Optimal Debt to GDP: A Quantitative Theory*

Johannes Brumm Jakob Hußmann

Karlsruhe Institute of Technology (KIT)

February 2023

Abstract

We analyze public debt policies within a calibrated stochastic OLG model with distortionary taxation. The risk-free interest rate is realistically sensitive to government debt and lower than the growth rate. The risky rate is substantially higher, due to convenience benefits of government debt, idiosyncratic return risk, aggregate risk, and, potentially, market power and wealth inequality. We analyze deficit-maximizing debt (DMD) and welfare-maximizing debt (WMD). Although free-lunch deficits can reduce tax distortions, we find that DMD tends to exceed WMD. Both rise substantially if the risk-free rate falls due to increases in risk, convenience benefits, or longevity, yet not necessarily if it falls due to lower productivity growth or fertility. Taking market power into account barely changes DMD, but substantially reduces WMD. When wealth inequality is included, the poor favor lower public debt than the WMD in the representative agent case, while the rich favor much higher debt-to-GDP ratios.

Keywords: public debt, debt-to-GDP ratio, free-lunch deficits, real interest rate, risk premium, risk-sharing, convenience yield, distortionary taxation, market power, wealth inequality.

JEL Classification Codes: E43, E62, H62, H63.

*We thank Larry Kotlikoff, Felix Kubler, Ludwig Straub, and seminar participants at KIT for helpful comments. We acknowledge financial support from the ERC project SOLG for Policy. Contact details: Johannes Brumm, Karlsruhe Institute of Technology, johannes.brumm@kit.edu. Jakob Hußmann, Karlsruhe Institute of Technology, jakob.hussmann@kit.edu.

1 Introduction

What level of debt to GDP should a country aim for in the long run? Higher public debt crowds out private capital and thereby reduces wages, while it raises rates of return including the government borrowing rate. A higher supply of public debt can also reduce households' consumption risk, in particular for the retired, and provide convenience benefits due to liquidity or regulatory advantages. Finally, at interest rates lower than growth rates, debt provides free-lunch deficits that can alleviate distortionary taxation. These mechanisms jointly determine which debt-to-GDP ratio maximizes welfare. We provide an overlapping generations (OLG) model that features these mechanisms and calibrate the model to the US. We find that deficit-maximizing debt (DMD) is about 100 percent of GDP, yet welfare-maximizing debt (WMD) is only about half as large. It is even lower if market power is taken into account. When wealth inequality is included in the model, the poor favor lower government debt than the WMD in the representative agent case. The wealthy, in contrast, favor debt-to-GDP ratios even above the DMD level.

Our baseline model is just rich enough to capture and quantify the mechanisms most important for assessing the implications of debt levels for welfare. The model can be thought of as an extension of the two-period stochastic OLG model with Epstein–Zin preferences in Blanchard (2019). Our welfare measure is, also following that seminal paper, ex ante utility in the stochastic steady state.[1] As a first step in making the model quantitatively more meaningful, we calibrate the risk-free rate not only to be low but also to be realistically sensitive to government debt levels. That sensitivity stems from two forces. First, from the convenience benefits of government debt, which we, following Mian et al. (2022), include in households' utility and calibrate to empirical estimates of its level and sensitivity. Second, from the crowding out of capital, which we pin down by calibrating the production function to satisfy the overall sensitivity of the risk-free rate. The risky rate of return to capital is calibrated to be realistically higher than the government's borrowing rate, by six percentage points. That gap is partly due to the convenience yield, yet it mainly reflects a risk premium that households demand for idiosyncratic and aggregate risk. Idiosyncratic return risk is calibrated based on cross-sectional data from Snudden (2021). Aggregate risk stems from shocks to productivity and to depreciation that match the historical variation and correlation of returns to labor and capital, as in Krueger and Kubler (2006). Further ingredients of our baseline model include a pay-as-you-go social security system, government spending, and distortionary taxation of labor. The last of these features

[1]Ex ante utility, in contrast to ex interim utility, takes into account the risk that unborn generations face with respect to the state they will be born into. See Brumm et al. (2021) and Mankiw (2022) for a detailed discussion.

implies an important link between deficits and welfare since free deficits can reduce distortionary taxation. Indeed, as long as debt to GDP is below DMD, increasing debt lowers the tax burden and thus reduces the distortion imposed on the economy. Despite including this important mechanism, we find that WMD is substantially below DMD. Thus, at debt-to-GDP ratios in between those two maxima, free-lunch deficits are possible, yet they harm households in the long run.

To further analyze our welfare results we decompose the impact of debt-to-GDP changes on welfare into three effects: first, the convenience benefit of government debt; second, the risk-neutral effect that captures the impact on average consumption levels; third, the risk-sharing effect, which reflects the fact that government debt can help agents to partly insure against idiosyncratic and aggregate risk. Our baseline model and sensitivity analysis show that at reasonable rates of return — a risk-free rate two percentage points below the growth rate and a risky rate four percentage points above it — DMD is roughly 100 percent of GDP, while WMD is below 50 percent. This result is due to the fact that, at the DMD, the positive effects of convenience benefits and risk-sharing are out-weighted by the negative risk-neutral effect. Only at debt-to-GDP ratios much lower than DMD does the risk-neutral effect become substantially weaker and the positive effects dominate. To put this result in perspective, note that the simpler models by Blanchard (2019) and Brumm et al. (2021) focus on lower risky returns than we do. Insofar as they consider realistically high risky returns as calibration targets, they find strongly negative implications of public debt. Our model is more favorable to public debt, mainly due to three of its features. First, a positive convenience benefit of government debt. Second, a realistically lower elasticity of the risk-free rate. Third, endogenous labor supply, which allows free deficits to alleviate tax distortions.

As a next step we scrutinize the widely held notion that low real interest rates imply lower fiscal and welfare costs of public debt, thus speaking in favor of higher debt-to-GDP ratios. We consider various scenarios that all result in a fifty basis point drop in the risk-free rate relative to our baseline. These scenarios differ with regard to the cause of lower rates, and include many causes that have been discussed as drivers of the low rates experienced in developed countries over recent decades. We find that DMD and WMD rise substantially, although to different degrees, following increases in risk, convenience benefits, or longevity. However, they do not rise if risk-free rates fall due to lower productivity growth or reduced fertility. We thus provide a word of caution against interpreting low real rates as an invitation to increase debt-to-GDP ratios, in particular for countries with low fertility and/or low productivity growth.

Until this point in our analysis, we make the standard simplifying assumption that firms operate under perfect competition and thus that factor prices equal marginal products. Yet market power can substantially alter the welfare implications of public debt policy, as Ball and Mankiw (2022) show. To evaluate this nexus, we embed the production sector of their model in our, otherwise richer, OLG model. In the aggregate, there are only two key changes relative to our baseline. First, real factor prices are reduced by the aggregate markup. Second, part of aggregate income accrues as profits. As a consequence of these changes, we now have to distinguish between three measures of the risky rate of return: first, the rental rate of capital; second, the net return per unit of capital, which includes profits and corresponds to the calibration target from national accounts; finally, the social return to capital, which represents the actual marginal product of capital. In the baseline model there is no difference between these three measures. In our calibrated model with moderate levels of market power, the rental rate is exceeded by the net return, which in turn is exceeded by the social return to capital — by about 20 basis points. The higher social return to capital implies a stronger crowding-out impact on wages and labor supply than in the baseline model. As a consequence, WMD decreases very substantially when market power is taken into account. In contrast, DMD is effectively unchanged compared to the baseline — illustrating, once again, that focusing on free-lunch deficits can be misleading.

As a final extension of our model, we include ex ante heterogeneity between households. We assume that there are high-income and low-income households within each generation. With non-homothetic preferences, as in Straub (2019), the high-income households save a larger fraction of their income than the low-income households, resulting in wealth inequality that is even higher than income inequality, just as in real-world data. To generate this pattern in a simple way, we assume, as a stand-in for non-homothetic preferences, that income is positively correlated with patience. In the resulting model, DMD is, once again, basically the same as in the baseline. However, agents now differ strongly in their preferred levels of debt to GDP. The reason for this becomes apparent when comparing the composition of lifetime income of different types. Low-income households save a substantially smaller share of their wages than high-income agents. The impact of higher public debt — lower wages and higher returns — is thus less favorable for low-income households than for high-income households. As a result, poor households prefer a debt-to-GDP ratio even lower than in the baseline model. Rich households, in contrast, want the debt-to-GDP ratio increased even beyond DMD. This raises the return and reduces the risk of their large savings, but comes at the expense of tighter government budgets that imply more distortionary taxation.

While our exact quantitative results are naturally sensitive to modeling choices and calibration targets, our analysis demonstrates that the following insights are of quantitative relevance. First, it is rather the rule than the exception that DMD and WMD strongly differ. Second, WMD can be substantially lower than DMD, implying that it may not be desirable to take advantage of all available free-lunch deficits. Third, lower risk-free rates may or may not, depending on the root cause, speak in favor of higher debt-to-GDP ratios: increased risk, longevity, and convenience benefits do; reduced growth not necessarily. Forth, market power — even if it is conservatively calibrated and plays only a small role in driving rates of return — tilts the welfare evaluation strongly in favor of lower public debt. Finally, higher debt has a quite heterogeneous impact on households, which strongly depends on their reliance on different factors of production — the wealthy stand to benefit from higher debt even beyond maximizing free-lunch deficits, while such debt levels are quite detrimental to the working poor.

The remainder of this paper is organized as follows. Section 2 provides a short literature review. Section 3 describes our baseline model, its calibration, and the solution method. Section 4 presents our analysis of that model. Section 5 includes market power. Section 6 includes income and wealth inequality. Section 7 concludes.

2 Related Literature

This paper builds on the extensive literature on dynamic (in-)efficiency and intergenerational transfers in OLG models. It is most closely related to the recent literature assessing the feasibility of free-lunch deficits and the welfare implications of public debt in a low interest rate environment — reviewed in Reis (2022) and made accessible to a wider audience by Blanchard (2023).

Intergenerational Transfers. Samuelson (1958) and Diamond (1965) show in deterministic OLG models that competitive equilibria may be inefficient when the interest rate is below the growth rate and that intergenerational transfer schemes may be Pareto improving. In stochastic models, welfare assessment is much more difficult for two reasons. First, both the risk-free and the risky rate of return matter. Second, when evaluating welfare one has to take a stand on whether agents born at a given time under different shocks are considered as one single agent or as separate agents — resulting in the concepts of ex ante or ex interim Pareto efficiency; see, e.g., Abel et al. (1989) and Ball and Mankiw (2007), respectively. Several quantitative studies provide welfare evaluation of pay-as-you-go social security systems in OLG models, in the presence of idiosyncratic risk, e.g., İmrohoroglu et al. (1995), aggregate risk, e.g., Krueger and Kubler (2006) , or both, as in Harenberg and Ludwig (2019). In contrast to these papers, we take the scale of the US social security system as given, and focus on the optimal level of government debt.

Free-Lunch Deficits. The recent debate on government debt under low real interest rates prominently features Blanchard (2019), who argues that deficits may entail no fiscal costs and might even be welfare improving. These two claims are scrutinized in several recent papers. Both Reis (2021) and Mian et al. (2022) show — in models with idiosyncratic risk and liquidity benefits of public debt, respectively — that an interest rate below the growth rate indeed implies free-lunch deficits yet not unlimited fiscal space. We follow Mian et al. (2022) in quantifying DMD based on matching the sensitivity of the real interest rate to government debt, for which they provide a thorough overview of empirical estimates. Relative to that paper we include distortionary taxation and a larger set of drivers for the gap between the risky and the risk-free rate. This has the drawback that we lose analytical tractability,[2] yet the benefit that it allows a reasonable welfare analysis. Other papers focusing on the convenience benefit arising from safety and liquidity services of government debt include Mehrotra and Sergeyev (2021), Angeletos et al. (2021), Bayer et al. (2022), Domeij and Ellingsen (2018), and Brunnermeier et al. (2020). Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence on the functional form and spread of the convenience yield, which we use for our calibration.

Public Debt and Welfare. Turning to the question of the welfare assessment of public debt in low interest rate environments, there are several recent papers that add to the perspective of Blanchard (2019). Brumm et al. (2022a) provide stylized counterexamples showing that free-lunch deficits may not be Pareto improving. Barro (2022) considers an infinite-horizon neoclassical growth model where disaster risk generates a realistic risk premium, and shows that the model is dynamically efficient as long as the expected risky return is greater than the growth rate. Kocherlakota (2022) shows in a model with idiosyncratic tail risks that public debt bubbles can be welfare improving. Brumm et al. (2021) consider closed and open economy variants of the Blanchard (2019) model and show that welfare improvements through pay-as-you-go policy stem, if they arise at all, from risk-sharing.[3] Ball and Mankiw (2022) include market power in deterministic neoclassical growth models and find that government debt may reduce welfare even at low risk-free rates.[4] Motivated by this study, we extend our model to include market power as one of many factors driving rates of return, and find that it substantially lowers WMD. Finally, note that none of these recent papers analyzes the interaction of deficits and distortionary taxation. A seminal paper that does, although

[2]We solve the model globally via time iteration and interpolate on the four dimensional state space using sparse grids; see Brumm and Scheidegger (2017).

