Dollar Debt and the Global Financial Cycle[*]

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Abstract

This paper proposes a tractable model of the Global Financial Cycle and study its welfare implications for emerging market economies (EMEs). When local firms issue debt denominated in dollars, central banks must increase their policy rate when the U.S. tightens in order to offset balance sheet effects stemming from the depreciation of their currency. When global financial markets are imperfect, this synchronized policy response has negative spillovers: all individual countries seek to attract capital inflows at the expense of one another, exacerbating the Global Financial Cycle. This requires further tightening and results in inefficiently low levels of employment in EMEs. This congestion externality generates gains from coordination. A coordinated response by central banks dampens the output losses caused by the Global Financial Cycle.

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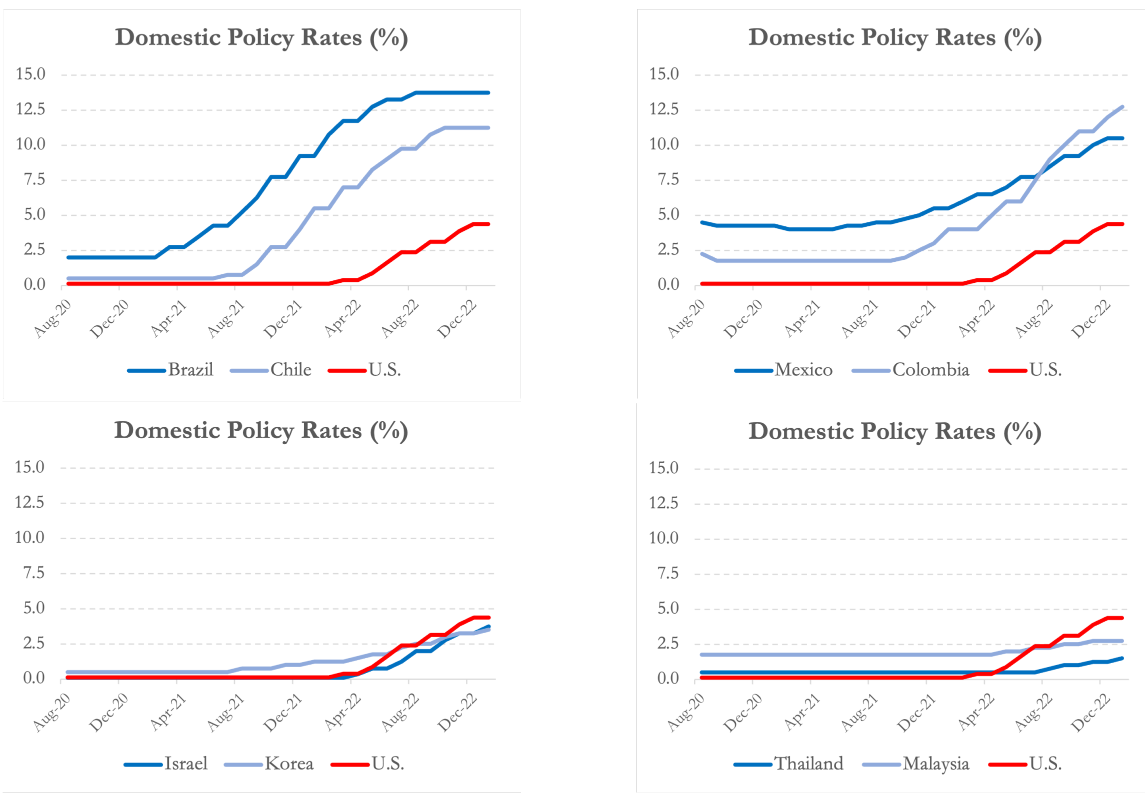
1 Introduction

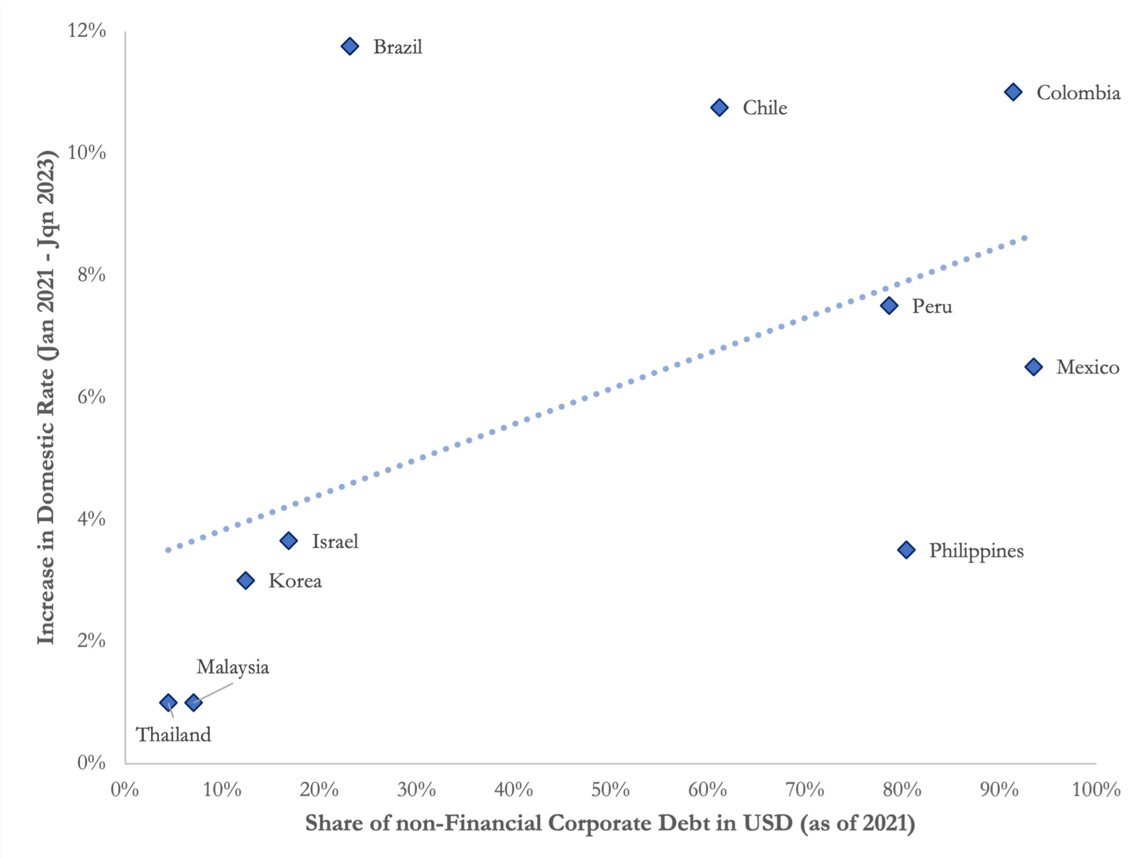
In May 2013, the U.S. Federal Reserve announced it would start tapering its large scale asset purchases. Financial conditions in emerging market economies (EMEs) immediately deteriorated: currencies depreciated, stock markets fell, and bond yields rose. This “taper tantrum” episode highlighted how EMEs may be severely affected by U.S. domestic policy decision: when private debt is denominated in dollars, a depreciation of the currency weakens balance sheets, which hurts financially constrained corporates. To fight such depreciations, central banks in EMEs usually rely on interest rates hikes, putting a drag on aggregate demand. The recent round of interest rate tightenings in EMEs (see Figures 1 and 2) revived this debate.

Figure 1: Interest Rates in Selected Emerging Economies and in the U.S.



While it is now understood that central banks in EMEs are constrained by the actions of the Federal Reserve (Rey 2015), their synchronized response to the Global Financial Cycle raises new questions. First, how should individual policymakers respond when private dollar debt is prevalent in all EMEs? Second, under

Figure 2: Cumulative Interest Rates in Selected Emerging Economies, as a function of non-Financial Corporate Debt in USD. Source: BIS, World Bank.



which conditions are there spillovers from EMEs' monetary policy response to the actions of the Federal Reserve? And third, are there eventual coordination gains for central banks in EMEs?

This paper proposes a tractable model that allows to answer these questions. The central result of the paper is that, when global financial markets are imperfect, a "congestion externality" appears in response to policy decisions in the U.S., exacerbating the Global Financial Cycle: central banks in EMEs raise domestic policy rates to counter depreciationary pressures and balance sheet effects, by attracting more capital inflows. This change in global capital flows, if happening in all EMEs at the same time, increases the world interest rate because of frictions in international financial markets. This feeds back into domestic conditions by creating further depreciationary pressures in emerging economies, requiring another round of tightening. A coordinated response from central banks solves this congestion externality by tightening less in response to a Fed shock, resulting in higher employment and higher output in all EMEs.

I start by developing in Section 2 a model of a small open economy featuring the different forces at play. The model is characterized by two key departures from the neo-classical benchmark: financial frictions and nominal rigidities. The presence of financial frictions implies that the net worth of entrepreneurs plays a crucial role (Tirole 2010 ; Bernanke and Gertler 1990): increasing this net worth allows entrepreneurs to level up more and invest more into productive assets. This channel naturally interacts with the existence of debt denominated in foreign currency — here in dollars. When entrepreneurs' revenues are in local currency, any movement in the exchange rate *vis-à-vis* the dollar impacts the net worth of entrepreneurs, giving rise to balance sheet effects. An increase in the U.S. interest rate provokes capital outflows that depreciate the local currency, weakening the balance sheet of entrepreneurs, forcing them to delever and invest less in productive capital, leading to lower output later on.

The central bank can counter these depreciationary pressures by raising its domestic policy rate. But the existence of nominal rigidities — modeled as rigid wages — implies that there is a monetary policy trade-off, fleshed out in Section 3. By increasing its interest rate, the EME is able to attract capital inflows that will appreciate its currency, lowering the repayment burden imposed on entrepreneurs, and thus leading to higher investment through the net worth effect described above. This increase in the interest rate, however, also leads to a rebalancing of households' demand away from non-tradable goods, eventually leading to involuntary unemployment and lower output in this sector because of rigid wages. This policy analysis provides a closed-form formula for the optimal interest rate. As expected, this optimal interest rate is increasing in the size of dollar debt held by entrepreneurs, and in the U.S. interest rate. The higher the Fed rate, the more difficult it is for the EM central bank to achieve full employment for a given level of dollar debt.

Furthermore, since several EMEs are characterized by high level of dollar debt, all of them will hike in response to a Fed tightening at the same time. Section 4 looks at the general equilibrium effects of this synchronized policy response. In particular, I show that monetary policy spillovers in this context are a cause of concern, but only when global financial markets are imperfect. If global capital flows have to go through financial intermediaries (or arbitrageurs) that face costs of intermediation (Gabaix and Maggiori 2015 ; Fanelli and Straub 2021), then the absolute size of capital flows impact the equilibrium determination of the interest rate for all countries. When central banks seek to counter depreciationary pressures and balance sheet effects, they need to attract more capital inflows. This change in global capital flows, if happening in all EMEs at the same time, increases the world interest rate because of the intermediation friction. This feeds back into domestic conditions by creating further depreciationary pressures in emerging economies: a higher world interest rate is depreciationary for all EMEs, weakening balance sheets, and thus requiring another round of tightening. At the heart of this feedback is thus what I call a congestion externality: all individual EMEs seek to attract capital inflows at the expense of one another when the Fed tightens, since all of their foreign-currency debt is denominated in the same currency: the dollar. The Global Financial Cycle is thus exacerbated, resulting in inefficiently low levels of employment.

