Underinvestment and Capital Misallocation Under Sovereign Risk*

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Abstract

Capital and its sectoral allocation affect sovereign risk. I show that, under general assumptions, default incentives are decreasing in the total stock of capital and increasing in the share of capital allocated to non-tradable production. This implies two externalities from private investment: a capital-stock externality and a portfolio externality. These externalities hamper the ability of a benevolent government to make optimal borrowing and default decisions and are exacerbated during crises. Competitive equilibria feature aggregate underinvestment, over-investment in non-tradable sectors, slower recovery from crises, weaker real exchange rates, higher spreads, and lower consumption than the constrained efficient allocation. Optimal investment subsidies are differentiated between sectors and larger in periods of distress.

Keywords: Sovereign default, Underinvestment, Investment externalities.

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1 Introduction

Output dynamics are at the core of the study of sovereign default risk. Default probabilities depend on expectations about future output and directly affect the borrowing terms governments face. In environments with capital accumulation, future output depends on investment decisions made in advance and, if productivity is affected by sovereign default, expectations about future default also affect current investment decisions.[1]

This feedback between default risk and investment has important implications for the dynamics of output, capital accumulation, and the allocation of capital in different sectors. The interaction of sovereign debt with investment has been widely studied by the literature on "debt overhang". Starting with the work of Krugman (1988) and Sachs (1989), and followed by Aguiar, Amador, and Gopinath (2009), this literature has focused on the negative effect that debt has on private investment. Regarding the feedback from investment to debt, Gordon and Guerron-Quintana (2018) and Arellano, Bai, and Mihalache (2018) study how investment and the sectoral allocation of capital affect default risk. However, as is the case with most of the sovereign default literature, these papers study environments where a sovereign makes all borrowing and investment decisions on behalf of households.[2] In this paper, I link these strands of literature by studying the feedback effects between debt and investment in an environment with sovereign debt, private investment, and endogenous default.

The main contribution of this paper is to show how—when the private sector behaves competitively—the feedback between capital allocations and sovereign risk gives rise to two pecuniary externalities: a *capital-stock externality*, which generates inefficient levels of investment, and a *portfolio externality*, which generates inefficient sectoral allocations of capital. These externalities are reminiscent to those studied by the literature on financial crises and macroprudential policies (e.g. Lorenzoni (2008); Bianchi (2011); Bianchi and Mendoza (2018); Bianchi and Mendoza

[1]In the data, default episodes are accompanied by significant declines in output. However, identification of the effect of default on output and productivity is elusive because low levels of either also increase default incentives. Herbert and Schreger (2017) use legal rulings from a case between private bond holders and the Argentinean government to identify causal effects of default on equity returns. They find that an increase in default probability causes a decline in the value of Argentinean equities, which favors the hypothesis that default carries output and productivity costs.

[2]Some exceptions are the work of Aguiar and Amador (2011), Galli (2021), and, more recently, Seoane and Yurdagul (2022) (see the literature review below).

(2020)). The models in this literature feature credit constraints linked to market prices, which give rise to a pecuniary externality as private agents do not internalize how borrowing decisions affect future collateral prices. In this paper, I study environments in which the economy's ability to borrow is endogenously restricted by the market price of government debt, which depends on future default incentives. The externalities arise from private agents not internalizing the effect of their investment decisions on this price.[3] I show that these externalities generate aggregate underinvestment, overinvestment in non-tradable sectors, weaker real exchange rates, higher spreads, and lower consumption than the constrained efficient allocation.

First, I develop two two-period models of sovereign default with foreign debt that flesh out both externalities and allow me to prove that—under standard assumptions for preferences, production technologies, and productivity costs of default—default incentives are decreasing in the aggregate stock of capital and increasing in the share of capital in the non-tradable sector. Both results are consistent with the intuition that capital increases production possibilities and, thus, the ability to repay debt in the future.

For the case of the aggregate stock of capital, the result relies on assuming a positive and increasing cost to productivity. Capital improves both the value of defaulting and the value of repaying the debt; however, the marginal effect on the value of repaying is higher because capital is less productive in default. Moreover, if the cost of default is increasing then with higher capital productivity must decrease in order for default to remain attractive. This implies that the default set shrinks as the productivity cutoff decreases when capital increases. These assumptions also generate an asymmetric cost of default, which is larger in "good" than in "bad" times. Exogenous costs of default with this property have been used in the quantitative literature because they allow models to generate countercyclical trade balances and default rates, which are consistent with the data. Mendoza and Yue (2012) develop a general equilibrium model with production that endogenously generates such a cost of default on TFP. They assume that some imported intermediate materials require working capital financing. When the government defaults, the economy loses access to all credit markets, which implies an efficiency loss as these materials are replaced by imperfect substitutes. Since financing of working capital is static in their environment (i.e. it happens within the same period), then the assumptions that drive my results can be rationalized by a model such

[3] Arce (2021) studies a similar externality of private borrowing on sovereign default risk.

as theirs.

In the case of the sectoral allocation of capital, I study its effect for a fixed stock of capital that has to be split between a tradable and a non-tradable sector. The result relies on debt being denominated in terms of the tradable good (foreign debt) and on the tradable and non-tradable goods being "complementary enough". Complementarity is a sufficient condition for the result because the portfolio allocation of capital has an income and a substitution effect on default incentives that counteract each other. The income effect relates to the intuition mentioned above: increasing the share of capital in the tradable sector increases the ability to service foreign debt. The substitution effect follows from the fact that the optimal default action changes the composition of the consumption bundle: it decreases consumption of non-tradable goods (through lower productivity) and increases that of tradable goods (through not servicing the debt). Having a high share of capital in the non-tradable sector unambiguously increases the cost of a potential default which, in a sense, could "buy" the sovereign some commitment and reduce default incentives. However, when tradable and non-tradable goods are complements then this potential gain from commitment is overwhelmed by the gain from balancing the consumption bundle in default, especially when debt payments are high—because consumption of tradable goods in repayment would be low. Thus, with enough complementarity the income effect dominates the substitution effect (dampened by complementarity), and default incentives unambiguously increase with the share of capital in the non-tradable sector.

I then develop a quantitative sovereign default model with production in two sectors, capital accumulation, and long-term debt. The model is overall standard and builds on the literature following the seminal work of Eaton and Gersovitz (1981). The main innovation is to solve for a competitive equilibrium in which competitive households make all investment decisions and compare it to a constrained efficient equilibrium that arises from solving the problem of a benevolent central planner. Both externalities studied in the two-period models arise in this quantitative version and the theoretical results described above hold for the chosen calibration.

In model simulations, I find that the decentralized equilibrium features aggregate underinvestment and a higher share of capital allocated to the non-tradable sector. I define wedges that are akin to investment subsidies that implement the constrained efficient allocation and study their cyclical behavior. These wedges are, in general, positive and larger during periods of distress, indicating that the externalities are amplified by crises. I also find that the cost of these subsidies is low, both compared to GDP and to potential gains in consumption. This suggests that similar instruments could be implemented at a low cost with large potential benefits, however more exhaustive quantitative and empirical work would be required to make that case.

Finally, I use the model to study the European debt crisis from the early 2010's. The model does a good job in replicating the paths of main macroeconomic variables during the crisis, both their direction and magnitude. I find that both externalities played a key role in deepening the crisis and slowing down the recovery of investment, GDP, and consumption.

Related literature.—This paper is closely related to the literature that focuses on disagreements between governments and households in environments where the government lacks commitment and there is default risk. Aguiar and Amador (2011) study an open economy that emphasizes political economy and contracting frictions. In their environment, the government can default on its debt and expropriate capital, which gives rise to slow growth driven by low rates of capital accumulation. This result is similar to the underinvestment that I study during debt crises. In my environment, the cause is the household's inability to internalize how higher investment improves borrowing terms in the present, while in theirs the cause is the risk of future expropriation. Galli (2021) studies an economy in which low investment from the private sector can be the result of self-fulfilling beliefs about high default risk. He builds on the work by Cole and Kehoe (2000) in an environment with production and capital accumulation. I make crucial timing assumptions that allow me to rule out the sources of multiplicity introduced by these two papers, which highlights that the externalities I study are orthogonal to that studied by Galli (2021). Finally, in a recent working paper Seoane and Yurdagul (2022) study an environment with production in one-sector, endogenous sovereign default risk, and private corporate investment. In their environment, there is a similar externality from aggregate investment on default risk, which amplifies the procyclicality of investment. The main difference is that I study an environment with production in multiple sectors and the interaction of both externalities. Another important difference is that they allow the government to levy income taxes and introduce consumption of public goods. My theoretical results regarding the capital-stock externality are complementary to their findings and their quantitative findings are consistent with mine.

As mentioned before, this paper builds on the sovereign debt literature following Eaton and

Gersovitz (1981). Aguiar and Gopinath (2006) and Arellano (2008) developed quantitative models to study the relation between default risk and output fluctuations. Later work by Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012) extended the framework to feature long-term debt and showed how this improved the model's ability to match business cycle data of debt, spreads, and default risk. Hatchondo, Martinez, and Sosa-Padilla (2016) show that with long-term debt the government's inability to commit not to dilute the value of future debt increases present borrowing costs, an inefficiency that is present in all Markov equilibria in sovereign debt models with long-term bonds. Gordon and Guerron-Quintana (2018) study an environment with long-term debt and capital accumulation in a single tradable sector, and Arellano, Bai, and Mihalache (2018) study an environment with capital accumulation in tradable and non-tradable sectors. My quantitative model mostly builds on the two latter papers, which provide a natural starting point for a quantitative model to study the externalities of interest.

Layout.—Section 2 presents the two two-period models that introduce both externalities. Section 3 presents the quantitative analysis with an infinite-horizon model. Section 4 concludes.

2 Two-period models

The models in this section flesh out two externalities from private investment on sovereign default risk: a *capital-stock externality* and a *portfolio externality*. Both models share the environment laid out below and only differ in the production technology for the final consumption good.

There is a small-open economy populated by a measure one of households, competitive firms, and a benevolent government. Households have preferences for consumption of a final good in each of the two periods represented by $U(c_0, c_1) = u(c_0) + \beta \mathbb{E}_0 [u(c_1)]$, where u is strictly increasing, concave and invertible, and $\beta \in (0, 1)$ is a discount factor. The final good is produced by competitive firms using capital.

The only source of uncertainty is a productivity shock $z \in \mathbb{R}_+$, which is realized at the beginning of period 1 and has CDF $G(z)$. Productivity in the initial period is normalized to $z_0 = 1$. Households own all the capital and firms in the economy, but do not have access to foreign borrowing.

The benevolent government can borrow on behalf of the households in international financial markets. At the beginning of period 0, the budget constraint of the government is $T_0 =$

$q(x_1) B_1 - B_0$, where B_0 is legacy debt that cannot be defaulted on, T_0 is a lump-sum transfer to the households, B_1 is non-contingent defaultable debt that matures in period 1, and $q(x_1)$ is the price schedule for B_1. Here, x_1 is a vector that contains all payoff-relevant variables for period 1 that are observable to the lenders when they purchase the debt.[4] Lenders are competitive, risk-neutral, have deep pockets, and have access to a risk-free bond that pays interest rate r^*.

At the beginning of period 1, the government observes z and can choose to repay B_1 by levying a lump-sum tax $-T_1 = B_1$ to the households. Alternatively, the government can default on B_1, in which case no tax is levied but real resources are lost in the form of a productivity penalty. I assume productivity in default is characterized by a function $z_D(z) \leq z$, which is differentiable, has $\frac{\partial z_D}{\partial z} \leq 1$ for all z, and $\lim_{z \to 0} [z - z_D(z)] = 0$. Also, suppose that there is a $\hat{z} > 0$ such that for $z \geq \hat{z}$ the inequalities are strict.[5]

The timing of events in period 0 is as follows. First, given B_0, the government chooses B_1 to maximize the lifetime utility of households subject to its budget constraint. The government takes into account how this choice affects household behavior and all the prices in the economy. Then, households observe B_1 and make all of their decisions. Finally, lenders observe x_1 and purchase the debt for an actuarially fair price

$$q(x_1) = \frac{\int_0^\infty [1 - d(x_1, z)]\, dG(z)}{1 + r^*} \tag{1}$$

where d is the government's default decision at the beginning of period 1, and x_1 is already pinned down by the time of the debt auction. This timing assumption allows me to rule out the multiplicities of equilibria studied by Cole and Kehoe (2000) and by Galli (2021) (see discussion below).

