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Capital Flows and Monetary Policy Trade-offs in Emerging Market Economies*

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Abstract

We lay out a small open economy model incorporating key features of EME economic and financial structure: high exchange rate pass-through to import prices, low pass-through to export prices and shallow domestic financial markets giving rise to occasionally binding leverage constraints. As a consequence of the latter, a sudden stop with large capital outflows can give rise to a financial crisis. In the sudden stop, the central bank faces an intratemporal trade-off as output declines while inflation rises. In normal times, there is an intertemporal trade-off as the risk of a future sudden stop forces the central bank to factor financial stability considerations into its policy conduct. The optimal monetary policy leans against capital flows and domestic leverage. Macroprudential, capital flow management and central bank balance sheet policies can help to mitigate both intra- and intertemporal trade-offs. Fiscal policy also plays a key role. A higher level of public debt and a weaker fiscal policy imply greater leverage and hence greater tail risk for the economy.

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Introduction

Over the past two decades, many emerging market economies (EMEs) have adopted price-stability oriented monetary policy regimes with floating exchange rates, catching up with prevailing practice in advanced economies. At the same time, EMEs have commonly combined these monetary policy regimes with systematic resort to foreign exchange (FX) intervention, macroprudential tools and capital flow management measures. This more eclectic set up of EME policy frameworks is explained by the enduring challenges from capital flow swings, which raise difficult trade-offs for monetary policy (BIS (2019), BIS (2022), IMF (2020)).

What is still missing from the literature is a clear characterization of the nature of the monetary policy trade-offs faced by EMEs and how complementary policy tools can address them. In particular, a key distinction that has so far not been explored is that between the policy challenges in times of stress when capital flows out rapidly and at large scale, and in normal times when capital flows are stable and vulnerabilities build up. In this paper, we aim to fill this gap. We lay out a small open economy model incorporating key features of EME economic and financial structure: high exchange rate pass-through to import prices, imperfect pass-through to export prices and shallow financial markets giving rise to occasionally binding leverage constraints. Based on this model, we analyze how capital flow swings affect macroeconomic and financial stability in EMEs. The model features three periods. At time 1, there is the risk of a strong tightening in global financial conditions which triggers a sudden stop in capital inflows and which might cause a credit crunch. To keep the analysis simple, we take period 2 to be the long run. This allows us to focus on the short-run dynamics of the model in period 1, when the shock occurs, and in period 0, the normal, tranquil time before the shock.

Our objective is to characterize optimal monetary policy in the sudden stop, at time 1, and to understand how the risk of a large future capital flow reversal affects the conduct of monetary policy during normal times, at time 0. We investigate the trade-offs faced by the central bank and how additional policy tools such as macroprudential and capital flow management measures as well as FX intervention can mitigate these trade-offs and enhance macro-financial stability. In this vein, we also explore how structural features of the economy, in particular the depth of FX markets, affect the trade-offs faced by monetary authorities and the effectiveness of complementary policy tools.

Our main findings are as follows. First, capital flow shocks have nonlinear effects in the model as a result of the occasionally binding financial constraint. Trade effects dominate when financial constraints are not binding, while financial effects prevail when they do. Small capital outflows have little impact on economic activity and inflation, while big outflows that push the economy against the financial constraint have a disproportionately larger adverse impact. Furthermore, in response to large outflows, output and inflation tend to move in opposite directions, with the former dropping and the latter rising, giving rise to an intratemporal trade-off for monetary policy that aims to stabilize both variables.

Second, the severity of a financial crisis depends on initial conditions inherited from the pre-sudden stop period (period 0). The lower the leverage accumulated by banks in that period, the greater their ability to cushion a large outflow shock in the future. Tighter monetary policy reduces bank leverage and hence future tail risk. As a result, monetary policy faces in normal times an intertemporal trade-off. It is trading off worse macroeconomic outcomes today against the future tail risk of a financial crisis. Our analysis suggests that optimal monetary policy entails a "leaning-against-the-wind" element. In the face of risks of large capital outflows in the future, optimal monetary policy is unconditionally tighter and leans against high credit demand and low capital inflows in order to contain bank leverage and mitigate tail risk.

Third, complementary policy tools can improve monetary policy trade-offs and enhance macroeconomic stability. In case of a large outflow, central bank bond purchases as well as a loosening of macroprudential and capital flow management measures can cushion the impact of the shock and mitigate the intratemporal trade-off faced by monetary policy. In normal times, tighter macroprudential and capital flow management policies can contain leverage and reduce the build up of vulnerabilities. Macroprudential policy is more effective in mitigating tail risks than capital flow measures as the latter limit the inflow of capital in the economy. Complementary tools also take some of the burden off monetary policy, allowing the central bank to give greater weight to current period macroeconomic stability, thus alleviating the intertemporal trade-off.

Finally, monetary policy trade-offs and the choice of complementary policy tools depend on structural country characteristics. In particular, FX intervention is an effective crisis management tool when FX markets are shallow, but not so when they are deep. When FX markets are shallow, FX intervention can influence the exchange rate, rendering it an effective complementary tool for macro-financial stabilization policy. Specifically, in that case, FX sales can cushion the impact of capital outflows and FX purchases can limit tail risk in normal times, working in a similar fashion as capital flow management measures. Another important country characteristic is the level of public debt. A higher level of public debt in normal times implies greater leverage and hence greater tail risk for the economy, inducing a tighter stance of optimal monetary policy to lean against these effects.

The remainder of the paper is structured as follows. The next section provides a short review of the related literature. Section 2 lays out the model. Section 3 characterizes optimal monetary and optimal complementary macro-financial stability policies in a sudden stop and in normal times. In that section, we also provide illustrative simulations, demonstrating the operation of the optimal policies and their interaction in response to external financial and to domestic shocks. Section 4 concludes.

1 Literature review

This paper contributes to three main related strands of literature.

First, it relates to the literature on the vulnerability of EMEs to capital flow and exchange rate swings. The vulnerability of EMEs' is commonly linked to their inability to issue external debt in domestic currency, referred to as original sin by Eichengreen and Hausmann (1999). Foreign currency borrowing gives rise to balance sheet vulnerabilities from currency mismatches. The exchange rate may then not play the stabilizing role through the standard trade channel that is at the core of the traditional Mundell-Fleming framework. In this vein, the third generation of currency crisis models highlighted how the interplay between collateral constraints and currency mismatches can give rise to self-fulfilling currency runs (Krugman (1999), Aghion et al. (2001), Aghion et al. (2004)). The subsequent literature has focused on the implications of currency mismatches in the presence of financial amplifications mechanisms based on small open economy models with financial frictions (e.g. Mendoza (2010), Akinci and Queralto (2018), Aoki et al. (2016) and Mendoza and Rojas (2019)). Also the recent papers by Basu et al. (2020), Adrian et al. (2020) focus on currency mismatches on EME borrowers' balance sheets.

The vulnerability of EMEs however goes beyond borrower currency mismatches. Even when borrowing from abroad in their local currency, exchange rate fluctuations can influence the risk capacity and hence the credit supply of foreign lenders to EMEs (Hofmann et al. (2022b)). Even in the absence of any balance sheet effects of exchange rate fluctuations, e.g. as a result of hedging, external financial conditions impact EMEs through the credit supply of foreign lenders and investors (Hofmann et al. (2022a)). In this paper, the vulnerability of EMEs to capital flow swings is not tied to the existence of currency mismatches but arises from the shallowness of their financial markets.

Second, our paper is related to the literature on monetary policy trade-offs raised by capital flow fluctuations. Céspedes et al. (2004), Christiano et al. (2004) and Gourinchas (2018) analyse the monetary policy trace-offs in a sudden stop in the presence of currency mismatches and credit constraints. The trade-off there is between supporting demand through monetary easing and supporting the exchange rate and limiting balance sheet disruption through monetary tightening.

More recently, Adrian et al. (2020) highlight a trade-off between inflation and output stabilization in EMEs that arises because of weakly anchored inflation expectations. As a result, exchange rate depreciation in the wake of an external financial tightening has larger inflationary consequences, inducing a tighter monetary policy stance that depresses output. Cavallino and Sandri (2020) highlight the existence of an expansionary lower bound below which monetary easing becomes contractionary because of capital outflows. Our paper features an intratemporal trade-off between stabilizing output and inflation in a sudden stop as well as an intertemporal trade-off in normal times between macroeconomic stability today and tomorrow. Such an intertemporal trade-off is absent in the canonical sudden-stop model featuring financial constraints and borrower currency mismatches (e.g. Devereux et al. (2019)).

Third, our paper is related to a recent literature that analyzes the role of macroprudential policies, capital flow management measures and FX intervention as complementary policy tools in open economies. Rey (2013) prominently highlighted that financial globalization has transformed the classical trilemma into a dilemma, so that monetary control can only be reestablished through the active management of capital flows. In this vein, Jeanne and Korinek (2010), Bianchi (2011), Benigno et al. (2013), Farhi and Werning (2014), Benigno et al. (2016) and Korinek and Sandri (2016) demonstrate how macroprudential and capital flow management measures can enhance macro-financial stability. Aoki et al. (2016) show how these policies can also improve monetary policy trade-offs.

Cavallino (2019) and Fanelli and Straub (2021) show how FX intervention by the central bank can mitigate the constraints faced by monetary policy from capital flow fluctuations. Hofmann et al. (2019)) develop a simple model and provide empirical evidence of the macroprudential effects of FX intervention on domestic credit. Adrian et al. (2020) demonstrate how FX intervention and capital controls can mitigate the intratemporal trade-off faced by EMEs when global financial conditions tighten, while Basu et al. (2020) analyze how different tools can improve macro-financial stability in face of different types of shocks and different types of frictions. In this paper, we analyze the joint operation of the full range of policies, including monetary policy (i.e. interest rate policy), macroprudential policy, capital flow management measures as well as FX and bond market interventions, deriving welfare maximizing policy reaction functions.

2 The model

We consider a small open economy which we call Home. The economy is inhabited by a measure one of households and produces tradable goods which are consumed domestically and exported to the rest of the world. Households consume domestic and imported goods and supply labor which is combined with physical capital to produce domestic tradable goods. The production of physical capital requires risky investment and is financed by domestic banks. Banks collect short-term deposits from households and invest in long-term assets. They issue loans to capital producers and purchase long-term government bonds. However, an agency problem affects their ability to raise deposits and limits their leverage, constraining the domestic supply of credit. Fortunately, banks are not the only source of credit for the Home economy. Foreign investors lend to the domestic government by purchasing Home-currency government bonds. Capital inflows, by affecting the size and composition of the balance sheet of the banking sector, are a critical determinant of the domestic cost of credit. When capital inflows are abundant, the leverage of the banking sector falls allowing banks to channel funds to the private sector. However, when inflows retrench domestic leverage rises and banks are forced to curtail their supply of credit.

Time is discrete and there are three periods, $t = 0, 1, 2$. At time 1, there is the risk of a strong tightening in global financial conditions which triggers a capital outflow and might cause a credit crunch. To keep the analysis simple, we take period 2 to be the long run. This allows us to easily focus on the short-run dynamics of the model.[1]

Households

Home is inhabited by a measure one of identical households, each composed of a unit mass of family members. Within each household, there are two types of members. Some of them are workers and some are bankers. Workers supply labor while bankers manage domestic financial intermediaries. Both agents return their earnings to the household within which there is perfect consumption insurance. Individuals switch across occupations stochastically. In particular, in each period a fraction $1 - \sigma$ of randomly selected bankers become workers in the following period.[2] An exiting banker pays out all retained earnings as dividends to her households and is replaced by a former worker, which becomes a new banker.

Each household maximizes

$$\mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \left\{ \log C_t - \frac{(H_t)^{1+\varphi}}{1+\varphi} \right\} \right]$$

where C_t is consumption, H_t is the aggregate amount of labor supplied by its workers,

[1] Formally, we assume that periods have unequal length. While period 0 and 1 have length one, period 2 lasts for a length T, with $T \to \infty$.

[2] This assumption implies that bankers have a finite horizon. This, in turn, insures that they do not accumulate enough earnings to fund their balance sheets entirely through own capital.

β is the intertemporal discount factor and φ the inverse of the Frisch elasticity of labor supply. The consumption index is a composite of Home and imported goods, given by $C_t = (C_{H,t})^{1-\alpha} (C_{F,t})^{\alpha} (1-\alpha)^{-1+\alpha} \alpha^{-\alpha}$,where $\alpha \in [0,1]$ measures the degree of Home bias in consumption. The optimal allocation of expenditure between domestic and foreign goods yields the demand function

$$C_{H,t} = (1-\alpha)(P_{H,t})^{-1} C_t$$

where $P_{H,t}$ is the real price of the Home good, that is its price in units of domestic consumption. To avoid confusion, we express all prices in real terms, that is in units of domestic consumption, except for inflation rates which represent the growth rate of nominal prices. This implies $P_{H,t} = (P_{F,t})^{-\frac{\alpha}{1-\alpha}}$, where $P_{F,t}$ denotes the domestic price of the imported good. We assume that the foreign good is priced in foreign currency and that there is full exchange rate pass-through to import prices. This implies that $P_{F,t} = E_t P_F^*$, where E_t denotes the real exchange rate, defined as the relative price of foreign consumption in terms of domestic consumption such that an increase in E_t represents a real depreciation for the domestic economy. The foreign price of the imported good is constant and normalized to 1, $P_F^* = 1$. Let Π_t and $\Pi_{H,t}$ denote CPI and PPI inflation, respectively, which are related by $\Pi_t = \Pi_{H,t}\frac{P_{H,t-1}}{P_{H,t}}$.

Domestic households can save by holding bank deposits which pay the nominal gross risk-free rate R_t^d. The budget constraint of the representative household is

$$C_t + D_t = D_{t-1}\frac{R_{t-1}^d}{\Pi_t} + W_t H_t + \Omega_t$$

where D_t denotes the aggregate real amount of deposits. W_t is the real wage rate and Ω_t denotes her net non-labor income which is the sum of dividends from firms, entrepreneurs, and financial intermediaries, minus lump-sum taxes: $\Omega_t = \Omega_t^y + \Omega_t^k + \Omega_t^f - T_t$. The households' labor supply decision is $(H_t)^{\varphi} C_t = W_t$, while their Euler equation is

$$1 = \mathbb{E}_t\left[\Lambda_{t|t+1}\frac{R_t^d}{\Pi_{t+1}}\right]$$

where $\Lambda_{t|t+1} = \beta C_t / C_{t+1}$ is the households stochastic discount factor.

Banks

Financial firms channel funds from savers to borrowers. In doing so, they engage in maturity transformation since they issue short-term liabilities and invest in long-term assets. Financial intermediaries finance two types of activities. First, they make loans to non-financial firms to finance the production of physical capital. Loans are claims to a stream of earnings associated with the underlying capital. Let P_t^l denote the market value of a loan that is financing one unit of capital and Q_t its cash flow. Then the real rate of return to the bank is

$$\frac{R_{t+1}^l}{\Pi_{t+1}} = \frac{Q_{t+1} + (1-\delta) P_{t+1}^l}{P_t^l}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. In addition, banks lend to the domestic government by purchasing long-term bonds. Government bonds are inflation-indexed perpetuities that pay one unit of domestic consumption per period indefinitely. Their gross real rate of return is

$$\frac{R_{t+1}^b}{\Pi_{t+1}} = \frac{1 + P_{t+1}^b}{P_t^b}$$

where P_t^b denotes the price of a new bond issued in period t.

