How Can Asset Prices Value Exchange Rate Wedges?*

Karen K. Lewis† Edith X. Liu‡

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Abstract

When available financial securities allow investors to optimally diversify risk across countries, standard theory implies that exchange rates should reflect this behavior. However, exchange rates observed in the data deviate from these predictions. In this paper, we develop a framework to value the welfare costs of these exchange rate wedges, as disciplined by asset returns. This framework applies to a general class of asset pricing and exchange rate models. We further decompose the value of these wedges into components, showing that the ability of goods markets to respond to financial markets through exchange rate adjustment has significant implications for welfare.

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†University of Pennsylvania, NBER, and CEPR; Email: lewisk@wharton.upenn.edu

‡Federal Reserve Board of Governors; Email: edith.x.liu@frb.gov

Resource dislocations across countries generated by recent events ranging from the Covid pandemic to increasing inflation rates raise important concerns about how individuals have been impacted. In principle, financial markets can mitigate these concerns by allowing for efficient sharing of risks. However, a long line of research has demonstrated that investors are not sufficiently diversified across countries.[1] Furthermore, disparities in inflation rates across countries point to the importance of exchange rates adjustment as a mechanism to reallocate resources. Thus, whether financial markets provide channels to minimize potential losses due to adverse exchange rate movements is a significant and enduring economic issue.

In this paper, we confront this important issue by asking: can asset prices be used to value the differences between observed real exchange rates and those implied by optimally diversified financial markets? If so, how? To answer these questions, we develop a general framework that conveniently nests a key building block in many macroeconomic and financial models. Thus, the framework provides an approach for using asset pricing data to value the potential under-diversified risk due to exchange rate misalignment.

The essential insight combines two well-known features. First, for a general class of preferences, welfare can be uniquely measured from the value of currrent wealth and consumption. Moreover, this value is implied by the budget constraint on lifetime consumption disciplined by asset return observations. Second, marginal utilities of consumption across individuals are equalized when available financial market securities span all of the economically important sources of risk; that is, when asset markets are complete. By contrast, when financial markets are incomplete, individuals value returns according to their own distinct intertemporal marginal utilities, or "stochastic discount factors." Combining these two features implies that the welfare differentials perceived by individuals across countries may be valued using asset returns together with standard techniques.

These two features have roots in two research traditions, in turn. The first tradition uses asset markets to uncover the implied "costs" of aggregate risk. This tradition has a long history including Lucas (1987), Alvarez and Jermann (2004), and others. The second tradition focuses upon international investor asset price deviations, or "wedges," in the presence of incomplete financial markets. This wedge was identified by D. Backus, Foresi,

[1] We discuss these papers in the Related Literature below.

and Telmer (2001) as the difference between the exchange rate in the data and the exchange rate implied by the ratio of stochastic discount factors across countries.[2]

To motivate our analysis, we briefly summarize here this measure of the international valuation wedge from the second literature as a starting point. For simplicity, assume there are two representative investors; a home investor with stochastic discount factor at state-time t defined as M_t and a foreign investor with counterpart $\widetilde{M}_t$. Further, denote S_t as the real exchange rate given by the relative price of foreign goods in units of domestic goods. Then, denoting with asterisk $*$ the complete markets counterparts to the variables above, the relationship between stochastic discount factors and prices can be written:[3]

$$M^*_{t+1}\left(S^*_{t+1}/S^*_t\right) = \widetilde{M}^*_{t+1}. \tag{1}$$

Analyzing this relationship requires data counterparts for these variables. While exchange rates are observed directly in the data, counterparts for stochastic discount factors are typically inferred indirectly using Euler equations disciplined with asset return data. It will be useful to distinguish these data-inferred variables from their complete markets counterparts with a "D" superscript. Using this notation, the complete markets relationship above may be rewritten as a deviation or "wedge" from the unique complete markets equation as:

$$\frac{S^D_{t+1}}{S^D_t}\frac{M^D_{t+1}}{\widetilde{M}^D_{t+1}} = \frac{S^*_{t+1}}{S^*_t}\frac{M^*_{t+1}}{\widetilde{M}^*_{t+1}}\exp(\eta_{t+1}) = \exp(\eta_{t+1}), \tag{2}$$

where the last equality follows from the identity in equation (1). Note that the variable η captures the difference between incomplete markets valuations across investors.

This paper combines these insights from the literature to consider asset market-implied costs of the deviation from a counterfactual equilibrium of complete markets. We begin by considering the value of the η wedge. For this purpose, we compare the two investor stochastic discount factors as in equation (2) to compare the valuations of a return implied by the data. Given that wealth is a sufficient statistic for welfare, we choose the return on wealth, defined

[2] The positive implications of these wedges have been examined by Sandulescu, Trojani, and Vedolin (2021), Lustig and Verdelhan (2019) and Bakshi, Cerrato, and Crosby (2018), as described in Related Literature below.

[3] On this equivalence between price-adjusted marginal utilities under complete markets, see Debreu (1959).

as R_c, as a candidate asset return to value across international investors. To illustrate the value of this return described in detail below, Table 1 considers a simple two-country example of this valuation using financial and macro data moments for two countries with close financial and trade ties: the U.S. and Canada. Given these close connections, the means and volatilities of consumption growth are similar across countries and their cross country correlations are greater than one half, as shown in Panel A. Despite these similarities, Panel B shows that the valuations of the η wedge, assuming that consumption growth in each country is identically and independently distributed (i.i.d.), is 31% of permanent consumption.[4] As discussed below, the size of this value is a reflection of the low correlation across countries in stochastic discount factors, a feature familiar from standard international diversification puzzles.

The η wedge provides a valuation comparison across investors under incomplete markets, but is silent on any deviation from complete markets. Therefore, it clearly does not answer the important question of how well asset markets may help buffer international shocks. We next illustrate this point by using the same data moments to infer the complete financial markets solution under the lens of three canonical exchange rate views: Non-tradeables vs Tradeables prices, Home Bias in preferences, and Sticky prices. We choose these three exchange rate examples to highlight a range of assumptions about how well goods markets function, ranging from fully flexible to fixed prices. While these cases provide useful examples, the framework can be applied more generally to other exchange rate approaches as well.[5] Calculating the value of the wedge compared to this counterfactual complete markets value of wealth implies more modest values of 1.76% and 1.93%.

Why are the η wedges in this example large when the wedges derived relative to complete markets are not? The η wedge definition in equation (2) shows that this wedge measures the difference in valuations between the two stochastic discount factors, M and $\widetilde{M}$, once converted into a common price as given by the data. Thus, given the imperfect correlation between consumption and prices in Table 1, even for two relatively integrated countries such as the U.S. and Canada, the derived benefit to a domestic investor of hypothetically

[4]Details of this simple example are described in Section 1.5 below as well as Appendix **??**.

[5]In Section 4 and Appendix **??**, we discuss these broader applications.

consuming the foreign wealth return corresponds to a less volatile consumption profile with the same growth rate, thereby generating large valuation differences. By contrast, when asset markets are complete, resources are redistributed across countries so that investors instead pool their wealth returns. They therefore internalize the effect of pooling that is absent in the η valuation. As such, these complete markets wedges generate valuation differences closer to business cycle costs.

Moreover, in contrast to values of the η wedge, the complete market wedges are dependent upon how exchange rates are determined. Intuitively, the value of completing financial markets depends on any goods market constraints because they impact the ability for prices to adjust to redistributed consumption. For example, when some goods are nontradeable, then a redistribution of tradeables across countries due to greater access to tradeable securities will alter the relative price of nontradeables and the exchange rate.

While i.i.d consumption provides a useful initial example, disciplining the stochastic discount factors with asset returns requires some persistence in investor risk. For example, this persistence may arise due to fears of a disaster (e.g., Barro and Ursúa (2008), Nakamura, Steinsson, Barro, and Ursúa (2013)), habit persistence (Campbell and Cochrane (1999)), and long run risk (Bansal and Yaron (2004)), to name a few. Although all of these models could be used in our framework, for parsimony we show the basic approach using only the long run risk version. As an illustration of this approach, Panel C reports the results of fitting the financial data moments for the two countries to a long run risk version using Simulated Method of Moments. In particular, the panel provides financial data moments such as real equity returns and risk-free rates for the two countries measured in real domestic price units.[6] To discipline measures of the stochastic discount factors with asset returns, we report our valuations throughout the rest of the paper using these implied measures.[7]

Our paper provides another important contribution by showing how the wedges can be decomposed. For example, as equation (2) highlights, the η wedge not only depends upon the exchange rates, but also upon the ratio of stochastic discount factors across countries. Thus, η may be comprised of three different wedges corresponding to each variable derived

[6]Appendix **??** describes these data and the Simulated Method of Moments matching approach.

[7]For this simplified simulation, we treat volatility as homoskedastic, although including time-varying volatility would improve the fit further.

from the data. More generally, as a novel implication of our framework, we decompose the value of the wedges into the components due to exchange rates, and to the stochastic discount factors of domestic versus foreign investors. Using only standard preferences and asset pricing assumptions, our simple decomposition example suggests that much of the value of wealth under complete markets relative to data measures comes from wedges in stochastic discount factors, instead of the exchange rate.

Finally, this wedge decomposition also provides another insight that is novel to this paper. That is, we show that the returns on wealth are equalized under complete markets once adjusted by the impact of exchange rates. Therefore, we can solve for the lifetime value of the effects of exchange rates on this common complete markets wealth return. We term this new wedge the "Total S-Wedge." Using our same data moments, this wedge implies that exchange rates have a negative impact on welfare.

