Monetary and Fiscal Policy According to HANK-IO

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Preliminary, comments welcome

Abstract

This paper studies monetary and fiscal policy transmission in a multi-sector heterogeneous-agent New Keynesian model with an input-output network ("HANK-IO"). We calibrate the model to capture systematic household-sector linkages observed in the data. To identify when these linkages have implications for policy transmission, we analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, novel earnings and expenditure heterogeneity channels emerge that govern the propagation of demand and supply shocks. Our quantitative analysis shows that these channels shape the transmission of policy to aggregates and its distributional consequences across households and sectors.

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1 Introduction

Economists have made substantial progress in documenting and modeling the implications of cross-sectional heterogeneity for policy transmission in disaggregated economies. Recent work on household behavior has emphasized the importance of heterogeneity in spending propensities and the incidence of shocks. On the production side, a long tradition of research has documented sectoral heterogeneity and studied its importance for the propagation of shocks in multi-sector models. This paper takes as its starting point a set of empirical regularities that point to systematic linkages *between* households and sectors at a disaggregated micro level.

Motivated by these stylized facts this paper proposes a quantitative framework to study the implications of household-sector linkages for the transmission of monetary and fiscal policy. We develop a multi-sector heterogeneous-agent New Keynesian model with an input-output production network, which we call the "HANK-IO" model.[1] Our framework is inspired by both the burgeoning heterogeneous-agent New Keynesian ("HANK") literature (Kaplan et al., 2018) and the long tradition of multi-sector business cycle models (Long and Plosser, 1983). We show analytically that an interaction between household and sectoral heterogeneity can shape the propagation of demand and supply shocks through novel earnings and expenditure heterogeneity channels. After disciplining our model empirically to match the household-sector linkages observed in micro data, we quantitatively assess their importance for monetary and fiscal policy transmission.

Our baseline economy departs from a canonical HANK model (Auclert et al., 2018) and enriches its production side by introducing multiple sectors and input-output linkages (Baqaee and Farhi, 2020). Households face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (ex-post heterogeneity). We also allow for permanent differences in household characteristics or *types* (ex-ante heterogeneity). In particular, household types may differ in their preferences over consumption goods and their labor endowments. Our key point of departure from the canonical HANK framework is that we allow for systematic links between household types and production sectors in terms of both earnings and expenditure patterns. We later discipline these household-sector linkages empirically using micro data in Section 4. Production in our economy takes place across a rich network of sectors. Firms use factors and intermediate inputs in production. Nominal price rigidity gives rise to endogenous and time-varying markups that introduce a wedge between prices and marginal costs. We allow for sectoral heterogeneity across factor and input shares, input-output linkages, competitiveness, and price rigidity.

As-if benchmark. We begin our analysis with an instructive *as-if* benchmark to illustrate conceptually under what conditions household-sector linkages may play a role for policy transmission. Our as-if benchmark assumes that household types all share the same homothetic preferences

[1] We choose this name in memory of the late Emmanuel Farhi whose work with David Baqaee on heterogeneous-agent economies with input-output production networks ("HA-IO") has inspired this paper (Baqaee and Farhi, 2018).

over consumption goods and there is a single labor factor. Under these two assumptions, there is consequently no role for *earnings* or *expenditure heterogeneity* as households are symmetrically exposed to price inflation and changes in labor demand. Our first main result is that the HANK-IO model features a strict decoupling between household and sectoral heterogeneity under these two assumptions. We show that the macroeconomic dynamics of our as-if benchmark admit an intertemporal IS-LM representation as a system of two forward-looking equations: a dynamic IS curve that determines output as a function of the time path of real interest rates, and a dynamic LM curve that pins down real interest rates as a function of future aggregate demand. The dynamic IS curve is shaped by household but not by sectoral heterogeneity: Given a path of real interest rates, it takes the same form as the IS equation of a canonical one-sector HANK model. Conversely, the dynamic LM curve is shaped by sectoral but not by household heterogeneity: It maps a given path of aggregate demand to the same path of real interest rates as a canonical representative-household multi-sector model. The as-if benchmark therefore highlights starkly that household and sectoral heterogeneity are decoupled in this sense in the absence of earnings and expenditure heterogeneity.

Our as-if economy also serves as an instructive benchmark to study the propagation of demand and supply shocks. These results serve as a reference point when we later revisit the transmission of policy in the presence of earnings and expenditure heterogeneity and meaningful household-sector linkages.

Monetary and fiscal policy transmission is characterized by an Intertemporal Keynesian Cross. When the monetary authority adopts a rule that neutralizes real interest rate effects, fiscal policy transmission is governed by the same Keynesian multiplier as in the one-sector HANK model of Auclert et al. (2018). Intertemporal marginal propensities to consume (iMPCs) remain a sufficient statistic for the output effects of government spending. This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. For a given path of real interest rates, the aggregate fiscal multiplier is therefore unaffected by sectoral heterogeneity and, in fact, identical to that in any HANK economy that admits the same IS curve representation.

Likewise, for a given change in the path of real interest rates, monetary policy transmission is solely governed by household heterogeneity. The sufficient statistics for policy are again identical to those in a one-sector HANK economy. Sectoral heterogeneity does, however, affect the transmission from nominal to real rates, which is governed by the LM curve. Our result shows that once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for policy transmission—the path of real rates is a summary statistic for the effect of monetary policy on aggregate activity. These results are manifestations of our decoupling result under the IS-LM representation: Demand shocks interact with the IS curve and are shaped by household but not by sectoral heterogeneity.

Finally, we unpack the LM curve and derive an aggregation result that traces the macroeconomic effects of sectoral technology shocks in our as-if benchmark. We decompose the impact on real GDP into a pure technology effect—accounting for increased productivity of resources at a given allocation—and changes in allocative efficiency—summarizing the effects on output from a reallocation of resources across firms and workers. Remarkably, our aggregation result for the as-if benchmark is identical to that of Baqaee and Farhi (2020). This is despite our economy featuring rich and dynamic household heterogeneity, whereas theirs is a static representative-household setting. For given changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks are not directly shaped by household heterogeneity. In other words, changes in sectoral markups and the labor income share $d \log \Lambda_t$ are sufficient statistics for the implications of household heterogeneity.

The role of household-sector linkages. Away from the as-if benchmark, household-sector linkages meaningfully shape the transmission of policy and shocks through novel earnings and expenditure heterogeneity channels.

Our next analytical result characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy away from the as-if benchmark. When households supply different labor factors, an *earnings heterogeneity channel* emerges. It is captured by a cross-sectional covariance across household types between iMPCs and changes in households' earnings shares. Earnings heterogeneity amplifies the aggregate effects of policy when it leads to a redistribution of income shares to factors (and in states of the world) with large spending propensities.

When households consume different consumption baskets, an *expenditure heterogeneity channel* emerges that comprises two distinct effects. A change in the price path of a household's basket elicits income and intertemporal substitution effects. If relative prices increase for households (and in states of the world) with large spending propensities, then the resulting drop in their effective purchasing power leads to a fall in aggregate consumer spending. This effect is captured by a covariance across household types between iMPCs and changes in relative bundle prices. Moreover, when households experience different rates of inflation in their respective bundle prices, then their effective real rates of return on savings differ as well. The expenditure heterogeneity channel of policy then also comprises a covariance across households between their relative rates of inflation and intertemporal interest rate response elasticities. Intuitively, if relative real savings rates increase for those household types that have large intertemporal elasticities of substitution, then the aggregate effect of policy is dampened.

Finally, we derive an aggregation result for sectoral technology shocks. Unlike in the as-if benchmark, gains from allocative efficiency are now shaped by household-sector linkages. Expenditure heterogeneity has implications for allocative efficiency because changes in purchasing power affect household labor supply and consequently firms' cost structure. This new force is governed by a covariance across household types between factor shares and changes in the relative price of households' consumption baskets: If consumer prices increase especially for those households whose labor factors have large cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, when the price of a household's consumption bundle increases, a given nominal wage has less purchasing power and the real wage falls. If prices increase for households whose factors play a dominant role in firms' cost structures (large cost-based Domar weights), then output falls. Crucially, the relevant covariance is with respect to *cost-based* factor shares because markups drive a wedge between prices and marginal costs. Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency. When the overall income share falls, households receive less labor income, which elicits a positive labor supply response and increases output. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the largest cost-based Domar weights experience a relative decrease in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is governed by households' labor supply elasticities.

Our results point to an important conceptual distinction between demand and supply shock propagation in HANK-IO. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share pricely captures those markups that drive a wedge between prices and marginal costs, and it is the appropriate measure to capture whether resources find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs. Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices. Mechanically, this distinction emerges because our results on demand propagation take as their starting point the goods market clearing condition and the aggregate consumption function, whereas our aggregation result for sectoral shocks focuses on the supply and production equations.

Quantitative analysis. To evaluate the importance of household-sector linkages quantitatively, we enrich the analytical baseline model by introducing capital. Allowing households to trade an illiquid asset helps us match intertemporal MPCs, which our analytical results show are crucial determinants of the earnings and expenditure heterogeneity channels (Kaplan and Violante, 2022). Explicitly modeling capital investment also allows us to capture the concentrated investement network documented by Vom Lehn and Winberry (2022). We calibrate our model to match the income and wealth distribution of households, as well as key moments of sectoral heterogeneity across a 66-sector production network. As it is well understood that production networks give rise to non-linearities in the propagation of shocks, we solve this model globally (Baqaee and Farhi, 2019).

Our quantitative analysis yields three main results. First, household-sector linkages dampen the transmission of both monetary and fiscal policy to aggregate activity. Intuitively, households who are most sensitive to policy shocks command lower spending propensities, resulting in a fall in aggregate consumer spending. Second, household-sector linkages have distributional implications and imply substantial dispersion in the cross-sectional incidence of policy transmission. Finally, fiscal multipliers vary considerably across sectors, suggesting a role for targeted fiscal interventions. Together, these results underscore the importance of household-sector linkages and the resulting earnings and expenditure heterogeneity channels of policy transmission.

Related literature. Our paper builds on much previous work that has documented and studied the implications of household and sectoral heterogeneity.

Sectoral heterogeneity. We contribute to a long tradition of research on the transmission and propagation of shocks in multi-sector business cycle models. Starting with Long and Plosser (1983), much work has employed structural real business cycle models to assess quantitatively whether sectoral technology shocks can account for observed business cycles patterns.[2] An important strand of this literature characterizes the aggregation properties of these models.[3] It has long been appreciated that Hulten's theorem applies to first order around efficient allocations and the aggregation of sectoral shocks is governed by sales shares (Domar weights). This literature has recently been reinvigorated with renewed focus on misallocation and potential gains in allocative efficiency in inefficient economies with distortions.[4] In an important contribution, Baqaee and Farhi (2020) derive an aggregation result for inefficient multi-sector economies with network linkages and arbitrary distortions.

Another strand of the multi-sector business cycle literature introduces nominal rigidities in the New Keynesian tradition.[5] Many of these papers investigate the implications of sectoral heterogeneity and input-output linkages for monetary policy and inflation dynamics. Baqaee et al. (2021) show that monetary policy can have supply-side effects and operate through a misallocation channel when policy redirects resources towards more monopolized sectors.

Relative to existing work, we develop a dynamic structural model featuring rich heterogeneity across both households and sectors. We emphasize the role of systematic household-sector linkages observed in micro data that give rise to novel transmission channels. Analytically, we derive an Intertemporal Keynesian Cross to characterize monetary and fiscal policy in the presence of household-sector linkages, and we extend the aggregation result of Baqaee and Farhi (2020) to a multi-sector heterogeneous-agent New Keynesian model. Quantitatively, we assess the importance of earnings and expenditure heterogeneity for policy transmission.

Household-sector linkages. Our paper contributes to research studying the interactions of house-

[2] Among many others, see Bak et al. (1993), Horvath (2000), Foerster et al. (2011), Atalay (2017). While these papers focus on sectoral input-output linkages, recent work by Vom Lehn and Winberry (2022) studies the role of the investment network in propagating sectoral shocks.

[3] See for example Hulten (1978), Horvath (1998), Dupor (1999), Gabaix (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Acemoglu et al. (2017). Also see Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for recent surveys.

[4] See Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Baqaee and Farhi (2019), Liu (2019), Bigio and La'O (2020), Clayton and Schaab (2022), and many more.

[5] See among others Aoki (2001), Bouakez et al. (2009), Pasten et al. (2017, 2020), and Baqaee et al. (2021).

hold and sectoral heterogeneity. Indeed, we take as our starting point and motivation the large body of empirical work that studies earnings and expenditure heterogeneity across households.[6] A strand of this literature explicitly documents systematic linkages between households and sectors through earnings and expenditure patterns. Clayton et al. (2018) show that richer and more educated households work in and buy from relatively price-rigid sectors. Relative to this empirical literature, we document several novel facts. Motivated by this body of empirical regularities, the main focus of our paper, however, is to assess their importance for policy transmission quantitatively in a structural model.

Our paper is naturally related to a small but growing body of research that studies disaggregated economies featuring both household and sectoral heterogeneity (Baqaee and Farhi, 2018; Flynn et al., 2022; Schaab, 2022; Guerrieri et al., 2022; Andersen et al., 2022). Most of these papers study static environments. The contribution of our paper is to develop a dynamic structural framework that is apt for quantitative analysis. We leverage the sequence-space representation of our economy to obtain analytical results even though our framework features rich and dynamically evolving heterogeneity.

HANK. A fast-growing literature studies the implications of household heterogeneity for business cycles and, in particular, policy transmission.[7] Taking as our starting point a canonical HANK framework, we unbundle the model's production side by introducing multiple sectors, an input-output network structure, and systematic household-sector linkages motivated by empirical regularities. We revisit the transmission mechanism of stabilization policy and extend the Intertemporal Keynesian Cross to this environment (Auclert et al., 2018). Monetary and fiscal policy transmission is governed not only but novel earnings and expenditure heterogeneity channels but also by a misallocation channel through which policy directly affects allocative efficiency. We unbundle the distributional and sectoral consequences of stabilization policy.

2 A Baseline HANK-IO Model

In this section, we develop a new multi-sector heterogeneous-agent New Keynesian framework with input-output linkages, which we call the "HANK-IO" model. Time is discrete and indexed by $t \in \{0, 1, \ldots\}$. We abstract from aggregate uncertainty and focus on one-time, unanticipated shocks. Our model features heterogeneous households and multiple sectors. Households differ in terms of their permanent characteristics (ex-ante heterogeneity), and they face idiosyncratic uncertainty that leads them to make different consumption, savings, and portfolio decisions (ex-post heterogeneity).

In our description of household behavior for a given type i, we deliberately stay close to McKay et al. (2016) and Auclert et al. (2018). Our key point of departure from the canonical

[6] See among many others Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Cravino et al. (2020), Jaravel (2021), Comin et al. (2021), and Andersen et al. (2022).

[7] Important contributions include, among many others, McKay and Reis (2016), McKay et al. (2016), Kaplan et al. (2018), Auclert (2019), and Auclert et al. (2018).

HANK framework is that we allow for systematic links between household types i and different production sectors n—in terms of both earnings and expenditure patterns. We later discipline these *household-sector linkages* empirically using micro data in Section 4. The quantitative HANK-IO model we bring to the data in Section 4 also features illiquid assets in the tradition of Kaplan et al. (2018), which we abstract from here.

2.1 Households

The economy is populated by a set of types $i \in \mathcal{I}$ of households. We denote their measure by μ_i and assume $\sum_i \mu_i = 1$. Household types differ in terms of their permanent characteristics. In addition to ex-ante heterogeneity across types, our baseline model allows for ex-post heterogeneity in earnings potential (z) and wealth (a) within types. We can therefore uniquely identify a household of type i with the two state variables (a, z), and we denote the cross-sectional income and wealth distribution for i by $g_{i,t}(a, z)$.

All households purchase consumption goods from and supply labor across N production sectors. The preferences of a household of type i are ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_i\left(\left\{ c_{ij,t} \right\}_j \right) - v\left(n_{i,t} \right) \right], \tag{1}$$

where $c_{ij,t}$ denotes the household's consumption of good j and $n_{i,t}$ denotes hours worked. All households of type i are endowed with and supply a distinct labor factor, which we refer to as factor i for simplicity. Preferences are separable across time and between consumption and work. Crucially, we allow preferences to vary across household types but not across households within a type.

Expenditure heterogeneity. We assume that households' consumption preferences can be represented by a homothetic aggregator. Abusing notation slightly, utility from consumption for a household of type i can then be written as $u_i(c_{i,t})$, where $c_{i,t} = \mathcal{D}_i(c_{i1,t}, \ldots, c_{iN,t})$ is the household's consumption basket, which we assume is CES. This allows us to construct an ideal price index $P_{i,t}$ for each household type i, satisfying

$$P_{i,t} c_{i,t} = \sum_j p_{j,t} c_{ij,t},$$

where $p_{j,t}$ is the price of good j in terms of numeraire. While households of the same type i share the same homothetic aggregator $\mathcal{D}_i$, we allow for differences across types that will give rise to expenditure heterogeneity.

Budget constraint. Households can trade a single asset, a bond, and face a nominal budget constraint of the form

$$\tilde{P}_t \tilde{a}_{i,t} = (1 + i_{t-1}) \tilde{P}_{t-1} \tilde{a}_{i,t-1} + E_{i,t} - P_{i,t} c_{i,t},$$

where $\tilde{P}_t$ is the price in which bonds are denominated, i_{t-1} is the nominal interest rate between periods $t-1$ and t, and $E_{i,t}$ denotes the household's nominal non-financial income, which we discuss below.

It will be convenient to work with a real budget constraint in terms of the household's effective purchasing power, which is proportional to $P_{i,t}$. To that end, we introduce the household's effective real rate of return on savings,

$$R_{i,t} = (1 + i_t) \frac{P_{i,t}}{P_{i,t+1}} \tag{2}$$

as well as real non-financial income, $e_{i,t} = \frac{1}{P_{i,t}} E_{i,t}$. The household budget constraint can then be rewritten as

$$a_{i,t} = R_{i,t-1} a_{i,t-1} + e_{i,t} - c_{i,t}, \tag{3}$$

where $a_{i,t} = \frac{\tilde{P}_t}{P_{i,t}} \tilde{a}_{i,t}$ is a measure of wealth that reflects household purchasing power. Households also face a borrowing constraint of the form

$$a_{i,t} \geq \underline{a}_i.$$

While borrowing constraints may potentially vary across types, we assume that they are specified in terms of $a_{i,t}$ rather than $\tilde{a}_{i,t}$ and reflect household purchasing power.

The household budget constraint (3) is useful because it directly encodes households' expenditure heterogeneity in two places: First, real rates of return on saving differ across households in our model. While all households are exposed symmetrically to changes in nominal interest rates and aggregate inflation, each household's optimal consumption-savings decision will also depend on inflation in the *relative* price of her consumption basket. Second, equation (3) directly deflates the household's nominal non-financial income by the price of her consumption basket, $\frac{1}{P_{i,t}}$. As will become clear in Section 3, expenditure heterogeneity is a key driving force of monetary and fiscal policy transmission through both of these channels.

Labor markets. Household labor supply decisions are intermediated by labor unions (Erceg et al., 2000; Auclert et al., 2018). While we allow for flexible nominal wage adjustments, we maintain the standard assumption of labor rationing. Concretely, all households of type i supply the same hours of work in equilibrium, which we denote for all (a, z) by

$$n_{i,t}(a, z) = N_{i,t}.$$

We present a detailed discussion of our model's labor market structure in Appendix A.1 and show there that the effective labor supply schedule of household type (labor factor) i is

$$v'(N_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w} w_{i,t} u'(C_{i,t}), \tag{4}$$

where $C_{i,t}$ denotes aggregate consumption of all households of type i, $C_{i,t} = \iint c_{i,t}(a,z) g_{i,t}(a,z)\, da\, dz$. Finally, ϵ^w is an elasticity of substitution that governs unions' desired markup of real wages over marginal rates of substitution. It is therefore a measure of monopsony in the labor market.

Earnings heterogeneity. Households' non-financial pre-tax income comes from three sources: labor income, corporate dividends, and fiscal taxes and transfers. We follow Heathcote et al. (2017) and Auclert et al. (2018) in modeling post-tax income in terms of a retention function that empirically captures U.S. tax progressivity and takes the form

$$E_{i,t} = \tau_t \Big(zW_{i,t}N_{i,t} + z\Pi_t \Big)^{1-\lambda}, \tag{5}$$

where (λ, τ_t) parameterize the implied tax system and are exogenously given as a part of fiscal policy. A household of type i and with idiosyncratic labor productivity z earns nominal pre-tax labor income $zW_{i,t}N_{i,t}$, where $W_{i,t}$ is the nominal wage paid to labor factor i. Notice that all households of type i work $N_{i,t}$ hours due to labor rationing. At the same time, we also assume that aggregate nominal corporate profits Π_t are paid to households in proportion to z.[8]

We further discuss earnings and expenditure heterogeneity below in Section 2.5.

2.2 Production with Input-Output Linkages

Our economy comprises N production sectors. To introduce nominal price rigidity in a tractable manner, we model each sector following the standard New Keynesian model as comprising a retailer and intermediate firms whose dynamic pricing problem gives rise to sectoral Phillips curves. For expositional convenience, we frontload discussion of firms' pricing problem and the resulting Phillips curve, before discussing production function and network details.

Retailer. Each sectoral good j is bundled by a retailer using varieties k from a continuum of monopolistically competitive firms according to a CES aggregation technology,

$$y_{j,t} = \left(\int_0^1 y_{jk,t}^{\frac{\epsilon_j - 1}{\epsilon_j}}\, dk \right)^{\frac{\epsilon_j}{\epsilon_j - 1}},$$

[8] We could allow for a more general dividend ownership structure of the form $\phi_{i,t}(z, \Pi_t)$ at the expense of some expositional clarity.

where the elasticity of substitution ϵ_j is a measure of monopolistic competition in sector j. The retailer's demand for inputs from firm k in sector j is given by

$$y_{jk,t} = \left(\frac{p_{jk,t}}{p_{j,t}}\right)^{-\epsilon} y_{j,t}, \tag{6}$$

where $p_{jk,t}$ is the price of firm k in sector j, and $p_{j,t}$ is the price of sector j's "final" good, which we simply refer to as good j going forward.

Dynamic pricing decision. Each firm k in sector j is monopolistically competitive and sets its price $p_{jk,t}$ to maximize the net present value of future profits. The firm faces a Rotemberg (1982) adjustment cost when changing its price, which is given by $\frac{\chi_j}{2}(\frac{p_{jk,t}}{p_{jk,t-1}} - 1)^2 p_{j,t}$. The firm's problem is then to maximize the net present value of appropriately discounted profits net of adjustment costs and subject to equation (6). We state this problem formally in Appendix **??** and derive its solution, which can be summarized by a set of linearized sectoral Phillips curves,

$$\pi_{j,t} = \beta\pi_{j,t+1} + \frac{\epsilon_j}{\chi_j}\left(mc_{j,t} - \frac{\epsilon_j - 1}{\epsilon_j}\right), \tag{7}$$

that characterize the price dynamics in sector j, with $\pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}} - 1$. The sectoral Phillips curves (7) express current inflation in terms of (expected) future inflation and current real marginal cost $mc_{j,t}$, which we further discuss below.

In the derivation of the sectoral Phillips curve, we leverage symmetry across all firms within a sector: With Rotemberg adjustment costs, all firms in a sector remain symmetric ex post as long as they are all initialized with the same price $p_{jk,-1}$. We make this assumption as part of our definition of competitive equilibrium and use it in the remainder of this paper. In particular, symmetry within sectors allows us to represent the entire production structure of the economy at the sectoral level.

Production structure. By symmetry, all firms k in sector j are identical. This allows us to characterize the production network at the sectoral rather than the firm level. For expositional convenience, we proceed *as if* production decisions in sector j were taken by a representative firm, taking as given the evolution of sectoral prices in accordance with (7).