The aggregate balance sheet of the banking sector is

$$P_t^l L_t + P_t^b B_t + S_t = N_t + D_t$$

where N_t is banks' end-of-period net worth, L_t their holding of loans, B_t their holding of government bonds, and S_t are reserves held by banks on accounts at the central bank which pay the gross nominal interest rate R_t. The bankers' objective is to maximize the present discounted value of their dividends to their households. Since dividends are paid only upon exiting, this is tantamount to maximize their terminal wealth. Therefore. the problem of the representative banker is

$$V_t(N_t) = \max_{L_t, B_t, D_t} \mathbb{E}_t \left[\Lambda_{t|t+1} \left\{ (1-\sigma) \dot{N}_{t+1} + \sigma V_{t+1} \left(\dot{N}_{t+1} \right) \right\} \right]$$

where $\dot{N}_{t+1}$ is the beginning-of-period net worth, that is before the exit and entry of bankers take place, which is given by

$$\dot{N}_{t+1} = N_t \frac{R_t}{\Pi_{t+1}} + \left(\frac{e^{\tau_t^l} R_{t+1}^l}{\Pi_{t+1}} - \frac{R_t}{\Pi_{t+1}} \right) P_t^l L_t + \left(\frac{R_{t+1}^b}{\Pi_{t+1}} - \frac{R_t}{\Pi_{t+1}} \right) P_t^b B_t - \left(\frac{R_t^d}{\Pi_{t+1}} - \frac{R_t}{\Pi_{t+1}} \right) D_t + T_{t+1}^l$$

where τ_t^l is a macroprudential tax on lending to non-financial firms. The proceeds from the tax are rebated lump-sum to banks, that is $T_{t+1}^l = \left(1 - e^{\tau_t^l}\right) \frac{R_{t+1}^l}{\Pi_{t+1}} P_t^l L_t$, such that the tax does not affect the evolution of their net worth. We write the banks' problem directly in terms of the representative bank since, as will be clear in a moment, their value function is linear in net worth and therefore can be aggregated across bankers.

Following Gertler and Karadi (2011), we assume that banks are subject to a moral hazard problem which might limit their ability to raise deposits. At the beginning of each period, a banker can divert a fraction of the assets held by the bank she manages and transfer the proceeds to her household. In particular, we assume that the banker can divert a fraction $\zeta_l \in (0,1)$ of its claims on physical capital and a fraction $\zeta_b \in (0,1)$ of its government bond holdings. Reserves held at the central bank cannot be diverted. When diversion occurs, depositors force the bank into bankruptcy and recover the remaining fraction of assets. Anticipating the incentives of the banker, depositors are willing to lend her if and only if the value of the bank exceeds the value of divertable assets. Hence, the following incentive compatibility constraint must be satisfied

$$\mathcal{V}_t\left(N_t\right) \geq \zeta_l P_t^l L_t + \zeta_b P_t^b B_t$$

where $\mathcal{V}_t$ denotes the value of the bank for the households.

Let μ_t be the Lagrange multiplier associated with the incentive compatibility constraint. The first order conditions of the banks problem give rise to the following pricing equations

$$\zeta_l \mu_t = \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\frac{\Lambda_{t|t+1}}{\Pi_{t+1}}\left(e^{\tau_t^l} R_{t+1}^l - R_t\right)\right]$$
$$\zeta_b \mu_t = \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\frac{\Lambda_{t|t+1}}{\Pi_{t+1}}\left(R_{t+1}^b - R_t\right)\right]$$

and $R_t^d = R_t$, where V_t is the marginal value of net worth for the banks, such that $\mathcal{V}_t\left(N_t\right) = V_t N_t$, and satisfies

$$\left(1-\mu_t\right)V_t = \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\Lambda_{t|t+1}\frac{R_t}{\Pi_{t+1}}\right]$$

Aggregate end-of-period net worth evolves as the sum of the net worth of the surviving bankers plus the start-up capital endowed to the new bankers by their households, denoted by $\hat{N}_t$, as follows:

$$N_t = \sigma \dot{N}_t + \hat{N}_t$$

while the net dividend paid from the banking sector to households is $\Omega_t^f = (1-\sigma)\dot{N}_t - \hat{N}_t$. We assume that $\hat{N}_t = N - \sigma N_{t-1} R_{t-1}/\Pi_t$ such that end-of-period net worth evolves as

$$N_t = N + \sigma\left[\left(\frac{R_t^l}{\Pi_t} - \frac{R_{t-1}}{\Pi_t}\right)P_{t-1}^l L_{t-1} + \left(\frac{R_t^b}{\Pi_t} - \frac{R_{t-1}}{\Pi_t}\right)P_{t-1}^b B_{t-1}\right]$$

where N is steady-state net worth. This simple specification allows us to focus on the amplification effect of asset prices on the leverage constraint of the banks. When asset prices fall, net worth falls and the leverage constraint becomes tighter, causing asset prices to fall further.

Tradable good firms

In the Home economy, there is measure one of monopolistically competitive firms that produce tradable goods. Each firm, indexed by $j \in [0,1]$, produces a different variety of the tradable good using the technology

$$Y_t(j) = (H_t(j))^{\gamma} (K_t(j))^{1-\gamma}$$

with $\gamma \in (0,1)$. Firms rent labor from households at the wage rate $(1-\tau_t^w) W_t$ and physical capital from entrepreneur at the rental rate $(1-\tau_t^q) Q_t$, where τ_t^w and τ_t^q are wage and capital subsidies, respectively.[3] Their cost minimization problem yields the following (aggregate) input demands

$$H_t = \gamma \frac{MC_t Y_t}{(1-\tau_t^w) W_t}$$
$$K_{t-1} = (1-\gamma) \frac{MC_t Y_t}{(1-\tau_t^q) Q_t}$$

where the real marginal cost of production is

$$MC_t = \frac{1}{A_t} \left(\frac{1-\tau_t^w}{\gamma} W_t \right)^{\gamma} \left(\frac{1-\tau_t^q}{1-\gamma} Q_t \right)^{1-\gamma}.$$

Producers sell their varieties to domestic retailers which aggregate them according to a constant elasticity of substitution technology $Y_{H,t} \equiv \left[\int_0^1 Y_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$. Retailers sell final output to consumers, capital producers and exporters in perfectly competitive markets. Their cost minimization problems give rise to the demand schedule $Y_t(j) = (P_{H,t}(j)/P_{H,t})^{-\epsilon} Y_t$, where $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$ is the domestic price index. Each exporter transforms the domestic good into different varieties and sells them to foreign retailers which aggregate them according to $Y_{H,t}^* \equiv \left[\int_0^1 Y_{H,t}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$.

In line with the empirical evidence (Gopinath et al. (2020)), we assume a dominant currency pricing paradigm. Producers set their prices in domestic currency while exporters set their prices in foreign currency. To simplify the analysis, we assume that prices are rigid at time 0 and 1 while they are fully flexible at time 2. Let P_H and P_H^* be firms' prices

[3] We assume that these subsidies are finances through taxes levied lump-sum on firms and we will set them to simplify some of the algebra. Specifically, the wage subsidy will be set to remove the wealth effect on labor supply while the capital subsidy will be set to stabilize the expected rental rate of capital. See Appendix A for details. Removing these assumptions would not alter the main results of the paper.

at the beginning of time zero. Then, $P_{H,0} = P_{H,1} = P_H$ and $P^*_{H,0} = P^*_{H,1} = P^*_H$, while $P_{H,2} = \mathcal{M}MC_2$ and $P^*_{H,2} = \mathcal{M}MC_2/E_2$, where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ is the the firms' desired markup. Aggregate profits are $\Omega^y_t \equiv P_{H,t}Y_t - Q_tK_{t-1} - W_tH_t + E_tP^*_{H,t}Y^*_{H,t} - P_{H,t}Y^*_{H,t}$, where we used the market clearing conditions $\int_0^1 H_t(j)\,dj = H_t$ and $\int_0^1 K_t(j)\,dj = K_{t-1}$.

Capital good firms

In the Home economy, there is also a unit mass of homogeneous firms that produce physical capital. Capital good firms combine domestic and imported goods to produce new capital and rent it to tradable good producers at rate Q_t. Capital production is subject to adjustment costs. Producing I_t new units of capital costs $I_t\left[1 + \frac{\iota}{2}\left(\frac{I_t}{I} - 1\right)^2\right]$ units of the domestic consumption basket, where I is the steady-state level of investment. To finance production, capital producers must borrow from banks. For each new unit of capital produced, they issue a state-contingent claim to the future stream of earnings from the unit, as described above, which is then sold to banks at price P^l_t. Capital producers maximize the present discounted value of their profits for the households. Their problem is

$$\max_{S_{t+j}} \sum_{j=0}^{\infty} \mathbb{E}_0 \left[\Lambda_{t|t+j}\left\{P^l_{t+j}I_{t+j} - I_{t+j} - \frac{\iota}{2}\left(\frac{I_{t+j}}{I} - a\right)^2 I_{t+j}\right\}\right].$$

The first order condition of the problem yields

$$P^l_t = 1 + \frac{\iota}{2}\left(\frac{3I_t}{I} - 1\right)\left(\frac{I_t}{I} - 1\right).$$

Aggregate profits of entrepreneurs are $\Omega^k_t \equiv P^l_t I_t - \left[1 + \frac{\iota}{2}\left(\frac{I_t}{I} - 1\right)^2\right] I_t$, while physical capital evolves according to the law of motion

$$K_t = I_t + (1-\delta)\,K_{t-1}.$$

Foreign investors

The country can attract foreign capital by selling government bonds internationally. Foreign investors demand for Home government bonds gives rise to the following uncovered interest rate parity condition

$$\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\frac{R^b_{t+1}}{\Pi_{t+1}}\frac{e^{-\tau^b_t}}{R^*_t\eta^*_t}\right] = 1$$

where τ_t^b is a tax on capital inflows. The return demanded by foreign investors to hold Home government bonds is determined by the foreign interest rate, R_t^*, and an external premium, η_t^*. The foreign interest rate is assumed to be stochastic and follows $R_t^* = e^{\varepsilon_t^{r^*}}/\beta$, where $\varepsilon_t^{r^*}$ is a mean-zero shock. The external premium is given by

$$\eta_t^* = \left[e^{\varepsilon_t^\kappa} + \varpi \left(e^{\frac{P_t^b B_t^* - P^b B^*}{P_H Y_H}} - 1 \right) \right] e^{(\varsigma + \zeta_b)\mu_t}.$$

We assume that the external premium paid by the Home economy is determined by three distinct elements. The first element, $e^{\varepsilon_t^\kappa}$, is a stochastic component which captures fluctuations in the premium which are exogenous to the Home country. This shock is a proxy for changes in global financial conditions induced by shifts in foreign investors' risk sentiment. The second term, $\varpi \left(e^{\frac{P_t^b B_t^* - P^b B^*}{P_H Y_H}} - 1 \right)$, is a component related to capital flows, where $\varpi \geq 0$ captures the depth of the foreign exchange market as in Gabaix and Maggiori (2015). When $\varpi = 0$, the foreign exchange market is "deep" and capital flows have no impact on the value of the domestic currency. When $\varpi > 0$ the foreign exchange market is "shallow" and the relative demand for domestic and foreign currency generated by the foreign investors have a material impact on the exchange rate. Finally, $e^{(\varsigma + \zeta_b)\mu_t}$ is an endogenous component that is associated with domestic financial conditions. This term captures the nonlinear effects induced by a fall in foreign investors confidence, or panics, when the country experiences a financial crisis.

Government sector

The public sector is composed of a central bank and a fiscal authority. The central bank has multiple tools at its disposal. First, it controls the risk-free rate of the economy by setting the nominal interest rate on reserves, R_t. This interest rate is the marginal rate at which banks can invest and determines the deposit rate as well as the borrowing rates in the economy. Second, the central bank can engage in balance sheet operations in local and foreign currency. By issuing reserve deposits to domestic banks, the central bank can fund the purchase of government bonds, B_t^c, and foreign reserves $X_t^\$$. Without loss of generality, we assume that the central bank operates without capital and turns over to the households any profits generated by its portfolio of assets. Therefore, the balance sheet of the central bank is

$$P_t^b B_t^c + E_t X_t^\$ = S_t.$$

The fiscal authority finances interest payments from its stock of long-term debt by taxing households and, through capital controls, foreign investors. Let B_t^g denote the outstanding stock of government debt. The budget constraint of the government is

$$P_t^b B_t^g = \frac{R_t^b}{\Pi_t} P_{t-1}^b B_{t-1}^g - T_t - \tau_t^b P_t^b B_t^* + G_t.$$

where $G_t = e^{P_H Y \varepsilon_t^g} - 1$ and ε_t^g is a mean-zero fiscal shock. To close the model, we assume that the tax policy follows the simple fiscal rule

$$T_t = \left(\frac{1}{\beta} - \varrho\right)\left(P^b B_{t-1}^g - P^b B^g\right) - \tau_t^b P_t^b B_t^*$$

where ϱ captures the response of fiscal policy to the deviation of public debt from its steady state level, with a low value meaning a strongly debt-stabilizing conduct of fiscal policy.

Equilibrium

Goods market clearing requires $Y_t = Y_{H,t} + Y_{H,t}^*$, where

$$Y_{H,t} = (1-\alpha)\left(P_{H,t}\right)^{-1}\left[C_t + I_t + \frac{\iota}{2}\left(\frac{I_t}{I} - 1\right)^2 I_t + G_t\right]$$
$$Y_{H,t}^* = \alpha^*\left(P_{H,t}^*\right)^{-1}\left(\beta R_t^*\right)^{-\chi} Y^*$$

with χ denoting the foreign elasticity of intertemporal substitution.

Market clearing in the labor and capital markets require $\int_0^1 H_t(j)\, dj = H_t$ and $\int_0^1 K_t(j)\, dj = K_{t-1}$, while market clearing in the asset markets require

$$L_t = K_t$$
$$B_t^g = B_t + B_t^c + B_t^*$$

Finally, the Home country aggregate budget constraint is

$$e^{\tau_t^b} P_t^b B_t^* = \frac{R_t^b}{\Pi_t} P_{t-1}^b B_{t-1}^* + \frac{\alpha}{1-\alpha} P_{H,t} Y_t + X_t - X_{t-1}\frac{E_t}{E_{t-1}} R_{t-1}^*$$
$$- \left(1 + \frac{\alpha}{1-\alpha}\frac{P_{H,t}}{P_{H,t}^* E_t}\right)\alpha^* Y^* \left(\beta R_t^*\right)^{-\chi} E_t$$

where $X_t \equiv E_t X_t^{\$}$ is the value of foreign reserves in units of the domestic consumption basket.

3 Optimal policies

Our objective is to characterize optimal monetary policy in a crisis, at time 1, and to understand how the risk of a large capital flow reversal affects the conduct of monetary policy during normal times, at time 0. We will highlight the trade-offs faced by the central bank and analyze how additional policy tools such as balance sheet policies, both in domestic and foreign currency, macroprudential policies, and capital flow management policies can mitigate these trade-offs.

To make the problem of the Home policymaker tractable while preserving the nonlinearities introduced by the occasionally binding leverage constraint, we transform the model into a linear Markov-switching one. We assume that the economy can be in two regimes: one in which the leverage constraint is not binding and one in which the leverage constraint is binding. We then log-linearize the equations that describe the equilibrium of the model in the two regimes and assume that the global financial condition shock at time 1 is sufficiently large to move the equilibrium from the first to the second regime.[4] This structure, coupled with a quadratic objective function for the planner, delivers a linear-quadratic optimization problem which yields closed-form linear policies.