This congestion externality generates gains from coordination. I show that the optimal interest rate implemented by central banks is lower when the response to an U.S. tightening is coordinated, and that the difference with the un-coordinated interest rate is increasing in the severity of the friction on global financial markets. This naturally leads to higher employment and higher output in EMEs, and dampens the Global Financial Cycle.[1]

Related Literature: The starting motivation of this paper is the conjunction of two well-established facts: corporate debt issuance in dollar in EMEs, and the Global Financial Cycle. First, a large quantity of corporate borrowing in emerging markets is denominated in dollars, and in outsized proportion relative to the wealth share of the U.S. in the world (Bruno and Shin 2015 ; McCauley, McGuire and Sushko 2015 ; Maggiori, Neiman and Schreger 2020). Second, the domestic monetary policy of the U.S. drives a Global Financial Cycle in capital flows, asset prices and in credit growth (Rey 2015 ; Miranda-Agrippino and Rey 2020 ; Miranda-Agrippino and Rey 2022). My paper explains the latter fact with the former: the presence of dollar debt ties the hands of central banks in emerging countries. Being forced to respond in a synchronized manner to interest rates movements in the U.S., an (inefficient) Global Financial Cycle appears.

The literature has proposed several explanations for why firms in emerging markets tend to issue in dollars rather than in their domestic currency, exposing

[1] The last part of the paper, Section 5, shows that entrepreneurs optimally issue a large share of their debt in dollars when they expect the central bank to tighten aggressively in the face of a U.S. tightening, creating a moral hazard problem. This problem is reduced if entrepreneurs believe that the policy response will be coordinated across countries. Finally, optimal macroprudential policy taxes debt issuance in dollars to solve the moral hazard problem.

themselves to currency mismatches. McKinnon and Pill (1998), Burnside, Eichenbaum and Rebelo (2001), and Schneider and Tornell (2004) argue that the excessive use of foreign currency debt stems from bailout guarantees for foreign creditors, creating a moral hazard problem. Caballero and Krishnamurthy (2003) show that limited financial development in emerging markets makes agents undervalue insuring against an exchange rate depreciation, so that agents choose excessive level dollar debt. Jeanne (2002) proposes that lack of monetary credibility is a source of risk, and that the optimal hedging strategy for firms is to issue a large share of debt in foreign currency. My paper does not take a stance on why so many firms in emerging markets issue in dollars: it rather takes this fact as given and explores its general equilibrium consequences for the global financial cycle.[2]

The presence of dollar debt generates powerful balance-sheet effects. This has been studied in response of the East Asian Crisis of the 1990s by, e.g., Krugman (1999), Céspedes, Chang and Velasco (2004), Aghion, Bacchetta and Banerjee (2004), and Chamon and Hausmann (2005). Recent papers have focused on the determination of optimal policy under foreign-denominated debt in modern models. Matsumoto (2021) and Coulibaly (2021) show that discretionary monetary policy is contractionary during crises, in order to mitigate balance sheet effects originating from exchange rate depreciations. Wang (2019) shows that incomplete exchange rate pass-through to goods prices leads to a new form of balance sheet effects, and derives the associated optimal macroprudential policy. More generally, Bianchi and Lorenzoni (2021) reviews the literature on optimal policy under "fear-of-floating."[3] A closely related paper is the recent work of Akinci and Queralto (2021). They develop a quantitative two-country model that can account for powerful spillovers of U.S. monetary policy on EMEs, but do not study the optimal policy response. My paper builds on their insights, and pushes their implications further: the optimal response of EMEs to these U.S. spillovers itself has spillover effects on other countries and requires coordination.[4]

My results are thus linked to a vast literature on international policy cooper-

[2] Relatedly, there is also a large literature on why sovereign debt is often issued in dollars — the so-called "original sin." See Eichengreen, Hausmann and Panizza (2007) for a review. My paper is only concerned with private debt.

[3] This is related to a large literature, which I build upon, studying optimal monetary policy under financial fragility (Boissay, Collard, Galí and Manea 2021 ; Farhi and Werning 2020 ; Asriyan, Fornaro, Martin and Ventura 2021).

[4] Jiang, Krishnamurthy and Lustig (2021) develop a model of the Global Financial Cycle that starts from the global demand for dollar-denominated safe assets, and highlight in particular the spillovers from U.S. monetary policy. My work is complementary as they are not looking at optimal policy in EMEs affected by this cycle and its consequences.

ation, starting with Obstfeld and Rogoff (2002) and Benigno and Benigno (2006). Importantly, the seminal work of Korinek (2017) lays out the conditions that need to be violated to generate inefficiency and scope for cooperation. In my paper, this stems from the use of a single instrument (monetary policy) in order to control both employment and the exchange rate. Fornaro and Romei (2019) show that, when monetary policy is constrained by the zero lower bound, non-cooperative financial and fiscal policies can lead to global output losses. Fornaro and Romei (2022) study monetary policy when there is excessive demand for tradable goods. They show that the optimal response is to implement expansionary monetary policy, but that the non-cooperative equilibrium is not expansionary enough. Closer to the mechanism highlighted in my paper, Caballero and Simsek (2020) develop a model with fire sales where domestic authorities want to restrict capital inflows in order to increase fire-sale prices in their countries. This reduces global liquidity, which in general equilibrium exacerbates fire sales.[5]

Similar to Akinci and Queralto (2021), I also show that imperfections in domestic and international financial markets are necessary to generate my spillover results. I build on the work of Gabaix and Maggiori (2015), Fanelli and Straub (2021), and Itskhoki and Mukhin (2021), who provide models of such imperfections that micro-found deviations from the UIP condition.

2 A Small Open Economy Model

Structure We consider a small open economy that can be thought of as an emerging economy.[6] Time is discrete and indexed by $t \in \{1, 2, 3\}$. Since the presence of risk only obfuscates my results, agents have perfect foresight. There are two key types of agents. Households consume and provide labor in period 2 and 3. Entrepreneurs issue debt in period 1 in order to finance investment in a capital stock that will produce domestic goods in period 2 and 3. Entrepreneurs simply seek to maximize profits, which are fully rebated to households.

The main insights of the paper come from the behavior of the equilibrium in the intermediate period, when entrepreneurs have some dollar debt to repay and

[5] A different literature also emphasized the role of terms-of-trade manipulation in the analysis of optimal tariffs and its implications for the trade agreements (see, e.g. Bagwell and Staiger 1999 ; Broda, Limao and Weinstein 2008 ; Costinot, Lorenzoni and Werning 2014). These effects are absent in my model because there is a single homogeneous traded goods.

[6] In Section 4, there is a continuum of infinitesimal countries and the world interest rate is endogenous.

need to make investments. I thus start by describing the intermediate period, and will present period $t = 1$ for completeness in Section 5.[7]

2.1 The economy at $t = 2$

Entrepreneurs Entrepreneurs enter period 2 each with a stock of capital K_1, as well as dollar and peso debts to pay back. Their existing stock of capital produces η_2 units of non-tradables goods per unit of capital. The net worth of entrepreneurs is thus denoted by:

$$n_2 = \eta_2 K_1 - b_1 - e_2 b_1^* \tag{1}$$

After η_2 is realized, a random fraction κ of firms are still productive and can produce in period 3 if they maintain their capital stock, and the remaining fraction $1 - \kappa$ is unproductive: their capital depreciates entirely and they stop producing. Unproductive firms repay their debt, lend to other firms, and rebate the rest of their profits to households.

To maintain their existing stock of capital in order to keep producing non-tradables goods in period 3, productive entrepreneurs must invest s units of non-tradables goods per unit of capital: to maintain k_2 they need to pay $s \cdot k_2$, which will pay off ρk_2 units of non-tradables at $t = 3$. Un-maintained capital fully depreciates. To finance this investment, entrepreneurs can borrow b_2 from other unproductive firms but are subject to a classic monitoring problem (Tirole 2010) that limits the amount they can borrow:

$$b_2 \leq \rho_0 k_2 \tag{2}$$

where ρ_0 is the pledgeable part of the project, with $\rho_0 < s < 1$. Since entrepreneurs seek to maximize future output, their budget constraint is:

$$n_2 + b_2 = s k_2 \quad \text{s.t.} \quad k_2 \leq K_1 \,;\, b_2 \leq \rho_0 k_2 \tag{3}$$

The case of interest will be when entrepreneurs are constrained by the plegdeability limit, which will imply that:

$$k_2 = \frac{n_2}{s - \rho_0} \tag{4}$$

[7] What ultimately matters for my model is that entrepreneurs find it optimal to issue at least some of their initial debt in dollars. This can be for a variety of reasons already highlighted by previous works (see the literature review above). In Section 5, the level of the interest rate on dollar debt depends on the size of the loan, such that entrepreneurs issue in dollars and in the domestic currency, up to the point where they are indifferent between both on the margin.

As is common in these models, net worth plays a crucial role. Entrepreneurs can lever their wealth with a multiplier $1/(s-\rho_0)$. By improving entrepreneurs net worth, monetary policy will thus be able to prop up investment in the capital stock. Since only a fraction ϕ of entrepreneurs are productive, the aggregate stock of capital used for production at $t = 3$, when entrepreneurs are constrained, will be given by:

$$K_2 = \kappa \frac{n_2}{s - \rho_0} \tag{5}$$

while the amount of non-tradable goods used for maintaining capital is $s \cdot K_2$.

Households Households receive an endowment of tradables goods y_2^T at time $t = 2$. They only consume starting at $t = 2$ and have the following utility function:

$$U_2 = \frac{1}{1-\sigma} \left(\phi (c_2^T)^{1-\sigma} + (1-\phi)(c_2^N)^{1-\sigma} \right) + \beta \left(c_3^N + c_3^T \right) \tag{6}$$

Households have an inelastic supply of labor $\bar{n}$. They can save and borrow in peso-denominated bonds (a_3) or dollar-denominated bonds (a_3^*), at the respective interest rates i_2 and i_2^*. The central bank sets the domestic interest rate i_2. We keep the same convention as for entrepreneurs: a positive position $a_3^* > 0$ means that households are borrowing in dollars. They thus have the following budget constraint:

$$p^T c_2^T + p^N c_2^N = e_2 y^T + w_2 l_2 + \frac{1}{1+i_2} a_3 + \frac{1}{1+i_2^*} e_2 a_3^* \tag{7}$$

Under these conditions, the standard UIP condition holds:

$$1 + i_2 = (1 + i_2^*) \frac{e_3}{e_2} \tag{8}$$

We assume that peso-denominated bonds are only traded domestically. Since households are symmetric, and cannot lend to entrepreneurs, we have $a_3 = 0$ in equilibrium.

Production Perfectly competitive firms produce non-tradables goods using a linear technology $y_2^N = l_2$. Wages are fully rigid at $\bar{w}$, so that involuntary unemployment arises when the interest rate is too high.

2.2 The economy at $t = 3$

In the last period, productive entrepreneurs produce and rebate profits to households. Households provide labor to fully competitive firms, settle their foreign currency debt, and consume. Since there is no savings decisions to be made, there is full employment $l_3 = \bar{l}$. The budget constraint is simply:

$$p_3^N c_3^N + p_3^T c_3^T + a_3 + e_3 a_3^* = p_3^T y_3^T + \bar{w}\bar{l} + \Pi_3 \tag{9}$$

We can now formally define the competitive equilibrium.

Definition 1. *A competitive equilibrium is a path of real allocations $\{c^N_, c_t^T, l_t\}_{(t=1,2)}$, capital K_2 and capital flows a_3^*, such that, given a domestic policy rate i_2, a world interest rate i_2^* and legacy debt b_1 and b_1^*: (i) households maximize (6) under the constraints (7) and (9) ; and (ii) entrepreneurs invest according to (5).*

In what follows, we restrict ourselves to situations where: (i) there is a unique equilibrium ; (ii) productive entrepreneurs are against their borrowing constraint (2) ; and (iii) y_2^T is large enough such that the SOE lends to the rest of the world.[8] Unless stated otherwise, all derivations and proofs are in Appendix A.

