since we are interested in the comparative statics with respect to π_i, we manipulate the equations to produce two 2D-diagrams in the $(m_i a_i, a_i)$ and (a_i, h_i) space. Also, since after characterizing the contract with $\xi < 1$ we noticed that nothing of substance hinges on $\xi < 1$, here we present the version with $\xi = 1$ that allows for a more compact exposition.

The first diagram follows from using (25) to eliminate h_i, and isolating π_i to only one equation. After the required algebra, we obtain the two functions $m_i a_i = 1 - \frac{\varepsilon(1-a_i)}{a_i \pi_i - \varepsilon}$ and $m_i a_i = \frac{\mu a_i^2 - (1+2\mu) a_i + 1}{2-(2+\mu) a_i}$, depicted in the top panel of Figure 1.[8] As both functions

[8] The first function has a vertical asymptote at $a_i = \varepsilon / \pi_i$, starts negative, is increasing in a_i, becomes positive at $a_i = 2/(1 + \pi_i/\varepsilon)$ and takes value 1 at $a_i = 1$. The second function has a vertical asymptote at $a_i = 2/(2+\mu)$, starts at $+\infty$, is decreasing in a_i and becomes negative at a value of a_i in the interval

intersect the horizontal axis, a unique interior solution with $a_i > 0$ and $m_i a_i > 0$ exists if $\pi_i > \pi_D$, where π_D is the value of π_i at which the zeros of the two functions coincide. Setting $m_i a_i = 0$ in both equations and solving the first for a_i and substituting the result in the second, the value of π_D is the positive root of the quadratic equation $\left(\pi_i \frac{1}{\varepsilon} + 1\right)^2 - (1+2\mu)\, 2\left(\pi_i \frac{1}{\varepsilon} + 1\right) + \mu 4 = 0$, which is equal to

$$\pi_D = \varepsilon \left(2\mu + \sqrt{(1+2\mu)^2 - \mu 4}\right). \tag{26}$$

The interior solution has the property that as π_i rises, the first curve shifts up and generates a movement along the second curve. Accordingly, a_i falls and $m_i a_i$ rises; see Figure 1. Note that according to this analysis, a_i has a lower bound: as π_i goes to infinity, a_i converges to $a_i = 2/\left(2+\mu\right)$. Similarly, $m_i a_i$ has an upper bound: as π_i goes to infinity, it converges to 1. The associated behavior of employment is described by

$$m\left(\pi_i\right) = \frac{1}{a\left(\pi_i\right)} \left[\frac{\left(1 - a\left(\pi_i\right)\right)\left(1 - \mu a\left(\pi_i\right)\right) - \mu a\left(\pi_i\right)}{2\left(1 - a\left(\pi_i\right)\right) - \mu a\left(\pi_i\right)}\right]. \tag{27}$$

Since the right-hand side is decreasing in $a\left(\pi_i\right)$ and $a'\left(\pi_i\right) < 0$, we have $m'\left(\pi_i\right) > 0$. Moreover, since $\lim_{\pi\to\infty} a\left(\pi\right) = 0$, we have $\lim_{\pi\to\infty} m\left(\pi\right) < \infty$.

The second diagram follows from using (23) to eliminate m_i, and isolating π_i to only one equation. Again, after the required algebra, we obtain the two functions $a_i = \frac{2-3h_i}{\mu+2(1-h_i)}$ and $a_i = \frac{\varepsilon}{\pi_i(1-h_i)}$, depicted in the mid panel of Figure 1.[9] A unique interior solution with $a_i > 0$ and $h_i > 0$ always exists whenever $\varepsilon/\pi_i < 2/\left(2+\mu\right)$. As Figure 1 illustrates, the reason why delegation does not occur when $\pi_i \leq \pi_D$ is that given the positive values of a_i and h_i the owner wants to set $m_i = 0$. That is, the optimal mass of managers paid optimally and who exert optimal effort is zero. The solution has the property that as π_i rises, the second curve shifts downward and generates a movement along the first curve. Accordingly, a_i falls while h_i rises.

The analysis of this section can be summarized in the following proposition.

Proposition 1 *The manager's effort is a function $h\left(\pi_i\right)$ defined over the domain $\pi_i \in \left[\pi_D, \infty\right)$ with the properties $h\left(\pi_D\right) > 0$, $h'\left(\pi_i\right) > 0$, $\lim_{\pi_i\to\infty} h\left(\pi_i\right) = 1$. The compensation of the*

$\left(2/\left(2+\mu\right), 1\right)$.

[9] The first function starts at $a_i = 2/\left(2+\mu\right)$, is decreasing in a_i and cuts the horizontal axis at $h_i = 2/3$. The second function starts at the positive value $a_i = \varepsilon/\pi_i$, is increasing in a_i and has a vertical asymptote at $h_i = 1$.

typical manager is a function $a(\pi_i)$ defined over the domain $\pi_i \in [\pi_D, \infty)$ with the properties $a(\pi_D) > 0$, $a'(\pi_i) < 0$, $\lim_{\pi_i \to \infty} a(\pi_i) > 0$. Employment of managers is a function defined over the same domain with the properties $m(\pi_D) > 0$, $m'(\pi_i) > 0$, $\lim_{\pi_i \to \infty} m(\pi_i) < \infty$. The total compensation to managers is a function defined over the same domain with the properties $a(\pi_D)m(\pi_D) > 0$, $d(a(\pi_i)m(\pi_i))/d\pi_i > 0$, $\lim_{\pi_i \to \infty}(a(\pi_i)m(\pi_i)) = 1$.

Incentive and innovation The owner internalizes the dependence of the manager's effort on the firm's cash flow, Π_i, and its dependence on the firm's knowledge stock, Z_i, thus exploiting the positive feedback-loop profitability→effort→profitability. To see this more formally, consider the first-order condition for the manager's effort, $a_i \frac{\partial \Pi_i}{\partial M_i} \frac{\partial M_i}{\partial h_i} \frac{1}{Z} = s'(h_i)$ and differentiate it to get $a_i \frac{\partial^2 \Pi_i}{\partial M_i \partial Z_i} \frac{\partial M_i}{\partial h_i} \frac{1}{Z} dZ_i = s''(h_i)\, dh_i$, where $\frac{\partial^2 \Pi_i}{\partial M_i \partial Z_i} = \frac{1}{M_i} \frac{\partial \Pi_i}{\partial Z_i}$ and $\frac{\partial \Pi_i}{\partial M_i} = \frac{\Pi_i}{M_i}$. Using the manager's decision to eliminate a_i, we obtain an expression describing how the manager's effort depends on the firm's stock of knowledge:

$$\frac{dh_i}{dZ_i} = \frac{s'(h_i)}{\frac{\Pi_i}{M_i} \frac{\partial M_i}{\partial h_i} \frac{1}{Z}} \frac{1}{M_i} \frac{\partial \Pi_i}{\partial Z_i} \frac{\partial M_i}{\partial h_i} \frac{1}{Z} \frac{1}{s''(h_i)} = \frac{1}{\Pi_i} \frac{\partial \Pi_i}{\partial Z_i} \frac{s'(h_i)}{s''(h_i)} \geq 0. \tag{28}$$

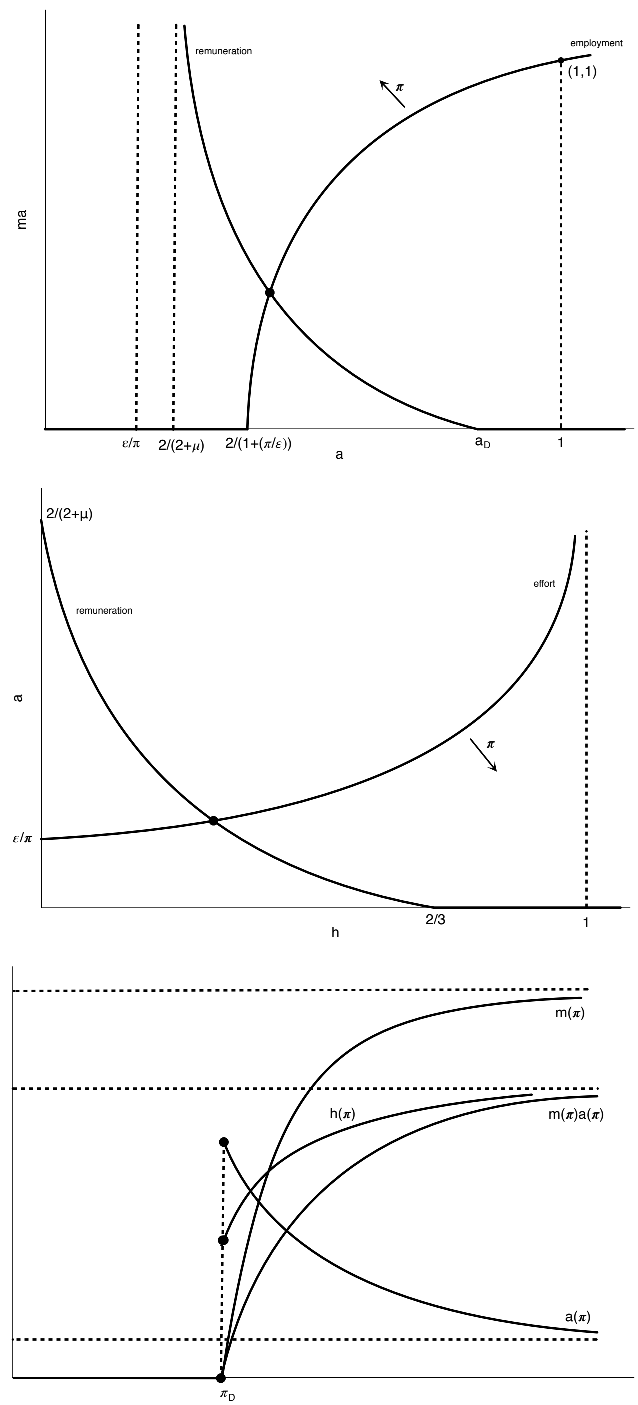
Next, to calculate the rate of return to in-house innovation under delegation, reduce the first-order conditions for investment to $r = (1 - m_i a_i) \left[1 + \frac{\partial M_i}{\partial h_i} \frac{h_i}{M_i} \frac{s'(h_i)}{h_i s''(h_i)}\right] \frac{\partial \Pi_i}{\partial Z_i} - \phi$, use the two elasticities from the owner's decisions about (m_i, a_i), and obtain

$$r = (1 - m_i a_i) \left[1 + \frac{m_i a_i}{1 - m_i a_i} \frac{h_i s''(h_i)}{s'(h_i)} \frac{s'(h_i)}{h_i s''(h_i)}\right] \frac{\partial \Pi_i}{\partial Z_i} - \phi = \frac{\partial \Pi_i}{\partial Z_i} - \phi, \tag{29}$$

where $\frac{\partial \Pi_i}{\partial Z_i} = \alpha \frac{\Pi_i}{Z_i} = \alpha \pi_i M_i \frac{Z}{Z_i}$. This result implies that the return to in-house quality innovation under delegation has a form similar to that obtained under non delegation. In both cases the return is driven by the "raw" marginal gross profit of the unit of knowledge, $\partial \Pi_i / \partial Z_i$. The difference is the value taken by the managerial services term, M_i. Remarkably, accounting for the strategic term dh_i/dZ_i offsets the term $(1 - m_i a_i)$ that multiplies the raw profitability of the firm in the objective function of the owner.

