CBDC and Financial Stability*

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Abstract

What is the effect of Central Bank Digital Currency (CBDC) on financial stability? We answer this question by studying a global games model of financial intermediation with an endogenously determined probability of a bank run. As an alternative to bank deposits, consumers can also store their wealth in remunerated CBDC issued by the central bank. Consistent with widespread concerns among policymakers, higher CBDC remuneration increases the withdrawal incentives of consumers, and thus bank fragility. However, the bank optimally responds to the additional competition by offering better deposit rates to retain funding, which reduces fragility. Thus, the overall relationship between CBDC remuneration and bank fragility is U-shaped. We explore the efficacy of holding limits on CBDC and discuss the sensitivity of our results to imperfect competition for deposits and risk-taking on the asset side.

Keywords: Central Bank Digital Currency, Bank Fragility, Financial Stability, CBDC Remuneration, Global Games.

JEL Codes: D82, G01, G21.

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1 Introduction

Central banks around the globe (Boar and Wehrli, 2021) are researching the costs and benefits of central bank digital currency (CBDC). These efforts are a response to the declining importance of cash as means of payment and the challenges associated with the proliferation of new forms of private digital money (e.g., stablecoins). While CBDC aims to preserve the role of public money and fend off threats to monetary sovereignty, some policy makers are concerned about its potentially adverse effects on the financial system (Ahnert et al., 2022). One issue that has received particular attention is the effect of CBDC on financial stability (Bank for International Settlements, 2020). Its status as safe asset with potentially positive remuneration—a key difference to physical cash—could render it an attractive store of value and thus increase the risk of bank runs during crisis episodes.

This paper aims to inform this debate by developing a two-period bank-run model with remunerated CBDC. Initially, a profit-maximizing bank with access to profitable but risky long-term investment opportunities raises uninsured deposits. At the interim date, consumers receive a noisy private signal about the investment's profitability ("economic fundamentals") and decide whether to withdraw their balances or roll them over. When funds are not kept in the bank, consumers can hold them in cash or as (possibly remunerated) CBDC.[1]

We solve for the unique equilibrium at the withdrawal stage using global-games methods. When making their withdrawal decision, consumers trade off the value of keeping their funds in the bank and the outside option of converting them into CBDC (converting private into public money). Accordingly, our model allows us to study how the terms of the deposit contract and CBDC remuneration affect the probability of a bank run (our measure of bank fragility). As in Morris and Shin (1998), Goldstein and Pauzner (2005), and Carletti et al. (2022), the equilibrium is characterized by a threshold strategy: when the economic fundamentals are below a certain value, all depositors run on the bank (Proposition 1).

[1] In this model, the only difference between cash and CBDC is their remuneration. Accordingly, a positively remunerated CBDC is always preferred to cash.

In this economy, an increase in the CBDC remuneration has two effects. First, it makes withdrawals at the interim date more attractive by increasing the payoff from storing funds with the central bank for consumption at the final date. This "direct effect" makes the bank more fragile (Proposition 1), consistent with the line of argument underlying the ongoing policy debate. Second, a higher CBDC remuneration induces the bank to offer more attractive deposit contracts because consumers would otherwise not provide any funding at the initial date. As a consequence, consumers have lower incentives to withdraw their funds at the interim date. This "indirect effect" renders the bank more stable (Proposition 2).

In equilibrium, the total effect of CBDC remuneration on bank fragility depends on the relative strengths of these two countervailing forces. The indirect effect dominates if and only if the elasticity of the failure threshold with respect to the bank deposit rate exceeds one (Lemma 1). A sufficient condition for this is a high enough profitability of the bank's investment opportunity relative to the remuneration on CBDC (Proposition 3). In this case, fragility is minimized for a strictly positive level of CBDC remuneration.

We consider various extensions of our model. First, policymakers have advanced limits on individual holdings as a possible tool to reduce the financial stability concerns associated with CBDC (Bindseil et al., 2021).[2] In our model, holding limits reduce the effective remuneration of deposit withdrawals because only part of the proceeds can be stored in CBDC, with the remainder being held as cash. Accordingly, holding limits can help to attain the optimal level of bank fragility if remuneration cannot be set freely (e.g. because it is aimed at monetary policy objectives outside of our model). However, and in line with our previous results, holding limits have an ambiguous impact on bank fragility if the indirect effect is sufficiently strong. In this case, holding limits increase (decrease) fragility for low (high) levels of CBDC remuneration (Proposition 4).

Second, we explore the role of market power in the deposit market. We

[2]Other proposals include tiered remuneration, as suggested in Bindseil (2020). If the second tier is remunerated at zero or below, this is equivalent to holding limits in our model because consumers would prefer to invest in cash any amount not covered by the first tier.

have so far assumed a monopolist bank that responds strongly to the introduction of a competitor (the central bank). With several banks competing for deposits, the ability of a bank to attract funding by raising rates may be more limited, suggesting a more prominent role of the direct effect of CBDC remuneration on bank fragility. Intuitively, the size of the elasticity of the failure threshold with respect to deposit rates is smaller in more competitive environments. We study this issue in a spatial model of deposit competition (Salop, 1979). We show that the indirect effect is dominated as the number of banks grows large, so higher CBDC remuneration increases bank fragility under perfect competition (Proposition 6).

Third, our analysis thus far is limited to fragility on the liability side of the bank balance sheet. A large literature in banking is concerned with the risk-taking on their asset side via risk choices and asset substitution (e.g., Dell'Ariccia and Marquez, 2004, 2006; Martinez-Miera and Repullo, 2017). Our setup can be naturally extended along this dimension. We show that higher CBDC remuneration reduces risk-taking on the bank's asset side (Proposition 7). Thus, the financial stability implications for the bank's asset and liability sides are opposite, and the overall effect can be positive.

Fourth, we study a setting where early liquidation of the long-term investment does not lead to losses. In this case (Proposition 8), bank runs are never driven by panics, they can only stem from fundamental insolvencies. While the indirect effect (higher CBDC remuneration induces banks to offer more attractive deposit rates) is still present, it leads to a higher likelihood of bank insolvency by reducing the interest rate margin. This result suggests that panic runs are an essential ingredient to our main result.

Literature. Our paper is part of a fast-growing literature on CBDC. An overview of recent work is provided by Ahnert et al. (2022). A key feature of our model is that the bank is not passive, but instead adjusts its behaviour (here its deposit rates) in response to the introduction to CBDC. This channel is also present in recent papers that examine the effects of CBDC on credit supply credit supply (Keister and Sanches, 2021; Andolfatto, 2021; Chiu et al., 2022).

Several other papers connect CBDC to financial stability. Using a Diamond and Dybvig (1983) model, Fernández-Villaverde et al. (2020, 2021) study the implications for bank runs. They show that, by fostering a flow of deposits out of the banking system into the central bank, the introduction of CBDC completely removes the risk of bank runs, as also shown in Skeie (2020), while creating a trade-off for the central bank between efficiency and price stability. Keister and Monnet (2020) also consider the implications of CBDC for bank runs, but focus on the efficacy of government interventions. In their framework, CBDC allows the central bank to have more accurate information about the health of the banking sector and thus to intervene promptly to mitigate the risk of a run. A difference relative to those papers is our use of global-games methods to uniquely pin down the probability of a bank run. This approach allows us to study how changes in CBDC design affect bank fragility (both directly and via changes in deposit rates).

Endogenizing the probability of runs relies on the use of global-games techniques. It originates from the seminal paper of Carlsson and van Damme (1993) and has been largely applied to finance (e.g., Corsetti et al. (2004), Goldstein and Pauzner (2005), Rochet and Vives (2004), Bebchuk and Goldstein (2011), Vives (2014), Ahnert (2016), Eisenbach (2017), Ahnert et al. (2019), Juelsrud and Nenov (2019), and Carletti et al. (2022)). See Morris and Shin (2003) and Vives (2005) for excellent surveys on the theory and application of global games.

Keister and Monnet (2020) is a closely related paper but differs considerably from our analysis. In their model, the bank offers the constrained-efficient allocation. The strategic complementarity in withdrawal decisions arises from the interaction of depositors with bank resolution authority (the anticipation of depositors of a future haircut), not directly from the deposit contract. They show how the introduction of CBDC remuneration affects the constraint-efficient level of risk-sharing, whereby higher remuneration reduces the short-term deposit rate and, thus, maturity transformation. This risk-sharing effect is absent in our model. They also study an information channel whereby CBDC enables more timely policy interventions, also absent in our model. Our work, instead, focuses on analytically deriving the effect of CBDC on bank fragility when banks maximize profits.

2 Model

The economy extends over three dates $t = 0, 1, 2$ and is populated by a bank and a unit continuum of consumers indexed by $i \in [0, 1]$. There is a single divisible good for consumption and investment. All agents are risk neutral and do not discount the future. Consumers are endowed with one unit of funds at $t = 0$ only.

At $t = 0$, the bank has access to a profitable but risky investment technology. Investment returns $L \in (0, 1)$ if liquidated at $t = 1$ (the liquidation value) and $R\theta$ upon maturity at $t = 2$, where $\theta \sim U[0, 1]$ represents the fundamentals of the economy and $R > 2$ is a constant that reflects the return from lending. To finance investment, the bank raises funds from consumers in exchange for demandable deposit contracts.[3] The bank chooses the deposit contract that maximizes expected profits. The contract specifies a repayment $r_1 \geq 1$ at $t = 1$ and r_2 at $t = 2$.

Since debt is demandable, depositors can withdraw their funds before the maturity of the bank's investment. At $t = 1$, each depositor receives a noisy private signal about the economic fundamental,

$$s_i = \theta + \varepsilon_i, \tag{1}$$

with $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. In addition to being informative about the profitability of the bank's investment project, it also provides information about the signals (and withdrawal actions) of other depositors. As is standard in much of the global-games literature, we assume vanishing noise, $\varepsilon \to 0$, to simplify the analysis.