The format of the paper is as follows. Section 1 sets up the valuation framework for the real costs of η and also shows that this wedge framework matches various exchange rate puzzles that are not targeted by our approach. Section 2 describes how the same framework can be used to evaluate the complete markets wedge and its decomposition into exchange rates and stochastic discount factors. Section 3 shows a new complete markets relationship implied by wealth returns and its Total S-Wedge effect. Section 4 describes various generalizations of the approach including multiple countries, alternative exchange rate views, and asset pricing models. Concluding remarks are in Section 5.

Related Literature: Since this paper is related to a number of important literatures in macroeconomics and finance, we mention only representative papers within each literature.

First, our paper is related to the literature on consumption, exchange rates, and complete markets. Brandt, Cochrane, and Santa-Clara (2006) use the η wedge to illustrate the risk-sharing puzzle implied by consumption and exchange rate data. Lustig and Verdelhan (2019) analyze risk-free rates to consider implications of the wedge for exchange rates while Bakshi et al. (2018) evaluate exchange rates and a portfolio of international returns. Sandulescu et al. (2021) demonstrate that the ratio of stochastic discount factors do not correspond to the equilibrium exchange rate in complete financial markets. Burnside and Graveline (2020) show that evaluating the costs from imperfect financial market risk-sharing inherent in these data requires an economic framework that depends upon goods markets, but do not specify that framework. Instead, our approach provides a framework that connects the implicit goods market conditions to financial markets.[8] Moreover, in contrast to the literature, our approach further allows a decomposition of these wedges into their respective components.

Second, this paper relates to the growing international asset pricing literature based upon complete markets that identifies the exchange rate with the ratio of stochastic discount factors. A number of studies have used this identity including, among others, Colacito and Croce (2011) and Colacito, Croce, Gavazzoni, and Ready (2018) with long run risk, Farhi and Gabaix (2016) with disaster risk, and Lustig, Stathopoulos, and Verdelhan (2019) for the term structure of exchange rate returns. In contrast with these papers, this paper begins with the presumption that markets may be incomplete and uses data to value its importance.

Third, our paper is related to the literature on consumption insurance and its welfare costs. These studies include household level analysis as in Mace (1991) and Cochrane (1991), and the welfare costs of business cycles as in Lucas (1987), and Alvarez and Jermann (2005). It is also related to the implications of that consumption insurance across countries noted by Obstfeld (1994), Tesar (1995), and Kalemli-Ozcan, Sørensen, and Yosha (2003) as well as the financial market integration literature including Bekaert, Harvey, Lundblad, and Siegal (2011) and Carrieri, Chaieb, and Errunza (2013). However, these papers do not consider goods market price effects on the international risk-sharing.

Finally, this paper is connected to the literature that examines the general connection between exchange rates and consumption aggregators used in international macroeconomics and finance. In highlighting the role of exchange rates, these papers range from D. K. Backus, Kehoe, and Kydland (1992), Coeurdacier and Rey (2012), and Berka, Devereux, and Engel (2018) in macroeconomics to Pavlova and Rigobon (2007), Verdelhan (2010), and Ready, Roussanov, and Ward (2017) in financial economics. This literature focuses upon understanding or explaining regularities in the data. By contrast, we provide a framework to ask what these models would imply about the costs of international financial market wedges.

[8] This aspect of the framework therefore builds on the seminal work by Cole and Obstfeld (1991).

1 Valuing the η Wedge

We now describe the general framework for valuing the Table 1 costs using standard Euler equation solutions in a model-free environment. Specifically, we show how to calculate the cost of the η wedge as well as the implied cost of a total wedge deviation from complete markets. For now, we assume that there exist data measures of all key economic variables, although the end of this section describes identifying data assumptions.

1.1 Domestic and Foreign Investor Valuations

To understand the valuation approach, we briefly summarize the relationship described in D. Backus et al. (2001), hereafter BFT, who pointed out a connection between the standard Euler equation based upon complete markets and its counterpart used in empirical analysis. This connection arises when a domestic investor who consumes in local goods units evaluates an asset with return payouts that are denominated in the units of a foreign good. Without loss of generality, we will treat the foreign price level as the numeraire and define the return of any asset that provides payouts denominated in those units at time t as $\widetilde{R}_{a,t}$. Then, the return on this foreign-denominated asset measured in units of domestic goods is: $\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)$ where S_t is the real exchange rate, defined by the price of foreign goods in units of domestic goods. Then the Euler equation for that foreign asset return valued by domestic investors and by foreign investors, respectively, can be written as:

$$E_t\left\{\widetilde{M}_{t+1}\widetilde{R}_{a,t+1}\right\} = 1 \tag{3}$$

$$E_t\left\{M_{t+1}\widetilde{R}_{a,t+1}\left(S_{t+1}/S_t\right)\right\} = 1 \tag{4}$$

Given these valuations, BFT combine two relationships. The first is that the stochastic discount factors measured in common price units are the same under complete markets as in equation (1). In this case, the above two Euler equations (3) and (4) are equivalent. The second relationship is that these complete markets identities can be rewritten as deviations from their counterparts in the data, if markets are not complete. Rewriting the data-implied counterparts for exchange rates and stochastic discount factors relative to the complete markets alternative defines η as in equation (2). Although these relationships have been combined with various asset returns to infer properties of η, we will focus on the implied wealth returns to measure the welfare implications of incomplete markets.

1.2 Wedges and Life-Time Consumption

The η wedge valuation above can be recast in welfare terms by replacing the return on any asset, R_a, with the return on wealth.[9] Specifically, defining C_t^D as consumption measured in the data, the value of wealth or lifetime consumption as measured by data, W_t^D, can be written as: $W_t^D \equiv C_t^D + \Gamma_t^D$ where Γ_t^D is the present value of future consumption discounted by the stochastic discount factors in the data.[10] Moreover, it will be useful to define the price ratio of this future wealth to consumption as: $Z_t \equiv \Gamma_t / C_t$. Therefore, wealth in the data can be written with this definition as:

$$W_t^D \equiv C_t^D (1 + Z_t^D) \tag{5}$$

Using this definition, the value of the future consumption sequence may then be calculated using the Euler equations above by treating the realization of consumption as the return on an asset R_c^D for the domestic investor and $\widetilde{R}_c^D$ for the domestic investor given as:

$$R_{c,t+1}^D \equiv (C_{t+1}^D / C_t^D)(1 + Z_{t+1}^D)/Z_t^D; \; \widetilde{R}_{c,t+1}^D \equiv (\widetilde{C}_{t+1}^D / \widetilde{C}_t^D)(1 + \widetilde{Z}_{t+1}^D)/\widetilde{Z}_t^D \tag{6}$$

We call these variables the wealth returns because they measure the per-period value of payouts on a claim to future lifetime consumption to each investor. The value of wealth can then be priced with data on consumption and the stochastic discount factors using the Euler equation for the wealth return. Specifically, substituting equation (6) into:

$$E_t \left\{ M_{t+1}^D R_{c,t+1}^D \right\} = 1 \tag{7}$$

[9] The value of lifetime consumption is equal to wealth through the intertemporal constraint. See for example, Cochrane (2005) for financial economics and Rogoff and Obstfeld (1996) for international finance. Campbell (1993) uses this condition to replace consumption with wealth.

[10] In particular, $\Gamma_t^D \equiv E_t \sum_{\tau=1}^{\infty} Q_{t+\tau}^D C_{t+\tau}^D$ where the discount factors are the intertemporal stochastic discount factors between $t+1$ and future periods $t+\tau$. That is, $Q_{t+\tau}^D \equiv \Pi_{j=1}^{\tau} M_{t+j}^D$ and $Q_{t+\tau}^* \equiv \Pi_{j=1}^{\tau} M_{t+j}^*$.

and solving for Z_t^D provides the value to the domestic investor of the future wealth-to-consumption ratio. For simplicity, throughout the rest of the paper, we refer to this ratio as the "price ratio". Further substituting Z_t^D into the wealth definition in equation (5) and repeating the process for the foreign investor provides the values of wealth, W_t^D and $\widetilde{W}_t^D$.

We can then use this insight to measure the value of the wedges. For example, the η wedge relative to wealth is captured in the two valuations given in the equations (3) and (4) by setting the return $\widetilde{R}_{a,t+1} = \widetilde{R}_{c,t+1}$ and denoting the variables with "D", implying the set of Euler equations:[11]

$$E_t \left\{ \widetilde{M}_{t+1}^D \widetilde{R}_{c,t+1}^D \right\} = 1 \tag{8}$$

$$E_t \left\{ M_{t+1}^D \widetilde{R}_{c,t+1}^D \left(S_{t+1}^D / S_t^D \right) \right\} = 1 \tag{9}$$

Clearly, solving equation (8) for $\widetilde{Z}^D$ in $\widetilde{R}_{c,t+1}^D$ gives the foreign investor's internal valuation of their own wealth price ratio as described above. However, solving equation (9) for the value of that same foreign wealth payout to the domestic investor implies a different price ratio, when markets are incomplete. We define this price ratio from domestic investors as $\widetilde{Z}^\eta$ and next connect these different values of foreign wealth to welfare measures.[12]

1.3 Certainty Equivalent Consumption Wedges

We can now consider the relative value to an investor consuming their own home country wealth relative to a foreign investor's valuation of that same wealth payout, but measured in foreign price units. For this purpose, consider a general utility function $U(C_t)$ for an agent with a set of resources that determines an intertemporal budget constraint. The optimization of preferences given this constraint then implies a sequence of lifetime consumption $\{C_t\}$ and a value function, $V(W_t)$. Assuming that preferences are homogeneous, this value function can be written as: $V(W_t) = C_t V(W_t/C_t)$.