Delegation decision: dynamic interpretation To compare equilibria and characterize the delegation decision we determine which organizational structure delivers the higher return to investment (and, under no arbitrage, firm ownership). The comparison requires us to study $r_i^{ND} = \alpha \mu \pi_i \frac{Z}{Z_i} - \phi$ versus $r_i^{D} = \alpha M(\pi_i) \pi_i \frac{Z}{Z_i} - \phi$. The trade-off is clear: by hiring managers the owner can obtain a better return but must compensate them to

Figure 1: The Incentive Contract



incentivize individual effort. For given π_i, the comparison reduces to

$$r_i^{ND} \gtreqless r_i^D \quad \text{for} \quad \mu \gtreqless M(\pi_i) = \mu + m(\pi_i)\, h(\pi_i)\,. \tag{30}$$

To summarize, the main results from the analysis above are: (i) there is a threshold π_D such that for $\pi_i \leq \pi_D$ there is no interior solution for the delegation contract and the managerial input is $M(\pi_i) = \mu$; (ii) for $\pi_i > \pi_D$ there is an interior solution for the delegation contract and the managerial input $M(\pi_i) = \mu + m(\pi_i)\, h(\pi_i)$ is increasing in π_i. The intuition for this mechanism is that hiring managers is worthwhile when they work sufficiently hard. As their incentive to exert effort is scaled by π_i, they do so only when they work for a sufficiently large operation.

5 Equilibrium

We now turn to the general equilibrium of the model. Since the equilibrium is symmetric, henceforth, we will drop the i subscript so that, for example, $X \equiv X_i$ denotes both firm-level and average intermediate good production.[10] We begin by discussing the structure of the equilibrium, then turn to examine the properties of the dynamical system.

5.1 Firm's Profitability and Market Size

In the model, market size, defined as total sales of intermediate good producers, is a key determinant of the dynamics of market structure and thereby of the incentives to delegate managerial functions and undertake innovative investments. The model yields a clear mapping between population size, total output, and a measure of firm's profitability, where the latter will turn out to be the state variable of the dynamical system describing equilibrium dynamics.

Output and firm's profitability Intermediate producers set the unit price as a constant markup over the marginal cost, $p = 1/\theta$, and in equilibrium they collect $pX = \theta Y/N$ as sales, which are proportional to output per firm, Y/N. Evaluating the production

[10]Two conditions ensure symmetry of equilibrium: (i) firm-specific rate of return to quality innovation is decreasing in its own quality; (ii) entrant firms enter at the average level of quality. The first implies that if one holds constant the mass of firms and starts the model from an asymmetric distribution of firm sizes, then the model converges to a symmetric distribution. The second requirement simply ensures that entrants do not perturb such symmetric distribution. See Peretto (1998) and Peretto (1999) for more discussion.

function (4) at the symmetric equilibrium and using $X = \theta Y / pN$ yields total output as

$$Y = \theta^{\frac{2\theta}{1-\theta}} NM(\pi) Z \left[\frac{\Lambda}{N} - m(\pi)\right]^{\gamma} \Omega^{1-\gamma}, \tag{31}$$

where we make explicit that managerial services, $M(\pi)$, and the mass of managers per firm, $m(\pi)$, are (implicit) functions of π.

In symmetric equilibrium, π reduces to

$$\pi = \left(\frac{1}{\theta} - 1\right) \frac{X}{Z} \frac{1}{M(\pi)} = \left(\frac{1}{\theta} - 1\right) \theta^{\frac{2}{1-\theta}} \left[\frac{\Lambda}{N} - m(\pi)\right]^{\gamma} \Omega^{1-\gamma}, \tag{32}$$

In this expression, three objects relate to market structure and market size: (i) the Lerner index, $p - 1 = 1/\theta - 1$, reflecting market power in the product market; (ii) the price decision, which determines the volume of sales, $(\theta/p)^{\frac{1}{1-\theta}} = \theta^{\frac{2}{1-\theta}}$; (iii) downstream employment per intermediate good, Λ/N.

Note that (31) and (32) encompass the link between population, Λ, output, Y, and firm's profitability, π, which is at the heart of endogenous growth models. Everything else equal, an increase in population raises total output of the final good producers, which in turn increases demand for intermediate goods and sales. In this sense, intermediate producers face a larger market size, which in turn raises the rates of return to knowledge accumulation and entry.

Equalization of rates of return Looking ahead, it is useful to write the rates of return as function of the state variable π as:

$$r = \alpha M(\pi) \pi - \phi; \tag{33}$$

$$r = \frac{[1 - m(\pi) a(\pi)] M(\pi) \pi - \phi - z}{\beta} + z. \tag{34}$$

Note first that the rate of return to knowledge accumulation (33) rises more than linearly with π. This positive relationship captures the property that managerial services, $M(\pi)$, are increasing in firm's profitability. As discussed above, the incentive contract mandates that both managers' employment and effort per manager increase with π.

Note next that, everything else equal, the relationship between the rate of return to entry (34) and π is potentially U-shaped. This non-monotonicity is the key model's property that raises the possibility of multiple steady states, which we explain further below.

It comes from the fact that employing managers raises sales, but it reduces distributed dividends as a share of the gross cash flows is paid to managers as compensation. In equilibrium, the two rates of return (33)-(34) must be the same for the households to willingly hold equity shares of both entrants and incumbents.[11]

5.2 The Timing of Delegation and Innovation

We now turn to the conditions under which the economy activates innovation before delegation, and the opposite case in which delegation precedes innovation. The equilibrium features a threshold of profitability π_Z such that for $\pi \leq \pi_Z$ we have $z = 0$, i.e., no investment in knowledge is undertaken by firms. Depending on the ordering of the delegation and investment thresholds π_D and π_Z, we have two cases.

Case #1: Innovation before delegation If parameters are such that $\pi_D > \pi_Z$, the delegation decision occurs *after* firms activate in-house innovation. In this case, the growth rate of knowledge is

$$z(\pi) = \begin{cases} 0 & \phi/\mu \leq \pi \leq \pi_Z \\ \frac{\beta\alpha-1}{\beta-1}\mu\pi - \phi & \pi_Z < \pi \leq \pi_D \\ \frac{\beta\alpha-1+m(\pi)a(\pi)}{\beta-1} M(\pi)\pi - \phi & \pi > \pi_D \end{cases}, \tag{35}$$

where $\pi_Z = \frac{\beta-1}{\beta\alpha-1}\frac{\phi}{\mu} < \pi_D$. Given its construction, we refer to this equation as the *no-arbitrage locus*. We assume $\beta\alpha > 1$ to ensure that the equilibrium with no delegation eventually delivers positive firm growth (i.e., the threshold π_Z is positive).

Associated to the piece-wise function describing in-house investment is the function describing the equilibrium rate of return to investment:

$$r(\pi) = \begin{cases} (\mu\pi - \phi)/\beta & \phi/\mu \leq \pi \leq \pi_Z \\ \alpha\mu\pi - \phi & \pi_Z < \pi \leq \pi_D \\ \alpha M(\pi)\pi - \phi & \pi > \pi_D \end{cases}. \tag{36}$$

In the first branch, firms do not invest in-house and the equilibrium rate of return to investment is governed only by the entry process (i.e., (34) evaluated at $z = a = 0$ and $M(\pi) = \mu$). After the economy crosses the threshold π_Z and firms invest in-house, the rate of return to investment is governed by the interaction of the entry process with the

[11] For $\beta > 1$ the equilibrium in the asset market is stable in the Nash sense.

firms' in-house investment decisions. The latter differ according to whether firms have already adopted or not the organizational structure with delegation of executive functions to specialized managers.

Case #2: Innovation after delegation We now turn to the case $\pi_D < \pi_Z$, which produces a path where the delegation decision occurs before firms activate in-house innovation. The activation threshold for in-house innovation is

$$\pi_Z \equiv \arg\text{solve}\left\{\frac{\beta\alpha - 1 + m(\pi)\,a(\pi)}{\beta - 1}M(\pi)\,\pi = \phi\right\}. \tag{37}$$

This solution is unique because the left-hand side of the argument of the arg solve function is increasing. The firm-level rate of quality growth is

$$z(\pi) = \begin{cases} 0 & \phi/\mu \leq \pi \leq \pi_D \\ 0 & \pi_D < \pi \leq \pi_Z \\ \frac{\beta\alpha - 1 + m(\pi)a(\pi)}{\beta - 1}M(\pi)\,\pi - \phi & \pi > \pi_Z \end{cases}. \tag{38}$$

The associated equilibrium rate of return to investment is

$$r(\pi) = \begin{cases} (\mu\pi - \phi)/\beta & \phi/\mu \leq \pi \leq \pi_D \\ (M(\pi)\,\pi - \phi)/\beta & \pi_D < \pi \leq \pi_Z \\ \alpha M(\pi)\,\pi - \phi & \pi > \pi_Z \end{cases}. \tag{39}$$

In the first two branches, firms do not invest in-house and the equilibrium rate of return to investment is governed only by the entry process. After the economy crosses the threshold π_Z and firms invest in-house, the rate of return to investment is governed by the interaction of the entry process with the firms' in-house investment decisions that obtain when firms have already adopted the organizational structure with delegation of executive functions to specialized managers.

5.3 The Two Equations Governing Equilibrium Dynamics

We now turn to examine the equilibrium dynamics of the model. While the general equilibrium of the model can be reduced to a dynamical system of two ODEs, the transition dynamics generated by such a system is rich enough to encompass qualitatively different types of equilibrium paths: the first type describes an economy in which firms remain owner-operated along all the transition, eventually converging to a low-growth steady state without a managerial class; the second type describes an economy that experiences the endogenous transition to delegation, converging to a high-growth steady state in which firms are operated with the help of salaried managers. The model also admits a third type of transition dynamics in which the economy moves away from an unstable mid-growth steady state.