Timing and multiplicity.—In their environment studied by Cole and Kehoe (2000), lenders first offer a price schedule and then the government chooses whether to issue B_1 and repay B_0 or to default. For certain regions of the state space (high levels of debt and low levels of output), this allows for two equilibria: one in which optimistic lenders offer a generous price schedule and the

[4] See the formal definitions for q in each model below.

[5] Aside from differentiability, these properties are satisfied by all commonly used functions for default penalties in the literature. The essential property is that the exogenous cost of defaulting is not symmetric and increasing in z, such that default happens in "bad times" (when z and the cost are small) and not in "good times" (when z and the cost are large). This property is captured by $\lim_{z \to 0} [z - z_D(z)] = 0$ and $\frac{\partial z_D}{\partial z} \leq 1$. Common functional forms for z_D feature a "kink" out of convenience of the parameterization, not as a necessary feature for the desired properties of the model.

government repays and one in which pesimistic lenders refuse to purchase B_1 and the government defaults on B_0. The fact that B_0 cannot be defaulted on is also crucial for me to rule out this type of multiplicity.

The above assumptions also rule out the type of multiplicity studied by Galli (2021) in environments with private investment. He assumes that lenders observe the amount of debt issued and offer a price schedule before investment is chosen. Under this timing assumption, lenders' beliefs about investment can be self-fulfilling due to the effect that the price of the debt has on household behavior through fiscal policy. To summarize, in my environment, multiplicity *a la* Cole-Kehoe is ruled out because I assume that lenders price B_1 after the government chooses it and commits to pay B_0; and multiplicity *a la* Galli is ruled out because lenders price B_1 after the capital allocation has been chosen.

2.1 Model 1: Capital-stock externality

With this model I study how, under standard general assumptions, the aggregate stock of capital affects default incentives. In particular, I prove that in this environment default incentives are decreasing in the stock of capital. That is, the larger the stock of capital accumulated for the next period, the weaker default incentives are and, thus, the cheaper it is for the government to issue new debt. In an economy where investment decisions are made by atomistic households, the equilibrium allocation is constrained inefficient because households fail to internalize this effect that capital has on default incentives and the ability of the government to borrow. Compared to a constrained efficient allocation chosen by a benevolent planner (who also lacks commitment to default), the equilibrium allocation features underinvestment.

2.1.1 Environment

The final good is produced by a competitive firm with technology $y_t = F(z_t, K_t)$, where z_t and K_t are productivity and the aggregate stock of capital in period t, respectively. The production function F is continuously differentiable, strictly increasing in both arguments, strictly concave in K, weakly convex in z, and the cross derivative is $F_{zK} \geq 0$. In each period, each household $i \in [0, 1]$ is endowed with $k_{i,t}$ units of capital, which they rent to the firm for a rate r_t. For simplicity, I assume that capital fully depreciates. Since the firm behaves competitively, the rental rate is $r_t = F_K(z_t, K_t)$, with $K_t = \int_0^1 k_{i,t} di$.

Households.—All households are ex ante identical and own k_0 units of capital, which implies $k_0 = K_0$. In period 0, a representative household observes B_1 and chooses consumption c_0 and how much capital to store for the next period k_1. The household maximizes its lifetime utility subject to its budget constraint:

$$\begin{aligned} &\max_{c_0, k_1} \{u(c_0) + \beta \mathbb{E}[u(c_1)]\} \qquad (2) \\ s.t. \quad & c_0 + k_1 \leq r_0 k_0 + \Pi_0 + q(x_1) B_1 - B_0 \\ & c_1 = r_1 k_1 + \Pi_1 + T_1 \\ & K_1 = \Gamma_H(B_1) \end{aligned}$$

where Π_t are profits made by the firm and $\Gamma_H(B_1)$ are the household's beliefs about the law of motion of aggregate capital. In period 1 the household consumes all available income, where T_1, r_1 and Π_1 are pinned down by the government's default decision.

Government.—At the beginning of period 1, the government observes $x_1 = (K_1, B_1)$ and the realization of z and decides whether to repay or default on the debt in order to maximize $u(c_1)$. The default set $\mathcal{D}(x_1) = [0, z^*(x_1))$ is characterized by a cutoff value $z^*(x_1)$ such that

$$F(z^*(x_1), K_1) - B_1 = F(z_D(z^*(x_1)), K_1) \qquad (3)$$

where the left-hand-side is consumption c_1 under repayment and the right-hand-side is consumption under default (note that this simplified expression follows from the assumption that the utility function u is invertible). Then, the problem of the government at the beginning of period 0 is

$$\begin{aligned} &\max_{B_1} \left\{ u(c_0) + \beta \int_0^{z^*(x_1)} u(F(z_D(z), K_1)) dG(z) + \beta \int_{z^*(x_1)}^{\infty} u(F(z, K_1) - B_1) dG(z) \right\} \qquad (4) \\ s.t. \quad & c_0 = F(1, K_0) - K_1 + q(x_1) B_1 - B_0 \\ & K_1 = k^*(B_1) \end{aligned}$$

where $k^*(B_1)$ is the capital policy function of the household's problem in (2). The government understands how B_1 affects the aggregate capital allocation, however, as I show below, the lump-sum transfer is insufficient to induce the desired household behavior.

2.1.2 Equilibrium and efficiency

An *equilibrium* is policy functions for the household $c_0(B_1)$, $k^*(B_1)$, household beliefs $\Gamma_H(B_1)$, a quantity of debt issued B_1^*, and a price schedule $q(x)$ such that: (i) given q, B_1^* solves the government's problem (4); (ii) given Γ_H, the policy functions $c_0(B)$ and $k^*(B)$ solve the household's problem (2) for any B; (iii) beliefs are consistent $\Gamma_H(B) = k^*(B)$ for any B; (iv) the price q satisfies

$$q(x) = \frac{1 - G(z^*(x))}{1 + r^*} \tag{5}$$

which is the version of (1) specific to this environment.

Using the above notation, we can define an *equilibrium allocation* as $\tilde{x} = \left(k^*\left(B_1^*\right), B_1^*\right)$. In order to characterize the constrained efficient allocation, I consider a benevolent central planner with the ability to choose x_1 at the beginning of period 0 and the ability to default at the beginning of period 1 after observing z. Note that, given x_1, the planner's default set is also characterized by the cutoff $z^*(x_1)$ defined in (3), which implies that the planner faces the same price schedule q as the government in the decentralized economy. The problem of the planner at the beginning of period 0 is

$$\max_{x_1} \left\{ u(c_0) + \beta \int_0^{z^*(x_1)} u(F(z_D(z), K_1))\, dG(z) + \beta \int_{z^*(x_1)}^{\infty} u(F(z, K_1) - B_1)\, dG(z) \right\} \tag{6}$$
$$s.t. \quad c_0 = F(1, K_0) - K_1 + q(x_1) B_1 - B_0$$

which is only different from the government's problem (4) in the sense that the planner chooses both B_1 and K_1 directly. Define the *constrained efficient allocation* as $\hat{x}_1$ that solves the planner's problem.

2.1.3 Discussion

In order to simplify notation, hereafter I will use "hat" variables $\hat{y}$ for variables (or functions) associated with (or evaluated at) the constrained efficient allocation, and "tilde" variables $\tilde{y}$ for the competitive equilibrium. The Euler equation associated with the problem of a representative household (2) is:

$$u'\left(\tilde{c}_0\right) = \mathbb{E}\left[\beta u'\left(\tilde{c}_1\right)\tilde{r}_1\right] \tag{7}$$

which, as is standard, equates the marginal expected return of capital in $t = 1$ to its marginal cost (foregone consumption in $t = 0$), in terms of marginal utility. The planner's Euler equation for capital is:

$$u'\left(\hat{c}_0\right)\left[1 - \widehat{\frac{\partial q}{\partial K}}\hat{B}_1\right] = \mathbb{E}\left[\beta u'\left(\hat{c}_1\right)\hat{r}_1\right] \tag{8}$$

which introduces an additional trade off in period 0.[6] An additional unit of capital K_1 has two effects on consumption in $t = 0$. As in the competitive equilibrium, it directly reduces c_0 because the resource constraint is binding; but it also affects default incentives in $t = 1$ and, thus, changes the price of newly issued debt:

$$\frac{\partial q}{\partial K} = -\frac{g\left(z^*\left(x\right)\right)}{1+r^*}\frac{\partial z^*\left(x\right)}{\partial K} \tag{9}$$

where $g > 0$ is the PDF of z and $\frac{\partial z^*(x)}{\partial K}$ is the derivative of the default cutoff with respect to K.

Proposition 1. The default set is shrinking in K_1. That is, $\frac{\partial z^*(x_1)}{\partial K_1} \leq 0$.

Proof: See Appendix A.□

The proof consists of taking the full derivative of equation (3) and using the assumptions on F and z_D to determine the sign of $\frac{\partial z^*(x_1)}{\partial K_1}$. Consider the Cobb-Douglas case $F\left(z,K\right) = zK^{\alpha}$ with $\alpha \in (0,1)$, then from fully differentiating (3) with respect to K_1 we get

$$\frac{\partial z^*\left(x_1\right)}{\partial K_1} = -\frac{\left[z^*\left(x_1\right) - z_D\left(z^*\left(x_1\right)\right)\right]}{\left[1 - \frac{\partial z_D\left(z^*\left(x_1\right)\right)}{\partial z}\right]}\frac{\alpha}{K_1} \leq 0$$

where the inequality follows from the assumptions $z_D\left(z\right) \leq z$ and $\frac{\partial z_D(z^*(x_1))}{\partial z} \leq 1$.

Intuitively, capital increases both the value of repayment and the value of defaulting because it increases production possibilities in both cases. However, the positive effect on the value of repayment dominates because marginal product in default is hindered by z_D. Thus, more capital increases both sides of (3), but increases the left-hand-side (consumption in repayment) more. For

[6] Note that there is no rental rate of capital in the planner's problem (the only price that the planner faces is $\hat{q}$). Here, $\hat{r}_1 = F_K\left(z,\hat{K}_1\right)$ only denotes the marginal product of capital evaluated at z and the planner's choice $\hat{K}_1$, which simplifies notation and makes the comparison of equations (7) and (8) more straightforward.

the equation to hold, then, z^* needs to adjust. A decrease in z^* decreases both sides of the equation, but decreases the repayment side more, since $\frac{\partial z_D}{\partial z} \leq 1$, which implies that for the equation to hold z^* must decrease as K increases.

Given Proposition 1, we can see from equation (9) that q is an increasing function of K. This implies a trade off between less consumption from setting resources aside for investment and more consumption from a higher ability to borrow. Under the constrained efficient allocation, the household's Euler equation would be

$$u'(\hat{c}_0) \geq \mathbb{E}\left[\beta u'(\hat{c}_1)\hat{r}_1\right]$$

which is inconsistent with optimal behavior. From the household's point of view, the constrained efficient amount of investment is too costly since they fail to internalize its effect on the ability to borrow. This illustrates the disagreement between the households and the benevolent government. Moreover, this disagreement is more severe when the desire to borrow is high and when lenders are more sensitive to small changes in default risk.

2.2 Model 2: Portfolio externality

In this model, the final consumption good is an aggregate of different intermediate goods and, crucially, debt is not denominated in the same units as consumption. The application laid out below is an environment in which consumption is a composite of tradable and non-tradable intermediates, but foreign debt is denominated in terms of the tradable good. Both intermediates are produced using capital, which has to be installed in each sector one period in advance. The sectoral allocation of capital affects default incentives in a non-trivial way because default—which only liberates tradable resources—affects final consumption differently than the productivity penalty does, which hits both sectors equally. In stark contrast with the model presented in the previous section, I assume that the aggregate stock of capital is fixed, which highlights the independent role of its sectoral allocation.

