Responsible Investing under Ambiguity Induced by Climate Uncertainty*

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Abstract

We propose a theory of responsible investing under conditions of ambiguity induced by climate uncertainty. We take steps from studying the portfolio allocation problem solved by a smoothly ambiguity averse representative agent. This new theory delivers three new insights. First, within this setting, we find that the returns ambiguity degree is a strictly increasing function of the environmental pollution scores of the assets in the menu of choice. Second, ambiguity-averse investors behave as environmentally motivated agents who allocate their wealth according to a mean-variance-ambiguity efficient frontier as well as their attitude towards risk and ambiguity. Third, the agents rationally choose "green" portfolios in order to reduce their exposition towards ambiguity and maximize their ambiguity-adjusted Sharpe ratio. Our theoretical predictions are consistent with the empirical literature on the realized rewards-to-risks trade-off of responsible investment.

Keywords: Ambiguity, Uncertainty, Asset Pricing, Portfolio Choice, Climate Uncertainty, Environmental Awareness, ESG, Sustainable Investing.

JEL Classification: D81, G11, G12, Q50.

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1 Introduction

The growing concerns about climate change and the ensuing worldwide plans to transition to more sustainable economies have motivated two recent streams of research, i.e., responsible investing and the economics of climate uncertainty[1]. In a broad definition, responsible investing refers to the practice of considering, in the evaluation of the investment opportunities available in the equity (and increasingly so also in the bond) markets, the attitudes and the manifested behavior by each issuing firm toward environmental, social, and governance (henceforth, ESG) issues. The literature on climate uncertainty focuses on the impact of the uncertainty about the parameters that rule climate change phenomena on financial markets and their connections to the real economy (see, for instance, Giglio, Kelly, and Stroebel, 2021). Our paper gives a contribution at the intersection of these two streams of literature and builds on their results to investigate the implications of responsible investing to an ambiguity-averse investor who optimizes her portfolio in a regime of ambiguity induced by climate change. In a model of risk- and ambiguity-averse investors, we prove that an agent should rationally sacrifice a portion of the maximum attainable reward-to-risk (Sharpe) ratio if and only if it is averse toward ambiguity and for the sole purpose of increasing his or her ambiguity-adjusted reward-to-risk performance. The existence of such a trade-off is a novel insight that is made possible by a framework in which climate uncertainty matters because it adds ambiguity to the economy.

The recent literature on responsible investing broadly agrees on the idea that the practice of seeking (or at least, weighting for) the ESG inclinations/practical performance of firms may have costs and benefits that need to be carefully assessed by rational decision-makers. In particular, the benefits are typically represented as being of a non-pecuniary type, such as the good feelings perceived by holding stocks issued by ESG-compliant firms (see, e.g., Baker et al., 2018, Pedersen, Fitzgibbons, and Pomorsky, 2021, Pastor, Stambaugh, and Taylor, 2021, Avramov et al., 2021), or the *potential*, indirect presumption that assets that score highly in a ESG dimension may offer a hedge against climate change shocks and the ensuing social tensions and required costly, structural transformation (see, e.g., Pastor, Stambaugh, and Taylor, 2021). To the contrary, the costs are directly associated with an immediate reduction in the Sharpe ratio of the holdings that include responsible investing-

[1]For reviews on these topics see e.g. Giglio, Kelly, and Stroebel (2021), and Hong, Karolyi, and Scheinkman (2021)

related assets. In fact, the dominant approach towards modeling the benefits of responsible investing consists of extending the utility function of the (representative) investor to include a non-pecuniary factor which measures any additional utility obtained from holding securities characterized by (relatively) high ESG scores. The immediate implication of introducing this non-monetary argument in the utility function of the agents is that those with a positive derivative of their utility function to the ESG scores of the securities in their portfolios will be willing to sacrifice higher risk premia in exchange for more responsible investing decisions. As a result, a "responsible" investor is likely to restrict her investment universe to include only assets that satisfy some lower bound constraint in terms of ESG scores or, at least, to steeply penalize their chances to enter any optimal portfolio. Therefore, she will allocate her wealth to portfolios that may be sub-optimal in the short-to-medium term (see, for instance, Pedersen, Fitzgibbons, and Pomorsky 2021, Fig. 1) and optimal only when the long-term, non-pecuniary aspects are duly taken into account.

This delicate balance between potential benefits and actual costs is crucial to understand why agents actually invest and, even more importantly, *should* invest in "green", socially-responsible assets. In particular, the arguments above naturally put on the table the question of whether agents who hold relatively "greener" portfolios may appear to be irrational in the short-medium term, and whether this seemingly sub-optimal behavior may find a rational explanation when further layers of uncertainty, such as climate-induced ambiguity, are considered. To address these issues, in this paper, we assume the presence of climate-induced ambiguity and take the perspective of a rational, ambiguity-averse agent. By introducing this assumption, we adopt an approach that, to the best of our knowledge, has been ignored so far in the responsible investing literature: that climate uncertainty is likely to create considerable uncertainty on the future path of the economy. Moreover, we avoid the issues arising from introducing non-monetary arguments in the utility that may naturally generate violations of first-order stochastic dominance and of the utility (over money or consumption) maximization principle[2]. This approach allows us to review the costs and benefits of responsible investing through the lenses of a model of rational, ambiguity-averse, utility-maximizing behavior.

Under the simplifying (yet, not critical) assumption that climate uncertainty represents the only source of ambiguity, our model has the following main implications for responsible

[2]In Section 3.2 we provide an example which proves that responsible investing preferences does not satisfy first order stochastic dominance.

investing. First, there exists a link between ambiguity in the probability distributions of returns and their associated environmental pollution rating (called EP score in what follows).[3] In particular, the higher the EP score, the higher the ambiguity of an asset. Second, a preference for less ambiguous assets (i.e., ambiguity aversion) *coincides* with a preference for greener assets. Therefore, in this setting, ambiguity averse investors *are* environmentally motivated agents and viceversa. Third, in the presence of climate uncertainty, all the assets in the economy, even those are individually not exposed to climate change uncertainty, share a minimum level of ambiguity that turns out to be a function of the global component of the consumption damage caused by climate change. This result is consistent with the intuition that when the effects of climate change materialize, all of the agents in the economy are likely to suffer a portion of the ensuing consumption damages, regardless of their specific asset holdings. Finally, ambiguity-averse investors are willing to sacrifice their standard (risk-based) Sharpe ratio to increase their ambiguity Sharpe ratio. Therefore, because it reflects ambiguity aversion, the behavior of an environmentally motivated agent is fully rational in our setting. Hence, the ambiguity concerning the effects of climate change seems to provide a powerful micro-foundation to the environmental motivations already featured in a growing body of applied literature, for instance, Pedersen, Fitzgibbons, and Pomorsky (2021).

Figure 1 graphically summarizes the main results described above with a stylized example. The picture on top plots the three dimensional efficient surface and color-differentiates the areas according to the EP score of the portfolios that lies on it. The picture below is obtained by projecting the efficient surface onto a standard two-dimensional risk-return plane. In the plot at the top, we highlight in red the portfolios with high EP scores (and hence, according to our results, most ambiguous) and in green those with low EP scores (low ambiguity). By assuming that the point M on the surface is the market portfolio, with an associated EP score of b_M, and R_f is the risk-free rate, the gray surface represents the plane tangent to the point M on the surface. This identifies the red risk-return frontier $\mathbb{F}_{b_M}$ in the plot below. Note that each risk-return frontier that belongs on the surface is efficient. Hence, the agent locates herself on the frontier according to her risk and ambiguity aversion propensities. In particular, as we shall formally prove in Section 3.2, any ambiguity-averse

[3] If s denotes the environmental pillar score, we define the environmental pollution rating as $1 - s$. The choice of working with environmental pollution rather than with environmental pillar ratings is exclusively for reasons of mathematical convenience. There is no influence of this choice on the results of the paper.

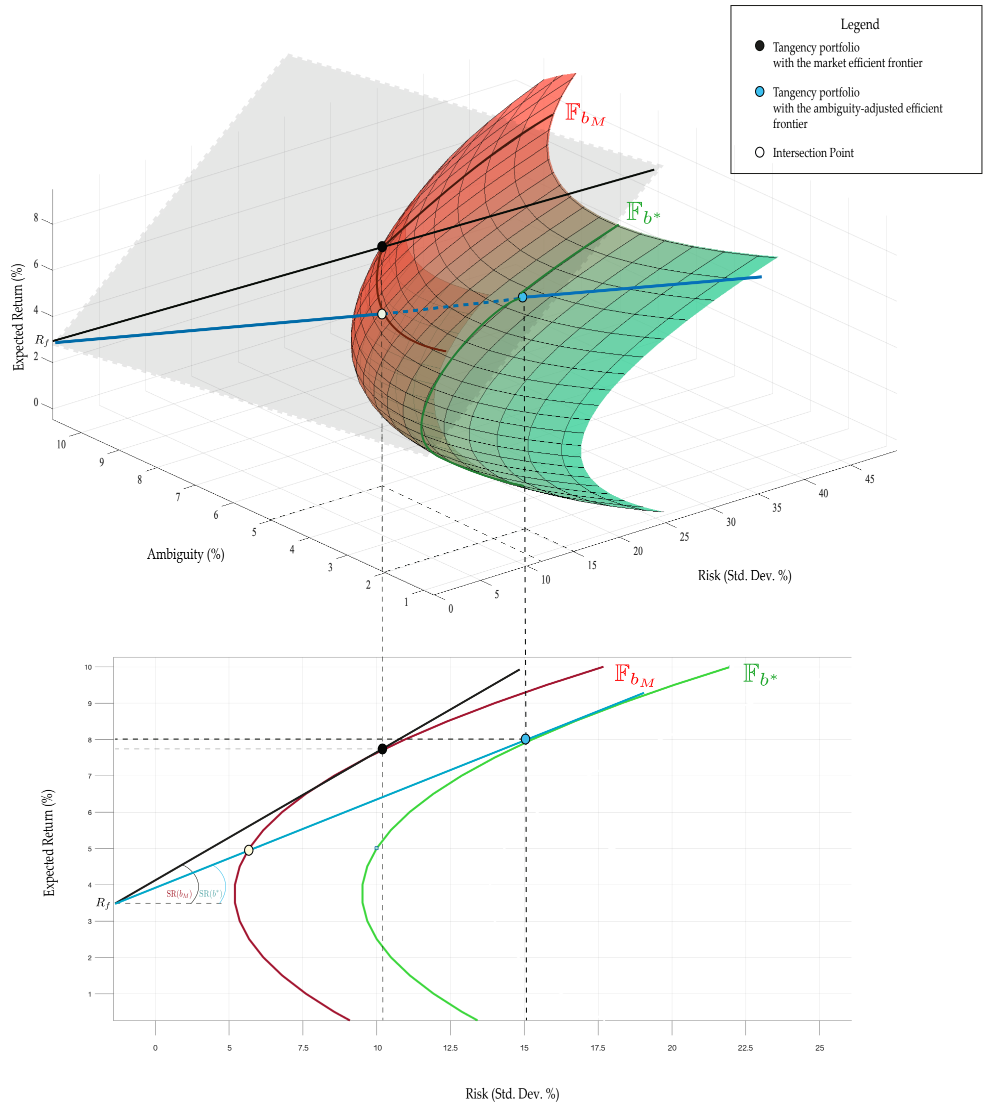

Figure 1
Graphical summary of the main results. In the top panel, we plot is the efficient surface determined by the ambiguity-risk-return trade-off. In the bottom panel, we plot its projection on the bi-dimensional risk-return plane. In the top panel, the different shadings represent the EP scores of the portfolios on the surface. Red areas represent portfolios with higher EP scores, green areas show portfolios with lower EP scores. R_f is the risk-free rate. M is the market portfolio, that lies on the red risk-return frontier identified by $\mathbb{F}_{b_M}$. P^* is the tangency portfolio that lies on the green risk-return frontier identified by $\mathbb{F}_{b^*}$, obtained by constraining the desired EP score to $b^* < b_M$. In the bottom panel, SR(b_M) and SR(b^*) are the Sharpe ratio measured at b_M and b^*, respectively.

agent will choose a portfolio located on a "greener" frontier (e.g., $\mathbb{F}_{b^*}$) in order to reduce her exposure to climate change-induced ambiguity. Indeed, the Sharpe ratio contraction visible in the bottom plot is then justified by an increase in the ambiguity Sharpe ratio, as one can see in the plot at the top.

