Greenflation?

Conny Olovsson[*] and David Vestin[†]

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Abstract

This paper examines the hypothesis of "Greenflation". We find that under flexible prices, the relative price adjustment of green and brown energy comes about without consequences for inflation. We extend the analysis to the case of sticky prices and wages and our findings continues to support the notion that a transition to a green economy may progress without too much worry about inflation, at least in the case where the fiscal measures are introduced in an orderly and well planned fashion.

Keywords: Inflation, green transition, monetary policy, climate change

JEL: E52; E58; Q43

[*]Olovsson: Sveriges Riksbank and ECB.

[†]Vestin: Sveriges Riksbank; the opinions expressed in this article are the sole responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank or the ECB.

1 Introduction

The temperature of our planet is gradually increasing and a growing consensus amongst climate researchers conclude that a big reduction of carbon dioxide emissions into the atmosphere is necessary in order to prevent dramatic effects on human activity (see e.g. IPCC, 2022). These insights has gradually begun to affect policy-making around the world, for example the European Union has adopted a "Fit for 55" program with the ambitious goal of zero net emissions in 2040. To achieve this, the relative price of "brown" energy, relative to "green" energy needs to increase.

In the central banking community, especially in the current environment of surging inflation at least partly due to supply-shocks to the energy sector, there is some unease about the inflationary consequences of the transition to a sustainable energy production.

The question is how monetary policy should, or should not, accommodate the transition, to which extent increasing energy prices will trigger a more broad increase in inflation and so forth.

This paper sets out to examine these questions in a New-Keynesian model extended with energy production where both green and brown energy is included. We consider a region such as the European Union and consider a green transition that will result from a policy similar to the Fit for 55 package, with the difference that, instead of modelling a quantity restriction, we focus on a carbon tax that is raised to the point where its use is substantially reduced.[1] We treat this tax, much like other fiscal policy measures, that monetary policy takes as exogenous but needs to adapt to due to the potential consequences for inflation. We model an externality by which current usage of brown energy will inflict a negative effect on future productivity, such that potential output becomes endogenous. The price of energy is flexible, whereas prices of other goods as well as wages may be sticky, to allow discussion of implications of transitory effects during the adjustment path.

In this environment, we conduct a number of experiments. First, we introduce a path for the adjustment of the tax in the flexible price version of the model and discuss the rather obvious result that in such a world, there are no inflationary consequences of a relative price adjustment. The reason is, of course, that in such a world there are no real effects of nominal variables and hence the inflation outcome is a purely nominal effect that depends only on monetary policy. If the latter targets inflation sufficiently strictly, the relative price adjustment of brown

[1] A quantity restriction and a carbon tax can theoretically be very similar.

energy comes about while meeting the inflation target. The price increases of brown energy is substantially above the inflation target and price increase of all other goods a bit below the inflation target.[2] [Alternatively, if monetary policy targets CPI excluding energy, the relative price change is achieved through an even larger increase in the price of brown energy which the central bank ignores, and all other prices in line with the inflation target.]

Next, we introduce nominal rigidities in the form of sticky prices and wages. In this case monetary policy will have real effects, and we will see how fighting the positive or negative inflation effects arising will come at a cost in terms of real variables.

But in our calibration, we find that these effects are quite limited, under the crucial assumption that the plan for the tax rate is announced well in advance of its implementation. We show that if instead, the tax-path is steep and implies almost immediate lift-off after it's announcement, the nominal as well as the real effects are pointedly stronger. We conclude that from a theoretical perspective it may well be possible to see a greening of the economy without too much fear of Greenflation. These theoretical results are in line with the empirical findings in Konradt and Weder di Mauro (2022), showing only limited effects on inflation from green policies.[3]

We proceed as follows. Section 2 introduces the RBC core of the model, including a discussion of how energy is introduced. Section 3 introduces sticky prices and wages, Section 4 discusses results and Section 5 concludes.

2 A model with sticky wages and prices

We now set up a relatively standard New-Keynesian model with two energy inputs and a representation of climate change. The basic framework builds upon Woodford (2003) and Gali (2015), whereas the energy sector and the climate module is taken from hassler, Krusell, and Olovsson (2020, 2021).

To understand our benchmark observation, that relative price adjustments may come about at various rates of aggregate inflation and that monetary policy will be instrumental in picking one of these possible equilibria, we will also consider a simple version of the New-Keynesian model that collapses to a RBC model with

[2] These results are in line with the findings in Ayoki (2001) that price changes in sectors with flexible prices can be ignored by the central bank.

[3] Airaudo, F, Pappa, Evi, and Hernan S. (2022) finds a relatively strong impact on inflation in the beginning of the transition, which could come from the fact that there is no announcement before the transition starts.

flexible nominal prices. Due to the absence of nominal rigidities, inflation and monetary policy will in this benchmark model be inconsequential to allocation and welfare, but nevertheless proves to be a useful starting point to understand the basic result.

2.1 Households

We consider an economy with a continuum of labor-types indexed by j.[4] The different types are organized in unions that sets wages. Given the wage, the respective types have to satisfy demand, which implies that the number of hours worked will generally not be optimal for each of the types[5] We further assume complete markets on the consumer side, which allows the income risk to be perfectly shared and consumption levels to be equalized. But the number of hours worked will not be identical and hence we will see fluctuations in the marginal disutility of labor. The agent derives utility from consumption, C, and disutility from providing labor. Denoting the amount of hours worked of a type j by $H_t(j)$, preferences for the household are a given by

$$\sum_{t=0}^{\infty} \beta^t \log(C_t) - \int_0^1 \varkappa \frac{H_t(j)^{1+\varphi}}{1+\varphi} dj \tag{1}$$

where β is the discount factor, $\varkappa$ and φ respectively determines the disutility of labor and the Frisch elasticity. Consumption is then an aggregate of a manufactured good—that is labelled C_g—and energy services—labelled C_e. Formally, we have

$$C_t = \left((1-\nu)^{\frac{1}{\eta}} C_{g,t}^{\frac{\eta-1}{\eta}} + \nu^{\frac{1}{\eta}} C_{e,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{2}$$

The price of the manufactured and the energy good is respectively denoted by $P_{g,t}$ and $P_{e,t}$, whereas the price of the aggregate consumption basket, C, is P. The budget constraint for the consumer is then given by

$$P_t C_t + B_t = \int_0^1 W_t(j) H_t(j) dj + \pi_t + I_{t-1} B_{t-1} + T_t, \tag{3}$$

where π_t is profits, W is the nominal wage rate, I_{t-1} is the gross interest rate paid in period t on funds B_{t-1} invested at time $t-1$, and T denotes the nominal tax

[4] Our modelling of sticky wages follows Erceg, Henderson, and Levin (2000).

[5] Just like the demand-determined output of the firm is sub-optimal in a given period when the price is fixed.

revenues that are rebated to the consumer.

2.2 Firms

Labor is used for three activities: the production of goods, fossil fuel, E_1, and green energy, E_2, where we use subscript 1 and 2 to respectively denote the fossil and the green sector. Because we want to allow for sticky wages, we assume that the labor employed by firm i consists of a of an aggregate (bundle) of different types of labor, indexed by j. The total labor supply in the economy is then given by

$$H_t = \left(\int_0^1 H_t\left(j\right)^{\frac{\xi_w - 1}{\xi_w}} dj \right)^{\frac{\xi_w}{\xi_w - 1}},$$

where $H_t\left(j\right)$ refers to the amount of hours worked by labor type j, and ξ_w is the elasticity of substitution between different labor types. Production functions in all activities are linear in labor. It is then straightforward to show that cost minimization by the firms will imply that the demand for the different types will satisfy

$$H_t\left(j\right) = \left(\frac{W_t\left(j\right)}{W_t} \right)^{-\xi_w} H_t,$$

where $W_t \equiv \left[\int_0^1 W_t\left(j\right)^{1-\xi_w} dj \right]^{\frac{1}{1-\xi_w}}$ is the aggregate wage.

