Consumption Heterogeneity and Monetary Policy in an Open Economy

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Abstract

We incorporate two types of agents (Ricardian versus Keynesian) into a standard open economy macro model. We find that consumption heterogeneity has major implications for the impact of monetary policy shocks, the international transmission mechanism, and the design of optimal monetary policy. With sticky prices, the existence of Keynesian agents causes a spillover of shocks across countries, and leads to the interdependence of optimal monetary targeting rules. In the case of local currency pricing, consumer heterogeneity leads an optimal monetary policy to generate currency misalignment and deviations from the law of one price. Theoretically, there are ranges of household heterogeneity in which monetary policy becomes ineffective, but this depends sensitively on the interaction of aggregate demand and relative price effects.

JEL classification: F3, F4

Keywords: Tank model, Optimal monetary policy, PCP, LCP, Consumption heterogeneity

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1 Introduction

A large recent literature studies the optimal monetary policy problem in New Keynesian open economy models. But almost all these studies assume that countries are populated by representative households with equal access to financial markets. This paper studies the positive and normative effects of monetary policy in an open economy model in the presence of within-country heterogeneity. Using a baseline open economy framework as in Clarida, Gali and Gertler (2002) or Engel (2011), we show that consumer heterogeneity has importance consequences for the impacts of monetary policy shocks, the spillovers of monetary policy, and the design of optimal monetary policy.

Our paper is motivated by observations on income inequality and heterogenous financial inclusion across the world. Over the last few decades, income inequality has increased in many countries. For example, the US Census Bureau reports that the Gini coefficient in the United States was 0.49 in 2021, rising from 0.4 in 1980. Likewise, according to the World Inequality Database, the share of the top 1 percent of US income earners went from 10 percent in 1980 to 19 percent in 2021. Similar trends have been seen in other OECD countries. At the same time, while World Bank (2022) estimates of the share of the population of who saved in high income countries rose from 70 to 76 percent between 2011 and 2021, it still suggests that a substantial minority of the population are effectively 'hand to mouth' households (see also Kaplan, Violante, and Weidner, 2014).

The analysis is particularly focused on the question of how monetary policy interacts with unequal access to financial markets, and how the presence of heterogeneity impacts on the optimal monetary policy problem in an open economy. We build on the contributions of many recent studies. A growing literature now incorporates heterogeneity into New-Keynesian macro models (see references below). In these models, households that are constrained in their access to financial markets have high marginal propensities to consume and their spending reacts strongly to changes in disposable income.

Heterogeneity in our model is introduced in a simple, analytically tractable manner. We allow for two types of households, namely, "Ricardian" and "Keynesian". Ricardian consumers can smooth consumption over time by borrowing and lending freely in financial markets, as in the standard New Keynesian model and have full ability to engage in international financial risk-sharing. Keynesian consumers however are hand-to-mouth agents who can only consume their disposable labor income every period. Our model is thus a version of a Two-Agent New Keynesian model (TANK) as in Debortoli and Gali (2018).

While the TANK model lacks some of the detailed heterogeneity features in the recent Heterogeneous Agent New Keynesian (HANK) literature,[1] an advantage is that it allows for a simple analytical exploration of the interaction of consumer heterogeneity with monetary policy, a clear illustration of how heterogeneity affects the international transmission of monetary policy shocks, and an exact analytical description of the optimal monetary policy in the presence of heterogeneity.

One central ingredient of our model is the interrelationship between measures of the output gap and consumer heterogeneity. In the absence of price stickiness, heterogeneity in our model is irrelevant. A rise in the output gap implies that Keynesian households consume more than Ricardian households. But with fully flexible prices, the output gap is closed, and therefore there is no consumption heterogeneity in the efficient flexible price equilibrium.

With sticky prices, monetary shocks will open an output gap, which leads to heterogenous consumption responses both within and across countries. In our baseline, assuming producer currency pricing (PCP), monetary shocks operate through aggregate consumption (expenditure changing) effects and terms of trade (expenditure switching) effects. But the strength of these effects are dependent on the degree of consumer heterogeneity. Up to a critical threshold, the aggregate impact of monetary policy shocks is increasing in the fraction of Keynesian households in a given country. In this region, an interest rate cut stimulates the consumption of Ricar-

[1]See, for example, Kaplan, Moll and Violante (2018).

dian households, but there is a magnified effect on the consumption of Keynesian households, leading to a larger impact on the output gap.

If the share of Keynesian agents is large enough, our model implies that there is the possibility of perverse effects of monetary policy, similar to the 'inverted aggregate demand' effect described by Bilbiie (2008). But in the open economy, we find the region of inverted aggregate demand is reduced by the presence of terms of trade adjustment.

Consumer heterogeneity has an important implication for the cross country spillovers of monetary policy shocks. In our baseline model, there are no spillover effects at all in the absence of Keynesian consumers. While an interest rate cut has an expenditure changing effect - directly increasing spending on foreign goods, this is exactly offset by the expenditure switching effects of an appreciation of the foreign currency. But with consumer heterogeneity, monetary policy shocks have positive spillover effects. This is because heterogeneity leads to a magnification of the direct spending (expenditure changing) effects due to the high marginal propensity to consume of Keynesian consumers, but it leaves the terms of trade response (expenditure switching) unchanged, relative to the model without heterogeneity. Thus consumer heterogeneity acts so as to propagate monetary policy shocks across borders, and the overall effects depend on heterogeneity in both the source and receiving countries. But crucially, international transmission requires consumer heterogeneity in the source country.

We extend the analysis to that of a small economy, where the rest of the world is arbitrarily large relative to the domestic economy. Again, in the baseline case without heterogeneity, this would not effect the results at all - economic size is irrelevant to the results, and there would be no spillover effects of policy shocks to or from the small economy. But with consumer heterogeneity, size becomes important. With heterogeneity, the effects of monetary shocks in the small economy are smaller than in the symmetric two country model, and the effects of shocks in the rest of the world are larger, so long as there is consumer heterogeneity in the outside world economy.

The baseline model allows for a simple second order welfare approximation where the presence of consumer heterogeneity plays an explicit role in the welfare analysis. The implications for optimal monetary policy depend on the composition of shocks. Assuming only productivity shocks, it turns out that an optimal monetary rule can remove both production and risk-sharing distortions. A monetary rule which stabilizes the PPI in each country ensures that the output gap is closed, and full risk sharing within and across countries is achieved simultaneously. But when we extend the model to allow for cost-push shocks, consumer heterogeneity plays a key role in optimal policy design. With Keynesian consumers, due to inefficient risk sharing across countries, optimal targeting rules are interdependent, and a domestic inflationary shock is transmitted to foreign inflation and output gaps. We find that increased heterogeneity cushions the impact of a cost-push shock on the domestic output gap, while exacerbating the effect on the foreign country.

The analysis easily extends to the alternative of local currency pricing (LCP). In this setting, consumer heterogeneity also plays a key role, but the effect is quite different from the PCP case. With LCP, there is also an upper limit on the share of Keynesian consumers in order to avoid perverse effects of monetary policy shock. But with LCP, there is always a cross country spillover of monetary policy shocks, even in the absence of Keynesian consumers. But again, the effect is magnified in the presence of consumer heterogeneity.

The presence of Keynesian consumers also affects the optimal monetary policy under LCP. When all consumers are Ricardian, we find that the optimal monetary policy rule under LCP is to stabilize the CPI, and to eliminate any currency misalignment (or deviations from the law of one price), as in Engel (2011). By contrast, with Keynesian consumers, due to the absence of within and across country risk-sharing, an optimal monetary policy allows for currency misalignment as a response to country specific productivity shocks. In response to a home country productivity shock, the optimal rule is to allow for a real exchange rate depreciation in the home country to stimulate higher consumption of Keynesian consumers.

1.1 Related Literature

Our paper is closely related to two strands of literature. The first is the work on new Keynesian open-economy macroeconomic models. Two papers are considered as benchmarks for comparison, Clarida, Gali and Gertler (2002, hereafter CGG) and Engel (2011), particularly for their explication of an optimal monetary policy in a global environment. CGG develops the canonical model for open-economy monetary policy analysis in a Keynesian framework. They show that if price setting is based on PCP, the central bank should target producer inflation (PPI). Engel (2011) examines optimal monetary policy under the LCP case and argues that due to currency misalignment, an optimal policy requires CPI targeting. These two papers assume a Calvo mechanism for price setting, which differs from other papers in which prices are sticky, but are set one period in advance. For example, see Devereux and Engel (2003), Corsetti and Pesenti (2005). Our paper incorporates two types of agents in the CGG and Engel model and studies how household heterogeneity affects the choice of optimal monetary policy. We illustrate the way in which consumption heterogeneity enters into the policy maker's loss function. We show that when the output gap cannot be closed, consumption heterogeneity should be a concern for the central bank, in addition to the output gap itself, inflation, and in the LCP environment, the degree of misalignment, as defined by Engel (2011).

The second relevant literature is growing work on heterogenous agent models that study the redistributive effects of fiscal and monetary policies. For example, Kaplan, Moll, and Violante (2018) embed heterogeneous agents and incomplete markets into the New Keynesian workhorse model. However, in the standard heterogeneous agent model, households face idiosyncratic labor income shocks that cannot be fully insured against. As a result, there exists a non-degenerate time-varying wealth distribution that needs to be tracked, as well as difficulties arising from the presence of occasionally binding borrowing constraints. To avoid these computational hurdles, Debortoli and Gali (2018) assess the comparative advantage of a simpler alternative heterogeneous agent model, namely the Two Agents New Keynesian (TANK) model, in understanding aggregate dynamics relative to fully heterogeneous agent models. In the TANK model, heterogeneity is characterized by two types of households, namely, "Ricardian" and "Keynesian" consumers, with a constant share over time. Similarly, Bilbiie (2008) introduces two types of households (asset holder vs non-asset holder) in dynamic general equilibrium and develops a simple analytical framework for monetary policy analysis. He argues that low asset participation may lead to inverted aggregate demand in response to a monetary shock, and it also affects the aggregate dynamics and stability of economy. In contrast to these papers, we focus on the implication of consumption heterogeneity for monetary policy in international dimensions.