The predictions delivered by our theory are consistent with the evidence presented in related empirical studies. Engle et al. (2020) build a mimicking portfolio based on the Sustainalytics E-Scores that can dynamically hedge climate uncertainty. They report evidence that green assets effectively provide a hedge against climate change innovations. This result is consistent with our theory in which, by including green assets in their portfolios, investors reduce their exposure towards climate-induced ambiguity. Bolton and Kacperczyk (2021) study the cross-section of U.S. stock returns and find a persistent and significant "carbon premium" that would be priced by the agents at a firm level. Therefore, their evidence is consistent with the hypothesis that the carbon premium is idiosyncratic. Maccheroni, Marinacci, and Ruffino (2013) show that the idiosyncratic *alpha* left unexplained by a single-factor equilibrium asset pricing model under the hypothesis of market efficiency reflects the ambiguity premia of the assets. Therefore, in this light, the existence of a carbon premium is consistent with the predictions of our theory. Indeed, in our model, portfolios composed of stocks with an above average carbon footprint would imply lower environmental scores, which also lead to a higher ambiguity (idiosyncratic) premium. Ilhan, Sautner, and Vilkov (2021) analyze options data to find that equity options price climate policy uncertainty. Also in this perspective, because climate policy uncertainty is one of the manifestations of climate-induced ambiguity, this is consistent with our the predictions from our model. Finally, Bauer, Ruof, and Smeets (2021) analyze households' preferences toward responsible investments and show that, when facing uncertainty about the expected returns of stocks with different sustainability scores, the majority selects the most sustainable assets. In our framework, because we reconcile this result with the empirical evidence that individuals are ambiguity averse, this is consistent with the hypothesis that ambiguity averse investors behave as agents who are environmentally motivated.

In summary, our paper contributes to the literature on responsible investing in at least two ways. First, we build a rational model of sustainable investing under conditions of climate-induced ambiguity, which, to the best of our knowledge, had not been developed before. Second, we provide a rational explanation to the empirical regularity that investors tend to invest or should invest responsibly even when this imposes short-run sacrifices to their realized reward-to-risk ratios, possibly due to the apparent over-pricing of green assets. In particular, our conclusions can be interpreted in two alternative ways. On the one hand, in our set up, responsible investors are ambiguity averse. In this sense, they sacrifice expected returns not because of a sentimental goodwill of theirs, but mainly to reduce their portfolio exposure to ambiguity. On the other hand, according to our model, every ambiguity-averse agent should invest responsibly to hedge against shocks to the uncertainty generated by climate change.

The remainder of the paper proceeds as follows. Section 2 presents the set up of the model. Section 3 reports our main theoretical results, while Section 4 concludes by presenting a few suggestions for additional research.

2 Setup

2.1 The Economy

Let $(\Omega, \mathscr{F}, P)$ be a probability space, $L^2 = L^2(\Omega, \mathscr{F}, P)$ be the space of square integrable random variables on Ω, and $L^\infty = L^\infty(\Omega, \mathscr{F}, P)$ be the subset of L^2 consisting of its almost surely bounded elements. With respect to an interval $I \subset \mathbb{R}$, we define the subset of L^∞ given by

$$L^\infty(I) = \{p \in L^\infty : \text{essinf}(p), \text{esssup}(p) \in I\}.$$

We take the space $L^\infty(I)$ as our reference set of investment alternatives because it can capture the potential ambiguity in asset returns whilst retaining the advantages of being mathematically tractable.

Next, let $\mathcal{T} = [0, t+1]$ denote a continuous time interval, where $t \in \mathcal{T}$ denotes the present date, $t+1$ denotes some future date in which divestment occurs, while any element of $[0, ..., t) \subset \mathcal{T}$ refers to a past date. We are interested in the portfolio allocation problem of an agent between time t and $t+1$. The economy is assumed to be characterized by n risky assets. With reference to the discrete time interval $\{t, t+1\}$, the set of risky assets is represented by a vector of gross returns $\mathcal{R} = (R_1, ..., R_n)'$, with $R_i \in L^\infty(I)$ for all $i = 1, ..., n$, and a vector of EP scores $\mathbf{b} = (b_1, ..., b_n)'$, with $b_i \in [0,1]$ for all $i = 1, ..., n$. Therefore, the set of assets available in the economy is defined by $\mathcal{V} = \{\{R_i, b_i\}, i = 1, ..., n\} \subset L^\infty(I) \times \mathbf{b}$, where the pair $\{R_i, b_i\}$ uniquely identifies asset i in the economy. Without loss of generality, we assume that $\mathcal{V}$ is sufficiently large to allow for the existence of assets with identical expectation and variance of returns but with different EP scores.

For each firm $i = 1, ..., n$, the EP score measures how much its activity harms the environment. Therefore, each EP score b_i is assumed to be determined as a function of the contribution of firm i to the average emissions in the economy.[4] In particular, for any firm i, let $E_{i,t}$ be the value of emissions at time t and $G_{i,t} = \frac{1}{t}\int_0^t E_{i,s}ds$ be its average over the interval $[0, t]$. In formal terms, we provide the following definition of the EP score.

Definition 1 (**Asset EP score**). *Let $\psi : \mathbb{R}^+ \to [0, 1]$ be a continuously differentiable function such that $\psi(G) = 0$ if and only if $G = 0$, is strictly increasing for $0 \leq G \leq \bar{G}$, and $\psi(G) = 1$ for all $G \geq \bar{G}$, for a certain upper threshold $\bar{G} > 0$. We define the EP score of firm i at time $t + 1$ as*

$$b_i = \psi(G_{i,t+1}), \tag{1}$$

where for simplicity of notation we drop the time subscript of b_i from its formal definition.

While our definition in Equation 1 is highly stylized, it shares the same intuition behind the methodology employed by ESG rating agencies to determine the score assigned to the environmental pillar.[5] For instance, MSCI determines the environmental pillar score by computing the average of different kinds of pollution factors, including carbon emissions, toxic waste, and electricity usage, weighted according to the expected time frame over which climate uncertainty is expected to materialize (MSCI Technical Report, 2020). Note that our choice of working with EP scores (which measure a "bad", i.e., an externality) instead of the separate environmental pillar scores (which capture how virtuous a firm may be), as traditionally done in most of the literature (see e.g. Pedersen, Fitzgibbons, and Pomorski (2021), Pastor, Stambaugh, and Taylor (2021), and Avramov et al. (2021)), is for pure analytical convenience. Indeed, all of the results presented in this paper can be also derived in terms of environmental pillar scores by considering instead $s_i = 1 - b_i$ for any $i = 1, ..., n$, where s_i denotes the "E" component of the ESG score associated to firm i.

Next, we characterize the uncertainty about the probability distributions of the returns by following Maccheroni, Marinacci, and Ruffino (2013). We denote as Δ the subset of L^2

[4] Here, for emissions, we mean every pollutant substance that can contribute to global warming. These include CO2 emissions, deforestation, toxic waste emissions, etc.

[5] The ESG (Environmental, Social, and Governance) score is typically computed through the aggregation of three individual scores named *pillars*.

containing all the square integrable probability density functions with respect to P:

$$\Delta = \left\{ q \in L_{+}^{2} : \int_{\Omega} q(\omega) dP(\omega) = 1 \right\}.$$

The corresponding probability measures Q on $\mathscr{F}$ are identified from the densities q via Radon-Nikodym derivative as $q = \frac{dQ}{dP}$. By assuming that Δ is the set of probability distributions faced by a representative agent, we have that the expected return $\mathbb{E}(.)$ and variance $\widetilde{\mathrm{VAR}}\ (.)$ are random functions defined by

$$\mathbb{E}(\{\tilde{R}_i, b_i\}) : q \to \int_{\Omega} \tilde{R}_i(\omega) q(\omega) dP(\omega),$$

and

$$\widetilde{\mathrm{VAR}}\ (\{\tilde{R}_i, b_i\}) : q \to \int_{\Omega} \tilde{R}_i(\omega)^2 q(\omega) dP(\omega) - \left(\int_{\Omega} \tilde{R}_i(\omega) q(\omega) dP(\omega) \right)^2,$$

respectively.

As in Maccheroni, Marinacci and Ruffino (2013), we assume that there exists a second order probability distribution (prior) that the agent associates to the first order probability distributions in the posited set Δ. Therefore, it is possible to compute a measure of the ambiguity faced by the agent as the variance of the random expected return function weighted by the prior. Formally, we define

$$\mathrm{VAR}_{\mu}(\mathbb{E}(\{\tilde{R}_i, b_i\})) = \int_{\Delta} \left(\int_{\Omega} \tilde{R}_i(\omega) q(\omega) dP(\omega) \right)^2 d\mu(q) - \left(\int_{\Delta} \left(\int_{\Omega} \tilde{R}_i(\omega) q(\omega) dP(\omega) \right) d\mu(q) \right)^2.$$

This measure quantifies, for each of the assets and at each point of time, how much the return is affected by ambiguity. In fact, Maccheroni, Marinacci and Ruffino (2013) prove the following:

Proposition 1. *The following statements are equivalent:*

1. *An asset* $\{R_i, b_i\} \in \mathcal{V}$ *is approximately unambiguous*

2. $\mathrm{VAR}_{\mu}(\mathbb{E}(\{\tilde{R}_i, b_i\})) = 0.$

Proof: The proof is in Maccheroni, Marinacci, and Ruffino (2013), Proposition 5 and Definition 4 for the set $L^{\infty}(I)$. By considering that $\mathcal{R} \subset L^{\infty}(I)$ and that each asset has a uniquely associated EP score b_i, the proof extends trivially to $\mathcal{V}$. ■

Next, we assume there exists a representative agent with an initial level of wealth denoted by W_t, who at $t+1$ consumes a single composite good in the amount C_{t+1}. The investor trades in combinations of the the risky assets to assemble a portfolio characterized by asset commitments (i.e., these are fractions of monetary wealth invested in each of the n assets) collected in the vector $\mathbf{x} = (x_1, ..., x_n)$, whose return is $\tilde{R} = \frac{\mathbf{x}'\mathcal{R}}{\mathbf{x}'\mathbf{1}}$. Therefore, the future level of wealth guaranteed by this portfolio at $t+1$ is $\tilde{W}_{t+1} = \mathbf{x}'\tilde{\mathcal{R}}$ and the budget constraint can be written as $C_t = W_t - \mathbf{x}'\mathbf{1}$, where $\mathbf{1} \equiv (1, ..., 1)_n$. Finally, to characterize the EP score associated with a given portfolio, we provide the following definition.

Definition 2 (EP score of a portfolio). *Let $\bar{b}$ denote a portfolio EP score and $\varphi : \mathbb{R}_+ \to [0,1]$ be a function such that $\varphi(\frac{\mathbf{x}'\mathbf{G}_{i,t+1}}{\mathbf{x}'\mathbf{1}}) = \frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}}$. The composite portfolio EP score is given by*

$$\bar{b} = \varphi\left(\frac{\mathbf{x}'\mathbf{G}_{i,t+1}}{\mathbf{x}'\mathbf{1}}\right) = \varphi\left(\frac{\mathbf{x}'\psi^{-1}(\mathbf{b})}{\mathbf{x}'\mathbf{1}}\right) = \frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}}, \tag{2}$$

where, $\mathbf{G}_{i,t+1} = \{G_{1,t+1}, G_{2,t+1}, ..., G_{n,t+1}\}$, $\mathbf{b} = [b_1, ..., b_n]$, and, by Definition 1, $\varphi(.)$ is strictly increasing in $\frac{\mathbf{x}'\mathbf{G}_{i,t+1}}{\mathbf{x}'\mathbf{1}} \in [0, \bar{G}]$, bounded, and differentiable.

