An Anticipatory Utility Model of Consumption and Savings

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Abstract

This paper builds a consumption-saving model of anticipatory utility. In addition to consumption-derived utility, an agent experiences gains-loss utility from two sources: from anticipating future consumption, and from comparing their current level of consumption with past-formed anticipation levels. The agent chooses optimally both their consumption and anticipation levels. We highlight the model's relevance for macroeconomics analyzing the behavior of two types of agents in three contexts: when income is certain, when income is risky, and when there are credit market imperfections. Agents with a limited planning horizon emerge as "impatient" – predisposed to borrow, while agents with an unlimited planning horizon emerge as "patient" – predisposed to save. Agents have an endogenous time-discount factor in all contexts. Our main results relate to agents' precautionary savings.

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1 Introduction

Neoclassical economic theory prescribes that individuals make consumption and savings decisions that maximize the present discounted sum of current and future consumption utility. This standard intertemporal optimization problem results in the simple solution of equating the marginal utility of consumption over time by smoothing consumption. Under this perspective, individuals value savings only indirectly, through the weight they put on future utility from consumption.

However, survey evidence suggests that consumers have a more complex relationship with their finances. Money ranks as one of the top sources of stress, ahead of health, family, and work, and the vast majority of households report experiencing financial stress. Saving money provides households with more than just enjoyment in the future when they will consume their savings—it directly affects them in the present. On the one hand, having savings provides peace of mind. On the other hand, most households report difficulties in saving money. Thinking about future consumption provides a source of enjoyment in the present, leading households to value savings beyond their discounted consumption utility. At the same time, consumers struggle to overcome immediate gratification.

Early economists conceptualized intertemporal choices as involving both the enjoyment of thinking about future consumption and the difficulty of abstaining from immediate enjoyment. Smith (1759) observes that "the sensation of the present instant, makes but a small part of our happiness, that our enjoyment chiefly arises either from the cheerful recollection of the past, or the still more joyous anticipation of the future," while Rae (1834) notes that "the actual presence of the immediate object of desire in the mind by exciting the attention, seems to rouse all the faculties, as it were to fix their view on it, and leads them to a very lively conception of the enjoyments which it offers to their instant possession." Loewenstein (1987) presents a model focusing on the anticipation channel, while most other modern theories of intertemporal choice ignore this channel and simply assume that decision makers put less weight on consumption utility experienced further in the future (Ainslie, 1975; Mazur, 1984; Loewenstein and Prelec, 1992; Harvey, 1994; Laibson, 1997).

This paper develops a consumption-saving model that captures both of these channels. In addition to experiencing utility from the level of current consumption, as in standard models, a decision maker (DM) derives utility from two more sources: utility from anticipating future consumption, and reference-dependent gain-loss utility compared to past levels of anticipation. The DM chooses optimally both their consumption and anticipation levels. Optimally choosing an anticipation level introduces a notion of *hedonic expectation*; comparing current utility with past anticipation levels precludes costless *self-deception* of the DM.

More specifically, the DM starts their life in period 0 and they experience a utility flow from anticipating future consumption. In period 1 they experience utility from consuming, from a gain or loss of how much they consume relative to how much they chose to anticipate, and from revising their anticipation level of their period 2 consumption. In period 2 they experience utility from consuming, and from a gain or loss of how much they consume relative to how much they chose to anticipate, and their life ends.

Within this setup, we consider the behavior of two types of agents. A "present-focused" DM bases their decisions on a limited planning horizon, that is, as if next period is their last period. A "non-present-focused" DM bases their decisions taking their whole planning horizon into account.

Our setup gives rise to an endogenous time discount factor and distinctive behavior for the two types of agents in three different contexts. We start by assuming there are no credit market imperfections and anayze the DMs' behavior when income is certain and when it is risky. Lastly, we analyze their behavior in the presence of credit market imperfections.

When income is certain the non-present-focused DM is patient: they choose an increasing consumption path and their time discount factor is greater than 1. The present-focused DM is impatient: they choose a decreasing consumption path and their time discount factor is less than 1. Thus the former DM is a "natural saver" in the economy, while the latter a "natural borrower."

We then analyze behavior when income is risky, and in particular when income in period 2 is a mean-preserving spread of income in period 1. Both agents have an endogenous stochastic discount factor in this case and a precautionary savings motive. Contrary to the benchmark model where the precautionary savings motive is second-order, in our model certainty equivalence is broken for both types: even with quadratic utility of consumption a DM has a willingness to save in the presence of risk.

The precautionary savings motive of the non-present-focused DM, combined with their patience makes them unambiguously wanting to save for period 2. A sufficient condition for the precautionary savings motive to be stronger than the one in the benchmark model is that the DM is patient, i.e. have a time-discount factor greater than 1, at the level of consumption chosen in the benchmark.

In the case of the present-focused DM, a trade-off emerges: the DM is impatient when income is certain, while they have a precautionary savings motive when period 2 income is risky. We show that as the risk of period 2 income increases, at some point the precautionary savings motive dominates.

Our analysis up to this point has assumed credit markets without imperfections, meaning the DM can borrow and save at the same rate up to their natural borrowing limit. In the final part of the paper we assume the borrowing rate is greater than the saving rate, a credit market imperfection that allows for the distinction between a liquid and an illiquid asset. A recent literature has highlighted the empirical relevance of this distinction (e.g., Angeletos et al., 2001; Kaplan and Violante, 2022).

The non-present-focused DM does not change their behavior under credit market imperfections. The present-focused DM has a demand for borrowing that is decreasing in the borrowing rate as one would expect, but importantly, we show there is an endogenous cut-off above which they don't want to either borrow or save.

Our paper contributes most directly to existing work that models utility from anticipation and its consequences. The seminal paper by Loewenstein (1987) presents a model in which decision makers experience utility from anticipation, where anticipatory utility is proportional to the discounted sum of future consumption utility. While the resulting utility function can explain the desire to postpone pleasure, the model also predicts that decision makers may prefer to perpetually defer consumption. In contrast with models in which anticipatory utility depends directly on the *level* of future consumption utility, Thakral (2022) introduces an alternative formulation in which decision makers optimally choose their anticipation of future consumption and experience utility from *changes* in their anticipation levels. While that paper studies choices over exogenous consumption streams, this paper extends the framework to consumption-saving problems and explores the model's macroeconomic implications.

Our model relates closely to the dynamic model of reference dependence due to Kőszegi and Rabin (2009). Their model has two key features. First, decision makers experience gain-loss utility from comparing current consumption to a reference point or when updating the reference point for future consumption. Second, "recently held" rational expectations determine the reference point. Our model retains the first feature but differs by endogenizing the updating of the reference point.[1] In particular, decision makers in our model optimally choose their reference point (how much to look forward to future consumption), and the level of anticipation of future consumption in a given period serves as their subsequent reference point. While our model contrasts with the expectations-based reference point of Kőszegi and Rabin (2009), our assumption resembles other models of belief-based utility such as the optimal expectations framework of Brunnermeier and Parker (2005).

As already discussed, in the case of income uncertainty in period 2, our model features a first-order precautionary savings motive for both types of agents. A first-order precautionary savings motive is highlighted by Kőszegi and Rabin (2009) as well. In the case of income uncertainty in period 1, comparing our "present-focused" agent to Kőszegi and Rabin (2009), we find that uncertainty makes the DM consume more than they would in its absence only when the low outcome is realized, and that the DM may become patient, choosing to consume more in period 2 when the high income is realized. Both results contrast the predictions of Kőszegi and Rabin (2009) in the same context.

