On-the-Job Search with Hidden Attributes

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Motivation

1. Many important economic decisions require individuals to search for and compare 'complex' (= multidimensional) alternatives.

2. When consumers are boundedly rational, they can be exploited by a provider offering attractive headline attributes and shrouding undesirable hidden attributes.

   - Ellison (2005), Gabaix & Laibson (2006), ... , Heidhues & Kőszegi (2018)
   - Examples: printer toner pricing, dietary supplements, credit card/banking/mutual fund/mortgage fee structures.

3. Even though consumers might be prone to exploitation initially, it is natural to imagine that they eventually realize they are being taken advantage of...
   - Costly switching permitted in industries cited as relevant applications.
   - Nonetheless, the behavioural IO literature has not yet accounted for consumers learning the hidden attribute and possibly switching providers.
     - Johnen (2019): Automatic renewal contracts with consumers learning about their idiosyncratic preference, but differing in propensity to act.
     - Johnen (2020): Dynamic competition between firms who learn private information about their customers naiveté.
     - Heidhues, Kőszegi & Murooka (2021): Consumers can switch to better terms over time (known in advance), but procrastinate.

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$\Longrightarrow$ How does the possibility to learn the hidden attribute and conduct 'on-the-job' search affect the consumers' optimal search strategy and the firms' optimal offers?

Outline

1. Ingredients of the model + Headline results
2. Literature
3. Search behaviour
4. Optimal offers
5. Discussion + Conclusion

Ingredients of the model

- Firms' offers consist of an observable and a hidden attribute.
  - 'Price' is continuous, 'quality' is binary.
- When searching, boundedly rational consumers compare only the observable attributes. They eventually learn the hidden attribute associated with the offer they selected and can search again if unhappy.
  - Costly search: Costs $\nearrow$ on the job generate stickiness of the initial choice.

Headline results

1. Search behaviour
   - To focus on the *learning & re-contracting* aspect, distinguish between sophisticates, who correctly anticipate future search costs, and naifs, who underestimate them.
     - Naifs overestimate the option value of searching on the job and accept 'too many' offers initially.
     - (In the paper: Comparison with a fully rational search rule. Comparative statics on the "naiveté wedge".)

2. Optimal offers (equilibrium)
   - Symmetric pure-strategy equilibria with inefficiently poor hidden attributes are easier to sustain than equilibria with socially desirable hidden attributes, even though the firms' profits suffer.
   - Asymmetric pure-strategy equilibria may exist, but only if poor hidden attributes are paired with attractive headline terms, and vice versa.
     - The logic is very different from the theory of compensating differentials (Rosen 1986).

Leading example: Wages & benefits

- In the last two decades, the generosity of social security benefits has been reduced in most of the developed countries.
  $\implies$ Greater importance of private provisions, often offered at a workplace.
  - In 2020, an average worker in the US received 69% of her compensation as a wage, with the remaining 31% reflecting various pecuniary benefits.
    - Most importantly: health insurance, paid leave, retirement benefits.

- At the same time, large literature in behavioural Household Finance indicates that people are ill-equipped to evaluate complicated insurance contracts and pension plans (Beshears et al. 2018; Campbell 2016; Lusardi & Mitchell 2014).
  - Thus, I will call the observable component of a compensation package 'wage' and its unobservable attribute 'benefits'.

Literature

1. **Behavioural IO $\rightarrow$ Products with shrouded attributes**
   - Study dynamics when consumers learn the hidden component over time and can search again.
   - Ellison (2005); Gabaix & Laibson (2006); Johnen (2019, 2020); Heidhues et al. (2021); **Gamp & Krähmer (2022)**

2. **Behavioural labour $\rightarrow$ Search models**
   - Treat benefits as a hidden component.
   - DellaVigna & Paserman (2005); Paserman (2008); Spinnewijn (2015); **Bubb & Warren (2020)**

Search behaviour

Set-up:

- $t = 1, 2$
- Random sequential search with perfect recall within each period.
- Search costs increase 'on the job', i.e. $0 < c_1 < c_2$.
  - Could be due to classical reasons (e.g., opportunity cost of time $\uparrow$, as in Burdett 1978) or behavioural ones (e.g., loss aversion, as in Karle et al. 2021).
- Each offer consists of an observable attribute w ('wage') and a hidden attribute θ. When sampling offers, an individual observes w, but learns θ only after accepting a particular offer.