This definition implies that the EP score of a portfolio is defined by a transformation that simply equates the weighted average of the EP scores of the assets included in the portfolio, where the weights are represented by the portfolio weights in the vector $\frac{\mathbf{x}}{\mathbf{x}'\mathbf{1}}$

2.2 Damages from Climate Change

In the same spirit as the current literature on climate finance, we conjecture the existence of damages to future consumption caused by climate change events. Following Barnett, Brock, and Hansen (2020), we model consumption damages as a function $\tilde{\Gamma}$ of the temperature changes induced by environmental pollution, plus an exogenous shock. The temperature is assumed to evolve by following Matthews et al.'s (2009) approximation, where emissions at time t permanently impact future temperatures. This model is obviously just a stylized approximation of the actual climate system, which should also consider, for instance, transient components in the temperature change dynamics. However, for the purposes of this paper, i.e., to emphasize the link between financial and climate uncertainty, such an approximation provides a suitable choice (See e.g. Barnett, Brock and Hansen, 2018, 2020 and 2022). Let T_t be the temperature at time t, F_t be the cumulative emissions in the economy, and $\tilde{\zeta}$ be the (ambiguous) parameter that measures climate sensitivity to emissions.

Matthews et al. (2009) model propose that

$$T_t - T_0 \simeq \tilde{\zeta} \int_0^t E_s ds = \tilde{\zeta} F_t.$$

Then, similarly to Denning et al. (2015), Dietz, Gollier and Kessler (2018), and Barnett, Brock, and Hansen (2020), we formalize the process followed by consumption damages as follows

$$D_{t+1} = \tilde{\Gamma}(\tilde{\zeta} F_{t+1}) + \tilde{\xi}_f \frac{\mathbf{x}'}{W_t} \mathbf{F}_{i,t+1} + \xi, \tag{3}$$

where $\mathbf{F}_{i,t+1} = (F_{1,t+1}, ..., F_{n,t+1})$ is the vector of the cumulative emissions of each firm, $\tilde{\xi}_{f,t+1}$ is the dimension of the exogenous shift associated to cumulative environmental pollution relative to the firm in the agent's portfolio, and ξ is a further exogenous shift that captures the direct and indirect effects of transient changes in temperature and other technological contributions that can affect damages. The damage function $\tilde{\Gamma}$ has been formalized in various ways in the literature. We adopt the functional form proposed by Barnett, Brock and Hansen (2020) in which $\tilde{\Gamma}$ is defined as

$$\tilde{\Gamma}(y) = \begin{cases} \gamma_1 y + \frac{1}{2}\gamma_2 y^2 & \text{if } 0 \leq y < \bar{\gamma}, \\ \gamma_1 y + \frac{1}{2}\gamma_2 y^2 + \frac{1}{2}\tilde{\gamma}_+ (y - \bar{\gamma})^2 & \text{if } y \geq \bar{\gamma}, \end{cases} \tag{4}$$

where $\gamma_1, \tilde{\gamma}_+ \in \mathbb{R}^+$, the parameter $\tilde{\gamma}_+ \geq 0$ measures the difference between a low-damage and a high-damage specification of the same function $\tilde{\Gamma}$, while $\bar{\gamma}$ takes the meaning of a "tipping point" after which the consequences of higher cumulative emissions become more severe.

The second term in Equation 3, which is new vs. the formalization by Barnett, Brock, and Hansen (2020), captures the local damage to consumption which is strictly linked with the portfolio held by the agent when the damage occurs. We deliberately introduce a component that depends on the agent's portfolio to accommodate the observed heterogeneity in emissions and, therefore, in estimated consumption damages (see, for instance, Chakravarty et al., 2009). Intuitively, the introduction of such heterogeneity finds motivation in observing that some climate events produce more significant damages to the agent's consumption when he or she holds stocks of the firms that contributed most to generate such an event. An example that we may consider is the case of an agent with a portfolio that includes stocks of high carbon footprint firms. When a bad climate event happens (e.g., the acceptable limit of emissions in the environment is approached), the consequences are suffered from all of the agents in the economy (global damage). However, an investor who holds the high carbon footprint portfolio undergoes a further reduction in wealth due to, for instance, the introduction of policies to curb carbon emissions (local damage caused by realized transition risk). While we take equation (3) as the benchmark damage specification for developing our theory, in Section 4, we discuss whether assuming alternative specifications for damages may influence the predictions of our model.

Barnett, Brock, and Hansen (2020) argue that two interconnected sources of uncertainty affect the evolution of damages. First, there is substantial uncertainty about the climate sensitivity parameter $\tilde{\zeta}$ as documented by MacDougall, Swart, and Knutti (2017). In particular, their empirical analysis shows that the estimated range of values at which $\tilde{\zeta}$ lies using a 95% confidence interval is extensive and fails to deliver predictions with sufficient precision on the evolution of temperature. Second, there is uncertainty about the damage function $\tilde{\Gamma}$ specification, and in particular about the parameter $\tilde{\gamma}_+$. This is carefully analyzed in Barnett, Brock, and Hansen (2020, Figure 3, p. 1035), where they compare the shapes of the damage function for different hypothesized values of $\tilde{\gamma}_+$, showing significant departures in terms of reduction in economic welfare of a representative agent. Consistently, we assume parameter uncertainty on the exogenous shifter $\xi_{f,t+1}$. Next, by considering that local and global damages are linked to the same parameter (i.e., emissions) in this framework, we can plausibly assume that, in most scenarios, global damages are higher when local damages are higher, and vice versa. Therefore, we impose

$$\mathrm{COV}_{\mu}(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta})), \mathbb{E}(\tilde{\xi}_f)) > 0. \tag{5}$$

Finally, because of the existence of consumption damages we have that the net level of consumption at time $t+1$ is given by

$$\bar{C}_{t+1} = C_{t+1} - \tilde{D}_{t+1}, \tag{6}$$

where C_{t+1} is the baseline future consumption level in the absence of climate change damages.

2.3 Preferences

We equip our representative agent with the smooth intertemporal preferences that account for ambiguity proposed by Klibanoff, Marinacci, and Mukerji (2009). In their framework, they consider the space of all observable paths generated by an event tree where each node k_j^t branches out into a vector of immediate successor nodes. Formally, at time t, if $\mathcal{K}^{t+1}$ is the set of possible observations at time $t+1$, the possible successor nodes are all the $k_j^{t+1} \in \mathcal{K}^{t+1}$. The agent chooses between future consumption plans c each of which associates a payoff to a node k_j^t on the event tree. The preferences are then represented recursively as

$$V_{k_j^t}(c) = u(c(k_j^t)) + \beta\phi^{-1}\left[\int_\Delta \phi\left(\int_{\mathcal{K}^{t+1}} V_{k_j^t,k_j^{t+1}}(c)q(k_j^{t+1}|k_j^t)dP\right)d\mu(k_j^t)\right],$$

where $V_{s_j^t}(c)$ is the recursive value function, $u(.)$ and $\phi(.)$ characterize the attitudes of the agent toward risk and ambiguity, respectively, and β is a time discount factor. $q(k_j^{t+1}|k_j^t)$ denotes the conditional probability density function associated with a node $k_j^{t+1} \in \mathcal{K}_{t+1}$, provided that node k_j^t is reached. Finally, $\mu(k_j^t)$ denotes the second order probability associated to the scenario $k_j^{t+1} \in \mathcal{K}_{t+1}$, defined as the successor node that is reached after k_j^t. In this setting, each scenario k_j^{t+1} associates to the function $V(.)$ a probability density function $q(k_j^{t+1}|k_j^t)$ and a value to the parameters $\tilde{\gamma}_+, \tilde{\zeta}$ and $\tilde{\xi}_f$ in the damage function. Therefore, the prior $\mu(k_j^t)$ represents the probability associated to each climate scenario as well as to the probabilistic models of the returns on the available stocks.

Since in our setting we take t as the present date and assume that the agent di-vest and consumes everything at $t+1$, the recursive value function is truncated at $t+1$ and $V_{s_j^t,s_j^{t+1}}(c) = u(\tilde{C}_{t+1}) = u(\tilde{W}_{t+1})$. Hence, we have

$$V(\tilde{W}_{t+1}) = u(C_t) + \beta\int_\Delta \phi\left(\int_\Omega u(\tilde{W}_{t+1}(\omega))q(\omega)dP(\omega)\right)d\mu. \tag{7}$$

Note that, as shown by Klibanoff, Marinacci, and Mukerji (2009), the recursive model in equation (7) embeds the atemporal smooth model of preferences in Klibanoff, Marinacci, and Mukerji (2005). In particular, let γ and θ denote the coefficients of absolute risk aversion and ambiguity aversion, respectively. As proven by Maccheroni, Marinacci, and Ruffino (2013), (7) implies that the agent ranks the assets in $\mathcal{V}$ according to the certainty equivalent function

$$C_a(\tilde{W}_{t+1}) = \mathbb{E}[\tilde{W}_{t+1}|\bar{Q}] - \frac{\gamma}{2}\text{VAR}[\tilde{W}_{t+1}|\bar{Q}] - \frac{\theta}{2}\text{VAR}_\mu(\mathbb{E}(\tilde{W}_{t+1})), \tag{8}$$

where $\bar{Q}$ is the ambiguity neutral probability measure[6], $\lambda = -\frac{u''(W)}{u'(W)}$, and $\theta = -\frac{(\phi \circ u)''(W)}{(\phi \circ u)'(W)} - \lambda$. It follows that the function in equation (8) represents a preference relationship $\succsim_a$ on $\mathcal{V}$ that satisfies the axioms of the smooth model of preferences under ambiguity.[7]

In our derivation of the main theoretical results, we use both the intertemporal and the atemporal smooth models in equations (7) and (8), respectively. In particular, we employ the intertemporal model to derive the implicit link between ambiguity and EP scores in the presence of climate uncertainty, whereas such results naturally hold in the atemporal setting. Next, we use the atemporal model to elicit first the connection between preferences over ambiguity and preferences over EP scores and, second, the implications of the existence of climate uncertainty for optimal portfolio choices.

3 Main Results

We begin the presentation of our results by deriving first the equilibrium pricing kernels. In a setting similar to ours, Klibanoff, Marinacci, and Mukerji (2009) and Collard et al. (2018) have shown that the maximization problem of the agent can be described in terms of the following recursive Bellman equation:

$$J(W_t, \mu_t) = \max_{C_t, x} u(C_t) + \beta\phi^{-1}[\mathbb{E}_{\mu_t}(\phi\mathbb{E}_q(J(\tilde{W}_{t+1}, \mu_{t+1})))],$$

where $J(W_t, \mu_t)$ indicates a recursively defined indirect value function, and the optimization is solved subject to the consumption budget constraint. We therefore follow Klibanoff, Marinacci and Mukerji and assume, just for the sake of tractability the specifications $u(C_t) = \log C_t$ and $\phi(x) = -\exp(-\theta x)$, where θ is the ambiguity aversion coefficient.

[6]The ambiguity neutral probability measure is defined as the measure $\bar{Q}$ such that, for any $\tilde{p} \in L^2, \mathbb{E}_\mu(\mathbb{E}(\tilde{p})) = \mathbb{E}(\tilde{p}|\bar{Q})$. Maccheroni, Marinacci and Ruffino (2013, Lemma 1) discuss the existence and uniqueness of such a probability measure.

[7]A binary preference relationship $\succsim_a$ on $\mathcal{V}$ is represented by a function $C_a(.)$ if and only if for any $\{R_1, b_1\}, \{R_2, b_2\} \in \mathcal{V}, \{R_1, b_1\} \succsim_a \{R_2, b_2\} \Leftrightarrow C_a(\{R_1, b_1\}) \geq C_a(\{R_2, b_2\})$.