[3]Blanchard (2019) and Brumm et al. (2021) both use ex ante utility as their welfare measure, yet Blanchard (2019) assumes a risk aversion equal to one with respect to birth risk, while Brumm et al. (2021) assess that risk with the same risk aversion as the risk of old-age consumption.

[4]Basu (2019) and De Loecker et al. (2020) provide empirical evidence on rising markups and corporate profits in the US. Barkai (2020) notices declining labor and capital shares and traces them back to rising profits. Farhi and Gourio (2018) explain the decline in interest rates partially by market power.

in an infinite-horizon model and not in a context of low rates and aggregate risk, is Aiyagari and McGrattan (1998).

3 An OLG Model for Debt Policy Analysis

This section presents and calibrates a stochastic two-period OLG model with multiple sources of risk, convenience benefits of government debt, and endogenous labor supply. We consider these to be the minimal ingredients for analyzing WMD, which we do in Section 4. Sections 5 and 6 extend this model further by including income and wealth inequality and market power, respectively.

3.1 Model

We first present households' decision problem, which is to choose labor, and savings in capital and government bonds. Next we characterize the convenience benefit of the latter. Then we turn to production and aggregate risk, which relates to productivity and depreciation. Finally, we describe the government, which consumes, taxes labor and capital, runs a pay-as-you-go social security system, and issues debt.

Households' Problem. Households live for two periods, working age and retirement. Young households elastically supply labor, ℓ_t, with Frisch elasticity v, at wage w_t, which is taxed at rate $\tau_l + \tau_p$, representing labor tax and pay-as-you-go pension contribution. From their net earnings, the young consume, $c_{y,t}$, and save for retirement. Savings are invested in risky physical capital, k_{t+1}, and risk-free government bonds, b_{t+1}, which provide convenience benefits, $V(b_{t+1}, y_t)$, where y_t is output. The old receive a pension from the pay-as-you-go system, $\tau_p \ell_t w_t$, and returns from their investment in physical capital, $R_t k_t$, and in government bonds, $R_t^f b_t$, which are both taxed at rate $\tau_{k,t}$. As there is no bequest motive, the old consume everything they own, $c_{o,t}$. While bonds are risk free, returns on physical capital are subject to aggregate and idiosyncratic risk. Aggregate risk arises from productivity and depreciation risk, which we specify when we describe the production sector below. Idiosyncratic return risk, which we calibrate based on cross-sectional data, is captured by the random variable ξ_i, which is household specific, equals one in expectation, and is i.i.d. across households.[5] Preferences over consumption are Epstein–Zin with an intertemporal elasticity of substitution (IES) of one, risk aversion γ, and discounting $\beta/(1-\beta)$.

[5]We assume a continuum of agents within each generation, $i \in [0, 1]$, yet suppress the individual-specific index whenever possible. Aggregation across agents is defined as L^2-Riemann integration; see Uhlig (1996).

Thus, households solve the following maximization problem; first-order conditions (FOCs) can be found in Appendix A.1.

$$\max_{k_{t+1},b_{t+1},\ell_t} \quad u_t = (1-\beta)\ln\left(c_{y,t} - \zeta\frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1},y_t)\right) + \frac{\beta}{1-\gamma}\ln\left(\mathbb{E}_t\left\{c_{o,t+1}^{1-\gamma}\right\}\right)$$

$$\text{s.t.} \qquad c_{y,t} = (1-\tau_{l,t}-\tau_p)w_t\ell_t - k_{t+1} - b_{t+1}$$

$$c_{o,t+1} = (1-\tau_{k,t+1})\cdot\left(\xi_i R_{t+1}k_{t+1} + R^f_{t+1}b_{t+1}\right) + \tau_p\ell_{t+1}w_{t+1}$$

Convenience Yield. Besides the risk channels driving a wedge between the risky and the risk-free rate, we include a convenience yield in the model—a spread between risk-free government bonds and risk-free private bonds—arising from (utility) benefits, $V(b_{t+1}, y_t)$, specific to holding government bonds, such as liquidity or regulatory advantages. Although we do not model the private bond explicitly, the respective (shadow) rate of return, $R^{f,N}$, can be derived by assuming it is traded in zero net supply, see Appendix A.1. The above specification of the utility function gives us a closed form expression of the convenience yield from the first order conditions of the household.

$$\frac{R^{f,N}_{t+1}}{R^f_{t+1}} = \frac{1}{1 - V'(b_{t+1}, y_t)}$$

Following Krishnamurthy and Vissing-Jorgensen (2012) and Mian et al. (2022), we assume $V' \geq 0$, $V'' \leq 0$, and that the annualized convenience yield is linear in debt to GDP.[6] To pin down the functional form of V we make three further assumptions. First, $V(0) = 0$; thus government bonds provide strictly positive (convenience) utility. Second, the spread at a given (initial) debt-to-GDP ratio ρ_{B_0} equals ψ. Third, the elasticity of the convenience yield regarding the government debt ratio is constant and parameterized by κ. The explicit functional form of V and its derivation from these assumptions is relegated to Appendix A.2.

Production and Aggregate Risk. The representative firm rents labor and physical capital from the young and the old households, respectively. The firm produces output y_t according to a general constant elasticity of substitution (CES) production function, parameterized by capital intensity α and elasticity of substitution $1/(1-\iota)$. We will calibrate ι to match the observed sensitivity of the risk-free rate to government debt — getting this driver of the crowding-out effect right is crucial for our quantitative

[6]Krishnamurthy and Vissing-Jorgensen (2012) find a negative, linear relationship between convenience yield and debt to GDP to be a reasonable fit to the data. Mian et al. (2022) assume the same linear relationship and provide a thorough overview of empirical estimates of the elasticity of the convenience yield, which we will use in our calibration.

analysis of public debt policies. Production is stochastic and faces two sources of uncertainty. First, total factor productivity, z_t, which is log-normally distributed with zero mean[7] and affects both returns to capital, R_t, and labor, w_t. Second, depreciation, δ_t, which is stochastic and distributed such that the returns to capital follow a log-normal distribution and that we can match the (imperfect) correlation between returns to capital and labor.[8] For now we assume production is perfectly competitive; thus, factor prices equal marginal products.

$$
\begin{aligned}
y_t &= z_t \left(\alpha k_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}} \\
w_t &= z_t(1-\alpha)\ell_t^{\iota-1}\left(\alpha k_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}-1} \\
R_t &= z_t \alpha k_t^{\iota-1}\left(\alpha k_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}-1} + (1-\delta_t) \\
\ln z_t &\sim N(0,\sigma_z),\ \ln \eta_t \sim N(\mu_d,\sigma_d),\ \ln \varepsilon_t \sim (1-\chi)\ln z_t + \chi \ln \eta_t \\
\delta_t &= 1 + \alpha z_t (1-\varepsilon_t) k_t^{\iota-1}\left(\alpha k_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1-\iota}{\iota}}
\end{aligned}
$$

Government Policies. The fiscal authority operates according to four simple rules. First, it collects pension contributions as a fixed share, τ_p, of labor income and it transfers them in a pay-as-you-go fashion to the old. Second, it engages in government consumption, g_t, totaling a fixed share, ρ_G, of GDP.[9] Third, it issues a constant share of GDP, ρ_B, in bonds and repays last period's debt. The parameter ρ_B will be key to our analysis, as it parameterizes the debt-to-GDP level.

$$
\begin{aligned}
g_t &= \rho_G y_t \\
b_{t+1} &= \rho_B y_t
\end{aligned}
$$

Finally, the government levies taxes on labor, $\tau_{l,t}$, and capital, $\tau_{k,t}$, to balance its budget, where the tax rates are pinned down by assuming that labor and capital pay fractions Δ and $1-\Delta$, respectively, of government net expenditures.

$$
g_t + R_t^f b_t - b_{t+1} = \tau_{l,t} w_t \ell_t + \tau_{k,t}\left(R_t k_t + R_t^f b_t\right)
$$

$$
\tau_{l,t} = \frac{g_t + R_t^f b_t - b_{t+1}}{w_t \ell_t}\Delta,\ \ \tau_{k,t} = \frac{g_t + R_t^f b_t - b_{t+1}}{R_t k_t + R_t^f b_t}(1-\Delta)
$$

[7] We assume that log-productivity is normally distributed with zero mean as we follow Blanchard (2019) in considering a detrended economy. To relate our results to real-world data, in particular when it comes to growth rates and rates of return, we need to consider an extension with labor-augmenting technological progress that exhibits a balanced growth path, as we show in Appendix A.3.

[8] To do so, we assume that depreciation is driven both by z_t and by another shock, η_t, their respective weights being captured by the parameter χ.

[9] Government consumption does not enter the households utility function. Note, however, that if it did, lower GDP (e.g., from higher government debt) would be welfare deteriorating through this additional channel.

Table 1: Externally calibrated parameters.

Parameter		Interpretation	Source
Government			
ρ_{B_0}	100%	debt-to-GDP ratio	stylized average
ρ_G	14%	government consumption	Mian et al. (2022), World Bank
τ_p	12%	pension contribution	US payroll tax
Δ	66%	labor share in tax revenue	IRS Statistics of Income 2020
Convenience Yield			
ψ	1%	convenience yield spread	FRED, Appendix B
κ	0.9%	conv. yield elasticity	d'Avernas and Vandeweyer (2021)
Labor Supply			
v	0.75	Frisch elasticity	Chetty et al. (2011)

3.2 Calibration

We calibrate our model to the US economy and consider the length of a model period to be $T = 25$ years. Conceptually, our calibration procedure consists of two steps. We start from a plausibly calibrated specification of the government sector, production process, and the households' exposure to different sources of risk. We then calibrate three key parameters — discounting, risk aversion, and the elasticity of substitution between capital and labor — to ensure that the model matches three aspects of the real world that are of crucial importance for debt policy analysis. These are the risk-free rate, the much higher risky rate, and the elasticity of the risk-free rate with respect to government debt. All parameters calibrated externally are available in Table 1, while parameters calibrated internally and the corresponding targets are given in Table 2.

Government Policies. We parameterize government consumption, ρ_G, to match the average government expenditure in the US over the previous ten years, which amounts to 14%, the same value that Mian et al. (2022) pick. Government debt to GDP, ρ_{B_0}, is set to a stylized 100 percent, even though the US surpassed this value during the Covid-19 pandemic. The pension contribution rate, τ_p, is set to 12% and the share of tax revenue attributable to labor Δ is set to 66%.[10]

Labor Share and Labor Supply. The Frisch elasticity of labor supply, v, is set to 0.75 following Chetty et al. (2011). The average labor supply is normalized to 0.3 using the disutility of labor, ζ. Lastly, we calibrate α to match a labor share of 63%.

[10]IRS Statistics of Income (SOI) Table 1.3, 2020. Available here.

Table 2: Internally Calibrated Parameters.

Parameter		Target		Source
Risk				
σ_I	0.26	$\mathbb{E}_0\{\xi_i R_t\}$	40%	Snudden (2021)
σ_z	0.14	$\text{CV}(w_t)$	13%	Jordà et al. (2019)
σ_d	0.10	$\text{CV}(R_t)$	25%	Jordà et al. (2019)
χ	2.12	$\text{Corr}(w_t, R_t)$	-7.5%	Jordà et al. (2019)
Production				
ζ	0.20	$\mathbb{E}_0\{\ell_t\}$	30%	normalization
α	0.70	$\mathbb{E}_0\{w_t\ell_t/y_t\}$	63%	stylized fact
μ_d	-0.08	$\mathbb{E}_0\{k_t/y_t\}$	300%	Ball and Mankiw (2022)
Rates of Return				
β	0.65	$\mathbb{E}_0\{R_t\}$	4%	Ball and Mankiw (2022)
γ	19.63	$\mathbb{E}_0\{R_t^f\}$	-2%	stylized fact
ι	0.29	$\mathbb{E}_0\{\varphi\}$	2.2%	Mian et al. (2022)

Idiosyncratic and Aggregate Risk. Our calibration of idiosyncratic return risk is empirically motivated by Snudden (2021) and Fagereng et al. (2020), who find heterogeneous returns on wealth for households in the US and Norway. Snudden (2021) provides quantitative evidence of heterogeneous returns in the US on an annual basis. He finds a standard deviation of 8% in returns, which we scale up to the 25-year time horizon, assuming a random walk, resulting in a value of 40%. We fit σ_i such that the portfolio return heterogeneity — including the government bond, which is not affected by ξ_i — matches this 40%. Aggregate shocks are calibrated to resemble US long-term data on volatility and correlation of labor and capital income. In line with Krueger and Kubler (2006), we calibrate the coefficient of variation of wages and risky returns and their correlation at the model's frequency. To get sufficient data points for 25-year aggregates, we use US data provided in the macrohistory database by Jordà et al. (2019) going back to the nineteenth century. We find a coefficient of variation of 13% for wages, 25% for risky returns, and a -7.5% correlation of the two — and calibrate σ_z, σ_d, and χ accordingly. The mean depreciation shock, μ_d, is chosen such that the ratio of capital to (annual) output equals 300%, the same target as in Ball and Mankiw (2022).