The paper proceeds as follows. In a two-period model, we illustrate the process of anticipation-level formation and its implications for consumption in three different contexts: when income is certain (Section 2), when income is risky (Section 3), and when the DM can distinguish between a liquid and an illiquid asset. (Section 4). Section 5 concludes.

[1]How quickly the reference point updates in response to changes in expectations is a degree of freedom in the Kőszegi and Rabin (2009) model, as highlighted by Thakral and Tô (2021).

2 Consumption when income is certain

2.1 Components of the model

We introduce our model in the simplest possible context for intertemporal choice. The decision maker (DM) lives for two periods $t = 1, 2$. They receive income y_1, y_2 with certainty in the two periods.

There is *no credit market imperfection*, that is the DM can borrow and save at the same interest rate, that is $r^b = r^s = r$, and they can borrow up to their lifetime income. As we are not interested in the effects of changes in the interest rate at the moment, we will also assume $r = 0$.

The DM is assumed to choose optimally their consumption path of c_t's as usual. In addition, they choose a path of anticipation levels of their future consumption. We denote by α_{t+1}^t the anticipation level formed in period t for consumption utility in period $t + 1$.

Timing: In period 0 the DM receives all information of the economy—their income process, and the interest rate—and choose their anticipation levels α_1^0, α_2^0 for the consumption utility they will enjoy in the next two periods. In period 1 they consume and they choose their anticipation level α_2^1 for the next period. In period 2 they consume.

Preferences: The DM's flow utility consists of two components, consumption utility denoted by $m(\cdot)$ and gain-loss utility denoted by $n(\cdot)$.

The function $m(\cdot)$ corresponds to a standard utility flow from the level of consumption. The function $n(\cdot)$ is a gain-loss value function of the type introduced by Kahneman and Tversky (1979), having a different branch in the gains, and different branch in the loss domain. Denote:

$$n(\alpha) = \begin{cases} n_+(\alpha) & \text{if } \alpha \geq 0 \\ n_-(\alpha) & \text{if } \alpha \leq 0 \end{cases}$$

We will assume the following throughout:

Assumption 1. The consumption-utility function satisfies $m(0) = 0$, $m'(\cdot) \geq 0$, and

$m''(\cdot) \leq 0$.

Assumption 2. The gain-loss function n satisfies the following properties: continuity, loss aversion ($0 < n_+(\alpha) < -n_-(-\alpha)$ and $0 < n'_+(\alpha) < n'_-(-\alpha)$ for all $\alpha > 0$), and diminishing sensitivity ($n''_+ \leq 0 \leq n''_-$).

The DM experiences utility according to the following flows in the respective periods:

$$u_0 = n\Big(\alpha_1^0\Big) + n\Big(\alpha_2^0\Big) \tag{1}$$

$$u_1 = m(c_1) + n\Big(m(c_1) - \alpha_1^0\Big) + n\Big(\alpha_2^1 - \alpha_2^0\Big) \tag{2}$$

$$u_2 = m(c_2) + n\Big(m(c_2) - \alpha_2^1\Big) \tag{3}$$

We highlight that the anticipation levels formed (chosen) in a previous period are taken as given in the next period, and thus affect the DM's decisions in that period. For example, α_1^0 which is formed in period 0, is taken as given in period 1.

In each period t, the DM chooses consumption c_t and their future anticipation levels to maximize $u_t + \tilde{V}_{t+1}$, where u_t is their current period utility flow, and $\tilde{V}_{t+1}$ a continuation value that depends on the DM's planning horizon, and will be explained momentarily.

2.2 The DM's problem – unlimited planning horizon

In each period t, the DM maximizes $u_t + \tilde{V}_{t+1}$, where in this case of unlimited planning horizon, $\tilde{V}_{t+1} = \sum_{t' \geq t+1} u_{t'}$.

Denote lifetime income by $W = y_1 + y_2$.[2] The DM solves their problem by backward induction.

At $t = 2$ they choose

$$c_2 = W - c_1$$

since the budget constraint binds. At $t = 1$ they jointly choose c_1, α_2^1. From Equa-

[2]Under credit markets without imperfections, all that matters is lifetime income, and not how this income is distributed across periods. We revisit this point in Section 4.

tions (2) and (3), the FOC for α_2^1 is

$$n'\Big(\alpha_2^1 - \alpha_2^0\Big) = n'\Big(m(c_2) - \alpha_2^1\Big),$$

yielding

$$\alpha_2^1 = \frac{\alpha_2^0 + m(c_2)}{2},$$

and the FOC implicitly characterizing c_1 is

$$m'(c_1)\Big[1 + n'\Big(m(c_1) - \alpha_1^0\Big)\Big] = m'(W - c_1)\Big[1 + n'\Big(m(W - c_1) - \alpha_2^1\Big)\Big]$$

At $t = 0$ they choose

$$\alpha_1^0 = \frac{m(c_1)}{2}$$
$$\alpha_2^0 = \frac{\alpha_2^1}{2} = \frac{m(W - c_1)}{3}$$

Combining the FOCs, we get the following Euler equation:

$$m'(c_1^*)\left[1 + n'\left(\frac{m(c_1^*)}{2}\right)\right] = m'(c_2^*)\left[1 + n'\left(\frac{m(c_2^*)}{3}\right)\right] \tag{4}$$

This condition uniquely pins down c_1^*, as the LHS is decreasing and the RHS is increasing in c_1.

2.3 Comparison to benchmark model

We can rearrange Equation (4) to write it as

$$m'(c_1^*) = \beta^{\text{AU}} m'(c_2^*)$$

where $\beta^{\text{AU}} := \frac{1+n'\left(\frac{m(c_2)}{3}\right)}{1+n'\left(\frac{m(c_1)}{2}\right)}$ is an endogenous time-discounting factor, determined in equilibrium.

Proposition 1. *When income is certain and there is no credit market imperfection, the DM exhibits "patience," that is* $c_1^* < c_2^*$*, and* $\beta^{\text{AU}} > 1$*.*

Proof. $c_1^* < c_2^*$: If $c_1 = c_2 = W/2$ in Equation (4), then the LHS < RHS. Thus it has to be that $c_1 < W/2$ for the equality to hold.

$\beta^{\mathrm{AU}} > 1$: Since $c_1^* < c_2^*$, it holds that $m'(c_1^*) > m'(c_2^*)$; thus for the equality to hold it has to be that $\beta^{\mathrm{AU}} > 1$. □

The benchmark model is recovered for $n(\cdot)$ constant, or in fact also $n(\cdot)$ linear. Then $\beta^{\mathrm{AU}} = 1$, and the DM perfectly smooths their consumption between the two periods.

2.4 A "present-focused" DM – limited planning horizon

In this case the DM is assumed to have a planning horizon of only 1 period ahead. Specifically, at time 0 they form their anticipation level *as if* tomorrow is the last period of their life.

Denote again by W lifetime income.