The first-order condition under this specification delivers the following Euler equation

$$\delta \mathbb{E}_{\mu_t^*}[\mathbb{E}(\tilde{R})u'(\bar{C}_{t+1}))] = u'(C_t), \tag{9}$$

where $\tilde{R}$ is the return on the portfolio and μ^* is the posterior second-order probability measure obtained as $\epsilon_t \times \mu_t$, where

$$\epsilon_t = \frac{\phi'(\mathbb{E}_q(J(W_{t+1}, \mu_{t+1})))}{\mathbb{E}_{\mu_t}[\phi'(\mathbb{E}_q(J(W_{t+1}, \mu_{t+1})))]}. \tag{10}$$

Finally, by equation (9), we define the stochastic discount factor m_{t+1} (henceforth, SDF) as identified by

$$1 = \mathbb{E}_{\mu*}[\mathbb{E}(\tilde{R}m_{t+1})]. \tag{11}$$

As pointed out by Klibanoff, Marinacci and Mukerji (2009), under the assumed specifications for $u(.)$ and $\phi(.)$, the SDF takes the same functional form as that derived in standard Bayesian frameworks. This allows the comparison with previous results in the literature (see e.g., Ju and Miao (2007, 2012), and Collard et al., 2018) and, hence, motivates our choice to prefer the tractability imposed by our specifications over full generality.

3.1 Climate-induced ambiguity and environmental pollution scores

This first set of results uncovers a relationship between ambiguity in returns and the firms' EP scores. To this purpose, we make the following identifying assumption:

Assumption 1. *Assets prices are affected by the uncertainty induced by climate change only.*

The intent of Assumption 1 is to switch off the ambiguity arising from sources different from climate uncertainty that in reality will naturally affect asset prices. In particular, it allows us to isolate the ambiguity strictly induced by climate uncertainty and, hence, reflected by the EP scores. Because the primary objective of this paper is to study the existing link between the ambiguity in asset returns and climate uncertainty, all the main results in this paper are derived under this assumption. Nevertheless, in Section 4, we discuss the consequences of removing this assumption and how our results are likely to be affected. As we shall argue, in case we were to remove Assumption 1, the substance of our main results remain unaltered.

Proposition 2. *In the economy built in Section 2, the stochastic discount factor is given by*

$$\tilde{m}_{t+1} = \frac{u'(C_{t+1} - \tilde{D}_{t+1})}{u'(C_t)}, \tag{12}$$

where

$$\tilde{D}_{t+1} = \tilde{\Gamma}\left(\tilde{\zeta}(t+1)\sum_{i=1}^{n}\psi^{-1}(b_i)\right) + \tilde{\xi}_f(t+1)\varphi^{-1}(\bar{b}) + \xi. \tag{13}$$

Proof: The proof follows immediately by substituting in equation (11) the definition of net consumption in equation (6), and by substitution in the definition of consumption damages in equation (3) of the definitions of asset EP score and of portfolio EP score from equations (1) and (2), respectively. Finally, by considering that $F_{t+1} = \sum_{i=1}^{n} F_{i,t+1} = \sum_{i=1}^{n}(t+1)G_{i,t+1}$, equation (13) follows. ■

Proposition 2 shows that, in our setting, the SDF is a function of the EP scores associated with each of the assets and of the portfolio EP score $\bar{b}$. Moreover, since $\tilde{\zeta}$, $\tilde{\Gamma}$ and $\tilde{\xi}_f$ are all affected by parameter uncertainty, it is immediate to see that if there is at least one i such that $b_i > 0$, the stochastic discount factor will be ambiguous as well. As argued earlier, in our intertemporal asset pricing model, an asset return is ambiguous if and only if it derives in equilibrium from a SDF that is ambiguous. It follows that, under Assumption 1, the return on the composite portfolio is ambiguous if and only if $b_i > 0$ for some i, i.e., if there exists at least an asset with negative environmental impact.

Although the connection established by Proposition 2 may seem far from surprising (portfolios will reflect ambiguity if and only if they contain at least one ambiguous asset), it is the following proposition that further explores this connection by analyzing the link between ambiguity in the probability distributions of the assets returns and the associated EP scores.

Proposition 3. *Under Assumption 1, the following statements hold:*

1. *All the assets traded in the economy are approximately unambiguous if and only if* $b_i = 0$ *for all* $i = 1, ..., n$.

2. *For any* $\{R_j, b_+\}$ *and* $\{R_h, b_-\} \in \mathcal{V}$, $b_+ > b_-$ *if and only if* $\mathrm{VAR}_\mu(\mathbb{E}(\{R_j, b_+\})) > \mathrm{VAR}_\mu(\mathbb{E}(\{R_h, b_-\}))$.

3. *For any* $\{R_i, b_i\} \in \mathcal{V}$ *with* $b_i = 0$, $\mathrm{VAR}_\mu(\mathbb{E}(\{R_i, b_i\})) = \mathrm{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta} F_{t+1}))$. *Moreover* $\mathrm{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta} F_{t+1})))$ *is the minimum obtainable ambiguity.*

Proof: See Appendix B.1. ■

Propositions 3 uncovers the relationship existing between ambiguity in returns and EP scores. In particular, Proposition 3.1 states that *any* firm with a non-zero environmental impact induces the presence of ambiguity in the economy *as a whole* and hence in all portfolios of traded assets. Indeed the claim in the theorem stresses that the absence of climate change risk loadings in all assets is not only sufficient but also necessary for the whole economy to escape the ambiguity caused by climate uncertainty. The economic intuition behind this conclusion is straightforward. Climate uncertainty is implicitly linked with the presence in the economy of firms with a positive EP score. The presence of climate uncertainty, among the other effects, naturally fosters the proposal of political measures the penalize brown assets, which therefore incur the risk of abrupt changes in their future expected returns, what is often called *transition risk*. Hence, the existence of even one single asset with positive EP score is sufficient to propagate the climate change-induced ambiguity throughout the entire economy and to make each individual asset exposed to such uncertainty source.[8]

The second claim in Proposition 3 shows that the ambiguity exposure and the individual EP score of each asset are positively related. In particular, under Assumption 1, assets characterized by a higher EP score (say, brown stocks) are more ambiguous than assets with lower EP scores (green stocks). Again, the intuition lies in the fact that firms with a relatively high EP score are more exposed to climate uncertainty and, therefore, more exposed to the effects of climate uncertainty on the probability distributions of their payoffs. Finally, Proposition 3.3 presents evidence that, in the presence of climate uncertainty, all the assets in the economy, even those are individually not directly exposed to climate change uncertainty, do share a minimum level of ambiguity that turns out to be a function of the global component of the consumption damage caused by climate change. This result is consistent with the intuition that when the effects of climate change materialize, all of the agents in the economy are likely to suffer a portion of the ensuing consumption damages,

[8]This is similar to the case in which even the stock with the minimum, but positive variance among all stocks, in a portfolio may end up having a large and positive exposure to risk because of its beta, its covariance with the overall portfolio risk

regardless of their specific asset holdings.[9]

3.2 The relationship between ambiguity aversion and environmental motivation

In this section, we further advance our analysis and demonstrate that ambiguity aversion and preferences for lower EP scores, i.e. responsible behavior, are connected. With this purpose, we consider an alternative representative agent equipped with a revised version of the ESG-adjusted expected utility function (here expressed in terms of certainty equivalent function) proposed by Pedersen, Fitzgibbons, and Pomorski (2021). In particular, let $f : [0,1] \to \mathbb{R}_+$ be a continuous strictly increasing function such that $f(0) = 0$. We define

$$C_b(\{\tilde{R}_i, b_i\}) = \mathbb{E}[\{\tilde{R}_i, b_i\}|\bar{Q}] - \frac{\gamma_b}{2}\text{VAR}[\{\tilde{R}_i, b_i\}|\bar{Q}] - f(b), \ \ \{\tilde{R}_i, b_i\} \in \mathcal{V}, \tag{14}$$

as the continuous certainty equivalent function of the environmentally motivated agent, where $\bar{Q}$ is the ambiguity-neutral probability measure, γ_b is the risk aversion coefficient and $f(b)$ is the EP preference function that captures any environmental motivations characterizing this representative investor. The preferences expressed by (14) incorporate three differences with respect to the function proposed by Pedersen, Fitzgibbons, and Pomorski (2021). First, we define the ESG preference function on the EP score, i.e. the complement to one of the environmental pillar, for consistency with our analysis, which explains the negative sign in front of $f(b)$. Second, and consistently with the purposes of our study, we also elect not to consider the social and governance pillars, which are instead accounted for in the original specification by Pedersen, Fitzgibbons, and Pomorski. Note that, from a mathematical perspective, the introduction of these two adjustments would not alter the nature of the results obtained by Pedersen, Fitzgibbons, and Pomorski (2021), as we demonstrate in Appendix A.1. Third, to extend the model proposed by Pedersen, Fitzgibbons, and Pomorski, which is defined over purely risky alternatives of investment, to an environment where the returns of the assets might be affected by ambiguity, the two moments featured in equation (14) are computed under the ambiguity neutral probability measure $\bar{Q}$. The intuition of this step is that an environmentally aware agent (14) is a decision-maker who cares only about expected returns, risk, and EP scores while not accounting for the

[9] Of course, stocks characterized by $b_i > 0$ are likely to carry a level of ambiguity that exceeds the minimum $\text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})))$.

uncertainty about the probability distributions of the returns. Therefore, the assumption that the agent evaluates the alternatives at $\bar{Q}$ is a natural choice. Notice that, in a setting with naturally ambiguous stocks and no climate uncertainty, this creates a natural wedge between the representative investor represented by equation 8 and the agent described by equation (14).

As discussed in the introduction, the preferences represented by the function in (14) violate first order stochastic dominance. Whilst Borch (1969) and Blavatskyy (2010) documented that standard mean-variance preferences allows as well for violations of F.O.S.D., adding a non-pecuniary argument to a mean-variance objective function increases the number of such violations. To see this, the next example provides a stylized proof of this statement.

Example 1. *Consider two lotteries, l_1 and l_2, defined as follows.*

$$l_1 = \begin{cases} 1 \text{ with prob. } 0.5 \\ 0 \text{ with prob. } 0.5 \end{cases} \qquad l_2 = \begin{cases} 1 \quad \text{with prob. } 0.5 \\ 0.5 \text{ with prob. } 0.5 \end{cases}$$

Clearly, l_2 statewise (and hence F.O.S.) dominates l_1, and $\mathbb{E}[l_2] - \frac{\gamma}{2}\text{VAR}[l_2] = 0.75 - \frac{\gamma}{2}0.0625 > 0.5 - \frac{\gamma}{2}0.25 = \mathbb{E}[l_1] - \frac{\gamma}{2}\text{VAR}[l_1]$ for any $\gamma > 0$. Next, assume $\gamma = 1$, $f(b) = 10b$ and that l_1 and l_2 are paired with $b_1 = 0$ and $b_2 = 0.04$, respectively. We have $C_b(l_1) \simeq 0.375 > 0.32 \simeq C_b(l_2)$, proving the claim.

Let $\succsim_b$ be the ranking on $\mathcal{V}$ induced by the certainty equivalent function in equation 14. For simplicity, and in order to allow the comparison in the rankings between the responsible agent and the ambiguity-averse agent, in what follows we assume that $\succsim_b$ is complete on $\mathcal{V}$.[10] Next, consider again our rational agent equipped with the smooth ambiguity model we have introduced in Section 2 and recall that we denote by $\succsim_a$ the order of preferences represented by the certainty equivalent function in equation (8). The following proposition compares $\succsim_b$ with the preferences induced by the smooth ambiguity model $\succsim_a$. With this purpose, we make the following, technical assumption.

Assumption 2. *For any $\{R_i, b_i\}, \{R_j, b_j\} \in \mathcal{V}$, $\{R_i, b_i\} \succsim_b \{R_j, b_j\}$ if and only if $\{R_i, b_i\} \succsim_a \{R_j, b_j\}$.*

[10]While we make this assumption for simplicity, it is possible to show that $\succsim_b$ satisfies weak order, and hence completeness, on $L^\infty(I) \times \mathbf{b} \supset \mathcal{V}$. Indeed, by assuming the continuity of the certainty equivalent function in (14) on $L^\infty(I) \times \mathbf{b}$, weak order follows immediately.

Assumption 2 simply states that the two agents rank any assets in the asset menu they face, in the same way. Note that this assumption is simply instrumental in the study of the attitude towards ambiguity that is implicit in the ranking of the investment alternatives according to environmentally motivated preferences. Consequently, we maintain this hypothesis only throughout this section and we will remove it in the portfolio analysis that follows in Section 3.3.