To measure the relative welfare cost of an η wedge, we consider the value to a domestic agent investor of the foreign investor's lifetime wealth, a measure defined as: $W_t^\eta \equiv$

[11] We could also calculate the value of domestic wealth return from the point of view of each investor, but since we assume symmetry in most of the paper only one view is reported for parsimony. Section 4 describes non-symmetric settings.

[12] Appendix **??** describes the connection between these and other price ratios in more detail.

$C_t^D(1+Z_t^\eta)$. This measure provides the implied wealth faced by the domestic investor using lifetime consumption measures in equation (7) given by W_t^D relative to an alternative lifetime consumption of the foreign wealth implied by W_t^η.

In permanent consumption units, what is the value to such an agent of consuming not their own wealth, but that of the foreign agent? Clearly, the values to the agents of the two economies are equivalent when markets are financially complete because the returns on lifetime wealth are equalized.[13] However, when markets are incomplete, the value of lifetime wealth by the domestic agent valuing foreign wealth relative to their own wealth will differ by $\Delta_{\eta,D}$ in the expression:

$$(1-\Delta_{\eta,D}) = \frac{V(W_t^D/C_t^D)}{V(W_t^\eta/C_t^D)} = \frac{V(1+Z^D)}{V(1+Z^\eta)} \tag{10}$$

In other words, $\Delta_{\eta,D}$ measures the certainty equivalent (CE) difference in permanent consumption of the hypothetical value to domestic households of consuming the foreign wealth return relative to value of their own domestic wealth return. We use this insight next.

1.4 Quantifying η Wedges

As the discussion above shows, the consumption processes in each country can be used to construct the relative value of wedges given preferences that are homogeneous in wealth. As an example, we consider Epstein-Zin-Weil recursive preferences for the value function as it conveniently nests common preferences used in macro-finance, including Constant-Relative Risk Aversion (CRRA). We also assume that the two countries have identical preferences over time and aggregate consumption. Thus, for the domestic investor, the utility at time t over the general consumption basket can be written:

$$U(C_t, U_{t+1}) = \left\{ C_t^{\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{11}$$

[13] When countries are not symmetric in initial wealth, the complete markets solution of the social planner must compensate some investors with differential initial levels of consumption. In this section, we only consider symmetric countries for simplicity but return to this issue in Section 4 below.

where U_{t+1} is the utility function at $t+1$; $0 < \beta < 1$ is the time discount rate; $\gamma \geq 1$ is the risk-aversion parameter; $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution; and where $\mathrm{E}_t(\cdot)$ is the expectation operator conditional on the information set at time t.[14] The foreign country preferences are identical with variables $\widetilde{C}$ and $\widetilde{W}$. However, we will allow below for country-specific preferences in individual goods within the consumption aggregates.

To compare these consumption processes in welfare units requires the value function solution for each country. Using the value function from Epstein and Zin (1991) and Weil (1990), equation (11) can be written with the future wealth ratio Z_t as:

$$V(C_t, W_t) = (1 + Z_t)^{\Psi} C_t \tag{12}$$

where W_t is the present value of all future expected consumption and where $\Psi \equiv \psi/(\psi - 1)$. Therefore, rewriting the solution for the cost of wedges in equation (10) using equation (12) provides a general form for the Certainty Equivalent value of the η wedge given as $\Delta_{\eta,D}$ in:

$$(1 - \Delta_{\eta,D}) = \frac{V(W^D/C^D)}{V(W^\eta/C^\eta)} = \left\{ \frac{1 + Z^D}{1 + Z^\eta} \right\}^{\Psi} \tag{13}$$

Clearly, valuing the difference in lifetime CE units for investors facing the η wedge depends upon the price-ratios Z given by the consumption processes.

1.5 A Simple Two-Country Example: η Wedge Explained

We now explain the η valuation calculations in Table 1, relegating details to Appendix **??**. Specifically, Panel B reports the results of $\Delta_{\eta,D}$ for the symmetric example of two countries, solved in equation (13). The required price ratios, Z^D and Z^η, are respectively determined by the valuation of the foreign wealth return by foreign investors in (8) and by domestic investors in (9). Note that the foreign wealth return, $\widetilde{R}_c$, is inferred from the consumption process of the foreign country. Thus the Euler equations represent internal investor valuations of consumption claims that in general need not correspond to an actual traded security. Observed asset return data are therefore used to determine these internal

[14] Specifically, we follow the form in Epstein and Zin (1989), equation (5.3) which specializes to standard time-additive CRRA preferences when $\gamma = \frac{1}{\psi}$.

valuations of wealth returns through the price ratios.

For the purpose of solving for these Z price ratios, we use the data moments in Panel A for the United States and Canada first assuming joint log normal i.i.d consumption growth. As detailed in Appendix **??**, calculating the relevant Euler equations then only requires the means, variances and co-variances of the two consumption and real exchange rate growth processes. Feenstra, Inklaar, and Timmer (2015) argue that for welfare comparisons, consumption and relative prices must be comparable across countries and time as in the Penn World Tables (PWT). Therefore, we discipline the valuations using PWT 9 data for the U.S. and Canada, and further extend to the United Kingdom and Australia in Section 4.

For the simple i.i.d consumption case reported in Table 1B, each country follows identical consumption processes given here for the domestic country along with the exchange rate as:

$$\ln(C^D_{t+1}/C^D_t) = \mu + \sigma_c \nu_{t+1} \; ; \; \ln(S^D_{t+1}/S^D_t) = \sigma_s \nu^s_{t+1} \tag{14}$$

where $\nu_t \sim N(0,1)$, $\nu^s_t \sim N(0,1)$, and where the cross-country correlations of consumption growth rates and exchange rates are defined as: $Corr(c,\widetilde{c}) \equiv Corr(\nu_t,\widetilde{\nu}_t)$ and $Corr(c,s) \equiv Corr(\nu^s_t,\nu_t)$, respectively. This simple example assumes random walk processes for consumption and the exchange rate, although we modify this assumption below.[15] To focus upon the implications of diversification, we assume that the two country mean growth rates, μ, and the standard deviations, σ_c, in Table 1 A are the same across the two countries.

Then, why is the value of the η-Wedge reported in Table 1 so large? Comparison of the two investor Euler equations (8) and (9) of the same foreign wealth return $\widetilde{R}_c$ in equation (6) helps answer that question. Noting that the stochastic discount factors, M and $\widetilde{M}$, depend upon each country's respective consumption growth rate, then the $\widetilde{Z}^D$ priced by equation (8) depends only on the variance of the local foreign consumption growth rate, σ_c. By contrast, when the domestic investor values foreign wealth return in equation (9), the price ratio Z^η depends on the second moments of M, $\widetilde{M}$, and S. These moments include the cross-country consumption correlations, $Corr(c,\widetilde{c})$, the correlation between consumption

[15] These assumptions are made only for simplicity. Alternatively, these processes could include a cointegrating error-correction term as in Colacito and Croce (2013). We discuss this possibility in Section 4, as well as alternative exchange rate processes that allow for long term mean reversion in the real exchange rate.

and exchange rates, $Corr(c, s)$, and the variance of the exchange rate. Clearly, if stochastic discount factors were equalized as under complete markets in equation (1), then either the correlations of consumption would be equal to one if the $\sigma_s = 0$ or else exchange rate movements would explain the observed consumption correlations, a point made by Brandt et al. (2006). However, the observed correlations of consumption across these two countries at 0.57 and of consumption with the exchange rate at -0.18 are relatively low. Therefore, the value to domestic investors of the foreign consumption process implies a lower variability, and thereby a high value of the η wedge in this example.[16] The high valuation reflects the well-known regularity that investors do not sufficiently diversify internationally, a phenomenon also captured in low consumption growth correlations.[17]

1.6 Matching Asset Return and Exchange Rate Regularities

The specification of i.i.d consumption growth as in Table 1B is inconsistent with asset return behavior.[18] Therefore, in Table 1C, we include a persistent risk component in consumption. SMM is then used to estimate the persistent "long run risk" component that provides the best fit for the reported equity and the risk-free rate moments. For parsimony, we do not include stochastic volatility and therefore the asset returns are somewhat less volatile than the data. However, the fit does a reasonable job of matching return levels. We therefore report results throughout the rest of the paper using this asset-return disciplined consumption process. This analysis also matches exchange rate regularities discussed in recent papers such as Lustig and Verdelhan (2019) and Jiang, Krishnamurthy, Lustig, and Jialu (2022). We therefore highlight them here.

A. Volatility The analysis in Table 1 is calibrated to the volatility of the real exchange rate and is therefore matched by construction. Similarly, the cross-country consumption correlations and standard deviations and asset returns are fit to data moments.

B. Foreign Exchange Risk Premium We do not target the foreign exchange return volatility. Nevertheless, the implied volatility at 1.40% matches the volatility in the data of

[16] Sufficiently high levels for exchange rate volatility can significantly reduce the value of η and even drive it negative. The range of plausible η values for our sample of OECD countries was positive both with i.i.d. and persistent consumption.

[17] See, e.g., Bekaert et al. (2011), Carrieri et al. (2013) for financial and Coeurdacier and Rey (2012) for macro variables.

[18] For example, the equity premium is too low and the risk-free rate is constant (e.g., Mehra and Prescott (1985), Weil (1990)).

2.77% reasonably well. As with the other asset returns, stochastic volatility would improve this fit further and could readily be input into the wedge analysis.

