This paper develops and estimates a DSGE model with stock market bubbles and nominal rigidities using Bayesian methods. Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs, and their movements are driven by a sentiment shock. This paper shows that stock market bubbles are an important factor in explaining volatility in investment, output, and also in inflation. Moreover, a monetary policy rule that targets stock prices can help to diminish the impact of bubble sentiment shocks, and thus stabilise the economy faster than a policy rule that does not react to asset prices.

Key Words: Bubbles, Monetary Policy, Bayesian estimation, DSGE.
JEL Reference Number: E22, E32, E43, E44, E52
1 Introduction

The economic upturn and downturn associated with the subprime mortgage bubble in the US revived a long debate on how monetary policy should react to asset price developments. The current consensus about monetary policy is that the main objective of Central Banks is to maintain price stability, that is to say keeping low and stable inflation. However, price stability generally concerns the stabilisation of the consumer prices index, which covers only a segment of prices in the economy. While the omission of asset prices for monetary policy is normally not considered as a problem, large movements in asset prices and bubbles’ bursts led many economists to reconsider if the focus of monetary policy on consumer prices alone is still pertinent.

Before the 2008 financial crisis, the conventional strategy often named the “Jackson Hole Consensus”, calls for Central Banks to focus on maintaining price stability and stabilizing the output gap. Thus, Central Banks should ignore asset price fluctuations unless they are a threat to price and output stability (e.g. Bernanke and Gertler, 1999, 2001; Kohn, 2006). One of the main reasons for this consensus is that instruments of monetary policy were judged ‘too blunt’ to successfully target asset prices. However, this strategy prescribes that Central Banks should take the necessary actions (e.g. via interest rate cuts) once a bubble collapsed in order to protect the economy against the harmful effects of the bubble’s burst.

This asymmetric strategy for reacting to asset price developments was challenged by many economists arguing that price stability would not guaranty financial stability (e.g. Cecchetti et al., 2000; Borio and Lowe, 2002). The opposing prominent strategy for Central Banks is ‘leaning against the wind’, which advocates that Central Banks should try to mitigate the risk associated with the developments and bursts of bubbles. In this case, central banks are required to tighten their monetary policy stance in the face of an inflating asset market, even if it creates a temporary deviation from their price stability objective.

The depression of the US economy following the 2008 banking crisis led many economists to agree that monetary policy should also focus on financial stability. A pure passive “cleaning up the mess” policy is likely to be more costly than a ‘leaning against the wind’ policy. Ikeda (2017) concluded that the optimal monetary policy should be tightened to control the output boom caused by the bubble at the expense of inflation stabilisation. Miao et al. (2019) argue that, under adaptive learning, monetary policy that leans against the wind can reduce the volatility of bubbles. Galichère (2021) found that monetary policy that targets asset price bubbles can deflate the bubble. However, using monetary policy to deflate a bubble can be costly in terms of output.
and a monetary policy that overreacts to asset prices can generate a recession. In contrast, Galí and Gambetti (2015) argued Central Banks should not lean against the wind. They found that tightening monetary policy would persistently increase stock prices. While there is no agreement on how Central Banks should react to bubbles, there is a consensus that monetary policy may have a role in addressing bubbles.

This paper investigates how monetary policy interacts with stock market bubbles and asks: Can lessen the impact of bubbles, How and at what cost. To answer these questions, I develop a New Keynesian model with rational bubbles, where bubbles can exist because of financial friction (e.g. Miao and Wang, 2018). In contrast to the literature that mainly focuses on pure bubbles (e.g. Martin and Ventura, 2012; Galí, 2014; Hirano and Yanagawa, 2017), the proposed model is based on Miao et al. (2015) where the stock market price of wholesale firms contains a bubble component in addition to the fundamental value. Unlike pure bubbles, stock market bubbles are attached to productive firms with positive dividends and are not separately tradable from firm stocks. The stock price bubbles can emerge in different firms or in different sectors, and their emergence or collapse may be unrelated to the emergence or collapse of pure bubbles.

Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. Precisely, households believe that the value of some wholesale firms may not be equal to their fundamentals. These firms, which pledge their assets as collateral in order to borrow funds, are able to relax their borrowing constraints because of the ‘optimistic’ beliefs of households on firms’ values. Consequently, firms are able to borrow more and increase profit, which in turn raise the value of these firms. In this sense, bubbles can exist because of self-fulfilling beliefs. Without the presence of bubbles, these firms would be unable to borrow extra funds and deliver higher profits. Finally, as in Miao et al. (2015), the beliefs of the households about the movement of the bubble is modelled with the introduction of a sentiment shock. I estimate this sentiment shock and evaluate its importance in explaining changes in real and nominal variables.

I find that bubbles can cause large fluctuations in aggregate variables such as investment or output, and can also be the cause of high inflation. Moreover, as in Miao et al. (2019) or Galichère (2021), I found that leaning against the wind can reduce the impact of the sentiment shock which drives bubbles. The latter finding is based on evaluation of two alternative policy rules that target stock prices: the first rule reacts to changes in stock prices and the second reacts

\[ \text{The value of an asset is equal to its market fundamental, that is to say, the expected and discounted present value of its dividends (or more generally its rents), plus a bubble component. Pure bubbles are defined as intrinsically useless assets, that it to say that have no fundamental value. However, these assets have a positive price. Such assets are often interpreted in the literature as money, gold or lands (e.g. Weil, 1987; Kocherlakota, 2009).} \]
to deviation of stock price from its trend. The first rule reacts to changes in stock prices while
the second reacts to the deviations from the stead-state of the stock price. I find that these rules
can reduce the impact of bubble sentiment shock, but not the volatility of the bubble size itself.
Nonetheless, these alternative policies can stabilise quicker aggregate output, investment and the
stock price than a policy rule that does not react to the stock price.

Finally, while these alternative policy rules can quickly stabilise the economy after a sentiment
shock, their specification matters for their reaction to inflation. My analysis shows that a policy
that reacts to changes in stock prices is not successful in promptly bringing back inflation to
steady-state. In contrast, the policy rule that reacts to deviation from steady-state of the stock
price will be more aggressive towards inflation. This type of rule will stabilise quicker inflation
than the traditional rule or the first alternative rule mentioned above.

The rest of the paper is structured as follows. Section 2 outlines the model. Section 3 presents
the calibration of structural parameters, the estimation procedure and the estimated parameter.
Section 4, which presents the main findings, is composed of four parts: i) an evaluation of the
model in explaining historical bubble episodes, ii) a counterfactual experiment, iii) a analysis
the transmission mechanism of monetary policy in a bubbly economy, and iv) an examination of
alternate policy rules that react to stock prices. Finally, section 5 concludes.

2 The Model

The model is based on the real model presented in Miao et al. (2015) and incorporates nominal
rigidities à la Calvo to study the interaction between monetary policy and stock market bubbles.
The main structure of proposed model is presented in this section, and all the details given in
Appendix A. The model represents a discrete time economy populated by households, capital
good firms, wholesale firms, retail firms and a Central Bank. Households, capital goods firms and
retail firms have infinite lives while wholesale firms operate on the market for a stochastic length
of time.

Three financial assets are available in the economy: loans, deposits and stocks of wholesale
firms. Households can deposit and invest in wholesale firms stocks but cannot borrow. Wholesale
firms can borrow funds for their production or save unused funds. Retail and capital goods firms
can neither borrow nor save. Finally, the stock price of the wholesale can contain a rational asset
price bubble because of financial friction. The size of these bubbles is stochastic.
2.1 Households

2.1.1 Decision Problem

Each household derives utility from consumption and leisure according to the expected utility function:

\[ U = E_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t/A_t - \theta C_{t-1}/A_{t-1})^{1-\sigma}}{1-\sigma} - \psi_t \frac{N_t^{1+\eta}}{1+\eta} \right] \xi_t, \]  

(1)

where \( C_t \) denotes consumption and \( N_t \) is the household’s labour supply. The household consumes \( C_t \) and provides labour \( N_t \) to wholesale firms for the nominal wage \( W_t \). The household can accumulate wealth by purchasing shares of aggregate stock price \( s_{t+1} \) at price \( P_t \) and by saving in deposits \( D_t \) (where \( D_t \geq 0 \)) at the deposit rate \( R_t^d \).