The log-linear equilibrium conditions can be summarized in a system of ten difference equations.The equilibrium equations can be simplified and reduced to ten (see Appendix A for details):

$$
\begin{aligned}
y_t &= (1-\alpha)\left(1-\tilde{i}-\tilde{nx}\right)c_t + (1-\alpha)\,\tilde{i}i_t + \alpha\left(1-\tilde{nx}\right)e_t + (1-\alpha)\,\varepsilon_t^g \\
&\quad - \chi\left(\alpha+\tilde{nx}-\alpha\tilde{nx}\right)\varepsilon_t^{r^*} \\
c_t &= \mathbb{E}_t c_{t+1} - r_t + \mathbb{E}_t p_{t+1} - p_t + \varepsilon_t^c \\
\iota i_t &= \iota\beta\,(1-\delta)\,\mathbb{E}_t i_{t+1} - r_t - \tau_t^l + \mathbb{E}_t p_{t+1} - p_t - \zeta_l \mu_t \\
p_t &= \frac{\alpha}{1-\alpha} e_t \\
k_t &= \delta i_t + (1-\delta)\,k_{t-1} \\
p_t^l &= \beta\,(1-\delta)\,\mathbb{E}_t p_{t+1}^l - r_t - \tau_t^l + \mathbb{E}_t p_{t+1} - p_t - \zeta_l \mu_t \\
p_t^b &= \beta\mathbb{E}_t p_{t+1}^b - r_t + \mathbb{E}_t p_{t+1} - p_t - \zeta_b \mu_t \\
e_t &= \mathbb{E}_t e_{t+1} - r_t + \mathbb{E}_t p_{t+1} - p_t + \varpi\left(\tilde{nb}_t^* + \tilde{x}_t\right) + \tau_t^b + \varepsilon_t^{r^*} + \varsigma\mu_t + \kappa_t \\
\tilde{nb}_t^* &= \frac{1}{\beta}\tilde{nb}_{t-1}^* - \frac{\beta\tilde{nx}}{1-\beta}\tau_t^b + \frac{\alpha}{1-\alpha}y_t - \frac{\alpha+(1-2\alpha)\,\tilde{nx}}{1-\alpha}e_t + \chi\frac{\alpha+\tilde{nx}\,(1-\alpha)}{1-\alpha}\varepsilon_t^{r^*} \\
\tilde{b}_t^g &= \varrho\tilde{b}_{t-1}^g + \varepsilon_t^g
\end{aligned}
$$

[4]In both regimes we approximate the equations around the unconstrained steady state of the model.

where $nb_t^* \equiv b_t^* - x_t$ denotes net foreign debt and variables with a tilde denote percentage of steady-state GDP while variables without a time index denote steady-state values. So, for example, $\tilde{i}$ denotes steady-state investment-over-GDP, while $\tilde{nx}$ denotes steady-state net exports-over-GDP. If the constraint is not binding, that is the model is in the first regime, then $\mu_t = 0$. If the constraint is binding, that is if the model is in the second regime, then the Lagrange multiplier is given by

$$\mu_t = -\bar{\mu} - (\gamma_k + \delta\omega_k)\, p_t^l + \omega_k k_t - \gamma_b p_t^b + \omega_b \left(\tilde{b}_t^g - \tilde{nb}_t^* - \tilde{x}_t - \tilde{b}_t^c\right).$$

The parameter $-\bar{\mu} \leq 0$ is the "shadow" value of the Lagrange multiplier in steady state. Since we are log-linearizing around a steady state in which the leverage constraint is not binding, this value must be weakly negative. Hence, $\bar{\mu}$ measures the steady-state distance of the banks to the leverage constraint and is a proxy for market depth. The parameters $\gamma_k \equiv (\sigma - \zeta_l)(1-\delta)\frac{P^l K}{N}$ and $\gamma_b \equiv (\sigma - \zeta_b)\frac{PB}{N}$ summarize the effect of loan and bond prices on the constraint. An increase in asset prices has two opposite effects on the leverage constraint. On the one hand, it increase banks' net worth. On the other, it increases the value of the demand for credit. We assume that $\sigma > \max\{\zeta_l, \zeta_b\}$ such the first effect dominates and an increase in asset prices relaxes the leverage constraints. The parameters $\omega_k \equiv \zeta_l \frac{P^l K}{N}$ and $\omega_b \equiv \zeta_b \frac{P_H Y}{N}$ capture the effect of loan and bond demand on the leverage constraint. An increase in the demand for loans, or a reduction in foreign investors' demand for bonds, increases banks leverage and tightens the constraint. To simplify the equations, in what follows we set $\iota = 1$ and $\varpi = 0$ (except when considering FXI, as explained below).

Let $\mathbf{z}_t$ be a vector of endogenous variables, $\mathbf{q}_t$ a vector of policy variables and $\boldsymbol{\varepsilon}_t$ a vector of iid mean-zero shocks defined as follows

$$\mathbf{z}_t \equiv \begin{bmatrix} y_t & i_t & p_t & c_t & k_t & p_t^l & p_t^b & e_t & \tilde{b}_t^* & \tilde{b}_t^g \end{bmatrix}^\top$$
$$\mathbf{q}_t \equiv \begin{bmatrix} r_t & \tilde{b}_t^c & \tilde{x}_t & \tau_t^l & \tau_t^b \end{bmatrix}^\top$$
$$\boldsymbol{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_t^\kappa & \varepsilon_t^{r^*} & \varepsilon_t^g \end{bmatrix}^\top$$

Then the equilibrium of the model can be written as

$$\begin{aligned} \mathbf{z}_t = {} & \mathbf{1}_\mu \bar{\mathcal{A}} + \left(\mathcal{B} + \mathbf{1}_\mu \bar{\mathcal{B}}\right) \mathbf{z}_{t-1} + \left(\mathcal{C} + \mathbf{1}_\mu \bar{\mathcal{C}}\right) \mathbb{E}_t \mathbf{z}_{t+1} \\ & + \left(\mathcal{D} + \mathbf{1}_\mu \bar{\mathcal{D}}\right) \mathbf{q}_{t-1} + \left(\mathcal{E} + \mathbf{1}_\mu \bar{\mathcal{E}}\right) \mathbf{q}_{t-1} + \left(\mathcal{F} + \mathbf{1}_\mu \bar{\mathcal{F}}\right) \boldsymbol{\varepsilon}_t \end{aligned} \tag{3.1}$$

where $\mathbf{1}_\mu$ is an indicator function that takes value one if the banks' leverage constraint is binding, that is $\mu > 0$, and zero if it is not binding, that is $\mu = 0$.

Consistent with the macroeconomic stabilization objective of many central banks, we assume that the objective of the policymaker is to stabilize prices and economic activity. In practice, we assume that the objective of the central bank is to stabilize the quadratic deviations of output and the price level around their stead-state values.[5]

The policy maker's loss function can be written as

$$\mathbb{L} = \frac{1}{2} \sum_{t=0}^{1} \beta^t \left(\mathbf{z}_t^\top \mathcal{Z} \mathbf{z}_t + \mathbf{q}_t^\top \mathcal{Q} \mathbf{q}_t \right) \tag{3.2}$$

where $\mathcal{Z}$ and $\mathcal{Q}$ are diagonal matrices with $\text{diag}\left(\mathcal{Z}\right) = \begin{bmatrix} \phi_y & 0 & 0 & \phi_p & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $\text{diag}\left(\mathcal{Q}\right) = \begin{bmatrix} \phi_r & \phi_x & \phi_s & \phi_k & \phi_b \end{bmatrix}$. The matrix $\mathcal{Q}$ captures the cost of using each tool that we introduce to obtain first order conditions which are linear in the policy instruments. The problem of the policymaker is to choose $\mathbf{q}_0$ and $\mathbf{q}_1$ to minimize 3.2 subject to 3.1 with $\mathbf{z}_2 = 0$, given initial conditions $\mathbf{z}_{-1}$ and $\mathbf{q}_{-1}$, and exogenous shocks $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\varepsilon}_1$. We assume that the policymaker cannot commit to future policies and we focus on the optimal discretionary policy. The details of the derivation of the equilibrium matrices as well as the optimal policies are reported in Appendix B.

To summarize, the Home central bank has five policy tools available:

- Monetary policy: The policymaker controls the short-term risk-free lending rate of the economy r_t.
- Macroprudential policy: The policymaker can tax/subsidize banks' loan issuance, τ_t^l.
- Capital flow management policy: The policymaker can tax/subsidize foreign investors' holding of government bonds, τ_t^b.
- Bond market intervention: The policymaker can purchase/sell long-term government bonds, B_t^c, in exchange for short-term central bank deposits, S_t.
- FX intervention: The policymaker can purchase/sell foreign exchange reserves, $X_t^\$$, in exchange for long-term government bonds, B_t^c.

[5] We choose to use the price level p_t instead of inflation π_t as the objective of the central bank to make the policy problem at time 0 similar to time 1. Our assumption that the economy returns to steady-state in period 2 implies that $p_2 = 0$. Hence, in both cases the problem of the central bank at time 1 is to stabilize p_1. The problem at time 0 is however different when π_t is the objective. Since at time 1 there is upside inflation risk due to a sudden stop, the central bank has an inflationary bias at time 0, as this raises the price level and mechanically reduces inflation at time 1.

We assume that all the tools are under the control of a single authority, the central bank. The tools are therefore fully integrated as their reaction functions are determined based on a common analytical framework and a common objective function.

3.1 Optimal policies in a sudden stop

At time 1 there is the risk that a tightening of external financial conditions, that is an increase in ε_1^κ, causes a domestic financial crisis. In particular, we assume that with probability ρ global financial conditions tighten, that is $\varepsilon_1^\kappa = \overline{\varepsilon}_1^\kappa > 0$, while with probability $1-\rho$ they relax, that is $\varepsilon_1^\kappa = \underline{\varepsilon}_1^\kappa < 0$.[6] A tightening of external financial conditions causes a depreciation of the domestic currency and capital outflows. Nonresidents pull out of the country and reduce their holdings of domestic assets, triggering a reallocation of bonds from foreign investors to domestic banks. Whether this reallocation occurs smoothly or not depends on the capacity of domestic banks to expand their balance sheets to replace the outflow of funds. This, in turn, depends on how close banks are to their leverage constraint.

Consider first the case of a "small" shock. When the capital outflow is smaller than the amount of slack on domestic balance sheets, banks can increase their leverage to absorb the assets sold by foreign investors. Their leverage constraint is not binding and the reallocation of bonds from nonresidents to residents does not affect domestic financial conditions. In such an equilibrium, the effects of a shock to external financial conditions on economic activity and prices are

$$\frac{\partial y_1}{\partial \varepsilon_1^\kappa} = 0$$
$$\frac{\partial p_1}{\partial \varepsilon_1^\kappa} = \alpha$$

. A small shock to global financial conditions raises inflation, through its impact on the exchange rate, but does not affect output. This is due to the fact that both export demand and domestic demand are unaffected by the depreciation of the domestic currency. The former effect is due to our assumption that exports are priced in foreign currency. The latter effect is due to the fact that, while domestic households demand relatively more domestic goods due to the increase in the price of imported goods, their overall consumption falls since the depreciation raises the real interest rate faced by them. The two forces cancel each other since the inter- and intratemporal elasticity of substitution are equal to one. Hence, domestic absorption remains constant.

Now consider a sudden stop with large capital outflows. That is, assume that $\overline{\varepsilon}_1^\kappa$ is

[6] Since shocks have mean zero, this implies that $\underline{\varepsilon}_1^\kappa = -\frac{\rho}{1-\rho}\overline{\varepsilon}_1^\kappa$.

sufficiently large to cause a capital outflow which is greater than the slack on domestic banks' balance sheets.[7] In this case, as banks try to expand their balance sheets to absorb the assets liquidated by foreigners, they hit their leverage constraint. Once the constraint binds, it curbs their demand for government bonds and causes bond prices to fall. The fall in bond prices transmits to other assets in the economy and raises credit spreads and lending rates. This process is further amplified by the impact of fire sales on banks' net worth. Falling asset prices erode their equity and force balance sheets to contract even more. Thus, a large capital outflow reduces the overall amount of credit available in the economy and tightens domestic financial conditions, giving rise to a full-blown financial crisis. As credit spreads rise, investment and economic activity fall. Furthermore, as foreign investors' confidence falls, the crisis amplifies the depreciation of the domestic currency and the impact of the shock on inflation. In this equilibrium, which we denote with a bar over the variables, the effect of the shock on output and inflation is

$$\frac{\partial \bar{y}_1}{\partial \varepsilon_1^\kappa} = -\nu \tilde{i} \zeta_l (1-\alpha) \left[\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\,\omega_b\right]$$
$$\frac{\partial \bar{p}_1}{\partial \varepsilon_1^\kappa} = \alpha + \nu\varsigma\alpha \left[\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\,\omega_b\right]$$

where the $\nu > 1$ captures the strength of the financial amplification mechanism at work when the leverage constraint binds. A high ν magnifies the impact of the capital outflow shock on domestic financial conditions and therefore on output and inflation. The equation of ν is reported in Appendix B.

Capital flow shocks thus have nonlinear effects in the model. While small outflows have little impact on economic activity and inflation, big outflows have a disproportionately larger effect. Furthermore, in response to large outflows, output and inflation tend to move in opposite directions as the former falls and the latter rises.

3.1.1 Monetary policy

The nonlinearity brought about by the occasionally binding leverage constraint gives rise to a nonlinear optimal monetary policy reaction function.

When the domestic banks' leverage constraint is not binding, i.e. if there is no sudden stop, the optimal monetary policy rule is

$$r_1 = (1-\alpha)(1-\tilde{nx})\frac{\phi_y}{\phi_r} y_1 + \alpha\frac{\phi_p}{\phi_r} p_1.$$

[7] Formally, this requires $\bar{\varepsilon}_1^\kappa > \frac{\bar{\mu}}{\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\omega_b}$.

The central bank increases its policy rate in response to an increase in economic activity and to an increase in prices. Thus, the central banks responds to a small capital outflow by increasing its interest rate to stabilize inflation.

In a sudden stop, when the financial constraint binds, the optimal monetary policy rule is

$$\bar{r}_1 = (1-\alpha)\left[1 - \tilde{nx} + \nu\tilde{i}\zeta_l (1-\alpha)(\gamma_b + \gamma_k - \tilde{nx}\omega_b)\right]\frac{\phi_y}{\phi_r}\bar{y}_1$$
$$+ \alpha\left[1 - \nu\varsigma(1-\alpha)(\gamma_b + \gamma_k - \tilde{nx}\omega_b)\right]\frac{\phi_p}{\phi_r}\bar{p}_1$$

The presence of the leverage constraint, captured by the parameter ν, affects the weights on output and inflation in the optimal monetary policy rule. This is because the central bank takes into account how the leverage constraint alters the monetary policy transmission mechanism. The policy rate affects the leverage constraint, and therefore domestic financial conditions, through two channels. First, policy rate changes affect asset prices. A reduction of the policy rates raises asset prices and improves banks' net worth, relaxing the leverage constraint. This price channel, which works through both loan and bond prices, is capture by the term $\gamma_b + \gamma_k$ in the equations above. Second, policy rate changes affect asset demand and supply. A reduction of the policy rate increases loan demand and reduces capital inflows, thereby tightening the leverage constraint. This quantity channel is captured by the term $-\delta\omega_k - \tilde{nx}\omega_b$ in the equations above. If the price channel dominates, then the optimal interest rate policy, compared to the unconstrained equilibrium, responds more aggressively to economic activity and less aggressively to inflation. By contrast, when the quantity channel dominates, the central bank responds less aggressively to economic activity and more aggressively to inflation.

Note that when one of the two channels is particularly strong, the impact of monetary policy on output and inflation can change sign. When the price channel is very strong, that is $\gamma_b + \gamma_k - \tilde{nx}\omega_b > \frac{1}{\nu\varsigma(1-\alpha)}$, an interest rate cut reduces inflation rather than increasing it. By contrast, when the quantity channel is very strong, that is $\gamma_b + \gamma_k - \tilde{nx}\omega_b < -\frac{1-\tilde{nx}}{(1-\alpha)\tilde{i}\nu\zeta_l}$, interest rate cuts are contractionary rather than expansionary.[8] In what follows, we focus on the situation in which monetary policy maintains its conventional effect on both output and inflation. In this case, a large capital outflow gives rise to a monetary policy trade-off, since output and inflation move in opposite directions. An interest rate cut mitigates the recession but raises inflation even further. An interest rate hike reduces inflation but deepens the recession. Whether the optimal policy is to increase or reduce the policy rate, that is to stabilize output or inflation, depends on the central bank's preferences, the impact of the

[8] This case is studied in Cavallino and Sandri (2020).

shock on the two objectives, and the relative strength of the price and quantity channels as just described. Either way, the central bank cannot improve one objective without worsening the other. The key source of this trade-off is the impact of the capital outflow on domestic financial conditions. Therefore, complementary policy tools that can stabilize the financial sector would mitigate the monetary policy trade-off and improve the allocation.

