Stock Market Bubbles and Monetary policy: a Bayesian Analysis*

Arthur Galichère†

University of Warwick

February 12, 2023

Abstract

This paper develops and estimates a DSGE model with stock market bubbles and nominal rigidities using Bayesian methods. Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs, and their movements are driven by a sentiment shock. This paper shows that stock market bubbles are an important factor in explaining volatility in investment, output, and also in inflation. Moreover, a monetary policy rule that targets stock prices can help to diminish the impact of bubble sentiment shocks, and thus stabilise the economy faster than a policy rule that does not react to asset prices.

Key Words: Bubbles, Monetary Policy, Bayesian estimation, DSGE.
JEL Reference Number: E22, E32, E43, E44, E52

*I am grateful to Tatiana Kirsanova for very helpful comments and suggestions. All errors are mine.

†Address: Office S0.70, Department of Economics, University of Warwick, Coventry, CV4 7AL, United Kingdom; e-mail: arthur.galichere.research@gmail.com

1 Introduction

The economic upturn and downturn associated with the subprime mortgage bubble in the US revived a long debate on how monetary policy should react to asset price developments. The current consensus about monetary policy is that the main objective of Central Banks is to maintain price stability, that is to say keeping low and stable inflation. However, price stability generally concerns the stabilisation of the consumer prices index, which covers only a segment of prices in the economy. While the omission of asset prices for monetary policy is normally not considered as a problem, large movements in asset prices and bubbles' bursts led many economists to reconsider if the focus of monetary policy on consumer prices alone is still pertinent.

Before the 2008 financial crisis, the conventional strategy often named the "Jackson Hole Consensus", calls for Central Banks to focus on maintaining price stability and stabilizing the output gap. Thus, Central Banks should ignore asset price fluctuations unless they are a threat to price and output stability (e.g. Bernanke and Gertler, 1999, 2001; Kohn, 2006). One of the main reasons for this consensus is that instruments of monetary policy were judged 'too blunt' to successfully target asset prices. However, this strategy prescribes that Central Banks should take the necessary actions (e.g. via interest rate cuts) once a bubble collapsed in order to protect the economy against the harmful effects of the bubble's burst.

This asymmetric strategy for reacting to asset price developments was challenged by many economists arguing that price stability would not guaranty financial stability (e.g. Cecchetti et al., 2000; Borio and Lowe, 2002). The opposing prominent strategy for Central Banks is 'leaning against the wind', which advocates that Central Banks should try to mitigate the risk associated with the developments and bursts of bubbles. In this case, central banks are required to tighten their monetary policy stance in the face of an inflating asset market, even if it creates a temporary deviation from their price stability objective.

The depression of the US economy following the 2008 banking crisis led many economists to agree that monetary policy should also focus on financial stability. A pure passive "cleaning up the mess" policy is likely to be more costly than a 'leaning against the wind' policy. Ikeda (2017) concluded that the optimal monetary policy should be tightened to control the output boom caused by the bubble at the expense of inflation stabilisation. Miao et al. (2019) argue that, under adaptive learning, monetary policy that leans against the wind can reduce the volatility of bubbles. Galichère (2021) found that monetary policy that targets asset price bubbles can deflate the bubble. However, using monetary policy to deflate a bubble can be costly in terms of output and a monetary policy that overreacts to asset prices can generate a recession. In contrast, Galí and Gambetti (2015) argued Central Banks should not lean against the wind. They found that tightening monetary policy would persistently increase stock prices. While there is no agreement on how Central Banks should react to bubbles, there is a consensus that monetary policy may have a role in addressing bubbles.

This paper investigates how monetary policy interacts with stock market bubbles and asks; Can lessen the impact of bubbles, How and at what cost. To answer these questions, I develop a New Keynesian model with rational bubbles, where bubbles can exist because of financial friction (e.g. Miao and Wang, 2018). In contrast to the literature that mainly focuses on pure bubbles (e.g. Martin and Ventura, 2012; Galí, 2014; Hirano and Yanagawa, 2017), the proposed model is based on Miao et al. (2015) where the stock market price of wholesale firms contains a bubble component in addition to the fundamental value.[1] Unlike pure bubbles, stock market bubbles are attached to productive firms with positive dividends and are not separately tradable from firm stocks. The stock price bubbles can emerge in different firms or in different sectors, and their emergence or collapse may be unrelated to the emergence or collapse of pure bubbles.

Bubbles emerge through a positive feedback loop mechanism supported by self-fulfilling beliefs. Precisely, households believe that the value of some wholesale firms may not be equal to their fundamentals. These firms, which pledge their assets as collateral in order to borrow funds, are able to relax their borrowing constraints because of the 'optimistic' beliefs of households on firms' values. Consequently, firms are able to borrow more and increase profit, which in turn raise the value of these firms. In this sense, bubbles can exist because of self-fulfilling beliefs. Without the presence of bubbles, these firms would be unable to borrow extra funds and deliver higher profits. Finally, as in Miao et al. (2015), the beliefs of the households about the movement of the bubble is modelled with the introduction of a sentiment shock. I estimate this sentiment shock and evaluate its importance in explaining changes in real and nominal variables.

I find that bubbles can cause large fluctuations in aggregate variables such as investment or output, and can also be the cause of high inflation. Moreover, as in Miao et al. (2019) or Galichère (2021), I found that leaning against the wind can reduce the impact of the sentiment shock which drives bubbles. The latter finding is based on evaluation of two alternative policy rules that target stock prices: the first rule reacts to changes in stock prices and the second reacts

[1] The value of an asset is equal to its market fundamental, that is to say, the expected and discounted present value of its dividends (or more generally its rents), plus a bubble component. Pure bubbles are defined as intrinsically useless assets, that it to say that have no fundamental value. However, these assets have a positive price. Such assets are often interpreted in the literature as money, gold or lands (e.g. Weil, 1987; Kocherlakota, 2009).

to deviation of stock price from its trend. The first rule reacts to changes in stock prices while the second reacts to the deviations from the stead-state of the stock price. I find that these rules can reduce the impact of bubble sentiment shock, but not the volatility of the bubble size itself. Nonetheless, these alternative policies can stabilise quicker aggregate output, investment and the stock price than a policy rule that does not react to the stock price.

Finally, while these alternative policy rules can quickly stabilise the economy after a sentiment shock, their specification matters for their reaction to inflation. My analysis shows that a policy that reacts to changes in stock prices is not successful in promptly bringing back inflation to steady-state. In contrast, the policy rule that reacts to deviation from steady-state of the stock price will be more aggressive towards inflation. This type of rule will stabilise quicker inflation than the traditional rule or the first alternative rule mentioned above.

The rest of the paper is structured as follows. Section 2 outlines the model. Section 3 presents the calibration of structural parameters, the estimation procedure and the estimated parameter. Section 4, which presents the main findings, is composed of four parts: i) an evaluation of the model in explaining historical bubble episodes, ii) a counterfactual experiment, iii) a analysis the transmission mechanism of monetary policy in a bubbly economy, and iv) an examination of alternate policy rules that react to stock prices. Finally, section 5 concludes.

2 The Model

The model is based on the real model presented in Miao et al. (2015) and incorporates nominal rigidities à la Calvo to study the interaction between monetary policy and stock market bubbles. The main structure of proposed model is presented in this section, and all the details given in Appendix A. The model represents a discrete time economy populated by households, capital good firms, wholesale firms, retail firms and a Central Bank. Households, capital goods firms and retail firms have infinite lives while wholesale firms operate on the market for a stochastic length of time.

Three financial assets are available in the economy: loans, deposits and stocks of wholesale firms. Households can deposit and invest in wholesale firms stocks but cannot borrow. Wholesale firms can borrow funds for their production or save unused funds. Retail and capital goods firms can neither borrow nor save. Finally, the stock price of the wholesale can contain a rational asset price bubble because of financial friction. The size of these bubbles is stochastic.

2.1 Households

2.1.1 Decision Problem

Each household derives utility from consumption and leisure according to the expected utility function:

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t/A_t - \theta C_{t-1}/A_{t-1})^{1-\sigma}}{1-\sigma} - \psi_t \frac{N_t^{1+\eta}}{1+\eta} \right] \xi_t, \tag{1}$$

where C_t denotes consumption and N_t is the household's labour supply. The household consumes C_t and provides labour N_t to wholesale firms for the nominal wage W_t. The household can accumulate wealth by purchasing shares of aggregate stock price s_{t+1} at price P_t^s and by saving in deposits D_t (where $D_t \geq 0$) at the deposit rate R_t^d.

The representative household's budget constraint is given by:

$$C_t + p_t^s s_{t+1} + d_{t+1} \frac{(1+\pi_{t+1})}{R_t^d} = w_t N_t + \Pi_t^I + \Pi_t^F + d_t + (d_t^s + p_t^s)\, s_t, \tag{2}$$

where p_t^s is the relative price of the stock, d_t is the real quantity of deposits, π_{t+1} is the inflation rate, w_t is the real wage and d_t^s is the real aggregate dividend on their stock investment. Household receives real profits from capital goods firms Π_t^I, profit from retailers Π_t^F. Moreover, the gross inflation rate $(1 + \pi_{t+1}) = P_{t+1}/P_t$, where P_t is price level of consumption.

The representative agent maximises (1) subject to (2).