Convenience Yield and Risk-Free-Rate Elasticity. The annualized convenience spread between treasury bonds and corporate bonds, ψ, at the debt-to-GDP ratio of 100% is set to 1pp, which fits the empirical spread over the past 20 years.[11] The elasticity of the

[11] We reconsidered the data sources of Krishnamurthy and Vissing-Jorgensen (2012) taking into account the additional data points for the past 20 years. See Appendix B for details.

convenience yield, $\kappa = \mathbb{E}_0\{\partial(R_t^{f,N}/R_t^f)/\partial b_t\}$, is chosen to be 0.9% following d'Avernas and Vandeweyer (2021). The elasticity of the risk-free rate, $\varphi = \mathbb{E}_0\{\partial R_t^f/\partial b_t\}$, is set to 2.2% based on Mian et al. (2022).[12] We select these elasticities over other estimates in the literature for two reasons: First, the estimates are based on (relatively) recent data — d'Avernas and Vandeweyer (2021) build upon data from 2014 to 2016, while Mian et al. (2022) refer to a political event in 2021. Second, both studies use exogenous events — d'Avernas and Vandeweyer (2021) a change in money market regulation, and Mian et al. (2022) a sudden increase in federal debt after the Georgia Senate election — to measure the elasticity; hence, the methods are consistent. To match the elasticity of the risk-free rate given the elasticity of the convenience yield, we use the parameter ι of the CES production function as it drives the crowding-out effect, which determines the overall elasticity of the risk-free rate together with the elasticity of the convenience yield. The parameter ι takes the value 0.29, implying an elasticity of substitution between labor and capital of 1.41, somewhat higher than in the Cobb–Douglas case. This can be thought of as capturing the effect of openness, which is absent from our model yet modeled in Brumm et al. (2021).

Rates of Return and Preferences. In the model we abstract from growth. Interest rates in the model therefore correspond to the interest–growth differential. Assuming an average growth rate in the US of 2%, our targets for $R^f = -2\%$ and $\mathbb{E}\{R\} = 4\%$ correspond to a real risk-free rate of 0% and a risky return of 6% in the US. While the target for the risk-free rate seems reasonable for recent decades, the risky interest rate is more difficult to measure.[13] We choose the relevant target to be capital income per unit of capital, R^m, which is in our baseline equivalent to R. Differences arise when we introduce market power, which drives a wedge between R, R^m, and the social return to capital. From US national accounts Ball and Mankiw (2022) infer $R^m = 6\%$, which corresponds to $\mathbb{E}\{R\} = 4\%$ in the baseline model. To meet our targets we calibrate discounting, $\beta/(1-\beta)$, and relative risk aversion, γ. Despite the rich sources of risk included in the model, a risk aversion of almost 20 is needed to match the large difference between risky and risk-free returns. We regard the high γ as a stand-in for risks not modeled in the paper — including disaster risk, which can reduce the required risk aversion substantially without changing the welfare results very much, as Brumm et al. (2021) show.

[12] The parameter φ is calculated numerically by increasing the debt-to-GDP ratio by 10 percentage points. For φ we estimate log elasticities. Both elasticities refer to annualized interest rates.

[13] See Blanchard (2019) for evidence on the risk-free rate as well as for a discussion on how to measure risky returns.

3.3 Solution Approach

This section briefly describes our approach to solving and simulating the model. More details can be found in Appendix A.

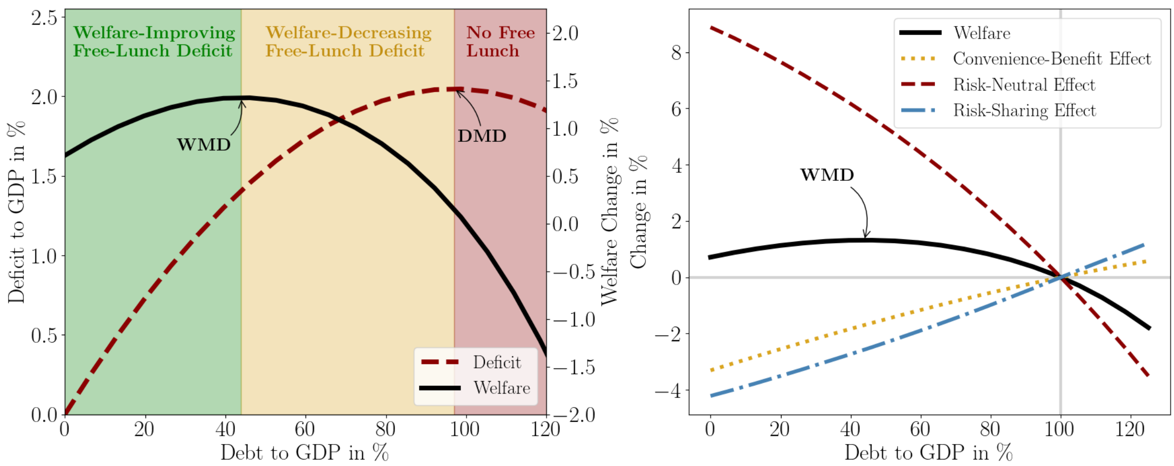
Time Iteration on Sparse Grids. Unlike the simpler models in Blanchard (2019) or Brumm et al. (2021) our model cannot be solved along the simulation since next period's capital returns now depend on endogenous labor supply in that period. We thus solve for the equilibrium policy functions of our model by iterating on the first order conditions — time iteration. The state is four-dimensional, consisting of a productivity shock, z_t, depreciation shock, ε_t, capital stock, k_t, and government debt burden relative to capital, $R_t^f b_t / k_t$. Already in four dimensions, conventional tensor-product grids imply considerable computational costs, which is why we employ sparse grids with hierarchical basis functions as in Brumm and Scheidegger (2017) and Brumm et al. (2022b). Expectations over shocks z_{t+1}, ε_{t+1}, ξ_{i+t+1} are approximated using Gauss–Hermite quadrature with several hundred quadrature points.

Simulation and Debt Diagrams. Given policies that solve the households' problem at a debt policy ρ_B, we approximate the ergodic distribution of the model by simulating for a sufficient amount of periods. For a given debt policy ρ_B we can then calculate unconditional expectations over endogenous outcomes on the ergodic set. When the model is solved and simulated for different debt policies $\rho_B \in \{0\%, \ldots, 120\%\}$ and the statistics are computed, we can then plot them as functions of the debt policy. These plots are the main vehicle of our analysis below.

4 Deficit-Maximizing and Welfare-Maximizing Debt

We now analyze debt policy using the model presented and calibrated above. First, we consider the size of deficits for different debt-to-GDP ratios. To do so we plot deficit–debt diagrams and identify DMD — the level of debt to GDP that allows for maximal (average) deficits. We then move beyond this narrow fiscal perspective and consider welfare–debt diagrams, using ex ante expected utility to measure welfare. We find that the presence of distortionary taxation creates a link between DMD and WMD, as higher deficits allow for lower taxes and less distortion. Nevertheless, WMD turns out to be much lower than DMD.

Figure 1: Deficits, Welfare, and Welfare Decomposition.



The left plot displays the deficit to GDP for different debt rules ρ_B. It also shows the percentage change in welfare compared to $\rho_b = 100\%$. The right plot provides a decomposition of welfare changes into the convenience-yield effect, the risk-neutral effect, and the risk-sharing effect.

4.1 Free-Lunch Policy: Deficit-Maximizing Debt

When the interest–growth differential is negative, as in our baseline where it equals -2%, the government can improve its budget by simply issuing debt and keeping a constant debt-to-GDP ratio. But can it increase debt without limit? And if not, what choice of debt to GDP maximizes free-lunch deficits?

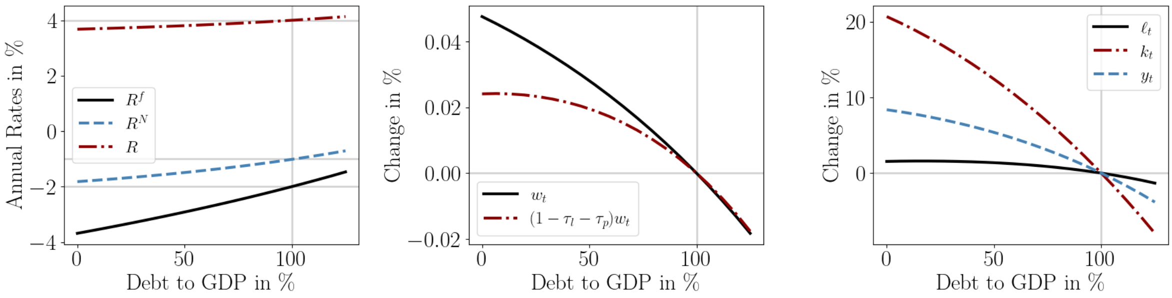
Deficit-Maximizing Debt. To determine the deficit-maximizing debt-to-GDP ratio one has to take into account not only the interest–growth differential but also the elasticity of the risk-free rate with respect to government debt, φ. Instead of $R^f - G < 0$ the necessary condition for free-lunch deficits is $R^f - G - \varphi < 0$, as pointed out by Mian et al. (2022). In our baseline at 100% debt to GDP with $R^f - G = -2\%$ and $\varphi = -2.2\%$ this condition is violated by a small margin and, indeed, the maximum deficit in our stochastic model is obtained at a debt-to-GDP ratio of 97%.[14] Below that debt level the government is able to run free-lunch deficits. The left panel of Figure 1 includes the deficit–debt diagram of our baseline model.[15] The (dashed) curve is hump shaped, starting at zero deficit without debt, monotonically rising up to 2.03% at the DMD, and then falling again as higher debt decreases deficits—the no-free-lunch region.

Limits of Debt. So why are there limits to free-lunch deficits? Because the government borrowing rate rises when debt to GDP increases. That happens in our model for

[14] The deterministic condition from Mian et al. (2022), while obviously no longer exact, still provides guidance in our stochastic setting.

[15] In order to put deficits in the right proportion to debt some adjustment to the time horizon is required, as explained in Appendix A.5.

Figure 2: Rates of Return, Wages, Labor, and Capital.



The left plot displays the (annualized) risk-free rate on government bonds, R^f, on private bonds, R^N, and the risky return, R, as a function of debt to GDP. The middle plot displays percentage changes, relative to 100% debt to GDP, of before-tax wages w_t and after-tax wages $(1 - \tau_{l,t} - \tau_p)w_t$. The right plot exhibits percentage changes in labor supply, ℓ_t, capital, k_t, and output, y_t.

two reasons; both are apparent in Figure 2. First, as displayed in the left plot, rising debt causes a decline in the convenience yield — that is to say, the gap between the government borrowing rate and the private risk-free rate narrows. Second, capital is crowded out (see right plot) lifting the risky rate of return and the safe rates along with it (see left plot). The elasticity of the risk-free rate with respect to government debt, φ, is calibrated such that these two forces together are as strong as they appear in the real world.[16]

4.2 Beyond Fiscal Arithmetic: Welfare-Maximizing Debt

We now understand when free-lunch deficits are possible and what level of debt to GDP allows for the largest average deficit. However, that does not tell us which debt policy is desirable. To try and answer that question we have to assess the welfare implications of debt policies.

Measuring Welfare. As in Blanchard (2019) we calculate the ex ante utility of agents born in the long run, i.e. in the stochastic steady state of the economy. We follow Brumm et al. (2021) in assessing risk with respect to the birth state with the same risk aversion as risk of old-age consumption. The resulting welfare measure, ex ante utility of agents in the long run, $\mathcal{U}_0$, is defined as follows.[17]

$$\mathcal{U}_0 = \mathbb{E}_0 \left\{ \exp(u_t)^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

[16]The resulting DMD does not depend much on the specific forces that determine this elasticity. However, the welfare analysis does, and heavily, as we show in Appendix C.

[17]Note that u_t as defined above needs to be transformed into $\exp(u_t)$ to make it homogeneous of degree one.

Here, $\mathbb{E}_0$ denotes expectations over the stochastic steady state, i.e. the ergodic distribution over exogenous and endogenous states. A debt policy that maximizes that measure, the WMD, can be thought of as the answer to the following question: Suppose you are waiting behind Rawls's veil of ignorance to enter the economy without knowing under which circumstances you will be born — what debt-to-GDP policy would you want the government to run? To better understand the answer to that question that our model and welfare measure deliver, we decompose changes in ex ante utility, building upon Brumm et al. (2021). We distinguish between the effect that originates from the convenience benefit, the effect of risk-sharing, and the effect that would be present even in the absence of risk aversion or convenience benefits, which we call the risk-neutral effect. Details on the decomposition are relegated to Appendix A.4.

