Time-varying risk aversion and the equity term structure

Martijn de Vries*

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ABSTRACT

I show that time-varying risk aversion can generate a term structure of equity risk premia that is upward sloping in bad times and downward sloping in good times. I derive three conditions that jointly generate this result. First, risk aversion is negatively correlated with consumption growth. Second, risk aversion is mean reverting and, third, preferences can become risk seeking in good states. Moreover, I propose a highly parsimonious model with one single state variable and a representative agent with utility over both consumption and wealth fluctuations that allows for states of risk-seeking preferences in equilibrium and, hence, can match the recent empirical findings on the equity term structure. In my model, the unconditional term structure of equity risk premia can be increasing, flat, or decreasing depending on the relative frequency of good and bad times. I show that time-varying risk aversion provides the first simultaneous explanation for the cyclicality of the equity term premia, the positive equity term risks, and, the predictability of stock returns without needing to assume predictability in cashflows. More broadly, my results show the relevance of time-varying risk aversion for explaining maturity-dependent risk premia.

JEL Codes: D81, G12, G41.

*Vrije Universiteit Amsterdam, Department of Finance. Email: m.vries@vu.nl. I am particularly grateful to Nicholas Barberis, Nikolaus Schweizer, Bas Werker and Ole Wilms for their invaluable guidance and advice. I also like to thank Anne Balter, Jules van Binsbergen, Stefano Cassella, Joost Driessen, Sebastian Ebert, Gleb Gertsman, Stefano Giglio, Emiel Jerphanion, Ralph Koijen, Albert Menkveld and Oliver Spalt as well as all seminar participants at Bayes Business School, Durham University, ESADE, Paris-Dauphine University, Radboud University, Tilburg University, University of Amsterdam and Vrije Universiteit Amsterdam for insightful comments.

1. INTRODUCTION

In nearly all models of aggregate stock market behavior investors are assumed to always be averse to risks. Yet the experimental economics literature demonstrates that decision makers can also act in a risk-seeking manner.[1] Such behavior may have important consequences as risk-seeking preferences invert the effects risks have on equilibrium prices relative to risk-averse preferences. Therefore, allowing investors to become risk-seeking in certain scenarios may explain more of the variability in aggregate stock prices and thereby may yield new insights in the cyclicality of financial markets. These insights are desirable given the puzzling empirical findings about the shape of the term structure of equity risk premia—the decomposition of the equity premium by maturity—and its cyclicality.

Recently, the literature on financial economics has provided new results that are informative about the cyclicality of the aggregate stock market. Namely, the results of **?** indicate that the term structure of equity risk premia is countercyclical.[2] More specifically, it is upward sloping in bad times and downward sloping in good times.[3] These findings are puzzling, as they are not supported by the predictions of both classical and more recent asset pricing models which generate either always upward sloping term structures of equity risk premia (**??**) or always downward sloping term structures of equity risk premia (**?**).

In this paper, I show that a model with time-varying risk aversion is able to generate the empirically observed state-dependent slope of the equity term structure if it meets three conditions. First, risk aversion needs to be negatively correlated with consumption growth shocks, and second, it needs to be mean reverting over time. These two assumptions are not new, on the contrary, they are already commonly satisfied in models of time-varying risk aversion (**??**). As I will explain in more detail below, these two assumptions jointly generate positive equity term risks without relying on any maturity-specific dividend risk.[4] Then, mind that a risk averse agent will demand a positive premium to hold any type of risk. Therefore, a model that only allows states of risk averse preferences will always translate such positive term risks into increasing term structures of equity risk premia. However, in case there exist states

[1]See **?** and **?** for a discussion of the literature.

[2]In the literature there is an ongoing debate on the shape of the unconditional term structure of equity risk premia (**???**). The findings of **?** are helpful as they suggest that the shape of the unconditional term structure is a result of the relative weighting of good and bad times in a specific time period.

[3]I follow the definition of **?**, who defines bad times as times where the ex-ante price-dividend ratio is in the historical bottom quintile, and good times as all other points in time.

[4]A detailed explanation of the mechanism is given in section **??**.

where risk-seeking preferences are allowed, in such states the positive term risks translate to a decreasing term structure of equity risk premia. Therefore, the third condition of models with time-varying risk aversion to fully match the state-dependent slope of the equity term structure is that investors can become risk-seeking in good times.

The key driver of the results is the inclusion of states of risk-seeking preferences. Even though models of aggregate stock market behavior tend to neglect the possibility of such preferences, risk-seeking preferences are supported by a wide range of papers in the experimental economics literature. In their seminal paper, **?** report that a significant part of their participants make decisions in a risk-seeking manner. They give individuals the choice between ten paired lotteries where the probabilities of the lotteries change while going down the list. These lotteries differ in expected payoff and riskiness. The expected payoff is first larger for lottery A, but at some point it is larger for lottery B. However, B is always the riskier lottery. This method allows to infer the level of risk aversion of the participants by analyzing their switching behavior. Their results show that some participants display risk-seeking behavior.[5]

In the experiments of **?** and in follow-up work by the literature, it remains difficult to assess where this type of behavior may originate from. One explanation for risk-seeking behavior is the *house money effect* (**?**). According to the house money effect, individuals tend to be risk-seeking after prior gains. In the context of financial markets, prices are high in good times such that on average investors are likely to have had prior gains. Therefore, the house money effect may imply that some investors become risk seeking in good times. However, incorporating the micro-foundation of risk preferences would add an additional layer of complexity to the model that is not of interest to the current analysis. For the ease of the analysis, risk preferences are thus chosen to be modeled directly.

The model I propose extends the general equilibrium model that was originally proposed by **?** and has been shown to explain aggregate stock market behavior (**?**), investor behavior (**?**), option pricing (**?**) and the cross-section of stocks (**?**). The innovation of their model is an alternative preference structure that separates consumption utility from utility of financial gains and losses. My model builds on this preference separation as it allows for domain specific risk preferences (**?**).[6] For instance, a person can be risk averse when making decisions about health

[5]**?** revise the experimental design used in **?** to solve problems raised in the literature concerning the menu of lotteries (**??**) and find even stronger support for the existence of risk-seeking behavior.

[6]I follow the literature in experimental economics and use the term *domain-specific* for differentiating decisions related to for instance health from those related to finances. The literature useses the closely related term *context-dependent* to differentiate between decisions to specific payoffs like gains and losses.

care, whereas at the same time having a different risk tolerance for purely financial decisions (**?**). It turns out that this finding is also very useful for incorporating states of risk-seeking preferences over financial decisions in an equilibrium model. The intuition is as follows. By separating consumption utility from utility obtained by financial wealth fluctuations, in specific states the representative agent can be risk-seeking for financial decisions and at the same time still imply market clearance. The equilibrium is assured by the degree of risk aversion over consumption decisions that counteracts the risk-seeking preferences over financial gains and losses. Including both utility sources generates a high degree of return volatility and a large expected return on equity. Moreover, the separation of preferences allows the model to be the first general equilibrium model to include states of risk-seeking preferences over financial decisions.

An important contribution of the analysis is the mechanism of how time-varying risk preferences induce *equity risks* and *equity risk premia*. The mechanism is the same as in other models of time-varying risk aversion (like, **??**), though a detailed explanation of the mechanism has not yet been provided. It works as follows. First consider the effect of the mean reversion of risk aversion. Mean reversion of risk aversion induces an agent to expect her future risk aversion to move towards its unconditional mean. The magnitude of this future change in risk aversion depends on the distance of the current level of risk aversion relative to its unconditional mean. Consider you currently have some level of risk aversion and suppose an economic growth shock materializes that increases your risk aversion. In case you started and stayed below your unconditional level of risk aversion, your risk aversion moves closer to the unconditional mean and thus its degree of mean reversion decreases. Considering that a higher level of risk aversion is a worse state of the world, your expectation on the future change of states is now less bad, i.e., the difference between the current state and the expected future state is now smaller compared to before the shock. Similarly, in case you started above your unconditional level of risk aversion, the shock moves your risk aversion away from its unconditional level, such that the mean reversion of risk aversion becomes stronger. This implies that in expectation, the future change of states has become better.

Summarizing, in both cases your expectations about future state changes improved. Now, consider the second condition: risk aversion is negatively correlated with consumption growth. In that case, the shock in the example before was actually a bad shock. Combined this implies that after a bad shock expectations about the future improve. Therefore, under the first two conditions, time variation in risk aversion causes expectations about the future to counteract current economic risks. Hence, as mean reversion creates a stronger force in the short-run, this counteracting effect is stronger for short maturity assets. This causes short maturity assets to be less risky compared to long maturity assets, i.e., an increasing term structure of equity *risk*.[7] Finally, in a model that only allows risk-averse preferences, investors always demand a positive premium for risks so that the positive term risks translate to an upward sloping term structure of equity *risk premia*. However, as I do in this paper, when introducing the possibility of states with risk-seeking preferences, the model is able to match the cyclicality that is found empirically. Risk-seeking preferences reverse the premia such that the term structure of equity *risk premia* becomes downward sloping in these states.

The approach of this paper gives an alternative insight in addition to recent work in the literature. Instead of analyzing the preference channel, **?**, **?** and **?** analyze the implications of alternative beliefs. One drawback of these adaptions to the belief process is that the economy in these models is no longer independent and identically distributed (IID) over time. The economy thus contains predictability in cashflows. This contradicts empirical results which show that even though there is predictability in returns, this does not come from predictability in cashflows (**?**). Another drawback is that these models will, by construction, generate time variation in the equity term risks, which is not supported by empirical work (**?**). On the contrary, the model below does satisfy these empirical findings as it generates predictability in returns by time-varying preferences whereas the economy is assumed to be a standard IID endowment economy. Furthermore, **?** states that models of alternative beliefs need at least two state variables to match the state-dependent slopes of the equity term structure. This is not the case for the model time-varying risk aversion which is parsimonious in the sense that it only uses a single state variable, namely risk aversion.

This paper makes several contributes to the literature. First, I explain the mechanism of how models with time-varying risk preferences generate the equity term risks. Additionally, I show that there are three conditions for such models to generate the empirically observed state-dependent slopes of the term structure of equity risk premia. Moreover, I provide the first simultaneous explanation for the shape of the equity term premia, the shape of the equity term risks and the existence of return predictability without needing to assume predictability

[7]The empirical literature investigating the dividend strips find that the betas of these assets are increasing with maturity (**?**). This increase of betas with maturity is also found by **?** who do not use the dividend strip assets to analyze the equity term structure, instead they estimate the term structure cross-sectionally using a principal component analysis. My model thus complies with the empirical increasing term risks.

in cashflows (**?**). Furthermore, I propose the first general equilibrium model that contains states of risk-seeking preferences over the full payoff space of the financial asset.[8] To do so, I show the relevance of domain-specific risk preferences. Additionally, this model is potentially helpful for an empirical estimation of risk-seeking preferences in the aggregate stock market as previous studies excluded risk-averse preferences ex-ante (**?**). Finally, the analysis derives the stochastic discount factor (SDF) explicitly. This is a helpful result as it may help us understand the mechanisms behind recently proposed models within this literature that use an exogenously specified SDF (**???**).

The paper is organized as follows. Section **??** introduces the model of time-varying risk preferences. Section **??** discusses some results for the experimental economics and empirical finance literature that support the assumptions of the model. The implications of the model are shown in **??** and section **??** concludes.

2. The Model

In this section I propose a model of time-varying risk aversion that is able to generate the empirically observed state-dependent slope of the term structure of equity risk premia. The term structure results are driven by the following three conditions,

(i) risk aversion is negatively correlated with consumption growth,

(ii) risk aversion is mean reverting,

(iii) states of risk-seeking preferences are allowed.

The novelty of the model comes from the fact that it allows states of risk-seeking preferences. The model I propose follows that of **?** by assuming the representative agent obtains utility from wealth fluctuations in addition to consumption utility, which is based on the findings of **?**. This additional source for utility is helpful for the model to include risk-seeking preferences. I follow the general framework first proposed by **?**, though the model that I propose below is simplified along several lines. I simplify the dynamics of risk aversion over gain-loss utility and I simplify the functional form of the gain-loss utility itself. Moreover, I impose one extension, namely, I allow for states of risk-seeking preferences such that the model also fulfills condition **??**.

[8] The model that I propose is the first to allow for *domain-specific* risk-seeking preferences such that in certain states the investor is risk-seeking over the full payoff space of the financial asset. In contrast, **?** incorporate *context-dependent* risk-seeking preferences in an equilibrium model, where the preference of the agent depends on whether the payoff is a gain or a loss. Their model therefore does not generate the empirical cyclicality of the term structure of equity risk premia.

2.1. Beliefs and Preferences

I assume a standard **?**-type endowment economy with I.I.D. log-normal consumption and dividend growth,

$$\log\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right) = g_c + \sigma_c \varepsilon_{c,t+1}, \qquad \log\left(\frac{D_{t+1}}{D_t}\right) = g_d + \sigma_d \varepsilon_{d,t+1}, \tag{1}$$

where the error terms of both growth processes are I.I.D. and standard normally distributed with positive correlation ω. There are two assets in the economy, the risk-free asset that pays the guaranteed return of $R_{f,t}$ and the risky asset that pays the risky return, R_{t+1}.

The preferences of the agent are enriched relative to the regular consumption utility models by adding utility obtained from fluctuations in financial wealth. More specifically, the investor obtains positive utility from gains and negative utility from losses. The gain-loss utility is

$$v\big(R_{t+1}, z_t\big) = \begin{cases} (1-\kappa_1 z_t)(R_{t+1}-R_{f,t}) & \text{for } R_{t+1} \geq R_{f,t} \\ (\lambda + \kappa_2 z_t)(R_{t+1}-R_{f,t}) & \text{for } R_{t+1} < R_{f,t} \end{cases}, \quad \kappa_1, \kappa_2 \geq 0, \tag{2}$$

which is motivated by the results of **?**.[9] The reference point is assumed to be the risk-free interest rate, $R_{f,t}$, such that the investor considers the return on the risky investments, R_{t+1}, as a gain or a loss if the realized return is larger or smaller than $R_{f,t}$. The unconditional level of risk aversion over gain-loss utility is denoted by λ and z_t denotes the zero-mean time-varying part. Figure **??** illustrates the gain-loss utility function for several levels of risk aversion and thereby shows how the risk preference changes as a function of z_t.

The figure shows that the level of risk aversion over the gain-loss utility depends on the slopes of the utility function in the gain and the loss region. The variation in the slope causes the utility function to be either concave, linear or convex utility function such that the investor is respectively, risk averse, risk neutral or risk seeking. I set $\kappa_2 = 0$ and thereby vary the level of risk aversion by varying the slope in the gain domain.[10] For negative values of z_t the utility

[9] The original characterization of **?** assumes investors are risk-averse over gains and risk-seeking over losses. However, most implications are caused by the kink at the reference point ($R_{f,t}$) such that risk neutrality is assumed within both regions to simplify the analysis.

[10] Intuitively one may expect that only the curvature of the function matters, not the individual slopes. However, which of the two "legs" of the function changes over time does matter for the equilibrium asset prices. Namely, in a **?**-tree endowment economy, the return distribution of the risky asset is generally asymmetric around the risk-free interest rate, which implies that the two parts of the gain-loss utility function differ in their importance for the asset prices in equilibrium. The results on the equity term structure are qualitatively unaffected when setting $\kappa_1 = 0$ instead.

