# Sovereign Bailouts: Are Ex-Ante Conditions Useful?\*

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#### Abstract

Bailout guarantees create moral hazard, even when full repayment can be enforced. In a strategic default model calibrated to the GIIPS countries, I show that, with unconditional bailout guarantees, government deficits are 6 to 15 percentage points higher than they would be in the absence of guarantees, for a given debt level. This results in a high frequency of bailouts, which is inefficient if bailouts are costly. Ex-ante fiscal conditions can be an effective way to make bailouts a time-consistent policy. The model provides a rationale to a recent reform of the European Stability Mechanism (ESM).

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## 1 Introduction

One of the key questions after the sovereign defaults in emerging markets of the 1990's was to what extent the availability of IMF bailouts could result in moral hazard by investors and borrowing countries. Rogoff (2002) casts doubt on the belief that moral hazard should be a big concern to taxpayers who finance IMF bailouts, noting that "IMF loans have had a stubborn habit of being repaid in full". Recently, the European debt crisis has renewed interest in the questions involving the pros and cons of bailouts and the way to design them in order to minimize their unintended consequences.

In January 2021, the Eurogroup members signed an agreement to reform the European Stability Mechanism (ESM). Part of the reform is an amendment of its Precautionary Conditioned Credit Line (PCCL), specifying that access to this credit line be possible only to countries respecting some ex-ante criteria in terms of debt and deficit levels.<sup>1</sup> Countries not fulfilling these criteria would instead have access, in case of a crisis, to the Enhanced Conditions Credit Line (ECCL), with stronger ex-post conditions, possibly including debt restructuring. Notice that even to access the ECCL, the debt of a country needs to be regarded as "sustainable". One might ask why are ex-ante conditions necessary, if the debt is deemed sustainable and is expected to be repaid in full. Indeed some European countries have viewed the ex-ante conditions attached to the PCCL as an unreasonable request for "austerity", and perceive abiding to them as a loss of sovereignty in fiscal matters (see e.g. Galli (2020)).

In this paper I build a dynamic stochastic model of strategic sovereign default that explains why moral hazard is a real concern, even when bailout institutions have the ability to enforce repayment, and provides a rationale for demanding ex-ante condi-

<sup>&</sup>lt;sup>1</sup>These rules, similar to those of the Stability and Growth Pact of 1997, include: fiscal deficit not higher than 3% and debt not higher than 60% (or, if higher than 60%, debt in a declining trajectory, approaching the 60% level at an average rate of 1/20 per year).

tionality in terms of fiscal policy. In a nutshell, the option of the government to default limits its ability to borrow from market investors. An International Financial Institution (IFI) has the ability to enforce repayment, and for this very reason it can bail out the government by lending more than the market is willing to do. However, in anticipation of a bailout, and in the absence of conditionality, the market and the government change their behavior: the market is willing to lend more and charges lower spread, and the government takes advantage of this and relaxes its fiscal policy. The result is a very high frequency of bailouts, despite a lower frequency of outright defaults. Ex-ante fiscal conditionality can mitigate the relaxation in the government's fiscal policy.

In the model, the government's fiscal policy and repayment decision, labor supply and production in the private sector, the market's and the bailout institution's willingness to lend are all endogenous and interacting decisions. The government enters a period with a certain debt level d, and, in the Eaton-Gersovitz (1981) tradition, decides whether or not to repay by comparing the value function in repayment and in default. If it repays, it decides how much to tax and how much to borrow. The only available taxes are distorting taxes on labor, implying that raising taxes has an increasing cost in terms of labor supply and output. Borrowing decisions are crucially constrained by the market's willingness to lend, which is related to the maximum the government is expected to repay next period.

The inability of the government to commit to future repayment implies that the market's willingness to lend is inferior to the present discounted value of the surpluses the government could collect in the future.<sup>2</sup> After a bad (fundamental) shock, the government might decide to default although it would be willing to repay if it was able to borrow more and commit to future repayments.

Although international investors are individually risk-neutral, they suffer an aggregate negative externality in case of default. Curbing the negative externality is the

<sup>&</sup>lt;sup>2</sup>This point is also made by Collard, Habib and Rochet (2015).

rationale for setting up an International Financial Institution (IFI) of the sort of the IMF and the ESM.

The defining characteristic of the IFI, in contrast to the markets, is its ability to enforce repayment.<sup>3</sup> Thanks to this, the IFI is willing to lend up to the maximum sustainable debt level, i.e. the maximum debt level consistent with the full intertemporal budget constraint of the government. A bailout occurs when the market cannot lend enough for the government to repay its past debt, but the IFI can. In this case I assume that the IFI imposes on the government a repayment schedule specified so the government is indifferent between defaulting or accepting the IFI's plan. This way, the government is willing to accept a bailout when (and only when) it would otherwise default on its debt. In the baseline model the IFI does not impose any conditionality on the access to a bailout, other than the sustainability of the debt level. This is meant to provide a benchmark against which the need for conditionality in terms of fiscal policy should be measured.

A crucial part of my analysis consists in analyzing how the presence of the IFI affects the optimal decisions of the government and the lenders. If the maximum investors were willing to lend did not change, the presence of the IFI would reduce the risk of a default to almost zero. But in fact I show that the market's willingness to lend dramatically increases in the presence of the IFI. As a consequence, default risk persists, to the point that, if the government borrows as much as it can from the markets, the probability that it defaults next period (by exceeding even the debt threshold that makes it eligible for a bailout) is unchanged relative to the model without the IFI. In turn, the optimal

<sup>3</sup>Historically, losses incurred by the IMF on its loans have been very small. As discussed for example by Aylward and Thorne (1998), the IMF employs a set of strategies to enforce repayment and to quickly resolve cases of overdue obligations. These strategies include conditionality on the use of resources, technical assistance in the design and implementation of adjustment programs, strong remedial measures in case of protracted arrear problems. response of the government changes: it borrows more, reduces its primary surplus for a given level of debt, and reduces the maximum surplus it is willing to raise before resorting to default (or asking for a bailout).

Importantly, I show that, although at the time of accepting the bailout the government is indifferent between a bailout and a default, ex-ante it is important for the government to contain the risk of a default, but it is not important to avoid a bailout. Indeed, the risk of a default entails paying higher interest in the states of repayment, and incurring a deadweight cost in case of default. If a bailout is available, instead, the government repays either the IFI (in the bailout states) or the investors directly, and no credit spreads are charged in the limit in which the outright default probability is negligible.

In order to quantify the change in behavior of the markets and the government when bailouts are available, I perform a numerical analysis of the model calibrated to the economy of the GIIPS countries. I solve for the optimal fiscal and default policy of the government and find that the primary surplus chosen by the government in the presence of the IFI, for any given level of debt, is 6 to 15 percentage points lower than in the absence of the IFI. The maximum primary surplus (MPS) is 5.8% in the absence of the IFI and 4.1% in the presence of the IFI. On the positive side, the IFI reduces the frequency of defaults: without the IFI, a default occurs with probability 27% in a century; with the IFI, this probability is reduced to around 6%. However, a bailout occurs on average every 20 years.

The policy implications of these numbers depend on the costs associated with bailouts. If bailouts were costless, this state of things would pose no problem, and the IFI would be an ideal solution to the inability of the government to commit to future repayments. If the cost of bailouts is significant, limiting their frequency becomes crucial, hence it becomes crucial to impose ex-ante access conditions to mitigate the change in the government's fiscal choices. Finally, I focus the effect of ex-ante fiscal rules, imposed as a precondition for the bailout guarantee. Any conditionality entails a tradeoff between the welfare of the country, which is maximal when no ex-ante conditions are imposed, and the welfare of the international agents, who bear the cost of the bailout and the spillover costs of a default. While not attempting to estimate these costs, I show that my framework allows for a simple evaluation of ex-ante conditions: a bailout guarantee with ex-ante conditions can be considered Pareto-improving if the value function of the borrowing country is higher than or equal to what it would be without the IFI for any debt level, and, on the side of the international agents, the expected value of the externality caused by possible future defaults plus the expected cost of future bailouts is lower than it would be without any bailout promise.

With ex-ante conditions similar to those contemplated in the recent ESM reform, the probability of default would decrease essentially to zero, and a bailout would occur on average every 115 years. I find that a bailout promise with such ex-ante conditions is Pareto improving if the ratio between the cost of a bailout and the cost of default is below 0.58. In contrast, a bailout promise without any ex-ante condition would be Pareto improving only if this ratio is below 0.1.

In conclusion, the assessment of my model is that, unless the cost of a bailout is smaller than 10% of the spillover costs of a default, a bailout promise without any ex-ante condition is inefficient, as it increases the total expected costs borne by the international agents. On the other hand, whenever the cost of a bailout is smaller than 58% of the spillover costs of a default, a bailout promise with ex-ante conditions similar to those in the ESM reform achieves the objective of reducing these costs.

#### Literature review

This paper builds on the seminal paper on strategic sovereign default by Eaton and Gersovitz (1981), and on the subsequent quantitative contributions by Aguiar and Gopinath (2006), by Arellano (2008), and by Cuadra, Sanchez and Sapriza (2010) (the latter also endogenize the government's fiscal policy). Relative to this literature, one novel aspect of my paper is the central role of the market's willingness to lend, which endogenously interacts with the decisions of the other agents, notably with the government's fiscal decisions.

This paper is also related to the "excusable default" models by Collard, Habib and Rochet (2015) and Ghosh, Kim, Mendoza, Ostry and Qureshi (2013), which provide an alternative to the Eaton-Gersovitz tradition. In these two papers a central role is given to the market's willingness to lend, which depends on the MPS achievable by the government; however, unlike in my paper, the MPS is exogenous.

Models about sovereign bailouts include Zettelmeyer, Ostry and Jeanne (2008), Fink and Scholl (2016), Roch and Uhlig (2018), Corsetti, Guimaraes and Roubini (2006), Corsetti, Erce and Uhlig (2018), Tirole (2015), Gourinchas, Martin and Messer (2020). The bailout agency takes different roles in these papers. In Zettelmeyer, Ostry and Jeanne (2008) the bailout agency can force the government to undertake ex-post fiscal reforms, and can therefore make solvent a previously insolvent government. In Roch and Uhlig (2018) and Corsetti, Erce and Uhlig (2018) the bailout agency can coordinate investors' expectations, and is therefore useful to avert self-fulfilling debt crises. In Corsetti, Guimaraes and Roubini (2006) the agency can provide liquidity support to solvent countries and is therefore useful in case of liquidity runs. In Fink and Scholl (2008) the government, even before a debt crisis, can decide to switch from market investors to official lenders, if it commits to undertake some fiscal reforms. In Pancrazi, Seoane and Vukotić (2020), the agency creates a tradeoff for the government between better borrowing conditions and harsher ex-post conditions. In Tirole (2015), bailouts are transfers that do not need to be repaid, emerging from fear of the spillover costs of a default; the paper investigates the conditions under which joint-liability agreements between countries (similar to Eurobond proposals) are optimal.