Welfare-Maximizing Debt. The left plot of Figure 1 shows the welfare–debt diagram (right scale) next to the deficit–debt diagram (left scale). Both are hump shaped, yet welfare peaks at a much lower debt-to-GDP ratio, 44% versus 97%. That means that even though free-lunch deficits are possible they harm households in the long run. Figure 1 shows that welfare falls by more than 1% from WMD to DMD and then falls even more steeply as debt to GDP is increased further. The welfare decomposition, displayed in the right plot of Figure 1, reveals the trade-off that WMD results from. There is a negative risk-neutral effect (RNE) that captures the impact on average consumption levels and there are two counteracting forces, the convenience-benefit effect (CBE) and the risk-sharing effect (RSE). The overall effect is concave, mainly due to the curvature in the RNE, and exhibits a distinct maximum, the WMD.

The Role of Distortionary Taxation. WMD and DMD are far apart in our baseline calibration. The sensitivity analysis in Appendix C shows that this is rather the rule than the exception. The main reason for this is simply that even free-lunch deficits crowd out capital, which hurts welfare if the marginal product of capital is realistically large. While there are risk-sharing benefits as well as convenience benefits that work in the other direction, it would certainly be pure chance if DMD and WMD were close. So is there any tight connection between the two maxima, if not quantitatively then at least in terms of an economic mechanism? In other words, is there an obvious welfare benefit of being able to run sustained deficits? In the model, and arguably in the real world, the answer is that free deficits can reduce distortionary taxation. Indeed, as long as we are to the left of DMD, increasing debt reduces the amount of taxes (as a share of GDP) that needs to be raised. This reduces the distortionary effect of taxation, which can be seen from the fact that the after-tax wage is flatter than the before-tax wage in Figure 2 if and only if debt to GDP is below DMD. This positive effect of government debt, which weakens the RNE in the region to the left of DMD, is not present in Blanchard (2019) or Brumm et al. (2021) as these studies do not consider endogenous labor supply.

4.3 Determinants of Optimal Debt to GDP

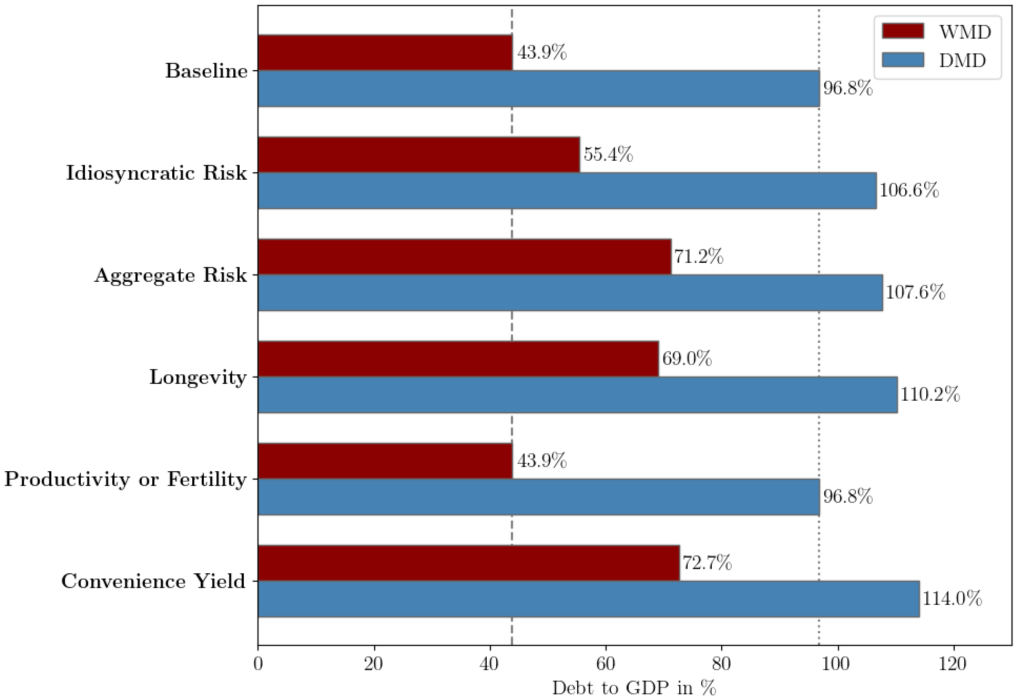
It is a widely held view, prominently and eloquently stated by Blanchard (2019) and elaborated on in Blanchard (2023), that low interest rates imply lower fiscal and welfare costs of public debt, thus speaking in favor of higher debt-to-GDP levels. Through the lens of our model, we now provide a differentiated analysis of this proposition, which confirms it with some qualification. We not only distinguish between DMD and WMD, but also between different causes of low rates. To do so, we consider various scenarios that all result in a fifty basis point drop in the risk-free rate relative to our baseline.[18] These scenarios differ in the cause of lower rates, including many causes that have been discussed and identified as partial drivers of the low rates experienced in developed countries over recent decades: increased idiosyncratic (return) risk, increased aggregate (depreciation) risk, increased longevity, reduced fertility, reduced productivity, or increased convenience benefits. In Figure 3 we list these interest rate drivers and report the implied WMD and DMD after recalibration.

Idiosyncratic and Aggregate Risk. By increasing the risk premium various sources of risk can reduce the risk-free rate. We consider an increase in idiosyncratic return risk (from $\sigma_I = 0.26$ to $\sigma_I = 0.3$) and aggregate depreciation risk (from $\sigma_d = 0.1$ to $\sigma_d = 0.12$) and find that both increase DMD by about ten percentage points. While WMD rises by about the same amount in the case of idiosyncratic return risk, it rises substantially more in the case of aggregate depreciation risk. This difference between the two scenarios indicates that government debt, in our model, does a better job of insuring against aggregate risk than insuring against idiosyncratic risk.

Demographics and Growth. Turning to demographic drivers of the real interest rate, we consider discounting as a proxy for longevity. For the drop in the risk-free rate of fifty basis points to materialize, yearly discounting needs to increase from $\beta = 0.65$ to $\beta = 0.73$. We find that increased longevity raises both WMD and DMD substantially, implying that low rates due to stronger incentives to save for retirement indeed speak in favor of higher public debt. This is in contrast to the scenarios of reduced productivity growth or lower fertility. Assuming that growth is labor augmenting and the convenience benefit is independent of growth, we find that interest rates move one

[18] The remaining parameters are kept constant; the other calibration targets are, hence, not satisfied after recalibration.

Figure 3: Causes of Low Risk-Free Rates — Impact on WMD and DMD.



We consider several scenarios that all result in a fifty basis point drop in the risk-free rate relative to our baseline: increased idiosyncratic (return) risk, increased aggregate (depreciation) risk, increased longevity, reduced fertility, reduced productivity growth, and increased convenience benefits.

for one with growth rates; see Appendix A.3 for details. Since the interest–growth differential thus stays constant in these scenarios (and the elasticity φ does not change), DMD is the same. Moreover, WMD also stays constant.[19]

Convenience Benefits and Overall Comparison. Finally, an increase in the convenience benefit (from $\psi = 1\%$ to $\psi = 1.5\%$) that delivers the same drop in the risk-free rate of fifty basis points results in the largest increase in both DMD and WMD. On the face of it, this result is, unfortunately, not very informative as the convenience benefit is a black box in our model. However, if we take our calibration seriously, this result tells us that convenience benefits of public debt are an important determinant of both the fiscal and the welfare implications of public debt. All in all, it is clear that any drop in the risk-free rate that decreases the interest–growth differential will increase DMD. Obviously a larger spread between interest and growth rates, at roughly the same risk-free-rate elasticity, implies larger fiscal space. Turning to WMD, we find that it qualitatively behaves like DMD in all our scenarios, while there is a substantial dif-

[19] In the case of the fertility scenario this conclusion requires the following additional assumption: welfare is derived from per capita utility in each generation, so without weighting generations by their size. More importantly, this result also rests on the assumption of a unit elasticity of intertemporal substitution (IES). If the IES were lower, the risk-free rate would fall more than one for one, making DMD and WMD rise; if the IES were greater than one, in contrast, DMD and WMD would even fall.

ference between the two in terms of quantitative changes. For instance, the longevity scenario implies a higher DMD than the aggregate risk scenario but a smaller WMD. What does that mean? Low rates that stem from longevity, as compared to aggregate risk, make it even easier from a fiscal perspective to run deficits, yet less desirable from a welfare perspective.

5 Market Power and Public Debt

So far we have maintained the standard simplifying assumption that firms are perfectly competitive and factor prices equal marginal products. Yet Ball and Mankiw (2022) show that market power can substantially alter the welfare implications of public debt policy. They model the impact of market power in deterministic neoclassical growth models (the Solow growth model and the Samuelson OLG model) and find that markups create a wedge between market rates of return and the marginal product of capital. This implies that the cost of crowding out may be higher than market rates of return suggest. We embed the production sector of Ball and Mankiw (2022) in our baseline model in order to test and quantify their conjecture about the role of market power in the welfare assessment of debt policies.

5.1 Including Market Power

We first describe how firms make profits, then how those profits are distributed and taxed, and finally how we calibrate the model with market power.

Firms with Market Power. Our specification of the firm sector closely follows Ball and Mankiw (2022).[20] Firms produce output using capital, labor, and intermediate goods supplied by other firms. Individual firms exert market power, which allows them to impose a markup over marginal costs. Since markups at the individual level are reflected in intermediate good prices, the economy-wide markup, μ, is higher than the individual markup. While markups imply profits, there are also overhead fixed costs, θ, that reduce profits. Together, markups and fixed costs determine the economy-wide pure profits, π. In the aggregate, there are only two key changes from our baseline model. First, real factor prices are reduced by the aggregate markup. Second,

[20] Their model is in turn based on Rotemberg and Woodford (2021). To save on notation, we summarize the micro-foundation verbally, and formulate the production problem only in its final, reduced form.

aggregate income consists not only of labor and capital income, but also of profits. Output, factor prices, and profits are as follows.

$$\begin{aligned}
y_t &= z_t \left(\alpha k_t^\iota + (1-\alpha)(\ell_t - \theta)^\iota\right)^{\frac{1}{\iota}} \\
w_t &= z_t(1-\alpha)\ell_t^{\iota-1}\left(\alpha k_t^\iota + (1-\alpha)(\ell_t-\theta)^\iota\right)^{\frac{1}{\iota}-1}\frac{1}{\mu} \\
R_t &= z_t \alpha k_t^{\iota-1}\left(\alpha k_t^\iota + (1-\alpha)(\ell_t-\theta)\right)^{\frac{1}{\iota}-1}\frac{1}{\mu} + (1-\delta_t) \\
\pi_t &= y_t + (1-\delta_t)k_t - w_t\ell_t - R_t k_t
\end{aligned}$$

In contrast to the model without market power, there are now different rates of return to capital. First, the rental rate of capital as defined above. Second, the net return per unit of capital, R^m, which includes profits, as is usual in national accounts. Following Ball and Mankiw (2022) we pick that rate of return as the model counterpart to the risky rate of return from national accounts. Finally, there is the marginal return to capital, which we, also following that paper, refer to as the social return to capital, R^s. The net return per unit of capital and the social return to capital are defined as follows.

$$\begin{aligned}
R_t^m &= \frac{R_t k_t + \pi_t}{k_t} \\
R_t^s &= z_t \alpha k_t^{\iota-1}\left(\alpha k_t^\iota + (1-\alpha)(\ell_t-\theta)\right)^{\frac{1}{\iota}-1} + (1-\delta_t)
\end{aligned}$$

Distributing Profits. Now that firms make non-zero profits, one has to take a stand on who they accrue to. We follow Ball and Mankiw (2022) and assume that profits flow to the young; one can interpret this as young entrepreneurs/managers starting/running businesses and retaining the profits, the old receiving nothing. Furthermore, we have to take a stand on taxation. We assume that the government levies the capital tax also on profits.[21] Under these assumptions the household problem reads as follows.

$$\begin{aligned}
\max_{c_{y,t}, c_{o,t+1}} \quad & u_t = (1-\beta)\ln\left(c_{y,t} - \zeta\frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)\right) + \frac{\beta}{1-\gamma}\ln\left(\mathbb{E}_t\left\{c_{o,t+1}^{1-\gamma}\right\}\right) \\
\text{s.t.} \quad & c_{y,t} = (1-\tau_{l,t}-\tau_p)w_t\ell_t + (1-\tau_{k,t})\pi_t - k_{t+1} - b_{t+1} \\
& c_{o,t+1} = (1-\tau_{k,t+1})\cdot\left(\xi_i R_{t+1}k_{t+1} + R_{t+1}^f b_{t+1}\right) + \tau_p \ell_{t+1} w_{t+1}
\end{aligned}$$

Obviously, assuming that all profits flow to the working-age population is a strong assumption. We provide an alternative specification in Appendix A.6, where firms and the associated claims to profits are traded and only a certain share of (new) firms

[21] We make a further assumption on taxation to ensure numerical stability in computing expectations. We cap the capital tax rate ad hoc at 50% and assign the remaining tax burden to labor. Fortunately, this only applies to cases with tiny probability and is thus without economic relevance.

Table 3: Maxima and Rates of Return — Model with Market Power.

Model	WMD	DMD	R	R^m	R^s
Baseline	43.9%	96.8%	4.0%	4.0%	4.0%
Market Power	15.8%	96.7%	3.7%	4.0%	4.2%

This table reports DMD and WMD for our baseline and the model with market power. For both models we report the rental rate of capital R, the net return to capital R^m, and the social return to capital R^s.

is owned by the young, the rest is owned by the old and sold to the young. We find that the welfare implications of this alternative model are almost identical to our original model with market power.