The bank satisfies interim withdrawals by liquidating investment. Let $n \in [0, 1]$ be the fraction of consumers who withdraw at $t = 1$. When the liquidation proceeds at $t = 1$ are insufficient to meet withdrawals, $n > \overline{n} \equiv \frac{L}{r_1}$, the bank is bankrupt due to illiquidity. Otherwise, it continues to operate until $t = 2$. If the

[3] Bank debt is assumed to be demandable, which arises endogenously with liquidity needs (Diamond and Dybvig, 1983) or as a commitment device to overcome agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). Accordingly, uninsured deposits refer to any short-term or demandable debt instrument, which includes uninsured retail deposits and insured deposits when deposit insurance is not credible (Bonfim and Santos, 2020). Three quarters of U.S. commercial bank funding are deposits, half of which are uninsured (Egan et al., 2017).

bank cannot meet the remaining withdrawals, $n > \widehat{n} \equiv \frac{R\theta - r_2}{R\theta \frac{r_1}{L} - r_2}$, it is bankrupt due to insolvency, where $\widehat{n}$ solves the insolvency condition

$$R\theta \left(1 - \frac{\widehat{n} r_1}{L}\right) = (1 - \widehat{n})\, r_2. \tag{2}$$

The left-hand side is the return on the part of the project that was not liquidated at $t = 1$, and the right-hand side represents the remaining withdrawals at $t = 2$. Bankruptcy is costly and we assume zero recovery for simplicity.[4]

As alternatives to bank deposits, consumers can store their wealth in CBDC or cash. A deep-pocketed central bank offers consumers deposits with a per-period gross return $\omega \geq 1$, while cash is unremunerated.[5] Accordingly, consumers strictly prefer CBDC over cash as long as $\omega > 1$. They are indifferent for $\omega = 1$, so that this case is equivalent to a model without CBDC.

Relative to an economy with only deposits and cash, the introduction of CBDC has two effects. First, it improves the outside option of consumers deciding at $t = 0$ whether to deposit funds with the bank from 1 to ω^2 (the compound return on CBDC over two periods). Second, it pays interest ω on funds withdrawn from the bank at $t = 1$. Table 1 summarizes the timeline of the economy.

$\boldsymbol{t = 0}$	$\boldsymbol{t = 1}$	$\boldsymbol{t = 2}$
1. CBDC design **2.** Bank sets deposit contract **3.** Consumers choose where to deposit **4.** Bank invests deposits	**1.** Fundamental shock **2.** Private signals **3.** Consumers withdraw **4.** Liquidation of investment	**1.** Investment matures **2.** Consumption

Table 1: Timeline

[4]Bankruptcy costs are large. For example, James (1991) measures the losses associated with bank failure as the difference between the book value of assets and the recovery value net of direct expenses associated with failure. These losses amount to about 30% of failed banks' assets.

[5]For simplicity and ease of exposition, we abstract from both raising funds (e.g. via taxation) and an investment choice of the central bank at $t = 0$.

3 Equilibrium

This section solves for the equilibrium. We proceed backwards. First, for a given deposit contract (r_1, r_2), we characterize a threshold $\theta^*(r_1, r_2; \omega)$ such that, for every $\theta < \theta^*$, all depositors withdraw their funds at $t = 1$ and the bank fails. Next, we solve for the profit-maximizing deposit contract $(r_1^*(\omega), r_2^*(\omega))$ as a function of CBDC remuneration. Finally, we analyze how CBDC remuneration affects bank fragility $\theta^*\left(r_1^*(\omega), r_2^*(\omega); \omega\right)$ in equilibrium.

3.1 Bank fragility

We use global-games methods to solve for the unique equilibrium at the withdrawal stage. To characterize individual withdrawal decisions, we start by establishing the dominance bounds that yield ranges of the fundamental θ for which consumers have a dominant strategy. Following Goldstein and Pauzner (2005), we assume that there exists an upper dominance bound $\overline{\theta}$ such that the liquidation value is high ($L = R$) for $\theta > \overline{\theta}$. In this case, a depositor will never withdraw irrespectively of the withdrawal decision of all other depositors. We assume $\overline{\theta} \to 1$ when analysing the bank's choice of deposit contract at $t = 0$.

Second, withdrawing is a dominant strategy when $\theta < \underline{\theta}$. This (lower dominance) bound solves

$$R\underline{\theta} - r_2 = 0, \tag{3}$$

so that $\underline{\theta} = \frac{r_2}{R} \in (0, 1)$.[6] The intuition is as follows. When no other depositor withdraws ($n = 0$), the bank is always liquid at $t = 1$ and insolvent at $t = 2$ for $R\theta < r_2$. Therefore, withdrawing yields a payoff of r_1, while not withdrawing returns zero. So running on the bank is a dominant strategy for $\theta < \underline{\theta}$ (bankruptcy).

In the intermediate range $(\underline{\theta}, \overline{\theta})$, an consumer's decision to withdraw depends on what she expects the other consumers to do. Using global-games techniques,

[6]Note that a profit-maximizing bank will always choose $r_2 < R$ since deposit-taking cannot be profitable otherwise.

we can solve for the bank failure threshold at $t = 1$.

Proposition 1. ***Failure threshold.*** *There exists a unique fundamental threshold $\theta^* \in (\underline{\theta}, \overline{\theta})$. Each consumer withdraws their deposits from the bank if and only if $\theta < \theta^*$, where*

$$\theta^* \equiv \underline{\theta} \frac{r_2 - \omega L}{r_2 - \omega r_1} > \underline{\theta}. \tag{4}$$

The threshold θ^* decreases in L and R, increases in ω and r_1, and is non-monotonic in r_2: $\frac{\partial \theta^*}{\partial L} < 0$, $\frac{\partial \theta^*}{\partial R} < 0$, $\frac{\partial \theta^*}{\partial \omega} > 0$, $\frac{\partial \theta^*}{\partial r_1} > 0$, $\frac{\partial \theta^*}{\partial r_2} < 0$ if and only if $r_2 < r_2^{max}$.

Proof. See Appendix A, which also defines the cutoff r_2^{max}. □

Under vanishing noise, the bank failure threshold θ^* corresponds to the probability of a bank run, which we thus use as our measure of bank fragility. A higher liquidation value L or higher profitability R reduce depositors' incentives to run.

The terms of the deposit contract (r_1, r_2) also affect the failure threshold. As in Diamond and Dybvig (1983); Goldstein and Pauzner (2005), higher higher short-term deposit rates increase fragility. Liquidity provision by the bank $(r_1 > L)$ gives rise to strategic complementarity in consumer withdrawal decisions, so that both panic runs and fundamental runs exist, $\theta^* > \underline{\theta}$.

Moreover, the relationship between the long-term deposit rate r_2 and bank fragility is non-monotonic: when the deposit rate is low, higher rates reduces fragility while the opposite holds for high deposit rates. On the one hand, a higher long-term deposit rate implies that depositors receive a higher payoff when they wait and the bank is solvent. On the other hand, a higher long-term rate makes it more likely for the bank to be insolvent.

Finally, all else equal, the probability of a bank run increases with CBDC remuneration, since it increases the payoff from storing wealth outside the bank between $t = 1$ and $t = 2$, and thus makes withdrawing more attractive. However, this direct effect, $\frac{\partial \theta^*}{\partial \omega}$, fails to capture the overall impact because r_1 and r_2 are held fixed. As we show below, changes in CBDC remuneration induce the bank to adjust the terms of the deposit contract, which in turn affects θ^*. To see this formally, we can use total differentiation:

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_1}\frac{dr_1^*}{d\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}. \tag{5}$$

We next study the indirect effects of CBDC remuneration on bank fragility via the equilibrium deposit rates r_1^* and r_2^*.

3.2 The deposit contract

Since bank runs lead to zero profits, the bank internalizes the effects of the deposit contract on fragility, $\theta^* = \theta^*(r_1, r_2)$. With vanishing noise, consumer behaviour is fully symmetric. For $\theta > \theta^*$, there are no interim withdrawals and the investment matures at $t = 2$ with a return $R\theta$. The banker pays the promised return r_2 to consumers and pockets the difference, $R\theta - r_2$. For $\theta < \theta^*$, all consumers withdraw at $t = 1$ and the bank makes zero profits. Using $\overline{\theta} \to 1$, the banker's problem at $t = 0$ is therefore[7]

$$\max_{r_1 \geq 1, r_2} \Pi \equiv \int_{\theta^*}^{1} (R\theta - r_2)\, d\theta \tag{6}$$

$$\text{s.t. } V \equiv \int_{\theta^*}^{1} r_2\, d\theta - \omega^2 \geq 0. \tag{7}$$

Equation (7) is the consumers' participation constraint. The first term is the expected payoff from keeping funds in the bank until $t = 2$, which is the long-term deposit rate in case there is no bank run. The second term reflects the outside option, which is to store wealth in remunerated CBDC for a per-period return ω.

The following proposition characterizes the bank deposit rates in equilibrium.

Proposition 2. ***Deposit rates.*** *Let $\omega < \widetilde{\omega}$ and $R > \underset{\sim}{R}$. Then, the equilibrium deposit rates are given by $r_1^* = 1$ and $r_2^* < r_2^{max}$. The long-term deposit rate solves $V(r_2^*) \equiv 0$ (the participation constraint binds), increases in CBDC remuneration,*

[7] Expected bank profits can be written as $\Pi = (1 - \theta^*)\left(\frac{R}{2}(1 + \theta^*) - r_2\right)$, which is naturally interpreted as the probability of no run times the expected bank profits conditional on no run.

and decreases in the liquidation value and investment profitability: $\frac{dr_2^*}{d\omega} > 0$, $\frac{dr_2^*}{dL} < 0$, *and* $\frac{dr_2^*}{dR} < 0$.

Proof. See Appendix B, which also defines the bounds $\widetilde{\omega}$ and $\underset{\sim}{R}$. □

A higher short-term deposit rate r_1 reduces expected bank profits because the bank is more fragile (see Proposition 1). This also tightens consumers' participation constraint, since they are repaid less often. Accordingly, the bank chooses the lowest possible value for r_1, which is independent of CBDC remuneration.

In general, the long-term deposit rate r_2^* is pinned down by either the bank's first-order condition or the consumer participation constraint. The bounds on R and ω are sufficient conditions for the participation constraint to bind. Intuitively, they ensure that the bank has a large enough margin to adjust the deposit contract. We henceforth assume that these conditions are met.

An increase in CBDC remuneration improves consumers' outside option, both at the initial and interim dates. Accordingly, to remain attractive and guarantee consumer participation, the bank needs to offer a more attractive long-term deposit rate r_2^*. A higher liquidation value or investment profitability has the opposite effect. Because they reduce bank fragility, consumer participation can be satisfied with a lower long-term deposit rate.

Combining Propositions 1 and 2, a change in CBDC remuneration ω has two opposing effects on bank fragility θ^*. On the one hand, a higher remuneration leads to a higher incentive to withdraw at $t = 1$ and thus a larger threshold θ^*. On the other hand, the bank responds to the increase in remuneration by increasing deposit rates r_2^*, which reduces bank fragility ceteris paribus. The overall effect of a change in ω on θ^* depends on which of these two effects dominates. The next result offers some insight into their relative strength.

Lemma 1. ***Elasticity of the failure threshold.*** *Let* $\eta \equiv -\frac{r_2}{\theta^*}\frac{\partial \theta^*}{\partial r_2}$ *be the elasticity of the failure threshold with respect to the deposit rate. Higher CBDC remuneration reduces bank fragility,* $\frac{d\theta^*}{d\omega} < 0$, *if and only if* $\eta > 1$.

