# MARKET CHOICE IN ASSET TRADING:

# THE ROLE OF COLLATERAL\*

Lukas Altermatt

Piero Gottardi

University of Essex

University of Essex

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## Abstract

Many financial assets are simultaneously traded on centralised exchanges and on bilateral over-the-counter (OTC) markets which are riddled with frictions. In this paper, we investigate why agents choose to trade on OTC markets, with a specific focus on collateral requirements. Our model suggests that if trade on centralised exchanges is restricted by collateral requirements and matching on OTC markets is sufficiently efficient, agents with large trading needs find it preferable to trade on OTC markets in order to avoid collateral constraints. However, their presence on OTC markets also attracts agents with small trading needs, as these agents can extract gains from trade by trading with an agent with a large trading need, while their gains from trade on centralised exchanges are near zero. Meanwhile, agents with medium-sized trading needs prefer centralised exchanges, since they are less severely affected by collateral requirements, while their trading needs are strong enough so that they prioritize efficency over uncertain, but potentially larger gains from trade on the OTC market.

Keywords: OTC markets, Financial assets, Collateral, Efficiency.

JEL codes: G0, G1.

Preliminary - please do not quote or circulate

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#### 1 Introduction

Many financial assets are traded bilaterally, in what is often referred to as overthe-counter (OTC) markets. Since these markets are generally opaque and hard to monitor, there is a sense that the prevalence of these markets engenders financial instability. This was an issue in particular during the financial crisis of 2007-09, when some assets were essentially not traded anymore as their markets froze. Since then, there has been a push by policymakers to move these trades to centralised exchanges instead. However, while many assets are now partially traded on centralised exchanges, sizeable trading volumes remain on OTC markets. This gives rise to the question why investors decide to trade on OTC markets when centralised exchanges are available: Centralised exchanges are typically thought of as operating near frictionless, while OTC markets are riddled with frictions and hence appear less efficient.

In this paper, we investigate a possible answer this question. One rationale which has been brought forward for the coexistence of OTC markets and centralised exchanges is the role of informational asymmetries (see further below for a discussion of the existing literature); while we believe that information certainly plays a relevant role, we focus on a different channel here. In particular, our starting point is the idea that OTC trades are easier to customise than trades on centralised exchanges. More specifically, we focus on the role of collateral requirements in transactions, by assuming that on centralised exchanges they are standardised, whereas they can be negotiated upon when trade occurs in bilateral matches. Following this approach, we find that if on the OTC market traders can direct their search so as to identify the most appropriate counterparty, in equilibrium investors with very large trading needs choose to trade on OTC markets, while all other investors trade on the centralised exchange. The reason for this is that the OTC market allows investors with large trading needs to bypass the collateral requirements which they face on the centralised exchange. Perhaps more interestingly, if they can only imperfectly direct their search for counterparties in the OTC market, in equilibrium both investors with large and small trading needs trade on the OTC market, while investors with medium trading needs choose the centralised exchange.

The intuition for this equilibrium is that if, with some probability, investors end up meeting a different type of counterparty from the one they selected, this makes it attractive for investors with small trading needs to visit the OTC market and hope for a match with an investor with large trading needs. If such a match is formed, they can then extract some of the surplus from trade by selling the asset their counterparty wants at a high price.

Model summary. In the model we analyze, investors hold an identical portfolio of three assets (A, B, Z). Investors are then hit with a preference shock that determines their relative utility of holding assets A and B; for simplicity, we assume that this shock is perfectly negatively correlated across the two assets. Such shock induces them to seek to rebalance their portfolio, they can do that by trading either in the OTC market or in the centralised exchange. Asset Z is a collateral asset, that is a relatively standard/liquid asset and the benefit of holding it is not subject to shocks. Such asset must be pledged as collateral on the centralised exchange to settle transactions regarding asset A or B. We implicitly assume that any revenue from selling one asset accrues too late to purchase the other asset. The centralised exchange is a competitive market, where investors can trade any quantity of assets A and B at a given price, subject to the constraint that all purchases they wish to make of any of the two assets must be backed by appropriate amounts of collateral Z. Hence, their holdings of Z limits the purchases they can carry out on this market.

In the OTC market, investors are able to find a counterpart only with probability  $1 - \alpha < 1$ . Conditional on finding one, with probability  $\gamma$  they are matched with a randomly selected counterparty from the pool of investors who are present in that market (and wish to carry out a transaction in the

opposite direction from them). With the remaining probability  $1-\gamma$  they are instead matched with an investor with the opposite preference shock to theirs. As we will argue, the latter obtains in a stable configuration of matches (under complete information) in this market and in this sense we will argue constitutes the "most appropriate" counterparty. Once a match is formed, terms of trade are determined through bargaining.

Trading outcomes in a centralised exchange when all investors are present in that market and the holdings of collateral are Z sufficiently high such that nobody is collateral constrained are exactly equivalent with the ones obtained in the OTC market when again all investors are present and  $\alpha = \gamma = 0$ . Hence, the limited holdings of collateral Z constitutes the friction present on the centralised exchange, whereas the values of  $\alpha$  and  $\gamma$  describe the extent of the matching frictions which operate in the OTC market.

We find that when  $\alpha$  and  $\gamma$  are positive but small, an equilibrium exists where investors with high and low preference shocks choose to trade on the OTC market, while those with medium preference shocks trade on the centralised exchange. Investors with high preference shocks have the largest trading needs and hence their trades on the centralised exchange are quite restricted by the collateral constraint. As long as  $\gamma$  is not too high, the OTC market offers an enticing alternative to them, since if they manage to meet the counterparty towards which they direct their search (the appropriate counterparty), they can trade more than on the centralised exchange at similar terms of trade. When  $\gamma$  is also not too small, the presence of these investors with large trading needs also attracts to the OTC market the investors with the least extreme preference shocks (who would trade almost nothing on the centralised exchange). The reason is that the chance of meeting an investor with large trading needs, though small, is quite attractive to them. When in fact such a match occurs, the investor with small trading needs sells the asset his counterparts desires at a high price and thus receives a high surplus from trade.

When instead  $\gamma$  is sufficiently close to zero, search in the OTC market is mostly direct and hence almost all meetings occur among appropriate counterparties. Hence investors with small trading needs are unlikely to meet investors with larger trading needs and thus have little benefit from visiting the OTC market. In that case we find that an equilibrium exists where only investors with large trading needs visit the OTC market, while investors with medium and small trading needs visit the centralised exchange.

Varying  $\alpha$  has straightforward consequences: If  $\alpha$  is high, the OTC market becomes unattractive for everyone as there is a high chance for investors in that market to remain unmatched, and only the centralised exchange remains open. When instead  $\alpha$  approaches zero (while  $\gamma$  remains small and positive), the OTC market becomes highly attractive for everyone except a handful of investors with medium trading needs.