- The hidden attribute consists of an idiosyncratic match quality ϵ_{ik} and a discretionary component b ('benefits') chosen by the employer:

$$\theta_{ik} = b_k + \epsilon_{ik}$$

- Suppose that ϵ_{ik} are i.i.d. with mean 0 and that $b_k \in \{\underline{b}, \bar{b}\}$ for some $0 < \underline{b} < \bar{b}$.
- The utility of individual i from accepting an offer k is:

$$u_{ik} = w_k + \theta_{ik}$$

- The individual maximises the sum of utilities obtained in periods 1 and 2, net of search costs. The value of her outside option is normalised to 0.

Behavioural frictions:

1. Searching individuals are analogy-based reasoners (Jehiel 2005).
   - A boundedly rational agent bundles the employers' choices of wages and benefits into two *analogy classes* and responds to an average behaviour within each class, essentially displaying correlation neglect.
   - These analogy-based reasoners do not infer b from w, but they don't ignore the fact that they will learn b upon acceptance. Moreover, they have correct beliefs about the unconditional distributions for w and b.

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2. Individuals can be either *sophisticated* or *naïve* about search costs $\nearrow$ on the job.
    - In period 1, sophisticates believe $\hat{c}_2 = c_2$, while naifs believe $\hat{c}_2 = c_1$.
    - Allows to focus attention on the dynamic aspect of the model.

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   - Allows to focus attention on the dynamic aspect of the model.

Partial equilibrium analysis $\Longrightarrow$ What is the impact of correlation neglect and naiveté on the search strategy? Are naifs necessarily worse off?

Solving backwards:

- If the worker engages in on-the-job search in period 2, her optimal *reservation wage* R_2 satisfies:

$$c_2 = \int_{w \geq R_2} (w - R_2)\phi(w)dw$$

equalising the marginal cost of search with the marginal benefit of further search.

- As in McCall (1970), we have:

Lemma 1: *In period 2, the worker stays in current employment (searches on the job) when* $u_{ik} = w_k + \theta_{ik} \ \geq (<) \ R_2 + \mathbb{E}[\theta]$.

- Since $c_1 < c_2$, $R_2^N > R_2^S = R_2$ and we have:

Lemma 2: *In period 1, a naïve agent overestimates her future propensity to search on the job. Specifically, for realisations* $u_{ik} \in [R_2^S + \mathbb{E}[\theta], R_2^N + \mathbb{E}[\theta])$ *she expects to search for a different job, but actually doesn't.*

- In period 1, the naif's perceived utility from accepting an offer w is:

$$u_1(w) = \big(w + \mathbb{E}[\theta]\big) \; + \; \mathbb{P}[w + \theta \geq \hat{R}_2] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w]\big)$$
$$+ \; \mathbb{P}[w + \theta < \hat{R}_2] \cdot \big(\hat{v}_2 - \hat{c}_2\big)$$

Three mistakes of a naif:

1. Underestimate the probability of staying in the initially accepted job.
2. Overestimate the average utility from a job they stay in.
3. Overestimate the option value of searching on the job.

Which together imply:

Lemma 3: *A naïve agent strictly overestimates the utility from accepting any job offer.*

- Finally, we arrive at:

  Lemma 4: *A naïve agent adopts a strictly lower reservation wage when searching initially, that is* $R_1^N < R_1^S$.

  Overestimating the option value of searching on the job, naifs accept 'too many' offers initially, but are they necessarily worse off as a result?

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  Overestimating the option value of searching on the job, naifs accept 'too many' offers initially, but are they necessarily worse off as a result?

- ... No.
  - One can imagine distributions of offers, under which the naïve search strategy coincidentally maximises actual welfare.
  - E.g., when lower wages are paired with higher benefits in a way that keeps the total utility constant, naifs maximise welfare by minimising total expected search cost.

- (In the paper: Comparison with a fully rational search rule. Comparative statics on the "naiveté wedge".)

Optimal offers

The game:

- Unit mass of workers and of firms.
- Worker's problem as above. The probability that worker is naïve is $\lambda \in (0, 1)$.
- Before workers start searching, firms simultaneously commit to the same offers for both periods. Firm's objective is to maximise the sum of profits in both periods.

Equilibrium concept: "Weak Perfect Bayesian equilibrium with correlation neglect".

For now, focus on a class of 'sticky' equilibria, where no worker searches on the job on the equilibrium path.