Proof. See Appendix C. □

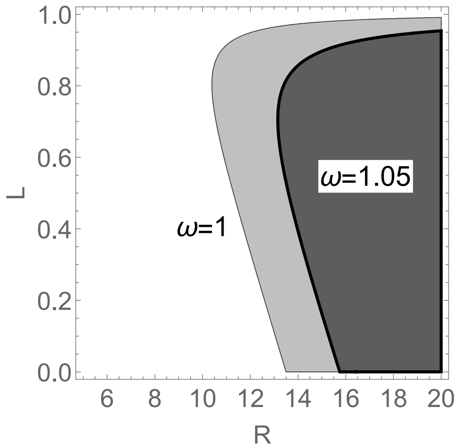

Figure 1: Elasticity of the failure threshold $\eta > 1$. The graph shows the range of parameters for investment profitability R and liquidation value L for which the elasticity of the failure threshold θ^* with respect to the deposit rate r_2 exceeds one. This range corresponds to the grey area on the right, drawn for two values of CBDC remuneration (light grey line for $\omega = 1$ and black solid line for $\omega = 1.05$).

Lemma 1 states that the indirect effect of higher CBDC remuneration dominates the direct effect whenever the failure threshold θ^* is very elastic to changes in the bank deposit rate r_2. That is, higher CBDC remuneration needs to induce a sufficiently strong increase in deposit rates for overall fragility to fall. The elasticity η depends on equilibrium deposit rates and, thus, ultimately on the investment return R and liquidation value L. In Figure 1, we plot the range of parameters in (R, L) for which it is high enough for two levels of CBDC remuneration ω.

We now state our main result on CBDC remuneration and bank fragility.

Proposition 3. ***CBDC remuneration and bank fragility.*** *Bank fragility is U-shaped in CBDC remuneration with a unique minimum* ω_{min}.

Proof. See Appendix C. □

Figure 2 shows the result: a positive remuneration of CBDC, $\omega > 1$, can be desirable in the sense of maximizing financial stability (minimizing fragility θ^*).

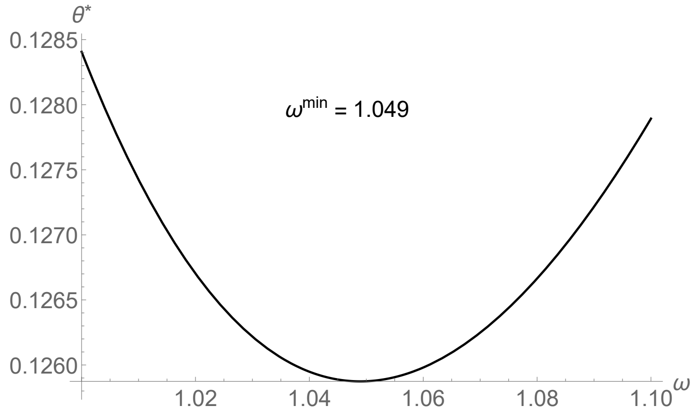

Figure 2: Bank failure threshold θ^* and CBDC remuneration ω. Minimum fragility is reached at roughly 4.9% interest on CBDC per period (where two periods correspond to the maturity of bank loans). Parameters: $L = 0.9$, $R = 15$.

The same main conclusion arises when we consider utilitarian welfare as objective function instead of bank fragility. Abstracting from a constant associated with the resources of the central bank, welfare W for $\epsilon \to 0$ and $\overline{\theta} \to 1$ comprises expected bank profits and the payments to consumers and can be written as:

$$W \equiv \int_{\theta^*}^{1} (R\theta - r_2) d\theta + \int_{\theta^*}^{1} r_2 d\theta = \frac{R}{2} \left[1 - (\theta^*)^2 \right]. \tag{8}$$

Thus, minimizing fragility is equivalent to maximizing welfare in our economy.

While consumers benefit from a low but positive CBDC remuneration, equilibrium bank profits are strictly below the level obtained in the absence of CBDC ($\omega = 1$). This effect is driven through a tightening of consumer's participation constraint and the bank's resulting choice of the deposit rate. Thus, rents are redistributed from the bank to consumers, with a net increase in welfare.

4 Extensions

We consider several extensions in this section. We first study the implications of holding limits and fundamental runs and then discuss imperfect competition in the market for bank deposits as well as risk-taking on the bank's asset side.

4.1 Holding limits

Widespread concerns about potentially adverse effects of CBDC on bank fragility have motivated a number of policy proposals aimed at limiting consumers' demand for CBDC, especially in the presence of bad news about fundamentals arrive. Although our previous analysis qualifies these concerns, it is relevant to gauge the effects of potential policy measures.

One of these proposals is the introduction of individual holding limits (Bindseil et al., 2021), whose effects we now analyze formally in the context of our model. Specifically, we assume that consumers can only hold a proportion γ of their wealth in a CBDC.[8] This changes the effective per-period remuneration on wealth held outside the bank from ω to

$$\omega^{HL} \equiv 1 + \gamma(\omega - 1), \tag{9}$$

because the remaining $1 - \gamma$ must be held in cash. Proposition 4 summarizes the consequences of an introduction of such holding limits.

Proposition 4. ***Holding limits.*** *Holding limits, $\gamma < 1$, increase (reduce) bank fragility for low (high) levels of CBDC remuneration. The fragility-minimizing level of remuneration, ω^{HL}_{min} decreases in γ.*

Proof. See Appendix D. □

The introduction of holding limits reduces the pass-through of CBDC remuneration to consumers' outside option. In line with our previous analysis, this leads to two opposing effects on bank fragility at the funding stage and the withdrawal stage. At $t = 1$, holding limits reduce the return that consumers earn on withdrawn funds. Since only part of their wealth held outside the bank can be stored in remunerated CBDC, the remainder must be held as cash and earns a

[8] In practice, policy makers are considering nominal limits (Bindseil et al., 2021). In our model, all consumers are identical at both $t = 0$ and $t = 1$ in equilibrium, so a proportional limit is equivalent to nominal limit. However, nominal and proportional holding limits may have different implications when consumers are heterogeneous.

return of 1. This corresponds to the "direct effect" and makes the bank less fragile. However, at $t = 0$, holding limits soften the competition that the bank faces at the funding stage, and thus imply a lower long-term equilibrium deposit rate r_2^*. This increases consumers' withdrawal incentives at $t = 1$ and thus makes the bank more fragile (the "indirect" effect).

The overall effect on bank fragility depends on which of these two effects dominates. As shown in Figure 3, holding limits have a beneficial effect on bank fragility when the level of CBDC remuneration is high, while a detrimental one otherwise. This latter effect emerges because holding limits reduce the responsiveness of the deposit rate to changes in CBDC remuneration, which enlarges the range of ω for which higher CBDC remuneration leads to more bank fragility.

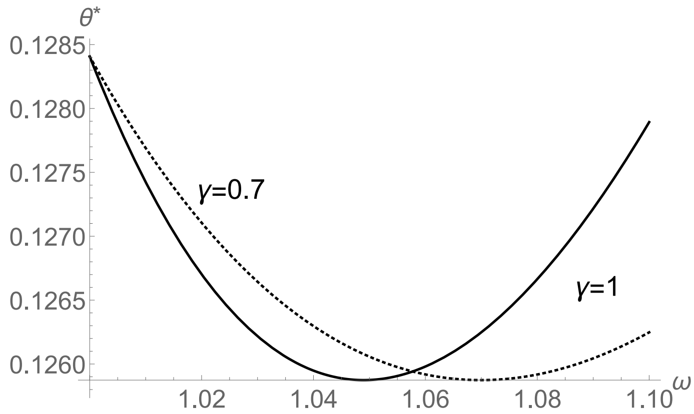

Figure 3: Bank failure threshold θ^*, CBDC remuneration ω, and holding limits γ. The solid line captures an economy without holding limits, while the dotted line captures an economy in which consumers can hold 70% of their funds in CBDC. Parameters: $L = 0.9$, $R = 15$.

This result can be interpreted as a note of caution for policymakers: holding limits and CBDC remuneration should be adequately designed to avoid inefficient outcomes via higher bank fragility. Still, holding limits are an additional and powerful policy instrument, particularly when CBDC remuneration is determined by other objectives, e.g. monetary policy considerations. In this case, the central bank limit holdings to achieve the fragility-minimizing CBDC remuneration.

Finally, the introduction of holding limits increases the expected profit of banks. The effective remuneration of the outside option of consumers decreases, so the bank's problem becomes more relaxed, leading to higher expected profits. When holding limits are appropriately designed such that the same level of financial stability is attained, $\theta^*(\omega_{min}) = \theta^*_\gamma(\omega^{HL}_{min})$, welfare is unchanged. As a result, holding limits redistribute surplus from consumers to banks in our model.

4.2 Market power in the deposit market

So far we have considered a bank that is monopolistic in the deposit market. In this subsection, we consider imperfect competition for bank deposits.[9] This is motivated by theoretical work on the effects of CBDC for bank credit supply, which comes to different conclusions for depending on the degree of deposit market competition (Keister and Sanches, 2021; Andolfatto, 2021; Chiu et al., 2022). In particular, we model imperfect competition for deposits in a spatial model as in Salop (1979). We follow the specification of Ahnert and Martinez-Miera (2021), whereby the withdrawal decision at $t = 1$ and the deposit decision at $t = 0$ can be separated.

There are $N \geq 2$ banks (indexed by j) competing for deposits from consumers symmetrically located on a unit-sized circle (Figure 4). Consumers incur a transportation cost $\mu > 0$ per unit of distance to a bank. Apart from the traditional interpretation as disutility from travel, they may also capture other aspects such as heterogeneity in consumer tastes for service bundles offered by different banks or pre-existing relationships.[10] We assume that the transport cost is sufficiently low such that the funding market is covered if at least two banks are active, and that banks are equidistantly located on the circle.

Banks offer deposit rates (r_{1j}, r_{2j}) to attract deposits h_j and invest the proceeds as described in the main analysis. The withdrawal stage is unchanged, so that bank j's failure threshold is $\theta^*(r_{1j}, r_{2j})$ as described in Proposition 1. Each

[9]A large literature documents imperfect competition in retail deposit markets, including Neumark and Sharpe (1992), Hannan and Berger (1997), and Drechsler et al. (2017).

[10]See Basten and Juelsrud (2022) for evidence on cross-selling of deposits and other financial services.