2.2.1 Environment

The final consumption good is non-tradable and is produced by a competitive firm which aggregates tradable and non-tradable intermediates, c_T and c_N, respectively, using technology $Y =$

$F(c_N, c_T)$, where F is strictly increasing and strictly concave in both arguments, has positive cross derivatives, and has constant returns to scale. The intermediate goods are produced by competitive firms using Cobb-Douglas production technologies $y_i = zf(K_i)$, where $i \in \{N, T\}$, $f(K) = K^{\alpha}$, $0 < \alpha < 1$, and productivity z is the same in both sectors in all periods. Intermediate firms rent capital from households at a rate r_i. Households own all the capital and firms in the economy. Debt is denominated in terms of the tradable good, which is the nummeraire. The resource constraints of the economy are $c_{N,t} = y_{N,t}$, $c_{T,t} = y_{T,t} + T_t$, and $c_t = Y_t$.

Households.—All households are ex ante identical and own a fixed stock of capital $\bar{k}$ that does not depreciate and cannot be increased. Capital can be allocated in either of the two sectors as long as $k_{N,t} + k_{T,t} = \bar{k}$, but this allocation has to be decided one period in advance. In what follows, I normalize $\bar{k} = 1$ to simplify notation; however, all the results in this section hold for any $\bar{k} > 0$. Let λ_t be the share of a representative household's capital stock that is allocated in the tradable sector in period t and let Λ_t be the corresponding share for the aggregate capital stock $\bar{K} = \bar{k}$. Households start period 0 with some given λ_0 and choose their portfolio λ_1 to maximize their lifetime utility taking all prices as given. The budget constraint of a representative household in period 0 is $P_0 c_0 = (1 - \lambda_0) r_{N,0} + \lambda_0 r_{T,0} + \Pi_0 + q(x_1) B_1 - B_0$, where P_0 is the relative price of the final good, $r_{N,0}$ and $r_{T,0}$ are the rental rates of capital in the non-tradable and tradable sectors, respectively, and Π_0 are profits from all firms. In period 1, the household consumes all available income such that $P_1 c_1 = (1 - \lambda_1) r_{N,1} + \lambda_1 r_{T,1} + \Pi_1 + T_1$, where P_1, $r_{N,1}$, $r_{T,1}$, Π_1, and T_1 are pinned down by the government's default decision. The problem of a representative household is then:

$$\max_{\lambda_1} \{u(c_0) + \beta \mathbb{E}[u(c_1)]\} \tag{10}$$

subject to the budget constraints in both periods and to $\Lambda_1 = \Gamma_H(B_1)$, where $\Gamma_H(B_1)$ are the household's beliefs about the law of motion of the aggregate capital allocation.

Government.—At the beginning of period 1, the government observes $x_1 = (\Lambda_1, B_1)$ and the realization of z and decides whether to repay or default on the debt in order to maximize $u(c_1)$. The default set $\mathcal{D}(x_1) = [0, z^*(x_1))$ is characterized by a cutoff value $z^*(x_1)$ such that

$$V^D(z^*(x_1), \Lambda_1) = V^P(z^*(x_1), x_1) \tag{11}$$

where the values of default and repayment are

$$V^{D}(z,\Lambda)=u\left(F\left(z_{D}(z)\,f(1-\Lambda),z_{D}(z)\,f(\Lambda)\right)\right) \tag{12}$$

$$V^{P}(z,x)=u\left(F\left(zf(1-\Lambda),zf(\Lambda)-B\right)\right) \tag{13}$$

, respectively, for any (z,x). Equations (12) and (13) highlight the trade off that the government faces when making its default decision: on one hand, consumption of tradable goods increases by not exporting B but, on the other, production of both the non-tradable and tradable goods decrease. Unlike in the case of a unique tradable good, here default has a non-homothetic effect on final consumption due to the potential change in the bundle of intermediate goods (c_N,c_T) used for final production. The problem of the government at the beginning of period 0 is:

$$\begin{aligned}
&\max_{B_1}\left\{u(c_0)+\beta\int_0^{z^*(x_1)}V^D\left(z_D(z),\Lambda_1\right)dG(z)+\beta\int_{z^*(x_1)}^{\infty}V^P(z,x_1)\,dG(z)\right\} \qquad (14)\\
&s.t.\quad c_0=F\left(z_0f(1-\Lambda_0),z_0f(\Lambda_0)+q(x_1)B_1-B_0\right)\\
&\qquad\quad \Lambda_1=\lambda^*(B_1)
\end{aligned}$$

where $\lambda^*(B_1)$ is the policy function of the household's problem in (10). As in Model 1, the government understands how B_1 indirectly affects the aggregate capital allocation.

2.2.2 Equilibrium and efficiency

The equilibrium definition is analogous to that in Model 1.[7] An *equilibrium allocation* is $\tilde{x}=\left(\lambda^*\left(B_1^*\right),B_1^*\right)$ and the constrained efficient allocation is $\hat{x}_1$ that solves the problem of a benevolent planner in period 0:

$$\begin{aligned}
&\max_{x_1}\left\{u(c_0)+\beta\int_0^{z^*(x_1)}V^D\left(z_D(z),\Lambda_1\right)dG(z)+\beta\int_{z^*(x_1)}^{\infty}V^P(z,x_1)\,dG(z)\right\} \qquad (15)\\
&s.t.\quad c_0=F\left(z_0f(1-\Lambda_0),z_0f(\Lambda_0)+q(x_1)B_1-B_0\right)
\end{aligned}$$

[7] An *equilibrium* is policy functions for the household $c_0(B_1)$, $\lambda^*(B_1)$, household beliefs $\Gamma_H(B_1)$, a quantity of debt issued B_1^*, and a price schedule $q(x)$ such that: (i) given q, B_1^* solves the government's problem (14); (ii) given Γ_H, the policy functions $c_0(B)$ and $\lambda^*(B)$ solve the household's problem (10) for any B; (iii) beliefs are consistent $\Gamma_H(B)=\lambda^*(B)$ for any B; (iv) the price q satisfies $q(x)=\frac{1-G(z^*(x))}{1+r^*}$ with $x=(\Lambda,B)$ and z^* as defined in (11).

which, as in Model 1, is only different from the government's problem (14) in the sense that the planner chooses both B_1 and Λ_1 directly. As it was the case in Model 1, the planner's default set is also characterized by the cutoff $z^*(x_1)$ defined in (11), which implies that the planner faces the same price schedule q as the government in the decentralized economy.

2.2.3 Discussion

As in Model 1, I will use "hats" for the efficient allocation and "tildes" for the competitive equilibrium. The Euler equation associated with the problem of a representative household (10) is:

$$0 = \mathbb{E}\left[\beta u'(\tilde{c}_1)\left(\tilde{R}_{N,1} - \tilde{R}_{T,1}\right)\right] \tag{16}$$

where $\tilde{R}_{i,1} = \tilde{r}_{i,1}/\tilde{P}_1$ for $i = T, N$. This resembles a no-arbitrage condition: in equilibrium, households allocate capital in each sector in a way such that the expected discounted marginal returns are equated. The planner's Euler equation for the sectoral allocation of capital Λ is:

$$u'(\hat{c}_0)\frac{\partial \hat{q}}{\partial \Lambda}\frac{\hat{B}_1}{\hat{P}_0} = \mathbb{E}\left[\beta u'(\hat{c}_1)\left(\hat{R}_{N,1} - \hat{R}_{T,1}\right)\right] \tag{17}$$

which illustrates the additional trade off for the planner in period 0.[8] On one hand, Λ_1 affects the aggregate capital portfolio and expected income for period 1, and, on the other, it affects the price of B_1 through its effect on default incentives.

Proposition 2. If the elasticity of substitution between tradable and non-tradable intermediates is $\eta < 1$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x_1)}{\partial \Lambda_1} \leq 0$.

Proof: See Appendix A.□

As with Proposition 1, the proof consists of taking the full derivative of equation (11) and using the assumptions on F and z_D to determine the sign of $\frac{\partial z^*(x_1)}{\partial \Lambda_1}$. The assumption of $\eta < 1$ is a sufficient condition for the result to hold and is in line with parameterizations and estimates used in the international macroeconomics literature.

To understand the role of this assumption, first note that Λ_1 has two effects on default incen-

[8] As in Model 1, the only relative price that the planner faces is q. To ease exposition, here I plug in for $\hat{R}_{N,1} = \hat{p}_{N,1} z_1 f'(1-\Lambda_1)$, $\hat{R}_{T,1} = z_1 f'(\Lambda_1)$, $\hat{p}_{N,t} = \frac{\partial F}{\partial c_N} / \frac{\partial F}{\partial c_T}$, $\hat{P}_t = 1/\frac{\partial F}{\partial c_T}$. These variables are akin to their decentralized counterparts because there is no "static inefficiency" in this model in the sense that, given the same (z_1, x_1) the planner would choose the same c_N and c_T as the decentralized economy.

tives: an *income* and a *substitution* effect. The *income effect* refers to the fact that the ability to service the debt increases with Λ_1 because debt is denominated in terms of the tradable good. This reduces default incentives as repaying becomes less painful. For the *substitution effect*, note that at z^* the default action reduces c_N and increases c_T.[9] As Λ_1 increases, the potential cost from default through lower c_N decreases and the potential benefit through higher c_T increases. In a sense, the substitution effect implies that choosing low values of Λ_1 gets the government (or the planner) some commitment by increasing the potential net losses from default.

Consider the extreme case in which c_N and c_T are perfect substitutes. Then, F is a linear combination of c_N and c_T and equation (11) becomes:

$$\omega z_D(z^*) f(1-\Lambda_1) + (1-\omega) z_D(z^*) f(\Lambda_1) = \omega z^* f(1-\Lambda_1) + (1-\omega)\left[z^* f(\Lambda_1) - B_1\right]$$

with $\omega \in (0,1)$. Taking the full derivative with respect to Λ_1 and rearranging we get:

$$\frac{\partial z^*}{\partial \Lambda} = -\frac{\left[z^* - z_D(z^*)\right]\left[(1-\omega) f'(\Lambda_1) - \omega f'(1-\Lambda_1)\right]}{\left[1 - \frac{\partial z_D(z^*)}{\partial z}\right]\left[\omega f(1-\Lambda_1) + (1-\omega) f(\Lambda_1)\right]} \tag{18}$$

where the denominator is clearly positive since $\frac{z_D}{\partial z} \leq 1$ by assumption. From concavity of f, it follows that for large enough values of Λ the numerator is negative, which implies that default incentives increase as Λ increases ($\frac{\partial z^*}{\partial \Lambda} > 0$). This is because the marginal product of capital in the non-tradable sector is so large that the marginal decrease in the cost of default from an increase in Λ—the *substitution effect*—overwhelms the marginal increase in the ability to pay—the *income effect*. The more complementary c_N and c_T are, then the less overwhelming the substitution effect becomes. This is because unbalanced bundles are less efficient than balanced ones. A sufficient condition for the income effect to always dominate is an elasticity of substitution that is less than 1 (see the proof in Appendix A).

Misallocation—Proposition 2 implies that q is an increasing function of Λ (for $\eta < 1$). This implies a trade off between increasing non-tradable consumption in period 1 (lower Λ_1) and increasing tradable consumption in period 0 through higher borrowing. Under the constrained efficient

[9] In general, default has a dual effect on c_T: it reduces tradable output through the productivity penalty, but it increases available resources by not having to export B_1. By definition, at z^* it must be the case that default increases c_T, otherwise repayment would be strictly preferred.

allocation, the household's Euler equation would be

$$0 \leq \mathbb{E}\left[\beta u'\left(\hat{c}_1\right)\left(\hat{R}_{N,1} - \hat{R}_{T,1}\right)\right]$$

which is inconsistent with optimal behavior. From the household's point of view, there are excess returns to capital in the non-traded sector under the constrained efficient allocation. This implies that, from the point of view of the planner, households underinvest in the tradable sector and overinvest in the non-tradable. As was the case with Model 1, the disagreement is more severe when the desire to borrow is high and when lenders are more sensitive to small changes in default risk.

3 Quantitative analysis

I now extend the environment from Section 2 to an infinite-horizon model of sovereign default with production and capital accumulation that features both externalities. The model builds on the existing literature that follows the seminal work of Eaton and Gersovitz (1981) and its main innovation is to contrast an economy in which the private sector makes all investment decisions with an economy where all allocations are chosen by a central planner. The constrained efficient allocation can be decentralized with appropriate wedges that are akin to investment subsidies in each sector. I analyze the properties of these wedges over the business cycle and during periods of distress using a standard parametrization and a calibration with values in line with those used in the literature.

