CENTRAL BANK EXIT STRATEGIES: DOMESTIC TRANSMISSION AND INTERNATIONAL SPILLOVERS*

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PRELIMINARY AND INCOMPLETE

Abstract. We study domestic and spillover implications of advanced economy central banks withdrawing monetary stimulus ("exit strategies"). We first show empirically that, compared to conventional monetary policy, large scale asset purchases have a relatively small impact on domestic aggregate demand, but strongly affect the exchange rate. We then build a two-country, New Keynesian model consistent with these facts, in which cognitive discounting additionally limits the efficacy of forward guidance, making quantitative easing the "instrument of last resort", whenever the lower bound constrains interest rate policy. We apply this framework to quantify the asymmetric effects of asset purchases and sales both at home and internationally, and we show how spillover size can vary markedly depending on the monetary framework of the "recipient" economy. Our results highlight international benefits of conventional tightening, suggest that entry and exit effects are unlikely to be mirror images, and demonstrate some scope for stabilizing via foreign exchange interventions.

Keywords: Monetary Policy, Quantitative Easing, International Spillovers

JEL Codes: C54, E52, E58, F41

1. Introduction

Figure 1 documents a marked increase in the size of US, euro area (EA), UK and Japanese central bank (CB) balance sheets relative to GDP, as well as the fact that major central banks ended up owning a significant fraction (20-45%) of the universe of bonds outstanding. Recent inflationary outcomes have led to a broad consensus arguing for interest rate tightening and an "exit" from the corresponding unconventional policies, with many jurisdictions already several "conventional" interest rate hikes along the way. Accordingly, our focus in this paper is on analyzing the likely domestic transmission and international spillovers of both types of policy interventions.

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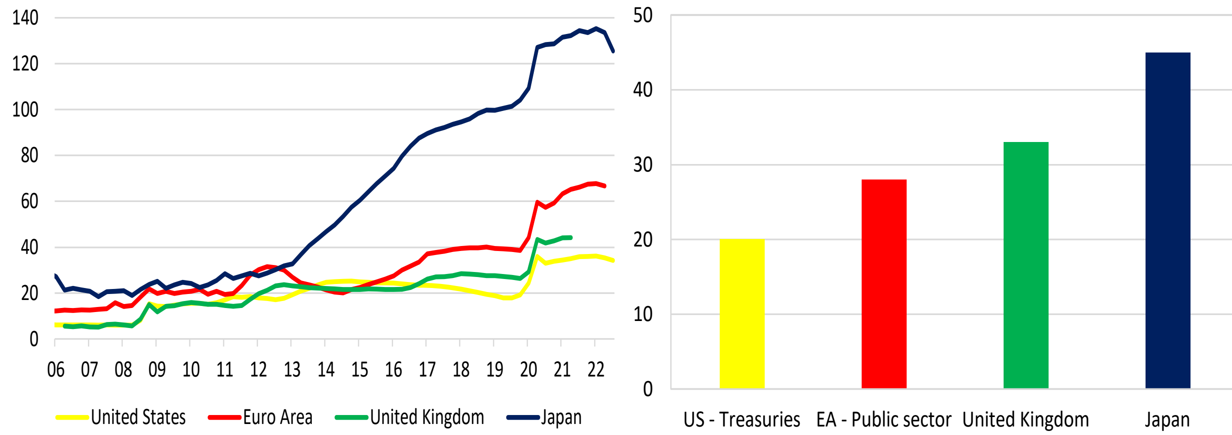
Figure 1. Central Bank Asset Holdings



(a) CB Assets as Share of GDP **(b)** CB Bond Holdings as % of Eligible Universe

Left panel source: TBC.
Right panel source: Graph IV.2 in https://www.bis.org/publ/mktc11.pdf [October 2019].

Our first contribution is to provide novel empirical evidence showing that – and contrary to commonly-held beliefs – conventional monetary policy and large scale asset purchases are not perfectly substitutable and transmit through different channels. More specifically, and broadly in line with the early findings of Kiley (2014), we show that unconventional policy has small effects on industrial production, and that these effects mainly arise via the exchange rate channel. Expressed differently, when normalized by their impact on long term yields, central bank asset purchases have *much* smaller effects on domestic demand than conventional policy.

To analyze domestic transmission and international spillovers, we then develop a two-country framework consistent with our empirical findings. Our open economy New Keynesian (NK) model extends that developed in Kolasa and Wesołowski (2020) (KW henceforth) and features home bias in consumption preferences, along with prices assumed to be sticky in local currency. One of its distinguishing features is asset market segmentation, implemented in line with Chen et al. (2012), wherein short- and long-term bonds are imperfect substitutes because of the presence of portfolio transaction costs. These costs depend on positions taken by agents and are influenced by central bank asset purchases, as those affect the outstanding bond supply. In addition, segmentation prevents some agents from trading in short-term bonds, limiting the extent to which changes in term premia can be arbitraged away. Accordingly, since a share of agents' consumption decisions are tied to the long-rate, policy-induced changes in the term-premium end up mattering for real allocations.

The fact that forward guidance is counterfactually potent in the baseline NK model (Del Negro et al., 2012) raises the question of why any central bank would want to rely on other demand management tools. To provide a rationale for quantitative easing, we modify the KW model and allow for a moderate degree of cognitive discounting, in the spirit of Gabaix (2020). This makes household consumption less sensitive to the future path of policy rates and means that quantitative easing essentially becomes "the policy lever of last resort" whenever the lower bound becomes a binding constraint on interest rate policy.

Our second key modification is predicated on the results of Harding et al. (2021), which show that strategic complementarities in price setting are crucial in accounting for the dynamics of inflation, and, in particular, that they help resolve the "missing deflation puzzle". Motivated by this evidence, we replace standard, log-linear Dixit and Stiglitz (1977) demand aggregators using quasi-kinked alternatives (Kimball, 1995).[1] This tends to flatten pricing curves, particularly in a recessionary environment with inflation below target.[2] Perhaps more importantly, however, the Phillips curve becomes non-linear. An almost immediate implication of this fact is that effects of large scale asset sales do not have to mirror those of asset purchases, because they may be occurring at different segments of the curve, with sizeable implications for monetary policy transmission.

We start our quantitative analysis by demonstrating that the resulting model can account for key empirical regularities established at the outset, and that it is consistent with the results in Kiley (2014), which point to important non-substitutabilities between conventional and unconventional monetary policy. We show, in particular, that compared to conventional monetary policy, large scale asset purchases have a relatively small impact on domestic aggregate demand, but strongly affect the exchange rate. We also highlight the role of the Phillips curve non-linearity, quantifying multidimensional asymmetries in the transmission of 1% worth of GDP in asset purchases relative to asset sales of equal size.

We then turn our attention to international spillovers. We show that the impact of advanced economy monetary policies – on both advanced and emerging market counterparts – can be quantitatively significant, and can crucially depend on which of the two monetary instruments is used as

[1] As shown by Harding et al. (2021), the Kimball aggregator conveniently nests the Dixit and Stiglitz one, meaning we can easily switch this additional feature on or off.

[2] Notably, this also augments cognitive discounting by making inflation expectations less sensitive to monetary announcements.

the primary tightening lever (see e.g., Brainard 2017, Gilchrist et al. 2017 or the Taper Tantrum-focused work of N'Diaye et al. 2014, and Moriyama et al. 2014). Notably, our findings are in line with empirical evidence of U.S. interest rate policy affecting financial conditions in EMDEs (e.g., Rey, 2013; Kalemli-Ozcan, 2019; Alter and Elekdag, 2020; IMF, 2021), possibly even exceeding the effects of domestic interventions (Cecchetti et al., 2020). Strong spillovers have also been documented for balance sheet policies (e.g., Chen et al., 2012), with notable cross country heterogeneity (Bowman et al., 2015; Caballero and Kamber, 2019).

Our analysis is also related to the debate on which international transmission channels dominate. Curcuru et al. (2018) and Deng et al. (2022), for example, focus on the role of exchange rate effects, while Albagli et al. (2019) stress the importance of term premia, particularly in the context of monetary policy spillovers. We note that, because of the lack of substitutability between conventional and unconventional policies, the role of variable chosen to normalize intervention size can play an important role in establishing dominant transmission channels. Overall, however, our results tend to favor conventional interest rate hikes, as these are associated with smaller negative spillovers.

Finally, our structural model also allows us to efficiently parse out the role of FX interventions and to address the role of macroeconomic conditions in both source and recipient economies. Here, we outline the stabilizing potential of FXI, which operate by muting exchange rate effects. Reinforcing the empirical results of Ahmed et al. (2021), we also show that tighter policy caused by stronger host country aggregate demand generates modestly positive spillovers to economic activity, while tightening driven by concerns about inflation tends to be contractionary abroad. Relatedly, we also confirm the findings in Ahmed et al. (2017), and show how stronger fundamentals in "recipient" countries translate into more muted responses of their financial variables.

The remainder of the paper is structured as follows: Section 2 discusses our empirical framework and the key results on conventional and LSAP transmission. Section 3 provides an overview of our baseline theoretical model, with Section 4 discussing key extensions relative to extant contributions. Section 5 presents our calibration choices, Section 6 outlines the transmission mechanism of conventional and unconventional policies, while Section 7 focuses on the spillovers of exit strategies and discusses the robustness of our findings. Section 8 concludes.

2. Empirical Results

Our empirical results are based on the following BVAR(12)

$$Y_t = c + \sum_{p=1}^{P} B_p Y_{t-p} + A_0 \varepsilon_t, \tag{1}$$

which includes the Federal Funds rate, 1 year bond yield, log of industrial production, log of CPI, the excess bond premium and the real effective exchange rate. The data is monthly and runs from 1990M1 to 2019M12.

Our aim is to identify a conventional and an unconventional (LSAP) policy shock. We use the proxies for conventional and LSAP shocks developed by Swanson (2021) as instruments to estimate the first two columns of the contemporaneous impact (A_0) matrix. Swanson (2021) estimates the proxies using factors extracted from high frequency data on the change in yields around FOMC meetings. The proxy for conventional monetary policy or the target factor (m_{1t}) is allowed to affect short-maturity yields. In contrast, the LSAP factor (m_{2t}) does not load on the short yield and is restricted to be small in terms of its variance before 2009.

We assume that these instruments are relevant and exogenous, i.e.,

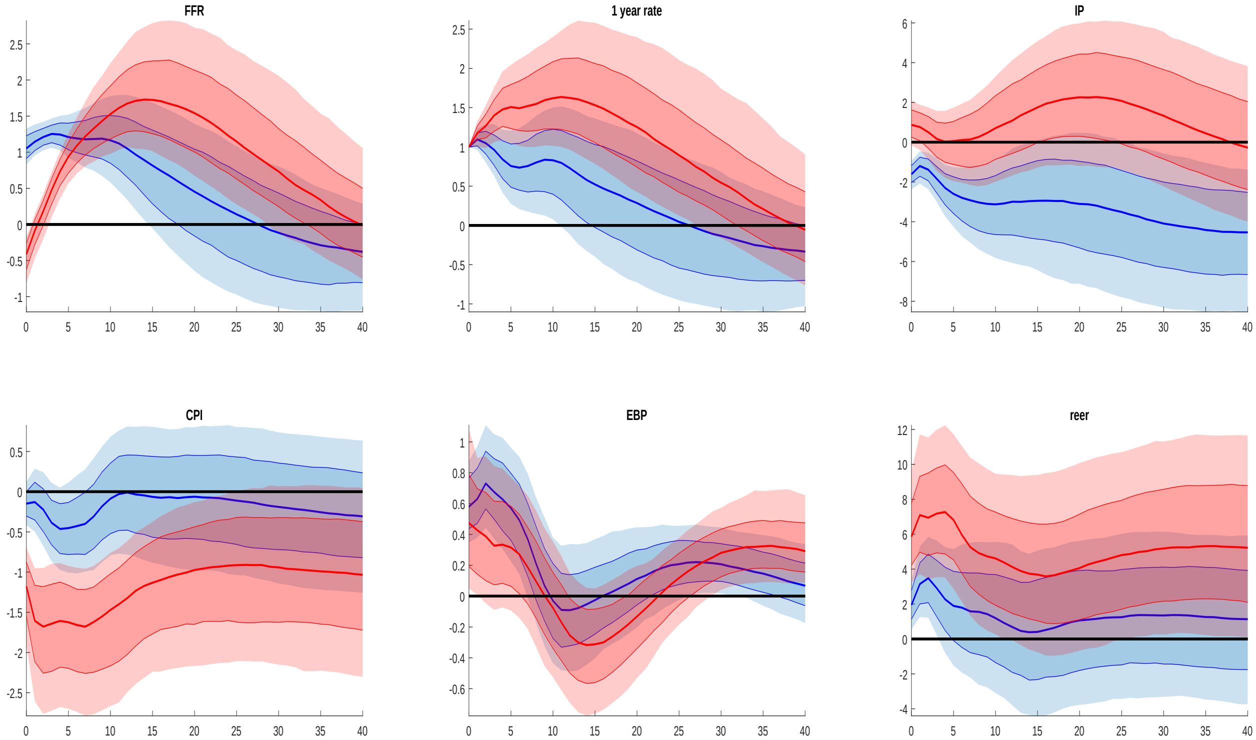
$$E\left(m_t \varepsilon_{[1:2]t}\right) = \alpha,$$

$$E\left(m_t \varepsilon_{\cdot t}\right) = 0,$$

where $m_t = (m_{1t}, m_{2t})$, $\varepsilon_{[1:2]t}$ denotes the two shocks of interest, and $\varepsilon_{\cdot t}$ are the remaining shocks. These equations identify a convolution of the structural shocks. To seperate the disturbances, we follow Lakdawala (2019) and Lunsford (2015) and impose restrictions on the covariance α. In particular, α is assumed to be upper triangular: $E\begin{pmatrix} m_{1t}\varepsilon_{1t} & m_{1t}\varepsilon_{2t} \\ m_{2t}\varepsilon_{1t} & m_{2t}\varepsilon_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ 0 & \alpha_{22} \end{pmatrix}$. That is, the LSAP instrument is assumed to be unrelated to the Federal Funds rate shock. This is consistent with the rotation applied by Swanson (2021) to estimate the yield curve factors.