The 2x2 dynamical system To derive the dynamical system, we define a stationary variable, i.e., quality-adjusted per capita consumption, $c \equiv C/Z\Lambda$. Log-differentiating c with respect to time, and using the expressions for the equilibrium rates of return and quality growth, we obtain a modified Euler equation linking the growth rate of c to the interest rate, $r(\pi)$, and the rate of quality growth, $z(\pi)$, which are both implicit functions of the state variable π:

$$\frac{\dot{c}}{c} = r\left(\pi\right) - z\left(\pi\right) - \rho. \tag{40}$$

Next, we use the fact that in equilibrium household wealth is $A = \beta NZ$ to write the household budget constraint as $\frac{\dot{A}}{A} = r + \frac{(1-\theta)Y}{A} - \frac{C}{Z\Lambda}\frac{Z\Lambda}{A}$, so that $n \equiv \dot{N}/N = r - z + \frac{\pi M(\pi)}{\beta\theta} - \frac{c}{\beta}\frac{\Lambda}{N}$. Using equation (32) to eliminate Λ/N, we obtain

$$n\left(\pi, c\right) = r\left(\pi\right) - z\left(\pi\right) + \frac{\pi M\left(\pi\right)}{\beta\theta} - \frac{c}{\beta}\left(\pi^{\frac{1}{\gamma}}\left[\left(1-\theta\right)\theta^{\frac{1+\theta}{1-\theta}}\Omega^{1-\gamma}\right]^{-\frac{1}{\gamma}} + m\left(\pi\right)\right). \tag{41}$$

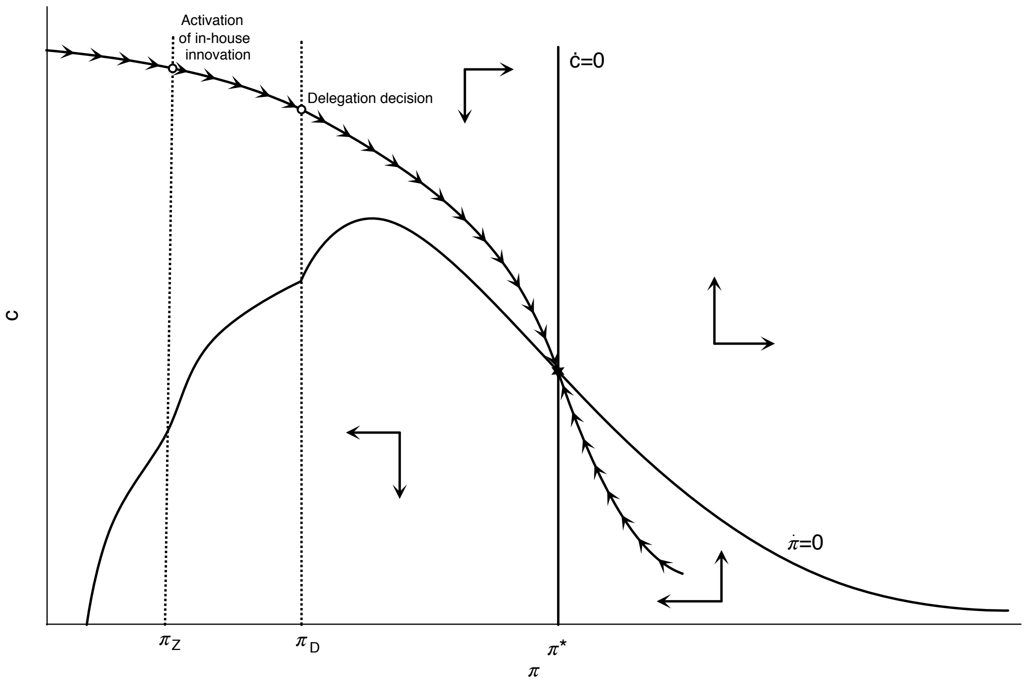
Finally, we differentiate (32) with respect to time to obtain

$$\dot{\pi} = \gamma\left[\lambda - n\left(\pi, c\right)\right]\frac{\pi^{\frac{1}{\gamma}}\left[\left(1-\theta\right)\theta^{\frac{1+\theta}{1-\theta}}\Omega^{1-\gamma}\right]^{-\frac{1}{\gamma}} + m\left(\pi\right)}{\pi^{\frac{1}{\gamma}-1}\left[\left(1-\theta\right)\theta^{\frac{1+\theta}{1-\theta}}\Omega^{1-\gamma}\right]^{-\frac{1}{\gamma}} + \gamma m'\left(\pi\right)}. \tag{42}$$

We thus have reduced the equilibrium dynamics of our model to a tractable system in (π, c) space.[12] Recall that the functions $z\left(\pi\right)$ and $r\left(\pi\right)$ have three branches delineated by the two profitability thresholds, π_Z, and π_D, that trigger the activation of in-house investment and the adoption of the organizational structure with managers. Hence, the

[12] The domain of π must take into account the non-negativity constraint on flow profit, i.e., when profit is too low, firms shut down R&D and for $\left[1 - m\left(\pi\right)a\left(\pi\right)\right]M\left(\pi\right)\pi - \phi \leq 0$, they exit. Moreover, as π becomes smaller and smaller, not only firms shut down R&D, but their owners do not delegate and run things themselves. This identifies the region $\pi \leq \pi^{exit} \equiv \phi/\mu$. The economy cannot be inside this region because there firms leave and N changes discretely moving the economy back at exactly the boundary $\pi = \pi^{exit}$ of the region.

Figure 2: An Illustration of Equilibrium Dynamics with a Unique Steady State



Notes: The figure illustrates the phase diagram for the case in which delegation occurs after the activation of in-house investment, i.e., $\pi_D > \pi_Z$, and the economy converges towards a unique steady state.

dynamics play out in three regions characterized by the different behavior of firms.

Phase diagram with unique steady state As an illustrative example, Figure 2 shows the phase diagram of the dynamical system (40)-(42) for the case in which delegation comes after the activation of innovative investments, and the economy converges to a unique steady state. The trajectories above (below) the $\dot{\pi} = 0$ schedule correspond to an increasing (decreasing) π. The $\dot{c} = 0$ schedule is given by a vertical line at π^*. It can be shown that the unique steady state is always a saddle. In this sense, the dynamics is qualitatively similar to those prevailing in a version of the model without endogenous delegation: given an initial condition for $\pi_0 \in (0, \infty)$, there is a unique equilibrium path that converges monotonically to the steady state. Any trajectory other than the saddle path violates the household's Euler equation or the transversality condition.

As we discuss further below, the model admits the possibility of multiple steady states. In this case, the characterization of the equilibrium is more involved since there are a number of cases to be considered with rather different qualitative properties, an issue to which we turn in the next subsection.

5.4 The Model Admits Multiple Steady States

Depending on parameter values, the model features either *one* or *three* steady states. We will examine the mechanism and the implications of such multiplicity in Sections 6 and 7. Here, instead, we discuss basic model properties, shared by all steady states, which depict an economy growing at a constant exponential rate. Along such an equilibrium path, (i) the mass of firms grows at the same rate of population growth, so that the ratio of the mass of firms to population is constant, (ii) per capita income and consumption grow at the same rate of knowledge, z, and (iii) the growth rate of knowledge with managers is higher than that without managers.

The 2x2 steady-state system Using equations (40)-(42), we obtain the $\dot{c} = 0$ and $\dot{\pi} = 0$ schedules:

$$\dot{c} = 0 : \quad r(\pi) - z(\pi) = \rho; \tag{43}$$

$$\dot{\pi} = 0 : \quad n(\pi, c) = \lambda. \tag{44}$$

To visualize the existence and the possibility of multiple steady states, it is useful to substitute the rate of return to in-house investment (33) into (43) to obtain an equilibrium relationship between knowledge growth, z, and π, which given its construction we refer to as *general equilibrium locus*:

$$z(\pi) = \begin{cases} 0 & \phi/\mu \le \pi \le \bar{\pi} \\ \alpha\mu\pi - \phi - \rho & \bar{\pi} < \pi \le \pi_D \\ \alpha M(\pi)\pi - \phi - \rho & \pi > \pi_D \end{cases}, \tag{45}$$

where $\bar{\pi} \equiv (\rho + \phi)/\mu\alpha$.

The steady state is then determined as the intersection in the (π, z) space of the no-arbitrage locus (35) with the general equilibrium locus (45):

$$\pi^* = \arg\text{solve}\left\{\frac{1 - \alpha - m(\pi)\,a(\pi)}{\beta - 1} M(\pi)\,\pi = \rho\right\}. \tag{46}$$

This expression says that an intersection always exists: the left-hand side of the argument of the arg solve function starts at zero and has a positive slope for values of π that are close to zero. Notably, for $\pi \leq \pi_D$, i.e., in the region of the state space in which the owner runs the firm, $m(\pi) = 0$, and $M(\pi) = \mu$, so that the term reduces to $\frac{1-\alpha}{\beta-1}\mu\pi$, which increases linearly with π. While a steady state always exists, it need not be unique. More than one intersection is possible as the term on the left-hand side of the argument of the arg solve function can be U-shaped for $\pi > \pi_D$, as opposed to monotonically increasing for all π. We have a unique solution $\pi^* > \pi_D$ if and only if at π_D the left-hand side of the argument of the arg solve function in (46) is below ρ,

$$\frac{1-\alpha}{\beta-1}\mu\pi_D < \rho. \tag{47}$$

The jump variable c is decreasing everywhere in the two regions $\pi < \pi_D$, so that the steady state can exist only in the region $\pi > \pi_D$. If condition (47) fails, the steady state occurs to the left of π_D, either in the region $\pi_Z < \pi \leq \pi_D$ with $z > 0$ or in the region $\phi \leq \pi \leq \pi_Z$ with $z = 0$.

Growth with and without managers The steady-state growth rate of knowledge with delegation is

$$z^* = \frac{\beta\alpha - 1 + m(\pi^*)\, a(\pi^*)}{1-\alpha - m(\pi^*)\, a(\pi^*)}\rho - \phi. \tag{48}$$

It is surely larger than the rate that would obtain under no delegation since

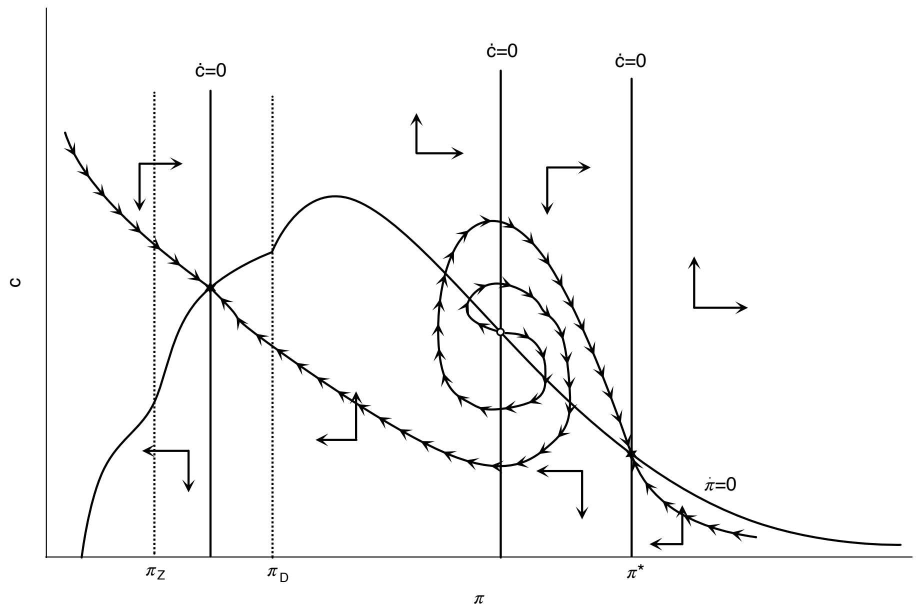
$$\frac{\beta\alpha - 1 + m(\pi^*)\, a(\pi^*)}{1-\alpha - m(\pi^*)\, a(\pi^*)}\rho - \phi > \frac{\beta\alpha - 1}{1-\alpha}\rho - \phi. \tag{49}$$

In other words, the transition to delegation delivers faster growth.