3.1 Environment

Time is discrete and runs forever. There is a small-open economy populated by a measure one of households and a benevolent government. Households own all capital and firms in the economy but lack access to foreign borrowing. The government borrows on behalf of the households in international financial markets and lacks commitment to repay its debt.

Preferences and technology.—Households have preferences for streams of consumption of a final non-tradable good represented by $U\left(\{c_t\}_{t=0}^{\infty}\right) = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right]$, where $0 < \beta < 1$ is a discount factor and $u(c)=\frac{c^{1-\sigma}}{1-\sigma}$. The final good is produced by a competitive firm using technology $F(c_N,c_T)=\left[\omega^{\frac{1}{\eta}}c_N^{\frac{\eta-1}{\eta}}+(1-\omega)^{\frac{1}{\eta}}c_T^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, where c_N and c_T are tradable and non-tradable intermediate goods and $\eta<1$ is the elasticity of substitution.[10] All prices are denominated in terms of the intermediate tradable good. The relative price of the non-tradable intermediate is $p_N=\left(\frac{\omega}{1-\omega}\frac{c_T}{c_N}\right)^{\frac{1}{\eta}}$ and $P=\left[\omega p_N^{1-\eta}+(1-\omega)\right]^{\frac{1}{1-\eta}}$ is the price index of the final good. Intermediate goods $j\in\{N,T\}$ are produced by competitive firms using technology $y_j=zK_j^{\alpha_j}$, where $\alpha_j\in(0,1)$, K_j is capital in sector j, and z is a productivity shock. Productivity follows an AR(1) process $\log z_t=\rho\log z_{t-1}+\epsilon_t$, where $\rho\in(0,1)$ is a persistence parameter and $\epsilon_t\sim N\left(0,\sigma_z^2\right)$. There are two stocks of capital in the economy—one for each sector—which depreciate at a rate δ. Capital is owned by the households and rented to the firms for a rental rate r_j, where $r_N=p_N\alpha_N zK_N^{\alpha_N-1}$ and $r_T=\alpha_T zK_T^{\alpha_T-1}$. The final good is purchased by the households and can be used for consumption and investment. Households make the investment goods and the cost, in units of the final good, of producing i_j units of the investment good j is $i_j+\Psi\left(i_j,k_j\right)$, where $\Psi(i_i,k_i)=\frac{\phi}{2}\frac{(i_i)^2}{k_i}$.[11] The shadow price of investment good j is $P_{k,j}=1+\phi\frac{I_j}{K_j}$. The budget constraint of a representative household is:

$$P_t\left(c_t+\sum_{j\in N,T}\left[i_{j,t}+\Psi\left(i_{j,t},k_{j,t}\right)\right]\right)=\sum_{j\in N,T}\left(r_{j,t}k_{j,t}\right)+\Pi_t+T_t \tag{19}$$

where T_t is a lump-sum transfer from the government and Π_t are the profits made by all firms in the economy. Note that all prices and Π_t are functions of the aggregate state. The law of motion of capital in sector j owned by a representative household is

$$k_{j,t+1}=i_{j,t}+(1-\delta)\,k_{j,t}\qquad j\in\{N,T\} \tag{20}$$

Government debt and default.—The government is benevolent and can issue long-term, non-contingent debt. Following Chatterjee and Eyigungor (2012), I assume that debt matures at a rate γ and the fraction $(1-\gamma)$ that remains outstanding pays a coupon κ. At the beginning of each period, the government observes the state of the economy and, if it is in good financial standing, decides

[10]Standard values for η used in the literature range between 0.4 and 0.83. See Stockman and Tesar (1995), Mendoza (2005), and Bianchi (2011).

[11]Thus, capital cannot be imported or exported directly. This assumption captures the idea that productive capital has a significant non-tradable component, usually in the form of construction or land.

whether to repay or default. If the government repays it gets to issue new debt $i_{b,t} = B_{t+1} - (1-\gamma) B_t$ for a price q_t. Debt is purchased by risk-neutral competitive lenders with deep pockets and discount factor e^{-r^*}. The government's budget constraint in repayment is $T_t = q_t i_{b,t} - [\gamma + \kappa(1-\gamma)] B_t$, where T_t is a lump-sum transfer (or tax) of the tradable good to the households. If the government defaults, then $T_t = 0$ and it gets excluded from financial markets. When the government is in autarky, it gets readmitted to financial markets with probability θ and zero debt. Also, when the government is in default productivity is $z_D(z) = z - \max\{0, d_0 z + d_1 z^2\}$, with $d_0 < 0 < d_1$.[12] This implies that all prices and profits in the budget constraint of the household (19) also depend on whether the government is in good standing or in default.

Timing within a period.—At the beginning of each period, after all shocks are realized, the government observes the state of the economy and decides whether to repay or default. If the government repays then it chooses a debt issuance and a lump-sum transfer to satisfy its budget constraint, taking as given the price schedule q_t and how households will respond to policy. The government can commit to policy within the same period. Then, households observe the government's policy and make their investment decisions. Finally, lenders observe borrowing and investment decisions and purchase the government debt.

3.2 Recursive formulation

The aggregate state of the economy is (x, z), where $x = (B, K)$ and $K = (K_N, K_T)$. Denote the government policy as $g = (d, B', T)$, where d is the default decision in the current period, B' debt chosen for the next period, and T is a lump-sum transfer. The individual state of a representative household is $k = (k_N, k_T)$. The value of a household when the government is in good financial standing is:

$$H^P(g, x; k, z) = \max_{c, i_N, i_T, k'} \left\{ u(c) + \beta \mathbb{E}\left[d' H^D(K'; k', z') | z \right] + \beta \mathbb{E}\left[(1-d') H^P(g', x'; k', z') | z \right] \right\} \tag{21}$$

[12]Here, I also follow Chatterjee and Eyigungor (2012). Note that, except for differentiability, this function for productivity in default satisfies all of the assumptions in Section 2.

where the maximization problem is subject to the household's budget constraint (19) in repayment, the laws of motion for capital (20), household's beliefs about the evolution of aggregate capital stocks in repayment $K' = \Gamma_K^P(g,x,z)$, and households beliefs about future government policy $g' = \Gamma_g(x',z')$. When the government is in default, the value of a representative household is:

$$H^D(K;k,z) = \max_{c,i_N,i_T,k'} \left\{ u(c) + \beta(1-\theta)\mathbb{E}\left[H^D(K';k',z')\,|z\right] \right. \\ \left. +\beta\theta\mathbb{E}\left[(1-d')H^P(g',x';k',z') + d'H^D(K';k',z')\,|z\right]\right\} \tag{22}$$

where the maximization problem is subject to the household's budget constraint (19) in default, the laws of motion for capital (20), household's beliefs about the evolution of aggregate capital stocks in default $K' = \Gamma_K^D(g,x,z)$, and households beliefs about future government policy $g' = \Gamma_g(x',z')$ with $x' = (0,K')$.

Given the above value functions, the value of the government at the beginning of a period in good financial standing is

$$G(x,z) = \max_{d\in\{0,1\}} \left\{dG^D(K,z) + (1-d)G^P(x,z)\right\} \tag{23}$$

, where d is the government's default decision. The value of default is

$$G^D(K,z) = u\left(c^D(K;K,z)\right) + \beta(1-\theta)\mathbb{E}\left[G^D(K',z')\,|z\right] \\ +\beta\theta\mathbb{E}\left[G(x',z')\right]$$

where $x' = (0,K')$, $K' = k^D(K;K,z)$, and k^D and c^D are the household's policy functions for consumption and both capital choices in default. The value of repaying is

$$G^P(x,z) = \max_{T,B'} H^P((0,B',T),x;K,z) \tag{24}$$
$$s.t. \quad T = q(x',z)\left[B' - (1-\gamma)B\right] - (\gamma + \kappa(1-\gamma))B$$
$$x' = \left(B', k^P(g,x;K,z)\right)$$

where k^P is the household's policy function for both capital choices in repayment, and q is the price schedule of B'. Denote the government's policy function as $g(x,z) = (d(x,z), B(x,z), T(x,z))$.

Since lenders are risk neutral, the price q is actuarially fair:

$$q(x',z) = \mathbb{E}\left[e^{-r^*}\{1-d'\}\{\gamma + (1-\gamma)(\kappa + q(x'',z'))\}\right] \tag{25}$$

where d' and x'' are lender's beliefs about default, capital, and debt choices in the next period. The dependence on x' follows from the timing assumption (i.e. the auction happens after all investment and borrowing choices have been made).

Competitive equilibrium.—A *competitive equilibrium* is value and policy functions for the household, value and policy functions for the government, household beliefs, and a price schedule $\tilde{q}$ such that: (i) given all prices and government policy functions, the value and policy functions for the household solve the problems in (21) and (21) for $g = g(x,z)$; (ii) given all prices and household's policy functions, the value and policy functions for the government solve the problems in (23) and (24); (iii) household beliefs are consistent $\Gamma_K^P = k^P(g,x;K,z)$, $\Gamma_K^D = k^D(K;K,z)$, $\Gamma_g = g(x,z)$; (iv) the price schedule $\tilde{q}$ satisfies equation (25) with $d' = d(x,z)$, $x'' = (K'',B'')$, $K'' = k^P\left(g^P(x',z'), x'; K', z'\right)$, and $B'' = B(x',z')$.

Note that the above definition only requires conditions to hold along the equilibrium path, but not necessarily off it. For instance, $k^P(g,x;K,z)$ is only required to solve the maximization problem in (21) when $g = g^P(x,z)$. Given this, let $\tilde{K}^P(x,z) = k^P(g(x,z),x;K,z)$ and $\tilde{K}^D(K,z) = k^D(K;K,z)$ be the functions that describe the evolution of capital along the competitive equilibrium path.

3.3 Efficiency and decentralization

Consider now a benevolent social planner that can choose all allocations in the economy. The value of the planner in good financial standing is

$$V(x,z) = \max_{d\in\{0,1\}}\left\{dV^D(K,z) + (1-d)V^P(x,z)\right\}$$

, where $\hat{d}(x,z)$ as the default policy function that solves the above maximization problem. The value of default is:

$$V^{D}(K,z) = \max_{c,I_N,I_T,K'} \left\{ u(c) + \beta(1-\theta)\mathbb{E}\left[V^{D}(K',z')\right] + \beta\theta\mathbb{E}\left[V(x',z')\right] \right\}$$

where the maximization problem is subject to the laws of motion for capital $K_j' = I_j + (1-\delta)K_j$ for $j = N,T$, and the resource constraints in default $c + \sum_{j\in N,T}\left[I_j + \Psi\left(I_j,K_j\right)\right] \leq F(c_N,c_T)$, $c_N = z_D(z)K_N^{\alpha_N}$ and $c_T = z_D(z)K_T^{\alpha_T}$. Denote $\hat{K}^{D}(K,z)$ as the planner's policy function for capital in default. The value of repayment is

$$V^{P}(x,z) = \max_{c,I_N,I_T,x'} \left\{ u(c) + \beta\mathbb{E}\left[V(x',z')\right] \right\}$$

where the maximization problem is subject to the laws of motion for capital, and the resource constraints in repayment $c + \sum_{j\in N,T}\left[I_j + \Psi\left(I_j,K_j\right)\right] \leq F(c_N,c_T)$, $c_N = zK_N^{\alpha_N}$ and $c_T = zK_T^{\alpha_T} + \hat{q}(x',z)\left[B' - (1-\gamma)B\right] - (\gamma + \kappa(1-\gamma))B$. Here, $\hat{q}$ is the price schedule for bonds issued by the planner. Denote $\hat{K}^{P}(x,z)$ and $\hat{B}(x,z)$ as the capital and debt policy functions for the planner in repayment.

A *constrained efficient equilibrium* is value and policy functions for the planner and a price schedule $\hat{q}$ such that: (i) given $\hat{q}$, the value and policy functions solve the planner's problem; and (ii) the price schedule $\hat{q}$ satisfies equation (25) with $d' = \hat{d}(x,z)$ and $x'' = \left(\hat{K}^{P}(x',z'), \hat{B}(x',z')\right)$.