At $t = 0$ the DM chooses α_1^0 so that

$$\alpha_1^0 = \arg\max \Big\{ u_0 + \tilde{V}_1 \Big\}$$

where in this case

$$\begin{aligned} u_0 &= n(\alpha_1^0) \\ \tilde{V}_1 &= m(W) + n\Big(m(W) - \alpha_1^0\Big) \end{aligned}$$

It follows that

$$\alpha_1^0 = \frac{m(W)}{2}$$

At $t = 1$ the DM takes α_1^0 as given and jointly chooses c_1, α_2^1 so that

$$(c_1, \alpha_2^1) = \arg\max \Big\{ u_1 + \tilde{V}_2 \Big\}$$

where

$$u_1 = m(c_1) + n\Big(m(c_1) - \alpha_1^0\Big) + n\Big(\alpha_2^1\Big)$$
$$\tilde{V}_2 = m(W - c_1) + n\Big(m(W - c_1) - \alpha_2^1\Big)$$

The FOC for α_2^1 yields

$$\alpha_2^1 = \frac{m(W - c_1)}{2}$$

The FOC for c_1 yields

$$m'(c_1)\left[1 + n'\left(m(c_1) - \frac{m(W)}{2}\right)\right] = m'(W - c_1)\Big[1 + n'\Big(m(W - c_1) - \alpha_2^1\Big)\Big]$$

Combining the two FOCs yields the condition characterizing c_1^*

$$m'(c_1^*)\left[1 + n'\left(m(c_1^*) - \frac{m(W)}{2}\right)\right] = m'(W - c_1^*)\left[1 + n'\left(\frac{m(W - c_1^*)}{2}\right)\right] \tag{5}$$

Re-arranging the above we can write it again as

$$m'(c_1^*) = \beta^{\text{AU,PF}} m'(c_2^*)$$

where $\beta^{\text{AU,PF}} := \frac{1+n'\left(\frac{m\left(W-c_1^*\right)}{2}\right)}{1+n'\left(m\left(c_1^*\right)-\frac{m(W)}{2}\right)}$ is an endogenous time-discounting factor, determined in equilibrium.

Proposition 2. *When income is certain, there is no credit market imperfection, and the DM is "present-focused," the DM exhibits "impatience," that is $c_1^* > c_2^*$, and $\beta^{\text{AU,PF}} < 1$.*

Proof. $c_1^* > c_2^*$: If $c_1 = c_2 = W/2$ in Equation (5), then the LHS > RHS. Thus it has to be that $c_1 > W/2$ for the equality to hold.

$\beta^{\text{AU,PF}} < 1$: Since $c_1^* > c_2^*$, it holds that $m'(c_1^*) < m'(c_2^*)$; thus for the equality to hold it has to be that $\beta^{\text{AU,PF}} < 1$. □

We notice again that for $n(\cdot)$ constant, or in fact also $n(\cdot)$ linear, the FOC reduces to the benchmark model of two-period consumption under certainty.

2.5 Discussion

We saw that our model of anticipatory utility features an endogenous time-discounting factor. A *present-focused DM* is impatient and has a *decreasing consumption path*, while a *non-present-focused DM* is patient and has an *increasing consumption path*.

We note that the present-focused DM is subject to a type of error: the time-0 self thinks time-1 self will consume everything, while time-1 self doesn't do so when the time comes. In contrast, the non-present-focused DM's problem is not: time-0 self chooses their anticipation levels taking into account a consumption plan which is in fact the one the future selves will indeed follow.

Anticipation level and market completeness. We introduced the way the present-focused DM chooses their anticipation level "as if" they have a 1-period only planning horizon, and thus they will consume all their cash-in-hand next period.

There is a conceptual equivalence between present focus as introduced here and the degree of market (in-)completeness. More specifically, in our model the DM can be thought to form their anticipation levels conditional on the contracts they have available to sign right now. In period 0 the agent knows they can have all their income as cash-in-hand in period 1 (no credit market imperfections assumption). If they don't have access to a forward contract that will allow them to carry over anything left from period 1 to 2, they assume they will consume everything in period 1, and present focus emerges. If they do have access to such a contract, they can put some money aside for period 2, and thus anticipate that consumption already in period 0.

Mental accounting may be more intricately connected to market (in-)completeness than what the literature has realized.

Anticipatory utility and heterogeneous agent models. We notice that anticipatory utility, combined with present focus can give rise to two types of agents: an impatient-borrower and a patient-saver. The degrees of patience and impatience are endogenously determined based on lifetime income, and there can thus be within-goup heterogeneity in the two types.

Such a distinction of agent types has been shown to be important for a general equilibrium analysis (e.g., Eggertsson and Krugman, 2012) and anticipatory utility could offer a behavioral foundation for it.

3 Consumption when income is risky

3.1 Choice of anticipation level

To illustrate how anticipation level is formed in the presence of uncertainty, we start with the simplest possible case: the DM lives for a single period, and their income in that period is risky. More specifically the DM's income is a lottery with two outcomes: income can be y^h with probability p, or $y^\ell < y^h$ with probability $1-p$.

The consumption decision is of course trivial, as the DM will consume the income they receive; but the formation of the anticipation level α_1^0 is meaningful, and this is what we focus on for now. As $m(\cdot)$ would only complicate notation in this case, we present the special case of $m(c) = c$.

The DM's anticipation-formation problem in period 0 is

$$\alpha_1^0 = \arg\max_{\alpha} \Big\{ p f^h(\alpha) + (1-p) f^\ell(\alpha) \Big\}$$

where

$$f^i(\alpha) \coloneqq n(\alpha) + n\Big(y^i - \alpha\Big)$$

for $i \in \{\ell, h\}$. Note that $f^i(\cdot)$ has a unique maximum at $y_i^* \coloneqq y^i/2$, being increasing on $(-\infty, y_i^*)$ and decreasing thereafter, and is concave on $(0, y^i)$.

Letting $g(\alpha) = (1-p)f^\ell(\alpha) + pf^h(\alpha)$, we can show that $g(\alpha)$ has a maximum, and the maximizer is in the interval $[y_\ell^*, y_h^*]$.

Proof. Since f^ℓ and f^h are both increasing for $\alpha < y_\ell^*$ and decreasing for $\alpha > y_h^*$, we have $g(y_\ell^*) > g(\alpha)$ for all $\alpha < y_\ell^*$ and $g(y_h^*) > g(\alpha)$ for all $\alpha > y_h^*$. Moreover, since $g(\cdot)$ is continuous, it attains a maximum value on the closed and bounded interval $[y_\ell^*, y_h^*]$. □

The above holds for *any* specification of $n(\cdot)$ satisfying Assumption 2. We will also show this maximizer is unique. It helps to distinguish two sub-cases.

Sub-case 1: $y^\ell > y_h^*$. In this case there is a unique maximizer: We have

$$g'(y_\ell^*) = p[n'(y^\ell/2) - n'(y^h - y^\ell/2)] > 0$$
$$g'(y_h^*) = (1-p)[n'(y^h/2) - n'(y^\ell - y^h/2)] < 0$$

and $g'' < 0$ in (y_ℓ^*, y_h^*) since both f^ℓ, f^h are concave in this region. Thus g' has a unique root and this is the maximizer.

What we show next is that when $y^\ell < y_h^*$, g' has again a unique root but there is a cut-off $\overline{p}$ such that for $p \geq \overline{p}$ the root is in the $\left[y^\ell, y_h^*\right)$ region, while for $p < \overline{p}$ the root is in the $\left(y_\ell^*, y^\ell\right)$ region.

Sub-case 2: $y^\ell < y_h^*$. Let us work with an $n()$ that is linear in losses, in particular take

$$n(\alpha) = \begin{cases} n_+(\alpha) & \text{if } \alpha \geq 0 \\ \lambda\alpha & \text{if } \alpha < 0 \end{cases}$$

where $n_+()$ is as usual increasing and concave, and it is such that $n'(0_+) < \lambda$.

Now, for $a = y_-^\ell$ we have

$$g'(y_-^\ell) = (1-p)[n'(y^\ell) - n'(0_+)] + p[n'(y^\ell) - n'(y^h - y^\ell)] \tag{6}$$

For $\alpha = y_+^\ell$ we have

$$g'(y_+^\ell) = (1-p)[n'(y^\ell) - n'(0_-)] + p[n'(y^\ell) - n'(y^h - y^\ell)] \tag{7}$$
$$= (1-p)\underbrace{[n'(y^\ell) - \lambda]}_{<0} + p\underbrace{[n'(y^\ell) - n'(y^h - y^\ell)]}_{>0} \tag{8}$$

Finally, for $\alpha = y_h^*$

$$\begin{aligned} g'(y_h^*) &= (1-p)[n'(y_h^*) - n'(y^\ell - y_h^*)] + p[n'(y_h^*) - n'(y^h - y_h^*)] \\ &= (1-p)[n'(y_h^*) - \lambda] \\ &< 0 \end{aligned}$$

The last line follows from $n'()$ being decreasing thus $n'(y_h^*) < n'(0_+)$, and having assumed that $n'(0_+) < \lambda$.