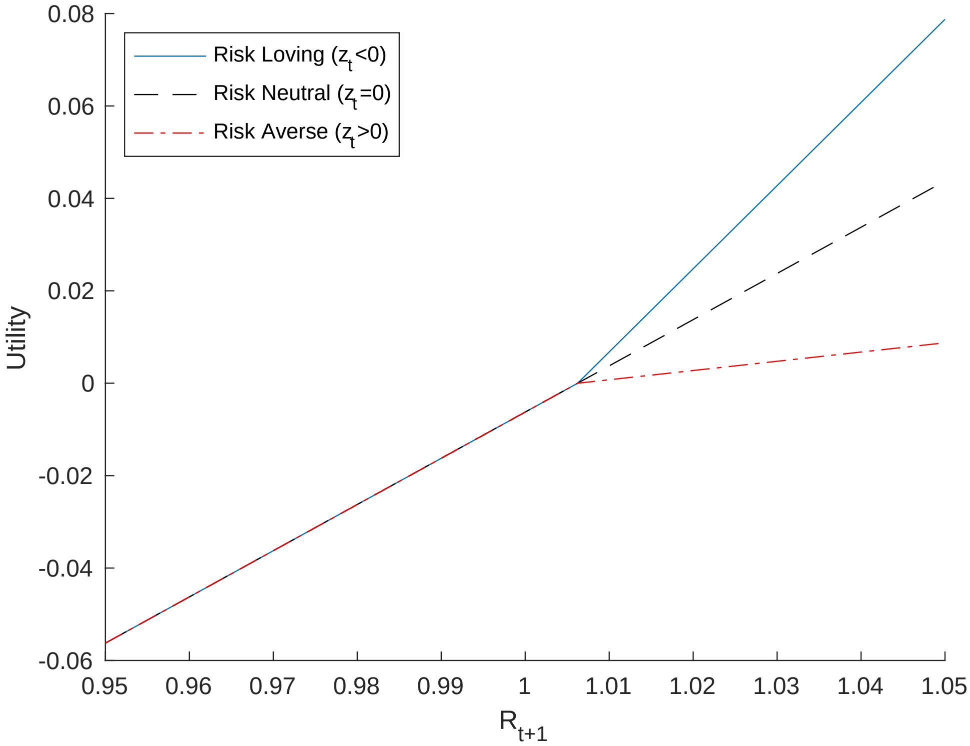

FIGURE 1:
GAIN-LOSS UTILITY FUNCTION
A static representation of the monthly gain-loss utility function for parameter values; $g_c = 0.0015$, $\sigma_c = 0.0078$, $\lambda = 1$, $\kappa_1 = 1$ and $\kappa_2 = 0$.

function is concave and for positive values of z_t it is convex, such that the level of risk aversion of the gain-loss utility is increasing in z_t.

The time variation in risk aversion is assumed to be mean reverting and negatively correlated with the economic growth shocks. A general dynamics that fulfills both these conditions is the following AR(1) process

$$z_{t+1} = \rho_z z_t - \sigma_z \varepsilon_{d,t+1}, \tag{3}$$

where conditions **??** and **??** are satisfied for $0 \leq \rho_z \leq 1$ and $\sigma_z > 0$. The dynamics imply that the level of risk aversion over gain-loss utility decreases (increases) after positive (negative) shocks to dividend growth. The specification in (**??**) is very similar to letting the dynamics of risk aversion be a direct function of returns, which one might consider more intuitive.[11] However, the latter specification yields a relation between risk aversion and dividend growth shocks that is not straightforward as it is non-linear and state-dependent. Such a structure makes

[11]This is the case for the model of **?**. The motivation for the dynamics of risk aversion in (**??**) is however similar to that of **?** as the returns highly comove with the dividend growth shocks.

it rather complex to interpret the mechanism driving the results of the equity term structures. Introducing the dividend growth shocks directly into (**??**) yields a more simple structure which is very convenient for the analysis. Moreover, as dividend shocks highly correlate with returns the intuition that risk aversion drops after good returns and vice versa is largely satisfied.

2.2. Equilibrium

The representative agent maximizes the expected discounted lifetime utility by optimally choosing consumption (C_t) and the proportion of wealth that he or she invests in the risky asset (α_t). Based on the beliefs and preferences specified above, the agent faces the following maximization problem:

$$\max_{\{C_t,\alpha_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty}\left(\beta^t u(C_t)+\alpha_t(W_t-C_t)\beta^{t+1}u'(\bar{C}_t)v(R_{t+1},z_t)\right)\right] \tag{4}$$

$$W_{t+1}=(W_t-C_t)\left\{R_{f,t}+\alpha_t(R_{t+1}-R_{f,t})\right\} \tag{5}$$

$$z_{t+1}=\rho_z z_t-\sigma_z\varepsilon_{d,t+1}, \tag{6}$$

where β is the parameter of time discounting, $u(C_t)$ is the CRRA-utility function over consumption, α_t is the proportion of wealth invested in the risky asset, W_t-C_t is the wealth of the representative investor after consumption and $v(R_{t+1},z_t)$ is the gain-loss utility function as specified in (**??**), where the dynamics of risk aversion of the gain-loss utility are specified by (**??**). I follow **?** and scale the gain-loss utility by $u'(\bar{C}_t)$, which ensures that the gain-loss utility is stationary even though wealth increases over time. Note that $\bar{C}_t$ is aggregate consumption, which is therefore not a choice variable for the investor. The utility over gains and losses materializes in the subsequent period after observing the return realization, such that the corresponding utility is time-discounted for one additional period. The budget equation of the agent is specified in (**??**).

The central topic of the analysis is the relation between time variation in risk preferences over gain-loss utility and the shape of the term structure of equity risk premia. It is thus convenient to let the other elements of the model follow the baseline assumptions in the literature. I thus assume consumption utility, $u(C_t)$, to be CRRA with (static) risk aversion γ. The equilibrium risk-free interest rate and risky return are obtained by solving the Euler equations that correspond to the model setup shown in (**??**) through (**??**). Proposition **??** formalizes these results.

Proposition 1. *Let there be a representative agent who wants to maximize his or her expected* *lifetime utility in (**??**) subject to the budget constraint in (**??**) with CRRA(γ)-utility over consumption and gain-loss utility over financial wealth fluctuations as in (**??**) and where the dynamics of risk aversion of gain-loss utility follows (**??**). The risk-free interest rate then equals*

$$R_f = \frac{1}{\beta} e^{\gamma g_c - \frac{1}{2}\gamma^2 \sigma_c^2}, \tag{7}$$

where the subscript t is dropped as the risk-free interest rate does not depend on state z_t. Then, the SDF equals

$$M_{t,t+1} = \frac{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \frac{v(R_{t+1}, z_t)}{R_{t+1} - R_f}}{\mathbb{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \frac{v(R_{t+1}, z_t)}{R_{t+1} - R_f}\right] R_f}. \tag{8}$$

Lastly, the equilibrium price-dividend ratio function (and thereby the distribution of the risky return) follows from

$$\beta e^{g_d - \gamma g_c + \frac{1}{2}\gamma^2 \sigma_c^2 (1-\omega^2)} \mathbb{E}_t \left[\frac{1 + f(z_{t+1})}{f(z_t)} e^{(\sigma_d - \gamma \sigma_c \omega)\varepsilon_{d,t+1}} + v(R_{t+1}, z_t) \right] = 1, \tag{9}$$

which needs to be solved numerically for the price-dividend ratio function $f(z_t)$ when replacing the returns by,

$$R_{t+1} = \frac{1 + f(z_{t+1})}{f(z_t)} e^{g_d + \sigma_d \varepsilon_{d,t+1}}. \tag{10}$$

Proof. See Appendix **??**. □

Proposition **??** shows that the equilibrium risk-free interest rate is solely determined by the utility the agent obtains from consumption. Therefore, the risk-free interest rate is the same as for a standard CRRA consumption utility model without gain-loss utility. The Euler equation for the risky asset in (**??**) cannot be solved in closed form and needs to be solved numerically. The solution is obtained using the collocation projection method (**?**) as proposed by **?** for solving asset pricing models. The assumed economy is I.I.D. such that the equilibrium price-dividend ratio function is solely a function of z_t. The price-dividend ratio that solves (**??**) is thus a function of the level of risk aversion over gain-loss utility.

2.3. Equity Term Structure

The term structure of equity risk premia is defined as the premia on one-period holding returns of dividend strips (**??**). A dividend strip is an asset that pays a one-time dividend, namely, the dividend in n periods from time t and its price is $P_t^{(n)}$. Readers unfamiliar with the equity term structure may want to read the derivation in Appendix **??** which follows the procedure first introduced in **?**. The return an investors obtains from holding a dividend strip with maturity n from period t to period $t+1$ equals

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}. \tag{11}$$

The term structure of equity risk and the term structure of equity risk premia are respectively defined as

$$\sqrt{\mathrm{Var}\left(R_{t+1}^{(n)}\right)} \tag{12}$$

and,

$$\mathbb{E}_t\left[R_{t+1}^{(n)}\right] - R_{f,t} = -R_{f,t}\mathrm{Cov}_t\left(M_{t,t+1}, \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right). \tag{13}$$

The shape of these term structures is a result of the three main assumptions listed above. Their influence is best explained visually by comparing the term structures when changing the parameters corresponding to the assumptions. Figure **??** shows the term structure of equity *risk* when adapting the parameters corresponding to the assumption **??** and **??**. The top left graph shows the term structure of equity *risk* for parameter values that satisfy the assumptions first two assumptions and the other graphs show the results when either **??** violated (bottom-right), **??** is violated (top-left), or both (bottom-right).

The upper left graph shows the results when risk aversion has a negative correlation with consumption growth ($\sigma_z > 0$), and is mean reverting ($0 < \rho_z < 1$). The results illustrate that when conditions **??** and **??** are satisfied, the term structure of equity risk is upward sloping for all three domains of risk aversion. In contrast, when we assume the opposite of **??**, namely risk aversion is positively correlated with consumption growth shocks ($\sigma_z < 0$), the term structure is downward sloping as is illustrated in the top right graph. The intuition for these results will

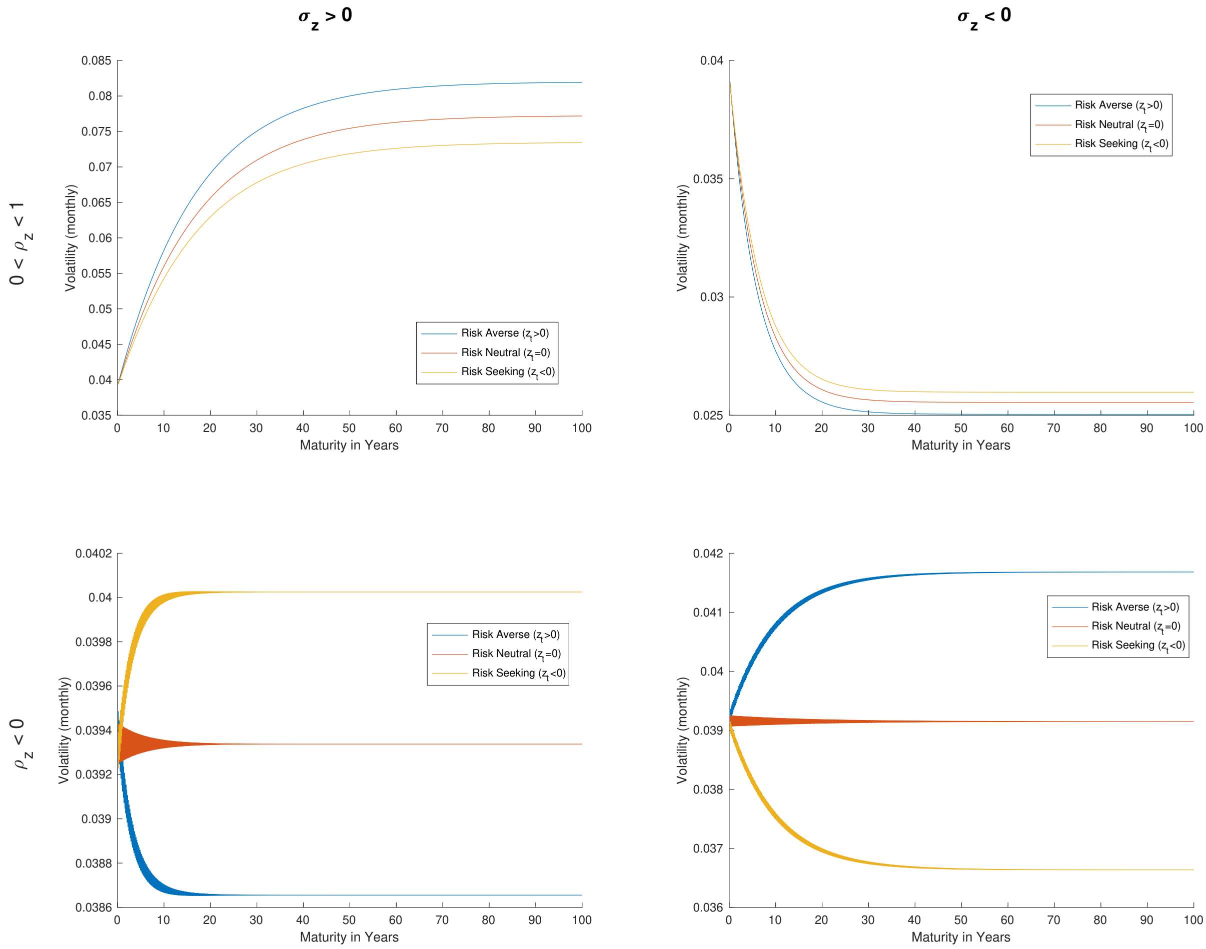

FIGURE 2:
CONDITIONAL TERM STRUCTURES OF EQUITY RISK

be discussed in detail in the next section. Finally, the lower two graphs show results when risk aversion is negatively autocorrelated ($\rho_z < 0$). This specification is included for the sake of completeness though it is rather counter-intuitive as it implies that the agent is alternating between risk-averse and risk-seeking preferences every period. This alternating behavior also causes alternating pricing effects as can be seen from the thickness of the graphs.

The term structure of equity *risk premia* is a result of applying the risk preference to the term risks. Namely, a risk-seeking agent likes to hold risks and thus implies lower premia for higher risks, whereas a risk-averse agent dislikes risks and demands higher premia for holding larger risks. Applying the states of risk preference to the top graphs of Figure **??** gives the term structures of equity risk premia that are shown in Figure **??**.