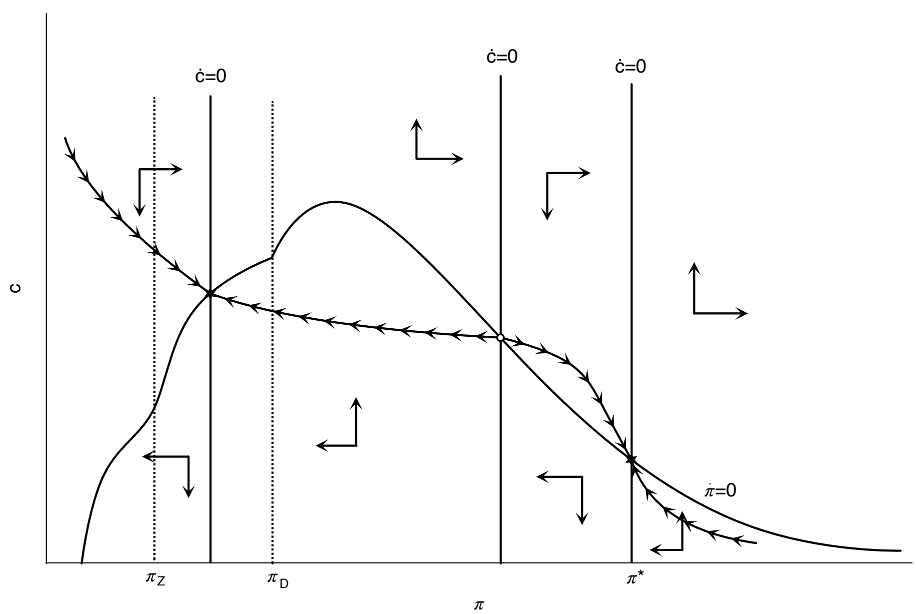
6 Management as an Amplification Mechanism

As mentioned above, the model possibly has three steady states of which the two outer ones are saddle-path stable and the inner one is unstable. Using standard terminology in the theory of dynamical systems, the unstable steady state can be either a "focus" or a "source," see Figure 3 for an illustration of these two configurations. Such multiplicity is the extreme manifestation of the complementarity between management and innovative investment at work in the model. If such complementarity is sufficiently strong, multiple equilibria arise; however, even in the absence of multiple equilibria, the complementarity remains as an *amplification* mechanism.

Figure 3: An Illustration of Equilibrium Dynamics with Multiple Steady States



(a) Focus



(b) Source

Notes: The figures show two possible configurations of the dynamical system (40)-(42) in which the unstable steady state is either a "focus" (top panel) or a "source" (bottom panel).

6.1 Complementarity between Management and Innovation

The model's features that raise the possibility of multiple steady states is the interaction between market structure and the extensive margin of managerial employment. There exists a positive feedback loop such that the incentives to delegate managerial functions and undertake innovative investments reinforce each other resulting in an "escalation effect" that produces a convex relationship between firm's profitability and investment. This convexity, in turn, yields that for a thick set of parameter values there can be more than one, but at most three steady states.

When firms employ more managers, they invest more, which leads to a higher stock market valuation. At the same time, more managers subtract resources from dividend distribution. If the negative effect on distributed dividends dominates that on market valuation, in free-entry equilibrium there is less entry. This in turn implies that firm size is larger, which reinforces the incentives to invest. Following the same logic, a smaller number of managers per firm reduces incentive to invest, which lowers stock market valuation, even though it raises dividends. If the latter effect prevails, there is more entry and smaller firm size, which further reduces the incentives to invest.

To organize the discussion, it is useful to reproduce here the nonlinear equation that determines the steady-state value of π^*, i.e., $(1-\alpha-m(\pi^*)\,a(\pi^*))\,M(\pi^*)\,\pi^*/\,(\beta-1)=\rho$, keeping in mind that there is a positive relationship between π and the growth rate of knowledge, z. The possibility of multiple roots comes from the interplay of two offsetting mechanisms. The first mechanism works through the term $M(\pi^*)$, which is increasing in π^*, and captures the positive relationship between managerial services and profitability: both the number of managers, $m(\pi^*)$, and the effort per manager, $h(\pi^*)$, are increasing in π^*. The second mechanism works through the term $-m(\pi^*)a(\pi^*)$, which is decreasing in π^*, and captures the fact that managers are compensated and so their remuneration is subtracted from gross cash flows. Which of these two effects dominates depends of course on parameter values, and more subtly, on the shape of the functions that map π^* into $m(\pi^*)$ and $a(\pi^*)$.

Note that in the version of the model with a fixed number of managers, multiple steady states are not possible. To see this, fix the mass of managers to $m(\pi^*)=\bar{m}$, so that we shut down the extensive margin of management altogether. In this case, there are no longer offsetting effects at work: $a(\pi^*)$ is decreasing in π^*, and the left-hand side of the equation reads $(1-\alpha-\bar{m}a(\pi))\,M(\pi^*)\,\pi/\,(\beta-1)$, which is monotonically increasing in π. Since it starts below ρ, a unique solution is guaranteed to exist. In a nutshell, multiple equilibria live and die with *endogenous* managerial employment.

Sunk entry costs play a critical role as well. As the entry cost parameter, β, becomes arbitrarily large, multiple equilibria vanish. In other words, everything else equal, it is always possible to find a high enough value for the entry cost that rules out multiplicity. This suggests that it is the interaction between endogenous firm entry and endogenous managerial employment that generates multiple equilibria.

6.2 Amplification without Multiplicity

The mechanism that generates complementarity between management and investment can substantially amplify the effects of policy even without multiplicity. To see this formally, let us consider the case in which the steady state is unique and the economy is in the neighborhood of such steady state so that we can safely rely on local analysis. Around the steady state implied by (45)-(46), a simple expression describes how changes in the state variable π map into changes in the growth rate of quality, $z(\pi)$:

$$\begin{aligned} \frac{dz(\pi^*)}{d\pi^*} &= \underbrace{\alpha\left\{\left[m'(\pi^*)f\left(h(\pi^*)\right)+m(\pi^*)f'\left(h(\pi^*)\right)h'(\pi^*)\right]\pi^*+m(\pi^*)f(h(\pi^*))\right\}}_{\text{amplification from management}} \\ &+ \alpha\mu \geq 0. \end{aligned} \tag{50}$$

From (50) it is evident that management act as an amplification mechanism of changes in π. The first term on the right-hand side is always positive, since both the mass of managers per firm, $m(\pi)$, and effort per manager, $h(\pi)$, are increasing in π, and the second term $\alpha\mu$ is the value of $dz(\pi)/d\pi$ that would prevail in a steady state without managers.

Any government policy that alters the steady state value of π^* has a larger effect on the long-run growth rate in the economy with managers than in an alternative economy without managers. Importantly, the type of policies that can affect π^* is large and include business income taxation and distortions affecting firms' entry decisions. For example, changes in the parameters governing sunk entry costs, β, and fixed operating costs, ϕ, lead to different steady-state values of π^* and thereby $z(\pi^*)$. Thus, to the extent that the regulatory environment affects the cost of setting up and running a business, as captured by β and ϕ, the presence of management can substantially amplify the impact of entry regulations and subsidies. Further, business taxation in the form of distortionary taxes on dividend, capital gains, and profit taxes can have a potentially large impact on π^*, too, as they operate through changes in the rate of return to investment and entry (Ferraro, Ghazi and Peretto, 2020; Peretto, 2003, 2007). A similar logic applies to business tax credits and/or expensibility of R&D expenditures as well. Overall, management plays an important role in the model above and beyond multiple equilibria.

7 Growth Traps

As mentioned in the introduction, when multiple equilibria occur the model provides a novel rationale for the theory of "convergence clubs" according to which countries with similar fundamentals converge to different steady states, generating bimodal or multimodal distributions in relative per capita income levels (see Quah, 1996*a*,*b*, 1997). An intriguing feature of these dynamics is the existence of a "growth trap," defined as a self-reinforcing steady state of low growth and no delegation. The incentives to delegate managerial functions and to undertake innovation reinforce each other in a feedback loop: the higher the managerial services, the higher the rate of return to innovative investment; in turn, the higher the investment, and so the level of knowledge, the higher the effect of managerial services on product quality and sales. As discussed, such feedback loop generates a strong complementarity between management and innovation.

Figure 3 shows two alternative phase diagrams describing equilibrium dynamics consistent with the dynamical system (40) and (42) when the economy has three steady states. Recall that the two outer steady states are saddle-path stable, whereas the mid one is unstable, with either spiraling or monotone trajectories moving away from it. In both cases, the model exhibits a growth trap, that is, there is a thick set of initial conditions on the state variable such that the economy converges to the steady state with no delegation and low growth. In this sense, the economy can get "stuck" in an equilibrium in which firms are run by owners, and because of that they do little or no innovation.

7.1 Indeterminacy and Self-Fulfilling Expectations

In Figure 4(a) the unstable steady state is a focus and trajectories spiral away from it. There is thus a region of overlapping trajectories resulting in indeterminacy: for given initial condition on the state variable, there are infinitely many choices of the initial value of the jumping variable that put the economy on a path that satisfies the equilibrium conditions. In this case whether the economy converges to the high-growth or the low-growth steady state depends on which of those equilibrium trajectories is selected. Equilibrium selection in turn depends on how agents coordinate their initial expectations on the future path, such expectations will be self-fulfilling given the perfect foresight nature of the model. In a nutshell, the initial condition for π, or history, is not sufficient to pin down the time path of the economy; the latter is a function of agents' expectations as well (see Matsuyama, 1991; Gali, 1995). Note that the range of initial conditions for which multiple equilibrium paths exist depends on the extent of the overlap of the manifolds associated with the two stable steady states.