The representative household’s budget constraint is given by:

\[ C_t + p_t^s s_{t+1} + d_{t+1} \left( \frac{1 + \pi_{t+1}}{R_t^d} \right) = w_t N_t + \Pi_t^I + \Pi_t^F + d_t + (d_t^s + p_t^s) s_t, \]  

(2)

where \( p_t^s \) is the relative price of the stock, \( d_t \) is the real quantity of deposits, \( \pi_{t+1} \) is the inflation rate, \( w_t \) is the real wage and \( d_t^s \) is the real aggregate dividend on their stock investment. Household receives real profits from capital goods firms \( \Pi_t^I \), profit from retailers \( \Pi_t^F \). Moreover, the gross inflation rate \((1 + \pi_{t+1}) = P_{t+1}/P_t\), where \( P_t \) is price level of consumption.

The representative agent maximises (1) subject to (2).

2.1.2 Optimal Behaviour

The remainder of the household’s problem is standard. The first order conditions for habit-adjusted consumption, labour and deposits are given by:

\[ \Lambda_t = \frac{\xi_t}{(C_t/A_t - \theta C_{t-1}/A_{t-1})^{\sigma}} - \theta \beta E_t \left[ \frac{\xi_{t+1}}{(C_{t+1}/A_{t+1} - \theta C_{t+1}/A_{t+1})^{\sigma}} \right], \]  

(3)

\[ \frac{N_t^\eta}{\Lambda_t} = \frac{w_t}{\xi_t} (1 - \tau_t), \]  

(4)

\[ \Lambda_t = \beta E_t \left[ \frac{\Lambda_{t+1}}{(1 + \pi_{t+1})} \right] R_t^d \text{ if } d_t > 0, \]  

(5)

where \( \Lambda_t \) represents the marginal utility of consumption. The first order condition for share of aggregate stock investment is:

\[ \Lambda_t = \beta E_t [\Lambda_{t+1} s_t^s] \text{ if } s_{t+1} > 0, \]  

(6)
where the expected real rate of stock return is defined as $r_s^t \equiv E_t \left[ \left( d_{t+1}^s + p_{t+1}^s \right) / p_t^s \right]$. Equations (6) and (5) set the non-arbitrage condition between stock investments and deposits, which is given by:

$$E_t \left[ \frac{\Lambda_{t+1}}{(1 + \pi_{t+1})} \right] R_d^t = E_t [\Lambda_{t+1} r_t^s].$$

2.2 Capital Producers

2.2.1 Decision Problem

The households own capital producers and receive the profit $\Pi_t^I$. A representative capital goods firm produces new capital using input of final output and subject to adjustment costs. It sells new capital $I_t$ to wholesales firms at price $P_t^I$. The objective of a capital producer is to choose $I_t$ to maximise:

$$V^I = \max_{I_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left[ p_t^I I_t - \left( 1 + \frac{\Omega}{2} \left[ \frac{I_t}{I_{t-1}} - \lambda^I \right] \right) \frac{I_t}{Z_t} \right],$$

where $p_t^I$ is the relative price of capital goods, $\lambda^I$ is the growth rate of aggregate investment, $\Omega > 0$ is the adjustment cost parameter and $Z_t$ represents an investment cost shock that follow the exogenous process:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_t^Z$$

where $\epsilon_t^Z$ is an independent and identically distributed shock (IID) over time.

2.2.2 Optimal Behaviour

The optimal level of investment goods satisfies the first order condition with respect to $I_t$:

$$Z_t p_t^I = 1 + \frac{\Omega}{2} \left[ \frac{I_t}{I_{t-1}} - \lambda^I \right]^2 + \Omega \frac{I_t}{I_{t-1}} \left[ \frac{I_t}{I_{t-1}} - \lambda^I \right] - \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{I_{t+1}}{I_t} \right)^2 \Omega \left[ \frac{I_{t+1}}{I_t} - \lambda^I \right] \frac{Z_t}{Z_{t+1}} \tag{7}$$

and household receive the real profit:

$$\Pi_t^I = p_t^I I_t - \left( 1 + \frac{\Omega}{2} \left[ \frac{I_t}{I_{t-1}} - \lambda^I \right] \right) \frac{I_t}{Z_t} \tag{8}$$

2.3 Wholesale Firms

Following Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Gertler and Kiyotaki (2010), and Miao et al. (2015), exogenous entry and exit of firms is assumed because of
non-arbitrage condition. To understand the necessity of this mechanism, suppose that households believe that each wholesale firm’s stock may contain a bubble and that this bubble may burst with some probability. Because of rational expectations, a bubble cannot re-emerge in the same firm after bursting. Otherwise there would be an arbitrage opportunity. This means that none of the firms would contain any bubble once all bubbles have burst if no new firms enter the economy.

A firm may exit with an exogenously given probability $\delta_e$ each period. After exiting the economy, its value is zero and a new firm enters the economy without costs so that the total measure of firms is fixed at unity in each period. A new firm entering at date $t$ starts with an initial capital stock $K_{0t}$ and then operates in the same way as older firms. Moreover, each new firm may bring a new bubble into the economy with probability $\omega$.

Wholesale firms make investment decisions that maximize their cum-dividend stock market value of the firms. They can purchase investment goods $I_t$ from capital producers at price $P_{\text{it}}$ and they sell their good $Y_{jt}$ to retail firms at price $P_{wt}$.

### 2.3.1 Decision Problem

A wholesale firm $j \in [0,1]$ combines capital $K_{jt}^j$ and labour $L_{jt}$ to produce intermediate goods $Y_{jt}^j$ using the production function:

$$Y_{jt}^j = \left( u^j_t K_{jt}^j \right)^\alpha \left( A_t N_{jt}^j \right)^{1-\alpha},$$

(9)

where $\alpha \in (0,1)$, $u^j_t$ denotes the capacity of utilisation rate and $A_t$ denotes the labour-augmenting technology shock (or total factor productivity (TFP) shock given the Cobb-Douglas production function). For a new firm entering at date $t$, I set $K_{jt}^i = K_{0t}$.

Assume that the capital depreciation rate between period $t$ and period $t + 1$ is given by $\delta_t^j = \delta(u_t^j)$, where $\delta$ is a twice continuously differentiable convex function that maps a positive number into $[0,1]$. The function $\delta(\cdot)$ does not need to be parametrised because the model will be solved using the log-linearisation solution method, where the steady-state capacity utilisation rate will be normalized to 1.

The capital stock evolves according to:

$$K_{jt+1}^j = \left( 1 - \delta_t^j \right) K_{jt}^j + \varepsilon_t^j I_t^j,$$

(10)

where $I_t^j$ denotes investment and $\varepsilon_t^j$ is an idiosyncratic shock that measures the efficiency of the investment. Investment is assumed to be irreversible at the firm level so that $I_t^j \geq 0$. Moreover, $\varepsilon_t^j$ is an IID shock across firms and over time, and is drawn from the fixed cumulative distribution
Φ over \([\varepsilon_{\text{min}}, \varepsilon_{\text{max}}] \subset (0, \infty)\) with mean 1 and probability density function \(\phi\). This shock induces firm heterogeneity in the model. For tractability, assume that the capacity utilisation decision is made before the observation of investment efficiency shock \(\varepsilon_j^t\). Consequently, the optimal capacity utilisation does not depend on the idiosyncratic shock \(\varepsilon_j^t\).