The study of the heterogeneous agent model in an open economy is still relatively new. There is a growing literature on how heterogeneity changes the effects of external shocks in small open economies. Recent examples include Auclert, Rognile, Souchier, and Straub (2021), de Ferra, Mitman, and Romei (2020), Guo, Ottonello, and Perez (2022) , Oskolkov (2022), and others.[2]

Auclert, Rognile, Souchier, and Straub (2021) introduce heterogeneous households to a New-Keynesian small open economy model and find that heterogeneity amplifies the real income channel of exchange rates: the rise in import prices from a depreciation lowers households' real incomes, and leads them to cut back on spending. de Ferra, Mitman and Romei (2020) study the role of heterogeneity in an small open model economy that experiences a current account reversal. They highlight the effect of heterogeneous portfolio composition, finding that the contraction is more severe when poorer households with higher MPC are leveraged and owe debt in foreign currency. Guo, Ottonello and Perez (2022) also explore the distributional consequences of sudden stops in small open economies, but emphasize household heterogeneity in income, wealth, and real and financial integration with international markets. They show that there is a trade-off between aggregate stabilization and inequality in consumption responses to external shocks. Oskolkov (2022) studies the

[2]For example, see Prasad, and Zhang (2015), Cugat (2019), Hong (2020), and Zhou (2022).

role of exchange rate regimes in shaping the distributional effects of external monetary shocks in a small open economy with heterogeneity in wealth and exposure to international trade. He shows that the response of wage and interest rate, and thus consumption equality will be very different under different exchange rate regimes.

These papers, however, are based on a small open economy model, and do not explicitly investigate the role of heterogeneity in the optimal monetary problem in such a setting. By contrast with their work, we investigate the role of household heterogeneity in the international monetary transmission mechanism in a two-country model, and explore the optimal monetary policy in an environment of heterogeneity. Compared to the small open economy setting, we show that the spillover effect of domestic monetary shocks depends on the consumption heterogeneity in both the home and foreign countries.

The rest of the paper is organized as follows. Section 2 lays out the basic model and defines an equilibrium. Section 3 analyzes the transmission of monetary shocks under PCP with alternative assumptions about consumer heterogeneity. Section 4 derives an optimal policy under PCP. Section 5 examines the alternative pricing strategy of LCP in the presence of consumer heterogeneity. Conclusions are in Section 6.

2 A two-country model with household heterogeneity

The benchmark model extends the existing New Open Economy Macroeconomics (NOEM) literature by introducing household heterogeneity. The baseline model, is standard and similar to the classic Clarida, Gali and Gertler (2002) (CGG) two-country model, which assumes producer currency pricing. The only difference is that we model two types of agent in each country. Following Bilbiie (2008) and Debortoli and Gali (2018), we adopt a Two-Agent New Keynesian (TANK) model, in which the *within group* difference for different types of agents is ignored but the *between* *group* difference (across two groups) is emphasized.

2.1 Household

There is a continuum of households $[0,1]$, all having the same utility function in the home country. A constant measure $1-n$ of households is labelled Ricardian and have unconstrained access to financial markets. A fraction n of households, referred to as Keynesian just consume their labor income and lump-sum transfers each period. n^* is the fraction of Keynesian households in the Foreign country.

Let $s \in \{R, K\}$ specify the household type (Ricardian and Keynesian). Utility is

$$E\sum_{t=0}^{\infty}\beta^t[\ln(C_t^s) - \eta\frac{(L_t^s)^{1+\omega}}{1+\omega}] \tag{1}$$

where $C_t^s = [C_{ht}^s]^{\frac{v}{2}}[C_{ft}^s]^{1-\frac{v}{2}}$ is the aggregate consumption of home and foreign goods with home bias $v \geq 1$, which is assumed to be identical in both types of agent.[3] This implies that $P_t^s = P_t$, so the consumption price index is identical for both types of households. So we omit the superscript "R" or "K" for the price variables.

2.1.1 Ricardian Households

Ricardian households have access to both state-contingent bonds in the domestic market and foreign market. They are also equity holders who claim the ownership of firms. Their period budget constraint is

$$\begin{aligned} & P_t^R C_t^R + B_{t+1} + Q_t^e B_t^e + \sum_{\zeta^{t+1}\in Z_{t+1}} Q(\zeta^{t+1}|\zeta^t)D_{hh}(\zeta^{t+1}) + S_t \sum_{\zeta^{t+1}\in Z_{t+1}} Q^*(\zeta^{t+1}|\zeta^t)D^*_{hf}(\zeta^{t+1}) \\ = \;& W_t L_t^R + (1+i_{t-1})B_t + (D_t^e + Q_t^e)B_{t-1}^e + D_{hh}(\zeta^t) + S_t D^*_{hf}(\zeta^t) \end{aligned} \tag{2}$$

where B_{t+1} and B_t^e are the holdings of domestic non-state-contingent bonds and equity. Q_t^e and D_t^e are the price and dividend of the equity. i_{t-1} is the domestic bond's nominal interest rate. S_t is the nominal exchange rate, defined as the price

[3] The case of asymmetric home bias is explored in Technical Appendix Section 8.

of 1 unit of foreign currency in terms of domestic currency. $D_{hh}(\zeta^{t+1})$ and $D^*_{hf}(\zeta^{t+1})$ are home's holding of the state-contingent domestic and foreign securities. Here we are assuming complete international financial markets for Ricardian agents. L_t^R is the labor supplied by the Ricardian household and $W_t L_t^R$ is wage income.

Let Λ_t represents the Lagrangian multiplier associated with the flow budget constraint, then the stochastic discount factor between t and $t+i$ is $\Lambda_{t,t+i} = \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} = \beta(\frac{C^R_{t+i}}{C^R_t})^{-1}\frac{P_t}{P_{t+i}}$. From the first order conditions of the Ricardian households,[4] we have:

$$\frac{1}{1+i_t} = E_t \Lambda_{t,t+1} \tag{3}$$

$$W_t \frac{1}{C_t^R P_t} = \eta (L_t^R)^{\omega} \tag{4}$$

In equilibrium, we also have the risk-sharing condition as follows:

$$\frac{C_t^R}{C_t^{R*}} = e_t \tag{5}$$

where $e_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate, C_t^{R*} and P_t^* represent the consumption and price level of foreign Ricardian households.

2.1.2 Keynesian Households

Keynesian households simply consume their labor income each period. The budget constraint is

$$P_t C_t^K = W_t L_t^K \tag{6}$$

where C_t^K and L_t^K denote consumption and labor supply for Keynesian households. We abstract away from idiosyncratic shocks as in Bilbiie (2008) and Debortoli and Gali (2018). Optimal labor supply by Keynesian households is given by:

$$W_t \frac{1}{C_t^K P_t} = \eta (L_t^K)^{\omega} \tag{7}$$

Using Equations (6) and (7), Keynesian household's labor supply is $L_t^K = \eta^{-\frac{1}{1+\omega}}$.

[4]First order conditions with respect to equity are reported in the Technical Appendix Section 1.2.

2.1.3 Demand for Goods

Given the aggregate consumption $C_t^s = [C_{ht}^s]^{\frac{v}{2}}[C_{ft}^s]^{1-\frac{v}{2}}$, the demand for the home and Foreign good is

$$C_{ht}^s = \frac{v}{2}\frac{P_tC_t^s}{P_{hht}},\ C_{ft}^s = (1-\frac{v}{2})\frac{P_tC_t^s}{P_{fht}} \tag{8}$$

where $P_t = \overline{\Theta}[P_{hft}]^{\frac{v}{2}}[P_{fht}]^{1-\frac{v}{2}}$, and P_{hht} and P_{fht} are the prices of domestic and foreign goods sold in the Home market, respectively. $\overline{\Theta}$ is a constant. Given the demand structure $C_{ht}^s = [\int_0^1 C_{ht}^s(i)^{\frac{\varepsilon-1}{\varepsilon}}di]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{ft}^s = [\int_0^1 C_{ft}^s(i)^{\frac{\varepsilon-1}{\varepsilon}}di]^{\frac{\varepsilon}{\varepsilon-1}}$, we can solve for the demand for varieties, which are reported in Technical Appendix Section 1.2.

2.1.4 Consumption Heterogeneity

Aggregate consumption is $C_t = (1-n)C_t^R + nC_t^K$. We define the index of heterogeneity between the Ricardian and Keynesian households, H_t, as follows

$$H_t \equiv \frac{C_t^R}{C_t} = (1-n+n\frac{C_t^K}{C_t^R})^{-1}$$

When $C_t^R = C_t^K, H_t = 1$, there is no consumption heterogeneity.

With this, we can rewrite the Euler equation of Ricardian households and the risk-sharing condition in terms of aggregate consumption.

$$\frac{1}{1+i_t} = \beta E_t(\frac{C_{t+1}^R}{C_t^R})^{-1}\frac{P_t}{P_{t+1}} = \beta E_t(\frac{C_{t+1}}{C_t})^{-1}(\frac{H_{t+1}}{H_t})^{-1}\frac{P_t}{P_{t+1}} \tag{9}$$

$$\frac{(C_tH_t)^{-1}}{P_t} = \frac{(C_t^*H_t^*)^{-1}}{S_tP_t^*} \tag{10}$$

2.2 Firms, Price Setting, and Equilibrium

Each firm i in the home economy has the production technology:

$$Y_t(i) = Z_tL_t(i) \tag{11}$$

where $Z_t = exp(\theta_t)$ is a country-specific productivity shock, and θ_t is distributed with mean zero and variance σ_θ^2.

Firms adjust prices following a standard Calvo mechanism. In the home country, a firm may reset its prices with probability $1-\kappa$ each period. In the baseline analysis, we limit discussion to a producer currency pricing (PCP) strategy. The optimization problem of firms is standard and is defined in Technical Appendix Section 1.1.1.