Our key empirical findings are summarized in Figure 2, which compares IRFs to a large scale asset sale / quantitative tightening, to those of a conventional interest rate shock. In line with the strategy adopted in the subsequent model section, both shocks are normalized by their impact on longer-term interest rates, which, in this case, is the 1 year rate. As can be seen in the top

Figure 2. Quantitative Tightening vs Policy Rate Transmission



Note: This figure compares IRFs to a quantitative tightening shock (red line and shaded area corresponding to the median, 68%, and 90% error bands) to a conventional monetary policy shock (blue line and shaded areas). Both shocks are normalised to increase the 1 year rate by 100BP on impact. The shocks are identified via the Swanson (2021) proxies and the assumption that the proxy for LSAP is unrelated to FFR (conventional) shock.

left panel, conventional monetary policy translates into a higher, and slowly decaying, level of the federal funds rate, while unconventional asset sales do very little on impact, with a hump-shaped impulse response peaking at around fifteen quarters.

Intuitively, tighter policy translates into a dollar appreciation (positive values of the REER), and tighter financial conditions – as evidenced by the dynamics of the excess bond premium in the bottom middle panel. In our view, however, the most striking aspects of Figure 2, are the fact that the real exchange rate is more responsive to unconventional monetary policy, as well as the response of industrial production (a proxy for output, which is unavailable at monthly frequency). Specifically, following a quantitative easing shock, the industrial production IRF (top-right panel) is not significantly different from zero, in stark contrast to significant "conventional" falls depicted in blue. The relatively larger exchange rate response and missing output response following an unconventional intervention will be used as key litmus tests when assessing the performance of the theoretical model, which we now turn to describing.

3. The Baseline Model

As alluded to in the introduction, the two-country DSGE model used in our analysis nests the one developed in Kolasa and Wesołowski (2020). We start by providing an overview of its key elements before documenting our two extensions. Since the model structure is largely symmetric, in what follows we focus on problems faced by agents populating the home economy, except when foreign problems are non-trivially different. In all cases, foreign variables are indicated with an asterisk and for any variable X_t: X denotes X_t's steady state.

Broadly, global population is normalized to unity and the relative size of the domestic economy is $\omega \in (0, 0.5)$. Each country is populated by two types of households, as well as final and intermediate goods producers that supply domestic and foreign markets. The two types of households are labelled "restricted" and "unrestricted", and indexed with $j \in \{r, u\}$, respectively, with $\omega_r \in (0, 1)$ denoting the share of restricted households.[3] Both types offer labor services to firms at the nominal wage rate W_t, receive dividends from monopolistically competitive firms D_t^j, and pay lump sum taxes T_t^j, with lifetime utility maximized by household of type j given by

$$U_t^j = E_t \sum_{s=0}^{\infty} \beta_j^s \exp\{\varepsilon_{t+s}^d\} \left[\frac{(c_{t+s}^j)^{1-\sigma}}{1-\sigma} - \frac{(n_{t+s}^j)^{1+\varphi}}{1+\varphi} \right], \tag{2}$$

where ε_t^d is the preference shock, $\beta_j \in (0; 1)$ is the subjective discount factor, $\sigma > 0$ denotes relative risk aversion, and $\varphi > 0$ is the (inverse) Frisch elasticity of labor supply.

Following Woodford (2001), we model long-term bonds as perpetuities paying an exponentially decaying coupon $1, \kappa, \kappa^2, \ldots$ starting in the period following issuance, where $\kappa \in (0, 1]$. By absence of arbitrage, $P_{L-s,t}$, i.e., the current price of a long term bond issued s periods ago then has to be related to the price of a newly issued perpetuity $P_{L,t}$, via $P_{L-s,t} = \kappa^s P_{L,t}$. A convenient implication is that we only need to keep track of the long term bond price issued contemporaneously, as prices of all past vintages can be easily recovered using the preceding formula. With that as background, Table A.2 provides an overview of permissible asset holdings, with its entries denoting *units* of the underlying short- or long-term bonds, respectively.

As Table A.2 makes clear, restricted households trade only in long-term bonds, which is meant to proxy for longer-horizon investors, such as pension funds. Relatedly, unrestricted households

[3] These name choices are largely for consistency with previous literature (Andres et al., 2004; Chen et al., 2012; Kiley, 2014; Alpanda and Kabaca, 2020) even though, as per Table A.2, "unrestricted" agents cannot hold all asset types.

Table 1. Asset Holding Notation

Bond Issuer:	Home	Foreign	Home	Foreign
Bond Type:	Short-term		Long-term	
Home unrestricted	$B^u_{H,t}$	-	$B^u_{H,L,t}$	$B^u_{F,L,t}$
Foreign unrestricted	-	$B^{u,\star}_{F,t}$	$B^{u,\star}_{H,L,t}$	$B^{u,\star}_{F,L,t}$
Home restricted	-	-	$B^r_{H,L,t}$	-
Foreign restricted	-	-	$B^{r,\star}_{H,L,t}$	$B^{r,\star}_{F,L,t}$

Note: An entry means that an agent has access to a given bond, with the symbol denoting time t holdings (units of the security), while a gray background indicates that these holdings are associated with transaction costs.

have to pay transaction costs to trade in those markets, with corresponding costs for specialized, "restricted" investors assumed to be negligible.

Letting P_t be the aggregate price level, and with $R_{L,t} \equiv P^{-1}_{L,t} + \kappa$ denoting the gross yield to maturity on domestic long-term bonds, the restricted households' budget constraint becomes[4]

$$P_t c^r_t + \left[P_{L,t} B^r_{H,L,t}\right] + T^r_t = \left[P_{L,t} R_{L,t} B^r_{H,L,t-1}\right] + W_t n^r_t + D^r_t, \tag{3}$$

where the bond accumulation parts are denoted using square brackets. In line with Table A.2, unrestricted agents additionally have access to domestic short-term bonds and long-term bonds issued by the foreign government, i.e., they trade a total of three assets, and they are required to pay upfront transaction costs $\zeta_{H,t}$ and $\zeta_{F,t}$ when acquiring home and foreign long term assets, respectively. These considerations translate into the following, "unrestricted" budget constraint

$$\begin{aligned} P_t c^u_t + [B^u_{H,t}] + [\left(1+\zeta_{H,t}\right) P_{L,t} B^u_{H,L,t}] + [\left(1+\zeta_{F,t}\right) S_t P^\star_{L,t} B^u_{F,L,t}] + T^u_t \\ = \left[R_{t-1} B^u_{H,t-1}\right] + \left[P_{L,t} R_{L,t} B^u_{H,L,t-1}\right] + \left[S_t P^\star_{L,t} R^\star_{L,t} B^u_{F,L,t-1}\right] + W_t n^u_t + D^u_t + \Xi^u_t, \end{aligned} \tag{4}$$

where R_t is the short-term policy rate and S_t is the home nominal exchange rate. All the bond holding costs are rebated lump sum through Ξ^u_t and they are assumed to satisfy

$$\frac{1+\zeta_{H,t}}{1+\zeta_H} = \left(\frac{P_{L,t} b^u_{H,L,t}}{P_L b^u_{H,L}}\right)^{\xi_H} \quad \text{and} \quad \frac{1+\zeta_{F,t}}{1+\zeta_F} = \left(\frac{P_{L,t} b^u_{F,L,t}}{P_L b^u_{F,L}}\right)^{\xi_F},$$

[4] The current period yield to maturity is, by definition, $R_{L,t}$ satisfying

$$P_{L,t} = \frac{1}{R_{L,t}} + \frac{\kappa}{R^2_{L,t}} + \frac{\kappa^2}{R^3_{L,t}} + \ldots \equiv \frac{1}{R_{L,t}} \left(\frac{1}{1-\frac{\kappa}{R_{L,t}}}\right) \iff R_{L,t} - \kappa = \frac{1}{P_{L,t}} \iff R_{L,t} = P^{-1}_{L,t} + \kappa.$$

with $\xi_H, \xi_F > 0$, and where

$$b^u_{H,L,t} \equiv \frac{B^u_{H,L,t}}{P_t} \qquad \text{and} \qquad b^u_{F,L,t} \equiv \frac{B^u_{F,L,t}}{P_t}.$$

As implied by Table A.2, households in the foreign economy are modeled analogously, *except that* restricted agents trade in both domestic and foreign long-term bonds, which gives rise to a slightly different budget constraint

$$\begin{aligned} P^\star_t c^{r,\star}_t + [P^\star_{L,t} B^{r,\star}_{F,L,t}] + [\left(1+\Gamma^{r,\star}_t\right) \frac{P_{L,t}}{S_t} B^{r,\star}_{H,L,t}] + T^{r,\star}_t \\ = \left[P^\star_{L,t} R^\star_{L,t} B^{r,\star}_{F,L,t-1}\right] + \left[\frac{P_{L,t}}{S_t} R_{L,t} B^{r,\star}_{H,L,t-1}\right] + W^\star_t n^{r,\star}_t + D^{r,\star}_t + \Xi^{r,\star}_t \end{aligned} \tag{5}$$

where $\Gamma^{r,\star}_t$ is an adjustment cost rebated lump sum through Ξ^{r*}_t and assumed to follow[5]

$$1+\Gamma^{r,\star}_t = \exp\left\{\xi^\star_r \left(\frac{P_{L,t} B^{r,\star}_{H,L,t}}{S_t P^\star_{L,t} B^{r,\star}_{F,L,t}} - \kappa^{r,\star}\right)\right\}. \tag{6}$$

For more on the rationale and merits of our chosen approach to introducing segmentation we refer the reader to Appendix A.

On the production side, in the *baseline* version of our model, perfectly competitive final goods producers combine homogeneous home-made goods $y_{H,t}$ and imported goods $y_{F,t}$ according to the following technology

$$\tilde{y}_t = \left(\eta^{\frac{1}{\nu}} y_{H,t}^{\frac{\nu-1}{\nu}} + (1-\eta)^{\frac{1}{\nu}} y_{F,t}^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}, \tag{7}$$

where $\eta \in (0;1)$ is the home-bias parameter, and $\nu > 0$ is the elasticity of substitution between domestic and imported goods. Inputs into final goods production are aggregated according to

$$y_{h,t} = \left(\int_0^1 y_{h,t}(i)^{\frac{1}{\mu}} di\right)^{\mu}, \tag{8}$$

for $h = \{H, F\}$, where $\mu > 1$ controls the degree of substitution between intermediate inputs $y_{h,t}(i)$. As usual, the resulting demand schedules are taken as given in the intermediate goods' producers optimization problems (outlined below), and one of our key extensions described in Section 4 is to replace the Dixit and Stiglitz (1977) setup just described, with a more general alternative due to Kimball (1995).

[5] The parameter $\kappa^{r*} > 0$ captures the steady state ratio of restricted households' holdings of bonds issued by the small and large economies.