In each period \(t\), firm \(j\) can make investment \(I_j^t\) by purchasing investment goods from capital producers at the price \(P_l^t\). Its real flow-of-funds constraint is given by:

\[
d^e_j + p^i_l I^t_j - l^t_{j+1} \frac{(1 + \pi_{t+1})}{R^t_l} = p^w_l Y^t_j - w_t A_t N^t_j - l^t_j,
\]  

where \(l^t_{j+1} > 0\) \((< 0)\) represents the real quality of borrowing (savings) at time \(t\), \(R^t_l\) represents the lending rate, \(p^w_l\) is the relative price of wholesale firms’ goods and \(d^e_j > 0\) \((< 0)\) represents dividends (new equity issuance). Assume that external financial markets are imperfect so that firms are subject to the constraint on new equity issuance:

\[
d^e_j \geq -\phi_t K^t_j,
\]

where \(\phi_t\) is an exogenous stochastic shock to equity issuance. The demand for capital good is constrained such that:

\[
0 \leq p^i_l I^t_j \leq p^w_l Y^t_j - w_t A_t N^t_j + \phi_t K^t_j - l^t_j + l^t_{j+1} \frac{(1 + \pi_{t+1})}{R^t_l}.
\]

In addition, external borrowing is subject to the credit constraint:

\[
\beta_E \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \bar{V}_{t+1,a+1} \left( K^t_j, l^t_j, \varepsilon^t_j \right) \right] \geq \beta E \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \bar{V}_{t+1,a+1} \left( K^t_{j+1}, 0 \right) \right]
\]

where \(\bar{V}_{t,a} \left( K^t_j, l^t_j \right) \equiv \int V_{t,a} \left( K^t_j, l^t_j, \varepsilon^t_j \right) d\Phi(\varepsilon)\) represents the ex-ante value after integrating out \(\varepsilon^t_j\) and \(V_{t,a} \left( K^t_j, l^t_j, \varepsilon^t_j \right)\) represents the cum-dividends stock market value of the firm of age \(a\) with with assets \(K^t_j\), debt \(l^t_j\) and idiosyncratic investment shock \(\varepsilon\) at time \(t\). In equation (15), \(\gamma_t\) represents the collateral shock that reflects the frictions in the credit market. Note that \(a\) represents the age of the firm. The equity value depends on the age of the firm because it contains a bubbles component that is age dependent.
Following Miao and Wang (2018), equation (15) is an incentive constraint in a contracting problem between the firm and the lender which ensures firm \( j \) has no incentive to default in equilibrium. The firm has limited commitment and can default on debt \( l_{j,t+1}^l \) at the beginning of period \( t + 1 \). If the firm does not default, its continuation value is given by \( \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1} \left( K_{t+1}^j, l_{j,t+1}^l \right) \). If the firm defaults, the debt is renegotiated, the repayment is relieved and the value of the firm is \( \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1} \left( \gamma_t K_t^j, 0 \right) \). Then the RHS of equation (15) is the value of the firm if it chooses to default.

An intermediate goods producer \( j \) with age \( a \) chooses labour, \( N_{j,t}^l \geq 0 \), investment, \( I_{j,t}^l \geq 0 \), debt, \( l_{j,t+1}^l \geq 0 \), and capacity of utilisation, \( u_{j,t}^l \geq 0 \), to maximize its value:

\[
V_{t,a} \left( K_{j,t}^l, l_{j,t+1}^l, \varepsilon_{j,t}^l \right) = \max_{N_{j,t}^l, I_{j,t}^l, l_{j,t+1}^l, u_{j,t}^l} p_t^w Y_{j,t}^l - (w_t A_t N_{j,t}^l + I_{j,t}^l p_t^l) - l_{j,t}^l + l_{j,t+1}^l \left( 1 + \pi_{t+1} \right) R_t^l + (1 - \delta_a) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1} \left( K_{t+1}^j, l_{t+1}^l, \varepsilon_{t+1}^l \right),
\]

subject to the production function (9), the law of motion of capital (10), the constraint on new equity issuance (12), the borrowing (15) and the flow of funds (11).

As in Miao et al. (2015), I conjecture and verify that the value function takes the form:

\[
V_{t,a} \left( K_{j,t}^l, l_{j,t+1}^l, \varepsilon_{j,t}^l \right) = Q_t \left( \varepsilon_{j,t}^l \right) K_{j,t}^l + B_{t,a} \left( \varepsilon_{j,t}^l \right) - Q_t^L \left( \varepsilon_{j,t}^l \right) l_{j,t}^l.
\]

### 2.3.2 Optimal Behaviour

The first order condition of the wholesale firm’s problem with respect to labour given the wage rate \( W_t \) and the capacity utilisation rate \( u_{j,t}^l \) yields the labour demand of the wholesale firm \( j \):

\[
N_{j,t}^l = \frac{u_{j,t}^l}{A_t} \left[ \frac{(1 - \alpha) p_t^w}{w_t} \right]^{\frac{1}{\alpha}} K_{j,t}^l.
\]

Given the wage rate \( w_t \) and the capacity utilisation rate \( u_{j,t}^l \), the production problem can be simplified such that:

\[
\max_{N_{j,t}^l} p_t^w Y_{j,t}^l - w_t A_t N_{j,t}^l = u_{j,t}^l \Psi_t K_{j,t}^l
\]

\( \gamma_t \) may be interpreted as an efficiency parameter in the sense that lender may not be able to efficiently use the firm’s assets \( K_{t+1} \) (Miao and Wang, 2018).
where \( \Psi_t \) is given by:

\[
\Psi_t = \alpha \left[ \frac{1 - \alpha}{w_t} \right]^{\frac{1 - \alpha}{\alpha}} (p_t^w)^{\frac{1}{\alpha}}
\]

(18)

after substituting out the labour decision (17) of the production problem.

Using (6), the date-\( t \) ex-dividend stock relative price of the firm \( j \) of age \( a \) can be rewritten as:

\[
p_t^{s,j} = (1 - \delta_e) \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1} \left( K_{t+1}^{j}, l_{t+1}^{j}, \varepsilon_{t+1}^{j} \right) \right].
\]

Given the above conjecture (16), stock relative price can be rewritten in the form:

\[
p_t^{s,a} = q_t K_{t+1}^{j} + b_{t,a} - q_t^{L} l_{t+1}^{j},
\]

where \( q_t, b_{t,a} \) and \( q_t^{L} \) define such that:

\[
q_t = (1 - \delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} \left( \varepsilon_{t+1}^{j} \right),
\]

(19)

\[
b_{t,a} = (1 - \delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} B_{t+1,a+1} \left( \varepsilon_{t+1}^{j} \right),
\]

(20)

\[
q_t^{L} = (1 - \delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}^{L} \left( \varepsilon_{t+1}^{j} \right),
\]

(20)

Note that \( q_t, b_{t,a} \) and \( q_t^{L} \) do not depend on future idiosyncratic shocks \( \varepsilon_{t+1}^{j} \) because they are integrated out.

The first order condition for \( l_{t+1}^{j} \) using the guess of the value function (16) gives:

\[
q_t^{L} = \frac{(1 + \pi_{t+1})}{R_{t+1}^{l}},
\]

and the credit constraint can be rewritten such that:

\[
q_t \gamma_t K_{t}^{j} + b_{t,a} \geq \frac{(1 + \pi_{t+1})}{R_{t+1}^{l}} l_{t+1}^{j}.
\]

The investment level \( l_t^{j} \) of a wholesale firm \( j \) with a bubble depends on the efficiency shock of the investment \( \varepsilon_t^{j} \) being greater that the threshold \( \varepsilon_t^{*} \). Consequently, the optimal investment level \( l_t^{j} \) of a wholesale firm \( j \) respects that:

\[
l_t^{j} = \begin{cases} 
  q_t \Psi_t K_{t}^{j} + \varphi_t K_{t}^{j} - l_{t}^{j} + q_t \gamma_t K_{t}^{j} + b_{t,a}, & \text{if } \varepsilon_t^{j} \geq \varepsilon_t^{*} \\
  0, & \text{otherwise}. 
\end{cases}
\]

(21)
where the investment threshold, $\varepsilon^*_t \equiv \frac{p_t}{q_t}$, is given by the first order condition for $I_t^j$ using the guess of the value function (16).