Goods in sector j are produced using a constant-returns technology represented by the sectoral production function

$$y_{j,t} = A_{j,t} F_j\Big(\Big\{x_{jk,t}\Big\}_k, \Big\{N_{ji,t}\Big\}_i\Big) \tag{8}$$

where $A_{j,t}$ is a Hicks-neutral technology shifter, and $x_{jk,t}$ and $N_{ji,t}$ respectively denote firm j's uses of intermediate inputs from sector k and labor factor i. We define sectoral profit as total revenue

$p_{j,t}y_{j,t}$ net of operating expenses, that is

$$\Pi_{j,t} = p_{j,t}y_{j,t} - \sum_k p_{k,t}x_{jk,t} - \sum_i W_{i,t}N_{ji,t} = p_{j,t}y_{j,t} - \boldsymbol{C}_{j,t},$$

where we denote by $\boldsymbol{C}_{j,t}$ the sector's total cost.[9] The real marginal cost of sector j is then given by

$$mc_{j,t} = \frac{\boldsymbol{C}_{j,t}}{y_{j,t}},$$

and is a function of technology and prices, $mc_{j,t} = mc_j(A_{j,t}, \{p_{k,t}\}_k, \{W_{i,t}\}_i)$.

2.3 Government

Monetary policy. The monetary authority sets the path of nominal interest rates. We take $\{i_t\}_{t\geq 0}$ as exogenously given and study monetary policy shocks, i.e., perturbations di_t.

Fiscal policy. The fiscal authority sets an exogenous path for aggregate government spending $\{G_t\}_{t\geq 0}$ and tax revenue $\{T_t\}_{t\geq 0}$. Like households, the government spends on goods from all N sectors. We assume that aggregate government spending is a homothetic aggregator of sectoral expenditures given by $G_t = \mathcal{G}(\{G_{j,t}\}_j)$, which we assume is CES. There then exists a fiscal price index P_t^G that satisfies

$$P_t^G G_t = \sum_j p_{j,t}G_{j,t}.$$

Fiscal policy may be debt-financed, subject to an intertemporal budget constraint that ensures fiscal sustainability. Nominal government debt outstanding B_t then evolves according to

$$B_t = (1 - i_{t-1})B_{t-1} + \sum_j p_{j,t}G_{j,t} - T_t. \tag{9}$$

To raise desired revenue T_t, the government sets tax policy τ_t according to

$$T_t = \sum_i \mu_i \iint \left(zW_{i,t}N_{i,t} + z\Pi_t - \tau_t(zW_{i,t}N_{i,t} + z\Pi_t)^{1-\lambda}\right) g_{i,t}(a,z)\, da\, dz.$$

2.4 Markets and Equilibrium

Equilibrium in our economy requires that the markets for each sectoral good j, the markets for each labor factor i, and the market for bonds clear. Goods market clearing in sector j requires that total

[9] In the appendix, we formally characterize the dual of the firm's profit maximization problem in terms of the cost function $\boldsymbol{C}_{j,t}$. See also Baqaee and Farhi (2020).

production be equal to total use, in real terms, so that

$$y_{j,t} = C_{j,t} + \sum_{i=1}^{N} x_{ij,t} + G_{j,t}, \tag{10}$$

where $C_{j,t} = \sum_i \iint c_{ij,t}(a,z) g_{i,t}(a,z)\, da\, dz$ is total consumer spending on good j and $\sum_{i=1}^{N} x_{ij,t}$ denotes all spending on good j as an intermediate input in production by all other sectors. Labor markets clear when hours worked by households of type i are equal to firms' labor demand for factor i. That is, when

$$\mu_i N_{i,t} = \sum_j N_{ji,t}, \tag{11}$$

where $N_{i,t}$ denotes hours worked by a single household of type i, whose total mass in the economy is μ_i. Finally, the bond market clears when the nominal net asset position of the household sector is equal to the government's outstanding nominal debt, that is

$$\sum_i P_{i,t} \iint a g_{i,t}(a,z)\, da\, dz = B_t. \tag{12}$$

We conclude the description of our baseline model with the definition of competitive equilibrium.

Definition (Competitive Equilibrium)**.** *Given a symmetric initial price distribution $p_{j,-1}$, initial government debt B_{-1}, and an initial cross-sectional distribution $g_{i,-1}(a,z)$, and taking as exogenously given paths for sectoral technology $\{A_{j,t}\}$ as well as monetary and fiscal policy $\{i_t, G_t, T_t\}$ satisfying (9), the competitive equilibrium of the HANK-IO model consists of sequences of prices $\{r_t, p_{j,t}, W_{i,t}\}$, sectoral allocations $\{y_{j,t}, x_{jk,t}, N_{ji,t}\}$, individual allocation rules $\{c_{ij,t}(a,z), n_{i,t}(a,z), a_{i,t}(a,z)\}$, and cross-sectional distributions $\{g_{i,t}(a,z)\}$ such that: households optimize, unions optimize and labor is rationed, firms optimize, and markets clear.*

2.5 GDP, Network Objects, and Sufficient Statistics

Nominal and real GDP. We define nominal GDP as the sum of all expenditures on goods for final use, that is

$$\text{nominal GDP} \equiv Y_t^n = \sum_j p_{j,t}(C_{j,t} + G_{j,t}).$$

Final expenditures shares are then defined as the share of a good j in nominal GDP,

$$b_{j,t} = \frac{p_{j,t} C_{j,t} + p_{j,t} G_{j,t}}{Y_t^n}.$$

It is conceptually challenging to define real GDP in levels in the presence of expenditure heterogeneity. We therefore characterize our analytical results in Section 3 in terms of changes in real GDP, given by

$$d\log Y_t = \sum_j b_{j,t}\Big(d\log C_{j,t} + dG_{j,t}\Big)$$

around an initial stationary equilibrium where government spending is zero. Changes in the GDP deflator are then given by

$$d\log P_t = \sum_j b_{j,t} d\log p_{j,t}.$$

Finally, we take as our numeraire nominal GDP in steady state $Y^n_{ss} = 1$.

Domar weights and input-output matrices. We define the revenue-based Domar weight of sector j as

$$\lambda_{j,t} = \frac{p_{j,t} y_{j,t}}{Y^n_t}.$$

For factor Domar weights, we use $\Lambda_{i,t}$ instead, which is defined as the income share in nominal GDP,

$$\mu_i \Lambda_{i,t} = \frac{\mu_i W_{i,t} N_{i,t}}{Y^n_t},$$

which corresponds to all firms' total spending on labor factor i relative to nominal GDP.

Sufficient statistics for earnings and expenditure heterogeneity. We now introduce two objects that we later show are sufficient statistics for the implications of expenditure and earnings heterogeneity for policy transmission. We introduce a measure of relative household purchasing power (relative price), which is

$$\rho_{i,t} = \frac{P_{i,t}}{P_t},$$

so that we have $d\log \rho_{i,t} = d\log P_{i,t} - d\log P_t$. We also introduce a measure of relative household earnings given by

$$\xi_{i,t} = 1 + \Lambda_{i,t} - \mathbb{E}_i(\Lambda_{i,t}),$$

where $\Lambda_{i,t}$ is household (factor) i's income share. Notice that $\mathbb{E}_i(\xi_{i,t}) = 1$, so $\xi_{i,t}$ is a cross-sectional dispersion measure of households' non-financial income. Also, $\mathbb{E}_i(\Lambda_{i,t}) = \sum_i \mu_i \Lambda_{i,t}$ denotes the average Domar weight of labor factors in the economy, so that $1 - \mathbb{E}_i(\Lambda_{i,t})$ corresponds to the profit share in nominal GDP and is thus a measure of the aggregate markup. There are two useful interpretations of our earnings heterogeneity measure $\xi_{i,t}$. First, $1 + \Lambda_{i,t} - \mathbb{E}_i(\Lambda_{i,t})$ is the aggregate markup plus the Domar weight for factor i. Second, we can also interpret it as 1 plus the Domar weight of factor i relative to the mean Domar weight for labor.

Intertemporal MPCs and interest rate elasticities. Following Auclert et al. (2018), we define the intertemporal marginal propensity to consume (iMPC) for household type i as

$$M_{i,ts} = \frac{\partial \mathcal{C}_{i,t}}{\partial e_{i,s}},$$

which is the propensity to consume at time t out of a marginal increase in *unearned income* at time s for households i. Likewise, we define the interest rate response

$$M^r_{i,ts} = \frac{\partial \mathcal{C}_{i,t}}{\partial r_{i,s}}$$

as the propensity to consume at time t out of a change in the *effective real borrowing rate* of households i at time s. We collect iMPCs and interest rate responses in the matrices $\boldsymbol{M}_i$ and $\boldsymbol{M}^r_i$, whose tsth elements are respectively given by $M_{i,ts}$ and $M^r_{i,ts}$.

3 Analytical Results

Our results in this section make reference to two instructive benchmarks that are nested by the model of Section 2. We briefly describe these now.

HANK benchmark. Our multi-sector model nests a canonical heterogeneous-agent New Keynesian model with a single production sector, $N = 1$, and a single labor factor, which we refer to as the "HANK benchmark". With a single good, all households face the same consumption price index $P_{i,t} = P_t = p_{1,t}$, which is also equal to the price at which the good is sold by retailers. Similarly, with a single labor factor, all households work the same hours, $N_{i,t} = N_t$. As a result, there is neither earnings nor expenditure heterogeneity in the sense that $\rho_{i,t} = \xi_{i,t} = 1$. Auclert et al. (2018) show that equilibrium in this HANK benchmark can be summarized by the equation

$$Y_t = \mathcal{C}^{\text{HANK}}_t(\{Y_s - T_s, r_s\}) + G_t, \tag{13}$$

where $\mathcal{C}^{\text{HANK}}_t(\cdot)$ denotes the aggregate consumption function.

RANK-IO benchmark. The model of Section 2 similarly nests a canonical multi-sector representative-agent New Keynesian model, which we refer to as the "RANK-IO benchmark" (Bouakez et al., 2009; Pasten et al., 2020; Baqaee et al., 2021). With a representative household, there is only one labor factor and a single consumption price index P_t. Aggregate consumption satisfies a standard Euler equation of the form $u'(C_t) = \beta R_{t+1} u'(C_{t+1})$. The real interest rate in this benchmark admits a sequence-space representation

$$r_t = \mathcal{R}^{\text{RANK-IO}}_t(\{Y_s, G_s, i_s, A_{j,s}\}), \tag{14}$$

which determines the real interest rate at time t as a function of the time paths of aggregate activity and policy shocks.

3.1 An "As-If" Benchmark

We start our discussion of policy transmission in the HANK-IO model by considering an instructive *as-if* benchmark that obtains when we shut down earnings and expenditure heterogeneity across types. The as-if benchmark highlights starkly under which conditions the interaction between household and sectoral heterogeneity matters and household-sector linkages affect policy transmission. Formally, we make two assumptions in this subsection.

Assumption A1. The consumption preferences of all household types are represented by the same homothetic consumption aggregator. That is, $u_i(\cdot) = u(\cdot)$ and $\mathcal{D}_i(\cdot) = \mathcal{D}(\cdot)$.

Under Assumption A1, all households aggree on the same consumption bundle price index, which we can denote by $P_{i,t} = P_t$. This also implies that there is no expenditure heterogeneity and $\rho_{i,t} = 1$. Under symmetric consumption preferences, we can also use P_t to unambiguously define real GDP in levels, i.e., $Y_t^n = P_t Y_t$.

Assumption A2. All households are endowed with and supply a single labor factor.

Under Assumption A2, households of all types work the same hours, which we can denote $N_{i,t} = N_t$, and face the same nominal wage, $W_{i,t} = W_t$. There is consequently no earnings heterogeneity and $\xi_{i,t} = 1$.

These two assumptions together imply that the consumption function of households in equilibrium admits the sequence-space representation $c_{i,t}(a,z) = c_i(a, z, \{Y_s - T_s, r_s\})$. We denote by $r_s = i_s - \pi_s$ the net real interest rate defined in terms of inflation in the price index P_t and by $Y_s - T_s$ the aggregate after-tax real income received by households. In other words, all households face the same price of intertemporal substitution, r_t, and the exposure of their non-financial income to aggregate activity can be summarized simply by $Y_s - T_s$. As a result, intertemporal MPCs are symmetric, with $\boldsymbol{M}_i = \boldsymbol{M}$ for all types i.

Proposition 1 (Intertemporal IS-LM Representation)**.** *Assume that all households have the same homothetic preferences over consumption goods (A1) and there is a single labor factor (A2). Then, our HANK-IO economy admits an intertemporal IS-LM representation, given by*

$$Y_t = \mathcal{C}^{HANK}\Big(\{Y_s - T_s, r_s\}_{s \geq t}\Big) + G_t \tag{15}$$

$$r_t = \mathcal{R}^{RANK\text{-}IO}\Big(\{Y_s, G_s, T_s, i_s, A_{j,s}\}_{s \geq 0}\Big), \tag{16}$$

where $\{G_s, T_s, i_s\}$ are exogenous policy perturbations and $\{A_{j,s}\}$ denotes exogenous technology shocks.

Our as-if benchmark features a complete decoupling between household and sectoral heterogeneity in the following sense: Household heterogeneity only matters for the determination of aggregate demand Y_t, while sectoral heterogeneity only affects the determination of the real interest rate r_t.

Under assumptions A1 and A2, the HANK-IO model's competitive equilibrium admits an intertemporal IS-LM representation as a system of two forward-looking equations: the dynamic IS curve (15) and the dynamic LM curve (16). According to Proposition 1, the dynamic IS curve is shaped by household but not by sectoral heterogeneity: Given a path of real interest rates $\{r_s\}_{s\geq t}$, it takes the same form as the IS equation of the one-sector HANK model (13). Conversely, the dynamic LM curve is shaped by sectoral but not by household heterogeneity: It maps a given path of aggregate demand $\{Y_s\}_{s\geq t}$ to the same path of real interest rates as the representative-household multi-sector RANK-IO model. In this sense, the determination of aggregate demand is unaffected by sectoral heterogeneity taking as given a path of real interest rates, while the evolution of real interest rates is independent from household heterogeneity taking as given a path of aggregate activity. In other words, household and sectoral heterogeneity respectively shape the demand and supply sides of the as-if economy but do not interact with each other beyond that.

We now leverage Proposition 1 to characterize the transmission of stabilization policy and the aggregate effects of sectoral shocks in the as-if economy. These results will serve as important reference points when we revisit the same questions in the presence of earnings and expenditure heterogeneity in Sections 3.2 and 3.3. Here and throughout, we use bold-faced notation to denote time paths, so $d\boldsymbol{Y} = \{dY_s\}_{s\geq 0}$.

Corollary 2 (Fiscal Policy)**.** *Under a monetary policy rule that stabilizes the real interest rate, $d\boldsymbol{r} = 0$, the effect of an untargeted fiscal policy shock $\{d\boldsymbol{G}, d\boldsymbol{T}\}$ on activity $d\boldsymbol{Y}$ is characterized by the Intertemporal Keynesian Cross*

$$d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T} + \boldsymbol{M}d\boldsymbol{Y}. \tag{17}$$

If $\mathcal{M}$ is a linear map with $(\boldsymbol{I} - \boldsymbol{M})\mathcal{M} = \boldsymbol{I}$, then the solution of (17) is $d\boldsymbol{Y} = \mathcal{M}(d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T})$.[10]

The as-if benchmark of our economy admits the same Intertemporal Keynesian Cross characterization of untargeted fiscal policy shocks as the canonical one-sector HANK model (Auclert et al., 2018).[11] In particular, the iMPC matrix $\boldsymbol{M}$ remains a sufficient statistic for the effects of fiscal policy. It is observationally equivalent to that in the one-sector HANK model.

[10] We assume throughout that all sequences are bounded and that GE multipliers such as $\mathcal{M}$ exist.

[11] Untargeted fiscal policy refers to a shock to aggregate spending $d\boldsymbol{G}$, holding constant spending weights, as distinct from a shock to sectoral spending $d\boldsymbol{G}_j$. The sectoral expenditure responses under untargeted fiscal policy implied by $d\boldsymbol{G}$ are governed by the homothetic government spending aggregator $\mathcal{G}(\cdot)$. We study targeted fiscal spending shocks in our quantitative analysis in Section 4.

This result obtains in spite of substantial sectoral heterogeneity that is masked by the dynamic LM equation. When monetary policy neutralizes indirect effects through the real interest rate, untargeted fiscal policy is entirely unaffected by sectoral heterogeneity. In particular, fiscal multipliers in our as-if benchmark are identical to any HANK economy without sectoral heterogeneity that admits a representation of the aggregate consumption function as in (15).

Sectoral heterogeneity does, however, affect the monetary policy response $d\boldsymbol{i}$ that is necessary to neutralize real rates since this policy rule is governed by the dynamic LM equation. In fact, the nominal interest rate rule $d\boldsymbol{i}$ is determined solely by the dynamic LM curve, taking as given aggregate demand $\{Y_s\}$, which implies that it is governed by sectoral but not by household heterogeneity.

Corollary 3 (Monetary Policy)**.** *The response of real GDP $d\boldsymbol{Y}$ to a real interest rate perturbation $\{d\boldsymbol{r}\}$ effected by monetary policy is characterized by the Intertemporal Keynesian Cross*

$$d\boldsymbol{Y} = \boldsymbol{M}^r d\boldsymbol{r} + \boldsymbol{M} d\boldsymbol{Y}. \tag{18}$$

If $\mathcal{M}$ is a linear map with $(\boldsymbol{I} - \boldsymbol{M})\mathcal{M} = \boldsymbol{I}$, then the solution of (18) is $d\boldsymbol{Y} = \mathcal{M}(\boldsymbol{M}^r d\boldsymbol{r})$.

For a given change in the real interest rate, $d\boldsymbol{r}$, the effect of monetary policy on aggregate activity is again completely independent of sectoral heterogeneity and solely shaped by household heterogeneity. In particular, the iMPC and interest rate response matrices that appear in Corollary 3, $\boldsymbol{M}$ and $\boldsymbol{M}^r$, are equivalent to their counterparts in the one-sector HANK economy.

Sectoral heterogeneity does, however, affect the transmission of nominal interest rate shocks, $d\boldsymbol{i}$, to the real interest rate, $d\boldsymbol{r}$. In particular, heterogeneity in price rigidities and other sectoral variables does shape the strength of interest rate policy, i.e., monetary non-neutrality, but only through its transmission to real rates. Corollay 3 demonstrates that the path of real interest rates is a summary statistic for the effect of monetary policy on aggregate activity. Once the monetary authority has implemented a desired path of real rates, the production network structure of the economy no longer matters for transmission.

Corollary 4 (Aggregating Sectoral Technology Shocks)**.** *The response of real GDP to sectoral technology shocks is*

$$d \log Y_t = \underbrace{\frac{1+\eta}{\gamma+\eta} \tilde{\lambda} \, d \log A_t}_{\text{Pure Technology Effect}} \underbrace{- \frac{1+\eta}{\gamma+\eta} \tilde{\lambda} \, d \log \mu_t - \frac{\eta}{\gamma+\eta} \, d \log \Lambda_t}_{\text{Change in Allocative Efficiency}} \tag{19}$$

where $\gamma = -\frac{C_{ss} u''(C_{ss})}{u'(C_{ss})}$ and $\eta = \frac{N_{ss} v''(N_{ss})}{v'(N_{ss})}$.

Corollary 4 is an aggregation result that traces the macroeconomic effects of microeconomic sectoral technology shocks, with $A_t = (A_{1,t}, \ldots, A_{N,t})$. As in Baqaee and Farhi (2020), equation (19) decomposes the impact on real GDP into two effects. The pure technology effect holds fixed the resources employed by firms and workers, and measures the change in output that results from the increased productivity of given resources. The second effect represents the change in allocative efficiency and summarizes the effect on output from a reallocation of resources across firms and workers. We denote by $\mu_{j,t} = \frac{p_{j,t}}{mc_{j,t}}$ the time-varying markup in sector j, so that $d \log \mu_t$ captures the endogenous response of sectoral markups. Finally, since there is a single labor factor in our as-if benchmark, $d \log \Lambda_t$ is the change in the labor income share.

It is remarkable that equation (19) is virtually identical to the aggregation result of Baqaee and Farhi (2020).[12] This is despite our economy featuring rich and dynamic household heterogeneity, whereas theirs is a static representative-household setting. Corollary 4 therefore underscores further that the as-if baseline of our HANK-IO economy features a strict decoupling of household and sectoral heterogeneity: For given changes in markups and factor shares, the aggregate consequences of microeconomic technology shocks do not directly interact with household heterogeneity. In other words, changes in sectoral markups $d \log \mu_t$ and the labor income share $d \log \Lambda_t$ are sufficient statistics for the implications of household heterogeneity. Our key result in Section 3.3 is that this simple benchmark is no longer valid in the presence of systematic household-sector linkages: With earnings and expenditure heterogeneity, the Baqaee and Farhi (2020) aggregation result must be augmented to account for household heterogeneity.

The importance of allocative efficiency is tightly linked to nominal rigidities. In the flexprice limit of our as-if benchmark, where markups are positive but constant, equation (19) becomes

$$d \log Y_t^{\text{flex}} = \frac{1+\eta}{\gamma+\eta} \tilde{\lambda}' \, d \log A_t. \tag{20}$$

This flexprice aggregation result still features two key departures from the canonical Hulten's theorem, according to which the macroeconomic effect of a sectoral technology shock is proportional to that sector's revenue-based Domar weight, λ. First, the importance of sectoral technology shocks is governed by the cost-based Domar weight $\tilde{\lambda}_j = \mu_j \lambda_j = \frac{\epsilon_j}{\epsilon_j - 1} \lambda_j$. Second, the standard Hulten's theorem applies in settings with inelastic factor supply. The multiplier $\frac{1+\eta}{\gamma+\eta}$ accounts for elastic labor supply, with the elasticities η and γ governing the household labor supply curve. When we allow for appropriate sectoral employment subsidies to offset steady state markups, $\mu_j = 1$ for all j, a variant of Hulten's theorem extended to elastic labor supply applies in the as-if benchmark with flexible prices. In this case, production is efficient and aggregation on the production side of the

[12] In their baseline model, Baqaee and Farhi (2020) derive the aggregation result $d \log Y = \tilde{\lambda} \, d \log A - \tilde{\lambda} \, d \log \mu - \tilde{\Lambda} \, d \log \Lambda$. There are minor differences between our result and theirs. First, our setting is dynamic but equation (19) holds in each period. Second, we allow for elastic factor supply, which accounts for the presence of the elasticities η and γ. Baqaee and Farhi (2020) extend their main result to elastic factor supply in Appendix H.2 and equation (19) mirrors their extended formula. Finally, markups in our setting are endogenous and result from nominal rigidities. Nonetheless, Corollary 4 underscores that the importance of changes in markups are captured in reduced form by $\frac{1+\eta}{\gamma+\eta} \tilde{\lambda}$, as in Baqaee and Farhi (2020). Also notice that cost-based factor shares always sum to 1, and so $\tilde{\Lambda} = 1$ with a single factor.

economy is again governed by revenue-based Domar weights despite rich household heterogeneity.

3.2 An Intertemporal Keynesian Cross with Earnings and Expenditure Heterogeneity

Section 3.1 characterizes policy transmission and the aggregation of sectoral shocks in an *as-if* benchmark that shuts off earnings and expenditure heterogeneity. We are now ready to study the implications of household-sector linkages in the full HANK-IO model of Section 2.

Our main result in this subsection characterizes an Intertemporal Keynesian Cross for monetary and fiscal policy. We derive our result to first order around a stationary equilibrium with no government spending, $G_{ss} = 0$, and we take as our numeraire initial nominal GDP, $Y^n_{ss} = 1$. For notational simplicity, we drop time subscripts for variables evaluated at steady state, so that $\xi_i = \xi_{i,ss}$ denotes the earnings share of type i in stationary equilibrium. For tractability, we assume that $\lambda = 0$ and we denote iMPC and interest rate response matrices in terms of log changes, i.e., $M_{i,ts} = \frac{\partial \log C_{i,t}}{\partial \log e_{i,s}}$.