3.1.2 Macroprudential and capital flow management policies

We start by considering macroprudential tools. The optimal macroprudential tax policy is

$$\bar{\tau}_1^l = (1-\alpha)\,\tilde{i}\left[1+\nu\zeta_l\left(\gamma_k+\alpha\tilde{i}\omega_b\right)\right]\frac{\phi_y}{\phi_l}\bar{y}_1 - \alpha\nu\varsigma\left(\gamma_k+\alpha\tilde{i}\omega_b\right)\frac{\phi_p}{\phi_l}\bar{p}_1$$

.

The rule implies that, in response to a large capital outflow, the policymaker would reduce macroprudential taxes. By reducing the cost of borrowing, this mitigates the fall in investment and the recession. This, in turn, sustains the price of physical capital and mitigates the outflow of capital. Both forces tend to relax the leverage constraint and mitigate domestic financial conditions, increasing output and reducing inflation. The optimal macroprudential tax policy has an unambiguous sign and improves simultaneously both policy objectives.

The optimal capital inflow tax policy is

$$\begin{aligned}\bar{\tau}_1^b = {} & \nu(1-\alpha)\,\tilde{i}\zeta_l\left[\alpha\gamma_b+\alpha\gamma_k+\left(\alpha-2\alpha\tilde{nx}+\frac{\tilde{nx}}{1-\beta}\right)\omega_b\right]\frac{\phi_y}{\phi_b}\bar{y}_1 \\ & -\alpha\left\{1+\nu\varsigma\left[\alpha\gamma_b+\alpha\gamma_k+\left(\alpha-2\alpha\tilde{nx}+\frac{\tilde{nx}}{1-\beta}\right)\omega_b\right]\right\}\frac{\phi_p}{\phi_b}\bar{p}_1.\end{aligned}$$

Not surprisingly, when faced with a large outflow of capital, the policymaker would reduce inflow taxes. By increasing the return of domestic bonds for foreigners, this reduces the outflow of capital and improves domestic financial conditions. Furthermore, a reduction of the inflow tax sustains the exchange rate and mitigates the depreciation. This, in turn, reduces inflation and increases domestic asset prices, thereby improving financial conditions further. Also the sign of the optimal capital inflow tax policy has an unambiguous sign and improves both policy objectives at the same time.

3.1.3 Bond market intervention and FX intervention

We start by considering central bank balance sheet operations in domestic currency. Consider an asset purchase policy in which the central bank buys long-term government bonds and finances these purchases by issuing reserve deposits. By purchasing government bonds, the central bank absorbs some of the assets sold by foreign investors. This relaxes the leverage constraint of the banking sector both directly, by reducing the overall amount of bonds that banks must hold in equilibrium, and indirectly, by sustaining asset prices and banks' net worth. This policy stabilizes domestic financial conditions and mitigates the outflow of capital which, in turn, sustains economic activity and reduces inflation.

The optimal asset purchase policy is

$$\bar{b}_1^c = -(1-\alpha)\,\tilde{i}\nu\omega_b\zeta_l\frac{\phi_y}{\phi_s}\bar{y}_1 + \alpha\nu\varsigma\omega_b\frac{\phi_p}{\phi_s}\bar{p}_1.$$

Unlike interest rate policy, the asset purchase policy does not face any trade-off and has an unambiguous sign. During a sudden stop, central bank purchases of government bonds simultaneously improve both policy objectives.

FX intervention can affect the equilibrium of the economy both through its FX leg and its sterilization leg. Through the FX leg, FX intervention can affect the exchange rate and domestic inflation. Through the sterilization leg, that is the purchase/sale of government bonds, FX intervention can affect domestic financial conditions, as just described. However, the effectiveness of both channels depends on the depth of the foreign exchange market. When the foreign exchange market is deep, i.e. if $\varpi = 0$, which would by typical for advanced economies, FX intervention has no impact on the exchange rate since investors would simply absorb the central bank trade without any impact on the exchange rate. Interestingly, also the sterilization leg of the FX intervention would be ineffective in this case. This is because when the central bank sells foreign reserves and purchases government bonds to relax the leverage constraint of the banking sector, gross capital inflows fall by an equivalent amount. This implies that the overall amount of bonds that banks need to absorb is unchanged. In other words, FX intervention does not affect net capital inflows. Hence, when the foreign exchange market is deep, FX intervention does not relax the leverage constraint and is ineffective in stabilizing domestic financial conditions.

When the foreign exchange market is shallow, that is $\varpi > 0$, which would be typical for EMEs, foreign exchange intervention affects the exchange rate and thereby inflation, capital inflows and loan demand. Hence, the central bank has one additional tool at its disposal to mitigate the impact of a capital outflow shock at time 1 and to manage the build-up of risk at time 0. To simplify the algebra and to facilitate the comparison with the results in the previous section, we focus on a "small" value of ϖ. That is, we linearize the weights in the optimal foreign exchange intervention policies around $\varpi = 0$. This captures the first order effect of the intervention on the exchange rate.[9]

[9]The higher order, or second round, effect of the foreign exchange intervention on the exchange rate is the

The optimal FX intervention policy in a sudden stop is

$$\bar{x}_1 = \varpi (1-\alpha) \tilde{i} \nu \zeta_l \left[\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\,\omega_b\right] \frac{\phi_y}{\phi_x} \bar{y}_1 \\ - \varpi\alpha \left\{1 + \nu\varsigma \left[\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\,\omega_b\right]\right\} \frac{\phi_p}{\phi_x} \bar{p}_1.$$

The central bank intervenes in the FX market to sell foreign reserves and mitigate the depreciation of the domestic currency. This policy achieves two objectives. First, it mitigates inflation. Second, it relaxes the leverage constraint and stabilizes domestic financial conditions. While gross capital inflows still fall in response to a foreign exchange sale, they do so less than proportionally since the intervention mitigates the depreciation and the reversal of the trade balance. This implies that, by selling foreign reserves to buy government bonds, the central bank reduces the amount of bonds that banks need to absorb. This relaxes their leverage constraint and stabilizes domestic financial conditions. Furthermore, by mitigating the depreciation, the central bank stabilizes the real interest rate and sustains asset prices, relaxing the leverage constraint further. Hence, both the foreign exchange and the sterilization leg of the foreign exchange intervention policy contributes to stabilize domestic financial conditions.

Notice that both balance sheet operations in domestic and foreign currency involve the purchase of government bonds. In the former case, these purchases are financed by the issuance of reserve deposits. In the latter, they are financed by the sale of foreign reserves. A natural question is which of the two policies is more effective in stabilizing domestic financial conditions. Compared to bond market intervention, foreign exchange intervention is less effective in reducing the bonds held by domestic banks, since it causes a simultaneous decline in capital inflows, but it is more effective in sustaining asset prices due to its impact on inflation and the real interest rate in the economy. The net effect depends on the depth of the foreign exchange market. When

$$\varpi > \frac{\omega_b}{\alpha\gamma_b + \alpha\gamma_k + (\alpha + \tilde{nx} - 2\alpha\tilde{nx})\,\omega_b}$$

the second channel dominates and FX intervention is more effective than bond market intervention.

one that goes through its impact on capital inflows.

3.2 Optimal ex-ante policies

We next solve for the time-0 equilibrium of the model to characterize how the conduct of monetary policy in normal times is affected by the possibility of a large capital outflow at time 1. In doing so, we assume that the economy is in time 0 not in a financial crisis, that is the leverage constraint of the banks is not binding. Notice that while the time-1 shock has mean zero, its impact on the equilibrium variables is nonlinear due to the asymmetry induced by the leverage constraint. This nonlinearity allows us to characterize policies even in a linear-quadratic optimal control setting.

Before looking at optimal policies, it is instructive to understand how the equilibrium at time 0 affects financial conditions at time 1. The balance sheet of the banks at time 0 determines the leverage with which they enter time 1. The lower their leverage, the farther they are from their leverage constraint. This implies that, in the event of a large capital outflow, banks will be able to absorb more assets on their balance sheets before the leverage constraint starts binding. This reduces the tightness of the constraint and lowers credit spreads, mitigating the depth of the financial crisis. Hence, by affecting the distance of the financial sector to the leverage constraint, ex-ante policies can mitigate tail risk.

3.2.1 Monetary policy

We start by looking at optimal monetary policy. With a slight abuse of notation, we denote with μ_r the derivative of μ_1, the Lagrange multiplier on the time-1 leverage constraint, with respect to r_0. Since the leverage constraint is not binding at time 0, there is no possibility of confusion so we can drop the time index. Its equation is

$$\mu_r = -\nu\left(1-\alpha\right)\frac{\beta\delta\left(1-\delta\right)\omega_k + \tilde{nx}\omega_b}{\Upsilon}$$

where $\Upsilon > 0$ and its equation is reported in the Appendix B.

Time-0 interest rate policy affects the tightness of the time-1 leverage constraint through two channels. The first one works through its impact on loan demand, and is captured by the term multiplying ω_k in the equation above. A higher interest rate reduces the demand for loans at time 0 and the leverage of the banks. Since loans are long-term assets, this also reduces the leverage of the banks at time 1, increasing their distance from the leverage constraint. The second channel works through the impact of the interest rate on capital inflows, and is captured by the term multiplying ω_b in the equation above. A higher interest rate increases capital inflows at time zero and reduces the leverage of the banks. Since capital inflows are a persistent variable, this also reduces the leverage of the banks at time

1, increasing their distance from the leverage constraint. Hence, both channels work in the same direction. Tighter ex-ante monetary policy reduces tail risk.

It is interesting to note how monetary policy can have a different impact on current and future (expected) financial conditions. As we have seen in the previous section, a higher interest rate tightens the collateral constraint and worsens domestic financial conditions if its impact on asset prices is stronger than its impact on quantities. In this case, tighter monetary policy tightens the leverage constraint today but relaxes the leverage constraint tomorrow.

The optimal interest rate policy at time zero is

$$\begin{aligned} r_0 = {} & (1-\alpha)\left(1-\tilde{nx}+\tilde{i}\rho\mu_r\Delta\right)\frac{\phi_y}{\phi_r}y_0+\alpha\left(1-\rho\varsigma\mu_r\right)\frac{\phi_p}{\phi_r}p_0 \\ & -\rho\beta\mu_r\frac{\Gamma}{\phi_r}-\rho\beta\mu_r\frac{\phi_\mu}{\phi_r}\left[\omega_k\left(1-\delta\right)k_0+\varrho\omega_b\tilde{b}_0^g\right]+\rho\mu_r\omega_b\frac{\phi_\mu}{\phi_r}\tilde{b}_0^*\end{aligned}$$

The risk of a future large capital outflow changes optimal monetary policy along two dimensions. First, it alters its effectiveness and therefore the optimal weights that the central banks put on the contemporaneous policy objectives. Tail risk increases the responsiveness of inflation to policy rate changes and hence its weight in the optimal interest rate rule. An interest rate hike lowers current inflation not only by reducing aggregate demand, but also by mitigating tail risk and reducing inflation expectations. The impact of tail risk on the responsiveness of output is, however, ambiguous and depends on the sign of $\Delta \equiv \beta(1-\delta)\zeta_l - \alpha\varsigma[1-\beta(1-\delta)]$. On the one hand, lower tail risk increases expected consumption which increases current aggregate demand. On the other hand, it reduces expected inflation which increases the real interest rate and decreases current aggregate demand. If the first channel dominates, that is $\Delta > 0$, monetary policy is less effective in stabilizing output and therefore its weight in the optimal interest rate rule falls.

Most importantly, the risk of a future large capital outflow induces the central bank to deviate from its contemporaneous policy objectives to mitigate tail risk and improve future macroeconomic stability. First of all, the optimal monetary policy stance is unconditionally tighter. This is captured by the constant term in the reaction function. In Appendix B we show that Γ is strictly positive and is negatively correlated with the depth of the domestic financial markets, as proxied by the distance of banks from their leverage constraint in steady state. This implies that the central bank wants to keep the interest rate above its steady-state level in order to reduce tail risk, even though this comes at the cost of lower output and inflation at time zero.

Second, the desire to mitigate tail risk introduces additional variables, or targets, into the optimal interest rate rule. Note that we use the word “targets” to highlight that these variables are not objectives of the policy *per se*, but they affect future policy objectives in certain states of the world. Indeed, $\phi_{\mu} \equiv \nu \left[(1-\alpha)^2 \tilde{i}^2 \zeta_l \phi_y + \alpha^2 \varsigma^2 \phi_p \right]$ measures the marginal effect on time-1 welfare of relaxing the leverage constraint, in the event of a large capital outflow. In setting its policy rate, the central bank does not only take into account its contemporaneous policy objectives, but also financial variables as they affect the tail risk of a future financial crisis in case of a sudden stop. In particular, it takes into account credit demand and capital inflows. When domestic credit rises, either due to an increase in the private or in the public demand for credit, banks leverage increases, raising tail risk. Hence, the central bank reacts by increasing the policy rate.

This finding also highlights the role of the level of public debt as well as of the set up of fiscal policy as important country fundamentals determining vulnerabilities to capital flow reversals. A higher level of public debt in normal times $\tilde{b}_0^g$ implies greater leverage and hence greater tail risk for the economy, inducing a tighter stance of optimal monetary policy to lean against these effects. In the same vein, a higher level of ϱ, reflecting a weakly debt-stabilizing conduct of fiscal policy, increases tail risk and gives rise to tighter ex-ante monetary policy to compensate for this effect.

Similarly, when capital inflows are low, domestic leverage rises, increasing tail risk. Hence, the central bank reacts by increasing the policy rate. Note that the weights on these additional targets, as well as the unconditional policy stance, are increasing in the effectiveness of monetary policy in mitigating tail risk, $|\mu_r|$, and in the probability of a large capital outflow at time 1, ρ.

Due to the presence of these additional target variables in the optimal reaction function, the central bank faces an intertemporal trade-off. It is trading off worse macroeconomic outcomes today to improve resilience in the event that a large capital outflow hits the economy in the future. Complementary policy tools that could mitigate the build up of financial vulnerabilities ex ante would mitigate the intertemporal monetary policy trade-off in period 0 and enhance macro-financial stability.

3.2.2 Macroprudential and capital flow management policies

Macroprudential policies are the natural candidate to deal with tail risk and mitigate the intertemporal trade-off faced by monetary policy. Denote by μ_l the derivative of $\mu_1$with respect to τ_0^l. The impact of the time-0 macroprudential tax on the tightness of the leverage constraint is then given by

$$\mu_l = -\nu \frac{\beta (1-\delta) \delta \omega_k - \alpha \tilde{i} \omega_b}{\Upsilon}.$$

On the one hand, the macroprudential tax reduces the demand for credit and the leverage of the financial sector. This increases its capacity to buffer a large capital outflow. Through this channel, captured by the term multiplying ω_k in the equation above, an increase in the macroprudential tax reduces tail risk. On the other hand, the macroprudential tax increases domestic savings and reduces capital inflows. Through this channel, captured by the term multiplying ω_b in the equation above, an increase in the macroprudential tax increases tail risk. While the first channel is likely to dominate in any reasonable calibration of the model, such that the overall effect of the macroprudential tax on the tightness of the constraint is negative, the presence of second channel reduces its effectiveness in mitigating tail risk. This is particularly true when ω_b is large compared to ω_k, that is when the financial sector has large holdings of government debt. In the following discussion, we assume that $\mu_l \leq 0$.