Each firm chooses the same capacity utilisation rate $u_t$ satisfying:

$$
\Psi_t (1 + G_t) = q_t \delta'(u_t),
$$

where $G_t$ satisfies:

$$
G_t = \int_{\varepsilon \geq \varepsilon^*_t} \left( \frac{\varepsilon}{\varepsilon^*_t} - 1 \right) d\Phi(\varepsilon).
$$

The price of installed capital, the bubble, and the lending rate satisfy:

$$
q_t = (1 - \delta e) \beta E_t \Lambda_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \left[ u_{t+1} \Psi_{t+1} + q_{t+1} (1 - \delta_{t+1}) + G_{t+1} \left( u_{t+1} \Psi_{t+1} + q_{t+1} \gamma_{t+1} + \varphi_{t+1} \right) \right],
$$

$$
b_{t,a} = (1 - \delta e) \beta E_t \Lambda_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}) b_{t+1,a+1},
$$

$$
\frac{(1 + \pi_{t+1})}{R_t^i} = (1 - \delta e) \beta E_t \Lambda_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}).
$$

where $\delta_t = \delta(u_t)$.

### 2.3.3 Sentiment Shock

The household beliefs on the movement of bubbles is modelled with the introduction of a sentiment shock $\kappa_t$. Denote $b_{t,a}$ the real value of the bubble attached to a wholesale firms with age $a$ at time $t$. Households believe that a new firm in period $t$ may contain a bubble of real size $b_{t,0} = b^*_t$ with probability $\omega$. Then the total value of emerging bubble in the economy at date $t$ is given by $\omega \delta_e b^*_t$. Moreover, they believe that the relative size of the bubbles at date $t + a$ for any two firm born at date $t$ and $t + 1$ is given by $\kappa_t$ such that:

$$
\kappa_t = \frac{b_{t+a,a}}{b_{t+a,a-1}}, \quad t \geq 0, \quad a \geq 1.
$$

The relative size of bubbles $\kappa_t$ follows an exogenously given process:

$$
\ln \kappa_t = (1 - \rho_\kappa) \ln \kappa + \rho_\kappa \ln \kappa_{t-1} + \epsilon_\kappa^t
$$

where $\rho_\kappa$ is the persistence parameter and $\epsilon_\kappa^t$ is an IID normal random variable with mean zero and variance $\sigma^2_\kappa$. This process reflects the beliefs of households about the fluctuations of bubbles and is interpreted as the sentiment shock.
2.4 Retailers

2.4.1 Decision Problem

There is a continuum of differentiated retail firms of measure one, each indexed by \( i \). Each retail firm \( i \) buys a wholesale good \( Y_t^j \) at price \( P_t^w \) and repackage into a specialized retail good \( Y_t(i) \). Retailers sell their specialized retail good \( Y_t(i) \) to competitive final firms at price \( P_t(i) \).

Firm’s optimisation problem is standard: a firm \( i \) chooses prices to maximise its profit:

\[
V_t(i) = \max_{\{P_t(i)\}_{s=t}} E_t \sum_{s=t}^{\infty} (\vartheta \beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[ \left( \frac{P_t(i)}{P_{t+s}} - p_{t+s}^w \right) Y_{t+s}(i) \right],
\]

subject to the competitive demand constraint for good \( Y_t(i) \):

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\kappa_t} Y_t,
\]

where \( p_t^w \) is the real marginal cost of the retailer and \( \kappa_t > 1 \) governs the elasticity of substitution between any two specialized retail goods. Retailers are subject to cost push shocks that affect the elasticity of substitution between any two retail goods, where \( \kappa_t \) follows an exogenously given process:

\[
\ln \kappa_t = (1 - \rho) \ln \kappa + \rho \ln \kappa_{t-1} + \epsilon_t^\kappa
\]

Price rigidity and price indexation are introduced as following: i) like in Calvo (1983), a firm \( i \) at time \( t \) has the opportunity to reset its price \( P_t(i) \) with probability \( \vartheta \); ii) when it has the chance of resetting its price, it chooses price optimally, \( P^*_t(i) \), with probability \( 1 - \varpi \), or chooses its new price with probability \( \varpi \) according to the simple rule of thumb

\[
P^b_t = \frac{P^R_t}{1 + \pi_t} - \frac{1}{1 - \kappa_t}
\]

where \( P^R_t \) is given by:

\[
P^R_t = \left[ (1 - \varpi) P^*_t(i)^{1-\kappa_t} + \varpi (P^b_t)^{1-\kappa_t} \right]^{1/1-\kappa_t}
\]

2.4.2 Optimal Behaviour

The first order condition of the retail firms with respect to \( P_t(i) \) yields the following system for aggregate inflation:

\[
H_t = \Lambda_t p_t^w Y_t \left( \frac{\kappa_t}{\kappa_t - 1} \right) + \vartheta \beta E_t \left( 1 + \pi_{t+1} \right)^{\kappa_t} H_{t+1}
\]

\[
F_t = \Lambda_t Y_t + \vartheta \beta E_t \left( 1 + \pi_{t+1} \right)^{\epsilon - 1} F_{t+1}
\]

\[
1 - \vartheta \left( 1 + \pi_t \right)^{\kappa_t - 1} = (1 - \varpi) \left( \frac{H_t}{F_t} \right)^{1-\kappa_t} + \varpi \left[ 1 - \vartheta \left( 1 + \pi_{t-1} \right)^{\kappa_t - 1} \right] \left( \frac{1 + \pi_{t-1}}{1 + \pi_t} \right)^{1-\kappa_t}
\]
The system (28)-(30), once log-linearised, gives us the log-linearise Phillips curve:
\[ \pi_t = \kappa_c \hat{p}_t^w + \hat{\chi}_t + \chi_f \beta \pi_{t+1} + \chi_b \pi_{t-1} \tag{31} \]
where \( \chi_f = \frac{\bar{\varphi}}{\Upsilon}, \chi_b = \frac{\bar{\omega}}{\Upsilon}, \kappa_c = (1-\omega)(1-\varphi)(1-\varphi \beta)\), \( \Upsilon = \varphi + \omega (1 - \varphi + \varphi \beta) \), and where the cost push shock has been normalised.

2.5 Equilibrium

2.5.1 Aggregation and Market Clearing

The aggregation is characterised as follow. Let \( K_t^A \) denote the aggregate capital stock after the realisation of the exit shock of the wholesale firms, but before new investments and depreciation take place. Thus:
\[ K_t^A = (1 - \delta_e) K_t + \delta_e K_{0t}, \tag{32} \]
where \( K_t = \int_0^1 K_t^j \, dj \) is the aggregate capital stock of all firms at the end of of period \( t - 1 \) before the realization of the exit shock, and \( K_{0t} \) is the aggregate capital stock brought by new entrants.

In equilibrium, the labour market clears so the labour demand, \( \int_0^1 N_t^j \, dj \), must be equal to its supply, \( N_t \). Then, the aggregate labour demand of wholesale firms is given by:
\[ N_t = u_t A_t \left[ \frac{(1 - \alpha) p_t^w}{w_t} \right]^{\frac{1}{\alpha}} K_t^A, \tag{33} \]
where \( u_t \) is the capacity utilisation rate which is the same across firms, because wholesale firms have the same capital-labour ratio.