Proposition 4. *Let $\succsim_a$ and $\succsim_b$ be the rankings on $\mathcal{V}$ induced by the preferences in equation (8) and (14), respectively. If $\succsim_b$ is complete, and assumptions 1 and 2 are satisfied, then $\succsim_a$ is ambiguity averse.*

Proof: See Appendix B.2. ■

Proposition 4 states that if an agent has a preference for lower EP scores, i.e., $f(b) > 0$ for $b > 0$, then he or she behaves as an ambiguity averse agent, i.e., it prefers all else equal assets characterized by lower ambiguity over assets characterized by higher ambiguity. Note that this is consistent with the results in Proposition 3, where we show that, under Assumption 1, assets with higher EP score are characterized by higher ambiguity. An immediate consequence of this is that non-strictly rational (in the sense that these are lacking axiomatic foundations) preferences for lower environmental impact, such as those employed by Pedersen, Fitzgibbons, and Pomorski (2021) and Pastor, Stambaugh, and Taylor (2021), can be reinterpreted as rational in the light of the ambiguity implied by climate uncertainty.[11] In fact, in our setting, ambiguity averse agents rationally choose greener assets because of their lower uncertainty by obtaining a concrete benefit in terms of the exposition of their portfolio to climate uncertainty-induced ambiguity. Hence, they always act by maximizing their expected utility according to their tastes over risk and uncertainty but end up featuring environmental motivations similar to those featured in equation (14).

To complete this line of reasoning, we next proceed to formally compare the attitudes toward EP scores and ambiguity and show that agents who are comparatively more averse toward EP scores need also to be comparatively more averse toward ambiguity. First, we provide a definition of comparatively higher EP score aversion.

[11]Technically, for non-rational, we mean choices that allows for violation of first-order stochastic dominance and profit maximization principles

Definition 3. *Let* $\succsim_b$ *and* $\succsim'_b$ *be two binary relations over* $\mathcal{V}$ *induced by the certainty equivalent function in equation (14) that shares the same risk aversion. We say that* $\succsim'_b$ *is more EP score averse than* $\succsim_b$ *if, for any* $\{R_i, b\}, \{R_j, 0\} \in \mathcal{V}$ *with* $b>0$*,*

$$\{R_i, b\} \succsim'_b \{R_j, 0\} \Longrightarrow \{R_i, b\} \succsim_b \{R_j, 0\}. \tag{15}$$

The definition of comparatively higher EP score aversion is inspired after the definition of comparative higher ambiguity aversion by Klibanoff, Marinacci, and Mukerji (2005, Definition 5). The intuition is that if two agents identified by $\succsim_b$ and $\succsim'_b$, respectively, share the same risk aversion but $\succsim_b$ prefers a lower EP scores asset over a green asset whenever $\succsim'_b$ does, then this must be because $\succsim_b$ is less EP score averse than $\succsim'_b$.

Proposition 5. *Consider two pairs of alternative preference orderings* $\succsim_a$*,* $\succsim'_a$*, that satisfy the smooth ambiguity model axioms and share the same prior* μ*, and* $\succsim_b$*,* $\succsim'_b$*, with same risk aversion and represented by the function in equation (14). Assume that, for any* $\{R_i, b\}, \{R_j, 0\} \in \mathcal{V}$ *with* $b > 0$*,* $\{R_i, b\} \succsim'_b \{R_j, 0\}$ *if and only if* $\{R_i, b\} \succsim'_a \{R_j, 0\}$*. Under assumptions 1 and 2,* $\succsim'_b$ *is more EP score averse than* $\succsim_b$ *if and only if* $\succsim'_a$ *is more ambiguity averse than* $\succsim_a$*.*

Proof: See Appendix B.3. ■

Therefore, the consistency in the choices between EP score preferences and smooth ambiguity preferences holds both in absolute terms and when comparing different degrees of aversion toward environmental pollution and ambiguity. In particular, Proposition 5 states that, under Assumption 1, comparatively more ambiguity averse agents behave as comparatively more environmentally motivated agents and viceversa. This establishes that, even though it always acts by maximizing its expected utility according to tastes over risk and uncertainty, the representative investor that is driven by (8) always ends up featuring environmental motivations similar to those featured by the investors driven by the certainty equivalent function in (14). Hence, the ambiguity concerning the effects of climate change seems to provide a powerful micro-foundation to the environmental motivations already featured in a growing body of applied literature.

3.3 Portfolio allocation

We now proceed with the last step of our analysis and draw a number of logical deductions concerning the link between EP scores and ambiguity derived from the portfolio allocation problem of our smooth ambiguity averse representative agent. Next, we compare our results to those by Pedersen, Fitzgibbons, and Pomorski (2021) and show that environmentally motivated agents rationally choose sub-optimal positions in a mean-variance setting but are efficient in an extended mean-variance-ambiguity setup.

Consider again the certainty equivalent function in equation (8), and recall that $\tilde{W}_{t+1} = \mathbf{x}'\mathcal{R}$. We can write the maximization problem solved by the smooth ambiguity-averse representative agent as

$$\max_{\mathbf{x}\in X} C_a(\tilde{W}_{t+1}) = \max_{\mathbf{x}\in X} \left\{ \mathbf{x}'\mathbf{m} - \frac{\gamma}{2}\mathbf{x}'\Sigma\mathbf{x} - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) \right\}. \tag{16}$$

where the a in C_a stands for (smooth preferences under) ambiguity our agent is equipped with, $\mathbf{m} = \mathbb{E}[\mathcal{R}|b]$, $\Sigma = \text{VAR}[\mathcal{R}|b]$, $\bar{b}$ is the portfolio aggregate EP score, and $\sigma_\mu^2(\bar{b}) = \text{VAR}_\mu(\mathbb{E}(\{x'\mathcal{R}, \bar{b}\}))$ is the portfolio ambiguity level. By splitting the maximization problem in parts, (16) can be re-written as

$$\max_{\mathbf{x}\in X} C_a(W_{t+1}) = \max_{\mathbf{b}} \left\{ \max_{\sigma} \left[\max_{\substack{\mathbf{x}\in X \\ \bar{b}=\frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}} \\ \sigma^2=\mathbf{x}'\Sigma\mathbf{x}}} \left(\mathbf{x}'\mathbf{m} - \frac{\gamma}{2}\sigma^2 - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) \right) \right] \right\}. \tag{17}$$

At this point, for a given portfolio EP score, $\bar{b}$, we define the maximum Sharpe ratio as

$$\text{SR}(\bar{b}) = \max_{\substack{\mathbf{x}\in X \\ \bar{b}=\frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}}}} \left\{ \frac{\mathbf{x}'\mathbf{m}}{\sqrt{\mathbf{x}'\Sigma\mathbf{x}}} \right\}, \tag{18}$$

which can be used to re-write the maximization problem in equation (17) as

$$\max_{\mathbf{x}\in X} C_a(W_{t+1}) = \max_{\mathbf{b}} \left\{ \max_{\sigma} \left[\left(\text{SR}(\bar{b})\sigma - \frac{\gamma}{2}\sigma^2 - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) \right) \right] \right\}. \tag{19}$$

Finally, following Mukerji, Ozsoylev and Tallon (2020), we define the ambiguity Sharpe ratio as:

$$\text{SR}^a(\bar{b}) = \max_{\mathbf{x}\in X} \left(\frac{\mathbf{x}'\mathbf{m}}{\sigma_\mu(\bar{b})} \right). \tag{20}$$

Intuitively, in the same way in which the conventional Sharpe ratio measures the premium for unit of risk, the ambiguity Sharpe ratio measures the return premium required by the agent to hold a certain position in uncertain (and risky) assets per unit of ambiguity, as measured by $\sigma_\mu(\bar{b})$. Within this setting, the theorem that follows presents our main result concerning the portfolio allocation for an ambiguity-averse representative agent in an economy characterized by climate change uncertainty:

Theorem 1. *Consider a representative agent with preferences represented by (8). Under Assumption 1, the following statements hold:*

1. *The agent optimization problem is equivalent to*

$$\max_{\bar{b}} \left\{ \frac{\mathrm{SR}(\bar{b})^2}{2\gamma} - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) \right\}. \tag{21}$$

2. *The maximum Sharpe ratio that can be achieved for a given, desired portfolio EP score $\bar{b}$ is*

$$\mathrm{SR}(\bar{b}) = \sqrt{c_{mm} - \frac{(c_{bm} - \bar{b}c_{1m})^2}{c_{bb} - 2\bar{b}c_{1b} + \bar{b}^2 c_{11}}}, \tag{22}$$

where $c_{xy} = \mathbf{x}'\Sigma\mathbf{y}$, i.e., the covariance of the returns of portfolios defined by $\mathbf{x}$ and $\mathbf{y}$, and where the maximum potential Sharpe ratio, $\sqrt{c_{mm}}$, is achieved at an EP score of $\bar{b}^N = c_{bm}/c_{1m}$.

3. *The maximum ambiguity Sharpe ratio is*

$$\mathrm{SR}^a(\bar{b}) = \frac{\mathrm{SR}(\bar{b})^2}{\gamma\sigma_\mu(\bar{b})}. \tag{23}$$

4. *The optimal portfolio is*

$$\mathbf{x} = \frac{1}{\rho}\Sigma^{-1}\left(\mathbf{m} - \frac{c_{bm} - c_{1m}\bar{b}}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}(\mathbf{b} - \mathbf{1}\bar{b})\right). \tag{24}$$

Proof: See Appendix B.4. ■

We can compare these results with those reported in Pedersen, Fitzgibbons, and Pomorski (2021) for an environmentally motivated agent, recalled in Theorem 2, Appendix

A.1 for a Reader's convenience. It is immediate to see that the conventional Sharpe ratio and the optimal portfolio composition for the agent are the same as those of the environmentally motivated agent. However, if the optimal allocation for the environmentally motivated agent results from a trade-off between risk and preference for higher environmental scores, here it comes from a tradeoff between risk and ambiguity, represented by equation (21). In particular, (21) defines the efficient frontier faced by the agent, which is a function of the portfolio return, risk, and ambiguity. Note that this is consistent with the theoretical results on standard portfolio analysis under ambiguity (see, e.g., Maccheroni, Marinacci, and Ruffino, 2013; and Mukerji, Ozsoylev, and Tallon, 2020).

Next, we compare our ambiguity-averse investor's willingness to sacrifice a higher Sharpe ratio to maximize its ambiguity Sharpe ratio with that displayed by an environmentally motivated agent. In particular, recall that in Pedersen, Fitzgibbons, and Pomorski's model, the environmentally motivated agent chooses a portfolio with a lower Sharpe ratio to obtain a higher, desirable average environmental score. The two propositions that follow show that, in our framework, the agent sacrifices a higher Sharpe ratio if and only if it is averse toward ambiguity and for the sole purpose of increasing his or her ambiguity Sharpe ratio. The existence of such a trade-off is a novel insight that is made possible by our novel framework in which climate change matters because it adds ambiguity in the economy.

Proposition 6. *Consider a representative agent with preferences represented by equation (8). Let $\bar{b}^*$ be the optimal EP score, i.e., the EP score associated to the optimal portfolio allocation displayed in equation (24). Under Assumption 1, the following statements hold:*

1. *$\bar{b}^* < \bar{b}^N$ if and only if the agent is ambiguity averse.*

2. *$\bar{b}^* = \bar{b}^N$ if and only if the agent is ambiguity neutral.*

3. *$\bar{b}^* > \bar{b}^N$ if and only if the agent is ambiguity lover.*

Proof: See Appendix B.5. ■

Proposition 7. *Under Assumption 1, the following statements hold:*

1. *For any $\bar{b}^* > \bar{b}^N$, $\mathrm{SR}(\bar{b}^*) < \mathrm{SR}(\bar{b}^N)$ and $\mathrm{SR}^a(\bar{b}^*) < \mathrm{SR}^a(\bar{b}^N)$.*

2. *For any $\bar{b}^* < \bar{b}^N$, $\mathrm{SR}(\bar{b}^*) < \mathrm{SR}(\bar{b}^N)$ and $\mathrm{SR}^a(\bar{b}^*) > \mathrm{SR}^a(\bar{b}^N)$. In particular, the maximum obtainable ambiguity Sharpe ratio is achieved at $\bar{b}^* = 0$.*

Proof: See Appendix B.6. ■

Proposition 7 completes our portfolio analysis. It shows that, in a mean-variance-ambiguity setting, EP scores higher than the threshold $\bar{b}^N$ are always inefficient and only selected by ambiguity-seeking agents. Regarding the EP scores lower than $\bar{b}^N$, these are chosen by ambiguity averse agents to maximize their ambiguity Sharpe ratio, although this implies the cost of having to partially sacrifice their conventional risk Sharpe ratio. Therefore, choosing lower EP scores is rationally justified in our setting by the goal of reducing the exposition to ambiguity.