Finally, intermediate inputs are produced by monopolistically competitive firms indexed by i and operating a production function that is linear in (local) labor

$$y_{H,t}(i) + y^*_{H,t}(i) = \exp\{\varepsilon^z_t\} n_t(i) - \phi, \tag{9}$$

where ε^z_t denotes a productivity shock, and ϕ is a fixed cost of production. Every period these firms face a fixed probability θ_H of domestic price reoptimization, and probability θ^*_H of export price reset, with firms unable to reoptimize indexing prices to steady state CPI. Assuming local firm ownership – by "restricted" and "unrestricted" agents and in proportion to their shares in the population – the problem of reoptimizing firms becomes to maximize

$$E_t \sum_{s=0}^{\infty} (\theta_H)^s \Lambda_{t+s} \left(P_{H,t}(i)\, \pi^s - \frac{W_{t+s}}{\exp\{\varepsilon^z_{t+s}\}} \right) y_{H,t+s}(i), \tag{10}$$

$$E_t \sum_{s=0}^{\infty} (\theta^*_H)^s \Lambda_{t+s} \left(S_{t+s} P^*_{H,t}(i)\, (\pi^*)^s - \frac{W_{t+s}}{\exp\{\varepsilon^z_{t+s}\}} \right) y^*_{H,t+s}(i), \tag{11}$$

where $\Lambda_{t+s} \equiv P^{-1}_{t+s}[\omega_r \beta^s_r (c^r_t)^{-\sigma} + (1-\omega_r)\beta^s_u (c^u_t)^{-\sigma}]$ is the nominal stochastic discount factor, $P_{H,t}(i)$ is the price set by intermediate producer i for the domestic market, $P^*_{H,t}(i)$ is the price set for the foreign market, while $\pi_t \equiv P_t/P_{t-1}$ and $\pi^*_t \equiv P^*_t/P^*_{t-1}$ are the domestic and foreign final good inflation rates.

Finally, "conventional" monetary policy is assumed to follow a Taylor-type feedback rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y} \right]^{1-\gamma_r} \exp\{\varepsilon^r_t\}, \tag{12}$$

where ε^r_t is the monetary policy shock, $\gamma_r \in (0,1)$ controls the degree of interest rate smoothing, while γ_π and γ_y determine, respectively, the strength of interest rate responses to deviations of inflation and output from their steady state values. For subsequent reference, and along the lines of Chen et al. (2012) and Kolasa and Wesołowski (2020), we can also define the term premium on long-term bonds as

$$TP_t = R_{L,t} - R^{EH}_{L,t}, \tag{13}$$

where $R^{EH}_{L,t}$ is the counterfactual yield to maturity on a long-term bond in the absence of transaction costs, which we price using unrestricted households' stochastic discount factor, and κ^{EH} is chosen to ensure that the counterfactual bond has the same steady state duration D_L as the actual one.[6] Notably, and as shown by Chen et al. (2012), the term premium can be approximated up to first order as the discounted sum of expected values of transaction costs $\zeta_{H,t}$ associated with trade in domestic long-term bonds, i.e.,

$$TP_t \approx D_L^{-1} \sum_{s=0}^{\infty} \left(\frac{D_L - 1}{D_L} \right)^s E_t \zeta_{H,t+s}. \tag{14}$$

This relationship shows that fluctuations in the term premium reflect portfolio rebalancing decisions, both current and planned, and thus links the term premium directly to unconventional monetary policy.

3.1. **Market Clearing Conditions.** Equilibrium in the goods market requires

$$\tilde{y}_t = \omega_r c_t^r + (1 - \omega_r) c_t^u + g_t, \tag{15}$$

and

$$y_t \equiv y_{H,t} \Delta_{H,t} + \frac{1-\omega}{\omega} y_{H,t}^* \Delta_{H,t}^* = \exp\{\varepsilon_t^z\} n_t - \phi, \tag{16}$$

where $n_t \equiv \omega_r n_t^r + (1 - \omega_r) n_t^u$ is aggregate labor input, y_t defines aggregate output while

$$\Delta_{H,t} = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{\frac{\mu}{1-\mu}} di, \tag{17}$$

captures price dispersion arising on account of staggered price setting in the intermediate goods sector.

Complementing these, we also have market clearing conditions for bonds issued by the home economy's government

$$(1 - \omega_r) B_{H,t}^u = B_{H,t}^g, \tag{18}$$

and

$$\omega_r B_{H,L,t}^r + (1 - \omega_r) B_{H,L,t}^u + \frac{1-\omega}{\omega} \omega_r^* B_{H,L,t}^{r*} + \frac{1-\omega}{\omega} (1 - \omega_r^*) B_{H,L,t}^{u*} = B_{H,L,t}^g. \tag{19}$$

[6] Expressed alternatively, κ^{EH} is set to ensure the following equality

$$D_L = \frac{R_L}{R_L - \kappa} = \frac{R_L^{EH}}{R_L^{EH} - \kappa^{EH}}.$$

See also Appendix B for a discussion.

Using these market clearing conditions together with the budget constraints of households and the government, as well as the zero-profit condition of the final goods producers and aggregators, we obtain the following law of motion for the small economy's net foreign assets position

$$A_t = R^*_{L,t}\frac{S_t}{S_{t-1}}\frac{P^*_{L,t}}{P^*_{L,t-1}}\left[(1-\omega_r)S_{t-1}P^*_{L,t-1}B^u_{F,L,t-1}\right] \\ - R_{L,t}\frac{P_{L,t}}{P_{L,t-1}}\left[\frac{1-\omega}{\omega}P_{L,t-1}\left((1-\omega^*_r)B^{u*}_{H,L,t-1}+\omega^*_r B^{r*}_{H,L,t-1}\right)\right] + NX_t \quad (20)$$

where $NX_t = \frac{1-\omega}{\omega}S_t P^*_{H,t}y^*_{H,t} - P_{F,t}y_{F,t}$ are net exports and the terms in the square brackets are, respectively gross foreign assets and liabilities.

3.2. **Exogenous shocks.** Aside from QT-induced shifts in the composition of government debt in the large economy – discussed in more detail in Section 4 – the model also features a set of standard shocks used in open economy DSGE models. These are the country pairs of shocks to productivity (ε^z_t and ε^{z*}_t), time preferences (ε^d_t and ε^{d*}_t), government spending (ε^g_t and ε^{g*}_t) and monetary policy (ε^r_t and ε^{r*}_t). Unless otherwise stated, all shocks are modeled as independent first-order autoregressions.

4. Extensions

This section discusses the two main areas in which our model extends on the framework of Kolasa and Wesołowski (2020), i.e., i) behavioral discounting, and ii) Kimbal aggregation. Note that we also have a slightly more granular model of central bank balance sheets, which is discussed in Appendix E.

4.1. **Behavioral Discounting.** [TBC]

4.2. **Kimbal Aggregation.** [TBC]

5. Calibration

Our calibration comes in two flavors. The large foreign economy is always assumed to represent the US. The smaller, home country is meant to proxy either an advanced (AE), or emerging market economy (EME). Broadly, and apart from having smaller relative size, i.e., ω set to 0.014, the advanced economy is calibrated symmetrically to the large one. Conversely, and as discussed in detail below, the emerging market economy features larger pass-through from exchange rates to inflation, in line with the estimates provided by Brandao-Marques et al. (2021). Overall, the calibration for the emerging market economy implies that its policymakers face a more difficult monetary trade-off, in line with the discussion in Adrian et al. (2021).

Following standard practice, we either set parameters to match key steady state proportions observed in the data, or we rely on extant literature for parameter values. Table 2 shows the values we adopted, with Table 3 presenting the targeted steady state ratios. The time period throughout is meant to correspond to a quarter. The home bias parameter η is calibrated to 0.75 to capture the average share of imports in GDP. The elasticity of substitution between domestically produced goods and imports is set to 3, which can be seen as a compromise between the micro and macro estimates found in the literature.

Table 2. Calibrated Parameters

Parameter	Symbol(s)	Value(s)
Size of the Small Economy	ω	0.01
Share of Restricted Households	ω_r, ω_r^*	0.15
Inv. Elasticity of Intertemporal Substitution	σ, σ^*	1
Inv. Frisch Elasticity of Labor Supply	φ, φ^*	2
Discount factor, unrestricted households	β^u, β^{u*}	0.9975
Discount factor, restricted households	β^r, β^{r*}	0.99875
Consol duration (quarters)		40
Transaction cost on long-term bonds	ξ_H, ξ_F	0.015
Portfolio adjustment cost	ξ_r^*	10^{-5}
Calvo probability for domestic production	θ_H, θ_F^*	0.75
Calvo probability for exports	θ_H^*, θ_F	0.667
Price markup	μ, μ^*	1.15
Kimball parameter	ψ, ψ^*	-12
Elast. of subst. between home and foreign goods	μ	-4
Home-bias	η	0.75
Steady-state inflation	π, π^*	1.005
Interest rate smoothing	γ_r, γ_r^*	0.9
Interest rate response to inflation	γ_π, γ_π^*	2
Interest rate response to output gap	γ_y, γ_y^*	0.125
Smoothing parameter in debt composition	γ_L, γ_L^*	0.95, 0.985

In our model the key transmission channel of international policy spillovers relies on gross bond holdings and their adjustment. Hence, the crucial part of our calibration concerns the steady state composition of the bond portfolios held by agents in the small and large economy. Our targets for these are calculated by combining several data sources. The shares of sovereign bonds in quarterly GDP are calibrated to 1.25 and 2.65, respectively, while the share of long-term bonds in total sovereign bonds is set at 0.71 for the home economy and 0.65 for the foreign one, mirroring choices in Kolasa and Wesołowski (2020). The steady state share of resident holdings in total long-term bonds issued by the small economy is set to 0.76. The ratio of foreign bonds to total bonds held by small economy's agents is calibrated to 0.05. Finally, we assume that the share of small economy's bonds in the long-term bond portfolio held by foreign households is the same for their two types, which pins down the value of κ^{r*} at 0.0018.

Table 3. Targeted Steady State Ratios

Steady state ratio	Expression	Value
Share of Govt. Spending in GDP	$\frac{g}{y}, \frac{g^*}{y^*}$	0.2
Share of Govt. Bonds in GDP	$\frac{b^g+P_L b^g_{H,L}}{y}, \frac{b^{g*}+P^*_L b^{g*}_{H,L}}{y^*}$	1.25, 2.65
Share of Long-Term (LT) bonds in Total	$\frac{P_L b^g_{H,L}}{b^g+P_L b^g_{H,L}}, \frac{P^*_L b^{g*}_{H,L}}{b^{g*}+P^*_L b^{g*}_{H,L}}$	0.71, 0.65
Residents' share in SOE LT bonds	$\frac{P_L(\omega_r b^r_{H,L}+(1-\omega_r)b^u_{H,L})}{b^g_{H,L}}$	0.76
Share of Foreign Bonds in SOE portfolio	$\frac{(1-\omega_r)sP^*_L b^u_{F,L}}{\omega_r P_L b^r_{H,L}+(1-\omega_r)(sP^*_L b^u_{F,L}+P_L b^u_{H,L})}$	0.05

Another important group of parameters determines the degree of market segmentation and sensitivity of transaction costs, and hence the term premia, to adjustments in agents' portfolios. We set the share of restricted households ω_r and ω^*_r to 0.15 for both economies. While calibrating transaction costs on long-term bonds ξ_H and ξ_F, our primary goal is to make sure that, in response to a quantitative tightening (QT) scenario that we describe subsequently, our model generates a reaction of bond prices in the large economy consistent with the results of Chen et al. (2012).[7] Finally, the smoothing coefficient in the debt composition rule for the large economy γ^*_L is calibrated at 0.99 to reflect high persistence of asset purchase programs.

The following parameters determine the steady state levels of the interest rates and bond prices, and hence the term premia and bond duration. We set the inflation targets π and π^* to 1.005 (2% annualized) so that they are consistent with those targeted by the Federal Reserve.

[7] Chen et al. (2012) estimate a reduction of the term premium by 10 bps following the LSAP II in the United States. However, the ultimate scale of this program (600 bn USD, i.e. 4% of the US GDP) is about three times smaller than in our case (12% of the combined GDP of the US, the UK and the euro area), so we target the fall in the term premium in the large economy by 30 bps.

These, together with the discount factors for restricted and unrestricted households β_r and β_u, pin down the steady state long and short-term interest rates. Again, we target US averages of 5.2% and 4%, respectively, symmetrically between the two regions. Since long-term bonds are modeled as perpetuities, we specify their coupons κ and κ^* to arrive at a duration of ten years. The Calvo probabilities for domestic sales θ_H and θ_F^* are calibrated at 0.75, which results in a slope of the Phillips curve consistent with empirical macro literature (Erceg and Linde, 2014). We assume that the price duration for foreign sales is two times lower and set θ_H^* and θ_F to 0.6.

The remaining parameters are either relatively well-established in the literature or do not have important effects on our key results. The steady state government spending in both countries is set to 20% of GDP, roughly in line with the long-run averages observed in the data. The elasticity of intertemporal substitution σ, the Frisch elasticity of labor supply φ, price markups μ, interest rate rule coefficients γ_r, γ_π and γ_y, as well as their large economy counterparts, are all set to standard values considered in the DSGE literature. Finally, the portfolio adjustment cost ξ_r^* is set to a number that ensures determinacy, but which does not markedly affect model dynamics otherwise.