Energy services, $C_{e,t}$, are then produced by a competitive representative firm that combines the two energy sources as inputs, but this activity does not require any labor. In line with the data, we assume, the prices of energy inputs and energy services to be fully flexible, but that the price of the non-energy good, good g is sticky.

2.2.1 The goods sector

Sector g is assumed to consist of a continuum of price setting firms that are optimizing under monopolistic competition. The price of individual good i is then $P_t\left(i\right)$. The production function for goods is linear in labour, i.e.,

$$Y_{g,t}\left(i\right) = A_{g,t} H_{g,t}\left(i\right), \tag{4}$$

where $H_{g,t}\left(i\right)$ is the labor demanded by firm i (not to be confused with that is the labor supplied by labor type j).

There also exists a perfectly competitive firm that buys the individual goods and package them to produce a manufactured final good, using the following aggregator

$$Y_{g,t} = \left(\int_0^1 \left(Y_{g,t}(i)^{\frac{\xi-1}{\xi}} di \right) \right)^{\frac{\xi}{\xi-1}}. \tag{5}$$

The cost minimization problem for the packaging firm delivers the individual demand functions for goods of type i, i.e.,

$$Y_{g,t}(i) = \left(\frac{P_{g,t}(i)}{P_{g,t}} \right)^{-\xi} Y_{g,t}, \tag{6}$$

as well as the price index for good g:

$$P_{g,t} = \left(\int_0^1 \left(P_{g,t}(i)^{1-\xi} di \right) \right)^{\frac{1}{1-\xi}}. \tag{7}$$

We assume Calvo pricing, which implies that the probability that a firm is allowed to change its price in each period is $1-\theta$. Since all firms that are allowed to change their price, will choose the same price, the maximization problem for firm i in period t is given by

$$\max_{P_{g,t}^*} \mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(P_{g,t}^* Y_{g,t+j}(i) - Tc_{t+j}\left(Y_{g,t+j}(i) \right) \right), \tag{8}$$

s.t.

$$Y_{g,t+j}(i) = \left(\frac{P_{g,t}^*}{P_{g,t+j}} \right)^{-\xi} Y_{g,t+j}, \tag{9}$$

and where where Tc_{t+j} is the total cost of production in period $t+j$.

2.2.2 Energy production and supply

The energy sector features two types of firms: those that produce the different energy inputs and the energy-service provider that buys these inputs and use them as inputs to produce and supply energy services, $C_{e,t}$. Both energy inputs are produced using linear technologies. Price of energy inputs and energy services are assumed to be fully flexible, and only fossil-fuel production is subject to taxation. The profit-maximization problem in the fossil, and the green sector is then, respectively, given by

$$\pi_{1,t} = \max_{H_{e,t}} (1-\tau_t) P_{1,t} A_{1,t} H_{1,t} - W_t H_{1,t}, \tag{10}$$

and

$$\pi_{2,t} = \max_{H_{e,t}} P_{2,t} A_{2,t} H_{2,t} - W_t H_{2,t}, \tag{11}$$

where τ denotes the tax on fossil fuel production (and $H_{1,t}$ and $H_{2,t}$ respectively, denote the labor demand by energy producer 1 and 2).

Energy services, $C_{e,t}$, are then produced by a competitive representative firm that combines the two energy sources as inputs using the following aggregator.

$$C_{e,t} = \left((1-\lambda)^{\frac{1}{\varepsilon}} E_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} E_{2,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{12}$$

The cost-minimization problem for the energy service provider is given by

$$\min_{E_{1,t},E_{2,t}} P_{1,t} E_{1,t} + P_{2,t} E_{2,t} - P_{e,t} \left[C_{e,t} - \left((1-\lambda)^{\frac{1}{\varepsilon}} E_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} E_{2,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right]. \tag{13}$$

2.3 Unions

The union is setting the wage that maximizes the expected utility of its members. Since the wage only is allowed to change with a certain probability, the intertemporal problem becomes

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\log(C_{t+k}) - \int_0^1 \frac{H_{t+k|t}(j)^{1+\varphi}}{1+\varphi} dj \right) \tag{14}$$

s.t.

$$P_{t+k} C_{t+k} + B_{t+k} = \int_0^1 W_t^*(j) H_{t+k}(j)\, dj + \pi_{t+k} + I_{t+k-1} B_{t+k-1} + T_{t+k},$$

and

$$H_{t+k|t}(j) = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\xi_w} H_{t+k},$$

and where $1-\theta_w$ is the probability that the wage for type j can be changed its price in each period.

2.4 Monetary policy

We start by considering a monetary policy rule of the form

$$I_t = \beta^{-1} \left(\frac{\Pi_t}{\Pi^*} \right)^{\alpha_\pi}, \tag{15}$$

where Π^* is the gross inflation target, and

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \tag{16}$$

is the inflation rate. This is the simplest possible Taylor-rule without response to the output-gap, which makes perfect sense since due to the assumption about flexible prices output will always equal potential.

2.5 Climate change

The use of fossil fuel generates carbon dioxide emissions that produce climate damages. This is a pure externality that is not internalized by the market. The stock of carbon dioxide in the atmosphere, S, follows the process

$$S_{t+1} = (1 - \delta_s) \sum_{j=0}^{\infty} \left(E_{1,t-j} + E_{1,t-j}^{ROW} \right),$$

where δ_s is the depreciation of carbon dioxide from the atmosphere and $E_{1,t}^{ROW}$ denotes the exogenous emissions from the rest of the world. Following Golosov et al. (2014), the damages are modelled as reducing the total factor productivity in each production sector, i.e.,

$$A_{g,t} = \overline{A}_{g,t} \exp\left(-\gamma S_t\right), \tag{17}$$

$$A_{1,t} = \overline{A}_{1,t} \exp\left(-\gamma S_t\right), \tag{18}$$

$$A_{2,t} = \overline{A}_{2,t} \exp\left(-\gamma S_t\right), \tag{19}$$

here $\overline{A}$, $\overline{A}_1$, and $\overline{A}_2$ denote the total factor productivities that would prevail without any climate damages, and γ determines the size of the externality. Note that we assume the same damages in all sectors.

2.6 Equilibrium

We now describe the equilibrium conditions.

2.6.1 Households

All derivations of the optimality conditions are displayed in the Appendix. Using the definitions $p_{g,t} \equiv P_{g,t}/P_t$, $p_{e,t} \equiv P_{e,t}/P_t$, the maximization problem for the consumer can be written as follows.

$$\begin{aligned} C_{g,t} &= (1-\nu)\, p_{g,t}^{-\eta} C_t, &(20)\\ C_{e,t} &= \nu p_{e,t}^{-\eta} C_t, &(21)\\ \frac{1}{C_t} &= \beta \mathbb{E}_t \left(\frac{1}{C_{t+1}} \frac{I_t}{\Pi_{t+1}} \right), &(22)\\ P_t &= \left[(1-\nu)\, P_{g,t}^{1-\eta} + \nu P_{e,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. &(23) \end{aligned}$$

where (22) is the Euler equation and (23) gives the aggregate price level, i.e., the CPI. The optimality conditions for fossil and green fuel are respectively given by

$$E_{1,t} = (1-\lambda) \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1-\tau_t} \frac{1}{h_t} \right)^{-\varepsilon} C_{e,t}, \tag{24}$$

and

$$E_{2,t} = \lambda \left(\frac{1}{h_{1,t}} \right)^{-\varepsilon} C_{e,t}, \tag{25}$$

where $h_t \equiv \left[(1-\lambda) \left(\frac{A_{2,t}}{A_{1,t}} \frac{1}{1-\tau_t} \right)^{1-\varepsilon} + \lambda \right]^{\frac{1}{1-\varepsilon}}$.