3 Dollar Debt and Monetary Policy

This Section studies the optimal policy problem, when the only instrument available is conventional monetary policy.[9] The central bank seeks to maximize the welfare of the representative consumer:

$$\mathcal{W} = \frac{1}{1-\sigma}\left(\phi(c_2^T)^{1-\sigma} + (1-\phi)(c_2^N)^{1-\sigma}\right) + \beta\left(c_3^N + c_3^T\right) \tag{10}$$

since entrepreneurs are rebating all of their profits to households. The key premise of this model is that the presence of dollar debt creates a trade-off for the central bank. The first channel works through aggregate demand: changing the domestic interest rate rebalances demand between non-tradable and tradable goods, as can

[8] None of these assumptions are crucial for the results, but the trade-offs are starker in this situation.

[9] Korinek (2017) lays out the conditions that need to be violated to generate inefficiency and scope for cooperation. Here, this stems from the use of a single instrument (monetary policy) in order to control both employment and the exchange rate. If the policymaker was also able to use foreign exchange intervention at zero cost, we would be back to the Korinek (2017) benchmark of the "first welfare theorem."

be seen from the following optimality condition:

$$c_2^N = \left(\frac{\phi}{1-\phi} \frac{(1+i_2)}{(1+i_2^*)} \right)^{-1/\sigma} c_2^T \tag{11}$$

When i_2 decreases, the demand for non-tradables rises relative to tradables (since $\sigma > 0$) which can increase non-tradable output (i.e. lower unemployment) since wages are rigid.

A decrease in i_2, for instance to increase employment and reach potential output, has an impact on the exchange rate through the usual UIP condition:

$$1 + i_2 = (1 + i_2^*) \frac{e_3}{e_2} \tag{12}$$

which mechanically increases e_2. Indeed, a fall in the interest rate creates capital outflows from the small open economy to the rest of the world, depreciating the exchange rate to restore equilibrium in global capital markets.

Because of dollar debt repayments, however, this change in the exchange rate weakens the balance sheet of entrepreneurs that need to borrow subject to the financial friction (2) in order to maintain their capital stock:

$$\frac{dK_2}{di_2} = \frac{e_2 \kappa b_1^*}{s - \rho_0} \tag{13}$$

Thus, when entrepreneurs are constrained a depreciation of the domestic currency *vis-'a-vis* the dollar results in a lower capital stock at $t = 2$. Finally, this decrease in capital has a negative impact on welfare, by lowering output at $t = 3$. The next proposition characterizes, in closed-form, how central banks should trade-off the aggregate demand and net worth effects.

Proposition 1 (Optimal Monetary Policy at $t = 2$)**.** *There exists a unique level of dollar debt $\tilde{b}^*$ such that:*

1. *When $b_1^* > \tilde{b}^*$, optimal monetary policy trades off aggregate demand and balance sheet effects according to:*

$$1 + i_2^{opt} = \Omega \left(\frac{(1 + i_2^*) b_1^*}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma - 1}} \tag{14}$$

where $\Omega = \left(\sigma \rho \kappa \bar{w} \beta^{1/\sigma} \right)^{\sigma/(2\sigma-1)}$. The optimal interest rate is thus strictly increasing in the level of dollar debt, and we have involuntary unemployment: $l_2 < \bar{l}$.

2. *When $b_1^* \leq \tilde{b}^*$, the central bank implements full employment, choosing nominal rates according to:*

$$1 + i_2^{full} = \frac{(1+i_2^*)s\kappa b_1^* \bar{w}}{\left(\frac{\kappa \frac{b_1^*}{s-\rho_0} + \beta\rho}{s(1-\phi)}\right)^{-\frac{1}{\sigma}} - \bar{l} - \eta_2 K_1 + \frac{b_1}{\bar{w}} + \frac{s\kappa}{s-\rho_0}(\eta_2 K_1 - b_1)} \tag{15}$$

The first part of the proposition naturally ties together the forces at play. The level of dollar debt directly matters for monetary policy. Its is amplified by the net worth multiplier $1/(s-\rho_0)$: when $s-\rho_0$ is low, a shock to net worth transmits to investment in capital more strongly, thus inflating the effects of a policy hike. At the same time, aggregate demand is hurt by an increase in the interest rate, and here this effect is disciplined by the elasticity of substitution σ that relates how changes in interest rates impact demand for non-tradable goods. Finally, the level of U.S. interest rates matters: the domestic central bank is forced to follow the actions of the Fed to prevent excessive devaluation of the peso that results in adverse balance sheet effects, which is of course costly for aggregate demand.

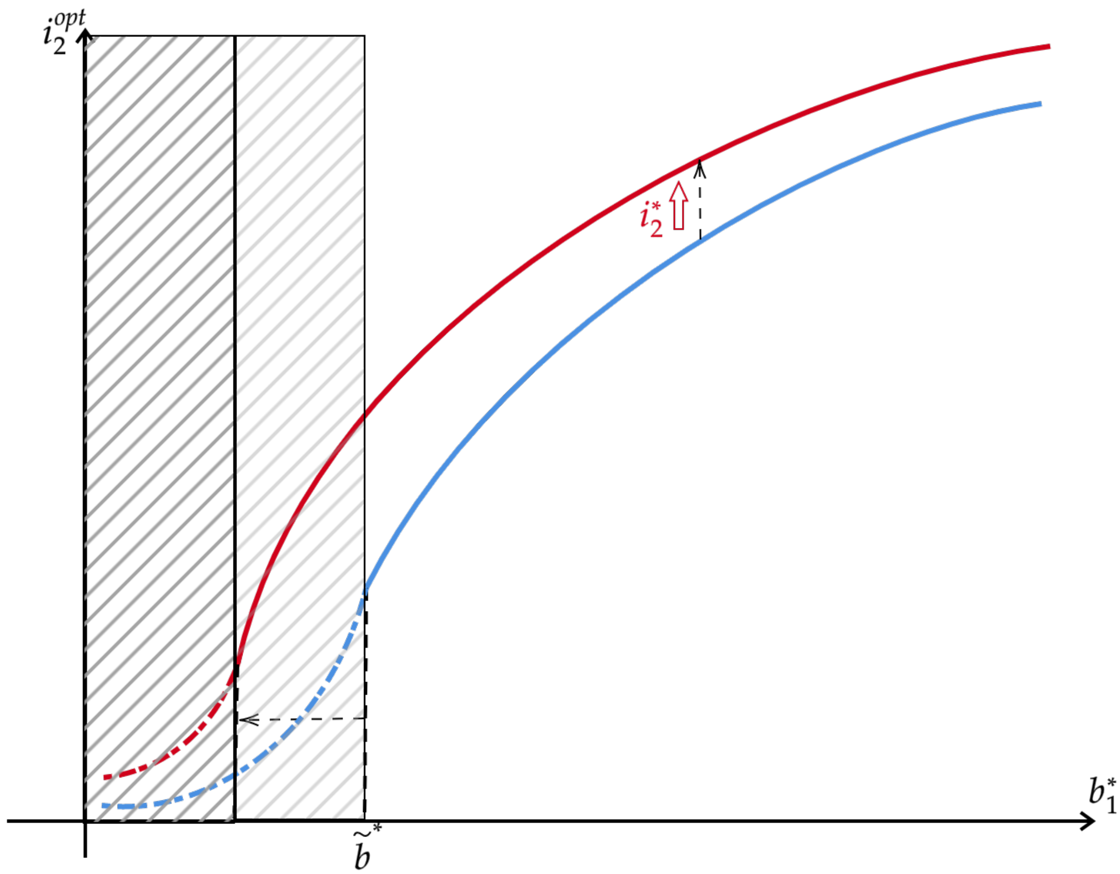
Finally, notice how the second part of the proposition links to the large literature studying aggregate demand externalities (Korinek and Simsek 2016 ; Farhi and Werning 2016 ; Guerrieri and Lorenzoni 2017 ; Fornaro and Romei 2019). The interest rate necessary to achieve full employment is decreasing in b_1, the amount of domestic debt issued by entrepreneurs in the first period. It is then further assumed in this literature that a zero lower bound constraint binds at period 2: in such a case, a higher debt at $t = 1$ translates into weaker aggregate demand at $t = 2$, and the policymaker is unable to stimulate the economy enough, resulting in unemployment and low output. In my paper, the ZLB constraint does not play any role: the presence of foreign debt makes the policymaker more likely to hike interest rates.[10]

The optimal interest rate chosen by the central bank, as a function of dollar debt, is pictured on Figure 3. The red dashed line corresponds to the case where the U.S. interest rate is higher (an increase in i_2^*). As can be seen graphically or from Proposition 1, an increase in the Federal Reserve rate worsens the emerging market's monetary policy dilemma: it becomes harder to achieve full employment

[10] In a full-fledged model, the central bank would overheat the economy below the threshold $\tilde{b}^*$. I focus on the under-employment issue in this paper since the trade-offs are starker, but the intuitions are similar.

because of balance sheet effects.

Figure 3: Optimal interest rate chosen by the central bank, as a function of dollar debt b_1^*. The shaded grey areas correspond to regions where the central bank is able to achieve full employment. A shock to the U.S. interest rate i_2^* moves the full employment threshold to the left, meaning that it becomes harder to achieve full employment.



The Global Financial Cycle An immediate implication of Proposition 1 is that the presence of dollar debt creates a synchronization between the domestic policy decisions of emerging markets. Irrespective of their own aggregate demand shocks, all central banks fearing balance sheet effects from dollar debt optimally tighten in the face of tighter financial conditions in the U.S. For instance, Proposition 1 illustrates the "taper tantrum" episode of 2013, where emerging markets' central banks aggressively hiked after the Fed hinted that it would raise rates in the near future. The fact that all countries privately act in a manner consistent with 1 can create coordination issues, however. This is the focus of the next Section.

4 Spillovers

The previous analysis studies a small open economy in isolation, taking U.S. interest rates as given. In practice, many emerging economies are characterized by high level of corporate debt dollarization. This raises questions about coordination issues and possible spillovers: when the Federal Reserve hikes U.S. interest rates, each country faces depreciationary pressures. Each government would then find it optimal to increase their domestic rates in order to counter the net worth effects, as highlighted in Proposition 1. If global financial markets are not frictionless, this general movement towards higher rates will backfire and amplify even further depreciationary pressures.

4.1 The World Economy

We consider a similar setup as in Section 2 but this time with a continuum of identical and symmetric small open economies. Each country is indexed by j. In particular, country j at time $t = 2$ sets its nominal interest rate at $i_{2,j}$, taking all other world interest rates as given. Small open economies are in mass of 1, and we denote the aggregate variables without the subscript j: $b^*_{1,j}$ thus refers to the dollar debt of country j, and b^*_1 to the aggregate dollar debt of emerging economies.

4.2 Global Financial Markets

We assume that global financial markets are not frictionless in the spirit of Gabaix and Maggiori (2015). Each country can only trade dollar-denominated bonds with a global arbitrageur, at rate i^*_2. The global arbitrageur can borrow directly on U.S. financial markets at the rate set by the Fed, $i^{\$}_2$, but faces costs of intermediation. The rate it offers individual countries will thus depend on the aggregate flow of foreign debt it has to intermediate.