The novel role of the bailout agency in my paper lies in its ability to lend more

than the markets, due to the fact that it can enforce repayment.<sup>4</sup>

## 2 Model

### 2.1 Households

A small open economy is populated by a representative household with preferences

$$\sum_{t=0}^{\infty} \beta^t E_0[\delta_t u(c_t, L_t)] \tag{1}$$

where  $c_t$  is consumption,  $L_t$  hours worked,  $\beta$  is the discount factor and  $\delta_t$  is a variable that incorporates households' disutility in case of a government default. More details on the variable  $\delta_t$  will be given in the next subsection. Production is

$$Y_t = A_t L_t \tag{2}$$

 $A_t$  is the stochastic productivity, following a process

$$\frac{A_t}{A_{t-1}} = g_t \tag{3}$$

In this paper I assume that the factors  $g_t$  are i.i.d. lognormally distributed:  $g_t \sim \mathcal{LN}(\mu, \sigma^2)$ .<sup>5</sup>

Having no access to private saving instruments, the household decision is a purely static consumption/leisure choice: the budget constraint is simply

$$c_t = (1 - \tau_t) A_t L_t \tag{4}$$

where  $\tau_t$  is the labor tax rate, imposed by the government and taken as given by the household. I use a Greenwood, Hercovitz and Huffman (1988) specification u(c, L) =

<sup>5</sup>The model could be generalized to the case in which  $g_t$  depends on the history  $g^{t-1} \equiv (g_0, g_1, \dots, g_{t-1})$ .

<sup>&</sup>lt;sup>4</sup>In Boz (2011), the main characteristic of the IFI is also its ability to enforce repayments. However this is treated essentially as a cost for the borrowing government, which, given the choice, prefers to borrow from the market to preserve its option to default. In my model, debt enforceability allows the government to borrow more and commit to repay.

u(c-g(L)), and in particular

$$u(c_t, L_t; A_t) = \frac{\left(c_t - A_t \frac{L_t^{1+\psi}}{1+\psi}\right)^{1-\gamma}}{1-\gamma}$$
(5)

which results in the following policy functions

$$L(\tau) = (1-\tau)^{\frac{1}{\psi}} \tag{6}$$

$$c(\tau) = A_t (1-\tau)^{\frac{1+\psi}{\psi}}$$
(7)

We see that this specification of the utility function, in which labor disutility increases in the technology level  $A_t$ , results in a labor supply that reacts to changes in tax rates but not to changes in technology.<sup>6</sup> Using the policy functions we can express the felicity function u as a function of  $\tau$  only (given the exogenous  $A_t$ ):

$$u(\tau; A_t) = \left(\frac{\psi A_t}{1+\psi}\right)^{1-\gamma} \frac{1}{1-\gamma} (1-\tau)^{\frac{(1+\psi)}{\psi}(1-\gamma)} \equiv \left(\frac{\psi A_t}{1+\psi}\right)^{1-\gamma} \tilde{u}(\tau) \tag{8}$$

where the rescaled felicity function  $\tilde{u}$  is given by  $\tilde{u} \equiv \frac{1}{1-\gamma}(1-\tau)^{\frac{(1+\psi)}{\psi}(1-\gamma)}$ . Finally, using (3) and (8) the utility function (1) can be written as

$$\left(\frac{\psi A_0}{1+\psi}\right)^{1-\gamma} \Sigma_{t=0}^{\infty} \beta^t E_0 \left[ \left( \Pi_{t'=1}^t g_{t'} \right)^{1-\gamma} \delta_t \tilde{u}(\tau_t) \right]$$
(9)

With no loss of generality I normalize  $A_0$  so that  $\left(\frac{\psi A_0}{1+\psi}\right)^{1-\gamma} = 1$  and neglect this factor in the following.

### 2.2 Government

The government sets the tax rate  $\tau_t$  and consumes  $G_t = \tau_g Y_t$ . I assume for simplicity that  $\tau_g$  is constant, i.e. that government spending is a constant fraction of output. The

<sup>&</sup>lt;sup>6</sup>Models with the technological factor in the utility function include for example Rudebusch and Swanson (2012). A microfoundation is provided by Campbell and Ludvigson (2001): leisure is not valuable per se, but to the extent that it allows time for the production of non-market goods (home production); such production is also proportional to the technological factor.

government is benevolent and shares the same utility function (1) as the household. While the household makes the intratemporal decision between leisure and consumption, the government, which has access to international debt markets at the constant international rate r, is responsible for the intertemporal decision.

More precisely the government starts period t with outstanding debt  $D_t$ . The first decision is whether or not to honor this debt. If it defaults, it exits the debt market and from then on the tax rate will be a constant  $\tau = \tau_g$  (I assume the recovery rate to be zero for simplicity). In addition to exclusion from the international markets, defaulting also entails an extra loss of utility: the variable  $\delta_t$  in (1) is equal to 1 as long as the government has never defaulted, but if the government defaults at time  $t^{def}$ ,  $\delta_t$ is equal to a constant  $\xi$  for every period  $t \geq t^{def}$ .<sup>7</sup> The value function in default is thus

$$V^{def} = \xi \Sigma_{t=0}^{\infty} \beta^{t} E_{t} \left[ \left( \Pi_{s=0}^{t} g_{s} \right)^{1-\gamma} \right] \tilde{u}(\tau_{g}) \\ = \frac{\xi (1-\tau_{g})^{(1+\psi)(1-\gamma)}}{(1-\gamma) \left( 1-\beta exp \left( \mu (1-\gamma) + \frac{\sigma^{2}}{2} (1-\gamma)^{2} \right) \right)}$$
(10)

If it does not default, the government needs to decide how much to borrow and how much to tax, in order to finance its own expenditure and repay the outstanding debt, i.e. to satisfy the budget constraint

$$D_t = (\tau_t - \tau_g) A_t L_t + B_{t+1}$$
(11)

 $B_{t+1}$  is the amount borrowed at  $t,\,{\rm that}$  results in debt to be repaid at t+1

$$D_{t+1} = (1 + r + x_{t+1})B_{t+1} \tag{12}$$

where  $x_{t+1}$  is the credit spread demanded by investors.

Defining

$$d_t \equiv \frac{D_t}{A_t} \tag{13}$$

$$b_{t+1} \equiv \frac{B_{t+1}}{A_t} \tag{14}$$

<sup>&</sup>lt;sup>7</sup>Assuming  $\gamma > 1$ , the value function is negative, and loss of utility occurs for  $\xi > 1$ .

and using the policy function (6), (11) and (12) can be rewritten as

$$d_t = (\tau - \tau_g)(1 - \tau)^{\frac{1}{\psi}} + b_{t+1}$$
(15)

$$d_{t+1} = (1+r+x_{t+1}) \frac{b_{t+1}}{g_{t+1}}$$
(16)

 $d_t$  and  $b_{t+1}$  can be interpreted as outstanding debt and borrowing as a fraction of potential output (the maximum output level for a given technology level, which occurs when  $\tau = 0$  and L = 1).

The choice of how much to tax vs how much to borrow is constrained by the fact that the government has limited borrowing capacity:  $b_{t+1} \leq \hat{b}$ . With i.i.d. growth factors the borrowing capacity  $\hat{b}$  is a constant, whose determination is addressed in Section 2.3.

The amount that can be raised through taxes is also limited: given the household policy function for labor supply (6), the primary surplus  $S_t$ , i.e. the difference between tax revenues and government expenditure (all scaled by  $A_t$ ) is

$$S_t = (\tau_t - \tau_g) L_t = (\tau - \tau_g) (1 - \tau)^{\frac{1}{\psi}}$$
(17)

Thus the primary surplus follows a Laffer curve whose peak is reached at  $\tau^{peak} = (\psi + \tau_g)/(1 + \psi)$ .<sup>8</sup> The MPS is the value of the surplus at the peak of the Laffer curve:

$$\mathcal{S}^{peak} = \psi \left(\frac{1 - \tau_g}{1 + \psi}\right)^{\frac{1 + \psi}{\psi}} \tag{18}$$

Maximal labor supply,  $L_{max}$ , occurs for zero taxes, and minimal labor supply,  $L_{min}$  occurs for  $\tau = \tau^{peak}$  (higher tax rates will never be chosen by the government):

$$L_{max} = 1 \tag{19}$$

$$L_{min} = \left(\frac{1-\tau_g}{1+\psi}\right)^{\frac{1}{\psi}} \tag{20}$$

<sup>&</sup>lt;sup>8</sup>Total fiscal revenues  $\tau A_t L_t = A_t \tau (1 - \tau)^{\frac{1}{\psi}}$  also follow a Laffer curve, but with peak at a different tax rate  $\tau = \frac{\psi}{1+\psi}$ .

#### 2.2.1 The government's problem

When collecting the MPS and borrowing the maximum from the market is insufficient to repay, the government has no choice but to default. Otherwise, the government chooses to repay when its value function in repayment is higher than the value function in default. In summary the government's problem in recursive form is

For  $d_t > \hat{b} + S^{peak}$ 

$$V(d_t) = V^{def} \tag{21}$$

otherwise:

$$V(d_t) = max(V^{def}, V^{no \ def}(d_t))$$
(22)

$$V^{no\ def}(d_t) = max_{\{b_{t+1},\tau_t\}} \left( \tilde{u}(\tau_t) + \beta E_t[g_{t+1}^{1-\gamma} V(d_{t+1})] \right) (23)$$

s.t.

$$d_t - b_{t+1} = (\tau_t - \tau_g)(1 - \tau_t)^{\frac{1}{\psi}}$$
(24)

$$d_{t+1}g_{t+1} = (1+r+x_{t+1})b_{t+1}$$
(25)

$$b_{t+1} \leq \tilde{b} \tag{26}$$

If imposing the maximum tax rate  $\tau^{peak}$  and exhausting the borrowing capacity is preferable to default, i.e.

$$\tilde{u}(\tau^{peak}) + \beta E_t[g_{t+1}^{1-\gamma} V(d_{t+1})|b_{t+1} = \hat{b}] > V^{def}$$
(27)

then there is never a strategic default: the government only defaults when it is impossible to repay. In this limit model is effectively equivalent to a model of "excusable default", similar to the model of Collard, Habib and Rochet (2015).