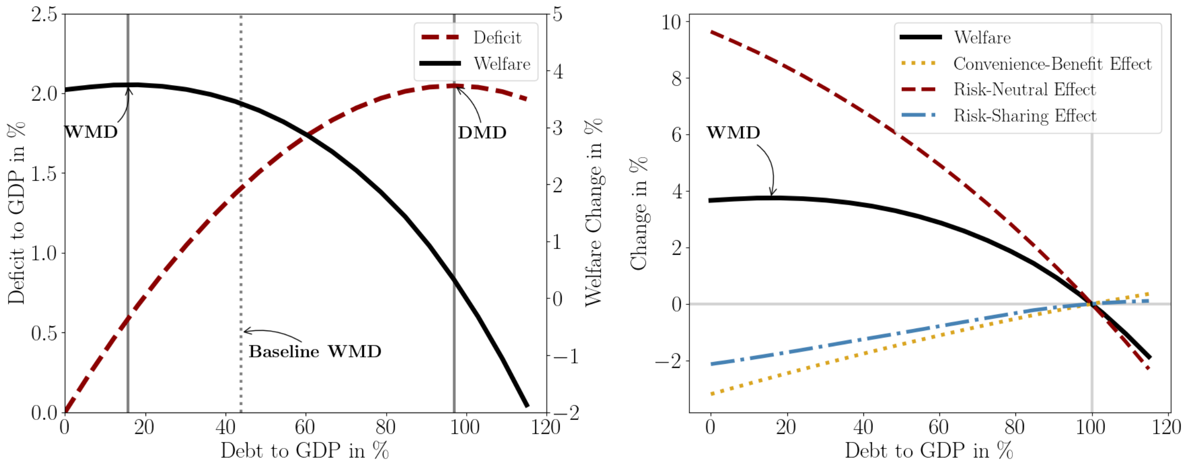
Calibration with Market Power. To calibrate our model with market power, we can keep all externally calibrated parameters and calibration targets as in the baseline, and only have to assign values to two new parameters: the aggregate markup, μ, and overhead fixed costs, θ. We make very conservative choices with respect to these two parameters to get a conservative estimate of how market power changes our baseline results. For the aggregate markup we assume $\mu = 1.1$, a 10% aggregate markup over marginal cost. To pin down overhead fixed costs we calibrate the profit share π/y to 2%. This implies overhead fixed costs equal to 11% of labor costs. Compared to the values reported in De Loecker et al. (2020), these numbers are all at or below the lower end of plausible values for the decades since 1980. Table 9, in Appendix B, summarizes the internally calibrated parameters, while externally calibrated parameters besides μ are equivalent to the baseline calibration and therefore are not listed explicitly.

5.2 Optimal Debt to GDP with Market Power

For an understanding of the impact of market power on welfare, rates of return to capital are key.

Rates of Return. In the model with market power we now have to distinguish between the three measures of the risky rate of return that all amount to the same in the model without market power. While the net return per unit of capital R^m is calibrated to 4%, the effective return to capital amounts to only 3.7% in our conservative calibration as the factor price is suppressed by firms' market power. In contrast, the social return to capital lies above R^m, at 4.2% — meaning that focusing on the net return to capital, R^m, in fact underestimates the marginal product of capital. For our calibration to retrieve the same elasticity of the risk-free rate under a higher social return (that

Figure 4: Deficit, Welfare, and Decomposition — Model with Market Power.



The left plot displays the deficit-to-GDP ratio for different debt-to-GDP rules ρ_B. It also shows the percentage change in welfare compared to $\rho_B = 100\%$. The right plot provides a decomposition of welfare changes into the convenience-benefit effect, the risk-neutral effect, and the risk-sharing effect.

naturally raises crowding out), the production technology needs to be more linear, as can be seen from Table 9 in the Appendix.

DMD and WMD. Although we keep the risk-free-rate elasticity fixed via recalibration, the higher social return to capital implies a stronger crowding-out impact on wages and labor supply than in the baseline model. As a consequence, WMD decreases substantially, from 44% to 16%. Moreover, welfare can be increased by about 4% when reducing debt to GDP from 100% to the WMD level. That increase is about three times as large as in the baseline case — as can be seen when comparing Figures 1 and 4. Deficit-maximizing debt, in contrast, is effectively unchanged compared to the baseline. That is because DMD is, consistent with Mian et al. (2022), a function of only the growth-interest rate differential and the elasticity of the risk-free rate, irrespective of their origin. All in all, we find that taking market power into account can substantially tilt our welfare assessment toward lower debt levels, while it makes virtually no difference for a purely fiscal assessment of debt policy—illustrating, once again, that focusing on DMD alone can be misleading.

6 Inequality and Public Debt

So far we have maintained the simplifying assumption that households within a generation do not differ in any respect. In the real world, households differ, of course, not only with respect to income but even more so with respect to wealth; see, e.g., Kuhn et al. (2020). For our analysis of debt policy, inequality matters mainly for two reasons.

First, inequality can reduce real interest rates as Mian et al. (2021b) and Mian et al. (2021a) argue. Second, and even more importantly, households that differ in income and wealth might differ substantially in how they benefit or suffer from increases in public debt, which is what we find.

6.1 Including Income and Wealth Inequality

We first describe what type of ex ante heterogeneity we include in the model and then go on to specify how we calibrate it.

Heterogeneous households. We extend our model to include two stylized facts: income inequality, which we take as extraneously given, and wealth inequality in excess of income inequality, which we explain by heterogeneous discounting. Using heterogeneous discount rates to match wealth inequality is an often used modeling device; see, e.g., Krusell and Smith (1998). In the two-period OLG model heterogeneous discount rates that are (negatively) correlated with income can be thought of as a simple shortcut for non-homothetic preferences.[22] We consider two types of households, $\{h, l\}$, h denoting the high-income households and l the low-income households. The high-income households represent a fraction λ_h of the population but a share $s_h > \lambda_h$ of labor income. Household types also differ in their discount rate $\beta_j/(1-\beta_j)$. The optimization problem of household $j \in \{h, l\}$, stated in per capita terms, is as follows.

$$\max_{c_{y,t;j}, c_{o,t+1;j}} \quad u_{t;j} = (1-\beta_j) \ln \left(c_{y,t;j} - \zeta_j \frac{\ell_{t;j}^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1;j}, y_t) \right) + \frac{\beta_j}{1-\gamma} \ln \left(\mathbb{E}_t \left\{ c_{o,t+1;j}^{1-\gamma} \right\} \right)$$

$$\text{s.t.} \qquad c_{y,t;j} = \frac{s_j}{\lambda_j}(1-\tau_{l,t}-\tau_p) w_t \ell_t - k_{t+1;j} - b_{t+1;j}$$

$$c_{o,t+1;j} = (1-\tau_{k,t+1}) \cdot \left(\xi_i R_{t+1} k_{t+1;j} + R^f_{t+1} b_{t+1;j} \right) + \tau_p \ell_{t+1} w_{t+1}$$

Note that households also differ with respect to their disutility of labor, ζ_j, an assumption we make to be able to normalize average labor supply for both groups despite differing wages and discount rates. The convenience benefit is drawn from individual bond holding, $b_{t+1,j}$, which is consistent with the representative agent case. Production and government sector are unchanged.

Calibration with Inequality. We set the share of top earners, λ_h, to 10% and their income share, s_h, to a stylized 20%. As a calibration target for β_h we set the wealth share of high income households to 30%. In the US economy, according to Kuhn et

[22]For a model with non-homothetic preferences proper, see Straub (2019).

Table 4: Maxima — Model with Inequality

	WMD l	WMD h	DMD
Baseline	43.9%		96.8%
Inequality	20.2%	130.5%	97.0%

al. (2020), both inequality moments are significantly higher, but our stylized model still gives decent intuition on the differential welfare effects of government debt on heterogeneous groups. The convenience-yield elasticity, parameterized by κ, which is no longer given analytically, is calibrated to 0.9% consistent with the baseline model. Last we calibrate the disutility of labor parameters for both groups such that their labor supply does not differ. Table 10 summarizes the internally calibrated parameters.

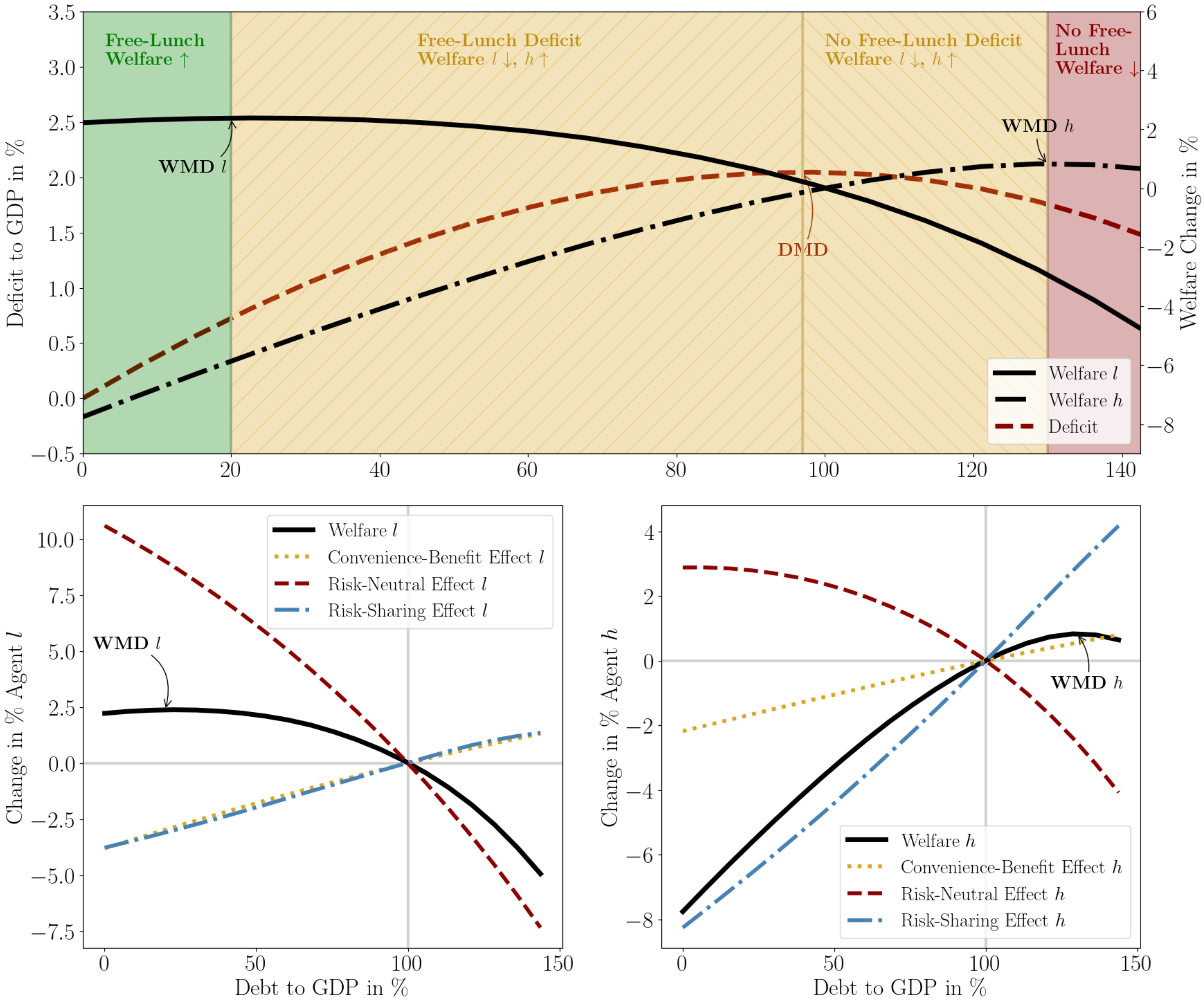
6.2 Optimal Debt to GDP with Inequality

We find that the welfare impact of public debt varies strongly across the income spectrum, a fact that we trace back to different risk-sharing needs and varying reliance on wage versus capital income.

DMD and WMD. In the model with inequality, DMD amounts to 97%, which is basically equal to baseline DMD due to the mechanism described by Mian et al. (2022) and already discussed. However, WMD differs substantially across the wealth distribution, as reported in Table 10 and displayed in Figure 5. Low-income households favor a debt-to-GDP ratio of 20%, much lower than DMD, thus preferring to forgo free-lunch deficits. Rich households, in turn, want the government not simply to reap all the available free lunch, but rather to forgo some of it by raising debt beyond DMD.