7.2 History Dependence and Hysteresis

In Figure 4(b) the unstable steady state is a source and trajectories move away from it monotonically. There is thus a unique equilibrium path for given initial condition on the state variable. In this case, indeterminacy is ruled out and the model becomes a threshold model, in which history determines the future path of the economy. For example, consider two alternative economies that only differ by their initial conditions on π_0. Suppose further that one economy has initial condition π_0 just on the right of the middle steady state π_M^*, whereas the other has initial condition just on the left of π_M^*. This is a case of extreme magnification of small initial differences as the former economy converges to the high-growth steady state, while the latter converges to the low-growth steady state. This scenario describes the possibility of "history dependence" or "hysteresis," in which the initial condition has permanent effects on the long-behavior of the economy in terms of both levels and growth rates of real per capita income.

8 Income Distribution Dynamics

In the model, the functional distribution of income is intimately linked to the internal organization of the firm. The managerial delegation decision determines the level of pay of production workers and managers, as well as the employment share of managers. In the United States, the relative pay of managers and their share of total employment have been steadily increasing over time (see Acemoglu and Newman, 2002, and Appendix B for more recent evidence).

An appealing feature of the model is that all variables regarding employment shares and the functional income distribution only depend on π, a model object that encompasses market forces driving firm's profitability. In this precise sense, π is a "sufficient statistic," i.e., given a time path for π, one can back out the dynamics of managerial income and employment shares independently of other general equilibrium objects. While this property is appealing in general, it is even more so given the possibility of multiple steady states and equilibrium paths.

For future reference, it is useful to recall that total managerial income is the sum of the managers' shares of gross cash flows, i.e., $\int_0^N m_i a_i \Pi_i di = Nma\Pi$, where Nm is the total mass of managers, and $a\Pi$ is the compensation of a typical manager.

8.1 The Marginal Product of Managerial Effort

Before turning to the model's implications for managerial compensation and relative pay, it is useful to analyze the link between firm's sales—a commonly used measure of firm size—and managerial effort. To this aim, we will use a notion of *"marginal product of effort,"* defined as the change in sales implied by an extra unit of managerial effort.

In the model, sales are proportional to product quality, i.e., $p_i X_i = \theta^{\frac{1+\theta}{1-\theta}} Q_i L_i^{\gamma} \Omega^{1-\gamma} = \theta^{\frac{1+\theta}{1-\theta}} M_i Z_i^{\alpha} Z^{1-\alpha} L_i^{\gamma} \Omega^{1-\gamma}$, where we used $Q_i = M_i Z_i^{\alpha} Z^{\alpha}$. The marginal product of effort is then equal to

$$\frac{\partial(p_i X_i)}{\partial h_i} = \theta^{\frac{1+\theta}{1-\theta}} Z_i^{\alpha} Z^{1-\alpha} L_i^{\gamma} \Omega^{1-\gamma} \frac{\partial M_i}{\partial h_i}, \tag{51}$$

where the last term on the right-hand side $\partial M_i / \partial h_i = f'(h_i)$ captures the assumption that managerial services M_i are increasing and concave in effort (i.e., $f'(h_i) > 0$ and $f''(h_i) < 0$).

So, the marginal product of effort (51) is increasing in the firm's own stock of knowledge, Z_i^{α}, and in symmetric equilibrium it becomes linear in the average stock of knowledge, Z. In a steady state with positive growth, then, the marginal contribution of a manager to firm's sales grows at the same constant rate of knowledge. In this sense, managers, as production workers, become increasingly more productive as firms accumulate knowledge. This is the mechanism of creative accumulation at work in this class of models that makes the productivity of the labor input grow without bounds.

The gains from delegation are eventually chocked off by decreasing returns to effort, so that the finiteness of managerial services is a natural limit to the growth of the firm. The mass of managers per firm, $m_i(\pi)$, and effort per manager, $h_i(\pi)$, do not grow forever. Eventually, π will settle on a finite steady-state value, and so will $m_i(\pi)$ and $h_i(\pi)$. At the firm level, sustained growth of firm's sales can only come from knowledge accumulation, not from management. In this sense, management is not a growth engine. However, while the contractual variables related to managerial incentives are not the source of long-run growth, they do affect the steady-state growth rate because they affect the incentives to accumulate knowledge. Moreover, they affect the relative pay of managers and workers, and thus the functional distribution of income.

8.2 Managers' Relative Pay

The relative pay of managers to production workers is

$$\frac{a\Pi}{w} = \frac{\theta}{\gamma}\left[\left(\frac{1}{\theta}-1\right)\theta^{\frac{2}{1-\theta}}\Omega^{1-\gamma} =\right]^{-\frac{1}{\gamma}} a(\pi)\pi^{\frac{1}{\gamma}}. \tag{52}$$

As evident from (52), π affects relative pay through two channels: (i) the contractual term $a(\pi)$ captures the strength of incentive provision, which indirectly depends on firm size, as implied by the solution to the optimal contracting problem; (ii) the term $\pi^{1/\gamma}$, which is *convex* in π given $\gamma < 1$, captures the direct effect of firm size. The convexity captures the "chain letter" effect on the value of the firm highlighted by Rosen (1990). This comes from the fact that the marginal contribution of the manager's effort to the firm's gross cash flow scales with firm size. The larger the firm size, the larger the marginal impact of an extra unit of effort on the gross cash flow. Such a positive relationship between managers' pay and firm size is one of the best documented empirical regularities in executive compensation, a regularity that is remarkably stable across time and industries (Baker, Jensen and Murphy, 1988; Edmans, Gabaix and Landier, 2009; Edmans and Gabaix, 2016; Gabaix and Landier, 2008; Gabaix, Landier and Sauvagnat, 2014; Hall and Liebman, 1998; Murphy, 1999, 2013).

Along an equilibrium path with rising firm size—the empirically relevant case for the United States and other developed countries (see, e.g., Berry, Rodriguez and Sandee, 2002; Lucas Jr., 1978; Nugent and Yhee, 2002; Tybout, 2000; Urata and Kawai, 2002; Wiboonchutikula, 2002)—there are two forces at play. Since $a(\pi)$ is decreasing in π, the contractual term pushes relative pay down. As the economy goes through a period of expanding firm size, it becomes easier to provide incentives to managers in the sense that owners can afford to forgo a smaller percentage of gross cash flows. This is counteracted by the direct firm size effect that pushes relative pay up. Which of the two forces dominates is in general ambiguous. However, because of the convexity of the direct firm size effect, the latter will likely dominate as π rises. Two possible histories are thus possible: a monotonically increasing path of relative pay, if the direct firm size effect dominates along all the transition, or a U-shaped time profile if the contractual term dominates early on.

8.3 Managers' Share of Labor Income

Let labor income be the sum of managers' income, $Nma\Pi$, and production workers' income, $w\left(\Lambda - mN\right)$. The managers' *share* of labor income can be written in a compact form as $\frac{1}{1+\frac{w(\Lambda-mN)}{Nma\Pi}}$. Thus, the higher the income of production workers relative to managers, the smaller the share of labor income accruing to managers. In the model, the relative income of production workers to managers takes the simple form:

$$\frac{w\left(\Lambda - mN\right)}{Nma\Pi} = \frac{\gamma/\theta}{m(\pi)a(\pi)}. \tag{53}$$

This income ratio is unambiguously decreasing in π, since the product $m(\pi)a(\pi)$ at the denominator is monotonically increasing in π. Hence, along a transition path with rising π, the managers' share of labor income steadily increases. Data for the United States confirms this model's prediction (see Appendix B).

Note also that the left-hand side of (53) can be expressed as the product of relative pay and the inverse of the employment share of managers, i.e., $\frac{w}{a\Pi} \times \frac{\Lambda-mN}{mN}$. Here, the first term is the inverse of (52), and the second term can be written as $\frac{\Lambda}{mN} - 1$, where the ratio of managers to total employment is

$$\frac{mN}{\Lambda} = \frac{m\left(\pi\right)}{\left[\left(1-\theta\right)\theta^{\frac{1+\theta}{1-\theta}}\Omega^{1-\gamma}\right]^{-\frac{1}{\gamma}}\pi^{\frac{1}{\gamma}} + m\left(\pi\right)}, \tag{54}$$

which is hump-shaped in π since we showed that $m\left(\pi\right)$ is bounded above. Specifically, the ratio becomes positive at $\pi = \pi_D$, rises initially and the becomes decreasing in π, converging back to zero as π grows large.

9 Conclusion

Business historians view technological progress and market size as major driving forces of the transition from an economy with small owner-operated businesses to a modern economy with managers. While all developed economies have made such a transition, the extent to which they have done so varies substantially across countries. Indeed, large cross-country differences in market structure and in the internal organization of the firm, still persist today, more than a century after the Second Industrial Revolution. While one would reasonably expect differences to persist based on the level of economic development, they remain nontrivial even among developed countries.

In this paper we provide a theory in which the internal organization of the firm is endogenous and driven by market size. Importantly, the emergence of managers is not hardwired into the theory: equilibria in which firms remain owner-managed and small forever are possible. The theory is embedded in a general equilibrium model of endogenous technological progress in which firms' innovative investments determine transitional and long-run growth in aggregate productivity.

We identify a new channel through which the interaction of endogenous firm entry and managerial employment generates a growth acceleration and, possibly, multiple steady states. When the model admits multiple steady states, there are multiple equilibrium paths converging to different steady states for given initial conditions. Under a thick set of parameter values, the economy can be stuck in a growth trap in which firms are small, are operated by owners, and invest little if at all. The dynamics admit indeterminacy, and so sunspot equilibria, as well as history dependence or hysteresis. This property suggests a possible explanation of why some countries failed to become innovative, knowledge-based economies with large and complex organizations, or why countries with arguably similar fundamentals converged to very different growth paths. The mechanism driving these outcomes in our model provides a novel rationale for the observed bimodal or multi-modal distributions in relative per capita income levels.

An important lesson from our analysis is that management, and its complementarity with innovative investment, is a powerful amplification mechanism of policy and fundamentals above and beyond the multiplicity of equilibria. The quantification of this mechanism in relation to entry regulation or business income taxation is a promising area of future research. All in all, opening the black box of the firm, and studying how the internal structure of the firm evolves with, while at the same time affects, macroeconomic outcomes remains an under-studied research topic. Devoting time and energy to it will likely provide novel insights on the evolution of market structure and growth over time and across countries.