C. Cyclicality Another exchange rate puzzle is the low correlation between consumption and exchange rates, as pointed out by D. Backus and Smith (1993). Indeed, the correlation between exchange rates and consumption growth is -0.18 in Table 1, a number that is within the range reported by Lustig and Verdelhan (2019). Thus, overall, the wedge analysis below incorporates the essential exchange rate puzzles from the literature.

2 Valuing the Complete Financial Markets Wedge

The prior section described a framework for measuring the valuation wedge between investors across countries using data measures of returns. However, this η wedge does not capture any deviation from complete markets if observed markets are truly incomplete. As a result, the deviation of the exchange rate from its complete market counterpart cannot be directly observed. Therefore, we now show how the same data can be used to infer the value of the wedge deviation from incomplete markets. Details are relegated to Appendix **??**.

2.1 Certainty Equivalent Costs of Incomplete Financial Markets

To motivate this investigation, we return to the insight in the previous section that the value of lifetime wealth can be implied by the Euler equation (7). By this reasoning, we define the difference in the valuation of the state price of current wealth relative to an alternative complete markets wealth signified as $*$ as:

$$M^D_{t+1}R^D_{c,t+1} \equiv \zeta_{t+1}M^*_{t+1}R^*_{c,t+1} \equiv \zeta_{t+1}\widetilde{M}^*_{t+1}\left(S^*_{t+1}/S^*_t\right)^{-1}R^*_{c,t+1} \tag{15}$$

where the second identity follows from the complete markets relationship between stochastic discount factors in equation (1). This ζ wedge measures the state price deviation of the return on wealth relative to its complete markets alternative. In this case, the value of the deviation ζ could be measured by solving for the return in Euler equation (7) replacing the consumption process in the data with a consumption process for an investor with access to complete financial markets. Defining the domestic consumption process for an investor facing complete markets as C^* and the wealth price-to-consumption ratio as Z^*, then the implied value of wealth under complete financial markets is:

$$W_t^* \equiv C_t^*(1 + Z_t^*) \tag{16}$$

Then the certainty equivalent loss for the domestic country, $\Delta_{D,*}$, is given by the following:

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi} \tag{17}$$

where we have used the fact that under symmetry, $C^D = C^*$.[19] Since this cost compares wealth in the data to its counterpart under complete markets, it must be non-negative.

Measuring the counterfactual consumption and prices under complete markets requires deriving the resource allocations implied by spanning a set of Arrow-Debreu securities. These allocations can be derived as the outcome of a planner's problem facing any relevant goods market constraints. To see that outcome in the general context, first define the world aggregate consumption and world wealth both measured in the foreign numeraire country price units as: $\widetilde{C}_t^w \equiv S_t^* C_t^* + \widetilde{C}_t^*$ and $\widetilde{W}_t^{w*} \equiv S_t^* W_t^* + \widetilde{W}_t^*$ where $S_t^* \equiv (P_t^*/\widetilde{P}_t^*)$ is the real exchange rate between the domestic and foreign country when markets are complete. Then optimal consumption for the domestic and the foreign investors are given by a sharing rule:

$$C_t^* = \omega_t \widetilde{C}_t^w; \; \widetilde{C}_t^* = (1 - \omega_t)\widetilde{C}_t^w \ . \tag{18}$$

where $\widetilde{C}_t^w$ is aggregate world consumption measured in numeraire units and where ω_t is the share for the domestic country implied by complete asset market spanning.

This sharing rule depends upon any goods market restrictions that impact the ability of the real exchange rates to adjust. Therefore, these total ζ wedge valuations differ by exchange rate view because the shares include relative goods price effects generated by consumption reallocations. The implications for these differences were reported in Table 1 Panel C for certainty equivalent consumption based upon three different exchange rate views discussed

[19] These consumption levels will differ when countries have asymmetric resource processes, as described in Section 4.

below: Nontradeables price, Home Bias, and Sticky Prices. As these numbers showed the deviations are around 2% of permanent consumption, which are far lower than the implied numbers for the η wedge. We next describe these examples to illustrate the role of goods market constraints in impacting the costs of financial market frictions.

2.2 The Role of Goods Market Frictions

We choose these three examples as representatives of a range of goods market responses. The "Sticky Price" case considers a counterfactual extreme in which prices do not adjust to reallocations in consumption, and thereby provides a metric to compare other endogenous exchange rate versions. By contrast, the "Home Bias" case represents a different extreme in which goods markets adjust frictionlessly to clear commodity prices. In this case, completing financial markets can impact exchange rates by reallocating resources across investors who have different preferences for goods. Finally, the "Non-Tradeables" case provides an intermediate goods market version in which prices adjust to clear the market for tradeable goods, but not for nontradeable goods. Greater access to securities that reallocate tradeables then impacts the real exchange rate through the relative price of nontradeables.[20]

To understand the effects of the different exchange rate views, we describe next how the complete markets exchange rates can be recovered from the data under the lens of these three goods market views. Moreover, viewing the data through the goods market equilibrium condition disciplines the quantity of resources that the planner can reallocate. We briefly summarize in turn each of these solutions as they relate to data measures.

A general feature of this approach can be seen by rewriting the relationship between exchange rates and stochastic discount factors in equation (1) using the solution for the stochastic discount factor for Epstein-Zin preferences given as: $M_{t+1} = \beta^{\theta}(C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)}(R_{c,t+1})^{\theta-1}$. Thus, combining this complete markets condition with the sharing rule, the relationship

[20] Appendix ?? details the implications for the sharing rule ω for these different views of exchange rates determination, which is based upon allowing the planner to reallocate consumption although we can in principle based our analysis on reallocations of income as described in Section 4.

between stochastic discount factors can be restated as:

$$\frac{\widetilde{M}^*_{t+1}}{M^*_{t+1}} = \left(\frac{\widetilde{C}^*_{t+1}/\widetilde{C}^*_t}{C^*_{t+1}/C^*_t}\right)^{-\gamma}\left(\frac{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}{(1+Z^*_{t+1})/Z^*_t}\right)^{(\theta-1)} = \frac{S^*_{t+1}}{S^*_t}. \tag{19}$$

We use this first-order condition to the planner's optimization to illustrate how the goods market restrictions impact the cases below. In each case, we infer the existing quantity of commodities to be reallocated by financial markets using data on exchange rates and consumption aggregates.[21]

Sticky Prices This version provides a simple benchmark to consider the case when complete financial markets cannot alter the exchange rate. The social planner is then constrained to take prices as given by the data. Thus, calculating the complete markets shares of aggregate consumption for each country simply requires reallocating aggregate consumption at the given prices. Based upon this assumption, the aggregate resource constraint is just given by the aggregate world consumption in numeraire units at the data exchange rate; that is, $\widetilde{C}^w_t \equiv S^*_t C^*_t + \widetilde{C}^*_t = S^D_t C^D_t + \widetilde{C}^D_t$. Since this case solves for financial market adjustment without any goods market response, by construction, the exchange rate in the data is the same under complete financial markets so that: $S^* = S^D$.

The optimal consumption growth in this case has an intuitive form. For example, in the case where consumption growth is i.i.d. and the countries are symmetric, then, equation (19) can be rewritten in log growth terms to imply:

$$g^*_{c,t+1} = \widetilde{g}^*_{c,t+1} + \frac{1}{\gamma}g^*_{s,t+1} = \frac{1}{2}g^{wD}_{c,t+1} - \frac{1}{2}(1-\frac{1}{\gamma})g^D_{s,t+1}. \tag{20}$$

Thus, the complete markets domestic consumption growth rate is an equal share of aggregated world consumption adjusted by the intertemporal effect of the exchange rate, both measured from aggregate consumption and exchange rate growth in the data.

Home Bias This version assumes that goods markets function frictionlessly so that prices clear individual commodity markets. According to this view, the real exchange rate

[21] We treat consumption allocations in the data as the outcome of an unspecified incomplete markets equilibrium and therefore are agnostic about the nature of financial market integration in the data. As a result, the analysis is isomorphic to the "portfolio autarky" condition studied in i.i.d. by Cole and Obstfeld (1991) and in a long-run risk setting by Colacito and Croce (2013).

varies due to a greater preference for the home goods produced in each respective country.[22] In this case, suppose there are two goods, indexed by 1 for the good produced in the domestic country and 2 for the good produced in the foreign country. Following much of the literature, we assume that the consumption aggregator over these two goods is Cobb-Douglas. Then, since the domestic investor prefers their own good, the domestic and foreign consumption aggregators can be written as:

$$C_t = (C_{1,t})^a (C_{2,t})^{1-a}; \ \widetilde{C}_t = \left(\widetilde{C}_{1,t}\right)^{1-a} \left(\widetilde{C}_{2,t}\right)^a \tag{21}$$

where $a > 1/2$. In this case, the real exchange rate implied by goods market equilibrium is: $S_t = (P_t/\widetilde{P}_t) = (P_{1,t}/P_{2,t})^{2a-1}$ where $P_{i,t}$ is the price of good i. Thus, as the planner reallocates domestic and foreign goods across countries, the exchange rate under complete markets will generally differ from the exchange rate data.

To quantify the goods that can be distributed, we follow the assumption of this exchange rate view that goods market equilibrium holds within any given period. By contrast, if financial markets are incomplete, intertemporal consumption decisions will not be optimized in general. In this case, we can uncover the relative consumption in the domestic country for each good depending on its relative price in the data, a decision that in turn depends on the observed exchange rate. Specifically, this first-order condition can be written:

$$\left(C^D_{1,t}/C^D_{2,t}\right) = a(1-a)^{-1} \left(S^D_t\right)^{(1-2a)}$$

A similar relationship holds for the foreign country but with preferences for good 2. Using these conditions allows us to measure the global resource constraint for each good as a function of observed exchange rates and consumption aggregates. The implied complete markets solutions generate sharing rules that allocates the world supplies of each good to optimize risk-sharing over time. The complete markets exchange rate, S^*_t, will be changed as a result of these reallocations and, consequently, will differ from the exchange rate in the data.