Since the value function is a decreasing function of d – the only state variable –

default occurs for  $d_t$  bigger than a fixed threshold  $d^*$ :<sup>9</sup>

$$d^* = \min\{d \ s.t. \ V(d) = V^{def}, \mathcal{S}^{peak} + \hat{b}\}$$

$$(28)$$

Given the relationship (16) between borrowing and next period's debt, it follows that, for a given  $b_{t+1}$ , default at t + 1 occurs if the growth factor  $g_{t+1}$  is below a threshold value  $g^{th}(b_{t+1})$ , where

$$g^{th}(b_{t+1}) = \frac{b_{t+1}(1+r+x(b_{t+1}))}{d^*}$$
(29)

Given the default threshold  $d^*$  and thus the function  $g^{th}(b_{t+1})$ , equations (21)-(23) can be collapsed into

$$V(d_{t}) = max_{\{b_{t+1}\},\{\tau_{t}\}}\tilde{u}(\tau_{t}) + \beta \int_{g^{th}(b_{t+1})}^{\infty} g^{1-\gamma}V\left(\frac{b_{t+1}(1+r+x(b_{t+1}))}{g}\right) f(g)dg + \beta V^{def} \int_{-\infty}^{g^{th}(b_{t+1})} g^{1-\gamma}f(g)dg$$
(30)

where f(g) is the probability density of the growth factor.

### 2.3 Investors

International investors are risk neutral and perfectly competitive. Knowing that the government can default, and given the borrowing level b, they set the credit spread x(b) so that

$$(1 - P^{def})(1 + r + x(b)) = (1 + r)$$
(31)

As discussed in Section 2.2, default occurs if the growth factor is smaller than a threshold value  $g^{th}(b) = \frac{b(1+r+x(b))}{d^*}$ . Hence

$$P^{def}(d^e) = F\left(\frac{d^e}{d^*}\right) \tag{32}$$

where  $d^e \equiv b(1+r+x(b))$  and F(g) is the probability distribution of the growth factor.

 $<sup>^{9}</sup>$ see also Arellano (2008)

The borrowing capacity  $\hat{b}$  is the highest value of borrowing b for which (31) has a solution. It is then

$$\hat{b} = \frac{1}{1+r} max_{d^e} d^e (1 - P^{def})$$
(33)

which, using (32), can be rewritten as

$$\hat{b} = \frac{d^*}{1+r} max_g \ g(1-F(g)) \equiv \theta \frac{d^*}{1+r}$$
 (34)

The borrowing capacity is therefore proportional to the default threshold debt level  $d^*$ , and the proportionality factor contains the constant  $\theta \equiv max_g(g(1 - F(g)))$ , which is related to the distribution of the growth factor.

Two cases can be distinguished:

#### • Case 1 – "Strategic default"

In this case the default threshold  $d^*$  is determined by the condition  $V(d^*) = V^{def}$ . The borrowing capacity  $\hat{b}$  and default threshold  $d^*$  need to be jointly solved for: given  $\hat{b}$ , we can solve for the government's value function V(d), and find  $d^*$  as the solution to  $V(d^*) = V^{def}$ . Given  $d^*$ ,  $\hat{b}$  solves (34).

#### • Case 2 – "Excusable default"

In this case we know the default threshold to be  $d^* = S^{peak} + \hat{b}$ . Using (34) we obtain the borrowing capacity

$$\hat{b} = \frac{\theta}{1+r-\theta} \mathcal{S}^{peak} \tag{35}$$

Given (18) we can fully express the borrowing capacity in terms of the model's exogenous parameters:

$$\hat{b} = \frac{\theta}{1+r-\theta}\psi\left(\frac{1-\tau_g}{1+\psi}\right)^{\frac{1+\psi}{\psi}}$$
(36)

### 2.4 Equilibrium

An equilibrium is given by:

- household policy functions  $c(\tau)$ ,  $L(\tau)$
- government policy functions: default decision  $\mathcal{D}(d_t)$  ( $\mathcal{D} = 1$  is default,  $\mathcal{D} = 0$  is no default), tax and borrowing decisions in case of no default  $\tau(d_t)$ ,  $b_t(d_t)$
- borrowing capacity  $\hat{b}$ , spread  $x(d_t)$

such that: given  $\tau$ , the labor/leisure policy functions  $c(\tau)$  and  $L(\tau)$  maximize intratemporal households' utility; given the exogenous shocks, given  $\hat{b}$  and x chosen by investors, and given the household's choices,  $\mathcal{D}$  and, in case of repayment,  $\tau$  and b, maximize the government's value function; given the government's policy functions the credit spread x is such that that the investors' participation constraint is satisfied and  $\hat{b}$  is the highest borrowing level for which the participation constraint can be satisfied.

### 2.5 Euler Equation and Policy Functions

The Euler equation resulting from the government problem (22) is

$$\frac{\tilde{u}'(\tau_t)}{\chi(\tau_t)} + \lambda_t = \beta (1 + r + P^{surv}(b_{t+1})b_{t+1}x'(b_{t+1}))E_t \left[ g_{t+1}^{-\gamma} \frac{\tilde{u}'(\tau_{t+1})}{\chi(\tau_{t+1})} \right| g_{t+1} \ge g^{th}(b_{t+1}) \\ + \beta \left( g^{th}(b_{t+1}) \right)^{1-\gamma} (V^{def} - V(d^*))f(g^{th}) g^{th'}(b_{t+1})$$
(37)

with 
$$\chi(\tau) = \frac{dS}{d\tau} = (1-\tau)^{\frac{1}{\psi}} - \frac{1}{\psi}(\tau-\tau_g)(1-\tau)^{\frac{1}{\psi}-1}$$
 (38)

$$P^{surv}(b_{t+1}) = 1 - P^{def}(b_{t+1}) = 1 - F(g^{th}(b_{t+1}))$$
(39)

Here  $\lambda_t$  is the Lagrange multiplier associated to the condition  $b_t \leq \hat{b}$ ;  $x'(b_{t+1})$  is the derivative of the credit spread;  $g^{th'}$  is the derivative of the function (29); f(g) is the probability density and F(g) is the cumulative probability distribution of the growth factor.

The intuition conveyed by the Euler equation is the following: suppose the government at time t borrows  $b_t$  and imposes the tax rate  $\tau_t$ , satisfying the budget constraint (24). What would be the marginal effect of increasing borrowing by an amount  $\Delta$ ? At time t, the tax rate could be decreased by an amount  $\frac{\Delta}{\chi(\tau_t)}$ , which would increase today's utility by an amount  $\frac{u'(\tau_t)}{\chi(\tau_t)}\Delta$ . Next period, two things would occur: first, default would occur in more states, as the probability of default would increase by  $f(g^{th})g^{th'}\Delta$ . In the new default states (those just below the default threshold  $d^*$ , where default would not have occurred without extra borrowing  $\Delta$ ) utility changes by an amount  $(g^{th})^{1-\gamma}(V^{def} - V(d^*))$ , which is zero in the "strategic default" case and negative in the "excusable default" case. Second, in the survival states, the amount the government needs to repay would increase from  $(1 + r + x(b_{t+1}))b_{t+1}$  to  $(1 + r + x(b_{t+1} + \Delta))(b_{t+1} + \Delta)$ . To first order in  $\Delta$ , and using (31), the increase can be written as  $\frac{1+r+P_{surv}(b_{t+1})x'(b_{t+1})}{P_{surv}(b_{t+1})}\Delta$ .

The increase in default risk, reflected by a positive x'(b), clearly makes the extra  $\Delta$  amount borrowing less attractive: repayment in the (fewer) survival states increases by more than  $\frac{(1+r)\Delta}{P_{surv}}$ , while utility is at best unchanged in the (more numerous) default states.

A solution of the government's problem consists in finding the default threshold  $d^*$ and the policy functions  $\tau(d)$  and b(d) – tax rate and borrowing level for  $d \leq d^*$ , when debt is repaid – satisfying the Euler equation (37). While a full solution needs to be computed numerically, the following properties of the policy functions and the default threshold can be proved analytically (all detailed proofs are in Appendix):

# **Proposition 1** If, for a debt level d, $b(d) < \hat{b}$ , then $\tau(d) < \tau^{peak}$ .

This proposition tells us that the government would never want to raise a surplus corresponding to the peak of the Laffer curve if it has not already exhausted its borrowing capacity. As the tax rate approaches the peak of the Laffer curve, raising taxes is more and more costly in terms of consumption, but the increase in the surplus gradually approaches zero. In fact  $f(\tau)|_{\tau=\tau^{peak}} = 0$ , making the LHS of the Euler equation infinitely negative at this point. The only case in which the government would want to reach this tax rate is in the limit case in which outstanding debt is exactly equal to the maximum repayable amount,  $d = S^{peak} + \hat{b}$ , and defaulting is still more expensive than repaying. In this case the Lagrange multiplier is infinite.

**Proposition 2** As long as  $b(d) < \hat{b}$ , both  $\tau(d)$  and b(d) are strictly increasing in d. This proposition implies that the surplus collected by the government at debt level equal to the default threshold is the MPS:  $MPS = \tau(d^*) - \tau_g$ . The intuition is that, if the government needs to repay a higher debt, it has to either raise taxes, or raise borrowing, or both. The first option involves more pain today, the second involves more pain next period. When constraints are not binding, the optimal choice is the third, which consists in spreading the extra pain between today and next period.

Propositions 1-2 imply that the solution of the government problem falls in one of three cases, depending on model parameters. In a first case,  $\tau(d)$  and b(d) both increase for every debt level  $d \leq d^*$ , and we have  $b(d^*) < \hat{b}$ ,  $\tau(d^*) < \tau^{peak}$ . In this case the government defaults before exhausting its fiscal and borrowing capacity. In a second case  $\tau(d)$  and b(d) both increase until b(d) reaches the borrowing capacity  $\hat{b}$  for a debt level  $\tilde{d} < d^*$ ; for  $\tilde{d} < d \leq d^* \tau(d)$  continues to increase while b(d) is constant at  $\hat{b}$ . At the default threshold we have  $b(d^*) = \hat{b}$  and  $\tau(d^*) < \tau^{peak}$ . In both these first two cases default is strategic. In a third case (the "excusable default" case) default occurs above the debt level  $\hat{b} + S^{peak}$ , with  $b(d^*) = \hat{b}$  and  $\tau(d^*) = \tau^{peak}$ , so that all the government's resources are exhausted.