Finally, if  $\gamma$  is large (while  $\alpha$  is small and positive), the OTC market unravels from the top, as the investors with the most extreme trading needs no longer find it attractive since the probability of getting bad terms of trade becomes too high for them. Once they leave the market, it becomes much less attractive for everyone else as well, since the matches which promise the highest surplus are not available anymore. In that case, an equilibrium exists where everyone except the investors with the smallest trading needs enter the enters the centralised exchange.

Existing literature. To the best of our knowledge, only a few other recent papers study the coexistence of OTC markets and centralised exchanges with an endogenous market choice: Dugast et al. (2022), Idem (2022), Lee and Wang (2022), and Yoon (2019). Among these, the latter two focus on the role of information, an aspect we abstract from. In Lee and Wang (2022), dealers on the OTC market quote a larger spread to traders who appear to be

better informed, e.g., investment funds who typically are well informed about the assets they trade. Hence, these traders prefer to trade on a centralised exchange, whereas only traders that are unlikely to have superior information trade on OTC markets in equilibrium. In Yoon (2019), the trade-off between OTC markets and the centralised exchange has two dimensions. First, traders have private information, and the markets differ in terms of how information is revealed to participants. Second, traders have different preferences over assets, and while price impacts are typically larger on the OTC market, traders may find an ideal counterparty with opposite trading preferences there such that the price impacts basically cancel each other out. Thus, the mechanism in this paper shares some similarities with ours, but there are also important differences: As mentioned earlier, we abstract from incomplete information and from traders having market power on the centralised exchange. Instead, we highlight the role of collateral, an aspect that Yoon (2019) does not consider. Idem (2022) follows a mechanism-design approach and assumes that the central exchange is run by a profit-maximising agent. He shows that the profit of the centralised exchange increases with search frictions on the OTC market.

The paper most closely related to ours is Dugast et al. (2022). As in our paper, in this work the centralised exchange operates as a Walrasian market, and the OTC market features bargaining between the two matched counterparties. The authors assume that traders are heterogeneous in their trading capacities. They find an equilibrium where traders with small and large trading capacities visit the OTC market, while traders with medium capacities visit the centralised exchange. This equilibrium resembles the one we find, for some parameter values, but note that the forces driving the result are rather different: In Dugast et al. (2022), traders with large capacities can act as intermediaries for traders with small capacities on the OTC market, which is the reason they choose to operate in this market. In turn, traders with small capacities exploit the chance of meeting these other traders with large capacit-

ies to expand their trading ability and compensate then such traders for this service. In our paper, the traders with large trading would prefer instead that those with small trading needs were not present in the OTC market and the latter exploit the chance to meet the first ones to extract some rents.

A number of papers also exist that compare outcomes on OTC markets and centralised exchanges, but do not have an endogenous participation decision. Recent examples are Geromichalos and Herrenbrueck (2016), Liu et al. (2018), Li and Song (2019), Vogel (2019), Glode and Opp (2020), Colliard et al. (2021), or Allen and Wittwer (2022). However, we believe that allowing for an endogenous participation decision is crucial to understand the coexistence of these two markets and has also substantial impact on trade outcomes.

Finally, there is also a literature on the market fragmentation versus centralisation. This literature studies how outcomes differ when an asset is traded on several (identical) segmented markets, or a single centralised market. Recent examples include Malamud and Rostek (2017), Chen and Duffie (2021), Babus and Hachem (2021), and Babus and Parlatore (2022). This question is clearly closely related to ours as it also studies how market structures affect trading outcomes; however, we do not focus on fragmentation, but on competing market structures instead.

**Outline.** The rest of this paper is structured as follows: Section 2 presents the environment, and Section 3 discusses the equilibrium of the model. In Section 4, we present parametrical examples to discuss the equilibrium in more detail, and finally, Section 5 concludes.

## 2 Environment

There are three time periods, indexed by t = 0, 1, 2. There is a unit measure of ex-ante identical agents. There are three storable assets: Illiquid assets A and B, and asset Z, which we call the collateral. The precise sense in which

Z serves as a collateral will be discussed further below. We should think of Z as a more standard, easily recognisable asset, while assets A and B are more specialised assets. Each agent is endowed with a, b and z units of the three assets.

At the beginning of period 1, each agent is hit by an i.i.d. shock  $\varepsilon$ , with cdf  $F(\varepsilon)$  and support (0,1). As a consequence, the agent's utility from holding the assets she is endowed with is

$$U(a, b, z; \varepsilon) = \varepsilon u(a) + (1 - \varepsilon)u(b) + z,$$

We assume that u'(z) > 0 > u''(z), and  $u'(0) = \infty$ . The perfectly negative correlation between the utility from holding assets A and B is not crucial for our results, but helps us to keep the model tractable. We also believe that there are numerous examples in reality where agents have negatively correlated utility from holding two assets, in particular for hedging instruments like swaps. We assume that a = b and the distribution of the shock  $F(\varepsilon)$  is symmetric around 1/2, so the situation regarding the two assets is perfectly symmetric.

During period 1, after the shock is realised, agents may trade with each other. They can either visit a decentralised over-the-counter (OTC) market or a centralised exchange. On the centralised exchange, assets A and B are traded separately, in exchange of the (collateral) asset Z. The exchange is a competitive market where the prices in terms of Z, denoted  $p^A$  and  $p^B$ , respectively, are determined by market clearing. Importantly, we assume that trading occurs simultaneously; thus, if an agent wants to sell asset A and purchase asset B she is not able to use the revenue generated from the sale of A in order to purchase a greater amount of B. Various interpretations are possible for the role of asset Z on the centralised exchange. Perhaps the simplest one is that asset buyers turn over Z for immediate payment of their purchase, which makes Z resemble a means of payment, e.g., money. An alternative interpretation is that the settlement of the transaction will occur in the next period, but to ensure that buyers do not renege they need to pledge

collateral. It has been shown that these two interpretations can be represented by the same set of equations<sup>1</sup>; since we think that the collateral interpretation fits our application better in terms of how these markets work in reality, we refer to Z as collateral.<sup>2</sup>

We assume that agents entering the OTC market find a counterparty with probability  $1-\alpha$ . In a match, the terms of trade are determined by Kalai bargaining with equal bargaining power.<sup>3</sup> Further, letting  $\bar{\varepsilon}$  denote the preference shock of the modal agent entering the OTC market, we consider the case where agents who find a match are always matched with someone from the other side of the market, that is, an agent with shock  $\varepsilon < \bar{\varepsilon}$  is matched with an agent with shock  $\varepsilon > \bar{\varepsilon}$  and viceversa. Finally, we assume the matching technology in the OTC market operates as follows: conditionally on finding a match (a counterpart), an agent is matched with somebody with an identical, opposite shock, with probability  $1-\gamma$ ; i.e., an agent with shock  $\varepsilon$  is matched with an agent with shock  $\tilde{\varepsilon} = 1 - \varepsilon$ . With the remaining probability  $\gamma$ , an agent is instead randomly matched with someone from the other side of the market. We will justify these features further below when discussing the terms of trade in the OTC market.