[22] Studies that have used this approach to explain exchange rate behavior include Colacito and Croce (2011), Coeurdacier and Rey (2012), and Stathopoulos (2021), among many others.

Non-Tradeables While these two extreme perspectives on goods markets provide useful benchmarks, the standard non-tradeables view of exchange rates treats goods markets somewhere in between. That is, only the traded goods market is assumed to clear internationally while the nontradeables markets do not.[23] Continuing the two country Cobb Douglas example and defining $C_{T,t}$ and $C_{N,t}$ as the consumption of tradeables, T, and nontradeables, N, respectively, the consumption aggregator for both countries would be:

$$C_t \equiv C(C_{T,t}, C_{N,t}) = (C_{T,t})^{\alpha} (C_{N,t})^{1-\alpha} \tag{22}$$

Thus, defining $P_{T,t}$ as the price of tradeables and $P_{N,t}$ and $\widetilde{P}_{N,t}$ as the price of nontradeables in the domestic and foreign countries, respectively, the real exchange rate becomes $S_t = (P_{N,t}/\widetilde{P}_{N,t})^{1-\alpha}$, the relative price of non-tradeables across countries.

Given that the goods market does not equilibrate for nontradeables goods, the financial market preserves this condition as a resource constraint. In order to identify the quantity of tradeables implied by goods market equilibrium, we again use the insights of the goods market condition implying that the domestic country Tradeables and Nontradeables consumption can be measured as:

$$\left(C_{T,t}^{D}/C_{N,t}^{D}\right) = \alpha(1-\alpha)^{-1}\widetilde{\rho}_{N,t}\left(S_t^{D}\right)^{(1-\alpha)^{-1}}$$

where $\widetilde{\rho}_{N,t}$ is the relative price on nontradeables in the foreign country. A similar counterpart relationship holds for the foreign country. Using this condition with consumption allows the aggregate world tradeables to be inferred from the data. Optimizing the sharing rule for tradeables across investors implies that the planner will redistribute according to nontradeables shocks as well as tradeables. As such, the real exchange rate will again be different under complete markets then the exchange rate implied by the data.

[23]See for example Cole and Obstfeld (1991), Engel (1999), and Asea and Mendoza (1994).

2.3 The Exchange Rate Wedge and Decomposition

Since complete markets implies a reallocation of existing consumption, the implied real exchange rate in general differs from the exchange rate in the data, as noted above. We therefore call this deviation the S-Wedge, as given by the exchange rate wedge between the data and its complete markets alternative as:

$$\left(\frac{S_{t+1}^{D}}{S_{t}^{D}}\right) \equiv \zeta_{S,t+1}\left(\frac{S_{t+1}^{*}}{S_{t}^{*}}\right) \tag{23}$$

The approach to value wealth inferred from the data and under complete markets suggest that this S-Wedge may be valued in the same way. In particular, we can compare the value of complete markets wealth distorted by the exchange rate in the data as the wealth price ratio-consumption implied by solving the Euler equation:

$$E_t\left\{\widetilde{M}_{t+1}^{*} R_{c,t+1}^{*}\left(S_{t+1}^{D}/S_{t}^{D}\right)\right\} = 1 \tag{24}$$

where Z^S is the wealth-consumption ratio that solves equation (24) for the consumption process under the risk-sharing rule in equation (18). Given this price ratio, we can again consider the welfare loss for an investor with lifetime consumption under complete markets relative to an investor with consumption distorted by the exchange rate wedge:

$$1 - \Delta_{S,*} = \left\{\frac{1+Z^{S}}{1+Z^{*}}\right\}^{\Psi} \tag{25}$$

where $\Delta_{S,*}$ is the certainty equivalent cost of consuming wealth under complete markets but distorted by the exchange rate in the data.

The two wedges given by η in equation (13), and by the S Wedge in equation (25) suggests a general decomposition of the total welfare costs relative to complete markets given by ζ in equation (17). Specifically, defining the value of future wealth-to-consumption of any two arbitrary sets of lifetime consumption streams labeled X and Y as, respectively, Z^X and Z^Y, the certainty equivalent cost of an investor consuming wealth X instead of Y is given by:

$$1 - \Delta_{X,Y} \equiv \left\{ \frac{1 + Z^X}{1 + Z^Y} \right\}^{\Psi} \tag{26}$$

Then the wedges for η and for the exchange rate S Wedge can be used to decompose the effect on the overall cost Δ. That is, since the total wedge can be rewritten according to this decomposition as: $1-\Delta_{D,*} = (1 - \Delta_{D,\eta})(1 - \Delta_{\eta,S})(1 - \Delta_{S,*})$, and because $\ln(1-\Delta) \approx -\Delta$, then the impact of a particular wedge relative to other wedges can be decomposed as:

$$\Delta_{D,*} = \Delta_{D,\eta} + \Delta_{\eta,S} + \Delta_{S,*} \tag{27}$$

Figure 1 illustrates the structure of this decomposition, subsuming the time subscripts in the labels for clarity. The top row illustrates the total cost, $\Delta_{*,D}$ between Complete Markets and the Data Inference. This cost compares the difference between the price ratios that value the complete markets wealth, M^*R^* and the standard data-inferred wealth, M^DR^D. However, the vertical arrows provide an "under-the-hood" evaluation of the impact of, alternatively, the S-Wedge and the η-Wedge. That is, the first vertical column demonstrates the relative valuations of wealth to an investor under complete markets M^*R^* relative to one distorted only by the effect of exchange rates, $\widetilde{M}^*R^*S^D$. This comparison is measured by $\Delta_{*,S}$. By contrast, the impact of the η-Wedge compares two valuations inferred directly from data as the difference in valuations of M^DR^D and $\widetilde{M}^DR^DS^D$. The remaining term, $\Delta_{S,\eta}$, compares the value of these distortions on the measured wealth returns.

2.4 Exchange Rate Wedge Decomposition Quantified

Table 2 shows the effects of the costs of incomplete markets for the three goods market conditions along with their decompositions for the i.i.d. case in Panel A and the persistent case in Panel B. The total cost measure, $\Delta_{\eta,S}$, is around 2% for the i.i.d. case, but near 1% for the persistent consumption case. This pattern reflects the assumption that the cross-country correlation of the persistent "long run risk"component is assumed to be equal to one, in order to provide a conservative case.[24] The next two columns report the costs of the wedges given by the S-Wedge certainty equivalent wedges, $\Delta_{D,\eta}$ and $\Delta_{S,*}$, respectively. By construction, the "Sticky Price" version has no S-Wedge and we therefore refer to this case as also the "No S-Wedge" case below. By contrast, the Non-Tradeables case implies that the cost of distorting the exchange rate as in the data rather than complete markets is about 3% and 2.5% of permanent consumption for the i.i.d. and persistent case, respectively.

Table 2 also reports the decompositions as noted in equation (27). While the total cost measure $\Delta_{D,*}$ is positive, as noted earlier, the components need not be. Therefore, to highlight the relative absolute contribution of each component to the total, we follow Nagel and Xu (2022) in measuring the shares in absolute values. These shares are reported under "Absolute Shares." To focus on the comparisons to either the Data or Complete markets, we report $\Delta_{D,\eta}$ and $\Delta_{S,*}$, subsuming the other component, $\Delta_{D,\eta}$, as the residual. As the shares show, the distortions implied by the exchange rate wedges are relatively small compared to the implicit wedges due to the other components of η.

Finally, under "Returns", we report the implied wedges in return form. Specifically, we use the wealth return as implied by the consumption processes in the data from Table 1 adjusted by a leverage parameter used to match equity returns. This calibration provides an estimated return of 7.8% for the i.i.d. case and 9.74% for the persistent risk case. The next two columns present differentials for hypothetical assets that pay out the value of the η wedge under the column headed $E[r^D - r^\eta]$, and the $S - Wedge$ under $E[r^D - r^S]$. Since the value of η is high, the η returns are lower than the consumption asset ranging from 32 to 23 basis points. By contrast, the $S - Wedge$ for Nontradeables commands a premium of 1 to 3 basis points while the No $S - Wedge$ and Home bias cases suggest a discount.

The small relative share of the S Wedge relative to η as a contribution of welfare raises questions about the other components of η. Therefore, the next section examines the impact of the implicit wedges in these components more carefully.

[24]For example, Lewis and Liu (2015) show that the benefits of risk-sharing are minimized when the correlation of persistent risk is equal to one.

3 Valuing the Total Exchange Rate Wedge

So far, we have focused upon the role of the exchange rate in impacting welfare through the stochastic discount factors. However, the deviation of wealth under complete markets compared to standard data measures in equation (15) depends critically on the return to future wealth itself. This section therefore also considers the role of wedges in the return on future wealth, implying a new relationship between wealth returns under complete markets. This observation highlights a new wedge between how the exchange rate impacts the total history of future returns compared to wealth returns under complete markets.