We denote by $\int_j a^*_{3,j} dj$ the aggregate capital flow from the continuum of SOEs to the U.S. To intermediate global arbitrageur has to pay costs equal to (Bianchi and Lorenzoni 2021):

$$\Phi\left(\int_j a^*_{3,j} dj, 1 + i^*_2\right) = \frac{1}{2\gamma}\left(\frac{\int_j a^*_{3,j} dj}{1 + i^*_2}\right)^2 \tag{16}$$

Profit maximization for the global arbitrageur thus leads to the following expression, that determined the interest rate faced by each emerging economy, as a function of global capital flows:

$$i_2^* = i_2^\$ + \frac{\int_j \frac{a_{3,j}^*}{1+i_2^*} dj}{\gamma} \tag{17}$$

4.3 Benchmark: a No-Spillover Result

Before introducing a key friction that creates spillovers in response to the Global Financial Cycle, it is instructive to look at the benchmark case that does *not* create spillovers, and to understand why.

Since each country is taking i_2^* as given, the optimal policy program is entirely unchanged from the perspective of a single monetary authority. We thus know, thanks to Proposition 1, that country j reacts to the interest rate it faces, i_2^*, with a domestic rate of:

$$1 + i_{2,j} = \Omega \left(\frac{(1+i_2^*) b_{1,j}^*}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma - 1}} \tag{18}$$

Now, however, the part $(1 + i_2^*)$ is endogenous. It must be determined by the aggregation of all capital flows from EMEs, as explicit in the next Lemma.

Lemma 1 (Benchmark Equilibrium Capital Flows)**.** *The aggregate capital flow from emerging economies towards the U.S. at period $t = 2$ is given by:*

$$\frac{1}{1+i_2^*} a_3^* = \left(\beta\phi \frac{1}{1+i_2^*} \right)^{\frac{1}{\sigma}} + b_1^* - y_2^T \tag{19}$$

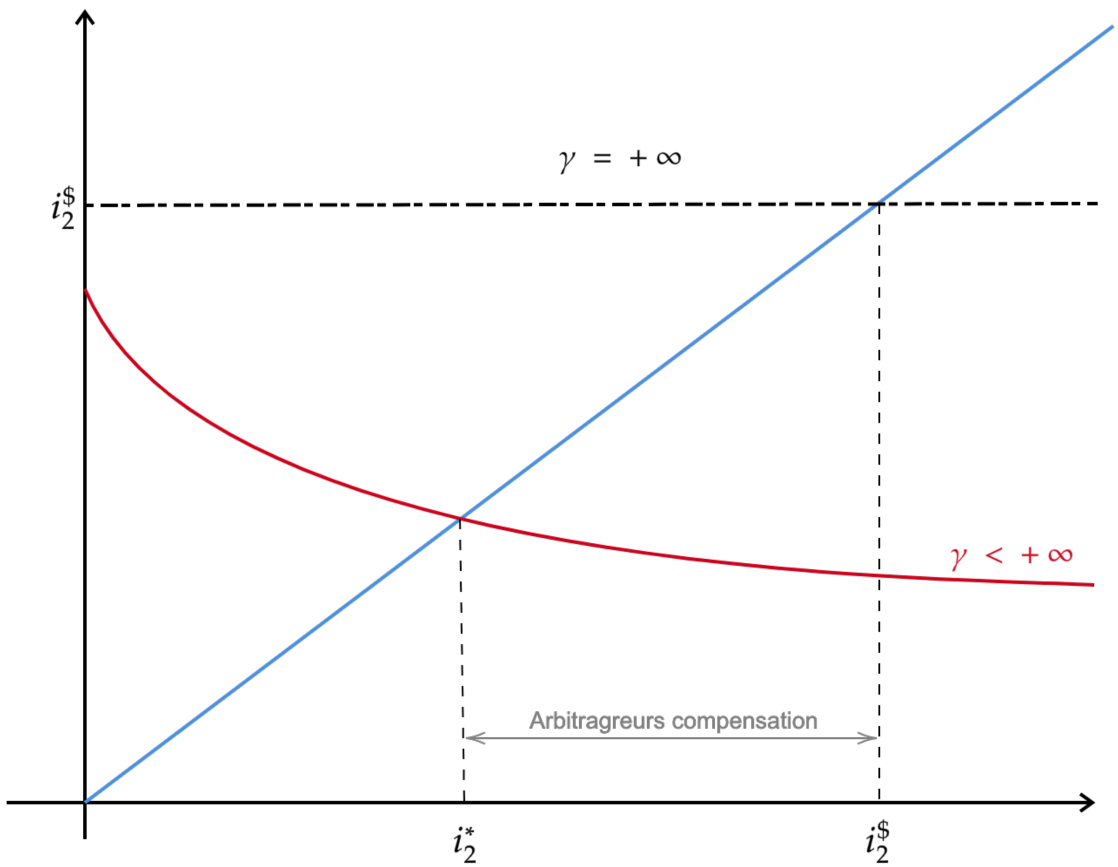
where the interest rate i_2^* is the implicitly defined according to:

$$i_2^* = i_2^\$ + \gamma^{-1} \left(\left(\beta\phi \frac{1}{1+i_2^*} \right)^{\frac{1}{\sigma}} + b_1^* - y_2^T \right) \tag{20}$$

The determination of the equilibrium interest rate at which emerging economics can save can be graphically seen on Figure 4. A low level of γ indicates large friction on global financial markets, such that global entrepreneurs must be compensated more for intermediating capital flows from emerging economies. This result in a lower i_2^* compared to the Fed nominal rate.

The key feature of Lemma 1, however, is that equation (19) (determining aggregate capital flow for an emerging economy) does not depend on the domestic rate of interest, i_2. This, in turn, means that any change in the domestic policy rates of emerging economies will not create spillovers effects on the exchange rates of

Figure 4: Aggregate Capital Flows and Equilibrium Interest Rate. The blue line is the 45° line. The black line corresponds to the case where global markets are perfectly elastic: the size of capital flows has no impact on the equilibrium rate. The red curve depicts the right-hand side of equation (20), i.e. the size of global capital flows from emerging economies to the U.S. as a function of the interest rate charged by global arbitrageurs.



other emerging economies. As a result, all emerging economies hike their interest rate in a synchronized fashion in response to an increase in the U.S. policy rate, but their is no need for coordination since domestic actions do not spillover other international financial conditions.

4.4 Global Capital Flow and the Financial Wedge

As hinted just previously, the benchmark ”no-spillover” result hinges on the peculiar fact that capital flows are independent of the policy rate. There are, however, numerous ways to depart from this condition. In the spirit of this paper, I focus on a financial friction that breaks this irrelevance result.[11] The financial friction view favored here is simply an example of why such spillovers would arise, but serves two purposes. First, it allows me to relate to a large literature on financial shocks and exchange rate puzzles (Itskhoki and Mukhin 2021). Second, it also relates to the widespread use of dollar savings in emerging countries that are intermediated by domestic banks (Montamat 2020).

We thus assume that, to save in dollar bonds, households in emerging economies need to go through domestic banks, which are perfectly competitive. However, banks have to incur a cost to intermediate dollar savings that is proportional to the domestic rate, which can naturally be understood as the costs of holding reserves at the central bank. Specifically, banks in country j have to incur a unit opportunity cost of:

$$c_{\$,j} = (1 + i_2^*)^{\psi} \tag{21}$$

with $\psi \in [0, 1[$.[12] Such a financial friction yields the following modified UIP condition:

$$1 + i_2 = \left((1 + i_2^*) \frac{e_3}{e_2} \right)^{\frac{1}{1+\psi}} \tag{22}$$

where the ψ plays the role of a "financial wedge." This can equivalently be seen as writing the effective interest rate at which the emerging economy can save as:

$$(1 + \hat{i}_2^*) = (1 + i_2^*)(1 + i_2)^{-\psi} \tag{23}$$

which is equivalent to saying that the perceived rate of return on dollar savingsi is decreasing in the domestic policy rate.[13]

The full structure of this stylized global financial system is depicted on Figure 5. This leads naturally to capital flows from country j to be equal to:

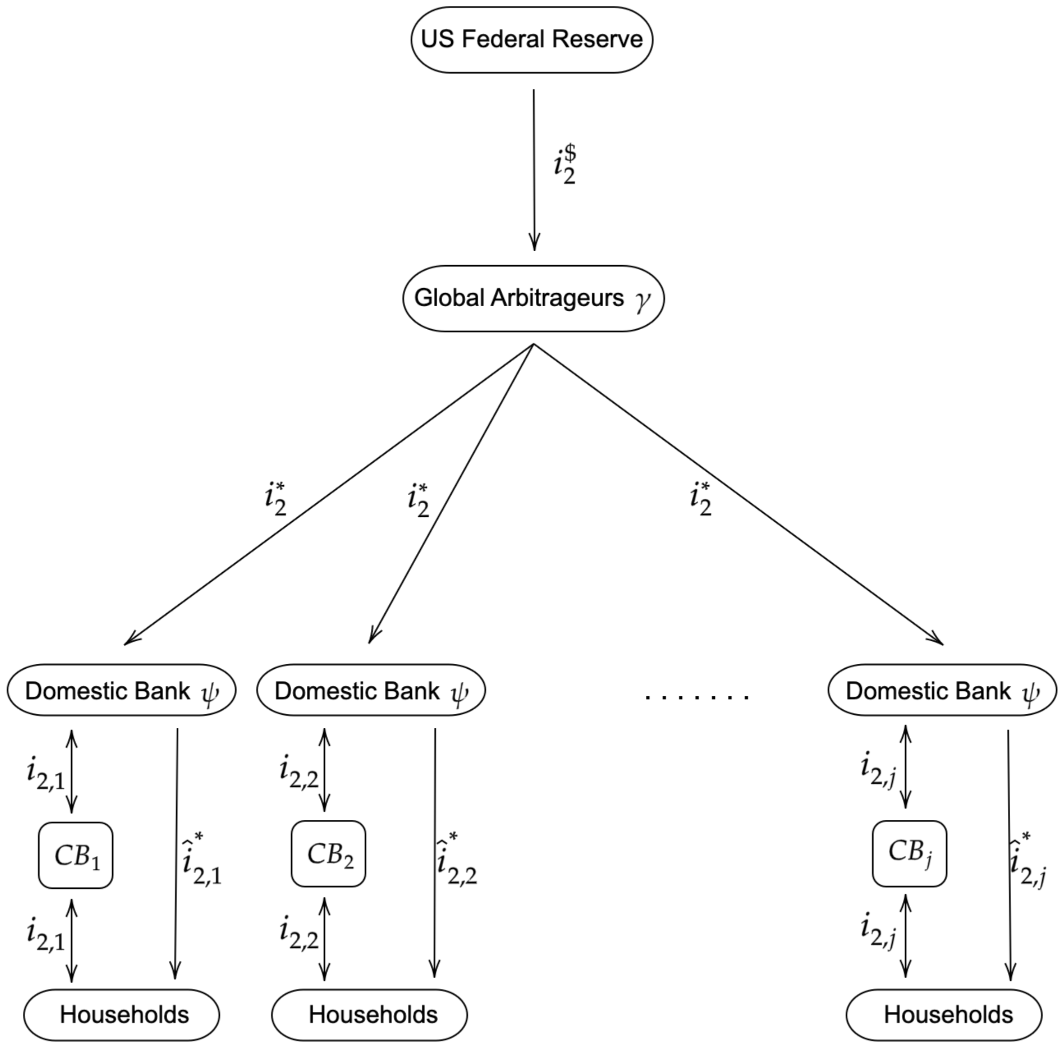
$$\frac{1}{1 + \hat{i}_2^*} a_{3,j}^* = \left(\beta \phi \frac{(1 + i_2)^{\psi}}{1 + i_2^*} \right)^{\frac{1}{\sigma}} + b_{1,j}^* - y_{2,j}^T \tag{24}$$

[11] For instance, one could break this benchmark result by using non-separable preferences, a form of external habit formation, or a reach-for-yield type of mechanism.

