CBDC design and bank runs in non-cashless society: information channel

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Abstract

I develop a model of bank runs where supply of CBDC is endogenous: depositors face a trade-off between CBDC withdrawals, which are faster and more suitable for digital goods, and cash withdrawals which are slower and more suitable for physical goods. We show that presence of CBDC mitigates runs by making banks react faster to runs with the penalties on withdrawals. However, it leads to over-mining of CBDC during stress to the levels above what is needed for the consumption, leading to extra transaction costs. Lastly, if given a choice, banks are shown to have incentives to offer mixed currency

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deposit products in the interest of depositors. We calibrate the model empirically and evaluate necessary policies to prevent runs.

1 Introduction

As a response to the worldwide digitization, majority of the central banks indicated their desire to research and possibly issue the central bank digital currency. While the scope and specifics of such issuance is still an open question for many, it is certain that introduction of CBDC will not replace cash overnight, as well as not entire economy will immediately become digital with the introduction of the digital coin. We consider a case of when CBDC is introduced as a complement to cash in the economy that trades both digital and physical goods. We study how the introduction of CBDC will impact usage of cash and bank deposits during stress and non-stress episodes in the context of banking and financial stability. For this, we develop a Diamond and Dybvig (1983) type model with sequential bank runs (as in Ennis and Keister (2006) and Green and Lin (2003)) to study whether introduction of CBDC will impact the bank runs intensity and specifics, and what can be done to prevent the panic-like behaviors. We assume that currency and CBDC money creation are driven by the demand of consumers for such payment instruments and show why CBDC is mined disproportionally more than cash during the stressful events than during peaceful times.

Our main findings are the following. If CBDC is introduced in the environment of no stress, depositors prefer to use it for digital goods purchases, and remain using cash for physical goods purchases. As such, CBDC improves welfare of consumers that pay for digital goods and otherwise would need to exchange cash to digital methods of payment. However, with stress, the depositors that consume physical goods become more inclined to withdraw in CBDC because digital withdrawals are faster than the withdrawals in cash (think of the physical bank locations such as branches and ATMs). The degree to which bank runs shift towards CBDC depends on the equilibrium outcome.

We find that, for any parameters values, there are equilibria with the run taking place fully in CBDC. In this case, withdrawals happen faster than in the case of no CBDC and harm depositors with preferences for physical goods because they have to bear exchange costs. Thus, in the equilibria with fully-digital runs, welfare is improving whenever the digital transactions are more common than the non-digital ones in the economy.

This however does not prevent existence of equilibria where runs take place in both CBDC and cash. The equilibria of such kind arise in the presence of strong preferences of consumers for cash as a method of payment and higher exchange transactions costs between cash and CBDC. In the equilibria with mixed currency runs, we find that withdrawals are less likely to occur and deliver higher welfare to the depositors than in the no-CBDC case. To explain how this works, we rely on the findings of the previous bank run literature that assume that banks may respond to runs with the additional charges on the withdrawals, also punishing the remaining depositors. Clearly, such preventive measures are only applied when the bank is certain that the state of the world is such that the depositors withdraw for the reasons of stress and panic and not for regular consumption. Without CBDC, the bank would infer about the state of the world by observing the withdrawals of all depositors, those who consume in digital and those who consume in physical stores. In the case when CDBC co-exists with cash, a signal is received faster: by only observing the digital withdrawals, the targeted bank may reveal the preferences of consumers sooner and respond to the run faster by placing additional withdrawal barriers and preventing more panic runs. Without CBDC, bank would have a higher threshold on the withdrawals to say with certainty that the depositors are driven by negative sentiment (sunspot) and the panic is taking place. Prompt response of the bank discourages withdrawals motivated by panics, and allows for higher returns on investments, thus, higher welfare.

The model can be extended in multiple directions. While we focus on the CBDC without re-numeration, the results can be extended to positive rates paid by the central bank. Also, our results do not depend on the distribution model of CBDC, but rely on CBDC payments being much faster than the wire payments, with latter would be another way to withdraw deposits, which we assume to be slower and more costly than withdrawing in either cash and CBDC.

As an extension, we consider the case when bank offers two different deposit products instead of one: deposit with only CBDC withdrawals, and deposit with only cash withdrawals. We find when it is welfare improving for the bank to offer withdrawals in both currencies rather than to create two different account types. This result helps motivating CBDC design with the bank-centered distribution model, as our paper illustrates that banks may be interested in offering consumers an option to withdraw in CBDC for two reasons: convenience of digital payments, and an efficient bank run management.