Discussion.—As with the two-period models, capital allocations in the competitive equilibrium are inefficient because households fail to internalize how these affect future default incentives and, thus, present borrowing costs. The Euler equations of a representative household when the government is in good financial standing are:

$$u'(\tilde{c})\tilde{P}_{k,j} = \beta\mathbb{E}\left[\tilde{d}'u'(\tilde{c}')\tilde{R}_j'|z\right] + \beta\mathbb{E}\left[\left(1-\tilde{d}'\right)u'(\tilde{c}')\tilde{R}_j'|z\right] \quad j \in \{N,T\} \tag{26}$$

where $\tilde{R}_j = \frac{\tilde{r}_j}{\tilde{P}} + (1-\delta)\tilde{P}_{k,j} - \tilde{\Psi}_{2,j}$ is the return to capital in sector j in terms of the final consumption good; $\tilde{\Psi}_{2,j}$ is the derivative of the adjustment cost function with respect to its second argument evaluated at choices for capital and investment in sector j; $\tilde{c}$ is the household's policy functions for consumption; and $\tilde{d}$ is the government's policy function for default. In order to ease exposition, primes indicate variables dependent on the state in the next period. I also use tildes to denote prices evaluated at the states induced by the functions $\tilde{d}$, $\tilde{K}^D$, $\tilde{K}^P$, and $\tilde{B}$ defined above, as well as other competitive equilibrium policy functions.[13]

Using similar notation, the Euler equations for capital from the planner's problem in repayment can be written as

$$u'(\hat{c})\left[\hat{P}_{k,j} - \frac{\partial \hat{q}}{\partial K'_j}\frac{\hat{B}' - (1-\gamma)\hat{B}}{\hat{P}}\right] = \beta\mathbb{E}\left[\hat{d}' u'(\hat{c})\hat{R}_j|z\right] + \beta\mathbb{E}\left[\left(1-\hat{d}'\right)u'(\hat{c})\hat{R}_j|z\right] \quad j \in \{N,T\} \tag{27}$$

where prices have the same functional form described above but the hat indicates that they are evaluated at allocations induced by the planner's policy functions.

The above Euler equations only differ in the presence of the terms $-\frac{\partial \hat{q}}{\partial K'_j}\frac{\hat{B}'-(1-\gamma)\hat{B}}{\hat{P}^P}$ on the left-hand-side of equation (27), which indicate how borrowed resources change with investment—the margin that is ignored by the households since they take the evolution of aggregate capital as given. The magnitude of the disagreement depends on the planner's desire to borrow (i.e. the optimal borrowing choice) given the state, on the real exchange rate (defined as $1/P$), and on the sensitivity to investment of the planner's price schedule $\hat{q}$ (note that, absent default risk, $\hat{q}$ would be constant and the disagreement would vanish).

Proposition 3. (*First-best subsidies*) The constrained efficient equilibrium can be implemented as a competitive equilibrium with state-contingent subsidies to investment in repayment equal to $\tau_j(x,z) = \frac{\partial \hat{q}(\hat{x}',z)}{\partial K'_j}\frac{\hat{B}'-(1-\gamma)\hat{B}}{\hat{P}(x,z)}$, where $\hat{x}' = \left(\hat{K}^P(x,z), \hat{B}(x,z)\right)$.

Proof: Obvious from equations (26) and (27).□

Note that Proposition 3 does not require these subsidies to satisfy the government's budget constraint, which implies that their implementation is feasible if and only if the subsidy to one type of investment is perfectly offset by a tax (negative subsidy) to the other. This is unlikely to be the case. However, studying the properties of τ_j is a useful first-step to understand the degree of inefficiency and to shed light on desired characteristics for feasible policy recommendations, in particular their sign and cyclical properties.[14]

[13] For instance $\tilde{r}_N$ is really a function of the aggregate state and government policy. If $\tilde{d} = 1$ then $\tilde{r}_N(g,x,z) = p_N(g,x,z)\alpha_N z_D(z) K_N^{\alpha_N - 1}$, with $p_N((1,0,0),x,z) = \left(\frac{\omega}{1-\omega}\frac{z_D(z)K_T^{\alpha_T}}{z_D(z)K_N^{\alpha_N}}\right)^{\frac{1}{\eta}}$.

[14] As can be seen in the following subsection, due to the dimensionality of the state space, the computation of the competitive equilibrium is extremely demanding. Using it to do a calibration exercise or to attempt to solve a Ramsey

The following two subsections illustrate how, under a standard calibration, the signs and magnitudes of $\frac{\partial \hat{q}}{\partial K'_N}$ and $\frac{\partial \hat{q}}{\partial K'_T}$ are consistent with the intuition implied by Propositions 1 and 2 in Section 2: (i) the price $\hat{q}$ that the planner faces is increasing in the total stock of capital ($K' = K'_N + K'_T$) keeping the portfolio fixed (the *capital-stock externality*), and (ii) for a fixed amount of aggregate capital $K' = K'_N + K'_T$, $\hat{q}$ is increasing in the share Λ' of K' allocated to the tradable sector (the *portfolio externality*). I also use this quantitative exercise to analyze the cyclical properties of τ_j and of the total cost, as a fraction of GDP, of implementing the subsidies $s_t = \left(\tau_{N,t} K'_{N,t} + \tau_{T,t} K'_{T,t}\right)/GDP$.

3.4 Computation and calibration

I solve both the competitive and the constrained efficient equilibrium using value function iteration. Following Hatchondo, Martinez, and Sapriza (2010), I compute the limit of the finite-horizon version of the economy in both cases. In the constrained efficient case, I jointly solve for optimal investment and borrowing decisions using a non-linear optimization routine in each iteration. In the competitive case, I use Newton methods to find investment decisions that jointly solve the household's Euler equations for a given borrowing level. To find the optimal borrowing choice, I use a non-linear optimization routine where the objective function takes into account how each potential choice affects the solution to the household's Euler equations. I approximate value functions and the price schedule for bonds using linear interpolation, and compute expectations over the productivity shock using a Gauss-Legendre quadrature.

A period in the model corresponds to one quarter. There are two sets of parameters: one with values taken from the literature and another chosen to match some stylized facts from the data. I use the planner's problem for the moment-matching exercise.[15] The calibration is summarized in Table 1.

problem for optimal and feasible subsidies is computationally impractical. However, studying some properties of a Ramsey allocation is an exciting future avenue of research in this topic.

[15]The numerical solution of the model is computationally demanding given the dimensionality of the state space. In particular, the computation of the competitive equilibrium—which would ideally be used in a moment-matching calibration exercise—is an order of magnitude slower than that of the central planner—which is typically used in moment-matching calibration exercises in the literature. The last two columns in the second part of Table 1 suggest that parameters chosen to match moments in the decentralized equilibrium may not be too different from the ones chosen here.

Table 1: Parameter values

Parameter	Value	Parameter	Value	Parameter	Value
σ	2	r^*	0.01	ρ	0.95
η	0.83	ω	0.6	σ_z	0.017
α_N	0.33	α_T	0.33	γ	0.05
δ	0.07	θ	0.0625	κ	0.03
Parameter	**Value**	**Moment**	**Target**	**Planner (targeted)**	**Decentralized (untargeted)**
β	0.97	$\frac{B}{GDP}$	0.53	0.52	0.56
ϕ	2.61	$\frac{\sigma_i}{\sigma_y}$	2.0	2.0	3.4
d_0	-0.19	Av (spread)	2.0%	2.0%	3.8%
d_1	0.266	Std (spread)	2.0%	1.25%	1.4%

To compute the model moments I draw 300 samples of 1,050 periods and drop the first 1,000. Each sample is chosen to start at least 25 periods after the most recent default. Spreads are computed as $r_t - r^*$, where $1 + r_t = \left[1 - \log\left(\frac{q_t}{\gamma+(1-\gamma)(\kappa+q_t)}\right)\right]^4$.

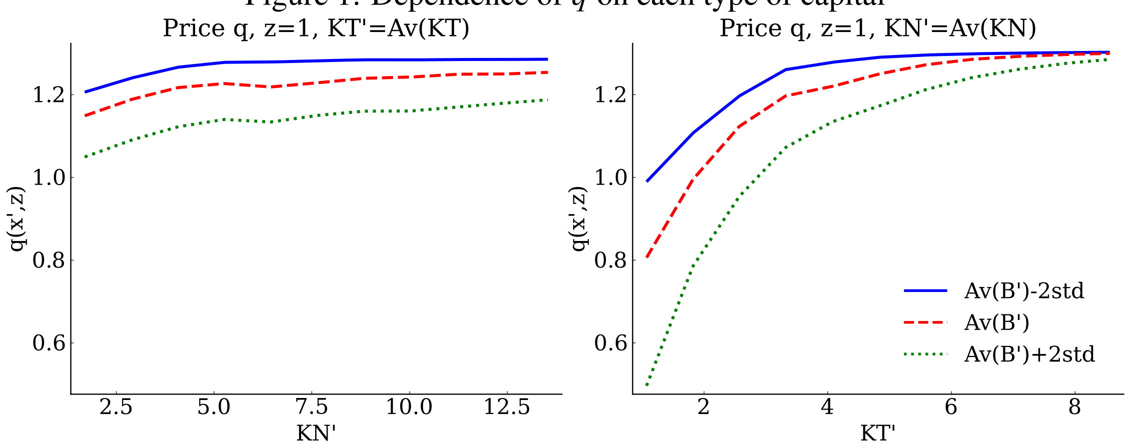
The risk-free interest rate is $r^* = 0.01$ and the CRRA parameter is $\sigma = 2$, which are standard values in business cycle and sovereign default studies. The elasticity of substitution between traded and non-traded goods is $\eta = 0.83$ and the share of non-traded is $\omega = 0.6$; both of which I take from Bianchi (2011). The capital shares are $\alpha_N = \alpha_T = 0.33$, the capital depreciation rate is $\delta = 0.07$, and the parameters governing the stochastic process for productivity are $\rho = 0.95$ and $\sigma_z = 0.017$, which are all standard values. The probability of reentry $\theta = 0.0625$ is set so that the average exclusion period after default is 4 years, which is the median duration documented by Gelos, Sahay, and Sandleirs (2011). I take the debt duration parameter $\gamma = 0.05$ and the coupon rate $\kappa = 0.03$ from Chatterjee and Eyigungor (2012). The discount factor β, productivity loss parameters d_0 and d_1, and the capital adjustment cost parameter ϕ are set to jointly match an average debt-to-GDP ratio of 0.53, relative volatility of total investment to GDP of 2, average spreads of 2%, and standard deviation of spreads of 2%.

The lower part of Table 1 reports these moments for the planner's problem (used in the moment-matching exercise) and in the decentralized equilibrium, both using the exact same parametrization and calibration. The decentralized economy experiences higher and more volatile spreads, a higher relative volatility of investment, and a slightly higher debt-to-GDP ratio.

3.5 Underinvestment and sectoral misallocation

Similar to the two-period models, equations (26) and (27) show that the signs of the derivatives of $\hat{q}$ determine whether the competitive equilibrium features over- or under-investment in each sector. Figure 1 illustrates how $\hat{q}$ is increasing in K'_T (right pannel) and mostly increasing in K'_N (left pannel). Moreover, $\hat{q}$ is more sensitive to capital in the traded sector than to capital in the non-traded sector. To understand why this is the case, it is useful to borrow some intuition from Propositions 1 and 2. Keeping everything else constant, an increase in K'_T increases both the aggregate stock of capital and the share of capital in the tradable sector, both of which lower default incentives for the next period—recall that debt is denominated in terms of the tradable good. In contrast, an increase in K'_N increases the aggregate stock of capital, but reduces the share of capital in the tradable sector. Propositions 1 and 2 suggest that these have opposite effects on default incentives, which explains why $\hat{q}$ is "flatter" on K'_N and more sensitive to K'_T.