Denote \( Y_t^W = \int_0^1 Y_t^j \, dj \) the aggregate output of wholesale firms. The aggregation of their production functions yields:
\[ Y_t^W = (u_t K_t^A)^{\alpha} (A_t N_t)^{1-\alpha}. \tag{34} \]

In equilibrium, the aggregate supply of the wholesale goods \( Y_t^W \) has to be equal to the demand of the retailers \( \int_0^1 Y_t (i) \, di \). Thus, the final output is given by:
\[ Y_t \equiv \int_0^1 \left[ \frac{P_t^i}{P_t} \right]^{\kappa} Y_t (i) \, di = (u_t K_t^A)^{\alpha} (A_t N_t)^{1-\alpha}, \tag{35} \]
using (34), where \( Y_t^W = \int_0^1 Y_t (i) \, di \).
Let $b_t$ denote the aggregate real bubble at time $t$. When adding up the bubbles of the firms of all ages, the total real value of the bubble in the economy at time $t$ is given by:

$$b_t = \sum_{a=0}^{t} (1 - \delta_e)^a \omega \delta_e b_{t,a} = m_t b^*_t,$$  \hspace{1cm} (36)$$

where $b^*_t$ is the size of new emerging bubbles at date $t$ and where $m_t$ satisfies the recursion:

$$m_t = m_{t-1} (1 - \delta_e) \kappa_{t-1} + \delta_e \omega,$$ \hspace{1cm} (37)$$

with $m_0 = \delta_e \omega$. Using the law of motion of the bubble (25), restriction on the size of the new bubble is given by:

$$b^*_t = (1 - \delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}) \kappa_t b^*_{t+1}.$$ \hspace{1cm} (38)$$

Finally, substituting total real bubble at date $t$ (36) into the restriction on the size of the new bubble (38) yields the non arbitrage condition for the total bubble in the economy:

$$b_t = (1 - \delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{m_t}{m_{t+1}} (1 + G_{t+1}) \kappa_t b_{t+1}.$$ \hspace{1cm} (39)$$

The market clearing conditions for credits implies that the demand for loans is equal to supplies of savings, $L_t = \int_0^1 L^j_t \, dj = D_t = 0$. Moreover, competitive financial intermediaries require that the deposit rate is equal to the lending rate. Therefore $R^d_t = (1 - \delta_e) R^l_t$, taking into account that firms exits the market in each period with probability $\delta_e$. However, from (26) and $G_{t+1} > 0$ that follow:

$$\frac{(1 + \pi_{t+1})}{(1 - \delta_e) R^l_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + G_{t+1}) > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{(1 + \pi_{t+1})}{R^l_t},$$

where the RHS of the inequality comes from the FOC of the households (5). Consequently, households do not have the incentive to save, and prefer to borrow until their borrowing constraint binds (i.e. $D_t = 0$).[^1] Only firms that receive low efficiency shocks save and lend funds to productive firms.

Aggregating the value of all firms, the aggregate relative stock price is equal to:

$$p^*_t = q_t K_{t+1} + b_t,$$ \hspace{1cm} (40)$$

[^1]: Without borrowing constraints, no arbitrage implies that $G_{t+1} = 0$. In this case, (25) and the transversality condition would rule out bubbles.
where the aggregate stock holding of the household is normalised to a unit, \( s_{t+1} = 1 \). This equation reveals that the aggregate price of the stock has two components, the fundamental \( q_tK_{t+1} \) and the aggregate bubble \( b_t \).

The total capacity of external financing is given by:

\[
\varphi_t K_t + q_t \gamma_t K_t + b_t,
\]

which reflects the overall financial market conditions. Following Miao et al. (2015), a financial shock \( \zeta_t \) is introduced to capture the disturbance of the overall financial constraints, which is defined as:

\[
\zeta_t = \frac{\varphi_t}{q_t} + \gamma_t,
\]

so that the total capacity of external financing can be rewritten such as \( \zeta_t q_t K_t + b_t \).

Aggregating over the idiosyncratic shock \( \varepsilon_t \), the aggregate investment is given by:

\[
I_t^p = \left[ (u_t \Psi_t + \zeta_t q_t) K_t^A + b_t \right] \int_{\varepsilon > \varepsilon_t^*} d\Phi(\varepsilon),
\]

using the financial shock \( \zeta_t \), where \( \Psi_t = \alpha \left( \frac{1-\alpha}{u_t} \right) \left( p_t^w \right)^{1-\alpha} \) and \( \varepsilon_t^* = \frac{p_t^I}{q_t} \). Furthermore, the law of motion of capital is given by:

\[
K_{t+1} = (1 - \delta_{t+1}) K_t^A + I_t \int_{\varepsilon > \varepsilon_t^*} d\Phi(\varepsilon),
\]

and the aggregate price of installed capital follows:

\[
q_t = \left( 1 - \delta_e \right) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ u_{t+1} \Psi_{t+1} q_{t+1} (1 - \delta_{t+1}) + G_{t+1} (u_{t+1} \Psi_{t+1} q_{t+1}) + \zeta_{t+1} q_t \right]
\]

The resource constraint is given by:

\[
C_t + \left( 1 + \Omega \frac{I_t}{Z_t} - \lambda^I \right)^2 \frac{I_t}{Z_t} = Y_t
\]

using the budget constraint of the households (2), the flow-of-funds of the wholesale firms (11), the profit of the capital good producers (8) and the profit of the retailers.

Finally, the policy rule that I will consider is the following Taylor-type rule:

\[
R_t^I = R \left[ (1 + \pi_t)^{\phi_{\pi}} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \left( \frac{R_{t-1}^I}{R} \right)^{\phi_R} \exp(\epsilon_t^R) \right]
\]

where \( R \) is the natural rate at the zero inflation steady-state, and \( \phi_{\pi}, \phi_y \) and \( \phi_R \) are the three policy feedback coefficients on inflation, evolution of output and past interest rate. Empirical rules in this form are often used in empirical literature and are shown to behave well.
2.5.2 Equilibrium

Equations (3), (4), (7), (18), (22), (23), (26), (28), (29), (30), (32), (33), (35), (37), (39), (41), (42), (43), (44) and (45) jointly determine the 20 aggregate endogenous variables $C_t$, $N_t$, $p^I_t$, $p^w_t$, $u_t$, $K_t^A$, $K_t$, $w_t$, $\Psi_t$, $I_t$, $q_t$, $G_t$, $R_t$, $b_t$, $m_t$, $Y_t$, $H_t$, $F_t$ and $\pi_t$ where $\delta_t = \delta(u_t)$ and $\varepsilon^*_t = \frac{p^I_t}{q_t}$.

3 Bayesian Estimation

The model, presenting no occasionally binding condition in equilibrium, is log-linearised around the bubbly non-stochastic steady-state, and fit the US data using Bayesian estimation methods.

3.1 Data and Shocks

The model has seven shocks: 1) a TFP shock, $g_{dt}$, 2) an investment adjustment cost shock, $Z_t$, 3) an elasticity of substitution between any two specialised retail goods shock, $\kappa_t$, 4) a financial shock, $\zeta_t$, 5) a sentiment shock, $\kappa_t$, 6) a household taste shock $\xi_t$ and finally, 7) a labour supply shock $\psi_t$.

These shocks are identified by using seven time series to estimate the parameters of the model: 1) the Federal Funds Rate, 2) the industrial inflation rate, 3) the US real GDP, 4) the US real investment, 5) the relative price of investment, 6) the S&P 500 composite index and 7) Chicago Fed’s National Financial Conditions Index (NFCI). I compute the quarterly growth rates of real GDP, real investment, relative price of investment and the relative stock price for the estimation.