Proposition 5 (Intertemporal Keynesian Cross in HANK-IO)**.** *The aggregate effects of fiscal and monetary policy are characterized by*

$$\underbrace{(\boldsymbol{I} - \bar{\boldsymbol{M}}_i)}_{\text{Multiplier}} d\log \boldsymbol{Y} = \overbrace{P_G d\boldsymbol{G} - \bar{\boldsymbol{M}}_i d\boldsymbol{T} + \bar{\boldsymbol{M}}^r_i d\boldsymbol{r}}^{\text{As-If Benchmark}}$$

$$+ \underbrace{\mathbb{C}ov_i\left(\frac{\boldsymbol{M}_i}{\xi_i}, d\boldsymbol{\xi}_i\right)}_{\text{Earnings Heterogeneity}} - \underbrace{\mathbb{C}ov_i\left(\boldsymbol{M}_i, d\boldsymbol{\rho}_i\right)}_{\substack{\text{Expenditure Heterogeneity:} \\ \text{Income Effect}}} - \underbrace{\mathbb{C}ov_i\left(\boldsymbol{M}^r_i, d\log \Delta\boldsymbol{\rho}_i\right)}_{\substack{\text{Expenditure Heterogeneity:} \\ \text{Intertemporal Substitution Effect}}}$$

where $\bar{\boldsymbol{M}}_i = \mathbb{E}_i(P_i C_i \boldsymbol{M}_i)$ and $\bar{\boldsymbol{M}}^r_i = \mathbb{E}_i(P_i C_i \boldsymbol{M}^r_i)$ are the cross-sectional weighted average iMPC and interest rate response matrices.

Proposition 5 characterizes the implications of household-sector linkages for policy transmission. Three new effects emerge relative to the transmission channels already operative in our as-if benchmark.

The first new effect is an *earnings heterogeneity channel* that emerges when households supply different labor factors. It is captured by a cross-sectional covariance across household types between iMPCs $\boldsymbol{M}_i$ and changes in earnings shares $d\boldsymbol{\xi}_i$. Using its definition, we can rewrite the change in earnings shares as $d\boldsymbol{\xi}_i = d(\boldsymbol{\Lambda}_i - \mathbb{E}_i\boldsymbol{\Lambda}_i)$, i.e., the change in factor i's income share net of the change in the profit share or aggregate markup in the economy. And since the earnings heterogeneity channel is governed by a covariance across household types, we are then simply left with $\mathbb{C}ov_i(\frac{\boldsymbol{M}_i}{\xi_i}, d\boldsymbol{\Lambda}_i)$. Intuitively, this is because we assume all household types have symmetric exposure to corporate dividends. Changes in earnings shares are thus entirely explained by changes in factors income shares. Earnings heterogeneity therefore amplifies the aggregate effects of monetary and fiscal policy when policy redistributes income share to factors with large iMPCs.

The next two terms of our decomposition capture the implications of *expenditure heterogeneity*. A change in the price path $P_{i,t}$ of household type i's consumption basket elicits income and substitution effects. If the price of i's basket increases holding fixed the current allocation, then households' effective purchasing power falls and they are poorer in real terms. Households respond to such an income effect in proportion to their marginal propensities to consume. The first new channel due to expenditure heterogeneity is therefore a covariance across household types between iMPCs $\boldsymbol{M}_i$ and changes in relative bundle prices $d\boldsymbol{\rho}_i = d\boldsymbol{P}_i - d\boldsymbol{P}$. When policy leads to a relative price increase in the consumption basket of households (and in states of the world) with large iMPCs, then aggregate consumer spending falls.

A change in bundle price $P_{i,t}$ also elicits an *intertemporal substitution effect*. The price of a household's consumption basket governs her decision between total consumption in period t and consumption in all future periods, i.e., savings. Given our assumption of homothetic preferences, this effect does not capture substitution across goods for a given amount of total expenditures. In other words, the bundle price $P_{i,t}$ govern's the household's intertemporal substitution. In fact, we can rewrite this channel in terms of households' effective real interest rate, noting that $d\log\Delta\boldsymbol{\rho}_i = d\log\boldsymbol{R} - d\log\boldsymbol{R}_i$. Therefore, we have

$$-\mathbb{C}ov_i\left(\boldsymbol{M}_i^r,\, d\log\Delta\boldsymbol{\rho}_i\right) = -\mathbb{C}ov_i\left(\boldsymbol{M}_i^r,\, d\log\boldsymbol{R} - d\log\boldsymbol{R}_i\right) = \mathbb{C}ov_i\left(\boldsymbol{M}_i^r,\, d\log\boldsymbol{R}_i\right).$$

When households experience *relative* price inflation, their effective real rate of return on savings $d\log\boldsymbol{R}_i$ falls disproportionately. And when relative price inflation is especially large for those households with strong intertemporal interest rate responses $\boldsymbol{M}_i^r$, then aggregate demand increases.

Finally, we refer to Proposition 5 as an Intertemporal Keynesian Cross because changes in consumption and government spending are amplified by a Keynesian-cross-like multiplier (Auclert et al., 2018). As in our as-if benchmark, this multiplier is given by the linear map $\mathcal{M} = (\boldsymbol{I} - \bar{\boldsymbol{M}}_i)^{-1}$, where $\bar{\boldsymbol{M}}_i = \mathbb{E}_i(P_iC_i\boldsymbol{M}_i)$ is the average iMPC across household types weighted by their nominal outlays. Therefore, this Keynesian multiplier is not directly shaped by household-sector linkages.

3.3 Beyond Hulten: An Aggregation Result for Sectoral Shocks

In this subsection, we derive our aggregation result for the macroeconomic effects of sectoral technology shocks, $d\log A_t$. For expositional clarity, we state this result in terms of reduced-form sufficient statistics as in Baqaee and Farhi (2020). Concretely, we decompose the aggregate effect on real GDP into a pure technology effect and changes in allocative efficiency, which we capture by endogenous markup responses, $d\log\mu_t$, and changes in factor shares, $d\log\Lambda_t$. Unlike in our as-if benchmark, allocative efficiency is now also shaped by earnings and expenditure heterogeneity.

Proposition 6 (Aggregation Result for Sectoral Technology Shocks in HANK-IO)**.** *Assuming symmetric $\gamma_i = \gamma$ and $\eta_i = \eta$, the aggregate effect of sectoral technology shocks is given by*

$$d\log Y_t = \overbrace{\frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log A_t - \frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log \mu_t - \frac{\eta}{\gamma+\eta}\, d\log \Lambda_t}^{\text{As-If Benchmark}}$$
$$- \underbrace{\frac{1}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_i,\, d\log \rho_{i,t}\right)}_{\text{Expenditure Heterogeneity}} - \underbrace{\frac{\eta}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_i,\, d\log \xi_{i,t}\right)}_{\text{Earnings Heterogeneity}} - \underbrace{\frac{\gamma}{\gamma+\eta}\mathbb{C}ov_i\left(\tilde{\Lambda}_i,\, d\log \delta_{i,t}\right)}_{\text{Income Effect on Labor Supply}}$$

where $d\log \delta_{i,t} = d\log C_{i,t} - d\log C_t$.

Proposition 6 generalizes the aggregation result we derived for our as-if benchmark (Corollary 4). The three terms in the first line correspond to that benchmark.

The aggregate output response is again governed by a pure technology effect and changes in allocative efficiency. The pure technology effect, given by $\frac{1+\eta}{\gamma+\eta}\tilde{\lambda}\, d\log A_t$, summarizes the increased productivity of resources holding fixed the current allocation. It is remarkable that pure technology gains are again unaffected by household heterogeneity. They are proportional to sectors' cost-based Domar weights, $\tilde{\lambda}$, and to the elasticities governing household labor supply, γ and η. Our assumption that these elasticities are symmetric across household types is critical: When the income and substitution effects governing labor supply are heterogeneous at a given allocation, then gains from pure technology are also shaped by household heterogeneity.[13]

The remaining terms of our decomposition summarize changes in allocative efficiency. As in the as-if benchmark, changes in endogenous markups and factor shares have implications for allocative efficiency. These terms are exactly as in Corollary 4, and we discuss their economic intuition there.

More surprisingly, household heterogeneity and, in particular, cross-sectional household-sector linkages now also shape allocative efficiency. Three additional channels emerge that all work

[13] Assuming heterogeneous γ_i and η_i, the aggregation result can instead be stated as

$$\sum_i \tilde{\Lambda}_i \frac{\gamma_i+\eta_i}{1+\eta_i}\, d\log Y_t \;=\; \tilde{\lambda}\, d\log A_t - \tilde{\lambda}\, d\log \mu_t - \sum_i \tilde{\Lambda}_i \frac{\eta_i}{1+\eta_i}\, d\log \Lambda_t$$
$$- \sum_i \tilde{\Lambda}_i \frac{1}{1+\eta_i} d\log \rho_{i,t} - \sum_i \tilde{\Lambda}_i \frac{\eta_i}{1+\eta_i}\, d\log \xi_{i,t} - \sum_i \tilde{\Lambda}_i \frac{\gamma_i}{1+\eta_i}\, d\log \delta_{i,t}.$$

Gains from pure technology are then given by

$$\left[\mathbb{E}_i\left(\frac{\gamma_i+\eta_i}{1+\eta_i}\right) + \mathbb{C}ov_i\left(\tilde{\Lambda}_i,\, \frac{\gamma_i+\eta_i}{1+\eta_i}\right)\right]^{-1} \tilde{\lambda}\, d\log A_t.$$

through the factor supply equations given by

$$d\log W_{i,t} - d\log P_{i,t} = \eta\, d\log N_{i,t} + \gamma\, d\log C_{i,t}.$$

Households of type i adjust their labor supply either in response to changes in their real wage, $d\log W_{i,t} - d\log P_{i,t}$, or when their income and consequently their consumption changes, $d\log C_{i,t}$.

The first novel determinant of allocative efficiency is the effect of expenditure heterogeneity on household labor supply. Firms' cost structure is governed by nominal factor prices $W_{i,t}$. In other words, what matters for the production side is the marginal cost at which firms can hire additional labor. Labor supply, however, is governed by real wages. When the price of a household's consumption bundle increases, $d\log \rho_{i,t} > 0$, a given nominal wage for household i has less purchasing power and the real wage falls. At a given nominal wage, this household now supplies fewer hours and labor becomes more expensive for firms. The new effect on allocative efficiency is proportional to the cross-sectional covariance across household types or labor factors $\mathbb{C}ov_i(\tilde{\Lambda}_i,\, d\log \rho_{i,t})$: If consumer prices *increase* for those households whose labor factors have *large* cost-based Domar weights, then output falls and allocative efficiency deteriorates. Intuitively, labor factors that play a dominant role in firms' cost structures become relatively more expensive because those household types experience a relative drop in their purchasing power. Crucially, the relevant covariance is with respect to *cost-based* factor shares $\tilde{\Lambda}_i$ because markups drive a wedge between prices and marginal costs.

Earnings heterogeneity also shapes the transmission of technology shocks to allocative efficiency. The overall effect of changes in factor shares on allocative efficiency is captured by

$$-\frac{\eta}{\gamma+\eta}\sum_i \tilde{\Lambda}_i\, d\log \Lambda_{i,t} = \underbrace{-\frac{\eta}{\gamma+\eta} d\log \Lambda_t}_{\text{Labor Income Share}} - \underbrace{\frac{\eta}{\gamma+\eta}\mathbb{C}ov_i(\tilde{\Lambda}_i,\, d\log \Lambda_{i,t})}_{\text{Earnings Heterogeneity}}$$

where we denoted the average change in factor share by $d\log \Lambda_t = \mathbb{E}_i(d\log \Lambda_{i,t})$. Notice that the aggregate labor income share is not weighted by cost-based Domar weights because $\mathbb{E}_i\tilde{\Lambda}_i = 1$: All production costs must ultimately be accounted for by factor costs. The aggregate effect is also operative in the as-if benchmark. Intuitively, when the total labor income share falls, $d\log \Lambda_t < 0$, then households receive less labor income. This elicits a positive labor supply response governed by the elasticities $\frac{\eta}{\gamma+\eta}$. Heterogeneity in factor share responses can amplify or dampen the resulting change in allocative efficiency. When factors with the *largest* cost-based Domar weights $\tilde{\Lambda}_i$ experience a relative *decrease* in their income share, then resources are reallocated towards the more monopolized and distorted sectors. The strength of this effect is again proportional to $\frac{\eta}{\gamma+\eta}$, which governs the endogenous labor supply response to the fall in income.

Finally, there may be a heterogeneous income effect on labor supply across factors. This effect is summarized by a covariance across household types between cost-based factor shares $\tilde{\Lambda}_i$ and changes in consumption dispersion $d\log \delta_{i,t}$. Intuitively, if households i experience a relative drop in consumption, $d\log\delta_{i,t} < 0$, then they supply more labor for a given real wage. This income effect is governed by the elasticities $\frac{\gamma}{\gamma+\eta}$. As a result, firms can hire a given amount of labor factor i more cheaply and marginal cost falls. If the factors that become relatively cheaper also have high cost-based Domar weights, then output increases due to gains in allocative efficiency. Intuitively, marginal costs then fall in the most monopolized sectors.

Our discussions in Sections 3.2 and 3.3 point to an important conceptual takeaway. What matters for the aggregation of sectoral technology shocks—in particular their transmission through changes in allocative efficiency—are covariances with respect to cost-based factor shares $\tilde{\Lambda}_i$. Gains from allocative efficiency result from a reallocation of resources to relatively more distorted and monopolized sectors. The cost-based factor share $\tilde{\Lambda}_i$ pricely captures those markups that drive a wedge between prices and marginal costs. The covariance terms with respect to $\tilde{\Lambda}_i$ that appear in Proposition 6 therefore indicate whether resources and labor find more productive uses. The transmission of monetary and fiscal policy (demand) shocks, on the other hand, is determined by covariances with respect to iMPCs $\boldsymbol{M}_i$. Intuitively, demand propagation is governed by spending propensities in response to changes in income and prices. Mechanically, this distinction emerges because our results in Section 3.2 take as their starting point the goods market clearing condition and the aggregate consumption function, whereas our derivations in Section 3.3 focus on the supply and production equations.

4 Taking a Quantitative HANK-IO Model to the Data

This section develops a quantitative HANK-IO model that matches the key empirical regularities that suggest systematic household-sector linkages in the micro data. We enrich our baseline environment of Section 2 by adding capital as an additional production factor and as a second, illiquid asset households can trade. Our analytical results highlight that the novel policy transmission channels that emerge in the presence of household-sector linkages are governed by intertemporal marginal propensities to consume. It is well understood that matching the empirically observed (i)MPCs in a HANK-type model is difficult with a single liquid asset (Kaplan and Violante, 2022). Adding an illiquid asset allows us to overcome this challenge. Adding capital in production also allows us to capture the importance of the investment network that is an important part of the economy's network structure (Vom Lehn and Winberry, 2022).

We introduce these key new model elements and discuss our calibration in Section 4.1. We then present reduced-form evidence in Section 4.2 to show that our model matches the key household-sector linkages we observe in the data. Appendix B presents a self-contained description of our quantitative model.

4.1 Key Model Elements and Calibration

4.1.1 Production Network

We calibrate the production network of our quantitative model to match data on 66 sectors. As described in Section 2.2, each sector is modeled as comprising a retailer and intermediate firms that face price adjustment costs. While this structure allows us to derive sectoral Phillips curves (7), we otherwise focus on a symmetric equilibrium in which all firms within a sector are identical. We can therefore proceed as if sectoral production decisions are taken by a representative firm.

The production function of sector j is CES over intermediate inputs and a primary factor that combines capital and labor, given by

$$y_{j,t} = A_{j,t}\left((1-\theta_j)^{\frac{1}{\eta_{f,j}}} f_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} + \theta_j^{\frac{1}{\eta_{f,j}}} x_{j,t}^{\frac{\eta_{f,j}-1}{\eta_{f,j}}}\right)^{\frac{\eta_{f,j}}{\eta_{f,j}-1}}, \tag{21}$$

where $A_{j,t}$ is a Hicks-neutral technology shifter. We denote by θ_j the CES weight on intermediate inputs in sector j's production and by $\eta_{f,j}$ the elasticity of substitution between the primary factor and intermediate inputs.

The primary factor is a Cobb-Douglas aggregate of capital and labor, given by

$$f_{j,t} = K_{j,t}^{\alpha_j} N_{j,t}^{1-\alpha_j}, \tag{22}$$

where α_j is the share of capital in total factors.[14] We denote by $K_{j,t}$ the capital rented by sector j in period t and by $N_{j,t}$ a CES aggregate over all I labor factors used by sector j in production, given by

$$N_{j,t} = \left(\sum_i \left(\Gamma_{ji}^w\right)^{\frac{1}{\eta_{w,j}}} N_{ji,t}^{\frac{\eta_{w,j}-1}{\eta_{w,j}}}\right)^{\frac{\eta_{w,j}}{\eta_{w,j}-1}}, \tag{23}$$

where $N_{ji,t}$ is sector j's demand for labor factor i, Γ_{ji}^w is the relative CES weight on factor i, and $\eta_{w,j}$ is a sector-specific elasticity of substitution across labor factors in production.

Sector j uses a CES basket of intermediate inputs, $x_{j,t}$, given by

$$x_{j,t} = \left(\sum_k \left(\Gamma_{jk}^x\right)^{\frac{1}{\eta_{x,j}}} x_{jk,t}^{\frac{\eta_{x,j}-1}{\eta_{x,j}}}\right)^{\frac{\eta_{x,j}}{\eta_{x,j}-1}}, \tag{24}$$

where $\eta_{x,j}$ is sector j's constant elasticitiy of substitution across intermediate inputs. Γ_{jk}^x denotes the CES weight on good k in sector j's production function. As in Section 2, $x_{jk,t}$ is the demand for

[14] Horvath (2000), Carvalho (2014), Atalay (2017), Carvalho et al. (2021) and Ferrante et al. (2022) all aggregate primary factors using a Cobb-Douglas calibration. Vom Lehn and Winberry (2022) use Cobb-Douglas in their main calibration and explore deviations from Cobb-Douglas in a sensitivity analysis, where they show there are no sizable quantitative implications.

good k as an input by sector j.

Firms rent capital in an integrated and competitive market at the nominal rental rate i_t^K. Likewise firms hire labor of each factor in non-segmented labor markets at nominal wage rates $W_{i,t}$. Sectoral profits are now given by

$$\Pi_{j,t} = p_{j,t} y_{j,t} - \sum_i W_{i,t} N_{ji,t} - i_t^K K_{j,t} - \sum_k p_{k,t} x_{jk,t}.$$

Elasticities. We follow much of the literature and set the elasticity between the primary factor and intermediate inputs to $\eta_{f,j} = 1$ across sectors.[15] Consensus on the appropriate calibration of $\eta_{x,j}$, the elasticity of substitution across intermediate inputs, has evolved over time. While most prior work uses a Cobb-Douglas calibration, Atalay (2017) argues that this elasticity should be much smaller. We follow Atalay (2017) and Vom Lehn and Winberry (2022) and set $\eta_{x,j} = 0.1$. Finally, we calibrate the elasticity of substitution between different labor factors to $\eta_{w,j} = 1$.

Factor shares. Sectors differ in the share of intermediate inputs in production, θ_j, and the share of capital in primary factors, α_j. We compute 66 sector-specific factor shares from the BEA GDP-by-Industry dataset. The intermediate input share θ_j is computed as input expenditures as a percentage of gross output, averaged over 1997-2021. We compute the labor share $1 - \alpha_j$ as total compensation of employees as a percentage of value added, adjusted for taxes and subsidies, averaged over the same period. We show in Appendix **??** that there is substantial heterogeneity in factor shares across sectors.

Input-output network. We calibrate the CES weights on intermediate inputs Γ_{jk}^x so that our model's production network is consistent with the BEA's input-output table. We use the industry input-ouptut "use" table and compute sector j's nominal expenditures on intermediate inputs from sector k as a share of its total expenditure on inputs, averaged over 1997-2021.

Sectoral price rigidities. To measure sectoral price rigidities, we use data made publicly available by Nakamura and Steinsson (2008) who estimate the frequency of price changes using the U.S. Bureau of Labor Statistics (BLS) micro data underlying the Consumer Price Index (CPI). Nakamura and Steinsson (2008) estimate the frequency of price changes for 272 product categories. Since these estimates are more granular than our production network, we follow Clayton et al. (2018) and use their "many-to-one" merge from product categories to our 66 production sectors.

The price adjustment frequency estimated by Nakamura and Steinsson (2008) naturally maps into a Calvo parameter. Pinning down the Rotemberg adjustment cost parameter χ_j in our model therefore requires a formal mapping between the two models. In the Calvo model, a randomly selected fraction of firms, $1 - \theta_j$, can adjust their price in a given period. The slope of the resulting

[15] Atalay (2017), Vom Lehn and Winberry (2022), and most prior work use this calibration.

NKPC is $(1-\theta_j)(1-\theta_j\beta)/\theta_j$. The slope of sectoral Phillips curves in our model with Rotemberg adjustment costs is $\frac{\epsilon_j-1}{\chi_j}$. To first order, we can therefore calibrate $\chi_j = \frac{(\epsilon_j-1)\theta_j}{(1-\theta_j)(1-\theta_j\beta)}$, where we take θ_j from the data as the probability that prices remain unchnaged for a quarter.[16]

Wage share. To measure sectoral labor shares—and consequently the share of salary expenditures paid to different household types i—we use data from the American Community Survey (ACS), made available by IPUMS (Ruggles et al., 2015). The ACS survey provides data on income, education, and sector of occupation. Following Clayton et al. (2018), we compute the average payroll share of college-educated households over the sample period from 2000 to 2015, and use a "many-to-one" merge from ACS industry identifiers to our 66 sector production network.

Markups. Steady state markups across sectors are given by $\mu_j = \frac{\epsilon_j}{\epsilon_j-1}$. While our baseline model sets $\epsilon_j = 11$ for all sectors, we explore the implications of markup heterogeneity in several of our quantitative experiments. For these exercises, we calibrate ϵ_j directly to match sectoral markups using data from Baqaee and Farhi (2020). They use three alternative approaches to estimate sectoral markups for 66 sectors from 1997 to 2015. The average markup for each sector in any particular year is computed as the harmonic sales-weighted average of firm markups, which are taken from Compustat and assigned to BEA sectors. We use the average of their benchmark estimates following the accounting profits approach because the average markup is then around 10% and thus closer to the standard markup assumed in the HANK literature. We assign the average markup of 10% for missing values. Appendix **??** shows that there is substantial heterogeneity in the mark-ups across sectors.

4.1.2 Households

Our quantitative model features two household types—and consequently two labor factors—that we associate with and calibrate to match data about college educated and non-college educated households, $i \in \{C, NC\}$. We allow these types to differ in their permanent characteristics along three dimensions: (i) marginal propensities to consume, (ii) earnings shares across sectors, and (iii) expenditure shares across sectors.

Illiquid asset. We allow households to trade two assets, a liquid checking account a and an illiquid investment account b held with a bank. Households can move funds between these two

[16] There is a strand of literature that studies the difference between Rotemberg and Calvo models. Our approach follows most closely Sims and Wolff (2017). Nistico (2007) and Lombardo and Vestin (2008) compare the welfare implications of the two models. Ascari et al. (2011) and Ascari and Rossi (2012) investigate the differences between the two models under a positive trend inflation rate. Ascari and Rossi (2011) study the effect of a permanent disinflation in the Rotemberg and Calvo models. More recently, Boneva et al. (2016), Richter and Throckmorton (2016), Eggertsson and Singh (2019), and Miao and Ngo (2021) investigate the differences in the predictions of the Rotemberg and Calvo models with the zero lower bound for the nominal interest rate. Born and Pfeifer (2020) discuss the mapping between Rotemberg and Calvo wage rigidities.

Table 1. Calibrated Moments

	Parameters	Model	Target	Source
	Aggregates			
K/Y	Mean illiquid assets to GDP ratio	2.9	2.92	KMV (2018)
B/Y	Mean liquid assets to GDP ratio	0.23	0.23	KMV (2018)
G/Y	Government spending to GDP ratio	0.16	0.16	ARS (2021)
MPC	Quarterly marginal propensity to consume	0.09	0.10	BJS (2022)
	Wealth Distribution			
HtM^{poor}	Fraction with $a = 0$ and $k = 0$	0.12	0.1	KMV (2018)
HtM^{rich}	Fraction with $a = 0$ and $k > 0$	0.2	0.2	KMV (2018)
$Borrower$	Fraction with $a < 0$	0.12	0.15	KMV (2018)

accounts subject to a transaction cost paid out of the liquid account. The liquid account bears a relatively low real rate of return $r^a_{i,t}$. Households can accumulate liquid debt up to a borrowing constraint, $a_{i,t} \geq \underline{a}$. The illiquid account bears a higher return $r^b_{i,t}$ and is subject to a short-sale constraint $b_{i,t} \geq 0$. As in our baseline model, real rates of return (in terms of households' purchasing power) depend on the relative rates of inflation households face in their type-specific consumption bundles.