[12] This function form is simply taken to keep the optimal policy problem tractable. What matters is that this relation between costs and the interest rate is increasing. Notice that for small ψ, this cost function can equivalently be expressed as $c_{\$,j} = e^{\psi i_2}$, which yields an UIP condition similar to the one proposed by Itskhoki and Mukhin (2021).

[13] This is reminiscent of Drechsler, Savov and Schnabl (2017). They present a model with market power in deposit markets, and show that when the Fed funds rate rises, banks widen the spreads they charge on deposits .

Figure 5: Architecture of the Global Financial System



In the case where households are saving in dollars ($a_3^* < 0$) an increase in the domestic rate $i_{2,j}$ makes it more costly to save, reducing the size of dollar savings and thus decreasing the absolute magnitude of capital outflows.

This functional form allows for a description of the optimal domestic policy rate in closed form. This is fleshed out in the following lemma.

Lemma 2 (Optimal Monetary Policy with an Endogenous Financial Wedge)**.** *With a financial wedge as posited in equation* (21)*, optimal monetary policy above the threshold* $\tilde{b}^*$ *is now given by:*

$$1 + i_2^{opt} = \Omega_\psi \left(\frac{b_1^*(1 + i_2^*)}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \tag{25}$$

with a coefficient defined by:

$$\Omega_\psi = \left(\sigma(1+\psi)\rho\kappa\bar{\omega}\beta^{\frac{1-\sigma\psi}{\sigma}}\right)^{\frac{\sigma}{2\sigma-1+\sigma\psi}} \tag{26}$$

Equation (25) quantifies how the ψ friction modifies the optimal interest rate implemented by the central bank. When ψ is higher, the response of the central bank to any change in dollar debt b_1^* or U.S. interest rate i_2^* is inhibited compared to the frictionless case. Indeed, because of intermediation frictions a rise in the interest rate helps appreciates the currency through two independent channels: first through the usual expenditure switching mechanism, and second through the spreads charged by banks that makes it less attractive to save in dollars and thus reduces capital outflows.

4.5 Congestion Externalities

We are now ready to develop the main result of this paper. The intuition for this result comes from the juxtaposition of the four main equilibrium conditions, linking the Fed policy rate to the domestic policy rate of each emerging economy:

$$i_2^* = i_2^\$ + \frac{\int_j \frac{a_{3,j}^*}{1+\hat{i}_{2,j}^*} dj}{\gamma} \tag{27}$$

$$\frac{a_{3,j}^*}{1+\hat{i}_{2,j}^*} = \left(\frac{\beta\phi}{1+\hat{i}_{2,j}^*}\right)^{\frac{1}{\sigma}} + b_{1,j}^* - y_{2,j}^T \tag{28}$$

$$1+\hat{i}_{2,j}^* = (1+i_2^*)(1+i_{2,j})^{-\psi} \tag{29}$$

$$1+i_{2,j} = \Omega_\psi \left(\frac{b_{1,j}^*(1+i_2^*)}{s-\rho_0}\right)^{\frac{\sigma(1+\psi)}{2\sigma(1+\psi)-1}} \tag{30}$$

The first equation, (27), links the U.S. domestic rate to the world interest rate charged by global arbitrageurs give the aggregate size of capital flows. The second equation, (28), gives the size of the flows given the interest rate charged by domestic banks. The third equation, (29), links this rate offered by domestic banks to the financial wedge and the domestic rate of the emerging economy. The final equation (30) links the domestic policy rate to the world interest rate by trading-off balance sheet effects and aggregate demand.

A shock to the U.S. domestic policy rate then transmits through EMEs by trickling up these equilibrium conditions. The central bank from the emerging economy increases its domestic policy rate to counter depreciationary pressures and balance sheet effects, and attract more capital inflows as a result. This change in global capital flows, if happening in all small emerging economies at the same time, increases the world interest rate because of frictions in international financial markets. This feeds back into domestic conditions by creating further depreciationary pressures in emerging economies, requiring another round of tightening. At the heart of this feedback is an externality: individual emerging countries do not internalize that they their domestic policy rate decisions have spillovers and impact the equilibrium determination of the world interest rate i_2^*.

Proposition 2 (Monetary Policy Spillovers)**.** *Individual central banks in emerging economies do not internalize that their domestic decisions spill over to the equilibrium determination of the world interest rate:*

$$\mathscr{C}(i_2, i_2^*) = \frac{d\ln(1+i_2^*)}{d\ln(1+i_2)} = \psi \frac{1}{\frac{\gamma\sigma}{(\beta\phi)^{\frac{1}{\sigma}}} \frac{(1+i_2^*)^{\frac{\sigma+1}{\sigma}}}{(1+i_2)^{\frac{\psi}{\sigma}}} + 1} \tag{31}$$

The result in Proposition 2 highlights why the two frictions on the international financial market are necessary to create spillovers. Fist, if $\psi = 0$, then changes in the domestic rate do not impact the world interest rate since global capital flows are constant. Second, if $\gamma = +\infty$, global arbitrageurs do not face intermediation costs and changes in flows do not impact the world interest rate. It is the combination of those two ingredients that yield the spillover result, and create a need for coordination.

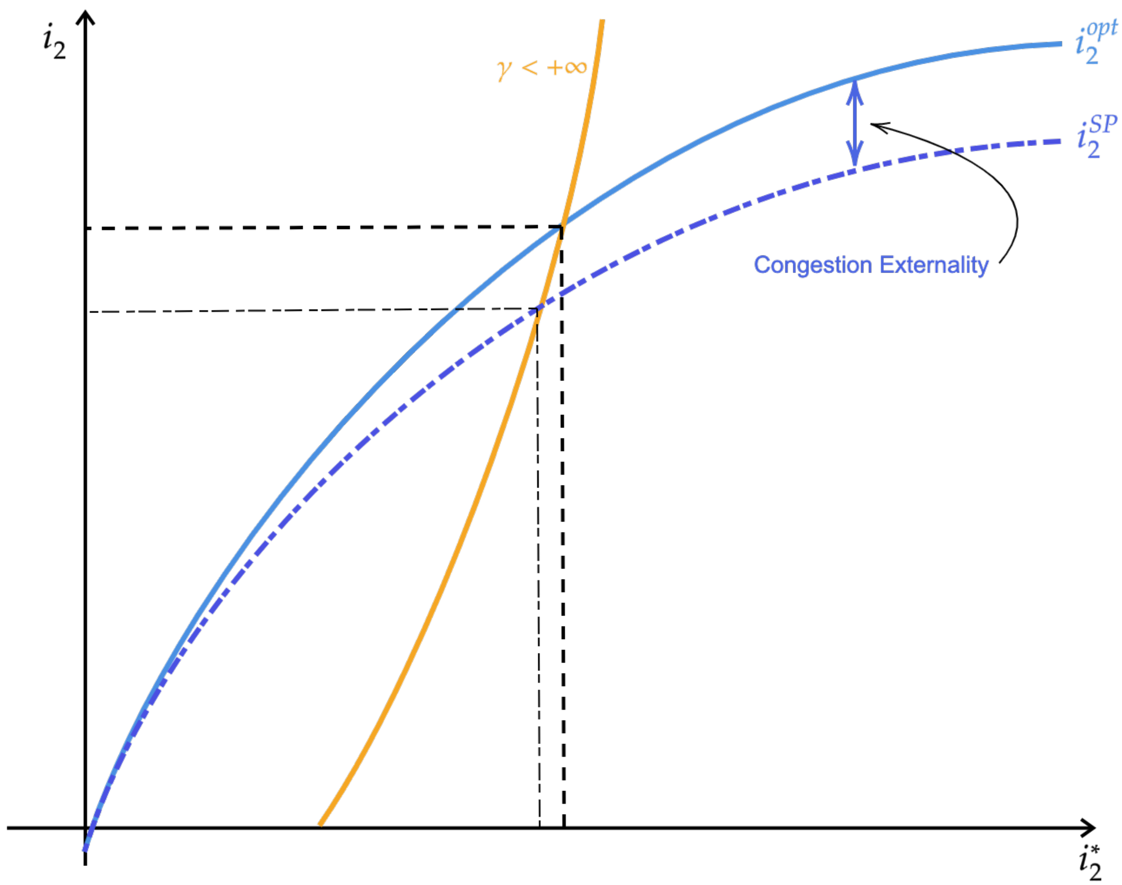
Proposition 3 (Coordinated Monetary Policy)**.** *A Social Planner that coordinates monetary policy across emerging economies implements a lower interest rate than in the decentralized case, and the difference between the two interest rates is exactly quantified by the congestion externality:*

$$1 + i_2^{SP} = \Omega_\psi \left(\frac{b_1^*(1+i_2^*)}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \left(1 - \frac{1}{1+\psi} \mathscr{C}(i_2^{SP}, i_2^*) \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \tag{32}$$

Employment and output are higher in each emerging country in the coordinated equilibrium than in the un-coordinated one.

This proposition and its implications for the global equilibrium can be understood graphically on Figure 6. This Figure pictures the best responses of central banks in the uncoordinated and coordinated equilibria. The difference between the two is the congestion externality highlighted above. The equilibrium is at the intersection of central banks' best response, and the "γ locus" that traces the relation between the world interest rate and the individual domestic rates in emerging countries, given the intermediation friction given by equation (28). By internalizing how their capital inflow will create congestion and result in a higher world interest rate, central bank in the coordinated equilibrium raise rates by less (in proportion to the externality in Proposition 2) which leads to less depreciation, and an equilibrium with higher employment and output.

Figure 6: Coordinated and Uncoordinated Equilibria. The blue line is the individual best response of an individual central bank for a given world interest rate i_2^*. The dashed blue line is the best response of central banks taking into account the effect of their rate setting on aggregate capital flows and the resulting world interest rate. The orange line depicts the world interest rate determination for a given $\gamma < +\infty$.



5 Moral Hazard and Excessive Dollar Debt Issuance

Having solved the equilibrium nominal rate – and thus the equilibrium exchange rate – at $t = 2$, we now turn to the optimal debt issuance of entrepreneurs at $t = 1$. We assume that there is also in the background a mass of domestic savers between period 1 and 2, with deep pockets.

5.1 The economy at $t = 1$

Investment Entrepreneurs must issue debt to finance an investment of fixed size, K_1. They can either issue peso debt to domestic savers, or to international intermediaries. Domestic savers have a (large) endowment of y_1^N tradable goods in the initial period, and have the following linear utility function:

$$c_{1,s}^N + \beta_s c_{2,s}^N \tag{33}$$

We denote by b_1 the amount borrowed from savers, at the gross interest rate $1 + i_1 = 1/\beta_s$. Because of these assumptions, domestic savers are irrelevant for welfare: their only purpose is to be available to lend in peso to entrepreneurs.

Supply of Dollar Funds International intermediaries lend to entrepreneurs in dollars, but charge a premium over the U.S. interest rate. As usual in the literature on limited asset participation (see Bacchetta and Van Wincoop 2010 ; Gabaix and Maggiori 2015 ; Fanelli and Straub 2021), this leads to an equilibrium interest rate on dollar borrowing that depends on the size of the flow:

$$\frac{b_1^*}{1 + \hat{i}_1^*} = \frac{\hat{i}_1^* - i_1^*}{\Gamma} \tag{34}$$

Entrepreneurs then issue debt to minimize repayments, taking into account the equilibrium exchange rate at $t = 2$, e_2:

$$\min_{b_1, b_1^*} \quad b_1 + e_2 b_1^* \tag{35}$$

$$\text{s.t.} \quad \frac{b_1}{1 + i_1} + \frac{e_1 b_1^*}{1 + \hat{i}_1^*} = K_1 \tag{36}$$

Taking as given the interest rates on peso and dollar debt, the optimal amount issued in dollars by entrepreneurs is characterized in the following lemma.