**Proposition 3**  $d^* \ge \hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}$  where  $\tau^o$  is such that  $\tilde{u}(\tau^o) = \xi \tilde{u}(\tau_g)$ This proposition gives us a lower bound for the default threshold  $d^*$ . The lower bound comes as a consequence of the fact that the government does not default if it can repay by collecting a surplus such that its current-period utility is equal to utility in default, and borrowing the rest. By defaulting it would lose optionality without any utility gain.

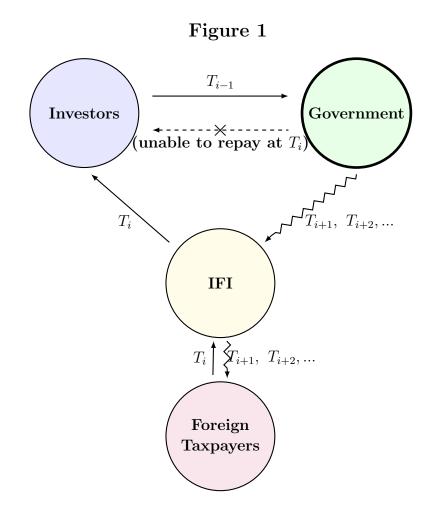
## 3 Bailouts

Why bailouts? I posit that investors, although risk-neutral, suffer a utility loss  $\kappa^{def}$  in case of default. This does not affect their willingness to lend, since default is not something they individually can affect. However, the disutility they suffer in case of default induces them to establish an International Financial Institution (IFI).

The IFI purports to avoid the deadweight cost associated with default by acting as an intermediary between a pool of (risk-neutral) foreign taxpayers and the government, as graphically represented in Figure 1.

Differently from uncoordinated investors, the IFI can enforce government repayment and can propose an equity-like contract to the government: the latter will repay a fraction of its output in each period after the bailout, so that the expected value of all future repayments equals the bailout amount.<sup>10</sup> I assume that the IFI incurs a cost  $\kappa^{bail}$ , such that  $0 < \kappa^{bail} < \kappa^{def}$ , for each bailout, and that the government is free to accept or reject the bailout contract. The equilibrium between these two forces implies that the IFI offers the government a contract such that the government is indifferent between accepting it and rejecting it, when rejecting it would mean defaulting on the debt. Indeed, if the utility of the government after accepting a bailout,  $V^{bail}$ , was lower than  $V^{def}$ , the government would prefer default, and if  $V^{bail} > V^{def}$  the government

<sup>10</sup>Although repayment schedules to bailout agencies such as the IMF are usually defined in dollar terms, rather than in percentages of the debtor country's output, many countries experiencing difficulties, often political problems or civil unrests, have been able to run even protracted arrears without losing the IMF financial support. IMF loans thus contain in practice an element of state-contingency that is absent from ordinary market debt.



would accept a bailout also in situations where it would otherwise have repaid the debt, which would impose an unnecessary cost on the IFI.

**Proposition 4** Assuming  $\xi > 1$  (i.e. assuming that default entails a loss of utility, on top of the exclusion from international markets), the maximum the IFI is willing to lend to the government is bigger than  $d^*$ 

(Detailed proof is in Appendix). This is the crucial proposition which tells us that, while both individual investors and investors intermediated by the IFI are risk-neutral and want a fair return for their investment, "IFI investors" can lend more than individual ones (by Proposition 3,  $d^* > \hat{b}$  if  $\xi > 1$ ).

One way to see this is the following. Consider the following "base scenario": we are in the absence of the IFI and debt at the beginning of a period  $t_0$  is equal to  $d^*$ .

The government is willing to repay by imposing a tax rate  $\tau(d^*)$  (and borrowing  $b(d^*)$ ) in the same period, and a sequence of tax rates in the following periods, contingent on the realization of the productivity shock, until default. The investor participation constraint implies that the present discounted value of the sequence of surpluses (including the one at  $t_0$ ) is equal to  $d^*$  After default, the tax rate would be  $\tau_g$  but the felicity function would only be  $\xi \tilde{u}(\tau_g)$ .

The IFI could propose a contract to the government, so that, contingent on the realization of the shock, the latter would collect the same surpluses to repay the IFI as in "base scenario" (including the one at the bailout time  $t_0$ ). When the sequence of shocks is such that the government would be in default in "base scenario", the IFI would impose a payment equal to a fraction  $\tau_o$  of income, with  $\tau_o$  such that  $\tilde{u}(\tau_o) = \xi \tilde{u}(\tau_g)$ . If  $\xi > 1$ , the present discounted value of this sequence of surpluses is higher than the one in "base scenario", hence must be higher that  $d^*$ . I will call  $b^{bail}$  the maximum that the IFI can lend.

The above discussion shows that the IFI is able to lend the government more than the markets would, under a contract that respects the participation constraint of "foreign taxpayers" and that the government would take only if it is unable to repay by borrowing from the markets. The higher willingness to lend of the IFI is due to its enforcement ability, and to the fact that it can propose equity-like contracts to the government, whereas individual investors are restricted to 1-period, non-contingent bonds.

The core question I want to address is how agents change their behavior after the IFI is established. In particular:

- 1. Would investors be willing to lend more to the government and/or at lower spreads?
- 2. Would the government's fiscal decisions, in particular the MPS, change?

- 3. How often would the IFI need to intervene by bailing out the government?
- 4. Finally, how would the frequency of defaults change?

#### 3.1 The market's willingness to lend

This subsection addresses the first of the above-listed questions: how the market's willingness to lend is affected by IFI's bailout guarantee.

The first observation is that market investors want to be repaid, regardless whether they are repaid by the government directly or through a bailout. The only case in which they are not repaid is if the country's debt at the beginning of next period exceeds  $b^{bail}$ . By the same reasoning used in Section 2.3, the credit spread demanded by investors for a borrowing level b solves

$$(1+r+x(b))\left(1-F\left(\frac{(1+r+x(b))b}{b^{bail}}\right)\right) = 1+r$$
(40)

and the market's willingness to lend  $b^{mkt}$  is

$$b^{mkt} = \frac{b^{bail}}{1+r} max \ g(1-F(g)) \equiv \theta \frac{b^{bail}}{1+r}$$
(41)

**Proposition 5**  $\hat{b} < b^{mkt} < b^{bail}$ . For a given borrowing level  $b \leq \hat{b}$ , the spread x(b) charged by the market is lower in the presence if the IFI.

The first inequality,  $\hat{b} < b^{mkt}$ , telling us that the market is willing to lend more in the presence of the IFI, is obvious if we compare (34) and (41), given that  $d^* < b^{bail}$ . The second inequality,  $b^{mkt} < b^{bail}$ , telling us that the increased willingness to lend of the market is still lower than the willingness to lend of the IFI, is proved in Appendix. Finally, for a given borrowing level, default next period occurs for fewer realizations of g in the presence of the IFI, therefore the spread charged by investors is lower. The latter is also evident when comparing (31) with (40), given  $b^{bail} > d^*$ .

In the next subsection, and especially in the numerical analysis in Section 4, I will explore how the government changes its fiscal behavior in the presence of the IFI. However, the increase in the market's willingness to lend already allows us to draw one conclusion:

**Proposition 6**: Suppose that an impatient (or constrained) government borrows the maximum it can from the market. Whether we are in the absence of the IFI (so that the impatient government borrows  $\hat{b}$ ), or in the presence of the IFI (so that it borrows  $b^{mkt}$ ), the probability of default next period does not change.

While we still don't know how the government changes its fiscal behavior in the presence of the IFI, this proposition tells us that, given the increased willingness to lend of the market, the promise of an IFI intervention does not guarantee a decrease in the probability of default.

### 3.2 The value function and the Euler equation

In the presence of the IFI, in any period the government repays its outstanding debt dwhen d is lower than a threshold value  $d^*$ ; asks for a bailout when d is higher than  $d^*$ but lower than  $b^{bail}$  – the maximum the IFI can lend – and defaults when debt is higher than  $b^{bail}$ . As discussed at the beginning of this section, I assume that the repayment schedule imposed by the IFI is such that the government's value function is the same in bailout and in default. With these assumptions, the government's problem can be written as

$$V(d_{t}) = max_{\{b_{t+1}\},\{\tau_{t}\}}\tilde{u}(\tau_{t}) + \beta \int_{g^{th}(b_{t+1})}^{\infty} g^{1-\gamma}V\left(\frac{b_{t+1}(1+r+x(b_{t+1}))}{g}\right) f(g)dg$$
  
+  $\beta V^{def} \int_{g^{th}(b_{t+1})}^{g^{th}(b_{t+1})} a^{1-\gamma}f(g)dg$  (42)

+ 
$$\beta V^{def} \int_{-\infty} g^{1-\gamma} f(g) dg$$
 (42)  
s.t.  $d_t = (\tau - \tau_a)(1 - \tau_t)^{\frac{1}{\psi}} + b_{t+1}$  (43)

$$d_{t+1}g_{t+1} = (1+r+x_{t+1})b_{t+1}$$
(44)

$$g^{th}(b_{t+1}) = \frac{(1+r+x_{t+1})b_{t+1}}{d^*}$$
(45)

$$d^* = \min(d \ s.t. \ V(d) = V^{def}, \ \mathcal{S}^{peak} + b^{mkt})$$

$$(46)$$

$$b_{t+1} \le b^{mkt} \tag{47}$$

The differences between the government's problem without the IFI, summarized by (21)-(26) and (30), and the one above are the following: the borrowing constraint (47) is looser than (26) thanks to the higher willingness to lend of the IFI (Proposition 5); the credit spread appearing in (42) is lower, for the same level of borrowing, than the one in (30) (Proposition 6). These two facts imply that, for a given level of debt, the value function with the IFI is higher than the one without the IFI. Since the value function in default (or bailout)  $V^{def}$  is unchanged – it is the same as in (10) – it follows that the default threshold debt level  $d^*$  is higher in the presence of the IFI.

The Euler equation in the presence of the IFI is

$$\frac{u'(\tau_t)}{\chi(\tau_t)} + \lambda_t = \beta (1 - F(g^{th}(b_{t+1})))(1 + r + x(b_{t+1}) + bx'(b_{t+1}))E_t \left[ g_{t+1}^{-\gamma} \frac{u'(\tau_{t+1})}{\chi(\tau_{t+1})} \mid g_{t+1} \ge g^{th}(b_{t+1}) \right] + (V^{def} - V(d^*))f(g^{th}(b_{t+1}))g^{th}((b_{t+1}))^{1-\gamma} g^{th'}(b_{t+1})$$

$$(48)$$

Again,  $\chi(\tau)$  is defined as in (38) and  $\lambda_t$  is the Lagrange multiplier associated with the borrowing constraint.