When transfering funds $\iota_{i,t}$ from the liquid to the illiquid account, households incur a transaction cost $\psi(\iota_{i,t}, b_{i,t})$ that may be proportional to the size of the illiquid account. We adopt the functional form for $\psi(\cdot)$ used in Kaplan et al. (2018) and calibrate this transaction cost to match important moments of the household wealth distribution, including the shares of hand-to-mouth and wealth hand-to-mouth households. We summarize the moments our model targets in Table 1.

Expenditure heterogeneity. Households consume a CES basket of goods given by

$$c_{i,t} = \left(\sum_j (\Gamma^c_{ij})^{\frac{1}{\eta_c}} c_{ij,t}^{\frac{\eta_c - 1}{\eta_c}} \right)^{\frac{\eta_c}{\eta_c - 1}},$$

where $c_{ij,t}$ denotes consumption of good j by a household of type i. η_c is the elasticity of substitution across consumer goods produced in different sectors. We allow household of different types to differ in their preferences for goods across sectors, which is captured by Γ^c_{ij}.

We use data from the Consumer Expenditure Survey (CEX) to calibrate consumption preferences across households. Since we associate household types with education groups (labor factors), we use data on spending patterns across education groups. We follow Borusyak and Jaravel (2021) and add imputed rents to the measure of expenditures on housing services—CEX categories include rents and mortgage interest but not principal payments. Finally, we implement Clayton et al. (2018)'s "many-to-one" merge to map the 650 consumption categories featured in the CEX to the sectors in our production network by summing up spending in all categories linked to the same sector.

Earnings heterogeneity. Following much recent work in the HANK literature, we assume that the labor supply decision of each household type is intermediated by a union as in our baseline model Auclert et al. (2018). Unions ration labor, so that $n_{i,t}(a,b,z) = N_{i,t}$. Earnings heterogeneity emerges because production sectors differ in their demand for different labor factors, as discussed in Section 4.1.1.

4.1.3 Financial Sector

We model a financial sector that consists of a representative financial intermediary, the "bank", which has two activities: (1) a banking activity, performing maturity transformation by collecting real liquid assets from households and investing them in government bonds, subject to an intermediation spread; and (2) a mutual fund activity, collecting illiquid funds and intermediating them in the form of physical capital to firms.

The bank owns the economy's capital stock and makes capital investments. It rents capital to firms in a competitive rental market. We assume the bank operates an investment technology that transforms sectoral goods into gross capital investment. The capital stock then evolves according to $K_{t+1} = I_t + (1-\delta)K_t$, where investment is given by the CES aggregator

$$I_t = \left(\sum_j \Gamma_{I,j}^{\frac{1}{\eta_I}} I_{j,t}^{\frac{\eta_I - 1}{\eta_I}} \right)^{\frac{\eta_I}{\eta_I - 1}},$$

where η_I is the elasticity of substitution across sectoral goods used for capital investment. Our quantitative model takes seriously the sectoral and network implications of investment spending, as emphasized by Vom Lehn and Winberry (2022). We calibrate the "investment use" parameter $\Gamma_{I,j}$ to match data from the BEA 1997 Capital Flow table, where we compute sector j's total distribution of capital as a share of the economy's total capital distribution.

4.1.4 Government

Government spending takes the form of a homothetic CES aggregate over sectoral goods given by

$$G_t = \left(\sum_k \left(\Gamma_{Gk} \right)^{\frac{1}{\eta_g}} G_{j,t}^{\frac{\eta_g - 1}{\eta_g}} \right)^{\frac{\eta_g}{\eta_g - 1}}.$$

We calibrate Γ_j^g to match the share of government spending across sectors using the BEA industry input-output "use" table, computing government expenditures in sector j as a percentage of total government spending. Government expenditures include spending by the federal government, federal government enterprises, state and local government, and state and local government enterprises.

Table 2. List of Calibrated Parameters

	Parameters	Value	Target / Source
	Preferences		
$\bar{\rho}$	Average discount rate (p.q.)	4 %	Internally calibrated
γ	Relative risk aversion	3	Standard
ϕ	Inverse Frisch elasticity	1	Standard
η_c	Elasticity of substitution between sectors in consumption	1	Standard
η_x	Elasticity of substitution between sectors in intermediate inputs	0.1	Atalay (2017)
η_l	Elasticity of substitution between types in labor supply	1	Standard
η_f	Elasticity of substitution between factors in production	1	Standard
η_c	Elasticity of substitution between sectors in consumption	1	Standard
η_i	Elasticity of substitution between sectors in capital investment	1	Standard
	Household portfolio choice		
$\underline{a}$	Borrowing constraint	-0.25	1.273 quarter of average income
δ	Capital depreciation (p.q.)	1.4 %	Internally calibrated
ψ_0	Linear adjustment cost	0.002	Internally calibrated
ψ_1	Convex adjustment cost 1	0.1	Internally calibrated
ψ_2	Convex adjustment cost 2	2	Internally calibrated
	Financial Intermediary		
ω	Intermediation cost	0.5 %	Bank transaction fee
ϑ	Borrowing wedge	2 %	Internally calibrated
	Firms		
κ	Aggregate capital adjustment cost	2	Δy after shock
	Nominal rigidities		
ϵ	Elasticity of substitution for goods in retailer bundling	11	CEE (2005)
χ_j	Average price adjustment cost	11.74	ACEL (2011)
ϵ^w	Elasticity of substitution for labor	10	CEE (2005)
χ^w	Avg. duration of wage contracts	0	Flexible-wage limit
	Government		
λ_π	Taylor rule weight on inflation	1.5	Standard
λ_Y	Taylor rule weight on output	0	Standard
τ^{lab}	Income tax rate	15 %	Standard

We assume the government balances its budget. Any surplus or deficit is rebated to households according to a rescaling rule that is designed to neutralize the quantitative implications of potentially counterfactual lump-sum transfers. The proportion of the aggregate rebate distributed to households of type i is equal to their income share in stationary equilibrium.

The monetary authority follows a standard Taylor rule with weights λ_π and λ_y on inflation and output, respectively. We assume the monetary authority uses the counterpart of the empirical CPI to measure inflation in this context.

4.1.5 Calibration Summary

We summarize the calibration of our two-type 66-sector HANK-IO model in Table 2.

In addition to the parameters already discussed in this section, we calibrate the household discount rate ρ and the capital depreciation rate δ to match average MPCs and the aggregate

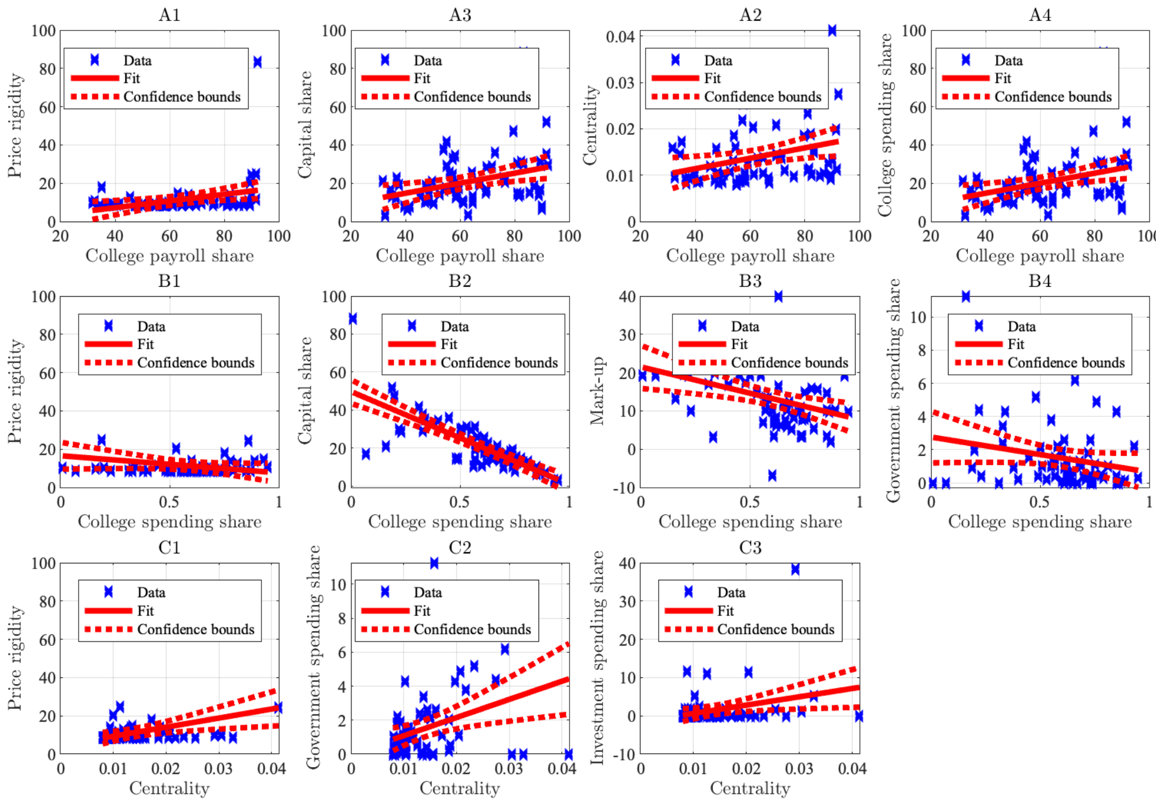

Figure 1. Linkage Between the Household Heterogeneity and the Sectoral Heterogeneity

Note. We plot the OLS regression of sectors features on college payroll share (Panel A1-A4), on college spending share (Panel B1-B4), and on centrality (Panel C1-C3). The red solid lines are the OLS fit line, and the red dashed lines show the 95% confidence interval. All OLS coefficients are statistically significant at 5% level.

capital-output ratio. We set the capital adjustment cost parameter κ so that our baseline model matches VAR evidence on the relative response of investment and output to a monetary policy shock. We take the remaining parameters—including Taylor rule coefficients and the labor income tax rate—from the literature.

4.2 Reduced-Form Evidence

We now present reduced-form evidence to show that our model captures important household-sector linkages observed in the data.

Household earnings and expenditure shares. Drawing on much previous work, our reduced-form evidence suggests that different households are systematically exposed to different sectors through both earnings and expenditure patterns. In the main text, we focus on the earnings and spending shares of college educated households. Panel A4 of Figure 1 visualizes an OLS regression of the college spending share on college payroll share. Sectors that pay more to college graduates are also those that college graduates spend in.

Sectoral heterogeneity. A reduced-form measure of a sector's centrality in the input-output production network is the Katz-Bonacich centrality measure discussed by Carvalho (2014). We compute centrality as $c = \eta(\boldsymbol{I} - \lambda\Gamma^x_{ij})^{-1}\mathbf{1}$, where we set $\eta = \frac{1-\theta}{N} = \frac{1-0.5}{66}$ and $\lambda = 0.5$. Panels C1-C3 of Figure 1 plot the regressions of price rigidity, government spending share, and investment spending share on centrality. Sectors that are more central in the production network are more rigid, account for a larger share of government spending, and are used more in the capital production process.

Household-sector linkages. Panels A1-A3 of Figure 1 display the regression of price rigidity, capital share, centrality, and spending share by college graduates on college payroll share. Sectors that pay more to college graduates are also more rigid, more capital intensive, and more central in the production network.

Panels B1-B4 of Figure 1 plot the regression of price rigidity, capital share, mark-ups, and government spending share over the spending share of college graduates across sectors. Sectors that sell to college graduates are also more price-flexible, less capital intensive, have lower markups, and account for a smaller share of government spending.

4.3 Numerical Solution Strategy

It is well understood that multi-sector production networks can give rise to non-linearities in the transmission and propagation of shocks (Baqaee and Farhi, 2019). First-order approximations may therefore be inaccurate. To overcome such concerns, we solve our quantitative model globally. To that end, we employ the efficient quasi-Newton routine that is part of the SparseEcon toolbox developed by Schaab and Zhang (2022).

5 Quantitative Results

5.1 Monetary Policy

Aggregate effects. Figure 2 plots impulse responses of output and inflation to a 25bps expansionary monetary policy shock with a half-life of 1 quarter. We compare between the representative-household (Panel A) and heterogeneous-household (Panel B) models, as well as several network structures.[17]

The input-output structure has significant implications in both models. Accounting for spending on intermediate inputs amplifies the effects of the monetary policy shock—even in the one-sector model where such spending takes place within-sector. Furthermore, a more disaggregated

[17] We recalibrate the RANK model to be in line with the matched moments discussed in Section 4 as much as possible. The model abstracts from financial intermediation and portfolio adjustment costs.

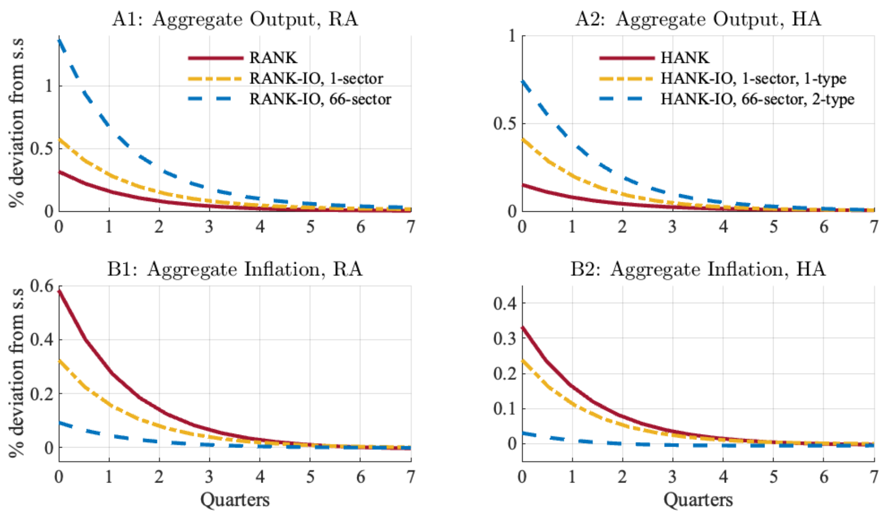

Figure 2. Transition Dynamics of an Expansionary Monetary Policy Shock

Note. Figure 2 plots the impulse responses (in % deviations from steady state) of aggregate real output (Panels A1 and A2) and aggregate inflation (Panels B1 and B2) following a 25 bps expansionary monetary policy shock with a half-life of 1 quarter.

network structure implies larger monetary non-neutrality. While the output effects are amplified, the response of inflation is dampened: As is well understood, accounting for input-output linkages increases the effective price rigidity of the production sector as we see here.

Sectoral implications. The effects of monetary policy across production sectors vary substantially. In Figure 3, we plot the responses of output, inflation, and profits for the economy's most price-rigid and most price-flexible sectors. Since monetary non-neutrality is rooted in the presence of nominal rigidities in the New Keynesian framework, it is unsurprising that relatively more price-rigid sectors are more sensitive to monetary shocks. Output in the most rigid sector increases nearly twice as much as in the most flexible sector. In fact, price-flexible sectors have an incentive to adjust prices and capture market share from relatively more rigid sectors.

Figure 3 also plots the distribution of sectoral responses across all 66 production sectors in the form of a two-dimensional bin-scatter plot. Lighter colors correspond to a clustering of sectors. The bottom panel of Figure 3 illustrates the monetary policy shock's effects on sectors with different significance in the production network. The largest responses are concentrated in sectors that are also central in the production network. Such cross-sectional implication contributes to the amplification of shock impact by the production network. The first panel highlights that there is a relative clustering of sectors that have a low network centrality score and relatively small output responses.

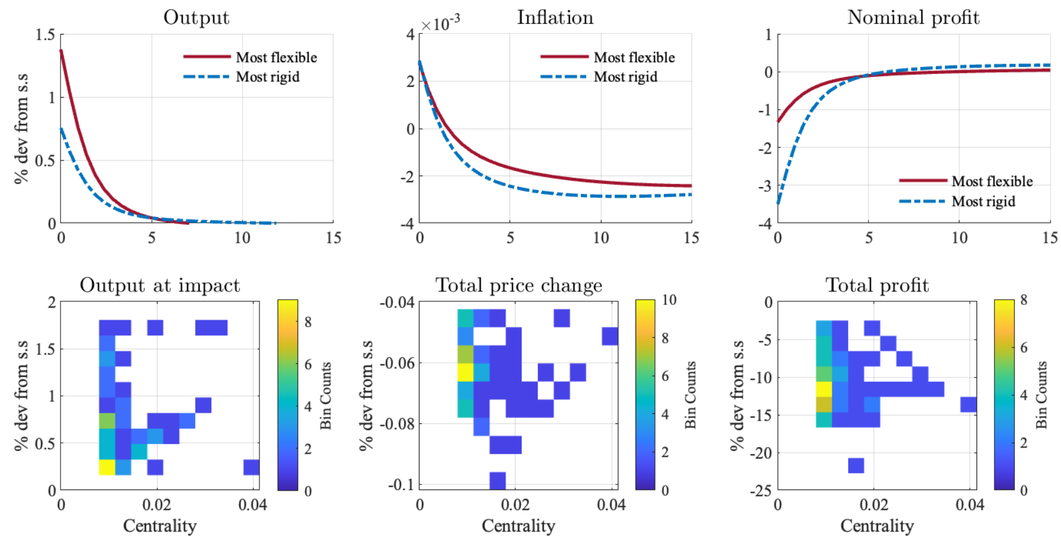

Figure 3. Sectoral Implications of Expansionary Monetary Policy Shock

Note. Figure 3 plots the effects of a monetary policy shock across production sectors. In the top row of panels, we plot the impact of policy on the most price-rigid (dashed blue line) and the most price-flexible (solid red line) sectors. In the bottom row of panels, we show two-dimensional binscatter plots that depict the relationship between sectoral centrality and on-impact output gap, total price, and total profit responses across all sectors.

Distributional effects. The monetary policy shock entails distributional implications. Figure 4 shows that the consumption response for college graduates is more persistent than that for non-college graduates, despite that on impact non-college graduates increase their consumption more.

5.2 Fiscal Policy

Balanced-budget aggregate fiscal shock. Figure 5 shows what happens to real aggregate output when the government surprises the economy with an increase in aggregate government spending G_t by 1% of steady state real output, with a persistence of $log(2)$. The government uses this increased budget to purchase final goods from sectors according to its existing government spending network Γ_j^g. We call such a shock the "aggregate fiscal shock" throughout the rest of our discussion. Such an aggregate fiscal stimulus is financed by a balanced budget, which leads to a negative wealth effect. Similar to what we observe with the monetary policy shock, In either the RA or the HA setting, the I/O structure amplifies fiscal policy shock transmission, leading to fiscal mutipliers in both settings to be above 1 at impact.

Our output response results in Figure 5 are consistent with the empirical literature. Both SVAR and EVAR identification methods estimate an increase in GDP and hours in response to a

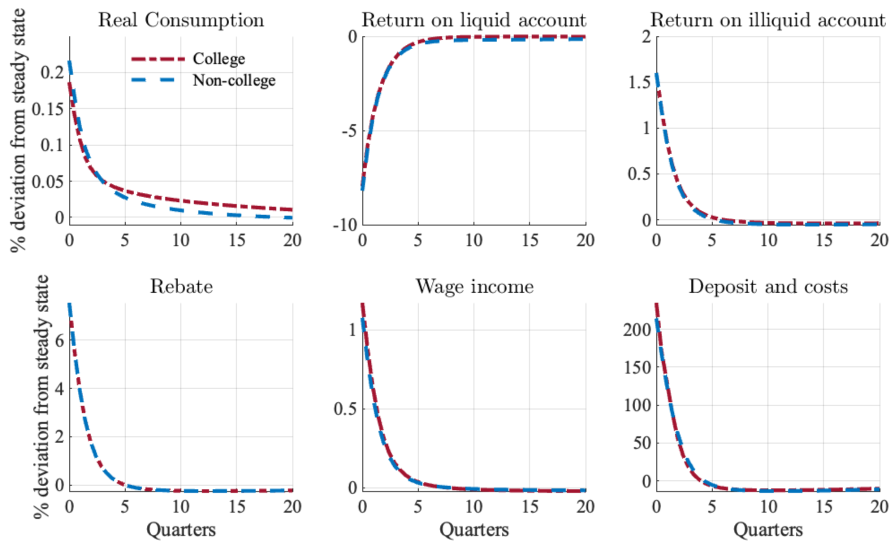

Figure 4. Distributional Implications of Expansionary Monetary Policy

Note. Figure 4 plots the impulse responses (in % deviations from steady state) of type-specific aggregate consumption and its contributing factors to a 25 bps expansionary monetary policy shock with half life of 1 quarter.

positive government spending shock. The empirical literature are less in consensus with regard to the real wage, and consumption. SVAR studies imply a rise in consumption and real wages, whereas the EVAR studies point to a fall in consumption and real wages ([Ramey handbook]). While our current baseline model indicates a rise in consumption for both household types, the direction of the response is sensitive to assumptions we make in the model. Our aggregate fiscal shock is financed by a reduction in transfer to households so that the government can maintain a balanced budget. Therefore, whether the government spending stimulus will crowd out private consumption depends on the tax rate, the utility curvature, the discount rate, and the monetary authority's response to inflation.

The aggregate fiscal shock has different impact on different household types. Our reduced-form evidence makes clear non-college graduates spends more in flexible sectors than college graduates, and government spends more in rigid sectors. In a nutshell, inflation is more responsive to non-college private consumption than to college private consumption when compared with the inflation response to government spending. Such household heterogeneity amplifies the monetary response to the fiscal stimulus for non-college than for college graduates, leading to more intertemporal substitution, more crowding out, and lower consumption.

Self-financed targeted fiscal shock. We further investigate the potential of targeted fiscal shocks. Instead of increase aggregate government spending, which purchases final goods from sectors according to the government spending share Γ_j^g, we now assume that the government reallocate

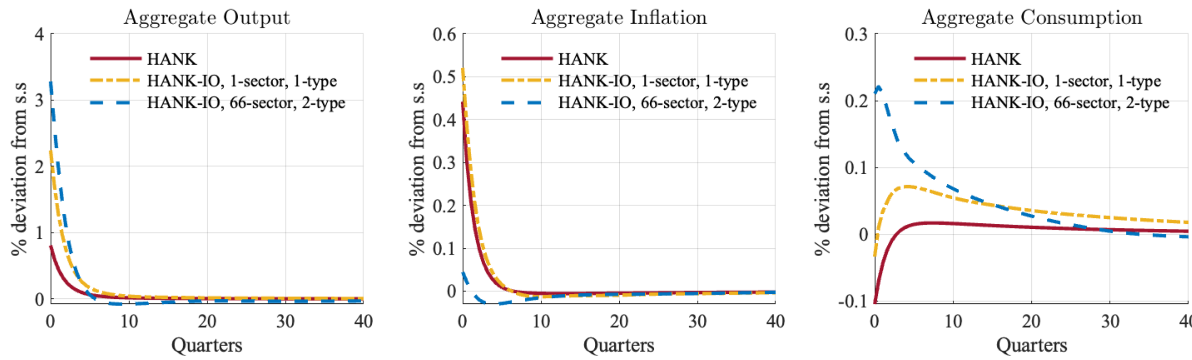

Figure 5. Fiscal Policy Shock

Note. Figure 5 plots the impulse responses (in % deviations from steady state) of aggregate real output to an aggregate fiscal policy shock. The model specifications are the same as those in Figure 2.

some of the funds from the fiscal budget to an individual sector. In other words, this targeted fiscal shock is self-financed without increasing the aggregate government spending. To do so, the government would first reduce aggregate fiscal spending by 1% of steady state output with a half life of 2 quarters, and then allocate such funds to a single sector.

6 Conclusion

This paper is motivated by empirical evidence of systematic household-sector linkages in disaggregated micro data. We build a quantitative framework to assess their implications for policy transmission and the aggregation of sectoral shocks. Our "HANK-IO" model brings together a heterogeneous-agent New Keynesian model with a multi-sector business cycle model with input-output linkages in the tradition of Long and Plosser (1983). We analytically characterize an as-if benchmark that features a strict decoupling between household and sectoral heterogeneity. Away from this benchmark, however, novel earnings and expenditure heterogeneity channels emerge through which household-sector linkages have important implications for policy transmission.