2.6.2 Firms

The first order condition to (8) can, after some manipulation, be written as

$$p_{g,t} = \frac{\xi}{\xi-1} \left[\frac{1-\theta \Pi_{g,t}^{\xi-1}}{1-\theta} \right]^{\frac{1}{\xi-1}} \frac{N_{2,t}}{N_{1,t}}, \tag{26}$$

where $N_{1,t}$ and $N_{2,t}$ respectively are given by

$$N_{1,t} = U'(C_t)\, Y_{g,t} + \beta \theta E_t \Pi_{t+1}^{-1} \left(\Pi_{t+1}^{g} \right)^{\xi} N_{1,t+1}, \tag{27}$$

$$N_{2,t} = U'(C_t)\, \widehat{W}_t \frac{Y_{g,t}}{A_{g,t}} + \beta \theta E_t \left(\Pi_{t+1}^{g} \right)^{\xi} N_{2,t+1}, \tag{28}$$

and where $\widehat{W}_t \equiv W_t / P_t$ is the real wage. We consistently use hats to refer to real variables, i.e., $\widehat{X}_t \equiv X_t / P_t$.

2.6.3 Unions

The solution to the union's problem (14) is

$$\widehat{W}_t^* = \left[\frac{\xi_w}{\xi_w - 1}\frac{M_{1,t}}{M_{2,t}}\right]^{\frac{1}{1+\xi_w\varphi}}, \tag{29}$$

where $M_{1,t}$ and $M_{2,t}$ respectively are given by

$$M_{1,t} = \left(\widehat{W}_t^{\xi_w} H_t\right)^{1+\varphi} + \beta\theta_w E_t \Pi_{t+1}^{\xi_w(1+\varphi)} M_{1,t+1}, \tag{30}$$

$$M_{2,t} = \frac{1}{C_t}\widehat{W}_t^{\xi_w} H_t + \beta\theta_w E_t \Pi_{t+1}^{\xi_w-1} M_{2,t+1}. \tag{31}$$

The real wage is then given by

$$\widehat{W}_t = \left[(1-\theta_w)\left(\widehat{W}_t^*\right)^{1-\xi_w} + \theta_w \Pi_t^{\xi_w-1}\left(\widehat{W}_{t-1}\right)^{1-\xi_w}\right]^{\frac{1}{1-\xi_w}}. \tag{32}$$

2.6.4 Market clearing and prices

Labor market clearing requires that the demand for labor equals the supply, i.e.,

$$H_{g,t} + H_{1,t} + H_{2,t} = H_t. \tag{33}$$

Using the first order condition to (11) in combination with (53), the marginal cost of labor, $\widehat{W}_t$, has to equal the marginal product of labor in the green sector, i.e.,

$$\widehat{W}_t = A_{2,t} p_{e,t} \frac{1}{h_t}. \tag{34}$$

Rewriting (23), we get the relation between the relative prices of energy and non-energy prices implicitly defined by

$$\left[(1-\nu) p_{g,t}^{1-\eta} + \nu p_{e,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} = 1. \tag{35}$$

The budget constraint written in real terms becomes

$$C_t + \widehat{B}_t = \widehat{W}_t H_t + \widehat{\pi}_{g,t} + \frac{I_{t-1}}{\Pi_t}\widehat{B}_{t-1} + \widehat{T}_t, \tag{36}$$

where

$$\widehat{T}_t \equiv \frac{T_t}{P_t} = \frac{A_{2,t}}{h_t} p_{e,t} H_{1,t} \frac{\tau_t}{1-\tau_t} - \frac{I_{t-1}}{\Pi_t}\widehat{B}_{t-1}, \tag{37}$$

and $\widehat{\pi}_{g,t} = Y_{g,t}\left(\frac{P_{g,t}}{P_t} - \frac{W_t}{P_t A_{g,t}}\right)$. The first term in (37) is the tax revenues from taxing the fossil fuel and the second term is the payments to $\widehat{B}$. The supply of goods, fossil and green fuel must all equal the demand for these goods. We have

$$\underbrace{\frac{A_{g,t}H_{g,t}}{D_{g,t}}}_{Y_{g,t}} = C_{g,t} \tag{38}$$

$$\underbrace{A_{1,t}H_{1,t}}_{Y_{1,t}} = E_{1,t} \tag{39}$$

and

$$\underbrace{A_{2,t}H_{2,t}}_{Y_{2,t}} = E_{2,t}, \tag{40}$$

where

$$D_{g,t} = (1-\theta)\left[\frac{1-\theta\left(\Pi_{g,t}\right)^{\xi-1}}{1-\theta}\right]^{\frac{\xi}{\xi-1}} + \theta\Pi_{g,t}^{\xi}D_{g,t-1}. \tag{41}$$

Finally, the inflation rate for goods and energy services can respectively be written as

$$\Pi_{g,t} = \Pi_t \frac{P_{g,t}}{P_t}\frac{P_{t-1}}{P_{g,t-1}} \tag{42}$$

and

$$\Pi_{e,t} = \Pi_t \frac{P_{e,t}}{P_t}\frac{P_{t-1}}{P_{e,t-1}}. \tag{43}$$

The equations that define the equilibrium are (15), (17)-(19), (20)-(22), (24)–(43), and the unknowns are $A_{g,t}$, $A_{1,t}$, $A_{2,t}$, $H_{g,t}$, $H_{1,t}$, $H_{2,t}$, H_t, C_t, $C_{g,t}$, $C_{e,t}$, $E_{1,t}$, $E_{2,t}$, $\widehat{B}_t$, $N_{1,t}$, $N_{2,t}$, $M_{1,t}$, $M_{2,t}$, D_t, $p_{g,t}$, $p_{e,t}$, Π_t, $\Pi_{g,t}$, $\Pi_{e,t}$, I_t, $\widehat{W}_t^*$, $\widehat{W}_t$, and $\widehat{T}_t$. Note that the aggregate price level is indeterminate, but that the change in the level is determined.

2.7 Calibration

The model is now calibrated. A period is one quarter. There are five preference parameters: β, $\varkappa$, φ, ν, and η. The discount factor β is set to 0.995 to generate a real steady-state annual interest rate of about two percent. Parameter φ determines the disutiity of labor and it is set so that the labor supply, in steady state about 1/3. We then set $\varphi = 3$ to generate a Frisch elasticity of about 0.5. The weight on energy in the consumption bundle, ν, is set to 0.05 to match that about five percent of the total income is spent on energy. The parameter η is the elasticity of substitution (EOS) between energy services and other goods and we set it to 0.2 to match the low substitution elasticity found in Hassler, Krusell, and Olovsson (2021).

Parameters ε and λ determines the elasticity of substitution between fossil and green energy in the energy bundle. These parameters are respectively set to 1.5 and 0.36, which is roughly in line with the values in Hassler, Krusell, and Olovsson (2021).

We consider different values for the price and wage rigidities. However, we follow Gali (2015) and set $\theta = 0.75$ for the case with sticky prices and $\theta_w = 0.75$ for sticky wages. The EOS between, ξ, firms output varieties is set to 10, which generates a steady state markup of about 11 percent, which is similar to in Gali (2015). The EOS between ξ_w labor types is set to 2.