The data are available quarterly and cover the period from 1975Q1 to 2019Q4. Data for the industrial inflation (2), US GDP (3) and investment (4) are from the BEA website. The Federal Funds Rate (1), the relative price of investment (5) and the Chicago Fed’s National Financial Condition Index are retrieved on the FRED website. Finally, the stock price data (6) are the S&P composite index downloaded from Robert Shiller’s website. Figure 1 presents the transformed data used for the estimation.

3.2 Solution and Estimation Procedure

As in Miao et al. (2015), there is no need to to parametrise the depreciation function $\delta(\cdot)$ and the distribution function $\Phi(\cdot)$ because of the log-linearisation method. These terms will be components of estimated parameters. Yet, knowing the steady-state values of the following parameters is necessary: $\delta(1)$, $\delta'(1)$, $\delta''(1)$, $\Phi(\varepsilon^*)$ and $\mu = \frac{\delta(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$ where the capacity of utilisation is equal
Figure 1: Plots of the data from 1975Q1 to 2019Q4. This figure illustrates the different time series used for the Bayesian estimation.

to 1 in steady-state and $\varepsilon^*$ is the steady-state investment threshold for the idiosyncratic shock $\varepsilon_t$. These parameters will be estimated, except for $\delta(1)$ which will be calibrated.
The quarterly real gross rate of interest is calibrated using the means of inflation and Federal Funds Rate times series, \( R = R^l - \pi = 1.0048 \). As is standard in the literature, the quarterly subjective discount rate is calibrated to 0.995 using the quarterly real gross rate of interest \( R \). The inverse Frisch labour supply elasticity is set to 1/5 which is in the range of macroeconomics estimates. The capital share in production is also set to its traditional value 0.3. The coefficient of relative risk aversion is set at 2 and the elasticity of substitution between any two specialised retail goods is set to 11, yielding a steady-state mark-up value of 1.1 on intermediate good relative price \( p^w_t \). Finally, I set the exit parameter \( \delta_e \) to 2% as in Miao et al. (2015).

The steady-state depreciation rate \( \delta(u) \) where \( u = 1 \) in steady-state is calibrated to 0.025, and the steady-state investment to output ratio \( I/Y \) is set to 0.2. I use the mean of the growth rate of output to compute the steady-state quarterly gross growth rate of output \( g_a \), which is equal to 1.0068. Finally, using the real rate \( R \) and the output growth rate \( g_a \) to compute the steady-state relative size of the old bubble to the new bubble, \( \kappa = R/g_a \), which yields 0.9980.

The parameter \( K_0/\bar{K} \) is also estimated through the model. The mean prior on this parameter was 0.005 with a standard deviation of 0.001. I found that it converged to zero, thus I fixed it \( K_0/\bar{K} \).

Table 1 below summarises the calibrated parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.995</td>
<td>Quarterly subjective discount rate</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>Risk aversion coefficient</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1/5</td>
<td>Inverse Frisch labour supply elasticity</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.3</td>
<td>Capital share in production</td>
</tr>
<tr>
<td>( \delta_e )</td>
<td>0.02</td>
<td>Probability of exiting the economy</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>11</td>
<td>Elasticity of substitution between any two specialised retail goods</td>
</tr>
<tr>
<td>( g_a )</td>
<td>1.0068</td>
<td>Steady-state quarterly gross growth rate of output</td>
</tr>
<tr>
<td>( R )</td>
<td>1.0048</td>
<td>Quarterly natural gross rate of interest at the zero inflation steady-state</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.9980</td>
<td>Steady-state relative size of the old bubble to the new bubble</td>
</tr>
<tr>
<td>( u )</td>
<td>1</td>
<td>Steady-state capacity of utilisation rate</td>
</tr>
<tr>
<td>( \delta(1) )</td>
<td>0.025</td>
<td>Steady-state depreciation rate</td>
</tr>
<tr>
<td>( I/Y )</td>
<td>0.2</td>
<td>Steady-state investment-output ratio</td>
</tr>
<tr>
<td>( K_0/\bar{K} )</td>
<td>0.001</td>
<td>Ratio of capital endowment for a entering firms to total capital stock</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameters

The estimation was first initiated using a Markov jump-linear-quadratic (MJLQ) model à la Svensson (2005), where uncertainty takes the form of different “modes” (or regimes) that follow
a Markov process. The estimation was done with 4 different modes; *Dry Monetary Policy, Wet Monetary Policy, High Shock Volatility and Low Shock Volatility*. The analysis reflected that monetary policy was always dry and the volatility of shocks was always low. Therefore, I continue the estimation process without using any modes.

As in Miao et al. (2015), I use the NFCI time series to better identify the financial shock $\zeta_t$. The estimation of the model without the NFCI index produces a coherent smoothed financial shock series, yet the financial shocks are very persistent and $\rho_\zeta$ converges to 1. The introduction of the NFCI index helps to reduce the persistence of the financial shock. The financial shock is identified using the following measurement equation:

$$\text{NFCI}_t = -f_\zeta \hat{\zeta}_t - f_q \hat{q}_t - f_{Kb}(\hat{b}_t - \hat{K}_t)$$

which describes movements in the financing capacity of wholesale firms. An increase in $\hat{\zeta}_t$, $\hat{q}_t$, $\hat{b}_t$ or a decrease in $\hat{K}_t$ reduces the NFCI, in turn relaxes the financial constraint of the wholesale firms. However, the coefficient on marginal Tobin’s Q, $f_q$, converged to zero in every estimated specification. Thus I set it to zero ($f_q = 0$).

### 3.3 Estimated Parameters

Table 2 presents the prior and posterior distributions of the estimated parameters. Most prior distributions for the parameters are based on the posterior of Miao et al. (2015) the structural similarities between this study and theirs. The priors for parameters related to the structural parameters of New-Keynesian models are as follows: the mean prior of the Calvo parameter $\vartheta$ is set to 0.75 with a standard deviation of 0.01, and the mean prior of the price indexation parameter $\varpi$ is set to 0.2 with a standard deviation of 0.02. The feedback coefficient parameter on inflation $\phi_\pi$ has a prior mean of 1.5 and a standard deviation of 0.1, the feedback coefficient parameter on change in output $\phi_y$ has a prior mean of 0.4 and a standard deviation of 0.15, and the feedback coefficient parameter on past nominal interest rate $\phi_R$ has a prior mean of 0.4 and a standard deviation of 0.1.

The main differences between our estimates come from the estimation of the steady-state values for the financial constraint parameter $\zeta$ and the investment productivity distribution parameter $\mu$. Miao et al. (2015) had a relatively loose prior on these two parameters and explained that $\zeta$ was not particularly sensitive to the prior distribution. Using the same priors as Miao et al. (2015) for these two parameters, the posteriors of these two were significantly different and higher. The mean posterior of the financial shock $\zeta$ peaked at 0.7 and the elasticity of the
### Table 2: Prior and posterior distributions

<table>
<thead>
<tr>
<th>Param.</th>
<th>Distr.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>5%</th>
<th>95%</th>
</tr>
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<tbody>
<tr>
<td>( \theta )</td>
<td>Beta</td>
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<td>0.9882</td>
<td>0.0021</td>
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<td>0.1</td>
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<td>1.4024</td>
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<td>0.0121</td>
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<td>( \mu )</td>
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<td>0.7530</td>
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<td>0.0208</td>
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<td>( \sigma_{g_\pi} )</td>
<td>Inv-Gamma</td>
<td>0.015</td>
<td>0.005</td>
<td>0.0154</td>
<td>0.0010</td>
<td>0.0139</td>
<td>0.0170</td>
</tr>
<tr>
<td>( \sigma_\xi )</td>
<td>Inv-Gamma</td>
<td>0.012</td>
<td>0.005</td>
<td>5.6266</td>
<td>0.6558</td>
<td>4.5567</td>
<td>6.7090</td>
</tr>
<tr>
<td>( \sigma_\psi )</td>
<td>Inv-Gamma</td>
<td>0.03</td>
<td>0.005</td>
<td>0.0379</td>
<td>0.0044</td>
<td>0.0313</td>
<td>0.0456</td>
</tr>
<tr>
<td>( \sigma_R )</td>
<td>Inv-Gamma</td>
<td>0.005</td>
<td>0.0015</td>
<td>0.0074</td>
<td>0.0004</td>
<td>0.0068</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