The term structures of equity risk premia in the left graph are the result of an increasing term structure of equity risk. Under risk-averse preferences we observe that indeed the positive

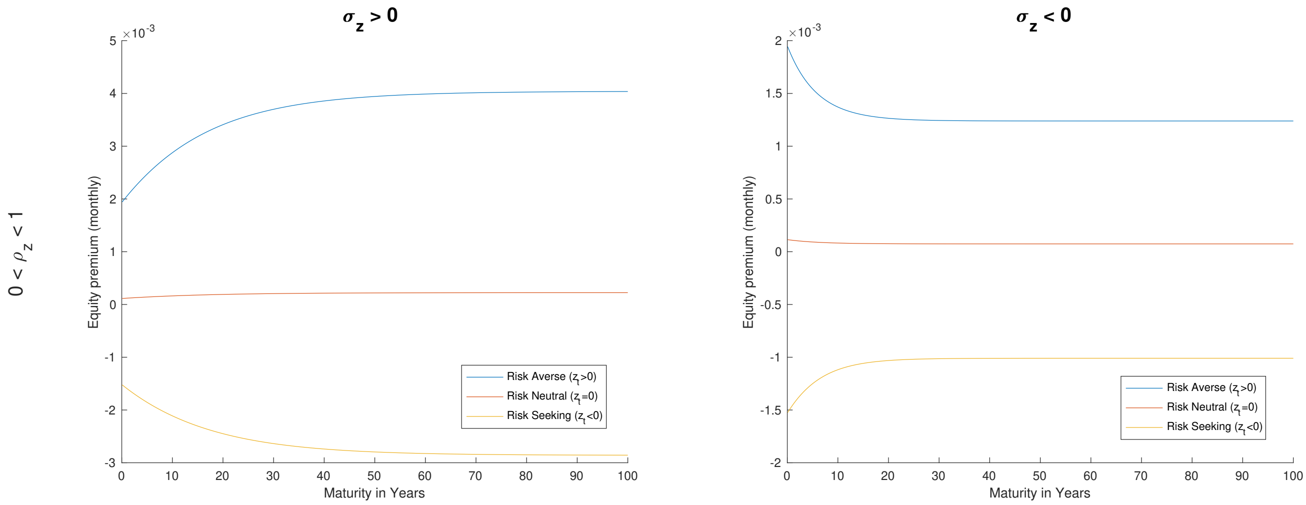

FIGURE 3:
CONDITIONAL TERM STRUCTURES OF EQUITY RISK PREMIA

term risks are translated to positive term premia. However, for the case of risk neutrality there is no premium for term risks and in the case of risk-seeking preferences the positive term risks translate to negative term premia.[12] The graph on the right are the result of a decreasing term structure of equity risk. These negative term risks are the opposite of the risks underlying the left graph. The risk-averse agent thus demands a negative term risk premium as risks are lower for long maturities. The risk-seeking agent demands a positive term premium due to the negative term risks and, similar to the results for the left graph, the risk-neutral agent demands no term premium.

The results in Figure **??** and Figure **??** illustrate the implications of assumptions **??**, **??** and **??**. Empirically we observe that the term structure of equity risk premia is decreasing in good times and increasing in bad times. The figures above show that these results are qualitatively obtained when the dynamics of risk aversion are positively autocorrelated, negatively correlated with consumption growth shocks and in good times states of risk-seeking preferences are possible.

2.4. MECHANISM

The mechanism driving the results of the previous section is best understood when first addressing the term structure of equity *risk*. I provide the intuition for the class of models that allow time-varying risk aversion. Models of time-varying risk aversion always generate an upward sloping

[12]The agent is assumed to always be risk averse over consumption. For the case of risk-neutral gain-loss utility this results in a pricing effect on the first period which causes the shape to deviate slightly from a flat line.

term structure of equity *risk* and I find that this is because they generally assume that

(i) risk aversion is negatively correlated with consumption growth,

(ii) risk aversion is mean reverting.

For risk-averse preferences these positive term risks translate to an increasing term structure of equity *risk premia*, though, when allowing states of risk-seeking preferences, the agent likes risks such that the positive term risks imply negative term premia.

Let me now provide a general framework to explain the intuition. Consider a model of time-varying risk aversion that excludes states of risk-seeking preferences. For ease of discussion, let γ denote the unconditional level of risk aversion and let z_t denote its zero-mean time-varying part. Consider the case where z_t is larger than zero. The intuition for z_t below zero directly follows. Assume that there is a negative shock to consumption growth. This shock causes an increase in risk aversion due to their assumed negative correlation ($\sigma_z > 0$). The left graph of Figure **??** illustrates the general result that prices decrease relative to dividends when risk aversion increases (see for instance, **?**). At this larger value of z_t, the mean reversion ($\rho_z > 0$) causes an increase of the expected future change of risk aversion compared to the lower mean reversion of the previous period. This is shown in the middle graph of Figure **??**, where the absolute future change of risk aversion increases when moving further away from the unconditional mean. In response to the negative shock to consumption growth, the expected future decrease in risk aversion becomes larger. Such a strong expected future decrease in risk aversion implies an immediate increase in prices. The mean reversion of risk aversion (ρ_z) combined with the negative correlation assumption ($\sigma_z > 0$) therefore imply a counteracting (hedging) effect of the economic growth shock, which reduces the riskiness of equity.

This counteracting effect is not the same for all maturities. More precisely, it is stronger for short maturities. The reason for this is that mean reversion implies a larger *absolute value change* at short horizons. The role that mean-reversion plays here is not straightforward. Intuitively one might expect that mean reversion has stronger implications for long-horizons, as for long-horizons risk aversion is expected to equal its unconditional mean. However, the crucial determinant here are expected *changes* in risk aversion and not the *level* of risk aversion. Any shift of risk aversion away from its unconditional mean implies, by means of mean reversion, a strong change in absolute value for short maturities, though this change today barely effects the expected changes far into the future. The right graph of Figure **??** provides an illustration

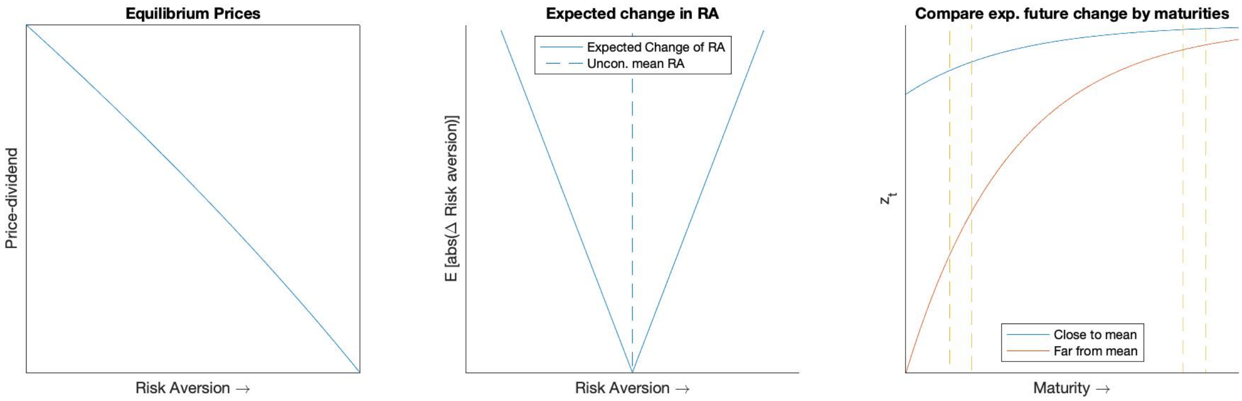

FIGURE 4:
GENERAL ILLUSTRATIONS IN SUPPORT OF MAIN MECHANISM

of this effect. The graph shows the expected changes per maturity for a state that is close to the unconditional mean and for a state that is far away from the unconditional mean. It follows that, when moving from a state close to the unconditional mean to a state far away from the unconditional mean, the one-period expected change in risk aversion increases more for short maturities than for long maturities. The counteracting effect described above is thus stronger for short maturities compared to long maturities.

We established that a negative shock to consumption growth is counteracted by an increase in expected returns. Similarly, a positive shock to consumption growth is counteracted by a decrease in expected returns. This is because when the currently value of z_t is larger than zero, a good consumption growth shock brings the level of risk aversion (z_t) closer to the unconditional mean. The absolute change of risk aversion induced by mean-reversion after the shock therefore drops, which implies the expected future price increase to become smaller.

Combining the implications of the positive and the negative shock implies that economic growth shocks are counteracted by future expected returns through the time-varying risk aversion. This causes the riskiness of equity to decrease. Lastly, mean reversion implies a larger one-period absolute value change for short maturities, therefore equity risk is lowest for short maturities and increases by maturity. Overall this results in the upward sloping term structure of equity *risk*. This finding is formalized in Proposition **??**.

For z_t smaller than zero the intuition is similar. After a negative shock to consumption growth, the absolute value change of risk aversion decreases, and after a positive consumption growth shock the future change in absolute value increases. This is the exact opposite of the case where z_t is larger than zero. However, the effect of mean reversion on prices also reverses. In this case mean reversion of z_t implies that z_t is expected to increase, which means prices are expected to decrease. In sum the effect is similar as for the case where z_t is above the unconditional mean (zero), namely, equity risks increase by maturity.

Finally, the equity term risks are translated to equity term premia depending on the state of risk preferences. The positive term risks are translated to positive term premia for risk-averse preferences and to negative term premia for risk-seeking preferences. Models of time-varying risk aversion generally do impose the first two assumptions on the dynamics, but do not allow for states of risk-seeking preferences (see for instance, **??**). In those models the term risks are positive, though the term premia are also always positive. Proposition **??** formalizes the results discussed above.

Proposition 2. *Let the economy follow a standard* **?***-type endowment economy,*

$$log\left(\frac{C_{t+1}}{C_t}\right) = g_c + \sigma_c \varepsilon_{c,t+1}, \qquad log\left(\frac{D_{t+1}}{D_t}\right) = g_d + \sigma_d \varepsilon_{d,t+1}, \tag{14}$$

where $\varepsilon_{c,t+1}$ and $\varepsilon_{d,t+1}$ are positively correlated and are both I.I.D. and standard normally distributed. Assume there exists an equilibrium (prices are finite) and let the bond term structure be flat. Moreover, let risk aversion be time varying as indicated by state variable z_t. High values of z_t imply states of risk-averse behavior and low values of z_t imply risk-seeking behavior. Finally, risk aversion is negatively correlated with consumption growth and is mean reverting over time. This implies that

(i) the term structure of equity risk, $\sqrt{Var\left(R^{(n)}_{t+1}\right)}$, is upward sloping for all values of z_t,

(ii) the term structure of equity risk premia, $\mathbb{E}_t\left[R^{(n)}_{t+1}\right] - R_{f,t}$, is upward sloping in states of risk-averse preferences and downward sloping in states of risk-seeking preferences.

Proof. See Appendix **??** □

3. Supporting Evidence

The asset pricing model proposed Section **??** incorporates two insights of experimental economics that have not received a lot of attention in financial economics. The first addition, which is the main driver of the cyclicality results of the model, is to allow states of risk-seeking preferences. In such states, these risk-seeking preferences cause the premia of risks to reverse and thereby better match the empirical findings on the cyclicality of the term structure of equity risk premia. In the section below, I provide a more detailed review of the experimental economics literature that finds support for the existence of risk-seeking preferences.[13] The second insight of experimental economics that is incorporated in the model is that of domain-specific risk preferences. In other words, the risk preference of an individual tends to be specific to the domain that can for instance be in the domain of personal health, consumption or finances. This finding suggests that when modeling investor behavior, we should separate the risk preferences concerning consumption decisions from those concerning financial decisions. The previous section illustrated how this finding helps to improve our theoretical models of investor behavior. Below I provide more details on the findings supporting these assumptions.

3.1. Risk-seeking preferences

In financial economics, individuals are predominantly assumed to be averse to risks. This seems intuitive given the size of the insurance market[14] and the positive unconditional equity premium for holding equity risks (**?**). However, there are also indications that individuals (at times) like to hold risks. For instance, the results of **?** are also suggestive of periods of a negative risk premium and, more generally, there is a large interest in gambling,[15] there is excessive risk taking in tournaments (**??**), and there are many individuals active in the very volatility asset class of cryptocurrencies (**??**). Regardless of the motivation behind the behavior of each of these individuals, these observations suggestive of individuals being risk seeking in their decision making.

The experimental economics literature has taken up the task to improve our understanding of risk preferences by producing several risk elicitation methods. One of the most familiar methods is the multiple price lists method by **?**. Their work is a cornerstone in the literature one of the reasons for this being that they were able to give participants relatively large monetary payouts.[16] These large monetary payouts allowed them to tackle the critique that small stake decisions in experiments are not representative of real-world decisions (**?**).

In the experiments of **?**, each participant chooses between two binary lotteries with different

[13] **?** find that preferences elicited from experiments with monetary payments are reliable for natural choices when there is minimal *background risk*. However, in case the risks of the experiment correlate with other risks, the elicited risk preferences are overstated (biased towards higher risk aversion).

[14] **?** report that, according to the Board of Governors of the Federal Reserve System, the market of life insurance in the US alone was already worth $4068 billion in 2012.

[15] **?** reports that, according to the American Gambling Association, in 2007 in the US a total of 55 million people made 376 million visits to the casinos.

[16] After the experiment, one of the gambles in the list was chosen at random and the participants received the outcome of the gamble that they chose. In the high payoff experiments, the payoffs were multiplied by 90.

payoffs and equal probabilities (10% chance to obtain the high payoff and 90% chance to obtain the low payoff). Then, the participant chooses between two bets that are similar to the first round where only the payoffs probabilities are adjusted by 10%. This sequence of choice tasks goes on until the first payoff is guaranteed. By changing the probabilities of the payoffs, the difference of the expected payoff of both lotteries changes, however, the second lottery is always the more riskier choice. An overview of this choice experiment is given in Table **??**, which is an augmented version of Table 1 from **?**.

TABLE 1:
MULTIPLE PRICE LISTS METHOD

Option A	Option B	Expected payoff difference	Volatility Option A	Volatility Option B
1/10 of \$2.00, 9/10 of \$1.60	1/10 of \$3.85, 9/10 of \$0.10	\$1.17	\$0.12	\$1.13
2/10 of \$2.00, 8/10 of \$1.60	2/10 of \$3.85, 8/10 of \$0.10	\$0.83	\$0.16	\$1.50
3/10 of \$2.00, 7/10 of \$1.60	3/10 of \$3.85, 7/10 of \$0.10	\$0.50	\$0.18	\$1.72
4/10 of \$2.00, 6/10 of \$1.60	4/10 of \$3.85, 6/10 of \$0.10	\$0.16	\$0.20	\$1.84
5/10 of \$2.00, 5/10 of \$1.60	5/10 of \$3.85, 5/10 of \$0.10	-\$0.18	\$0.20	\$1.88
6/10 of \$2.00, 4/10 of \$1.60	6/10 of \$3.85, 4/10 of \$0.10	-\$0.51	\$0.20	\$1.84
7/10 of \$2.00, 3/10 of \$1.60	7/10 of \$3.85, 3/10 of \$0.10	-\$0.85	\$0.18	\$1.72
8/10 of \$2.00, 2/10 of \$1.60	8/10 of \$3.85, 2/10 of \$0.10	-\$1.18	\$0.16	\$1.50
9/10 of \$2.00, 1/10 of \$1.60	9/10 of \$3.85, 1/10 of \$0.10	-\$1.52	\$0.12	\$1.13
10/10 of \$2.00, 0/10 of \$1.60	10/10 of \$3.85, 0/10 of \$0.10	-\$1.85	\$0.00	\$0.00

For each participant, the risk preference can be computed based on their preferred lotteries. Specifically, the level of risk aversion is computed based on whether and when the participant switches from option A to B. A risk-neutral individual would choose option A for the first four lotteries and B for lottery 5 up to 10. In the extreme, an infinitely risk-averse person would choose option A in the first task and at least for the last choice switch over to option B. The switching behavior is therefore insightful about the risk preferences of the decision maker. Individuals that make an early switch to option B (before the 5th decision) fall in the domain of risk-seeking preferences.[17] The analysis does not distinguish between the origins for such risk-seeking behavior, but merely establishes that some individuals prefer a more risky bet (higher volatility) even if it has a lower expected payoff.

The work of **?** popularized the multiple price lists method, which lead others to perform similar experiments (**???**). In addition to the multiple price lists method, also other approaches have been performed to elicit risk preferences. Some of the most familiar methods are: the balloon analogue risk task (**?**), the **??** method, the **?** method, the adaptive lottery method (**?**), the

[17]The results excluded a few participants who switched more than once.

marbles task method (**?**) and the Columbia card task (**?**).[18] All methods excluding the BART method have identified a substantial proportion of their participants as risk seeking (**?**).