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Online Appendix

A Proofs and Additional Model Details for Sections 2 and 3

A.1 Labor Market Structure and Union Problem

We follow closely Erceg et al. (2000) and Auclert et al. (2018). Each household of type i provides $n_{ik,t}$ hours of work to each of a continuum of unions indexed by $k \in [0,1]$. Total labor hours supplied by this single household are

$$n_{i,t} = \int_k n_{ik,t} dk.$$

Each union aggregates effective labor units provided by each household into a union-type-specific task $N_{ik,t}$, given by

$$N_{ik,t} = n_{k,t}.$$

A labor packer then further aggregates these labor services into aggregate supply of factor i

$$N_{i,t} = \left(\int_k N_{ik,t}^{\frac{\epsilon^w - 1}{\epsilon^w}} dk \right)^{\frac{\epsilon^w}{\epsilon^w - 1}}$$

and sells it to firms at the nominal wage $W_{i,t}$. Importantly, unions ration labor so that all households of a type work the same hours.

Labor union k sets a common wage $W_{ik,t}$ for each of its members. It can adjust this wage flexibly—unlike Auclert et al. (2018), our paper focuses on price stickiness, for which we have good sectoral data. Under flexible wage adjustments, we obtain a type-specific labor supply curve

$$v'(N_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w} w_{i,t} u'(C_{i,t}),$$

where $w_{i,t} = \frac{W_{i,t}}{P_{i,t}}$ is the real wage. Crucially, we assume that unions maximize an objective specified in terms of average consumption for each household type.

A.2 Proof of Proposition 1

A.2.1 Dynamic IS Curve

We impose assumptions A1 and A2, and we also assume that the household consumption and government spending aggregators are the same CES, $\mathcal{D} = \mathcal{G}$. We drop household i subscripts to emphasize that all types are effectively symmetric under these assumptions.

Consumption is then given by $c_t = \left[\sum_j \kappa_j c_{j,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$ and $G_t = \left[\sum_j \kappa_j G_{j,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$, where κ_j is the

CES weight on good j. The ideal price index then solves $P_t = \left[\sum_j \kappa_j P_{j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, and the standard sectoral demand function is then given by $c_{j,t} = \kappa_j \left(\frac{P_{j,t}}{P_t}\right)^{-\eta} c_t$.

Under assumption A2, we also have $n_t(a,z) = N_t$ for all (a,z), where N_t is aggregate labor. Using the GDP deflator P_t, the household budget constraint can then be written as $a_t = R_{t-1}a_{t-1} + e_t + T_t - c_t$, where R_t is the real interest rate. Real earnings are given by $e_t(z) = \tau_t\left(zw_tN_t + z\frac{1}{P_t}\sum_j \Pi_{j,t}\right)^{1-\lambda}$, where $w_t = \frac{W_t}{P_t}$ is the real wage.

Notice that aggregate corporate profits satisfy $\frac{1}{P_t}\Pi_t = \frac{1}{P_t}\sum_j(p_{j,t}y_{j,t} - W_tN_{j,t} - \sum_k p_{k,t}x_{jk,t})$. Given the price index P_t, we can unambiguously define real GDP in levels,

$$Y_t^n = P_tY_t.$$

Therefore, we are left with

$$\begin{aligned}
\frac{1}{P_t}\Pi_t &= \frac{1}{P_t}\left(\sum_j p_{j,t}y_{j,t} - \sum_j\sum_k p_{k,t}x_{jk,t}\right) - w_tN_t \\
&= \frac{1}{P_t}\sum_j\left(p_{j,t}y_{j,t} - \sum_k p_{j,t}x_{kj,t}\right) - w_tN_t \\
&= Y_t - w_tN_t
\end{aligned}$$

where we used the labor market clearing condition $N_t = \sum_j N_{j,t}$ and the sectoral goods market clearing conditions. Household earnings can then be written in terms of real GDP as

$$e_t = \tau_t z_t^{1-\lambda} Y_t^{1-\lambda}.$$

Sequence-space recursive representation. The household problem admits a recursive representation. The consumption policy function can be written as $c_t(a,z)$, and likewise the post-tax earnings function of a household is given by $e_t(a)$.

As in Auclert et al. (2018), we now define

$$Z_t = \int e_t(a,z)g_t(a,z)\,da\,dz = \tau_tY_t^{1-\lambda}\int z^{1-\lambda}g_t(a,z)\,da\,dz,$$

and we define the government's tax revenue as $T_t = Y_t - \int e_t(a,z)g_t(a,z)\,da\,dz$. Total income is split into tax revenue and post-tax payouts to households via labor income and dividends according to $Y_t = Z_t + T_t$. Finally, notice that we can define a given household's post-tax income as

$$e_t(a,z) = e_t(z) = z^{1-\lambda}\tau_tY_t^{1-\lambda} = \frac{z^{1-\lambda}}{\int \tilde{z}^{1-\lambda}g_t(a,\tilde{z})\,da\,d\tilde{z}}Z_t,$$

where we used

$$\tau_t Y_t^{1-\lambda} = \frac{Z_t}{\int z^{1-\lambda} g_t(a,z)\, da\, dz}$$

from above. Crucially, the normalization here $\int \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\, da\, d\tilde{z}$ is a constant even though $g_t(\cdot)$ moves around. Therefore, we have

$$e_t(z) = e(z; Z_t).$$

Summarizing: the household problem. In conclusion, the household problem can be written as

$$\max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_t) - v(N_t) \Big],$$

subject to

$$c_t + a_{t+1} = (1 + r_{t-1}) a_t + \frac{z_t^{1-\lambda}}{\int \tilde{z}^{1-\lambda} g_t(a,\tilde{z})\, da\, d\tilde{z}} Z_t$$

$$a_{t+1} \geq \underline{a},$$

and taking as given the law of motion for idiosyncratic earnings risk z_t. It then follows that the household's consumption policy function at time t can be represented as

$$c_t = c\Big(a, z; \Big\{Z_s, r_s\Big\}_{s\geq t}\Big).$$

Similarly, we define the aggregate consumption function as

$$\mathcal{C}_t = \int c\Big(a, z; \Big\{Z_s, r_s\Big\}_{s\geq t}\Big)\, g_t(a,z)\, da\, dz,$$

which consequently also admits the representation

$$\mathcal{C}_t\Big(\Big\{Z_s, r_s\Big\}_{s\geq t}\Big),$$

taking as given the cross-sectional distribution at time t. Crucially, g_t is invariant to shocks (since it's pre-determined at time t), so we don't explicitly need to take it into account.

Sectoral demand and goods market clearing. The key observation so far is that the *aggregate consumption function* of this economy, $\mathcal{C}_t(\cdot)$, admits the same sequence space representation as in the standard HANK model. Now using household sectoral demand, we can also characterize an aggregate sectoral consumption function, given by

$$\begin{aligned}\mathcal{C}_{j,t} &= \int c_{j,t}(a,z)\, g_t(a,z)\, da\, dz \\ &= \int \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} c\left(a,z;\left\{Z_s, r_s\right\}_{s\geq t}\right) g_t(a,z)\, da\, dz \\ &= \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right).\end{aligned}$$

The sectoral demand equations implied by CES preferences aggregate conveniently. In particular, sectoral demand is a function of aggregate consumption and the *relative price*. Only the contemporaneous relative price matters, while the forward-looking sequences $\{Z_s, r_s\}$ matter to pin down the aggregate consumption level.

We can now use this in goods market clearing, which becomes

$$y_{j,t} = \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right) + \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} G_t.$$

Plugging into the definition of real GDP, we now have

$$\begin{aligned}Y_t &= \sum_j \frac{p_{j,t}}{P_t} \kappa_j \left(\frac{p_{j,t}}{P_t}\right)^{-\eta} \left(\mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right) + G_t\right) \\ &= \sum_j \kappa_j p_{j,t}^{1-\eta} P_t^{-(1-\eta)} \left(\mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right) + G_t\right)\end{aligned}$$

Using the definition of the ideal CPI $P_t = \left[\sum_j \kappa_j p_{j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, we arrive at

$$\begin{aligned}Y_t &= \sum_j \kappa_j P_{j,t}^{1-\eta} \left(\sum_j \kappa_j P_{j,t}^{1-\eta}\right)^{-1} \left(\mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right) + G_t\right) \\ &= \mathcal{C}_t\left(\left\{Z_s, r_s\right\}_{s\geq t}\right) + G_t.\end{aligned}$$

This concludes the proof of the dynamic IS curve representation.

A.2.2 Dynamic LM Curve

The real interest rate is given by the Fisher relation

$$r_t = i_t - \pi_t,$$

where i_t is the exogenous nominal interest rate and $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is CPI inflation in the as-if benchmark. The CPI P_t is a function of sectoral prices $p_{j,t}$, which are themselves determined by sectoral Phillips curves. In particular, sectoral Phillips curves in our setting admit a sequence-space representation

$$p_{j,t} = \mathcal{P}_{j,t}\Big(\big\{mc_{j,s}\big\}_{s\geq 0}\Big).$$

Furthermore, it follows directly from firm cost minimization that marginal cost takes the form

$$mc_{j,t} = mc_j\Big(A_{j,t}, \{p_{k,t}\}_k, W_t\Big).$$

Thus, sectoral prices $p_{j,t}$ depend on the time paths of all past and future sectoral prices, $\boldsymbol{p}_j = \{p_{j,s}\}_{s\geq 0}$, as well as the time paths of sectoral technology shocks and nominal wages.

Finally, the nominal wage of the single labor factor (assumption A2) is determined by the (union) labor supply schedule

$$v'(N_t) = \frac{\epsilon^w - 1}{\epsilon^w}\frac{W_t}{P_t}u'(C_t)$$
$$v'(\sum_j N_{j,t}) = \frac{\epsilon^w - 1}{\epsilon^w}\frac{W_t}{P_t}u'(Y_t - G_t).$$

This equation solves for the nominal wage W_t as a function of real GDP and government spending, $Y_t - G_t$, sectoral prices $\{p_{k,t}\}_k$, and sectoral labor demand $N_{j,t}$. It then follows directly from the firm's cost minimization problem that optimal labor demand $N_{j,t}$ must be a function of the wage W_t, input prices $\{p_{k,t}\}_k$, technology $A_{j,t}$ and output $y_{j,t}$. Output $y_{j,t}$ is determined by demand, which is again pinned down by prices and aggregate demand.

The real interest rate r_t then admits a sequence-space representation as a function of the time paths of aggregate demand $\{Y_s\}_{s\geq 0}$, technology shocks $\{A_{j,s}\}_{j,s\geq 0}$, and policy $\{G_s, T_s, i_s\}_{s\geq 0}$. This sequence-space representation is equivalent to its analog in the representative-household RANK-IO model, taking as given a path of aggregate demand. The assumption of labor rationing is key for this.

A.2.3 Proof of Corollary 2

Starting with the dynamic IS equation, we have

$$dY_t = dG_t + \sum_{s=t}^{\infty} \frac{\partial \mathcal{C}_t}{\partial(Y_s - T_s)}(dY_s - dT_s),$$

where monetary policy holds the real interest rate constant, $dr_t = 0$. Notice that

$$\frac{\partial \mathcal{C}_t}{\partial(Y_s - T_s)} = \frac{\partial \mathcal{C}_t}{\partial e_s} = M_{ts}$$

using our notation in the main text: this is the spending propensity with respect to a marginal increase in unearned income. In matrix form,

$$d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T} + \boldsymbol{M}d\boldsymbol{Y},$$

which concludes the proof.

A.2.4 Proof of Corollary 3

We again start with the dynamic IS equation. Differentiating,

$$dY_t = \sum_{s=t}^{\infty}\left[\frac{\partial \mathcal{C}_t}{\partial (Y_s - T_s)} dY_s + \frac{\partial \mathcal{C}_t}{\partial r_s} dr_s\right].$$

Recalling our definition of M^r_{ts} from the main text and stacking, the matrix form becomes

$$d\boldsymbol{Y} = \boldsymbol{M}d\boldsymbol{Y} + \boldsymbol{M}^r d\boldsymbol{r}.$$

A.2.5 Proof of Corollary 4

From the definition of factor shares (revenue-based Domar weights), $\Lambda_t = \frac{W_t N_t}{P_t Y_t}$, we have

$$d\log\Lambda_t = d\log W_t + d\log N_t - d\log P_t - d\log Y_t.$$

Also notice that $d\log w_t = d\log W_t - d\log P_t$. Also, from the labor supply schedule, we have

$$d\log W_t - d\log P_t = \eta d\log N_t + \gamma d\log Y_t.$$

Next, we define the time-varying markup $\mu_{j,t}$ to represent the sectoral Phillips curve in reduced form,

$$p_{j,t} = \mu_{j,t} MC_{j,t}.$$

This yields $d\log p_{j,t} = d\log\mu_{j,t} + d\log MC_{j,t}$. From cost minimization (and Shephard's lemma) it follows that marginal cost can be unpacked as

$$d\log p_{j,t} = d\log\mu_{j,t} + \sum_{k=1}^{N}\tilde{\Omega}_{jk}\, d\log p_{k,t} + \tilde{\Omega}_{jN+1}\, d\log W_t - d\log A_{j,t}$$

$$d\log p_t = \left(\boldsymbol{I} - \tilde{\Omega}^p\right)^{-1}\left(\tilde{\Omega}_{[:,L]}\, d\log W_t + d\log\mu_t - d\log A_t\right)$$

Now notice that

$$\tilde{\Psi}_{jL} = \sum_k \tilde{\Psi}_{jk}\tilde{\Omega}_{kL} = 1$$

because $\tilde{\Omega}_{jL} + \sum_k \tilde{\Omega}_{jk} = 1$. Therefore, we have

$$d \log p_t = \mathbf{1}\, d \log W_t + \tilde{\Psi}\, d \log \mu_t - \tilde{\Psi}\, d \log A_t.$$

Now, notice that we can write

$$d \log P_t = b' d \log p_t = d \log W_t + \tilde{\lambda}\, (d \log \mu_t - d \log A_t)$$

And we have

$$d \log \Lambda_t = d \log W_t - d \log P_t + d \log N_t - d \log Y_t$$

$$d \log W_t - d \log P_t = \eta d \log N_t + \gamma d \log Y_t.$$

Thus, $d \log N_t = d \log P_t + d \log Y_t + d \log \Lambda_t - d \log W_t$. Plugging in for real wages

$$d \log W_t - d \log P_t = \tilde{\lambda}\, (d \log A_t - d \log \mu_t),$$

we have

$$d \log \Lambda_t = \tilde{\lambda}\, (d \log A_t - d \log \mu_t) + d \log N_t - d \log Y_t$$

$$\tilde{\lambda}\, (d \log A_t - d \log \mu_t) = \eta d \log N_t + \gamma d \log Y_t,$$

and solving out for $d \log N_t$ yields

$$\tilde{\lambda}\, (d \log A_t - d \log \mu_t) = \eta \Big(d \log Y_t + d \log \Lambda_t - \tilde{\lambda}\, (d \log A_t - d \log \mu_t) \Big) + \gamma d \log Y_t.$$

Rearranging, we arrive at our result.

A.3 Proof of Proposition 5

We start from the definition of nominal GDP $Y_t^n = \sum_j p_{j,t}(C_{j,t} + G_{j,t})$. We then decompose changes in nominal GDP into real GDP and changes in the GDP deflator according to

$$dY_t^n = \sum_j p_{j,t}(dC_{j,t} + dG_{j,t}) + \sum_j (C_{j,t} + G_{j,t}) dp_{j,t}.$$

Notice that we can write

$$d\log Y_t^n = \frac{1}{Y_t^n}\sum_j p_{j,t}(dC_{j,t} + dG_{j,t}) + \frac{1}{Y_t^n}\sum_j p_{j,t}(C_{j,t} + G_{j,t})d\log p_{j,t}$$
$$= \sum_j b_{j,t}\frac{dC_{j,t} + dG_{j,t}}{C_{j,t} + G_{j,t}} + \sum_j b_{j,t} d\log p_{j,t}$$
$$= \sum_j b_j d\log C_{j,t} + \sum_j \frac{b_j}{C_j} dG_{j,t} + \sum_j b_j d\log p_{j,t}.$$

where the last line evaluates around a steady state with $G_{j,ss} = 0$. Notice that

$$\frac{b_j}{C_j} = \frac{p_j(C_j + G_j)}{Y^n C_j} = \frac{p_j}{Y^n} = p_j,$$

where the last equality uses our numeraire assumption that $Y_{ss}^n = 1$. Thus, we arrive at the following expression for real GDP changes:

$$d\log Y_t = \sum_j b_j d\log C_{j,t} + \sum_j p_j dG_{j,t}.$$

Fiscal policy. Under our CES assumption, we have

$$G_{j,t} = \kappa_{Gj}\left(\frac{p_{j,t}}{P_{G,t}}\right)^{-\eta_G} G_t,$$

where

$$P_{G,t} = \left[\sum_j \kappa_{Gj} p_{j,t}^{1-\eta_G}\right]^{\frac{1}{1-\eta_G}}.$$

Thus, we have

$$dG_{j,t} = \kappa_{Gj}\left(\frac{p_{j,t}}{P_{G,t}}\right)^{-\eta_G}\left[dG_t - \eta_G\left(d\log p_{j,t} - d\log P_{G,t}\right)G_t\right].$$

We evaluate our result to first order around a steady state with $G_{ss} = 0$. So we are simply left with, to first order,

$$dG_{j,t} = \kappa_{Gj}\left(\frac{p_j}{P_G}\right)^{-\eta_G} dG_t.$$

Sectoral consumption. Next, we unpack sectoral consumption. Recall that

$$C_{j,t} = \sum_i \mu_i \kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\eta_i} \mathcal{C}_{i,t}\left(\left\{e_{i,s}, R_{i,s}\right\}_{s\geq t}\right).$$

We have

$$d\log C_{j,t} = \frac{1}{C_{j,t}}\sum_i \mu_i d\left[\kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\eta_i} C_{i,t}\right]$$
$$= \frac{1}{C_{j,t}}\sum_i \mu_i \kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\eta_i}\left[dC_{i,t} - \eta_i\left(d\log p_{j,t} - d\log P_{i,t}\right)C_{i,t}\right]$$

Notice that real GDP is weighted by

$$b_j d\log C_{j,t} = p_j \sum_i \mu_i\kappa_{ij}\left(\frac{p_j}{P_i}\right)^{-\eta_i} dC_{i,t} - p_j\sum_i \mu_i\kappa_{ij}\left(\frac{p_j}{P_i}\right)^{-\eta_i}\eta_i\left(d\log p_{j,t} - d\log P_{i,t}\right)C_i$$

where we now evaluated around steady state and used our numeraire assumption $Y^n = 1$, which implies

$$b_j = \frac{p_j(C_j + G_j)}{Y^n} = p_j C_j.$$

Next, we work out the contribution to real GDP. Evaluating around steady state, the consumption is given by

$$\sum_j b_j d\log C_{j,t} = \sum_j p_j\left[\sum_i \mu_i\kappa_{ij}\left(\frac{p_j}{P_i}\right)^{-\eta_i} dC_{i,t} - \sum_i \mu_i\kappa_{ij}\left(\frac{p_j}{P_i}\right)^{-\eta_i}\eta_i\left(d\log p_{j,t} - d\log P_{i,t}\right)C_i\right]$$
$$= \sum_j\sum_i \mu_i\kappa_{ij}p_j\left(\frac{p_j}{P_i}\right)^{-\eta_i} dC_{i,t} - \sum_j\sum_i \mu_i\kappa_{ij}p_j\left(\frac{p_j}{P_i}\right)^{-\eta_i}\eta_i\left(d\log p_{j,t} - d\log P_{i,t}\right)C_i.$$

The first term becomes

$$\sum_j\sum_i \mu_i\kappa_{ij}p_j\left(\frac{p_j}{P_i}\right)^{-\eta_i} dC_{i,t} = \sum_i \mu_i P_i^{\eta_i}\sum_j \kappa_{ij}p_j^{1-\eta_j} dC_{i,t}$$
$$= \sum_i \mu_i P_i^{\eta_i} P_i^{1-\eta_i} dC_{i,t}$$
$$= \sum_i \mu_i P_i dC_{i,t}.$$

Now notice that

$$d\log P_{i,t} = \sum_j \kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{1-\eta_i} d\log p_{j,t}$$

Thus, we get

$$
\begin{aligned}
&-\sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \eta_i \Big(d \log p_{j,t} - d \log P_{i,t}\Big) C_i \\
=&-\sum_j \sum_i \mu_i \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \eta_i \Big(d \log p_{j,t} - \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t}\Big) C_i \\
=&-\sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} d \log p_{j,t} - \sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t}\right] \\
=&-\sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j \left(\frac{p_j}{P_i}\right)^{-\eta_i} d \log p_{j,t} - P_i \sum_j \kappa_{ij} \left(\frac{p_j}{P_i}\right)^{1-\eta_i} d \log p_{j,t}\right] \\
=&-\sum_i \mu_i \eta_i C_i \left[\sum_j \kappa_{ij} p_j^{1-\eta_i} P_i^{\eta_i} d \log p_{j,t} - \sum_j \kappa_{ij} p_j^{1-\eta_i} P_i^{\eta_i} d \log p_{j,t}\right] \\
=&0.
\end{aligned}
$$

Under CES, there is no composition effect.

Real GDP. We are thus left with

$$
\begin{aligned}
d \log Y_t &= \sum_i \mu_i P_i C_i d \log C_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i \sum_{s \geq t} \left(\frac{\partial \log \mathcal{C}_{i,t}}{\partial \log e_{i,s}} d \log e_{i,s} + \frac{\partial \log \mathcal{C}_{i,t}}{\partial \log R_{i,s}} d \log R_{i,s}\right) + P_G dG_t.
\end{aligned}
$$

Next, we have $e_{i,s} = d(\frac{1}{P_{i,s}} \tau_s (z Y_s^n \xi_{i,s})^{1-\lambda})$, and so

$$
d \log e_{i,s} = -d \log P_{i,s} + d \log \tau_s + (1-\lambda) d \log Y_s^n + (1-\lambda) d \log \xi_{i,s}.
$$

Notice that around the steady state with $G = T = 0$ and numeraire $Y^n = 1$, we have

$$
\begin{aligned}
\tau_t Y_t^n &= Y_t^n - T_t \\
d\log\tau_t + d\log Y_t^n &= d\log(Y_t^n - T_t) \\
d\log\tau_t + d\log Y_t^n &= \frac{dY_t^n - dT_t}{Y_t^n - T_t} \\
d\log\tau_t + d\log Y_t^n &= \frac{dY_t^n}{Y_t^n} - \frac{dT_t}{Y_t^n} \\
d\log\tau_t &= -\frac{dT_t}{Y_t^n} \\
d\log\tau_t &= -dT_t.
\end{aligned}
$$

Finally, using $\lambda = 0$, we have

$$
d\log e_{i,s} = -dT_t + d\log Y_s - d\log\rho_{i,s} + d\log\xi_{i,s}.
$$

Thus, we have

$$
\begin{aligned}
d\log Y_t &= \sum_i \mu_i P_i dC_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i d\log C_{i,t} + P_G dG_t \\
&= \sum_i \mu_i P_i C_i \sum_{s\geq t}\left(M_{i,ts}\Big[-dT_t + d\log Y_s - d\log\rho_{i,s} + d\log\xi_{i,s}\Big] + M^r_{i,ts} dr_{i,s}\right) + P_G dG_t.
\end{aligned}
$$

where $M_{i,ts} = \frac{\partial \log C_{i,t}}{\partial \log e_{i,s}}$. Now notice that by definition

$$
\begin{aligned}
d\log P_t &= \sum_j p_{j,t} C_{j,t} d\log p_{j,t} \\
&= \sum_j p_j \sum_i \mu_i \kappa_{ij}\left(\frac{p_j}{P_i}\right)^{-\eta_i} C_i d\log p_{j,t} \\
&= \sum_i \mu_i P_i C_i \sum_j \kappa_{ij}\left(\frac{p_j}{P_i}\right)^{1-\eta_i} d\log p_{j,t} \\
&= \sum_i \mu_i P_i C_i\, d\log P_{i,t}.
\end{aligned}
$$

We next show that under our (steady state) nominal GDP numeraire, we have

$$\sum_i \mu_i P_i C_i = 1.$$

This follows because we have $1 = Y_n = \sum_j p_j C_j$, and

$$\sum_i \mu_i P_i C_i = \sum_i \mu_i \sum_j p_j c_{ij} = \sum_j p_j \sum_i \mu_i c_{ij} = \sum_j p_j C_j.$$

Our result follows after applying a covariance decomposition using the operator $\mathbb{E}_i = \sum_i \mu_i$. Note that

$$\begin{aligned}
\sum_i \mu_i P_i C_i \boldsymbol{M}_i \, d\log \boldsymbol{\rho}_i &= \sum_i \mu_i \boldsymbol{M}_i \sum_i \mu_i P_i C_i d\log \boldsymbol{\rho}_i - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right) \\
&= \bar{\boldsymbol{M}}_i \sum_i \mu_i P_i C_i (d\log P_{i,t} - d\log P_t) - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right) \\
&= \bar{\boldsymbol{M}}_i \left(\underbrace{\sum_i \mu_i P_i C_i d\log P_{i,t}}_{=d\log P_t} - d\log P_t \underbrace{\sum_i \mu_i P_i C_i}_{=1} \right) - \mathbb{C}ov_i\left(\boldsymbol{M}_i, P_i C_i \, d\log \boldsymbol{\rho}_i\right)
\end{aligned}$$

so the average term cancels.