3.1 The Stochastic Discount Factor versus the Return on Wealth

As noted above, the ζ wedge defined in equation (15) captures the deviation in state prices between their data-inferred values and complete markets counterparts through the lens of different exchange rate models. These state prices depend upon the data-implied returns on wealth for the domestic and foreign investor, given as R_c^D and $\widetilde{R}_c^D$, respectively. Therefore, to look at the potential contribution of each component, we define partial wedges for each of these components as:

$$\begin{aligned} M_{t+1}^D &\equiv \zeta_{M,t+1} M_{t+1}^*; \ \widetilde{M}_{t+1}^D \equiv \zeta_{\widetilde{M},t+1} \widetilde{M}_{t+1}^* \\ R_{c,t+1}^D &\equiv \zeta_{R_c,t+1} R_{c,t+1}^*; \ \widetilde{R}_{c,t+1}^D \equiv \widetilde{\zeta}_{R_c,t+1} \widetilde{R}_{c,t+1}^* \end{aligned} \tag{28}$$

Following our terminology above, we call $\zeta_{M,t+1}$ the "M Wedge" and $\zeta_{R_c,t+1}$ the "R Wedge", and similarly for the foreign investor. Using these definitions to replace the state price of domestic wealth in the data implies that the complete markets wedge in equation (15) can be written as:

$$M_{t+1}^D R_{c,t+1}^D \equiv \zeta_{M,t+1} \zeta_{R_c,t+1} M_{t+1}^* R_{c,t+1}^*$$

Moreover, these two partial wedges may be valued separately following the same approach as above, calculating the price ratios Z that value the wealth returns at the wedge-distorted levels. Using the general definition of the value of these wedges in equation (26), we can similarly decompose the total cost as in equation (27), replacing η and S with the M-Wedge and R-Wedge, respectively.[25] Appendix **??** describes the implied valuations for each version.

Table 3 reports the certainty equivalent costs to a domestic investor of consuming wealth measured in the data compared to complete markets implied by the exchange rate model examples. Under "Wedges", the first column reports the cost to a domestic investor who values their wealth at the state price value $M_t^D R_{c,t}^D$ relative to the complete markets alternative only distorted by the M Wedge, implied by $M_t^D R_{c,t}^*$. Following the terminology above, we define this certainty equivalent difference as: $\Delta_{D,M}$. As this first column shows, the results are quite different from those of the η wedge reported in Table 2. In contrast to the η wedge, the size and even the sign of the M Wedge cost depends upon the exchange rate view. When consumption is i.i.d. as shown in Panel A, the size varies from 2.88% for the "No S-Wedge" case when prices are fixed to -1.00% when prices adjust as in the Nontradeable case. Indeed, the negative sign implies that maintaining the stochastic discount factor in the data to value future wealth implied by complete markets would detract from welfare.

This varied impact of the stochastic discount factor together with the overall benefits of complete markets implies that the effects of the return to wealth, the R Wedge, must be important. Thus, the second column under Wedges reports the "cost" relative to distorting complete markets by the return in the data R^D, rather than the counterfactual complete markets return, R^*. In all exchange rate versions, this "cost" is negative, meaning that investors would prefer the return in the data compared to complete markets. This finding is not surprising because, as shown in Table 1, the value of returns in the data provide a better diversification benefit than the one implied by pooling total world resources. Indeed, the next column under "Wedges" reports the certainty equivalent gains of the returns in the data relative to this R-Wedge, $\Delta_{D,R}$, showing the sizes are indeed large in all cases. By contrast, the effects of the wedge on M as measured by $\Delta_{M,*}$ are more modest.

This pattern is highlighted by the absolute value shares reported under the columns with heading "Absolute Shares." As the first decomposition shows, the R-Wedge relative to the data-inferred wealth accounts for over 40% of the share, while the deviation due to the M-Wedge from complete markets only measures 12% or less. Similarly, the last two columns show that same pattern when the R-Wedge is compared to the data and the M-Wedge to

[25]That is, $\Delta = \Delta_{D,M} + \Delta_{M,R} + \Delta_{R,*}$ where "M" and "R" denote the state price distorted by respective M and R Wedges.

complete markets.

Overall, the impact of the M Wedge is lower than the counterparts from the R Wedge. This finding suggests that the wedge due to future wealth return is more important than the current valuations represented in the standard η wedge given in equation (2). If so, the effects of the current exchange rate wedge described in Table 2 may be missing important impacts on *future* returns. We consider this possibility next.[26]

3.2 A New Complete Markets Condition and The Total S Wedge

The results in Table 3 suggest that future wealth may be a primary component of the overall costs from incomplete markets. By contrast, the analysis above has focused only upon the current period first-order condition relating exchange rates and stochastic discount factors in equation (1). However, when markets are complete, the domestic and foreign investors share the world consumption aggregates and thereby a common component of wealth returns. In the special case of recursive preferences used in this paper the relationship across countries under complete markets has a convenient form as shown in Appendix **??**.[27] That is,

$$R^*_{c,t+1} = \left(\frac{\omega_{t+1}C^w_{t+1}}{\omega_t C^w_t}\right)\left(\frac{S^*_{t+1}}{S^*_t}\right)^{-1}\left[\frac{1+Z^*_{t+1}}{Z^*_t}\right];\ \widetilde{R}^*_{c,t+1} = \left(\frac{\widetilde{\omega}_{t+1}C^w_{t+1}}{\widetilde{\omega}_t C^w_t}\right)\left[\frac{1+\widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t}\right]$$

where as above, ω is the domestic country share of world consumption and $\widetilde{\omega} = 1-\omega$.

Moreover, when countries are symmetric and consumption is i.i.d., this relationship simplifies to: $R^*_{c,t+1} = \widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{\frac{1}{\gamma}}$. Combining this observation with the standard complete markets exchange rate relationship in equation (1) leads to a new complete markets relationship between the wealth returns across countries. From equation (1) and the above, we have:

$$M^*_{t+1}R^*_{c,t+1} \equiv \widetilde{M}^*_{t+1}R^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}(S^*_{t+1}/S^*_t)^{-(1-\frac{1}{\gamma})} \tag{29}$$

[26] The effect of the M Wedge relative to the R Wedge on welfare also depends critically on the assumption of the intertemporal elasticity of substitution. In this section, we have maintained the assumption that this parameter is greater than one in order to match asset pricing relationships. However, as described in the appendix and in the next section, this pattern may be reversed if the intertemporal elasticity of substitution is less than one.

[27] In particular, $\frac{R^*_{c,t+1}}{\widetilde{R}^*_{c,t+1}} = \left(\frac{S^*_{t+1}}{S^*_t}\right)^{\frac{1}{\gamma}}\left[\frac{(1+Z^*_{t+1})/Z^*_t}{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}\right]^{\frac{\theta}{\gamma\psi}}$.

Note that this relationship implies that under complete markets, wealth returns are equalized except for the impact of the exchange rate. This observation leads to measurement of a different exchange wedge that considers the impact of relative prices alone on future wealth. That is, we can consider the deviation from complete markets by replacing the right-hand side variables in equation (29) with their data counterparts to define a total wedge, η^T, as:

$$M^*_{t+1} R^*_{c,t+1} \;\; \equiv \;\; \widetilde{M}^D_{t+1} \widetilde{R}^D_{c,t+1} (S^D_{t+1}/S^D_t)^{-(1-\frac{1}{\gamma})} \eta^T \tag{30}$$

Furthermore, separating the effect of the exchange rate wedge and using the notation for the S Wedge in equation (23), we can rewrite the relationship in equation (29) as:

$$M^*_{t+1} R^*_{c,t+1} = \widetilde{M}^*_{t+1} \widetilde{R}^*_{c,t+1} \left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})} \left(\zeta_{S,t+1}\right)^{-(1-\frac{1}{\gamma})} \tag{31}$$

Therefore, we can examine the effects of an exchange rate wedge on the return to future wealth through this "Total S-Wedge", denoted as S^T.

Table 4 shows these measures using the same parameters and assumptions as in Table 2 and therefore the "Total Cost" measures and S-Wedge measures are repeated for reference in the first three columns. The next column under "Total S" labeled Δ_{D,η^T} reports the value for certainty equivalent differences due to η^T as in equation (30) relative to the basic value of wealth in the data. This relationship compares the hypothetical value of the foreign wealth in the data but valuing all future wealth at the exchange rate in the data. According to complete markets, domestic and foreign consumption growth rates are equalized except for these exchange rate valution effects. Therefore, foreign consumption no longer provides diversification to domestic investors, but instead these investors now face exchange rate risk. As such, the benefit of investing in this foreign wealth that only differs by the exchange rate will be slightly negative at -0.05% and -0.02% of permanent consumption for i.i.d. and persistent risk, respectively.

The next column compares the effect of the Total S-Wedge on complete markets. In all cases, the effects are closer to zero than for the S-Wedge itself. However, given that the overall impact of the difference in valuation of relative wealth measured in Δ_{D,η^T} is negative, the relative contribution of the Total S-Wedge is larger as can be seen by comparing the

Absolute Shares in the final four columns. For example, in Panel A, the Absolute Shares of Δ_{D,S^T} range from 5.3% to 9% compared to 1.5% and 4.6% and a similar pattern holds in Panel B. Thus, overall, the relative contribution of the Total S-Wedge has a larger impact that the current S-wedge.

4 Exchange Rate Wedge Generalizations

Many of the insights so far are based upon risk and intertemporal parameters that were seen to fit the asset returns in Table 1. The importance of the future wealth relative to the current stochastic discount factors therefore depends on these parameters. Furthermore, the analysis so far has focused upon a two country case to highlight the basic features of measuring exchange rate wedges in a parsimonious way. However, this framework can easily be formulated to consider the impact of different international valuations in other models and applications. Therefore, in this section, we illustrate the approach with various generalizations. First, we consider how the basic patterns above are altered with different assumptions about exchange rate adjustment and risk preferences. Second, we show that the framework may be extended naturally to multiple countries, illustrated with data for four countries. Third, we point to a number of other applications including different asset pricing models, exchange rate processes and production economies.