We define the terms of trade as the relative price of foreign to home goods

$$Q_t = \frac{S_t P^*_{fft}}{P_{hht}} \tag{12}$$

The goods market clearing condition for the home good is:

$$Y_t = \frac{v}{2}\frac{P_t C_t}{P_{hht}}\Delta_{hh,t} + (1-\frac{v}{2})\frac{P^*_t C^*_t}{P^*_{hft}}\Delta^*_{hf,t} \tag{13}$$

where $\Delta_{hh,t} = \int(\frac{P_{hht}(i)}{P_{hht}})^{-\varepsilon}di$ and $\Delta^*_{hft} = \int(\frac{P^*_{hf,t}(i)}{P^*_{hft}})^{-\varepsilon}di$ are price dispersion terms.

The labor market clearing condition of home is:

$$L_t = (1-n)L^R_t + nL^K_t \tag{14}$$

Markets for domestic non state-contingent bonds, state-contingent bonds, and equities clear in the model. Details are given in the Technical Appendix Section 1.2. Monetary authorities use a nominal interest rate as the policy instrument. The policy rule will be specified below.

Definition. *Given the stochastic process of the productivity shocks and monetary policy rules and monetary shocks, an equilibrium is characterized by a collection of variables on consumption, productions and prices level such that: (a) Ricardian households optimally choose consumption, state-contingent assets, domestic non-state-contingent bonds, equity and labor supply; (b) Keynesian households optimally choose consumption and labor supply; (c) Firms set their prices to maximize profits; (d) Home and Foreign goods market, labor market, and financial markets all clear.*

3 Transmission mechanism of monetary shocks

3.1 The Flexible Price Equilibrium

To highlight the role of monetary policy, we first illustrate the flexible price equilibrium where money is neutral. Since there is no monopoly distortion in the model,[5] the flexible price allocation is also the efficient allocation if it can assure full risk-sharing between Keynesian and Ricardian consumers.

We normalize the wage, marginal cost, prices and dividend by the CPI price level in the relevant country. The normalized variables are labelled as lower case letters. That is, $w_t = \frac{W_t}{P_t}, mc_t = \frac{MC_t}{P_t}, p_{hht} = \frac{P_{hht}}{P_t}$,and $d_t^e = \frac{D_t^e}{P_t}$. In our notation, variables with hat refer to the deviation of the log of corresponding variables from steady state, and variables with a superscript "fb" are defined as variables in flexible price equilibrium. The solution to the flexible price equilibrium is summarized in Proposition 1.

Proposition 1. *With household heterogeneity ($n > 0, n^* > 0$), the solution to the flexible price equilibrium is identical to the efficient equilibrium in the standard new open macro model (i.e., CGG).*

$$\begin{aligned}
\widehat{H}_t^{fb} &= \widehat{H}_t^{fb,*} = 0 \\
\widehat{q}_t^{fb} &= (\theta_t - \theta_t^*) \\
\widehat{Y}_t^{fb} &= \theta_t, \qquad \widehat{Y}_t^{fb*} = \theta_t^* \\
\widehat{C}_t^{fb} &= \widehat{C}_t^{R,fb} = \widehat{C}_t^{K,fb} = \frac{v}{2}\theta_t + \frac{2-v}{2}\theta_t^* \\
\widehat{C}_t^{*,fb} &= \widehat{C}_t^{R*,fb} = \widehat{C}_t^{K*,fb} = \frac{v}{2}\theta_t^* + \frac{2-v}{2}\theta_t
\end{aligned}$$

The proof is presented in the Technical Appendix Section 1.4. In the model, the heterogeneity of Ricardian to Keynesian consumption depends critically on their difference in labor supplies, which is negatively related to the country's output gap.

[5] The markup is eliminated by constant production subsidy and the dividend is zero in the flexible price equilibrium, so households are indifferent as to their asset holdings.

In particular, using Equations (4), (7), (11), and (14), and the analogous conditions for the foreign country, we can show

$$\widehat{H}_t = \frac{-n\omega}{1-n}(\widehat{Y}_t - \theta_t) \tag{15}$$

$$\widehat{H}^*_t = \frac{-n^*\omega}{1-n^*}(\widehat{Y}_t - \theta^*_t) \tag{16}$$

An increase in the output gap in either country must be associated with a rise in the relative consumption of Keynesian households. A rise in the output gap $\widehat{Y}_t - \theta_t$ means an increase in hours worked for Ricardian households, then by Equation (4), this implies a lower consumption response than Keynesian households, whose labor supply is constant. But in the flexible price equilibrium, labor supply of Ricardian households is also constant, due to exactly offsetting income and substitution effects, so that consumption responses are identical for both households.

We note that conditions (15) and (16) are derived from the household side, independent of the firm's pricing policy, whether prices are sticky or not.

3.2 Monetary policy shocks and spillovers under PCP

We now discuss the PCP case with sticky prices. To highlight the key mechanisms that channel consumer heterogeneity, we first focus on a symmetric case where $n = n^*$, and assume symmetric preferences without home bias (so that $v = 1$).

Without home bias, purchasing power parity holds, so that the real exchange rate is constant, i.e. $\widehat{e}_t = 0$, and the risk-sharing condition implies that Ricardian households have equal consumption responses; $\widehat{C}^R_t = \widehat{C}^{*,R}_t$. Then goods market clearing conditions in the home and foreign countries can be rewritten as:

$$\widehat{Y}_t = \widehat{C}_t + \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}^*_t) = \widehat{C}^R_t - \widehat{H}_t + \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}^*_t) \tag{17}$$

$$\widehat{Y}^*_t = \widehat{C}^*_t - \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}^*_t) = \widehat{C}^{R*}_t - \widehat{H}^*_t - \frac{1}{2}(\widehat{q}_t + \widehat{H}_t - \widehat{H}^*_t) \tag{18}$$

Equations (17) and (18) indicate that home and foreign GDP are driven by changes in demand by Ricardian households, changes in the terms of trade, and changes in consumption heterogeneity among home and foreign consumers. Holding constant the terms of trade and Ricardian consumption, a rise in Keynesian consumption in either country (a fall in $\widehat{H}_t$ or $\widehat{H}_t^*$), will raise demand in both countries. But by (15) and (16), $\widehat{H}_t$ and $\widehat{H}_t^*$ are themselves driven directly by changes in the output gap.

Equations (17) and (18) together yield

$$\widehat{q}_t = \widehat{Y}_t - \widehat{Y}_t^* \tag{19}$$

which implies that the terms of trade under PCP is simply determined by relative output levels. The terms of trade does not directly depend on the presence of consumer heterogeneity. This feature of the model is important for understanding how heterogeneity leads to cross country spillovers. Moreover, this property continues to hold even in the case of home bias in consumption, as shown in the Technical Appendix Section 2.1.3.

Following CGG, we define the PPI-based real interest rates in the home and foreign countries as $\widehat{r}_t = \widehat{i}_t - E_t\pi_{hh,t+1}$ and $\widehat{r}_t^* = \widehat{i}_t^* - E_t\pi_{ff,t+1}^*$. Then taking a log linearization of (3) we obtain the home country Euler equation:[6]

$$\widehat{C}_t^R = E_t(\widehat{C}_{t+1}^R) - [\widehat{r}_t - \frac{1}{2}E_t\Delta\widehat{q}_{t+1}] \tag{20}$$

where we define $\Delta x_t = x_t - x_{t-1}$ as the first difference of a variable x_t.

Using (20) along with (17) and (18), along with the two equations of consumption heterogeneity (15) and (16) we arrive at the following intuitive characterization for the dynamics of home and foreign output in terms of real interest rate and productivity shocks.

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \frac{1+\delta}{2\delta}\widehat{r}_t - \frac{1-\delta}{2\delta}\widehat{r}_t^* + \frac{1-\delta}{2\delta}(E_t\Delta\theta_{t+1} + E_t\Delta\theta_{t+1}^*) \tag{21}$$

[6]The dynamics of CPI inflation and PPI inflation are determined by

$$\pi_t = \pi_{hh,t} + \frac{1}{2}\Delta\widehat{q}_t$$

$$\pi_t^* = \pi_{ff,t}^* - \frac{1}{2}\Delta\widehat{q}_t$$

$$\widehat{Y}_t^* = E_t\widehat{Y}_{t+1}^* - \frac{1+\delta}{2\delta}\widehat{r}_t^* - \frac{1-\delta}{2\delta}\widehat{r}_t + \frac{1-\delta}{2\delta}(E_t\Delta\theta_{t+1} + E_t\Delta\theta_{t+1}^*) \tag{22}$$

where $\delta = 1 - \frac{n\omega}{1-n}$ is a term controlling the degree of household heterogeneity, with $\delta = 1$ representing a fully Ricardian economy, and δ falls as the measure of Keynesian consumers rise. Moreover, $\delta < 0$ for $n > \frac{1}{1+\omega}$.

From the Euler equation in the flexible price equilibrium, we define the domestic natural interest rate as: $\widehat{r}_t^{fb} = E_t(\widehat{Y}_{t+1}^{fb}) - \widehat{Y}_t^{fb} = E_t\Delta\theta_{t+1}$. Therefore, we can rewrite (21) and (22) in terms of deviations from the flexible price equilibrium:

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{1+\delta}{2\delta}(\widehat{r}_t - \widehat{r}_t^{fb}) - \frac{1-\delta}{2\delta}(\widehat{r}_t^* - \widehat{r}_t^{*fb}) \tag{23}$$

$$\widehat{Y}_t^* - \widehat{Y}_t^{fb*} = E_t(\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^{fb*}) - \frac{1+\delta}{2\delta}(\widehat{r}_t^* - \widehat{r}_t^{*fb}) - \frac{1-\delta}{2\delta}(\widehat{r}_t - \widehat{r}_t^{fb}) \tag{24}$$

In order to highlight the role of heterogeneity in the response to money shocks, we abstract away from dynamics by assuming the monetary authorities in both countries follow a rule whereby the expected PPI based real interest rate is constant (a similar assumption is made in Auclert, Rognile, Souchier, and Straub, 2021, and in a closed economy context by Woodford, 2011). Thus, we assume that the real interest rates equal their natural rate plus a monetary shock. $\widehat{i}_t = E_t\pi_{hh,t+1} + u_t + \widehat{r}_t^{fb}$, $\widehat{i}_t^* = E_t\pi_{ff,t+1}^* + u_t^* + \widehat{r}_t^{*fb}$ where monetary shocks u_t, u_t^* are i.i.d. This assumption ensures that the expected effects of monetary policy are purely transitory, and the expected real interest rate is constant. We can then solve for the equilibrium allocations under PCP as follows:

$$\begin{aligned}
\widehat{Y}_t - \widehat{Y}_t^{fb} &= -\frac{1+\delta}{2\delta}u_t - \frac{1-\delta}{2\delta}u_t^* \\
\widehat{Y}_t^* - \widehat{Y}_t^{fb*} &= -\frac{1+\delta}{2\delta}u_t^* - \frac{1-\delta}{2\delta}u_t \\
\widehat{q}_t &= \widehat{Y}_t - \widehat{Y}_t^* = \widehat{Y}_t^{fb} - \widehat{Y}_t^{fb*} + u_t^* - u_t \\
\widehat{C}_t^R &= \widehat{C}_t^{R*} = -\frac{1}{2}(u_t + u_t^*) + \frac{1}{2}E_t(\widehat{Y}_t^{fb} + \widehat{Y}_t^{fb*})
\end{aligned}$$

We first summarize these results in the following proposition, and then provide an intuitive discussion.