Finally, we research the preventive measures for mitigating CBDC bank runs. For policy purposes, such measured should be considered in light of the last century regulatory changes for banks and other financial institutions, including deposit insurance, Basel III, implicit government guarantees for too-big-to-fail banks, recovery and resolution plans reforms, and money market reforms. In this paper, we limit our analysis to the stylized calibration for the largest Canadian banks and generally held theoretical results. We ask whether CBDC runs should be addressed by the policy makers in the same way as fiat currency runs. In the example of liquidity gates, we show that regulation of fiat currency alone is insufficient to prevent the digital run. However, similar intuition holds for controlling cash and CBDC withdrawals. We first show that imposing liquidity gates on withdrawals in both cash and CBDC is not effective when depositors do not believe in the commitment of banks to maintain their rates throughout the run. As such, additional measures should be imposed. We find that if deposit insurance is in place for the withdrawals of both kind, it is sufficient to prevent the bank runs in our environment. However, the price of such insurance should be different in the case with CBDC than without CBDC due to the different nature of runs highlighted above. The last result indicates that introduction of CBDC may require revisiting regulatory bank policies such as deposit insurance and liquidity regulation with the introduction of CBDC.

The structure of the paper is the following: in section 2 we provide a literature review, in section 3 we present the model, in section **??** we provide policy recommendations, and in section **??** we calibrate the model for Canadian economy.

2 Literature review

We base our research on the bank run literature: Diamond and Dybvig (1983), Green and Lin (2003), Ennis and Keister (2006), Goldstein and Pauzner (2005), Peck and Shell (2003), Ennis and Keister (2009), Andolfatto *et al.* (2017), Andolfatto and Nosal (2008). Keister and Monnet (2022) make similar predictions with respect to the information channel of the digital currency, however without cash being the second alternative. Optimal deposit insurance was also considered by Dávila and Goldstein (2021). Experimental literature on bank runs that looks at the sunspot and coordination of depositors Arifovic *et al.* (2013), Arifovic and Jiang (2014), Kiss *et al.* (2012). More generally, literature on CBDC and banking includes: Andolfatto (2020), Kiester and Sanches (2021), Chiu et al. (2020, 2022), Parlour et al (2020), Whited et al. (2022), Garratt et al. (2022), Brunnermeier and Niepelt (2019). Classic bank runs: Diamond and Dybvig (1983), Ennis and Kiester (2009,2010), Peck and Shell (2003), Goldstein and Pauzner (2005), Arifovic et al. (2013), Allen and Gale (1988). CBDC bank runs specifically: Kiester and Monnet (2022), Ahnert et al (2022), Williamson (2022), Skeie (2019), Kumhof and Noone (2021).

3 Model without CBDC

3.1 Timing and types of transactions

Consider interactions of one bank with its depositors. As in the standard Diamond and Dybvig (1983) model, the bank operates within three periods: $t = 0, 1, 2$. At $t = 0$, there is a unit mass of depositors, each of whom supplies 1 unit of funds to the bank. The bank invests on behalf of depositors. The investment pays off gross return R in the last period. At $t = 1$ and $t = 2$ the bank processes the withdrawals of depositors. Such withdrawals may arise spontaneously due to the strategic response of some of depositors to consumption shocks. Ex-ante, depositors do not know whether they will receive the shock or nor. As such, the bank serves as an insurance device for the depositors who want to invest but cannot guarantee their commitment to the financial market.

In the interim period, fraction L of the bank's assets can be liquidated to repay withdrawals. Variable L can be interpreted as the amount of liquid assets in the investment portfolio of the bank; liquidation itself induces no transaction cost, so revenue of the bank that liquidates L remains $R(1-L)$.

Depositors place funds in the bank to eventually withdraw it and use for consumption. For simplicity, we assume that each depositor will eventually consume one type of good: digital if the depositor's type is $\theta^\delta = 1$ or physical if the depositor's type is $\theta^\delta = 0$. There is no aggregate uncertainty in the model about the share of digital depositors: the probability of depositor consumption to be digital is δ and the probability of it being physical is $1-\delta$. . The preferences for consumption are revealed at $t = 1$.[1] In this way δ measures the level of digitalization of the economy rather than the heterogeneity of the consumer tastes in the economy, which would be known ahead of time.