6. Transmission of Asset Purchases and Interest Rate Policy in the Model

We start our analysis by comparing the transmission channels of quantitative tightening and conventional interest rate hikes. To that effect, we first observe that the macroeconomic effects of asset purchases depend on the effect these purchases have on the term structure of interest rates and on the exchange rate. To clarify how central bank asset purchases propagate within the model, it is therefore instructive to examine a linearized arbitrage condition linking the expected one-period rate of return from holding long-term bonds to the risk-free interest rate on one-period bonds

$$E_t R_{L,t+1} = R_t + \zeta_F B_{L,t}^u. \tag{21}$$

As Equation 21 makes clear, transaction costs – captured by the final term on the right hand side – effectively drive a wedge between the two rates of return. These costs are an increasing function of long-term bond holdings of unrestricted agents (i.e., of those who can hold both types of assets). It follows that, for a given short-term policy rate, asset sales by the central bank increase the supply of long-term bonds available to private agents and so increase the associated expected return. Intertemporal smoothing then implies that the consumption of agents exclusively trading long-term bonds falls, contributing to a contraction in aggregate outptut.

The second key equilibrium relationship is associated with portfolio choices of unrestricted agents, and it is best summarized by the linearized "long-UIP" condition

$$E_t R^{L1}_{t+1} = E_t R^{L1,*}_{t+1} + E_t \Delta S_{t+1}. \tag{22}$$

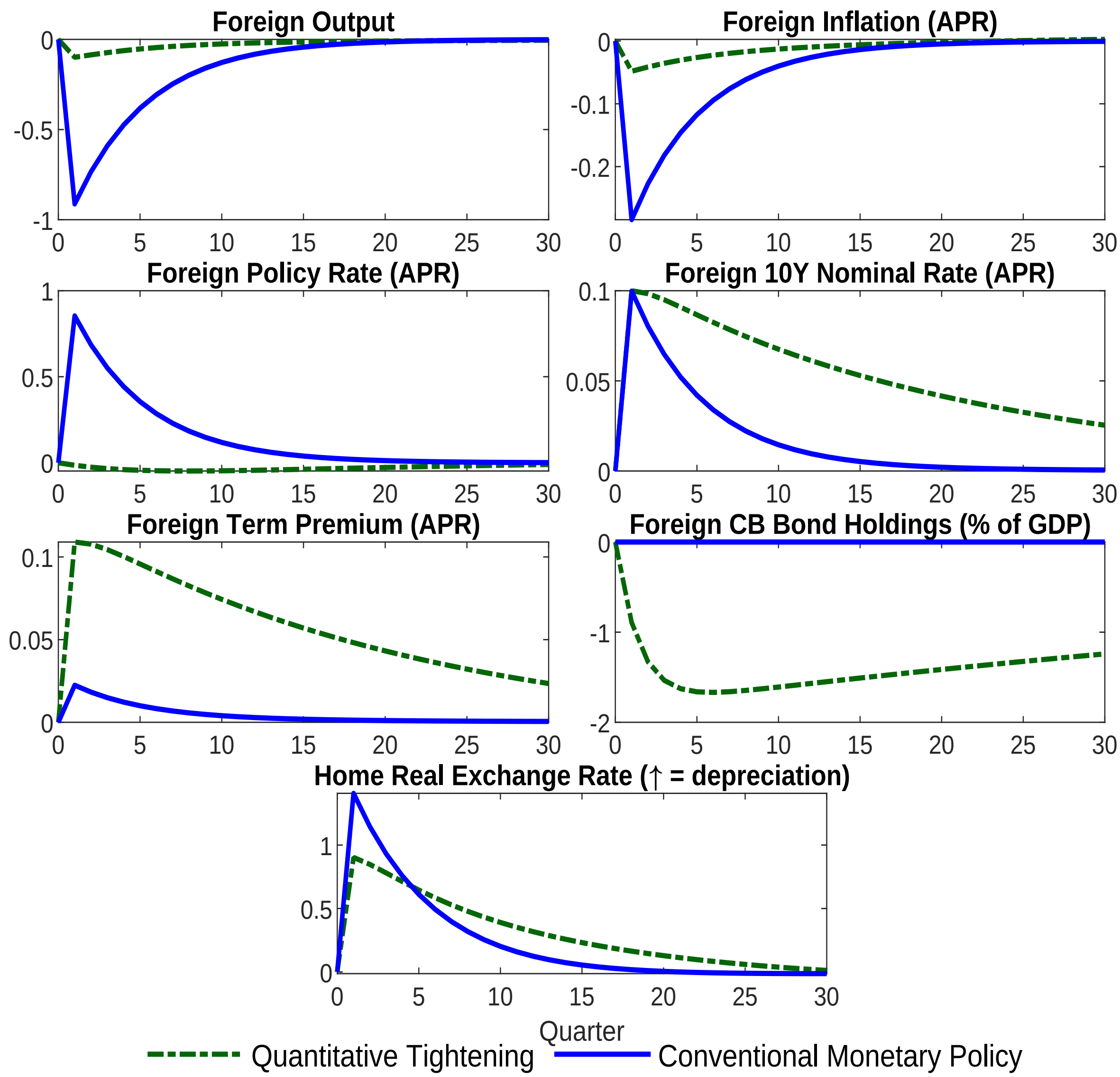
This identity postulates the equalization of one-period holding returns on long-term bonds denominated in different currencies, with S_t denoting the nominal exchange rate. Its immediate implication is that an increase in the expected return on long-term bonds issued by one country, possibly associated with quantitative tightening conducted by the central bank, must generate an increase in the other country's expected bond return or expected exchange rate depreciation. Expressed alternatively, the exchange rate of the country implementing QT will tend to appreciate on impact. Notably, the exact extent to which the increase in long-term rates translates into domestic appreciation and an increase in foreign long-term rates depends on the policies endogenously pursued by the neighboring central bank. For example, in the case in which the monetary authority operates an exchange rate peg, the effects of foreign QT would be fully transmitted to domestic long-term rates. If, on the other hand, the central bank adopted a standard Taylor-type instrument rule, our model implies that a 100bp hike in foreign long-term rates would increase the same maturity domestic rates by about 35bp.

To show the basic workings of the model under our baseline parameterization, Figure 3 compares the effects of a short-term policy shock and quantitative tightening, both sized to provide a 10-basis point increase in the long-term nominal rate. We predominantly report variables for the foreign economy, i.e., the economy implementing the policy changes, but also show the transmission to the small open economy (home) real exchange rate.

Broadly, conventional monetary tightening in the large economy contracts aggregate demand, which leads to a fall in output and demand for other countries' exports. As in the case of quantitative tightening, the associated depreciation of the exchange rate improves the price competitiveness of the small economy.[8] Both standard and unconventional monetary tightening in the large economy increase the return on foreign long-term bonds. However, foreign long-term rates decline much more following QT compared to conventional interest rate hikes.

[8] For QT, however, the adjustment in the exchange rate is weaker, owing to which the net effects on the small economy's trade balance and GDP turn out to be negative. It is worth stressing that this negative output spillover occurs despite no impact on absorption. In fact, the latter actually increases because imported disinflation leads to some loosening of monetary policy.

Figure 3. Transmission of Balance Sheet and Short-Term Policy Rate Tightening



Note: This figure compares IRFs to quantitative tightening (dashed green line) to those corresponding to a conventional monetary policy tightening (solid blue line). Both types of intervention are scaled to ensure a 10bp increase in the large economy (foreign) 10Y nominal interest rate. All variables are plotted as deviations from their steady states.

In the former case, the short-term rate in the small economy does increase, thus increasing domestic long-term rates. It also turns out that the scale of monetary policy accommodation produced by a standard Taylor rule proves insufficient to equalize foreign and domestic bond returns. Accordingly, a large change in foreign long-term bond prices generated by QT leads to a large exchange rate adjustment and a contemporaneous increase in the home term premium that increases the long-term interest rate. Both developments are a mirror reflection of a massive capital outflow from the small economy's bond markets as foreign investors search for yield at home.

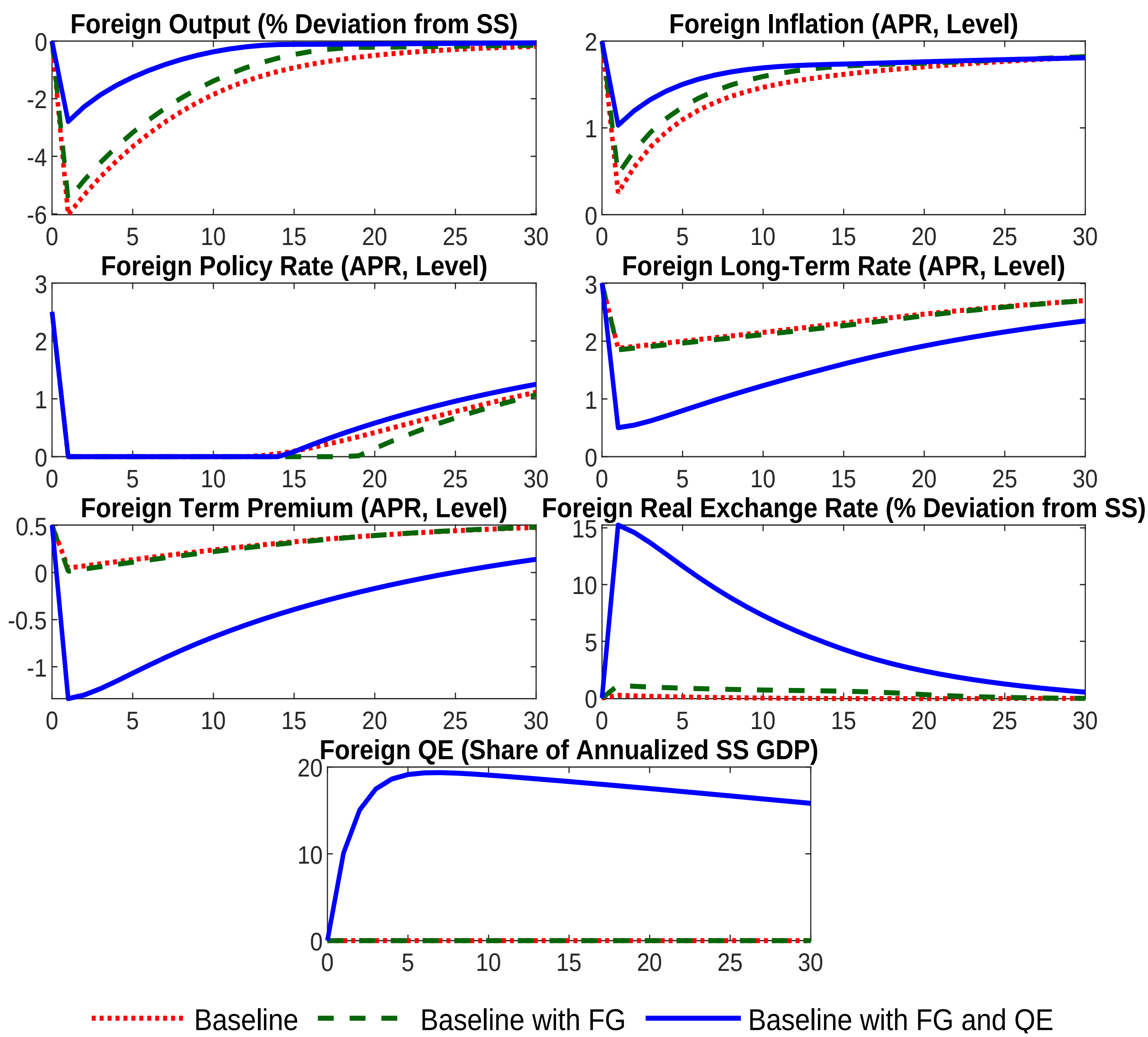
Crucially, the model matches key aspects of the IRFs derived and documented in Section 2. Adopting a similar on-impact normalization, here by the ten year rate, we find that unconventional monetary policy has negligible effects on output, in stark contrast to interest rate interventions.[9] Unconventional policy also has little impact on the foreign policy rate and is associated with a more persistent appreciation. In summary, and consistent with the empirical evidence, compared to conventional monetary policy, large scale asset purchases have a relatively small impact on domestic aggregate demand, but affect the exchange rate more strongly in the medium to long-run.

Our calibrated model suggests that policy rate hikes lower output by about 1 percent, and inflation by 0.35 percent. These effects are also broadly consistent with VAR and DSGE evidence for euro area and the US, albeit admittedly a bit too frontloaded for inflation and output. The latter arguably occurs because we abstract from real rigidities such as habit persistence, as well as intrinsic persistence in the pricing equations. For quantitative tightening, a 1.75 percent sell-off of CB bond holdings affects output by an order of magnitude less, although the long-term nominal rate increases more persistently. The effects on inflation are also smaller, though in line with the US estimates of Chung et al. (2012), if slightly below the median estimates in Fabo et al. (2021).

In the context of our model, the larger output effects of conventional policy compared to QT reflect the fact that 85 percent of consumers are assumed to be financially unconstrained and hence respond relatively more strongly to changes in the short-term policy rate than the term premium-driven changes in the long rate. Even so, an important implication of Figure 3 is that the real exchange rate responds quite strongly to QT. This finding is driven by Equation 22, which stipulates that the exchange rate is primarily driven by the long-term interest rate differential. Since QT has a more persistent influence on the long-term nominal rate, it therefore exerts a relatively larger effect on the real exchange rate, particularly further out. As the next section will show, the sizeable exchange rate impact of QT documented here will play an important role in the international propagation of unconventional policies.