Proposition 2. *a) For $\delta = 1$, a domestic monetary expansion (interest rate cut) increases the output gap one for one, and has zero spillover effects on the foreign output gap.*

b) For $1 > \delta > 0$, a domestic monetary expansion increases the output gap by more than one for one, and has a positive spillover to the foreign output gap.

c) For $0 > \delta > -1$, a domestic monetary expansion has a negative effect on the domestic and foreign output gaps.

d) When $-1 > \delta$, a domestic monetary expansion raises the domestic output gap, but reduces the foreign output gap.

Proof: Please see Technical Appendix Section 2.1.2.

We first discuss cases a) and b) of the Proposition, which we argue below is the presumptive case. The key intuition is that the presence of Keynesian consumers increases the demand effects of an interest rate cut (the expenditure changing effect), but has no effect on the terms of trade (the expenditure switching) response. In the model without consumer heterogeneity, an interest rate cut increases the home and foreign country consumption one for one, as there is full risk sharing and all agents are Ricardian. At the same time, the home terms of trade deteriorates, which leads to an expenditure switching away from foreign goods and towards home goods. The combination of the positive demand effect and the negative relative price effect exactly cancels out for the foreign country, leaving the foreign output gap unchanged.

In the model with Keynesian consumers, there is a magnification effect on aggregate demand, as the rise in the output gap following the interest rate cut leads (through conditions (15) and (16)) to a more than proportional rise in the consumption of Keynesian agents. This raises the home output gap by more than proportionately, but also spills over into higher demand for the foreign good, leading to a further magnification effect. But since the response of the terms of trade is the same as in the full Ricardian model, the demand effect exceeds the relative price effect and the foreign output gap rises. This leads to a second round effect on home demand, as a 'spillback' term, and the final outcome is that both home and foreign output rise by more than in the model without heterogeneity.

These channels may be illustrated more clearly by re-writing the goods market clearing condition for the home and foreign country in the following way (assuming no productivity shocks)

$$\widehat{Y}_t = \frac{1}{\delta}\widehat{C}_t^R + \frac{1}{2}\widehat{q}_t = \frac{\frac{1-n}{1+\omega}}{\frac{1}{1+\omega} - n}\widehat{C}_t^R + \frac{1}{2}\widehat{q}_t \tag{25}$$

$$\widehat{Y}_t^* = \frac{1}{\delta}\widehat{C}_t^{R*} - \frac{1}{2}\widehat{q}_t = \frac{\frac{1-n}{1+\omega}}{\frac{1}{1+\omega} - n}\widehat{C}_t^{R*} - \frac{1}{2}\widehat{q}_t \tag{26}$$

The first term on the right hand side of (25) relates to the aggregate consumption effect. As shown in the equilibrium solution, both $\widehat{C}_t^R$ and the terms of trade $\hat{q}_t$ are independent of consumer heterogeneity and increase in response to a domestic interest rate cut. Specifically, if we assume no productivity shocks and foreign monetary shocks, we have $\hat{C}_t^{R*} = \hat{C}_t^R = -\frac{\mu_t}{2}$ and $\hat{q}_t = -\mu_t$. The term $0 < \delta < 1$ thus imposes a magnification effect on home output. That is, the response of output is amplified by the presence of Keynesian consumers ($0 < n < \frac{1}{1+\omega}$). Likewise, Equation (26) is written in the same way, but the terms of trade effect is negative. When $0 < \delta < 1$, the spillovers to foreign output must be positive.

Cases c) and d) of the proposition are associated with a feature of the TANK model first highlighted by Bilbiie (2008) whereby if the share of Keynesian agents rises above a threshold, there is no finite equilibrium in which a cut in interest rates can raise output, since the second round demand effects from Keynesian agents will always exceed the first round output gap. Bilbiie (2008) identifies this as a situation of 'inverted aggregate demand', and technically, it implies an interest rate cut should reduce both home and foreign output when $0 > \delta > -1$ (case c). From Equation (25), the first term is negative and would dominate the positive terms of trade effect. By contrast, under case d), where $-1 > \delta$ the interest rate cut should raise home output, but have a negative spillover to foreign output. In this case, the (negative) impact on consumption demand from (25) is offset by the positive impact from the terms of trade deterioration, and since the terms of trade deterioration reduces foreign demand, the foreign output gap falls.

Section 3 of Technical Appendix discusses the case c) and d) of the Proposition in further detail, and provides a graphical illustration of the determination of the output gap in the two countries in cases a-d). But we also show that cases c) and d) are likely to exhibit indeterminancy of equilibrium under a standard monetary policy rule. Moreover, we argue that these cases rely on empirically implausible values for the share of Keynesian agents in the economy. In particular, for an expansionary monetary policy to have a contractionary effect, the share of Keynesian consumers should range from $\frac{1}{\omega+1}$ to $\frac{1}{\frac{\omega}{2}+1}$. The value for ω, the inverse elasticity of labour supply, is usually set at unity. In this case, the range for perverse effects of monetary policy is from 0.5 to 0.67. The consensus in the literature suggests that the share of Keynesian households in the economy is much less than 0.5. For example, Debortili and Gali (2018) argue that about 21%-27% of households are financially constrained, while Kaplan Violante and Weidner (2014) estimate that the share of hand-to-mouth households was 30%.[7] Thus, we view the possibility of an inverse relationship between monetary expansion and aggregate demand in the open economy model as of limited empirical relevance. As a result, in what follows we discuss only the results where $\delta > 0$, and the extension of this condition to the case of differences in country heterogeneity.

3.3 Asymmetric household heterogeneity and spillovers

Now we discuss effects of monetary policy shocks and the spillovers for the case with asymmetric household heterogeneity where $n \neq n^*$, and we assume general but symmetric home bias, so that $v \geq 1$. Again, we focus only on monetary shocks. As shown in Technical Appendix Section 2.1.1, we solve for equilibrium allocations as

[7] This accords well with the data on the fraction of the population who engage in savings reported by the World Bank, as cited in the Introduction.

below.

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{\delta^* + 1 + (v-1)(\delta^* - 1)}{\Delta_0}(\widehat{r}_t - \widehat{r}_t^{fb}) - \frac{(1-\delta^*)(2-v)}{\Delta_0}(\widehat{r}_t^* - \widehat{r}_t^{fb*}) \tag{27}$$

$$\widehat{Y}_t^* - \widehat{Y}_t^{fb*} = E_t(\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^{fb*}) - \frac{\delta + 1 + (v-1)(\delta - 1)}{\Delta_0}(\widehat{r}_t^* - \widehat{r}_t^{fb*}) - \frac{(1-\delta)(2-v)}{\Delta_0}(\widehat{r}_t - \widehat{r}_t^{fb}) \tag{28}$$

$$\widehat{q}_t = -(r_t - r_t^*) \tag{29}$$

where $\Delta_0 = (\delta + \delta^*)(2 - v) + (v - 1)2\delta\delta^*$. Again, we note that absent consumption heterogeneity, an interest rate cut increases domestic output one for one, and there are no cross country spillovers. But with different degrees of heterogeneity across countries, the size of domestic output and spillover responses may vary considerably. Despite that, the terms of trade is again independent of heterogeneity, and depends only on the relative size of the interest rate shocks.

From Equations (27) to (29), we establish the following proposition.

Proposition 3. *For values of parameters n and ω satisfying $n < \frac{1}{1+\omega}$ and $n^* < \frac{1}{1+\omega}$, the following set of results hold:*

(a) An interest rate cut in either the home or foreign countries increases domestic output. The output response is increasing in the share of Keynesian households in both the source country and the receiving country.

(b) If the source country has no consumption heterogeneity, then the direct effect of interest rate shocks is to reduce output in the source country one for one, and there are no spillover effects to the other country.

(c) The spillover effects are increasing in the size of Keynesian households in both source country or host country ; the spillover of monetary shocks is more sensitive to the size of Keynesian households in the source country than that in the host country.

(d) In the absence of Keynesian households, the output response to an interest rate cut is independent of home bias in preferences. But when $\delta < 1$ and $\delta^* < 1$,

home bias increases the own country response, and reduces the spillover response to interest rate shocks.