4 Discussion and Conclusions

In this paper, we have developed a theory of responsible investing under conditions of ambiguity induced by climate change uncertainty. By considering this novel feature, our paper differs from the existing literature by providing four, key findings. First, we find that the ambiguity in the distribution of returns is a strictly increasing function of the environmental pollution scores of the assets. Next, we consider an ambiguity-averse agent and show that, ceteris paribus, she behaves as an environmentally motivated agent who benefits from holding green(er) assets. Third, investors allocate their wealth according to a mean-variance-ambiguity efficient frontier and to their attitude towards risk and ambiguity. Finally, investors rationally choose to invest in less environmentally damaging firms with the purpose of reducing their exposure to ambiguity.

In developing our theory, we employ some simplifications that could, in principle, be removed to broaden the generality and range of applicability of the results presented in the paper. This remainder of this section briefly discusses the consequences of removing such simplifications/restrictive hypotheses to provide a flavor on how our results are most likely to change as a result of employing alternative assumptions or of introducing variations in the framework assumed.

First, we have deliberately limited our attention to the study of the link between environmental scores and ambiguity. However, a complete analysis would require extending our investigation to also include the S and G factors of the ESG phenomenon. Indeed, the social and governance dimensions may plausibly influence the uncertainty of the assets, as climate change does. Nevertheless, while to explicitly entertain such an analysis is beyond the scope of this paper, we focus on the E scores also because of the current unavailability of commonly accepted theoretical models and empirical evidence that would link governance and social scores to assets return ambiguity.

Second, we explicitly rule out other sources of uncertainty that are likely to affect asset prices. These include the natural ambiguity affecting returns distributions (see, e.g., Collard et al., 2020) and the very ESG scores (Avramov et al., 2021). By considering other sources of uncertainty, it should be straightforward to verify that some of the results will be weakened, while others will need some more structure to be added in the present setting to hold. For instance, the first claim in Proposition 1 will continue to hold, but only in one direction. Indeed, if there is no ambiguity, then, necessarily, there must be no climate change uncertainty. However, in the presence of other sources, the absence of climate uncertainty does not imply the absence of ambiguity. As for all of the other results, these continue to hold but only with respect to one source of ambiguity, i.e. climate change, and not in general terms. To see this, consider that, for instance, it is possible to substitute the assumed smooth model with the source-dependent preferences under ambiguity introduced by Cappelli et al. (2021). In such a setting, for each source of uncertainty $z \in \mathcal{Z}$, with $\mathcal{Z}$ denoting the collection of all sources, there exists an intra-source ranking $\succsim_z$ on $\mathcal{V}_z$ and an associated certainty equivalent function C_z that represents $\succsim_z$, where $\mathcal{V}_z$ is the subset of $\mathcal{V}$ containing all of the prospects affected by source z. In this setting, it is straightforward to demonstrate that the substance of our result remains unaltered, although at the cost of the introduction of little more structure in preferences.[12]

Finally, we must acknowledge that alternative specifications of the damage function in equation (3) have been proposed in the literature. Among the others, Barnett, Brock, and Hansen (2020) propose a damage function that includes only the global damages borne by all the agents in the economy. Specifically, the specification they adopt is the following:

$$D_{t+1} = \tilde{\Gamma}(\tilde{\zeta} F_{t+1}) + \tilde{\xi}_f F_{t+1} + \xi. \tag{25}$$

Using the damage specification in (25) instead of that in equation (3) implies that the ambiguity induced by climate change is shared equally by all of the assets in the economy. Obviously, this does not allow to discriminate between assets that are in principle more

[12]For instance, within this setting we would get that environmentally motivated agent are averse to climate change induced ambiguity, but not necessarily ambiguity averse with respect to all the uncertainty sources (i.e., w.r.t. to all $z \in \mathcal{Z}$]).

sensitive to climate uncertainty than others. Nevertheless, such simpler global damage specifications may still be useful in models that include ambiguity and that, for instance, are geared towards studying households' participation in the stock market during spells of climate change. Again, this is beyond the immediate objectives of our paper and therefore left for future research.

APPENDIX: PROOFS AND RELATED ANALYSIS

A Preliminaries: environmental motivated preferences

A.1 Optimal portfolio allocation

Let $\mathbf{s} = (s_1, ..., s_n)$ be the vector of environmental pillar scores associated to the n assets in the economy, where $s_i = 1 - b_i$. We denote by $\bar{s}$ the average environmental pillar score of the composite portfolio chose by the agent, i.e. $\bar{s} = \frac{\mathbf{x}'\mathbf{s}}{\mathbf{x}'\mathbf{1}}$. In this first appendix we show that the results in terms of portfolio allocation obtained by Pedersen, Fitzgibbons, and Pomorski (2021) through the certainty equivalent function

$$C_s(W_{t+1}) = \mathbb{E}[W_{t+1}|s] - \frac{\gamma}{2}\text{VAR}[W_{t+1}|s] + W_t f_s(\bar{s}), \tag{26}$$

where $f_s : [0, 1] \to \mathbb{R}_+$, continuous and strictly increasing, coincides with those obtained by equipping the agent with preferences represented by

$$C_b(W_{t+1}) = \mathbb{E}[W_{t+1}|b] - \frac{\gamma}{2}\text{VAR}[W_{t+1}|b] - W_t f(\bar{b}). \tag{27}$$

Throughout the rest of this subsection we assume there is no uncertainty. Let $\mathbf{m} = \mathbb{E}[\mathcal{R}|s]$ and $\Sigma = \text{VAR}[\mathcal{R}|s]$. First, next theorem recalls the results on the portfolio allocation obtained by Pedersen, Fitzgibbons, and Pomorski (2021). Then, Theorem 3 shows the equivalence between the two sets of results.

Theorem 2. *Consider an agent with preferences represented by Equation (26). The following statements are true:*

1. *The agent optimization problem coincide with* $\max_{\bar{s}}\{\mathrm{SR}(\bar{s})^2 + 2\rho f(\bar{s})\}$, *where* ρ *is the coefficient of relative risk aversion.*

2. *The maximum Sharpe ratio that can be achieved for a desired score* $\bar{s}$ *is*

$$\mathrm{SR}(\bar{s}) = \sqrt{c_{mm} - \frac{(c_{sm} - \bar{s}c_{1m})^2}{c_{ss} - 2\bar{s}c_{1s} + \bar{s}^2 c_{11}}},$$

where $c_{xy} = x\Sigma y$, *and where the maximum potential Sharpe ratio,* c_{mm}, *is achieved at* $s^* = c_{sm}/c_{1m}$.

3. *The optimal portfolio for a given environmental pillar score* $\bar{s}$ *is*

$$\mathbf{x}_s = \frac{1}{\rho}\Sigma^{-1}(\mathbf{m} + \lambda_{1,s}(\mathbf{s} - \mathbf{1}\bar{s})),$$

where

$$\lambda_{1,s} = -\frac{(\mathbf{s} - \mathbf{1}\bar{s})\Sigma^{-1}\mathbf{m}}{(\mathbf{s} - \mathbf{1}\bar{s})\Sigma^{-1}(\mathbf{s} - \mathbf{1}\bar{s})}.$$

Proof: The proof for all of the statements is provided by Pedersen, Fitzgibbons, and Pomorsky (2021, propositions 1-3). ■

Theorem 3. *Consider an agent with preferences represented by equation (27). The following are true:*

1. *The agent optimization problem coincides with* $\max_{\bar{b}}\{\mathrm{SR}(\bar{b})^2 - 2\rho f(\bar{b})\}$, *where* ρ *is the coefficient of relative risk aversion.*

2. *The maximum Sharpe ratio that can be achieved for a desired score* $\bar{b}$ *is*

$$\mathrm{SR}(\bar{b}) = \sqrt{c_{mm} - \frac{(c_{bm} - \bar{b}c_{1m})^2}{c_{bb} - 2\bar{b}c_{1b} + \bar{b}^2 c_{11}}},$$

where the maximum potential Sharpe ratio, c_{mm}, *is achieved at* $\bar{b}^N = c_{bm}/c_{1m} = 1 - \bar{s}^N$.

3. *The optimal portfolio for a given EP-score is*

$$\mathbf{x}_b = \frac{1}{\rho}\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}(\mathbf{b} - \mathbf{1}\bar{b})) = \frac{1}{\rho}\Sigma^{-1}(\mathbf{m} + \lambda_{1,s}(\mathbf{s} - \mathbf{1}\bar{s})) = \mathbf{x}_s.$$

Proof: First note that, for a given $\mathbf{s}$, $\mathbf{m} = \mathbb{E}[r|b] = \mathbb{E}[r|s]$ and $\Sigma = \text{VAR}[r|b] = \text{VAR}[r|s]$. For statement (1), by equation (27) the optimization problem is given by

$$\max_{\mathbf{x}\in X} \left\{ \mathbf{x}'\mathbf{m} - \frac{\rho}{2}\mathbf{x}'\Sigma\mathbf{x} - f\left(\frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}}\right) \right\}. \tag{28}$$

Define

$$\text{SR}(\bar{b}) = \max_{\substack{\mathbf{x}\in X \\ \mathbf{x}'\mathbf{1}=1 \\ \mathbf{x}'\mathbf{b}=\bar{b}}} \left\{ \frac{\mathbf{x}'\mathbf{m}}{\sqrt{\mathbf{x}'\Sigma\mathbf{x}}} \right\}, \tag{29}$$

where, clearly, $\text{SR}(\bar{b}) = \text{SR}(\bar{s})$. Equation (28) can be rewritten as

$$\max_{\mathbf{b}} \left\{ \max_{\sigma} \left[\max_{\substack{\mathbf{x}\in X \\ \bar{b}=\frac{\mathbf{x}'\mathbf{b}}{\mathbf{x}'\mathbf{1}} \\ \sigma^2=\mathbf{x}'\Sigma\mathbf{x}}} \left(\mathbf{x}'\mathbf{m} - \frac{\rho}{2}\sigma^2 - f(\bar{b}) \right) \right] \right\}. \tag{30}$$

where, by Pedersen, Fitzgibbons, and Pomorsky (2021, equation 35), $\text{SR}(\bar{b})$ is independent from σ. Hence, we have

$$\max_{\mathbf{b}} \left\{ \max_{\sigma} \left[\text{SR}(\bar{b})\sigma - \frac{\rho}{2}\sigma^2 - f(\bar{b}) \right] \right\}, \tag{31}$$

that is solved for $\sigma = \text{SR}(\bar{b})/\rho$. By substituting σ in equation (31) we have

$$\max_{\mathbf{b}} \left\{ \frac{1}{2}\frac{\text{SR}(\bar{b})^2}{\rho} - f(\bar{b}) \right\}, \tag{32}$$

which by multiplying for 2ρ proves the first statement.