When digital vs physical types are revealed, fraction π of depositors also receives a consumption preference shock to consume immediately. Following the literature, we call these depositors impatient, with their type $\theta^\pi = 1$. The remaining $1 - \pi$ depositors, called patient have type $\theta^\pi = 0$. The patient depositors have no preferences over when to consume. However, if there is a bank run and likely insolvency of the bank, it may be in their interest not to hold on to the last period. Realizations of impatience type θ^π and digital type θ^δ are independent.

3.2 Withdrawals

In the middle period, each depositor is given a chance to withdraw funds. We assume that withdrawals in CBDC are faster than and take place before

[1] We assume that withdrawals happen slightly before consumption, so the depositor cannot withdraw by making a direct payment in either digital or physical store (as with a debit card). Also, we look at banks in isolation and interbank payments, such as wire transfers, are not considered by the depositor as a way of withdrawal, because the empirical evidence suggests that interbank transactions take on average more time and cost than withdrawals in cash and CBDC. In the Canadian context, majority of depositors hold a single deposit account only, so an interbank transfer for them would be done either through online banking or in person with additional time spent on opening a new bank account. In addition, for the cases of severe crisis, multiple banks may experience bank runs at the same time.

withdrawals in cash. The order in which depositors decide on withdrawals is ex-ante random, and follows a sequential service constraint (as defined in Wallace (1988)) with model specification from Ennis and Keister (2006). One by one depositors are selected for their turn to make a decision. By the time depositor i is selected, it is informed about its order and the total number of withdrawals that have already taken place. Without loss of generality, assume that the decision order of depositor i is i. The depositor then decides whether to withdraw or not. We will track the strategy to withdraw with variables $y_i^c \in \{0,1\}(\theta, s)$ and $y_i^d(\theta, s) \in \{0,1\}$, which are conditional on the type of the depositor $\theta = (\theta^\pi, \theta^\delta)$ and the signal s.

The number of withdrawals with depositor i given the realization of type $\theta_{i'}$ for each depositor $h' \in [0, i]$ can be determined recursively:

$$\mu^d(i) = \int_0^i y_h^d(\theta_h, \lambda_h)\varepsilon dh$$

$$\mu^c(i) = \int_0^i y_h^c(\theta_h, \lambda_h)\varepsilon dh$$

At time $t = 1$, the strategy of the bank is to setup the payments to each depositor conditional on its order of withdrawals in either CBDC or cash lines. This strategy can be summarized by interim period payment functions $x^d(\mu^d, \mu^c)$ and $x^c(\mu^d, \mu^c)$. Because we model CBDC and cash runs sequentially, it is enough to focus on the cases of strategies $x^d(\mu^d)$ and $x^c(\mu^c)$, where the number $x^d(\mu^{dc})$ is the payment given to the μ^{dc}-th depositor who arrives to withdraw CBDC, and $x^c(\mu^c)$ is the payment given to the μ^c-th depositor who arrives to withdraw cash. By design, not all depositors will withdraw in both cash and CBDC, so $\mu^c \leq i$ and $\mu^d \leq i$ after each depositor i makes a decision.

The strategies are limited by the budget constraint in the interim period:

$$x^1_{all} = \int_0^1 x^d(\mu)d\mu + \int_0^1 x^c(\mu)d\mu \leq \alpha$$

At time $t = 2$, when the bank run is over and some funds are left over at the bank, the strategy of the bank can be set up to pay equal amount to each withdrawing depositor. Thus, assuming bank keeps no profit, the repayments at time $t = 2$ are determined by the strategies at time $t = 1$. The strategies of i given umber of withdrawals μ^{dc} and μ^c are x^c and x^{dc}. The strategies of the bank are c_1^c, c_1^d, c_2^c, c_2^d.

$$\int_0^1 x^d(\mu)d\mu + \int_0^1 x^c(\mu)d\mu \leq \alpha$$

In the last period $t = 2$, the bank repays equal fraction of the total remaining profit x^2_{all}.

$$x^2_{all} = R(1 - x^1_{all})$$

The consumption of depositor i in the game is determined by its type $\theta = (\theta^\pi, \theta^\delta)$, withdrawals preceding i, denoted as $\mu = (\mu^d, \mu^c)$, and bank's repayment strategies $x = (x^d, x^c)$.