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Appendix

A Local Stability Analysis

To study the local stability of the dynamical system governing equilibrium dynamics, it is useful to rewrite the system in a more compact form as:

$$\dot{c} = f(\pi, c); \tag{A.1}$$

$$\dot{\pi} = g(\pi, c); \tag{A.2}$$

where f and g are generic functions of the state variable π and "jump" variable c. Letting $k \in \{\mathbf{L}, \mathbf{M}, \mathbf{H}\}$ denote the three steady states, linearization of (A.1) and (A.2) around *each* steady state (c_k^*, π_k^*) yields (omitting the subscript k for convenience):

$$\begin{bmatrix} \dot{c} \\ \dot{\pi} \end{bmatrix} = \begin{bmatrix} \underbrace{f_c(\pi^*, c^*)}_{a_{11}} & \underbrace{f_\pi(\pi^*, c^*)}_{a_{12}} \\ \underbrace{g_c(\pi^*, c^*)}_{a_{21}} & \underbrace{g_\pi(\pi^*, c^*)}_{a_{22}} \end{bmatrix} \begin{bmatrix} c - c^* \\ \pi - \pi^* \end{bmatrix}. \tag{A.3}$$

The eigenvalues of $A_k = [a_{ij}]_k$ determine local stability. The characteristic polynomial of A_k takes the form $J_k = \lambda_k^2 - [\text{trace}(A_k)]\,\lambda_k + \det(A_k)$. The eigenvalues are given by the roots of the characteristic equation:

$$\lambda_k = \frac{1}{2}\left[\text{trace}(A_k) \pm \sqrt{[\text{trace}(A_k)]^2 - 4\det(A_k)}\right]. \tag{A.4}$$

Since managerial employment $m(\pi)$ and managerial compensation $a(\pi)$ are implicit functions of π, we cannot characterize the eigenvalues analytically. Below we distinguish then between 2 cases:

1. **Unique steady state.** The unique steady state is saddle-path stable: the eigenvalues are real and have opposite signs. Specifically, the eigenvalue associated with the jump variable c is positive; the eigenvalue associated with the state variable π is negative.

2. **Multiple steady states.** For the two *odd* steady states **L** and **H**, saddle-path stability requires the eigenvalues to be real and of opposite sign. For the *even* steady state

M, there are two cases. In the first case, the unstable steady state is a "source": both eigenvalues are real and positive. In the second case, the unstable steady state is a "focus": both eigenvalues are complex and positive. Complex eigenvalues occur when $4\,\det(A) > [\text{trace}(A)]^2$.

B Data and Evidence

In this appendix, we report evidence on trends in managerial compensation, and income and employment shares of managers in the United States for 1950-2017.

B.1 Data Sources and Variables' Construction

Data are from the Decennial Census 1 percent samples for 1950, 1960, and 1970, from the Decennial Census 5 percent samples for 1980, 1990, and 2000, and from the American Community Survey (ACS) for 2001-2017. Data were extracted from the Integrated Public Use Microdata Series (IPUMS) USA, on March 29, 2019.

Sample selection The data sample consists of individuals of 16+ years old. As standard, we exclude individuals who fall in the following categories: (i) Residents of institutional group quarters, prisons and psychiatric institutions (GQ = 3-4); (ii) Self-employed (CLASSWKR = 1); (iii) Armed forces (CLASSWKRD = 26); (iv) Unpaid family workers (CLASSWKRD = 29); (v) Unknown occupations (OCC1990 = 999); (vi) agriculture (OCC1990 $\geq$ 473 & OCC1990 $\leq$ 498).

Income and managerial occupations The income variable is *total pre-tax wage and salary income* (INCWAGE):

> "INCWAGE reports each respondent's total pre-tax wage and salary income - that is, money received as an employee - for the previous year. [...] Sources of income in INCWAGE include wages, salaries, commissions, cash bonuses, tips, and other money income received from an employer. Payments-in-kind or reimbursements for business expenses are not included."

INCWAGE is adjusted for top coding based on the procedure in Autor and Dorn (2013). Hourly wages are defined as yearly wage and salary income divided by the product of weeks worked (WKSWORK1) times usual hours worked per week (UHRSWORK).

To determine the employment status, we use the variable EMPSTAT. Averages are calculated using person weight (PERWT). To convert dollar figures to constant 1999 dollars, we use CPI99.

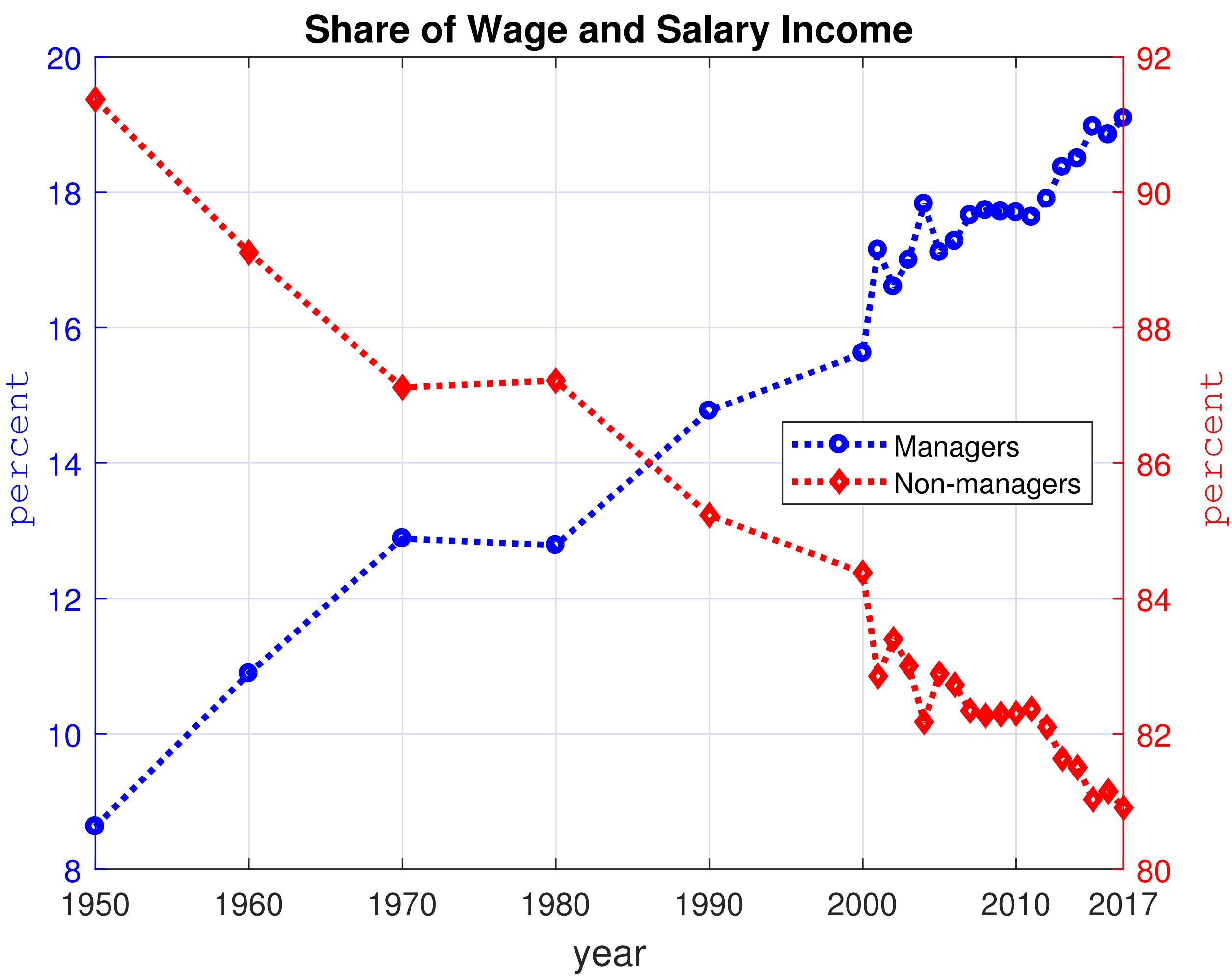
To classify managers, we use the occupation variable OCC1990:

Code	Description
004	Chief executives and public administrators
007	Financial managers
008	Human resources and labor relations managers
013	Managers and specialists in marketing, advertising, etc.
014	Managers in education and related fields
015	Managers of medicine and health occupations
016	Postmasters and mail superintendents
017	Managers of food-serving and lodging establishments
018	Managers of properties and real estate
019	Funeral directors
021	Managers of service organizations, n.e.c.
022	Managers and administrators, n.e.c.

B.2 Evidence

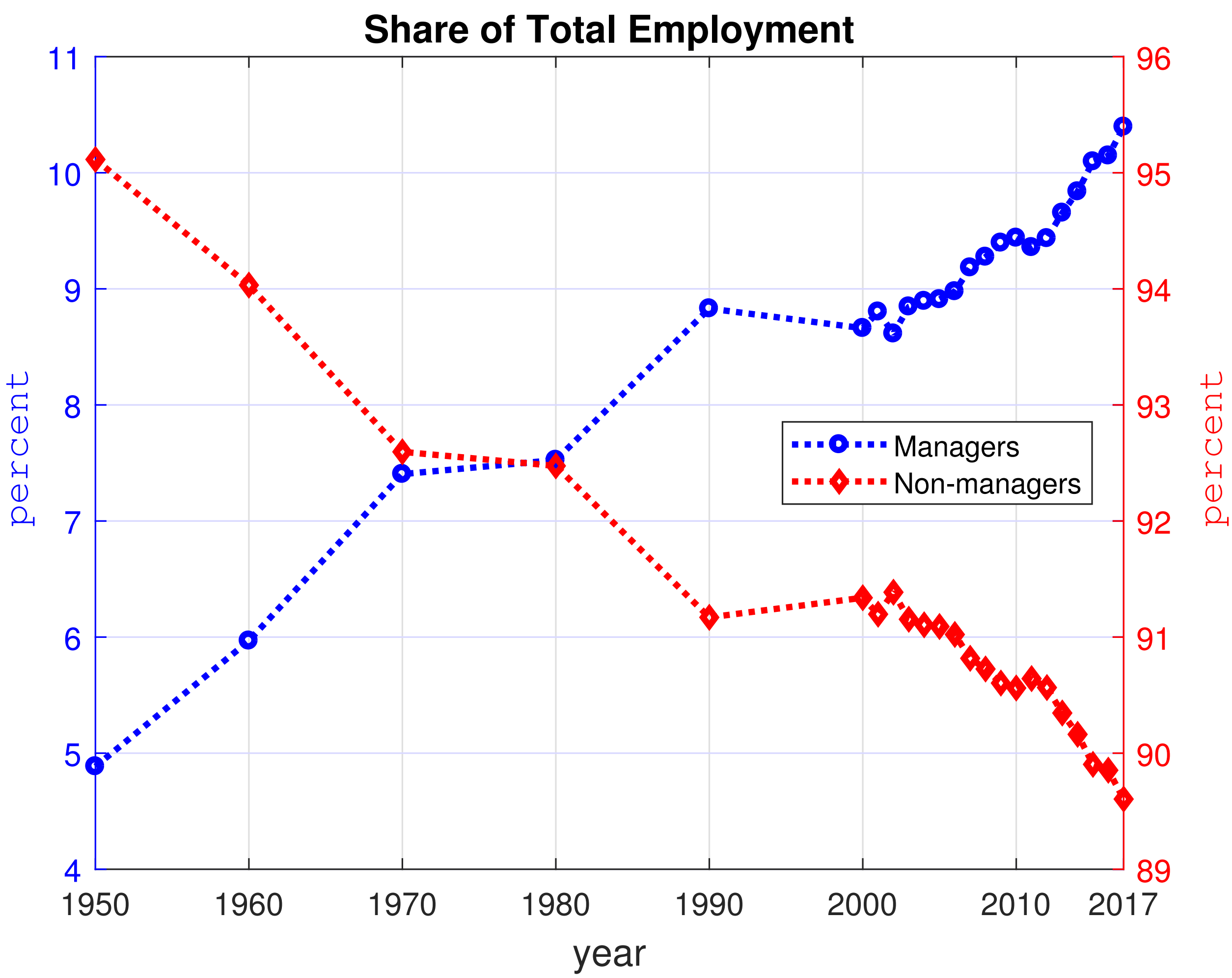
Figures B.1-B.2 show trends in the wage and salary income and employment shares of managers and non-managers, respectively. Two facts stand out. First, the managers' income share rose from 8.6% to 19%. Second, the employment share of managers rose from 5% to 10%. Similar patterns hold across several managerial occupations (Figure B.3); by different definitions of managers (Figures B.4-B.5); and for the manufacturing and service sector (Figures B.6-B.7).