Following these findings of risk-seeking preferences by the experimental literature, one may wonder why such preferences have been rarely analyzed in asset pricing models. There can be multiple reasons. First, models used to analyze asset prices have predominantly targeted the equity premium puzzle of **?** for which they need a substantial level of risk aversion. Consider for example the optimal investment fraction of **?**

$$\frac{\mathbb{E}[R_{t+1}] - R_f}{\gamma \mathrm{Var}(R_{t+1})}, \tag{15}$$

where γ denotes the level of risk aversion. In the case where the representative agent invests according to this fraction, it directly follows that risk-seeking preferences imply negative risk premia. It is thus clear that considering risk-seeking preferences would not help a model to solve the equity premium puzzle of **?**.

Another reason may be the difficulty of incorporating risk-seeking preferences in an equilibrium asset pricing model. Consider for instance CRRA-preferences over consumption. These preferences come with the restriction that the level of risk aversion needs to be strictly larger than one which excludes states of risk-seeking preferences. A problem that arises with risk-seeking preferences is that for commonly used parameter values, there does not exist a positive price that solves the pricing equation of the risky asset. To illustrate this, consider a **?**-tree endowment economy, where for simplicity consumption and dividends are identical both with growth g_c and volatility σ_c, and where the representative agent has constant CRRA(γ)-preferences and a subjective time discount factor, β. The equilibrium return on the risky asset then follows from the following equilibrium price-dividend ratio[19]

$$\frac{P_t}{D_t} = \frac{1}{\frac{1}{\beta} e^{-(1-\gamma)g_c - \frac{1}{2}\sigma_c^2(1-\gamma)^2} - 1}. \tag{16}$$

Positive prices are thus obtained if the denominator on the right of (**??**) is positive. The empirical estimates of the consumption growth imply an annual growth of about 1.8% with an annualized volatility of 2.7% (**?**). Assuming risk neutrality ($\gamma = 0$) the parameter of time discounting, β, can be at most 98.18% in order for the price dividend ratio to remain positive.[20] Values of γ

[18] See **?** or **?** for an overview.

[19] For a derivation see Appendix **??**.

[20] The results become stronger when separating dividends and consumption. Following the literature and assuming

that imply risk-seeking preferences ($\gamma < 0$) rapidly reduce this upper-bound on the value of time-discounting. Since the finance literature commonly assumes time discounting to equal 98% annually, the model cannot have positive prices for risks seeking preferences under commonly used parameters.

Combining the above, there seems to have been little incentive to incorporate the experimental findings in asset-pricing models. This subsequently also carries over to the empirical literature in finance that attempts to estimate risk preferences. Compared to the controlled setting of experiments, it is difficult to estimate risk preferences empirically (**?**). Moreover, as there has been little interest in risk-seeking preferences and it complicates financial models, risk-seeking preferences have been excluded ex-ante from the estimation analysis. For instance, **?** finds strong dispersion in risk aversion among analysts, though, the findings are truncated to non-negative risk aversion such that risk-seeking preferences are prohibited ex-ante.

3.2. Domain-specific risk preferences

The findings in experimental economics illustrate the existence of risk-seeking preferences. However, even if one wants to analyze such preferences in an asset pricing model, the previous section showed that it is non-trivial to implement.

It turns out that another finding of experimental economics is particularly helpful for this problem. Namely, the finding that risk preferences are not universal but are domain-specific (**?**). The domains that have been shown to yield different risk taking behavior are: financial, health/safety, recreational, ethical and social decisions (**?**). The differences in behavior per domain seem to originate from differences in perceived riskiness. Even for similar risks, individuals tend to perceive risks concerning their health to be a lot larger compared to financial decisions.

The model in Section **??** is similar to the model of **?** and **?** that was originally proposed in **?**. The important aspect of the original model is that it enforces domain-specific risk preferences by separating the preferences for financial decisions from those for consumption decisions.[21] This separation is done by assuming the investor obtains utility both from consumption and from

a dividend growth equal to consumption growth, increasing dividend volatility by five times and a correlation of 15% with consumption growth, yields an upper-bound for time discounting of 97.33% for a risk-neutral representative agent.

[21]The micro-foundation of this behavior is left out as it goes into a level of detail that is not appropriate for the target of the analysis, which is the shape and cyclicality of the equity term structures. One could introduce perceived riskiness through a preference probability distortion similar to probability weighting. By letting this be domain-specific one would also obtain the domain-specific risk taking behavior.

financial gains and losses. This separation is key for the model to have an equilibrium when allowing states where the agent is risk seeking for financial decisions. Namely, by simultaneously allowing a sufficient degree of risk aversion for consumption decisions, the agent is always overall risk averse even in those states where he or she is risk-seeking for financial decisions.[22]

Adding gain-loss utility leads to a separation of the SDF into two terms, which is formally stated in Proposition **??**. This is convenient as it creates the opportunity to explain the equity premium and the conditional equity term structures simultaneously. The intuition for this result is as follows. The SDF is separated into two terms, one of which is linked to consumption utility and the other is linked to gain-loss utility, $M_{t,t+1} = M^{(c)}_{t,t+1} + M^{(gl)}_{t,t+1}$. Now consider the common result that the risk premium of an asset is equal to the covariance of the return with the SDF multiplied with minus the risk-free interest rate. This implies for the excess return on the stock and on a dividend strip with maturity n:

$$\mathbb{E}_t[R_{t+1} - R_{f,t}] = -R_{f,t}\Big\{\mathrm{Cov}\big(M^{(c)}_{t,t+1}, R_{t+1}\big) + \mathrm{Cov}\big(M^{(gl)}_{t,t+1}, R_{t+1}\big)\Big\} \tag{17}$$

$$\mathbb{E}_t[R^{(n)}_{t+1} - R_{f,t}] = -R_{f,t}\Big\{\mathrm{Cov}\big(M^{(c)}_{t,t+1}, R^{(n)}_{t+1}\big) + \mathrm{Cov}\big(M^{(gl)}_{t,t+1}, R^{(n)}_{t+1}\big)\Big\}. \tag{18}$$

The equations directly show that the separation of the stochastic discount factor yields one more degree of freedom to fit the $n+1$ equations. A positive equity premium is generated when the SDF has a negative correlation with consumption (and dividends), which is the case for risk-averse preferences. Though for obtaining the downward sloping term structure of equity risk premia, the SDF needs to have a positive correlation with dividends. By seperating the SDF into two terms, both effects can hold simultaneously. Following the model of **?**, the time variation in risk preferences comes from gain-loss utility such that $M^{(gl)}_{t,t+1}$ takes the role of the SDF that generates the term premia. At the same time, $M^{(c)}_{t,t+1}$ is time-invariant and can thus be calibrated to match the equity premium. More specifically, the agent can become risk-seeking over his or her gain-loss utility, while at the same time being risk-averse over consumption utility. States where the investor is risk-seeking over gain-loss utility then induce a downward sloping term structure of equity risk premia, whereas the unconditional risk aversion over consumption utility induces a positive equity premium. The risk aversion over consumption utility counteracts states where the investor is risk-seeking over his or her gain-loss utility, such that it needs to

[22]The proposed model is unique as it allows states where the representative agent is risk-seeking for all financial payoffs. The model thereby differs from models that incorporate *contect-dependent* risk preferences that imply risk-seeking solely over losses (and risk aversion over gains) as in prospect theory (**?**).

be sufficiently large to be able to counteract states of risk-seeking gain-loss utility and thereby allow for markets to clear.

4. Empirical Implications

4.1. Model Calibration

In line with the literature I specify the model on a monthly frequency and use parameter values of consumption and dividend growth that have been calibrated to match the US post-war data as in **?**. Both economic growth processes have the same expected growth, $g_c = g_d = 0.0015$ and dividend growth is substantially more volatile than consumption growth, $\sigma_c = 0.0078$ and $\sigma_d = 5\sigma_c$. The shocks to consumption growth and dividend growth are positively correlated $\omega = 0.15$. Table **??** shows the annualized moments of economic growth compared to the empirical estimates (**?**).[23].

Table 2:
Annualized simulated moments

Moments	Data	Model
$\mathbb{E}(\Delta c)$	1.93	1.80
$\sigma(\Delta c)$	2.16	2.18
$AC1(\Delta c)$	0.45	0.23
$\mathbb{E}(\Delta d)$	1.15	1.80
$\sigma(\Delta d)$	11.05	10.89
$AC1(\Delta d)$	0.21	0.23

The preference structure of the model consists of both consumption utility and gain-loss utility. I use a standard calibration for the risk aversion over consumption by assuming $\gamma = 5$ as well as the subjective time discount factor $\beta = 0.998$. For the sake of simplicity of the model, I have chosen consumption utility to be CRRA, which is commonly used in **?**-tree models. This assumption does come at a cost. Namely, the model will face difficulty matching the common moments of both the risky and the risk-free asset (**?**).[24] The level of risk aversion over consumption plays an important role in the model, as its value needs to be sufficiently high to counteract states where the investor is risk seeking over gain-loss utility.

[23]In Table **??**, lowercase letters describe logs

[24]Under the simplifying assumptions of the model, the model has difficulty to match the risk-free interest rate and the level of equity premium, for common parameter values. These moments can potentially be matched by adding features to the model that have shown to match these moments, like for adding example disaster risks (**??**), long-run risks (**?**) or **?**-preferences.

In addition to the utility over consumption, I also introduced the utility over financial wealth fluctuation or gain-loss utility. I follow **?** and set the monthly autocorrelation of risk aversion to 99.13%, which equals 90% on an annual basis. Then, I choose to vary the risk aversion over gain-loss utility through the gain domain such that $\kappa_1 = 1$ and $\kappa_2 = 0$. Appendix **??** shows the results when choosing the loss domain instead, where the results for the equity term structure are qualitatively unaffected. Furthermore, I choose σ_z and λ to match the empirical findings on the equity term structure. For the baseline results I therefore use $\lambda = 1$ and $\sigma_z = \frac{0.12}{\sqrt{12}}$. This implies that the agent is unconditionally risk neutral over gain-loss utility. Additionally the value of σ_z implies that the unconditional distribution of z_t is normal with mean zero and standard deviation of 0.26. This implies moderate values of concavity and convexity of the gain-loss utility function such that the model uses relatively conservative values of risk-seeking preferences to obtain the main results. Table **??** provides an overview of the baseline parameters for a monthly frequency.

TABLE 3:
MODEL PARAMETERS

Fundamentals	Symbol	Value
Expected log-consumption growth	g_c	0.0015
Volatility of log-consumption growth	σ_c	0.0078
Expected log-dividend growth	g_d	0.0015
Volatility of log-dividend growth	σ_d	$5\sigma_c$
Correlation of log-consumption and log-dividend growth	ω	0.15
Preferences	Symbol	Value
Time discount factor	β	0.998
Risk aversion (consumption-utility)	γ	5
Time-variation valuation of gains	κ_1	1
Time-variation valuation of losses	κ_2	0
Unconditional level of risk aversion (gain-loss utility)	λ	1
Autocorrelation of risk aversion (gain-loss utility)	ρ_z	$0.9^{(1/12)}$
Variability of risk aversion (gain-loss utility)	σ_z	$\frac{0.12}{\sqrt{12}}$

4.2. Pricing Implications for the Equity Term Structure

The numerical solution to the price-dividend ratio is shown in Figure **??**, where the interval of state variable z_t covers +/- three standard deviations.

The price-dividend ratio decreases with z_t, that is, it decreases with the degree of risk aversion. This is in line with previous results that show for higher risk aversion the price-dividend ratio is lower **??**. Furthermore, the graph illustrates the importance of gain-loss utility as can be observed from the significant changes in the price dividend ratio when varying the level

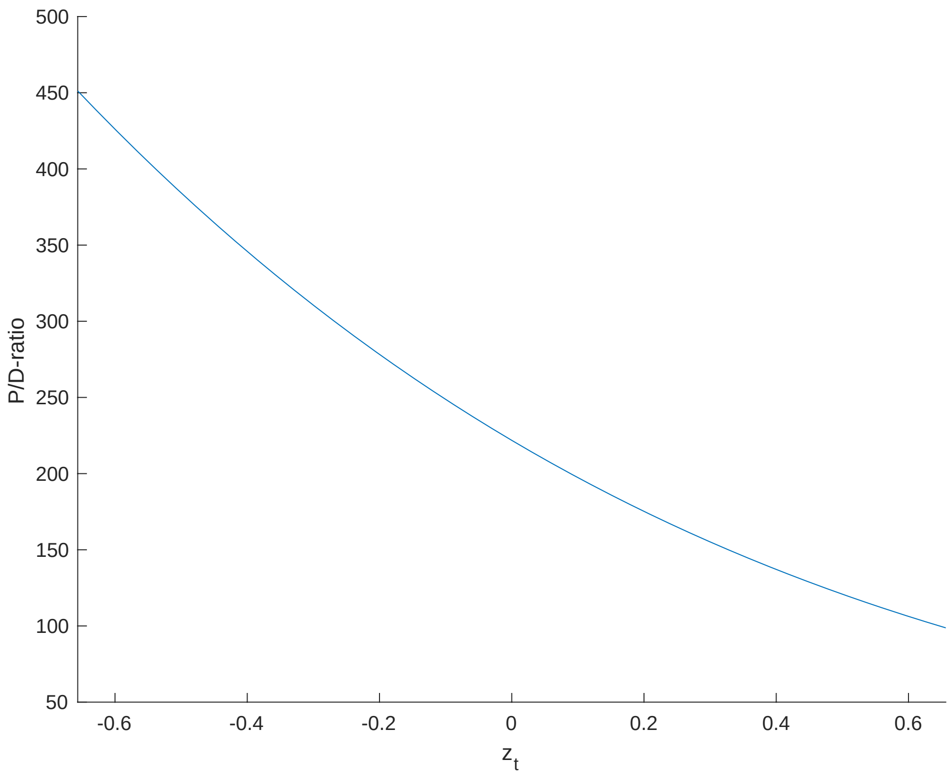

FIGURE 5:
EQUILIBRIUM PRICE-DIVIDEND RATIO

of risk aversion. Considering the general pricing implications in the next section, it shows that this variation in the price-dividend ratio translates to a large level of excess volatility on financial markets, which is an empirical findings that is generally difficult for consumption-based asset pricing models to produce.

Using the equilibrium price-dividend ratio function, the individual price-dividend ratios of the dividend strips can be computed for all maturities. With the price-dividend ratios of the dividend strips we can then compute the one-period holdings returns to construct the term structure of equity risk premia. These term structures of equity risk premia conditional on current state z_t are shown in left graph of Figure **??**.