2.1.2 Optimal Behaviour

The remainder of the household's problem is standard. The first order conditions for habit-adjusted consumption, labour and deposits are given by:

$$\Lambda_t = \frac{\xi_t}{(C_t/A_t - \theta C_{t-1}/A_{t-1})^{\sigma}} - \theta \beta E_t \left[\frac{\xi_{t+1}}{(C_{t+1}/A_{t+1} - \theta C_t/A_t)^{\sigma}} \right], \tag{3}$$

$$\frac{N_t^{\eta}}{\Lambda_t} = \frac{w_t}{\xi_t \psi_t} (1 - \tau_t), \tag{4}$$

$$\Lambda_t = \beta E_t \left[\frac{\Lambda_{t+1}}{(1+\pi_{t+1})} \right] R_t^d \qquad \text{if } d_t > 0, \tag{5}$$

where Λ_t represents the marginal utility of consumption. The first order condition for share of aggregate stock investment is:

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} r_t^s \right] \qquad \text{if } s_{t+1} > 0, \tag{6}$$

where the expected real rate of stock return is defined as $r_t^s \equiv E_t\left[\left(d_{t+1}^s + p_{t+1}^s\right)/p_t^s\right]$. Equations (6) and (5) set the non-arbitrage condition between stock investments and deposits, which is given by:

$$E_t\left[\frac{\Lambda_{t+1}}{(1+\pi_{t+1})}\right]R_t^d = E_t\left[\Lambda_{t+1}r_t^s\right].$$

2.2 Capital Producers

2.2.1 Decision Problem

The households own capital producers and receive the profit Π_t^I. A representative capital goods firm produces new capital using input of final output and subject to adjustment costs. It sells new capital I_t to wholesales firms at price P_t^I. The objective of a capital producer is to choose I_t to maximise:

$$V^I = \max_{I_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0}\left[p_t^I I_t - \left(1 + \frac{\Omega}{2}\left[\frac{I_t}{I_{t-1}} - \lambda^I\right]^2\right)\frac{I_t}{Z_t}\right],$$

where p_t^I is the relative price of capital goods, λ^I is the growth rate of aggregate investment, $\Omega > 0$ is the adjustment cost parameter and Z_t represents an investment cost shock that follow the exogenous process:

$$\ln Zt = \rho_Z \ln Z_{t-1} + \epsilon_t^Z$$

where ϵ_t^Z is an independent and identically distributed shock (IID) over time.

2.2.2 Optimal Behaviour

The optimal level of investment goods satisfies the first order condition with respect to I_t:

$$Z_t p_t^I = 1 + \frac{\Omega}{2}\left[\frac{I_t}{I_{t-1}} - \lambda^I\right]^2 + \Omega\frac{I_t}{I_{t-1}}\left[\frac{I_t}{I_{t-1}} - \lambda^I\right] - \beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{I_{t+1}}{I_t}\right)^2 \Omega\left[\frac{I_{t+1}}{I_t} - \lambda^I\right]\frac{Z_t}{Z_{t+1}} \quad (7)$$

and household receive the real profit:

$$\Pi_t^I = p_t^I I_t - \left(1 + \frac{\Omega}{2}\left[\frac{I_t}{I_{t-1}} - \lambda^I\right]^2\right)\frac{I_t}{Z_t} \quad (8)$$

2.3 Wholesale Firms

Following Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Gertler and Kiyotaki (2010), and Miao et al. (2015), exogenous entry and exit of firms is assumed because of non-arbitrage condition. To understand the necessity of this mechanism, suppose that households believe that each wholesale firm's stock may contain a bubble and that this bubble may burst with some probability. Because of rational expectations, a bubble cannot re-emerge in the same firm after bursting. Otherwise there would be an arbitrage opportunity. This means that none of the firms would contain any bubble once all bubbles have burst if no new firms enter the economy.

A firm may exit with an exogenously given probability δ_e each period. After exiting the economy, its value is zero and a new firm enters the economy without costs so that the total measure of firms is fixed at unity in each period. A new firm entering at date t starts with an initial capital stock K_{0t} and then operates in the same way as older firms. Moreover, each new firm may bring a new bubble into the economy with probability ω.

Wholesale firms make investment decisions that maximize their cum-dividend stock market value of the firms. They can purchase investment goods I_t from capital producers at price P_t^I and they sell their good Y_t^j to retail firms at price P_t^w.

2.3.1 Decision Problem

A wholesale firm $j \in [0,1]$ combines capital K_t^j and labour L_t^j to produce intermediate goods Y_t^j using the production function:

$$Y_t^j = \left(u_t^j K_t^j\right)^\alpha \left(A_t N_t^j\right)^{1-\alpha}, \tag{9}$$

where $\alpha \in (0,1)$, u_t^j denotes the capacity of utilisation rate and A_t denotes the labour-augmenting technology shock (or total factor productivity (TFP) shock given the Cobb-Douglas production function). For a new firm entering at date t, I set $K_t^j = K_{0t}$.

Assume that the capital depreciation rate between period t and period $t+1$ is given by $\delta_t^j = \delta(u_t^j)$, where δ is a twice continuously differentiable convex function that maps a positive number into $[0,1]$. The function $\delta(\cdot)$ does not need to be parametrised because the model will be solved using the log-linearisation solution method, where the steady-state capacity utilisation rate will be normalized to 1.

The capital stock evolves according to:

$$K_{t+1}^j = \left(1 - \delta_t^j\right) K_t^j + \varepsilon_t^j I_t^j, \tag{10}$$

where I_t^j denotes investment and ε_t^j is an idiosyncratic shock that measures the efficiency of the investment. Investment is assumed to be irreversible at the firm level so that $I_t^j \geq 0$. Moreover, ε_t^j is an IID shock across firms and over time, and is drawn from the fixed cumulative distribution

Φ over $[\varepsilon_{min}, \varepsilon_{max}] \subset (0, \infty)$ with mean 1 and probability density function ϕ. This shock induces firm heterogeneity in the model. For tractability, assume that the capacity utilisation decision is made before the observation of investment efficiency shock ε_t^j. Consequently, the optimal capacity utilisation does not depend on the idiosyncratic shock ε_t^j.

In each period t, firm j can make investment I_t^j by purchasing investment goods from capital producers at the price P_t^I. Its real flow-of-funds constraint is given by:

$$d_t^{sj} + p_t^I I_t^j - l_{t+1}^j \frac{(1+\pi_{t+1})}{R_t^l} = p_t^w Y_t^j - w_t A_t N_t^j - l_t^j, \tag{11}$$

where $l_{t+1}^j > 0$ (< 0) represents the real quatity of borrowing (savings) at time t, R_t^l represents the lending rate, p_t^w is the relative price of wholesale firms' goods and $d_t^{sj} > 0$ (< 0) represents dividends (new equity issuance). Assume that external financial markets are imperfect so that firms are subject to the constraint on new equity issuance:

$$d_t^{sj} \geqslant -\varphi_t K_t^j, \tag{12}$$

where φ_t is an exogenous stochastic shock to equity issuance. The demand for capital good is constrained such that:

$$0 \leqslant p_t^I I_t^j \leqslant p_t^w Y_t^j - w_t A_t N_t^j + \varphi_t K_t^j - l_t^j + l_{t+1}^j \frac{(1+\pi_{t+1})}{R_t^l}. \tag{13}$$

In addition, external borrowing is subject to the credit constraint:

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \underbrace{\bar{V}_{t+1,a+1}\left(K_{t+1}^j, l_{t+1}^j\right)}_{\text{continuation value of the firm}} \geqslant \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \underbrace{\bar{V}_{t+1,a+1}\left(K_{t+1}^j, 0\right)}_{\substack{\text{value of the firm if it defaults}\\ \text{when the repayment is relieved}\\ \text{and debt is renegotiated}}} \tag{14}$$

$$-\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \underbrace{\bar{V}_{t+1,a+1}\left(\gamma_t K_t^j, 0\right)}_{\text{the threat value to the lender}} \tag{15}$$

where $\bar{V}_{t,a}\left(K_t^j, l_t^j\right) \equiv \int V_{t,a}\left(K_t^j, l_t^j, \varepsilon_t^j\right) d\Phi(\varepsilon)$ represents the ex-ante value after integrating out ε_t^j and $V_{t,a}\left(K_t^j, l_t^j, \varepsilon_t^j\right)$ represents the cum-dividends stock market value of the firm of age a with with assets K_t^j, debt l_t^j and idiosyncratic investment shock ε at time t . In equation (15), γ_t represents the collateral shock that reflects the frictions in the credit market. Note that a represents the age of the firm. The equity value depends on the age of the firm because it contains a bubbles component that is age dependent.

Following Miao and Wang (2018), equation (15) is an incentive constraint in a contracting problem between the firm and the lender which ensures firm j has no incentive to default in equilibrium. The firm has limited commitment and can default on debt l_{t+1}^{j} at the beginning of period $t+1$. If the firm does not default, its continuation value is given by $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,a+1}\left(K_{t+1}^{j}, l_t^{j}\right)$. If the firm defaults, the debt is renegotiated, the repayment is relieved and the value of the firm is $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,a+1}\left(K_{t+1}^{j}, 0\right)$. The lender can seize the collateralised asset $\gamma_t K_t^{j}$ and keep the firm running with these assets by reorganizing the firm. Thus the threat value to the lender is $\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \bar{V}_{t+1,a+1}\left(\gamma_t K_t^{j}, 0\right)$.[2] Then the RHS of equation (15) is the value of the firm if it chooses to default.

An intermediate goods producer j with age a chooses labour, $N_t^{j} \geqslant 0$, investment, $I_t^{j} \geqslant 0$, debt, $l_{t+1}^{j} \geqslant 0$, and capacity of utilisation, $u_t^{j} \geqslant 0$, to maximize its value:

$$\begin{aligned} V_{t,a}\left(K_t^{j}, l_t^{j}, \varepsilon_t^{j}\right) &= \max_{I_t^{j}, N_t^{j}, l_{t+1}^{j}, u_t^{j}} p_t^{w} Y_t^{j} - \left(w_t A_t N_t^{j} + I_t^{j} p_t^{I}\right) - l_t^{j} + l_{t+1}^{j} \frac{(1+\pi_{t+1})}{R_t^{l}} \\ &\quad + (1-\delta_e)\, \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1}\left(K_{t+1}^{j}, l_{t+1}^{j}, \varepsilon_{t+1}^{j}\right), \end{aligned}$$

subject to the production function (9), the law of motion of capital (10), the constraint on new equity issuance (12), the borrowing (15) and the flow of funds (11).