Turning to production parameters, we abstract from technical change and set $\overline{A}_t \equiv \overline{A} = 1$, and $\overline{A}_{e,t} \equiv \overline{A_e} = 1$. As a starting point, we set γ to zero and abstract from the direct effects of climate change. This also implies that the considered region cannot improve welfare by increasing the carbon tax, which is quantitatively correct when the region is taken to be the EU.[6]

Finally, for monetary policy we consider several different values for α_{pi}, but use 1.5 as a benchmark value.

2.8 The considered experiments

We think of the region as the European Union and consider a green transition that will result from the Fit for 55 policy proposal. Specifically, this proposal dictates how the EU ETS cap and trade system will evolve over the coming decades. The reason for why we consider this policy rather than an optimal fiscal policy is that we are interested in the effects of policies that actually are considered to be implemented rather than theoretical constructs. In addition, global optimallity requires all countries to have the same carbon tax. If all regions but one chooses to not implement the gloablly optimal policy, then it is individually optimal also for the last country to not implement the policy. Finally, the Fit for 55 package is likely not too far from optimal if it were to be implemented globally.

A cap and trade system such as the EU ETS, places a limit on the quantity of aggregate emissions that are allowed from the region. The companies can then trade emission rights within the area. The Fit for 55 package states that the

[6] See Hassler, Krusell, and Olovsson (2020) that shows that the global laissez fair is basically quantitatively the same when only the EU implements a carbon taxx.

newly emitted emission rights are linearly reduced up to 2040, where no additional emission rights will be emitted. We will instead consider a carbon tax that is linearly increased up to 2040, after which it remains constant. A carbon tax is, in all important aspects that we are concerned with, virtually identical to a cap and trade system. We consider two cases, one where the transition is announced 5 years in advance and one where the transition starts immediately.

It is important to note that, a green transition that is coming from a carbon tax or a quantity restriction will have real effects. Because the externality is global, a tax or quantity restriction in just one relatively small region such as the EU will only have a marginal effect on the externality, but the policy will increase the real costs of producing and consuming fossil energy. This implies that the climate policy will reduce consumption and output in the implementing region. Only if all (or most of the) regions implement the carbon tax will consumption and output increase. These implications are important for trends for consumption and output as we will see in the next section.

The transition between the steady states are solved with the "Perfect foresight solver" in Dynare, which implies that the transition path is not linearized.

3 Results

3.1 Case 1: Flexible prices

To understand the main results, we start by considering a simple RBC model with flexible prices and where the transition is announced 5 years in advance. In this setting, monetary policy has no allocative effects, but we can still study both the real and the nominal effects of the the green transition. Specifically, if we assume that all labor types and firms are identical so that they are perfect substituted, then the utility function becomes $\sum_{t=0}^{\infty} \beta^t \log(C_t) - \varphi \frac{H_t^{1+\varphi}}{1+\varphi}$, and the the production function for goods becomes $Y_{g,t} = A_{g,t} H_{g,t}$. In this case, the model collapses to a RBC model with flexible prices that can be solved in closed form. The price level is indeterminate, but the inflation rates are not.[7]

The results are presented in figures (1)-(2).

We first note that ïf the central bank follows a Taylor rule and prices are flexible, then the green transition is marginally deflationary. Specifically, inflation on goods falls whereas energy inflation increases substantially during the transition, but the effect on overall inflation—the CPI—is negative but quantitatively small.

[7] The model is solved in the Appendix.

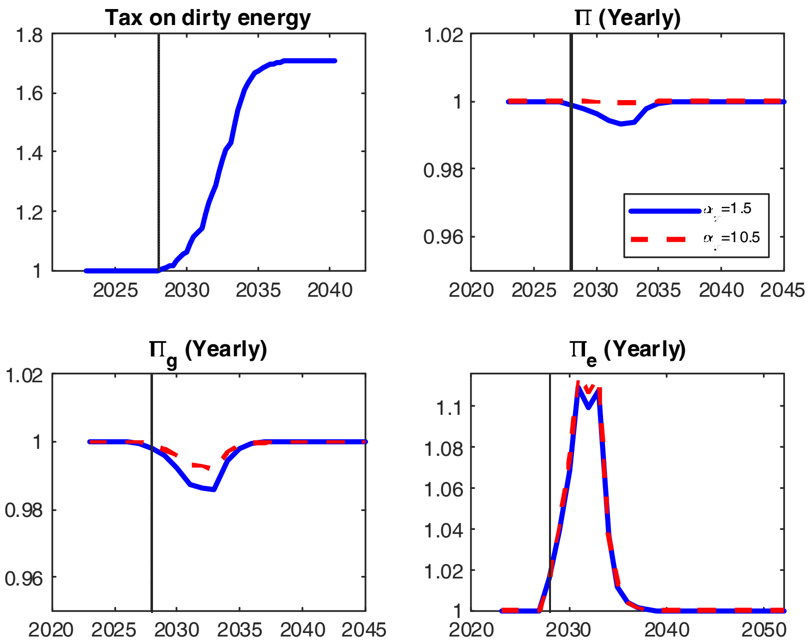

Figure 1: The effects of green policy (the carbon tax) on the economy when there are no nominal rigidities. The transition is announced 5 years in advance, and the vertical line denotes the first increase in the tax rate.

The intuition for this result comes from that the introduction of the tax gives a negative trend in consumption during the transition as explained above and as can be seen in Figure (2). This reduction in consumption translates into an increasing marginal utility of consumption along the transition path. In our perfect foresight experiment, the ratio $U'(C_t)/U'(C_{t+1})$ is declining. The real interest rate can be derived from the Euler equation $R_{t+1} = (1/\beta) E_t [U'(C_t)/U'(C_{t+1})]$ and it will, thus, be lower during the transition relative to before, which also is plotted in Figure (2). Once the tax rate reaches its new level, of course, the real interest rate reverts back to the inverse of the discount factor. Turning now to the implications for nominal variables, the nominal rate is linked to the real rate through a Fisher equation that in our perfect foresight experiment, simplifies to[8]

$$R_t = \frac{I_t}{\Pi_{t+1}}.$$

[8] Equation (**??**) can be derived by combining the real Euler equation with its nominal counterpart (22) to get $R_t = E_t \frac{\frac{U'(C_{t+1})}{\Pi_{t+1}}}{(U'(C_{t+1}))} I_t$, and then noting that with perfect foresight the expectation term can be dropped.

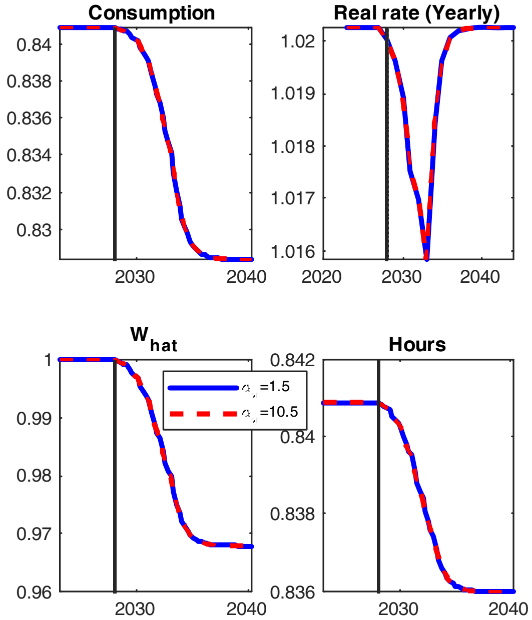

Figure 2: The effects of green policy (the carbon tax) on the economy when there are no nominal rigidities. The transition is announced 5 years in advance, and the vertical line denotes the first increase in the tax rate.