4.1 Valuing Wedges with Different Preference Parameters

The analysis above depended upon two sets of preference parameters. One set impacts risk assessment and intertemporal decisions as characterized by the intertemporal elasticity of substitution ψ, risk aversion γ, and the time discount factor β. The second set determines the goods market clearing, including the preference for Tradeables relative to Nontradeables α, and the preference for Home Goods relative to Foreign Goods a. We find that our results are most sensitive to variations in IES and in the value of Tradeables, so we focus on these parameters as examples.

Table 5 then repeats the analysis in Table 2 for different measures of α in Panel A and of ψ in Panel B taking the i.i.d. consumption case as a benchmark. As Panel A shows, the overall costs of incomplete versus complete markets decline with the preference for tradeables. Intuitively, as α approaches 0.5, the two goods become perfect substitutes and since the planner can only reallocate tradeables, any benefits from this potential reallocation declines.

By contrast, Panel B shows that the intertemporal elasticity of substitution, ψ, has a strong impact on the η wedge. When ψ is great than 1 as typically assumed to fit asset return behavior, investors easily substitute current for future consumption. Therefore, relative valuations by domestic investors of the foreign wealth become very appealing. However, when ψ is less than one, these investors prefer current consumption and the possible returns on foreign wealth become less attractive.

4.2 The Costs of Wedges with Multiple Countries

It is straightforward to generalize the approach above to a set of countries $j = 1, ..., J$. Appendix **??** describes this extension along with basic data moments that include the United Kingdom (UK) and Australia (Aus). We now denote the variables specific to a given country j with a superscript. Arbitrarily choosing the last country J to be the numeraire, the aggregate consumption level and real exchange rate for country j can be rewritten as C_t^j and $S_t^j = (P_t^j/\widetilde{P}_t^J)$, respectively. This multi-country version requires extending the resource constraints to J countries so that world consumption units measured in the numeraire country must be written as a sum across countries. As an example, for the Sticky Price version, this resource constraint becomes: $\sum_{j=1}^{J} S_t^{jD} C_t^{j,D} = \widetilde{C}_t^w$ and the optimal consumption policy for each country is a sharing rule that depends upon the exchange rate versions that can be written in general form as: $C_t^{j*} = \omega_t^j \widetilde{C}_t^w$ where ω_t^j is country j share of world wealth depending upon the goods market restriction as seen above.

To illustrate the implications of this generalization, we focus on the No S-Wedge case. We also extend the data series from the United States (US) and Canada (Can) to include the United Kingdom (UK) and Australia (Aus). To focus upon the impact of multiple countries, we only report the total cost $\Delta_{D,*}$. When countries have non-symmetric consumption and price processes as implied by these data, then the value of future wealth differs. Therefore, the planner reallocates initial consumption across countries to incentivize countries with better wealth returns to be willing to pool their assets in complete markets. These reallocations are given in the table as "Weights" given by the ratio of initial C^*/C^D.

To illustrate the impact of exchange rate variability, we first assume counterfactually that the exchange rate volatility is zero; that is, $g_{s,t}^{j,D} = 0$. Table 6 Panel A reports the results from these calculations using the correlations for real consumption growth. These levels are shown for a base case set of typical CRRA macro parameters of $\gamma = 2$ and $\psi = 0.5$, as well as the parameters used to match asset returns above of $\gamma = 10$ and $\psi = 1.5$. In all cases, the benefits of risk-sharing imply positive certainty equivalent consumption wedge $\Delta_{D,*}$. Moreover, these costs increase with risk aversion and the intertemporal elasticity of substitution, IES. By contrast, Panel B shows the impact of exchange rate volatility when the variance of $g_{s,t}^{j,D}$ is given by the price data. Comparing the final two rows of Table 6 reports the results with "Asset Pricing" parameters with exchange rate volatility. These rows show a benefit of 1.28% of permanent consumption for the Canadian investor who is willing to give up some initial consumption level at 0.985 of their initial level in order to participate in a better lifetime consumption growth path. After this initial compensation, all countries have non-negative welfare gains.

4.3 Other Extensions

Our framework is general and applicable to a number of other assumptions and models that we only briefly highlight here. Appendix **??** provides more details for each of these cases.

Alternative Asset Pricing Models To illustrate how our framework can use consumption and exchange rates that match asset price data, we chose a long run risk process as an example of persistent risk. However, the approach could be also consider alternative consumption processes that match asset returns. For example, a number of papers have shown the importance of disaster risk in matching financial markets including Barro (2009), Wachter (2013), and D. Backus, Chernov, and Martin (2011). Moreover, the importance of disaster risk in global valuations has been shown in Nakamura et al. (2013) and Gourio, Siemer, and Verdelhan (2013) and Lewis and Liu (2017). Much of this literature uses recursive preferences with consumption processes disciplined by disaster events, an approach conveniently nested in our framework.

Persistence in Exchange Rates The endogenous price versions of the exchange rate that we have studied above require assumptions about its process in the data. This process is then used to uncover the commodity quantities that are implicit within the consumption data aggregate. In the analysis above, we have taken the simplifying assumption that the real exchange rate in the data is a random walk. However, studies of the longer run behavior of the real exchange rate suggest that prices cannot diverge indefinitely. Thus, while the random walk assumption provides convenient closed form solutions in order to investigate the basic wedge relationships, a more realistic approach would allow for persistence in these relative prices. Again, this approach can be readily incorporated by amending the implied persistence in relative goods supplies across countries.

The Costs of Wedges using Production The analysis in this paper has focused upon using consumption data because it is the driver in many asset pricing models, and is related to a large literature on consumption risk-sharing. Nevertheless, our approach can alternatively accommodate the implications of inefficient allocations in production. As described in Feenstra et al. (2015), the PWT data provide country output and absorption price measures that can be used for international comparisons. Thus, these data can be used along with our framework to consider the reallocation due to asset markets that span production risks rather than consumption risks.

One approach would be to suppose that production is linear in technology. For example, consider a model in which output in each firm is produced with linear technology: $y_t(z) = Y_t z l_t(z)$ where $l_t(z)$ is the amount of labor employed by the firm and where Y_t is a stochastic process generating aggregate productivity. In this case, if domestic consumption depends upon claims to this output across countries, the total world consumption, C_t^w would be replaced to total world output, Y_t^w in the data. In this way, the same analysis of the wedges due to real price differences across countries can be calculated for production-side risks. Rather than reallocating consumption, the complete markets solution would then reallocate output and consumption would become endogenous.

Alternative Exchange Rate Versions In this paper, we focused upon three different examples of exchange rate approaches. However, our approach is general enough to allow for other determination models. For example, Obstfeld and Rogoff (2001) suggested that transactions costs could potentially explain the disconnect between exchange rates and fundamentals. They use a proportional transactions cost specification with a homogeneous consumption aggregator that directly nests within our approach above. More recently, Itskhoki and Mukhin (2021) show that a friction process outside of fundamentals that they call "financial shocks" is needed to explain exchange rates. They parameterize these shocks with interest parity, similar to our foreign exchange returns. Therefore, these measured returns could be included as alternative measures of exchange rate wedges using our approach.

5 Concluding Remarks

How much financial market frictions affect the welfare of economic agents across countries is clearly an important question. In this paper, we develop an approach to help answer this question by connecting standard frameworks that address both exchange rates and asset return behavior. These connections provide four main contributions.

First, we show how financial and macroeconomic data can be used to measure the welfare costs implied by the wedge in global investor valuations of returns. The implied cost of this wedge is independent of the exchange rate determination view. However, this wedge only compares incomplete market valuations and cannot evaluate optimality.

Therefore, as a second contribution, we show how the same data can be used to consider the welfare value of financial market completeness. We demonstrate how to measure this wedge using examples of several off-the-shelf exchange rate approaches, although the setting is general enough to consider other such exchange rate views.

Third, our paper provides a framework to decompose the valuation of the implied wedge from complete markets into individual components, such as wedges in the exchange rate and the stochastic discount factors of investors. For parameters that match standard asset pricing moments, we find that the effect of the exchange rate wedge is small relative to the stochastic discount factors of investors.

Fourth, we highlight a new complete markets relationship across countries based upon the wealth returns. Using this insight, we construct a "Total" exchange rate wedge that measures the impact of exchange rates over future wealth.

In exploring these new relationships, we simplify much of our analysis by considering a two country symmetric framework with stylized assumptions. However, we illustrate how our framework is sufficiently rich to allow for further work and extensions to multiple countries, a stationary long run real exchange rate, production risk, and broader asset pricing models. Thus, the paper provides an important step toward connecting the behavior of asset returns, exchange rates, and the value of deviations from optimal financial market risk insurance.

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Table 1: **Data Moments and Wedges Example**

Panel A: Data Moments

	Consumption Growth			Exchange Rate	
	Mean	Std Dev	Correlation	Std Dev	Corr with Consn
US	1.91	1.56	0.574	N/A	N/A
Canada	1.89	1.52	0.574	1.12	-0.18

Panel B: Data η Wedge and Total Complete Markets Wedge for US and Canada

	Data η Wedge	Total Wedge
Non-Tradeable ($\alpha = 0.7$)	31.34	1.76
Home Bias (a = 0.9)	31.34	1.93
Sticky Prices	31.34	2.10

Panel C: Asset Return Data and Model Moments

	Equity Mean	Equity Vol	Risk Free Mean	Risk Free Vol	Equity Premium
(i) Data Moments					
US	8.32	18.62	2.88	1.48	5.44
Canada	7.65	21.22	0.74	2.06	6.91
(ii) SMM Model Moments					
US	7.13	17.99	2.17	0.47	4.96
Canada	8.47	21.12	1.55	0.67	6.92

Notes: Panel A under "Data Moments" gives the sample moments of real consumption growth and the relative price levels for the U.S. and Canada measured in local prices using the expenditure benchmark in Feenstra et al. (2015). Panel B provides the percentage deviation in certainty equivalent units for the i.i.d. case of the η wedge deviation in equation (2) and the total wealth deviation to be described in Section 2. Panel C under "Data Moments" reports the sample moments of real equity returns and the risk free rate measured in local prices with the same deflator as Panel A. "SMM Model Moments" gives the implied Simulated Method of Moments (SMM) counterparts assuming perfectly correlated long run risk consumption growth across countries, as detailed in Appendix **??**. Estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5.