The graph shows the premia demanded by the investor for different maturities for states of risk-seeking preferences ($z_t < 0$), risk-neutral preferences ($z_t = 0$) and risk-averse preferences ($z_t > 0$). The shapes illustrate that the model predicts term premia to be changing up to a maturity of about 60 years. Due to data limitations, empirically the long-maturity claim is commonly proxied by the stock. As the average duration of the stock in the model is about 27 year, the empirical analysis focuses on the relative short-end of the term structure. Moreover, the duration

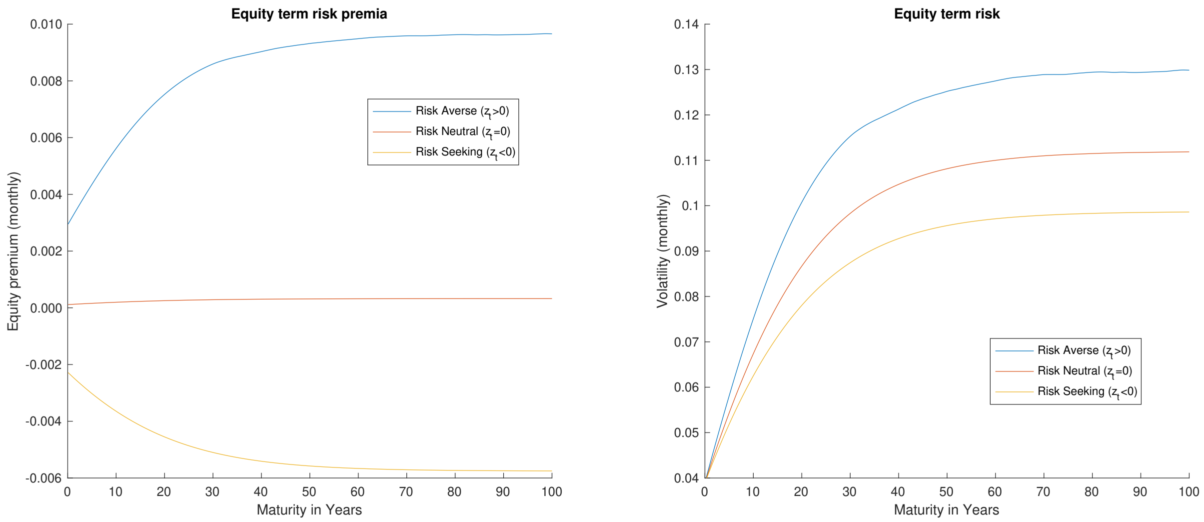

FIGURE 6:
SIMULATED EQUITY TERM RISKS AND TERM RISK PREMIA

of the stock is state-dependent and varies from 14 years to 38 years for respectively +/- three standard deviations of the unconditional mean. The variation of the stocks duration across states illustrates a problem for using the stock as long maturity claim in the definition of term premia as is done in the empirical literature (**??**). Therefore, in the quantitative analysis below, I also show the results when using long maturity dividend strips instead.

The differences between the slope of the equity term *risk premia* in the left graph of Figure **??** are best explained from the term structure of equity *risk* in the right graph. Namely, as explained in Section **??**, the term structure of equity *risk* is upward sloping for any value of z_t as the mechanism that drives the riskiness of short versus long maturity claims depends on the dynamics of z_t. However, when translating the term structure of equity *risk* to that of *risk premia*, it is crucial whether the current state of z_t implies risk-averse, risk-neutral or risk-seeking preferences. In case of risk aversion, the agent dislikes risks and demands a larger premium for the long maturity claims. Alternatively, a risk-neutral agent does not demand any premium and a risk-seeking agent is willing to pay a premium, even more so for the risky long maturity claims. Even though we are mostly interested in their slope, the level of the graphs is related to the equity premium and will be discussed in more detail in the next section.

The results reported in the empirical literature are annualized ex post premia, hence, for a direct comparison I simulate the model and compute the annualized ex post premia. To obtain accurate estimates of the moments I perform monthly simulations of 1000 years and take the average of these moments over 10 of such simulations. Figure **??** provides an overview of the simulated state variable z_t. Which confirms that the state variable z_t is normally distributed with mean zero and variance $\frac{\sigma_x^2}{1-\rho_z^2}$. Subsequently, I follow **?** and define bad times as states where the

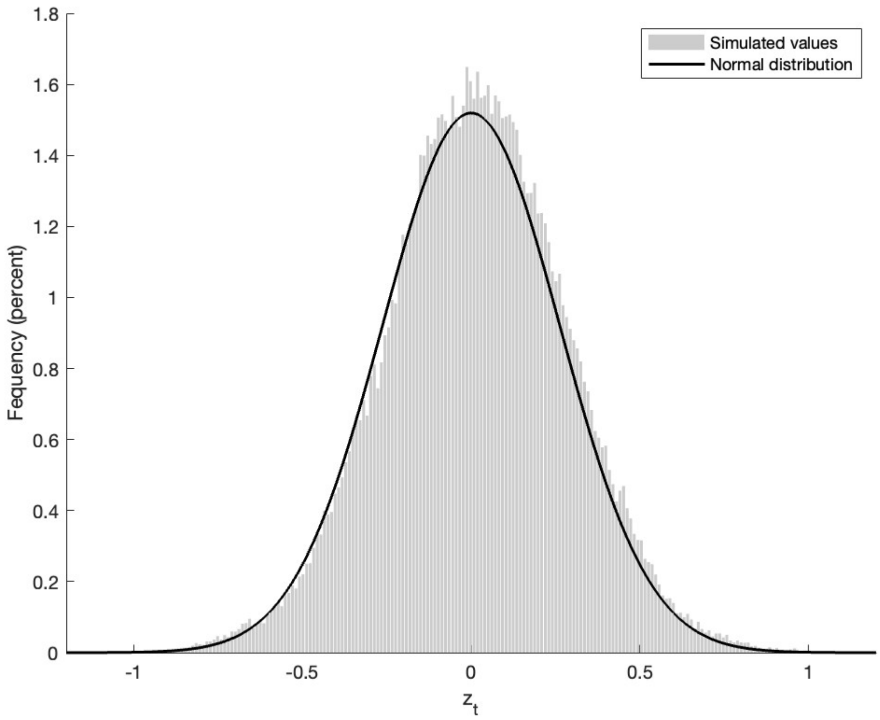

FIGURE 7:
SIMULATED STATE VARIABLE

price-dividend ratio is in the bottom quintile of its historical distribution. I perform ten monthly simulations of 1000 years and take the average to obtain robust estimates of the term structures. The resulting estimates of the unconditional and conditional term structure of equity risk premia are shown in Figure **??**.

The graphs in Figure **??** show the equity premia for various maturities and how these differ between good and bad times. During bad times the agent is likely to be risk averse and thus translates the upward sloping term structure of equity risk to an upward sloping term structure of equity risk premia. Also, because bad times contain relatively many extreme realizations relative to good times, the term premia are a lot larger in magnitude. In good times the preferences of the agent are a mixture of risk-averse and risk-seeking preferences. Their relative frequency determines the slope of the equity term structure in good times. **?** compares the average return of the short horizon claims (dividend strips from maturities of 1 up to 7 years) with that of the stock and finds an upward slope in bad times of roughly 5% and a downward slope in good times of roughly 4%. The equivalent values for the model are an upward slope of 6.49% in bad times

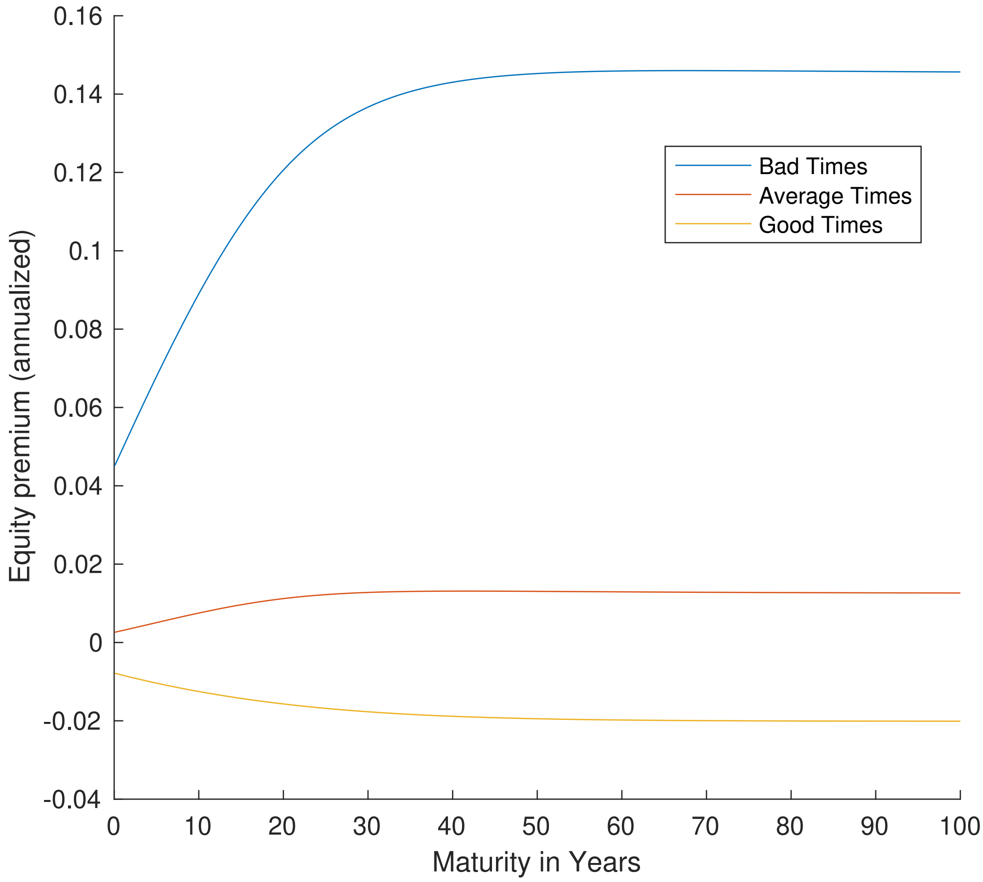

FIGURE 8:
SIMULATED ANNUALIZED EQUITY TERM RISK PREMIA

and a downward slope of 1.29% in good times. Figure **??** also illustrates that the empirically induced metric used by **?** does not reveal the strong term premia and the cyclicality therein that the model predicts. For the quantitative analysis below I therefore also consider maturities beyond that of the stock.

? raise an important concern on the empirical findings of cyclicality found by **?** as the main findings are based on only 16 years of data. They argue that this relatively short time period may provide biased results as the frequency of good and bad times during this specific time period potentially deviates from the historical average. Although **?** shows robustness of his results using both option data and performing a cross-sectional analysis that allows him to use a longer time period, I illustrate the concern raised by **?**. To do so, I use the common procedure of simulating the model on a time period equivalent to the data and comparing the data with the outer percentiles of the simulations (**???**). Figure **??** in Appendix **??** illustrates that indeed such a time period may give very different results then the actual moments shown in Figure **??**.[25]

The results up to now show the model is able match the qualitative empirical results on the cyclicality of the equity term structure. Next, I analyze whether the predictions of the model are also quantitatively in line with the empirical findings. For this quantitative comparison I perform the following regressions,

$$R^{long}_{t,t+12} - R^{short}_{t,t+12} = \alpha_1 + \beta_1 (p_t - d_t) + \varepsilon_{t,t+12}, \tag{19}$$

$$sgn\left(R^{long}_{t,t+12} - R^{short}_{t,t+12}\right) = \alpha_2 + \beta_2 (p_t - d_t) + \varepsilon_{t,t+12}, \tag{20}$$

where a significant negative value of β_1 in (**??**) shows the countercyclicality of the term premium. This regression does however not distinguish between upward and downward slopes. For a quantitative analysis of the inversion of the slope, I also perform the regression in (**??**), where a negative value of β_2 implies the inversion of the slope of the equity term structure.

? shows the empirical results for (**??**) when approximating the term premium by the difference of the stock and the two year dividend strip. However, as the model shows, the equity term structure can be substantially upward sloping beyond the maturity of the stock. In such a case the term premium is more accurately estimated using longer maturity equity claims. I therefore consider the specification used in **?** but also analyze the results when using either the 10 year or the 20 year dividend strip as the long maturity claim. The results of both regressions are in Table **??**.

TABLE 4:
REGRESSION ESTIMATES

	Data - β_1	Model - β_1	Model - β_2
Stock	-0.50	-0.01	-0.09
	(-3.28)	(-1.79)	(-3.22)
10-years	-	-0.05	-0.43
		(-17.43)	(-13.97)
20-years	-	-0.10	-0.42
		(-16.62)	(-13.32)

Results show the estimated value of β_1 of (**??**) and β_2 of (**??**) where the short maturity asset is the two-year dividend strip and the long maturity asset is denoted in the first column. The t-statistics are within parentheses. The empirical estimates are taken from **?**, who does not report the empirical values of (**??**). The model is simulated 100 times at a monthly frequency for 10.000 years.

[25] Figure **??** in Appendix **??** shows the outer percentiles of simulations of 1000 years instead and illustrates that the estimated term premia shown in Figure **??** are sizable and different from zero.

The results show that for long maturity claims, the term premium is significantly countercyclical. However, when using the stock as the long maturity claim the results are of the same sign, though, no longer significant. This result highlights that the term premium estimated by the difference of the stock with the two year dividend strip gives a relatively poor estimate of the term premium. The longer maturity claims provide more power to test the cyclicality. Additionally, the estimates of (**??**) show that the term premium is significantly inverting over the business cycle and thereby show the model also quantitatively matches the empirical findings.

4.3. General Pricing Implications

The previous section illustrated the model implications for the equity term structures. Mind that the value of the stock is obtained by aggregating all dividend strips and thereby the previous results about the dividend strips carry over to that of the stock. Let us therefore now consider the implications the model has for the general stock market moments.

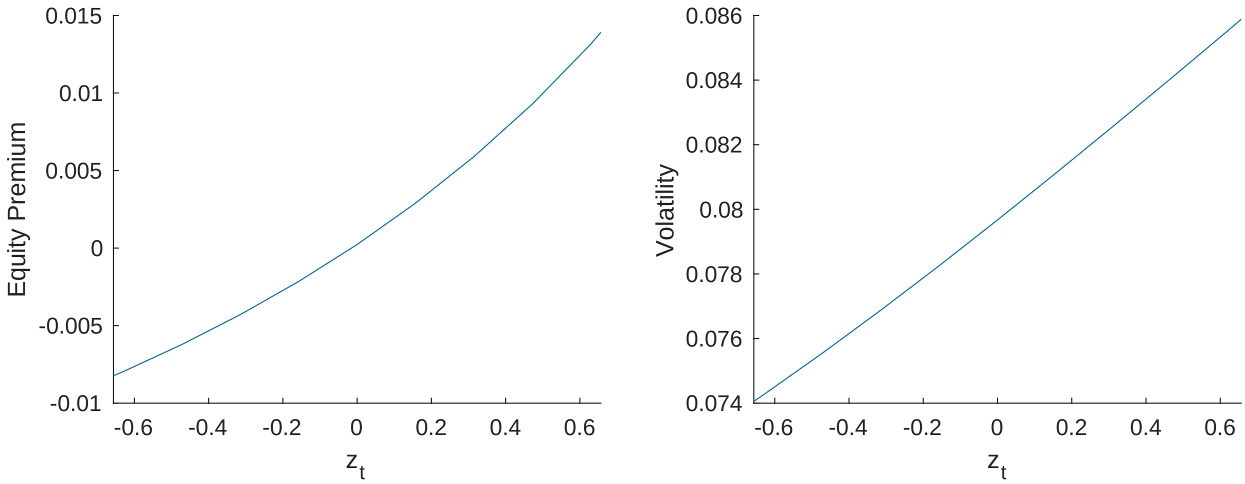

Figure 9:
Monthly equilibrium moments

Figure **??** displays the equity premium and volatility of the stock conditional on state z_t. The left panel shows that the risk premium is increasing with risk aversion, and for low values of z_t, which imply risk-seeking preferences, the equity premium becomes negative. The equity premium is low on average because of the CRRA consumption utility. Specifically, due to CRRA consumption utility the risk free interest rate, R_f, is relatively high which causes a relatively small equity premium. The graph does however show that there is substantial variation of the monthly equity premium, which again highlights the relevance of gain-loss utility in explaining financial moments. The strong time variation of the model is also illustrated in the right graph of Figure **??**, which shows that the monthly volatility induced by the model is relatively large.