Finally, to pin down out of equilibrium beliefs in a market that is non active, that is not selected by any trader, we assume that there is a fringe of agents with measure n close to 0, captive on each market. Thus such agents cannot choose in which market to operate and are only able to trade on either the OTC market or the centralised exchange. They are also hit by a preference

<sup>&</sup>lt;sup>1</sup>See e.g. Altermatt et al. (2023) for a discussion.

<sup>&</sup>lt;sup>2</sup>In reality, the amount of collateral required for transactions may be more or less than 100% of the value of the asset trade. In our model, since the initial holding of Z is exogenous, allowing for different collateral requirements is equivalent to varying the initial endowment of Z.

<sup>&</sup>lt;sup>3</sup>Our theory can easily be extended to allow for unequal bargaining power, but since distinguishing the two sides in a match is somewhat difficult, we think that equal bargaining power is the most natural assumption.

shock at the beginning of period 1, drawn from the same distribution  $F(\varepsilon)$ .

## 3 Equilibrium

After an  $\varepsilon$  realisation of the shock, the choice of the market of a trader is determined by

$$\max\{V^D(\varepsilon), V^C(\varepsilon)\},\tag{1}$$

where  $V^D(\varepsilon)$  (resp.  $V^C(\varepsilon)$ ) denotes the value function of the agent if she were to enter the OTC market (centralised exchange) and trade in that market (starting from her initial portfolio (a, b, z)).

#### 3.1 OTC market

The value function in the OTC market is

$$V^{D}(\varepsilon) = (1 - \alpha) \left( (1 - \gamma)U \left( a + \chi_{\varepsilon, 1 - \varepsilon}^{a}, b + \chi_{\varepsilon, 1 - \varepsilon}^{b}, z + \chi_{\varepsilon, 1 - \varepsilon}^{z}; \varepsilon \right) + \gamma \mathbb{E}_{\tilde{\varepsilon}} \left[ U \left( a + \chi_{\varepsilon, \tilde{\varepsilon}}^{a}, b + \chi_{\varepsilon, \tilde{\varepsilon}}^{b}, z + \chi_{\varepsilon, \tilde{\varepsilon}}^{z}; \varepsilon \right) \right] \right) + \alpha U \left( a, b, z; \varepsilon \right)$$

$$(2)$$

where  $1 - \alpha$  denotes the probability of finding a match, and  $\chi^j_{\varepsilon,\tilde{\varepsilon}} \in \mathbb{R}$  denotes the amount of asset  $j = \{a, b, z\}$  that is traded in a meeting of type  $\varepsilon$  with type  $\tilde{\varepsilon}$ .

#### Bargaining solution

In any match between agents  $\varepsilon$  and  $\tilde{\varepsilon}$ , the bargaining problem is given by

$$\begin{aligned} \max_{\chi_{\varepsilon,\tilde{\varepsilon}}^{a},\chi_{\varepsilon,\tilde{\varepsilon}}^{b},\chi_{\varepsilon,\tilde{\varepsilon}}^{z}} & \left\{ U\left(a + \chi_{\varepsilon,\tilde{\varepsilon}}^{a},b + \chi_{\varepsilon,\tilde{\varepsilon}}^{b},z + \chi_{\varepsilon,\tilde{\varepsilon}}^{z};\varepsilon\right) - U\left(a,b,z;\varepsilon\right) \right\} \\ \text{s.t.} & \left\{ U\left(a + \chi_{\varepsilon,\tilde{\varepsilon}}^{a},b + \chi_{\varepsilon,\tilde{\varepsilon}}^{b},z + \chi_{\varepsilon,\tilde{\varepsilon}}^{z};\varepsilon\right) - U\left(a,b,z;\varepsilon\right) \right\} \\ & = \left\{ U\left(a - \chi_{\varepsilon,\tilde{\varepsilon}}^{a},b - \chi_{\varepsilon,\tilde{\varepsilon}}^{b},z - \chi_{\varepsilon,\tilde{\varepsilon}}^{z};\tilde{\varepsilon}\right) - U\left(a,b,z;\tilde{\varepsilon}\right) \right\}, \end{aligned}$$

with  $\chi_{\varepsilon,\tilde{\varepsilon}}^j \in [-j,j]$  for  $j \in \{a,b,z\}$ . In words, the outcome of the bargaining process maximises an agent's surplus from trade, given by the utility of

the asset portfolio after trading minus the utility of the asset portfolio before trading, subject to the constraint that the agent's surplus from trade must be equal to her counterparty's surplus.

The above problem can be rewritten as follows:

$$\begin{aligned} \max_{\chi_{\varepsilon,\tilde{\varepsilon}}^a,\chi_{\varepsilon,\tilde{\varepsilon}}^b,\chi_{\varepsilon,\tilde{\varepsilon}}^z} & \left\{ \varepsilon \Big[ u \big( a + \chi_{\varepsilon,\tilde{\varepsilon}}^a \big) - u (a) \Big] + (1 - \varepsilon) \Big[ u \big( b + \chi_{\varepsilon,\tilde{\varepsilon}}^b \big) - u (b) \Big] + \chi_{\varepsilon,\tilde{\varepsilon}}^z \right\} \\ \text{s.t.} & \varepsilon \Big[ u \big( a + \chi_{\varepsilon,\tilde{\varepsilon}}^a \big) - u (a) \Big] + (1 - \varepsilon) \Big[ u \big( b + \chi_{\varepsilon,\tilde{\varepsilon}}^b \big) - u (b) \Big] + \chi_{\varepsilon,\tilde{\varepsilon}}^z \\ & = \Big( \tilde{\varepsilon} \Big[ u \big( a - \chi_{\varepsilon,\tilde{\varepsilon}}^a \big) - u (\tilde{a}) \Big] + (1 - \tilde{\varepsilon}) \Big[ u \big( b - \chi_{\varepsilon,\tilde{\varepsilon}}^b \big) - u (\tilde{b}) \Big] - \chi_{\varepsilon,\tilde{\varepsilon}}^z \Big) \\ & - a \le \chi_{\varepsilon,\tilde{\varepsilon}}^a \le a \\ & - b \le \chi_{\varepsilon,\tilde{\varepsilon}}^b \le b \\ & - z \le \chi_{\varepsilon,\tilde{\varepsilon}}^z \le z. \end{aligned}$$

Now, rearranging the first constraint yields

$$\chi_{\varepsilon,\tilde{\varepsilon}}^{z} = \frac{1}{2} \left( \tilde{\varepsilon} \left[ u \left( a - \chi_{\varepsilon,\tilde{\varepsilon}}^{a} \right) - u(a) \right] + (1 - \tilde{\varepsilon}) \left[ u \left( b - \chi_{\varepsilon,\tilde{\varepsilon}}^{b} \right) - u(b) \right) \right] - \varepsilon \left[ u \left( a + \chi_{\varepsilon,\tilde{\varepsilon}}^{a} \right) - u(a) \right] - (1 - \varepsilon) \left[ u \left( b + \chi_{\varepsilon,\tilde{\varepsilon}}^{b} \right) - u(b) \right] \right)$$
(3)