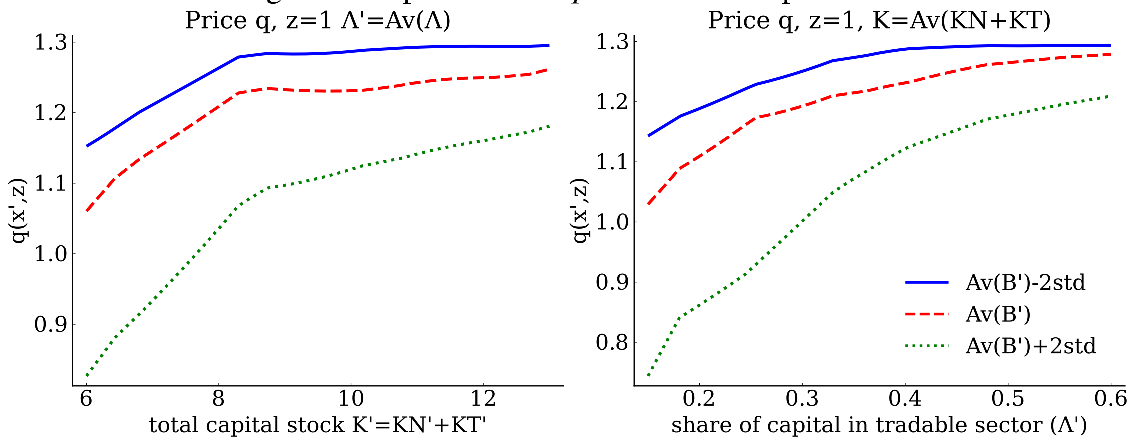
Figure 1: Dependence of $\hat{q}$ on each type of capital



In both graphs, the shock is set to $z = 1$. On the left panel, K'_T is set to its average in the ergodic distribution. On the right panel, K'_N is set to its average. Finally, B' is set to its average minus two standard deviations in the blue-solid lines, to its average in the red-dashed lines, and to its average plus two standard deviations in the green-dotted lines.

To make the above point clearer, Figure 2 shows how $\hat{q}$ depends on the total stock of capital $K' = K'_N + K'_T$ while keeping the portfolio constant (left panel), and how it depends on the capital portfolio Λ' while keeping the aggregate stock constant (right panel). These suggest that, as it was the case in the two-period models, households in the decentralized equilibrium underinvest overall and allocate a smaller share of capital in the tradable sector (both relative to what the central planner would choose).

Figure 2: Dependence of $\hat{q}$ on total K and portfolio



In both graphs, the shock is set to $z = 1$ and the price $\hat{q}(x', z)$ is interpolated in order to evaluate it at $\hat{q}(B', \Lambda' K', (1-\Lambda')K', 1)$ for certain values for K' and Λ'. On the left panel, Λ' is set to its average in the ergodic distribution. On the right panel, $K' = K'_N + K'_T$ is set to its average. Finally, B' is set to its average minus two standard deviations in the blue-solid lines, to its average in the red-dashed lines, and to its average plus two standard deviations in the green-dotted lines.

The above graphs show that the capital externalities in this model have the same qualitative properties as in the two-period models from Section 2. In order to have a notion of how quantitatively relevant these inefficiencies are, Table 2 compares the average values of different variables for the planner and the decentralized economy over a long time series. Columns (1) and (2) show how the planner accumulates more capital, both in absolute terms and as a fraction of GDP. In absolute terms, the planner accumulates almost twice as much capital.

Table 2: Underinvestment and misallocation

	$K_N + K_T$	$\frac{P*(K_N+K_T)}{GDP}$	$\Lambda = \frac{K_T}{K_N+K_T}$	B	$\frac{B}{GDP}$	c	$rer = \frac{1}{P}$	σ_{rer}
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Decentralized	6.3	3.5	0.38	1.17	0.56	1.50	0.84	1.36
Planner	10.2	4.9	0.41	1.32	0.52	1.66	0.83	1.24

To compute the above moments, I draw a long time series of 11,000 periods and drop the first 1,000. The reported numbers are the averages for each variable along the 10,000 periods except for those regarding borrowing, which are the averages conditional on being in good standing, and those in Column (8), which are the standard deviations of the real exchange rate expressed in log deviations from its mean.

Column (3) shows how the planner allocates a higher share of capital in the tradable sector. This induces a weaker real exchange rate in the decentralized economy, as can be seen in Column (7). Column (4) shows how the planner's investment decisions allow it to sustain a higher level of debt. These results highlight how the higher debt-to-GDP ratio in Column (5) is misleading, because the planner's GDP is much higher. Column (6) shows how consumption is, on average, around ten percent higher under the constrained efficient allocation. Finally, Column (8) shows how the real exchange rate is also more volatile in the decentralized economy, which suggest larger swings in the composition of the consumption basket.

Table 3 shows average values for first-best subsidy rates and costs relative to GDP. All values are expressed in percentage units. As expected, both sectors require a subsidy given the level of aggregate underinvestment. However, the fist-best subsidy to tradable investment is much larger than the one to non-tradable investment.

Table 3: First-best subsidies over the business cycle

τ_N	τ_T	$\frac{\tau_N I_N}{Av(GDP)}$	$\frac{\tau_T I_T}{Av(GDP)}$	$\frac{\tau_N I_N + \tau_T I_T}{Av(GDP)}$
0.18	0.88	0.02	0.08	0.10

To compute these moments I draw 300 samples of 1,050 periods and drop the first 1,000. Each sample is chosen to start at least 25 periods after the most recent default. I use the decentralized equilibrium to draw the samples of the states and the price and policy functions of the planner to compute first-best $\tau_{N,t}$ and $\tau_{T,t}$ given the state at t. All values are expressed in percentage units.

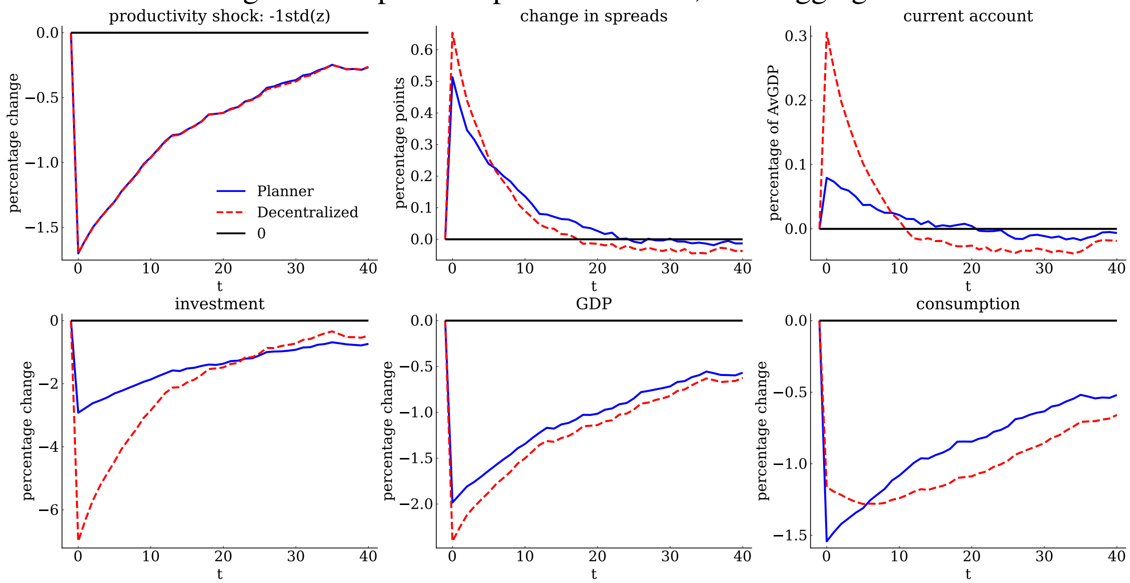
The magnitude of the above subsidies and their costs appears small, in particular if compared to the differences in consumption and capital stocks from Table 2. It is important to note that Table 2 compares averages over two different ergodic distributions. Very small distortions in each period—such as those suggested by Table 3—can amount to big differences in the long-run, as can be attested by the values in Table 2. To complement the above analysis, the following subsections study the behavior of macro variables after small shocks and around debt crises.

3.6 Response to shocks

Figure 3 shows the responses to a negative productivity shock of spreads, the current account, total investment, GDP, and final consumption for both the planner and the decentralized economy. On impact, spreads increase, there is a current account reversal, and investment, GDP and consumption drop. All of these responses are stronger in the decentralized equilibrium, except for that of consumption, which drops less on impact than in the planner's allocation.

The smaller drop in consumption from the decentralized economy is a direct consequence of the *capital-stock externality*. The planner understands that a smaller drop in investment tames the increase in default risk, which allows it to partially smooth the shock with a smaller current account reversal—which comes from its ability to borrow at better prices. Households fail to realize this effect and use lower investment to smooth the effect of the shock. Given the smaller drop in investment, the planner's GDP and consumption recover much faster.

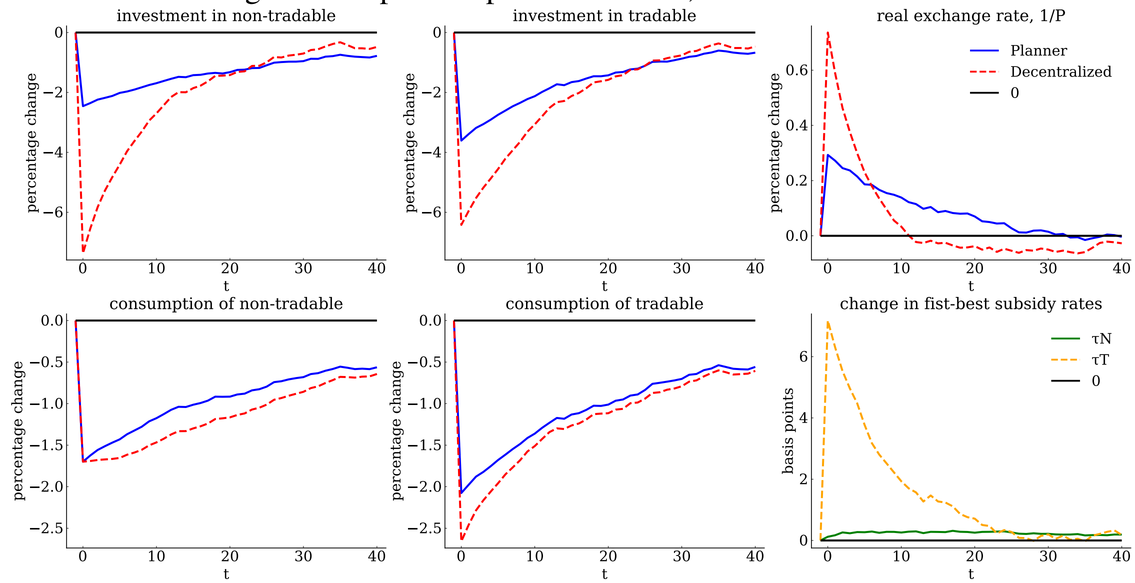
Figure 3: Impulse-response functions, main aggregates



Each line is the average of 10,000 simulated paths following a negative productivity shock in period $t = 0$ of one standard deviation $\Delta \log(z_t) = -\sigma_\epsilon$. From period $t = 1$ onward, z follows its normal Markov process. The aggregate state in $t = -1$ is taken from the ergodic distribution after dropping the initial 1,000 periods. I only consider initial periods for which the economy has been in good financial standing for at least 25 consecutive periods. I only consider paths without default episodes from $t = 0$ onward.

Figure 4 shows the responses of investment in each sector, consumption of intermediates, the real exchange rate, and first-best subsidy rates to the same shock. In the decentralized economy, investment in the non-tradable sector falls more than investment in the tradable sector. The reverse is true for the planner. The planner understands that, during the recovery in the subsequent periods, it will have more tradable resources available. Having a lower drop in non-tradable investment allows the planner to recover aggregate consumption faster. The larger drop in future tradable output due to the drop in investment is partially off-set by the planner's ability to borrow more.

Figure 4: Impuse-response functions, sectoral variables



Each line is the average of 10,000 simulated paths following a negative productivity shock in period $t = 0$ of one standard deviation $\Delta \log(z_t) = -\sigma_\epsilon$. From period $t = 1$ onward, z follows its normal Markov process. The aggregate state in $t = -1$ is taken from the ergodic distribution after dropping the initial 1,000 periods. I only consider initial periods for which the economy has been in good financial standing for at least 25 consecutive periods. I only consider paths without default episodes from $t = 0$ onward. I compute fist-best subsidy rates using the paths of states from the decentralized equilibrium and the price and policy functions from the central planner.