As in Miao et al. (2015), I conjecture and verify that the value function takes the form:

$$V_{t,a}\left(K_t^{j}, l_t^{j}, \varepsilon_t^{j}\right) = Q_t\left(\varepsilon_t^{j}\right) K_t^{j} + B_{t,a}\left(\varepsilon_t^{j}\right) - Q_t^{L}\left(\varepsilon_t^{j}\right) l_t^{j}. \tag{16}$$

2.3.2 Optimal Behaviour

The first order condition of the wholesale firm's problem with respect to labour given the wage rate W_t and the capacity utilisation rate u_t^{j} yields the labour demand of the wholesale firm j:

$$N_t^{j} = \frac{u_t^{j}}{A_t} \left[\frac{(1-\alpha)\, p_t^{w}}{w_t}\right]^{\frac{1}{\alpha}} K_t^{j}. \tag{17}$$

Given the wage rate w_t and the capacity utilisation rate u_t^{j}, the production poblem can be simplified such that:

$$\max_{N_t^{j}} p_t^{w} Y_t^{j} - w_t A_t N_t^{j} = u_t^{j} \Psi_t K_t^{j}$$

[2]The variable γ_t may be interpreted as an efficiency parameter in the sense that lender may not be able to efficiently use the firm's assets K_{t+1} (Miao and Wang, 2018).

where Ψ_t is given by:

$$\Psi_t = \alpha \left[\frac{1-\alpha}{w_t} \right]^{\frac{1-\alpha}{\alpha}} (p_t^w)^{\frac{1}{\alpha}} \tag{18}$$

after substituting out the labour decision (17) of the production problem.

Using (6), the date-t ex-dividend stock relative price of the firm j of age a can be rewritten as:

$$p_t^{s,j} = (1-\delta_e)\,\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1,a+1}\left(K_{t+1}^j, l_{t+1}^j, \varepsilon_{t+1}^j\right) \right].$$

Given the above conjecture (16), stock relative price can be rewritten in the form:

$$p_{t,a}^{sj} = q_t K_{t+1}^j + b_{t,a} - q_t^L l_{t+1}^j,$$

where q_t, $b_{t,a}$ and q_t^L define such that:

$$q_t = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}\left(\varepsilon_{t+1}^j\right), \tag{19}$$

$$b_{t,a} = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} B_{t+1,a+1}\left(\varepsilon_{t+1}^j\right), \tag{20}$$

$$q_t^L = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}^L\left(\varepsilon_{t+1}^j\right).$$

Note that q_t, $b_{t,a}$ and q_t^L do not depend on future idiosyncratic shocks ε_{t+1}^j because they are integrated out.

The first order condition for l_{t+1}^j using the guess of the value function (16) gives:

$$q_t^L = \frac{(1+\pi_{t+1})}{R_t^l},$$

and the credit constraint can be rewritten such that:

$$q_t \gamma_t K_t^j + b_{t,a} \geqslant \frac{(1+\pi_{t+1})}{R_t^l} l_{t+1}^j.$$

The investment level I_t^j of a wholesale firm j with a bubble depends on the efficiency shock of the investment ε_t^j being greater that the threshold ε_t^*. Consequently, the optimal investment level I_t^j of a wholesale firm j respects that:

$$I_t^j p_t^I = \begin{cases} u_t^j \Psi_t K_t^j + \varphi_t K_t^j - l_t^j + q_t \gamma_t K_t^j + b_{t,a}, & \text{if } \varepsilon_t^j \geqslant \varepsilon_t^*, \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

where the investment threshold, $\varepsilon_t^* \equiv \frac{p_t^I}{q_t}$, is given by the first order condition for I_t^j using the guess of the value function (16).

Each firm chooses the same capacity utilisation rate u_t satisfying:

$$\Psi_t \left(1 + G_t\right) = q_t \delta^{'}(u_t), \tag{22}$$

where G_t satisfies:

$$G_t = \int\limits_{\varepsilon \geqslant \varepsilon_t^*} \left(\frac{\varepsilon}{\varepsilon_t^*} - 1\right) d\Phi\left(\varepsilon\right). \tag{23}$$

The the price of installed capital, the bubble, and the lending rate satisfy:

$$q_t = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \begin{bmatrix} u_{t+1}\Psi_{t+1} + q_{t+1}\left(1-\delta_{t+1}\right) \\ +G_{t+1}\left(u_{t+1}\Psi_{t+1} + q_{t+1}\gamma_{t+1} + \varphi_{t+1}\right) \end{bmatrix}, \tag{24}$$

$$b_{t,a} = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left(1 + G_{t+1}\right) b_{t+1,a+1}, \tag{25}$$

$$\frac{\left(1+\pi_{t+1}\right)}{R_t^l} = (1-\delta_e)\,\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left(1 + G_{t+1}\right). \tag{26}$$

where $\delta_t = \delta(u_t)$.

2.3.3 Sentiment Shock

The household beliefs on the movement of bubbles is modelled with the introduction of a sentiment shock κ_t. Denote $b_{t,a}$ the real value of the bubble attached to a wholesale firms with age a at time t. Households believe that a new firm in period t may contain a bubble of real size $b_{t,0} = b_t^*$ with probability ω. Then the total value of emerging bubble in the economy at date t is given by $\omega\delta_e b_t^*$. Moreover, they believe that the relative size of the bubbles at date $t+a$ for any two firm born at date t and $t+1$ is given by κ_t such that:

$$\kappa_t = \frac{b_{t+a,a}}{b_{t+a,a-1}}, \quad t \geqslant 0,\ a \geqslant 1.$$

The relative size of bubbles κ_t follows an exogenously given process:

$$\ln \kappa_t = (1-\rho_\kappa)\ln\kappa + \rho_\kappa \ln \kappa_{t-1} + \epsilon_t^\kappa \tag{27}$$

where ρ_κ is the persistence parameter and ϵ_t^κ is an IID normal random variable with mean zero and variance σ_κ^2. This process reflects the beliefs of households about the fluctuations of bubbles and is interpreted as the sentiment shock.

2.4 Retailers

2.4.1 Decision Problem

There is a continuum of differentiated retail firms of measure one, each indexed by i. Each retail firm i buys a wholesale good Y_t^j at price P_t^w and repackage into a specialized retail good $Y_t(i)$. Retailers sell their specialized retail good $Y_t(i)$ to competitive final firms at price $P_t(i)$.

Firm's optimisation problem is standard: a firm i chooses prices to maximise its profit:

$$V_t(i) = \max_{\{P_t(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} (\vartheta\beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\left(\frac{P_t(i)}{P_{t+s}} - p_{t+s}^w \right) Y_{t+s}(i) \right],$$

subject to the competitive demand constraint for good $Y_t(i)$:

$$Y_t(i) = \left[\frac{P_t^i}{P_t} \right]^{-\varkappa_t} Y_t,$$

where p_t^w is the real marginal cost of the retailer and $\varkappa_t > 1$ governs the elasticity of substitution between any two specialized retail goods. Retailers are subject to cost push shocks that affect the elasticity of substitution between any two retail goods, where $\varkappa_t$ follows an exogenously given process:

$$\ln \varkappa_t = (1 - \rho_\varkappa) \ln \varkappa + \rho_\varkappa \ln \varkappa_{t-1} + \epsilon_t^\varkappa$$

Price rigidity and price indexation are introduced as following: i) like in Calvo (1983), a firm i at time t has the opportunity to reset its price $P_t(i)$ with probability ϑ ; ii) when it has the chance of resetting its price, it chooses price optimally, $P_t^*(i)$, with probability $1 - \varpi$, or chooses its new price with probability ϖ according to the simple rule of thumb $P_t^b = P_{t-1}^R \pi_{t-1}$, where P_t^R is given by:

$$P_t^R = \left[(1 - \varpi) P_t^*(i)^{1-\varkappa_t} + \varpi (P_t^b)^{1-\varkappa_t} \right]^{\frac{1}{1-\varkappa_t}}$$

2.4.2 Optimal Behaviour

The first order condition of the retail firms with respect to $P_t(i)$ yields the following system for aggregate inflation:

$$H_t = \Lambda_t p_t^w Y_t \frac{\varkappa_t}{(\varkappa_t - 1)} + \vartheta\beta E_t (1 + \pi_{t+1})^{\varkappa_t} H_{t+1} \tag{28}$$

$$F_t = \Lambda_t Y_t + \vartheta\beta E_t (1 + \pi_{t+1})^{\varepsilon-1} F_{t+1} \tag{29}$$

$$\frac{1 - \vartheta (1 + \pi_t)^{\varkappa_t - 1}}{1 - \vartheta} = (1 - \varpi) \left(\frac{H_t}{F_t} \right)^{1-\varkappa_t} + \varpi \frac{\left[1 - \vartheta (1 + \pi_{t-1})^{\varkappa_t - 1} \right]}{1 - \vartheta} \left(\frac{1 + \pi_{t-1}}{1 + \pi_t} \right)^{1-\varkappa_t} \tag{30}$$

The system (28)-(30), once log-linearised, gives us the log-linearise Phillips curve:

$$\pi_t = \kappa_c \hat{p}_t^w + \hat{\varkappa}_t + \chi_f \beta \pi_{t+1} + \chi_b \pi_{t-1} \tag{31}$$

where $\chi_f = \frac{\vartheta}{\Upsilon}$, $\chi_b = \frac{\varpi}{\Upsilon}$, $\kappa_c = \frac{(1-\varpi)(1-\vartheta)(1-\vartheta\beta)}{\Upsilon}$, $\Upsilon = \vartheta + \varpi(1 - \vartheta + \vartheta\beta)$, and where the cost push shock has been normalised.