For statement (2), consider the Lagrangian

$$\mathscr{L} = \mathbf{x}'\mathbf{m} - \frac{\rho}{2}\sigma^2 - f(\bar{b}) - \lambda_{1,b}(\mathbf{x}'\mathbf{b}) - \lambda_{2,b}(\mathbf{x}'\Sigma\mathbf{x} - \sigma^2),$$

where we denote by $\lambda_{1,b}$ and $\lambda_{2,b}$ the Lagrange multipliers. The first order condition delivers

$$\mathbf{x}_b = \frac{1}{\lambda_{2,b}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}}), \tag{33}$$

where $\hat{\mathbf{b}} = \mathbf{b} - \mathbf{1}\bar{b}$. By looking at the constraints, the first one delivers

$$0 = \frac{1}{\lambda_{2,b}} \hat{\mathbf{b}}' \Sigma^{-1} (\mathbf{m} - \lambda_{1,b} \hat{\mathbf{b}}), \tag{34}$$

from which the first multiplier is

$$\lambda_{1,b} = \frac{(\mathbf{b} - \mathbf{1}\bar{b})\Sigma^{-1}\mathbf{m}}{(\mathbf{b} - \mathbf{1}\bar{b})'\Sigma^{-1}(\mathbf{b} - \mathbf{1}\bar{b})} = \frac{c_{bm} - c_{1m}\bar{b}}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}. \tag{35}$$

The second constraint delivers

$$\sigma^2 = \left[\frac{1}{\lambda_{2,b}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}})\right]' \Sigma \left[\frac{1}{\lambda_{2,b}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}})\right] = \frac{1}{\lambda_{2,b}^2}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}})'\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}}), \tag{36}$$

that by employing the first constraint defined in equation (35) can be simplified as

$$\sigma^2 = \frac{1}{\lambda_{2,b}^2} \mathbf{m}'\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}}). \tag{37}$$

Then, by substituting $\lambda_{1,b}$ in equation (37) we get

$$\lambda_{2,b} = \frac{1}{\sigma}\sqrt{c_{mm} - \left(\frac{c_{bm} - c_{1m}\bar{b}}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}\right)}. \tag{38}$$

Next, by considering that

$$\mathrm{SR}(\bar{b}) = \frac{\mathbf{m}'\mathbf{x}}{\sigma} = \frac{\mathbf{m}'\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}\hat{\mathbf{b}})}{\lambda_{2,b}\sigma} = \sigma\lambda_{2,b}, \tag{39}$$

we have

$$\mathrm{SR}(\bar{b}) = \sqrt{c_{mm} - \frac{(c_{bm} - c_{1m}\bar{b})^2}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}}, \tag{40}$$

where $\bar{b}^N = \frac{c_{bm}}{c_{1m}} = \frac{c_{1m} - c_{sm}}{c_{1m}} = 1 - \bar{s}^N$ is obtained by taking the first order condition with respect to b.

For statement (3), starting from equation (33) and recalling that $\lambda_{2,b} = \mathrm{SR}(\bar{b})/\sigma$, $\sigma = \mathrm{SR}(\bar{b})/\rho$, and $\hat{\mathbf{b}} = \mathbf{b} - \mathbf{1}\bar{b}$, we have

$$\mathbf{x}_b = \frac{1}{\rho}\Sigma^{-1}(\mathbf{m} - \lambda_{1,b}(\mathbf{b} - \mathbf{1}\bar{b})). \tag{41}$$

Finally, by considering that $-\lambda_{1,b} = \lambda_{1,s}$ we have $\mathbf{x}_b = \mathbf{x}_s$, which completes the proof of the theorem. ∎

B Proofs of the main results

B.1 Proof of Proposition 3

Proof of Statement (1). First, we prove that (1) ⇒ (2). By Proposition 1, an asset $\{R_i, b_i\}$ is approximately unambiguous if $\text{VAR}_\mu(\mathbb{E}(\{\tilde{R}_i, b_i\})) = 0$. It follows that, if all of the assets in the economy are approximately unambiguous, the composite portfolio is approximately unambiguous. Therefore, $\text{VAR}_\mu(\mathbb{E}(\{\tilde{\bar{R}}, \bar{b}\})) = 0$. Next, since the agent consumes everything at time $t+1$ and, by Assumption 1, there is no uncertainty affecting baseline consumption, we must have

$$\text{VAR}_\mu(\mathbb{E}(\{\tilde{\bar{R}}, b\})) = \text{VAR}_\mu(\mathbb{E}(C_{t+1} - \tilde{D}_{t+1})) = \text{VAR}_\mu(\mathbb{E}(\tilde{D}_{t+1})) = 0.$$

that, by equation (3), coincides with

$$\begin{aligned}\text{VAR}_\mu(\mathbb{E}(\tilde{D}_{t+1}) =& \text{VAR}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})) + \mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) \\ =& \text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})) + \text{VAR}_\mu\left(\mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) \\ &+ 2\text{COV}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}), \mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) = 0. \end{aligned} \tag{42}$$

Since by equation (5) all the three terms in equation (42) are not negative, the condition is satisfied if and only if, respectively,

$$\text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})) = 0, \tag{43}$$

$$\text{VAR}_\mu\left(\mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) = 0, \tag{44}$$

and

$$2\text{COV}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}), \mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) = 0. \tag{45}$$

Consider equation (44). We have that

$$\text{VAR}_\mu\left(\mathbb{E}\left(\tilde{\xi}_f \frac{\mathbf{x}}{W_t}\mathbf{F}_{i,t+1}\right)\right) = \text{VAR}_\mu(\mathbb{E}(\tilde{\xi}_f))\frac{1}{W_t^2}\left(\sum_{i=1}^{n} x_i F_{i,t+1}\right)^2 = 0,$$

for any x_i if and only if $F_{i,t+1} = 0 = b_i$ for all $i = 1, ..., n$. Such condition trivially satisfies equations (44) and (45), and therefore equation (42), completing the proof. For the other direction the proof is trivial and hence omitted. ■

Proof of Statement (2). For any pair of assets $\{R_j, b_+\}$ and $\{R_h, b_-\}$ consider the two investment decisions $\mathcal{I}_j : \{x_j = W_t, x_i = 0 \ \forall \ i \neq j\}$ and $\mathcal{I}_h : \{x_h = W_t, x_i = 0 \ \forall \ i \neq h\}$. We have,

$$\begin{aligned}
&\text{VAR}_\mu(\mathbb{E}(\{R_j, b_+\})) > \text{VAR}_\mu(\mathbb{E}(\{R_h, b_-\})) \iff && (46)\\
&\text{VAR}_\mu(\mathbb{E}(W_{t+1}|\mathcal{I}_j)) > \text{VAR}_\mu(\mathbb{E}(W_{t+1}|\mathcal{I}_h)) \iff \\
&\text{VAR}_\mu(\mathbb{E}((C_{t+1} - \tilde{D}_{t+1})|\mathcal{I}_j) > \text{VAR}_\mu(\mathbb{E}((C_{t+1} - \tilde{D}_{t+1})|\mathcal{I}_h) \iff \\
&\text{VAR}_\mu(\mathbb{E}(\tilde{D}_{t+1}|\mathcal{I}_j)) > \text{VAR}_\mu(\mathbb{E}(\tilde{D}_{t+1}|\mathcal{I}_h)). && (47)
\end{aligned}$$

By plugging the specifications of $\tilde{D}_{t+1}$ considering the two alternative investments opportunities we have

$$\begin{aligned}
&\text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})) + \text{VAR}_\mu(\mathbb{E}(\tilde{\xi}_f F_{j,t+1})) + 2\text{COV}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}), \mathbb{E}(\tilde{\xi}_f F_{j,t+1})\right) > \\
&\text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})) + \text{VAR}_\mu(\mathbb{E}(\tilde{\xi}_f F_{h,t+1})) + 2\text{COV}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}), \mathbb{E}(\tilde{\xi}_f F_{h,t+1})\right), \qquad (48)
\end{aligned}$$

that can be rearranged as

$$(F_{j,t+1}^2 - F_{h,t+1}^2)\text{VAR}_\mu(\mathbb{E}(\tilde{\xi}_f)) + 2(F_{j,t+1} - F_{h,t+1})\text{COV}_\mu\left(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}), \mathbb{E}(\tilde{\xi}_f)\right) > 0 \qquad (49)$$

which is satisfied if and only if $F_{j,t+1} > F_{h,t+1}$, and therefore $b_+ > b_-$. ■

Proof of Statement (3). Fix $b_i = 0$ and consider the investment opportunity $\mathcal{I}_i : \{x_i = W_t, x_j = 0 \ \forall \ j \neq i\}$. By Definition 1, we have that $G_{i,t+1} = F_{i,t+1} = 0$. Hence, by following the same steps in the proof of Statement 2 we have

$$\text{VAR}_\mu(\mathbb{E}(D_{t+1}|\mathcal{I}_i) = \text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1})).$$

Since by Proposition 3 $\text{VAR}_\mu(\mathbb{E}(.))$ is strictly increasing in b, $\text{VAR}_\mu(\mathbb{E}(\tilde{\Gamma}(\tilde{\zeta}F_{t+1}))$ is the minimum obtainable ambiguity, completing the proof. ■

B.2 Proof of Proposition 4

First, recall that by Klibanoff, Marinacci and Mukerji (2005) and Maccheroni, Marinacci and Ruffino (2013), $\succsim_a$ is ambiguity averse if and only if $\theta > 0$. Next, consider two alternative investment opportunities $\{R_i, 0\}$ and $\{R_j, b_j\}$ with $b_j > 0$, $\mathbb{E}_{\bar{Q}}(\{R_i, 0\}) = \mathbb{E}_{\bar{Q}}(\{R_j, b_j\})$, and $\text{VAR}_{\bar{Q}}(\{R_i, 0\}) = \text{VAR}_{\bar{Q}}(\{R_j, b_j\})$. By equation (14) we have that $\{R_i, 0\} \succ_b \{R_j, b_j\}$ and, by Assumption 2, $\{R_i, 0\} \succ_a \{R_j, b_j\}$. Therefore, by Equation 8 we must have

$$\begin{aligned}
&C_a(\{R_i, 0\}) > C_a(\{R_j, b_j\}) \iff \\
&\mathbb{E}_{\bar{Q}}(\{R_i, 0\}) - \frac{\gamma}{2}\text{VAR}_{\bar{Q}}(\{R_i, 0\}) - \frac{\theta}{2}\text{VAR}_\mu(\mathbb{E}(\{R_i, 0\})) > \\
&\mathbb{E}_{\bar{Q}}(\{R_j, b_j\}) - \frac{\gamma}{2}\text{VAR}_{\bar{Q}}(\{R_j, b_j\}) - \frac{\theta}{2}\text{VAR}_\mu(\mathbb{E}(\{R_j, b_j\})) \iff \\
&\frac{\theta}{2}\Big(\text{VAR}_\mu(\mathbb{E}(\{R_j, b_j\})) - \text{VAR}_\mu(\mathbb{E}(\{R_i, 0\}))\Big) > 0.
\end{aligned}$$

By Assumption 1 and Proposition 3, $\text{VAR}_\mu(\mathbb{E}(\{R_j, b_j\})) > \text{VAR}_\mu(\mathbb{E}(\{R_i, 0\}))$. Hence, last equation is satisfied if and only if $\theta > 0$, completing the proof. ■

B.3 Proof of Proposition 5

First, recall that by Statement 3 of Proposition 3 $\text{VAR}_\mu(\mathbb{E}\{\tilde{R}, 0\}) \leq \text{VAR}_\mu(\mathbb{E}\{\tilde{R}, b\})$ for any $b > 0$. Therefore, by Klibanoff, Marinacci and Mukerji (2005, Definition 5) a decision-maker with ranking $\succsim'_a$ and same prior of a decision-maker with ranking $\succsim_a$ is more ambiguity averse than $\succsim_a$ if $\{\tilde{R}_i, b\} \succsim'_a \{R_j, 0\} \Longrightarrow \{\tilde{R}_i, b\} \succsim_a \{R_j, 0\}$ for all $\{\tilde{R}_i, b\} \in \mathcal{V}$.