Lemma 3 (Dollar Debt Issuance)**.** *The amount issued in dollar is given by:*

$$\frac{b_1^*}{1+\hat{i}_1^*} = \max\left(\min\left(\frac{(1+i_1)\frac{e_1}{e_2} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{37}$$

This expression is intuitive: entrepreneurs issue more in dollars when they expect a stronger currency next period (low e_2), and when the world interest rate is attractive (low i_1^* compared to the domestic rate i_1).

Remark 1. Although I used the same class of financial frictions at time $t = 1$ and $t = 2$, their modeling purpose is entirely different. In the initial period where firm make their currency issuance choices, the point of the Γ friction is to avoid corner solutions such that firms are indifferent on the margin between issuing in dollars or in domestic currency. In the second period, the γ friction serves to introduce strategic complementarities in the actions of small countries: aggregate flows drive the wedge between i_2^* and the U.S. domestic policy rate.

5.2 Externalities

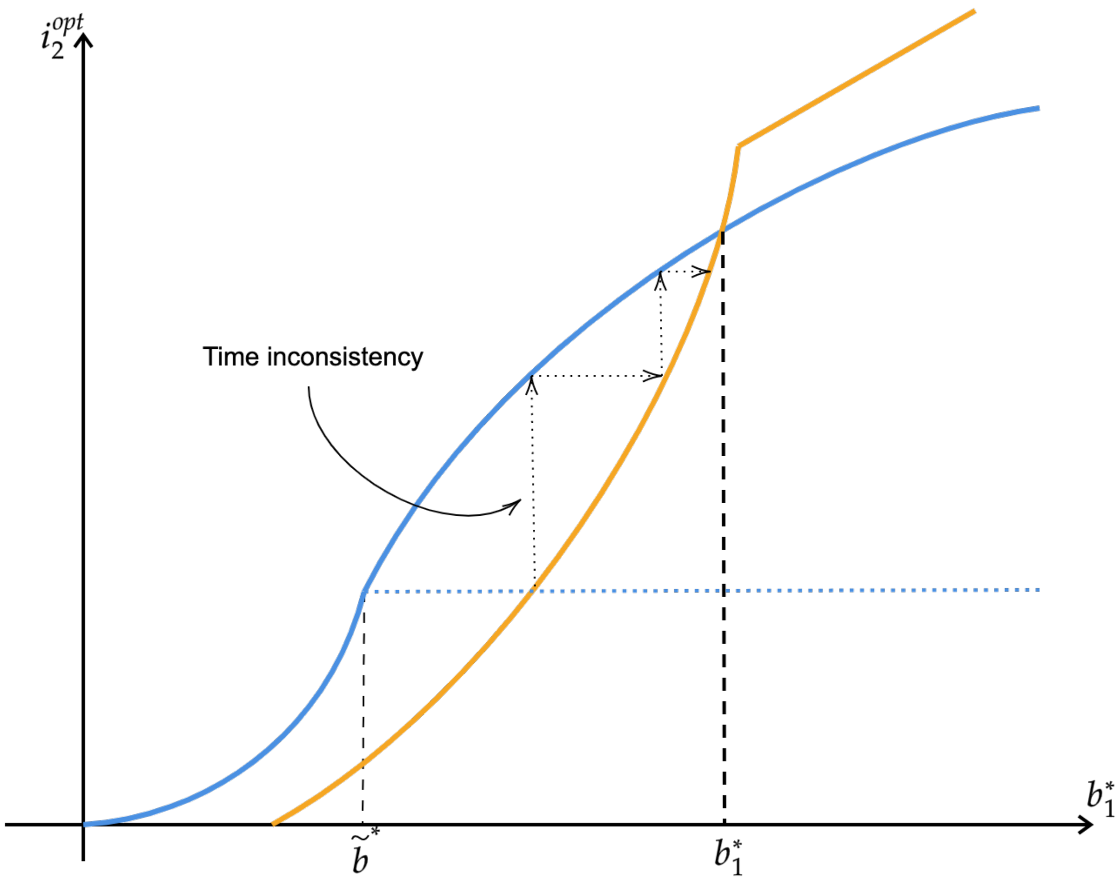
The main result of this section comes from recalling Proposition 1: the equilibrium exchange rate at $t = 2$, implemented by a welfare-maximizing central bank, also depends on the size of dollar debt b_1^*. This can be seen from the following condition:

$$b_1^* = (1+i_1)\frac{1+i_2}{1+i_2^*}\max\left(\min\left(\frac{(1+i_1)\frac{1+i_2}{1+i_2^*} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{38}$$

The amount of foreign debt that needs to be repaid at $t = 2$ is clearly an increasing function of the interest rate i_2. This is because a higher interest rate at $t = 2$ appreciates the currency, which makes it more attractive to issue in dollar. Conversely, as we demonstrated earlier, the optimal interest rate at $t = 2$ is also an increasing function of b_1^*: the more foreign debt outstanding there is in the economy, the stronger the incentive for the central bank to appreciate the currency in order to allow entrepreneurs to finance their productive investment more easily. The equilibrium determination of b_1^* is depicted in Figure 7.[14]

[14] As is apparent in Figure 1, we can find parameters such that the issuance at $t = 1$ exhibits multiple equilibria. This will happen if strategic complementarities are strong enough: if everyone expects the central bank to tighten strongly in the future, all debt will be issued in dollars and the central

Figure 7: Equilibrium Dollar Debt Issuance



An intuitive way to understand the time inconsistency problem faced by the central bank is to look at the blue dashed line in Figure 7. This line represents the hypothetical case where the central bank tries to commit to implement at time $t = 2$ a domestic rate that would be consistent with full employment for the threshold level of debt $\tilde{b}^*$. But even if entrepreneurs believe that this policy rate will be implemented, they still choose an equilibrium dollar debt level higher than this $\tilde{b}^*$. When time $t = 2$ comes, it is then optimal for the central bank to deviate from that planned interest rate, as can be seen from the dotted arrow going up to the line tracing the optimal policy rate, leading to an equilibrium with potentially large unemployment.

Proposition 4 (Dollar Debt Issuance Externalities)**.** *Entrepreneurs do not internalize*

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bank will have to tighten aggressively. And if everyone expects the central bank to implement full employment, all issuance will be in peso and the central bank will find it optimal to implement full employment. This possibility has been studied by Chang and Velasco (2006), which is why we focus here on the case where the equilibrium is unique.

that issuance denominated in dollars has a pecuniary effect on future interest rates, which then reduce aggregate demand in equilibrium. This uninternalized effect is quantified by the following expression:

$$\frac{dl_2}{db_1^*} = -\frac{c_2^N}{b_1^*(2\sigma - 1)} \tag{39}$$

5.3 Optimal Macroprudential Policy

These externalities can be internalized through a simple tax on dollar debt issuance.

Proposition 5 (Optimal Dollar Debt Issuance)**.** *The Social Planner's optimal allocation is to issue dollar debt exactly up to the threshold point where the $t = 2$ optimal interest rate implements full employment:*

$$b_1^{*SP} = \tilde{b}^* \tag{40}$$

This allocation can be obtained by taking dollar debt issuance at the rate τ, with τ implicitly defined by:

$$\tilde{b}^* = (1 + i_1)\frac{1 + i_2^{opt}(\tilde{b}^*)}{1 + i_2^*}\frac{(1 + i_1)\frac{1+i_2^{opt}(\tilde{b}^*)}{1+i_2^*} - (1 + i_1^*)}{\Gamma} \tag{41}$$

and where we used $1 + i_2^{opt}(\tilde{b}^*)$ as the optimal discretionary interest rate when the level of dollar debt is exactly $\tilde{b}^*$, as defined in Proposition 1. Tax proceeds are rebated lump-sum to entrepreneurs.

6 Conclusion

This paper shows that the presence of dollar debt in emerging markets has profound normative and positive implications, not only for individual emerging markets themselves, but also for the global financial system. The key result of this paper is that the presence of dollar debt makes all central banks acting in the same direction when the Federal Reserve changes its interest rates. This in turn initiates congestion externalities, since all central banks seek to maximize capital inflows in order to appreciate their currency, at the expense of other countries. This leads to inefficiently high interest rates in emerging economies, and inefficiently low levels of employment, highlighting the need for coordination amongst central banks in the face of the Global Financial Cycle. Finally, I showed that the anticipation of this (then optimal) behavior by individual central banks encourages even more dollar debt issuance in emerging countries, amplifying the Global Financial Cycle and worsening central banks' dilemma. Macroprudential policy, by discouraging dollar issuance and encouraging issuance in other currencies, must be used to counter this issuance externality.

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A Proofs and Derivations

A.1 Equilibrium at $t = 2$

$$U_2 = \frac{1}{1-\sigma}\left(\phi(c_2^T)^{1-\sigma} + (1-\phi)(c_2^N)^{1-\sigma}\right) + \beta\left(c_3^N + c_3^T\right) \tag{A.1}$$

and call the consumption index C_2: $C_2 = \phi^\eta(c_2^T)^{1-\sigma} + (1-\phi)^\eta(c_2^N)^{1-\sigma}$.

Budget constraints (and Lagrange Multipliers):

$$p_2^T c_2^T + p_2^N c_2^N = e_2 y_2^T + w_2 l_2 + \Pi_2 + \frac{1}{1+i_2}a_3 + \frac{1}{1+i_2^*}e_2 a_3^* \qquad (\lambda_2) \tag{A.2}$$

$$p_3^N c_3^N + p_3^T c_3^T + a_3 + e_3 a_3^* = p_3^T y_3^T + \bar{w}\bar{l} + \Pi_3 \qquad (\lambda_3) \tag{A.3}$$

with $p_t^T = e_t$ and $p_t^N = \bar{w}$. First-order conditions for households are:

$$\frac{\lambda_2}{1+i_2} = \beta\lambda_3 \tag{A.4}$$

$$\frac{\lambda_2}{1+i_2^*}e_2 = \beta\lambda_3 e_3 \tag{A.5}$$

$$\phi(c_2^T)^{-\sigma} = \lambda_2 p_2^T \tag{A.6}$$

$$(1-\phi)(c_2^N)^{-\sigma} = \lambda_2 p_2^N \tag{A.7}$$

$$1 = \lambda_3 p_3^N \tag{A.8}$$

$$1 = \lambda_3 p_3^T \tag{A.9}$$

which can be used to write non-tradables demand as:

$$c_2^N = \left(\frac{\phi}{1-\phi}\frac{p_2^N}{p_2^T}\right)^{-1/\sigma} c_2^T = \left(\frac{\phi}{1-\phi}\frac{\bar{w}}{e_2}\right)^{-1/\sigma} c_2^T \tag{A.10}$$

The savings/borrowing decisions in peso and dollar yield the standard UIP condition since there is no uncertainty:

$$1 + i_2 = (1+i_2^*)\frac{e_3}{e_2} \tag{A.11}$$

Using the fact that the price of tradables is equal to the exchange rate, and that the price of non-tradables is the wage since firms are perfectly competitive, we have the following demand function for non-tradables:

$$c_2^N = \left(\frac{\phi}{1-\phi} \frac{w_2}{e_2} \right)^{-1/\sigma} c_2^T \tag{A.12}$$

and plugging the UIP condition:

$$c_2^N = \left(\frac{\phi}{1-\phi} \frac{(1+i_2)w_2}{(1+i_2^*)e_3} \right)^{-1/\sigma} c_2^T \tag{A.13}$$

which shows how monetary policy can shift demand between T and NT.