Ignoring the constraints on  $\chi^z_{\varepsilon,\tilde{\varepsilon}}$  for now, we can rewrite the maximisation problem as

$$\max_{\substack{\chi_{\varepsilon,\tilde{\varepsilon}}^a,\chi_{\varepsilon,\tilde{\varepsilon}}^b,\chi_{\varepsilon,\tilde{\varepsilon}}^z\\ \varepsilon}} \frac{1}{2} \left( \tilde{\varepsilon} \left[ u \left( a - \chi_{\varepsilon,\tilde{\varepsilon}}^a \right) - u(a) \right] + (1 - \tilde{\varepsilon}) \left[ u \left( b - \chi_{\varepsilon,\tilde{\varepsilon}}^b \right) - u(b) \right) \right] \\
+ \varepsilon \left[ u \left( a + \chi_{\varepsilon,\tilde{\varepsilon}}^a \right) - u(a) \right] + (1 - \varepsilon) \left[ u \left( b + \chi_{\varepsilon,\tilde{\varepsilon}}^b \right) - u(b) \right] \right) \\
\text{s.t.} \quad - a \le \chi_{\varepsilon,\tilde{\varepsilon}}^a \le a \\
- b \le \chi_{\varepsilon,\tilde{\varepsilon}}^b \le b.$$

The first-order conditions of this problem are

$$\varepsilon u'(a + \chi^a_{\varepsilon,\tilde{\varepsilon}}) = \tilde{\varepsilon} u'(a - \chi^a_{\varepsilon,\tilde{\varepsilon}}) \tag{4}$$

$$(1 - \varepsilon)u'(b + \chi_{\varepsilon,\tilde{\varepsilon}}^b) = (1 - \tilde{\varepsilon})u'(b - \chi_{\varepsilon,\tilde{\varepsilon}}^b)$$
 (5)

The first condition requires that the marginal rate of substitution between asset A and the collateral asset Z is equalized among the two agents. The second one imposes the same condition for asset B. Equations (4) and (5) determine the reallocation of assets A and B among the two agents. The reallocation of asset Z is then obtained after substituting the quantity traded of the two assets in equation (3). This ensures the equalisation of surplus from trade among the two agents. Note that the constraints on  $\chi^a_{\varepsilon,\tilde{\varepsilon}}$  and  $\chi^b_{\varepsilon,\tilde{\varepsilon}}$  never bind because of the Inada conditions.

To investigate the constraints we ignored, concerning asset z, let  $\chi^z_{\varepsilon,\tilde{\varepsilon}}(\chi^a_{\varepsilon,\tilde{\varepsilon}},\chi^b_{\varepsilon,\tilde{\varepsilon}})$  denote the value obtained from equation (3) for any  $\chi^a_{\varepsilon,\tilde{\varepsilon}},\chi^b_{\varepsilon,\tilde{\varepsilon}}$ . If  $-z \leq \chi^z_{\varepsilon,\tilde{\varepsilon}}(\chi^a_{\varepsilon,\tilde{\varepsilon}},\chi^b_{\varepsilon,\tilde{\varepsilon}}) \leq z$ , the constraints are satisfied and so equations (4) and (5) yield the solutions to the bargaining problem. If instead the constraint  $-z > \chi^z_{\varepsilon,\tilde{\varepsilon}}(\chi^a_{\varepsilon,\tilde{\varepsilon}},\chi^b_{\varepsilon,\tilde{\varepsilon}})$   $(z < \chi^z_{\varepsilon,\tilde{\varepsilon}}(\chi^a_{\varepsilon,\tilde{\varepsilon}},\chi^b_{\varepsilon,\tilde{\varepsilon}}))$  is violated, the trade of the collateral asset is  $\chi^z_{\varepsilon,\tilde{\varepsilon}} = -z$   $(\chi^z_{\varepsilon,\tilde{\varepsilon}} = \tilde{z})$ . In these cases, the asset trades  $\chi^a_{\varepsilon,\tilde{\varepsilon}}$  and  $\chi^b_{\varepsilon,\tilde{\varepsilon}}$  are obtained as the joint solution to

$$\frac{\tilde{\varepsilon}u'(a-\chi_{\varepsilon,\tilde{\varepsilon}}^a)}{\varepsilon u'(a+\chi_{\varepsilon,\tilde{\varepsilon}}^a)} = \frac{(1-\tilde{\varepsilon})u'(b-\chi_{\varepsilon,\tilde{\varepsilon}}^b)}{(1-\varepsilon)u'(b+\chi_{\varepsilon,\tilde{\varepsilon}}^b)}$$
(6)

and equation (3), with  $\chi_{\varepsilon,\tilde{\varepsilon}}^z = -z$  or  $\chi_{\varepsilon,\tilde{\varepsilon}}^z = z$ , depending on which constraint is violated in the unconstrained solution. In this case, the solution to (4) and (5) would require a larger transfer of collateral than the agents hold and is not feasible. Thus the solution of the bargaining problem features one agent giving up all her amount of collateral, and the amounts of A and B traded are set so that the marginal rates of substitution between asset A and B are equalised across the two agents (the marginal rates of substitution between the two assets and the collateral are instead not equalised).

Analysing the solution to the bargaining problem reveals that every agent prefers matching with someone with a more extreme utility shock than the shock hitting the agent. The reason is that agents with extreme values of  $\varepsilon$  gain the most from trading. Since the total gains from trade are split equally across both agents, this benefits their counterparty as well. For example, an

agent with  $\varepsilon$  close to 1 gains a lot from acquiring as much as possible of A, and for this to happen the agent is willing to give up a large amount of B and possibly also of Z to compensate her counterparty. This will happen when the value of the shock  $\varepsilon'$  hitting the counterparty is less extreme. Hence, if given the choice, all agents would like to match with the most extreme agents from the other side of the market, i.e., agents with  $\varepsilon < 1/2$  (resp.  $\varepsilon > 1/2$ ) would like to match with an agent with  $\varepsilon = 1$  ( $\varepsilon = 0$ ). To put this differently, consider (without loss of generality) an agent with some  $\varepsilon < 1/2$ , and vary  $\tilde{\varepsilon}$ , i.e., the counterparty's preference shock. It is then easy to show that the total surplus of the match is increasing in  $\tilde{\varepsilon}$  and, since the total surplus is split equally, an agent with some  $\varepsilon < 1/2$  strictly prefers meeting a counterparty with the highest value of  $\tilde{\varepsilon}$ .

Since this is also true for the agents hit by the most extreme preference shocks, a stable matching outcome in the market (with a symmetric distribution of agents) is one where each agent is matched with a counterparty with a symmetric, equally extreme value of  $\varepsilon$ . That is, an agent with shock  $\varepsilon < 1/2$  is matched with a counterparty with shock  $1 - \varepsilon$ . This outcome is stable in the sense that there is no pair of agents that would agree to dissolve their respective matches in order to be matched instead with each other. In the match between  $\varepsilon$  and  $1-\varepsilon$  the solution to the bargaining problem is given by  $\chi^a_{\varepsilon,\tilde{\varepsilon}} = \chi^b_{\varepsilon,\tilde{\varepsilon}}$  and  $\chi^z_{\varepsilon,\tilde{\varepsilon}} = 0$  (thus the collateral constraint does not bind).