The probability of undertaking investment at the steady-state cutoff \( \mu \) could range between between 5 and 12. Jointly or individually, the values of these parameters yold counter-intuitive results and did not match the range in previous studies. Covas and Den Haan (2011) reported that the financial constraint parameter ranges between 0.1 and 0.4, and Miao et al. (2015) estimated...
For the elasticity $\mu$, Miao et al. (2015) obtained a posterior mean of $\mu^M = 2.58$ and Wang and Wen (2012) found a similar estimate equal to 2.4. Consequently, I restricted the standard deviation priors for these two parameters to 0.01 around mean 0.15 for $\zeta$ and 2.3 for $\mu$. It seems that these mean posteriors of these two parameters do not correspond to their priors because I use data on the interest rate in addition to data on investment and stock prices. The introduction of the interest rate as a decision variable by the Central Bank and using it as an observable variable increases the posterior means of $\zeta$ and $\mu$. A higher $\zeta$ relaxes the borrowing constraint and makes the mean value of $\zeta$ less sensitive to changes in the interest rate.

The results of the estimation indicates that habits are very persistent, i.e. $\theta = 0.9882$, and that the adjustment cost parameter for price of investment is equal to 0.1781. These two parameters are significantly higher than in Miao et al. (2015), who found $\theta^M = 0.54$ and $\Omega^M = 0.03$. For the ‘curvature’ of the depreciation function $\delta(\cdot)$, I found that $\delta'' = 12.33$ which is similar to Miao et al. (2015). The posterior distributions of the feedback coefficients for the policy rule are in the usual ranges of the literature with posterior means of 1.637 for $\phi_\pi$, 0.32 for $\phi_y$, and 0.89 for $\phi_R$. The persistence and standard deviations posteriors are conventional, except for the taste shock $\hat{\xi}_t$ which tends to 1. Implications of the finding of this estimate is discussed in Section 4.

4 Results

The Result Section is composed of four analyses: i) an evaluation of the model in explaining historical bubble episodes, ii) a counterfactual experiment, iii) a analysis the transmission mechanism of monetary policy in a bubbly economy, and iv) an examination of alternate policy rules that react to stock prices.

4.1 Model Evaluation: Sentiment Shock, Bubbly Firms and Aggregate Bubble

During the last 50 years, the US economy experienced two major bubble episodes, the dot-com bubble and the subprime mortgage bubble. In this subsection, I investigate the explanatory power of the presented model about these two events.

As previously established, bubbles emerge because of by self-fulfilling beliefs about the value of wholesale firms. Moreover, movements in bubbles can be driven by household sentiment shocks $\hat{\kappa}_t$ about the relative size of bubbles between two firms of different ages. A positive sentiment shock increases the bubble size of young firms and increases the total value of the bubble. Finally,

\footnote{In the case of Miao et al. (2015), the mean posterior of the financial shock $\zeta$ did not deviate from the mean prior.}
the total value of the bubble depends on the aggregation of all bubbles in the economy. Due to stochastic lives of the wholesale firms, the aggregation of the bubble depends on the variable $m_t$ whose dynamics is described by equation (37). As in Miao et al. (2015), I interpret $m_t$ as the mass of firms having bubbles. The log-linearized version of $m_t$ is given by:

$$\hat{m}_t = (1 - \delta_e) \kappa (\hat{m}_{t-1} + \hat{\kappa}_{t-1})$$

(46)

This equation (46) establishes that fluctuations in the mass of bubbly firms depend on past fluctuations in the mass of bubbly firms plus fluctuations in past sentiment shock. Therefore, the sentiment shock affects $\hat{m}_t$ with a lag. However, the bubble law of motion, i.e. equation (39), depends on both $\hat{m}_t$ and expected $\hat{m}_{t+1}$. The latter implies that fluctuations in the aggregate bubble depend on both current and lagged sentiment shocks.

Figure 2: Sentiment shock, bubbly firms and aggregate bubbles. This figure illustrates the log-deviations from the steady-state of the sentiment shock $\hat{\kappa}_t$, the mass of bubbly firms $\hat{m}_t$, and the aggregate value of the bubble $\hat{b}_t$ estimated from the model.

Figure 2 illustrates the estimation of the sentiment shock (Panel A), the mass of bubbly firms (Panel B) and the total value of aggregate bubble (Panel C). How well does the model describe the boom and bust of the dot-com bubble? Panel B shows that the mass of bubbly firms slowly but constantly increased from the 90s until 2001 due to small positive sentiment shocks. This
increase in the mass of bubbly firms seems to have a positive effect on the development of the aggregate bubble (Panel C). During Q1 2001, Panel A clearly depicts a relatively persistent, but moderate, fall in sentiment shocks during the burst of dot-com bubble. We can see that this movement in the sentiment shock implies a net sustained fall in the mass of bubbly firms. This fall also seems to pass on to the total value of the aggregate bubble with a strong fall from 2000Q1 to 2002Q4.

Concerning the subprime mortgage bubble, we can see in Panel A of Figure 2 that the sentiment shock was relatively stable around steady-state between 2003 and 2007. This explains why the mass of bubbly firms slightly recovered but remained significantly below steady-state during the ‘booming’ bubble episode. In other words, the model may not be able to capture well the boom of the subprime mortgage bubble. One possible explanation for this pitfall could be that this boom is relatively localised to the housing market and thus did not strongly affect the S&P 500. Therefore, the boom is not captured in the data. However, the burst of the bubble is clearly depicted by a strong fall in sentiment shock which results in a large and substantial decrease in the mass of the bubbly firms. Panel C also shows that the total value of the bubble plummets from 2007Q4 to 2009Q1.

In the next subsection, I will show that the sentiment shock is not the most significant factor to explain volatility of the bubbles which contrasts with the results of Miao et al. (2015). The sentiment shock is able to explain the volatility in the bubble, but its effect is dominated by changes in the investment threshold $\hat{\varepsilon}_t^*$ and the taste shock $\xi_t$.

4.2 Counterfactual Experiment: No Sentiment Shock Pre-Financial Crisis

This section presents a counterfactual experiment in which the US economy would not be hit by a strong sentiment shock before the financial crisis. Figure 3 illustrates the counterfactual experiment with $\hat{\kappa}_t = 0$ from 2007Q2.

As established, the sentiment shock $\kappa_t$ only affects the mass of bubbly firms in the economy $m_t$, which in turn only affects the law of motion for the aggregate bubble $b_t$. Panel C shows that, while the sentiment shock is muted, the value of the aggregate bubble still plummets.

The change in the aggregate value of the firm, as small as it is, has large implications on the real variables and on some nominal variables (see Figure 3). A small change in the aggregate value of the bubble will have a direct effect on investment. We can see in Panel G in Figure 3 that investment falls far less. This change in aggregate investment has a net a positive effect on output, but this effect is relatively marginal during the crisis as exhibited in Panel FF.
Figure 3: No bubble sentiment shock pre-financial crisis. This figure illustrates the response of the economy from 2007Q2 with $\kappa_t = 0$.

The change in the sentiment shock results in high inflation, and thus in a high reaction of the nominal interest rate. Because of the higher cost of labour, the price of wholesale firms’ goods increases, which raises the marginal cost of retailers. Consequently, the Central Bank reacts to the increase in inflation and raises the nominal interest rate.