The factor  $(1 - F(g^{th}(b_{t+1})))$  is the probability that the government repays its debt next period (with no need for a bailout), whereas the credit spread  $x(b_t)$  reflects the probability of default, i.e. the probability of debt exceeding the maximum lending capacity of the IFI  $b^{bail}$ . This implies that the product  $(1 - F(g^{th}(b_{t+1}))(1 + r + x(b_{t+1})))$ is smaller than (1+r). Without IFI, the Euler equation is very similar, but the factor  $(1 - F(g^{th}(b_{t+1}))(1 + r + x(b)))$  could be simplified to (1 + r) by the investor participation constraint.

With the IFI, in the limit in which the probability of a default can be neglected (hence x(b) = x'(b) = 0) but the probability of a bailout cannot be neglected (hence  $F(g^{th}(b_{t+1})) > 0$ ), the government has an incentive to increase its borrowing as much as it can, since by borrowing one extra unit it will have to repay less than 1 + runits next period directly to the investors. The investor participation constraint is satisfied because in the bailout states investors will be repaid by the IFI. Of course the government will then have to repay the IFI, but this payment does not affect utility in this states, which is always  $V^{def}$ . In sum, what may prevent the government from borrowing one extra unit is only the possibility of default, in particular the term proportional to  $x'(b_{t+1})$ , which may become very high close to  $b^{mkt}$ , rather than the possibility of a bailout.

A solution of the government's problem, given  $b^{bail}$  and  $b^{mkt}$  related by (41), consists in finding the default threshold  $d^*$  and the policy functions  $\tau(d)$  and b(d) satisfying the Euler equation (48). Propositions 1-3 still hold for the solution with the IFI (provided that we reinterpret  $\hat{b}$  as  $b^{mkt}$ ). A full solution, and a comparison between the solution with and without the IFI, needs to be obtained numerically.

## 4 Quantitative Analysis

To fully address how the government chooses its fiscal behavior – in particular the MPS and the default decision – with and without the IFI; how effective is the IFI to reduce the frequency of defaults, and how often the IFI would have to actually intervene with a bailout (Questions 2-4 listed in Section 3), I will now turn to a numerical analysis.

## 4.1 Calibration

I choose parameters consistent with the economy of the GIIPS countries. For these countries, the mean and standard deviation of the growth factor, defined as in the model as the ratio of GDP in two subsequent years, are shown in Table 1.<sup>11</sup> The baseline parameters that I adopt are shown in Table 2.

Table 1: Moments of the economies of the GIIPS countries						
	Greece	Italy	Ireland	Portugal	Spain	
mean growth rate 1950-2019	3.1%	3.0%	4.4%	3.5%	3.9%	
st. dev. growth rate 1950-2019	4.6%	3.0%	4.3%	3.3%	3.8%	
mean growth rate 1999-2019	0.9%	0.7%	5.5%	1.5%	1.0%	
st. dev. growth rate 1999-2019	4.3%	1.9%	6.2%	2.1%	2.5%	

Table 2: Baseline model parameters				
Parameter	Description			
$\mu = 0.015$	average growth rate			
$\sigma = 0.04$	volatility of growth rate			
$\psi = 2.5$	inverse labor elasticity			
$\gamma = 2$	relative risk aversion			
r = 0.02	risk-free rate			
$\xi = 1.015$	default cost			
$\tau_g = 0.4$	government spending (fraction of GDP)			

<sup>11</sup>Real GDP data for the GIIPS countries is from FRED database.

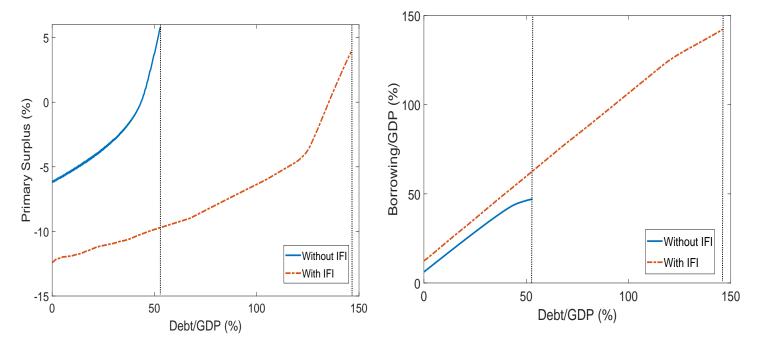
An average growth rate of 1.5% and a standard deviation of 4% are consistent with the range of values in Table 1, especially with the values corresponding to the last 20 years. Notice that the average growth rate needs to be lower than the risk-free rate (here set at 2%), otherwise any debt value would be sustainable. The inverse elasticity of labor supply is 2.5, consistent with the micro estimates (see e.g. Chetty, Guren, Manoli and Weber (2011)), and also with several macro model, such as, for example, Rotemberg and Woodford (1999) and Martinez-Garcia, Vilan and Wynne (2012). In the context of this model, this inverse Frisch elasticity implies that the peak of the Laffer curve occurs at a tax rate of 71%, consistent with the estimates of the Laffer curve in the EU-14 countries by Trabandt and Uhlig (2011). A relative risk aversion parameter equal to 2 is standard in the literature. The default cost  $\xi$  is the most difficult to estimate. A higher value of this cost implies a lower default rate and a higher borrowing capacity. The value I choose, given the other parameters, implies a default probability in the absence of the IFI around 27% in a century, and a realistic borrowing capacity, at least in the presence of the IFI, as we see in the next section.<sup>12</sup> This value of  $\xi$  represents a permanent utility loss of 63 bps in consumption terms after a default.

The government's problem without the IFI, defined by (21)-(26), and with the IFI, defined by (42)-(47), is solved by value-function iteration. More details about the solution method are given in Appendix. I now turn to discussing the results.

<sup>12</sup>As documented by Reinhart and Rogoff(2008), in the  $20^{th}$  century there was only one default by the GIIPS countries, by Greece in 1932. On the other hand, in the  $19^{th}$  century, Spain, Portugal and Greece defaulted 8, 5 and 4 times, respectively, while there were no defaults by Italy and Ireland. A default probability close to 30% in a century seems reasonable, giving more weight to the experience of the  $20^{th}$  century but also considering the possibility that the incidence of defaults in the future could be somewhat higher.

### 4.2 Borrowing capacity and default decisions

The government's fiscal decisions with and without the IFI, for debt levels below the respective thresholds, are shown in Figure 2.



**Figure 2: Fiscal Policy Functions** 

Figure 2: The optimal the primary surplus (left panel) and borrowing level (right panel) chosen by the government as a function of debt, without and with the IFI. the dashed vertical lines correspond to the default thresholds in the two cases.

Without the IFI, default occurs for debt higher than 53% of GDP. When debt is below 43.4% the government chooses a tax rate below the government expenditure rate  $\tau_g$ , i.e. the primary surplus is negative. When debt is between 43.4% and 53% the primary surplus is positive, reaching a MPS of 5.8% for debt equal to 53% of GDP.

As for the borrowing capacity, remember that the quantity  $\hat{b}$  is defined as the maximum borrowing amount as a fraction of potential output  $A_t$ , i.e. of GDP at 0 tax

rate (see (14)). Actual GDP depends on the tax rate: from (2) and (6),  $Y_t = A_t(1-\tau)^{\frac{1}{\psi}}$ , hence the minimum value of GDP given a technology level  $A_t$  is reached when the government collects the MPS. Defining

$$Y_t^{min} \equiv A_t (1 - \tau_{MPS})^{\frac{1}{\psi}} \tag{49}$$

the borrowing capacity as a fraction of  $Y^{min}$  can therefore be found as  $\hat{b}/(1 - \tau_{MPS})^{\frac{1}{\psi}}$ . The latter quantity is more relevant than simply  $\hat{b}$ , as the government gets close to the borrowing constraint when outstanding debt is high, which is also when tax rates are high, i.e. when the surplus is close to the MPS and output is close to  $Y^{min}$ .

I find that the government's borrowing capacity is 37.3% of potential GDP, or 47.7% of  $Y^{min}$ . The government however never competely exhausts the borrowing capacity: it borrows a maximum of 45.3% of GDP just below the default theshold.

What happens in the presence of the IFI? For simplicity, for this analysis I assume that the IFI can only offer a simple contract to the government, in which the latter is required to repay the IFI a constant fraction  $\tau$  of its income for N periods, starting in the period when the bailout occurs. Under these condition, the present value at bailout time of all the future government repayments (considering that the government still needs to finance its expenditure  $\tau_g$ ) is

$$\tau (1 - (\tau + \tau_g))^{\frac{1}{\psi}} \left( 1 + \sum_{i=1}^{N} \frac{(E[g_{t+1}g_{t+2}...g_{t+i}])}{(1+r)^i} \right)$$
(50)

The maximum value of (50) that is consistent with the government's participation constraint  $V^{def} = V^{bail}$  is obtained when  $N = \infty$  and the total tax rate the government is required to collect,  $\tau + \tau_g$ , equals  $\tau_o$ , the tax rate that makes the felicity function equal to the one in default (i.e.  $\tau_o$  is such that  $u(\tau_o) = \xi u(\tau_g)$ ). The maximum the IFI can lend is therefore

$$b^{bail} = (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}} \left(1 + \sum_{i=1}^{\infty} \frac{(E[g])^i}{(1+r)^i}\right) = \frac{(\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}}{1 - \frac{exp\left(\mu + \frac{\sigma^2}{2}\right)}{1+r}}$$
(51)

With the baseline parameters, I find  $\tau_o = 0.4063$  and  $b^{bail} = 129\%$  of potential GDP, or 163% of  $Y^{min}$ . In the presence of the IFI willing to lend so much, the borrowing capacity from the market increases to 116% of potential GDP, or 146% of  $Y^{min}$ .

In this environment, the primary surplus is negative for debt below 140% of GDP, and positive only in the narrow region of debt between 140% and 148% of GDP, above which the government asks for a bailout. The government, taking advantage of the higher borrowing capacity and less willing to raise taxes, changes its fiscal behavior, and the MPS is 4.1%, 1.7 percentage points lower than in the absence of the IFI.

For a given level of debt, below the default threshold without the IFI, the primary surplus with the IFI is 6 to 15 percentage points lower than without the IFI.