Welfare Assessment, Risk-Sharing, and Factor Income. To understand the stark difference in preferred debt-to-GDP ratios across income groups, it is helpful to compare the welfare decompositions provided in Figure 4. There is not much difference with respect to the convenience-benefit effect across agents, yet a huge difference with respect to the other two effects — working in the same direction. For the rich, the (positive) risk-sharing effect is much larger than for the poor. This is because the risk-free government bond is more important for them to smooth their old-age consumption, given that they hold a lot of risky capital and receive little social security income in relative terms. The difference with respect to the (negative) risk-neutral effect is even bigger — it hurts the rich much less. The reason for that lies mainly in the composition of lifetime income. Young, low-income households consume a substantially larger share of their wages than young, high-income agents. Hence, even relative to income,

Figure 5: Deficit, Welfare, and Decompositions — Model with Inequality.



The upper plot displays deficit to GDP for different debt rules ρ_B. It also shows the percentage change in welfare compared to $\rho_B = 100\%$ for low income households l and high income households h. The lower left and right plot provide a decomposition of welfare changes into convenience-benefit effect, risk-neutral effect, and the risk-sharing effect for the two types of households.

low-income households hold fewer assets, receive less capital income, and finance less old-age consumption from their savings as opposed to social security payments. Simply put, low-income households rely more on wage income and less on capital income. Yet government debt crowds out capital thereby decreasing wages and raising risk-free and risky returns. These consequences are, obviously, much more favorable for wealthy households than for poor households. As a result, poor households prefer a debt-to-GDP ratio of 20%, much lower than DMD. Rich households, in contrast, want the debt-to-GDP ratio increased even beyond DMD. To understand way, we take a closer look at their portfolios. Just going from DMD to the high-income WMD (i.e. from 97% to 131%) their portfolio share of bonds increases from 19 to 23 percent, reducing the standard deviation of their returns. However, despite having a saver portfolio, the average returns on that portfolio increase from 3.3 to 3.4 percent. That prospect of having higher returns at lower risk makes the rich favor higher public debt — even at the expense of tighter government budgets that imply more distortionary taxation

7 Conclusion

We analyze public debt policies in a stochastic OLG model with various drivers of low interest rates — aggregate risk, idiosyncratic risk, and convenience benefits. We carefully match the risk-free-rate elasticity and its drivers — the crowding-out of capital and the elasticity of the convenience yield. In line with Mian et al. (2022) we find that the debt-to-GDP ratio that maximizes free-lunch deficits, the DMD as we call it, is solely determined by the interest–growth differential and the risk-free-rate elasticity. The composition of interest-rate and elasticity drivers matters substantially, however, for the debt-to-GDP ratio that maximizes ex ante utility of agents in the stochastic steady state — what we refer to as the WMD. We find WMD to be significantly lower than DMD for the US. Thus, even if free-lunch deficits are feasible, they are not necessarily desirable. Even less so when market power is taken into account, as we show in one of our extensions. When inequality in income and wealth is included in the model, we find that low-income households are averse to high public debt while high-income households prefer the government to increase the debt-to-GDP ratio even above DMD — because that increases the returns and reduces the risk of their savings.

There are several directions for future research to build on our analysis. The most obvious limitation of our model is the two-period OLG structure. A finer generational structure naturally suits a more realistic and nuanced calibration. Moreover, such a model would allow for a reasonable modeling and analysis of (optimal) debt rules. Other aspects that might be interesting to include — separately or in combination — are disaster risk, long-run risk, demographic risk, corporate bonds, state-contingent government bonds, long-lived assets, housing and mortgages, bequest motives, and political economy considerations. Investigating the interaction of debt policy with other policy instruments is, of course, also of great significance. For this study, however, our aim is to make the model and its analysis just complex enough to capture and quantify the mechanisms most important for assessing welfare-maximizing debt and its relation to deficit-maximizing debt. One of our robust findings is that despite the benefits of public debt — in particular at low interest–growth differentials where it can alleviate distortionary taxation — long-run welfare maximization requires lower debt-to-GDP ratios than free-lunch deficits might lead us to believe.

A Model Details

This appendix comprises details regarding the baseline model and its two extensions.

A.1 First Order Conditions

The optimality conditions for households' decisions in the baseline model from Section 3 are given by the following first order conditions (FOCs). The FOCs pin down policies for labor supply, ℓ_t, physical saving, k_{t+1}, the risk free rate, R^f_{t+1}, and the shadow interest rate on risk-free private bonds, $R^{f,N}_{t+1}$.

$$
\begin{aligned}
\zeta \ell_t^{\frac{1}{v}} &= (1 - \tau_{l,t} - \tau_p) w_t \\
\frac{1-\beta}{c_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta \frac{\mathbb{E}_t \left\{ \xi_i R_{t+1} (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}} \\
\frac{(1-\beta)(1 - V'(b_{t+1}, y_t))}{c_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta R^f_{t+1} \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}} \\
\frac{1-\beta}{c_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta R^{f,N}_{t+1} \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}}
\end{aligned}
$$

We solve for policies $\ell_t(s_t), k_{t+1}(s_t), R^f_{t+1}(s_t), R^{f,N}_{t+1}(s_t)$ that satisfy the above optimality conditions for all states s_t using time iteration as explained in Section 3.3.

A.2 Convenience Yield

We adopt a linear specification of the convenience yield — the spread between returns on risk-free corporate bonds $R^{f,N}$ and treasury bonds R^f — suggested by Krishnamurthy and Vissing-Jorgensen (2012) and also applied by Mian et al. (2022). The annualized convenience yield CY_a is characterized by a linear relationship between convenience yield and the debt-to-GPD ratio, b/y, with ψ being the spread at the initial debt-to-GDP ratio, ρ_{B_0}, and κ the convenience yield elasticity.

$$
\text{CY}_a = \psi - \kappa \frac{\frac{b_{t+1}}{y_t} - \rho_{B_0}}{\rho_{B_0}}
$$

We characterize the relationship between the 25-year convenience yield CY_{25} and V' by dividing the first order condition for government bonds by the respective condition for private bonds, and rearranging the equation.

$$\mathrm{CY}_{25} = 1 - \frac{R^f_{t+1}}{R^{f,N}_{t+1}} = V'(b_{t+1}, y_t)$$

To break down the 25-year convenience yield into the annual convenience yield we discount periods and rearrange the last equation.

$$\mathrm{CY}_a = 1 - \left(\frac{R^f_{t+1}}{R^{f,N}_{t+1}}\right)^{\frac{1}{T}} = 1 - \left(1 - V'(b_{t+1}, y_t)\right)^{\frac{1}{T}}$$

Inserting the expression for the annualized convenience from above and rearranging gives us V'.

$$V'(b_{t+1}, y_t) = 1 - \left(1 - \left(\psi - \kappa \frac{\frac{b_{t+1}}{y_t} - \rho_{B_0}}{\rho_{B_0}}\right)\right)^T$$

Imposing $V(0, y_t) = 0$ and integrating with respect to b_{t+1} gives us the final expression of the utility drawn from government bond holding.

$$V(b_{t+1}, y_t) = b_{t+1} - \frac{y_t \rho_{B_0}}{\kappa(T+1)} \left(\left(\frac{\kappa b_{t+1}}{y_t \rho_{B_0}} + 1 - \psi - \kappa \right)^{T+1} - (1 - \psi - \kappa)^{T+1} \right)$$

With $V(0, y_t) = 0$ we ensure utility from government bonds is strictly positive. In the market power extension we use the exact same convenience utility function. In the inequality extension we assume $V(b^j_{t+1}, y_t)$ depends on households' government bond holding per capita b^j_{t+1} where $j \in \{l, h\}$ is the household type.

A.3 Balanced Growth Path

Throughout the paper we assumed the economy is stationary with zero growth, and that interest rates from the model can be translated to interest–growth differentials in the real economy. This assumption builds upon the existence of a balanced growth path — an equilibrium in which all endogenous state variables grow at the same constant rate. In this section we show there exists such a balanced growth path, thus the underlying assumption of our baseline model and its extensions is valid.

Let A_t be non-stochastic labor augmented productivity and $A_{t+1}/A_t = 1+n$ trend growth. Write the optimization problem with trend as follows. Note that labor disutility is scaled by trend productivity.

$$
\begin{aligned}
\max_{c_{y,t},c_{o,t+1}} \quad & u_t = (1-\beta)\ln\left(c_{y,t} - \zeta A_t \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)\right) + \frac{\beta}{1-\gamma}\ln\left(\mathbb{E}_t\left\{c_{o,t+1}^{1-\gamma}\right\}\right) \\
\text{s.t.} \quad & c_{y,t} = (1-\tau_{l,t}-\tau_p)w_t A_t \ell_t - k_{t+1} - b_{t+1} \\
& c_{o,t+1} = (1-\tau_{k,t+1})\cdot\left(\xi_i R_{t+1}k_{t+1} + R^f_{t+1}b_{t+1}\right) + \tau_p \ell_{t+1}A_{t+1}w_{t+1}
\end{aligned}
$$

By $\hat{x}_t = x_t/A_t$ we denote variables per productivity unit. Rewrite the budget constraint in per efficiency units.

$$
\begin{aligned}
\hat{c}_{y,t} &= (1-\tau_{l,t}-\tau_p)w_t\ell_t - (1+n)(\hat{k}_{t+1}+\hat{b}_{t+1}) \\
\hat{c}_{o,t+1} &= (1-\tau_{k,t+1})\cdot\left(\xi_i R_{t+1}\hat{k}_{t+1} + R^f_{t+1}\hat{b}_{t+1}\right) + \tau_p\ell_{t+1}w_{t+1}
\end{aligned}
$$

Gross production now features labor augmented technological progress $A_t\ell_t$. Which gives us the following characterization of the production process.

$$
\begin{aligned}
y_t &= z_t\left(\alpha k_t^{\iota} + (1-\alpha)(\ell_t A_t)^{\iota}\right)^{\frac{1}{\iota}} \\
w_t &= z_t(1-\alpha)(\ell_t A_t)^{\iota-1}\left(\alpha k_t^{\iota} + (1-\alpha)(\ell_t A_t)^{\iota}\right)^{\frac{1}{\iota}-1} \\
R_t &= z_t\alpha k_t^{\iota-1}\left(\alpha k_t^{\iota} + (1-\alpha)(\ell_t A_t)^{\iota}\right)^{\frac{1}{\iota}-1} + (1-\delta_t)
\end{aligned}
$$

We replace capital by capital per productivity unit $\hat{k}_t = k_t/A_t$ and rewrite production in per capita terms. Factor prices w_t and R_t are naturally normalized, GDP must be detrended to $\hat{y}_t$.

$$
\begin{aligned}
\hat{y}_t &= z_t\left(\alpha\hat{k}_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}} \\
w_t &= (1-\alpha)\ell_t^{\iota-1}\left(\alpha\hat{k}_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}-1} \\
R_t &= \alpha\hat{k}_t^{\iota-1}\left(\alpha\hat{k}_t^{\iota} + (1-\alpha)\ell_t^{\iota}\right)^{\frac{1}{\iota}-1} + (1-\delta_t)
\end{aligned}
$$

Next we write government budget in per capit terms.

$$
\begin{aligned}
(1+n)\hat{b}_{t+1} &= \rho_B \hat{y}_t \\
\hat{g}_t &= \rho_G \hat{y}_t \\
\hat{g}_t + R_t^f \hat{b}_t - (1+n) b_{t+1} &= \tau_{l,t} w_t \ell_t + \tau_{k,t} \left(R_t \hat{k}_t + R_t^f \hat{b}_t \right) \\
\tau_{l,t} &= \frac{\hat{g}_t + R_t^f \hat{b}_t - (1+n)\hat{b}_{t+1}}{w_t \ell_t} \Delta \\
\tau_{k,t} &= \frac{\hat{g}_t + R_t^f \hat{b}_t - (1+n)\hat{b}_{t+1}}{R_t \hat{k}_t + R_t^f \hat{b}_t} (1-\Delta)
\end{aligned}
$$

To make convenience benefits independent from growth we define a slightly modified convenience yield. For $n = 0$ the expression collapses to the convenience yield presented in the last section.

$$
V'(b_{t+1}, y_t) = 1 - \left(1 - \left(\psi - \kappa \frac{\frac{b_{t+1}}{y_t(1+n)} - \rho_{B_0}}{\rho_{B_0}} \right) \right)^T
$$

$$
V(b_{t+1}, y_t) = b_{t+1} - \frac{y_t(1+n)\rho_{B_0}}{\kappa(T+1)} \left(\left(\frac{\kappa b_{t+1}}{y_t(1+n)\rho_{B_0}} + 1 - \psi - \kappa \right)^{T+1} - (1-\psi-\kappa)^{T+1} \right)
$$

For the derivative of convenience benefits it holds that $V'(b_{t+1}, y_t) = V'(\hat{b}_{t+1}, \hat{y}_t)$, and $V(b_{t+1}, y_t)/A_t = (1+n)V(\hat{b}_{t+1}, \hat{y}_t)$. Further the first order conditions in gross terms are given by.

$$
\begin{aligned}
\zeta A_t \ell_t^{\frac{1}{v}} &= (1 - \tau_{l,t} - \tau_p) A_t w_t \\
\frac{1-\beta}{c_{y,t} - \zeta A_t \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta \frac{\mathbb{E}_t \left\{ \xi_i R_{t+1} (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}} \\
\frac{(1-\beta)(1 - V'(b_{t+1}, y_t))}{c_{y,t} - \zeta A_t \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta R_{t+1}^f \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}} \\
\frac{1-\beta}{c_{y,t} - \zeta A_t \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} &= \beta R_{t+1}^{f,N} \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) c_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}}
\end{aligned}
$$