2.5 Equilibrium

2.5.1 Aggregation and Market Clearing

The aggregation is characterised as follow. Let K_t^A denote the aggregate capital stock after the realisation of the exit shock of the wholesale firms, but before new investments and depreciation take place. Thus:

$$K_t^A = (1 - \delta_e) K_t + \delta_e K_{0t}, \tag{32}$$

where $K_t = \int_0^1 K_t^j \, dj$ is the aggregate capital stock of all firms at the end of of period $t-1$ before the realization of the exit shock, and K_{0t} is the aggregate capital stock brought by new entrants.

In equilibrium, the labour market clears so the labour demand, $\int_0^1 N_t^j \, dj$, must be equal to its supply, N_t. Then, the aggregate labour demand of wholesale firms is given by:

$$N_t = \frac{u_t}{A_t} \left[\frac{(1-\alpha) p_t^w}{w_t} \right]^{\frac{1}{\alpha}} K_t^A, \tag{33}$$

where u_t is the capacity utilisation rate which is the same across firms, because wholesale firms have the same capital-labour ratio.

Denote $Y_t^W = \int_0^1 Y_t^j \, dj$ the aggregate output of wholesale firms. The aggregation of their their production functions yields:

$$Y_t^W = \left(u_t K_t^A\right)^{\alpha} (A_t N_t)^{1-\alpha}. \tag{34}$$

In equilibrium, the aggregate supply of the wholesale goods Y_t^W has to be equal to the demand of the retailers $\int_0^1 Y_t(i) \, dj$. Thus, the final output is given by:

$$Y_t \equiv \int_0^1 \left[\frac{P_t^i}{P_t} \right]^{\varkappa} Y_t(i) \; di = \left(u_t K_t^A\right)^{\alpha} (A_t N_t)^{1-\alpha}, \tag{35}$$

using (34), where $Y_t^W = \int_0^1 Y_t(i) \, dj$.

Let b_t denotes the aggregate real bubble at time t. When adding up the bubbles of the firms of all ages, the total real value of the bubble in the economy at time t is given by:

$$\begin{aligned} b_t &= \sum_{a=0}^{t} (1-\delta_e)^a \, \omega \delta_e b_{t,a} \\ &= m_t b_t^*, \end{aligned} \tag{36}$$

where b_t^* is the size of new emerging bubbles at date t and where m_t satisfies the recursion:

$$m_t = m_{t-1} (1-\delta_e) \kappa_{t-1} + \delta_e \omega, \tag{37}$$

with $m_0 = \delta_e \omega$. Using the law of motion of the bubble (25), restriction on the size of the new bubble is given by:

$$b_t^* = (1-\delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1+G_{t+1}) \kappa_t b_{t+1}^*. \tag{38}$$

Finally, substituting total real bubble at date t (36) into the restriction on the size of the new bubble (38) yields the non arbitrage condition for the total bubble in the economy:

$$b_t = (1-\delta_e) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{m_t}{m_{t+1}} (1+G_{t+1}) \kappa_t b_{t+1}. \tag{39}$$

The market clearing conditions for credits implies that the demand for loans is equal to the supplies of savings, $L_t = \int_0^1 L_t^j \, dj = D_t = 0$. Moreover, competitive financial intermediaries require that the deposit rate is equal to the lending rate. Therefore $R_t^d = (1-\delta_e) R_t^l$, taking into account that firms exits the market in each period with probability δ_e. However, from (26) and $G_{t+1} > 0$ that follow:

$$\frac{(1+\pi_{t+1})}{(1-\delta_e) R_t^l} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1+G_{t+1}) > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} = \frac{(1+\pi_{t+1})}{R_t^d}.$$

where the RHS of the inequality comes from the FOC of the households (5). Consequently, households do not have the incentive to save, and prefer to borrow until their borrowing constraint binds (i.e. $D_t = 0$).[3] Only firms that receive low efficiency shocks save and lend funds to productive firms.

Aggregating the value of all firms, the aggregate relative stock price is equal to:

$$p_t^s = q_t K_{t+1} + b_t, \tag{40}$$

[3] Without borrowing constraints, no arbitrage implies that $G_{t+1} = 0$. In this case, (25) and the transversality condition would rule out bubbles.

where the aggregate stock holding of the household is normalised to a unit, $s_{t+1} = 1$. This equation reveals that the the aggregate price of the stock has two components, the fundamental $q_t K_{t+1}$ and the aggregate bubble b_t.

The total capacity of external financing is given by:

$$\varphi_t K_t + q_t \gamma_t K_t + b_t,$$

which reflects the overall financial market conditions. Following Miao et al. (2015), a financial shock ζ_t is introduced to capture the disturbance of the overall financial constraints, which is defined as:

$$\zeta_t = \frac{\varphi_t}{q_t} + \gamma_t,$$

so that the total capacity of external financing can be rewritten such as $\zeta_t q_t K_t + b_t$.

Aggregating over the idiosyncratic shock ε_t^j, the aggregate investment is given by:

$$I_t p_t^I = \left[\left(u_t \Psi_t + \zeta_t q_t \right) K_t^A + b_t \right] \int_{\varepsilon > \varepsilon_t^*} d\Phi\left(\varepsilon\right), \tag{41}$$

using the financial shock ζ_t, where $\Psi_t = \alpha \left[\frac{(1-\alpha) z_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} \left(p_t^w \right)^{\frac{1}{\alpha}}$ and $\varepsilon_t^* = \frac{p_t^I}{q_t}$. Furthermore, the law of motion of capital is given by:

$$K_{t+1} = \left(1 - \delta_t\right) K_t^A + I_t \frac{\int_{\varepsilon > \varepsilon_t^*} \varepsilon \, d\Phi\left(\varepsilon\right)}{\int_{\varepsilon > \varepsilon_t^*} d\Phi\left(\varepsilon\right)}, \tag{42}$$

and the aggregate price of installed capital follows:

$$q_t = \left(1 - \delta_e\right) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\begin{array}{c} u_{t+1} \Psi_{t+1} + q_{t+1} \left(1 - \delta_{t+1}\right) \\ + G_{t+1} \left(u_{t+1} \Psi_{t+1} + \zeta_{t+1} q_{t+1} \right) \end{array} \right] \tag{43}$$

The resource constraint is given by:

$$C_t + \left(1 + \frac{\Omega}{2} \left[\frac{I_t}{I_{t-1}} - \lambda^I \right]^2 \right) \frac{I_t}{Z_t} = Y_t \tag{44}$$

using the budget constraint of the households (2), the flow-of-funds of the wholesale firms (11), the profit of the capital good producers (8) and the profit of the retailers.

Finally, the policy rule that I will consider is the following Taylor-type rule:

$$R_t^l = R \left[\left(1 + \pi_t\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\phi_R} \left(\frac{R_{t-1}^l}{R} \right)^{\phi_R} \exp(\epsilon_t^R) \tag{45}$$

where R is the natural rate at the zero inflation steady-state, and ϕ_π, ϕ_y and ϕ_R are the three policy feedback coefficients on inflation, evolution of output and past interest rate. Empirical rules in this form are often used in empirical literature and are shown to behave well.

2.5.2 Equilibrium

Equations (3), (4), (7), (18), (22), (23), (26), (28), (29), (30), (32), (33), (35), (37), (39), (41), (42), (43), (44) and (45) jointly determine the 20 aggregate endogenous variables C_t, N_t, p_t^I, p_t^w, u_t, K_t^A, K_t, w_t, Ψ_t, I_t, q_t, G_t, R_t^l, Λ_t, b_t, m_t, Y_t, H_t, F_t and π_t where $\delta_t = \delta(u_t)$ and $\varepsilon_t^* = \frac{p_t^I}{q_t}$.

3 Bayesian Estimation

The model, presenting no occasionally binding condition in equilibrium, is log-linearised around the bubbly non-stochastic steady-state, and fit the US data using Bayesian estimation methods.

3.1 Data and Shocks

The model has seven shocks: 1) a TFP shock, g_{at}, 2) an investment adjustment cost shock, Z_t, 3) an elasticity of substitution between any two specialised retail goods shock, $\varkappa_t$, 4) a financial shock, ζ_t, 5) a sentiment shock, κ_t, 6) a household taste shock ξ_t and finally, 7) a labour supply shock ψ_t.

These shocks are identified by using seven time series to estimate the parameters of the model: 1) the Federal Funds Rate, 2) the industrial inflation rate, 3) the US real GDP, 4) the US real investment, 5) the relative price of investment, 6) the S&P 500 composite index and 7) Chicago Fed's National Financial Conditions Index (NFCI). I compute the quarterly growth rates of real GDP, real investment, relative price of investment and the relative stock price for the estimation.

The data are available quarterly and cover the period from 1975Q1 to 2019Q4. Data for the industrial inflation (2), US GDP (3) and investment (4) are from the BEA website. The Federal Funds Rate (1), the relative price of investment (5) and the Chicago Fed's National Financial Condition Index are retrieved on the FRED website. Finally, the stock price data (6) are the S&P composite index downloaded from Robert Shiller's website. Figure 1 presents the transformed data used for the estimation.