Figure B.1: Income Share of Managers



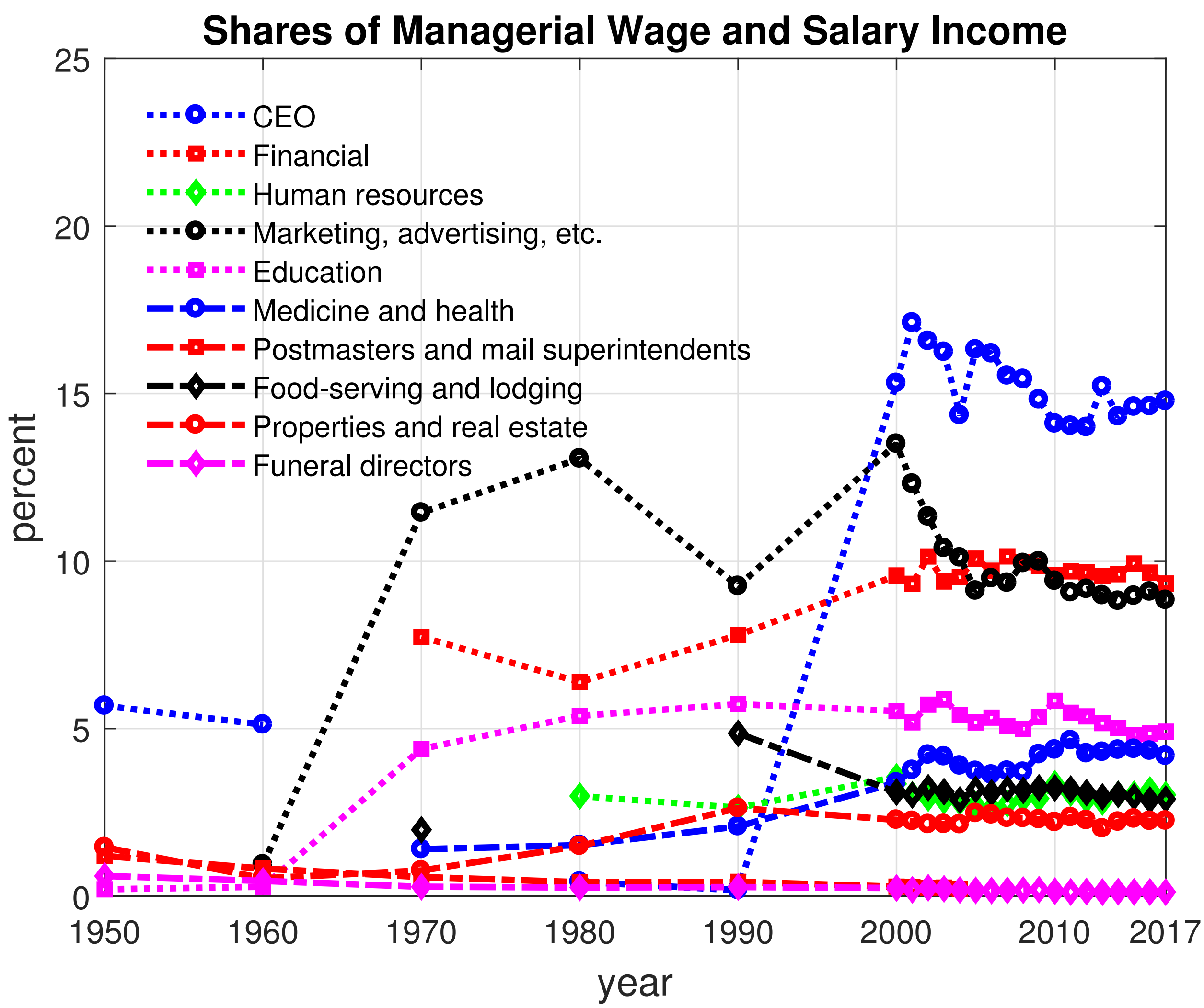
Source: Authors' own calculations using data from IPUMS-USA.

Figure B.2: Employment Share of Managers



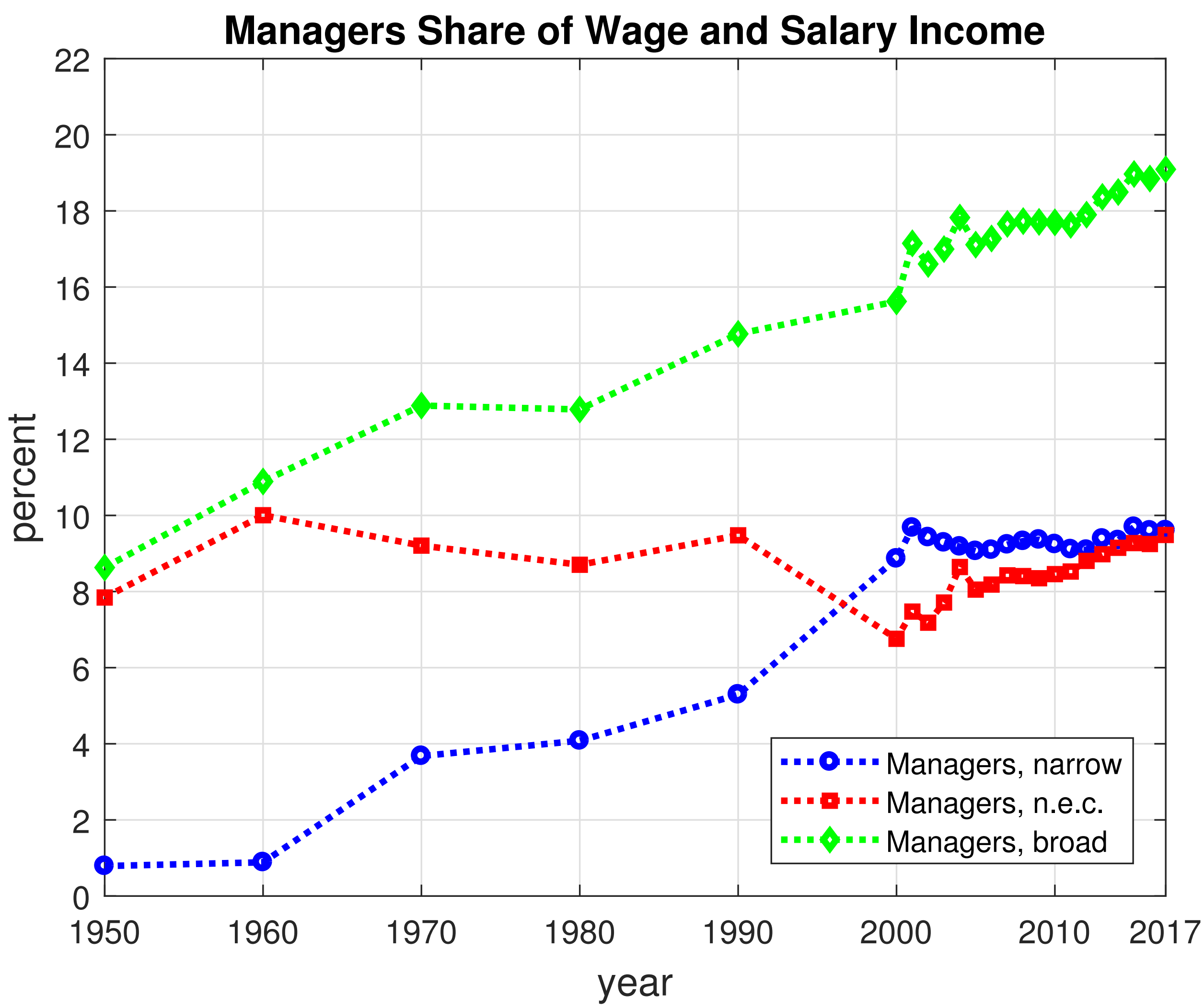
Source: Authors' own calculations using data from IPUMS-USA.