This translates to the following FOCs under trend growth.

$$\zeta \ell_t^{\frac{1}{v}} = (1 - \tau_{l,t} - \tau_p) w_t$$

$$\frac{1-\beta}{\hat{c}_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + (1+n)V(\hat{b}_{t+1}, \hat{y}_t)} = \beta \frac{\mathbb{E}_t \left\{ \xi_i R_{t+1} (1 - \tau_{k,t+1}) \hat{c}_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ \hat{c}_{o,t+1}^{1-\gamma} \right\}} \frac{1}{1+n}$$

$$\frac{(1-\beta)(1 - V'(\hat{b}_{t+1}, \hat{y}_t))}{\hat{c}_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + (1+n)V(\hat{b}_{t+1}, \hat{y}_t)} = \beta R_{t+1}^{f} \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) \hat{c}_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ \hat{c}_{o,t+1}^{1-\gamma} \right\}} \frac{1}{1+n}$$

$$\frac{1-\beta}{\hat{c}_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + (1+n)V(\hat{b}_{t+1}, \hat{y}_t)} = \beta R_{t+1}^{f,N} \frac{\mathbb{E}_t \left\{ (1 - \tau_{k,t+1}) \hat{c}_{o,t+1}^{-\gamma} \right\}}{\mathbb{E}_t \left\{ \hat{c}_{o,t+1}^{1-\gamma} \right\}} \frac{1}{1+n}$$

The interest rates — R_{t+1}, R_{t+1}^{f}, and $R_{t+1}^{f,N}$ — enter the nominator of the right-hand sides of all these FOCs linearly, while the trend growth rate, $1+n$, enters the denominator linearly. Thus, these equations only depend on the (log-) difference between the two rates. Therefore analyzing a stationary economy is a valid modelling choice — provided that one interprets rates of return from the model as interest-growth differentials in the real world. Note that interest and growth rates moving in a one-for-one fashion depends on the unit IES assumption that we share with Blanchard (2019).

A.4 Welfare Decomposition

To get a more detailed understanding of the welfare implications of debt policy, we define a decomposition of ex-ante utility, building upon Brumm et al. (2021). We distinguish between effects originating from convenience benefits, risk-sharing, and a risk-neutral effect, which captures the welfare effects of crowding out. For the decomposition we define welfare without convenience yield, $\widetilde{\mathcal{U}}_0^t$, and ex-ante risk-neutral welfare without convenience yield, $\overline{\mathcal{U}}_0^t$.

$$\widetilde{\mathcal{U}}_0^t = \mathbb{E}_0 \left\{ \left(c_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} \right)^{(1-\beta)(1-\gamma)} \mathbb{E}_t \left\{ c_{o,t+1}^{1-\gamma} \right\}^{\beta} \right\}^{\frac{1}{1-\gamma}}$$

$$\overline{\mathcal{U}}_0^t = \mathbb{E}_0 \left\{ \left(c_{y,t} - \zeta \frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} \right)^{1-\beta} \mathbb{E}_t \left\{ c_{o,t+1} \right\}^{\beta} \right\}$$

Using these expressions we can rewrite ex-ante welfare by the convenience benefit effect (CBE), the risk-sharing effect (RSE) and the risk neutral effect (RNE).

$$\mathcal{U}_0 = \underbrace{\frac{\mathcal{U}_0}{\widetilde{\mathcal{U}}_0^t}}_{\text{CBE}} \cdot \underbrace{\frac{\widetilde{\mathcal{U}}_0^t}{\overline{\mathcal{U}}_0^t}}_{\text{RSE}} \cdot \underbrace{\overline{\mathcal{U}}_0^t}_{\text{RNE}}$$

Percentage changes in ex-ante utility are approximately given by the changes in these three components.

A.5 Deficit Calculation

We measure debt, b_{t+1}, in terms of market value at the beginning of the period t. To put the deficits Ω_t, that accrue continuously throughout the $T = 25$ years into the right relation to debt and GDP, we calculate their market value at the beginning of period t, which we denote by Ω_t^M. For that we have to integrate and discount using the instantaneous interest rate r_{t+1}^f, which relates to the 25-period interest rate R_{t+1}^f as given below (where we drop time indices).

$$R^f = e^{\int_0^T r^f \tau d\tau} \Leftrightarrow r^f = \frac{\ln R^f}{T}$$

$$\Omega_t^M = \int_0^T \frac{\Omega_t}{T} e^{-r_{t+1}^f \tau} d\tau = \frac{\Omega_t}{T} \frac{\left(1 - e^{-r^f T}\right)}{r^f} = \Omega_t \frac{\left(1 - \frac{1}{R^f}\right)}{\ln R^f}$$

The latter formula provides a correcting factor to transform the deficit in the 25-year-period model, Ω_t, into a deficit that is comparable to debt and GDP in the same way as it would be in a model with short period length. Note that the correcting factor is bigger than one and close to one when R^f is.

A.6 Alternative Model with Market Power

In this section we present an extension to the market power model in which a share ω of profits accrues to young households — interpreted as entrepreneurial gains — the rest, $1 - \omega$ is given to shareholders. Then young households buy shares ϑ_t from the old at price p_t, in the second period the share pays a dividend π_{t+1}, then the old sell their profits to the young at a price p_{t+1}. The share price is endogenous and state dependent. We solve for the price policy using time iteration.

Model. The household optimization changes slightly compared to the model with market power from Section 5.

$$\max_{c_{y,t},c_{o,t+1}} \quad u_t = (1-\beta)\ln\left(c_{y,t} - \zeta\frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)\right) + \frac{\beta}{1-\gamma}\ln\left(\mathbb{E}_t\left\{c_{o,t+1}^{1-\gamma}\right\}\right)$$

$$\text{s.t.} \qquad c_{y,t} = (1-\tau_{l,t}-\tau_p)w_t\ell_t + (1-\tau_{k,t})\omega\pi_t - k_{t+1} - b_{t+1} - \vartheta_t p_t$$

$$c_{o,t+1} = (1-\tau_{k,t+1})\cdot\left(\xi_i R_{t+1}k_{t+1} + R^f_{t+1}b_{t+1} + \vartheta_t(\pi_{t+1}+p_{t+1})\right) + \tau_p\ell_{t+1}w_{t+1}$$

Production is the same as in the basic market power model. Share prices p_t are chosen such that the corresponding market clears, that is $\vartheta_t = 1-\omega$. The optimality conditions are extended by an optimality condition for the choice of shares ϑ_t.

$$\frac{(1-\beta)p_t}{c_{y,t} - \zeta\frac{\ell_t^{1+\frac{1}{v}}}{1+\frac{1}{v}} + V(b_{t+1}, y_t)} = \beta\frac{\mathbb{E}_t\left\{(1-\tau_{k,t+1})(\pi_{t+1}+p_{t+1})c_{o,t+1}^{-\gamma}\right\}}{\mathbb{E}_t\left\{c_{o,t+1}^{1-\gamma}\right\}}$$

Calibration and Results. We follow the calibration of the model in Section 5 with just one exception. We reduce the share of profits allocated to the young from $\omega = 1$ to $\omega = 0.9$. All externally calibrated parameters are unchanged. The calibrated parameters to meet the targets are summarized in Table 5. Compared to the basic market power model, where the young receive all the profits and $\omega = 1$, we find WMD unchanged up to the first digit in the model with $\omega = 0.9$. Even though we believe allocating profits purely to the young is not a sufficient reflection of reality, the story told by both models is the same: market power depresses welfare maximizing debt.

B Calibration Details

This Appendix provides details on our choice of aggregate risk targets, the convenience spread, and the calibrations of the models in Sections 5 and 6.

B.1 Aggregate Risk Data

In quantifying long-term aggregate risk our methodology closely follows Krueger and Kubler (2006). However, we differ with respect to data source and time horizon of our estimation. While Krueger and Kubler (2006) deal with 6-year periods we must account for a period length of 25-years. We use data from Macrohistory Database[23] by Jordà et al. (2019), covering the time horizon from 1880 to 2020. This gives us a total of

[23] https://www.macrohistory.net/database/

Table 5: Internally Calibrated Parameters — Alternative Model with Market Power.

Parameter		Target		Source
Risk				
σ_I	0.27	$\mathbb{E}_0\{\xi_i R_t\}$	40%	Snudden (2021)
σ_z	0.14	$\text{CV}(w_t)$	13%	Jordà et al. (2019)
σ_d	0.09	$\text{CV}(R_t)$	25%	Jordà et al. (2019)
χ	1.98	$\text{Corr}(w_t, R_t)$	-7.5%	Jordà et al. (2019)
Production				
ζ	0.1	$\mathbb{E}_0\{\ell_t\}$	30%	normalization
α	0.77	$\mathbb{E}_0\{w_t \ell_t / y_t\}$	63%	stylized fact
μ_d	-0.09	$\mathbb{E}_0\{k_t / y_t\}$	300%	Ball and Mankiw (2022)
Market Power				
μ	1.1	no target		stylized, De Loecker et al. (2020)
θ	0.03	$\mathbb{E}_0\{\pi_t / y_t\}$	2%	stylized, De Loecker et al. (2020)
Rates of Return				
β	0.63	$\mathbb{E}_0\{R_t^m\}$	4%	Ball and Mankiw (2022)
γ	18.62	$\mathbb{E}_0\{R_t^f\}$	-2%	stylized average
ι	0.32	$\mathbb{E}_0\{\varphi\}$	2.2%	Mian et al. (2022)

(still only) five subsequent 25-year periods for estimation. From the complete dataset we extract features on the year (year), consumer price index (cpi), wages (wage) and returns on risky assets (risky_tr). The return on risky assets is a weighted average of housing and equity — excluding safe assets like government bonds. Krueger and Kubler (2006) construct the risky return from a stock portfolio, which makes their data naturally more volatile. Since the risky rate in our model represents a broad class of assets we find an average of asset classes to be the best fit. Wages are adjusted by CPI, returns are discounted by the inflation rate. We aggregate 25-year real returns $\hat{r}_{25y}$ using the logarithmic sum. In line with Krueger and Kubler (2006), we transform

Table 6: Maxima — Alternative Model with Market Power.

Model	WMD	DMD
Baseline Model	43.9%	96.8%
Model with Market Power	15.8%	96.7%
Alternative Model with Market Power	15.8%	96.8%

Table 7: Aggregate Risk Data.

	1-year	5-year	10-year	25-year
CV($\hat{r}_j$)	131.9%	55.0%	47.6%	23.8%
CV($\hat{w}_j$)	15.6%	15.5%	15.2%	13.2%
Corr($\hat{r}_j, \hat{w}_j$)	-2.1%	-6.7%	-10.8%	-7.5%

This table presents the coefficient of variation of real-returns on risky-assets, the coefficient of variation of de-trended real wages and their correlation for time horizons of 1, 5, 10 and 25-years. Data is taken from Jordà et al. (2019).

wages, w_t, into de-trended real wages. We estimate a linear time trend, $(1+\hat{n})$, and compute de-trended wages, $\hat{w}_t$, as follows.

$$\hat{w}_t = \exp\left(\ln(w_t) - t \cdot \ln(1+\hat{n})\right)$$

Finally, we compute coefficients of variation and the correlation between aggregate wages and aggregate risky returns. The data for time horizons of 1, 5, 10 and 25 years is summarized in Table 7. We report these measures for higher frequencies in order to make sure that the data we actually use as calibration targets are reasonable despite the very low frequency. Comparing the five-year aggregate moments to Krueger and Kubler (2006)'s six year data we find the volatility of wages to be a close fit. We find a coefficient of variation of 15%, Krueger and Kubler (2006) 11%. Krueger and Kubler (2006), however, find significantly higher coefficient of variation in returns of 115%, relative to 55%, which can be explained by our different choice of risky-rate data. Finally they find a correlation of -38% — our correlation is also negative, yet closer to zero.

Table 8: Convenience Yield Spread 1960 - 2020.

1960 - 1980		1980 - 2000		2000 - 2020	
60s	70s	80s	90s	00s	10s
0.53		0.66		1.03	
0.40	0.66	0.65	0.66	0.95	1.10

This table reports the average convenience yield spread between returns on 20-year AAA-corporate-bonds and 20-year US-treasury-bonds in percentage points. The last line reports 10-year averages, while the second-to-last line reports 20-year averages.

Table 9: Internally Calibrated Parameters — Model with Market Power.