3.2 Solution and Estimation Procedure

As in Miao et al. (2015), there is no need to to parametrise the depreciation function $\delta(\cdot)$ and the distribution function $\Phi(\cdot)$ because of the log-linearisation method. These terms will be components of estimated parameters. Yet, knowing the steady-state values of the following parameters is necessary: $\delta(1)$, $\delta'(1)$, $\delta''(1)$, $\Phi(\varepsilon^*)$ and $\mu = \frac{\phi(\varepsilon^*)\varepsilon^*}{1-\Phi(\varepsilon^*)}$ where the capacity of utilisation is equal

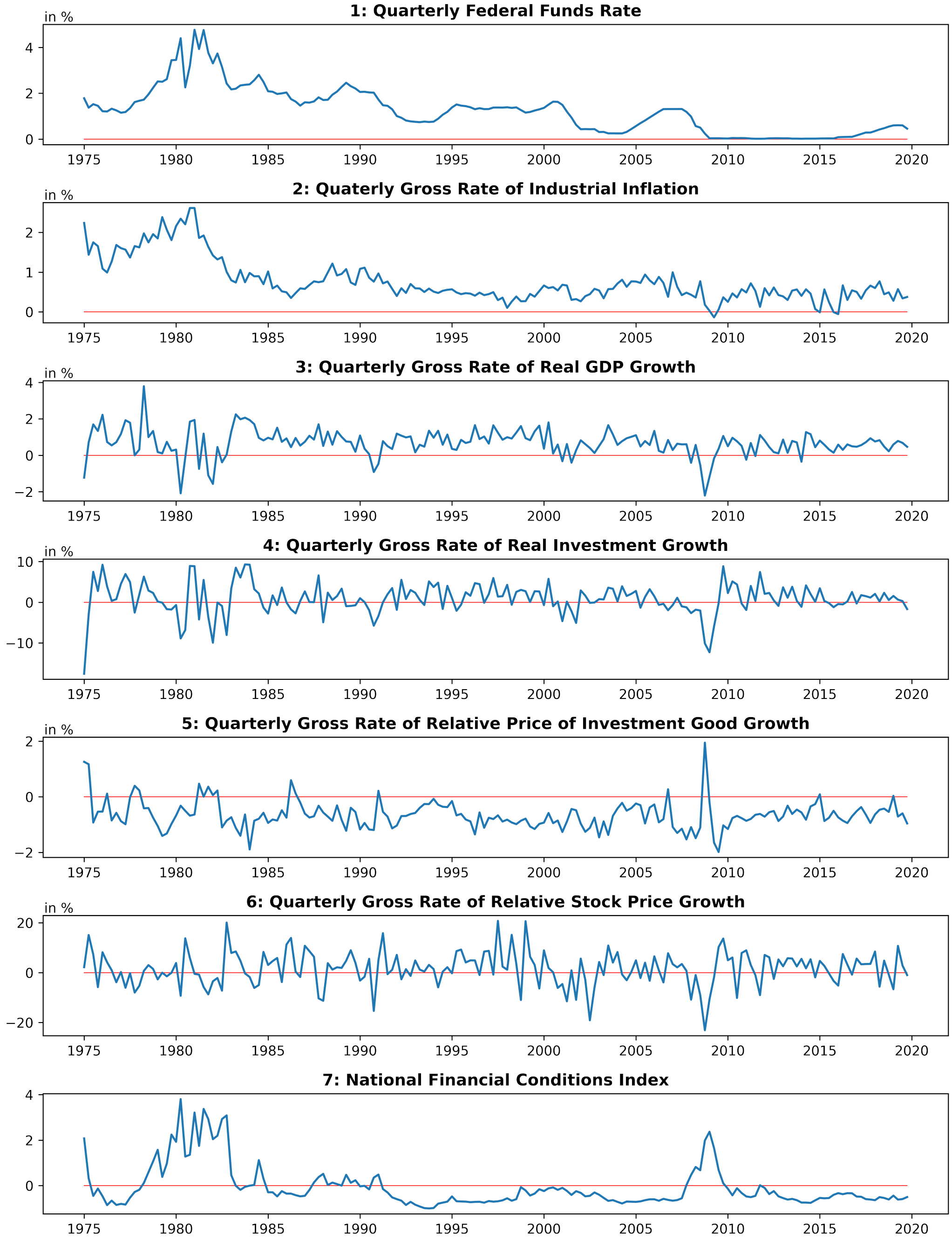

Figure 1: Plots of the data from 1975Q1 to 2019Q4. This figure illustrates the different time series used for the Bayesian estimation.

to 1 in steady-state and ε^* is the steady-state investment threshold for the idiosyncratic shock ε_t. These parameters will be estimated, except for $\delta(1)$ which will be calibrated.

The quarterly real gross rate of interest is calibrated using the means of inflation and Federal Funds Rate times series, $R = R^l - \pi = 1.0048$. As is standard in the literature, the quarterly subjective discount rate is calibrated to 0.995 using the quarterly real gross rate of interest R. The inverse Frisch labour supply elasticity is set to 1/5 which is in the range of macroeconomics estimates. The capital share in production is also set to its traditional value 0.3. The coefficient of relative risk aversion is set at 2 and the elasticity of substitution between any two specialised retail goods is set to 11, yielding a steady-state mark-up value of 1.1 on intermediate good relative price p_t^w. Finally, I set the exit parameter δ_e to 2% as in Miao et al. (2015).

The steady-state depreciation rate $\delta(u)$ where $u = 1$ in steady-state is calibrated to 0.025, and the steady-state investment to output ratio I/Y is set to 0.2. I use the mean of the growth rate of output to compute the steady-state quarterly gross growth rate of output g_a, which is equal to 1.0068. Finally, using the real rate R and the output growth rate g_a to compute the steady-state relative size of the old bubble to the new bubble, $\kappa = R/g_a$, which yields 0.9980.

The parameter $K_0/\tilde{K}$ is also estimated through the model. The mean prior on this parameter was 0.005 with a standard deviation of 0.001. I found that it converged to zero, thus I fixed it $K_0/\tilde{K}$.

Table 1 below summarises the calibrated parameters of the model.

Parameter	Value	Description
β	0.995	Quarterly subjective discount rate
σ	2	Risk aversion coefficient
η	1/5	Inverse Frisch labour supply elasticity
α	0.3	Capital share in production
δ_e	0.02	Probability of exiting the economy
$\varkappa$	11	Elasticity of substitution between any two specialised retail goods
g_a	1.0068	Steady-state quarterly gross growth rate of output
R	1.0048	Quarterly natural gross rate of interest at the zero inflation steady-state
κ	0.9980	Steady-state relative size of the old bubble to the new bubble
u	1	Steady-state capacity of utilisation rate
$\delta(1)$	0.025	Steady-state depreciation rate
I/Y	0.2	Steady-state investment-output ratio
$K_0/\tilde{K}$	0.001	Ratio of capital endowment for a entering firms to total capital stock

Table 1: Calibrated parameters

The estimation was first initiated using a Markov jump-linear-quadratic (MJLQ) model à la Svensson (2005), where uncertainty takes the form of different "modes" (or regimes) that follow a Markov process. The estimation was done with 4 different modes; *Dry Monetary Policy*, *Wet Monetary Policy*, *High Shock Volatility* and *Low Shock Volatility*. The analysis reflected that monetary policy was always dry and the the volatility of shocks was always low. Therefore, I continue the estimation process without using any modes.

As in Miao et al. (2015), I use the NFCI time series to better identify the financial shock ζ_t. The estimation of the model without the NFCI index produces a coherent smoothed financial shock series, yet the financial shocks are very persistent and ρ_ζ converges to 1. The introduction of the NFCI index helps to reduces the persistence of the financial shock. The financial shock is identified using the following measurement equation:

$$NFCI_t = -f_\zeta \hat{\zeta}_t - f_q \hat{q}_t - f_{Kb}(\hat{b}_t - \hat{K}_t)$$

which describes movements in the financing capacity of wholesale firms. An increase in $\hat{\zeta}_t$, $\hat{q}_t$, $\hat{b}_t$ or a decreases in $\hat{K}_t$ reduces the NFCI, in turn relaxes the financial constraint of the wholesale firms. However, the coefficient on marginal Tobin's Q, f_q, converged to zero in every estimated specifications. Thus I set it to zero ($f_q = 0$).

3.3 Estimated Parameters

Table 2 presents the prior and posterior distributions of the estimated parameters. Most prior distributions for the parameters are based on the posterior of Miao et al. (2015) the structural similarities between this study and theirs. The priors for parameters related to the structural parameters of New-Keynesian models are as follow: the mean prior of the Calvo parameter ϑ is set to 0.75 with a standard deviation of 0.01, and the mean prior of the price indexation parameter ϖ is set to 0.2 with a standard deviation of 0.02. The feedback coefficient parameter on inflation ϕ_π has a prior mean of 1.5 and a standard deviation of 0.1, the feedback coefficient parameter on change in output ϕ_y has a prior mean of 0.4 and a standard deviation of 0.15, and the feedback coefficient parameter on past nominal interest rate ϕ_R has a prior mean of 0.4 and a standard deviation of 0.1.