Our assumptions about matching in the OTC market should then be interpreted in the following way: conditionally on finding a match, with probability  $1-\gamma$  agents are matched with their stable counterparty - in the sense described above -, which we can argue is the situation which would arise if they could direct their search. With the remaining probability  $\gamma$  agents fail to find their stable counterparty and are instead randomly matched with someone from the other side of the market . Note, however, that failure here should not be taken as an always negative outcome: while an agent with an extreme value of the

shock  $\varepsilon$  always prefers to find a stable counterparty, an agent with  $\varepsilon$  close to 1/2 prefers the random outcome, as that gives her a chance to meet an agent with an extreme value of  $\varepsilon$  and thus to get favourable terms of trade. From now on, we will refer to the stable matches that occur with probability  $(1-\alpha)(1-\gamma)$  as directed meetings, while we refer to the other matches, that occur with probability  $(1-\alpha)\gamma$ , as random meetings.

Returning to the equilibrium analysis, the value of entering the OTC market is given by equation (2), with the terms of trade characterised by equations (3) - (6).

## 3.2 Centralised Exchange

Assets A and B are traded separately on the centralised exchange, against collateral Z. Let the prices of A and B against Z be  $p_A$  and  $p_B$ , respectively. Then, an agent's problem is given by

$$\begin{split} V^C(a,b,z;\varepsilon) &= \max_{\chi^a_{\varepsilon,C},\chi^b_{\varepsilon,C}} \quad \varepsilon u(a + \chi^a_{\varepsilon,C}) + (1-\varepsilon)u(b + \chi^b_{\varepsilon,C}) + z - p^A \chi^a_{\varepsilon,C} - p^B \chi^b_{\varepsilon,C} \\ \text{s.t.} \quad p^A \chi^a_{\varepsilon,C}|_{\chi^a_{\varepsilon,C}>0} + p^B \chi^b_{\varepsilon,C}|_{\chi^b_{\varepsilon,C}>0} \leq z \end{split}$$

Denote the unconstrained solutions to this problem as  $\hat{\chi}_{\varepsilon,C}^a$  and  $\hat{\chi}_{\varepsilon,C}^b$ . They satisfy the equations

$$\varepsilon u'(a + \hat{\chi}^a_{\varepsilon,C}) = p^A \tag{7}$$

$$(1 - \varepsilon)u'(b + \hat{\chi}^b_{\varepsilon C}) = p^B. \tag{8}$$

These values are the solutions of the agent's problem whenever the agent is not collateral constrained. Further, if an agent is selling one asset while buying the other, the unconstrained solution is achieved for the asset that is sold, while the agent spends all their collateral to purchase the other asset.

Finally, denote the optimal choices of an agent who wants to purchase both assets at the equilibrium prices, but is collateral constrained, as  $\check{\chi}^a_{\varepsilon,C}$  and  $\check{\chi}^b_{\varepsilon,C}$ .

These are given by the joint solutions to

$$\frac{p^B}{p^A} \varepsilon u' \left( a + \frac{z - p^B \check{\chi}_{\varepsilon,C}^b}{p^A} \right) = (1 - \varepsilon) u' (b + \check{\chi}_{\varepsilon,C}^b) \tag{9}$$

$$\check{\chi}_{\varepsilon,C}^a = (z - p^B \check{\chi}_{\varepsilon,C}^b)/p^A. \tag{10}$$

Summing up all these possibilities, the solutions are given by

$$\chi_{\varepsilon,C}^{a} = \begin{cases} \hat{\chi}_{\varepsilon,C}^{a} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} |_{\chi_{\varepsilon,C}^{a} > 0} + p^{B} \hat{\chi}_{\varepsilon,C}^{b} |_{\chi_{\varepsilon,C}^{b} > 0} \leq z \text{ or } \hat{\chi}_{\varepsilon,C}^{a} \leq 0 \\ \frac{z}{p_{A}} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} > z \text{ and } \hat{\chi}_{\varepsilon,C}^{b} \leq 0 \\ \hat{\chi}_{\varepsilon,C}^{a} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} |_{\chi_{\varepsilon,C}^{a} > 0} + p^{B} \hat{\chi}_{\varepsilon,C}^{b} |_{\chi_{\varepsilon,C}^{b} > 0} > z \text{ and } \hat{\chi}_{\varepsilon,C}^{j} > 0 \text{ for } j = a, b \end{cases}$$

$$\chi_{\varepsilon,C}^{b} = \begin{cases} \hat{\chi}_{\varepsilon,C}^{b} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} |_{\chi_{\varepsilon,C}^{a} > 0} + p^{B} \hat{\chi}_{\varepsilon,C}^{b} |_{\chi_{\varepsilon,C}^{b} > 0} \leq z \text{ or } \hat{\chi}_{\varepsilon,C}^{b} \leq 0 \end{cases}$$

$$\chi_{\varepsilon,C}^{b} = \begin{cases} \hat{\chi}_{\varepsilon,C}^{b} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} |_{\chi_{\varepsilon,C}^{a} > 0} + p^{B} \hat{\chi}_{\varepsilon,C}^{b} |_{\chi_{\varepsilon,C}^{b} > 0} \leq z \text{ or } \hat{\chi}_{\varepsilon,C}^{b} \leq 0 \\ \\ \hat{\chi}_{\varepsilon,C}^{b} & \text{if } p^{A} \hat{\chi}_{\varepsilon,C}^{a} |_{\chi_{\varepsilon,C}^{a} > 0} + p^{B} \hat{\chi}_{\varepsilon,C}^{b} |_{\chi_{\varepsilon,C}^{b} > 0} > z \text{ and } \hat{\chi}_{\varepsilon,C}^{j} > 0 \text{ for } j = a, b \end{cases}$$

$$(12)$$

Prices are determined by the market clearing conditions

$$\int \chi_{\varepsilon,C}^a d\Omega(\varepsilon) = 0 \tag{13}$$

$$\int \chi_{\varepsilon,C}^b d\Omega(\varepsilon) = 0, \tag{14}$$

where  $\Omega(\varepsilon)$  is the distribution of agents who visit the centralised exchange. We focus on symmetric equilibria, which we define as equilibria where  $\Omega(\varepsilon)$  is symmetric around 1/2. Then, we will have  $p^A = p^B$  in equilibrium.

We denote the price realised on the centralised exchange in a symmetric equilibrium if nobody is collateral constrained as  $p^*$ . Note that at this price, agents with  $\varepsilon < 1/2$  buy asset B and sell asset A, while agents with  $\varepsilon > 1/2$  do the opposite. Agents with  $\varepsilon = 1/2$  find it optimal not to trade at these prices.