Figure B.3: Income Shares by Managerial Occupations



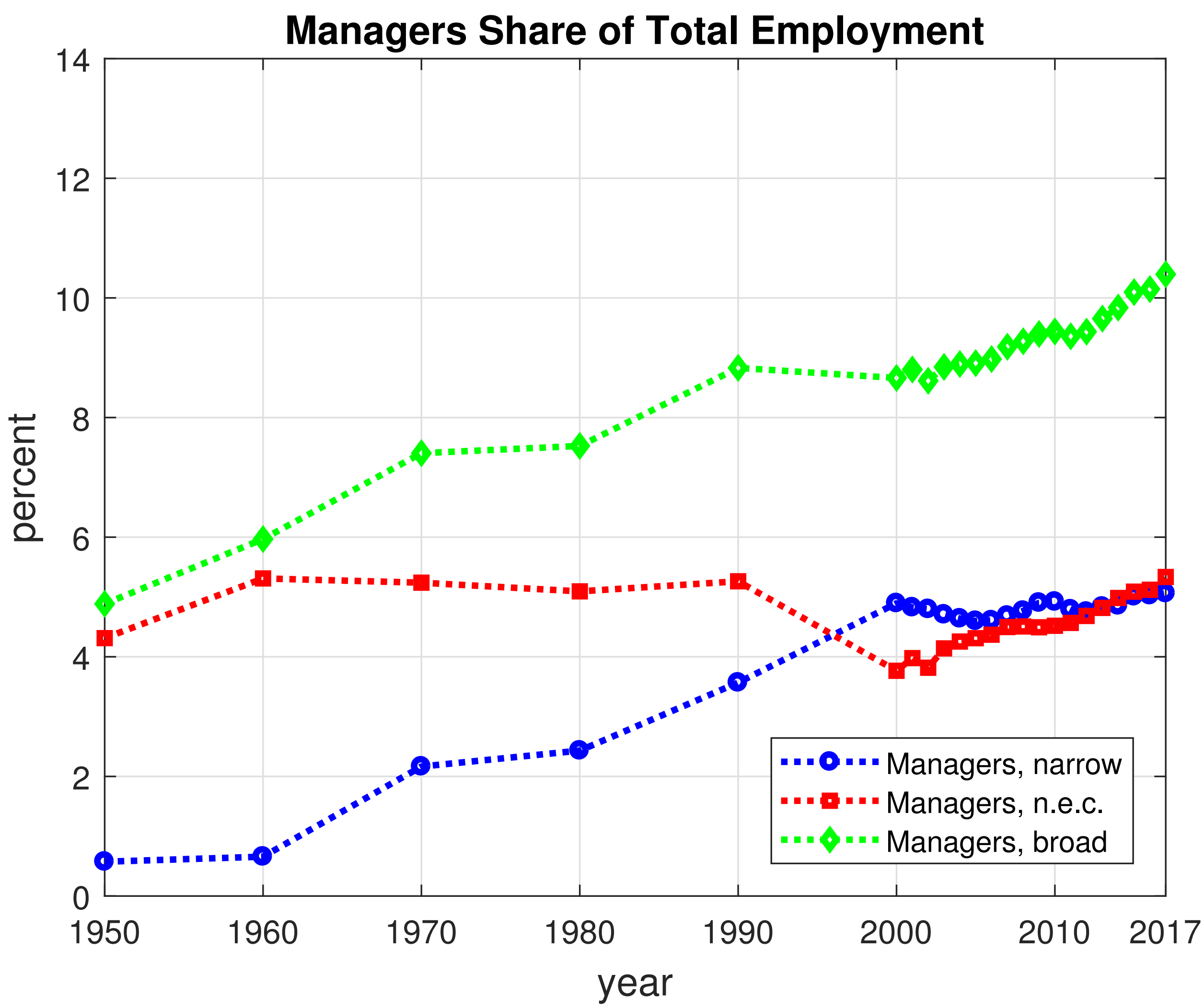
Source: Authors' own calculations using data from IPUMS-USA.

Figure B.4: Income Shares of Managers – Narrow vs. Broad Definition



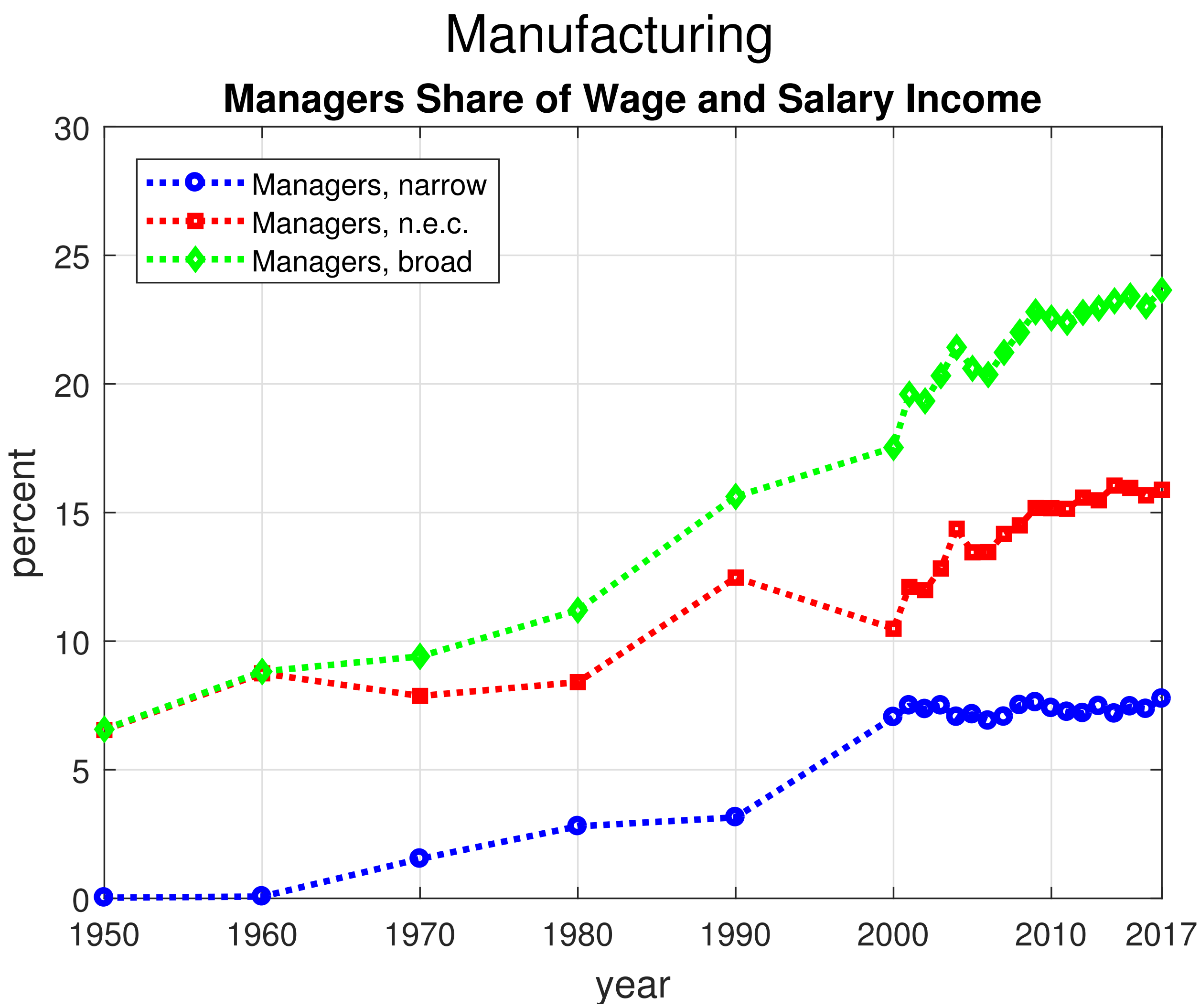
Source: Authors' own calculations using data from IPUMS-USA.

Figure B.5: Employment Shares of Managers – Narrow vs. Broad Definition



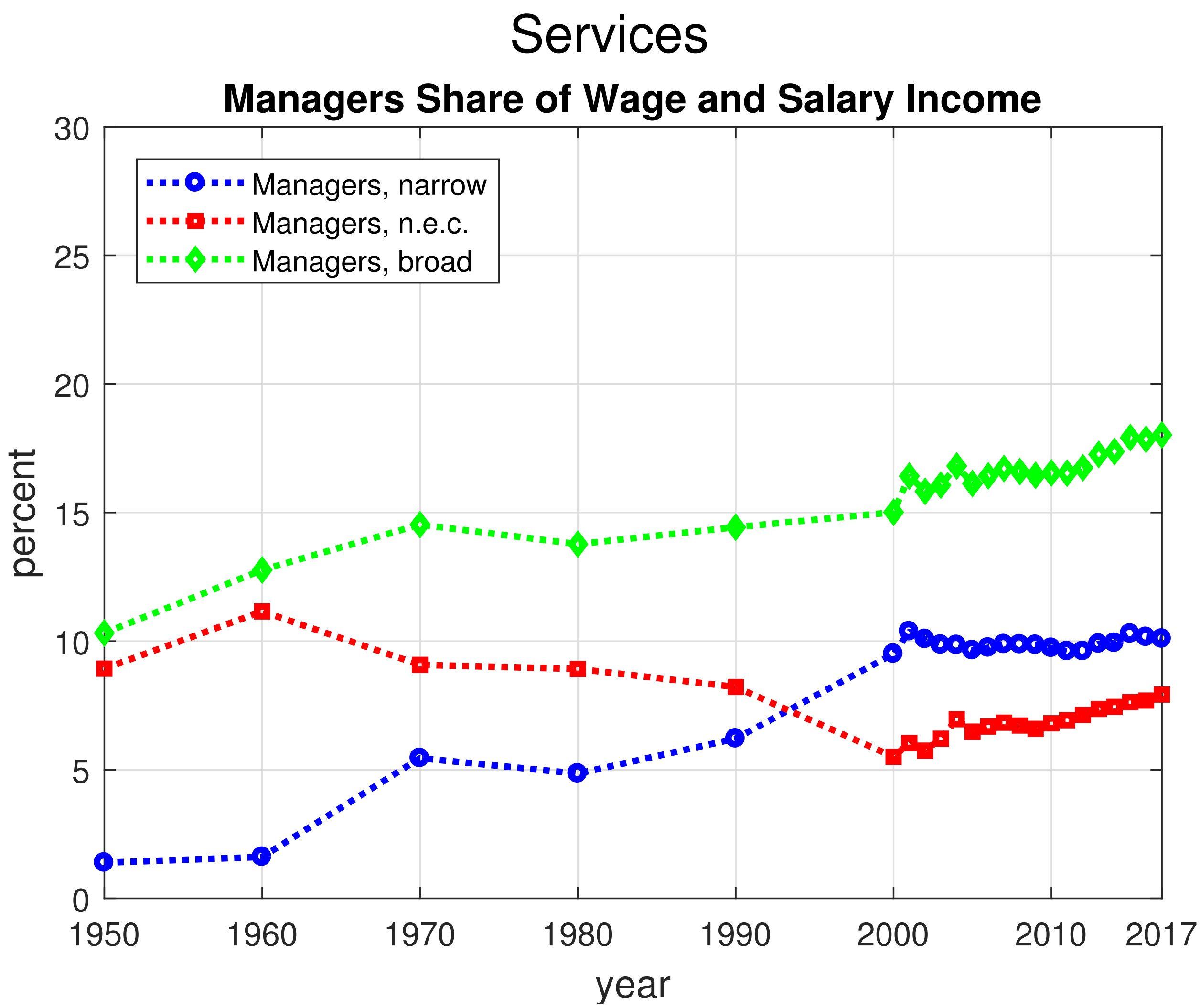
Source: Authors' own calculations using data from IPUMS-USA.

Figure B.6: Income Shares of Managers in Manufacturing



Source: Authors' own calculations using data from IPUMS-USA.

Figure B.7: Income Shares of Managers in Services



Source: Authors' own calculations using data from IPUMS-USA.