The optimal macroprudential tax policy is

$$\begin{aligned}\tau_0^l &= (1-\alpha)\,\tilde{i}\,(1+\rho\mu_l\Delta)\,\frac{\phi_y}{\phi_l}y_0 - \alpha\rho\varsigma\mu_l\frac{\phi_p}{\phi_l}p_0 \\ &\quad - \rho\beta\mu_l\frac{\Gamma}{\phi_l} - \rho\beta\mu_l\frac{\phi_\mu}{\phi_l}\left[\omega_k\,(1-\delta)\,k_0 + \varrho\omega_b\tilde{b}_0^g\right] + \rho\omega_b\mu_l\frac{\phi_\mu}{\phi_l}\tilde{b}_0^*.\end{aligned}$$

Not surprisingly, the use of the macroprudential tax is geared toward mitigating tail risk. First, the unconditional tax is positive. Second, the policymaker increases the macroprudential tax when tail risk increases. This occurs when credit demand, either private or public, rises or when capital inflow falls. In setting the optimal tax, the policymaker takes into consideration not only its impact on future risk, but also its effect on current macroeconomic conditions. As we have seen before, through the expectations channel, a reduction in tail risk raises current output and reduces current inflation. However, an increase in the macroprudential tax has also a direct negative effect on contemporaneous output, as it reduces investment and aggregate demand. Hence, when macroprudential policy is used in conjunction with monetary policy to mitigate tail risk, the optimal interest rate is lower. This is because the macroprudential policy reduces the burden of financial variables in the optimal monetary policy rule, and it allows the central bank to be more aggressive in sustaining economic activity.[10]

Turning to capital flow management tools, the impact of the time-0 capital inflow tax on

[10] It is worth noting, however, that it could be the case that the central bank finds it optimal to use the macroprudential tax to stabilize output and monetary policy to deal with tail risk. That is, it could be the case that in response to an increase in tail risk, the central bank raises the interest rate and reduces the loan tax. This situation occurs, for example, when the two channels described above balance each other and μ_l is close to zero. In such a case in fact, the macroprudential policy is ineffective in mitigating tail risk but it retains its impact on economic activity.

the tightness of the time-1 leverage constraint is

$$\mu_b = -\nu \frac{\alpha\beta\delta\left(1-\delta\right)\omega_k - \omega_b\left(\alpha - 2\alpha\tilde{nx} + \frac{\tilde{nx}}{1-\beta}\right)}{\Upsilon}.$$

The sign of this derivative is ambiguous. On the one hand, a positive tax reduces capital inflows and forces banks to increase their leverage, moving them closer to their leverage constraint. Through this channel, captured by the term multiplying ω_b in the equation above, an increase in the capital inflow tax increases tail risk. On the other hand, a tax on capital inflows reduces loan demand and lowers domestic leverage. Through this channel, captured by the term multiplying ω_k in the equation above, a higher capital inflow tax reduces tail risk. In the discussion that follows, we assume that $\mu_b \leq 0$, as this seems to be the empirically relevant case. Compared to the macroprudential tax, the capital inflow tax impacts capital inflows relatively more and loan demand relatively less. In fact, it is easy to show that $\mu_l < \mu_b$. This implies that a capital inflow tax is less effective in mitigating tail risk than a macroprudential tax.[11] This, however, does not imply that capital controls are less useful than macroprudential policies, as we discuss next.

The optimal capital inflow tax policy is

$$\begin{aligned}\tau_0^b &= \rho\left(1-\alpha\right)\tilde{i}\mu_b\Delta\frac{\phi_y}{\phi_b}y_0 - \alpha\frac{\phi_p}{\phi_b}\left(1+\rho\varsigma\mu_b\right)p_0 \\ &\quad - \rho\beta\mu_b\frac{\Gamma}{\phi_b} - \rho\beta\left(1-\delta\right)\omega_k\mu_b\frac{\phi_\mu}{\phi_b}\left[\omega_k\left(1-\delta\right)k_0 + \varrho\omega_b\tilde{b}_0^g\right] + \rho\omega_b\mu_b\frac{\phi_\mu}{\phi_b}\tilde{b}_0^*\end{aligned}$$

As for the macroprudential tax, the use of the capital inflow tax is geared toward mitigating tail risk. First, the unconditional tax is positive. Second, the policymaker increases the capital inflow tax when tail risk increases. This occurs when credit demand, either private or public, rises or when capital inflow falls. In setting the optimal tax, the policymaker takes into consideration not only its impact on future risk, but also its effect on current macroeconomic conditions. As we have seen before, through the expectations channel, a reduction in tail risk raises current output and reduces current inflation. However, an increase in the capital inflow tax has also a direct negative effect on contemporaneous inflation, as it appreciates the domestic currency and reduces import prices. Hence, when capital flow management policy is used in conjunction with monetary policy to mitigate tail risk, the optimal interest rate is lower. This is because the capital inflow tax, like the macroprudential tax, reduces the burden of financial variables in the optimal monetary policy rule.[12]

[11] However, if $\mu_b > 0$ it could be the case that capital inflow subsidies are more effective than macroprudential taxes in mitigating tail risk.

[12] As before, it is worth noting that it could be the case that the central bank finds it optimal to use the

3.2.3 Bond market intervention and FX intervention

Bond market interventions have no impact on the equilibrium variables in time 0. The reason is very simple. Since the leverage constraint is not binding, changing the portfolio of assets held by domestic banks does not have any impact on asset prices nor on equilibrium quantities.

The effectiveness of FX intervention depends also in time 0 on the depth of the foreign exchange market. As we have seen in the previous section, when foreign exchange markets are deep, FX intervention does not affect the exchange rate nor capital flows. When foreign exchange markets are shallow, so in the typical case of an EME, the impact of a time-0 FX purchase on the tightness of the time-1 leverage constraint is

$$\mu_x = -\nu \frac{\alpha\beta\delta\left(1-\delta\right)\omega_k - \omega_b\left(\alpha + \tilde{nx} - 2\alpha\tilde{nx}\right)}{\Upsilon}$$

Notice that the impact of an FX purchase is similar to the impact of a positive capital inflow tax, as they both work by depreciating the exchange rate. On the one hand, the depreciation increases the real interest rate and reduces loan demand. On the other, a weaker exchange rate reduces capital inflows. When the first channel dominates, that is $\mu_x < 0$, a depreciation of the exchange rate reduces domestic leverage and the tightness of the time-1 leverage constraint.

The optimal foreign reserves policy is

$$\begin{aligned} x_0 = {} & \rho\left(1-\alpha\right)\tilde{i}\varpi\mu_x\Delta\frac{\phi_y}{\phi_b}y_0 - \alpha\varpi\frac{\phi_p}{\phi_b}\left(1+\rho\varsigma\mu_x\right)p_0 \\ & - \rho\beta\varpi\mu_x\frac{\Gamma}{\phi_b} - \rho\beta\left(1-\delta\right)\omega_k\varpi\mu_x\frac{\phi_\mu}{\phi_b}\left[\omega_k\left(1-\delta\right)k_0 + \varrho\omega_b\tilde{b}_0^g\right] + \rho\omega_b\varpi\mu_x\frac{\phi_\mu}{\phi_b}\tilde{b}_0^*. \end{aligned}$$

The central bank uses FX intervention at time zero to achieve two objectives. First, to stabilize inflation. When domestic inflation is high, the central bank reduces domestic reserves to appreciate the exchange rate and reduce import prices. Second, to mitigate tail risk. When $\mu_x < 0$, the unconditional level of foreign reserves is positive. The central bank accumulates foreign reserves to depreciate the exchange rate and reduce steady-state tail risk. Furthermore, the central bank intervenes to depreciate the exchange rate further when tail risk increases, that is when credit demand is high or capital inflows are low. Notice that, while in our model the accumulation of foreign reserves is driven by precautionary motives the mechanism is different from canonical models. In our model, reserve accumulation is beneficial because

capital inflow tax to stabilize inflation and monetary policy to deal with tail risk. This situation occurs, for example, when μ_b is close to zero.

of its impact on the state variables of the model, which in turn affect risk, and not because the nonnegativity constraint on reserves is binding in some future states of the world. The addition of that constraint would further incentivize the accumulation of reserves at time zero.

3.3 Shocks

We now study how these policies operate in practice in response to various shocks at time 0. We are particularly interested in understanding how each shock affects tail risk and the intertemporal trade-off faced by the central bank, and to characterize the optimal joint use of the different policies. Although we are able to solve for the allocation in closed form, the expressions quickly become hard to handle. We therefore perform a numerical simulation. This simulation is meant to be an example and should not be thought of as a serious calibration exercise.[13]

Figure 3.1 plots the impulse response functions (IRFs) of output, inflation, credit demand, net capital inflows and real exchange rate to three types of shocks: (i) an increase in investor risk appetite, (ii) a foreign monetary policy easing and (iii) a domestic fiscal expansion. The sign and the size of each shock is chosen such that all shocks have the same impact on tail risk. That is, they all increase μ_1 by the same amount. To isolate the effect of each shock on the economy, we assume that all policy tools are at their steady state. The IRFs are in percent deviations from steady state.

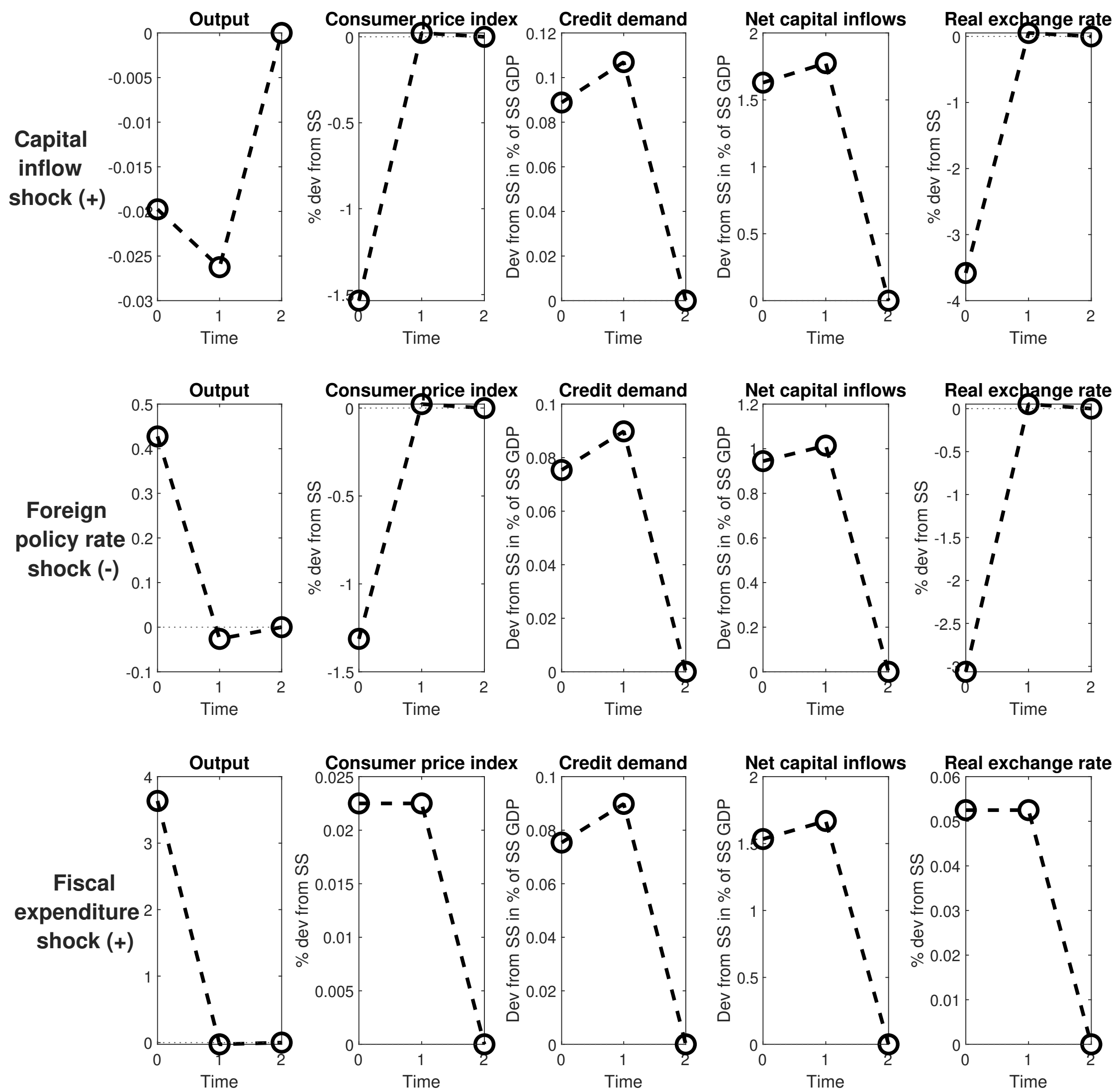
A capital inflow through an increase in foreign investors' risk appetite, shown in the top panels, impacts tail risk through two channels. On the one hand, the increase in the foreign demand for domestic assets generates an inflow of capital that reduces banks' leverage. On the other hand, it increases investment and loan demand which, in turn, increases domestic leverage. Notice that these are the same channels through which the capital inflow tax affects tail risk. Indeed, if $\mu_b \leq 0$ as we have assumed above, then the second channel dominates and a surge in capital inflows increases tail risk. As for monetary policy, it is interesting to note how a capital flow shock impacts differently current and future financial conditions. While a capital inflow today relaxes current financial conditions, it raises tail risk and tightens future financial conditions. Vice versa, a capital outflow today might trigger a financial crisis but, if that does not occur, it reduces the risk of this happening in the future. This is due to the fact that capital flows affect current financial conditions mostly through their impact on asset prices, while they affect future financial conditions mostly through their impact on

[13] We set the parameters of the model in line with a conventional calibration at annual frequency. Specifically, we set $\beta = 0.9$, $\alpha = 0.3$, $\varphi = 0.5$, $\gamma = 0.6$, $\delta = 0.1$, $\tilde{i} = 0.5$, $\tilde{nx} = 0.05$, $\chi = 0.1$, $\varpi = 0.1$, $\varrho = 1$, $\zeta^l = 0.3$, $\zeta^b = 0.15$, $\sigma = 0.99$, $\varsigma = 0.3$, $\rho = 0.5$ and $\bar{\varepsilon}_1^\kappa = 0.1$.

asset quantities, i.e. leverage.

Consider now an expansionary foreign monetary policy shock (centre panels). A reduction in the foreign interest rate causes an inflow of capital in the domestic economy which, similarly to an increase in foreign investors' risk appetite, raises tail risk and reduces inflation. However, an expansionary foreign monetary policy shock also increases export demand and boosts economic activity.

Figure 3.1: Time-0 Shocks: Impulse Response Functions



A domestic fiscal expansion (bottom panels) boosts output and prices by expanding aggregate demand. At the same time, it attracts capital inflows and increases credit demand. Thus, a domestic fiscal expansion increases tail risks in a way similar to the shocks to external financial conditions considered above. This finding mirrors the result of the previous subsection that a higher level of public debt raises tail risk by boosting leverage.

As the next step, we consider the impact of the same shocks for different combinations of policies. We assume that policies are determined in line with the optimal reaction functions described in the previous subsection. The effects of the different policies are illustrated by plotting the IRFs in deviation from the no-policy allocation.

A surge in capital inflows through an increase in investor risk appetite worsens the intertemporal trade-off faced by the domestic central bank. While tail risk rises, both output and inflation fall. Figure 3.2 plots the IRFs, in deviation from the no-policy allocation, under different policy scenarios. The optimal interest rate policy is expansionary, as the central bank tries to lift output and inflation. However, by doing so, the central bank raises credit demand and increases tail risk even further, amplifying the negative impact of capital inflows on financial stability. Complementary policies can help. In response to a surge in capital inflows, the policymaker can increase macroprudential and/or capital inflow taxes to stabilize credit demand and capital flows. When macroprudential policy is in place, the optimal monetary stance is more expansionary since the central bank must counteract the contractionary impact of the macoprudential tax. On the contrary, when capital flow management measures are in place the optimal monetary policy stance is less expansionary since the capital inflow tax depreciates the domestic currency and mitigates the fall in inflation. FX purchases can also be used to mitigate financial stability risk and to sustain the exchange rate. However, they might be less effective than a capital inflow tax.