The main differences between our estimates come from the estimation of the steady-state values for the financial constraint parameter ζ and the investment productivity distribution parameter μ. Miao et al. (2015) had a relatively loose prior on these two parameters and explained that ζ was not particularly sensitive to the prior distribution. Using the same priors as Miao et al. (2015) for these two parameters, the posteriors of these two were significantly different and higher. The mean posterior of the financial shock ζ peaked at 0.7 and the elasticity of the

Param.	Prior Distribution: Distr.	Mean	Std. Dev.	Posterior Distribution: Mean	Std. Dev.	5%	95%
θ	Beta	0.4	0.05	0.9882	0.0021	0.9846	0.9914
Ω	Normal	0.1	0.1	0.1781	0.0397	0.1174	0.2479
$\frac{\delta''}{\delta'}$	Normal	10	2	12.3344	1.4024	9.8474	14.4560
ζ	Beta	0.15	0.01	0.2970	0.0121	0.2770	0.3168
μ	Normal	2.3	0.01	2.3155	0.0098	2.2995	2.3313
ϑ	Normal	0.75	0.01	0.7671	0.0088	0.7530	0.7818
ϖ	Beta	0.2	0.02	0.2190	0.0208	0.1858	0.2543
ϕ_π	Normal	1.5	0.1	1.6374	0.0694	1.5256	1.7538
ϕ_y	Beta	0.4	0.15	0.3271	0.1175	0.1443	0.5302
ϕ_R	Beta	0.4	0.1	0.8929	0.0129	0.8722	0.9141
f_ζ	Beta	0.5	0.15	0.1465	0.0200	0.1158	0.1804
f_{Kb}	Beta	0.1	0.15	0.0034	0.0005	0.0027	0.0042
ρ_Z	Beta	0.5	0.1	0.8982	0.0204	0.8634	0.9299
$\rho_\varkappa$	Beta	0.7	0.1	0.9877	0.0036	0.9813	0.9930
ρ_ζ	Beta	0.5	0.1	0.7990	0.0329	0.7434	0.8512
ρ_κ	Beta	0.2	0.1	0.2297	0.0809	0.1225	0.3742
ρ_{g_a}	Beta	0.3	0.025	0.3450	0.0264	0.3016	0.3884
ρ_ξ	Beta	0.5	0.1	0.9966	0.0005	0.9956	0.9973
ρ_ψ	Beta	0.5	0.1	0.3390	0.0800	0.2144	0.4782
σ_Z	Inv-Gamma	0.01	0.005	0.0115	0.0009	0.0101	0.0131
$\sigma_\varkappa$	Inv-Gamma	0.005	0.005	0.0113	0.0010	0.0098	0.0129
σ_ζ	Inv-Gamma	0.02	0.005	0.0320	0.0043	0.0257	0.0400
σ_κ	Inv-Gamma	0.02	0.005	0.2937	0.1284	0.1243	0.5284
σ_{g_a}	Inv-Gamma	0.015	0.005	0.0154	0.0010	0.0139	0.0170
σ_ξ	Inv-Gamma	0.012	0.005	5.6266	0.6558	4.5567	6.7090
σ_ψ	Inv-Gamma	0.03	0.005	0.0379	0.0044	0.0313	0.0456
σ_R	Inv-Gamma	0.005	0.0015	0.0074	0.0004	0.0068	0.0081

Table 2: Prior and posterior distributions

probability of undertaking investment at the steady-state cutoff μ could range between between 5 and 12. Jointly or individually, the values of these parameters yold counter-intuitive results and did not match the range in previous studies. Covas and Den Haan (2011) reported that the financial constraint parameter ranges between 0.1 and 0.4, and Miao et al. (2015) estimated

$\zeta^M = 0.3$.[4] For the elasticity μ, Miao et al. (2015) obtained a posterior mean of $\mu^M = 2.58$ and Wang and Wen (2012) found a similar estimate equal to 2.4. Consequently, I restricted the standard deviation priors for these two parameters to 0.01 around mean 0.15 for ζ and 2.3 for μ. It seems that these mean posteriors of these two parameters do not correspond to their priors because I use data on the interest rate in addition to data on investment and stock prices. The introduction of the interest rate as a decision variable by the Central Bank and using it as an observable variable increases the posterior means of ζ and μ. A higher ζ relaxes the borrowing constraint and makes the mean value of ζ less sensitive to changes in the interest rate.

The results of the estimation indicates that habits are very persistent, i.e. $\theta = 0.9882$, and that the adjustment cost parameter for price of investment is equal to 0.1781. These two parameters are significantly higher than in Miao et al. (2015), who found $\theta^M = 0.54$ and $\Omega^M = 0.03$. For the 'curvature' of the depreciation function $\delta(\cdot)$, I found that $\frac{\delta''}{\delta'} = 12.33$ which is similar to Miao et al. (2015). The posterior distributions of the feedback coefficients for the policy rule are in the usual ranges of the literature with posterior means of 1.637 for ϕ_π, 0.32 for ϕ_y, and 0.89 for ϕ_R. The persistence and standard deviations posteriors are conventional, except for the taste shock $\hat{\xi}_t$ which tends to 1. Implications of the finding of this estimate is discussed in Section 4.

4 Results

The Result Section is composed of four analyses: i) an evaluation of the model in explaining historical bubble episodes, ii) a counterfactual experiment, iii) a analysis the transmission mechanism of monetary policy in a bubbly economy, and iv) an examination of alternate policy rules that react to stock prices.

4.1 Model Evaluation: Sentiment Shock, Bubbly Firms and Aggregate Bubble

During the last 50 years, the US economy experienced two major bubble episodes, the dot-com bubble and the subprime mortgage bubble. In this subsection, I investigate the explanatory power of the presented model about these two events.

As previously established, bubbles emerge because of by self-fulfilling beliefs about the value of wholesale firms. Moreover, movements in bubbles can be driven by household sentiment shocks $\hat{\kappa_t}$ about the relative size of bubbles between two firms of different ages. A positive sentiment shock increases the bubble size of young firms and increases the total value of the bubble. Finally,

[4] In the case of Miao et al. (2015), the mean posterior of the financial shock ζ did not deviate from the mean prior.

the total value of the bubble depends on the aggregation of all bubbles in the economy. Due to stochastic lives of the wholesale firms, the aggregation of the bubble depends on the variable m_t whose dynamics is described by equation (37). As in Miao et al. (2015), I interpret m_t as the mass of firms having bubbles. The log-linearized version of m_t is given by:

$$\hat{m}_t = (1 - \delta_e)\, \kappa\, (\hat{m}_{t-1} + \hat{\kappa}_{t-1}) \tag{46}$$

This equation (46) establishes that fluctuations in the mass of bubbly firms depend on past fluctuations in the mass of bubbly firms plus fluctuations in past sentiment shock. Therefore, the sentiment shock affects $\hat{m}_t$ with a lag. However, the bubble law of motion, i.e. equation (39), depends on both $\hat{m}_t$ and expected $\hat{m}_{t+1}$. The latter implies that fluctuations in the aggregate bubble depend on both current and lagged sentiment shocks.

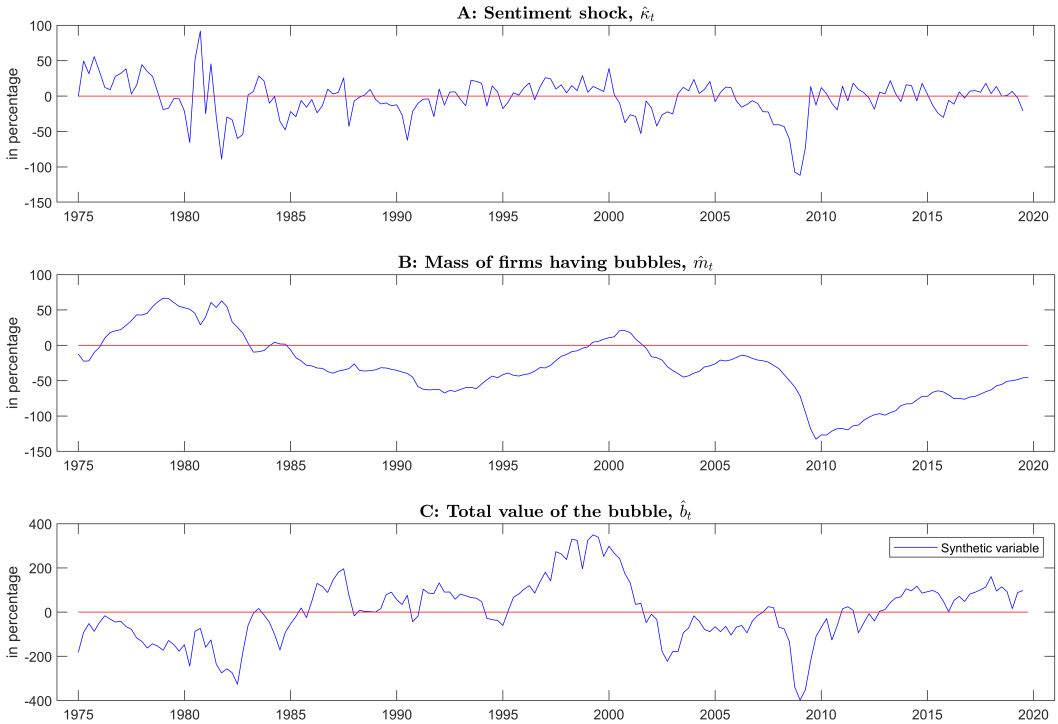

Figure 2: Sentiment shock, bubbly firms and aggregate bubbles. This figure illustrates the log-deviations from the steady-state of the sentiment shock $\hat{\kappa}_t$, the mass of bubbly firms $\hat{m}_t$, and the aggregate value of the bubble $\hat{b}_t$ estimated from the model.

Figure 2 illustrates the estimation of the sentiment shock (Panel A), the mass of bubbly firms (Panel B) and the total value of aggregate bubble (Panel C). How well does the model describe the boom and bust of the dot-com bubble? Panel B shows that the mass of bubbly firms slowly but constantly increased from the 90s until 2001 due to small positive sentiment shocks. This increase in the mass of bubbly firms seems to have a positive effect on the development of the aggregate bubble (Panel C). During Q1 2001, Panel A clearly depicts a relatively persistent, but moderate, fall in sentiment shocks during the burst of dot-com bubble. We can see that this movement in the sentiment shock implies a net sustained fall in the mass of bubbly firms. This fall also seems to pass on to the total value of the aggregate bubble with a strong fall from 2000Q1 to 2002Q4.

Concerning the subprime mortgage bubble, we can see in Panel A of Figure 2 that the sentiment shock was relatively stable around steady-state between 2003 and 2007. This explains why the mass of bubbly firms slightly recovered but remained significantly below steady-state during the 'booming' bubble episode. In other words, the model may not be able to capture well the boom of the subprime mortgage bubble. One possible explanation for this pitfall could be that this boom is relatively localised to the housing market and thus did not strongly affect the S&P 500. Therefore, the boom is not captured in the data. However, the burst of the bubble is clearly depicted by a strong fall in sentiment shock which results in a large and substantial decrease in the mass of the bubbly firms. Panel C also shows that the total value of the bubble plummets from 2007Q4 to 2009Q1.