When monetary policy follows a Taylor-type rule as in (15), we get

$$R_t = \frac{\beta^{-1}\Pi_t^{\alpha_\pi}}{\Pi_{t+1}},$$

or

$$\Pi_t = \prod_{j=0}^{\infty} (\beta R_{t+j})^{\frac{1}{\alpha_\pi^j}} \tag{44}$$

Since all the entries in the sum in (44) will be below or equal to its pre-transitional values, it follows that there is a tendency for inflation to decline. However, we also note that raising the response of the nominal interest rate to inflation, i.e., α_π, this effect may be reduced. In fact, with a high enough α_π inflation can be made arbitrarily small. In this simple RBC model with flexible prices, this has no cost but that may change when prices and wages are sticky. We now turn to the model with nominal rigidities. As mentioned above, if all regions were to implement the carbon tax, then the consumption path would be increasing rather than decreasing. This would deliver greenflation, i.e., the CPI would be positive instead of negative but, again, this effect could be completely offset by a tight

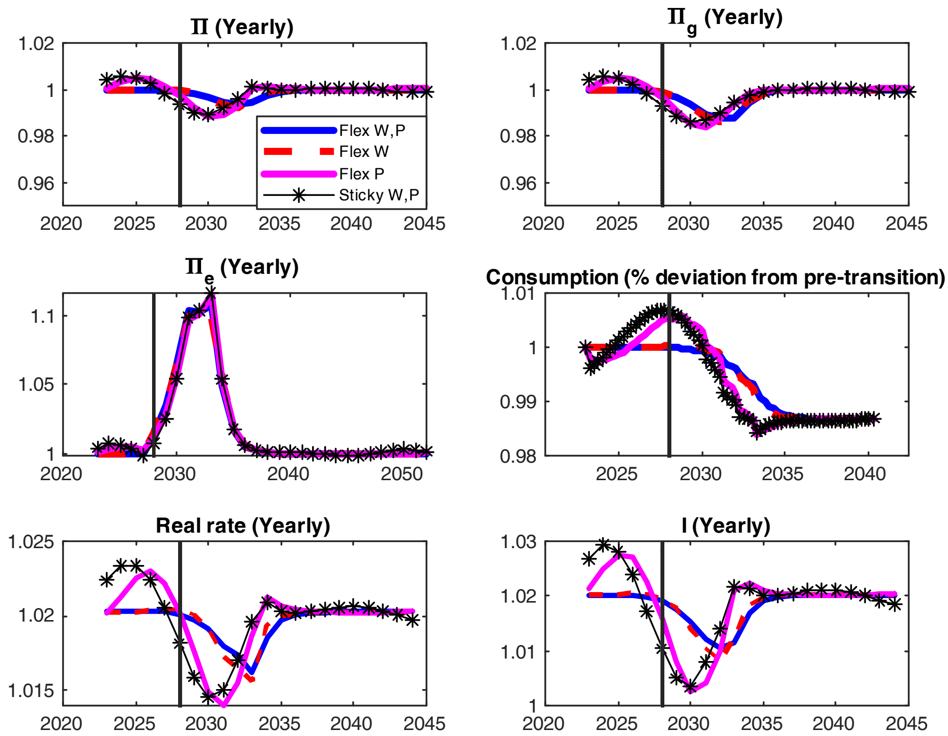

Figure 3: The effects of a carbon tax on the economy with sticky prices and/or wages. The coefficient in the Taylor rule is set to $\alpha_\pi = 1.5$. The transition is announced 5 years in advance, and the vertical line denotes the first increase in the tax rate.

monetary policy rule.

3.2 Nominal Rigidities

We now turn to the implications with nominal rigidities. We consider four cases: no rigidities (Flex W,P), sticky prices and flexible wages (Flex W), flexible prices and sticky wages (Flex P), and sticky prices and wages. We also consider two cases for the transition: one where it is announced 5 years in advance and one where it starts immediately. The first case gives the agents in the economy a chance to adjust to the transition before it starts.

3.2.1 An announced transition

The results for these different cases are presented in Figure 3.

Overall, the results are similar to the case with flexible prices. A difference is that with sticky wages and prices, we actually get some inflation at the star of the transition even though the first tax increase occur after five years. Quantitatively, however, this effect is small: less than 0.5 percent. CPI and goods inflation then both falls around the time of the first tax hike, whereas the energy inflation increases in a similar fashion as with flexible prices. The central bank can achieve this modest effect on inflation by hiking the nominal interest rate in the beginning of the transition, after which is should be reduced to offset the fall in inflation later in the transition.

Note that the case with sticky prices and flexible wages is very close to the case with flexible prices and wages, whereas the flexible prices and sticky wages generate substantially more deviations from the flex-price benchmark. This is particularly true for consumption, and interest rates. As in the example with flexible prices, the central bank can offset these inflation tendencies by using a stricter Taylor rule, but now this comes at a cost in terms of lower output. We address this trade-off below.

3.2.2 A transition that starts immediately

Figure 4 shows that the adjustment period to the transition is crucial. The case where the transition starts immediately has fundamentally different features than those from an announced transition. All economies with nominal rigidities now generate inflation between two and four percent. The transition also generate a serious energy-price of up to 60 percent.

The announced transition requires high nominal rates and with sticky wages and prices the drop in consumption is drastic, suggesting that such case is very costly.

4 Conclusions

We have analyzed the inflationary consequences of introducing a tax on brown energy with the purpose of reducing emissions of carbon dioxide and prevent further global warming. In our flexible price benchmark, we have shown that it is perfectly possible to achieve a desired relative price adjusment of the price of brown energy without any inflationary consequences. Our extensions to sticky prices (of other goods than energy) and wages tends to support the same basic message, in that if the measures are announced sufficiently far in advance the real and nominal consequences of the transition are limited. We are working to test the robustness of