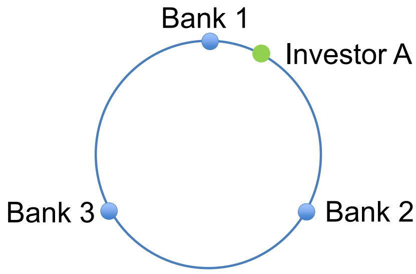

Figure 4: Location of banks on the Salop circle (for $N = 3$). Consumer A has a lower transport cost to bank 1 than to bank 2. However, consumer A prefers bank 2 over bank 1 if the former offers a sufficiently better deposit rate.

bank j maximizes expected profits:

$$\max_{r_{1j}, r_{2j}} \Pi_j \equiv h_j \int_{\theta_j^*}^{1} (R\theta - r_{2j})\, d\theta. \tag{10}$$

We focus on the unique symmetric equilibrium, so that $(r_{1j}^*, r_{2j}^*) = (r_1^*, r_2^*)$, $\theta_j^* = \theta^*$, and $h_j = \frac{1}{N}$ for all j.

Proposition 5. ***Deposit rates with imperfect competition.*** *Each bank offers $r_1^* = 1$, and r_2^* is the solution to*

$$\frac{1}{\mu}\left(1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}\right) \int_{\theta^*}^{1} (R\theta - r_2^*)\, d\theta = \frac{1}{N}\left(1 - \theta^* + (R\theta^* - r_2^*) \frac{\partial \theta^*}{\partial r_2}\right). \tag{11}$$

Proof. See Appendix E. □

Equation (11) illustrates banks' trade-off in choosing the equilibrium long-term deposit rate r_2^*. The left-hand side is the marginal benefit: a higher deposit rate increases consumers' expected payoff and thus attracts more deposits, which is weighted by the per-unit profitability of the bank. This effect is stronger when transportation costs are low because in this case many consumers are willing to switch banks. The right-hand side is the marginal cost. Higher deposit rates reduce the bank's profit margin one-for-one, conditional on the bank surviving $(1 - \theta^*)$. Moreover, a change in r_2 also affects the probability of survival, but the impact on the profit margin dominates. The overall effect is weighted by the equilibrium amount of deposits, $\frac{1}{N}$.

As in the main analysis, a change in CBDC remuneration has two effects on bank fragility. While it incentives interim withdrawals, it also induces banks to adjust their deposit contracts. However, the second effect is dampened by competitive forces, since depositors no longer face a monopolist. The following proposition summarizes the results in the polar opposite to the main model, the case of perfect competition.

Proposition 6. ***Equilibrium with perfect competition.*** *An increase in CBDC remuneration increases bank fragility,* $\frac{d\theta^*}{d\omega} > 0$.

Proof. See Appendix E. □

Under perfect competition, $N \to \infty$, the equilibrium deposit rate r_2^* is pinned down by bank's maximizing the expected value of the deposit claim subject to non-negative profits. Such fierce competition result in high deposit rates, so that higher deposit rates always increase the failure threshold, $\frac{\partial\theta^*}{\partial r_2} > 0$. In fact, deposit rates are so high that consumers prefer no further increase because the impact on bank fragility would be severe and the value of the entire claim would actually fall. Feasibility is ensured by strictly positive expected bank profits, $\pi(r_2^*) > 0$.

To sum up, higher CBDC remuneration again has two effects on bank fragility. The direct effect via the failure threshold is positive, as in the main model. The indirect effect via the equilibrium deposit rate can be either positive or negative, depending on parameters. In any case, rates are already so high under perfect competition that the indirect effect has a small impact and the overall effect on bank fragility is unambiguously detrimental.

4.3 Bank risk-taking on the asset side

So far we have considered a fragile liability side of banks (uninsured deposits) as a source of financial instability. However, financial instability can also be the result of banks' risk-taking decisions on their asset side (e.g., risk choices and asset substitution). In this extension, we allow for such risk-taking on the asset side.

Accordingly, we extend the baseline model assuming that at $t = 0$, it chooses its monitoring effort, consistent with an influential literature (e.g., Holmstrom and Tirole (1997), Hellmann et al. (2000), Morrison and White (2005), Dell'Ariccia and Marquez (2006), Allen et al. (2011), DellAriccia et al. (2014).) The effort q fully determines the probability of success of bank investment, whose return changes to

$$P = \begin{cases} R\theta & \text{w.p. } q \\ 0 & \text{w.p. } 1-q \end{cases}.$$

Higher monitoring leads to a higher success probability, but it entails a non-pecuniary cost $\frac{c}{2}q^2$. To keep the analysis tractable, we consider an exogenous deposit contract (r_1, r_2).[11] This assumption shuts down the channel along which higher CBDC remuneration improves financial stability in the main text and allows us to focus on how CBDC remuneration ω affects bank risk choices q^* instead.[12]

To solve the model, we proceed as in the main text.[13] We begin by deriving the endogenous run threshold θ_q^*, so that all depositors run on the bank at $t = 1$ if and only if $\theta < \theta_q^*$. Following the same steps as in Section 3.1, we deduce that

$$\theta_q^* = \frac{r_2}{R}\frac{qr_2 - \omega L}{qr_2 - \omega r_1}. \tag{12}$$

Better monitoring increases the probability that the bank is able to repay depositors at $t = 2$, and therefore reduces incentives to run ($\frac{\partial \theta_q^*}{\partial q} < 0$). This means that lower risk on the asset side of the bank leads to lower risk on its liability side.

Taking the run threshold θ_q^* into account, we then solve for the bank's optimal choice of monitoring effort q at $t = 0$. The bank solves

$$\max_q \Pi_q \equiv q\int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - \frac{cq^2}{2}. \tag{13}$$

[11]The deposit contract is such that the participation constraint of investors holds.

[12]Given that we assume an exogenous deposit contract (partly for tractability), we cannot really distinguish between monitoring and screening.

[13]We continue to assume that $\overline{\theta} \to 1$ and $\epsilon \to 0$. Moreover, we require that $qr_2 > \omega r_1$ to rule out a certain bank run. See also the discussion about dominance bounds in Section 3.1.

Provided that c is sufficiently high, there exists a unique and interior solution q^*, which is given by the solution to the following first-order condition

$$FOC_q \equiv \int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - q\frac{\partial \theta_q^*}{\partial q}\left(R\theta_q^* - r_2\right) - cq = 0. \tag{14}$$

The bank's risk choice at $t = 0$ reflects a trade-off. The last term in Equation (14) reflects the marginal cost of monitoring effort. The other two terms represent the marginal benefit of higher monitoring. First, more monitoring increases the probability that the project is successful, so that the bank reaps the residual claim $R\theta - r_2$ more often (provided there is no bank run, $\theta > \theta_q^*$). Second, the bank benefits from the interaction between the bank's asset and liability sides. An increase in monitoring reduces depositors' incentives to run, so that costly bankruptcy can be avoided.

Since we allow for risk-taking on the asset side of the balance sheet, it is important to note that there are now two potential sources of bank failure: bank runs and an unsuccessful investment project. We can therefore measure financial stability by the overall probability that the bank survives, which we define as

$$\Phi^* \equiv q^*\left(1 - \theta_q^*\right).$$

The following proposition shows that an increase in CBDC remuneration affects its two separate components in opposite ways.

Proposition 7. *Risk taking on the asset side.* *Higher CBDC remuneration improves monitoring, $\frac{dq^*}{d\omega} > 0$, but also increases fragility, $\frac{d\theta_q^*}{d\omega} > 0$.*

Proof. See Appendix F. □

Changes in CBDC remuneration affect the marginal benefit of bank monitoring, which follows directly from the first-order condition 14. The direction of this effect depends both on the direct effect of CBDC remuneration on the run threshold ($\frac{\partial \theta^*}{\partial \omega}$) as well as the threshold's sensitivity to changes in monitoring

($\frac{\partial^2\theta^*}{\partial q \partial \omega}$). Proposition 7 states that the overall effect is always positive, that is an increase in CBDC remuneration always leads to higher bank monitoring ($\frac{dq^*}{d\omega} > 0$) and thus renders the bank's asset side more stable.

The effect of CBDC remuneration of the probability of a bank run can be written as

$$\frac{d\theta_q^*}{d\omega} = \frac{\partial \theta_q^*}{\partial \omega}\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right]. \tag{15}$$

Just like in the main text, an increase in CBDC remuneration affects the run threshold θ_q^* both directly and indirectly. However, in this case, the indirect operates through bank monitoring, $\frac{dq^*}{d\omega}$, and not through the deposit contract (which is assumed to be exogenous). While these two effects go in opposite directions, Proposition 7 states that the direct effect always dominates, so that a higher CBDC remuneration always increases the risk of bank runs.

Since CBDC affects both aspects of financial stability in opposite ways, its overall effect on financial stability is ambiguous. While it is difficult to derive sufficient conditions analytically, Figure 5 provides a numerical example for which the beneficial effect of higher bank monitoring dominates. Accordingly, under these parameters, an increase in CBDC remuneration leads to an increase in financial stability, consistent with the main text.

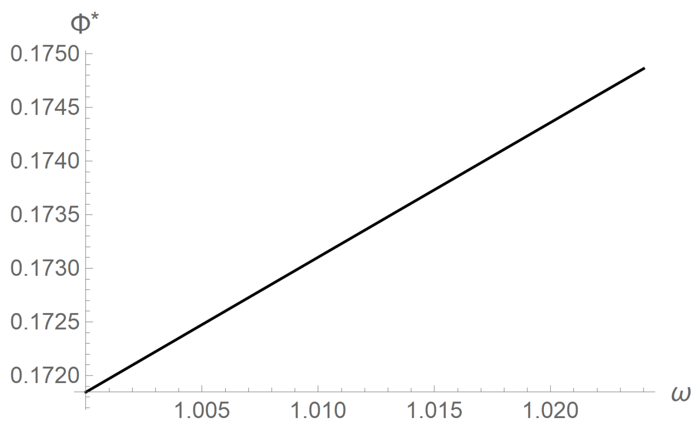

Figure 5: Financial stability Φ^* and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 20$, $c = 0.1$, $r_1 = 1$, and $r_2 = 6$.

4.4 Fundamental runs

So far we have considered panic runs arising from the short-term deposit return exceeding the liquidation value of bank assets. In this subsection, we limit attention to fundamental runs. This can be studied by considering $L \to 1$ (so the liquidation value of assets equals the promised short-term return). Thus, the strategic complementarity among depositors vanishes, $\theta^* \to \underline{\theta}$, and the threshold $\underline{\theta}$ becomes the relevant measure of bank instability. The following proposition summarizes.

Proposition 8. ***Fundamental runs only.*** *If $L \to 1$, then higher CBDC remuneration again raises the deposit rate, $\frac{dr_2}{d\omega} > 0$, but lowers bank stability, $\frac{d\underline{\theta}}{d\omega} > 0$.*

Proof. See Appendix G. □

The impact of CBDC remuneration on deposit rates is unchanged relative to the main text. The participation constraint of consumers binds in equilibrium, so higher CBDC remuneration increases deposit rates. What changes, however, is how higher deposit rates affect the relevant measure of bank instability. When the asset is perfectly liquid and no panic runs occur, the only concern about bank health is fundamental solvency at $t = 2$. Since higher deposit rates lower the chance for bank solvency, higher CBDC remuneration reduces bank stability.