When capital inflows surge due to a loosening in foreign monetary policy (Figure 3.3), the optimal monetary policy stance depends on the combination of tools deployed. When interest rate policy is combined with macroprudential policy, the latter is tighter, to reduce output and mitigate financial stability risks, while the former is looser, to sustain inflation. On the contrary, when combined with capital flow management tools, the optimal monetary policy stance is tighter, since the capital inflow tax raises inflation, allowing the central bank to focus on stabilizing economic activity. FX purchases have again similar effects as a capital flow tax, with a somewhat lower effectiveness.

Finally, in the case of a domestic fiscal expansion (Figure 3.4), optimal monetary policy is also tightened to dampen current period output and price increases as well tail risk from higher capital inflows and credit demand. Macroprudential and capital flow policies as well as FX intervention can reinforce the effect of monetary policy tightening on future tail risk.

Figure 3.2: Time-0 Policies: Capital Flow Shock (+)

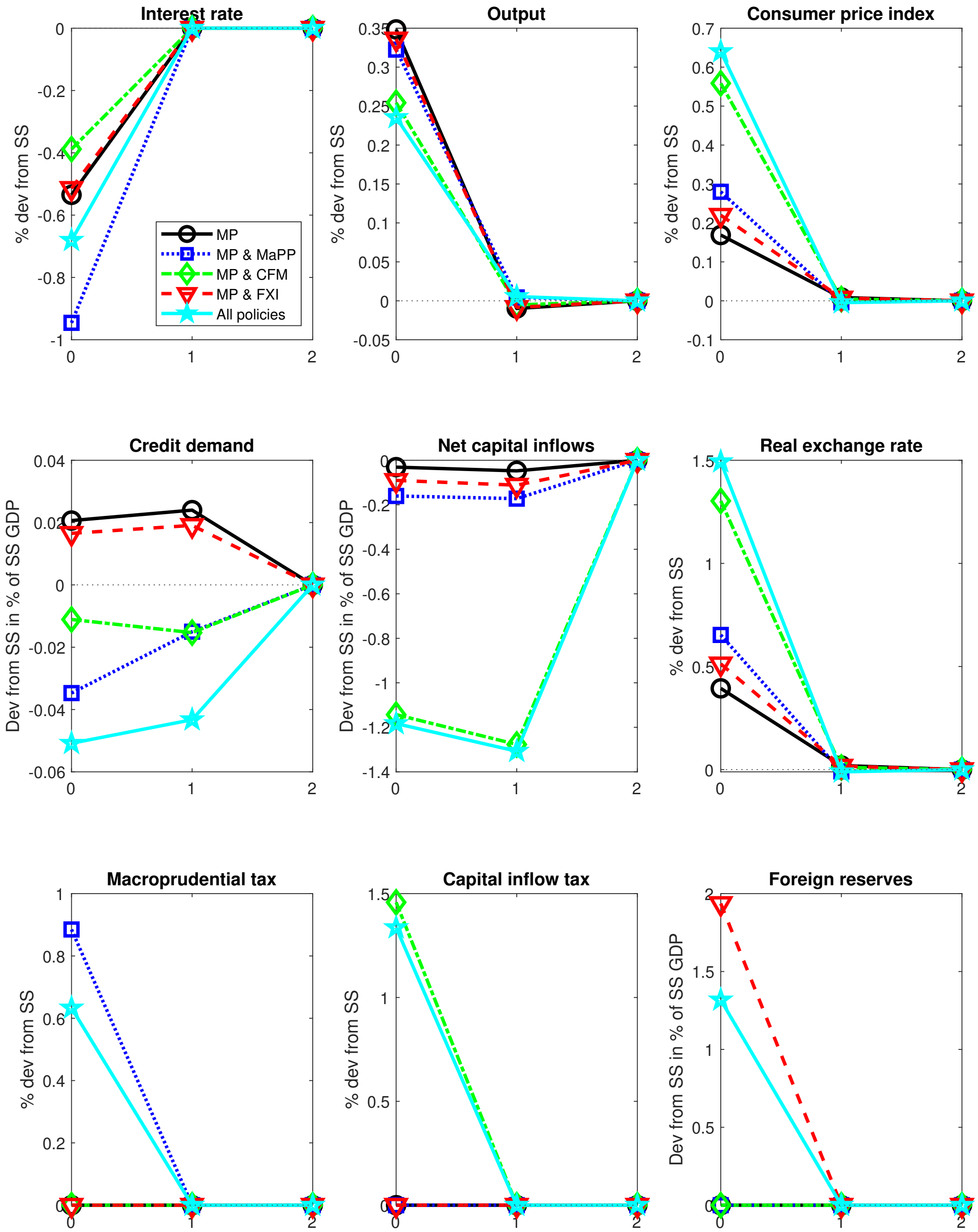


Figure 3.3: Time-0 Policies: Foreign Monetary Policy Shock (-)

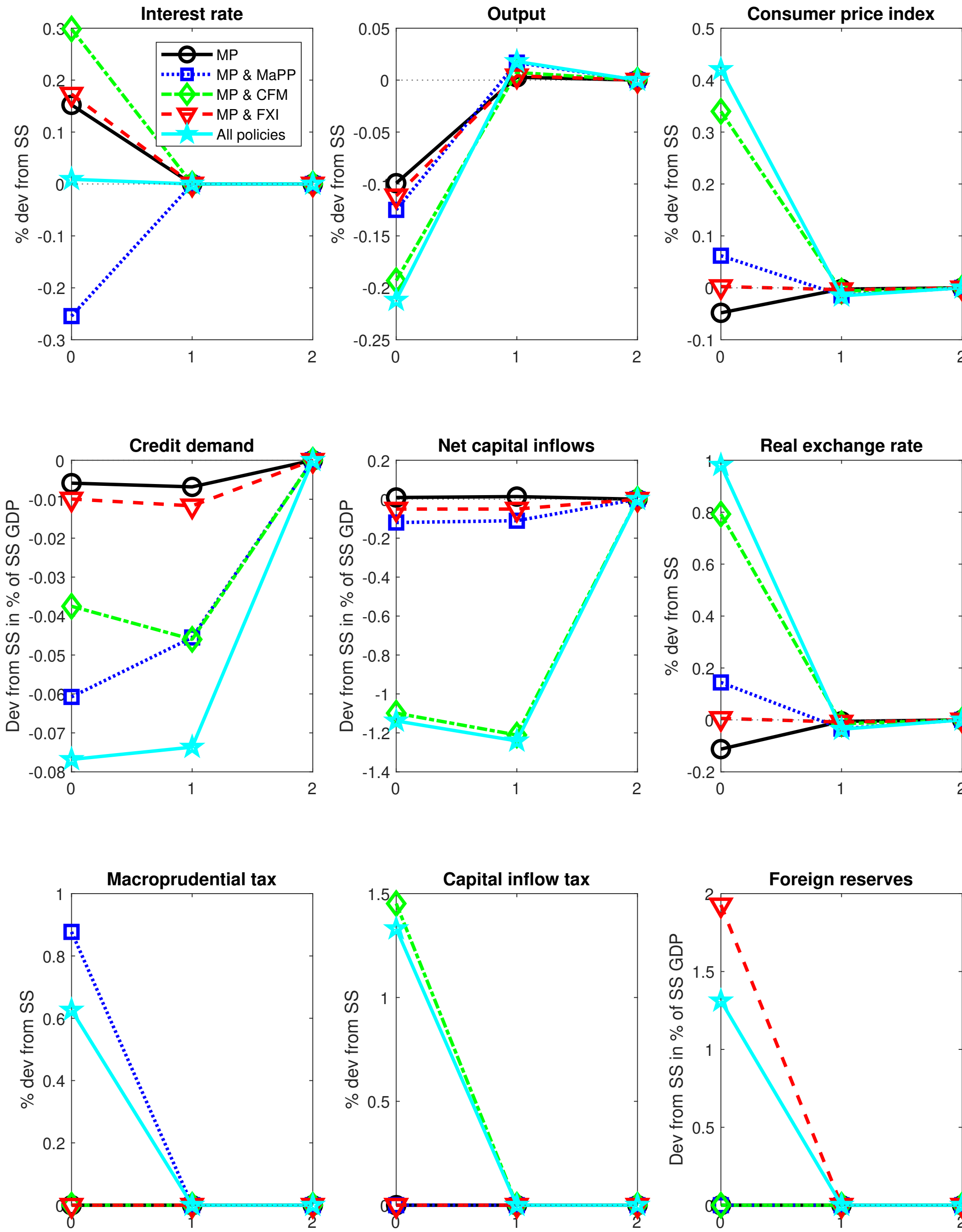
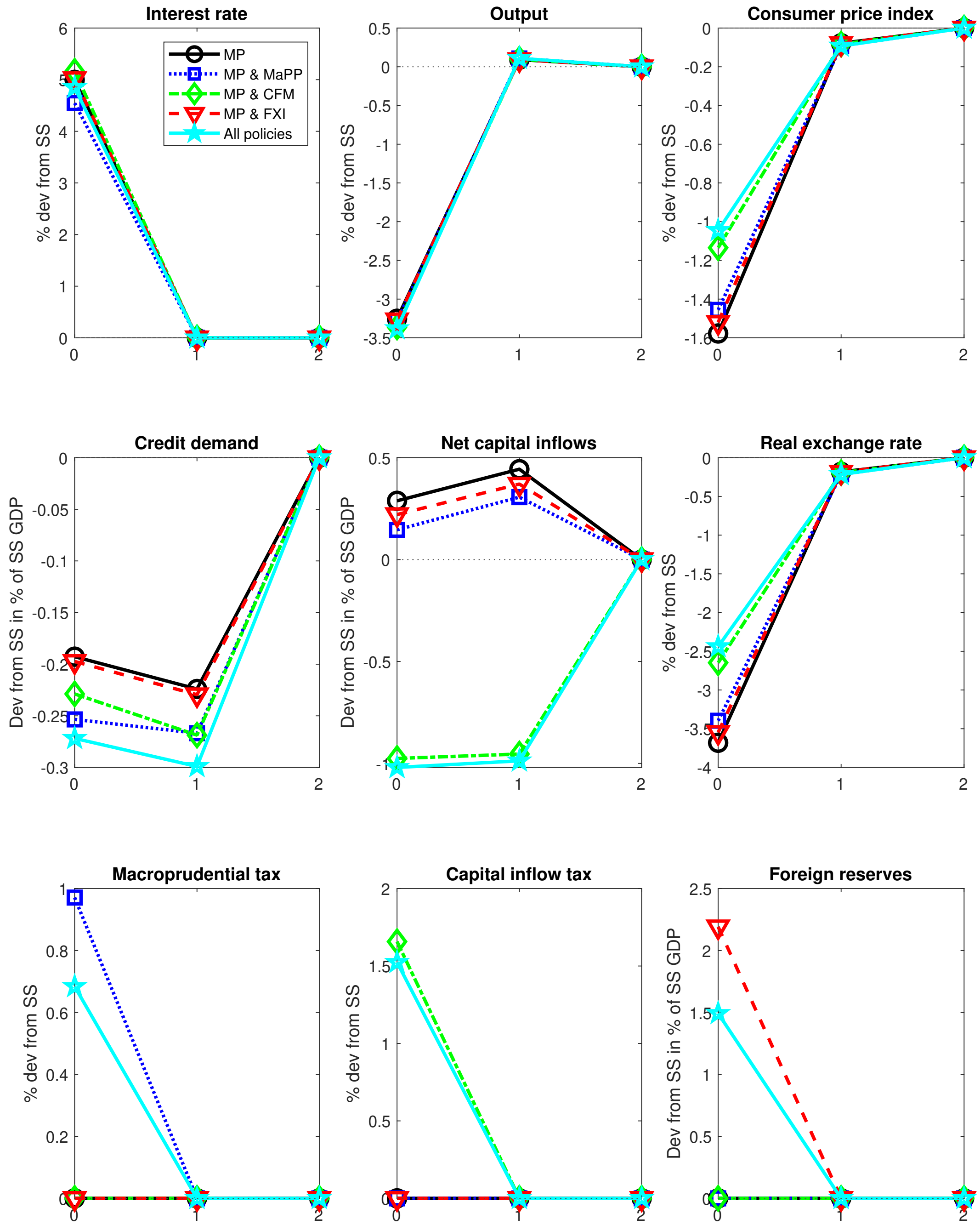


Figure 3.4: Time-0 Policies: Domestic Fiscal Expenditure Shock (+)



A key result cutting across all simulations is that the joint activation of all the tools (turquoise lines) reduces the burden on monetary policy to address tail risk to financial stability. It also reduces the intensity of the use of the other tools, as multiple tools share the burden of addressing the risks to stability and monetary policy trade-offs. Moreover, the joint use of the instruments enhances macro-financial stability. It achieves a greater stability of output and prices in period 0 as well as lower tail risk in period 1 by stabilizing capital inflows and credit demand.

4 Conclusion

We have laid out a small open economy model incorporating key features of EME economic and financial structure, featuring in particular shallow financial markets that give rise to occasionally binding leverage constraints. As a consequence of the latter, capital flows affect EMEs nonlinearly. Trade effects dominate when financial constraints are not binding while financial effects prevail when they do. The model features an intratemporal monetary policy trade-off when there is a sudden stop as output declines while inflation rises. At the same time, there is an intertemporal trade-off in normal times as the risk of a future sudden stop forces the central bank to factor financial stability considerations into its policy conduct. FX intervention and macroprudential and capital flow management measures help to improve both trade-offs. Bond market interventions can further alleviate constraints in a sudden stop.

Our model captures several key features of a macro-financial stability framework (MFSF) as outlined in BIS (2022). It incorporates the broader nature of global macro-financial interlinkages and channels of financial risk-taking beyond borrower currency mismatches in global financial spillovers. The concept of risk, particularly the risk associated with capital flow swings, plays a prominent role in the model, making the case for preventive policies that limit the build up of vulnerabilities ex ante. The model also captures key elements of an MFSF as well as their interactions. The integration of the tools is achieved through the assumption that all tools are under the control of a single authority, the central bank. The different policies are set based on a common analytical framework and a common objective function. As as consequence, the stance of the other policies is implicitly taken into account in the calibration of each individual policy.

That said, important conceptual and practical challenges remain. On the conceptual side, the development of a full quantitative analytical framework that could be used for the calibration of the various policies is still in progress (Cavallino et al. (2022)). On the practical side, there are important constraints for the deployment and integration of the various tools in a holistic framework. The different temporal dimensions of the various policies limit the extent to which synergies across instruments can be realized (Borio and Disyatat (2021)). At the same time, control over the different instruments is generally dispersed across different authorities. Designing appropriate coordinating mechanisms with strong institutional safeguards that respect the assignment of different responsibilities remains a challenge. Finally, the activation and calibration of the different policy tools will in practice need to be guided by a comprehensive cost-benefit analysis, properly taking into account the associated short- and long-run costs.