We also note that $g'(y^\ell_+) < g'(y^\ell_-)$ for any $p \in [0,1)$, and that inside the two intervals (y^*_ℓ, y^ℓ) and (y^ℓ, y^*_h), the function g is concave. This implies that g' is a decreasing function with $g'(y^*_\ell) > 0$ and $g'(y^*_h) < 0$, thus it has a unique critical point, and $g()$ a unique maximum.

Specifically, we have two cases:

- If (*)< 0 the maximum of g is inside the left interval
- If (*)> 0 the maximum of g is inside the right interval, including the possibility to be it's left boundary y^ℓ, which is the case when (**)< 0.

The existence of a cut-off $\overline{p}$: The transition occurs at the point where (**) is 0. Thus $\overline{p}$ is (implicitly) defined from the equation

$$(1-\overline{p})[n'(y^\ell) - \lambda] + \overline{p}[n'(y^\ell) - n'(y^h - y^\ell)] = 0$$

which yields the unique cut-off[3]

$$\overline{p} = \frac{\lambda - n'(y^\ell)}{\lambda - n'(y^h - y^\ell)}$$

Let us summarize the DM's optimal anticipation in the presence of income uncertainty, shown graphically in Figure 1. As the probability p of the good outcome increases from 0 to 1, the DM increases their anticipation level continuously. Whether the decision maker experiences a loss in the low-consumption state depends on the difference in consumption across states and the probability of the good state.

First, consider the case $y^\ell > y^h/2$. Even if the DM knew that the good state will occur with certainty ($p = 1$), their optimal anticipation level would be $y^h/2$; for any $p < 1$, their optimal anticipation level would be even lower. This implies the DM's anticipation level will be lower than y^ℓ in this case, and hence the DM will not experience a loss even if the bad state is realized.

Second, consider the case $y^\ell < y^h/2$. If the probability of the good outcome is sufficiently high (i.e., $p > \overline{p}$), the DM's anticipation level exceeds y^ℓ, and thus they will experience a loss if the bad outcome is realized. Otherwise, the DM's anticipation

[3]We can confirm as a sanity check that $\overline{p} \in (0,1)$

level will not exceed y^ℓ, so they will not experience a loss even if the bad outcome is realized.

And an inaction band: Finally, we note there is actually a range of probabilities p for which the DM will choose the same anticipation level, y^ℓ. Specifically, this is the case for any $p \in \left(\underline{p}, \overline{p}\right)$, where $\underline{p}$ is the root of (*),[4]

$$\underline{p} = \frac{n'(0_+) - n'(y^\ell)}{n'(0_+) - n'(y^h - y^\ell)}$$

We solve for α_1^0 as a function of p numerically, and notice it is piecewise linear:

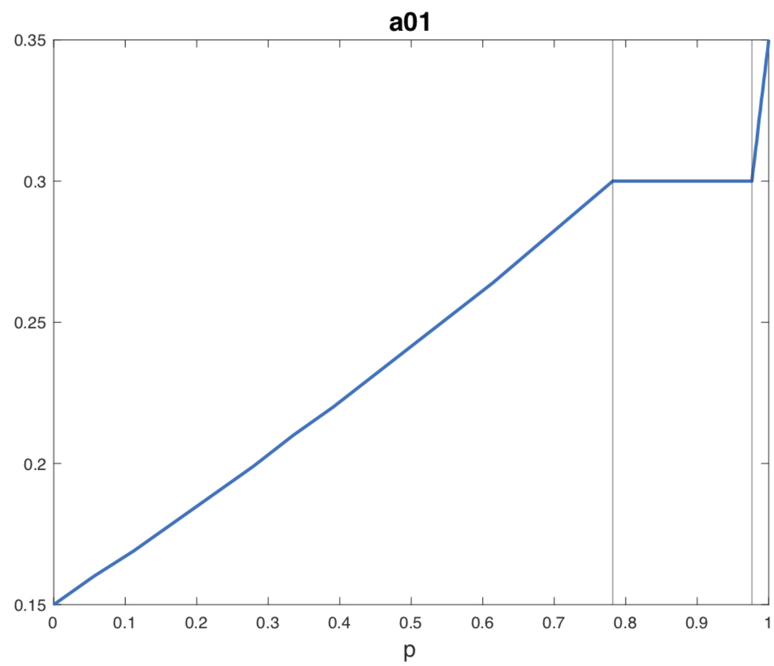

Figure 1: Numerical solution of $\alpha_1^0(p)$. Parameters: $y^\ell = 0.3$, $y^h = 0.7$, $n(x \geq 0) = \frac{(x+2)^{0.05} - 2^{0.005}}{0.05}$, $n(x \leq 0) = \lambda x$, $\lambda = 1.2$, implying a loss aversion of $\frac{n'(0_-)}{n'(0_+)} \approx 2.32$. The vertical lines show the analytically computed $\underline{p}$, $\overline{p}$.

To summarize, the DM needs to be compensated by a high enough probability of the good outcome in order to overcome their loss aversion and risk experiencing a loss ex-post, if the bad outcome is realized.

[4] Since (**) < (*) for any $p \in [0, 1)$ as we have already pointed out, it follows that $\underline{p} < \overline{p}$.

3.2 Precautionary savings

Having introduced how anticipation is formed under risky income, we now solve the same setup as the benchmark precautionary savings model. Suppose the DM receives some income $\bar{y}$ with certainty in period 1. In period 2 they receive a mean preserving spread of that income, that is risky income of the same expectation. Specifically, period 2 income can be y^h w.p. p, and $y^\ell < y^h$ w.p. $1-p$, such that $py^h + (1-p)y^\ell = \bar{y}$.

As the key object of interest is the amount of saving between periods 1 and 2, we express the problem in terms of that. Denote by s the amount of net saving out of period's 1 income; $s > 0$ means the DM saves, while $s < 0$ means the DM borrows.

Naturally, $s \in [-y^\ell, \bar{y}]$. It follows that $c_1 = \bar{y} - s$, $c_2 = y_2 + s$, where y_2, and hence c_2 are random variables.

In period 1, the DM jointly chooses

$$\left(s^*, \alpha_2^1\right) = \arg\max_{s,\alpha} \left\{ m(\bar{y}-s) + n\left(m(\bar{y}-s) - \alpha_1^0\right) + n\left(\alpha - \alpha_2^0\right) + \mathbb{E}[m(y_2+s) + n(m(y_2+s) - \alpha)] \right\}$$

taking α_1^0, α_2^0 as given. The expectation is taken over the realizations of y_2.

The FOC implicitly characterizing α_2^1 is

$$n'\left(\alpha_2^1 - \alpha_2^0\right) = \mathbb{E}\left[n'(m(y_2+s) - \alpha_2^1)\right]$$

and the FOC for s^* is

$$m'(\bar{y}-s)\left[1 + n'\left(m(\bar{y}-s) - \alpha_1^0\right)\right] = \mathbb{E}\left[m'(y_2+s)\left[1 + n'\left(m(y_2+s) - \alpha_2^1\right)\right]\right]$$

In period 0 the DM will choose

$$\alpha_1^0 = \frac{m(\bar{y}-s)}{2}$$

and

$$\alpha_2^0 = \frac{\alpha_2^1}{2}$$

For concreteness we start with some numerical examples that illustrate our results, following closely the textbook treatment of Jappelli and Pistaferri (2017) for the benchmark model. We then explain all results analytically.