Next, note that the collateral constraint affects the demand for assets, while their supply is always unconstrained. Hence, if some agents are collateral constrained, the equilibrium prices are such that  $p^A = p^B < p^*$ . In this case,

agents with  $\varepsilon=1/2$  find it optimal to purchase both assets. If z is relatively large, equilibrium trades will be determined by the first two lines in equations (11) and (12), as only agents with relatively extreme values of  $\varepsilon$  are affected by the collateral constraint, and these agents still find it optimal to purchase one asset and sell the other. When instead z is quite small the equilibrium value of  $p^A=p^B$  may be sufficiently smaller than  $p^*$  that agents with shock  $\varepsilon$  close to 1/2 would like to purchase large quantities of both assets, but are also constrained in doing so. In this case, their optimal trades are given by  $\check{\chi}^a_{\varepsilon,C}$  and  $\check{\chi}^b_{\varepsilon,C}$ .

Summarising, the value of entering the centralised exchange is given by

$$V^{C}(\varepsilon) = \varepsilon u(a + \chi_{\varepsilon,C}^{a}) + (1 - \varepsilon)u(b + \chi_{\varepsilon,C}^{b}) + z - p^{A}\chi_{\varepsilon,C}^{a} - p^{B}\chi_{\varepsilon,C}^{b},$$
 (15)

with the quantities traded determined by equations (11) and (12), and prices given by equations (13) and (14).

## 3.3 Market Choice

As mentioned above, we denote the distribution of agents visiting the centralised exchange with  $\Omega(\varepsilon)$ , while we use  $\Omega^{-1}(\varepsilon)$  to denote the distribution of agents visiting the OTC market. Note that  $\Omega(\varepsilon) \cup \Omega^{-1}(\varepsilon) = F(\varepsilon)$ , i.e., all agents visit either market, as the value of entering either market is weakly higher than autarky for all  $\varepsilon$ .  $\Omega(\varepsilon)$  and  $\Omega^{-1}(\varepsilon)$  are determined by each agent's optimal choice from (1), with the values from entering either market given by equations (2) and (15). Throughout the paper, we focus on symmetric equilibria, which implies both  $\Omega(\varepsilon)$  and  $\Omega^{-1}(\varepsilon)$  are symmetric around 1/2 and thus  $\overline{\varepsilon} = 1/2$ . Note that this is a fixed point problem, as the partition of agents across the two markets affects the value of entering the OTC market through the expectation of which counterparty will be met, and the value of entering the centralised exchange through the pricing equations (13) and (14).

## 4 Parametrical Example

Assume that  $u(\cdot) = \ln(\cdot)$ , and that  $F(\varepsilon)$  is the uniform distribution over (0, 1). Further, we assume without loss of generality that a = b = 0.5, as this implies  $p^* = 1$ . This is wlog since all that matters economically is the supply of collateral Z relative to the supply of assets A and B.

#### 4.1 OTC Market

Given our assumptions, the unconstrained solutions  $\chi^a_{\varepsilon,\tilde{\varepsilon}}$  and  $\chi^b_{\varepsilon,\tilde{\varepsilon}}$  from equations (4) and (5) are

$$\chi_{\varepsilon,\tilde{\varepsilon}}^a = a \frac{\varepsilon - \tilde{\varepsilon}}{\varepsilon + \tilde{\varepsilon}} \tag{16}$$

$$\chi_{\varepsilon,\tilde{\varepsilon}}^b = b \frac{\tilde{\varepsilon} - \varepsilon}{2 - \varepsilon - \tilde{\varepsilon}},\tag{17}$$

while

$$\chi_{\varepsilon,\tilde{\varepsilon}}^{z} = \lambda \left( \tilde{\varepsilon} \left[ \ln \left( 2a \frac{\tilde{\varepsilon}}{\varepsilon + \tilde{\varepsilon}} \right) - \ln(a) \right] + (1 - \tilde{\varepsilon}) \left[ \ln \left( 2b \frac{1 - \tilde{\varepsilon}}{2 - \varepsilon - \tilde{\varepsilon}} \right) - \ln(b) \right] \right) - (1 - \lambda) \left( \varepsilon \left[ \ln \left( 2a \frac{\varepsilon}{\varepsilon + \tilde{\varepsilon}} \right) - \ln(a) \right] + (1 - \varepsilon) \left[ \ln \left( 2b \frac{1 - \varepsilon}{2 - \varepsilon - \tilde{\varepsilon}} \right) - \ln(b) \right] \right).$$

$$(18)$$

Given our parametric assumptions, the highest value  $\chi^z_{\varepsilon,\tilde{\varepsilon}}$  may take is less than 0.08, thus the unconstrained bargaining solutions can be implemented as long as z > 0.08. Throughout this section, we assume that z satisfies this condition.<sup>4</sup>

 $<sup>^4\</sup>chi^z_{\varepsilon,\tilde{\varepsilon}}$  attains the highest value in a meeting of an agent with  $\varepsilon\approx 0.25$  and an agent with  $\varepsilon$  close to 1 (conversely it attains the lowest value in a meeting of an agent with  $\varepsilon\approx 0.75$  and an agent with  $\varepsilon$  close to 0). This is because there are two opposing forces affecting  $|\chi^z_{\varepsilon,\tilde{\varepsilon}}|$ : On the one hand,  $|\chi^z_{\varepsilon,\tilde{\varepsilon}}|$  increases if the two agents in a match benefit unequally from the redistribution of A and B, i.e., if the distance of their preference shock from 1/2 is very different. On the other hand, the surplus from trade increases as both agent's  $\varepsilon$  is further away from 1/2, and thus  $|\chi^z_{\varepsilon,\tilde{\varepsilon}}|$  attains the largest value if agents benefit unequally from the asset redistribution while the total surplus is also relatively large.

If  $\tilde{\varepsilon} = 1 - \varepsilon$ , the bargaining solutions reduce to

$$\chi_{\varepsilon,1-\varepsilon}^a = a(2\varepsilon - 1) \tag{19}$$

$$\chi_{\varepsilon,1-\varepsilon}^b = b(1-2\varepsilon) \tag{20}$$

$$\chi_{\varepsilon,1-\varepsilon}^z = 0. \tag{21}$$

Hence, the expected value from entering the OTC market given an agent's preference shock  $\varepsilon$  is

$$V^{D}(\varepsilon) = z + \alpha \left( \varepsilon \ln(a) + (1 - \varepsilon) \ln(b) \right)$$

$$+ (1 - \alpha) \left( (1 - \gamma) \left( \varepsilon \ln(2a\varepsilon) + (1 - \varepsilon) \ln(2b(1 - \varepsilon)) \right) \right)$$

$$+ \gamma \int_{\underline{\tilde{\varepsilon}}}^{\underline{\tilde{\varepsilon}}} \varepsilon \ln\left( 2a \frac{\varepsilon}{\varepsilon + \tilde{\varepsilon}} \right) + (1 - \varepsilon) \ln\left( 2b \frac{1 - \varepsilon}{2 - \varepsilon - \tilde{\varepsilon}} \right) + \chi_{\varepsilon, \tilde{\varepsilon}}^{z} d\Omega^{-1}(\tilde{\varepsilon}) \right),$$

$$(22)$$

where  $\chi^z_{\varepsilon,\tilde{\varepsilon}}$  is fully characterised by (18), and  $\tilde{\underline{\varepsilon}} = 0$  ( $\tilde{\underline{\varepsilon}} = 1/2$ ) while  $\bar{\tilde{\varepsilon}} = 1/2$  ( $\bar{\tilde{\varepsilon}} = 1$ ) for  $\varepsilon > 1/2$  ( $\varepsilon < 1/2$ ).