As in the previous case, the presence of Keynesian consumers induces an amplification of the effects of interest rate shock, given their higher marginal propensity to consume out of wage income. But the differences in heterogeneity across countries has an important implication for the own and spillover effects of shocks. While the impact of monetary policy shocks, both on own country and spillovers, is increasing in the degree of household heterogeneity, this is only the case if there is heterogeneity in the source country. If the source country has no Keynesian consumers, then there is no amplification of monetary shocks on domestic output, and there are no cross country spillovers. The intuition for this can be seen from an analysis of the market clearing condition, including the definition of consumption heterogeneity, the risk sharing condition, and for this example, assuming that $\delta = 1$ for the home country, so there is no consumption heterogeneity in the home country. We may then express this as:

$$\widehat{Y}_t = \widehat{C_t^R} + (1 - \frac{v}{2})(\delta^* - 1)\widehat{H}_t^* + (1 - \frac{v}{2})\widehat{q_t} \tag{30}$$

The home output response is determined by the direct influence of the interest rate on Ricardian consumption, the response of foreign consumption heterogeneity, and the terms of trade effect. The interest rate cut leads to a rise in Ricardian consumption $\widehat{C_t^R}$ equal to $-\frac{v}{2}\widehat{r}_t$ and a terms of trade appreciation of $-\widehat{r}_t$ which increases demand by $-(1 - \frac{v}{2})\widehat{r}_t$, so the combination of these effects implies that home output rises by $-\widehat{r}_t$, and there is no domestic amplification through Keynesian consumers. In addition, the analogous case for foreign output ensures that the foreign terms of trade appreciation exactly offsets the rise in Ricardian consumption, ensuring that foreign output, and hence $\widehat{H}_t^*$, is unchanged, hence the absence of cross country spillovers, given the absence of heterogeneity in the home country

A corollary of this Proposition is that monetary policy is more powerful in countries with greater heterogeneity in consumption. Since the own and spillover effects of an interest rate cut are greater for a country with more Keynesian agents, world

GDP is more sensitive to monetary policy shocks from these countries.

3.4 Country size and spillovers

Some recent papers analyzing heterogeneous agent open economy models focus on small open economies (see e.g. Auclert, Rognile, Souchier and Straub, 2021, Guo, Ottonello and Perez, 2022). In this subsection, we allow asymmetries in country size in the two-country model setting. We normalize the foreign country size (population) as 1 and denote $0 \leq \varkappa \leq 1$ as home's country size relative to foreign's. Following Sutherland (2005) and De Paoli (2009), the consumption aggregators in both the home and foreign countries are given by

$$C_t^s = [C_{ht}^s]^{\frac{1}{2}}[C_{ft}^s]^{\frac{1}{2}},\ C_t^{s,*} = [C_{ht}^{s,*}]^{\frac{\chi}{2}}[C_{ft}^{s,*}]^{1-\frac{\chi}{2}}$$

When $\varkappa = 1$, our model will be the benchmark two-country setting, but when $\varkappa \to 0$, the two-country model reduces to a small open economy model.[8]

Technical Appendix Section 7 describes the solution for the two-country model with asymmetric country size. Following the same steps as before, we express the dynamics of domestic and foreign output as a function of home and foreign interest rate shocks in the following way.

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{2\delta^* - (\delta^* - 1)\varkappa}{\Delta_1}(\widehat{r}_t - \widehat{r}_t^{fb}) - \frac{1-\delta^*}{\Delta_1}(\widehat{r}_t^* - \widehat{r}_t^{fb*}) \quad (31)$$

$$\widehat{Y}_t^* - \widehat{Y}_t^{fb*} = E_t(\widehat{Y}_{t+1}^* - \widehat{Y}_{t+1}^{fb*}) - \frac{\varkappa(\delta - 1)}{\Delta_1}(\widehat{r}_t - \widehat{r}_t^{fb}) - \frac{1+\delta}{\Delta_1}(\widehat{r}_t^* - \widehat{r}_t^{fb*}) \quad (32)$$

[8]There are two reasons that the home consumption structure is assumed to be unchanged with country size. First, it allows home preferences to be the same in both the two-country model and the small open economy model. Secondly, if the home consumption also depended on country size, then when $\varkappa = 0$ we would need to allow for other home bias parameters to guarantee that home goods are demanded.

where $\Delta_1 = \delta^*(1+\delta) + \varkappa\delta(1-\delta^*)$, and for $\varkappa = 1$, we arrive at the same expressions as Equations (23) and (24) above.

In the small economy case, the condition that monetary policy has a presumptive impact on output is different than the case for Proposition 2 and requires that $n < \frac{1}{\frac{\omega}{2}+1}$. We assume this condition holds in what follows.[9]

Proposition 4. *(a) In the absence of consumer heterogeneity, the response of the output gap and the terms of trade to a domestic or foreign country monetary shock is identical to that of the two country model and independent of the size of each country.*

(b) For a given degree of heterogeneity, the output response to a domestic interest rate cut is smaller in the small open economy (when $\varkappa = 0$) than in the two country world economy.

(c) In the small open economy, the output response to a domestic interest rate cut is independent of the share of Keynesian households in the foreign economy.

(d) The output response to a foreign interest rate cut is greater in the small open economy than in the two country world economy.

(e) The response of the terms of trade to an interest rate shock in either country in the small open economy is identical to that in the two country world economy.

Proof: Please see Technical Appendix Section 7.5.1.

Part (a) of the Proposition reflects standard results indicating that the size of the economy does not affect the responses to monetary policy shocks. But size becomes important in the presence of consumer heterogeneity. First, as before, in the small economy, the impact of an interest rate cut is magnified by the presence of Keynesian households. But when $\varkappa = 0$, there are no spillover effects to the foreign economy, as

[9]Also note that in contrast to the two country case, from conditions (31) and (32), in the small open economy, there is no upper threshold for the value of n in which the impact of expansionary monetary policy moves from negative to positive, analogous to part (d) of Proposition 2. The reason is tied to the fact there is no spillover from the home country monetary policy to the rest of the world (see below). This means that for $\delta < 0$, the negative impact of a monetary expansion on consumption always offsets the terms of trade response, as in part (c) of Proposition 2.

established by Equation (32), hence no 'spillback' to the small economy that enhances the response to the interest rate cut, reducing the overall size of the expansion, thus accounting for part (b). Then part (c) of the Proposition must follow, since the amplification of output in the small economy depends only on domestic Keynesian households. By contrast, part (d) of the Proposition says the spillover response to a foreign interest cut is greater in the small economy than in the two country world economy. The intuition comes from the fact that the foreign economy expands more, since the terms of trade has no effect on the large foreign economy, so the consumer real interest rate $\widehat{r}_t^* + \frac{\varkappa}{2} E_t \Delta \widehat{q}_{t+1}$ responds by the full amount of the interest rate shock, i.e. $\widehat{r}_t^*$, when $\varkappa = 0$. Since the movement of home Ricardian consumption and the terms of trade are the same in the $\varkappa = 0$ and $\varkappa = 1$ case, this means that home output must expand by more in the case of the small open economy. Finally, the explanation behind part (e) of the Proposition comes from the combination of the risk sharing condition $\widehat{C}_t^R - \widehat{C}_t^{R,*} = \widehat{e}_t = \frac{1-\varkappa}{2} \widehat{q}_t$ and the response of $\widehat{C}_t^R$ and $\widehat{C}_t^{R,*}$ to interest rate shocks, which in the case of temporary shocks, the difference can be derived as $-(\widehat{r}_t - \widehat{r}_t^*) - \frac{1+\varkappa}{2} \widehat{q}_t$. Putting these to together, we arrive at the response $\widehat{q}_t = -(\widehat{r}_t - \widehat{r}_t^*)$, as before.

4 Optimal Monetary policy

In this section we explore the implications for optimal monetary policy under PCP. To compare with Engel (2011), we assume a cooperative optimal monetary policy designed to maximize world welfare, but allowing for separate home and foreign monetary policy targets, and incorporating within-country heterogeneity.[10]

[10] Here we are characterizing an optimal monetary policy problem that abstracts away from strategic interaction between monetary authorities. In this, we are following the approach of the literature (see e.g. Benigno and Benigno 2003, 2006, Corsetti Dedola Leduc 2010, 2020, Engel 2011, and others) The cooperative approach involves each country abstaining from attempting to manipulate interest rates or terms of trade in its favour. A full analysis of the strategic interaction in the monetary policy framework would involve a significant expansion of the state space and an

In Technical Appendix Section 4.4, we derive the global planner's objective function in the presence of household heterogeneity. The global planner's problem is:

$$L_0 = \left\{ \begin{array}{c} \frac{1+\omega}{2}(\widehat{Y}_t - \theta_t)^2 + \frac{1+\omega}{2}(\widehat{Y}_t^* - \theta_t^*)^2 + \frac{1-n}{2n}\frac{1}{\omega}\widehat{H}_t^2 + \frac{1-n^*}{2n^*}\frac{1}{\omega}(\widehat{H}_t^*)^2 \\ +\frac{v(2-v)}{4}(\widehat{H}_t^* - \widehat{H}_t)^2 + \frac{\varepsilon}{2\widetilde{\kappa}}\pi_{hh,t}^2 + \frac{\varepsilon}{2\widetilde{\kappa}}\pi_{ff,t}^{*2} \end{array} \right\} \tag{33}$$

where $\widetilde{\kappa} = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$. Heterogeneity matters to the planner in itself, both individual country heterogeneity as well as heterogeneity differences across countries, in addition to the standard loss from output gaps and inflation. Heterogeneity matters for two reasons. The presence of consumption heterogeneity in the first line in the expression (33) captures the loss associated with the deviation of output from its efficient level in the home and foreign country due to heterogeneity. But in addition, the difference in consumption heterogeneity, shown in the second line in Equation (33), represents the loss in cross country risk sharing, since only Ricardian consumers can directly share risk across countries. If there are no Keynesian consumers, so that $n = 0$, the loss function collapses to the standard PCP loss function evaluated by Engel (2011).

Despite that heterogeneity enters the loss function directly in the planner's problem, the solution to the optimal monetary policy problem under PCP still exhibits the 'divine coincidence' of the standard new Keynesian model. In the absence of cost-push shocks, the optimal policy stabilizes producer price inflation, and this closes all output gaps. Intuitively, this follows from Proposition 1 and the fact that we can replace $\widehat{H}_t$ and $\widehat{H}_t^*$ using the definitions of heterogeneity given by equations (15) and (16)

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accounting for the manipulation of goods and financial market prices that would complicate the analysis considerably. Engel (2016) provides an extensive discussion of the issues involved. He notes that in setting up the non-cooperative policy-makers 2nd order approximated loss function, it is necessary to characterize an 'optimal tariff' which removes the pre-existing 'distortion' that exists because each policy maker is not exploiting its market power over the terms of trade. He is able to characterize non-cooperative monetary policy in an environment of complete financial markets (which we have for Ricardian agents), but only in a static model with no price rigidity.