In the next subsection, I will show that the sentiment shock is not the most significant factor to explain volatility of the bubbles which contrasts with the results of Miao et al. (2015). The sentiment shock is able to explain the volatility in the bubble, but its effect is dominated by changes in the investment threshold $\hat{\varepsilon}_t^*$ and the taste shock ξ_t.

4.2 Counterfactual Experiment: No Sentiment Shock Pre-Financial Crisis

This section presents a counterfactual experiment in which the US economy would not be hit by a strong sentiment shock before the financial crisis.. Figure 3 illustrates the counterfactual experiment with $\hat{\kappa}_t = 0$ from 2007Q2.

As established, the sentiment shock κ_t only affects the mass of bubbly firms in the economy m_t, which in turn only affects the law of motion for the aggregate bubble b_t. Panel C shows that, while the sentiment shock is muted, the value of the aggregate bubble still plummets.

The change in the aggregate value of the firm, as small as it is, has large implications on the real variables and on some nominal variables (see Figure 3). A small change in the aggregate value of the bubble will have a direct effect on investment. We can see in Panel G in Figure 3 that investment falls far less. This change in aggregate investment has a net a positive effect on output, but this effect is relatively marginal during the crisis as exhibited in Panel FF.

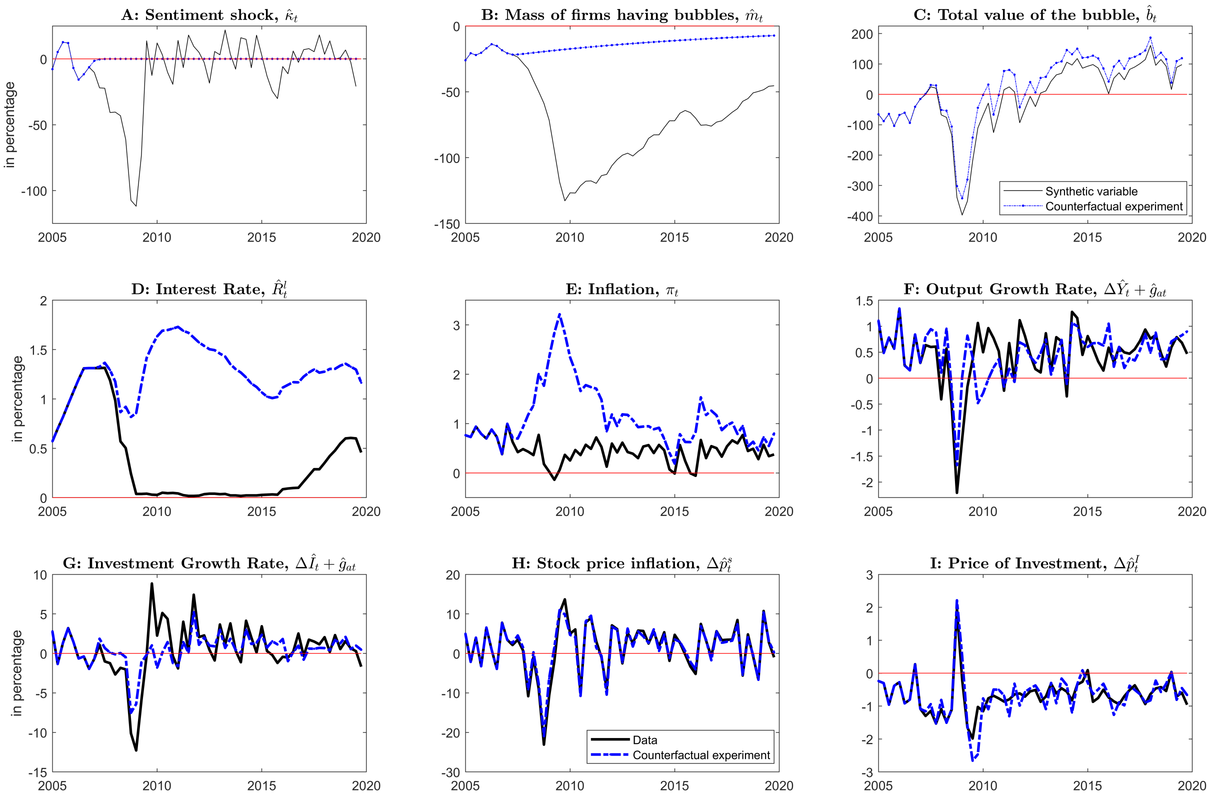

Figure 3: No bubble sentiment shock pre-financial crisis. This figure illustrates the response of the economy from 2007Q2 with $\hat{\kappa}_t = 0$.

The change in the sentiment shock results in high inflation, and thus in a high reaction of the nominal interest rate. Because of the higher cost of labour, the price of wholesale firms' goods increases, which raises the marginal cost of retailers. Consequently, the Central Bank reacts to the increase in inflation and raises the nominal interest rate.

Surprisingly, the growth rate of stock prices remains unaffected once the sentiment shock is muted. The log-linearised detrended stock relative price is given by:

$$\hat{p}^s_t = \overbrace{\frac{q\tilde{K}g_a}{\tilde{p}^s}(\hat{g}_{at+1}\hat{q}_t\hat{K}_{t+1})}^{\text{fundamental value, } V^f_t} + \frac{\tilde{b}}{\tilde{p}^s}\hat{b}_t$$

While the bubble component of the stock market bubble marginally changes, we could have expected changes in the fundamental value of the stock market. It appears that the taste shock ξ_t is the main driver of fluctuations in stock market prices and the bubble component. In contrast to Miao et al. (2015), I use data on the nominal interest rate and inflation in addition to the data on stock prices. The growth rate of stock prices appears to be too volatile while the interest rate is relatively smoother. Consequently, the taste shock ξ_t tends to capture the excess volatility of the data on the growth rate of stock price.

4.3 Monetary Policy Transmission in a Bubbly Economy

The monetary policy affects the entire economy by altering the borrowing cost for wholesale firms. By manipulating the credit market between wholesale firms, the Central Bank affects 1) wholesale firms' demands for investment goods and labour, 2) the supply of goods to the retailers and the real marginal cost p_t^w, and 3) the market value of the wholesale firms and in turn the households' return on the stock.

A contractionary policy of the Central Bank increases the borrowing cost due to a higher nominal rate. This policy has an intensive and extensive effect on the investment of wholesale firms. The intensive effect is a drop in the individual demand for investment goods. The extensive effect is a drop in the aggregate demand for investment goods because some wholesale firms with an efficiency shock close to the threshold will not have the incentive to borrow. This drop in demand implies a fall in the price of investment and a decrease in capital accumulation. Marginal q, price of installed capital, also falls because firms have too much capital.

This reduction in investment decreases aggregate capital and leads to a lower demand for labour, a lower utilisation rate and finally, a lower production. Nonetheless, despite a fall in output, the price of wholesale firm goods decreases because of the drop in the cost of labour. Consequently, inflation falls as retailers, facing a lower marginal cost, adjust their prices.

A noteworthy feature of the effect of monetary policy is an immediate fall of the need for bubbles. Since firms have too much capital, they do not need to make new investments. Therefore, bubbles are less needed and their value falls after an increase in the nominal interest rate. A drop in the aggregate value of the bubble implies a fall in the value of the firms, reflected by the value of stock price drops. This has an immediate negative wealth effect for the households, which leads to a drop in consumption.

4.4 Alternative Monetary Policies

In this section, I investigate the robustness of a monetary policy that reacts to stock prices in mitigating the impact of a sentiment shock on the economy. In this intent, I specify two alternative monetary policies than the one estimated in the benchmark model presented.

The first alternative policy reacts to the change in stock prices (Model 1). In its log-linearised