The annualized moments obtained from simulating the model are shown in Table **??**.[26] The empirical estimates are taken from **?**, lower-case letters reflect logs.

TABLE 5:
ANNUALIZED SIMULATED MOMENTS

Moments	Data	Model
$\mathbb{E}(r_e)$	5.47	0.63
$\sigma(r_e)$	20.17	30.12
$AC1(r_e)$	0.02	-0.05
$\mathbb{E}(r_f)$	0.56	11.06
$\mathbb{E}(p-d)$	3.36	2.91
$\sigma(p-d)$	0.45	0.29
$AC1(p-d)$	0.87	0.72

The results show indeed that the model with utility over financial wealth fluctuations is able to generate a large degree of excess volatility. Also, the price-dividend ratio and its variation are both relatively high compared to other asset pricing models (see **?**). The reason why this does not translate to a strong equity premium is because of the assumed CRRA-preferences over consumption. A risk aversion over consumption of $\gamma = 5$ leads to an excessive risk-free interest rate such that the equity premium is only slightly positive even-though the expected return on equity is fairly large. This is surprising considering the risk tolerance of the gain-loss utility function. As the target of the model is explaining the term premia, I do not aim to explain the risk free rate. However, one could extend the model by models that have shown to match these moments, like, disaster risks (**??**), long-run risks (**?**) or **?**-preferences.[27] The sensitivity analysis will show how, for different values of for instance λ and σ_z, the equity premium may change.

In addition to the moments in Table **??**, I analyze the predictability caused by the model. The empirical literature finds that the log price-dividend ratio predicting excess stock returns (**??**), whereas dividend growth or real interest rates do not. I therefore follow the approach of **?** and analyze the predictability of excess returns with the predictability of consumption and dividend growth. The results are obtained from simulations and shown in Table **??**.[28]

[26] I follow the approach of **?** and simulate time-series of 78 years, which is equivalent to that of the data. To initialize the state space the model is pre-run for 10 years. The moments are estimated for each simulated trajectory for 100,000 simulations. The median values of these moment estimates over all trajectories are shown in Table **??**.

[27] **?**-preferences seem particularly useful in this regard as they disconnect the risk-free interest rate from the level of risk aversion over consumption. Incorporating such preferences would strongly complicate the analysis and is thus left for further research.

[28] Again, I perform simulations of 78 years and perform the regressions per trajectory. The median over 100.000 simulated trajectories are given in Table **??**. Regressions are on annual frequency. The last column shows the percentage of estimated R^2's in the simulations that are below the corresponding empirical value.

TABLE 6:
RETURN AND CASHFLOW PREDICTABILITY

	$\hat{\beta}$ data	t data	$\hat{R}^2$ data	$\hat{\beta}$ model	t model	$\hat{R}^2$ model	$\%(\hat{R}^2)$ model
			$\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$				
1 year	-0.093	-1.803	0.044	0.063	0.559	0.004	0.998
3 years	-0.264	-3.231	0.170	-0.368	-1.913	0.048	0.980
5 years	-0.413	-3.781	0.269	-0.721	-3.000	0.122	0.924
			$\sum_{j=1}^{J}(\Delta c_{t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$				
1 year	0.011	1.586	0.060	0.001	0.125	0.009	0.928
3 years	0.010	0.588	0.013	-0.001	-0.061	0.020	0.417
5 years	-0.001	-0.060	0.000	-0.004	-0.147	0.030	0.000
			$\sum_{j=1}^{J}(\Delta d_{t+j}) = \alpha + \beta(p_t - d_t) + \varepsilon_{t+j}$				
1 year	0.074	1.977	0.092	0.039	0.803	0.010	0.985
3 years	0.107	1.330	0.059	-0.046	-0.474	0.015	0.828
5 years	0.089	1.214	0.039	-0.128	-1.017	0.029	0.566

Table **??** reveals the strong counter-cyclical nature of the model that matches the data especially well for relatively long maturities. With respect to explaining return predictability the model is doing well as the log price-dividend ratio predicts the excess market return on long horizons and does this with the correct sign. The model is thus able to match the substantial predictability that is observed empirically without assuming predictability of cash flows, which is not satisfied by the models addressing the cyclicality of the equity term structure by adapting the belief process (**????**). Overall, the model cannot be rejected based on the comparison of the empirical R^2 to the estimates of the simulations.

4.4. SENSITIVITY ANALYSIS

The baseline parameters are able to generate the empirical findings on the cyclicality of the term structure of equity risk premia. This section investigates how the predictions of the model depend on the preference parameters. The economic parameters $(g_c, g_d, \sigma_c, \sigma_d)$ are set equal to values commonly used to match the empirical moments of consumption and dividend growth and are therefore not of interest for this sensitivity analysis. The interest goes to all previously mentioned asset pricing implications. Appendix **??** provides overview tables that includes all these results for various parameter specifications.

First consider the unconditional level of risk aversion over the gain-loss utility, which is denoted by λ. The results are shown in Table **??**. A larger value for λ, implying a higher risk aversion over gain-loss utility, causes a shift in the realized levels of risk aversion. The frequency of risk-seeking states drops such that the average slope in good times moves more towards the states of risk-averse behavior. This effect causes the slope of the equity term structure in good times to become less negative. Also the slope of the equity term structure in bad times drops. This is because the volatility of z_t is now lower relative to the level of risk aversion. This change implies less relative variation in the level of risk aversion over gain-loss utility such that the term premum drop in magnitude also in bad times. Although a larger value of λ works against the term structure results, it does cause the equity premium and volatility of the stock to move closer to their empirical estimates. This result is in line with that of **?**, who find that adding gain-loss utility with loss averse preferences helps to explain the equity premium puzzle.

Now consider the degree of time variation of risk aversion over gain-loss utility, controlled by σ_z. Table **??** replicates the results when adapting the degree of time-variation. The results show that a larger variability of risk aversion over gain-loss utility helps to match the magnitude of the slopes of the equity term structures. When the dividend growth shock has a stronger effect on changes in risk aversion, future risks increase such that for a given value of risk aversion the term premia become larger. Additionally, increasing the variation in risk aversion also causes the level of stock volatility to increase to a fairly large value.

In the specification of the gain-loss utility function I have chosen to vary risk preferences through the gain domain by setting κ_1 equal to one. Table **??** shows the results when κ_2 is nonzero instead. The regression results show that the baseline specification where risk aversion of gain-loss utility changes through the gain-domain ($\kappa_2 = 0$) leads to a larger cyclicality of the equity term structure. Table **??**, however, also shows that the results give the same qualitative predictions when varying risk aversion through κ_2 instead of κ_1.

Lastly, I consider alternative values of risk aversion over consumption utility, γ. The level of risk aversion relative to the dynamics of the risk preferences of gain-loss utility is important for the existence of an equilibrium. The results for different values of γ are in Table **??**. Larger values of γ cause the overall risk aversion to increase such that, under the CRRA-specification of consumption utility used, both the return on equity and the risk-free interest rate raise substantially. Both effects seem to cancel eachother out when considering the equity premium. The risk-free interest rate and the equity premium can potentially be matched adding features to the model that have shown to match these moments, like for adding example disaster risks (**??**), long-run risks (**?**) or **?**-preferences.

In sum, I find that the results on the term premia provide insights in the preferences parameters used. The calibration of the model to the empirical observations of the term premia thereby illustrate the relevant values for the parameters. Although the model is motivated qualitatively on experimental findings, future work should estimate the parameters used to test the calibrated values.

5. CONCLUSION

In this paper I explain the role of time variation of risk aversion on the cyclicality of the slope of the equity term structure. I provide three conditions that collectively generate the empirical observed cyclicality, namely, that risk aversion is negatively correlated with consumption growth shocks; risk aversion is mean-reverting; and the agent can become risk seeking in some states of the world. I contribute to the theoretical asset pricing literature by explaining the mechanism behind models of time-varying risk aversion for the shape of the equity term structure.

Secondly, I contribute to the literature by proposing a parsimonious model of time-varying risk aversion that allows for states of risk seeking preferences. By extending the equilibrium model of **?**, this model is the first equilibrium model that allows states of risk-seeking preferences that are independent of payoff. The results of the model illustrate the importance of incorporating findings in experimental economics to models in financial economics. Specifically, this model is able to match the recent puzzling empirical findings on the equity term structure by allowing the possibility of having risk seeking preferences and of having domain-specific risk preferences (where domains refer to, for instance, health, consumption and financial decisions). The domain-specific risk preferences allow the agent to have states of risk-seeking preferences over his financing wealth, whereas he is always risk averse over his consumption decisions.

Thirdly, by deriving the stochastic discount factor from preferences and beliefs, this result can potentially help to understand the exogenous SDFs that have been proposed in the literature to generate the term structure results. Lastly, I contribute to the empirical asset pricing literature by providing a model that can potentially be used to estimate risk-seeking preferences on the aggregate market.

Appendices

A. The equity term structures

The term structure of equity risk premia is defined as the premia on one-period holding returns of dividend strips (**??**). For a formal definition I follow the procedure first introduced in **?**. Let $P_t^{(n)}$ be the current price of a dividend strip that pays a one-time dividend, namely, the dividend in n periods from time t. Let there exist a SDF, $M_{t,t+n}$, that determines the price at time t of an asset with payoffs n periods from t. Then, the price at current time t of a dividend strip with maturity n equals

$$P_t^{(n)} = \mathbb{E}_t \left[M_{t,t+n} D_{t+n} \right], \tag{A.1}$$

where the price follows from an conditional expectation, such that it depends on the current state variable z_t. Subsequently, we can rewrite (**??**) using the law of iterative expectations

$$P_t^{(n)} = \mathbb{E}_t \left[M_{t,t+1} P_{t+1}^{(n-1)} \right], \tag{A.2}$$

which in turn can be written as,

$$\frac{P_t^{(n)}}{D_t} = \mathbb{E}_t \left[M_{t,t+1} \frac{P_{t+1}^{(n-1)}}{D_{t+1}} \frac{D_{t+1}}{D_t} \right]. \tag{A.3}$$

This is a convenient way of computing the prices of dividend strips as this expression can be solved iteratively. A dividend strip with maturity $n = 0$ is an asset that pays today's dividend and therefore its price equals the current dividend, $P_t^{(0)} = D_t$. Following the iterative pricing equation in (**??**), the price-dividend ratio for $n = 1$ equals

$$\frac{P_t^{(1)}}{D_t} = \mathbb{E}_t \left[M_{t,t+1} \frac{P_{t+1}^{(0)}}{D_{t+1}} \frac{D_{t+1}}{D_t} \right] = \mathbb{E}_t \left[M_{t,t+1} \frac{D_{t+1}}{D_t} \right]. \tag{A.4}$$

For subsequent maturities one uses (**??**) and iterates forward using the previously found price-dividend ratio. Following this procedure one obtains the price-dividend ratios for all maturities n, each as a function of current state z_t. Subsequently, from these price-dividend functions one can compute the one-period holding returns of each dividend strip.[29] This is the return the investor

[29] In the literature also hold-to-maturity returns are considered for the equity term structure (**???**). The risk premia of the hold-to-maturity returns are however not the same as the one-period holding returns. The reason

obtains from holding a dividend strip of maturity n from period t to period $t+1$:

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}. \tag{A.5}$$

The *term structure of equity risk* is the variance of these holding-period returns plotted against maturity n and is thus defined as

$$\sqrt{\mathrm{Var}\big(R_{t+1}^{(n)}\big)}, \tag{A.6}$$

and the *term structure of equity risk premia* is the risk premium of these one-period holding returns plotted against maturity n:

$$\mathbb{E}_t\left[R_{t+1}^{(n)}\right] - R_{f,t} = -R_{f,t}\,\mathrm{Cov}_t\left(M_{t,t+1}, \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right). \tag{A.7}$$

B. PROOFS

Derivation of Equation **??**. Consumption and dividends are identical and follow a log normal growth process

$$\log\left(\frac{D_{t+1}}{D_t}\right) = \log\left(\frac{C_{t+1}}{C_t}\right) = g_c + \sigma_c \varepsilon_{c,t+1}. \tag{B.1}$$

The representative agent has CRRA(γ) utility, her subjective time discount factor is denoted by β and she maximizes her expected lifetime by optimally choosing consumption (C_t) and the proportion of wealth (W_t) that she wants to invest in the risky asset (α_t). She then faces the following maximization problem:

$$\max_{\{C_t,\alpha_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C_t)\right] \tag{B.2}$$

$$W_{t+1} = (W_t - C_t)\left\{R_{f,t} + \alpha_t (R_{t+1} - R_{f,t})\right\} \tag{B.3}$$

for using hold-to-maturity returns is that they provide a clear link to equity yields, which is the main focus of papers using such returns. However, hold-to-maturity returns are less convenient for determining the cyclicality of the equity term structure, because the resulting returns are averages over different maturities and thereby mix the maturity specific equity premia, which also complicates the interpretation of the cyclicality or risk premia. For this reason I follow **?** and use one-period holding returns.

For this framework the SDF, $M_{t,t+1}$, is commonly known to equal

$$M_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} = \beta e^{-\gamma(g_c+\sigma_c\varepsilon_{c,t+1})} \tag{B.4}$$

Then, the equilibrium return on the risky asset follows from,

$$1 = \mathbb{E}\big[M_{t,t+1}R_{t+1}\big]. \tag{B.5}$$

Plugging in the expression of the SDF and rewriting the return in terms of price dividend ratios, gives

$$1 = \mathbb{E}\Big[e^{-\gamma(g_c+\sigma_c\varepsilon_{c,t+1})}\frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}e^{g_c+\sigma_c\varepsilon_{c,t+1}}\Big] \tag{B.6}$$

Now, as there is no state variable, the model is I.I.D. over time, such that the price-dividend ratio is constant. We can therefore rewrite (**??**) as

$$\frac{P_t}{D_t} = \frac{1}{\frac{1}{\beta}e^{-(1-\gamma)g_c-\frac{1}{2}\sigma_c^2(1-\gamma)^2}-1}. \tag{B.7}$$

This completes the derivation. □

Proof of Proposition **??**. It is convenient to seperate the proposition in several steps. First we analyze the term structure of equity risk and subsequently the term structure of equity risk premia. We do this first for the risk-averse state and subsequently for the risk-seeking state. First consider the implications of the assumptions of the proposition.