A.4 Proof of Proposition 6

Firms. Firm cost minimization implies that

$$\begin{aligned}
d\log y_{j,t} &= d\log A_{j,t} + \sum_{k=1}^{N} \tilde{\Omega}_{jk} \, d\log x_{jk,t} + \sum_{i=1}^{I} \tilde{\Omega}_{jN+i} \, d\log N_{ji,t} \\
\Gamma_j \, d\log \tilde{x}_j &= d\log \tilde{p} - d\log MC_j - d\log A_j \\
d\log MC_{j,t} &= \sum_{k=1}^{N} \tilde{\Omega}_{jk} \, d\log p_{k,t} + \sum_{i=1}^{I} \tilde{\Omega}_{jN+i} \, d\log W_{i,t} - d\log A_{j,t}
\end{aligned}$$

We still have the Domar weight definition

$$\mu_i \Lambda_{i,t} = \frac{\mu_i W_{i,t} N_{i,t}}{P_t Y_t},$$

now accounting for household mass. From here, we have

$$d\log \Lambda_{i,t} = d\log W_{i,t} + d\log N_{i,t} - d\log P_t - d\log Y_t.$$

Next, we write

$$d\log p_{j,t} = d\log\mu_{j,t} + \sum_{k=1}^{N}\tilde{\Omega}_{jk}\,d\log p_{k,t} + \sum_{i=1}^{I}\tilde{\Omega}_{jN+i}\,d\log W_{i,t} - d\log A_{j,t}$$

$$d\log p_t = \left(\boldsymbol{I} - \tilde{\Omega}^p\right)^{-1}\left(\sum_{i=1}^{I}\tilde{\Omega}_{[:,\,N+i]}\,d\log W_{i,t} + d\log\mu_t - d\log A_t\right)$$

Now we have

$$d\log p_{j,t} = \sum_k\sum_i(\boldsymbol{I} - \tilde{\Omega}^p)^{-1}_{ji}\tilde{\Omega}_{jN+i}\,d\log W_{i,t} + \sum_k\tilde{\Psi}_{jk}\Big(d\log\mu_{k,t} - d\log A_{k,t}\Big)$$

or simply

$$d\log p_{j,t} = \sum_i\tilde{\Psi}_{jN+i}\,d\log W_{i,t} + \sum_k\tilde{\Psi}_{jk}\Big(d\log\mu_{k,t} - d\log A_{k,t}\Big)$$

Network objects and Domar weights. The goods market clearing condition for good j is

$$p_{j,t}y_{j,t} = \sum_i\mu_i p_{j,t}C_{ij,t} + \sum_k p_{j,t}x_{kj,t}.$$

Now aggregating households, we have

$$\begin{aligned} p_{j,t}y_{j,t} &= p_{j,t}C_{j,t} + \sum_k p_{j,t}x_{kj,t} \\ &= p_{j,t}\sum_i\mu_i b_{ij,t}C_{i,t} + \sum_k p_{j,t}x_{kj,t}. \end{aligned}$$

where

$$C_{j,t} = \sum_i\mu_i\kappa_{ij}\left(\frac{p_{j,t}}{P_{i,t}}\right)^{-\eta_i}C_{i,t}\Big(\Big\{e_{i,s}, R_{i,s}\Big\}_{s\geq t}\Big).$$

Now define

$$b_{j,t} = \frac{p_{j,t}C_{j,t}}{\sum_j p_{j,t}C_{j,t}} = \frac{p_{j,t}\sum_i\mu_i b_{ij,t}C_{i,t}}{\sum_j p_{j,t}\sum_i\mu_i b_{ij,t}C_{i,t}}$$

as the final (consumption) expenditure share of good j. Now we can rewrite the goods market clearing condition as

$$p_{j,t}y_{j,t} = b_{j,t}\left(\sum_j p_{j,t}\sum_i\mu_i b_{ij,t}C_{i,t}\right) + \sum_k\Omega_{kj,t}p_{k,t}y_{k,t}$$

Now notice that $\sum_j p_{j,t}C_{j,t} = Y_t^n$ is also nominal GDP. So defining the revenue-based Domar weight

$$\lambda_{j,t} = \frac{p_{j,t}y_{j,t}}{Y_t^n},$$

we have

$$\lambda_{j,t} = b_{j,t} + \sum_k \Omega_{kj,t}\lambda_{k,t}$$

$$\lambda_{j,t} = b_{j,t} + \Omega'_{[:,j],t}\lambda_t$$

$$\lambda_t = b_t + \Omega'_t\lambda_t$$

$$\lambda'_t = b'_t + \lambda'_t\Omega_t,$$

and thus

$$\lambda'_t = b'_t\Psi_t.$$

Similarly, we define

$$\tilde{\lambda}'_t = b'_t\tilde{\Psi}_t.$$

Putting it all together, we can write

$$d\log P_t = b'_t\, d\log p_{j,t} = \sum_i \tilde{\Lambda}_{i,t}\, d\log W_{i,t} + \sum_k \lambda_{k,t}\Big(d\log \mu_{k,t} - d\log A_{k,t}\Big)$$

Wages. The key change is that there is now a labor supply equation for each labor type. We have

$$\frac{W_{i,t}}{P_{i,t}} = \frac{\epsilon_i^w}{\epsilon_i^w - 1}\frac{v'(N_{i,t})}{u'(C_{i,t})}$$

and therefore

$$d\log W_{i,t} - d\log P_{i,t} = \eta_i\, d\log N_{i,t} + \gamma_i\, d\log C_{i,t}.$$

Now, crucially, we can no longer use the goods market clearing condition to just solve out for $d\log C_{i,t}$. Instead, we will use our sufficient statistics on the household side. We have

$$C_{i,t} = \mathcal{C}_{i,t}\Big(\Big\{e_{i,s}, r_{i,s}\Big\}_{s\geq t}\Big).$$

Thus, we have

$$\begin{aligned} d\log C_{i,t} &= \frac{1}{C_{i,t}}\sum_{s\geq t}\left(M_{i,ts}de_{i,s} + M^r_{i,ts}dr_{i,s}\right) \\ &= \frac{1}{C_{i,t}}\sum_{s\geq t} M_{i,ts}\left(-dT^n_s + d\log \xi_{i,s} + d\log Y_s - d\log \rho_{i,s}\right) + \frac{1}{C_{i,t}}\sum_{s\geq t} M^r_{i,ts}dr_{i,s} \end{aligned}$$

Domar weights still solve

$$d\log \Lambda_{i,t} = d\log W_{i,t} + d\log N_{i,t} - d\log P_t - d\log Y_t.$$

We now introduce our two sufficient statistics

$$\xi_{i,t} = 1 + \Lambda_{i,t} - \mathbb{E}_i \Lambda_{i,t}$$
$$d \log \rho_{i,t} = d \log P_{i,t} - d \log P_t$$

This yields

$$d \log N_{i,t} = d \log P_t + d \log Y_t + d \log \Lambda_{i,t} - d \log W_{i,t}$$

and plugging in we get

$$\begin{aligned}
d \log W_{i,t} - d \log P_{i,t} &= \eta_i \, d \log P_t + \eta_i \, d \log Y_t + \eta_i \, d \log \Lambda_{i,t} - \eta_i \, d \log W_{i,t} + \gamma_i \, d \log C_{i,t} \\
(1 + \eta_i) d \log W_{i,t} &= (1 + \eta_i) \, d \log P_{i,t} - \eta_i \, d \log \rho_{i,t} + \eta_i \, d \log Y_t + \eta_i \, d \log \Lambda_{i,t} + \gamma_i \, d \log C_{i,t} \\
d \log W_{i,t} &= d \log P_{i,t} - \frac{\eta_i}{1+\eta_i} \, d \log \rho_{i,t} + \frac{\eta_i}{1+\eta_i} \, d \log Y_t + \frac{\eta_i}{1+\eta_i} \, d \log \Lambda_{i,t} + \frac{\gamma_i}{1+\eta_i} \, d \log C_{i,t}.
\end{aligned}$$

Now I actually want to go in the other direction! I want to solve out for $d \log P_{i,t}$! So this yields

$$d \log W_{i,t} = \frac{1}{1+\eta_i} d \log \rho_{i,t} + d \log P_t + \frac{\eta_i}{1+\eta_i} \, d \log Y_t + \frac{\eta_i}{1+\eta_i} \, d \log \Lambda_{i,t} + \frac{\gamma_i}{1+\eta_i} \, d \log C_{i,t}$$

Now plugging into the pricing equation and noting that $\sum_i \tilde{\Lambda}_{i,t} = 1$, we have

$$\begin{aligned}
d \log P_t = \sum_i \tilde{\Lambda}_{i,t} \left(\frac{1}{1+\eta_i} d \log \rho_{i,t} + d \log P_t + \frac{\eta_i}{1+\eta_i} \, d \log Y_t + \frac{\eta_i}{1+\eta_i} \, d \log \Lambda_{i,t} + \frac{\gamma_i}{1+\eta_i} \, d \log C_{i,t} \right) \\
+ \sum_k \lambda_{k,t} \Big(d \log \mu_{k,t} - d \log A_{k,t} \Big)
\end{aligned}$$

and simplifying, we arrive at

$$\begin{aligned}
\sum_i \tilde{\Lambda}_{i,t} \frac{\eta_i}{1+\eta_i} \, d \log Y_t = - \sum_i \tilde{\Lambda}_{i,t} \frac{1}{1+\eta_i} d \log \rho_{i,t} - \sum_i \tilde{\Lambda}_{i,t} \frac{\eta_i}{1+\eta_i} \, d \log \Lambda_{i,t} - \sum_i \tilde{\Lambda}_{i,t} \frac{\gamma_i}{1+\eta_i} \, d \log C_{i,t} \\
+ \sum_k \lambda_{k,t} \Big(d \log A_{k,t} - d \log \mu_{k,t} \Big)
\end{aligned}$$

This concludes the proof.

B Quantitative HANK-IO: Model Details

This Appendix provides a self-contained description of the quantitative HANK-IO model that we implement and take to the data in Section 4.

B.1 Production Network

Production in this economy takes place in N distinct production sectors. Within each sector, we adopt the standard New Keynesian structure in which a retailer bundles intermediate varieties produced by a continuum of intermediate goods producers. Dimensions of heterogeneity include share of intermediate input bundle in production factors $\mu_{x,j}$, and share of capital in primary factor α_j. We detonate all variable in sector j with a underscript j.

B.1.1 Retailer

The retailer produces the final consumption good by bundling intermediate varieties according to the CES aggregation technology

$$y_{j,t} = \left(\int_0^1 y_{j,t}(k)^{\frac{\epsilon-1}{\epsilon}} dk \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $y_{j,t}$ denotes sectoral production output and $y_{j,t}(k)$ is the output produced by intermediate firm k in sector j. ϵ denotes the elasticity of substitution across intermediate inputs. Each retailer demands intermediate input j according to the standard demand function

$$y_{j,t}(k) = \left(\frac{P_{j,t}(k)}{P_{j,t}} \right)^{-\epsilon} y_{j,t},$$

where $P_{j,t}(k)$ is the price of intermediate good produced by firm k in sector j, and $P_{j,t}$ is the producer price index (PPI) in sector j,

$$P_{j,t} = \left(\int_0^1 P_{j,t}(k)^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}}.$$

B.1.2 Intermediate Goods Producers

Production function. Firms in each industry employ CES technology to transform intermediate inputs, capital and labor into final products.

$$y_{j,t} = A_{j,t} \cdot \left[(1-\mu_{x,j})^{\frac{1}{\eta_{f,j}}} \left(f_{j,t}\right)^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} + \left(\mu_{x,j}\right)^{\frac{1}{\eta_{f,j}}} \left(x_{j,t}\right)^{\frac{\eta_{f,j}-1}{\eta_{f,j}}} \right]^{\frac{\eta_{f,j}}{\eta_{f,j}-1}}$$

where $A_{j,t}$ is the factor-neutral total factor productivity of sector j at time t. $\mu_{x,j}$ is the share of intermediate inputs factor in sector j's production function. $\eta_{f,j}$ is the elasticity between primary factor and intermediate input factor. $f_{j,t}$ is the primary factor, and $x_{j,t}$ is the aggregate intermediate input bundle.

Primary factor. $f_{j,t}$ is aggregated by a Cobb-Douglas technology,

$$f_{j,t} = \left(K_{j,t}\right)^{\alpha_j} \left(N_{j,t}\right)^{1-\alpha_j}$$

α_j is the share of capital in the primary factor production in sector j. $K_{j,t}$ is capital in use for sector j, $N_{j,t}$ is the effective labor of sector j.

Intermediate inputs bundle. $x_{j,t}$ is aggregated by a CES technology,

$$x_{j,t} = \left(\sum_{i=1}^{S} \left(\Gamma^x_{i,j}\right)^{\frac{1}{\eta_{x,j}}} \left(x_{i\to j,t}\right)^{\frac{\eta_{x,j}-1}{\eta_{x,j}}} \right)^{\frac{\eta_{x,j}}{\eta_{x,j}-1}}$$

where $\eta_{x,j}$ parameterizes the elasticity of goods used in the intermediate input bundle from different sectors i. $\Gamma^x_{i,j}$ indicates the share of importance of industry i in the production of sector j's intermediate input bundle. $x_{i\to j,t}$ is the unit of final goods from sector i used in j's intermediate input bundle.

The standard demand functions for intermediate inputs from sector i is given by

$$x_{i\to j,t} = \Gamma^x_{i,j} \left(\frac{P_{i,t}}{P_{jx,t}} \right)^{-\eta_{x,j}} x_{j,t}$$

where P^i_t is the producer price index (PPI) in sector i and $P_{jx,t}$ is the price of intermediate input bundle in sector j. The relationship between intermediate input prices and the bundle price is given by

$$P_{jx,t} = \left[\sum_i \Gamma^x_{i,j} (P_{i,t})^{1-\eta_{x,j}} \right]^{\frac{1}{1-\eta_{x,j}}}$$

Nominal profit. There is an integrated and competitive market in which firms rent capital. The nominal rental rate of capital is i^K_t. Market clearing in the labor markets reallocation gives rise to a nominal wage rate $W_{j,t}$. The nominal price of the intermediate inputs bundle is $P_{jx,t}$. Firms in sector j have a nominal profit

$$\Pi_{j,t} = \underbrace{P_{j,t}y_{j,t}}_{\text{Revenue from sales}} - \underbrace{(1-\tau^{empl})W_{j,t}N_{j,t}}_{\text{Cost of labor}} - \underbrace{i_t^K K_{j,t}}_{\text{Cost of capital}} - \underbrace{P_{jx,t}x_{j,t}}_{\text{Cost of intermediate bundle}}$$

where τ^{empl} is the employment subsidy from the government to address the distortion resulted from monopolistic competition.

Optimization. The sector-specific salary expenditure is aggregated through the CES technology. The optimal composition between wage for different household types and effective labor is given by

$$N_{ij,t} = \left(\frac{W_{i,t}}{W_{j,t}}\right)^{-\eta_{l,j}} \Gamma_{ij}^w N_{j,t}$$

$$W_{j,t} = \left[\sum_i \Gamma_{ij}^w (W_{i,t})^{1-\eta_{l,j}}\right]^{\frac{1}{1-\eta_{l,j}}}$$

where $N_{ij,t}$ is the effective labor of household type i in sector j, $\eta_{l,j}$ is the elasticity of substitution between labor supply by different types of workers for sector j, $W_{i,t}$ is the type-specific wage, $W_{j,t}$ is the sector-specific wage, Γ_{ij}^w is the share of salaries earned by type i worker in sector j, and $N_{j,t}$ is the effective labor in sector j.

In our baseline model, we calibrate $\eta_l^s = 1$ across sectors with reasons discussed in Section **??**. The optimal labor factor composition for each sector can be re-written as

$$N_{ij,t}W_{i,t} = \Gamma_{ij}^w W_{j,t} N_{j,t}$$

$$W_{j,t} = \Pi_i (W_{i,t})^{\Gamma_{ij}^w}.$$

Furthermore, the optimization yields the relationship between marginal product of intermediate bundle and its nominal price as

$$\begin{aligned} P_{jx,t} &= MC_{j,t}\frac{\partial y_{j,t}}{\partial x_{j,t}} \\ &= MC_{j,t}(A_{j,t})^{\frac{\eta_{f,j}-1}{\eta_{f,j}}}(\mu_{x,j}y_{j,t})^{\frac{1}{\eta_{f,j}}}(x_{j,t})^{-\frac{1}{\eta_{f,j}}}; \\ (P_{jx,t})^{\eta_{f,j}} &= (MC_{j,t})^{\eta_{f,j}}(A_{j,t})^{\eta_{f,j}-1}\mu_{x,j}y_{j,t}(x_{j,t})^{-1}. \end{aligned}$$

Similarly, for the primary factor, we have

$$(i_t^K)^{\eta_{f,j}} = (MC_{j,t})^{\eta_{f,j}}(A_{j,t})^{\eta_{f,j}-1}(1-\mu_{x,j})y_{j,t}(f_{j,t})^{-1}\left(\alpha_j\frac{f_{j,t}}{K_{j,t}}\right)^{\eta_{f,j}}$$

$$\left((1-\tau^{empl})W_{j,t}\right)^{\eta_{f,j}} = (MC_{j,t})^{\eta_{f,j}}(A_{j,t})^{\eta_{f,j}-1}(1-\mu_{x,j})y_{j,t}(f_{j,t})^{-1}\left((1-\alpha_j)\frac{f_{j,t}}{N_{j,t}}\right)^{\eta_{f,j}}$$

In our baseline model, we calibrate $\eta_{f,j} = 1$ across sectors, for reasons discussed in Section **??**. Denote $\mu_{k,j} = (1-\mu_{x,j})\alpha_j$ to be the share of capital in total production, and $\mu_{l,j} = (1-\mu_{x,j})(1-\alpha_j)$. The production function is therefore given by

$$y_{j,t} = A_{j,t}\left(x_{j,t}\right)^{\mu_{x,j}}\left(K_{j,t}\right)^{\mu_{k,j}}\left(N_{j,t}\right)^{\mu_{l,j}}$$

The nominal marginal cost of production is given by

$$MC_{j,t} = \frac{1}{A_{j,t}}\frac{1}{(\mu_{x,j})^{\mu_{x,j}}(\mu_{k,j})^{\mu_{k,j}}(\mu_{l,j})^{\mu_{l,j}}}\left(P_{jx,t}\right)^{\mu_{x,j}}\left(i_t^K\right)^{\mu_{k,j}}\left((1-\tau^{empl})W_{j,t}\right)^{\mu_{l,j}}$$

The optimization yields the relationship between marginal product of factors and their nominal prices

$$P_{jx,t} = MC_{j,t}\mu_{x,j}\frac{y_{j,t}}{x_{j,t}}$$

$$i_t^K = MC_{j,t}\mu_{k,j}\frac{y_{j,t}}{K_{j,t}}$$

$$(1-\tau^{empl})W_{j,t} = MC_{j,t}\mu_{l,j}\frac{y_{j,t}}{N_{j,t}}$$

We define the real marginal cost in sector j as

$$mc_{j,t} = \frac{MC_{j,t}}{P_{j,t}}.$$

Given the definition of marginal cost, the firm's nominal profit can, as usual, be expressed as $\Pi_{j,t} = (1-mc_{j,t})P_{j,t}y_{j,t}$.

Dynamic price-setting. We have now discussed firms' optimal composition of factors of production. In this essentially static choice, the firm takes as given its price level $P_{j,t}(k)$, which is sticky in the short run, as well as the demand it faces at this price level, $y_{j,t}(k)$. We now turn to the dynamic choice of the optimal price level subject to an adjustment cost in the spirit of [Rotemberg 1982].

Define $\pi_{j,t}(k) = \dot{P}_{j,t}(k)/P_{j,t}(k)$ to be the instantaneous rate of inflation in the price of firm k. Firm k determines this rate of inflation subject to an adjustment cost (in utility units), in order to maximize an appropriately discounted sum of all future profits. The firm's problem, then, in real terms, is given by

$$\max_{\pi_{j,t}(k)} \int_0^\infty e^{-\int_0^t \rho ds} \frac{1}{P_{j,t}} \left[(1 - MC_{j,t}) P_{j,t}(k) y_{j,t}(k) - \Lambda(\pi_{j,t}(k)) \right] dt,$$

The cost of adjusting prices at rate $\pi_t(k)$ is given by $\Lambda(\pi_t(k))$. We assume the specific functional form

$$\Lambda(\pi_{j,t}(k)) = \frac{\chi_j}{2} \Big(\pi_{j,t}(k) \Big)^2 P_{j,t}.$$

Lemma 7. *The New Keynesian Phillips Curves for each production sector of the economy can be written as*

$$\dot{\pi}_{j,t} = \rho \pi_{j,t} - (mc_{j,t} - \frac{\epsilon - 1}{\epsilon}) \frac{\epsilon}{\chi_j} y_{j,t}$$

Proof of the Lemma is provided in Appendix B.6.

B.2 Government

B.2.1 Fiscal Policy

Employment subsidy. The government implements an employment subsidy $\tau^{empl} = \frac{1}{\epsilon}$. *On the household side*, the government pays a wage subsidy to households, and such outlays are funded by a lump-sum tax based on aggregate employment. That is, the household-side net fiscal rebate that a household with idiosyncratic labor productivity z receives is always zero, with

$$\int_0^1 \tau^{empl} (1 - \tau^{lab}) z_{i,t} W_{ik,t} n_{ik,t} dk - \tau^{empl} (1 - \tau^{lab}) z_{i,t} W_{i,t} n_{i,t} = 0$$

On the firm side, there is an employment subsidy in place to avoid distortion resulted from monopolistic competition. The government gives each firm k an employment subsidy $\tau^{empl} W_{j,t} N_{j,t}(k)$. And such subsidy to firms is funded a lump-sum tax based on aggregate subsidy. The aggregate subsidy is given by $\sum_j \tau^{empl} W_{j,t} N_{j,t}$.

Tax collected. The government's fiscal income comes from payroll tax collected, $\sum_j \tau^{lab} W_{j,t} N_{j,t}$.

Government spending. The government purchases final goods from all sectors $G_{j,t}$ according to the share of consumption Γ_j^g, therefore the aggregate government spending G_t can be written as

$$P_{j,t} G_{j,t} = \Gamma_j^g P_t G_t$$

Interest expenses. Another fiscal outflow for the government is nominal interest paid on the nominal government debt outstanding $i_t P_t A_t^g$.