[9] Notably, were we to plot effects on domestic demand, these would have been even more conspicuous by absence, as the small contraction in output is brought about by the appreciating exchange rate (aka depreciation of the home economy).

Figure 4. The Quantitative Irrelevance of Forward Guidance



Before moving on to discuss international spillovers of conventional and unconventional policies, we also wanted to draw attention to an additional desirable property of the model. Specifically, and as most clearly shown in Figure 4, the combination of cognitive discounting and Kimball aggregation means that our model does not suffer from the forward guidance puzzle (Del Negro et al., 2012). As the Figure makes clear, when confronted with a long-lasting zero lower bound, as in our baseline scenario, promising to keep rates lower for two years longer has very little stimulative effect on the economy. This is in contrast to asset purchases, similar to those implemented in response to the COVID pandemic, which can thus be argued to be the "policy lever of last resort".

7. International Spillovers from Central Bank Exit Strategies

7.1. **The Baseline Scenario.** In this section we apply our model to study the spillover effects of alternative exit strategies. To increase the relevance of the analysis presented here, and motivated by developments witnessed over the course of the last few years, we consider exit relative to a baseline, in which a negative demand shock drives the foreign policy rate to the zero lower bound for a prolonged period of time. This then prompts the central bank to announce a stimulative quantitative easing package, in which its holdings of long-term bonds increase by 20 percent of GDP, and which is illustrated using the blue line in Figure 5.

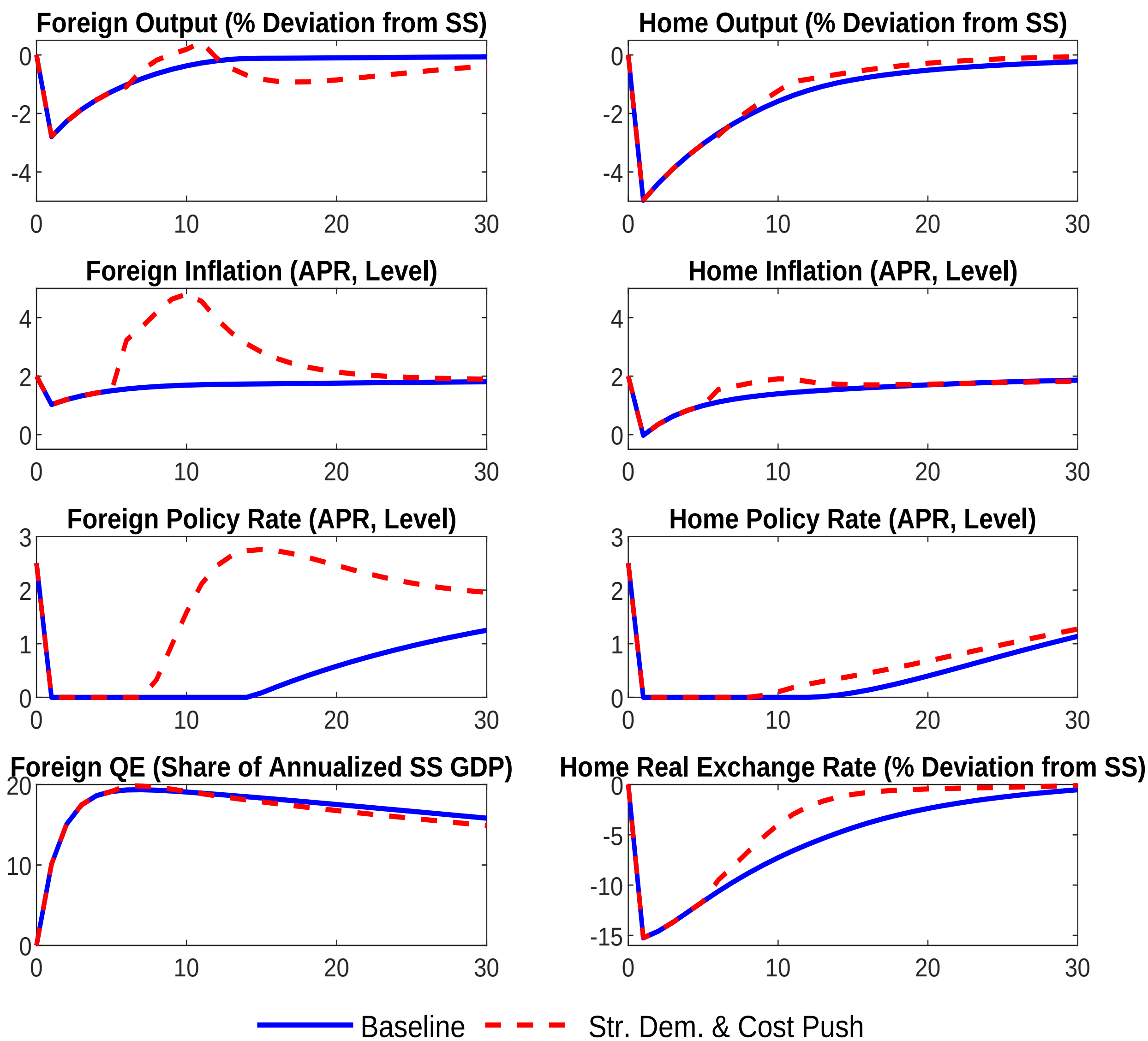
Against that backdrop, a mix of unanticipated positive demand and cost-push shocks hits the foreign economy in the fifth quarter, with the corresponding trajectories depicted using the red-dashed line. These unanticipated shocks cause foreign output to expand in the short run and contract slightly thereafter (relative to the no shock baseline). As the figure shows, they also push inflation well above the central bank's 2 percent target. In such circumstances, the Taylor rule calls for lift-off from the zero lower bound in period seven, two quarters after the occurrence of the unanticipated shock. As made clear by the bottom left panel, the scenario is also associated with a very gradual runoff of the foreign central banks' bond purchases, which we arrive at by setting the runoff parameter ϱ to 0.5525.

7.2. **Spillovers from Alternative Monetary Contractions.** As strongly suggested by the simultaneous overshoot of output and inflation, the foreign central bank can improve on outcomes by pursuing a tighter policy stance. In this section we therefore compare the domestic and foreign implications of effecting the contraction through quantitative tightening and via a conventional interest rate hike, with Figures 6 and 7 focusing on the advanced and emerging market calibrations respectively.[10] In both cases, the red-dashed lines illustrate the baseline scenario with adverse demand and cost push shocks (i.e., they correspond to the red-dashed lines in Figure 5), while the green-dotted and blue, dash-dotted lines capture paths under a more rapid asset sell-off and conventional monetary tightening, respectively.[11] Importantly, the two contractionary monetary strategies are sized to generate the same improvement in the expected discounted loss of inflation

[10] The fact that the evolution of foreign economy variables is almost completely unaffected by the calibration is a testament to the small role of policy spill-backs, mostly due to the relatively small size of the domestic economy.

[11] At the risk of belaboring the obvious, under quantitative tightening the central bank combines asset sales with an endogenously less aggressive short-term policy rate path. In addition, and henceforth, the vertical black dashed lines denote the period in which the unanticipated shocks occurred.

Figure 5. Baseline with Adverse Cost Push and Stronger Demand



and output gaps, which can be most easily observed in Figures 8 and 9, which show that they generate almost identical deviations of output and inflation from the baseline scenario.

By comparing the results for AEs and EMEs in Figures 8 and 9, we observe that both exit strategies lead to lower home output. Importantly, however, tightening via short-term policy rates is associated with notably smaller adverse output effects compared to the QT case, especially so in EMEs. As most readily seen in the bottom right panel of Figures 6 and 7, this occurs because the short-term policy rate tightening entails a notably smaller exchange rate depreciation than

Figure 6. Advanced Economy Effects of Alternative Exit Strategies

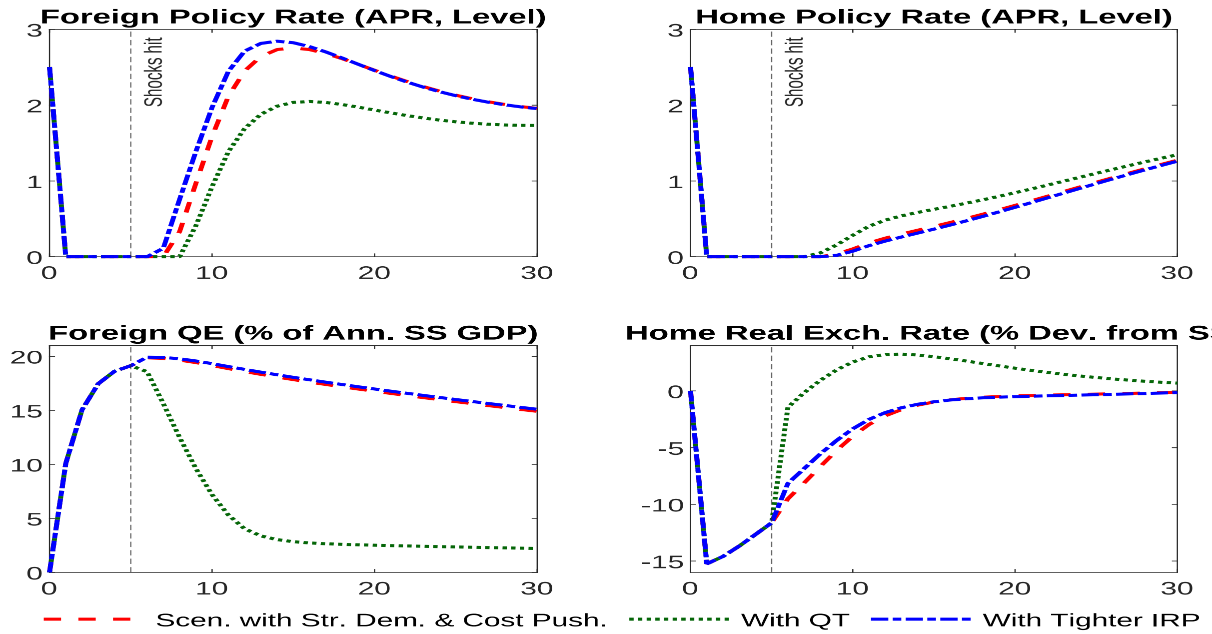
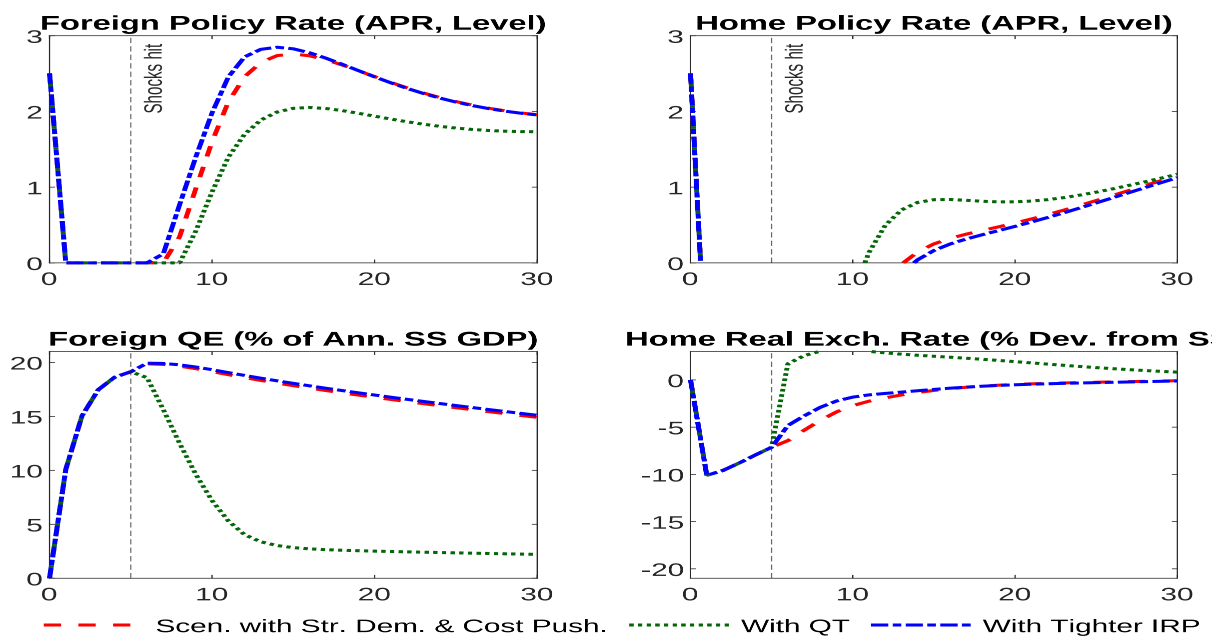