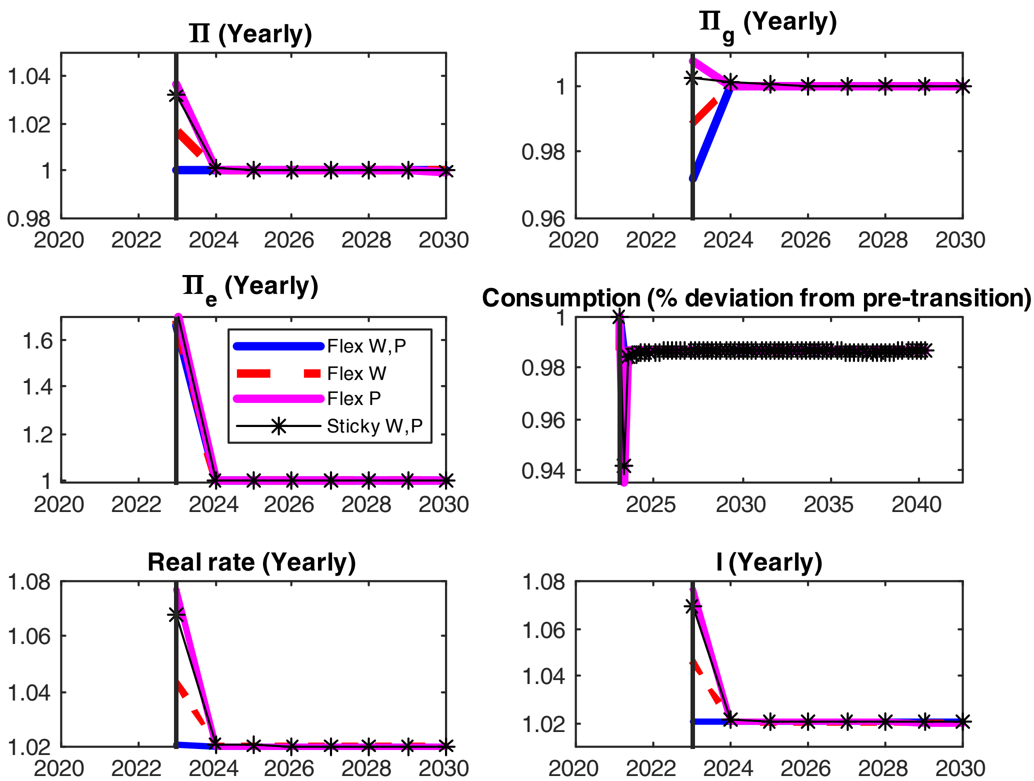

Figure 4: The effects of a carbon tax on the economy with sticky prices and/or wages. The transition starts immedeately, and the vertical line denotes the first increase in the tax rate.

these results to the inclusion of energy in the production of intermediate goods, as well as a proper welfare analysis going beyond the Taylor rule thus far employed.

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A Appendix

A.1 The consumer problem

The intra-temporal problem for the consumer is

$$\min_{C_{g,t},C_{e,t}} P_{g,t}C_{g,t} + P_{e,t}C_{e,t} - P_t \left[C_t - \left((1-\omega)^{\frac{1}{\eta}} C_{g,t}^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} C_{e,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \right].$$

The first-order condition w.r.t. $C_{g,t}$ and $C_{e,t}$ can, respectively, be written as (20) and (21). Inserting (20) and (21) into (2) delivers (23).

The intertemporal problem for the consumer is to maximize (1) subject to (3). The resulting first-order conditions gives (22).

A.2 Firms

A.2.1 The goods sector

The cost minimization problem for the firm that assembles the manufactured final good is

$$\min_{Y_{g,t}(i)} P_{g,t}(i) Y_{g,t}(i) - P_{g,t} \left[Y_{g,t} - \left(\int_0^1 \left(Y_{g,t}(i)^{\frac{\xi-1}{\xi}} di \right) \right)^{\frac{\xi}{\xi-1}} \right].$$

The first order condition w.r.t. $Y_{g,t}(i)$ delivers (6).

The first-order condition to (8) can be written as

$$\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(Y_{g,t+j}(i) + \left(P_{g,t}^* - Mc_{t+j}\left(Y_{g,t+j}(i)\right)\right) \frac{\partial Y_{g,t+j}(i)}{\partial P_{g,t}^*} \right) = 0,$$

where Mc_{t+j} is the marginal cost of production in period $t+j$. With the production function given by (4), the demand for labour by firm i is given by $H_{g,t}(i) = Y_{g,t}(i)/A_{g,t}$, which implies that the marginal cost of production is simply

$$Mc_t = \frac{W_t}{A_{g,t}}.$$

The marginal cost is thus independent of the level of production since W_t is exogenous from the viewpoint of the small firm. Noting that $\frac{\partial Y_{g,t+j}(i)}{\partial P_{g,t}^*} = -\frac{\xi}{P_{g,t}^*} Y_{g,t+j}(i)$, we get

$$\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{g,t+j}(i) \left[P_{g,t}^* - \frac{\xi}{\xi - 1} Mc_{t+j} \right] = 0.$$

Inserting the stochastic discount factor $Q_{t,t+j} = \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \frac{P_t}{P_{t+j}}$ and (9), we arrive at

$$\begin{aligned} & \tfrac{P_{g,t}^*}{P_{g,t}} E_t \textstyle\sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \tfrac{P_t}{P_{t+j}} \left(\tfrac{P_{g,t+j}}{P_{g,t}} \right)^{\xi} Y_{g,t+j} \\ = & \tfrac{\xi}{\xi-1} \tfrac{P_t}{P_{g,t}} E_t \textstyle\sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \tfrac{W_{t+j}}{P_{t+j}} \tfrac{1}{A_{t+j}} \left(\tfrac{P_{g,t+j}}{P_{g,t}} \right)^{\xi} Y_{g,t+j}. \end{aligned} \quad (45)$$

Now define

$$N_{1,t} = E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{P_t}{P_{t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}} \right)^{\xi} Y_{g,t+j},$$

which implies that

$$N_{1,t+1} = E_{t+1} \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j+1}) \frac{P_{t+1}}{P_{t+j+1}} \left(\frac{P_{g,t+j+1}}{P_{g,t+1}} \right)^{\xi} Y_{g,t+j+1}.$$

Then take expectations, using the law of iterated expectations and subtracting this from $K_{1,t}$ gives that

$$\begin{aligned} N_{1,t} &= U'(C_t) Y_{g,t} + E_t \frac{P_t}{P_{t+1}} \left(\frac{P_{g,t+1}}{P_{g,t}} \right)^{\xi} \beta\theta N_{1,t+1}, \text{ or} \\ N_{1,t} &= U'(C_t) Y_{g,t} + \beta\theta E_t \Pi_{t+1}^{-1} \Pi_{g,t+1}^{\xi} N_{1,t+1}. \end{aligned}$$

Similarly,

$$
\begin{aligned}
N_{2,t} &= E_t \sum_{j=0}^{\infty} (\beta\theta)^j U'(C_{t+j}) \frac{W_{t+j}}{P_{t+j}} \frac{1}{A_{g,t+j}} \left(\frac{P_{g,t+j}}{P_{g,t}}\right)^{\xi} Y_{g,t+j}, \text{ or} \\
N_{2,t} &= U'(C_t) \frac{W_t}{P_t} \frac{Y_{g,t}}{A_{g,t}} + \beta\theta E_t \Pi_{g,t+1}^{\xi} N_{2,t+1}.
\end{aligned}
$$

We can now express

$$
\frac{P_{g,t}^*}{P_{g,t}} N_{1,t} = \frac{\xi}{\xi - 1} \frac{P_t}{P_{g,t}} N_{2,t} \tag{46}
$$

Next, since $P_{g,t}^{1-\xi} = (1-\theta)\left(P_{g,t}^*\right)^{1-\xi} + \theta P_{g,t-1}^{1-\xi}$, we see that

$$
\frac{P_{g,t}^*}{P_{g,t}} = \left[\frac{1 - \theta\left(\Pi_{g,t}\right)^{\xi-1}}{1-\theta}\right]^{\frac{1}{1-\xi}}. \tag{47}
$$

Inserting (47) into (46) delivers (26).

Finally, use the demand function (9) and the market clearing condition that requires $Y_{g,t} = C_{g,t}$, with $C_{g,t}$ given by (20) and (47) to arrive at (27), (28), which allows us to write (**??**) as (26).