Parameter		Target		Source
Risk				
σ_I	0.26	$\mathbb{E}_0\{\xi_i R_t\}$	40%	Snudden (2021)
σ_z	0.14	$\text{CV}(w_t)$	13%	Jordà et al. (2019)
σ_d	0.09	$\text{CV}(R_t)$	25%	Jordà et al. (2019)
χ	1.97	$\text{Corr}(w_t, R_t)$	−7.5%	Jordà et al. (2019)
Production				
ζ	0.08	$\mathbb{E}_0\{\ell_t\}$	30%	normalization
α	0.79	$\mathbb{E}_0\{w_t \ell_t / y_t\}$	63%	stylized fact
μ_d	-0.09	$\mathbb{E}_0\{k_t / y_t\}$	300%	Ball and Mankiw (2022)
Market Power				
μ	1.1	no target		stylized, De Loecker et al. (2020)
θ	0.03	$\mathbb{E}_0\{\pi_t / y_t\}$	2%	stylized, De Loecker et al. (2020)
Rates of Return				
β	0.62	$\mathbb{E}_0\{R_t^m\}$	4%	Ball and Mankiw (2022)
γ	19.15	$\mathbb{E}_0\{R_t^f\}$	−2%	stylized average
ι	0.34	$\mathbb{E}_0\{\varphi\}$	2.2%	Mian et al. (2022)

B.2 Convenience Yield Spread Data

To find a suitable calibration target for the convenience spread ψ we explore empirical data on the convenience spread over the past 60 years. We quantify the convenience spread as the difference in returns between corporate AAA-bonds and US-treasury-bonds both with a maturity of 20 years, consistent with Krishnamurthy and Vissing-Jorgensen (2012). Data is taken from FRED database,[24] specifically time series AAA, GS20 and LTGOVTBD.[25] In Table 8 we report the average spread for each decade beginning in the 1960s as well as the 20-year average. For the time period from 2000 to 2020 we find an average spread of 1pp which we take as a calibration target for the baseline model. The average debt-to-GDP ratio during that period amounted to 81 percent.[26]

B.3 Calibration — Model with Market Power

Table 9 summarizes the calibration of the market power extension in section 5. Aggregate markups μ are set to 10% the profit share π/y is calibrated to 2%. All other

[24] https://fred.stlouisfed.org/

[25] For time series GS20 there are some values missing which we replace by the data from LTGOVTBD.

[26] FRED series GFDEGDQ188S.

Table 10: Internally Calibrated Parameters — Model with Inequality.

Parameter		Target		Source
Risk				
σ_I	0.22	$\mathbb{E}_0\{\xi_i R_t\}$	40%	Snudden (2021)
σ_z	0.14	$\text{CV}(w_t)$	13%	Jordà et al. (2019)
σ_d	0.10	$\text{CV}(R_t)$	25%	Jordà et al. (2019)
χ	2.12	$\text{Corr}(w_t, R_t)$	-7.5%	Jordà et al. (2019)
Production				
ζ_h	0.43	$\mathbb{E}_0\{\ell_t\}$	30%	normalization
ζ_l	0.19	$\mathbb{E}_0\{\ell_t\}$	30%	normalization
α	0.68	$\mathbb{E}_0\{w_t \ell_t / y_t\}$	63%	stylized fact
μ_d	-0.08	$\mathbb{E}_0\{k_t / y_t\}$	300%	Ball and Mankiw (2022)
Inequality				
β_h	0.84	Wealth Share h	30%	stylized, Kuhn et al. (2020)
Rates of Return				
β_l	0.61	$\mathbb{E}_0\{R_t\}$	4%	Ball and Mankiw (2022)
γ	23.65	$\mathbb{E}_0\{R_t^f\}$	-2%	stylized average
ι	0.29	$\mathbb{E}_0\{\varphi\}$	2.2%	Mian et al. (2022)

externally calibrated parameters and the remaining calibration targets stay unchanged compared to the baseline.

B.4 Calibration — Model with Inequality

Table 10 summarizes the calibration of the inequality extension in section 6. We assume the top 10% of earners account for 20% of labor income. We calibrate β_h such that these 10% however hold 30% of total wealth. Labor disutility for types $\{l, h\}$ is calibrated such that they supply 0.3 of their labor endowment on average.

C Sensitivity

This appendix provides a sensitivity analysis for our main results — DMD and WMD. We change values of externally calibrated parameters or calibration targets one at a time. Then we calibrate the model to otherwise unchanged targets and compute DMD and WMD. Table 11 reports the parameter / calibration target, its baseline value, the new assumption, and the resulting DMD and WMD.

Taxation. We increase the share of total taxes levied on labor income Δ from 66% to 70%. This mechanically increases the labor tax rate, thereby increasing labor supply distortions — and the potential to reduce those by using free-lunch deficits. Therefore, WMD rises moderately to 46%.

Convenience Spread. We assume the convenience yield spread ψ falls by 10bps compared to the baseline calibration. Risk aversion now needs to explain a larger portion of the spread between risky and risk-free rates, therefore γ rises. DMD barely moves as $R^f - G - \varphi$ does not change, while WMD increases mildly since risk-sharing becomes more beneficial with increased relative risk aversion.

Convenience Yield Elasticity. Next we reduce the convenience yield elasticity κ by 10bps, while keeping the overall risk-free rate elasticity φ constant. Now a higher portion of the risk-free-rate elasticity is explained by crowding out instead of changes in convenience benefits. To match φ and the higher crowding-out effect the elasticity of substitution in production decreases — moves closer to Cobb-Douglas production. Crowding out rises, therefore WMD decreases.

Frisch Elasticity. We reduce the Frisch-elasticity of labor supply from 0.75 to 0.5. Therefore, falling wages from crowding-out affect labor supply less. This has two implications: On the one hand the effect of crowding-out on the risk-neutral effect of welfare is smaller, on the other hand the risk-free rate elasticity decreases, which must be eliminated by reducing ι and enhancing crowding-out. The lower sensitivity of labor supply outweighs the effect of stronger crowding-out and WMD increases moderately to 50.4%.

Capital-Labor-Return Correlation. We increase the potential for risk sharing (between generations) by setting the correlation between wages and capital returns to -15% instead of -7.5%. Now the pension system provides better insurance against old-age consumption risk. That makes government bonds less attractive from a risk-sharing perspective, which is why WMD decreases to 42.6%.

Risky Return. The risky return R is reduced to 3.5% implying potentially lower crowding-out in production. Sticking to the same empirical risk-free rate elasticity implies production closer to Cobb-Douglas. After recalibration WMD decreases to 34.8%. The effect of adjusting the elasticity of substitution outweighs the reduced crowding out from the adjusted risky rate.

Table 11: Sensitivity Analysis — Baseline Model.

Target	Baseline	Sensitivity	WMD	DMD
Δ	0.66	0.7	46.0%	96.7%
ψ	1%	0.9%	46.8%	96.7%
κ	0.9%	0.8%	18.7%	97.2%
v	0.75	0.5	50.4%	97.3%
$\text{Corr}(R_t, w_t)$	-7.5%	-15%	42.6%	96.9%
$\mathbb{E}_0\{R_t\}$	4%	3.5%	34.8%	96.9%
$\mathbb{E}_0\{\varphi\}$	2.2%	2.3%	28.9%	95.4%
Baseline			43.9%	96.8%

The left column states the parameter or calibration target that we change in the considered sensitivity exercise. The columns "Baseline" and "Sensitivity" state the baseline value and the value chosen for the exercise. The last two columns report WMD and DMD for the sensitivity exercise, where we re-calibrated the model to all remaining baseline targets. In the bottom of the table we report WMD and DMD in the baseline model for comparison.

Risk-Free Rate Elasticity. Finally we assume a risk-free rate elasticity φ of 2.3% instead of 2.2%. To achieve the higher elasticity we need to raise crowding out in production compared to the baseline. We do so by lowering ι. We find that WMD decreases to 23.9%, mainly due to higher crowding out and its implications for the risk-neutral-effect on welfare.

References

Abel, A. B., Mankiw, N. G., Summers, L. H., & Zeckhauser, R. J. (1989). Assessing dynamic efficiency: Theory and evidence. *The Review of Economic Studies*, *56*(1), 1–19.

Aiyagari, S. R., & McGrattan, E. R. (1998). The optimum quantity of debt. *Journal of Monetary Economics*, *42*(3), 447–469.

Angeletos, G. M., Collard, F., & Dellas, H. (2021). Public debt as private liquidity: Optimal policy. *CEPR Discussion Paper*.

Ball, L., & Mankiw, N. G. (2007). Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design. *Journal of Political Economy*, *115*(4), 523–547.

Ball, L., & Mankiw, N. G. (2022). Market power in neoclassical growth models. *The Review of Economic Studies*.

Barkai, S. (2020). Declining labor and capital shares. *The Journal of Finance*, *75*(5), 2421–2463.

Barro, R. J. (2022). R minus g. *Review of Economic Dynamics*.

Basu, S. (2019). Are price-cost markups rising in the United States? A discussion of the evidence. *Journal of Economic Perspectives*, *33*(3), 3–22.

Bayer, C., Born, B., & Luetticke, R. (2022). The liquidity channel of fiscal policy. *Journal of Monetary Economics*.

Blanchard, O. (2019). Public debt and low interest rates. *American Economic Review*, *109*(4), 1197–1229.

Blanchard, O. (2023). *Fiscal policy under low interest rates*. MIT Press.

Brumm, J., Feng, X., Kotlikoff, L. J., & Kubler, F. (2021). When interest rates go low, should public debt go high? (Working Paper No. 28951). National Bureau of Economic Research.

Brumm, J., Feng, X., Kotlikoff, L. J., & Kubler, F. (2022a). Are deficits free? *Journal of Public Economics*, *208*, 104627.

Brumm, J., Krause, C., Schaab, A., & Scheidegger, S. (2022b). Sparse grids for dynamic economic models. In *Oxford research encyclopedia of economics and finance*.

Brumm, J., & Scheidegger, S. (2017). Using adaptive sparse grids to solve high-dimensional dynamic models. *Econometrica*, *85*(5), 1575–1612.

Brunnermeier, M. K., Merkel, S. A., & Sannikov, Y. (2020). Debt as safe asset: Mining the bubble. *Princeton University work in progress*.

Chetty, R., Guren, A., Manoli, D., & Weber, A. (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review*, *101*(3), 471–75.

d'Avernas, A., & Vandeweyer, Q. (2021). Treasury bill shortages and the pricing of short-term assets. *Working Paper*.

De Loecker, J., Eeckhout, J., & Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, *135*(2), 561–644.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, *55*(5), 1126–1150.

Domeij, D., & Ellingsen, T. (2018). Rational bubbles and public debt policy: A quantitative analysis. *Journal of Monetary Economics*, *96*, 109–123.

Fagereng, A., Guiso, L., Malacrino, D., & Pistaferri, L. (2020). Heterogeneity and persistence in returns to wealth. *Econometrica*, *88*(1), 115–170.

Farhi, E., & Gourio, F. (2018). Accounting for macro-finance trends: Market power, intangibles, and risk premia (Working Paper No. 25282). National Bureau of Economic Research.

Harenberg, D., & Ludwig, A. (2019). Idiosyncratic risk, aggregate risk, and the welfare effects of social security. *International Economic Review*, *60*(2), 661–692.

İmrohoroglu, A., Imrohoroglu, S., & Joines, D. H. (1995). A life cycle analysis of social security. *Economic theory*, *6*, 83–114.

Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M., & Taylor, A. M. (2019). The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics*, *134*(3), 1225–1298.

Kocherlakota, N. R. (2022). Public debt bubbles in heterogeneous agent models with tail risk. *International Economic Review*.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, *120*(2), 233–267.

Krueger, D., & Kubler, F. (2006). Pareto-improving social security reform when financial markets are incomplete!? *American Economic Review*, *96*(3), 737–755.

Krusell, P., & Smith, A. A., Jr. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, *106*(5), 867–896.

Kuhn, M., Schularick, M., & Steins, U. I. (2020). Income and wealth inequality in America, 1949–2016. *Journal of Political Economy*, *128*(9), 3469–3519.

Mankiw, N. G. (2022). Government debt and capital accumulation in an era of low interest rates (Working paper No. 30024). National Bureau of Economic Research.

Mehrotra, N. R., & Sergeyev, D. (2021). Debt sustainability in a low interest rate world. *Journal of Monetary Economics*, *124*, S1–S18.

Mian, A., Straub, L., & Sufi, A. (2021a). The saving glut of the rich. *Working Paper.*

Mian, A., Straub, L., & Sufi, A. (2021b). What explains the decline in r*? Rising income inequality versus demographic shifts. *Proceedings of the 2021 Jackson Hole Symposium*.

Mian, A., Straub, L., & Sufi, A. (2022). A goldilocks theory of fiscal deficits (Working Paper No. 29707). National Bureau of Economic Research.

Reis, R. (2021). The constraint on public debt when r< g but g< m. *Working Paper*.

Reis, R. (2022). The fiscal revenue from public borrowing. *Journal of Economic Perspectives*.

Rotemberg, J. J., & Woodford, M. (2021). Dynamic general equilibrium models with imperfectly competitive product markets. In *Frontiers of business cycle research* (pp. 243–293). Princeton University Press.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy*, *66*(6), 467–482.

Snudden, S. (2021). Idiosyncratic asset return and wage risk of US households. *Working Paper.*

Straub, L. (2019). Consumption, savings, and the distribution of permanent income. *Working Paper*.

Uhlig, H. (1996). A law of large numbers for large economies. *Economic Theory*, *8*, 41–50.