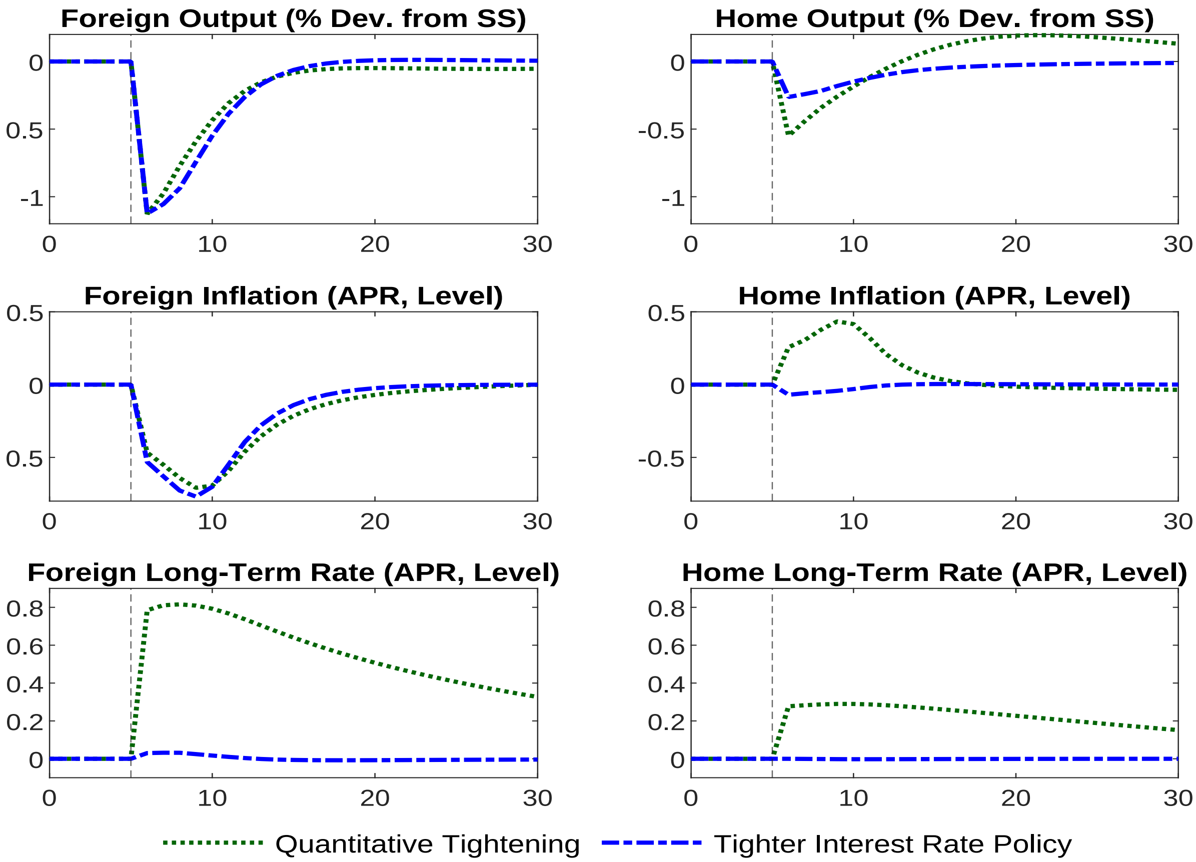
Figure 7. Emerging Market Effects of Alternative Exit Strategies



QT. Overall, and since both these policies achieve similar domestic objectives, our results therefore suggest using interest rates as the primary instrument for policy tightening.[12]

[12] This conclusion is broadly in the spirit of the IMF's Integrated Surveillance Decision (ISD) framework, which leans towards alternatives with fewer negative spillovers.

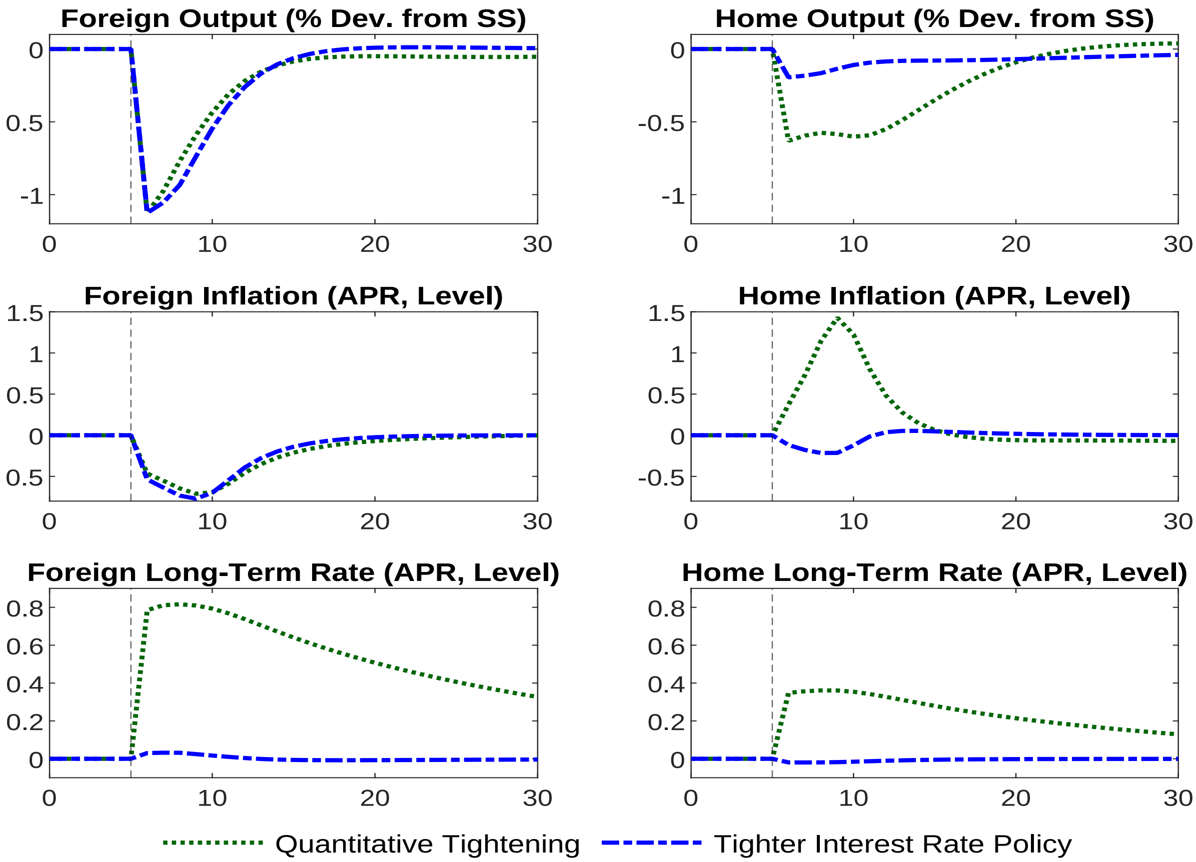
Figure 8. Advanced Economy Effects of Alternative Exit Strategies: Deviations from Adverse Scenario



The preceding result may seem surprising, particularly given our previous finding that short-term policy rates have a larger impact on the exchange rate than asset purchases (as in Figure 3) for a given movement in the 10Y nominal interest rate. However, the two results can be squared by recognizing that the alternative interventions depicted in Figures 6 - 9 are sized to give the same impact on foreign *output* and *inflation*. And because the short-term policy rate affects output and inflation by more than asset sales, larger sell-offs are required in the foreign economy. Accordingly, generating the same inflation and output paths in Figures 6 - 9 leads to larger long-term interest rate differentials, which tends to weaken the exchange rate more, particularly in the emerging market economy.

In turn, the relatively weaker EME exchange rate acts to exacerbate inflationary pressures, which causes the emerging market economy CB to tighten its policy stance more aggressively, translating into more notable output reductions under QT. Another source of negative spillovers from quantitative tightening is the larger transmission to long-term rates via term premia. This is

Figure 9. Emerging Market Effects of Alternative Exit Strategies: Deviations from Adverse Scenario

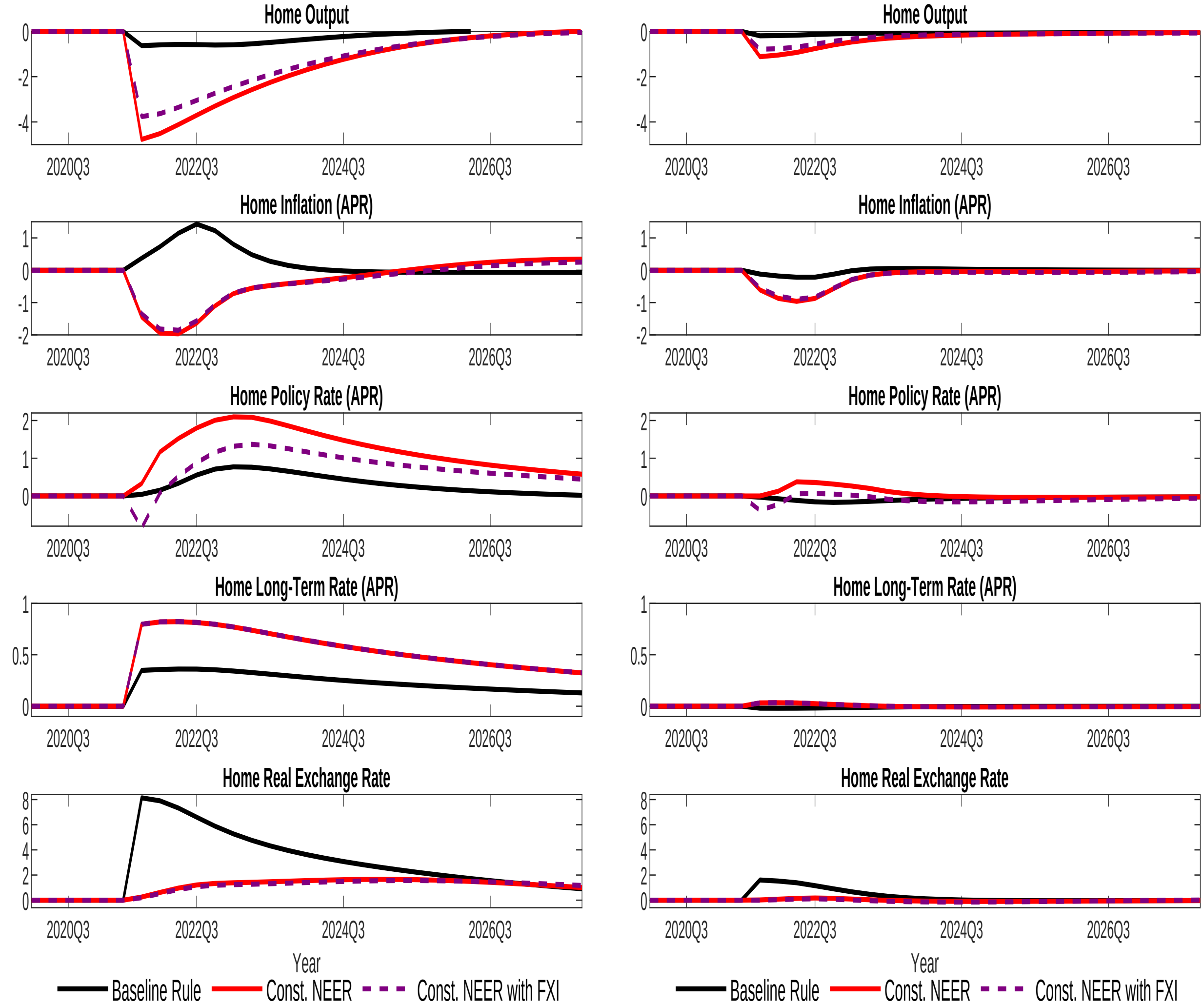


evident from the bottom rows of Figures 8 - 9, which show that long-term yields in AEs and EMEs rise notably more following QT compared to conventional tightening.

7.3. **The Role of Foreign Policy Conduct.** So far, we have assumed that the small advanced and emerging market economies follow the same interest policy rules as the one adopted in the foreign economy. Admittedly, this was meant to proxy flexible inflation targeting with no explicit exchange rate stabilization motive. Since the differences between spillovers of conventional monetary policy and QT hinge on the size of capital flows and exchange rate depreciation, we now investigate whether this result is robust to alternative economic policies designed to counteract at least one of them.

To that effect, in Figure 10 we study instead spillovers for alternative policy arrangements, purposefully restricting our analysis to the emerging market calibration, as that is where managed exchange rate regimes are most commonly observed. More specifically, the left (right) column

Figure 10. EM Spillovers for Alternative NEER Stabilization (Home)



(a) Quantitative Tightening **(b) Tightening via Short Rate**

of Figure 10 depicts spillovers for the same sized foreign QT (interest rate policy tightening) as previously, with the black line in both columns corresponding to the baseline interest rate rule analyzed in Figures 8 - 9.[13] In addition, Figure 10 also reports results for two alternative exchange rate regimes. The first (red solid lines) corresponds to a fixed exchange rate policy, which is solely implemented via the short-term policy rate. The second set of results (purple dashed lines) also pertains to a fixed exchange rate regime, but in this case we assume that the CB uses discretionary FX interventions to help stabilize its exchange rate.