Surprisingly, the growth rate of stock prices remains unaffected once the sentiment shock is muted. The log-linearised detrended stock relative price is given by:

$$
\hat{p}_t^s = \underbrace{\text{fundamental value, } V_t^f}_{\text{fundamental value, } V_t^f} - qKg_a(\hat{q}_t\hat{K}_t+1) + \tilde{b}_t \tilde{b}_t
$$

While the bubble component of the stock market bubble marginally changes, we could have expected changes in the fundamental value of the stock market. It appears that the taste shock $\xi_t$ is the main driver of fluctuations in stock market prices and the bubble component. In contrast to Miao et al. (2015), I use data on the nominal interest rate and inflation in addition to the data on stock prices. The growth rate of stock prices appears to be too volatile while the interest rate
is relatively smoother. Consequently, the taste shock $\xi_t$ tends to capture the excess volatility of the data on the growth rate of stock price.

## 4.3 Monetary Policy Transmission in a Bubbly Economy

The monetary policy affects the entire economy by altering the borrowing cost for wholesale firms. By manipulating the credit market between wholesale firms, the Central Bank affects 1) wholesale firms’ demands for investment goods and labour, 2) the supply of goods to the retailers and the real marginal cost $p_t^w$, and 3) the market value of the wholesale firms and in turn the households’ return on the stock.

A contractionary policy of the Central Bank increases the borrowing cost due to a higher nominal rate. This policy has an intensive and extensive effect on the investment of wholesale firms. The intensive effect is a drop in the individual demand for investment goods. The extensive effect is a drop in the aggregate demand for investment goods because some wholesale firms with an efficiency shock close to the threshold will not have the incentive to borrow. This drop in demand implies a fall in the price of investment and a decrease in capital accumulation. Marginal $q$, price of installed capital, also falls because firms have too much capital.

This reduction in investment decreases aggregate capital and leads to a lower demand for labour, a lower utilisation rate and finally, a lower production. Nonetheless, despite a fall in output, the price of wholesale firm goods decreases because of the drop in the cost of labour. Consequently, inflation falls as retailers, facing a lower marginal cost, adjust their prices.

A noteworthy feature of the effect of monetary policy is an immediate fall of the need for bubbles. Since firms have too much capital, they do not need to make new investments. Therefore, bubbles are less needed and their value falls after an increase in the nominal interest rate. A drop in the aggregate value of the bubble implies a fall in the value of the firms, reflected by the value of stock price drops. This has an immediate negative wealth effect for the households, which leads to a drop in consumption.

## 4.4 Alternative Monetary Policies

In this section, I investigate the robustness of a monetary policy that reacts to stock prices in mitigating the impact of a sentiment shock on the economy. In this intent, I specify two alternative monetary policies than the one estimated in the benchmark model presented.

The first alternative policy reacts to the change in stock prices (Model 1). In its log-linearised
Figure 4: Impulse responses after a positive sentiment shock and alternative monetary policies. This figure illustrates the response of the economy after a unit sentiment shock $\hat{\kappa}_t$ under the benchmark estimation (i.e. $\phi_{ps} = 0$) and under two alternative policy rules that reacts to changes on stock price (Model 1: $\phi_{ps} = 2$) and to stock price deviation from its steady-state (Model 2: $\phi_{ps} = 0.1$). The vertical axes represent the percentage deviations from the variables steady-state levels and the unit of time for the horizontal axes correspond to quarters.

In log-linearised form, this monetary policy follows the rule:

$$\hat{R}_t = (1 - \phi_R) \left[ \phi_\pi \pi_t + \phi_y \Delta \hat{Y}_t + \phi_{ps} \Delta \hat{p}_s + \phi_R \hat{R}_{t-1} \right]$$  \hspace{1cm} (47)

The second policy reacts to deviations of stock prices from its steady-states (Model 2). In its log-linearised form, this monetary policy follows the rule:

$$\hat{R}_t = (1 - \phi_R) \left[ \phi_\pi \pi_t + \phi_y \Delta \hat{Y}_t + \phi_{ps} \hat{p}_s + \phi_R \hat{R}_{t-1} \right]$$  \hspace{1cm} (48)

For this experiment, I calibrate $\phi_{ps} = 2$ for Model 1 and $\phi_{ps} = 0.1$ for Model 2.

Figure 4 presents the response of the economy after a unit sentiment shock $\hat{\kappa}_t$ under the benchmark estimation (i.e. $\phi_{ps} = 0$) and the two alternative policy rules specified above (i.e. Model 1 and 2). Under both alternative policies, the stock price does not increase as much
as under the benchmark model (Panel D). However, these smaller deviations are not due to a reduction in the value of the bubble (Panels B), which is only marginally affected by the policy reaction. The lower deviation of the stock price from its steady-state is the consequences of larger negative deviations in the fundamental value (Panels C).

The drop in the fundamental value of the stock represents a decrease in the aggregate value of the wholesale firms. This would imply a drop in investment, because of a contraction of the borrowing capacity of the firm. However, bubbles counter-balance this drop and the aggregate investment remains above its steady-state value after the sentiment shock (Panel G). A lower investment relative to the benchmark model implies a lower cost of labour and lower output.

Both alternative rules manage to stabilise investment and output faster than the benchmark model. The main difference between these two rules is that Model 2 implies a higher interest rate after the impact of the sentiment shock, which has consequently a more aggressive effect on inflation. This higher aggressiveness of the interest rate towards inflation permits to quickly stabilise inflation in contrast to Model 1 (Panel K).

Can monetary policy directly affect the total value of the bubble after a sentiment shock? The answer is mostly no. My finding shows that a rise in the interest rate has a direct negative effect on the aggregate value of the bubble. However, this effect is countered because there is a need for bubbles due to the contraction of the borrowing capacity of the wholesale firms; the fundamental value falls, so bubbles are needed to ease the borrowing constraints. Consequently, the interest rate is an adequate instrument to reduce the volatility of the value of the bubble but is still useful to quickly stabilise output (and inflation under Model 2) by reacting to stock prices.

5 Conclusion

In this paper, I developed and estimated a New Keynesian model with stock market bubbles. Moreover, I analysed the effects of bubbles on real and nominal variables and the transmission mechanism of monetary policy in a bubbly economy. Finally, I investigate if a monetary policy that leans against the wind can reduce the volatility of bubbles. Based on the presented analysis, I can draw the following conclusions.

First, the volatility of value of bubbles can explain a significant fraction of movements in investment, output and inflation. Bubbles, which exist in this economy because of self-fulfilling beliefs, allow firms to increase the borrowing capacity and increase investment and production. Thus, movements in bubbles directly affect the volatility of these variables as well as inflations.

Second, the sentiment shock is not the main cause for changes in bubble size. Because of tight
borrowing constraints, firms are very sensitive to changes in the cost of borrowing. Therefore, changes in the interest rates, in the price of installed capital, and in the price of capital goods can have significant intensive and extensive effects on the investment decisions of firms. Moreover, I found that the taste shock is also an important factor to explain variations in the volatility of the bubble’s value. Movements in these variables, or the taste shock, affect the number of investing firms and thus the volatility of bubbles. 

Finally, monetary policies that react to asset prices can mitigate the impact of the sentiment shock. In this paper, I present two alternative policies; i) a policy rule that reacts to the changes in stock prices and ii) a policy rule that reacts to deviations of the stock price from its steady-state. Such policies can stabilise quicker output, investment and stock prices. However, these policies have different effects on inflation. Reacting to changes in stock prices is not able to stabilise fast enough inflation and thus can reduce welfare because of persistent inflation. In contrast, a policy rule that reacts to the deviations of the stock price from steady-state can stabilise faster inflation than the estimated rule and the first alternative rule.

References