Numerical examples

We solve the model numerically, and compare its results to the benchmark model of no gain-loss utility, i.e. when $n(\cdot)$ is constant, for three different specifications of $m(\cdot)$: $m(c) = c^{1-\gamma}$, with $\gamma < 1$, so m' is convex; $m(c) = \zeta c - \eta c^2$, meaning m' is linear; $m(c) = c$.

The income process for these illustrations is chosen to be:

$$y_2 = \begin{cases} 0.2, & \text{w.p. } 0.4 \\ 1.2, & \text{w.p. } 0.6 \end{cases}$$

and thus $y_1 = \mathbb{E}[y_2] = 0.8$.

The results are summarized in table 1 that follows.[5]

		Saving (s^*)		
		$m(c) = c^{1-\gamma}$	$m(c) = \zeta c - \frac{\eta}{2}c^2$	$m(c) = c$
Anticipatory utility	no risk	0.0083	0.01	0.16
	risk	0.1633	0.04	0.30
Benchmark	no risk	0	0	-
	risk	0.1583	0	-

Table 1: Precautionary savings of anticipatory utility model.

Discussion of numerical examples

The entries of the last column are empty for the benchmark model, as for a linear $m(\cdot)$ the problem is not well defined.

[5]The remaining parameters and specifications used are $\gamma = 0.99$; $\zeta = 3, \eta = 2$; $n(x \geq 0) = \frac{(x+2)^{0.05} - 2^{0.05}}{0.05}$, $n(x \leq 0) = \frac{(-x+0.8254)^{0.05} - 0.8254^{0.05}}{0.05}$; implied loss aversion $\frac{n'(0_-)}{n'(0_+)} \approx 2.32$. They were chosen for expository purposes: the parameters of $n(\cdot)$ were chosen to give a reasonable loss aversion closely above 2; $\gamma \approx 1$ makes the CRRA specification comparable to the standard $\log(\cdot)$, as the two have first derivatives (almost) proportional to each other; ζ, η were chosen so that the maximum of the quadratic is to the right of the range of interest and so that $m(y^h - y^\ell)$ is the same in all three specifications, which we want for the comparability of the results of table 2.

The main message of the exercise is that the DM with anticipatory utility exhibits a stronger savings motive compared to the benchmark. This motive is partially due to the agent's patience (proposition 1), and partially to a motive to save to insure against the possible bad outcome of period 2 (precautionary savings motive).

We highlight that the anticipatory utility breaks certainty equivalence: there is a precautionary savings motive for the quadratic utility as well, meaning there is a *first-order* precautionary savings motive.

To unpack the results let us look back at the FOC of the DM's problem under anticipatory utility, which we can compare both to those of the problem under certainty, and the benchmark model.

$$\alpha_1^0 = \frac{m(\bar{y}-s)}{2} \tag{1}$$

$$\alpha_2^0 = \frac{\alpha_2^1}{2} \tag{2}$$

$$n'(\alpha_2^1 - \alpha_2^0) = \mathbb{E}\Big\{n'(m(y_2+s) - \alpha_2^1)\Big\} \tag{3}$$

$$m'(\bar{y}-s)\Big[1 + n'\Big(m(\bar{y}-s) - \alpha_1^0\Big)\Big] = \mathbb{E}\Bigg\{\Big[1 + n'\Big(m(y_2+s) - \alpha_2^1\Big)\Big]m'(y_2+s)\Bigg\} \tag{4}$$

By rearranging (4) we notice the DM effectively has an endogenous stochastic discount factor in this case

$$m'(\bar{y}-s) = \mathbb{E}\Bigg\{\underbrace{\frac{1 + n'(m(y_2+s) - \alpha_2^1)}{1 + n'(m(\bar{y}-s) - \alpha_1^0)}}_{\text{endog. stoc. disc. factor}} m'(y_2+s)\Bigg\}$$

Using that $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y] + Cov(X,Y)$, and (1) we can get

$$m'(\bar{y}-s) = \frac{\mathbb{E}\Big\{1 + n'(m(y_2+s) - \alpha_2^1)\Big\}}{1 + n'\Big(\frac{m(\bar{y}-s)}{2}\Big)}\ \mathbb{E}\Big\{m'(y_2+s)\Big\} + \frac{Cov(m', n')}{1 + n'\Big(\frac{m(\bar{y}-s)}{2}\Big)}$$

Now the covariance term is non-negative, as a covariance of two (weakly) decreasing functions of y_2 meaning,

$$m'(\bar{y}-s) \geq \frac{\mathbb{E}\Big\{1 + n'(m(y_2+s) - \alpha_2^1)\Big\}}{1 + n'\Big(\frac{m(\bar{y}-s)}{2}\Big)}\ \mathbb{E}\Big\{m'(y_2+s)\Big\}$$

with the equality holding when the covariance term is 0.

Combining with (2),(3) and re-arranging we get

$$m'(\bar{y}-s) \geq \frac{1+n'\left(\frac{\alpha_2^1}{2}\right)}{1+n'\left(\frac{m(\bar{y}-s)}{2}\right)}\ \mathbb{E}\Big\{m'(y_2+s)\Big\} \tag{A}$$

which has to be satisfied by the chosen α_2^1, s.

Finally, combining (2), (3), and given that n' is convex in our parameterization, from Jensen's inequality we get that the chosen α_2^1, s must also satisfy

$$n'\left(\frac{\alpha_2^1}{2}\right) > n'\left(\mathbb{E}\Big\{m(y_2+s)\Big\} - \alpha_2^1\right)$$

Since $n'(\cdot)$ is decreasing, this condition is equivalent to

$$\alpha_2^1 < \frac{2}{3}\mathbb{E}[m(y_2+s)]$$

and again by Jensen's inequality for $m(\cdot)$ that is concave we get

$$\alpha_2^1 < \frac{2}{3}m(\bar{y}+s) \tag{B}$$

This is a second inequality that has to be satisfied by the chosen α_2^1, s.

Remark 1. From (B) we have

$$\frac{1+n'\left(\frac{\alpha_2^1}{2}\right)}{1+n'\left(\frac{m(\bar{y}-s)}{2}\right)} > \frac{1+n'\left(\frac{m(\bar{y}+s)}{3}\right)}{1+n'\left(\frac{m(\bar{y}-s)}{2}\right)} \equiv \beta^{\mathrm{AU}}(s)$$

A *sufficient* condition for the DM to save more under anticipatory utility than what they save because of the precautionary motive in the benchmark is that

$$\beta^{\mathrm{AU}}(s^{prec.}) > 1$$

In that case it holds that

$$m'(\bar{y}-s^{prec.}) < \beta^{\mathrm{AU}}(s^{prec.})\ \mathbb{E}\Big\{m'(y_2+s^{prec.})\Big\}$$

and the DM will thus want to save more than $s^{prec.}$ for (A) to hold. The $\beta^{\mathrm{AU}}(s^{prec.}) > 1$ condition, which amounts to $\frac{m(\bar{y}+s^{prec.})}{3} < \frac{m(\bar{y}-s^{prec.})}{2}$ is indeed satisfied for both the CRRA and quadratic utilities in our examples, where $s^{prec.} = 0.1583$, and $s^{prec.} = 0$ respectively.