The figure also illustrates that, to a large extent, both policies succeed in moderating the inflationary spike under spike QT, and that they even generate a short-lived period of below-trend

[13] The black line is hence identical to the green dotted line in the QT case and the blue dash-dotted line in the tighter interest rate policy case, respectively.

price increases. However, the increase in the short-term rate required to achieve this outcome under the first policy is very large and capital outflows from the small economy turn out not to be much different than in the baseline case.[14] Expressed alternatively, smaller output volatility is not achieved by a reduction of external imbalances, but mainly by a stronger decrease in absorption, the side effect of which is lower inflation.[15] FX interventions, in turn, are effective at preventing the outflow of capital from the small economy as they increase the home term premium such that rates of return on domestic and foreign long-term bonds are equalized without sharp depreciations.

Overall, and in light of our findings in Figures 6 - 9, and specifically that QT is associated with larger depreciationary pressure on the EM exchange rate, it comes as no surprise that spillovers from QT are more marked under a fixed exchange rate regime than under the baseline policy. And while the adverse spillover effects of tighter interest rate policy are also higher when foreign exchange interventions are used, there is notably less amplification in that case. If follows that if the EM central bank has ample FX reserves and is able to curb capital outflow pressures through sizeable FX interventions, then it should pursue that path, as it creates more policy space. By and large, however, the results in Figure 10 are a stark reminder of the potential costs associated with a fixed exchange rate regime when the economy is exposed to large asymmetric shocks.

8. Conclusion

Our analysis shows that the international transmission of conventional and unconventional monetary contractions is not isomorphic, and that they engender different reactions in neighboring economies. Our analysis of the propagation of both types of policies also shows that spillover size can vary markedly depending on the monetary framework of the "recipient" economy, highlighting some scope for stabilizing foreign exchange interventions We quantify the effects of the two types of monetary contractions on both advanced and emerging market economies, and we study the relative importance of transmission via the exchange rate vis a vis term premia. Our results emphasize international benefits of conventional tightening, and they are a stark reminder of sizeable costs associated with exchange rate stabilization in the face of large and asymmetric shocks

[14] This is because the required increase in the short-term rate decreases household spending (in the small economy), which decreases import demand.

[15] The limited efficacy of the short-term rate in affecting capital inflows is not very surprising, given that it does not operate through the term premium.

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Appendices

Appendix A. The role of asset market segmentation

This Appendix briefly discusses the role of asset market segmentation, both across borders as well as maturities. The role of the latter, i.e., of preventing restricted households from trading in short-term bonds and subjecting unrestricted ones to portfolio adjustment costs has been extensively discussed by Chen et al. (2012) and Kolasa and Wesołowski (2020). In essence, limiting arbitrage between short- and long-term bonds results in fluctuations in term premia that have effects on real activity. Expressed alternatively, without the trading restrictions certainty equivalence would make short and long-term bonds perfectly substitutable, with restricted households then able to circumvent portfolio adjustment costs.

The international aspect, first introduced and studied in Kolasa and Wesołowski (2020), is arguably more important as it allows our model to match stylized facts on cross-border aspects of segmentation. Here, the key assumption is that unrestricted households cannot trade short-term bonds issued abroad. This, however, is broadly in line with the IMFs Coordinated Portfolio Investment Survey, which shows that short-term debt securities have a fairly stable market share typically oscilating below 5%, with foreign debt assets held by small open economies even less quantitatively relevant. In addition, restricting asset trade at the short end of the yield curve prevents counterfactual surges into the small economy, and generally helps the model generate realistic magnitudes of capital flows.

Appendix B. Term Premium and Duration

Typically, the 10Y term premium denotes the difference between the yield on a 10 year bond and the expected yield on a series of short term bonds. In a world with no costs, there would be no term premium, meaning that the yield on long term bonds would have to equal the expected yield of investing short term. Since the expected yield on the hypothetical long-term bond would equal the expected return on short term bonds, therefore we can define the term premium as

$$TP_t = R_{L,t} - R^{EH}_{L,t} \,,$$

where $R_{L,t}^{EH}$ is the counterfactual yield to maturity on a longer-term bond in the absence of transaction costs.

Duration is, by definition, equal to

$$D \equiv \frac{\sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s}}{P}$$

where R is the yield to maturity, and where, for a consol $R - \kappa = \frac{1}{P}$. Substituting the former into the numerator we end up with

$$D \equiv (R - \kappa) \sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s}.$$

Since

$$\sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s} = \frac{1}{R} \sum_{s=1}^{+\infty} s \left(\frac{\kappa}{R}\right)^{s-1},$$

therefore, letting $x \equiv \kappa/R$, we obtain

$$\frac{1}{R} \sum_{s=1}^{+\infty} s x^{s-1} = \frac{1}{R} \sum_{s=1}^{+\infty} \frac{d}{dx} x^s = \frac{1}{R} \frac{d}{dx} \sum_{s=1}^{+\infty} x^s = \frac{1}{R} \frac{d}{dx} \frac{x}{1-x} = \frac{1}{R} \frac{1}{(1-x)^2}.$$

Substituting back in for x we arrive at the formula used in the paper, i.e.,

$$D \equiv (R - \kappa) \left(\frac{1}{R} \frac{1}{\left(1 - \frac{\kappa}{R}\right)^2} \right) = (R - \kappa) \frac{R^2}{R (R - \kappa)^2} = \frac{R}{R - \kappa}.$$

Appendix C. The Government and Central Bank Balance Sheets

This section focuses on the interplay of unconventional monetary policy and conventional fiscal policy, both of which potentially entail issuance and purchases of local currency, long- and short-term bonds. In contrast to Kolasa and Wesołowski (2020), who proceed under the assumption of a consolidated government balance sheet, we account for the two policies separately, distinguishing issuance for fiscal purposes – superscript f – from issuance by the central bank – superscript c – and using the superscript g to denote the "consolidated" government fiscal position. This distinction facilitates a discussion of alternative central bank exit strategies, as it helps differentiate effects due to changes in fiscal plans from those induced by quantitative tightening. Finally, since the domestic central bank is assumed not to partake in unconventional monetary policy, therefore the ensuing discussion focuses only on the foreign economy.[16]

Fiscal Policy. The fiscal authority operates subject to the following constraint

$$B^f_{F,t} + P^\star_{L,t} B^f_{F,L,t} + T^\star_t = R^\star_{t-1} B^f_{F,t-1} + P^\star_{L,t} R^\star_{L,t} B^f_{F,L,t-1} + P^\star_t g^\star_t,$$

i.e., it finances its expenditures by issuing long-term $B^f_{F,L,t}$, and short-term $B^f_{F,t}$ bonds as well as through lump sum taxation $T^\star_t$. As the expression makes clear, the total amount requiring financing is a sum of the value of maturing obligations $R^\star_{t-1} B^f_{F,t-1} + P^\star_{L,t} R^\star_{L,t} B^f_{F,L,t-1}$ as well as government expenditures net of taxes $P^\star_t g^\star_t - T^\star_t$, with the total market value of all government debt given by

$$B^f_t \equiv B^f_{F,t} + P^\star_{L,t} B^f_{F,L,t}.$$

Government expenditures on final goods are assumed to follow $g_t \equiv g \exp\{\varepsilon^g_t\}$, where ε^g_t is the government spending shock. Taxes per capita are set equal across the two household types, which implies that they are levied in proportion to the restricted and unrestricted households' population shares ω_r and $1 - \omega_r$, respectively. In addition, in the baseline version of our model, the fiscal authority of the large country is assumed to keep the real market value of debt $b^f_{F,t} \equiv B^f_{F,t}/P^\star_t$ and its composition $\theta^L \equiv \left(P^\star_{F,t} B^f_{F,L,t}\right) / \left(P^\star_{L,t} B^f_{F,L,t} + B^f_{F,t}\right)$ constant.[17]

Unconventional Monetary Policy. The foreign central bank is assumed to be active in foreign asset markets, i.e., it can take positions $\tilde{B}^{c,\star}_{F,t}$ in short term bonds, and $B^{c,\star}_{F,L,t}$ in long term bonds.

[16] The domestic case can thus be thought of as a special case of the foreign one, with the size of unconventional monetary policy set to zero.

[17] In general, one can show that letting

$$\forall t : \frac{B^f_{F,t}}{P^\star_t} \equiv b^{f,\star} \qquad \text{and} \qquad \forall t : \frac{P^\star_{L,t} B^f_{F,L,t}}{P^\star_{L,t} B^f_{F,L,t} + B^f_{F,t}} \equiv \theta^L$$

implies

$$g^\star_t - \tau^\star_t = \frac{B^f_{F,t-1}}{P^\star_t} \left[\Pi^\star_t - \left(\left(1 - \theta^L\right) R^\star_{t-1} + \theta^L \Pi^\star_{L,t} R^\star_{L,t}\right)\right].$$

We thus see that if inflation exceeds the average rate of return on government debt (i.e., if government debt is deflated away) then the fiscal authority can run a primary fiscal deficit and still keep real debt constant. If, however, the average rate of return on government debt exceeds inflation, then the government has got to run a primary surplus to pay-off real interest accrual on its debt.

These positions are entirely financed by issuing central bank commercial paper $\tilde{B}^{cp,\star}_{F,t}$, which constitutes 100% of central bank liabilities, and which – from the perspective of private agents – is indistinguishable from short term government bonds. Accordingly the CB balance sheet is:

Table A.1. Central Bank Balance Sheet

Assets	Liabilities
$\tilde{B}^{c,\star}_{F,t}$	$\tilde{B}^{cp,\star}_{F,t}$
$P_{L,t}B^{c,\star}_{F,L,t}$	

Because of the perfect substitutability between central bank commercial paper and short term treasuries, purchases of the latter only affect the size of the CB balance sheet but not the quantity of short term assets outstanding.[18] For that reason we will only focus on purchases of long term assets and will "net out" shorter term assets, defining

$$B^{c,\star}_{F,t} \equiv \tilde{B}^{c,\star}_{F,t} - \tilde{B}^{cp,\star}_{F,t}.$$

This leads to the simplified CB balance sheet

Table A.2. Consolidated Central Bank Balance Sheet

Assets	Liabilities
$P_{L,t}B^{c,\star}_{F,L,t}$	$B^{c,\star}_{F,t}$

It also accounts for simplified definitions of government debt outstanding

$$B^{g,\star}_{F,t} \equiv B^{f,\star}_{F,t} - B^{c,\star}_{F,t} \qquad\qquad B^{g,\star}_{F,L,t} \equiv B^{f,\star}_{F,L,t} - B^{c,\star}_{F,L,t}.$$

[18] The total quantity of short and long term government securities (i.e., $B^{g,\star}_{F,t}$ and $B^{g,\star}_{F,L,t}$, respectively) equals

$$\begin{aligned} B^{g,\star}_{F,t} &\equiv B^{f,\star}_{F,t} - \tilde{B}^{c,\star}_{F,t} + \tilde{B}^{cp,\star}_{F,t} \\ B^{g,\star}_{F,L,t} &\equiv B^{f,\star}_{F,L,t} - B^{c,\star}_{F,L,t}. \end{aligned}$$

The first of these equations immediately implies that purchases of short term assets $\tilde{B}^{c,\star}_{F,t}$ financed by issuing commercial paper $\tilde{B}^{cp,\star}_{F,t}$ leave government supplied short term bonds $B^{g,\star}_{F,t}$ unchanged.

Importantly, because $B^{c,\star}_{F,t} = P_{L,t} B^{c,\star}_{F,L,t}$ therefore we shall refer to $P_{L,t} B^{c,\star}_{F,L,t}$ as the size of QE.[19] Relatedly, the holding period profits associated with unconventional monetary policy equal[20]

$$\Phi^{c,\star}_t \equiv R^\star_{t-1} B^{c,\star}_{H,t-1} + P^\star_{L,t} R^\star_{L,t} B^{c,\star}_{H,L,t-1}.$$

To keep matters simple, we assume that any QE "carry costs" are fully rebated to the treasury and that losses are rebated lump sum as well. As a consequence, the central bank budget constraint becomes

$$P^\star_{L,t} B^{c,\star}_{F,L,t} + B^{c,\star}_{F,t} = R^\star_{t-1} B^{c,\star}_{H,t-1} + P^\star_{L,t} R^\star_{L,t} B^{c,\star}_{H,L,t-1} - \Phi^{c,\star}_t,$$

or alternatively

$$P^\star_{L,t} B^{c,\star}_{F,L,t} + B^{c,\star}_{F,t} = 0,$$

which essentially states that the central bank starts every period with a "clean" balance sheet.

Runoff, Revaluation and Reinvestment. We now formally define runoff and reinvestment, the first or which describes the mechanical phenomenon of assets maturing and leaving the central bank balance sheet, while the second essentially pins down the baseline investment strategy of the central bank that we shall analyze deviations from.