With respect to consumption of intermediate goods, consumption of tradables drops more than consumption of non-tradables. As Arellano, Bai, and Mihalache (2018) explain for a similar environment, this larger drop of tradable consumption is due to the reversal in the current account: more tradable resources have to be exported to service the debt. This uneven response of c_N and c_T drives a depreciation of the real exchange rate—which I define as $1/P$.

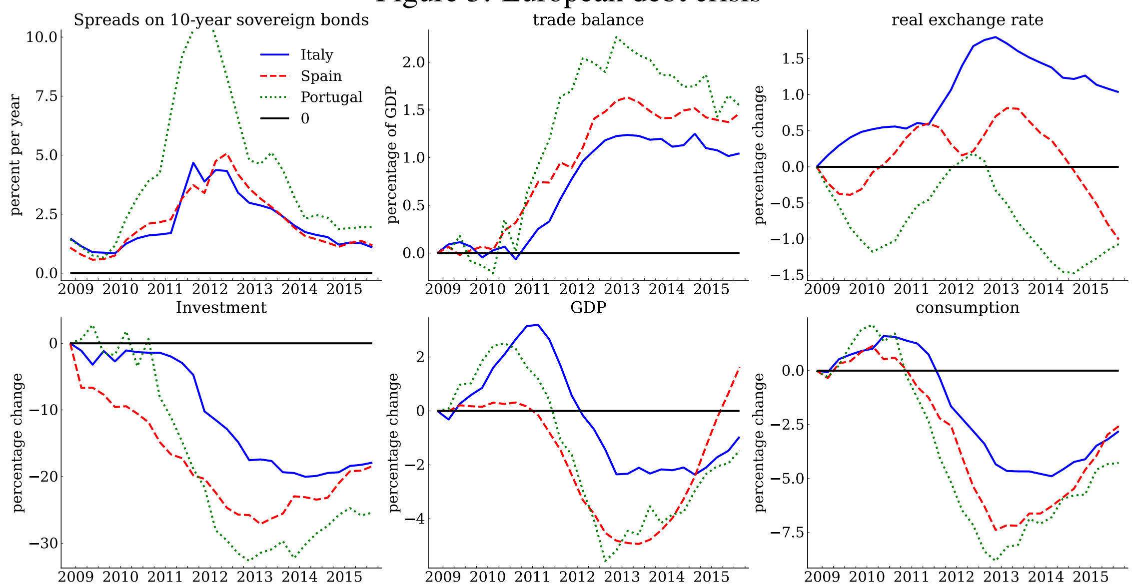
Fist-best subsidies increase, but do so substantially more for tradable investment than for non-tradable. It is important to note that these subsidies are computed using policy and price functions from the planner, as in Proposition 3, but evaluated at the state of the decentralized economy in each period. This means that the subsidies in the plot do not implement the responses of the planner in the plot. These subsidies implement the responses that the planner would have if the economy was in those particular states, which were induced by the policy functions from the decentralized equilibrium.

Finally, note that the real exchange rate, the current account, and spreads all recover faster in the decentralized economy than for the planner. These faster recoveries are detrimental for the households because they come at the expense of a slower recovery of consumption.

3.7 Application: European debt crisis

I now use the European debt crisis as a case study to analyze how the capital externalities in the model affect aggregate economic outcomes during realistic periods of distress. I simulate paths in the model so that spreads increase by three standard deviations without a default, which mimics the behavior of government spreads for Italy, Spain, and Portugal.[16] Then, I contrast the paths of other model variables to those in the data in order to validate the model's ability to generate a similar crisis. Figure 5 presents quarterly data for spreads, the trade balance, the real exchange rate, investment, GDP, and consumption from the first quarter of 2009 to the fourth quarter of 2015.

Figure 5: European debt crisis



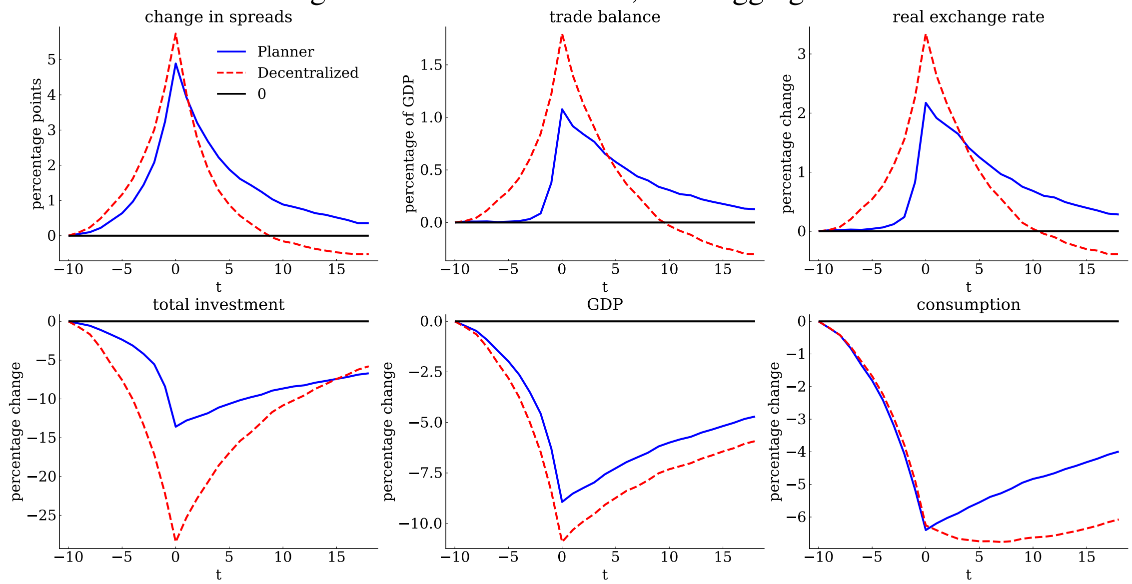
Data are quarterly. To compute spreads I use Maastricht criterion interest rates, whose selection guidelines require data to be based on central government bond yields on the secondary market, gross of tax, with a residual maturity of around 10 years. Spreads are relative to German interest rates. All panels show the cummulative change from the first quarter of 2009 except for the one for spreads, which show the level.

Spreads spike at around the second quarter of 2011, they increase by roughly 3 percentage points in Italy and Spain and 10 in Portugal. The trade surplus increases by around 1.5 percentage points of GDP in each country and the real exchange rate depreciates for Portugal and Italy. Investment, GDP, and consumption all drop significantly and experience a slow recovery. Figure 6 shows average paths of the same variables generated by the model. I choose paths for which spreads are three standard deviations above their mean in period $t = 0$ and for which there is no default episode. I plot 10 periods prior and 18 after the core of the crisis so that these paths have

[16] I exclude Greece from the sample because the Greek government actually defaulted. The data for Greece look similar to the data in Figure 5 but with changes of a much larger magnitude that dwarf those of the other countries.

the same length as the data.

Figure 6: Model debt crisis, main aggregates



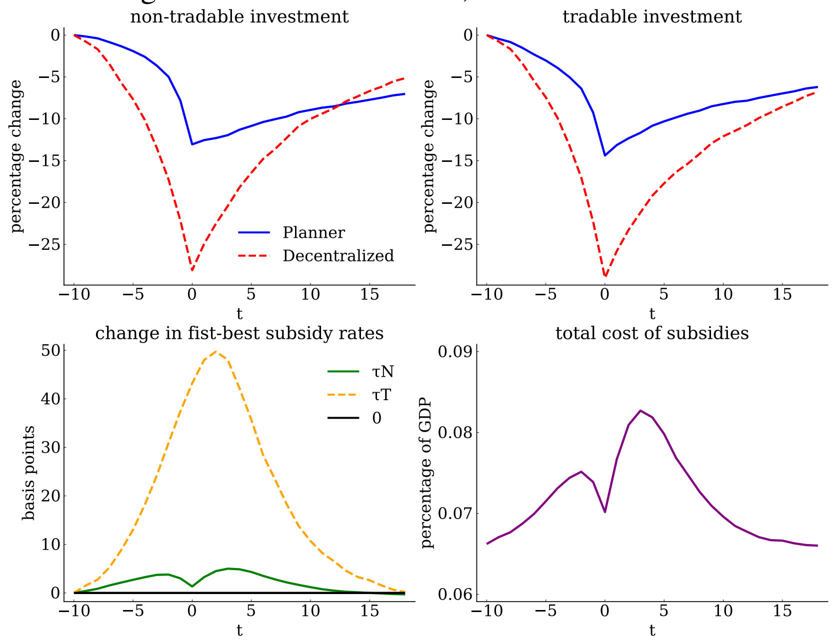
Each line is the average of 1,500 simulated paths for which spreads are three standard deviations above their mean in period $t = 0$ and for which there is no default episode. I plot 10 periods prior and 18 after the core of the crisis so that these paths have the same length as the data. The aggregate state in $t = -10$ is taken from the ergodic distribution after dropping the initial 1,000 periods. I only consider initial periods for which the economy has been in good financial standing for at least 25 consecutive periods.

For both the central planner and the decentralized equilibrium, all variables respond in the same direction as in the data. In addition, the magnitudes of the responses in the decentralized equilibrium are very close to those of the data, except for the real exchange rate and GDP, which are slightly larger in the model.

The top panels of Figure 7 show the paths of investment in each sector. The bottom panels show the first-best subsidy rates for the state of the decentralized economy in each period (left) and the total cost of implementing them (right).[17]

[17] See the discussion at the end of Subsection 3.6.

Figure 7: Model debt crisis, sectoral investment



Each line is the average of 1,500 simulated paths for which spreads are three standard deviations above their mean in period $t = 0$ and for which there is no default episode. I plot 10 periods prior and 18 after the core of the crisis so that these paths have the same length as the data. The aggregate state in $t = -10$ is taken from the ergodic distribution after dropping the initial 1,000 periods. I only consider initial periods for which the economy has been in good financial standing for at least 25 consecutive periods. I compute fist-best subsidy rates using the paths of states from the decentralized equilibrium and the price and policy functions from the central planner.

The percentage drop in investment is roughly the same for both sectors in the decentralized equilibrium and slightly larger for the tradable sector in the planner's case. However, fist-best subsidies substantially increase for tradable investment and mildly increase for non-tradable investment. This indicates how optimal policy considers both externalities by inducing the households to invest more, in general, but also by tilting investment more toward the tradable sector. The bottom-right panel shows that the cost of implementing fist-best subsidies increases during crisis periods; however, the magnitude suggests that implementing such policies comes at a relatively low cost in each period (less than 0.1 percent of GDP) but with large potential benefits given the substantial differences between the two equilibria.

4 Conclusion

I studied how capital and its allocation in different sectors affect default incentives. Using two two-period models, I showed that, under fairly general conditions, the aggregate stock of capital reduces default incentives and that the share of capital in the non-tradable sector increases them. Two externalities of private investment arise from these results: the *capital-stock externality* and the *portfolio externality*. These arise because private agents—who are price-takers—do not internalize how their investment decisions affect aggregate allocations and, through them, default incentives. I also show that the magnitude of the distortions is proportional to default risk, which implies that the externalities are amplified during debt crises.

I also developed a quantitative sovereign default model with production and private investment that featured both externalities. Under a standard calibration, the insights from the two-period models continue to hold in model simulations. The competitive equilibrium features underinvestment, a lower share of capital in the tradable sector, higher spreads, and lower consumption. All these relative to the constrained efficient allocation in which a benevolent planner makes borrowing and investment decisions directly. I show that the constrained efficient allocation can be implemented as a competitive equilibrium with appropriate distortions that are akin to investment subsidies.

I use the model to study the European debt crisis and find that it does a good job in reproducing its main features. I also find that first-best subsidies increase during the crisis as a result of the externalities being amplified by the larger default risk. This amplification implies that the competitive equilibrium features a deeper recession and a slower recovery.

The insights from this paper can be extended to richer production settings with private dynamic decisions. For example, frictional labor markets in which labor allocations persist through several periods would feature similar externalities. Another interesting extension would be to study how large endowments of natural resources affect the size and behavior of the portfolio externality through the classic Dutch disease mechanisms.

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A Proofs of Section 2

A.1 Proof of Proposition 1

Proposition 1. The default set is shrinking in K_1. That is, $\frac{\partial z^*(x_1)}{\partial K_1} \leq 0$.