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Table 2: **Wedge Value Decomposition:** S **Wedge**

Panel A: Wedge Decomposition for I.I.D. Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	$\Delta_{S},*$	Absolute Shares D,η	$S,*$	Levered Returns $E[r^D]$	$E[r^D - r_\eta]$	$E[r^D - r_S]$
Non Tradeable $\alpha = 0.7$	1.76	31.34	3.07	46.7%	4.6%	7.80	-0.32	0.01
Home Bias a = 0.9	1.93	31.34	-0.89	51.6%	1.5%	7.80	-0.32	-0.03
Sticky Price No S-Wedge	2.10	31.34	0.00	51.7%	0.0%	7.80	-0.32	-0.02

Panel B: Wedge Decomposition for Persistent Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	$\Delta_{S,*}$	Absolute Shares $\Delta_{D,\eta}$	$\Delta_{S,*}$	Returns $E[r^D]$	$E[r^D - r_\eta]$	$E[r^D - r_S]$
Non Tradeable $\alpha = 0.7$	0.78	13.41	2.49	43.2%	8.0%	9.74	-0.23	0.03
Sticky Price No S-Wedge	0.87	13.41	0.00	51.7%	0.0%	9.74	-0.23	-0.02

Notes: Panel A under "Wedge Decomposition for IID Consumption" reports certainty equivalent costs (Δ) when consumption growth is IID, while Panel B under "Wedge Decomposition for Persistent Consumption" displays results when consumption growth contains a persistent risk. "Total costs" measures the consumption equivalent loss from consuming the Data-implied domestic wealth (subscript D) instead of the complete markets wealth (subscript *). The columns under "Wedges" report the decomposition of the total cost into components of η and S-Wedge (subscript S) following Equation 27 with $\Delta_{\eta,S}$ excluded as the residual. "Absolute Shares" are the absolute values of the shares of contributions to total costs in equation (27); that is, for general contribution of Δ_i, the share is: $Abs(\Delta_i)/\Sigma_i^3 Abs(\Delta_i)$ where i indexes the Δ_i combinations that sum to $\Delta_{D,*}$. Levered returns are computed as the log of the return defined in Equation 6 multiplied by a leverage parameter of 3, as in Abel (1999). Estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5. The consumption growth parameters are calibrated at the annual mean of 1.7%, annual standard deviation of 1.52%, correlation of consumption growth across countries of 0.574, and correlation of exchange rate growth and consumption growth of -0.18. The correlation of the persistent consumption risk across countries in Panel B is set to 1 following Colacito and Croce (2011).

Table 3: **Wedge Value Decomposition in Local Prices: M-Wedge, R - Wedge**

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,M}$	Wedges $\Delta_{R,*}$	Wedges $\Delta_{D,R}$	Wedges $\Delta_{M,*}$	Absolute Shares $\Delta_{D,M}$	Absolute Shares $\Delta_{R,*}$	Absolute Shares $\Delta_{D,R}$	Absolute Shares $\Delta_{M,*}$
Non Tradeable $\alpha = 0.7$	1.76	-1.00	-6.31	7.88	2.76	6.2%	39.9%	40.4%	13.9%
Home Bias a = 0.9	1.93	0.94	-9.20	10.73	0.99	4.8%	47.3%	49.9%	4.6%
Sticky Price No S-Wedge	2.10	2.88	-12.21	13.62	-0.79	11.5%	48.4%	54.2%	3.2%

Notes: This table reports certainty equivalent consumption loss or wedges (Δ) from the stochastic discount rate (subscript M) and return to wealth (subscript R) in local prices. "Total costs" measures the consumption equivalent loss from the data economy (subscript D) to the risk sharing economy (subscript *). The columns under "Wedges" report the decomposition of the total cost wedge into components of M and R-Wedges with $\Delta_{M,R}$ excluded as the residual. "Absolute Shares" and parameter estimates are described in Table 2 notes.

Table 4: **Wedge Value Decomposition: Total S-Wedge**

Panel A: Wedge Decomposition for I.I.D. Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges S $\Delta_{D,\eta}$	Wedges S $\Delta_{S,*}$	Wedges Total S Δ_{D,η^T}	Wedges Total S $\Delta_{S^T,*}$	Absolute Shares S $\Delta_{D,\eta}$	Absolute Shares S $\Delta_{S,*}$	Absolute Shares Total S Δ_{D,η^T}	Absolute Shares Total S $\Delta_{S^T,*}$
Non Tradeable $\alpha = 0.7$	1.76	31.34	3.07	-0.05	0.17	46.7%	4.6%	2.8%	9.0%
Home Bias a = 0.9	1.93	31.34	-0.89	-0.05	-0.12	51.6%	1.5%	2.3%	5.3%

Panel B: Wedge Decomposition for Persistent Consumption

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges S $\Delta_{D,\eta}$	Wedges S $\Delta_{S,*}$	Wedges Total S Δ_{D,η^T}	Wedges Total S $\Delta_{S^T,*}$	Absolute Value Shares S $\Delta_{D,\eta}$	Absolute Value Shares S $\Delta_{S,*}$	Absolute Value Shares Total S Δ_{D,η^T}	Absolute Value Shares Total S $\Delta_{S^T,*}$
Non Tradeable $\alpha = 0.7$	0.78	13.41	2.49	-0.02	0.07	43.2%	8.0%	2.8%	9.0%

Notes: Panel A under "Wedge Decomposition for IID Consumption" reports certainty equivalent costs (Δ) when consumption growth is IID, while Panel B under "Wedge Decomposition for Persistent Consumption" displays results when consumption growth contains a small and persistent risk. "Total costs" measures the consumption equivalent loss from consuming wealth as inferred in the data (subscript D) instead of as in complete markets (subscript *). The columns under "Wedges" reports the decomposition of the total cost wedge into components of η^T and Total S-Wedge (subscript S^T) based on Equations 30 and 31 with Δ_{η^T,S^T} excluded as the residual. "Absolute Shares" and parameter estimates are described in Table 2 notes.

Table 5: **S-Wedge Value Decomposition: Varying Parameters**

Panel A: Non Tradeable (IID), Varying α

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	Wedges $\Delta_{S,*}$	Absolute Shares $\Delta_{D,\eta}$	Absolute Shares $\Delta_{S,*}$
$\alpha = 0.8$	1.99	31.34	4.83	44.6%	6.6%
$\alpha = 0.7$	1.76	31.34	3.07	46.7%	4.6%
$\alpha = 0.6$	1.23	31.34	6.44	42.2%	8.7%

Panel B: Non Tradeable (IID), Varying IES

Exchange Rate View	Total Cost $\Delta_{D,*}$	Wedges $\Delta_{D,\eta}$	Wedges $\Delta_{S,*}$	Absolute Shares $\Delta_{D,\eta}$	Absolute Shares $\Delta_{S,*}$
IES = 1.5	1.76	31.34	3.07	46.7%	4.6%
IES = 0.5	0.58	-3.36	-0.45	41.0%	5.5%

Notes: The table reports the certainty equivalent percentage difference between permanent consumption that equalizes welfare between the Data-inferred and Complete Markets along with the decompositions for different parameters. Panel A considers variations in the preference parameter α for tradeables given by the utility function in equation (22). Panel A estimates are based upon Epstein-Zin preferences with a Risk Aversion parameter of 10 and the Intertemporal Elasticity of Substitution of 1.5. Panel B results assume $\alpha = 0.7$ and Risk Aversion parameter of 10. All results assume symmetric countries in the data with i.i.d. consumption processes and parameters decribed in Table 2 notes.

Table 6: **The Total Costs for the No S-Wedge Case: Multiple Countries**

$\Delta_{D,*}$ Allowing Asymmetric Wealth: $(C^*/C^D) \equiv$ "$Weights$"					
Version		ConstantWeight Estimates			
A. Without Exchange Rate Volatility	Parameters	US	Can	UK	Aus
CRRA	$\gamma = 2, \psi = 0.5$	0.10	0.14	0.22	0.33
	Weights	1.001	1.000	1.000	0.999
Asset Pricing	$\gamma = 10, \psi = 1.5$	3.53	4.05	5.17	6.54
	Weights	1.003	0.993	1.006	0.998
B. With Exchange Rate Volatility					
CRRA	$\gamma = 2, \psi = 0.5$	0.00	-0.05	-0.01	0.23
	Weights	1.001	1.000	1.000	0.999
Asset Pricing	$\gamma = 10, \psi = 1.5$	2.52	1.28	-1.03	1.54
	Weights	0.991	0.985	1.017	1.007

Notes: This table reports certainty equivalent consumption loss ($\Delta_{D,*}$) for the wealth returns as described in Appendix **??**. Initial wealth values or "Weights" are reallocated so that all costs are non-negative. Parameter estimates are described in Table 2 notes.

Figure 1: Exchange Rate Wedge Decomposition