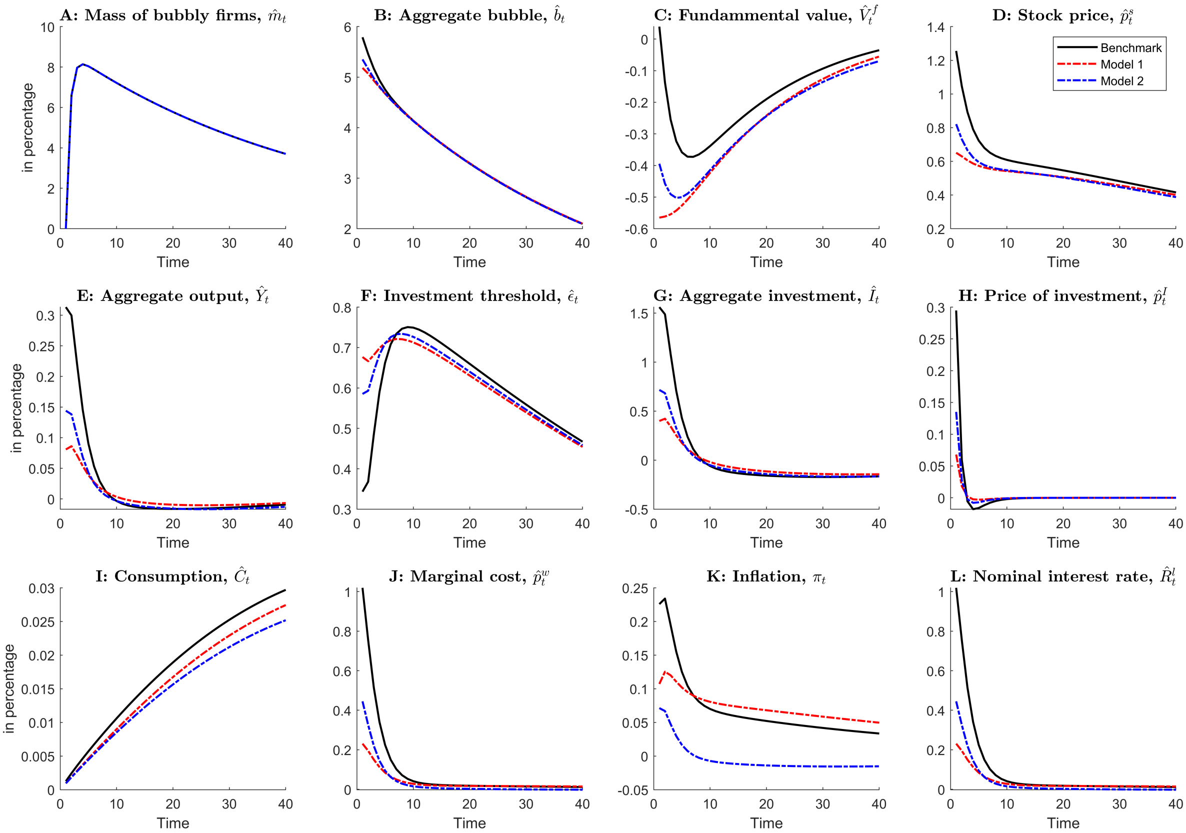

Figure 4: Impulse responses after a positive sentiment shock and alternative monetary policies. This figure illustrates the response of the economy after a unit sentiment shock $\hat{\kappa}_t$ under the benchmark estimation (i.e. $\phi_{p^s} = 0$) and under two alternative policy rules that reacts to changes on stock price (Model 1: $\phi_{p^s} = 2$) and to stock price deviation from its steady-state (Model 2: $\phi_{p^s} = 0.1$). The vertical axes represent the percentage deviations from the variables steady-state levels and the unit of time for the horizontal axes correspond to quarters.

form, this monetary policy follows the rule:

$$\hat{R}_t^l = (1 - \phi_R) \left[\phi_\pi \pi_t + \phi_y \Delta \hat{Y}_t + \phi_{p^s} \Delta \hat{p}_t^s \right] + \phi_R \hat{R}_{t-1}^l \tag{47}$$

The second policy reacts to deviations of stock prices from its steady-states (Model 2). In its log-linearised form, this monetary policy follows the rule:

$$\hat{R}_t^l = (1 - \phi_R) \left[\phi_\pi \pi_t + \phi_y \Delta \hat{Y}_t + \phi_{p^s} \hat{p}_t^s \right] + \phi_R \hat{R}_{t-1}^l \tag{48}$$

For this experiment, I calibrate $\phi_{p^s} = 2$ for Model 1 and $\phi_{p^s} = 0.1$ for Model 2.

Figure 4 presents the response of the economy after a unit sentiment shock $\hat{\kappa}_t$ under the benchmark estimation (i.e. $\phi_{p^s} = 0$) and the two alternative policy rules specified above (i.e. Model 1 and 2). Under both alternative policies, the stock price does not increase as much as under the benchmark model (Panel D). However, these smaller deviations are not due to a reduction in the value of the bubble (Panels B), which is only marginally affected by the policy reaction. The lower deviation of the stock price from its steady-state is the consequences of larger negative deviations in the fundamental value (Panels C).

The drop in the fundamental value of the stock represents a decrease in the aggregate value of the wholesale firms. This would imply a drop in investment, because of a contraction of the borrowing capacity of the firm. However, bubbles counter-balance this drop and the aggregate investment remains above its steady-state value after the sentiment shock (Panel G). A lower investment relative to the benchmark model implies a lower cost of labour and lower output.

Both alternative rules manage to stabilise investment and output faster than the benchmark model. The main difference between these two rules is that Model 2 implies a higher interest rate after the impact of the sentiment shock, which has consequently a more aggressive effect on inflation. This higher aggressiveness of the interest rate towards inflation permits to quickly stabilise inflation in contrast to Model 1 (Panel K).

Can monetary policy directly affect the total value of the bubble after a sentiment shock? The answer is mostly no. My finding shows that a rise in the interest rate has a direct negative effect on the aggregate value of the bubble. However, this effect is countered because there is a need for bubbles due to the contraction of the borrowing capacity of the wholesale firms; the fundamental value falls, so bubbles are needed to ease the borrowing constraints. Consequently, the interest rate is an adequate instrument to reduce the volatility of the value of the bubble but is still useful to quickly stabilise output (and inflation under Model 2) by reacting to stock prices.

5 Conclusion

In this paper, I developed and estimated a New Keynesian model with stock market bubbles. Moreover, I analysed the effects of bubbles on real and nominal variables and the transmission mechanism of monetary policy in a bubbly economy. Finally, I investigate if a monetary policy that leans against the wind can reduce the volatility of bubbles. Based on the presented analysis, I can draw the following conclusions.

First, the volatility of value of bubbles can explain a significant fraction of movements in investment, output and inflation. Bubbles, which exist in this economy because of self-fulfilling beliefs, allow firms to increase the borrowing capacity and increase investment and production. Thus, movements in bubbles directly affect the volatility of these variables as well as inflations.

Second, the sentiment shock is not the main cause for changes in bubble size. Because of tight borrowing constraints, firms are very sensitive to changes in the cost of borrowing. Therefore, changes in the interest rates, in the price of installed capital, and in the price of capital goods can have significant intensive and extensive effects on the investment decisions of firms. Moreover, I found that the taste shock is also an important factor to explain variations in the volatility of the bubble's value. Movements in these variables, or the taste shock, affect the number of investing firms and thus the volatility of bubbles.

Finally, monetary policies that react to asset prices can mitigate the impact of the sentiment shock. In this paper, I present two alternative policies; i) a policy rule that reacts to the changes in stock prices and ii) a policy rule that reacts to deviations of the stock price from its steady-state. Such policies can stabilise quicker output, investment and stock prices. However, these policies have different effects on inflation. Reacting to changes in stock prices is not able to stabilise fast enough inflation and thus can reduce welfare because of persistent inflation. In contrast, a policy rule that reacts to the deviations of the stock price from steady-state can stabilise faster inflation than the estimated rule and the first alternative rule.

References

Ajello, A. (2016). Financial Intermediation, Investment Dynamics, and Business Cycle Fluctuations †. *American Economic Review*, 106(8):2256–2303.

Bernanke, B. S. and Gertler, M. (1999). Monetary policy and asset price volatility. *Economic Review*, (Q IV):17–51.

Bernanke, B. S. and Gertler, M. (2001). Should Central Banks Respond to Movements in Asset Prices? *American Economic Review*, 91(2):253–257.

Blot, C., Hubert, P., Labondance, F., and Mislin, A. (2018). Monetary policy and asset price bubbles.

Borio, C. E. V. and Lowe, P. W. (2002). Asset Prices, Financial and Monetary Stability: Exploring the Nexus. *Bank for International Settlements (BIS) Working Paper*, (114).

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Cecchetti, S., Genberg, H., Lipsky, J., and Wadhwani, S. (2000). Asset Prices and Central Bank

Policy. Geneva Reports on the World Economy. *Geneva Reports on the World Economy 2*, 2:pp.1–160.

Covas, F. and Den Haan, W. J. (2011). The cyclical behavior of debt and equity finance. *American Economic Review*, 101(2):877–899.

Filardo, A. J. (2004). Monetary Policy and Asset Price Bubbles: Calibrating the Monetary Policy Trade-Offs. *BIS Working Papers*, (155).

Galí, J. (2014). Monetary Policy and Rational Asset Price Bubbles. *The American Economic Review*, 104(3):721–752.

Galí, J. and Gambetti, L. (2015). The Effects of Monetary Policy on Stock Market Bubbles: Some Evidence. *American Economic Journal: Macroeconomics*, 7(1):233–257.

Galichère, A. (2021). Asset Price Bubbles and Monetary Policy: Deflate the bubble?

Hirano, T. and Yanagawa, N. (2017). Asset Bubbles, Endogenous Growth, and Financial Frictions. *Review of Economic Studies*, 84:406–443.

Ikeda, D. (2017). Monetary Policy, Inflation and Rational Asset Price Bubbles. *Available at SSRN*.

Issing, O. (2009). Asset prices and monetary policy. *Cato Journal*, 29(1):45–51.

Jiménez, G., Ongena, S., Peydró, J. L., and Saurina, J. (2014). Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking? *Econometrica*, 82(2):463–505.

Kocherlakota, N. R. (2009). Bursting bubbles: Consequences and cures. *Federal Reserve Bank of Bank of Minneapolis*.

Kohn, D. L. (2006). Monetary policy and asset prices. In *Monetary Policy: A Journey from Theory to Practice*, page 43.

Martin, A. and Ventura, J. (2012). Economic Growth with Bubbles. *American Economic Review*, 102(6):3033–3058.

Miao, J., Shen, Z., and Wang, P. (2019). Monetary policy and rational asset price bubbles: Comment. *American Economic Review*, 109(5):1969–1990.

Miao, J. and Wang, P. (2018). Asset bubbles and credit constraints. *American Economic Review*, 108(9):2590–2628.

Miao, J., Wang, P., and Xu, Z. (2015). A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles. *Quantitative Economics*, 6(3):599–635.

Naqvi, H. and Acharya, V. (2012). Bank Liquidity and Bubbles: Why Central Banks Should Lean Against Liquidity. In Evanoff, D. D., Kaufman, G. G., and Malliaris, A. G., editors, *New perspectives on asset price bubbles*, chapter 9. Oxford University Press.

Svensson, L. E. O. (2005). Monetary Policy With Model Uncertainty :. (May).

Wang, P. and Wen, Y. (2012). Hayashi meets Kiyotaki and Moore: A theory of capital adjustment costs. *Review of Economic Dynamics*, 15(2):207–225.

Weil, P. (1987). Confidence and the Real Value of Money in an Overlapping Generations Economy. *The Quarterly Journal of Economics*, 102(1):1–22.