Transfer. The government finances such net flow through a transfer from the household. The nominal transfer to the household's budget constraint is

$$T_t = \sum_j (\tau^{lab} - \tau^{empl}) W_{j,t} N_{j,t} - P_t G_t - i_t P_t A_t^g + d(P_t A_t^g),$$

where $d(P_t A_t^g)$ is the change of nominal government debt outstanding. The total nominal transfer to the households is

$$T_t = \sum_j (\tau^{lab} - \tau^{empl}) W_{j,t} N_{j,t} - P_t G_t - i_t P_t A_t^g + P_t \dot{B}_t^g + \dot{P}_t A_t^g,$$

which in real terms is given by

$$\tau_{i,t} = \frac{\sum_j \Pi_{j,t} + T_t}{P_{i,t}}.$$

Rebate re-scaling. The government collect all the aggregate rebate then distribute them to different household types following a re-scaling rule. The proportion of aggregate rebate distributed to type i household is equal to the ratio of such type's total income over all types' total income at steady state. Total income is defined as the sum of wage earnings and interest income from savings.

B.2.2 Monetary Policy

The central bank in our model sets the nominal interest rate, i_t, according to a Taylor rule.

$$i_t = r_{ss} + \lambda_\pi \pi_t + \lambda_Y \triangle y_t + \varepsilon_t,$$

where r_{ss} is the real interest rate in the zero-inflation steady state, $\triangle y_t = \log(y_t/y^*)$ denotes the output gap, and ε_t is the monetary shock.

B.3 Households

The economy is populated by a set of types $i \in \mathcal{I}$ of households. We denote their measure by μ_i and assume $\sum_i \mu_i = 1$. Household types differ in terms of their permanent characteristics. In addition to ex-ante heterogeneity across types, our baseline model allows for ex-post heterogeneity in productivity (z) liquid assets (a) and illiquid assets (b) within types. We can therefore uniquely identify a household of type i with the three state variables (a, b, z), and we denote the cross-sectional income and wealth distribution for i by $g_{i,t}(a, b, z)$. All households purchase consumption goods from and supply labor across N production sectors.

Preference. The preferences of a household of type i are ordered according to

$$\mathbb{E}_0 \int_0^\infty e^{-\int_0^t (\rho_i+\xi) ds} u(c_{i,t}) - \Phi\left(\left\{n_{ik,t}, \pi^w_{ik,t}\right\}_{k\in[0,1]}\right) dt,$$

where

$$c_{i,t} = \mathcal{D}_i\Big\{c_{i1,t},\, c_{i2,t},\, \ldots,\, c_{iN,t}\Big\}$$

is a homothetic consumption (demand) aggregator and $c_{ij,t}$ denotes the household's consumption of goods from sector j at time t. $c_{i,t}$ depends on household's liquid asset holdings a, illiquid asset holdings b, and labor productivity z.

$n_{ik,t}$ is the labor hour supplied to union k. ρ is the discount rate. $\pi^w_{ik,t}$ is union k's wage inflation. Households die at rate ξ. The expectation operator is over future realizations of idiosyncratic earnings risk. We abstract from aggregate risk in this paper. Finally, we assume CRRA preferences, with

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}}{1-\gamma}.$$

The cost function of labor hour and wage inflation, $\Phi(\cdot)$, will be discussed in more detail below.

Heterogeneous consumption baskets. Households consume a CES basket of goods given by

$$c_{i,t} = \left[\sum_j (\Gamma^c_{ij})^{\frac{1}{\eta_c}} (c_{ij,t})^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}},$$

in which $c_{ij,t}$ denotes consumption by type i household of good produced in sector j. η_c is the elasticity of substitution between the goods produced in different sectors for consumption. To obtain the first of our two novel facts in our model, we allow different household types to have different preferences for goods produced in different sectors. This is captured here by Γ^c_{ij}, denoting the relative preference household type i has for goods produced in sector j.

The standard demand functions for sectoral goods consumption is given by

$$c_{ij,t} = \Gamma^c_{ij}\left(\frac{P_{j,t}}{P_{i,t}}\right)^{-\eta_c} c_{i,t},$$

where $P_{i,t}$ is the effective consumption price index (CPI) for household type i and $P_{j,t}$ is the producer price index (PPI) in sector j. Household consumer price index is given by

$$P_{i,t} = \left[\sum_j \Gamma^c_{ij}(P_{j,t})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}.$$

In our baseline model, we calibrate $\eta_c = 1$ with detailed discussion in Section **??**, therefore the demand function and nominal prices are given by

$$c_{ij,t} = \frac{\Gamma^c_{ij} P_{i,t} c_{i,t}}{P_{j,t}}$$
$$P_{i,t} = \prod_j (P_{j,t})^{\Gamma^c_{ij}}$$

The relationship between type-specific variables and aggregate variables is given by

$$\sum_i P_{i,t} c_{i,t} = P_t c_t,$$

where P_t is the aggregate price level and c_t is the aggregate consumption.

We denote the ratio of type-specific nominal consumption as Θ^c_i, and it is given by

$$\Theta^c_i = \frac{P_{i,t} c_{i,t}}{\sum_i P_{i,t} c_{i,t}} = \frac{P_{i,t} c_{i,t}}{P_t C_t}$$
$$P_t = \Pi_i P_{i,t}^{\Theta^c_i}$$

Death and birth process. Following Blanchard ([1985]), we introduce perfect annuity markets, in which households can trade claims on their remaining wealth at time of death. They pledge this wealth to a risk-neutral insurance company that, in turn, compensates households with a flow annuity payment at a rate ξ times their current asset positions. This is exactly the payment rate that makes the insurance company break even in expectation. Introducing household death rates is a commonly used technique to ensure stationarity in the wealth distribution.

Labor market. Following Auclert et al. (2018), we assume that household labor supply decisions are intermediated by labor unions. Each household type i provides $n_{ik,t}$ hours of work to each of a continuum of unions indexed by $k \in [0, 1]$. Total labor hours supplied is

$$n_{i,t} = \int_k n_{ik,t} dk.$$

Each union aggregates effective labor units provided by each household of type i, into a union-type-specific task $N_{ik,t}$, given by

$$N_{ik,t} = \bar{z} n_{ik,t},$$

where $\bar{z}$ is the average productivity of all type i households supplying labor to union k. A labor packer then further aggregates these labor services into aggregate labor supply

$$N_{i,t} = \left(\int_k N_{ik,t}^{\frac{\epsilon^w - 1}{\epsilon^w}} dk \right)^{\frac{\epsilon^w}{\epsilon^w - 1}}$$

and sell it to firms at the nominal wage $W_{i,t}$. Each household type i supply $n_{i,t}$ hours of work, which is the sum of labor hours supplied to N sectors

$$n_{i,t} = \sum_j n_{ij,t}$$

Wage subsidy. As is standard in the New Keynesian literature, we allow for an wage subsidy to avoid inefficiency resulted from monopolistic labor competition. Given union wage receipts $(1-\tau^{lab})z_{i,t}W_{ik,t}n_{ik,t}$ to a household with labor productivity $z_{i,t}$, the government pays the household a proportional wage subsidy $\tau^{empl}(1-\tau^{lab})z_{i,t}W_{ik,t}n_{ik,t}$ which the union internalizes when setting wages.

Wage rigidity. Labor union k sets a common wage $W_{ik,t}$ for each of its members, and regulates its members to supply the same hours of work. We assume that each union k faces a quadratic utility cost when adjusting its nominal wage $W_{ik,t}$. This cost is given by $\frac{\chi^w}{2}\left(\pi^w_{ik,t}\right)^2$, in which $\pi^w_{ik,t} = \frac{\dot{W}_{ik,t}}{W_{ik,t}}$ is the rate of nominal wage inflation by unions, χ^w modulates the strength of wage rigidity. $\Phi(\cdot)$ is given by

$$\Phi\left(\left\{n_{ik,t}, \pi^w_{ik,t}\right\}_{k\in[0,1]}\right) = v\left(\int_0^1 n_{ik,t}dk\right) + \frac{\chi^w}{2}\int_0^1 \left(\pi^w_{ik,t}\right)^2 dk,$$

where $v(\cdot)$ captures dis-utility from working, given by

$$v(n_{i,t}) = \frac{(n_{i,t})^{1+\phi}}{1+\phi}$$

Wage Phillips curve. Union k chooses wage in order to maximize stakeholder value, namely the sum of stakeholders' utilities. That is, union k solves

$$\max_{\pi^w_{ik,t}} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t \rho_s ds}\left[\int [u\left(c_{i,t}(a,b,z;W_{ik,t})\right) - v\left(\int_0^1 n_{ik,t}dk\right) - \frac{\chi^w}{2}\int_0^1 \left(\pi^w_{ik,t}\right)^2 dk] g_{i,t} d(a,b,z)\right] dt,$$

We further assume that union k is small, and it takes all the macroeconomic aggregates, including the cross-sectional household distribution. We solve the dynamic wage setting problem in Section **??** of the Appendix, where we derive the wage Phillips curve in continuous time. We show that in equilibrium the solution is symmetric, that is $W_{ik,t} = W_{i,t}$, $n_{ik,t} = n_{i,t}$, and $N_{ik,t} = N_{i,t}$. The New-Keynesian wage Phillips curve is given by

$$\dot{\pi}^w_{i,t} = \rho_t \pi^w_{i,t} + \frac{\epsilon^w}{\chi^w}\left[\frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab})w_{i,t}\Lambda_{i,t} - v'\left(n_{i,t}\right)\right] n_{i,t}$$

where we define $\Lambda_{i,t}$ as

$$\Lambda_{i,t} = \int z_{i,t} u'\left(c_{i,t}(a,b,z)\right) g_{i,t} d(a,b,z)$$

As χ^w approaches zeros when we assume flexible wage setting, for the wage Phillips curve to have stationary solution, we must have

$$v'\left(n_{i,t}\right) = \frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab})w_{i,t}\Lambda_{i,t}.$$

Two-account portfolio. Each household has two asset accounts, liquid $a_{i,t}$ and illiquid $b_{i,t}$. They can move funds between these two accounts, subject to a transaction cost. The liquid account has a relatively low real return of and they are subject to a borrowing constraint, $a_{i,t} \geq \underline{a}$. The illiquid account carries a higher return and is subject to a short-sale constraint $b_{i,t} \geq 0$. The household makes a real transfer decision in each period $\iota_{i,t}$, the deposit from liquid asset to illiquid account (or withdrawal from the illiquid account to the liquid account if $\iota_{i,t}$ is negative). The transfer is subject to a transaction cost $\psi(\iota_{i,t}, b_{i,t})$, which will be paid out from the liquid account. Households' two asset accounts are all held with the representative financial intermediary. We will discuss the transaction cost function below.

Portfolio adjustment costs. When households deposit funds into the illiquid account they incur adjustment costs given by $\psi(\iota_{i,t}, b_{i,t})$. We follow [Kaplan et al. (2018)] and use a functional form for adjustment costs given by

$$\psi(\iota_{i,t}, b_{i,t}) = \psi_0|\iota_{i,t}| + \psi_1\left(\frac{|\iota_{i,t}|}{\max\left\{b_{i,t}, \psi_3\right\}}\right)^{\psi_2}\max\left\{b_{i,t}, \psi_3\right\}.$$

in which $\psi_2 > 1$, $\psi_3 \geq 0$.

Such an adjustment cost function has a kink at $\iota = 0$. With $\psi_0 > 0$ and $\psi_1 > 0$, we have

$$\psi_\iota(\iota_{i,t}, b_{i,t}) = \begin{cases} \psi_0 + \psi_1\psi_2\left|\frac{\iota_{i,t}}{\max\{b_{i,t},\psi_3\}}\right|^{\psi_2-1}, & \iota_{i,t} > 0 \\ -\psi_0 - \psi_1\psi_2\left|\frac{\iota_{i,t}}{\max\{b_{i,t},\psi_3\}}\right|^{\psi_2-1}, & \iota_{i,t} < 0 \end{cases}.$$

The inverse of $\psi_{\iota_{i,t}}$ with respect to $\iota_{i,t}$ has the following function

$$\iota_t\left(\psi_{\iota_{i,t}}, b_{i,t}\right) = \begin{cases} \beta\left(\psi_{\iota_{i,t}} - \psi_0\right)^{\frac{1}{\psi_2-1}}\max\left\{b_{i,t}, \psi_3\right\}, & \psi_{\iota_{i,t}} > \psi_0 \\ \beta\left(-\psi_{\iota_{i,t}} - \psi_0\right)^{\frac{1}{\psi_2-1}}\max\left\{b_{i,t}, \psi_3\right\}, & \psi_{\iota_{i,t}} < \psi_0 \end{cases}.$$

in which $\beta = (\psi_1\psi_2)^{\frac{1}{1-\psi_2}}$, and $\iota_{i,t}$ cannot exceed the limit ι^{max}.

Budget constraints. Households faces two budget constraints, one for the liquid account and the other for the illiquid account.

Illiquid account. Household's illiquid account is denominated in the unit of capital, and it evolves according to

$$\dot{b}_{i,t} = \xi b_{i,t} + \iota_{i,t},$$

in which $\iota_{i,t}$ is the deposit from the liquid account in unit of capital, and ξ is death rate.

Liquid account. In nominal terms, the household's evolution of liquid wealth is given by

$$d(P_{i,t}a_{i,t}) = (i_t^a + \xi)P_{i,t}a_{i,t} + \sum_j \Pi_{j,t} + i_t^b P_t^K b_{i,t} + (1 - \tau^{\text{lab}})z_{i,t}n_{i,t}W_{i,t} + P_{i,t}\tau_{i,t} - P_{i,t}c_{i,t} - P_t^K \iota_{i,t} - P_{i,t}\psi(\iota_{i,t}, b_{i,t}),$$

where i_t^a is the nominal return on liquid account holdings. $\Pi_{j,t}$ is the nominal profits from intermediate producers in sector s which are all distributed to the liquid account. i_t^b is the nominal rate of return on illiquid account investment that is distributed to the liquid account. τ^{lab} is a constant income tax rate. $\tau_{i,t}$ is the real aggregate transfer. P_t^K is the nominal price per unit of capital. Rewriting this, we have

$$\dot{P}_{i,t}a_{i,t} + P_{i,t}\dot{a}_{i,t} = (i_t^a + \xi)P_{i,t}a_{i,t} + i_t^b P_t^K b_{i,t} + (1 - \tau^{\text{lab}})z_{i,t}n_{i,t}W_{i,t} + P_{i,t}\tau_{i,t} - P_{i,t}c_{i,t} - P_t^K \iota_{i,t} - P_{i,t}\psi(\iota_{i,t}, b_{i,t})$$

Let $\pi_{i,t} = \frac{\dot{P}_{i,t}}{P_{i,t}}$ denotes price inflation. Furthermore, we denote the household's effective real wage as $w_{i,t} = \frac{W_{i,t}}{P_{i,t}}$. Thus, the household's nominal wealth evolution equation becomes

$$\frac{\dot{P}_{i,t}}{P_{i,t}}a_{i,t} + \dot{a}_{i,t} = (i_t^a + \xi)a_{i,t} + \frac{i_t^b P_t^K}{P_{i,t}}b_{i,t} + (1 - \tau^{\text{lab}})z_{i,t}n_{i,t}w_{i,t} + \tau_{i,t} - c_{i,t} - q_{i,t}\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}),$$

Thus, a household's liquid wealth in real terms evolves according to

$$\dot{a}_{i,t} = (r_{i,t}^a + \xi)a_{i,t} + r_{i,t}^b b_{i,t} + (1 - \tau^{\text{lab}})z_{i,t}n_{i,t}w_{i,t} + \tau_{i,t} - c_{i,t} - q_{i,t}\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}).$$

in which

$$\begin{aligned}
r_{i,t}^a &= i_t^a - \pi_{i,t} \\
q_{i,t} &= \frac{P_t^K}{P_{i,t}} \\
r_{i,t}^b &= \frac{i_t^b P_t^K}{P_{i,t}} \\
\tau_{i,t} &= \frac{\sum_j \Pi_{j,t} + T_t}{P_{i,t}}.
\end{aligned}$$

$\tau_{i,t}$ is the total real transfer subject to rebate rescaling, including both firm profits and government transfers.

Optimization. The first-order conditions from HJB optimization are given by

$$
\begin{aligned}
\rho V_{i,t}(a,b,z) &= \max u(c_{i,t}) - \Phi(n_{ik,t}, \pi^w_{ik,t}) \\
&\quad + \Big((r^a_{i,t}+\xi)a_{i,t} + r^b_{i,t}b_{i,t} + (1-\tau^{\text{lab}})z_{i,t}n_{i,t}w_{i,t} + \tau_{i,t} - c_{i,t} - q_{i,t}\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t})\Big)\,\partial_a V_{i,t}(a,b,z) \\
&\quad + (\xi b_{i,t} + \iota_{i,t})\,\partial_b V_{i,t}(a,b,z) \\
&\quad + \mu_z \partial_z V_{i,t}(a,b,z) + \frac{\sigma_z^2}{2}\partial_{zz} V_{i,t}(a,b,z).
\end{aligned}
$$

The first-order conditions with respect to $c_{i,t}$ are given by

$$u'(c_{i,t}) = \partial_a V_{i,t}(a,b,z)$$

The first order condition of the HJB equation with respect to $\iota_{i,t}$ is given by

$$\partial_b V_{i,t}(a,b,z) = \partial_a V_{i,t}(a,b,z)(q_{i,t} + \psi_\iota(\iota_{i,t}, b_{i,t}))$$

Plugging in ψ_ι, we obtain the optimal deposit/withdraw being

$$\iota^*_{i,t} = \psi_\iota^{-1}\left(\frac{\partial_b V_{i,t}(a,b,z)}{\partial_a V_{i,t}(a,b,z)} - q_{i,t}, b_{i,t}\right).$$

We use an semi-implicit upwind finite different method following [Ben Moll], which we discuss in detail in the [Appendix]. Our upwind method splits the drift of a into two parts s^c and s^i, and splits the drift of b into m^k and m^i, and upwind them separately.

B.4 Financial Intermediary

A representative financial intermediary (the "bank") has two activities: (1) a banking activity, performing maturity transformation by collecting real liquid assets from households $a_{i,t}$ and invests them in government bonds, subject to an intermediation rate ω (2) a mutual fund activity, collecting illiquid funds $b_{i,t}$ and intermediate the funds in the form of physical capital to intermediate producers. The representative financial intermediary faces the following flow-of-funds constraints for the liquid and illiquid account respectively.

Liquid account. The financial intermediary takes the aggregate real liquid asset and invest them into government bond, net of an intermediation cost. On the liability side, it is obligated to deliver returns i^a_t on P_tA_t. On the asset side, the financial intermediary would own P_tA_t worth of government bond, which yields a return equal to the nominal interest rate i_t. The financial intermediary would take an intermediation cost ωP_tA_t from the return on government bonds in total. Under no arbitrage, the payment to households and the gain from owning government bonds net of intermediation cost must be equal, so we have:

$$i_t^a = i_t - \omega, a_{i,t} \geq 0$$

that is, in equilibrium, the financial intermediary fully pass through the cost of intermediation to the liquid account depositors.

Note that there is a borrowing wedge θ, that is the borrowing rate if $a_{i,t} < 0$ is,

$$i_t^a = i_t - \omega + \theta, a_{i,t} < 0$$

Given that $r_{i,t}^a = i_t^a - \pi_{i,t}$, the real return to liquid account investment $r_{i,t}^a$ is therefore given by

$$r_{i,t}^a = \mathbb{1}_{a_{i,t} \geq 0}(i_t - \omega - \pi_{i,t}) + \mathbb{1}_{a_{i,t} < 0}(i_t - \omega + \theta - \pi_{i,t}).$$

Illiquid account. The bank takes the illiquid assets in units of capital from all households $B_t = \sum_i \int b_{i,t} d(a,b,z)$, denominated in P_t^K. It is obligated to deliver a nominal return of i_t^b. The bank intermediate illiquid assets through an integrated and competitive rental market in the form of physical capital to intermediate producers in sectors as $K_{j,t}$. The sale price per unit of capitalis P_t^K and the rental rate is i_t^K. We have

$$B_t = \sum_i \int b_{i,t} d(a,b,z) = K_t = \sum_j K_{j,t},$$

and capital has a depreciation rate of δ.

Investment and capital stock. The bank owns capital, rents it to firms for final goods production, and makes investment decisions for capital stock replenishment. The bank has a technology that aggregate $I_{j,t}$ unit of final consumption goods across sectors indexed by j, through the economy's investment network, into $GI_t = I_t + \Phi K_t$ unit of gross investment priced at P_t^{GI}, which got turned into I_t unit of capital at price P_t^K and distributed to N sectors. Capital depreciates at rate δ. ΦK_t is a depreciation-offsetting quadratic adjustment costs in unit of final consumption goods

$$\Phi(\frac{I_t}{K_t})K_t = \frac{\kappa}{2}(\frac{I_t}{K_t} - \delta)^2 K_t$$

in which δ is the depreciation rate, and κ is the capital investment adjustment cost coefficient.

The capital stock evolution is given by

$$\dot{K}_t = I_t - \delta K_t$$

The total capital investment is aggregated by a CES technology,

$$GI_t = \left(\sum_{j=1}^{N} \left(\Gamma_j^I \right)^{\frac{1}{\eta_i}} \left(I_{j,t} \right)^{\frac{\eta_i - 1}{\eta_i}} \right)^{\frac{\eta_i}{\eta_i - 1}}$$

where η_i parameterizes the elasticity of goods used in the input bundle from different sectors for investment production. Γ_j^I indicates the share of importance of industry j in the production of the aggregate investment goods bundle. $I_{j,t}$ is the goods produced in sector j that is used in the production of the aggregate investment bundle.

The standard demand functions for investment goods from sector j is given by

$$I_{j,t} = \Gamma_j^I \left(\frac{P_{j,t}}{P_t^{GI}} \right)^{-\eta_i} GI_t$$

The relationship between sectoral prices and the capital price is given by

$$P_t^{GI} = \left[\sum_j \Gamma_j^{inv} (P_{j,t})^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}$$

In our baseline model, we calibrate $\eta_i = 1$, therefore we have the relationship for every j sector

$$P_{j,t} I_{j,t} = \Gamma_j^{inv} P_t^{GI} GI_t$$

$$P_t^{GI} = \prod_j (P_{j,t})^{\Gamma_j^{inv}}$$

The bank's nominal profit from capital investment per period is

$$\Pi_{b,t} = P_t^K I_t - P_t^{GI} \left(I_t + \Phi(\frac{I_t}{K_t}) K_t \right)$$

The static profit maximization problem is solved by the first-order condition with respect to I_t, given by

$$P_t^K = P_t^{GI}(1 + \Phi'(\frac{I_t}{K_t})) = P_t^{GI}(1 + \kappa(\frac{I_t}{K_t} - \delta))$$

Investment return. The bank conduct the following activities: (1) receives $i_t^K K_t$ for rent payment; (2) pays out $P_t^{GI} \left(I_t + \Phi(\frac{I_t}{K_t}) K_t \right)$ to generate new capital worth of $P_t^K I_t$, and (4) capital $P_t^K K_t$ depreciates at rate δ . The change in nominal value of the bank is given by

$$\dot{V}_t = i_t^K K_t + P_t^K I_t - P_t^{GI} GI_t - \delta P_t^K K_t$$

The initial value of the bank $V_t = P_t^K B_t = P_t^K \sum_j K_{j,t}$, and as we are left with the nominal return

i_t^b given by

$$i_t^b = \frac{i_t^K K_t + P_t^K I_t - P_t^{GI} G I_t - \delta P_t^K K_t}{P_t^K K_t}$$

and for r_t^b, we have

$$\begin{aligned} r_{i,t}^b &= \frac{i_t^b P_t^K}{P_{i,t}} \\ &= \frac{i_t^K K_t + P_t^K I_t - P_t^{GI} G I_t - \delta P_t^K K_t}{P_{i,t} K_t} \end{aligned}$$

B.5 Equilibrium

Definition 1. (Competitive Equilibrium) Given an initial capital level K_0, household variables $\{P_{i,t}, W_{i,t}, P_t\}$, sector-specific variables $\{y_{j,t}, N_{j,t}, K_{j,t}, I_{j,t}, P_{j,t}\}$, bank-related variables $\{P_t^G I, P_t^K, K_t, i_t^K\}$, individual decision rules $\{c_{i,t}, \iota_{i,t}, n_{i,t}\}$, such that households optimize, firms optimize, bank optimizes and markets clear.