Proposition 5. *The solution to the global planner's problem under PCP restores the economy to the flexible price equilibrium. PPI inflation stabilization can close both the output gap and eliminate the consumption heterogeneity even when $n \neq n^*$.*

See Section 5 of the Technical Appendix for proof. If the planner sets PPI inflation in each country to zero, this ensures the output gap is zero, and from Proposition 1, heterogeneity is zero in both countries.

4.1 Cost-Push Shocks and Targeting Rules

We can describe policy in the form of monetary targeting rules. The targeting rule is represented by the first order conditions implied by the global planner's problem of minimizing the loss function (33) subject to the following inflation equations

$$\pi_{hh,t} = \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t - (1+\omega)\theta_t + \frac{2-v}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi_{hh,t+1} + \nu_t \quad (34)$$

$$\pi^*_{ff,t} = \widetilde{\kappa}\left[(1+\omega)\widehat{Y}_t^* - (1+\omega)\theta_t^* - \frac{2-v}{2}(\widehat{H}_t^* - \widehat{H}_t)\right] + \beta E_t \pi^*_{ff,t+1} + \nu_t^* \quad (35)$$

Here, we allow for the possibility of temporary markup or cost-push shocks in each country, represented by the variables ν_t and ν_t^*.[11] With markup shocks, it is well known that monetary policy faces a trade off between stabilizing the output gap and stabilizing inflation.

Let ξ_t and ξ_t^* to be the shadow prices of (34) and (35). Assuming the 'timeless perspective' for the planner, the optimal monetary policy is characterized by the following first order conditions:

$$\begin{aligned} & (1+\omega)(\widehat{Y}_t - \theta_t) - \widehat{H}_t - \frac{v(2-v)}{2}(1-\delta)(\widehat{H}_t - \widehat{H}_t^*) \\ + \;& (1+\omega+\frac{2-v}{2}(1-\delta))\widetilde{\kappa}\xi_t - \frac{2-v}{2}(1-\delta)\widetilde{\kappa}\xi_t^* = 0 \end{aligned} \quad (36)$$

[11]Following Engel (2011), it is easy to show that these shocks can be modelled as temporary shocks to the elasticity of substitution across varieties.

$$\begin{aligned}&(1+\omega)(\hat{Y}_t^* - \theta_t^*) - \hat{H}_t^* - \frac{v(2-v)}{2}(1-\delta^*)(\hat{H}_t^* - \hat{H}_t)\\ + \quad &(1+\omega+\frac{2-v}{2}(1-\delta^*))\widetilde{\kappa}\xi_t^* - \frac{2-v}{2}(1-\delta^*)\widetilde{\kappa}\xi_t = 0 \qquad (37)\end{aligned}$$

$$\frac{\varepsilon}{\widetilde{\kappa}}\pi_{hh,t} + \xi_{t-1} - \xi_t = 0 \qquad (38)$$

$$\frac{\varepsilon}{\widetilde{\kappa}}\pi_{ff,t}^* + \xi_{t-1}^* - \xi_t^* = 0 \qquad (39)$$

The targeting rules involve an interaction between domestic and foreign output gaps, inflation rates and heterogeneity variables. In the absence of consumer heterogeneity in the domestic economy, (36)- (39) collapse to the standard targeting rules which are independent across countries and involve only a trade-off between a domestic output gap and domestic PPI inflation:

$$(\hat{Y}_t - \theta_t) - (\hat{Y}_{t-1} - \theta_{t-1}) + \varepsilon\pi_{hh,t} = 0 \qquad (40)$$

$$(\hat{Y}_t^* - \theta_t^*) - (\hat{Y}_{t-1}^* - \theta_{t-1}^*) + \varepsilon\pi_{ff,t}^* = 0 \qquad (41)$$

With Keynesian consumers, however, the optimal targeting rules are interdependent. The key insight is due to inefficient risk sharing, captured by the term in heterogeneity differences in the welfare function (33). Since a cost push shock must lead to a fall in the output gap and an increase in heterogenous consumption responses within a country, it leads to inefficient risk sharing across countries and requires a concomitant response in the partner country.

Figure 1 illustrates the response to a home country cost-push shock in the case with and without consumer heterogeneity. The calibrated parameters are presented in Table 1.[12] The labor supply elasticity parameter is set to be one. As shown in

[12] We follow Engel (2011) and Fujiwara and Wang (2017) in choosing the basic value of parameters. We assume that each period is one quarter; The discount factor is 0.99, while the preference weight on labor disutility is 2. The degree of price stickiness is 0.75 so that the average duration of price change is 4 quarters, the elasticity of substitution across individual goods is 11 so that the markup is 10%, the home bias parameter v is 1.5, the persistence of the productivity shock is 0.95.

Kaplan, Violante and Weidner (2014), a reasonable estimate for n in the United States is 0.3. We will follow their estimates for our benchmark model. A higher value of $n = 0.45$ is also considered for sensitivity analysis.

Table 1: Parameter values (Baseline)

Parameter	Description	Value
β	discount factor	0.99
η	preference weight on labor	2
ω	Inverse of the Frische elasticity in Labor supply	1
κ	degree of price stickiness	0.75
ε	elasticity of substitution across individual goods	11
v	home bias	1.5
n	the fraction of household whose is "hand-to-mouth"	0.3
ρ_θ	persistence of productivity shock	0.95
ρ_v	persistence of cost-push shock	0.5
σ	productivity shock size	0.01

As shown in Figure 1, in the absence of Keynesian consumers, an inflation shock leads to a fall in the home output gap and a rise inflation, and a rise in the home policy rate, but has no implications for the foreign country. But when $n = n^* > 0$, there is a spillover to the foreign country - the foreign output gap falls and inflation rises, while the foreign optimal policy rate falls. The spillover acts so as to partly ameliorate the impact of the shock in the home country, as the home policy rate rises by less, and the output gap falls by a smaller amount than in the model without consumer heterogeneity.

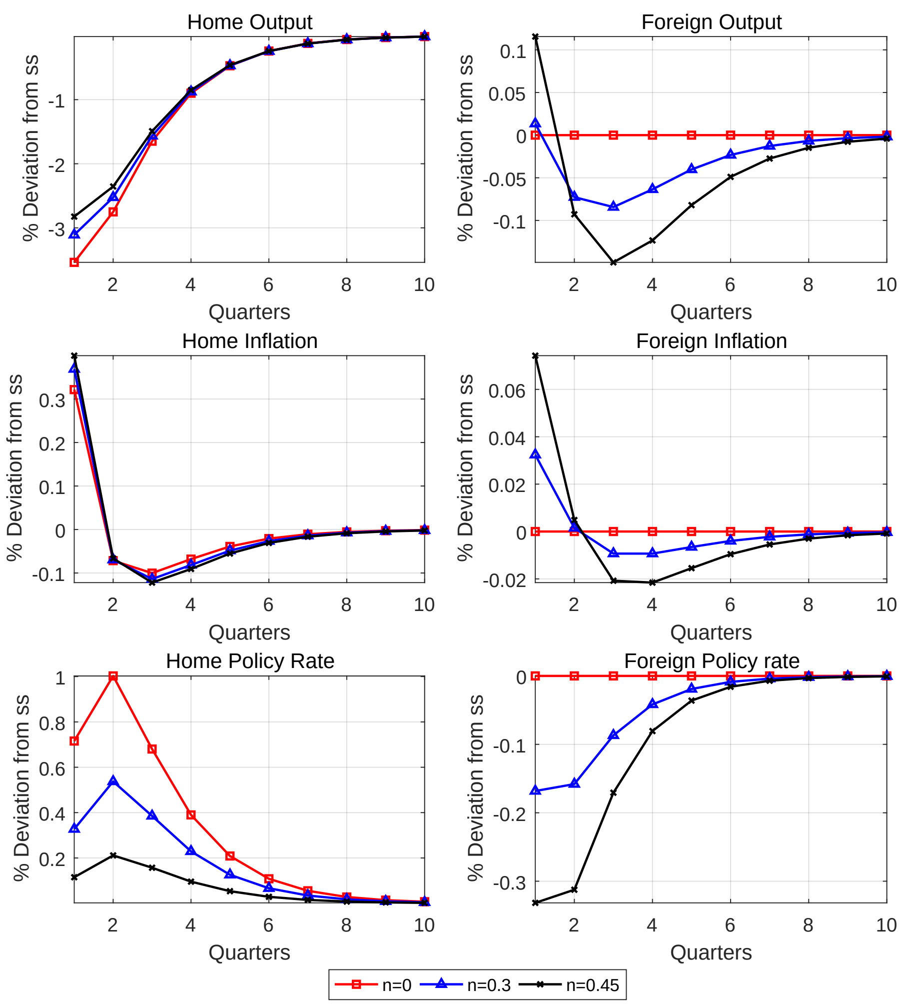

Figure 1: Impulse Response Functions for Home Cost-Push Shock

5 An alternative pricing strategy: LCP

The role of heterogeneity for the spillovers of monetary policy shocks is dependent on the currency of pricing. We now explore the model identical to that above except assuming that firms follow a local currency pricing strategy (LCP), so that exported goods prices are set in the currency of the buyer. As in the PCP case, a firm may reset its prices with probability $1-\kappa$ each period. But now the firm sets two prices, $P^{o}_{hht}(i)$ in home currency for sales in the home market, and $P^{o*}_{hft}(i)$ in foreign currency for sales in the foreign market. The optimization problem of firms is defined in Technical Appendix section 1.1.2.

Under LCP, the terms of trade is

$$Q_t = \frac{P_{fht}}{S_t P^*_{hft}} \tag{42}$$

The deviations from the law of one price (LOOP) for the home and foreign good are:

$$d_t = \frac{S_t P^*_{hft}}{P_{hht}}, \quad d^*_t = \frac{P_{fht}}{S_t P^*_{fft}}$$

5.1 Transmission mechanism of monetary shocks

The household side of model under LCP is identical to that of PCP case. We still have the equations (15) and (16) in the LCP case. We rely on the Euler equations as well as the goods market clearing condition to analyze the effect of monetary policy shocks. In the LCP case however, the deviation from the law of one price (LOOP) influences the equilibrium allocation. Therefore, we need to use the Phillips curves to characterize the dynamics of deviations from LOOP. This part differs from the PCP case and complicates the solution. We first consider the symmetric case ($n = n^*$) with no home bias ($v = 1$) and keep our discussion closely related to Engel (2011), but we highlight the role of household heterogeneity.