The right panel of Figure 2 shows the borrowing level chosen by the government as a function of debt. For a given debt level, borrowing and tax rates are linked by (11). So it is not surprising that, as we see a steepening of the surplus for debt levels close to the default threshold, we see a flattening of the borrowing level. As we see in Table 3, even at the default threshold the borrowing level  $(b(d^*))$  is slightly below capacity, both without and with the IFI.

Finally, Figure 3 shows that the MPS is decreasing in the bailout capacity of the IFI,  $b^{bail}$ , which again shows how a higher ability to borrow (from the market or the IFI) makes the government more reluctant to increase taxes.

### 4.3 Frequency of default and bailout

I first simulate the model without the IFI. I draw 10,000 paths for the productivity shock of 100 periods each, and use the previously obtained policy functions to observe the dynamic behavior of debt, and the frequency of default. I find the government enters a period with an average debt over GDP of 44%, and defaults with probability 7% over 20 years, and with probability 27% over a century.

Table 3					
Without IFI		With IFI			
$\hat{b}$	$47.7 \% Y^{min}$	$b^{bail}$	$163\% Y^{min}$		
		$b^{mkt}$	$146\% \ Y^{min}$		
$d^*$	$53\% Y^{min}$	$d^*$	$148\% Y^{min}$		
$b(d^*)$	$45.3\% \ Y^{min}$	$b(d^*)$	$144\% \ Y^{min}$		
MPS	5.8%	MPS	4.1%		

Figure 3: Maximum Primary Surplus

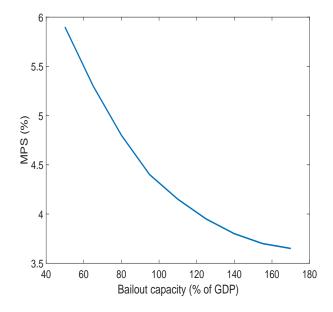


Figure 3: The MPS collected by the government as a function of the bailout capacity of the IFI.

The possibility of a bailout reduces the number of defaults: simulating the model with the IFI, I find that the default probability is 6.2% on a 100-year path. However, a bailout occurs on every path, with an average time to bailout of 20 years. The average debt with which the government enters a period is non-stationary and increasing in the time from inception.

If  $\kappa^{bail} = 0$ , this state of things would cause no problem. The IFI could be thought as a device to enforce government's repayment and reduce the incidence of default, and thus the incidence of the associated externality. Otherwise, if  $\kappa^{bail}$  is significant, clearly the IFI needs to find a way to reduce the frequency of bailouts. One obvious way would be to make bailout conditional on some prior "good behavior" on the part of the government.

#### 4.4 Alternative parameter choices

In Table 4 I show the main results obtained with a different choice for three key parameters: the mean and standard deviation of the growth factor, and the cost of default.

The new choices for the moments of the growth parameter are still within the range observed for the GIIPS countries in the last 20 years (see Table 1). Notice that the mean of the growth factor is kept in all cases below the risk-free rate r = 2%.

The major qualitative features of the results do not change. First, the inequalities  $\hat{b} < b^{mkt} < b^{bail}$  are always respected:  $b^{bail}$ , the amount that the IFI can lend, exceeds the borrowing capacity from the market (Proposition 5) and  $b^{mkt}$ , the amount that the market is willing to lend with the IFI, is bigger than  $\hat{b}$ , the amount it is willing to lend with the IFI, is bigger than  $\hat{b}$ , the amount it is willing to lend with the IFI (Proposition 6). The difference between the willingness to lend of the IFI and that of the market is especially big as the mean of the growth factor approaches the risk-free rate, and also when volatility of the growth factor is high. Second, in each scenario the MPS with the IFI is always smaller than without the IFI, showing that the presence of the IFI always indices a relation in the government's fiscal policy. Finally, the (outright) default probability is always smaller in the presence of the IFI, although in some cases the default probability with the IFI is very close to

Table 4								
	Without IFI			With IFI				
New	$\hat{b}$ MPS Def Prob $b^{bail}$ $b^{mkt}$	$b^{mkt}$	$b^{mkt}$ MPS	Def Prob	$T^{bail}$			
parameter	0		(per 100 years)		0 0	WII 5	(per 100 years)	(years)
$\sigma = 5\%$	39.4%	6.0%	32%	184.9%	162.9%	5.3%	1%	19.6
$\sigma = 3\%$	58.4%	5.6%	19%	148.4%	136.4%	2.9%	10%	20.6
$\mu = 1\%$	37.0%	4.5%	16%	71.7%	64.1%	2.4%	15%	18.4
$\mu = 1.7\%$	53.5%	6.5%	33%	330.2%	251.4%	6.3%	1%	23.1
$\xi = 1.012$	38.6%	4.9%	30%	129.4%	116.3%	2.8%	14%	18.1
$\xi = 1.018$	56.5%	6.9%	21%	196.1%	176.3%	5.1%	2.7%	21.5
baseline	47.7%	5.8%	27%	162.7%	146.3%	4.1%	6.2%	20.0

the one without the IFI (as in the case when  $\mu = 1$ ). In the rightmost column,  $T^{bail}$  denotes the average time to bailout in the presence of the IFI.

## 5 Conditionality

In this section I consider the scenario in which the IFI imposes some constraints on the government's fiscal policy, and denies access to a bailout to a government that violates the constraints.

As an extreme case, the IFI could impose the government to keep the same tax schedule that government chooses without the IFI. In this case, for every debt value below  $d^*$  – the default threshold without the IFI – the value function of the government would be the same as without the IFI. At  $d = d^*$  the value function would thus be equal to  $V^{def}$ , and the government would stop repaying and opt for a bailout. The probability of a bailout would be 27% within a century – the same as the probability of a default without the IFI – and the default probability would be essentially zero. In such a scenario the presence of the IFI would leave the welfare of the borrowing country unchanged, but result in a lower externality:  $\kappa^{bail}$  in place of  $\kappa^{def}$  whenever the debt of the borrowing country reaches the threshold  $d^*$ .

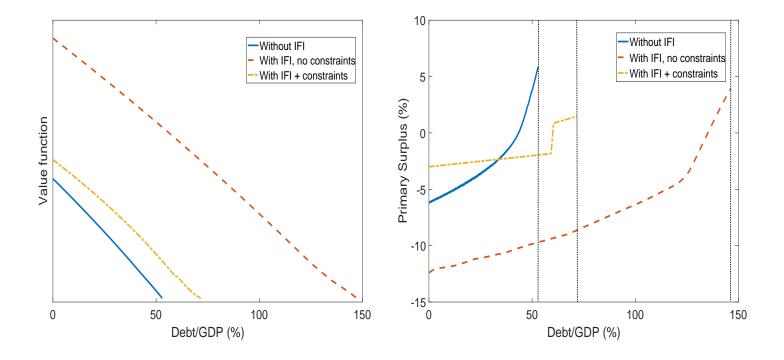


Figure 4: The effect of conditionality

Figure 4: The value function (left panel) and the optimal primary surplus (right panel) as a function of the debt label in three cases: without the IFI, with the IFI but without any conditionality, and with the IFI and ex-ante fiscal conditions.

Other, less extreme options would reduce less dramatically the bailout and default probability, but also would be more beneficial for the government's welfare.

I consider in particular the constraints set by the ESM to give access to the Precautionary Conditioned Credit Line. These include a track record of deficit below 3% and debt lower than 60% of GDP, or, in case of debt above 60%, debt in a lower trajectory, approaching the 60% level at an average rate of 1/20 per year. I compute the optimal fiscal policy of the government under these constraints. The results are shown in Figure 4, where the optimal fiscal policy chosen given the constraints is compared with the optimal fiscal policy without the IFI, and with the IFI but without any ex-ante conditions. With the ex-ante conditions described above, the country would ask for a bailout at a much lower debt level, relative to the case in which the bailout promise came with no conditionality (71.7% of GDP in place of 147%). For lower debt level, instead, the borrowing country would basically choose the minimum primary surplus compatible with the constraints.

A simulation shows that the default probability would be 0, and the probability of a bailout would be 12% in 20 years, or 55% in a century. A bailout would occur every 115 years on average. Table 5 summarizes the default and bailout probabilities, in a 20-year period and in a century, in all relevant cases.

Table 5						
	Without IFI	With IFI, no cond.	With $IFI + ex-ante cond.$			
Bailout prob.	_	55%	12%			
within 20 years		0070				
Default prob.	7%	1.8%	0			
within 20 years	170	1.070	0			
Bailout prob.		100%	56%			
within 100 years	-	10070	5070			
Default prob.	27%	6.2%	0			
within 100 years	2170	0.270				

The left panel of Figure 4 shows the government value function as a function of

the debt level, in the three cases without the IFI, with the IFI and no conditionality, and with the IFI and ex-ante conditions as described above. We clearly see that the government would accept the conditions in exchange for the bailout guarantee, as its value function (dash-dotted line in Figure 4) would still be higher under these condition than without the IFI (solid line) for every debt level.

A bailout promise with ex-ante conditions  $C^{ex\ ante}$  is Pareto-improving (i.e. increases the welfare of both the government and the international agents, relative to the case without IFI) if the value function of the government, given the presence of the IFI and the conditions  $C^{ex\ ante}$ , is higher than the value function without the IFI for every debt level, and at the same time that the expected value of the combined costs and externalities caused by the possible defaults and bailout is reduced:

$$P^{IFI}(bail; \mathcal{C}^{ex \ ante})\kappa^{bail} + P^{IFI}(def; \mathcal{C}^{ex \ ante})\kappa^{def} \leq P^{no \ IFI}(def)\kappa^{def}$$
(52)

$$V^{IFI}(d; \mathcal{C}^{ex \ ante}) \geq V^{no \ IFI}(d) \quad \forall d$$
 (53)

Inequality (52) is the participation constraint for the international agents. On the LHS we have the expected value of the externalities in the presence of the IFI with the ex-ante conditions, given by the probability of a bailout  $P^{IFI}(bail; C^{ex\ ante})$  times the externality of a bailout  $\kappa^{bail}$ , plus the probability of a default  $P^{IFI}(bail; C^{ex\ ante})$  times the externality of a default  $\kappa^{def}$ . On the RHS we have the expected value of the externality in the absence of the IFI, given by the probability of a default in this case,  $P^{no\ IFI}(def)$ , times the corresponding externality  $\kappa^{def}$ . Inequality (53) is the probabilities estimated by the simulations described above, the bailout and default the ex-ante constraints described above is Pareto improving if

$$\frac{\kappa^{bail}}{\kappa^{def}} \le 0.583 \tag{54}$$

In contrast, a bailout promise with no ex-ante condition would be Pareto-improving only if

$$\frac{\kappa^{bail}}{\kappa^{def}} \le 0.104 \tag{55}$$

While in this paper I do not attempt to estimate the cost of a default or of a bailout, this framework provides a clear criterion to evaluate the desirability of ex-ante conditions.