$$c(i, \theta, \mu, x) = (1-\theta^\pi)[(1-\tau)(1-\theta^d)+\theta^d]x^d(\mu)y^d(\mu^d(i)))$$

$$+(1-\theta^\pi)[(1-\tau)\theta^d+(1-\theta^d)]x^c(\mu)y^c(\mu^c(i))$$

$$+\theta^\pi[(1-\tau)(1-\theta^d)+\theta^d][x_1^d(\mu)+x_2^d(\theta,\mu)]y^d(\mu^d(i)))$$

$$+\theta^\pi[(1-\tau)\theta^d+(1-\theta^d)]x^c(\mu)y^c(\mu^c(i))$$

where μ^{dc} is the number of withdrawals that would take place in CBDC with i including if it runs via difgital currency, and μ^c is the number of withdrawals that would take place in cash with i including if it runs in cash.

The utility of the bank is the utility weighted by types:

$$u_i(c_1^d(\mu), c_1^c(\mu), c_2^d, c_2^c) = \int_0^1 \sum_{\theta_\pi, \theta_\delta \in \{0,1\}} \pi^{\theta_\pi}(1-\pi)^{1-\theta_\pi}\delta^{\theta_\delta}(1-\delta)^{1-\theta_\delta}u(\theta_\pi, \theta_\delta, \mu^{dc}, \mu^c)di$$

The equilibrium is defined as a set of strategies $y_i(\theta_i, \lambda_i)$ for all depositors $i \in [0, 1]$

3.3 Types of deposit contract

We will consider multiple deposit contracts in the game. First, we assume that the bank can change the deposit contract terms as the run goes. In this case, the bank will update two deposit rates: for those who want to withdraw earlier and those who stay until the last period.

In alternative setup, we will focus on the single deposit contract with commitment. This is when a bank offers the same pair of rates to any depositor: c_1 for period $t = 1$ withdrawals and c_2 for period $t = 2$ withdrawals despite bank run. Commitment does not necessarily mean that the bank does not have resources to monitor the withdrawals and respond accordingly. Instead, offering a contract with commitment may be an intentional strategy of the bank. As we will show in the following section, the key feature of the deposit contract with commitment is that depositors believe that the bank will not respond to the runs under any circumstances, which may peacify the panic runs. In the past literature, it was assumed that deposit contract with commitment is not realistic, meaning it is not supported by the expectations of depositors. However, in the age of smart contracts, it is possible that the bank may be interested in being committed.

4 Equilibrium in economy without CBDC

4.1 No run equilibrium

In our setup, bank runs are created by the coordination failure and are thus solely panic based. When the panic does not happen, the bank maximizes expected risk-adjusted payoff

$$V = (1 - \delta)[\pi u(c_{1,K+1}) + (1 - \pi)u(c_{2,K+1})]$$

$$+\delta[\pi u(\tau_d c_{1,K+1}) + (1-\pi)u(\tau_d c_{2,K+1})]$$

subject to budget constraint

$$(1-\pi)c_{2,K+1} = R(1-\pi c_{1,K+1}).$$

In this equilibrium, the optimal deposit rates offered by the bank are

$$c_{1,k} = \frac{1}{\pi + (1-\pi)R^{1-\sigma/\sigma}}$$

$$c_{2,k} = \frac{R^{1-\sigma/\sigma}}{\pi + (1-\pi)R^{1-\sigma/\sigma}}$$

4.2 Run equilibrium

We solve the model using backward induction. Each equilibrium is organized as a sequence of withdrawal waves. There are multiple equilibria. Each of them is characterized by the number of waves. One wave is the combination of withdrawals of deposits under the same repayment scheme. Consider and equilibrium with $K+1$ waves in the game without CBDC. In each wave k, fraction π of consumers withdraws to receive compensation $c_{1,k}$. Depositors know what wave is taking place, but not the bank. The bank observes the number of withdrawals, so it can infer how many waves have passed, but not whether the next wave will take place or not. It updates deposit rates with the beginning of each wave by offering terms $c_{1,k}$ and $c_{2,k}$ to depositors.