## 4.2 Centralised Exchange

With our parametric assumptions, we have

$$\hat{\chi}_{\varepsilon,C}^a = \frac{\varepsilon}{p^A} - a, \quad \hat{\chi}_{\varepsilon,C}^b = \frac{1 - \varepsilon}{p^B} - b \tag{23}$$

$$\check{\chi}_{\varepsilon,C}^a = \frac{\varepsilon}{p^A} (z + p^B b) - (1 - \varepsilon)a, \quad \check{\chi}_{\varepsilon,C}^b = \frac{1 - \varepsilon}{p^B} (z + p^A a) - \varepsilon b. \tag{24}$$

Given this, trading quantities are still given by equations (11) and (12), and equilibrium prices are determined by equations (13) and (14). From this, it can be seen that if  $\Omega(\varepsilon) = F(\varepsilon)$ ,  $p^A = p^B = p^*$  if and only if  $z \geq 0.5$ . In equilibrium, it is of course possible that only agents who are unconstrained visit the centralised exchange, and we may have  $p^A = p^B = p^*$  despite z < 1/2. Note further that at  $p^*$ , unconstrained trades on the centralised exchange are exactly equal to trades in a directed OTC meeting; that is, when type  $\varepsilon$  meets type  $1 - \varepsilon$ .

## 4.3 Equilibrium

To make things interesting, we assume z < 1/2 to ensure that some agents are constrained on the centralised exchange, should they decide to visit it.<sup>5</sup> In the remainder of this section, we will refer to agents with  $\varepsilon$  close to 1/2 as investors with small trading needs, to agents with  $\varepsilon$  close to either 0 or 1 as investors with large trading needs, and to the remaining agents as investors with medium trading needs.<sup>6</sup> The structure of the OTC market, i.e., the parameters  $\alpha$  and  $\gamma$ , play a crucial role in determining the type of equilibrium which exists. For  $\alpha$ , it is clear that a low value makes the OTC market more attractive for everyone. Things are less obvious for  $\gamma$  as a change in this parameters benefits certain investors but hurts others. We will discuss the equilibria that arise for certain values of  $\alpha$  and  $\gamma$  below, but we first go through the intuition behind the decision making for each investor group as this will help us explain the equilibria that arise.

Let us start with investors with small trading needs. Remember that at  $p^*$ , agents with  $\varepsilon = 1/2$  do not trade at all on the centralised exchange, and by continuity investors with small trading needs trade very little and hence get only small benefits from trade. If  $p < p^*$  however, investors with small trading needs can benefit from trading on the centralised exchange by purchasing both assets, as can be seen from (23). On the OTC market, investors with small trading needs gain very little if they are in a directed meeting, but they can benefit a lot from a random meeting with an investor with larger trading needs than themselves.

Next, let us turn to investors with large trading needs. On the centralised exchange, they are constrained by z in purchasing as much of the asset they

 $<sup>^5</sup>$ For  $z \ge 1/2$ , the centralised exchange becomes frictionless as the collateral constraint does not matter.

<sup>&</sup>lt;sup>6</sup>This description of the agents follows from the amount they trade on the centralised exchange at  $p^*$  or equivalently from their trades in a directed OTC meeting.

like, and if  $p < p^*$  they are hurt in addition by the fact that they need to sell the asset they do not like at a low price. On the OTC market, they benefit a lot from directed meetings, since these allow them to bypass the collateral constraint. In random meetings however, they might get worse terms of trade than on the centralised exchange, despite not being affected by the collateral constraint.

Finally, investors with medium trading needs generally like the centralised exchange because they can trade with certainty and are not heavily affected by collateral constraints. On the OTC market, consider first the directed meetings: investors with medium trading needs prefer them over the centralised exchange if they are collateral constrained there and  $p \approx p^*$ ; if instead they are not collateral constrained and  $p < p^*$ , the centralised exchange may be more attractive. Next, consider random meetings for such investors: the benefit from these is also similar to that of the centralised exchange, since the gains from meeting an investor with large trading needs more or less offset the losses generated by meeting an investor with small trading needs. Thus, it is hard to predict how the preference for the OTC market of investors with medium trading needs varies with  $\gamma$ . However, an important factor for these investors is the probability  $\alpha$  of not being able to trade, which can make the OTC market unattractive for them.<sup>7</sup>

In the remainder of this section, we present equilibria for certain values of  $\alpha$  and  $\gamma$  and discuss their economics. We set z=0.2 in all these examples.

#### Baseline Equilibrium with both Markets Active

We begin with  $\alpha = 0.05$  and  $\gamma = 0.1$ , since in this case, we get an example of what we refer to as our baseline equilibrium. For these parameter values, a

<sup>&</sup>lt;sup>7</sup>In contrast, investors with low trading needs do not mind a high  $\alpha$  as much since their gains from trade in either market are smaller to begin with.

symmetric equilibrium exists where  $\Omega(\varepsilon)$  is given by the uniform distribution over all agents with  $\varepsilon \in (0.2, 0.33) \cup (0.67, 0.8)$ , while the remaining agents visit the OTC market, i.e., those with  $\varepsilon \in (0, 0.2) \cup (0.33, 0.67) \cup (0.8, 1)$ . That is, investors with intermediate trading needs choose the centralised exchange, while investors with high and small trading needs choose the OTC market. In this equilibrium,  $p^A = p^B = 0.93$ . Figure 1 shows this equilibrium graphically.

This figure works in the following way: The value function of an agent with a given  $\varepsilon$  is given in blue for the case the agent were to enter the OTC market, while it is given in red for the case she enters the centralised exchange. As discussed earlier, both of these lines are affected by  $\Omega(\varepsilon)$  and  $\Omega^{-1}(\varepsilon)$ . The values of these for which the figure is drawn are depicted by solid segments of each line; i.e., the intervals for which agents visit a certain market are solid, while the intervals for which the agents visit the other market are dotted. A similar figure could be drawn for any values of  $\Omega(\varepsilon)$  and  $\Omega^{-1}(\varepsilon)$ ; it represents an equilibrium only if the solid intervals lie above the dotted intervals everywhere, as this implies that no agent has a profitable deviation. While it is difficult to see since the lines lie almost on top of each other for most of the graph, this is the case in Figure 1.