What is absent relative to the main model (and prevents us from generating a U-shape between bank instability and CBDC remuneration) is the beneficial effect of higher long-term deposit rates r_2^* on the withdrawal incentives of consumers that mitigates bank fragility when assets are illiquid ($L < 1$) and panic runs are possible. This results, thus, suggests the importance of panic runs for our results.

5 Conclusion

To be written.

A Proof of Proposition 1

The proof builds on the arguments developed on Carletti et al. (2022). The only difference is that our model exhibits global strategic complementarity in that a depositor's incentive to withdraw at $t = 1$ monotonically increases in the number of depositors withdrawing. The arguments in their proofs establish that, in the limit of $\epsilon \to 0$, there is a unique threshold value of the fundamental, denoted as θ^*, below which all consumers choose to withdraw from the bank. We first prove the existence of a unique equilibrium and then study its comparative statics.

Existence and uniqueness. For $\theta \in (\underline{\theta}, \overline{\theta})$, a depositor's decision to withdraw depends on the withdrawal choices of others. Suppose that all depositors use a threshold strategy s^*. Then, the fraction of depositors withdrawing at $t = 1$, $n(\theta, s^*)$, equals the probability of receiving a signal below s^*:

$$n(\theta, s^*) = \begin{cases} 1 & \text{if } \theta \leq s^* - \epsilon, \\ \frac{s^* - \theta + \epsilon}{2\epsilon} & \text{if } s^* - \epsilon < \theta \leq s^* + \epsilon, \\ 0 & \text{if } \theta > s^* + \epsilon. \end{cases} \tag{16}$$

Thus, a depositor's withdrawal decision is characterized by the pair of thresholds $\{s^*, \theta^*\}$, which solves the following system of equations:

$$R\theta^* \left(1 - \frac{n(\theta^*, s^*) r_1}{L}\right) - (1 - n(\theta^*, s^*)) r_2 = 0, \tag{17}$$

$$r_2 Pr(\theta > \theta^* | s^*) = \omega r_1 Pr(\theta > \theta_n | s^*), \tag{18}$$

where $\theta_n = s^* + \epsilon - 2\epsilon \frac{L}{r_1}$ is the solution to $n(\theta, s^*)\, r_1 = L$.

Condition (17) identifies the level of fundamentals θ at which the bank is just able to repay the promised repayment to non-withdrawing depositors. Hence, it pins down the cutoff θ^*. Condition (18), instead, states that at the signal threshold s^* a depositor is indifferent between withdrawing at $t = 1$ and waiting until $t = 2$, since the expected payoff at $t = 2$, as captured by the LHS in (18), is equal to the expected $t = 1$ payoff, which is captured by the RHS in (18). Hence, given θ^* from (17), it pins down the threshold signal s^*. Note that the payoff at $t = 1$ is received whenever the bank is liquid, while the payoff at $t = 2$ is received whenever the bank is solvent. Differentiating the LHS of (17) with respect to θ, we obtain

$$R\left(1 - \frac{n(\theta, s^*)r_1}{L}\right) - \frac{\partial n(\theta, s^*)}{\partial \theta}\left[R\theta\frac{r_1}{L} - r_2\right] > 0, \tag{19}$$

for any $\theta > \underline{\theta}$ since $r_1 > L$ and $\frac{\partial n(\theta,s^*)}{\partial \theta} \leq 0$. Taking the derivative of (17) with respect to $n(.)$, we obtain $-R\theta\frac{r_1}{L} + r_2 < 0$ for any $\theta > \underline{\theta}$ because $r_1 > L$. Overall, this implies that the LHS in (17) monotonically increases with θ and the signal s_i and so it does the LHS in (18). Furthermore, rearranging (17) as $R\theta^* - r_2 - n(\theta^*, s^*)\left[R\theta^*\frac{r_1}{L} - r_2\right] = 0$, it follows that (17) is negative at $\theta = \underline{\theta}$ and positive at $\theta = \overline{\theta}$. Using (18), this means that at $\theta = \underline{\theta}$, a depositor expects to receive 0 when waiting and thus strictly prefers to withdraw. At $\theta = \overline{\theta}$ such that the LHS in (17) is strictly positive, a depositor expects to receive $r_2 > \omega\, r_1$ when waiting. Since ωr_1 exceeds the RHS in (18), it follows that, at $\theta = \overline{\theta}$, a depositor strictly prefer not to withdraw.

Overall, the analysis above implies that $\underline{\theta} < \theta^* < \overline{\theta}$ and analogously that the threshold signal s^* falls within the range $\left(\underline{\theta} + \epsilon, \overline{\theta} - \epsilon\right)$. Given that $\underline{\theta} > 0$ and $\overline{\theta} \to 1$, it follows that the equilibrium pair $\{\theta^*, s^*\}$ falls in the range $(0, 1)$.

To obtain a closed-form expression, we perform a change of variable using (16) from which we obtain $\theta(n) = s^* + \epsilon(1 - 2n)$. At the limit, when $\epsilon \to 0$, $\theta(n) = s^*$, which identifies the run threshold and it is equal to the solution to

$$\int_0^{\widehat{n}(\theta^*)} r_2 dn = \int_0^{\overline{n}} \omega r_1\, dn \Rightarrow \widehat{n}\left(\theta^*\right) r_2 = \omega L. \tag{20}$$

Solving for θ^* yields the closed-form expression as stated in the proposition. And $\theta^* > \underline{\theta}$ directly follows from $L < 1 \leq r_1$.

Comparative statics. To complete the proof, we study how bank fragility θ^* changes with deposit rates r_1 and r_2, as well as CBDC remuneration ω, liquidation value L, and the investment profitability R. We have the following:

$$\frac{\partial\theta^*}{\partial r_1} = \frac{\omega\theta^*}{(r_2 - \omega r_1)} > 0, \tag{21}$$

$$\frac{\partial\theta^*}{\partial r_2} = \frac{1}{R}\frac{r_2 - \omega L}{r_2 - \omega r_1} - \frac{\underline{\theta}\omega(r_1 - L)}{(r_2 - \omega r_1)^2} = \frac{r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1}{R(r_2 - \omega r_1)^2}, \tag{22}$$

$$\frac{\partial\theta^*}{\partial\omega} = \underline{\theta}\frac{r_2(r_1 - L)}{(r_2 - \omega r_1)^2} > 0, \quad \frac{\partial\theta^*}{\partial L} = -\frac{\omega\underline{\theta}}{r_2 - \omega r_1} < 0, \quad \frac{\partial\theta^*}{\partial R} = -\frac{\theta^*}{R} < 0, \tag{23}$$

$$\tag{24}$$

where we used $r_1 > L$ and $r_2 > \omega r_1$.

To establish the sign of $\frac{\partial\theta^*}{\partial r_2}$, we need to determine the sign of the numerator since the denominator is positive. The numerator is negative whenever $r_2^A < r_2 < r_2^B$, where r_2^A and r_2^B denote the roots of the associated quadratic equation $r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1 = 0$ since $\Delta = 4\omega^2 r_1^2 - 4\omega^2 L > 0$. The two roots are equal to:

$$r_2^{A/B} = \omega r_1\left(1 \pm \sqrt{1 - \frac{L}{r_1}}\right). \tag{25}$$

The smaller root r_2^A is inadmissible as it implies $r_2 < \omega r_1$, a contradiction. Thus, only the bigger root $r_2^B > \omega r_1$ is admissible. Since this value is the maximum of the relevant deposit rates considered by the bank, as we will show shortly, we label it $r_2^{max} \equiv r_2^B$. To summarize, $\frac{\partial\theta^*}{\partial r_2} < 0$ if $r_2 < r_2^{max}$ and $\frac{\partial\theta^*}{\partial r_2} > 0$ if $r_2 > r_2^{max}$.

B Proof of Proposition 2

A higher short-term deposit rate r_1 increases fragility (Proposition 1), so it reduces expected bank profits because the bank is solvent less often, $\frac{\partial\Pi}{\partial r_1} = (R\theta^* - r_2)\frac{\partial\theta^*}{\partial r_1} < 0$. A higher short-term deposit rate also tightens the participation constraint of consumers because they are repaid less often, $\frac{\partial V}{\partial r_1} = -r_2\frac{\partial\theta^*}{\partial r_1} < 0$. Thus, $r_1^* = 1$.

The proof of the remaining claims is in several steps. We first derive sufficient conditions for the participation constraint of consumers to bind in equilibrium. Then, we derive comparative statics of the equilibrium deposit rate. But first, we state some second-derivatives (evaluated at r_1^*) that are useful in the next steps:

$$\frac{\partial \theta^*}{\partial r_2} = \frac{(r_2-\omega)^2 - \omega^2(1-L)}{R(r_2-\omega)^2} = \frac{1}{R} - \frac{\omega^2(1-L)}{R(r_2-\omega)^2}, \tag{26}$$

$$\frac{\partial^2 \theta^*}{\partial \omega \partial r_2} = -\frac{2(1-L)\omega r_2}{R(r_2-\omega)^3} < 0, \tag{27}$$

$$\frac{\partial^2 \theta^*}{\partial r_2^2} = \frac{2(1-L)\omega^2}{R(r_2-\omega)^3} > 0. \tag{28}$$

B.1 Binding participation constraint of consumers

Step 1: We derive bounds on the deposit rate chosen by the bank. A profit-maximizing bank never chooses a rate that entails $\theta^* = 1$. If a run is certain, the bank is certain to make zero (expected) profits. As a result, the bank chooses $r_2 > r_2^{min}$ where r_2^{min} solves $\theta^*(r_2^{min}) \equiv 1$, yielding an expression for the lower bound on the deposit rate:

$$r_2^{min} = \frac{R+\omega L}{2} - \sqrt{\left(\frac{R+\omega L}{2}\right)^2 - R\omega}. \tag{29}$$

We have shown in Proposition 1 that bank fragility decreases in the long-term deposit rate as long as $r_2 < r_2^{max}$. We now impose constraints on parameters to ensure that the participation constraint of consumers is slack at $r_2 = r_2^{max}$, that is $V(r_2^{max}) > 0$. Note that $\theta^*(r_2^{max}) = \frac{\omega}{R}\left(1+\sqrt{1-L}\right)^2$ and $V(r_2^{max}) = \omega\left(1+\sqrt{1-L}\right) - \frac{\omega^2}{R}\left(1+\sqrt{1-L}\right)^3 - \omega^2$, resulting in a lower bound on investment profitability:

$$R > \underline{R}_1 \equiv \frac{\omega\left(1+\sqrt{1-L}\right)^3}{1+\sqrt{1-L}-\omega}. \tag{30}$$

An upper bound on CBDC remuneration ensures that the denominator of $\underline{R}_1$ is always positive:

$$\omega < \widetilde{\omega} \equiv 1 + \sqrt{1-L}. \tag{31}$$

Note that $r_2^{min} < r_2^{max} < 1$, which justifies our labels, and ensures that the bank does not always fail, $\theta^*(r_2^{max}) < 1$, the economically interesting case.