# 6 Conclusion

In this paper I model the fiscal and default decisions of a government whose borrowing ability is limited by its inability to commit to future repayment. An International Financial Institution is established to avoid the externality associated with default. The IFI can lend more than the market because it can enforce repayment and because loans have equity-like features. In the presence of the IFI, the markets themselves are willing to lend more (about three times more than in the absence of the IFI, in a numerical analysis calibrated to the GIIPS economies), and the government borrows more for the same level of debt: its primary surplus for a given level of debt is 6 to 15 percentage points lower and its maximum primary surplus is 2 percentage points lower than in the absence of the IFI. In the presence of the IFI the frequency of defaults is reduced, from 27% in a century to 6% in a century, however a bailout occurs on average every 20 years. This might be a good outcome if the cost of providing a bailout is very small compared to the deadweight cost of an outright default. If this is not the case, providing a bailout would be a time-inconsistent policy: when a bailout is needed, it is optimal to provide it rather than letting a country default, assuming that the cost of a bailout is smaller than the spillover cost of a default. Ex-ante, however, the promise of a bailout results in such a high frequency of future bailouts that it would be better to commit to letting a country default.

The idea behind the recent ESM reform, consisting in having two different bailout mechanisms, with better ex-post conditions only for countries that respected ex-ante fiscal conditions, seems to go in the right direction to make bailouts a time-consistent policy.

# Appendix

### **Proposition** 1

If, for a debt level d,  $b(d) < \hat{b}$ , then  $\tau(d) < \tau^{peak}$ .

I prove this statement by contradiction. Suppose for some debt level d it is b(d) < band  $\tau(d) = \tau^{peak}$  (remember that tax rates bigger than  $\tau^{peak}$  are never chosen because always suboptimal). Then increasing borrowing by  $\epsilon$  is a feasible strategy, and I am going to show that it leads to an increase in utility.

Given the budget constraint (24), if borrowing b increases by  $\epsilon$ , then surplus  $S(\tau)$  can decrease by  $\epsilon$ . This leads to a utility gain in the current period

$$-\left(\tilde{u}'(\tau)\frac{d\tau}{d\mathcal{S}}\right)_{\tau=\tau^{peak}}\epsilon = -\left(\frac{\tilde{u}'(\tau)}{\chi(\tau)}\right)_{\tau=\tau^{peak}}\epsilon$$
(56)

Since  $\chi(\tau^{peak}) = 0$  and  $\tilde{u}'(\tau)$  is strictly negative and finite for every  $\tau$ , the utility gain in the current period is infinitely bigger than  $\epsilon$  in the limit  $\epsilon \to 0$ .

Now I show that the future utility loss is of order  $\epsilon$ , thus negligible relative to the current-period utility gain. The original borrowing level b leads to a debt level  $d_{t+1}$  next period, with  $d_{t+1} = \frac{b(1+r+x(b))}{g_{t+1}}$ . Increasing borrowing by  $\epsilon$  leads to an increase in next period's debt, which to first order becomes equal to  $d_{t+1} + \frac{(1+r+x(b))\epsilon}{g_{t+1}}$ . Now, there are three possibilities. First,  $d_{t+1} + \frac{(1+r+x(b))\epsilon}{g_{t+1}} < d^*$ , or equivalently,  $g_{t+1} > \frac{(b+\epsilon)(1+r+x(b))}{d^*}$ . In this case there is no default next period despite the increased level of current-

period borrowing. Future utility decreases from  $V(d_{t+1})$  to  $V\left(d_{t+1} + \frac{(1+r+x(b))\epsilon}{g_{t+1}}\right)$ . Since the value function is differentiable for  $V < d^*$  (see Stokey, Lucas and Prescott (1989) for proof), for each value of  $g_{t+1}$  satisfying the above condition the decrease in future utility is of order  $\epsilon$ . Second possibility,  $d_{t+1} > d^*$ , or equivalently,  $g_{t+1} < \frac{b(1+r+x(b))}{d^*}$ . In this case there would be default even without the extra borrowing, so that future utility is unchanged by the increase in borrowing. Third possibility,  $d_{t+1} < d^* < d_{t+1} + \frac{(1+r+x(b))\epsilon}{g_{t+1}}$ , or equivalently,  $\frac{b(1+r+x(b))}{d^*} < g_{t+1} < \frac{(b+\epsilon)(1+r+x(b)}{d^*}$ . In this case default occurs with the increased borrowing, and would not have happened otherwise. This means that the extra borrowing leads to a finite future utility loss if model parameters lead to excusable default (i.e. if  $V^{def} < V(d^*)$ ). However,  $g_{t+1}$  needs to lie in an interval whose size is of order  $\epsilon$  for this to occur, which happens with probability of order  $\epsilon$  (at most) if the probability distribution of  $g_{t+1}$  has no mass points. In conclusion the future expected utility loss is

$$(1+r+x(b))\epsilon \int_{\frac{(b+\epsilon)(1+r+x(b))}{d^*}}^{\infty} V'\left(\frac{b(1+r+x(b))}{g}\right) \frac{f(g)}{g} dg + \frac{(1+r+x(b))}{d^*}\epsilon (V^{def} - V(d^*))f(g)|_{g=\frac{b(1+r+x(b))}{d^*}}$$
(57)

Both terms are at most of order  $\epsilon$ . I thus showed that for any debt level d, deviating from the policy  $b(d) < \hat{b}$  and  $\tau(d) = \tau^{peak}$  leads to the a future utility loss which is infinitesimal relative to the current-period utility gain, so this policy cannot be an optimum.

#### **Proposition** 2

Call  $\tau(d)$  and b(d) the policy functions adopted by the government for  $d < d^*$ . As long as  $\tau(d) < \tau^{peak}$  and  $b(d) < \hat{b}$ , both  $\tau(d)$  and b(d) are strictly increasing in d.

For  $d < d^*$ , the value function is strictly concave in the argument d, as shown by

Stokey, Lucas and Prescott (1989). The envelope condition is

$$V'(d) = \left. \frac{u'(\tau)}{\chi(\tau)} \right|_{\tau=\tau(d)} \tag{58}$$

Given that  $u(\tau)$  is decreasing and strictly concave and that  $\chi(\tau)$  is positive and strictly decreasing, the RHS of (58) is strictly decreasing in  $\tau$ . Since V is strictly concave the LHS of (58) is strictly decreasing in d. For the equality to hold,  $\tau(d)$  must be strictly increasing.

Now I'll show that  $\tau(d)$  increasing implies b(d) increasing if  $b(d) < \hat{b}$ .  $\tau(d)$  strictly increasing implies that the LHS of the Euler equation (37) is strictly decreasing in the debt level  $d_t$  (strictly increasing in absolute value). Then also the RHS must be strictly increasing in absolute value. It is easy to see that the first term in the sum on the RHS is strictly increasing (in absolute value) in  $b_t$ ; the second term is zero if  $V^{def} = V(d^*)$ and is strictly increasing (in absolute value) in  $b_t$  otherwise. Then, for the RHS to be strictly increasing in absolute value in  $d_t$ , it must be that b(d) is strictly increasing.

### **Proposition** 3

$$d^* \geq \hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}} \text{ where } \tau^o \text{ is such that } \tilde{u}(\tau^o) = \xi \tilde{u}(\tau_g)$$

If  $d = \hat{b} + \tau^o (1 - \tau^o)^{\frac{1}{\psi}}$  the following strategy is available: repaying by taxing  $\tau^o$  today and borrowing  $\hat{b}$ . This strategy gives utility

 $V = u(\tau_o) + \beta C(\hat{b}) \ge u(\tau_o) + \beta E[g^{1-\gamma}]V^{def} = V^{def}$ , where C(b) is the continuation value after borrowing b.

The optimal strategy when  $d = \hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}$  must give at least this utility. Therefore, the threshold debt level  $d^*$ , which is such that  $V(d^*) = V^{def}$  cannot be lower than  $\hat{b} + (\tau^o - \tau_g)(1 - \tau^o)^{\frac{1}{\psi}}$ .

### **Proposition 4**

Assuming  $\xi > 1$ , the maximum the IFI is willing to lend to the government is bigger than  $d^*$ 

Consider what I called "base scenario" in Section 4.1: there is no IFI and outstanding debt at time t is equal to the default threshold level  $d^*$ . The government is willing to repay by imposing a tax rate  $\tau(d^*)$  and borrowing  $b(d^*)$ , with

$$d^* = S(d^*) + b(d^*)$$
(59)

where  $S(d) \equiv \tau(d)(1-\tau(d))^{\frac{1}{\psi}}$ . We can rewrite (59) as

$$d^* = \mathcal{S}(d^*) + \frac{g_{t+1}d_{t+1}}{1+r+x(b(d^*))} = \mathcal{S}(d^*) + P^{surv}(g_{t+1})\frac{g_{t+1}d_{t+1}}{1+r}$$
(60)

In the last equality in (60) I highlighted that, given the borrowing level at time t, the survival probability at t + 1 is a function of the shock  $g_{t+1}$ . Iterating forward I obtain

$$d^* = \mathcal{S}(d^*) + \frac{P_{t+1}^{surv}(g_{t+1})}{1+r}g_{t+1}\mathcal{S}(d_{t+1}) + \frac{P_{t+2}^{surv}(g_{t+1}, g_{t+2})}{(1+r)^2}g_{t+1}g_{t+2}\mathcal{S}(d_{t+2}) + \dots$$
(61)

(61) shows that outstanding debt at time t is equal to the present discounted value of future surpluses. Notice also that, given the initial outstanding debt, future debt  $d_{t+1}$ ,  $d_{t+2}$  and so on, and hence future surpluses and survival probabilities, are only functions of the realized shocks  $g_{t+1}$ ,  $g_{t+2}$ ....