Deterministic setting

- No idiosyncratic match quality, i.e. $\text{Var}[\epsilon_{ik}] \to 0$.
- Firms are identical and produce $y > 0$ units of a numeraire good per worker.
- Compensation package (w, b) costs the firm $w + (1 - \tau)b$.
  - $\tau \in [0, 1)$ represents tax advantages or public subsidies to workplace benefits.
  - Thus, offering generous benefits $\bar{b}$ is efficient.
- To hire the worker, the offer needs to satisfy $w_k \geq R_1$. But to keep the worker, it needs to satisfy $w_k + b_k \geq R_2 + \mathbb{E}[b]$.
  - The role of a 'decent wage' is to make the worker accept in the first place.
  - The role of 'generous benefits' is to make the worker stay.

Symmetric equilibria in pure strategies

- Suppose that all firms offer the same $w = w^*$, $b = b^*$.

- Both naifs and sophisticates correctly perceive the degenerate distribution of compensation packages and expect to learn nothing new on the job.
  $\Longrightarrow$ All workers adopt the same strategy.

- In period 1, a worker searches once and accepts with the intention of staying on.
  $\Longrightarrow$ She enters the labour market as long as $2(w^* + b^*) \geq c_1$.

- Conditional on b^*, all firms offer the same wage equal to R_1 (Diamond 1971).
  - Offering a higher wage gives no advantage. Offers with a lower wage get rejected.
  - In equilibrium, w^* makes the workers indifferent between entering the labour market and staying unemployed.

$$\Longrightarrow \quad w^* = c_1/2 - b^*$$

Case 1: $b^* = \bar{b}$

Can be sustained as an equilibrium, if a firm deviating to $b = \underline{b}$: (1) loses the worker, (2) although it would prefer to retain the worker for both periods.

- Need to take into account deviations where w is kept constant or raised.

Case 2: $b^* = \underline{b}$

There is no profitable deviation as long as firms make non-negative profits.

- Deviating to $b = \bar{b}$ makes the firm strictly worse off, because it still needs to offer $w^* = c_1/2 - \underline{b}$ in order to hire any workers.

Proposition 1: *In a deterministic setting, a symmetric pure-strategy equilibrium in which all firms offer low benefits ($b^* = \underline{b}$) exists as long as $y + \tau\underline{b} \geq c_1/2$. A symmetric pure-strategy equilibrium in which all firms offer high benefits ($b^* = \bar{b}$) exists if $c_2 \leq \tau(\bar{b} - \underline{b})$ and $y + \tau\bar{b} \geq (1 - \tau)(\bar{b} - \underline{b}) + c_1/2$. In all such equilibria, the distribution of wage offers is degenerate with $w^* = c_1/2 - b^*$.*

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- Equilibria with generous benefits are more difficult to sustain, even though firms make strictly higher profits (for $\tau > 0$).
- Illustrates the effects of hiddenness of b, as degenerate distribution of offers neutralises any impact of naiveté.

Asymmetric equilibria in pure strategies

- For a non-degenerate distribution of offers, naifs and sophisticates adopt search strategies with different reservation wages $R_1^N < R_1^S$.
- Firms have no incentive to post wages other than R_1^N or R_1^S (Albrecht & Axell 1984).
  - Firms that post the higher wage attract both worker types.
  - Firms that post the lower wage can hire only the naifs that contact them.
- Identical firms can post different offers, as long as the expected profits are equal.

What wage-benefit offers can arise in asymmetric equilibria?

Labelling $w_1 < w_2$, we can rule out:

Combo 1: $b(w_1) = b(w_2) = \bar{b}$
Combo 2: $b(w_1) = b(w_2) = \underline{b}$
Combo 3: $b(w_1) = \underline{b}$ and $b(w_2) = \bar{b}$

The only case not resulting in a contradiction is:

Combo 4: $b(w_1) = \bar{b}$ and $b(w_2) = \underline{b}$

- Both types of workers perceive the distribution of offers to be:

$$\begin{cases} (w_1, \bar{b}) & \text{prob. } p^2 \\ (w_1, \underline{b}) & \text{prob. } p(1-p) \\ (w_2, \underline{b}) & \text{prob. } (1-p)^2 \\ (w_2, \bar{b}) & \text{prob. } (1-p)p \end{cases}$$

- No worker expects to search further after receiving $(w_2, \bar{b})$. Both types have to expect and actually leave a job paying $(w_1, \underline{b})$.

- To have $R_1^N \neq R_1^S$, naifs have to wrongly believe that they will search on-the-job for at least one of realisations: $(w_1, \bar{b})$ and $(w_2, \underline{b})$. This gives 3 cases to consider.