A.2 Market clearing

Market clearing coupled with the linear production function for non-tradable goods imply that:

$$c_2^N = l_2 + \eta_2 K_1 - sK_2 - \frac{b_1}{\bar{w}} \tag{A.14}$$

Since households cannot lend to entrepreneurs, we must have $a_3 = 0$ (0 net supply of peso bonds for households). Unproductive entrepreneurs rebate profits to households equal to:

$$\Pi_2 = \bar{w}\eta_2 K_1 - (b_1 + e_2 b_1^*) - \bar{w} s K_2 \tag{A.15}$$

such that the budget constraint of households at $t = 2$ implies:

$$e_2 c_2^T = e_2 y_2^T - e_2 b_1^* + \frac{1}{1+i_2^*} e_2 a_3^* \tag{A.16}$$

Given the relation between the consumption of tradables and non-tradables, and the market clearing relation, this determines the amount of foreign borrowing by households:

$$\frac{1}{1+i_2^*} e_2 a_3^* = e_2 \left(-y_2^T + b_1^* + \left(\frac{\phi}{1-\phi} \frac{\bar{w}}{e_2} \right)^{1/\sigma} \left(l_2 + \eta_2 K_1 - sK_2 - \frac{b_1}{\bar{w}} \right) \right) \tag{A.17}$$

with a capital stock of:

$$K_2 = \kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s - \rho_0} \tag{A.18}$$

and the consumption level of tradables in the final period:

$$c_3^T = y_3^T - a_3^* \tag{A.19}$$

A.3 Proof of Proposition 1

We start by characterizing how a change in the interest translates into higher production through aggregate demand. Using the same condition as for the full employment interest rate, we get by differentiating in logs and approximating for i close enough to 0:

$$\frac{dc_2^N}{di_2} = \frac{dl_2}{di_2} - s\frac{dK_2}{di_2} = -\frac{c_2^N}{\sigma} \tag{A.20}$$

while at the same time, as long as entrepreneurs are constrained, capital moves according to:

$$\frac{dK_2}{di_2} = \frac{e_2\kappa b_1^*}{s-\rho_0} \tag{A.21}$$

through the appreciation of the currency and the UIP condition. Aggregate demand thus follows:

$$\frac{dl_2}{di_2} = -\frac{c_2^N}{\sigma} + s\kappa\frac{e_2 b_1^*}{s-\rho_0} \tag{A.22}$$

The first part of this expression is the usual aggregate demand channel. The second part comes from the crowding out of aggregate demand by entrepreneurs that use non-tradables good as an input to maintain their existing stock of capital.

The impact of i_2 on the consumption of non-tradables at time $t = 3$ is straightforward (no rigidities):

$$\frac{dc_3^N}{di_2} = \rho\frac{dK_2}{di_2} = \rho\kappa\frac{e_2 b_1^*}{s-\rho_0} \tag{A.23}$$

We are now ready to study the optimization of the planner. The problem of the central bank can thus be written as:[15]

$$\max_{l_2,e_2} \frac{1-\phi}{1-\sigma}\left(l_2 + \eta_2 K_1 - s\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}}\right)^{1-\sigma}$$

[15] Using the envelope theorem and separable preferences, monetary policy will not impact welfare when it comes to tradable consumption.

$$+\beta\left(\bar{l}+\rho\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0}\right) \quad \text{(A.24)}$$

with the constraints:

$$l_2 \leq \bar{l} \quad \text{(A.25)}$$

$$l_2 + \eta_2 K_1 - s\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}} = \left(\frac{e_2}{\beta\bar{w}(1+i_2^*)}\right)^{\frac{1}{\sigma}} \quad \text{(A.26)}$$

Let us denote by ν the Lagrange multiplier associated with the slackness condition (A.25), and ϵ the Lagrange multiplier on (A.26). Maximization implies:

$$(1-\phi)(c_2^N)^{-\sigma} = \nu + \epsilon \quad \text{(A.27)}$$

and

$$(1-\phi)\frac{s\kappa b_1^*}{s-\rho_0}(c_2^N)^{-\sigma} - \beta\rho\kappa\frac{b_1^*}{s-\rho_0} = \epsilon\left(\frac{s\kappa b_1^*}{s-\rho_0} - \frac{1}{\sigma\beta\bar{w}(1+i_2^*)}\left(\frac{e_2}{\beta\bar{w}(1+i_2^*)}\right)^{\frac{1}{\sigma}-1}\right) \quad \text{(A.28)}$$

Replacing this value for ϵ in the first condition yields:

$$(1-\phi)\left(c_2^N\right)^{-\sigma} = \nu + \frac{(1-\phi)\left(c_2^N\right)^{-\sigma} - \beta\rho}{1 - \frac{s-\rho_0}{\sigma s\kappa\beta\bar{w}b_1^*(1+i_2^*)}(\beta(1+i_2))^{\frac{\sigma-1}{\sigma}}} \quad \text{(A.29)}$$

If we are away from full employment, then $l_2 < \bar{l}$, and hence $\nu = 0$ which leads to:

$$(1-\phi)\left(c_2^N\right)^{-\sigma}\frac{s-\rho_0}{\sigma s\kappa\beta\bar{w}b_1^*(1+i_2^*)}(\beta(1+i_2))^{\frac{\sigma-1}{\sigma}} = \beta\rho \quad \text{(A.30)}$$

Next, use the optimality condition:

$$(1-\phi)\left(c_2^N\right)^{-\sigma} = \beta(1+i_2) \quad \text{(A.31)}$$

to finally get:

$$(\beta(1+i_2))^{\frac{2\sigma-1}{\sigma}} = \frac{\beta\rho}{\frac{s-\rho_0}{\sigma s\kappa\beta\bar{w}b_1^*(1+i_2^*)}} \quad \text{(A.32)}$$

So that the optimal interest rate is given by:

$$1 + i_2^{opt} = \beta^{\frac{1}{2\sigma-1}} \left(\frac{\rho s \sigma \kappa \bar{w} b_1^* (1 + i_2^*)}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma-1}} \tag{A.33}$$

where there is an increasing relationship between the level of dollar debt and the optimal interest rate. Define Ω as:

$$\Omega = \left(\sigma \rho \kappa \bar{w} \beta^{1/\sigma} \right)^{\sigma/(2\sigma-1)} \tag{A.34}$$

to end up with:

$$1 + i_2^{opt} = \Omega \left(\frac{b_1^* (1 + i_2^*)}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma-1}} \tag{A.35}$$

These calculations were valid only under the case where $\nu = 0$. When this is not satisfied, there is full employment and a change in the interest rate has no effect on the amount of labor supplied by households. That case is then equivalent to maximizing:

$$\max_{e_2} \frac{1-\phi}{1-\sigma} \left(\bar{l} + \eta_2 K_1 - s\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s - \rho_0} - \frac{b_1}{\bar{w}} \right)^{1-\sigma} + \beta \left(\bar{l} + \rho\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s - \rho_0} \right) \tag{A.36}$$

which leads to the following first-order condition:

$$\kappa \frac{b_1^*}{s - \rho_0} = s(1-\phi) \left(\bar{l} + \eta_2 K_1 - s\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s - \rho_0} - \frac{b_1}{\bar{w}} \right)^{-\sigma} - \beta\rho \tag{A.37}$$

Isolating e_2 from this expression yields:

$$e_2^{full} = (s - \rho_0) \frac{\left(\frac{\kappa \frac{b_1^*}{s-\rho_0} + \beta\rho}{s(1-\phi)} \right)^{-\frac{1}{\sigma}} - \bar{l} - \eta_2 K_1 + \frac{b_1}{\bar{w}} + \frac{s\kappa}{s-\rho_0}(\eta_2 K_1 - b_1)}{s\kappa b_1^*} \tag{A.38}$$

which leads to the optimal domestic interest rate in the full employment case using the UIP condition:

$$1 + i_2^{full} = \frac{(1 + i_2^*)\bar{w}}{e_2^{full}} \tag{A.39}$$

Finally, the regime-switching occurs when the two conditions intersect. When this is the case, there is full employment but agents are still against their Euler equation, hence:

$$\kappa \frac{b_1^*}{s-\rho_0} + \beta\rho = s(1-\phi)\left(c_2^N\right)^{-\sigma} \tag{A.40}$$

$$= s\beta(1+i_2) \tag{A.41}$$

$$= s\beta\Omega\left(\frac{b_1^*(1+i_2^*)}{s-\rho_0}\right)^{\frac{\sigma}{2\sigma-1}} \tag{A.42}$$

This equation has up to two solutions since $\sigma \geq 1$. We restrain ousrself to the case where there is only one solution (which only requires an assumption on the size of K_1: we want the second solution to be for greater foreign debt than if all initial investment in K_1 was made in foreign debt).

A.4 Proof of Lemma 3

Entrepreneurs' optimization program is given by:

$$\min_{b_1, b_1^*} \quad b_1 + e_2 b_1^* \tag{A.43}$$

$$\text{s.t.} \quad \frac{b_1}{1+i_1} + \frac{e_1 b_1^*}{1+\hat{i}_1^*} = K_1 \tag{A.44}$$

An interior solution exists when a simple UIP condition using the equilibrium interest rates is verified:

$$\frac{e_2}{e_1} = \frac{1+i_1}{1+\hat{i}_1^*} \tag{A.45}$$

Using the equilibrium relation (34), the equilibrium debt flow in dollar when we are at the interior solution is:

$$\frac{b_1^*}{1+\hat{i}_1^*} = \frac{(1+i_1)\frac{e_1}{e_2} - (1+i_1^*)}{\Gamma} \tag{A.46}$$

linking deviations from the frictionless UIP condition to the equilibrium flows. When this level of borrowing is negative or higher than K_1, there is no interior solution and entrepreneurs issue debt up to the corner solution:

$$\frac{b_1^*}{1+\hat{i}_1^*} = \max\left(\min\left(\frac{(1+i_1)\frac{e_1}{e_2} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{A.47}$$

so that the amount of foreign debt (in dollars) that needs to be paid back at $t = 2$ is:

$$b_1^* = (1+\hat{i}_1^*) \max\left(\min\left(\frac{(1+i_1)\frac{e_1}{e_2} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{A.48}$$

$$= (1+i_1)\frac{e_1}{e_2} \max\left(\min\left(\frac{(1+i_1)\frac{e_1}{e_2} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{A.49}$$

$$= (1+i_1)\frac{1+i_2}{1+i_2^*} \max\left(\min\left(\frac{(1+i_1)\frac{1+i_2}{1+i_2^*} - (1+i_1^*)}{\Gamma}, \frac{K_1}{e_1}\right), 0\right) \tag{A.50}$$

A.5 Proof of Lemma 1

Recall that consumption of tradables is given by the following expression for each country j:

$$c_{2,j}^T = \left(\frac{\phi}{1-\phi}\frac{1+i_2}{1+i_2^*}\right)^{\frac{1}{\sigma}} c_{2,j}^N \tag{A.51}$$

where we can replace NT consumption with the level of domestic rates:

$$c_{2,j}^T = \left(\frac{\phi}{1-\phi}\frac{1+i_2}{1+i_2^*}\right)^{\frac{1}{\sigma}} \left(\frac{1+i_2}{(1-\phi)\beta}\right)^{-\frac{1}{\sigma}} \tag{A.52}$$

Combining this with the market clearing expression for foreign debt (A.16):

$$\frac{1}{1+i_2^*} a_{3,j}^* = \left(\beta\phi\frac{1}{1+i_2^*}\right)^{\frac{1}{\sigma}} + b_{1,j}^* - y_{2,j}^T \tag{A.53}$$

And finally using the rate determination from global arbitrageurs (17):

$$i_2^* = i_2^{\$} + \gamma^{-1}\left(\left(\beta\phi\frac{1}{1+i_2^*}\right)^{\frac{1}{\sigma}} + b_1^* - y_2^T\right) \tag{A.54}$$