Appendix

A Equilibrium of the model

A.1 Equilibrium equations

The equations that describe the equilibrium of the model be divided in two blocks. The real block

$$
\begin{aligned}
\Lambda_{t-1|t} &= \beta \frac{e^{-\varepsilon_{t-1}^{c}} C_{t-1}}{C_t} \\
1 &= \mathbb{E}_t \left[\Lambda_{t|t+1} R_t \frac{P_{H,t+1}}{P_{H,t}} \right] \\
P_t &= \frac{1}{P_{H,t}} \\
P_{H,t} &= (E_t)^{-\frac{\alpha}{1-\alpha}} \\
P_t^{l} &= 1 + \frac{\iota}{2} \left(\frac{3 I_t}{I} - 1 \right) \left(\frac{I_t}{I} - 1 \right) \\
K_t &= I_t + (1-\delta) K_{t-1} \\
Y_t &= (1-\alpha)(P_{H,t})^{-1} \left[C_t + I_t + \frac{\iota}{2} \left(\frac{I_t}{I} - 1 \right)^2 I_t + e^{P_H Y \varepsilon_t^{g}} - 1 \right] + \alpha^* \left(e^{\varepsilon_t^{r^*}} \right)^{-\chi} \frac{E Y^*}{P_H} \\
1 &= \mathbb{E}_t \left[\frac{ \frac{E_t}{E_{t+1}} \frac{R_{t+1}^{b} e^{-\tau_t^{b}}}{\Pi_{t+1}} }{ \frac{e^{\varepsilon_t^{r^*}}}{\beta} \left\{ e^{\varepsilon_t^{\kappa}} + \varpi \left(e^{\frac{P_t^b B_t^* - P^b B^*}{P_H Y_H}} - 1 \right) \right\} e^{(\varsigma + \zeta_b)\mu_t} } \right] \\
e^{\tau_t^{b}} P_t^{b} B_t^{*} &= \frac{R_t^{b}}{\Pi_t} P_{t-1}^{b} B_{t-1}^{*} + \frac{\alpha}{1-\alpha} P_{H,t} Y_t - \left(1 + \frac{\alpha}{1-\alpha} \frac{P_{H,t}}{E_t} \frac{E}{P_H} \right) \alpha^* Y^* (\beta R_t^*)^{-\chi} E_t \\
&\quad + X_t - X_{t-1} \frac{E_t}{E_{t-1}} \frac{e^{\varepsilon_{t-1}^{r^*}}}{\beta}
\end{aligned}
$$

and the financial block

$$
\begin{aligned}
\frac{R_t^l}{\Pi_t} &= \frac{\frac{1-\gamma}{\gamma}\left(Y\right)^{\frac{1+\varphi}{\gamma}}\left(K\right)^{-\frac{1+\varphi-\gamma\varphi}{\gamma}}+\left(1-\delta\right)P_t^l}{P_{t-1}^l} \\
\frac{R_t^b}{\Pi_t} &= \frac{1+P_t^b}{P_{t-1}^b} \\
\zeta_l\mu_t &= \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\Lambda_{t|t+1}\left(\frac{e^{-\tau_t^l}R_{t+1}^l}{\Pi_{t+1}}-R_t\frac{P_{H,t+1}}{P_{H,t}}\right)\right] \\
\zeta_b\mu_t &= \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\Lambda_{t|t+1}\left(\frac{R_{t+1}^b}{\Pi_{t+1}}-R_t\frac{P_{H,t+1}}{P_{H,t}}\right)\right] \\
\left(1-\mu_t\right)V_t &= \mathbb{E}_t\left[\left(1-\sigma+\sigma V_{t+1}\right)\Lambda_{t|t+1}R_t\frac{P_{H,t+1}}{P_{H,t}}\right] \\
N_t &= \sigma\left[\left(\frac{R_t^l}{\Pi_t}-\frac{R_{t-1}}{\Pi_t}\right)P_{t-1}^l L_{t-1}+\left(\frac{R_t^b}{\Pi_t}-\frac{R_{t-1}}{\Pi_t}\right)P_{t-1}^b B_{t-1}\right] \\
P_t^b B_t^g &= \frac{R_t^b}{\Pi_t}P_{t-1}^b B_{t-1}^g-\left(\frac{1}{\beta}-\varrho\right)\left(P^b B_{t-1}^g-P^b B^g\right)+e^{P_H Y\varepsilon_t^g}-1 \\
0 &= \mu_t\left[V_t N_t-\zeta_l P_t^l K_t-\zeta_b P_t^b\left(B_t^g-B_t^*-B_t^c\right)\right] \\
0 &\leq V_t N_t-\zeta_l P_t^l K_t-\zeta_b P_t^b\left(B_t^g-B_t^*-B_t^c\right) \\
0 &\leq \mu_t
\end{aligned}
$$

where we used the fact that $\Pi_{H,t}=1$ and $P_{H,t}^*=\frac{P_H}{E}$ since prices are rigid at $t=0,1$ and flexible at $t=2$. Furthermore, we set the labor and capital subsidies as follows

$$
\begin{aligned}
1-\tau_t^w &= \frac{1}{C_t} \\
1-\tau_t^q &= \frac{Q_t}{Q}
\end{aligned}
$$

where $Q_t=\frac{1-\gamma}{\gamma}\left(Y_t\right)^{\frac{1+\varphi}{\gamma}}\left(K_{t-1}\right)^{-\frac{1+\varphi-\gamma\varphi}{\gamma}}$.

A.2 Log-linearization

We log-linearize (linearize with respect to B_t^g, B_t^*, B_t^c and $X_t^{\$}$) the equilibrium equations around a steady state in which the banks leverage constraint is not binding and $X=B^c=0$ and $\Pi=\Pi_H=1$. The log-linearized real block equations are

$$
\begin{aligned}
\lambda_{t-1|t} &= c_{t-1} - c_t - \varepsilon^c_{t-1} \\
0 &= \mathbb{E}_t \lambda_{t|t+1} + r_t + \mathbb{E}_t p_{H,t+1} - p_{H,t} \\
p_t &= -p_{H,t} \\
p_{H,t} &= -\frac{\alpha}{1-\alpha} e_t \\
p^l_t &= \iota i_t \\
k_t &= \delta i_t + (1-\delta) k_{t-1} \\
y_t &= -(1-\alpha)\frac{C+I}{P_H Y} p_{H,t} + (1-\alpha)\left(\frac{C}{P_H Y} c_t + \frac{I}{P_H Y} i_t + \varepsilon^g_t\right) - \chi \frac{\alpha^* Y^* E}{P_H Y} \varepsilon^{r^*}_t \\
e_t &= \mathbb{E}_t\left[e_{t+1} - r^b_{t+1} + \pi_{t+1}\right] + \tau^b_t + \varepsilon^{r^*}_t + (\varsigma + \zeta_b)\mu_t + \varepsilon^\kappa_t + \varpi \tilde{b}^*_t \\
\tilde{b}^*_t &= \frac{1}{\beta}\tilde{b}^*_{t-1} - \frac{P^b B^*}{P_H Y}\tau^b_t + \frac{\alpha}{1-\alpha}(p_{H,t} + y_t) - \left(e_t + \frac{\alpha}{1-\alpha} p_{H,t} - \frac{\chi}{1-\alpha}\varepsilon^{r^*}_t\right)\frac{\alpha^* Y^* E}{P_H Y} + \tilde{x}_t - \frac{1}{\beta}\tilde{x}_{t-1}
\end{aligned}
$$

$$
Y_t = (1-\alpha)(P_{H,t})^{-1}\left[C_t + I_t + \frac{\iota}{2}\left(\frac{I_t}{I} - 1\right)^2 I_t + P_H Y \varepsilon^g_t\right] + \alpha^*\left(e^{\varepsilon^{r^*}_t}\right)^{-\chi}\frac{EY^*}{P_H}
$$

The log-linearized financial block equations are

$$
\begin{aligned}
r^l_t - \pi_t &= \beta(1-\delta) p^l_t - p^l_{t-1} \\
r^b_t - \pi_t &= \beta p^b_t - p^b_{t-1} \\
\zeta_l \mu_t &= \mathbb{E}_t\left[r^l_{t+1} - \pi_{t+1} - \tau^l_t - r_t - p_{H,t+1} + p_{H,t}\right] \\
\zeta_b \mu_t &= \mathbb{E}_t\left[\left(r^b_{t+1} - \pi_{t+1} - r_t - p_{H,t+1} + p_{H,t}\right)\right] \\
v_t &= \mu_t + \sigma \mathbb{E}_t\left[v_{t+1}\right] \\
n_t &= \sigma(1-\delta)\frac{P^l K}{N} p^l_t + \sigma \frac{P^b B}{N} p^b_t \\
\tilde{b}^g_t &= \varrho \tilde{b}^g_{t-1} + \varepsilon^g_t \\
0 &\leq 1 - \zeta_l \frac{P^l K}{N} - \zeta_b \frac{P^b B}{N} + v_t + n_t - \zeta_l \frac{P^l K}{N}\left(p^l_t + k_t\right) - \zeta_b \frac{P^b B}{N} p^b_t - \zeta_b \frac{P_H Y}{N}\left(\tilde{b}^g_t - \tilde{b}^*_t - \tilde{b}^c_t\right) \\
0 &\leq \mu_t
\end{aligned}
$$

The equilibrium equations can be simplified and reduced to ten

$$
\begin{aligned}
y_t &= (1-\alpha)\left(1-\tilde{i}-\tilde{nx}\right)c_t + (1-\alpha)\tilde{i}i_t + \alpha\left(1-\tilde{nx}\right)e_t + (1-\alpha)\varepsilon_t^g \\
&\quad -\chi\left(\alpha+\tilde{nx}-\alpha\tilde{nx}\right)\varepsilon_t^{r^*} \\
c_t &= \mathbb{E}_t c_{t+1} - r_t + \mathbb{E}_t p_{t+1} - p_t + \varepsilon_t^c \\
\iota i_t &= \iota\beta(1-\delta)\mathbb{E}_t i_{t+1} - r_t - \tau_t^l + \mathbb{E}_t p_{t+1} - p_t - \zeta_l\mu_t \\
p_t &= \frac{\alpha}{1-\alpha}e_t \\
k_t &= \delta i_t + (1-\delta)k_{t-1} \\
p_t^l &= \beta(1-\delta)\mathbb{E}_t p_{t+1}^l - r_t - \tau_t^l + \mathbb{E}_t p_{t+1} - p_t - \zeta_l\mu_t \\
p_t^b &= \beta\mathbb{E}_t p_{t+1}^b - r_t + \mathbb{E}_t p_{t+1} - p_t - \zeta_b\mu_t \\
e_t &= \mathbb{E}_t e_{t+1} - r_t + \mathbb{E}_t p_{t+1} - p_t + \varpi\left(\tilde{nb}_t^* + \tilde{x}_t\right) + \tau_t^b + \varepsilon_t^{r^*} + \varsigma\mu_t + \kappa_t \\
\tilde{nb}_t^* &= \frac{1}{\beta}\tilde{nb}_{t-1}^* - \frac{\beta\tilde{nx}}{1-\beta}\tau_t^b + \frac{\alpha}{1-\alpha}y_t - \frac{\alpha+(1-2\alpha)\tilde{nx}}{1-\alpha}e_t + \chi\frac{\alpha+\tilde{nx}(1-\alpha)}{1-\alpha}\varepsilon_t^{r^*} \\
\tilde{b}_t^g &= \varrho\tilde{b}_{t-1}^g + \varepsilon_t^g
\end{aligned}
$$

where $\tilde{nb}_t^* \equiv \tilde{b}_t^* - \tilde{x}_t$ denotes net foreign debt and we defined $\tilde{i} \equiv \frac{I}{P_H Y}$, $\tilde{nx} \equiv \frac{\alpha^* E Y^* - \alpha(C+I)}{P_H Y}$ and used the steady-state relations

$$
\begin{aligned}
1 &= \frac{C}{P_H Y} + \frac{I}{P_H Y} + \frac{\alpha^* E Y^* - \alpha(C+I)}{P_H Y} \\
0 &= \frac{1-\beta}{\beta}\frac{P^b B^*}{P_H Y} + \frac{\alpha}{1-\alpha} - \frac{1}{1-\alpha}\frac{\alpha^* Y^* E}{P_H Y}
\end{aligned}
$$

Since the leverage constraint can bind only at time 1, then $v_1 = \mu_1$ and

$$
\mu_t = -\bar{\mu} - (\gamma_k + \delta\omega_k)p_t^l + \omega_k k_t - \gamma_b p_t^b + \omega_b\left(\tilde{b}_t^g - \tilde{nb}_t^* - \tilde{x}_t - \tilde{b}_t^c\right)
$$

with

$$
\begin{aligned}
\omega_k &\equiv \zeta_l\frac{P^l K}{N} \\
\gamma_k &\equiv (\sigma-\zeta_l)(1-\delta)\frac{P^l K}{N} \\
\gamma_b &\equiv (\sigma-\zeta_b)\frac{P^b B}{N} \\
\omega_b &\equiv \zeta_b\frac{P_H Y}{N}
\end{aligned}
$$

and $\bar{\mu} \equiv 1 - \zeta_l \frac{P^l K}{N} - \zeta_b \frac{P^b B}{N}$.

B Solution

Let $\mathbf{z}_t$ be a vector of endogenous variables, $\mathbf{q}_t$ a vector of policy variables and $\boldsymbol{\varepsilon}_t$ a vector of iid mean-zero shocks defined as follows

$$\mathbf{z}_t \equiv \left[\begin{array}{cccccccccc} y_t & i_t & p_t & c_t & k_t & p_t^l & p_t^b & e_t & \tilde{b}_t^* & \tilde{b}_t^g \end{array} \right]^\top$$
$$\mathbf{q}_t \equiv \left[\begin{array}{ccccc} r_t & \tilde{b}_t^c & \tilde{x}_t & \tau_t^l & \tau_t^b \end{array} \right]^\top$$
$$\boldsymbol{\varepsilon}_t \equiv \left[\begin{array}{ccc} \varepsilon_t^\kappa & \varepsilon_t^{r^*} & \varepsilon_t^g \end{array} \right]^\top$$

Then equilibrium equations derived above can be written in matrix form

$$\mathcal{K}\mathbf{z}_t = \hat{\mathcal{B}}\mathbf{z}_{t-1} + \hat{\mathcal{C}}\mathbb{E}_t\mathbf{z}_{t+1} + \hat{\mathcal{D}}\mathbf{q}_t + \hat{\mathcal{E}}\boldsymbol{\varepsilon}_t + \hat{\mathcal{F}}\mu_t$$
$$\mu_t = \max\left\{0, \hat{\mathcal{A}} + \hat{\mathcal{H}}\mathbf{z}_t + \hat{\mathcal{I}}\mathbf{q}_t + \hat{\mathcal{J}}\boldsymbol{\varepsilon}_t\right\}$$

or, in a more compact way

$$\mathbf{z}_t = \mathbf{1}_\mu\bar{\mathcal{A}} + \left(\mathcal{B} + \mathbf{1}_\mu\bar{\mathcal{B}}\right)\mathbf{z}_{t-1} + \left(\mathcal{C} + \mathbf{1}_\mu\bar{\mathcal{C}}\right)\mathbb{E}_t\mathbf{z}_{t+1} + \left(\mathcal{D} + \mathbf{1}_\mu\bar{\mathcal{D}}\right)\mathbf{q}_t + \left(\mathcal{E} + \mathbf{1}_\mu\bar{\mathcal{E}}\right)\boldsymbol{\varepsilon}_t$$
$$\mu_t = \mathbf{1}_\mu\nu\left[\hat{\mathcal{A}} + \hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{B}}\mathbf{z}_{t-1} + \hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{C}}\mathbb{E}_t\mathbf{z}_{t+1} + \left(\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{D}} + \hat{\mathcal{I}}\right)\mathbf{q}_t + \left(\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{E}} + \hat{\mathcal{J}}\right)\boldsymbol{\varepsilon}_t\right]$$

where $\mathbf{1}_\mu$ is an indicator function that takes value one if the banks' leverage constraint is binding, that is $\mu > 0$, and zero if it is not binding, that is $\mu = 0$, and

$$\bar{\mathcal{A}} \equiv \nu\mathcal{K}^{-1}\hat{\mathcal{F}}\hat{\mathcal{A}}$$
$$\mathcal{B} \equiv \mathcal{K}^{-1}\hat{\mathcal{B}}$$
$$\bar{\mathcal{B}} \equiv \nu\mathcal{K}^{-1}\hat{\mathcal{F}}\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{B}}$$
$$\mathcal{C} \equiv \mathcal{K}^{-1}\hat{\mathcal{C}}$$
$$\bar{\mathcal{C}} \equiv \nu\mathcal{K}^{-1}\hat{\mathcal{F}}\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{C}}$$
$$\mathcal{D} \equiv \mathcal{K}^{-1}\hat{\mathcal{D}}$$
$$\bar{\mathcal{D}} \equiv \nu\mathcal{K}^{-1}\hat{\mathcal{F}}\left(\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{D}} + \hat{\mathcal{I}}\right)$$
$$\mathcal{E} \equiv \mathcal{K}^{-1}\hat{\mathcal{E}}$$
$$\bar{\mathcal{E}} \equiv \nu\mathcal{K}^{-1}\hat{\mathcal{F}}\left(\hat{\mathcal{H}}\mathcal{K}^{-1}\hat{\mathcal{E}} + \hat{\mathcal{J}}\right)$$

Below we derive the optimal policies in matrix form. The analytical solutions are computed using Wolfram Mathematica. The code is available upon request.