Step 2: We can write marginal bank profits as

$$\frac{d\Pi}{dr_2} \equiv -\frac{\partial \theta^*}{\partial r_2}\left(R\theta^* - r_2\right) - \int_{\theta^*}^{1} d\theta. \tag{32}$$

Since $(R\theta^* - r_2) = r_2\omega\frac{(r_1-L)}{r_2-\omega r_1} > 0$ and $1-\theta^* > 0$ (given the bounds on r_2) as well as the parameter constraints ensuring that higher long-term deposit rates reduce bank fragility, there is a non-trivial trade-off for the bank: higher long-term deposit rates make the bank more stable but also reduce its profit margin.

Evaluating marginal profits at $r_2 = r_2^{\max}$ (where, by definition, $\frac{\partial \theta^*}{\partial r_2} = 0$), gives $\frac{d\Pi}{dr_2} < 0$. Moreover, $\frac{\partial \Pi}{\partial r_2} < 0$ for all $r_2 > r_2^{max}$. Thus, the bank chooses a deposit rate $r_2 < r_2^{max}$ if feasible (i.e. if the participation constraint of consumers holds). Given the parameter constraints on investment profitability and CBDC remuneration (see step 1), the participation constraint is indeed slack, so the bank chooses a rate $r_2^* < r_2^{max}$ (establishing an upper bound on the deposit rate).

Similarly, evaluating at $r_2 = r_2^{\min}$ (where, by definition, $\theta^* = 1$) gives $\frac{d\Pi}{dr_2} > 0$. Furthermore, at $r_2 = r_2^{\min}$, we also have $V < 0$ (i.e. the participation constraint is violated), so the bank always chooses a higher deposit rate, $r_2^* > r_2^{\min}$ (establishing a lower bound on the deposit rate).

Step 3: Next, we show that expected bank profits Π are globally concave. As a result, the unconstrained choice of deposit rate that ignores the participation constraint of consumers, denoted by r_2^{Π} and solving $\frac{d\Pi}{dr_2} \equiv 0$, is unique. To establish global concavity, we show that the second-derivative is always negative:

$$\frac{d^2\Pi}{dr_2^2} \equiv -\frac{\partial^2 \theta^*}{\partial r_2^2}\left(R\theta^* - r_2\right) - \left(\frac{\partial \theta^*}{\partial r_2}\right)^2 R + 2\frac{\partial \theta^*}{\partial r_2} < 0,$$

because $\frac{\partial \theta^*}{\partial r_2} < 0$ and $\frac{\partial^2 \theta^*}{\partial r_2^2} > 0$.

Consider $\underset{\sim}{r}_2 = \omega^2$, which solves the participation constraint investors in case of no bank failure. Since the bank sometimes fails, $\theta^* > 0$, $\underset{\sim}{r}_2$ is clearly a lower bound on the value that solves the binding participation constraint, $r_2^{PC} > \underset{\sim}{r}_2$. This bound is helpful in establishing sufficient conditions for the relevant equilibrium condition to be the binding participation constraint.

By global concavity of Π, a sufficient condition for $r_2^{\Pi} < \underset{\sim}{r}_2$ is $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$. Intermediate results are $\theta^*(\underset{\sim}{r}_2) = \frac{\omega^2(\omega-L)}{R(\omega-1)}$, $R\theta^*(\underset{\sim}{r}_2) - \underset{\sim}{r}_2 = \frac{\omega^2(1-L)}{\omega-1}$, and $\frac{\partial\theta^*(\underset{\sim}{r}_2)}{\partial r_2} = \frac{\omega^2-2\omega+L}{R(\omega-1)^2}$. Thus, we can express $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$ as a lower bound on profitability:

$$R > \underline{R}_2 \equiv \frac{\omega^2}{\omega-1}\left[-\frac{1-L}{(\omega-1)^2}\left(\omega^2-2\omega+L\right)+\omega-L\right] = \omega^2\left(1+\frac{(1-L)^2}{(\omega-1)^3}\right). \tag{33}$$

As a result, we have shown that $r_2^{PC} > r_2^{\Pi}$. Finally, we verify that $\underset{\sim}{r}_2 \geq r_2^{min}$. Rewriting $\theta^*(\underset{\sim}{r}_2) < 1$ yields another lower bound on profitability:

$$R > \underline{R}_3 \equiv \frac{\omega^2(\omega-L)}{\omega-1}. \tag{34}$$

Since $\omega < \widetilde{\omega}$, which implies $\omega^2 - 2\omega + L < 0$, we can rank these bounds $\underline{R}_2 > \underline{R}_3$. Thus, we can drop the bound $\underline{R}_3$. Taking stock, we define $\underset{\sim}{R}$ as the largest of all lower bounds on the investment returns (see below for the definition).

B.2 Existence of a unique deposit rate, comparative statics

Having established that the deposit rate r_2^* corresponds to the solution to the binding participation constraint, we next prove its existence and uniqueness.

Recall that the net value of the deposit claim is $V = \int_{\theta^*}^{1} r_2\, d\theta - \omega^2$. So, $V(r_2^*) \equiv 0$. Note that $V(r_2^{min}) = -\omega^2 < 0$ and $V(r_2^{max}) > 0$ given the parameter constraints on R and ω. Differentiating V with respect to r_2, we obtain

$$\frac{dV}{dr_2} = -\frac{\partial\theta^*}{\partial r_2} r_2 + (1-\theta^*) > 0, \tag{35}$$

so a higher (long-term) deposit rate increases the value of the deposit claim for two reasons: consumers receive a high payment in the absence of a bank run and the bank is less fragile (Proposition 1). Given the monotonicity of V in r_2 and its change of signs from the bound r_2^{min} to r_2^{max}, a solution for r_2^* exists and is unique.

Next, we study the comparative statics of r_2^*. First, consider CBDC remuneration ω, using the implicit function theorem, $\frac{dr_2}{d\omega} = -\frac{\frac{\partial V}{\partial \omega}}{\frac{\partial V}{\partial r_2}}$. The denominator is positive, as shown in Condition (35). Hence, the sign of $\frac{dr_2}{d\omega}$ is the opposite of the sign of the numerator:

$$\frac{\partial V}{\partial \omega} = -2\omega - \frac{\partial \theta^*}{\partial \omega} r_2 < 0. \tag{36}$$

It follows that r_2 monotonically increases in CBDC remuneration ω:

$$\frac{dr_2^*}{d\omega} = \frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2}{1 - \theta^* - \frac{\partial \theta^*}{\partial r_2} r_2} > 0. \tag{37}$$

Finally, we derive the comparative statics of the equilibrium deposit rate with respect to investment characteristics. Using the implicit function theorem again, the results $\frac{dr_2^*}{dL} < 0$ and $\frac{dr_2^*}{dR} < 0$ follow from $\frac{\partial V}{\partial L} = -r_2 \frac{\partial \theta^*}{\partial L} > 0$ and $\frac{\partial V}{\partial R} = -r_2 \frac{\partial \theta^*}{\partial R} > 0$.

C Proof of Lemma 1 and Proposition 3

We first prove the lemma and then the proposition. Using the expression for $\frac{dr_2}{d\omega}$ in Equation (37), we expand the expression for $\frac{d\theta^*}{d\omega}$:

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2^*}{1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}}. \tag{38}$$

Since the denominator of the second term is positive, we get $\frac{d\theta^*}{d\omega} < 0$ whenever $\frac{\partial \theta^*}{\partial \omega}\left(1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}\right) + \frac{\partial \theta^*}{\partial r_2}\left(2\omega + \frac{\partial \theta^*}{\partial \omega} r_2^*\right) < 0$. This inequality simplifies to

$$\frac{\partial \theta^*}{\partial \omega}(1 - \theta^*) + 2\omega \frac{\partial \theta^*}{\partial r_2} < 0. \tag{39}$$

Using the equilibrium deposit rate to replace $1 - \theta^* = \frac{\omega^2}{r_2^*}$ and the fact that $\frac{\partial \theta^*}{\partial r_2} = \frac{1}{r_2}\left[\theta^* - \omega \frac{\partial \theta^*}{\partial \omega}\right]$, we can re-express this condition as:

$$\theta^* + r_2^* \frac{\partial \theta^*}{\partial r_2} < 0, \tag{40}$$

which has the intuitive interpretation of an elasticity. In particular, the elasticity of the failure threshold with respect to deposit rate, $\eta = -\frac{r_2^*}{\theta^*}\frac{\partial \theta^*}{\partial r_2}$, has to exceed 1 for the indirect effect to dominate and thus $\frac{d\theta^*}{d\omega} < 0$, where r_2^* solves $V(r_2^*) = 0$.

Using $1-\theta^* = \frac{\omega^2}{r_2^*}$ to rewrite Condition (39) yields $\omega\frac{\partial\theta^*}{\partial\omega} + 2r_2\frac{\partial\theta^*}{\partial r_2} < 0$. Inserting the expressions for the partial derivatives dividing by the positive common term $\frac{r_2^*}{R(r_2^*-\omega)^2}$, we obtain $\eta > 1$ if and only if $\omega r_2^*(1-L) + 2\left[(r_2^*)^2 - 2\omega r_2^* + \omega^2 L\right] < 0$. Rewriting yields the following condition with a quadratic term:

$$h(r_2^*, \omega) \equiv (r_2^*)^2 - \frac{3+L}{2}\omega r_2^* + \omega^2 L < 0. \tag{41}$$

We turn to the proof of the proposition. First, we determine whether $\frac{d\theta^*}{d\omega} < 0$ when evaluated at $\omega = 1$ is possible. Using condition (41), this boils down to $(r_2^*)^2 - \frac{3+L}{2}r_2^* + L < 0$. Thus, we can find the roots $r_2^C \equiv \frac{\omega}{4}\left(3 + L - \sqrt{L^2 - 10L + 9}\right)$ and $r_2^D \equiv \frac{\omega}{4}\left(3 + L + \sqrt{L^2 - 10L + 9}\right)$ such that $h < 0$ if and only if $r_2^C < r_2^* < r_2^D$. Since $r_2^C < \omega$ is inadmissible, r_2^D is the relevant root, which is independent of R.