A.6 Proof of Lemma 2

We neglect the country subscript j for this part. Taking the financial wedge into account, the link between the domestic policy rate and the exchange rate is now given by:

$$(1+i_2) = \left((1+i_2^*)\frac{\bar{w}}{e_2}\right)^{\frac{1}{1+\psi}} \tag{A.55}$$

which is convenient for the aggregate demand condition using the interest rate since:

$$(1+i_2)^{-\frac{1}{\sigma}} = \left((1+i_2^*)\frac{\bar{w}}{e_2}\right)^{-\frac{1}{\sigma(1+\psi)}} \tag{A.56}$$

Going back to the optimal policy program, we can now write it as:

$$\max_{l_2,e_2} \frac{1-\phi}{1-\sigma}\left(l_2 + \eta_2 K_1 - s\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}}\right)^{1-\sigma} + \beta\left(\bar{l} + \rho\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0}\right) \tag{A.57}$$

with the constraints:

$$l_2 \leq \bar{l} \tag{A.58}$$

$$l_2 + \eta_2 K_1 - s\kappa\frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}} = \left(\frac{e_2}{\beta\bar{w}(1+i_2^*)}\right)^{\frac{1}{\sigma(1+\psi)}} \tag{A.59}$$

Thanks to this functional form, the resolution of the optimal policy problem is almost identical. A few steps of algebra yield the first-order condition:

$$(1-\phi)\left(c_2^N\right)^{-\sigma}\frac{s-\rho_0}{\sigma(1+\psi)s\kappa\beta\bar{w}b_1^*(1+i_2^*)}(\beta(1+i_2))^{\frac{\sigma(1+\psi)-1}{\sigma}} = \beta\rho \tag{A.60}$$

Next, use the optimality condition:

$$(1-\phi)\left(c_2^N\right)^{-\sigma} = \beta(1+i_2) \tag{A.61}$$

to finally get:

$$(\beta(1+i_2))^{\frac{2\sigma-1}{\sigma}+\psi} = \frac{\beta\rho}{\frac{s-\rho_0}{\sigma(1+\psi)s\kappa\beta\bar{w}b_1^*(1+i_2^*)}} \tag{A.62}$$

So that the optimal interest rate is given by:

$$1+i_2^{opt} = \beta^{\frac{2\sigma}{2\sigma-1+\sigma\psi}-1}\left(\frac{\rho s\sigma(1+\psi)\kappa\bar{w}b_1^*(1+i_2^*)}{s-\rho_0}\right)^{\frac{\sigma}{2\sigma-1+\sigma\psi}} \tag{A.63}$$

where there is an increasing relationship between the level of dollar debt and the optimal interest rate. Define Ω_ψ as:

$$\Omega_\psi = \left(\sigma(1+\psi)\rho\kappa\bar{w}\beta^{\frac{1-\sigma\psi}{\sigma}}\right)^{\frac{\sigma}{2\sigma-1+\sigma\psi}} \tag{A.64}$$

to end up with:

$$1 + i_2^{opt} = \Omega_\psi \left(\frac{b_1^*(1+i_2^*)}{s-\rho_0} \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \tag{A.65}$$

A.7 Proof of Proposition 3

We look at the case of a planner that takes into account how domestic policy rate decisions impact the equilibrium determination of the world interest rate. To do so, we manipulate this condition in order to only have e_2 and i_2^*:

$$c_2^T = \left(\beta\phi \frac{(1+i_2)^\psi}{1+i_2^*} \right)^{\frac{1}{\sigma}} \tag{A.66}$$

$$= \left(\beta\phi \frac{(1+i_2^*)^{\frac{\psi}{1+\psi}} (e_2)^{\frac{-\psi}{1+\psi}} (\bar{w})^{\frac{\psi}{1+\psi}}}{1+i_2^*} \right)^{\frac{1}{\sigma}} \tag{A.67}$$

$$= \left(\beta\phi \frac{(\bar{w})^{\frac{\psi}{1+\psi}}}{(1+i_2^*)^{1-\frac{\psi}{1+\psi}} (e_2)^{\frac{\psi}{1+\psi}}} \right)^{\frac{1}{\sigma}} \tag{A.68}$$

$$= \left(\beta\phi \frac{(\bar{w})^{\frac{\psi}{1+\psi}}}{(1+i_2^*)^{\frac{1}{1+\psi}} (e_2)^{\frac{\psi}{1+\psi}}} \right)^{\frac{1}{\sigma}} \tag{A.69}$$

The optimal policy problem can now equivalently be written:

$$\max_{l_2, e_2, i_2^*} \frac{1-\phi}{1-\sigma} \left(l_2 + \eta_2 K_1 - s\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}} \right)^{1-\sigma} + \beta \left(\bar{l} + \rho\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} \right) \tag{A.70}$$

with the constraints:

$$l_2 \leq \bar{l} \tag{A.71}$$

$$l_2 + \eta_2 K_1 - s\kappa \frac{\eta_2 K_1 - b_1 - e_2 b_1^*}{s-\rho_0} - \frac{b_1}{\bar{w}} = \left(\frac{e_2}{\beta\bar{w}(1+i_2^*)} \right)^{\frac{1}{\sigma(1+\psi)}} \tag{A.72}$$

$$i_2^* = i_2^\$ + \frac{1}{\gamma} \left(\frac{\beta\phi\bar{w}^{\frac{\psi}{1+\psi}}}{(1+i_2^*)^{\frac{1}{1+\psi}} e_2^{\frac{\psi}{1+\psi}}} \right)^{\frac{1}{\sigma}} + \frac{b_1^* - y_2^T}{\gamma} \tag{A.73}$$

We denote, respectively, the Lagrange multipliers on these three constraints as ν, ϵ and λ^*. Maximization then implies:

$$(1-\phi)(c_2^N)^{-\sigma} = \nu + \epsilon \tag{A.74}$$

and

$$(1-\phi)\frac{s\kappa b_1^*}{s-\rho_0}(c_2^N)^{-\sigma} - \beta\rho\kappa\frac{b_1^*}{s-\rho_0} = \epsilon\left(\frac{s\kappa b_1^*}{s-\rho_0} - \frac{1}{\sigma(1+\psi)}\frac{(\beta(1+i_2))^{\frac{\sigma(1+\psi)-1}{\sigma}}}{\beta\bar{w}(1+i_2^*)}\right) + \frac{\lambda^*\psi}{\gamma\sigma(1+\psi)}\frac{c_2^T}{e_2} \tag{A.75}$$

and finally:

$$\frac{\epsilon}{\sigma(1+\psi)}\frac{(\beta(1+i_2))^{-\frac{1}{\sigma}}}{1+i_2^*} = -\frac{\lambda^*}{\gamma\sigma(1+\psi)}\frac{c_2^T}{1+i_2^*} - \lambda^* \tag{A.76}$$

which yield the following relation between the Lagrange multipliers:

$$\lambda^* = -\frac{\epsilon(\beta(1+i_2))^{-\frac{1}{\sigma}}}{\sigma(1+\psi)(1+i_2^*) + \frac{c_2^T}{\gamma}} \tag{A.77}$$

And putting the last two condition together to eliminate λ^*:

$$(1-\phi)\frac{s\kappa b_1^*}{s-\rho_0}(c_2^N)^{-\sigma} - \beta\rho\kappa\frac{b_1^*}{s-\rho_0} = \epsilon\left(\frac{s\kappa b_1^*}{s-\rho_0} - \frac{1}{\sigma(1+\psi)}\frac{(\beta(1+i_2))^{-\frac{1}{\sigma}}}{e_2}\right) - \frac{\epsilon\psi}{\sigma(1+\psi)}\frac{(\beta(1+i_2))^{-\frac{1}{\sigma}}}{e_2} \tag{A.78}$$

we arrive at an expression for ϵ that resembles the one from the individual central bank optimization problem:

$$\epsilon = \frac{(1-\phi)(c_2^N)^{-\sigma} - \beta\rho}{1 - \frac{(s-\rho_0)(1+i_2)^{1+\psi}}{s\kappa b_1^* \sigma \bar{w}} \frac{c_2^N}{(1+\psi)(1+i_2^*) - \frac{\psi c_2^T}{\gamma\sigma + \frac{c_2^T}{1+i_2^*}}}} \tag{A.79}$$

which leads up to, when we are away from full employment:

$$\frac{(s-\rho_0)(1+i_2)^{1+\psi}}{s\kappa b_1^* \sigma \bar{w}} \frac{c_2^N}{(1+\psi)(1+i_2^*) - \frac{\psi c_2^T}{\gamma\sigma + \frac{c_2^T}{1+i_2^*}}} = \beta\rho \tag{A.80}$$

Using the equilibrium condition between domestic rates and consumption of non-tradables:

$$(1+i_2)^{\frac{2\sigma-1}{\sigma}+\psi} = \left(s\kappa\sigma\bar{w}(1+\psi)\right) \frac{b_1^*(1+i_2^*)}{s-\rho_0} \left(1 - \frac{\psi c_2^T}{(1+\psi)\left(\gamma\sigma(1+i_2^*) + c_2^T\right)}\right) \tag{A.81}$$

Finally, some algebra on the last congestion externality to express it as:

$$\frac{\psi c_2^T}{(1+\psi)\left(\gamma\sigma(1+i_2^*) + c_2^T\right)} = \frac{\psi}{1+\psi} \frac{1}{\gamma\sigma \frac{(1+i_2^*)}{c_2^T} + 1} \tag{A.82}$$

$$= \frac{\psi}{1+\psi} \frac{1}{\gamma\sigma \frac{(1+i_2^*)}{\left(\beta\phi \frac{(1+i_2)^{\psi}}{1+i_2^*}\right)^{\frac{1}{\sigma}}} + 1} \tag{A.83}$$

$$= \frac{\psi}{1+\psi} \frac{1}{\gamma\sigma \frac{(1+i_2^*)^{\frac{\sigma+1}{\sigma}}}{(\beta\phi(1+i_2)^{\psi})^{\frac{1}{\sigma}}} + 1} \tag{A.84}$$

$$= \frac{\psi}{1+\psi} \frac{1}{\frac{\gamma\sigma}{(\beta\phi)^{\frac{1}{\sigma}}} \frac{(1+i_2^*)^{\frac{\sigma+1}{\sigma}}}{(1+i_2)^{\frac{\psi}{\sigma}}} + 1} \tag{A.85}$$

The optimal interest rate thus verifies:

$$1 + i_2^{SP} = \Omega_\psi \left(\frac{b_1^*(1+i_2^*)}{s-\rho_0}\right)^{\frac{\sigma}{2\sigma-1+\sigma\psi}} \left(1 - \frac{\psi}{1+\psi} \frac{1}{\frac{\gamma\sigma}{(\beta\phi)^{\frac{1}{\sigma}}} \frac{(1+i_2^*)^{\frac{\sigma+1}{\sigma}}}{(1+i_2)^{\frac{\psi}{\sigma}}} + 1}\right)^{\frac{\sigma}{2\sigma-1+\sigma\psi}} \tag{A.86}$$

This condition is very similar to the original one, except for the additional congestion externality that work through the γ and ψ coefficients that denote the modified exchange rate sensitivity to interest rates. Finally, this last part is exactly the monetary policy spillover we identified in Proposition 2, leading to:

$$1 + i_2^{SP} = \Omega_\psi \left(\frac{b_1^*(1 + i_2^*)}{s - \rho_0} \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \left(1 - \frac{1}{1 + \psi} \mathscr{C}(i_2^{SP}, i_2^*) \right)^{\frac{\sigma}{2\sigma - 1 + \sigma\psi}} \tag{A.87}$$