B.1 Optimal policies at time 1

Since the model returns to steady state at $t = 2$, the problem of the central bank at time $t = 1$ is

$$\begin{aligned} &\min \frac{1}{2}\left(\mathbf{z}_1^\top \mathcal{Z} \mathbf{z}_1 + \mathbf{q}_1^\top \mathcal{Q} \mathbf{q}_1\right) \\ &\text{subject to } \mathbf{z}_1 = \mathbf{1}_\mu \bar{\mathcal{A}} + \left(\mathcal{B} + \mathbf{1}_\mu \bar{\mathcal{B}}\right) \mathbf{z}_0 + \left(\mathcal{D} + \mathbf{1}_\mu \bar{\mathcal{D}}\right) \mathbf{q}_1 + \left(\mathcal{E} + \mathbf{1}_\mu \bar{\mathcal{E}}\right) \boldsymbol{\varepsilon}_1 \end{aligned}$$

The first order condition is

$$\begin{aligned} \mathbf{q}_1 &= \mathcal{P}_1 \mathbf{z}_1 \\ \mathcal{P}_1 &= -\left[\mathcal{Z}\left(\mathcal{D} + \mathbf{1}_\mu \bar{\mathcal{D}}\right) \mathcal{Q}^{-1}\right]^\top \end{aligned}$$

and the equilibrium is

$$\begin{aligned} \mathbf{z}_1 &= \mathbf{1}_\mu \mathcal{S}_1 \bar{\mathcal{A}} + \mathcal{S}_1\left(\mathcal{B} + \mathbf{1}_\mu \bar{\mathcal{B}}\right) \mathbf{z}_0 + \mathcal{S}_1\left(\mathcal{E} + \mathbf{1}_\mu \bar{\mathcal{E}}\right) \boldsymbol{\varepsilon}_1 \\ \mathcal{S}_1 &= \left[\mathrm{I} - \left(\mathcal{D} + \mathbf{1}_\mu \bar{\mathcal{D}}\right) \mathcal{P}_1\right]^{-1} \end{aligned}$$

where I is a conformable identity matrix.

The optimal policies in the unconstrained regime are

$$\mathcal{P}_1 = \begin{bmatrix} (1-\alpha)(1-\tilde{nx})\frac{\phi_y}{\phi_r} & \frac{\alpha\phi_p}{\phi_r} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha\varpi\frac{\phi_p}{\phi_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\alpha)\tilde{i}\frac{\phi_y}{\phi_l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha\frac{\phi_p}{\phi_b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

while the optimal policies in the unconstrained regime are

$$\mathcal{P}_1 = \begin{bmatrix} (1-\alpha)\left[1-\tilde{nx}-(1-\alpha)\tilde{i}\nu\zeta_l\left(\gamma_b+\gamma_k-\tilde{nx}\omega_b\right)\right]\frac{\phi_y}{\phi_r} & \alpha\left[1-\nu\varsigma(1-\alpha)\left(\gamma_b+\gamma_k-\tilde{nx}\omega_b\right)\right]\frac{\phi_p}{\phi_r} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\alpha)\tilde{i}\nu\omega_b\zeta_l\frac{\phi_y}{\phi_s} & \alpha\nu\varsigma\omega_b\frac{\phi_p}{\phi_s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\alpha)\tilde{i}\nu\varpi\zeta_l\left[\alpha\gamma_b+\alpha\gamma_k+\omega_b\left(\alpha+\tilde{nx}-2\alpha\tilde{nx}\right)\right]\frac{\phi_y}{\phi_x} & -\alpha\varpi\left\{1+\nu\varsigma\left[\alpha\gamma_b+\alpha\gamma_k+\omega_b\left(\alpha+\tilde{nx}-2\alpha\tilde{nx}\right)\right]\right\}\frac{\phi_p}{\phi_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\alpha)\tilde{i}\left[1+\nu\zeta_l\left(\gamma_k+\alpha\tilde{i}\omega_b\right)\right]\frac{\phi_y}{\phi_l} & -\alpha\nu\varsigma\left(\gamma_k+\alpha\tilde{i}\omega_b\right)\frac{\phi_p}{\phi_l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\alpha)\tilde{i}\nu\zeta_l\left[\alpha\gamma_b+\alpha\gamma_k+\omega_b\left(\alpha-2\alpha\tilde{nx}+\frac{\tilde{nx}}{1-\beta}\right)\right]\frac{\phi_y}{\phi_b} & -\alpha\left\{1+\nu\varsigma\left[\alpha\gamma_b+\alpha\gamma_k+\omega_b\left(\alpha-2\alpha\tilde{nx}+\frac{\tilde{nx}}{1-\beta}\right)\right]\right\}\frac{\phi_p}{\phi_b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where
$$\nu \equiv \frac{1}{1-\gamma_b\left(\zeta_b+\alpha\varsigma\right)-\gamma_k\left(\zeta_k+\alpha\varsigma\right)-\omega_b\left[\alpha\tilde{i}\zeta_k+\varsigma\left(\alpha-2\alpha\tilde{nx}+\tilde{nx}\right)\right]}.$$

B.2 Optimal policies at time 0

To isolate the interaction the risk of a sudden stop and time-0 policies we assume that policies at time 1 are idle. Assume that $\boldsymbol{\varepsilon}_1$ can take two values. With probability ρ the realization of $\boldsymbol{\varepsilon}_1$, denoted with $\bar{\boldsymbol{\varepsilon}}_1$ is such that the banks' leverage constraint is binding. With probability $1-\rho$ the realization is such that the constraint is not binding. Since all shocks have mean zero, then the latter is $-\frac{\rho}{1-\rho}\bar{\boldsymbol{\varepsilon}}_1$. Then we have

$$\mathbb{E}_0\mathbf{z}_1=\rho\bar{\mathcal{A}}+\left(\mathcal{B}+\rho\bar{\mathcal{B}}\right)\mathbf{z}_0+\rho\bar{\mathcal{E}}\bar{\boldsymbol{\varepsilon}}_1$$

Since the leverage constraint is not binding at time $t=0$, then the equilibrium is

$$\mathbf{z}_0=\mathcal{C}\mathbb{E}_0\mathbf{z}_1+\mathcal{D}\mathbf{q}_0+\mathcal{E}\boldsymbol{\varepsilon}_0$$

Therefore
$$\mathbf{z}_0=\mathcal{T}+\mathcal{U}\mathbf{q}_0+\mathcal{V}\boldsymbol{\varepsilon}_0$$

where

$$\begin{aligned}
\mathcal{T} &\equiv \rho\left[\mathrm{I}-\mathcal{C}\left(\mathcal{B}+\rho\bar{\mathcal{B}}\right)\right]^{-1}\mathcal{C}\left(\bar{\mathcal{A}}+\bar{\mathcal{E}}\bar{\boldsymbol{\varepsilon}}_1\right)\\
\mathcal{U} &\equiv \left[\mathrm{I}-\mathcal{C}\left(\mathcal{B}+\rho\bar{\mathcal{B}}\right)\right]^{-1}\mathcal{D}\\
\mathcal{V} &\equiv \left[\mathrm{I}-\mathcal{C}\left(\mathcal{B}+\rho\bar{\mathcal{B}}\right)\right]^{-1}\mathcal{E}
\end{aligned}$$

and I is a conformable identity matrix. The problem of the central bank is

$$\begin{aligned}
&\min\frac{1}{2}\left\{\mathbf{z}_0^\top\mathcal{Z}\mathbf{z}_0+\mathbf{q}_0^\top\mathcal{Q}\mathbf{q}_0+\beta\mathbb{E}_0\left[\mathbf{z}_1^\top\mathcal{Z}\mathbf{z}_1\right]\right\}\\
&\text{subject to } \mathbf{z}_0=\mathcal{T}+\mathcal{U}\mathbf{q}_0+\mathcal{V}\boldsymbol{\varepsilon}_0
\end{aligned}$$

The first order condition is

$$\begin{aligned}
\mathbf{q}_0 &= \bar{\mathcal{P}}_0+\mathcal{P}_0\mathbf{z}_0\\
\bar{\mathcal{P}}_0 &\equiv -\rho\beta\left(\mathcal{Q}^\top\right)^{-1}\left\{\left[\mathcal{Z}\left(\mathcal{B}+\bar{\mathcal{B}}\right)\mathcal{U}\right]^\top\left\{\bar{\mathcal{A}}+\left(\mathcal{E}+\bar{\mathcal{E}}\right)\bar{\boldsymbol{\varepsilon}}_1\right\}-\left(\mathcal{Z}\mathcal{B}\mathcal{U}\right)^\top\mathcal{E}\bar{\boldsymbol{\varepsilon}}_1\right\}\\
\mathcal{P}_0 &\equiv -\left(\mathcal{Q}^\top\right)^{-1}\left\{\left(\mathcal{Z}\mathcal{U}\right)^\top+\beta\rho\left[\mathcal{Z}\left(\mathcal{B}+\bar{\mathcal{B}}\right)\mathcal{U}\right]^\top\left(\mathcal{B}+\bar{\mathcal{B}}\right)+\beta\left(1-\rho\right)\left(\mathcal{Z}\mathcal{B}\mathcal{U}\right)^\top\mathcal{B}\right\}
\end{aligned}$$

and the equilibrium is

$$\begin{aligned}
\mathbf{z}_0 &= \mathcal{S}_0 \left(\mathcal{T} + \mathcal{U}\bar{\mathcal{P}}_0\right) + \mathcal{S}_0 \mathcal{V} \boldsymbol{\varepsilon}_0 \\
\mathcal{S}_0 &= \left(\mathrm{I} - \mathcal{U}\mathcal{P}_0\right)^{-1}
\end{aligned}$$

where I is a conformable identity matrix.

The optimal policies are

$$\bar{\mathcal{P}}_0 = \begin{bmatrix} -\rho\beta\mu_r \frac{\Gamma}{\phi_r} \\ 0 \\ -\rho\beta\mu_x \varpi \frac{\Gamma}{\phi_x} \\ -\rho\beta\mu_l \frac{\Gamma}{\phi_l} \\ -\rho\beta\mu_b \frac{\Gamma}{\phi_b} \end{bmatrix}$$

$$\mathcal{P}_0 = \begin{bmatrix}
(1-\alpha) \frac{\phi_y}{\phi_r} \left(1 - \tilde{nx} + \tilde{i}\rho\Delta\mu_r\right) & \alpha \left(1 - \rho\varsigma\mu_r\right) \frac{\phi_p}{\phi_r} & 0 & -\rho\beta\mu_r (1-\delta)\,\omega_k \frac{\phi_\mu}{\phi_r} & 0 & 0 & 0 & \rho\omega_b\mu_r \frac{\phi_\mu}{\phi_r} & -\rho\beta\varrho\omega_b\mu_r \frac{\phi_\mu}{\phi_r} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\rho\tilde{i}\,(1-\alpha)\,\varpi\mu_x \frac{\phi_y}{\phi_x}\Delta & -\alpha\varpi \left(1 + \rho\varsigma\mu_x\right) \frac{\phi_p}{\phi_x} & 0 & -\rho\beta\varpi\mu_x\omega_k (1-\delta) \frac{\phi_\mu}{\phi_x} & 0 & 0 & 0 & \rho\varpi\omega_b\mu_x \frac{\phi_\mu}{\phi_x} & -\rho\beta\varpi\varrho\omega_b\mu_x \frac{\phi_\mu}{\phi_x} \\
\tilde{i}\,(1-\alpha)\left(1 + \rho\mu_l\Delta\right) \frac{\phi_y}{\phi_l} & -\alpha\rho\varsigma\mu_l \frac{\phi_p}{\phi_l} & 0 & -\rho\beta\mu_l\omega_k (1-\delta) \frac{\phi_\mu}{\phi_l} & 0 & 0 & 0 & \rho\omega_b\mu_l \frac{\phi_\mu}{\phi_l} & -\rho\beta\varrho\omega_b\mu_l \frac{\phi_\mu}{\phi_l} \\
\rho\tilde{i}\,(1-\alpha)\,\mu_b\Delta \frac{\phi_y}{\phi_b} & -\alpha \left(1 + \rho\varsigma\mu_b\right) \frac{\phi_p}{\phi_b} & 0 & -\rho\beta\mu_b (1-\delta)\,\omega_k \frac{\phi_\mu}{\phi_b} & 0 & 0 & 0 & \rho\omega_b\mu_b \frac{\phi_\mu}{\phi_b} & -\rho\beta\varrho\omega_b\mu_b \frac{\phi_\mu}{\phi_b}
\end{bmatrix}$$

with

$$\begin{aligned}
\mu_r &\equiv -\nu (1-\alpha) \frac{\beta\delta(1-\delta)\,\omega_k + \tilde{nx}\omega_b}{\Upsilon} \\
\mu_x &\equiv -\nu \frac{\alpha\beta\delta(1-\delta)\,\omega_k - \omega_b\left(\alpha + \tilde{nx} - 2\alpha\tilde{nx}\right)}{\Upsilon} \\
\mu_l &\equiv -\nu \frac{\beta(1-\delta)\,\delta\omega_k - \alpha\tilde{i}\omega_b}{\Upsilon} \\
\mu_b &\equiv -\nu \frac{\alpha\beta\delta(1-\delta)\,\omega_k - \omega_b\left(\alpha - 2\alpha\tilde{nx} + \frac{\tilde{nx}}{1-\beta}\right)}{\Upsilon}
\end{aligned}$$

and

$$\begin{aligned}
\Gamma &\equiv \beta + \beta^2\delta(1-\delta)^2\,\nu\rho\omega_k\left(\alpha\varsigma + \zeta_l\right) \\
&\quad + \rho\omega_b\left\{\alpha\varsigma\alpha\tilde{i}\left[1 - \beta(1-\delta)\right] - \alpha\varsigma - \alpha\beta(1-\delta)\,\tilde{i}\zeta_l + (2\alpha-1)\,\tilde{nx}\varsigma\right\} \\
\Upsilon &\equiv \beta\left[1 + \beta\delta(1-\delta)^2\,\nu\rho\omega_k\left(\alpha\varsigma + \zeta_l\right)\right] \\
&\quad + \nu\rho\omega_b\left\{\alpha\varsigma\left[-\alpha\beta(1-\delta)\,\tilde{i} + \alpha\tilde{i} - 1\right] - \alpha\beta(1-\delta)\,\tilde{i}\zeta_l + \tilde{nx}\varsigma(2\alpha-1)\right\} \\
\phi_\mu &\equiv \nu\left[(1-\alpha)^2\,\tilde{i}^2\zeta_l^2\phi_y + \alpha^2\varsigma^2\phi_p\right] \\
\Delta &\equiv \beta(1-\delta)\,\zeta_l - \alpha\varsigma\left[1 - \beta(1-\delta)\right]
\end{aligned}$$

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