When the bank has observed K waves, it maximizes expected risk-adjusted payoff V_{K+1} of remaining depositors, share π of which will withdraw at time $t = 1$, and remaining share $1 - \pi$ will withdraw at time $t = 2$:

$$V_{K+1}(\psi_K) = (1-\delta)[\pi u(c_{1,K+1}) + (1-\pi)u(c_{2,K+1})]$$

$$+\delta[\pi u(\tau_d c_{1,K+1}) + (1-\pi)u(\tau_d c_{2,K+1})]$$

subject to budget constraint

$$(1-\pi)c_{2,K+1} = R(\psi_K - \pi c_{1,K+1}).$$

Using backward induction, we can find the optimal contract in the middle waves. After $k = 1, ..., K-1$ waves, the probability that run ends is $1 - \frac{s_k}{s_{k-1}}$, probability that it continues is $\frac{s_k}{s_{k-1}}$. So the Bellman equation in the middle wave is:

$$V_k(\psi_{k-1}) = \left(1 - \frac{s_k}{s_{k-1}}\right)(1-\delta)[\pi u(c_{1,k}) + (1-\pi)u(c_{2,k})]$$

$$+\left(1 - \frac{s_k}{s_{k-1}}\right)\delta[\pi u(\tau_d c_{1,k}) + (1-\pi)u(\tau_d c_{2,k})]$$

$$+\left(\frac{s_k}{s_{k-1}}\right)(1-\delta)[\pi u(c_{1,k}) + (1-\pi)V_{k+1}(\psi_k)]$$

$$+\left(\frac{s_k}{s_{k-1}}\right)\delta[\pi u(\tau_d c_{1,k}) + (1-\pi)V_{k+1}(\psi_k)]$$

subject to budget constraint

$$(1-\pi)c_{2,k} = R(\psi_{k-1} - \pi c_{1,k}).$$

The equilibrium in the non-CBDC case can be found from the solution of each intertemporal problem:

$$c_{1,k} = \psi_{k-1} \frac{1}{\pi + \frac{1-\pi}{R}\frac{c_{2,k}}{c_{1,k}}}$$

$$c_{2,k} = \psi_{k-1} \frac{\frac{c_{2,k}}{c_{1,k}}}{\pi + \frac{1-\pi}{R}\frac{c_{2,k}}{c_{1,k}}}$$

for $k = 1, .., K+1$ (including the last period), such that the relative split between two contracts for each wave is

$$\frac{c_{2,k}}{c_{1,k}} = R^{1/\sigma} \left((1 - \frac{s_k}{s_{k-1}}) + \frac{s_k}{s_{k-1}} E_k \right)^{1/\sigma} \tag{1}$$

$$\frac{c_{2,K+1}}{c_{1,K+1}} = R^{1-\sigma/\sigma} \tag{2}$$

where unknown variables E_k are defined recursively as marginal contribution from the next period value function

$$E_k = \left(\pi R^{(\sigma-1)/\sigma} + (1-\pi) \left(1 - \frac{s_{k+1}}{s_k} + \frac{s_{k+1}}{s_k} E_{k+1} \right)^{1/\sigma} \right)^{\sigma}, \text{ for } k = 1, ..., K$$

$$E_{K+1} = (\pi R^{\sigma-1/\sigma} + 1 - \pi)^{\sigma}.$$

Theorem 1 *In the economy without CBDC, there exist a sequence of sunspot thresholds $s_1, ..., s_K$, such that for each realization of sunspot $s \in [s_k, s_{k-1})$, the withdrawals take place in $k \leq K$ waves. Within each of such waves, proportion π of withdrawals of depositors takes place. The bank offers deposit rates $(c_{1,k}, c_{2,k})$ during wave k:*

$$c_{1,k} = \left(\prod_{j=1}^{k} \frac{\frac{c_{2,j}}{c_{1,j}}}{\pi + \frac{1-\pi}{R} \frac{c_{2,j}}{c_{1,j}}} \right) \frac{R}{\frac{c_{2,j}}{c_{1,j}}} \tag{3}$$

$$c_{2,k} = \left(\prod_{j=1}^{k} \frac{\frac{c_{2,j}}{c_{1,j}}}{\pi + \frac{1-\pi}{R} \frac{c_{2,j}}{c_{1,j}}} \right) R \tag{4}$$

and the amount of funds available at the bank at wave k is

$$\psi_k = \prod_{j=1}^{k} \frac{\frac{c_{2,j}}{c_{1,j}}}{\pi + \frac{1-\pi}{R} \frac{c_{2,j}}{c_{1,j}}} \tag{5}$$

where $\frac{c_{2,j}}{c_{1,j}}$ are defined in equations (1) and (2).