1. The price to dividend ratio of the stock is finite in equilibrium. Therefore, the prices of the dividend strip assets should all converge to zero when the maturity goes to infinity. Given that $\frac{P_t^{(0)}}{D_t}(z_t) = 1$, we have that $\frac{P_t^{(n)}}{D_t}(z_t)$ is smaller than one for all n and z_t.
2. In a risk-averse state, the stochastic discount factor, $M_{t,t+1}$, has the following characteristics,

$$\frac{\partial M_{t,t+1}}{\partial \varepsilon_{d,t+1}} < 0, \qquad \frac{\partial^2 M_{t,t+1}}{\partial \varepsilon_{d,t+1}\partial z_t} < 0, \qquad \frac{\partial^3 M_{t,t+1}}{\partial \varepsilon_{d,t+1}\partial z_t^2} < 0 \tag{B.8}$$

3. In a risk-seeking state, the stochastic discount factor, $M_{t,t+1}$, has the following characteristics,

$$\frac{\partial M_{t,t+1}}{\partial \varepsilon_{d,t+1}} > 0, \qquad \frac{\partial^2 M_{t,t+1}}{\partial \varepsilon_{d,t+1}\partial z_t} < 0, \qquad \frac{\partial^3 M_{t,t+1}}{\partial \varepsilon_{c,t+1}\partial z_t^2} < 0 \tag{B.9}$$

First we consider the states of risk-averse preferences. For $n=0$ we have that $\frac{P_t^{(n)}}{D_t}=1$ for all z_t. Then, for $n=1$ we have,

$$\frac{P_t^{(1)}}{D_t}=\mathbb{E}\left[M_{t,t+1}\frac{D_{t+1}}{D_t}\right]=\frac{1}{R_{f,t}}e^{g_d+\frac{1}{2}\sigma_d^2}+\mathrm{Cov}\left(M_{t,t+1},\frac{D_{t+1}}{D_t}\right) \tag{B.10}$$

The first term is independent of z_t. The second term is driven by the correlation with the consumption growth shock. The dividend growth is positively correlated with $\varepsilon_{c,t+1}$. The stochastic discount factor is negatively correlated with $\varepsilon_{d,t+1}$ and this correlation is stronger for larger values of z_t according to (**??**). We thus have that the second term in (**??**) is negative and concave down decreasing in z_t.

Next consider $n=2$,

$$\frac{P_t^{(2)}}{D_t}=\mathbb{E}\left[M_{t,t+1}\frac{D_{t+1}}{D_t}\right]\mathbb{E}\left[\frac{P_{t+1}^{(1)}}{D_{t+1}}\right]+\mathrm{Cov}\left(M_{t,t+1}\frac{D_{t+1}}{D_t},\frac{P_{t+1}^{(1)}}{D_{t+1}}\right) \tag{B.11}$$

First consider the product term. The first term of the product is exactly equal to (**??**). It is therefore lower than 1 and a concavely decreasing function of z_t. The second part of the expectation is very similar. However, due to the dynamics of z_t the expectation is rotated such that it is less strongly decreasing in z_t compared to (**??**). Combining both effects gives that their product is more strongly decreasing in z_t. Due to both functions being between zero and one, there product is also lower compared to the individual functions. Then consider the covariance term. The first part of the covariance is positively correlated with $\varepsilon_{d,t+1}$ for reasonable risk aversion. Empirically the volatility of consumption growth is about 4.5 times smaller and the correlation is 15%. Considering standard CRRA utility the first term is negatively correlated with $\varepsilon_{d,t+1}$ for risk aversion up to 30. The second term of the covariance is the previous result $\frac{P_t^{(1)}}{D_t}(z_t)$ when z_t is replaced by z_{t+1}. The price-dividend function is concave down decreasing in z_t and z_{t+1} is a decreasing function of $\varepsilon_{d,t+1}$. Combined this gives a positive correlation with $\varepsilon_{d,t+1}$ that is stronger for large z_t. Overall the covariance term is therefore positive and concave down decreasing in z_t. The price dividend ratio of maturity $n=2$ is therefore lower than that of $n=1$ and more strongly decreasing in z_t.

For larger values of n, the results of $n=2$ iterate forward such that the price dividend ratio of dividend strips with longer maturities will all be lower than the ratio of the maturity before that and are more strongly decreasing in z_t. Now consider the term structure of equity risk,

$$\frac{1}{P_t^{(n)}/D_t}\sqrt{\text{Var}\left(\frac{P_{t+1}^{(n-1)}}{D_{t+1}}\frac{D_{t+1}}{D_t}\right)} \tag{B.12}$$

First consider the variance term. The price dividend ratio function is more decreasing in z_t for larger values of n. As z_{t+1} is decreasing in $\varepsilon_{d,t+1}$ we have that $\frac{P_{t+1}^{(n-1)}}{D_{t+1}}$ is positively correlated with $\varepsilon_{d,t+1}$ and more strongly for larger n. As dividend growth is by definition positively correlated with $\varepsilon_{d,t+1}$, the variance term increases with n. Then, the price dividend ratio function decreases in level when n increases. Therefore, the fraction in front on the squareroot decreases with n such that the term structure of equity risk increases with n.

For the result above we used that the investor has risk averse preferences. What happens in case of risk seeking preferences? For risk-seeking preferences we have that $\frac{\partial M_{t,t+1}}{\partial \varepsilon_{d,t+1}} > 0$. For (**??**) this implies that the covariance term is now positive instead of negative. Under the assumption of market equilibrium, the price dividend ratio must decrease and thus be lower than one. The conditions under which an equilibrium are obtained are however stricter than for the risk averse states. Similar to the previous case, the covariance term is still decreasing with z_t. We thus obtain similar results as for risk-averse preferences, namely, the price dividend ratio is concavely decreasing in z_t and lower than one. It immediately follows that the results of other maturities are also qualitatively similar and thus the term structure of equity risk in (**??**) is also increasing for risk-seeking preferences.

Next we are interested in the term structure of equity risk premia

$$\mathbb{E}_t\left[R_{t+1}^{(n)}\right] - R_{f,t} = -R_{f,t}\text{Cov}\left(M_{t,t+1}, \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right). \tag{B.13}$$

Decomposing the return in the price-dividend ratios and dividend growth gives,

$$\mathbb{E}_t\left[R_{t+1}^{(n)}\right] - R_{f,t} = -R_{f,t}\frac{1}{P_t^{(n)}/D_t}\text{Cov}_t\left(M_{t,t+1}, \frac{P_{t+1}^{(n-1)}}{D_{t+1}}\frac{D_{t+1}}{D_t}\right). \tag{B.14}$$

First, the risk-free interest rate is positive. Then, recall that the price dividend ratio decreases with maturity and thus the covariance is multiplied with a term that increases with maturity n. The second term of the covariance is positively correlated with dividend risk $\varepsilon_{d,t+1}$. This is by definition for dividend growth. Additionally, the price dividend ratio is decreasing in

z_{t+1}, though z_{t+1} is decreasing in $\varepsilon_{d,t+1}$. This term is thus also positively correlated with $\varepsilon_{d,t+1}$. Overall, the second term of the covariance is positively correlated with $\varepsilon_{d,t+1}$ and more strongly so for larger n. On the other hand, the SDF is negatively correlated with $\varepsilon_{d,t+1}$ in risk-seeking states and positively correlated with $\varepsilon_{d,t+1}$ in risk-averse states. The term structure of equity risk premia is thus downward sloping for states of risk-averse preferences and upward sloping for states of risk-seeking preferences. $\square$

Proof of Proposition **??**. We can rewrite the objective function in (**??**) recursively,

$$B_t(R_{t+1},W_t) \equiv \max_{C_t,\alpha_t} \left\{ u(C_t) + \beta \mathbb{E}_t \Big[\underbrace{\alpha_t(W_t - C_t)u'(\bar{C}_t)v\big(R_{t+1},z_t\big) + B_{t+1}(R_{t+2},W_{t+1})\big)}_{\text{define this as } \star} \Big] \right\} \tag{B.15}$$

By the envelope theorem we have that the derivative of next periods payoff with respect to wealth tomorrow equals the derivative of today's utility with respect to consumption C_t. Note that $\star$ is next periods payoff, then, the envelope theorem implies,

$$\frac{\partial \star}{\partial W_{t+1}} = \frac{\partial \Big\{ \alpha_t(W_t - C_t)u'(\bar{C}_t)v\big(R_{t+1},z_t\big) + B_{t+1}(R_{t+2},W_{t+1})\Big\}}{\partial W_{t+1}} = u'(C_{t+1}) \tag{B.16}$$

which is used in the derivations below. It is convenient to first optimize the value function in (**??**) with respect to consumption α_t. This gives,

$$\beta \mathbb{E}_t \left[\frac{\partial \star}{\partial W_{t+1}} \frac{\partial W_{t+1}}{\partial \alpha_t} + \frac{\partial \star}{\partial \alpha_t} \right] = 0 \tag{B.17}$$

$$\beta \mathbb{E}_t \big[u'(C_{t+1})(W_t - C_t)(R_{t+1} - R_f) + (W_t - C_t)u'(C_t)v(R_{t+1},z_t) \big] = 0 \tag{B.18}$$

$$\mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)}(R_{t+1} - R_f) + v(R_{t+1},z_t) \right] = 0 \tag{B.19}$$

Let us now consider the consumption choice. Optimizing (**??**) with respect to C_t gives,

$$u'(C_t) + \beta \mathbb{E}_t \left[\frac{\partial \star}{\partial W_{t+1}} \frac{\partial W_{t+1}}{\partial C_t} + \frac{\partial \star}{\partial C_t} \right] = 0. \tag{B.20}$$

Plugging in the results of the envelope theorem of (**??**) into (**??**) gives,

$$u'(C_t) - \beta \mathbb{E}_t \big[u'(C_{t+1})\big(R_{f,t} + \alpha_t(R_{t+1} - R_{f,t})\big) + \alpha_t u'(C_t)v(R_{t+1},z_t) \big] = 0 \tag{B.21}$$

$$\beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} R_{f,t} \right] + \alpha_t \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)}(R_{t+1} - R_{f,t}) + v(R_{t+1},z_t) \right] = 1. \tag{B.22}$$

Note that the second term of (**??**) equals (**??**) and thus is equals zero. It immediately follows from (**??**) that the risk-free interest rate equals

$$R_{f,t} = \frac{1}{\beta}\left(\mathbb{E}_t\left[\frac{u'(C_{t+1})}{u'(C_t)}\right]\right)^{-1} = \frac{1}{\beta}e^{\gamma g_c - \frac{1}{2}\gamma^2\sigma_c^2}, \tag{B.23}$$

which is actually independent of the state variable at time t. Combining the results of (**??**) and (**??**) reveals the SDF. The SDF should price this excess return as well as the risk-free interest rate. This is achieved by the following function,

$$M_{t,t+1} = \frac{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \frac{v(R_{t+1},z_t)}{R_{t+1}-R_f}}{\mathbb{E}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \frac{v(R_{t+1},z_t)}{R_{t+1}-R_f}\right]R_f}, \tag{B.24}$$

which is always positive and is thus a valid SDF. Finally, using (**??**), the pricing equation of excess returns in (**??**) can be simplified algebraically to

$$\beta e^{g_d - \gamma g_c + \frac{1}{2}\gamma^2\sigma_c^2(1-\omega^2)}\mathbb{E}_t\left[\frac{1+f(z_{t+1})}{f(z_t)}e^{(\sigma_d-\gamma\sigma_c\omega)\varepsilon_{d,t+1}} + v(R_{t+1},z_t)\right] = 1 \tag{B.25}$$

$\square$

C. Supporting Figures & Tables

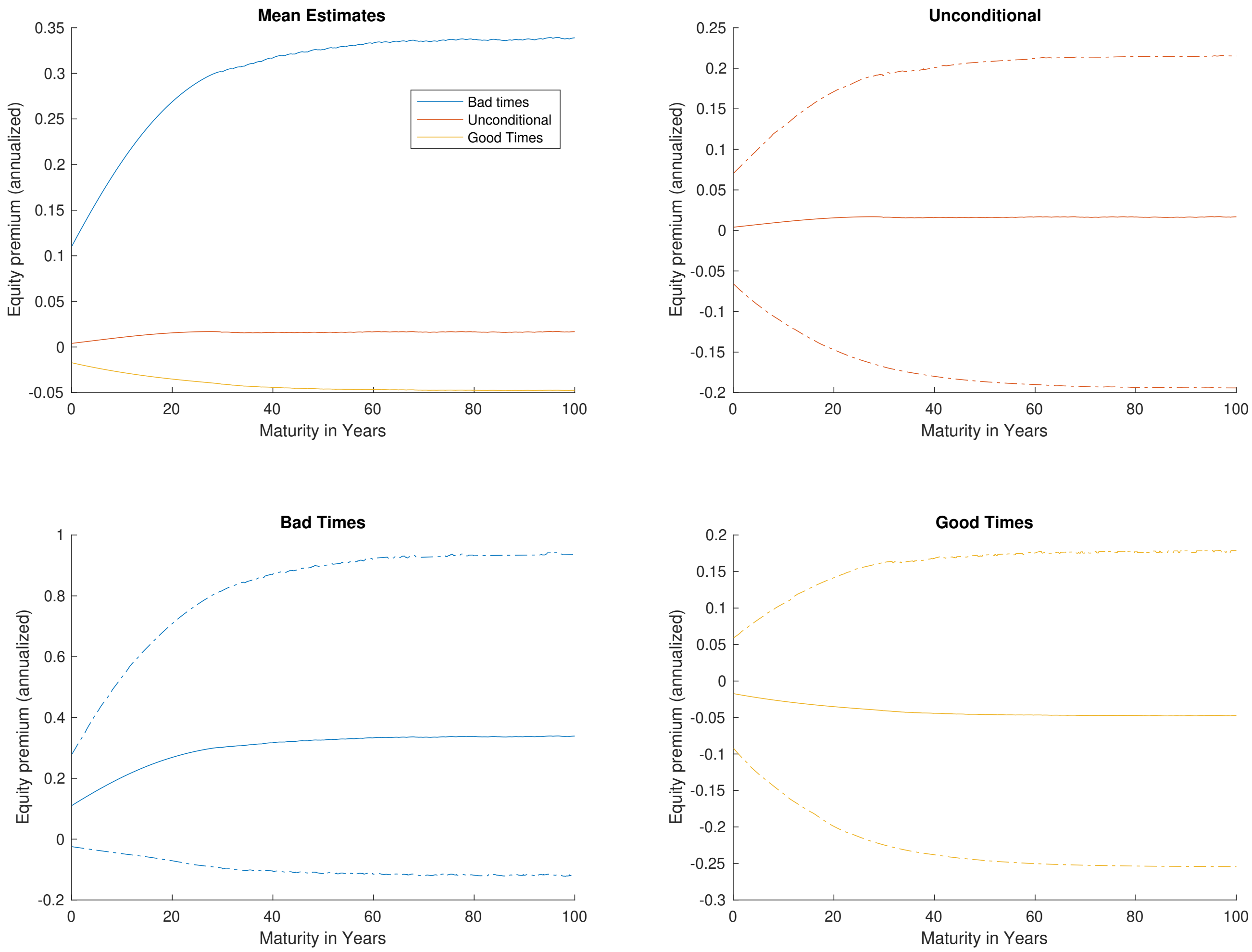

FIGURE C.1:
TERM STRUCTURES USING SIMULATIONS OF 16 YEARS.

Estimates of the conditional and unconditional term structure based on 16 years of data for 1000 simulations. The dotted lines show the 10th and 90th percentile of the simulated trajectories. The results show that for 16 years of data the realized conditional term structure and unconditional term structure can be very different.