Aggregation. The following equations characterize our definition of aggregation by type.

$$A_{i,t} = \int a g_{i,t} d(a, b, z)$$

$$B_{i,t} = \int b g_{i,t} d(a, b, z)$$

$$C_{i,t} = \int c_{i,t}(a, z) g_{i,t}(a, b, z) d(a, b, z)$$

$$\Lambda_{i,t} = \int z u' \left(c_{i,t}(a, b, z)\right) g_{i,t} d(a, b, z)$$

$$D_{i,t} = \int \iota_{i,t} g_{i,t} d(a, b, z)$$

$$\Psi_{i,t} = \int \psi_{i,t} g_{i,t} d(a, b, z)$$

The aggregation of the economy is given by

$$P_t A_t = \sum_i P_{i,t} A_{i,t}$$

$$B_t = \sum_i B_{i,t}$$

$$D_t = \sum_i D_{i,t}$$

$$P_t \Psi_t = \sum_i P_{i,t} \Psi_{i,t}$$

Market clearing. Capital market clears when capital owned by household is equal to the sum of capital demand by firms. Bond market clears when the household budget constraint is binding. Goods market clears when production in each sector is equal to the consumption and use. Note that we are assuming that the goods market finance all miscellaneous costs, including the borrowing wedge, the bank intermediation costs, and the portfolio adjustment costs, through the same consumption network and the dynamic type-specific consumption shares. The inputs are final goods from all different sectors $oc_{j,t}$. All market clearing conditions are given by

Bond market clears.

$$A_t = A_t^g$$

Capital market clears.

$$B_t = K_t = \sum_j K_{j,t}$$

Capital production clears.

$$D_t = I_t - \delta K_t$$

Labor market clears.

$$v'(n_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w}(1 + \tau^{empl})(1 - \tau^{lab}) w_{i,t} \Lambda_{i,t}$$

Goods market clears from Walras' Law in Appendix B.7.

$$y_{j,t} = c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t}$$

in which

$$P_{j,t} c_{ij,t} = \Gamma^c_{ij} P_{i,t} c_{i,t}$$

$$c_{j,t} = \sum_i c_{ij,t}$$

$$\Theta^c_i = \frac{P_{i,t} c_{i,t}}{\sum_i P_{i,t} c_{i,t}} = \frac{P_{i,t} c_{i,t}}{P_t C_t}$$

$$P_{j,t} oc_{j,t} = \sum_i \Theta^c_i \Gamma^c_{ij,t} \left(\Psi_t - r_t (A_t - A^g_t) + \omega A_t - \theta A^-_t - \frac{P^K_t}{P_t} (I_t - \delta K_t - D_t) \right)$$

$$G_{j,t} = \frac{\Gamma^g_j P_t G_t}{P_{j,t}}$$

Steady Equilibrium. At steady state, $\dot{c}_t = 0$, and $\dot{K}_t = 0$, $P_t = 1$, therefore we have

$$A_t = A^g_t$$

$$B_t = K_t$$

$$D_t = I_t - \delta K_t = 0$$

$$mc_{j,t} = \frac{\epsilon - 1}{\epsilon}$$

$$v'(n_{i,t}) = \frac{\epsilon^w - 1}{\epsilon^w} (1 + \tau^{empl})(1 - \tau^{lab}) w_{i,t} \Lambda_{i,t}$$

$$y_{j,t} = c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t}$$

Transition Equilibrium. During transition, we have the following conditions

$$
\begin{aligned}
S_t &= 0 \\
B_t &= K_t \\
v'(n_{i,t}) &= \frac{\epsilon^w - 1}{\epsilon^w}(1+\tau^{empl})(1-\tau^{lab})w_{i,t}\Lambda_{i,t} \\
\dot{\pi}_{j,t} &= \rho\pi_{j,t} - (mc_{j,t} - \frac{\epsilon - 1}{\epsilon})\frac{\epsilon}{\chi_j}y_{j,t} \\
y_{j,t} &= c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\to i,t} + oc_{j,t}
\end{aligned}
$$

B.6 Proof of Lemma ??

Proof. Since in equilibrium, all firms in sector s are symmetric, we will drop the j indexation for simplicity. Denote $p = P_t(j)$, $P = P_t$, $Y = Y_t$, $w = W_t/P_t$, taking P as given, the firm's problem in recursive form is

$$
\rho J(p,t) = \max_{\pi}\left\{\left(\frac{p}{P} - mc\right)\left(\frac{p}{P}\right)^{-\epsilon} Y - \frac{\chi}{2}\pi^2 + J_p(p,t)p\pi + J_t(p,t)\right\}
$$

where J is the corresponding value function of the maximization problem. The first order conditions of the recursive form are given by

$$
\begin{aligned}
J_p(p,t)p &= \chi\pi \\
(\rho - \pi)\, J_p(p,t) &= -\left(\frac{p}{P} - mc\right)\epsilon\left(\frac{p}{P}\right)^{-\epsilon-1}\frac{Y}{P} + \left(\frac{p}{P}\right)^{-\epsilon}\frac{Y}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)
\end{aligned}
$$

In a symmetric equilibrium we will have $p = P$, and hence

$$
\begin{aligned}
J_p(p,t) &= \frac{\chi\pi}{P} \\
(\rho - \pi)\, J_p(p,t) &= -(1 - mc)\epsilon\frac{Y}{P} + \frac{Y}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)
\end{aligned}
$$

Differentiating the first equation with respect to time, we get

$$
J_{pp}(p,t)\dot{p} + J_{pt}(p,t) = \frac{\chi\dot{\pi}}{P} - \frac{\chi\pi}{P}\frac{\dot{P}}{P}
$$

and plugging in the second equation, we get

$$
(\rho - \pi)\frac{\chi\pi}{P} = -(1 - mc)\epsilon\frac{Y}{P} + \frac{Y}{P} + \frac{\chi\dot{\pi}}{P} - \frac{\chi\pi}{P}\frac{\dot{P}}{P}
$$

Putting it together, we have

$$\dot{\pi}_{j,t} = \rho\pi_{j,t} - (mc_{j,t} - \frac{\epsilon-1}{\epsilon})\frac{\epsilon}{\chi_j}y_{j,t}$$

■

B.7 Derivation of the Walras' Law

Illiquid account. Given the capital production clearing condition $D_t = I_t - \delta K_t$, the aggregation of the illiquid account budget constraint is

$$M_t = -\xi B_t + \int dbd(a,b,z) = -\xi B_t + \xi \sum_i \int b_{i,t}d(a,b,z) + \sum_i \int \iota_{i,t}d(a,b,z) = D_t = I_t - \delta K_t$$

Liquid account. The law of motion for households' liquid weath is given by

$$\dot{a}_{i,t} = (r^a_{i,t} + \xi)a_{i,t} + r^b_{i,t}b_{i,t} + (1-\tau^{\text{lab}})z_t n_{i,t}w_{i,t} + rebate_{i,t} - c_{i,t} - q_{i,t}\iota_{i,t} - \psi(\iota_{i,t}, b_{i,t}).$$

Similarly, the aggregation of liquid account in real terms budget constraint is

$$\begin{aligned}
0 &= S_t \\
&= \frac{\sum_i P_{i,t}S_{i,t}}{P_t} \\
&= \frac{\sum_i P_{i,t}(-\xi A_{i,t} + \int da_{i,t}d(a,b,z))}{P_t} \\
&= -\xi A_t + \frac{1}{P_t}\sum_i P_{i,t}\left((r^a_{i,t}+\xi)A_{i,t} + r^b_{i,t}B_{i,t} + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t}N_{j,t}}{P_{i,t}} + \frac{\sum_j \Pi_{j,t} + T_t}{P_{i,t}} - C_{i,t} - q_{i,t}D_{i,t} - \Psi_{i,t}\right) \\
&= \frac{1}{P_t}\sum_i r^a_{i,t}P_{i,t}A_{i,t} + \frac{i^b_t P^K_t}{P_t}B_t + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t}N_{j,t}}{P_t} + \frac{\sum_j \Pi_{j,t} + T_t}{P_t} - C_t - \frac{P^K_t}{P_t}D_t - \Psi_t \\
&= \frac{1}{P_t}\sum_i r^a_{i,t}P_{i,t}A_{i,t} + \frac{i^K_t K_t + P^K_t I_t - P^{GI}_t GI_t - \delta P^K_t K_t}{P_t K_t}B_t + (1-\tau^{\text{lab}})\frac{\sum_j W_{j,t}N_{j,t}}{P_t} \\
&+ \frac{\sum_j(P_{j,t}y_{j,t} - (1-\tau^{empl})W_{j,t}N_{j,t} - i^K_t K_{j,t} - P_{jx,t}x_{j,t}) + \sum_j(\tau^{lab} - \tau^{empl})W_{j,t}N_{j,t} - P_tG_t - r_tP_tA^g_t}{P_t} - C_t - \frac{P^K_t}{P_t}D_t - \Psi_t \\
&= r_t(A_t - A^g_t) - \omega A_t + \theta A^-_t + \frac{P^K_t}{P_t}(I_t - \delta K_t - D_t) - \frac{P^{GI}_t}{P_t}GI_t + Y_t - G_t - C_t - \Psi_t - \frac{\sum_j(P_{jx,t}x_{j,t})}{P_t}
\end{aligned}$$

Government expenditure, investment expenditure, and consumption expenditure follow Γ^g_j, Γ^{inv}_j and Γ^c_{ij} respectively. We assume that all other expenditures, including the intermediate costs and the portfolio adjustment cost, follow the consumption preference and the consumption shares between different household types, Θ^c_i, are the same as that of the consumption expenditure.

Therefore the sector-specific goods market clearing condition is given by

$$y_{j,t} = c_{j,t} + G_{j,t} + I_{j,t} + + \sum_i x_{j\rightarrow i,t} + oc_{j,t}$$

in which $oc_{j,t}$ is the unit of final products from sector j which are used to finance all other costs for all types of households.

$$P_{j,t}oc_{j,t} = \sum_i \Theta_i^c \Gamma_{ij,t}^c \left(\Psi_t - r_t(A_t - A_t^g) + \omega A_t - \theta A_t^- - \frac{P_t^K}{P_t} \left(I_t - \delta K_t - D_t \right) \right)$$

Figure 6. Labor Share in the Primary Factor $1 - \alpha_j$

Note. We compute the sector-specific labor shares in the production of primary factor from the BEA GDP by Industry dataset. We remove Federal Reserve banks, credit intermediation, and related activities from the dataset. The labor share $1 - \alpha_j$ is the computed as the compensation of employees as a percentage of value added, adjusted for taxes and subsidies, averaged over 1997-2021.

C Data Appendix

This appendix provides details on the data we use to construct our empirical results.

C.1 Factor Shares

Figure 6 plots the labor share in the production of primary factors $1 - \alpha_j$ for each sector j. Given the Cobb-Douglas structure of our primary factor production function, the parameters are calculated as the ratio of compensation of employees to the value added adjusted for taxes and subsidies. We obtain this ratio for each year from 1997-2021, then take the average value.

Figure 7 plots the share of intermediate inputs in the production function $\mu_{x,j}$ for each sector j. Given the Cobb-Douglas structure of our production function, the parameters are calculated as the intermediate input expenditures as a percentage of gross output, averaged over 1997-2021. We obtain this ratio for each year from 1997-2021, then take the average value.

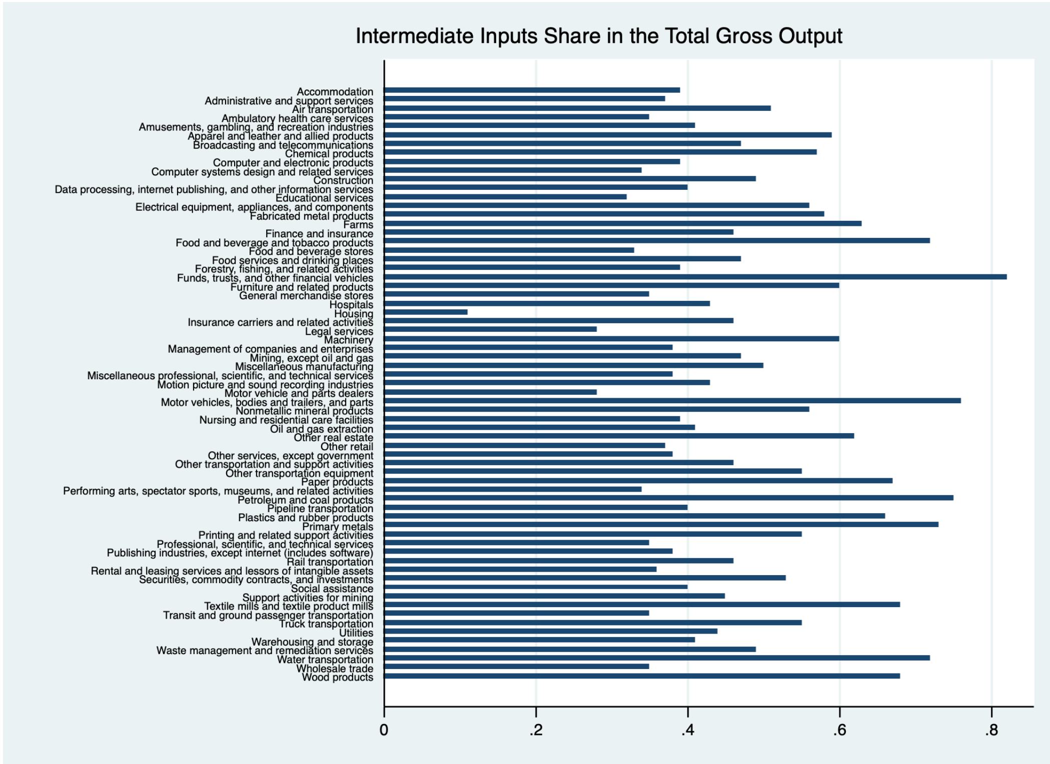

Figure 7. Intermediate Inputs Share in the Total Gross Output $\mu_{x,j}$

Note. We compute the sector-specific labor shares in the production of primary factor from the BEA GDP by Industry dataset. We remove Federal Reserve banks, credit intermediation, and related activities from the dataset. The intermediate input share $\mu_{x,j}$ is computed as the intermediate input expenditures as a percentage of gross output, averaged over 1997-2021.

C.2 Capital Investment

We use the BEA 1997 Capital Flows table to calculate the share of capital distribution from each sector Γ_j^{inv}. The Capital Flows table includes 180 commodities with corresponding NAICS codes. Our first step is to match the commodities used to the sector categories. Most of the matching are straightforward given the NAICS codes in both the Capital Flows table and the Input-Output table. Special attention should be paid to the following: (1) "Manufactured homes, mobile homes" in commodities is categorized under the "Housing" sector; (2) "Retail trade" in commodities include both 44 and 45 by NAICS codes, we divide and assign it to "Motor vehicle and parts dealers", "Food and beverage stores", "General merchandise stores", "Other retail" according to these four sectors' sizes measured by the gross output in 1997; (3) "Offices of real estate agents and brokers" in commodities is categorized under the "Rental and leasing services and lessors of intangible assets" sector; (4) "Noncomparable imports" is excluded. Table **??** summarizes the sectoral capital investment contributions and their corresponding shares of total capital investment. We can see that a substantial capital investment uses goods from "Construction", "Machinery", and "Computer

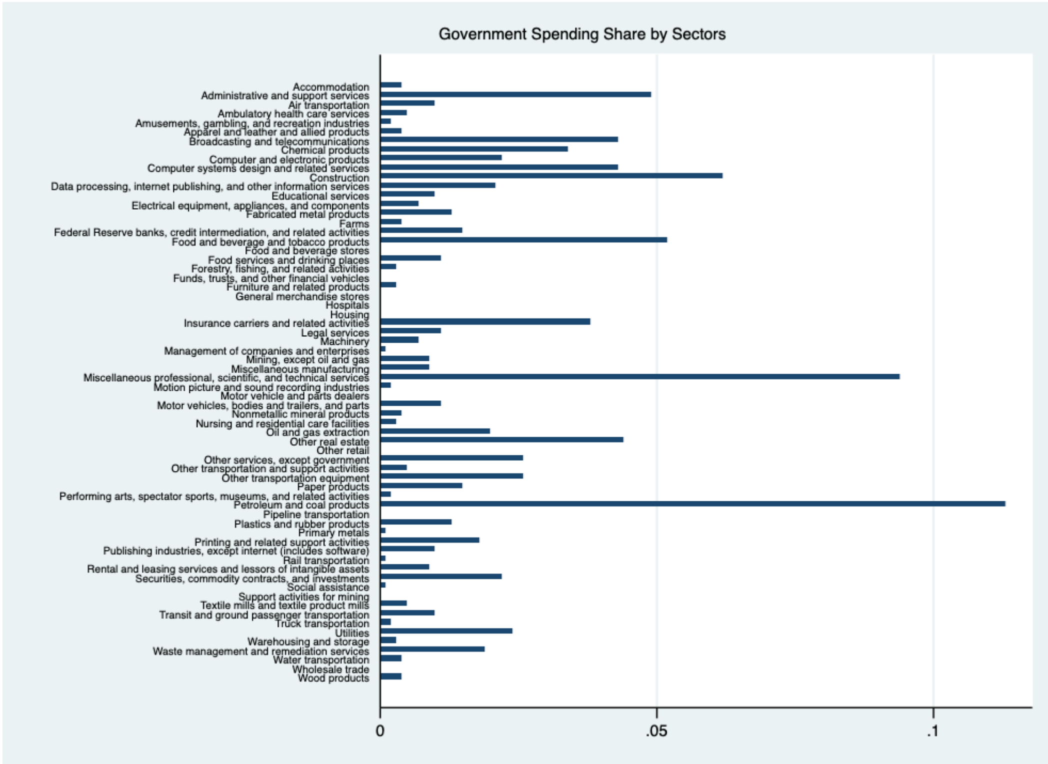

Figure 8. Government Spending Share on Sectoral Goods Γ_j^g

Note. We compute the share of government spending on goods from individual sector j in total government spending every year, then we average the ratio across 1997-2021.

and electronic products", while the service industries contribute little.

C.3 Government Spending

We use the BEA industry input-output "Use" table to compute the share of government spending on goods from different sectors Γ_j^g. The parameters are calculated as the government expenditures in sector j as a percentage of total government spending. "Government" includes federal government, federal government enterprises, state and local government, and state and local government enterprises. We average the annual share across 1997-2021. Figure 8 plots the government spending share, averaged across the sample period.

C.4 Intermediates input-output network.

For each year's BEA industry input-output "Use" table, we calculate the parameters of the intermediates input-output network Γ_{ij}^x as sector j (columns)'s nominal expenditure on intermediate inputs from sector i (rows) as a share of j's total expenditure on intermediate inputs. Then we average the

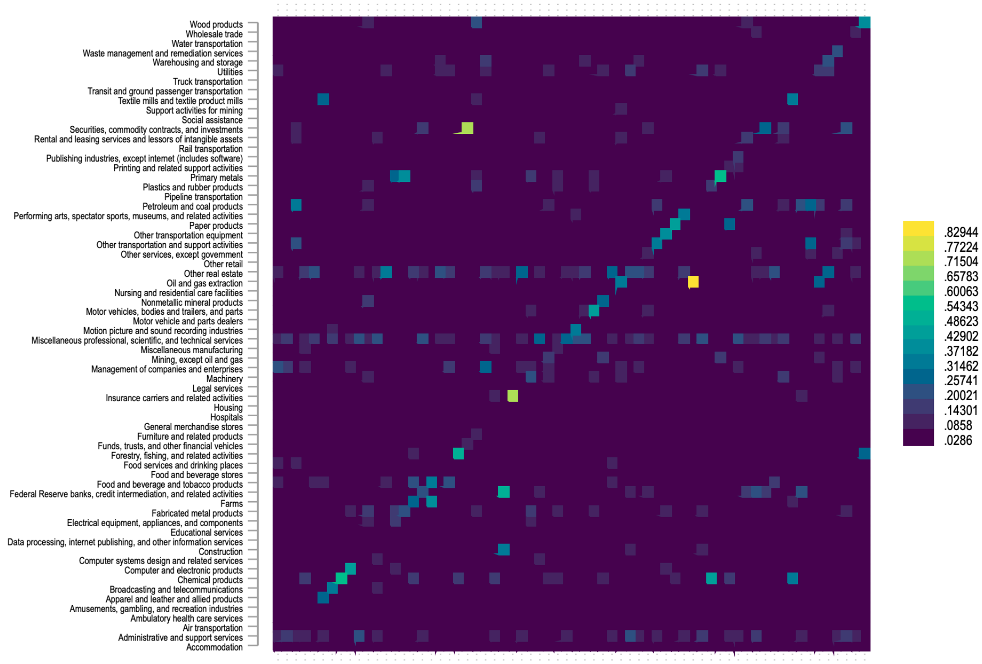

Figure 9. The Input-Output Network Γ^x_{ij}

Note. We compute the share of government spending on goods from individual sector j in total government spending every year, then we average the ratio across 1997-2021.

ratios across 1997-2021. Figure 9 plots the heatmap of the input-output network, averaged across the sample period.

C.5 Firm Adjustment Cost Parameters

To pin down parameter χ, We establish the relationship between our monthly price adjusting data, usually seen in Calvo models, and the adjustment cost parameter in Rotemberg-type setting. We derive the continuous-time firm adjustment cost parameters according to [Sims and Wolff (2017)].[18]

Calvo. In the Calvo model, a randomly selected fraction of firms, $1 - \theta$, can adjust their price in a given period. All updating firms adjust to the same price P^*, and the adjusting inflation is

[18] There is a strand of literature that studies the difference between Rotemberg and Calvo models. [Nistic'o (2007)] and [Lombardo and Vestin (2008)] compare the welfare implications of the two models. [Ascari et al. (2011)] and [Ascari and Rossi (2012)] investigate the differences between the two models under a positive trend inflation rate. [Ascari and Rossi (2011)] study the effect of a permanent disinflation in the Rotemberg and Calvo models. More recently, [Boneva et al. (2016), Richtera and Throckmorton (2016), Eggertsson and Singh (2018), and Miao and Ngo (2018)] investigate the differences in the predictions of the Rotemberg and Calvo models with the zero lower bound for the nominal interest rate. [Sims and Wolff (2017)] study the state-dependent fiscal multipliers in the two models under a Taylor rule in addition to periods where monetary policy is passive. Moreover, [Born and Pfeifer (2020)] discuss the mapping between Rotemberg and Calvo wage rigidities.

$1 + \pi_t^* = P_t^* / P_{t-1}$. Overall inflation π_t satisfies

$$\frac{1+\pi_t^*}{1+\pi_t} = \frac{\epsilon}{\epsilon-1}\frac{x_{1,t}}{x_{2,t}}$$
$$x_{1,t} = \frac{1}{C_t} mc_t Y_t + \theta\beta \mathbb{E}_t \left(1+\pi_{t+1}\right)^{\epsilon} x_{1,t+1}$$
$$x_{2,t} = \frac{1}{C_t} Y_t + \theta\beta \mathbb{E}_t \left(1+\pi_{t+1}\right)^{\epsilon-1} x_{2,t+1}$$

Inflation evolves according to

$$\left(1+\pi_t\right)^{1-\epsilon} = \left(1-\theta\right)\left(1+\pi_t^*\right)^{1-\epsilon} + \theta$$

The NKPC in the Calvo setting is given by

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\ln mc_t - \ln mc_t^*\right) \tag{25}$$

Rotemberg. We use an alternative of Rotemberg setting, where we denote the adjustment cost by $\Lambda(\pi_t) = \frac{\chi}{2}\left(\pi_t\right)^2 P_t Y_t$. The NKPC in Rotemberg is given by,

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\epsilon-1}{\chi}(\ln mc_t - \ln mc_t^*) y_t \tag{26}$$

First order equivalence. For the slopes of NKPC in equation 25 and in equation 26 to be equivalent, we would have

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = \frac{\epsilon-1}{\chi} y_t^*$$

Therefore, the adjustment cost parameter χ is given by

$$\chi_j = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\theta\beta)} y_{j,t}^*$$

in which θ is the probability that price remains unchanged for a quarter (3 months), $\theta = (1-\ominus)^3$, where $\ominus$ is the monthly price adjustment frequency. $y_{j,t}^*$ is the steady-state sectoral output