In our model, if the central bank purchased $P^\star_{L,t-1} B^{c,\star}_{F,L,t-1}$ worth of consols, then at the start of the following period it would have a coupon worth $B^{c,\star}_{F,L,t-1}$ and a stock of assets with a market value of $\kappa P^\star_{L,t} B^{c,\star}_{F,L,t-1}$.[21] Mechanically, the change in value of long term assets Ψ_t thus equals

$$\Psi_t \equiv \kappa^\star P^\star_{L,t} B^{c,\star}_{F,L,t-1} - P^\star_{L,t-1} B^{c,\star}_{F,L,t-1},$$

[19] Expressed alternatively, the model will be mute on the maturity composition of CB assets, because, in principle, we could have netted out any quantity of short term treasuries. In addition, since the central bank is assumed to purchase long-term assets when intervening, therefore the size and maturity composition of government debt determines the maximum size of unconventional stimulus. While interesting, the potential implications of such considerations for optimal issuance policies are beyond the scope of our paper.

[20] Since $\forall t : B^{c,\star}_{F,L,t} \geq 0 \Rightarrow \forall t : B^{c,\star}_{H,t-1} \leq 0$, i.e., if $\Phi^{c,\star}_t$ is *negative* then the central bank has made operating losses.

[21] A potentially helpful way of thinking about the consol is as a portfolio of zero coupon bonds with exponentially decaying nominal face value. Under that interpretation, it becomes clear that run-off would simply be equal to the face value of the first coupon, or $-B^{c,\star}_{F,L,t-1}$.

i.e., it can be written as a combination of runoff and revaluation as follows

$$\underbrace{\Psi_t}_{\text{passive change in portfolio value}} = \underbrace{-B^{c,\star}_{F,L,t-1}}_{\text{runoff}} + \underbrace{\left(\kappa^\star P^\star_{L,t} B^{c,\star}_{F,L,t-1} - P^\star_{L,t-1} B^{c,\star}_{F,L,t-1} + B^{c,\star}_{F,L,t-1}\right)}_{\text{revaluation}},$$

where runoff is defined as being negative, as it tends to decrease the, typically positive, value of long term bonds held by the central bank . Exploiting $R^\star_{L,t} \equiv \left(P^\star_{L,t}\right)^{-1} + \kappa^\star$, we can then simplify the expression for the revaluation component as

$$\begin{aligned}\left(1 + \kappa^\star P^\star_{L,t}\right) B^{c,\star}_{F,L,t-1} - P^\star_{L,t-1} B^{c,\star}_{F,L,t-1} &= P^\star_{L,t} \left(\frac{1}{P^\star_{L,t}} + \kappa^\star\right) B^{c,\star}_{F,L,t-1} - P^\star_{L,t-1} B^{c,\star}_{F,L,t-1} \\ &= \left(P^\star_{L,t} R^\star_{L,t} - P^\star_{L,t-1}\right) B^{c,\star}_{F,L,t-1} = \left(\Pi^\star_{L,t} R^\star_{L,t} - 1\right) P^\star_{L,t-1} B^{c,\star}_{F,L,t-1},\end{aligned}$$

which shows that positive inflation and yield to maturity will translate into positive *nominal* revaluation. Similarly, we can also express run-off in terms of the original value of the long term bond portfolio to arrive at

$$\underbrace{\Psi_t}_{\text{passive change in LT portfolio value}} = \left(\underbrace{-\frac{1}{P^\star_{L,t-1}}}_{\text{runoff}} + \underbrace{\Pi^\star_{L,t} R^\star_{L,t} - 1}_{\text{revaluation}}\right) P^\star_{L,t-1} B^{c,\star}_{F,L,t-1}.$$

Of course, typically, the central bank will also have a reinvestment strategy in place to counterbalance run-off and revaluation, and it may occasionally wish to deviate from that strategy as well. To capture these considerations, yet still keep the analysis tractable, we assume that the passive reinvestment strategy is expressed as a share of runoff and revaluation, and that it is governed by parameter ϱ, i.e., that total reinvestment Θ_t is given by

$$\underbrace{\Theta_t}_{\text{total reinvestment}} \equiv \underbrace{\varrho \left(\frac{1}{P^\star_{L,t-1}} - \Pi^\star_{L,t} R^\star_{L,t} + 1\right) P^\star_{L,t-1} B^{c,\star}_{F,L,t-1}}_{\text{passive reinvestment}} + \underbrace{\epsilon^{c,\star}_t}_{\text{active reinvestment}}.$$

Collecting terms, the expression for the evolution of the value of the central bank's portfolio becomes

$$\begin{aligned} P^{\star}_{L,t} B^{c,\star}_{F,L,t} &= \underbrace{P^{\star}_{L,t-1} B^{c,\star}_{F,L,t-1}}_{\text{previous value}} + \underbrace{\Psi_t}_{\text{mechanical change in value of QE portfolio}} + \underbrace{\Theta_t}_{\text{passive and active reinvestment}} \\ &= \left(1 + (1-\varrho)\left(-\frac{1}{P^{\star}_{L,t-1}} + \Pi^{\star}_{L,t} R^{\star}_{L,t} - 1\right)\right) P^{\star}_{L,t-1} B^{c,\star}_{F,L,t-1} + \epsilon^{c,\star}_t, \end{aligned}$$

which confirms that with ϱ set to one, and absent active reinvestment $\epsilon^{c,\star}_t = 0$, the nominal value of the long term bond portfolio would stay flat.[22] We conclude by presenting a real equivalent of the above expression, which we obtain by dividing through by $P^{\star}_t$ and simplifying to arrive at

$$P^{\star}_{L,t} b^{c,\star}_{F,L,t} = \frac{\left(1 + (1-\varrho)\left(-\frac{1}{P^{\star}_{L,t-1}} + \Pi^{\star}_{L,t} R^{\star}_{L,t} - 1\right)\right)}{\Pi^{\star}_t} P^{\star}_{L,t-1} b^{c,\star}_{F,L,t-1} + \frac{\epsilon^{c,\star}_t}{P^{\star}_t}.$$

This relationship confirms that positive inflation acts to erode the real value of the central banks' portfolio, even if the nominal value of debt outstanding is held fixed (i.e., even if ϱ is equal to one).[23]

Appendix D. Extended Model Equations

For completeness we conclude by outlining how the asset market specification of our model differs from the setup used in Kolasa and Wesołowski (2020). First we note that nothing changes in the home economy, where we continue to assume that the CB does not implement QE, and where we continue not to explicitly model the CB balance sheet (i.e., we only have g variables but no decomposition into f nor c ones). While in the foreign economy we continue to use g for the (consolidated government) bond position, we introduce four additional variables capturing short and long-term fiscal positions $b^{f,\star}_{F,t}$ and $b^{f,\star}_{F,L,t}$ respectively as well as the short and long term central bank positions $b^{c,\star}_{F,t}$ and $b^{c,\star}_{F,L,t}$. For the fiscal side of the economy, we simply copy the equations from the home economy, implicitly assuming that the real size of the budget is constant and that the treasury issues debt targeting a fixed maturity split of assets issued. This immediately gives us

[22] Conversely, with ϱ set to zero, corresponding to no reinvestment, the value of the portfolio would decrease at its fastest possible rate (barring active asset sales).

[23] See also Appendices E and D for a more detailed discussion of the underlying considerations.

two equations for $b_{F,t}^{f,\star}$ and $b_{F,L,t}^{f,\star}$: i) the equation for the debt composition becomes

$$\frac{\frac{b_{F,L,t}^{f,\star}}{R_{L,t}^{\star}-\kappa^{\star}}}{b_{F,t}^{f,\star}+\frac{b_{F,L,t}^{f,\star}}{R_{L,t}^{\star}-\kappa^{\star}}}=\frac{\frac{b_{F,L}^{f,\star}}{R_{L}^{\star}-\kappa^{\star}}}{b_{F}^{f,\star}+\frac{b_{F,L}^{f,\star}}{R_{L}^{\star}-\kappa^{\star}}},$$

while the equation for the size of the fiscal deficit can be written as

$$b_{F,t}^{f,\star}+\frac{b_{F,L,t}^{f,\star}}{R_{L,t}^{\star}-\kappa^{\star}}=b_{F}^{f,\star}+\frac{b_{F,L}^{f,\star}}{R_{L}^{\star}-\kappa^{\star}}.$$

For the central bank balance sheet evolution we also get two equations. The first is the "self-financing" assumption

$$P_{L,t}^{\star}B_{F,L,t}^{c,\star}+B_{F,t}^{c,\star}=0\Longleftrightarrow\frac{b_{F,L,t}^{c,\star}}{R_{L,t}^{\star}-\kappa^{\star}}=-b_{F,t}^{c,\star},$$

while the second is the real, non-linear QT specification discussed in the body of the paper

$$P_{L,t}^{\star}b_{F,L,t}^{c,\star}=\frac{\left(1+(1-\varrho)\left(-\frac{1}{P_{L,t-1}^{\star}}+\Pi_{L,t}^{\star}R_{L,t}^{\star}-1\right)\right)}{\Pi_t^{\star}}P_{L,t-1}^{\star}b_{F,L,t-1}^{c,\star}+\frac{\epsilon_t^{c,\star}}{P_t^{\star}},$$

which can be equivalently rewritten as

$$\frac{b_{F,L,t}^{c,\star}}{R_{L,t}^{\star}-\kappa^{\star}}=\frac{\left(1+(1-\varrho)\left(\kappa^{\star}-R_{L,t-1}^{\star}+\left(\frac{R_{L,t-1}^{\star}-\kappa^{\star}}{R_{L,t}^{\star}-\kappa^{\star}}\right)R_{L,t}^{\star}-1\right)\right)}{\Pi_t^{\star}}\frac{b_{F,L,t-1}^{c,\star}}{R_{L,t-1}^{\star}-\kappa^{\star}}+\varepsilon_t^{c,\star},$$

and where $\varepsilon_t^{c,\star}$ is a real, unconventional monetary policy shock.[24]

Appendix E. The Linear Run-Off Specification

In some circumstances having a linear expression for the run-off on the central bank's portfolio can be instructive. To derive one, we start by defining a variable called $\mathcal{CB}_t^{\star}$ which captures the

[24] For reference, in steady state this implies that

$$\frac{1+(1-\varrho)\left(\kappa^{\star}-R_{L,t-1}^{\star}+\left(\frac{R_{L,t-1}^{\star}-\kappa^{\star}}{R_{L,t}^{\star}-\kappa^{\star}}\right)R_{L,t}^{\star}-1\right)}{\Pi_t^{\star}}=\frac{1+(1-\varrho)\left(\kappa^{\star}-1\right)}{1.005}=\frac{0.982+0.018\varrho}{1.005},$$

which confirms that runoff is efficiently pinned down by long-bond duration.

size of the long-term bond position on the CBs balance sheet, and which follows

$$\mathcal{CB}_t^\star = \frac{(1-(1-\varrho)(\phi_t+\varphi_t))}{\Pi_t^\star}\mathcal{CB}_{t-1}^\star + \varepsilon_t^{c,\star},$$

where $\varepsilon_t^{c,\star} \equiv \epsilon_t^{c,\star}/P_t^\star$ is the real value of the "unscheduled" long-term bond purchases, and where ϕ_t and φ_t denote runoff and revaluation respectively[25]

$$\phi_t \equiv -\frac{1}{P_{L,t-1}^\star}, \qquad \text{and} \qquad \varphi_t \equiv \frac{R_{L,t}^\star}{\Pi_{L,t}^\star} - 1.$$

Linearizing this equation around a steady state with no unconventional monetary policy yields

$$\mathcal{CB}_t^\star \approx \underbrace{\frac{(1-(1-\varrho)(\phi+\varphi))}{\Pi^\star}}\mathcal{CB}_{t-1}^\star + \varepsilon_t^{c,\star},$$

which is essentially an AR(1) specification for the size of the central bank's balance sheet, and where ϕ, φ and $\Pi^\star$ denote steady state runoff, revaluation and inflation, respectively.

Of course, even though ϱ is a "free" policy parameter, ϕ and φ are not, and they can be expressed in terms of structural parameters. Using,

$$R_{L,t}^\star \equiv \frac{1}{P_{L,t}^\star} + \kappa^\star \qquad \text{and} \qquad DUR_L^\star \equiv \frac{R_L^\star}{R_L^\star - \kappa^\star},$$

we have that

$$\phi \equiv -\frac{1}{P_L^\star} - \kappa^\star + \kappa^\star = \frac{-R_L^\star + \kappa^\star}{R_L^\star}R_L^\star = -\frac{R_L^\star - \kappa^\star}{R_L^\star}R_L^\star = -\frac{R_L^\star}{DUR_L^\star},$$

where DUR_L denotes the consol duration, and where φ is given by

$$\varphi \equiv \frac{R_L^\star}{\Pi_L^\star} - 1.$$

[25] Both are expressed as shares of the $t-1$ long term portfolio value.