Remark 2. Starting again from (A) and applying Jensen's inequality for m' that is (weakly) convex in our first two specifications, we get

$$m'(\bar{y} - s) > \beta^{\mathrm{AU}}(s)\ m'(\bar{y} + s)$$

which has to be satisfied in equilibrium, while $s^{\text{no risk}}$ satisfies that condition with equality. Thus the DM will also be saving more than in the certainty case, meaning they have a precautionary saving motive under anticipatory utility.

Remark 3. In the case when $m(c) = c$, the covariance term is 0, and the problem reduces to

$$n'\left(\frac{\bar{y} - s}{2}\right) = \mathbb{E}\left\{n'(y_2 - \bar{y} + 2s)\right\}$$

We notice saving in this case is determined by a condition analogous to the one in the benchmark case but the role of $m'(\cdot)$ is played by $n'(\cdot)$. Therefore the precautionary saving motive in this case is determined by the prudence of n.[6]

3.3 Precautionary savings under present focus

Contrary to the case we studied above, when the DM is present-focused they have an incentive to over-consume in period 1 (proposition 2). Thus a trade-off emerges: on the one hand the DM wants to over-consume in period 1, according to their present-focused anticipation level formed in period 0, on the other hand, precautionary savings induce the DM to save for the bad outcome of period 2. We investigate the determinants of how this trade-off is resolved.

In period 0 now the DM thinks they will consume all their cash-in-hand in period 1, that is $\bar{y} + y^{\ell}$. This yields

$$\alpha_1^0 = \frac{m(\bar{y} + y^{\ell})}{2}$$

[6]Under certainty it is straightforward to solve the condition analytically, to get $s = \bar{y}/5$, which for $\bar{y} = 0.8$ gives $s = 0.16$, as we found numerically.

α_2^0 is determined by backward induction. $\alpha_2^1(\alpha_2^0;0)$ denotes the anticipation level α_2^1 *believed in period* 0, and is a function of α_2^0. It is characterized by

$$\alpha_2^1(\alpha_2^0;0) = \arg\max_{\alpha} \left\{ n(\alpha - \alpha_2^0) + \mathbb{E}\Big[n\Big(m(y_2 - y_\ell) - \alpha \Big) \Big] \right\}$$

and then

$$\alpha_2^0 = \arg\max_{a} \alpha \left\{ n(\alpha) + n(\alpha_2^1(\alpha;0) - \alpha) + \mathbb{E}\Big[n\Big(m(y_2 - y_\ell) - \alpha_2^1(\alpha;0) \Big) \Big] \right\}$$

Now in period 1, taking α_1^0, α_2^0 as given, the DM can decide their desired net saving s and their anticipation level, which are jointly determined from

$$\Big(\alpha_2^1, s^*\Big) = \arg\max_{\alpha,s} \left\{ m(\bar{y}-s) + n\Big(m(\bar{y}-s) - \alpha_1^0\Big) + n(\alpha - \alpha_2^0) + \mathbb{E}\Big[m(y_2+s) + n\Big(m(y_2+s) - \alpha \Big) \Big] \right\}$$

We note that in this case as well, for $n(\cdot)$ constant, we recover the benchmark model.

As before, we start with with some numerical examples for concreteness, and then explain all results analytically.

Numerical examples

We solve the model numerically for the same cases we solved it for the non-present-focused DM. The results are summarized in table 2 that follows.

		Saving (s^*)		
		$m(c) = c^{1-\gamma}$	$m(c) = \zeta c - \frac{\eta}{2}c^2$	$m(c) = c$
Anticipatory utility (PF)	no risk	-0.0002	-0.0074	-0.27
	risk	0.1575	0.0245	0.050
Benchmark	no risk	0	0	-
	risk	0.1583	0	-

Table 2: Precautionary savings of anticipatory utility model, under present focus.

Discussion of numerical examples

As expected, under no risk the DM borrows under all specifications. We also notice that in all cases precautionary savings are weaker compared to to non-present-focused DM, and in the case of linear utility significantly so.

In the case of quadratic utility, precautionary savings of the anticipatory utility DM appears again stronger than in the benchmark model, while for the CRRA specification the balance has flipped. So impatience can put enough downward pressure for precautionary savings to become weaker than the benchmark case. This is also confirmed in the next table.

Interestingly, for a narrower mean-preserving spread the present-focused DM can be inclined to borrow even in the presence of risk.

		Saving (s^*)		
		$m(c) = c^{1-\gamma}$	$m(c) = \zeta c - \frac{\eta}{2}c^2$	$m(c) = c$
Anticipatory utility (PF)	no risk	-0.0002	-0.0074	-0.27
	risk	0.040	-0.0015	-0.158
Benchmark	no risk	0	0	-
	risk	0.041	0	-

Table 3: Precautionary savings of anticipatory utility model, under present focus, for less risky outcome, $y^h = 1, \text{w.p. } 0.6, y^\ell = 0.5, \text{w.p. } 0.4$

To unpack the results of the present-focused DM, let us again write the FOC characterizing the solution in this case.

$$\alpha_1^0 = \frac{m(\bar{y} + y^\ell)}{2} \tag{1'}$$

$$\alpha_2^0 = \frac{\alpha_2^1(0)}{2} \tag{2a'}$$

$$n'(\alpha_2^1(0) - \alpha_2^0) = \mathbb{E}\Big\{n'(m(y_2 - y^\ell) - \alpha_2^1(0))\Big\} \tag{2b'}$$

$$n'(\alpha_2^1 - \alpha_2^0) = \mathbb{E}\Big\{n'(m(y_2 + s) - \alpha_2^1)\Big\} \tag{3'}$$

$$m'(\bar{y} - s)\Big[1 + n'\Big(m(\bar{y} - s) - \alpha_1^0\Big)\Big] = \mathbb{E}\left\{\Big[1 + n'\Big(m(y_2 + s) - \alpha_2^1\Big)\Big]m'(y_2 + s)\right\} \tag{4'}$$

We notice one key difference in the case of present focus: the appearance of condition $(2b')$. This condition pins down the belief of α_2^1 that self 0 holds, hence denoted by

$\alpha_2^1(0)$, to distinguish it from the anticipation level actually formed in period 1, denoted by α_2^1 and given by $(3')$.

Re-arranging (4') we notice again the DM can be thought to effectively have an endogenous stochastic discount factor,

$$m'(\bar{y}-s) = \mathbb{E}\left\{ \underbrace{\frac{1+n'(m(y_2+s)-\alpha_2^1)}{1+n'(m(\bar{y}-s)-\alpha_1^0)}}_{\text{endog. stoc. disc. factor}} m'(y_2+s) \right\}$$

but this discount factor will be different, as conditions (1')-(3') are different from (1)-(3).

Now, from the introductory discussion of this section, we know that the DM will not choose an as high anticipation level as to incur a possible loss ex-post, unless the probability of the good outcome is very high. It turns out this is not the case in any of the examples we are considering, thus (2a') and (2b') reduce to $\alpha_2^0 = 0$ (2") (and $\alpha_2^1(0) = 0$).

We are thus effectively left with a system of two equations and two unknowns (s, α_2^1):

$$n'(\alpha_2^1) = \mathbb{E}\Big\{ n'(m(y_2+s)-\alpha_2^1) \Big\}$$

$$m'(\bar{y}-s)\left[1+n'\left(m(\bar{y}-s)-\frac{m(\bar{y}+y^\ell)}{2}\right)\right] = \mathbb{E}\left\{\Big[1+n'\Big(m(y_2+s)-\alpha_2^1\Big)\Big]m'(y_2+s)\right\}$$

We linearize it around the certainty solution where $y_2 = \bar{y}$, which we denote by $(\bar{s}, \bar{\alpha})$. From section 2.4 we know that $\bar{\alpha} = \frac{m(\bar{y}+\bar{s})}{2}$, and $\bar{s} < 0$.