To solve the model, we make use of the following manipulation

$$
\begin{aligned}
D_{g,t} &\equiv \int_0^1 \left(\frac{P_{g,t}(i)}{P_{g,t}}\right)^{-\xi} di, \\
D_{g,t} &= \sum_{j=0}^{\infty} (1-\theta)\theta^j \left(\frac{P_{g,t-j}^*}{P_{g,t}}\right)^{-\xi}, \\
D_{g,t-1} &= \sum_{j=0}^{\infty} (1-\theta)\theta^j \left(\frac{P_{g,t-1-j}^*}{P_{g,t-1}}\right)^{-\xi} \\
\theta\left(\frac{P_{g,t-1}}{P_{g,t}}\right)^{-\xi} D_{g,t-1} &= \sum_{j=1}^{\infty} (1-\theta)\theta^j \left(\frac{P_{g,t-j}^*}{P_{g,t}}\right)^{-\xi}, \\
D_{g,t} - \theta\left(\frac{P_{g,t-1}}{P_{g,t}}\right)^{-\xi} D_{g,t-1} &= (1-\theta)\left(\frac{P_{g,t}^*}{P_{g,t}}\right)^{-\xi}, \\
D_{g,t} &= (1-\theta)\left(\frac{P_{g,t}^*}{P_{g,t}}\right)^{-\xi} + \theta\Pi_{g,t}^{\xi} D_{g,t-1},
\end{aligned}
$$

and using (47) we arrive at (41).

Similarly, we have

$$H_{g,t}\left(i\right) = \frac{1}{A_{g,t}} \left(\frac{P^*_{g,t}}{P_{g,t}}\right)^{-\xi} Y_{g,t},$$

or

$$H_{g,t} = \frac{Y_{g,t}}{A_{g,t}} D_{g,t}.$$

A.3 Unions

Cost minimization for the different labor types by the firms implies the following problem

$$\int_0^1 W_t\left(j\right) N_t\left(j\right) dj - W_t \left[Y_t - A_t \left(\int_0^1 N_t\left(j\right)^{\frac{\xi_w - 1}{\xi_w}} dj\right)^{\frac{\xi_w}{\varepsilon_{\omega} - 1}}\right].$$

The solution is

$$N_t\left(j\right) = \left(\frac{W_t\left(j\right)}{W_t}\right)^{-\xi_w} N_t.$$

To solve (14), substitute the constraints into the utility function and take the first order conditions to arrive at

$$\mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k \left(\frac{1}{C_{t+k}} \frac{1}{P_{t+k}} \left(H_{t+k|t}\left(j\right) + W_t^* \frac{\partial H_{t+k|t}(j)}{\partial W_t^*}\right) - H_{t+k|t}\left(j\right)^{\varphi} \frac{\partial H_{t+k|t}(j)}{\partial W_t^*}\right)$$
$$= 0.$$

Using

$$\frac{\partial H_{t+k|t}\left(j\right)}{\partial W_t^*} = -\frac{\xi_w}{W_t^*} H_{t+k|t}\left(j\right),$$

we get

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k \left(W_t^* \frac{1}{C_{t+k}} \frac{1}{P_{t+k}} H_{t+k|t}\left(j\right) - \frac{\xi_w}{\xi_w - 1} H_{t+k|t}\left(j\right)^{1+\varphi}\right).$$

Substitute out $H_{t+k|t}$ and manipulate to get an expression in terms of real wages instead of nominal

$$\mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k \left(\frac{(W_t^*)^{1+\xi_w\varphi}}{C_{t+k}P_{t+k}} \left(\frac{1}{P_{t+k}\widehat{W}_{t+k}}\right)^{-\xi_w} H_{t+k} - \frac{\xi_w}{\xi_w - 1} \left(\left(\frac{1}{P_{t+k}\widehat{W}_{t+k}}\right)^{-\xi_w} H_{t+k}\right)^{1+\varphi}\right)$$
$$= 0.$$

Now, multiply with P_t inside the parentheses with P_t/P_t to get

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\left(\widehat{W}_t^*\right)^{1+\xi_w\varphi} 1 \frac{P_t^{(1+\xi_w\varphi)+\xi_w-\xi_w(1+\varphi)}}{C_{t+k}P_{t+k}} \left(\frac{\Pi_{t,t+k}^{-1}}{\widehat{W}_{t+k}}\right)^{-\xi_w} H_{t+k} \right.$$
$$\left. - \frac{\xi_w}{\xi_w - 1} \left(\left(\frac{\Pi_{t,t+k}^{-1}}{\widehat{W}_{t+k}}\right)^{-\xi_w} H_{t+k} \right)^{1+\varphi} \right) = 0.$$

Because $(1+\xi_w\varphi) + \xi_w - \xi_w(1+\varphi) = 1$, we arrive at

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\left(\widehat{W}_t^*\right)^{1+\xi_w\varphi} \frac{1}{C_{t+k}} \Pi_{t,t+k}^{\xi_w-1} \widehat{W}_{t+k}^{\xi_w} H_{t+k} - \frac{\xi_w}{\xi_w-1} \left(\Pi_{t,t+k}^{\xi_w} \widehat{W}_{t+k}^{\xi_w} H_{t+k}\right)^{1+\varphi} \right)$$
$$= 0,$$

$$\left(\widehat{W}_t^{*w}\right)^{1+\xi_w\varphi} = \frac{\xi_w}{\xi_w - 1} \frac{M_{1,t}}{M_{2,t}}$$

with

$$M_{1,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\Pi_{t,t+k}^{\xi_w} \widehat{W}_{t+k}^{\xi_w} H_{t+k}\right)^{1+\varphi}$$
$$M_{2,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{1}{C_{t+k}} \Pi_{t,t+k}^{\xi_w-1} \widehat{W}_{t+k}^{\xi_w} H_{t+k}.$$

Recursify, take expectations at t, and subtract the result from $M_{1,t}$, to arrive at (30)-(31).

The aggregate wage index follows a similar logic as the price index:

$$W_t^{1-\xi_w} = (1-\theta_w)\left(W_t^*\right)^{1-\xi_w} + \theta_w W_{t-1}^{1-\xi_w}.$$

Dividing by $P_t^{1-\xi_w}$ to get (32).

A.3.1 Energy production and supply

The first-order conditions to (10), and (11) are given by

$$W_t = (1-\tau_t) P_{1,t} A_{1,t}, \tag{48}$$

and

$$W_t = P_{2,t} A_{2,t}. \tag{49}$$

$$P_{1,t} = P_{2,t}\frac{A_{2,t}}{A_{1,t}}\frac{1}{1-\tau_t}. \tag{50}$$

The cost-minimization problem for the energy service provider delivers the following equilibrium conditions

$$E_{1,t} = \left(\frac{P_{1,t}}{P_{e,t}}\right)^{-\varepsilon}(1-\lambda)\,C_{e,t}, \tag{51}$$

$$E_{2,t} = \left(\frac{P_{2,t}}{P_{e,t}}\right)^{-\varepsilon}\lambda C_{e,t}, \tag{52}$$

and

$$P_{e,t} = h_t P_{2,t}, \tag{53}$$

where

$$h_t \equiv \left[(1-\lambda)\left(\frac{A_{2,t}}{A_{1,t}}\frac{1}{1-\tau_t}\right)^{1-\varepsilon} + \lambda\right]^{\frac{1}{1-\varepsilon}}. \tag{54}$$

Using (50) and (53) in (51) and (52), these latter expressions can be formulated as (24) and (25).