To understand the intuition behind this equilibrium, let us go through each of the three investor groups separately. First, as discussed above, investors with high trading needs prefer the OTC market if the probability of having a directed meeting is high enough, which is the case in this example. Next, their presence on the OTC market attracts investors with small trading needs, as even though their probability of meeting an investor with large trading needs is relatively low, the benefits from such a meeting are enticing enough. Finally, investors with medium trading needs prefer the centralised exchange, since the probability of not finding any match on the OTC market is costlier for them than for the investors with small trading needs, while they are not (much) affected by the collateral constraint and hence do not mind using the

centralised exchange.

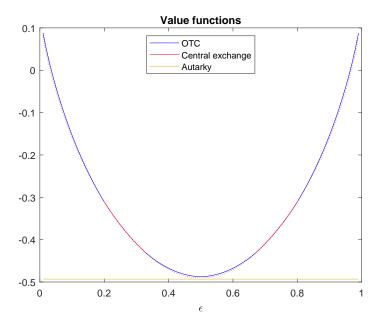


Figure 1: Equilibrium with  $\alpha = 0.05$  and  $\gamma = 0.1$ .

#### Comparative Statics with Respect to $\alpha$

Starting from our baseline equilibrium, we can now discuss how outcomes vary as we change  $\alpha$  and  $\gamma$ . First, consider a reduction in  $\alpha$ , while we keep  $\gamma$  unchanged. At  $\alpha = 0.04$ ,  $\gamma = 0.1$ , we have an equilibrium with  $\Omega(\varepsilon)$  uniform over  $\varepsilon \in (0.23, 0.31) \cup (0.69, 0.77)$ ; hence, the decrease in  $\alpha$  makes the OTC market more attractive and consequently, fewer agents visit the centralised exchange. If we reduce  $\alpha$  further, the group of investors with medium trading needs visiting the centralised exchange becomes smaller and smaller, until at  $\alpha = 0$ , only a handful of investors with medium trading needs visit the centralised exchange, while everyone else visits the OTC market.

If we increase  $\alpha$  instead, the centralised exchange becomes more attractive relative to the OTC market. E.g., at  $\alpha = 0.07$ ,  $\gamma = 0.1$ , we have an equilibrium with  $\Omega(\varepsilon)$  uniform over  $\varepsilon \in (0.13, 0.38) \cup (0.62, 0.87)$ ; i.e., fewer investors with large trading needs and consequently also fewer investors with small trading needs visit the OTC market. Further increasing  $\alpha$  further reduces these groups,

up to a point where all agents visit the centralised exchange and the OTC market is inactive. E.g., Figure 2 shows the equilibrium for  $\alpha = 0.1$ ,  $\gamma = 0.1$  where all agents prefer to visit the centralised exchange, and hence the payoff from visiting the OTC market is given by the probability of meeting someone from the fringe when deviating to it.

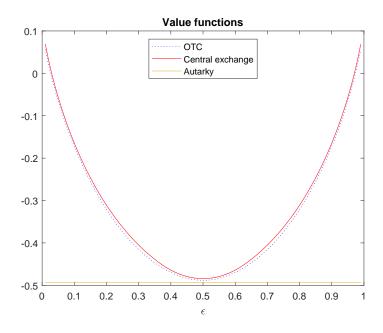


Figure 2: Equilibrium with  $\alpha = 0.1$  and  $\gamma = 0.1$ .

#### Comparative Statics with Respect to $\gamma$

Next, we revert  $\alpha$  back to 0.05 and vary  $\gamma$  instead. At  $\alpha=0.05$ ,  $\gamma=0.05$ , we have an equilibrium with  $\Omega(\varepsilon)$  uniform over  $\varepsilon\in(0.2,0.35)\cup(0.65,0.8)$ ; hence, the reduction in  $\gamma$  makes the OTC market less attractive for investors with small trading needs. To see why the market choice of investors with large trading needs is basically unaffected by the reduction in  $\gamma$ , consider the marginal investor with large trading needs. For them, the reduction in  $\gamma$  is somewhat attractive as it reduces the risk of meeting with an investor with small trading needs, but it also reduces their chance of meeting an investor with even larger trading needs than themselves; further, since some investors with small trading needs move to the centralised exchange, prices there increase

 $(p^A=p^B=0.946$  in this equilibrium), which makes the centralised exchange slightly more attractive for investors with large trading needs. It turns out that all of these effects basically cancel each other and the marginal investor with large trading needs stays put. If we reduce  $\gamma$  further, even fewer investors with small trading needs visit the OTC market, and this continues until none of them are left. E.g., at  $\alpha=0.05$ ,  $\gamma=0.001$ , we have an equilibrium with  $\Omega(\varepsilon)$  uniform over  $\varepsilon\in(0.18,0.82)$ ; thus, once no investors with small trading needs are left and the probability of a random meeting is basically zero, some of the marginal investors with large trading needs also migrate to the centralised exchange.

Finally, consider an increase in  $\gamma$ . At  $\alpha = 0.05$ ,  $\gamma = 0.15$ , we have an equilibrium with  $\Omega(\varepsilon)$  uniform over  $\varepsilon \in (0.18, 0.32) \cup (0.68, 0.82)$ . This shows that the attractiveness of the OTC market is non-monotonic in  $\gamma$ , as with an increase in the probability of random meetings, the value of visiting the OTC market relative to visiting the centralised exchange shrinks for the investors with the largest trading needs. In turn, if some of the marginal investors with large trading needs move to the centralised exchange, the OTC market becomes less attractive for marginal investors with small trading needs, as their probability of meeting someone with a more extreme preference shock now decreases. While at  $\alpha = 0.05$ ,  $\gamma = 0.15$ , there is still a sizeable share of agents on the OTC market, a further increase in  $\gamma$  actually moves all agents with large trading needs to the centralised exchange, as it tips the balance for those agents with the largest trading needs; and once these are gone, the OTC market becomes significantly less attractive for everyone else, which leads to an unravelling from the top. In this case, only some of the investors with the smallest trading needs remain on the OTC market, as the relatively high  $\gamma$ makes it attractive for them to hope for a meeting with an extreme  $\varepsilon$  from the fringe; as  $n \to 0$ , the measure of agents visiting the OTC market goes to zero as well, but it never quite vanishes because an individual deviation is profitable for several values of  $\varepsilon$  at any n when everyone is on the centralised exchange.

## 5 Conclusion

We have developed a model where agents can choose to trade either on OTC markets or on a centralised exchange. Trade on the centralised exchange is subject to collateral constraints, which can make the OTC market attractive for investors with large trading needs if their probability of finding a suitable counterparty is sufficiently high. However, unless investors are always able to find the suitable counterparty on the OTC market, the presence of investors with large trading needs also attracts investors with small trading needs to the OTC market. The reason is that if they are matched with an investor with large trading needs, they can extract some surplus from trade from these investors by selling them the asset they need at a high price. Thus, if matching on the OTC market is sufficiently, but not perfectly efficient, an equilibrium exists where investors with large and small trading needs visit the OTC market, while investors with medium trading needs visit the centralised exchange.

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