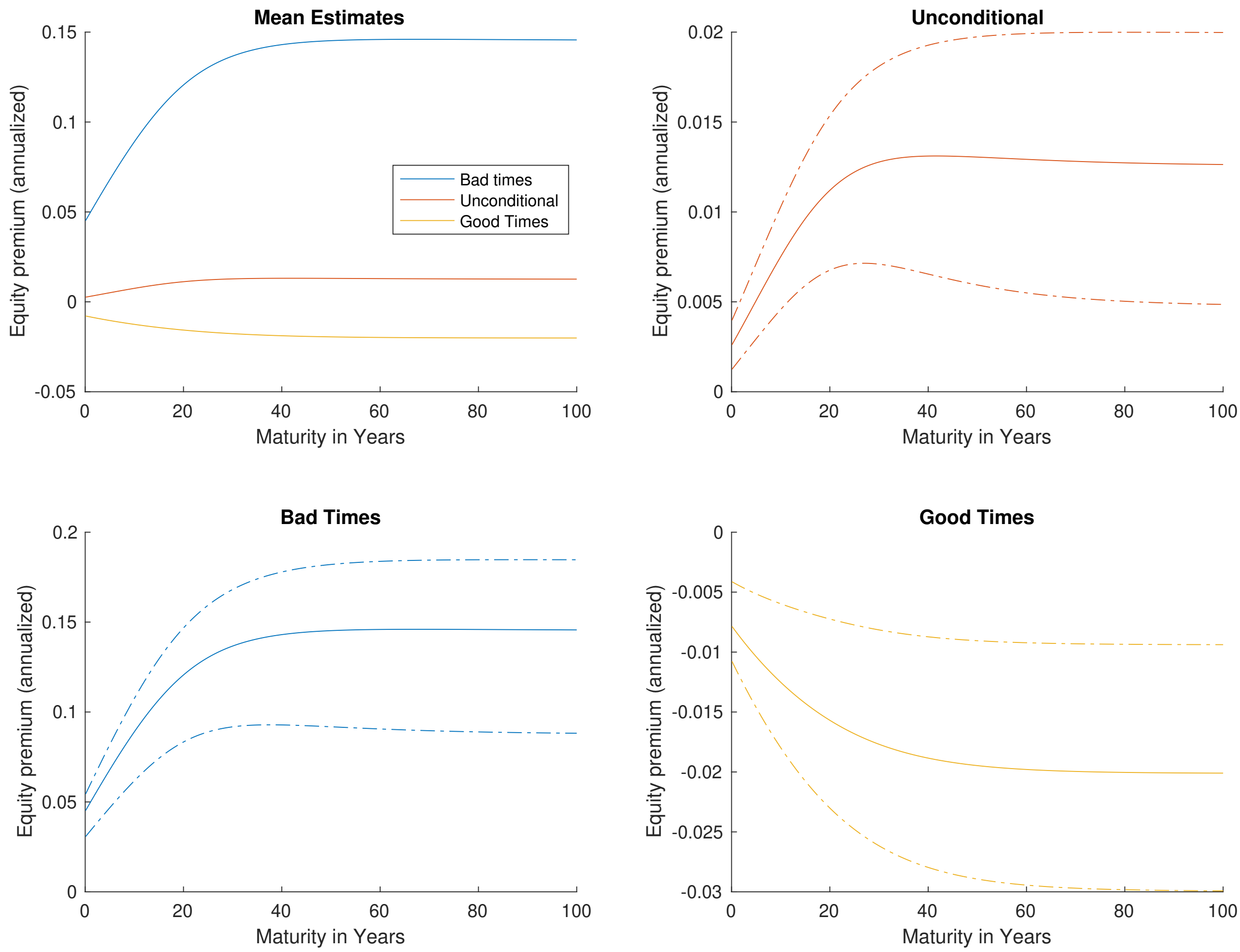

FIGURE C.2:
CONDITIONAL AND UNCONDITIONAL TERM STRUCTURES

Estimates of the conditional and unconditional term structure based on 1000 years of data for 50 simulations. The dotted lines show the 10th and 90th percentile of the simulated trajectories.

TABLE C.1:
SENSITIVITY ANALYSIS - λ

	Stock Moments							$\mathbb{E}[R-R^{(1:84)}]$			Regressions		
	$\mathbb{E}[r_e]$	$\sigma(r_e)$	$AC1(r_e)$	$\mathbb{E}[r_f]$	$\mathbb{E}[p-d]$	$\sigma(p-d)$	$AC1(p-d)$	Bad Times	Good Times	Unconditional		$\beta_1^{(long,2y)}$	$\beta_2^{(long,2y)}$
Data	5.47	20.17	0.02	0.56	3.36	0.45	0.87	5.00	-4.00	-0.87			
$\lambda = 1$													
Mean	0.63	30.35	-0.06	11.06	2.90	0.30	0.70	6.49	-1.29	0.15	Stock	-0.01 (-1.79)	-0.09 (-3.35)
Median	0.56	30.21	-0.07	11.06	2.90	0.29	0.72	5.52	-1.50	-0.04	10-years	-0.05 (-17.80)	-0.45 (-14.54)
p10	-1.24	26.70	-0.20	11.06	2.70	0.23	0.55	-0.87	-4.12	-2.44	20-years	-0.10 (-16.89)	-0.43 (-13.77)
p90	2.62	34.17	0.08	11.06	3.10	0.37	0.83	15.06	1.70	2.96			
$\lambda = 1.2$													
Mean	3.96	26.54	-0.06	11.06	2.36	0.22	0.64	4.74	0.05	0.92	Stock	0.04 (12.12)	0.08 (3.00)
Median	3.91	26.46	-0.06	11.06	2.36	0.22	0.66	4.22	-0.10	0.80	10-years	-0.09 (-19.87)	-0.62 (-14.70)
p10	2.38	23.53	-0.20	11.06	2.22	0.18	0.47	0.39	-1.71	-0.68	20-years	-0.21 (-20.70)	-0.64 (-15.19)
p90	5.63	29.66	0.08	11.06	2.50	0.27	0.78	9.96	1.93	2.61			
$\lambda = 1.4$													
Mean	6.72	25.25	-0.06	11.06	2.10	0.19	0.60	4.00	0.60	1.23	Stock	0.07 (25.57)	0.07 (4.32)
Median	6.66	25.16	-0.07	11.06	2.10	0.19	0.62	3.62	0.50	1.15	10-years	-0.14 (-21.54)	-0.73 (-14.79)
p10	5.23	22.44	-0.20	11.06	1.98	0.16	0.42	0.84	-0.70	0.04	20-years	-0.32 (-20.06)	-0.69 (-13.53)
p90	8.27	28.16	0.08	11.06	2.22	0.24	0.75	7.82	2.01	2.45			

The stock moments are derived from 1,000 simulations of 78 years, the term structure moments use 1,000 simulations of 16 years and the regression use 100 simulations of 10,000 years. The data estimates are from **?** and **?** respectively. $R^{(1:84)}$ denotes the average of the dividend strips up to a maturity of seven years. The regression results show the estimates of (**??**) and (**??**) with the t-statistics between brackets. The baseline results are those for $\lambda = 1$.

TABLE C.2:
SENSITIVITY ANALYSIS - σ_z

	Stock Moments							$\mathbb{E}[R-R^{(1:84)}]$			Regressions		
	$\mathbb{E}[r_e]$	$\sigma(r_e)$	$AC1(r_e)$	$\mathbb{E}[r_f]$	$\mathbb{E}[p-d]$	$\sigma(p-d)$	$AC1(p-d)$	Bad Times	Good Times	Unconditional		$\beta_1^{(long,2y)}$	$\beta_2^{(long,2y)}$
Data	5.47	20.17	0.02	0.56	3.36	0.45	0.87	5.00	-4.00	-0.87			
$\sigma_z = \frac{0.10}{\sqrt{12}}$													
Mean	0.38	25.02	-0.06	11.06	2.73	0.21	0.63	3.72	-0.78	0.05	Stock	0.02 (4.79)	0.04 (0.03)
Median	0.33	24.97	-0.06	11.06	2.73	0.21	0.64	3.24	-0.85	-0.04	10-years	-0.05 (-14.66)	-0.55 (-11.97)
p10	-1.41	22.21	-0.20	11.06	2.60	0.17	0.45	-0.61	-2.55	-1.53	20-years	-0.09 (-14.32)	-0.53 (-11.65)
p90	2.20	27.97	0.08	11.06	2.87	0.26	0.77	8.84	1.13	1.75			
$\sigma_z = \frac{0.12}{\sqrt{12}}$													
Mean	0.63	30.35	-0.06	11.06	2.90	0.30	0.70	6.49	-1.29	0.15	Stock	-0.01 (-1.79)	-0.09 (-3.35)
Median	0.56	30.21	-0.07	11.06	2.90	0.29	0.72	5.52	-1.50	-0.04	10-years	-0.05 (-17.80)	-0.45 (-14.54)
p10	-1.24	26.70	-0.20	11.06	2.70	0.23	0.55	-0.87	-4.12	-2.44	20-years	-0.10 (-16.89)	-0.43 (-13.77)
p90	2.62	34.17	0.08	11.06	3.10	0.37	0.83	15.06	1.70	2.96			
$\sigma_z = \frac{0.14}{\sqrt{12}}$													
Mean	1.72	46.86	-0.05	11.06	3.95	0.55	0.78	17.11	-2.70	0.95	Stock	-0.07 (-12.76)	-0.18 (-10.93)
Median	1.60	46.48	-0.06	11.06	3.96	0.53	0.79	14.44	-2.99	0.50	10-years	-0.05 (-21.99)	-0.28 (-17.93)
p10	-1.20	39.56	-0.19	11.06	3.58	0.41	0.65	-2.36	-9.13	-5.12	20-years	-0.09 (-19.73)	-0.26 (-16.39)
p90	4.79	54.89	0.09	11.06	4.33	0.70	0.87	40.52	3.77	7.51			

The stock moments are derived from 1,000 simulations of 78 years, the term structure moments use 1,000 simulations of 16 years and the regression use 100 simulations of 10,000 years. The data estimates are from **?** and **?** respectively. $R^{(1:84)}$ denotes the average of the dividend strips up to a maturity of seven years. The regression results show the estimates of (**??**) and (**??**) with the t-statistics between brackets. The baseline results are those for $\sigma_z = \frac{0.12}{\sqrt{12}}$.

TABLE C.3:
SENSITIVITY ANALYSIS - κ_1, κ_2

	Stock Moments							$\mathbb{E}[R - R^{(1:84)}]$			Regressions		
	$\mathbb{E}[r_e]$	$\sigma(r_e)$	$AC1(r_e)$	$\mathbb{E}[r_f]$	$\mathbb{E}[p-d]$	$\sigma(p-d)$	$AC1(p-d)$	Bad Times	Good Times	Unconditional		$\beta_1^{(long,2y)}$	$\beta_2^{(long,2y)}$
Data	5.47	20.17	0.02	0.56	3.36	0.45	0.87	5.00	-4.00	-0.87			
$\kappa_1 = 1$ and $\kappa_2 = 0$													
Mean	0.63	30.35	-0.06	11.06	2.90	0.30	0.70	6.49	-1.29	0.15	Stock	-0.01 (-1.79)	-0.09 (-3.35)
Median	0.56	30.21	-0.07	11.06	2.90	0.29	0.72	5.52	-1.50	-0.04	10-years	-0.05 (-17.80)	-0.45 (-14.54)
p10	-1.24	26.70	-0.20	11.06	2.70	0.23	0.55	-0.87	-4.12	-2.44	20-years	-0.10 (-16.89)	-0.43 (-13.77)
p90	2.62	34.17	0.08	11.06	3.10	0.37	0.83	15.06	1.70	2.96			
$\kappa_1 = 0$ and $\kappa_2 = 0.5$													
Mean	-0.02	19.87	-0.05	11.06	2.64	0.13	0.44	1.37	-0.36	-0.04	Stock	0.04 (15.02)	0.18 (7.16)
Median	-0.03	19.87	-0.05	11.06	2.64	0.13	0.44	1.07	-0.43	-0.13	10-years	-0.02 (-8.43)	-0.56 (-7.08)
p10	-2.03	17.61	-0.19	11.06	2.57	0.11	0.24	-0.48	-1.21	-0.82	20-years	-0.04 (-8.38)	-0.56 (-7.07)
p90	2.00	22.27	0.09	11.06	2.71	0.15	0.62	3.57	0.55	0.80			

The stock moments are derived from 1,000 simulations of 78 years, the term structure moments use 1,000 simulations of 16 years and the regression use 100 simulations of 10,000 years. The data estimates are from **?** and **?** respectively. $R^{(1:84)}$ denotes the average of the dividend strips up to a maturity of seven years. The regression results show the estimates of (**??**) and (**??**) with the t-statistics between brackets. The baseline results are those for $\kappa_1 = 1$ and $\kappa_2 = 0$. The used value of κ_2 differs from κ_1 as their relative impact on the kink differs.

TABLE C.4:
SENSITIVITY ANALYSIS - γ

	Stock Moments							$\mathbb{E}[R - R^{(1:84)}]$			Regressions		
	$\mathbb{E}[r_e]$	$\sigma(r_e)$	$AC1(r_e)$	$\mathbb{E}[r_f]$	$\mathbb{E}[p-d]$	$\sigma(p-d)$	$AC1(p-d)$	Bad Times	Good Times	Unconditional		$\beta_1^{(long,2y)}$	$\beta_2^{(long,2y)}$
Data	5.47	20.17	0.02	0.56	3.36	0.45	0.87	5.00	-4.00	-0.87			
$\gamma = 4$													
Mean	0.77	35.69	-0.06	9.44	3.59	0.39	0.74	10.01	-1.88	0.31	Stock	-0.04 (-9.14)	-0.19 (-8.22)
Median	0.68	35.51	-0.06	9.44	3.59	0.38	0.76	8.42	-2.08	0.08	10-years	-0.04 (-17.71)	-0.33 (-14.43)
p10	-1.48	31.07	-0.19	9.44	3.33	0.30	0.60	-1.58	-6.10	-3.57	20-years	-0.07 (-16.78)	-0.31 (-13.68)
p90	3.14	40.67	0.08	9.44	3.86	0.49	0.85	23.69	2.51	4.45			
$\gamma = 5$													
Mean	0.63	30.35	-0.06	11.06	2.90	0.30	0.70	6.49	-1.29	0.15	Stock	-0.01 (-1.79)	-0.09 (-3.35)
Median	0.56	30.21	-0.07	11.06	2.90	0.29	0.72	5.52	-1.50	-0.04	10-years	-0.05 (-17.80)	-0.45 (-14.54)
p10	-1.24	26.70	-0.20	11.06	2.70	0.23	0.55	-0.87	-4.12	-2.44	20-years	-0.10 (-16.89)	-0.43 (-13.77)
p90	2.62	34.17	0.08	11.06	3.10	0.37	0.83	15.06	1.70	2.96			
$\gamma = 10$													
Mean	0.77	24.50	-0.06	18.24	2.01	0.18	0.57	2.32	-0.45	0.06	Stock	0.09 (38.89)	0.02 (2.23)
Median	0.72	24.45	-0.07	18.24	2.01	0.17	0.59	2.02	-0.54	0.00	10-years	-0.11 (-18.14)	-0.80 (-14.78)
p10	-0.69	21.80	-0.20	18.24	1.90	0.15	0.39	-0.20	-1.48	-0.90	20-years	-0.20 (-17.15)	-0.77 (-13.99)
p90	2.30	27.30	0.08	18.24	2.12	0.22	0.73	5.37	0.66	1.05			

The stock moments are derived from 1,000 simulations of 78 years, the term structure moments use 1,000 simulations of 16 years and the regression use 100 simulations of 10,000 years. The data estimates are from **?** and **?** respectively. $R^{(1:84)}$ denotes the average of the dividend strips up to a maturity of seven years. The regression results show the estimates of (**??**) and (**??**) with the t-statistics between brackets. The baseline results are those for $\gamma = 5$. The lower level of γ is chosen to equal 4 as for lower levels the level of risk aversion over consumption is insufficient to counteract states of risk-seeking gain-loss preferences, such that an equilibrium does not exist.