5.1.1 Symmetric household heterogeneity ($n = n^*$)

In the LCP setting, the law of one price does not hold and the real exchange rate is variable, even when $v = 1$, so that $\widehat{e}_t \neq 0$. Then the risk-sharing condition for Ricardian consumers is

$$\widehat{C}_t^R = \widehat{C}_t^{R*} + \widehat{e}_t \tag{43}$$

The movement in the real exchange rate creates a gap between home and foreign Ricardian consumption. This will lead to different transmission mechanism of monetary shocks under LCP than under PCP.

Log-linearizing the deviation from the law of one price, terms of trade, and the real exchange rate, we obtain

$$\begin{aligned}
\widehat{d}_t &= \widehat{p}^*_{hft} + \widehat{e}_t - \widehat{p}_{hht} \\
\widehat{d}^*_t &= \widehat{p}_{fht} - \widehat{e}_t - \widehat{p}^*_{fft} \\
\widehat{q}_t &= \widehat{p}_{fht} - \widehat{p}^*_{hft} - \widehat{e}_t \\
m_t &= \frac{1}{2}(\widehat{d}_t - \widehat{d}^*_t) = \widehat{e}_t
\end{aligned}$$

where m_t is the average currency misalignment in the global economy, which is a key variable for optimal monetary policy.

The log-linearized goods market clearing is similar to that of (17) under PCP. In the symmetric case with $v = 1$, we can use the home and foreign goods market conditions to express the within-country terms of trade as the output difference between Home and Foreign countries.

$$\widehat{q}_t + \widehat{e}_t = \widehat{Y}_t - \widehat{Y}^*_t \tag{44}$$

where $\widehat{q}_t + \widehat{e}_t$ equals the average of the relative prices of foreign to home goods in both home and foreign markets.[13] In the LCP case, the relative prices includes two

[13] In particular, $\widehat{q}_t + \widehat{e}_t = \widehat{p}_{fh,t} - \widehat{p}^*_{hf,t}$, where we recall that each expression on the right hand side represents the price normalized by the local CPI. Note also that the relative price of foreign to home goods in the same currency is identical in the home and foreign country. This equivalence holds up to a first order approximation.

components: the terms of trade and the real exchange rate. Moreover, as shown in the Technical Appendix Section 2.2.1, $\widehat{Y}_t - \widehat{Y}_t^* = 0 + t.i.p$, and $\widehat{d}_t + \widehat{d}_t^* = 0 + t.i.p$ where t.i.p is a function of productivity shocks but independent of monetary policy shocks. Intuitively, because this represents a relative price expressed in the same currency, it cannot be manipulated by monetary policy in either country. Now we can use the Euler equations to solve for the effect of monetary shocks on output

$$\widehat{C}_t^R = E_t(\widehat{C}_{t+1}^R) - [\widehat{r}_t - \frac{1}{2}E_t\Delta\widehat{q}_{t+1} - \frac{1}{2}E_t\Delta\widehat{d}_{t+1}] \tag{45}$$

where $\widehat{r}_t = \widehat{i}_t - E_t\pi_{hh,t+1}$ is the real interest rate. The response of home output to monetary shocks and productivity shocks are given by,

$$\widehat{Y}_t = E_t(\widehat{Y}_{t+1}) - \frac{1}{2\delta}\widehat{r}_t - \frac{1}{2\delta}\widehat{r}_t^* + \Theta \tag{46}$$

where Θ is a function of productivity terms. Using the interest rate rules specified in the PCP case, we have

$$\widehat{Y}_t - \widehat{Y}_t^{fb} = E_t(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^{fb}) - \frac{1}{2\delta}\widehat{u}_t - \frac{1}{2\delta}\widehat{u}_t^* + \Theta \tag{47}$$

Proposition 6. *With household heterogeneity, under LCP an expansionary monetary shock in the home or foreign country will increase home and foreign output equally as long as* $n < \frac{1}{w+1}$.

Proof: See Technical Appendix Section 2.2.1 for proof.

As in the case of PCP, there is an upper limit on the size of the Keynesian sector in order for a monetary expansion to have positive effects on output. [14]

Just as in the case of PCP, the effect of heterogeneity is to magnify the impact of a monetary shock due to its effect on the demand of Keynesian consumers. But now the impact is uniform across countries. To see this, we may use the following relationship that is derived from the home goods market clearing condition,

$$\widehat{Y}_t = \frac{1}{\delta}(\widehat{C}_t^R - \frac{1}{2}\widehat{e}_t) + \frac{1}{2}(\widehat{q}_t + \widehat{e}_t) \tag{48}$$

[14] Unlike Proposition 2, due to the absence of relative price effects, there is no an upper region where the aggregate demand relationship reverts as n rises further.

In face of an interest rate cut in the home country, domestic Ricardian consumption increases. However, in the LCP case, foreign Ricardian consumption increases less than Home Ricardian consumption due to the real exchange rate depreciation. Here, $\widehat{C}_t^R - \frac{1}{2}\widehat{e}_t$ captures the total change in global Ricardian consumption and $\frac{1}{\delta}$ measures the response of aggregate consumption to Ricardian consumption.

Unlike the PCP case, the second term in the right hand side of (48) indicates that the relative price impact on demand involves changes in both terms of trade and the real exchange rate. But as we have just noted, the term $\widehat{q}_t + \widehat{e}_t$ is independent of monetary policy. The relative price effect is shut down in the LCP case.

5.2 Optimal monetary policy under LCP

Unlike the case with PCP, the monetary policy under LCP pricing cannot achieve the first best allocation, even in the absence of consumer heterogeneity. But as we show, the presence of Keynesian consumers has important implications for the optimal coordinated monetary policy under LCP. For ease of analysis, we focus on the case with symmetric household heterogeneity ($n = n^*$). It is convenient to define the planner's loss function in terms of global averages and global differences. Following Engel (2011), we define the "relative" and "world" value for output, heterogeneity and inflation. For variable $\hat{X}_t$, $\hat{X}_t^*$, define $\hat{X}_t^d = \frac{1}{2}(\widehat{X}_t - \widehat{X}_t^*)$ and $\hat{X}_t^W = \frac{1}{2}(\widehat{X}_t + \widehat{X}_t^*)$.

We can then define the global planner's loss function with household heterogeneity under LCP is given by:

$$L_0 = \left\{ \begin{array}{c} (1+\omega)(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb})^2 + (1+\omega)(\widehat{Y}_t^W - \widehat{Y}_t^{W,fb})^2 + \frac{1-n}{\omega}(\frac{1}{n})\left[(\widehat{H}_t^d)^2 + (\widehat{H}_t^W)^2\right] \\ +\frac{v(2-v)}{4}(\widehat{m}_t - 2\widehat{H}_t^d)^2 + \frac{\varepsilon}{\kappa}(\pi_t^d)^2 + \frac{\varepsilon}{\kappa}(\pi_t^W)^2 \\ +\frac{\varepsilon}{2\widetilde{\kappa}}\frac{v(2-v)}{2}\left[2(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb}) + (v-1)2\widehat{H}_t^d + (\theta_t - \theta_t^*) + (1-v)\widehat{m}_t\right]^2 \end{array} \right\} \tag{49}$$

Here, $\widehat{Y}_t^{d,fb} = \frac{1}{2}(\theta_t - \theta_t^*)$ and $\widehat{Y}_t^{W,fb} = \frac{1}{2}(\theta_t + \theta_t^*)$ represent the efficient relative and world value for output in the flexible price equilibrium. The loss function (49) indicates that the planner is concerned with average world inflation and the average output gap, as well as the differential in inflation and the output gap across countries, but also, as in Engel (2011), the deviation from the law of one price $\hat{m}_t$, which causes a deviation from efficient risk sharing. As in the case of PCP pricing, consumption inequality, represented by H_t and H_t^*, is also costly, both because it is associated with deviations of the output gap from the efficient level, but also because, given that only Ricardian consumers can engage in cross country risk sharing, consumption inequality leads to a deviation from efficient risk sharing across countries.

The constraints can also be rewritten as below:

$$\pi_t^d = (v-1)\widetilde{\kappa}B(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb}) + \widetilde{\kappa}\frac{v(2-v)}{2}\widehat{m}_t + \beta E_t \pi_{t+1}^d \tag{50}$$

$$\pi_t^W = \widetilde{\kappa}\,(1+\omega)\,(\widehat{Y}_t^W - \widehat{Y}_t^{W,fb}) + \beta E_t \pi_{t+1}^W \tag{51}$$

$$\begin{aligned} & A(\Delta\widehat{Y}_t^d - \Delta\widehat{Y}_t^{d,fb}) - \frac{(v-1)}{2}\Delta\widehat{m}_t + \frac{1}{2}(\Delta\theta_t - \Delta\theta_t^*) \\ = \;& \widetilde{\kappa}\left[-B(\widehat{Y}_t^d - \widehat{Y}_t^{d,fb}) + \frac{(v-1)}{2}\widehat{m}_t\right] \\ + \;& \beta E_t\left[A(\Delta\widehat{Y}_{t+1}^d - \Delta\widehat{Y}_{t+1}^{d,fb}) - \frac{(v-1)}{2}\Delta\widehat{m}_{t+1} + \frac{1}{2}(\Delta\theta_{t+1} - \Delta\theta_{t+1}^*)\right] \end{aligned} \tag{52}$$

where $A = 1-(v-1)(1-\delta), B = 1+\omega+(2-v)(1-\delta)$. Equation (51) indicates that the world output gap drives world inflation, and is equivalent to the standard closed economy representation. But from Equations (50) and (52), we see that output gap and inflation differences interact with $\hat{m}_t$, the deviations of the law of one price.

We first focus on the special case without home bias ($v = 1$). Then Equation (52) shows $\widehat{Y}_t^d$ is independent of monetary policy. Under LCP with $v = 1$, monetary policy can only influence the average world output gap, and cannot affect the division of the output gap between the Home and Foreign country. Then we can show from (52) that a positive relative productivity shock in the home country will lead the relative home output gap to fall. With sticky prices and LCP, the absence of expenditure switching means that after a home productivity shock, the home output response is less than is fully efficient.