Now, the IFI could write the following contract, that would clearly result in the same utility function for the government: conditional on the realized shocks, impose the same sequence of surpluses as in the "base scenario" when the shocks would imply survival in the "base scenario", and impose surplus  $S_o = (\tau_o - \tau_g)(1 - \tau_o)^{\frac{1}{\psi}}$ , with  $\tau_o$  defined as in Proposition 3, when the sequence of shocks would imply default in the "base scenario". The present value of such surpluses would be

$$S(d^{*}) + \frac{P_{t+1}^{surv}(g_{t+1})}{1+r}g_{t+1}S(d_{t+1}) + \frac{1 - P_{t+1}^{surv}(g_{t+1})}{1+r}g_{t+1}S_{o} + \frac{P_{t+2}^{surv}(g_{t+1}, g_{t+2})}{(1+r)^{2}}g_{t+1}g_{t+2}S(d_{t+2}) + \frac{1 - P_{t+2}^{surv}(g_{t+1}, g_{t+2})}{(1+r)^{2}}g_{t+1}g_{t+2}S_{0} + \dots$$
(62)

which would clearly be higher than  $d^*$ .

### **Proposition 5**

 $\hat{b} < b^{mkt} < b^{bail}$ . For a given borrowing level  $b \leq \hat{b}$ , the spread x(b) charged by the market is lower in the presence if the IFI.

The first inequality  $\hat{b} < b^{mkt}$  is obvious when comparing (34) with (41). To prove the second inequality,  $b^{mkt} < b^{bail}$ , I'll first show that  $\theta < \bar{g}$ , where  $\bar{g} = exp(\mu + \sigma^2/2)$  is the average value of the growth rate. Remember that  $\theta \equiv max_gg(1 - F(g))$ . But for every value of g

$$g(1 - F(g)) = g \int_g^\infty \mathbf{f}(g) dg < \int_g^\infty g \ \mathbf{f}(g) dg < \int_{-\infty}^\infty g \ \mathbf{f}(g) dg = \bar{g}$$
(63)

The inequality  $b^{mkt} < b^{bail}$  follows from (41), together with the inequality  $\theta < \bar{g}$  and the assumption  $\bar{g} < 1 + r$ .

I now show that, for a given borrowing level b, the credit spread in the presence of the IFI is lower than in its absence.

For a given borrowing level b, without the IFI the spread x(b) solves the equation

$$(1+r+x(b))\left(1-F\left(\frac{d^*}{b(1+r+x(b))}\right)\right) = (1+r)$$
(64)

and with the IFI

$$(1+r+x^{IFI}(b))\left(1-F\left(\frac{b^{bail}}{b(1+r+x^{IFI}(b))}\right)\right) = (1+r)$$
(65)

where F is the probability distribution of the growth factor, so that the second factor on the LHS of (64) and (65) is the survival probability given the borrowing level b, in the two cases without and with the IFI. I consider values of the borrowing level for which a solution exists in both the above cases. Given the lognormal probability distribution F, the LHS of both the above equations is bell-shaped as a function of the spread: it is increasing for low values of the spread and decreasing for very high values of the spread, with the possibility of one solution on each side. I assume that the solution on the increasing side is selected. Then, given a value of b, consider the spread x(b) solving (64). If we insert the same value in (65), since  $b^{bail} > d^*$ , the LHS of (65) would be higher than the LHS of (64), hence higher than 1 + r. Since the LHS of (65) is increasing in the spread in this region, the solution  $x^{IFI}(b)$  must be lower than x(b).

### Proposition 6

 $P^{def,IFI}(b^{mkt}) = P^{def}(\hat{b}).$ 

Let us start with the case without the IFI. (34) can be written as

$$\hat{b} = \frac{d^*}{1+r} max_d \frac{d}{d^*} \int_{\frac{d}{d^*}}^{\infty} \mathbf{f}(g) dg$$
(66)

Call  $\bar{d}$  the value that maximizes the above expression. The probability of default after borrowing  $\hat{b}$  is then  $F\left(\frac{\bar{d}}{d^*}\right)$ .

Let us consider now the case with the IFI. We can analogously write (41) as

$$b^{mkt} = \frac{b^{bail}}{1+r} max_d \ \frac{d}{b^{bail}} \int_{\frac{d}{b^{bail}}}^{\infty} f(g) dg$$
(67)

The value  $\bar{d}'$  that maximizes the above expression is clearly such that  $\frac{\bar{d}}{d^*} = \frac{\bar{d}'}{b^{bail}}$ . Therefore also the default probability after borrowing  $b^{mkt}$ , which is  $F\left(\frac{\bar{d}'}{b^{bail}}\right)$ , is the same as in the case without the IFI after borrowing  $\hat{b}$ , which is  $F\left(\frac{\bar{d}}{d^*}\right)$ .

### Solution Method

The government's problem without the IFI, defined by (30)-(??), and with the IFI, defined by (23), is solved by value-function iteration. Since outstanding debt is the only state variable when growth factors are i.i.d., the problem can be solved using a very dense grid (I use a grid with around 1500 points in debt space).

The algorithm to solve the problem without the IFI is the following:

- 1. Assume that the solution of the problem falls into the strategic default region and guess a value of the default threshold  $d^*$ .
- 2. Find the borrowing capacity (34) given  $d^*$ .
- 3. Find the government value function with a VFI method. At each iteration, and for each debt value d, a one-dimensional search is performed to find the policy functions  $\tau(d)$  and b(d) (connected via the budget constraint) maximizing the value function.
- 4. If the value function at the hypothesized default threshold  $d^*$  does not coincide with  $V^{def}$ , update the guess value of  $d^*$  and repeat the steps 2-4.
- 5. If a solution can't be found assuming strategic default, then the solution must fall into the excusable default region. In this case the borrowing capacity  $\hat{b}$  is given by (35) and  $d^* = S^{peak} + \hat{b}$ . Finally follow again step 3.

As for the problem with the IFI, the algorithm is

- 1. Find the bailout capacity  $b^{bail}$  using (51) and the willingness to lend of the market  $b^{mkt}$  using (41).
- 2. Find the value function with no default  $V^{no \ def}$  for debt between  $d = 0 \ d = b^{bail} + S^{peak}$  (beyond  $d = b^{bail} + S^{peak}$  there is default for sure). The procedure is similar as in step 3 in the problem without IFI.
- 3. Identify  $d^*$  as the debt level for which  $V^{no\ def} = V^{def}$ . If  $V^{no\ def} > V^{def}$  for any debt level  $d \le b^{bail} + S^{peak}$ , then the solution falls in the excusable default region and  $d^* = b^{bail} + S^{peak}$ .

# References

[1] Aguiar, Mark, and Gita Gopinath (2006), *Defaultable debt, interest rates and the current account*, Journal of International Economics 69.1: 64-83.

- [2] Arellano, Cristina (2008), Default risk and income fluctuations in emerging economies, American Economic Review 98.3: 690-712.
- [3] Aylward, Lynn, and Rupert Thorne (1998), Countries' Repayment Performance Vis-a-Vis the IMF: An Empirical Analysis, IMF Staff Papers 45.4: 595-618.
- [4] Boz, Emine (2011) Sovereign default, private sector creditors, a nd the IFIs, Journal of International Economics 83.1: 70-82.
- [5] Campbell, John Y., and Sydney C. Ludvigson (2001), Elasticities of substitution in real business cycle models with home production, Journal of Money, Credit and Banking, 33(4): 847-875.
- [6] Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011), Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins., American Economic Review 101, no. 3: 471-75.
- [7] Cole, Harold L., and Timothy J. Kehoe (2000), Self-fulfilling debt crises, The Review of Economic Studies 67, no. 1: 91-116.
- [8] Collard, Fabrice, Michel Habib, and Jean-Charles Rochet (2015), Sovereign debt sustainability in advanced economies, Journal of the European Economic Association 13.3: 381-420.
- [9] Corsetti, Giancarlo, Aitor Erce, and Timothy Uy (2018), Debt sustainability and the terms of official support.
- [10] Corsetti, Giancarlo, Bernardo Guimaraes, and Nouriel Roubini (2006), International lending of last resort and moral hazard: A model of IMF's catalytic finance, Journal of Monetary Economics 53.3: 441-471.
- [11] Cuadra, Gabriel, Juan M. Sanchez, and Horacio Sapriza (2010), Fiscal policy and default risk in emerging markets, Review of Economic Dynamics 13.2: 452-469.

- [12] Fink, Fabian, and Almuth Scholl (2016), A quantitative model of sovereign debt, bailouts and conditionality, Journal of International Economics 98: 176-190.
- [13] Galli, Giampaolo (2020), The reform of the ESM and why it is so controversial in Italy, Capital Markets Law Journal 15.3: 262-276.
- [14] Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D. and Qureshi, M. S. (2013), Fiscal fatigue, fiscal space and debt sustainability in advanced economies The Economic Journal, 123(566), F4-F30.
- [15] Gourinchas, Pierre-Olivier, Philippe Martin, and Todd E. Messer (2020), The economics of sovereign debt, bailouts and the Eurozone crisis, No. w27403, National Bureau of Economic Research.
- [16] Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988), Investment, capacity utilization, and the real business cycle, The American Economic Review: 402-417.
- [17] Martinez-Garcia, Enrique, Diego Vilán, and Mark A. Wynne (2012), Bayesian estimation of NOEM models: identification and inference in small samples, DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments. Emerald Group Publishing Limited.
- [18] Pancrazi, Roberto, Hernán D. Seoane, and Marija Vukotić (2020), Welfare gains of bailouts in a sovereign default model, Journal of Economic Dynamics and Control 113: 103867.
- [19] Roch, Francisco, and Harald Uhlig (2018), The dynamics of sovereign debt crises and bailouts, Journal of International Economics 114: 1-13.
- [20] Rogoff, Kenneth S. (2002), Moral hazard in IMF loans: how big a concern?, Finance and Development 39.3 (2002): 56-57.

- [21] Rotemberg, Julio J., and Michael Woodford (1999), Interest rate rules in an estimated sticky price model, Monetary policy rules, pp. 57-126. University of Chicago Press.
- [22] Rudebusch, Glenn D., and Eric T. Swanson (2012), The bond premium in a DSGE model with long-run real and nominal risks, American Economic Journal: Macroeconomics 4, no. 1: 105-43.
- [23] Tirole, Jean (2015), Country solidarity in sovereign crises, American Economic Review, 105(8), 2333-2363.
- [24] Stokey, Nancy L., Robert E. Lucas Jr. and Edward C. Prescott (1989), Recursive methods in economic dynamics, Harvard University Press.
- [25] Trabandt, Mathias, and Harald Uhlig (2012), How do Laffer curves differ across countries?. No. w17862, National Bureau of Economic Research.
- [26] Zettelmeyer, Jeromin, Jonathan David Ostry, and Olivier Jeanne (2008), A theory of international crisis lending and IMF conditionality, No. 8-236. International Monetary Fund.