Second, we impose parameter constraints to ensure $r_2^D \in (r_2^{min}, r_2^{max})$. Using the expression for r_2^{max} as given in (25) and evaluating it at $r_1 = 1$ and $\omega = 1$, $r_2^D < r_2^{max}$ can be expressed as $\frac{1-L}{4} + \sqrt{1-L} > \frac{1}{4}\sqrt{L^2 - 10L + 9}$. Squaring and rewriting yields $8(1-L)(1+\sqrt{1-L}) > 0$, which always holds for $L < 1$. Moreover, for $r_2^D > r_2^{min}$ to hold at $\omega = 1$, it suffices to show that $\theta^*(\omega = 1, r_2 = r_2^D) < 1$. This yields another lower bound on profitability:

$$R > \underline{R}_4 \equiv \frac{r_2^D(r_2^D - L)}{r_2^D - 1}. \tag{42}$$

Third, r_2^* decreases in R, while r_2^D is independent of it. Thus, there exists a critical value, $\underline{R}_5$, such that $r_2^* < r_2^D$ for all $R > \underline{R}_5$. Importantly, $\underline{R}_5 < \infty$. One can easily show that $r_2^D > 1 > L$ because $\sqrt{L^2 - 10L + 9} > 1 - L$ can be rearranged by squaring to $8(1-L) > 0$. By contrast, $r_2^* \to 1$ for $R \to \infty$ since $\theta^* \to 0$ and thus $r_2^* \to 1$ for a given $L < 1$ and $\omega = 1$.

The reader may notice that the bound $\underline{R}_2$ characterized in the proof of Proposition 2 converges to ∞ as $\omega \to 1$, thus becoming the binding bound on profitability. However, it is important to stress that this simple sufficient condition is quite restrictive. In fact, the numerical example in the main text shows that our results also hold for much lower levels of the investment profitability R.

Fourth, we show that $\frac{d\theta^*}{d\omega} > 0$ for large ω. Recall that $\frac{dr_2^*}{d\omega} > 0$ and $r_2^* < r_2^{max}$. Then, we can denote ω^{max} such that $r_2^* \to r_2^{max}$ when $\omega \to \omega^{max}$. In this limit, Condition (40) is violated because $\frac{\partial\theta^*}{\partial r_2} \to 0$ when $r_2 \to r_2^{max}$. Thus, $\frac{d\theta^*}{d\omega} > 0$.

Note that $\omega^{max} < \widetilde{\omega}$. To see this, recall that (i) $\underline{R}_1 = +\infty$ at $\omega = \tilde{\omega}$ and (ii) $\underline{R}_1 < \infty$ for any $\omega < \widetilde{\omega}$. That is, for any $\omega < \widetilde{\omega}$, there exists a finite $\underline{R}_1$ such that the participation constraint binds exactly at $r_2 = r_2^{max}$. Hence, $\frac{\partial \underline{R}_1}{\partial\omega} > 0$ implies that there exists an $\omega < \widetilde{\omega}$ and $R > \underline{R}_1$ for which $r_2^* = r_2^{max}$. We denote it as ω^{max}.

Taken these steps together, we have $\frac{d\theta^*}{d\omega} > 0\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\omega^{max}} > 0$. Hence, there is at least a value of ω, denoted as ω^{min}, at which θ^* is minimized.

Fifth, we show that ω^{min} is unique. The value ω^{min} solves $h(r_2^*, \omega_{min}) = 0$, where $h(r_2, \omega)$ is given in (41). Since r_2^* is a function of ω, $h(r_2(\omega), \omega)$ is a polynomial where ω is the main variable. The degree of the polynomial determines the number of possible values ω^{min}. Since $\frac{d\theta^*}{d\omega}\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\tilde{\omega}} > 0$, the number of solutions ω^{min} must be odd. To determine the degree of the polynomial $h(r_2(\omega), \omega)$, it is useful to characterize a closed-form solution for r_2^*. Since r_2^* solves $V(r_2^*, \omega) = 0$ given in (7). Substituting the expression for θ^* from (4), we obtain:

$$r_2^3 - r_2^2(R + \omega L) + r_2 R\omega(\omega + 1) - R\omega^3 = 0. \tag{43}$$

Equation (43) has three roots, which solve the corresponding depressed cubic equation

$$y^3 + Py + Q = 0, \tag{44}$$

where $y = r_2 - \frac{R+\omega L}{3}$, $P = \frac{3R\omega(1+\omega)-(R+\omega L)^2}{3}$ and $Q = \frac{-2(R+\omega L)^3+9(R+\omega L)R\omega(\omega+1)-27R\omega^3}{27}$. We focus on parameters such that $4P^3 + 27Q^2 > 0$. Thus, there is only one real root:

$$y = \sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}} + \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}}. \tag{45}$$

The expression pinning down y and, in turn, r_2^* is a function of ω. One can show that ω only appears at a power of 1. This implies that $h(r_2(\omega), \omega)$ has at most two roots, of which only one can be in the range $1 < \omega < \tilde{\omega}$. Since the derivative is initially negative and eventually positive, there must be an odd number of crossings with zero within $[1, \tilde{\omega}]$. Hence, ω_{min} is unique.

D Proof of Proposition 4

The introduction of holding limits affects consumers' decisions at $t = 0$ and $t = 1$. At $t = 0$, holding limits changes the consumers' participation constraint to:

$$\int_{\theta^*}^{1} r_2 \, d\theta \geq (\omega^{HL})^2 = [1 + \gamma(\omega - 1)]^2 . \tag{46}$$

The left-hand side is the value of the deposit claim to the consumer (unchanged relative to the main text). The right-hand side is the expected return of holding CBDC, which differs from the main text because only a fraction γ of funds can be held in CBDC. At $t = 0$ an consumer invests γ in CBDC and $1-\gamma$ in storage/cash. At $t = 1$, the initial investment returns ω on the γ units, whose a fraction γ is held in the CBDC account while the remainder is held in storage/cash. Thus, the analysis in the main text is a special case for no holding limits, $\omega^{HL}(\gamma = 1) = \omega$.

At $t = 1$, holding limits only affects a depositor's expected payoff from withdrawing, $r_1\omega^{HL}$, so they have the intended effect of directly reducing withdrawal incentives by lowering the remuneration of the withdrawn funds stored until $t = 2$. Thus, the effective CBDC remuneration with holding limits is $\omega^{HL} \equiv 1 + \gamma(\omega - 1)$. Once this transformation is made, the economy is identical to the one without holding limits with the only difference that ω is replaced by ω^{HL}.

The bank run threshold is $\theta^*_\gamma = \frac{r_2}{R}\frac{r_2 - L\omega^{HL}}{r_2 - r_1\omega^{HL}}$, where θ^*_γ increases in γ because $\frac{\partial \theta^*_\gamma}{\partial \gamma} = \frac{r_2}{R}\frac{r_2(\omega-1)(r_1-L)}{(r_2 - r_1(1+\gamma(\omega-1)))^2} > 0$ whenever $\omega > 1$. This result captures the "common wisdom" about holding limits: introducing them (i.e., setting $\gamma < 1$) reduces bank fragility, effectively mitigating the direct effect of CBDC remuneration on fragility.

However, the introduction of holding limits also affects the sensitivity of the run threshold to changes in r_2, thus leading to a potential ambiguous effect on fragility when banks respond to the introduction of CBDC. The derivative of the threshold θ^*_γ with respect to r_2 is now a function of γ and equal to $\frac{\partial\theta^*_\gamma}{\partial r_2} = \frac{\theta^*_\gamma}{r_2} - \frac{r_2}{R}\frac{(r_1-L)\omega^{HL}}{(r_2-r_1\omega^{HL})^2}$. Hence, the total effect of holding limits on bank fragility is thus not obvious and again depends on both a direct effect (via lower withdrawal incentives) and an indirect effect (via equilibrium deposit rates).

E Proof of Propositions 5 and 6

There is no change in the investment technology or information relative to the main text. Investment risk is aggregate, so there is no scope for diversification. Thus, withdrawal stage at $t = 1$ is unchanged and bank j's failure threshold is $\theta^*_j = \theta^*(r_{1j}, r_{2j})$, as given in Proposition 1.

Let $\rho_j = \rho(r_{1j}, r_{2j})$ denote the expected return of a deposit claim on bank j (before transport costs). Because of bankruptcy costs, we can write and obtain:

$$\rho_j = r_{2j}(1-\theta^*_j) \tag{47}$$

$$\frac{d\rho_j}{dr_{1j}} = -r_{2j}\frac{\partial\theta^*_j}{\partial r_{1j}} < 0 \tag{48}$$

$$\frac{d\rho_j}{dr_{2j}} = 1-\theta^*_j - r_{2j}\frac{\partial\theta^*_j}{\partial r_{2j}} > 0. \tag{49}$$

Let π_j denote the profit of bank j per unit of deposits. Thus,

$$\pi_j = \int_{\theta^*_j}^{1} (R\theta - r_{2j})\, d\theta \tag{50}$$

$$\frac{d\pi_j}{dr_{1j}} = -\left(R\theta^*_j - r_{2j}\right)\frac{\partial\theta^*_j}{\partial r_{1j}} < 0 \tag{51}$$

$$\frac{d\pi_j}{dr_{2j}} = -(1-\theta^*_j) - \left(R\theta^*_j - r_{2j}\right)\frac{\partial\theta^*_j}{\partial r_{2j}} < 0, \tag{52}$$

where the sign on the final derivative arises because the effect of higher deposit rates on a bank's profit margin dominates the effect of higher deposit rates on increased bank stability (see also the proof of Proposition 2). Moreover, a unique solution r_{2j}^* exists because Π_j is globally concave due to $\frac{\partial^2 h_j}{\partial r_{2j}^2} < 0$ and $\frac{\partial^2 pi_j}{\partial r_{2j}^2} < 0$.

Let d_{jk} the distance of consumer k to bank j. We assume that the competition between banks is fierce enough such that the deposit rates offered induce consumers to always prefer bank deposits over CBDC at $t = 0$. That is, $\rho_j - \mu d_{jk} \geq \omega^2$ for all consumers, where $d_{jk} \leq \frac{1}{2N}$. Since the market is covered, we can focus on the two banks nearest to k, whose distance is d_k and $\frac{1}{N} - d_k$ because banks are equidistant on the unit circle. Hence, the location at which consumer k is indifferent between either bank is $d_k^* = \frac{\rho^j - \rho^{-j}}{2\mu} + \frac{1}{2N}$, where ρ_{-j} is the expected return to consumers offered by bank $-j$. Total deposit funding comes from both sides relative to a bank's location, so

$$h_j^* = \frac{\rho^j - \rho_{-j}}{\mu} + \frac{1}{N}, \tag{53}$$

Thus, $\frac{dh_j}{d\rho_j} = \frac{1}{\mu}$, so offering a higher expected return attracts more deposits. Moreover, in symmetric equilibrium, $h_j^* = \frac{1}{N}$ for each bank.