The linearized system is given by

$$(\alpha_2^1 - \bar{\alpha}) - \frac{m'(\bar{y}+\bar{s})}{2}(s-\bar{s}) = 0 \tag{A'}$$

$$\phi_0 + \phi_a(\alpha_2^1 - \bar{\alpha}) + \phi_s(s-\bar{s}) = 0 \tag{B'}$$

where ϕ_0, ϕ_a, ϕ_s are the following constants, which are of determinate sign

$$\phi_0 = m'(\bar{y} - \bar{s})\left[1 + n'\left(m(\bar{y} - \bar{s}) - \frac{m(\bar{y} + y^\ell)}{2}\right)\right] - m'(\bar{y} + \bar{s})\left[1 + n'\left(\frac{m(\bar{y} + \bar{s})}{2}\right)\right] < 0$$

$$\phi_a = n''\left(\frac{m(\bar{y} + \bar{s})}{2}\right)m'(\bar{y} + \bar{s}) < 0$$

$$\phi_s = -\Bigg\{m''(\bar{y} - \bar{s})\Big[1 + n'\Big(m(\bar{y} - \bar{s}) - \tfrac{m(\bar{y}+y^\ell)}{2}\Big)\Big] + [m'(\bar{y} - \bar{s})]^2 n''\Big(m(\bar{y} - \bar{s}) - \tfrac{m(\bar{y}+y^\ell)}{2}\Big) + $$
$$+ m''(\bar{y} + \bar{s})\Big[1 + n'\Big(\tfrac{m(\bar{y}+\bar{s})}{2}\Big)\Big] + [m'(\bar{y} + \bar{s})]^2 n''\Big(\tfrac{m(\bar{y}+\bar{s})}{2}\Big)\Bigg\} > 0$$

Finally, plugging (A') into (B') we can solve for s,

$$s = \bar{s} + \varphi$$

where $\varphi \equiv \frac{-\phi_0}{\phi_s + \frac{m'(\bar{y}+\bar{s})}{2}\phi_a} > 0$.

Remark 4. The sign of ϕ_a is readily verified, as $n'' < 0$ and $m' > 0$.

The sign of ϕ_s is also readily verified from the properties of $n(\cdot)$ and $m(\cdot)$, as inside the curly brackets is a sum of negative terms.

The denominator of φ can be seen to be positive after collecting terms.

The sign of ϕ_0 is determined by the fact that in the certainty case, the same expression equals 0, but with $\alpha_1^0 = \frac{m(2\bar{y})}{2}$ in the place of $\alpha_1^0 = \frac{m(\bar{y}+y^\ell)}{2}$ here. Since $y^\ell < \bar{y}$, it follows from the properties of $m(\cdot)$ and $n(\cdot)$ that $\phi_0 < 0$.

Remark 5. The presence of $\phi_0 \neq 0$ is critical as it implies a *first-order* uncertainty effect. If we had $\phi_0 = 0$, it would mean that $\bar{s}, \bar{\alpha}$ would be the solution to the system, i.e. uncertainty would only have second-order effects, as is the case in the benchmark model.

Remark 6. We have $\varphi > 0$. This means that at a first-order uncertainty has a precautionary savings motive. However, $\bar{s} < 0$ thus whether the net effect of total saving is positive or negative depends on the parameterization, but also, critically, on the size of the spread.

The effect of the size of the spread is captured by the presence of y^ℓ. Keeping p fixed, a bigger spread has a lower y^ℓ.

Assuming prudence in $n(\cdot)$, that is $n''' > 0$, it can be shown that

$$\frac{\partial \varphi}{\partial y^\ell} < 0$$

thus a bigger spread implies a higher s.

In other words, the precautionary savings motive ($\varphi > 0$) dominates the impatience motive ($\bar{s} < 0$) at some point as uncertainty gets bigger, as we also saw in the numerical examples.[7]

3.4 Short-term income risk

To study precautionary savings we naturally focused on income uncertainty that is resolved further in the future, in period 2.

Another case of interest is how income uncertainty resolved in the short-term, in period 1, affects the DM's behavior. For this section only we limit attention on the present-focused DM for whom $c_1^* > c_2^*$ under no uncertainty. This is the empirically more relevant case, as well as the case more directly comparable with Kőszegi and Rabin (2009).

Assume that $y_2 = \bar{y}$, and y_1 can be y^h w.p. p, and $y^\ell < y^h$ w.p. $1-p$, such that $py^h + (1-p)y^\ell = \bar{y}$. In period 0, when the DM chooses their anticipation level α_1^0, they are uncertain about their income realization in period 1; uncertainty is resolved *before* the DM chooses how much to consume in period 1.

The following holds

Proposition 3. *If the DM knew their income realization from the beginning, they would consume less (more) in period* 1 *when* y^ℓ *(*y^h*) is realized, compared to what they consume under uncertainty.*

Proof. Anticipation level α_1^0 is formed from

$$n'\Big(\alpha_1^0\Big) = pn'\Big(m\Big(y^h + \bar{y}\Big) - \alpha_1^0\Big) + (1-p)n'\Big(m\Big(y^\ell + \bar{y}\Big) - \alpha_1^0\Big)$$

[7] Naturally if there is no uncertainty, meaning $y^\ell = \bar{y}$ $(= y^h)$, then $\phi_0 = \varphi = 0$ and we recover the certainty solution.

But $n'' < 0$ implies

$$\begin{aligned} n'\Big(\alpha_1^0\Big) &< pn'\Big(m\Big(y^\ell + \bar{y}\Big) - \alpha_1^0\Big) + (1-p)n'\Big(m\Big(y^\ell + \bar{y}\Big) - \alpha_1^0\Big) \\ &= n'\Big(m\Big(y^\ell + \bar{y}\Big) - \alpha_1^0\Big) \end{aligned}$$

And thus

$$\alpha_1^0 > \frac{m\Big(y^\ell + \bar{y}\Big)}{2} \equiv \alpha_1^0(y^\ell)$$

Analogously it is shown that

$$\alpha_1^0 < \frac{m\Big(y^h + \bar{y}\Big)}{2} \equiv \alpha_1^0(y^h)$$

In other words α_1^0 under uncertainty is between the anticipation levels the DM would choose if they knew they would receive y^ℓ or y^h in period 1, denoted $\alpha_1^0(y^\ell)$ and $\alpha_1^0(y^h)$ respectively (see also Figure 1). The result follows from the FOC characterizing c_1^*

$$m'(c_1^*)\Big[1 + n'\Big(m(c_1^*) - \alpha_1^0\Big)\Big] = m'(W - c_1^*)\left[1 + n'\left(\frac{m(W - c_1^*)}{2}\right)\right]$$

where $W = y^h + \bar{y}$, or $W = y^\ell + \bar{y}$ in each case respectively. □

This result is in contrast to Kőszegi and Rabin (2009) where the DM would want to consume less in both cases (proposition 6).

Furthermore, under uncertainty, the DM will be impatient – that is $c_1^* > c_2^*$, if y^ℓ is realized, while this is not necessarily the case when y^h is realized. This is also different from Kőszegi and Rabin (2009) where the DM would be impatient, consuming $c_1^* > c_2^*$ in both cases (proposition 5).

4 Consumption when both liquid and illiquid assets exist

A recent literature has highlighted the importance of considering both liquid and illiquid assets for consumption-saving problems as the one studied here (e.g., Kaplan and Violante, 2022).

Up to now we have worked with the assumption of credit markets with no imperfections, that is $r^b = r^s = r$ and there is no borrowing limit. Under this assumption, there can be no distinction between a liquid asset – money today, and an illiquid asset – money tomorrow. That is because income of y tomorrow is equivalent to income of $\frac{y}{1+r}$ today, and vice-versa.