  Consider parametrisations for which naifs wrongly believe to search on the job after discovering $(w_2, \underline{b})$ only:

$$R_2^S + \mathbb{E}[b] < R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b}$$
$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} < R_2^N + \mathbb{E}[b]$$

- When searching on the job, both types would (expect to) accept the first alternative offer:

$$c_2 > c_1 \geq (1-p)(w_2 - w_1)$$

- Then, we can derive $u_1^S(\cdot)$ and $u_1^N(\cdot)$.

  In period 1, sophisticates search until they find a high-wage offer w_2 if:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1))$$

  While naifs accept the first sampled offer if:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1))$$

- This simplifies to 5 inequalities, which together with the firms' equal profits condition and naifs' participation constraint describe the only possible pure-strategy equilibrium with differentiated offers.
  - The set of parameters satisfying all conditions simultaneously is indeed non-empty.
    Inequalities + Example

Proposition 2: *In a deterministic setting, a pure-strategy equilibrium with differential wage offers may only exist when lower wages are paired with high benefits and higher wages with low benefits. There does not exist an equilibrium in which lower wages are paired with lower benefits or in which different wage offers are bundled with the same benefits.*

- In the equilibrium characterised above, both sophisticates and naifs expect to stay upon discovering $\bar{b}$. While sophisticates expect (correctly) to stay in a job offering $(w_2, \underline{b})$, naifs expect (wrongly) to search further.

- Thus, naifs get 'stuck by surprise' in high-wage, low-benefit jobs, which are consciously accepted by sophisticates. At the same time, sophisticates reject all low-wage offers, although they would happily stick with $(w_1, \bar{b})$.

- Firms offering $(w_1, \bar{b})$ attract a smaller mass of workers, but generate more profit per worker.
  - Still, naïve workers who end up in a low-wage job are not necessarily worse off.

Discussion and conclusion

In a model of dynamic search with hidden attributes, I show that:

1. Symmetric equilibria with low benefits are easier to sustain than equilibria with socially desirable high benefits.
2. Asymmetric equilibrium can only exist, if low benefits are paired with a high wage and high benefits with a low wage.
   - Qualitatively similar to the prediction of the theory of compensating differentials (Rosen 1986), but the underlying logic very different.

Further questions:

- Non-sticky equilibria. Adjusting the offers in period 2.
- Uncertainty with *ex ante* vs. *ex post* resolution.

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Thank you for your attention!

Appendix

Search behaviour

Period 2 (on the job)

In the 2nd period, the individual i is employed at firm k earning $u_{ik} = w_k + \theta_{ik}$.

If the agent searches on the job, her value function from sampling an offer w is:

$$v_2(w) = \max \{ \underbrace{w + \mathbb{E}[\theta]}_{\text{accept}} \; ; \; \underbrace{v_2 - c_2}_{\text{keep searching}} \}$$

where $v_2 = \int_w v_2(w)\phi(w)dw$.

- $v_2(w)$ is piecewise linear in w, constant for low w and strictly increasing thereafter.

$\implies$ The optimal strategy is given by a *cutoff rule* with reservation wage R_2, which prescribes to keep searching until the first offer with $w \geq R_2$ is received.

Deriving the optimal cutoff R_2:

For all $w < R_2$, $\quad v_2(w) = v_2 - c_2$.

For all $w \geq R_2$, $\quad v_2(w) = w + \mathbb{E}[\theta]$.

Indifference at the cutoff: $R_2 + \mathbb{E}[\theta] = v_2 - c_2$.

$$
\begin{aligned}
\implies \int_w v_2(w)\phi(w)dw &= (R_2 + \mathbb{E}[\theta]) \cdot \mathbb{P}[w < R_2] \ + \ \int_{w \geq R_2} (w + \mathbb{E}[\theta])\phi(w)dw \\
&= R_2 \cdot \mathbb{P}[w < R_2] \ + \ \int_{w \geq R_2} w\phi(w)dw \ + \ \mathbb{E}[\theta] \\
&= R_2 \ + \ \mathbb{E}[\theta] \ + \ c_2
\end{aligned}
$$

Which solves for:

$$c_2 = \int_{w \geq R_2} (w - R_2)\phi(w)dw$$

- The optimal cutoff (reservation wage) R_2 equalises the marginal cost of search with its marginal benefit.
- RHS is strictly decreasing in $R_2 \rightarrow$ the solution is unique.

Lemma 1: *In period 2, the worker stays in current employment (searches on the job) when* $u_{ik} = w_k + \theta_{ik} \ \geq (<) \ R_2 + \mathbb{E}[\theta]$.