Proof: Taking the full derivative of equation (3) and rearranging terms we get

$$\frac{\partial z^*(x_1)}{\partial K_1} = -\frac{\frac{\partial F(z^*(x_1),K_1)}{\partial K} - \frac{\partial F(z_D(z^*(x_1)),K_1)}{\partial K_1}}{\frac{\partial F(z^*(x_1),K_1)}{\partial z} - \frac{\partial F(z_D(z^*(x_1)),K_1)}{\partial z}\frac{\partial z_D(z^*(x_1))}{\partial z}}$$

where both the numerator and denominator are positive.

For the numerator, note that, by assumption, the cross derivative is $F_{zK} \geq 0$. This implies that $\frac{\partial F}{\partial K}$ is weakly increasing in z. Since $z_D(z) \leq z$ for all z then we get that $\frac{\partial F(z^*(x_1),K_1)}{\partial K} - \frac{\partial F(z_D(z^*(x_1)),K_1)}{\partial K_1} \geq 0$ and, thus, the numerator is positive.

For the denominator, note that by assumption F is weakly convex in z, so by a similar argument $\frac{\partial F(z^*(x_1),K_1)}{\partial z} - \frac{\partial F(z_D(z^*(x_1)),K_1)}{\partial z} \geq 0$. In addition, $\frac{\partial z_D(z^*(x_1))}{\partial z} \leq 1$ so we get that the denominator is also positive.□

A.2 Proof of Proposition 2

Proposition 2 holds for any given $x_1 = (\Lambda_1, B_1)$. For convenience of notation, I will refer to c_N^D and c_T^D as consumption of the non-tradable and tradable goods, respectively, in default at $z = z^*(x_1)$. Similarly, c_N^P and c_T^P as consumption of the non-tradable and tradable goods, respectively, in repayment at $z = z^*(x_1)$. The following two lemmas are used throughout the proof of Proposition 2.

Lemma 1: $c_N^D \leq c_N^P$ and $c_T^D \geq c_T^P$.

Proof: First, note that since N is non-tradable $y_N = c_N$, so we get $c_N^D \leq c_N^P$ from $z_D(z) \leq z$. Then, note that at z^* we have $F\left(c_N^P, c_T^P\right) = F\left(c_N^D, c_T^D\right)$, since F is increasing in both arguments then it must be that $c_T^D \geq c_T^P$ at z^*.□

Lemma 2: $\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_T} \geq \frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_T}$ and $\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_N} \leq \frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_N}$.

Proof: Note that:

$$\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} \geq \frac{\partial F\left(c_N^P, c_T^D\right)}{\partial c_T} \geq \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T}$$

$$\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_N} \leq \frac{\partial F\left(c_N^D, c_T^P\right)}{\partial c_N} \leq \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_N}$$

where the first inequality follows from Lemma 1 and concavity of F, and the second also follows from Lemma 1 and positive cross derivatives.□

Proposition 2. If the elasticity of substitution between tradable and non-tradable intermediates is $\eta < 1$, then the default set is shrinking in Λ_1. That is, $\frac{\partial z^*(x_1)}{\partial \Lambda_1} \leq 0$.

Proof: Taking the full derivative of equation (11) and rearranging terms we get

$$\frac{\partial z^*}{\partial \Lambda_1} = -\frac{\frac{\partial V^P(z^*(x_1),x_1)}{\partial \Lambda} - \frac{\partial V^D(z^*(x_1),\Lambda_1)}{\partial \Lambda}}{\frac{\partial V^P(z^*(x_1),x_1)}{\partial z} - \frac{\partial V^D(z^*(x_1),\Lambda_1)}{\partial z}} \tag{28}$$

where $V^D(z,\Lambda) = u\left(F\left(z_D(z) f(1-\Lambda), z_D(z) f(\Lambda)\right)\right)$ and $V^P(z,x) = u\left(F\left(zf(1-\Lambda), zf(\Lambda) - B\right)\right)$. Lemma 3 below establishes that the denominator is positive. Lemma 4 below establishes that the numerator is positive. Both results imply that $\frac{\partial z^*}{\partial \Lambda_1} \leq 0$.□

Lemma 3. $\frac{\partial V^P(z^*(x_1),x_1)}{\partial z} - \frac{\partial V^D(z^*(x_1),\Lambda_1)}{\partial z} \geq 0$.

Proof: Note that V^D and V^P are increasing in z

$$\frac{\partial V^D}{\partial z} = u'\left(c^D\right)\left[\frac{\partial F}{\partial c_N} f(1-\Lambda) + \frac{\partial F}{\partial c_T} f(\Lambda)\right]\frac{\partial z_D}{\partial z} \geq 0$$
$$\frac{\partial V^P}{\partial z} = u'\left(c^P\right)\left[\frac{\partial F}{\partial c_N} f(1-\Lambda) + \frac{\partial F}{\partial c_T} f(\Lambda)\right] > 0$$

for all (z,Λ,B). From the assumption that $\lim_{z\to 0}\left[z - z_D(z)\right] = 0$ it follows that, for any $B > 0$ and any $\Lambda \in (0,1)$, there exists a z_- such that $V^D(z_-,\Lambda) > V^P(z_-,\Lambda,B)$. That is, for any positive level of debt, there is a value for productivity low enough such that it is more convenient to default. Similarly, note that since $\frac{\partial z_D}{\partial z} < 1$ and $z_D(z) < z$ for $z > \bar{z}$, then there exists $z_+ < \infty$ such that $V^D(z_+,\Lambda) < V^P(z_+,\Lambda,B)$. Then, by the intermediate value theorem there is z^* such that $V^D(z^*,\Lambda) = V^P(z^*,\Lambda,B)$. Since both V^D and $V^P$are increasing and V^P is strictly increasing, then z^* is unique. Note that for $z < z^*$ we have $V^D(z,\Lambda) > V^P(z,\Lambda,B)$ and for $z > z^*$ we have $V^D(z,\Lambda) < V^P(z,\Lambda,B)$, then at z^* we get $\frac{\partial V^P(z^*(x_1),x_1)}{\partial z} > \frac{\partial V^D(z^*(x_1),\Lambda_1)}{\partial z}$.□

Lemma 4. If the elasticity of substitution between tradable and non-tradable intermediates is $\eta < 1$, then $\frac{\partial V^P(z^*(x_1),x_1)}{\partial \Lambda} - \frac{\partial V^D(z^*(x_1),\Lambda_1)}{\partial \Lambda} \geq 0$.

Proof: The derivative of V^P with respect to Λ is:

$$\frac{\partial V^P(z,x)}{\partial \Lambda} = u'\left(c^P\right)\left[\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} z f'(\Lambda) - \frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_N} z f'(1-\Lambda)\right]$$

where $c^P = F\left(c_N^P, c_T^P\right)$, $c_N^P = z f(1-\Lambda)$, and $c_T^P = z f(\Lambda) - B$. Similarly, the derivative of V^D with respect to Λ is

$$\frac{\partial V^D(z,x)}{\partial \Lambda} = u'\left(c^D\right)\left[\frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T} z_D(z) f'(\Lambda) - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_N} z_D(z) f'(1-\Lambda)\right]$$

where $c^D = F\left(c_N^D, c_T^D\right)$, $c_N^D = z_D(z) f(1-\Lambda)$, and $c_T^D = z_D(z) f(\Lambda)$. Let $x_1 = (\Lambda_1, B_1)$, note that at $(z^*(x_1), x_1)$ we have that $c^P = c^D$, so subtracting and rearranging we get:

$$\begin{aligned}\frac{\partial V^P(z^*(x_1), x_1)}{\partial \Lambda} - \frac{\partial V^D(z^*(x_1), \Lambda_1)}{\partial \Lambda} &= u'\left(c^P\right)\left[\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} z - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T} z_D(z)\right] f'(\Lambda) \\ &\quad - u'\left(c^P\right)\left[\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_N} z - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_N} z_D(z)\right] f'(1-\Lambda)\end{aligned}$$

where $f' > 0$. This expression is the general version of the numerator in equation (18), which is the special case of perfect substitutes—where $\frac{\partial F(c_N, c_T)}{\partial c_T} = 1 - \omega$ and $\frac{\partial F(c_N, c_T)}{\partial c_N} = \omega$.

For the first term, note that

$$\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} z - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T} z_D(z) \geq \frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} z - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T} z \geq 0$$

where the first inequality follows from $z \geq z_D(z)$ and the second inequality follows from Lemma 2.

For the second term, first recall that $f(k) = k^\alpha$, so $f'(1-\Lambda) = \alpha \frac{f(1-\lambda)}{(1-\lambda)}$. Plugging in we get that the second term is

$$-\left[\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_N} c_N^P - \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_N} c_N^D\right] \frac{\alpha}{(1-\lambda)} \tag{29}$$

where we have used the fact that consumption of the non-tradable good equals production. Then, for the result to hold, it suffices to show that the term in the bracket of (29) is negative.

From Lemma 2 we have that $\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_N} \le \frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_N}$, but from Lemma 1 we have that $c_N^P \ge c_N^D$. Intuitively, the argument uses the fact that, when the elasticity of substitution is less than 1, the marginal rate of substitution changes more than the ratio of consumption on the same isoquant curve. This implies that the effect of higher marginal product of c_N from the default choice dominates the effect of the lower quantity and, thus, the term in brackets in negative. The formal argument follows below.

Note that F is homogeneous of degree 0 from the constant-returns-to-scale assumption. Then, applying Euler's theorem for homogeneous functions and using the fact that $F\left(c_N^P, c_T^P\right) = F\left(c_N^D, c_T^D\right)$ at z^* we get:

$$\frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_N} c_N^P + \frac{\partial F\left(c_N^P, c_T^P\right)}{\partial c_T} c_T^P = \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_N} c_N^D + \frac{\partial F\left(c_N^D, c_T^D\right)}{\partial c_T} c_T^D$$

which can be rearranged as

$$\frac{\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_N} c_N^P}{\frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_N} c_N^D} = \frac{1 + \frac{\frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_T}}{\frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_N}} \chi^D}{1 + \frac{\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_T}}{\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_N}} \chi^P} \tag{30}$$

where $\chi^D = \frac{c_T^D}{c_N^D}$ and $\chi^P = \frac{c_T^P}{c_N^P}$ are the consumption ratios in default and repayment. Now, note that since F is homogeneous of degree 1, its derivatives are homogeneous of degree 0. Then, we can define

$$MRS\left(\chi\right) = \frac{\frac{\partial F(1,\chi)}{\partial c_N}}{\frac{\partial F(1,\chi)}{\partial c_T}}$$

where the numerator is increasing and the denominator is decreasing (so MRS is increasing). Rewrite equation (30) as

$$\frac{\frac{\partial F\left(c_N^P,c_T^P\right)}{\partial c_N} c_N^P}{\frac{\partial F\left(c_N^D,c_T^D\right)}{\partial c_N} c_N^D} = \frac{1 + e\left(\chi^D\right)}{1 + e\left(\chi^P\right)} \tag{31}$$

where $e(\chi) = \frac{\chi}{MRS(\chi)}$. The derivative of e is

$$\begin{aligned} e'(\chi) &= \frac{d(\chi) MRS(\chi) - \chi d(MRS(\chi))}{MRS(\chi) MRS(\chi)} \\ &= \frac{\chi d(MRS(\chi))}{MRS(\chi) MRS(\chi)} \left[\frac{d(\chi)}{\chi} \frac{MRS(\chi)}{d(MRS(\chi))} - 1 \right] \\ &= \frac{\chi d(MRS(\chi))}{MRS(\chi) MRS(\chi)} [\eta - 1] < 0 \end{aligned}$$

where the inequality follows from $\eta < 1$ and from the observation that MRS is increasing. Note that Lemma 1 implies $\chi^D \geq \chi^P$, so we get that

$$\frac{\frac{\partial F(c_N^P, c_T^P)}{\partial c_N} c_N^P}{\frac{\partial F(c_N^D, c_T^D)}{\partial c_N} c_N^D} = \frac{1 + e(\chi^D)}{1 + e(\chi^P)} \leq 1$$

which implies that $\frac{\partial F(c_N^P, c_T^P)}{\partial c_N} c_N^P - \frac{\partial F(c_N^D, c_T^D)}{\partial c_N} c_N^D \leq 0.\square$