Perfect competition. We next study the case of perfect competition, $N \to \infty$. It implies that (i) the transport costs for consumers vanish and (ii) each bank maximizes the the expected return of its deposit claim ρ subject to non-negative profits, $\pi \geq 0$. Our approach will be to consider the unconstrained problem and then derive a condition sufficient for bank profits to be indeed non-negative. The first-order condition pins down the equilibrium deposit rate r_2^*:

$$H(r_2^*) \equiv \left. \frac{d\rho}{dr_2} \right|_{r_2 = r_2^*} = 0 \tag{54}$$

Since $H(r_2^{max}) > 0$, we deduct that $r_2^{max} < r_2^*$ and fragility increases in the deposit rate around the equilibrium, $\left. \frac{\partial \theta^*}{\partial r_2} \right|_{r_2 = r_2^*} > 0$. Since

$$\frac{\partial H}{\partial r_2} \equiv -2\frac{\partial \theta^*}{\partial r_2} - r_2 \frac{\partial^2 \theta^*}{\partial r_2^2} < 0, \tag{55}$$

a unique global maximum exists. Using the IFT and

$$\frac{\partial H}{\partial \omega} \equiv -\frac{\partial \theta^*}{\partial \omega} - r_2 \frac{\partial^2 \theta^*}{\partial r_2 \partial \omega}, \tag{56}$$

we obtain

$$\frac{dr_2^*}{d\omega} = \frac{\frac{\partial \theta^*}{\partial \omega} + r_2 \frac{\partial^2 \theta^*}{\partial r_2 \partial \omega}}{-2\frac{\partial \theta^*}{\partial r_2} - r_2 \frac{\partial^2 \theta^*}{\partial r_2^2}}. \tag{57}$$

Using the total derivative of bank fragility, $\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}$, we obtain $\frac{d\theta^*}{d\omega} > 0$ (after some rearrangement) whenever $-\frac{\partial \theta^*}{\partial \omega}\frac{\partial \theta^*}{\partial r_2} - r_2 \frac{\partial \theta^*}{\partial \omega}\frac{\partial^2 \theta^*}{\partial r_2^2} + r_2 \frac{\partial \theta^*}{\partial r_2}\frac{\partial^2 \theta^*}{\partial r_2 \partial \omega} < 0$, which always holds given the signs of the various partial derivatives already established.

Finally, we need to establish that $\pi(r_2^*) \geq 0$. Note that expected profits can be written as $\pi = (1-\theta^*)\left[\frac{R}{2}(1+\theta^*(r_2)) - r_2\right]$, where the first-order condition, $1-\theta^* = r_2 \frac{\partial \theta^*}{\partial r_2} > 0$ pins down r_2^* and ensures that the first factor is strictly positive. The second factor is also positive because $r_2^* \in (\omega, R)$ and the following result:

$$\frac{R}{2}(1+\theta^*) - r_2 = \frac{R}{2} + \frac{r_2(r_2 - \omega L)}{2(r_2-\omega)} - r_2 = \frac{R - r_2}{2} + \frac{r_2}{2}\frac{\omega(1-L)}{r_2 - \omega} > 0, \tag{58}$$

so expected profits at the equilibrium deposit rate are strictly positive, $\pi(r_2^*) > 0$.

F Proof of Proposition 7

This proof has three parts. First, we derive the run threshold. This part uses the same argument as the proof of Proposition 1. The threshold θ_q^* corresponds to the solution to

$$\int_0^{\widehat{n}(\theta)} q r_2 dn = \int_0^{\overline{n}} \omega r_1 dn,$$

because the bank repays depositors r_2 at $t = 2$ only when the project succeeds, where both $\widehat{n}(\theta)$ and $\overline{n}$ are independent of q and identical to the corresponding cutoffs in the main text. Some algebra yields the threshold θ_q^* stated in the proposition. Differentiating this threshold with respect to q and ω, we obtain:

$$\frac{\partial\theta_q^*}{\partial q} = \frac{r_2}{R}\frac{r_2 q r_2 - r_2\omega r_1 - q r_2 r_2 + \omega L r_2}{\left(q r_2 - \omega r_1\right)^2} = -\frac{r_2^2}{R}\frac{\omega\left(r_1 - L\right)}{\left(q r_2 - \omega r_1\right)^2} < 0, \quad (59)$$

$$\frac{\partial\theta_q^*}{\partial \omega} = \frac{r_2}{R}\frac{-L q r_2 + L\omega r_1 + q r_2 r_1 - \omega L r_1}{\left(q r_2 - \omega r_1\right)^2} = \frac{r_2}{R}\frac{q r_2\left(r_1 - L\right)}{\left(q r_2 - \omega r_1\right)^2} > 0. \quad (60)$$

Second, we solve for the bank's choice at $t = 0$. Differentiating the expected profits (13) with respect to q, we obtain Equation (14). A high enough c ensures that the solution q^* is interior and unique (because $SOC_q < 0$ for high c).

Third, and finally, we study how an increase in CBDC remuneration affects financial stability. Note that the monitoring effort q^* directly depends on CBDC remuneration. Formally, the overall effect of a change in CBDC remuneration on bank monitoring effort can be expressed as follows (because of the IFT):

$$\frac{\partial q^*}{\partial\omega} = -\frac{\frac{\partial FOC_q}{\partial\omega}}{SOC_q}. \quad (61)$$

Since $SOC_q < 0$, the sign of $\frac{dq^*}{d\omega}$is equal to the sign of $\frac{\partial FOC_q}{\partial\omega}$, which is equal to

$$\begin{aligned}\frac{\partial FOC_q}{\partial\omega} &= -\frac{\partial\theta_s^*}{\partial\omega}\left(R\theta_s^* - r_2\right) - q\frac{\partial\theta_s^*}{\partial q}\frac{\partial\theta_s^*}{\partial\omega}R - q\frac{\partial^2\theta_s^*}{\partial q\partial\omega}\left(R\theta_s^* - r_2\right) \\ &= -\left[\frac{\partial\theta_s^*}{\partial\omega} + q\frac{\partial^2\theta_s^*}{\partial q\partial\omega}\right]\left(R\theta_s^* - r_2\right) - q\frac{\partial\theta_s^*}{\partial q}\frac{\partial\theta_s^*}{\partial\omega}R \\ &= -\left[\frac{\partial\theta_s^*}{\partial\omega} + q\frac{\partial^2\theta_s^*}{\partial q\partial\omega}\right]\left(R\theta_s^* - r_2\right) + \omega\left(\frac{\partial\theta_s^*}{\partial\omega}\right)^2 R,\end{aligned}$$

where

$$\begin{aligned}\frac{\partial^2\theta_s^*}{\partial q\partial\omega} &= -\frac{r_2^2}{R}\left(1 - L\right)\frac{\left(q r_2 - \omega\right)^2 + 2\omega\left(q r_2 - \omega\right)}{\left(q r_2 - \omega\right)^4} = -\frac{r_2^2}{R}\left(1 - L\right)\frac{q r_2 + \omega}{\left(q r_2 - \omega\right)^3} < 0 \\ &= -\frac{\partial\theta_s^*}{\partial\omega} - 2\omega\frac{q r_2^2}{R}\frac{\left(1 - L\right)}{\left(q r_2 - \omega\right)^3}.\end{aligned}$$

Substituting the expressions for θ_s^*, $\frac{\partial\theta_s^*}{\partial\omega}$, $\frac{\partial^2\theta_s^*}{\partial q\partial\omega}$, and $\frac{\partial\theta_s^*}{\partial q}$, we obtain

$$\frac{\partial FOC_q}{\partial\omega} = 2\omega\frac{q r_2^2}{R}\frac{\left(1 - L\right)}{\left(q r_2 - \omega\right)^3}\left(R\theta_s^* - r_2\right) + \omega\left(\frac{\partial\theta_s^*}{\partial\omega}\right)^2 R > 0,$$

which implies, in turn, that $\frac{dq^*}{d\omega} > 0$.

Finally, we move on to the total effect of CBDC remuneration on bank fragility:

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega}\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] > 0, \tag{62}$$

where the sign arises because one can show that

$$\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] = 1 + \frac{\omega}{q}\frac{\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}(R\theta - r_2)d\theta - \frac{cq^2}{2}\right)}{\partial q\partial\omega}}{\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}(R\theta - r_2)d\theta - \frac{cq^2}{2}\right)}{\partial q^2}} > 0.$$

G Proof of Proposition 8

Consider the limit $L \to 1$. Note that $r_1^* = 1$ continues to hold and the fundamental run threshold continues to be $\underline{\theta} = \frac{r_2}{R}$, irrespective of CBDC remuneration ω. However, the fundamental run threshold becomes our measure of bank instability.

The banker's expected profit is $\Pi = \int_{\underline{\theta}}^{1}(R\theta - r_2)d\theta$, which is also irrespective of CBDC remuneration ω. Thus, $\frac{d\Pi}{dr_2} < 0$ and the banker chooses the lowest feasible level of r_2. This level is pinned down by the consumers participation constraint:

$$V_F \equiv r_2(1 - \underline{\theta}) - \omega^2 \geq 0, \tag{63}$$

where V_F is the value of the deposit claim in the case of fundamental runs only. CBDC remuneration affects the bank's problem only via the participation constraint (but no longer via the failure threshold).

The value of the deposit claim increases in r_2, $\frac{dV_F}{dr_2} > 0$, as long as $r_2 < \frac{R}{2}$. Supposing $r_2 < \frac{R}{2}$ for now, solving for the smallest root yields

$$r_2^* = \frac{R}{2} - \sqrt{\left(\frac{R}{2}\right)^2 - R\omega^2}, \tag{64}$$

which confirms the supposition, $r_2^* < \frac{R}{2}$. To ensure a non-negative radicand, we assume

$$R \geq \underline{R}_6 \equiv 4\omega^2. \tag{65}$$

Since $\underset{\sim}{R}$ collects the lower bounds on investment profitability, we define:

$$\underset{\sim}{R} \equiv \max\{\underline{R}_1, \underline{R}_2, \underline{R}_4, \underline{R}_5, \underline{R}_6\}. \tag{66}$$

In equilibrium, the relevant measure of bank instability is $\frac{r_2^*}{R}$. Thus, any change in deposit rates directly changes bank instability. In particular, higher CBDC remuneration increases deposit rates (as in the main text) but now also increases bank instability:

$$\frac{dr_2}{d\omega} = \frac{R\omega}{\sqrt{\frac{R^2}{4} - R\omega^2}} > 0. \tag{67}$$

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