4.3 Impact of digitalization on bank runs

We will say that economy experiences higher digitalization, if there there is more digital consumption δ.

Corollary 1 *Economy with higher digitization δ and no CBDC, has a lower efficiency, greater probability of run, and the same frequency of updates in deposit rates as in an economy with lower digitalization and no CBDC.*

This result can be explained with the following. In the economy without CBDC, the only safe haven for running depositors is cash, which is costly to hold if you spend digitally, and the digital depositors who withdraw need to be compensated for holding cash. Moreover, introduction of digital goods leads to intertemporal substitutions between two periods, with depositors favouring the immediate withdrawals.

As such, pretense of digital goods without a digital method of payment is a source of inefficincy. In this paper, CBDC aims at solving this inefficiency problem.

5 Model with CBDC

5.1 No run equilibrium with CBDC

When the panic does not happen, CBDC is welfare improving, because it allows to reduce transaction costs of depositors that withdraw not in the currency of their choice.

$$V = \pi u(c_{1,K+1}) + (1-\pi)u(c_{2,K+1})$$

subject to budget constraint

$$(1-\pi)c_{2,K+1} = R(1-\pi c_{1,K+1}).$$

In this equilibrium, the optimal deposit rates offered by the bank are identical to those without CBDC (due to CRRA utility of depositors)

$$c_{1,k} = \frac{1}{\pi + (1-\pi)R^{1-\sigma/\sigma}}$$

$$c_{2,k} = \frac{R^{1-\sigma/\sigma}}{\pi + (1-\pi)R^{1-\sigma/\sigma}}.$$

However, the social welfare is $(1-\delta+\delta\tau_d^{1-\sigma})$ larger, when CBDC is introduced.

Corollary 2 *In the absence of bank runs, CBDC improves welfare of depositors, and maintains the same deposit rates.*

5.2 Run equilibrium with CBDC

The intuition about the model can be best derived from the special case when the cost of exchanging funds from cash to CBDC and vice versa is positive but small: $\tau_d \to 1$, $\tau_c \to 1$ (general case is not in the draft yet, but it is solved).

During bank runs, depositors withdraw currency in two lines at $t = 1$: CBDC line proceeded by the cash line. In lines, there are K^d and K^{ph} waves correspondingly.

By the end of the last cash wave K^{ph}, funds $\psi_k \leq 1$ remain. The proportion of digital depositors is θ^d, which we will identify later on.

Then the bank optimizes withdrawals in cash in the last wave by maximizing:

$$V_{K^{ph}+1} = \pi(1-\theta^d)u(c_{1,K^{ph}+1}) + \left((1-\pi)(1-\theta^d)+\theta^d\right)u(c_{2,K^{ph}+1}) \tag{6}$$

subject to budget constraint

$$(1-\pi(1-\theta^d))c_{2,K^{ph}+1} = R(\psi_{K^{ph}} - \pi(1-\theta^d)c_{1,K^{ph}+1})$$

So the optimal deposit rates offered in the last cash wave at $t = 1$ and non-cash waves at $t = 2$ are:

$$c_{1,K^{ph}+1} = \psi_{K^{ph}} \frac{1}{\pi(1-\theta^d) + (1-\pi(1-\theta^d))R^{1-\sigma/\sigma}} \tag{7}$$

$$c_{2,K^{ph}+1} = \psi_{K^{ph}} \frac{R^{1-\sigma/\sigma}}{\pi(1-\theta^d) + (1-\pi(1-\theta^d))R^{1-\sigma/\sigma}} \tag{8}$$

We continue deriving equilibrium with CBDC using backward induction. For existence of at least two waves, $K^{ph}+1$ and K^{ph}, the incentives of depositors should be aligned such that patient physical depositors do not have incentives to run in the last post-wave, namely

$$c_{1,K^{ph}+1} < c_{2,K^{ph}+1}. \tag{9}$$

However, the same physical depositors should be interested in withdrawals in the earlier waves, when run takes place

$$c_{1,K^{ph}} > c_{2,K^{ph}+1}. \tag{10}$$

Condition (9) is also sufficient for digital patient depositors to stay until the maturity of the deposit contract and not withdraw earlier

$$\tau_p c_{1,K^{ph}+1} < c_{1,K^{ph}+1} < c_{2,K^{ph}+1}.$$

Interestingly that in the case when $\tau \to 1$, condition (10) guarantees that if a digital depositor given a chance to withdraw during or before wave K^{ph}, it will do so.