A.4 Steady state

The steady-state versions of (15) and (22) are

$$I = \frac{1}{\beta}\left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}$$

and

$$I = \Pi\frac{1}{\beta}.$$

Combining these two equations delivers

$$\Pi = (\Pi^*)^{\frac{\alpha_\pi}{\alpha_\pi - 1}}; \tag{55}$$

$$I = \frac{1}{\beta}\Pi. \tag{56}$$

We then have

$$\Pi_g = \Pi_e = \Pi; \tag{57}$$

$$D_g = \frac{1-\theta\Pi_g^{\xi-1}}{1-\theta\Pi_g^{\xi}}; \tag{58}$$

$$\widehat{W} = A_2 p_e \frac{1}{h}; \tag{59}$$

$$h \equiv \left[(1-\lambda)\left(\frac{A_2}{A_1}\frac{1}{1-\tau}\right)^{1-\varepsilon} + \lambda\right]^{\frac{1}{1-\varepsilon}}; \tag{60}$$

$$(1-\nu)(p_g)^{1-\eta} + \nu (p_e)^{1-\eta} = 1; \tag{61}$$

Using the steady-state expressions of (27) and (28) in (26), we get the steady-state version of the latter equation

$$p_g = \frac{\xi}{\xi-1}\frac{\widehat{W}}{A_g}\left[\frac{1-\theta\Pi_g^{\xi-1}}{1-\theta}\right]^{\frac{1}{\xi-1}}\frac{1-\beta\theta\Pi^{-1}\Pi_g^{\xi}}{1-\beta\theta\Pi_g^{\xi}}; \tag{62}$$

Inserting the steady-state versions of (30)-(31) into (29) and then into (32) and rearranging allows us to solve for H, we get

$$H = \left[\frac{1-\beta\theta_w\Pi^{\xi_w(1+\varphi)}}{1-\beta\theta_w\Pi^{\xi_w-1}}\frac{\xi_w-1}{\xi_w}\frac{\widehat{W}}{C}\left(\frac{1-\theta_w}{1-\theta_w\Pi^{\xi_w-1}}\right)^{\frac{1+\xi_w\varphi}{\xi_w-1}}\right]^{\frac{1}{\varphi}}; \tag{63}$$

$$M_1 = \frac{\left(\widehat{W}^{\xi_w}H\right)^{1+\varphi}}{1-\beta\theta_w\Pi^{\xi_w(1+\varphi)}}; \tag{64}$$

$$M_2 = \frac{\widehat{W}^{\xi_w}H}{C\left(1-\beta\theta_w\Pi^{\xi_w-1}\right)}; \tag{65}$$

$$\widehat{W}_t^* = \left(\frac{\xi_w}{\xi_w-1}\frac{M_{1,t}}{M_{2,t}}\right)^{\frac{1}{1+\xi_w\varphi}} \tag{66}$$

$$\begin{gathered} C\Bigg[(1-\nu)\tfrac{D_g}{A_g}p_g^{-\eta} \\ +\nu p_e^{-\eta}\left((1-\lambda)\tfrac{1}{A_1}\left(\tfrac{A_2}{A_1}\tfrac{1}{1-\tau}\tfrac{1}{h}\right)^{-\varepsilon} + \lambda\tfrac{1}{A_2}\left(\tfrac{1}{h}\right)^{-\varepsilon}\right)\Bigg] = H; \end{gathered} \tag{67}$$

$$C_e = \nu p_e^{-\eta} C; \tag{68}$$

$$C_g = (1-\nu)\,p_g^{-\eta} C; \tag{69}$$

$$E_1 = (1-\lambda)\left(\frac{A_2}{A_1}\frac{1}{1-\tau}\frac{1}{h}\right)^{-\varepsilon} C_e; \tag{70}$$

$$E_2 = \lambda \left(\frac{1}{h}\right)^{-\varepsilon} C_e; \tag{71}$$

$$A_g = \overline{A}_g \exp\left(-\gamma E_1\right); \tag{72}$$

$$A_1 = \overline{A}_1 \exp\left(-\gamma E_1\right); \tag{73}$$

$$A_2 = \overline{A}_2 \exp\left(-\gamma E_1\right); \tag{74}$$

$$N_{1,t} = \frac{1}{1-\beta\theta\Pi^{-1}\Pi_g^{\xi}}; \tag{75}$$

$$N_2 = \frac{\widehat{W}}{A_g\left(1-\beta\theta\Pi_g^{\xi}\right)}; \tag{76}$$

where we used the the market clearing condition that requires $Y_{g,t} = C_{g,t}$.

$$\widehat{B} = \frac{\widehat{W}_t + \widehat{\pi} + \widehat{T} - C}{1 - I/\Pi};$$

$$\widehat{T} = \frac{A_2}{h}\frac{P_e}{P}\frac{\tau}{1-\tau} - \frac{I}{\Pi}\widehat{B},$$

$$H_{g,t} = \frac{C_{g,t}D_{g,t}}{A_{g,t}},$$

$$H_{1,t} = \frac{E_{1,t}}{A_{1,t}}$$

$$H_{2,t} = \frac{E_{2,t}}{A_{2,t}}.$$

To solve for the steady state, note that (55)–(58) solves for Π, Π_g, I, and D_g in closed form. Then solve (59), (61)-(63), (67), (70), and (72)-(74) $\widehat{W}$, p_g, p_e, H, C, A, A_1, A_1, and E_1. The remaining equations can then be solved in closed form.

A.5 The RBC model with flexible prices

The equilibrium conditions are as follows.

$$C_t = g_t, \tag{77}$$

where g_t is given by

$$g \equiv \left[\frac{A_t A_{1,t} A_{2,t} \left(h_{1,t}^{-1/\eta} \frac{A_t}{\psi} \right)^{\frac{1}{\varphi}}}{(1-\nu)\, h_{1,t} A_{1,t} A_{2,t} + (1-\lambda)\, \nu h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{1,t}(1-\tau_t)}{A_t} \right)^{\varepsilon} A_t A_{2,t} + \lambda \nu h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{2,t}}{A_t} \right)^{\varepsilon} A_t A_{1,t}} \right]^{\frac{\varphi}{1+\varphi}} \tag{78}$$

Note that if $\gamma = 0$ or if regional emissions are negligible on the aggregate, then we have a closed-form solution for C_t, since A_t, $A_{1,t}$, and $A_{2,t}$ are then all exogenous.

$$C_{g,t} = (1-\nu)\, h_{1,t} g_t, \tag{79}$$

$$C_{e,t} = \nu \frac{h_{1,t}}{h_{2,t}^{\eta}} g_t. \tag{80}$$

$$E_{1,t} = \nu (1-\lambda)\, h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{1,t} (1-\tau_t)}{A_t} \right)^{\varepsilon} g_t, \tag{81}$$

and

$$E_{2,t} = \nu \lambda h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{2,t}}{A_t} \right)^{\varepsilon} g_t. \tag{82}$$

Market-clearing conditions:

$$C_{g,t} = A_t H_{g,t}$$

$$H_{g,t} = (1-\nu)\, h_{1,t} \frac{g_t}{A_t}. \tag{83}$$

$$H_{1,t} = \nu (1-\lambda)\, h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{1,t} (1-\tau_t)}{A_t} \right)^{\varepsilon} \frac{g_t}{A_{1,t}}. \tag{84}$$

$$H_{2,t} = \nu \lambda h_{1,t} h_{2,t}^{\varepsilon-\eta} \left(\frac{A_{2,t}}{A_t} \right)^{\varepsilon} \frac{g_t}{A_{2,t}} \tag{85}$$

The real wage,$\widehat{W}_t$, is given by

$$\widehat{W}_t \equiv \frac{W_t}{P_t} = \frac{A_t}{h_t^{1/\eta}}. \tag{86}$$

The budget constraint in real terms, we get

$$C_t + \widehat{B}_t = \widehat{W}_t H_t + \frac{I_{t-1}}{\Pi_t} \widehat{B}_{t-1} + \widehat{T}_t, \tag{87}$$

and

$$\widehat{T}_t \equiv \frac{T_t}{P_t} = \frac{A_t H_{1,t}}{h_t^{1/\eta}} \frac{\tau_t}{1-\tau_t} - \frac{I_{t-1}}{\Pi_t} \widehat{B}_{t-1} \tag{88}$$