In such a setup the DM can borrow their lifetime income (up to the natural borrowing limit) in the first period of their life, and only decide how much to save each period; the actual timing of a cash-flow is irrelevant. This is why up to now we only focused on lifetime income – $W = y_1 + y_2$.

We now make the distinction between a liquid and an illiquid asset meaningful, by assuming $r^b > r^s$. In such a setup borrowing, or equivalently liquidating an illiquid asset (tomorrow's income) is costly. For simplicity we will assume $r^s = 0$.

Non-present-focused DM. Clearly, the fact that borrowing is costly does not change the behavior of the non-present-focused DM, who is not borrowing anyway.

We thus limit attention to the present-focused DM.

Present-focused DM. We illustrate the present-focused DM's behavior when income is certain, i.e. when $y_1 = y_2 = \bar{y}$.

In this case, it will hold that

$$\alpha_1^0 = \frac{m(\bar{y} + \frac{\bar{y}}{1+r^b})}{2}$$

and net savings and the anticipation level in period 1 are jointly chosen from

$$\left(\alpha_2^1, s^*\right) = \arg\max_{\alpha,s} \Big\{ m(\bar{y} - s) + n\Big(m(\bar{y} - s) - \alpha_1^0\Big) + n(\alpha) + \\ + m\Big(\bar{y} + s(1 + r^b \mathbb{1}_{s<0})\Big) + n\Big(m\Big(\bar{y} + s(1 + r^b \mathbb{1}_{s<0}) - \alpha\Big)\Big\}$$

Lemma. *When $r^b > r^s = 0$ and income is certain, the present-focused DM will never want to save.*

Proof. Suppose it is the case, and $s > 0$. Then,

$$\alpha_2^1 = \frac{m(\bar{y} + s)}{2}$$

and the FOC pinning down s^* will be

$$m'(\bar{y}-s)\left[1+n'\left(m(\bar{y}-s)-\frac{m\left(\bar{y}+\frac{\bar{y}}{1+r^b}\right)}{2}\right)\right]=m'(\bar{y}+s)\left[1+n'\left(\frac{m(\bar{y}+s)}{2}\right)\right]$$

But for $s=0$, it holds that LHS $>$ RHS, thus the DM will want to borrow, i.e. $s<0$. □

Proposition 4. *When $r^b > r^s = 0$ and income is certain, then (a) the amount a present-focused DM wants to borrow is decreasing in r^b, and (b) there exists an endogenous cut-off $\bar{r}^b$, above which the DM chooses $s^* = 0$.*

Proof. Assuming $s<0$, then

$$\alpha_2^1=\frac{m\left(\bar{y}+s(1+r^b)\right)}{2}$$

and the FOC pinning down s^* will be

$$m'(\bar{y}-s)=(1+r^b)\frac{\left[1+n'\left(\frac{m\left(\bar{y}+s(1+r^b)\right)}{2}\right)\right]}{\left[1+n'\left(m(\bar{y}-s)-\frac{m\left(\bar{y}+\frac{\bar{y}}{1+r^b}\right)}{2}\right)\right]}m'\left(\bar{y}+s(1+r^b)\right)$$

Part (a) follows from applyting the implicit function theorem in the above condition to get

$$\frac{\partial s}{\partial r^b}>0$$

when $s<0$. Thus an increase in r^b decreases borrowing.

For part (b) we plug $s=0$ in the above. Now define $G(r^b)$ be the difference of the LHS-RHS in the above for $s=0$:

$$G(r^b)\equiv 1+n'\left(m(\bar{y})-\frac{m\left(\bar{y}+\frac{\bar{y}}{1+r^b}\right)}{2}\right)-(1+r^b)\left[1+n'\left(\frac{m(\bar{y})}{2}\right)\right]$$

We notice it holds that $G(0)>0$, and that $G(r^b)$ is decreasing in r^b. Thus there is a unique cut-off implicitly defined from

$$G(\bar{r}^b) = 0$$

above which the DM will be made worse-off by any borrowing, and thus they will choose $s^* = 0$.

The cut-off $\bar{r}^b$ depends on $m(\cdot), n(\cdot)$, and $\bar{y}$. For any $r^b < \bar{r}^b$ the DM borrows choosing $s^* < 0$; for $r^b \geq \bar{r}^b$ the DM chooses $s^* = 0$ □

We illustrate below the amount of borrowing for the present-focused DM with linear utility $m(c) = c$, and $\bar{y}, n(\cdot)$ same as in our numerical examples above.

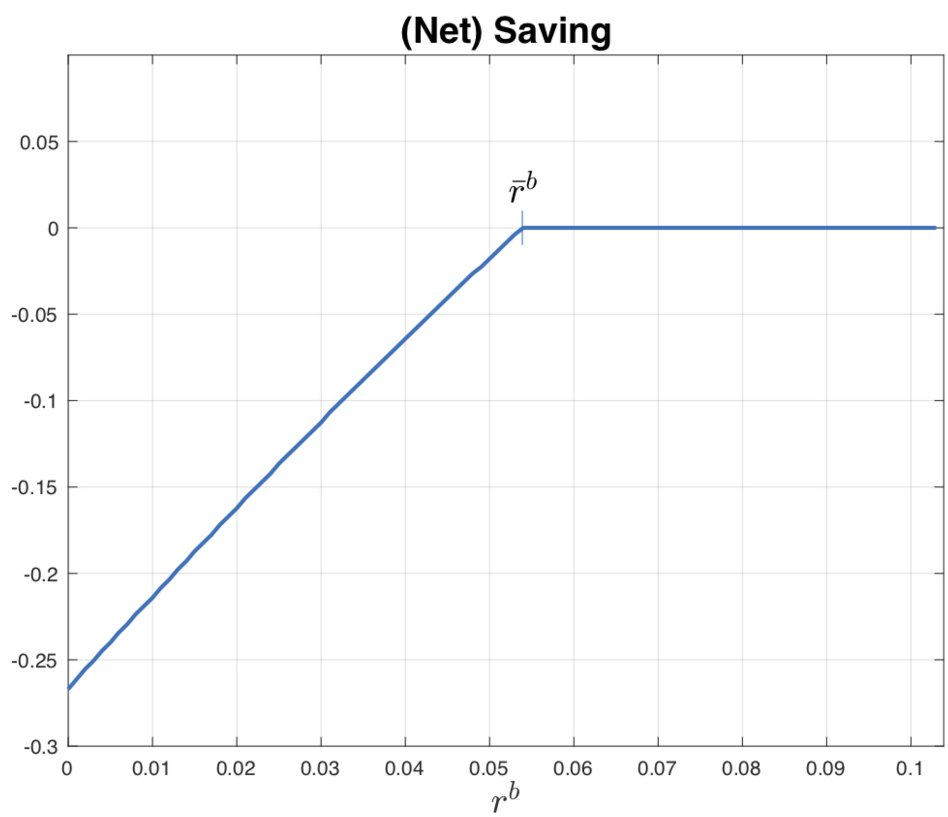

Figure 2: Present-focused DM's behavior under credit market imperfections. There is a cut-off $\bar{r}^b$ above which the DM, who is impatient and thus the natural borrower in the economy, is unwilling to borrow anything.

5 Conclusion

This paper has introduced a consumption-saving model of anticipatory utility. We derived the model's implications for a two-period horizon. We think there are two natural directions to extend the analysis: (a) to consider the two types of agents interacting in a general equilibrium setup, and (b) to extend the model to a multi-period horizon and calibrate it to assess it quantitatively.

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