$$\tau c_{1,K^{ph}} > c_{2,K^{ph}+1} \tag{11}$$

We reach a contradiction, because cash waves follow CBDC withdrawals, so each digital depositor is given a chance to withdraw at a rate above $c_{1,K^{ph}}$. That is why for digital depositors to stay until $K^{ph}+1$ (equivalent to all depositors to stay), there should exist only a single cash withdrawing post-wave $K^{ph} = K^d + 1$, not more than one panic wave as we originally assumed. Moreover, during CBDC waves, both types of depositors will withdraw funds.

Value function in the cash wave,

$$V_{K^{ph}+1} = \left(\pi(1-\delta) + (1-\pi(1-\delta))R^{1-\sigma/\sigma}\right)^{\sigma} \frac{(\psi_{K^{ph}})^{1-\sigma}}{1-\sigma}$$

is smaller than the value function for no CBDC case, ceteris paribus, due to high intertemporal sensitivity. There is also the same percentage split between patient and impatient depositors as in the K-th wave in no CBDC case: $\theta^d = \delta$ But the amount of funds that remain at the bank $\psi_{K^{ph}}$ can be different from ψ_K.

Value function in the digital waves of CBDC case is equivalent to the cash waves in the case of no-CBDC, however, now withdrawals take place in a different currency. Also, the wave structure is shorter. In the non-CBDC case, it is sufficient for the bank to observe a stop after share π of cash withdrawals to verify that the bank run is over, because end of bank run will lead to a non-panic based withdrawals of π % of depositors on cash. In the case of CBDC, end of panic runs in CBDC will be proceeded by $\pi\delta$ withdrawals of impatient depositors in CBDC (and $\pi(1-\delta)$ in cash), which is smaller value.

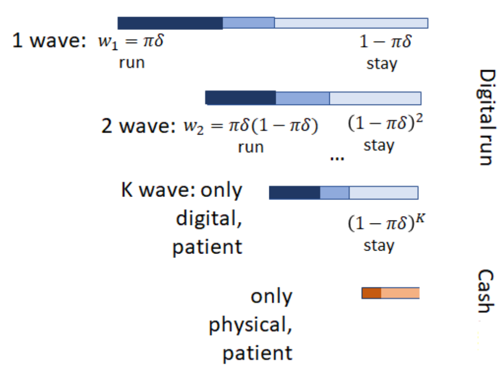

Figure 1: Schematic description of the wave structure in the economy with and without CBDC without cash waves.

$$
\begin{aligned}
V_k(\psi_{k-1}) = &\left(1 - \frac{s_k}{s_{k-1}}\right)(1-\delta)[\delta\pi u(c_{1,k}) + (1-\delta\pi)u(c_{2,k})] \\
&+ \left(1 - \frac{s_k}{s_{k-1}}\right)\delta[\delta\pi u(\tau_c c_{1,k}) + (1-\delta\pi)u(\tau_c c_{2,k})] \\
&+ \left(\frac{s_k}{s_{k-1}}\right)(1-\delta)[\delta\pi u(c_{1,k}) + (1-\delta\pi)V_{k+1}(\psi_k)] \\
&+ \left(\frac{s_k}{s_{k-1}}\right)\delta[\delta\pi u(\tau_d c_{1,k}) + (1-\delta\pi)V_{k+1}(\psi_k)]
\end{aligned}
$$

subject to budget constraint

$$(1-\pi)c_{2,k} = R(\psi_{k-1} - \pi c_{1,k}).$$

Without the information channel, the introduction of CBDC will be beneficial whenever $\delta > 1/2$. Shorter withdrawal waves lead to higher efficiency and more funds remaining on the account of the bank.

Theorem 2 *Introduction of CBDC in the economy with minimum transaction costs $\tau \to 1$, leads to an equilibrium with shorter withdrawal waves, lower average deposit rate paid at t=1 and higher average deposit rate paid at t=2.*

We also make specific predictions for the welfare analysis.

Theorem 3 *Introduction of CBDC in the economy with minimum transaction costs $\tau \to 1$ and the size of digital economy being greater than the size of physical economy, $\delta > 1/2$, always increases overall welfare of depositors.*

6 Other theory, policy, empirical applications

Contact author for the updates.

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