Next, assume that $\succsim'_b$ is more EP score averse than $\succsim_b$ and consider $\{\tilde{R}_i, b\}$ and $\{\tilde{R}_j, 0\}$ such that $\{\tilde{R}_i, b\} \succsim'_b \{\tilde{R}_j, 0\}$. By assumption, $\{\tilde{R}_i, b\} \succsim'_a \{\tilde{R}_j, 0\}$ and $\{\tilde{R}_i, b\} \succsim_b \{\tilde{R}_j, 0\}$. Finally, assume that $\{\tilde{R}_i, b\} \succsim'_a \{\tilde{R}_j, 0\}$ does not imply $\{\tilde{R}_i, b\} \succsim_a \{\tilde{R}_j, 0\}$. We would have that there exist some $\{\tilde{R}_i, b\}$ and $\{\tilde{R}_j, 0\} \in \mathcal{V}$ such that $\{\tilde{R}_j, 0\} \succ_a \{\tilde{R}_i, b\}$ and, by

Assumption 2, $\{\tilde{R}_j, 0\} \succ_b \{\tilde{R}_i, b\}$, a contradiction. A similar argument proves the result for the other direction. ■

B.4 Proof of Theorem 1

Proof of Statement (1). By starting from Equation (19) and deriving the first order condition with respect to σ, we find that $\sigma = \mathrm{SR}(\bar{b})/\gamma$. By substituting σ in equation (19) we have

$$\max_{\mathbf{b}} \left\{ \frac{1}{2}\frac{\mathrm{SR}(\bar{b})^2}{\gamma} - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) \right\}, \tag{50}$$

which by multiplying for 2γ proves the first statement. ■

Proof of Statement (2). Consider the Lagrangian

$$\mathscr{L} = \mathbf{x}'\mathbf{m} - \frac{\gamma}{2}\sigma^2 - \frac{\theta}{2}\sigma_\mu^2(\bar{b}) - \lambda_{1,a}(x'\hat{\mathbf{b}}) - \lambda_{2,a}(\mathbf{x}'\Sigma\mathbf{x} - \sigma^2),$$

where $\hat{\mathbf{b}} = \mathbf{1} - \mathbf{b}$ and where we denote by $\lambda_{1,a}$ and $\lambda_{2,a}$ the Lagrange multipliers. The first order condition delivers

$$\mathbf{x} = \frac{1}{\lambda_{2,a}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}}). \tag{51}$$

By looking at the constraints, the first one delivers

$$0 = \frac{1}{\lambda_{2,a}}\hat{\mathbf{b}}'\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}}), \tag{52}$$

from which the first multiplier is

$$\lambda_{1,a} = \frac{(\mathbf{b} - \mathbf{1}\bar{b})\Sigma^{-1}\mathbf{m}}{(\mathbf{b} - \mathbf{1}\bar{b})'\Sigma^{-1}(\mathbf{b} - \mathbf{1}\bar{b})} = \frac{c_{bm} - c_{1m}\bar{b}}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}. \tag{53}$$

The second constraint delivers

$$\sigma^2 = \left[\frac{1}{\lambda_{2,a}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}})\right]'\Sigma\left[\frac{1}{\lambda_{2,a}}\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}})\right] = \frac{1}{\lambda_{2,a}^2}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}})'\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}}), \tag{54}$$

that by employing the first constraint can be simplified as

$$\sigma^2 = \frac{1}{\lambda_{2,a}^2}\mathbf{m}'\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}}). \tag{55}$$

Then, by substituting $\lambda_{1,a}$ in equation (55) we get

$$\lambda_{2,a} = \frac{1}{\sigma}\sqrt{c_{mm} - \left(\frac{c_{bm} - c_{1m}\bar{b}}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}\right)}. \tag{56}$$

Next, by considering that

$$\text{SR}(\bar{b}) = \frac{\mathbf{m}'\mathbf{x}}{\sigma} = \frac{\mathbf{m}'\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}\hat{\mathbf{b}})}{\lambda_{2,b}\sigma} = \sigma\lambda_{2,a}, \tag{57}$$

we have

$$\text{SR}(\bar{b}) = \sqrt{c_{mm} - \frac{(c_{bm} - c_{1m}\bar{b})^2}{c_{bb} - 2c_{1b}\bar{b} + c_{11}\bar{b}^2}}, \tag{58}$$

where $\text{SR}(\bar{b}^N) = \sqrt{c_{mm}}$ is the maximum potential Sharpe ratio, and $\bar{b}^N = \frac{c_{bm}}{c_{1m}}$ is obtained by taking the first order condition with respect to **b**. completing the proof of Statement 2. ■

Proof of Statement (3). By equation (23) we have

$$\text{SR}^a(\bar{b}) = \frac{m'x}{\sigma_\mu(\bar{b})} = \frac{\sigma^2\lambda_{2,a}}{\sigma_\mu(\bar{b})} = \frac{\text{SR}(\bar{b})^2}{\gamma\sigma_\mu(\bar{b})}. \tag{59}$$

■

Proof of Statement (4). Starting from equation (51) and recalling that $\lambda_{2,a} = \text{SR}(\bar{b})/\sigma$, $\sigma = \text{SR}(\bar{b})/\gamma$, and $\hat{\mathbf{b}} = \mathbf{b} - \mathbf{1}\bar{b}$, we have

$$\mathbf{x}_a = \frac{1}{\gamma}\Sigma^{-1}(\mathbf{m} - \lambda_{1,a}(\mathbf{b} - \mathbf{1}\bar{b})). \tag{60}$$

The substitution of $\lambda_{1,a}$ from equation (53) completes the proof of the theorem. ■

B.5 Proof of Proposition 6

Let $\bar{b}^*$ be the optimal portfolio EP score resulting from the optimization in equation (21). We have

$$\begin{aligned} C_a(W_{t+1}|\bar{b}=\bar{b}^*) > C_a(W_{t+1}|\bar{b}=\bar{b}^N) \Longleftrightarrow \\ \frac{\mathrm{SR}(\bar{b}^*)^2}{2\gamma} - \frac{\theta}{2}\sigma_\mu^2(\bar{b}^*) > \frac{c_{mm}}{2\gamma} - \frac{\theta}{2}\sigma_\mu^2(\bar{b}^N) \Longleftrightarrow \\ \frac{\theta}{2}(\sigma_\mu^2(\bar{b}^N) - \sigma_\mu^2(\bar{b}^*)) > \frac{c_{mm} - \mathrm{SR}(\bar{b}^*)^2}{2\gamma} > 0, \end{aligned} \tag{61}$$

since $c_{mm} > \mathrm{SR}(\bar{b}^*)^2$. By the second statement of Proposition 3, $\bar{b}^* < b_N$ if and only if $\sigma_\mu^2(\bar{b}^N) - \sigma_\mu^2(\bar{b}^*)$, and by equation (61) $\theta > 0$. Statements 2 and 3 follow trivially from the previous considerations, as well as the other direction of each statement. ■

B.6 Proof of Proposition 7

Proof of Statement (1). Assume that $\bar{b}^* > \bar{b}^N$. By Statement 2 of Theorem 1 $SR(\bar{b}^*) < \sqrt{c_{mm}} = SR(\bar{b}^N)$. Next, consider Equation 23 and recall that by Proposition 3 $\sigma_\mu^2(\bar{b}^*) > \sigma_\mu^2(\bar{b}^N)$. Therefore, we have

$$\bar{b}^* > \bar{b}^N \Longrightarrow \frac{\mathrm{SR}(\bar{b}^*)^2}{\gamma\sigma_\mu(\bar{b}^*)} < \frac{\mathrm{SR}(\bar{b}^N)^2}{\gamma\sigma_\mu(\bar{b}^N)} \Longleftrightarrow \mathrm{SR}^a(\bar{b}^*) < \mathrm{SR}^a(\bar{b}^N). \tag{62}$$

■

Proof of Statement (2). Consider the first order condition of the function $\mathrm{SR}^a(.)$ with respect to $\bar{b}$. We have,

$$\frac{(\mathrm{SR}(\bar{b})^2)'\sigma_\mu(\bar{b}) - \sigma_\mu(\bar{b})'(\mathrm{SR}(\bar{b})^2)}{\gamma\sigma_\mu(\bar{b})^2} > 0 \Longleftrightarrow \tag{63}$$

$$\frac{(\mathrm{SR}(\bar{b})^2)'}{(\mathrm{SR}(\bar{b})^2)} > \frac{\sigma_\mu(\bar{b})'}{\sigma_\mu(\bar{b})} > 0. \tag{64}$$

by Proposition 3. Since $\mathrm{SR}(\bar{b})$ is strictly increasing in $\bar{b} \in [0, \bar{b}^N]$, (64) is always satisfied and the function $\mathrm{SR}^a(.)$ is continuous and strictly decreasing in $\bar{b}$. The rest is trivial. ■

References

[1] D. Avramov, S. Cheng, A. Lioui, and A. Tarelli. Sustainable investing with esg rating uncertainty. *Journal of Financial Economics*, 2021.

[2] M. Baker, D. Bergstresser, G. Serafeim, and J. Wurgler. Financing the response to climate change: The pricing and ownership of us green bonds. *National Bureau of Economic Research, Working Paper*, 2018.

[3] M. Barnett, W. Brock, and L. P. Hansen. Pricing uncertainty induced by climate change. *The Review of Financial Studies*, 33(3):1024–1066, 2020.

[4] R. Bauer, T. Ruof, and P. Smeets. Get real! individuals prefer more sustainable investments. *The Review of Financial Studies*, 34(8):3976–4043, 2021.

[5] P. Bolton and M. Kacperczyk. Do investors care about carbon risk? *Journal of Financial Economics*, 2021.

[6] V. Cappelli, S. Cerreia-Vioglio, F. Maccheroni, M. Marinacci, and S. Minardi. Sources of uncertainty and subjective prices. *Journal of the European Economic Association*, 19(2):872–912, 2021.

[7] S. Chakravarty, A. Chikkatur, H. De Coninck, S. Pacala, R. Socolow, and M. Tavoni. Sharing global co2 emission reductions among one billion high emitters. *Proceedings of the National Academy of Sciences*, 106(29):11884–11888, 2009.

[8] F. Collard, S. Mukerji, K. Sheppard, and J. M. Tallon. Ambiguity and the historical equity premium. *Quantitative Economics*, 9(2):945–993, 2018.

[9] F. Dennig, M. B. Budolfson, M. Fleurbaey, A. Siebert, and R. H. Socolow. Inequality, climate impacts on the future poor, and carbon prices. *Proceedings of the National Academy of Sciences*, 112(52):15827–15832, 2015.

[10] S. Dietz, C. Gollier, and L. Kessler. The climate beta. *Journal of Environmental Economics and Management*, 87:258–274, 2018.

[11] R. F. Engle, S. Giglio, B. Kelly, H. Lee, and J. Stroebel. Hedging climate change news. *The Review of Financial Studies*, 33(3):1184–1216, 2020.

[12] S. Giglio, B. Kelly, and J. Stroebel. Climate finance. *Working Paper*, 2021.

[13] Emirhan Ilhan, Zacharias Sautner, and Grigory Vilkov. Carbon tail risk. *The Review of Financial Studies*, 34(3):1540–1571, 2021.

[14] P. Klibanoff, M. Marinacci, and S. Mukerji. A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892, 2005.

[15] P. Klibanoff, M. Marinacci, and S. Mukerji. Recursive smooth ambiguity preferences. *Journal of Economic Theory*, 144(3):930–976, 2009.

[16] F. Maccheroni, M. Marinacci, and D. Ruffino. Alpha as ambiguity: Robust mean-variance portfolio analysis. *Econometrica*, 81(3):1075–1113, 2013.

[17] A. H. MacDougall, N. C. Swart, and R. Knutti. The uncertainty in the transient climate response to cumulative co2 emissions arising from the uncertainty in physical climate parameters. *Journal of Climate*, 30(2):813–827, 2017.

[18] H. D. Matthews, N. P. Gillett, P. A. Stott, and K. Zickfeld. The proportionality of global warming to cumulative carbon emissions. *Nature*, 459(7248):829–832, 2009.

[19] MSCI. Msci esg rating methodology. Technical report, 2020.

[20] S. Mukerji, H. N. Ozsoylev, and J.M. Tallon. Trading ambiguity: a tale of two heterogeneities. *Available at SSRN 3290605*, 2020.

[21] L. Pástor, R. F. Stambaugh, and L. A. Taylor. Sustainable investing in equilibrium. *Journal of Financial Economics*, 2020.

[22] L. H. Pedersen, S. Fitzgibbons, and L. Pomorski. Responsible investing: The esg-efficient frontier. *Journal of Financial Economics*, 2020.