Since $\widehat{H}_t^d$ and $\widehat{H}_t^W$ are functions of output gaps, the choice variables for the global planner are $\left\{ \widehat{Y}_t^W - \widehat{Y}_t^{W,fb}, \pi_t^d, \pi_t^W, m_t \right\}$.

Let $\xi_{1,t}$ and $\xi_{2,t}$ to be the shadow prices of (50) and (51). The optimal monetary policy is characterized by the following first order conditions:

$$2\left[1+\omega-\frac{n\omega}{1-n}\right](\widehat{Y}_t^W - \widehat{Y}_t^{W,fb}) + \widetilde{\kappa}\,(1+\omega)\,\xi_{2,t} = 0 \tag{53}$$

$$2\frac{\varepsilon}{\widetilde{\kappa}}\pi_t^d + \xi_{1,t} - \xi_{1,t-1} = 0 \tag{54}$$

$$2\frac{\varepsilon}{\widetilde{\kappa}}\pi_t^W + \xi_{2,t} - \xi_{2,t-1} = 0 \tag{55}$$

$$(\widehat{m}_t - 2\widehat{H}_t^d) + \widetilde{\kappa}\xi_{1,t} = 0 \tag{56}$$

We first note the following

Proposition 7. *If $n = 0$, then a solution to (53)-(56) satisfies $\pi_t^W = \pi_t^d = \widehat{m}_t = 0$.*

The proof is documented in Technical Appendix Section 6.2, and it can be seen by observation of (53)-(56). Although the planner cannot correct individual output gaps, the world output gap is closed, and an optimal policy means that monetary policy targets the CPI in each country, and eliminates all deviations from the law of one price.[15]

In the general case where $n > 0$, this property no longer holds. We establish the following Proposition on the optimal monetary policy under LCP based on the above solution.

Proposition 8. *In the model with LCP, when there is household heterogeneity ($n > 0$) but with no home bias ($v = 1$):*

[15] In Engel (2011), optimal monetary policy targets the CPI in each country when $\omega = 0$ (linear disutility of labour), even when $v \geq 1$. Here we show that the same property holds when $\omega > 0$ so long as $v = 1$, without consumer heterogeneity.

(a) Optimal monetary policy stabilizes the world average output gap and world average CPI.

(b) A home (foreign) relative productivity shock leads to a rise (fall) in relative home CPI inflation π^d.

(c) A home (foreign) relative productivity shock leads to a positive (negative) deviation from the law of one price $\hat{m}_t > 0,\ (< 0)$.

Proof: See Section 6.1 of Technical Appendix for proof.

To illustrate the nature of the optimal policy under LCP, we make use of the simplifying notation for the $\hat{H}_t^d$ process, noting that in this case ($v = 1$), $\hat{H}_t^d$ is driven solely by relative productivity shock and is independent of monetary policy. We let $\hat{H}_t^d = \rho \hat{H}_{t-1}^d + \eta_t$ where $E_{t-1}\eta_t = 0$, and η_t represents unanticipated shocks to relative productivity.

With this assumption, using (50), (54) and (56), we can show that the optimal monetary policy is characterized by a persistent process for $\hat{m}_t$ that satisfies

$$\hat{m}_t = \lambda \hat{m}_{t-1} + \gamma_0 \hat{H}_{t-1}^d + \gamma_1 \eta_t \tag{57}$$

where $1 > \lambda > 0$, and γ_0 and γ_1 are constant coefficients. In particular,

$$\begin{aligned} \gamma_0 &= \frac{2(\beta\rho^2 - \beta\rho - \rho + 1)}{\beta\lambda + \beta\rho - \varepsilon\widetilde{\kappa} - \beta - 1} < 0 \\ \gamma_1 &= -\frac{\beta\gamma_0 - 2\beta\rho + 2\beta + 2}{\beta\lambda - \varepsilon\widetilde{\kappa} - \beta - 1} > 0 \end{aligned}$$

Equation (57) implies that in response to a positive home relative productivity shock, the relative home output gap falls below zero. In this case, because Keynesian agents cannot share risk across countries, there is a failure of cross country risk sharing. The planner responds by allowing the home country a positive deviation from the law of one price, which generates a real exchange rate depreciation and a rise in relative home consumption.

Given (57), we may use Equations (54) and (56) to show that the world relative inflation rate π_t^d takes on the same dynamic process as $\hat{m}_t$. A positive home relative productivity shock is associated with a persistent positive relative CPI inflation rate as part of the optimal monetary policy response.

Now we consider a more general case with home bias. We assume symmetric household heterogeneity ($n = n^*$) but with home bias ($v \neq 1$). Equation (52) then shows $\widehat{Y}_t^d$ is now affected by monetary policy. The choice variables of policy makers are $\left\{\widehat{Y}_t^d - \widehat{Y}_t^{d,fb}, \widehat{Y}_t^W - \widehat{Y}_t^{W,fb}, \pi_t^d, \pi_t^W, m_t\right\}$. And the solution is shown in the Technical Appendix Section 6.3.

In Figure 2 we plot the impulse response functions for a home productivity shock, assuming policy makers implement optimal policy (see Table 1 for the calibrated values). We illustrate the difference in the response of the model under optimal policy in our TANK compared to the standard RANK model (RANK is simulated by setting $n = n^* = 0$). The Figure also shows the fully efficient response, assuming that monetary authorities in that case could stabilize PPI inflation rates in each country.

The rise in home country productivity causes a fall in the home output gap, even under the optimal policy, and a rise in the foreign output gap. As can be seen in the Figure, the optimal policy is to let Home CPI inflation rise, and foreign CPI inflation to fall. This is associated with a nominal and real exchange rate depreciation for the home country.

In the presence of consumer heterogeneity, the planner puts more weight on the response of the output gap in each country, since movements in the output gap generate heterogeneous responses between Ricardian and Keynesian consumers. The optimal policy is then to generate a larger nominal and real exchange rate depreciation for the home country, and a larger PPI and CPI inflation rate, and likewise larger deflation for the home country. As a result we see that a general message implied by optimal policy under LCP is that with consumer heterogeneity the planner wishes to reduce the responses of output gaps, while allowing larger deviations in inflation and exchange rates.

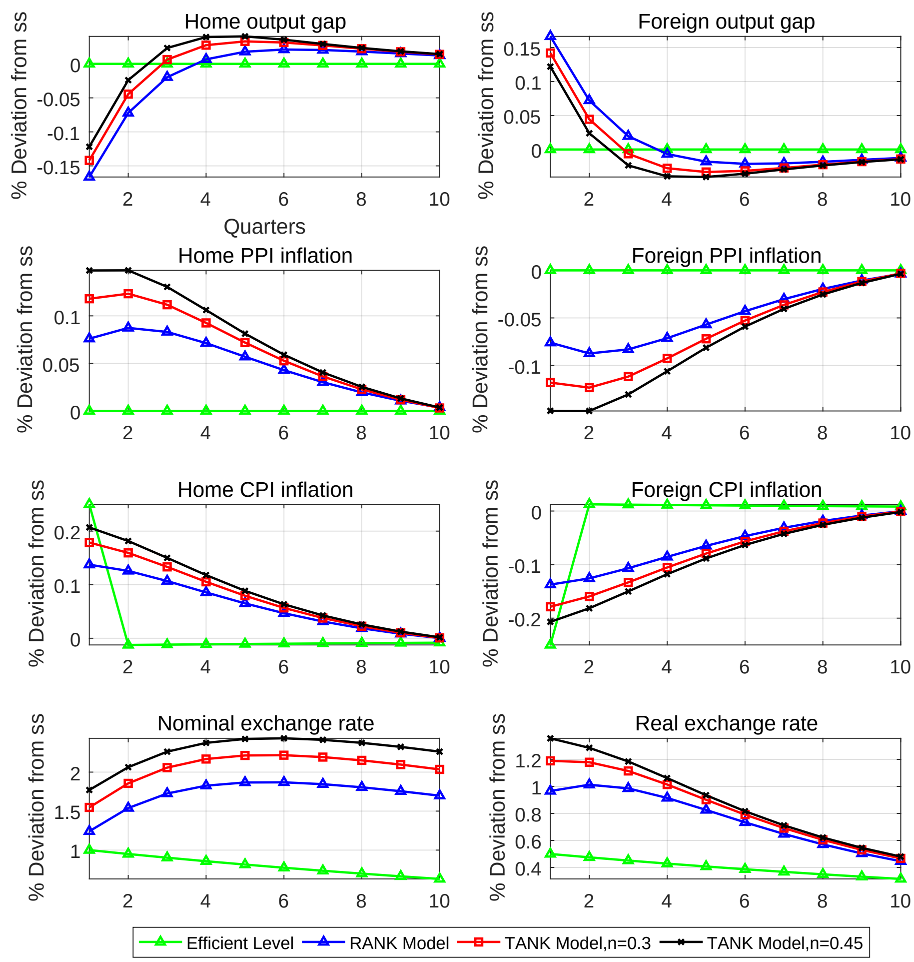

Figure 2: Impulse Response Functions for Home Productivity Shock

6 Conclusions

There has been an extensive debate about the question of whether central banks should be concerned with distributional effects of monetary policy. We extend this debate into the open economy in a simple but tractable model in which there is consumer heterogeneity due to the inability of a group of consumers to access financial markets. We show that consumer heterogeneity may have important effects on the outcome of monetary policy shocks and the transmission of monetary shocks across countries. We find that the optimal monetary policy rule should take account of consumer heterogeneity, and that optimal monetary targeting rules become interdependent in the context of heterogeneous consumers. We also show that the way prices are set matters a lot. In the situation of local currency pricing, an optimal monetary policy actively allows for deviations from the law of one price as a second best policy in face of consumer heterogeneity.

An interesting follow up from our results would be to investigate the empirical evidence for magnified spillovers of monetary policy shocks depending on the degree of heterogeneity within countries. Our model predicts that demand spillovers from monetary shocks are increasing in heterogeneity in the source and receiving country, but the terms of trade and real exchange rate effects should be relatively unaffected. We leave this exploration for further research.

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