- This is reminiscent of the definition of a *discouraged* worker in McCall (1970).
- For notational convenience, from now on normalise $\mathbb{E}[\theta] = 0$.

Period 1 (initial search)

Consider the following two types:

(i) A sophisticated agent internalises the fact that new information will be revealed once on the job and anticipates that her search costs will increase.

(ii) A naïve agent internalises the fact that new information will be revealed once on the job, but doesn't expect her search costs to increase.

Depending on her beliefs, the agent expects her future self to adopt $\hat{R}_2$:

$$\hat{c}_2 = \int_{w \geq \hat{R}_2} (w - \hat{R}_2)\phi(w)dw$$

Since $c_1 < c_2$, $R_2^N > R_2^S = R_2$ and we have:

Lemma 2: *In period 1, a naïve agent overestimates her future propensity to search on the job. Specifically, for realisations $u_{ik} \in [R_2^S, R_2^N)$ she expects to search for a different job, but actually doesn't.*

How is that biased perception reflected in the search strategy for period 1?

Let $v_1(w) = \max\{u_1(w), v_1 - c_1\}$, where

$$u_1(w) = \big(w + \mathbb{E}[\theta]\big) \; + \; \mathbb{P}[w + \theta \geq \hat{R}_2] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w]\big) \\ + \; \mathbb{P}[w + \theta < \hat{R}_2] \cdot \big(\hat{v}_2 - \hat{c}_2\big)$$

and $v_1 = \int_w v_1(w)\phi(w)dw$.

Three mistakes of a naif:

1. Underestimate the probability of staying in the initially accepted job.
2. Overestimate the average utility from a job they stay in.
3. Overestimate the option value of searching on the job.

The first two points are immediate, but more on statement 3.:

$$\text{For all } w \geq R_2^N, \quad v_2^N(w) = v_2^S(w) = w + \mathbb{E}[\theta]$$

$$\text{For all } w \in [R_2^S, R_2^N), \quad v_2^N(w) = v_2^N - c_1 = R_2^N + \mathbb{E}[\theta] > v_2^S(w) = w + \mathbb{E}[\theta]$$

$$\text{For all } w < R_2^S, \quad v_2^N(w) = v_2^N - c_1 > v_2^S(w) = v_2^S - c_2 = R_2^S + \mathbb{E}[\theta]$$

In the end, as $v_2^N > v_2^S$ and $c_1 < c_2$, $\left(v_2^N - c_1\right) > \left(v_2^S - c_2\right)$.

The sum of these three effects suggests strongly that for any w:

$$u_1^N(w) > u_1^S(w)$$

Formally:

$$u_1^N(w) > u_1^S(w) \iff$$

$$\mathbb{P}[w+\theta \geq R_2^N] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^N - w]\big) \;+\; \mathbb{P}[w+\theta < R_2^N] \cdot \big(v_2^N - c_1\big) >$$

$$> \mathbb{P}[w+\theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^S - w]\big) \;+\; \mathbb{P}[w+\theta < R_2^S] \cdot \big(v_2^S - c_2\big) =$$

Since $R_2^N > R_2^S$, expand RHS:

$$\begin{aligned}\text{RHS} = {} & \mathbb{P}[w+\theta \geq R_2^N] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^N - w]\big) \\ & + \mathbb{P}[R_2^N > w+\theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \,|\, R_2^N - w > \theta \geq R_2^S - w]\big) \\ & + \mathbb{P}[w+\theta < R_2^S] \cdot \big(v_2^S - c_2\big)\end{aligned}$$

Then, $u_1^N(w) > u_1^S(w) \iff$

$$\mathbb{P}[w + \theta < R_2^N] \cdot \big(v_2^N - c_1\big) >$$

$$\mathbb{P}[R_2^N > w + \theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \mid R_2^N - w > \theta \geq R_2^S - w]\big) \; + \; \mathbb{P}[w + \theta < R_2^S] \cdot \big(v_2^S - c_2\big)$$

$$\iff \big(v_2^N - c_1\big) > (1 - \rho) \underbrace{\big(w + \mathbb{E}[\theta \mid R_2^N - w > \theta \geq R_2^S - w]\big)}_{\in [R_2^S, R_2^N)} + \rho\big(v_2^S - c_2\big)$$

where $\rho = \mathbb{P}[w + \theta < R_2^S] \, / \, \mathbb{P}[w + \theta < R_2^N] \in (0, 1)$ as long as both probabilities are positive.

Since $\big(v_2^N - c_1\big) = R_2^N$ (under normalisation $\mathbb{E}[\theta] = 0$), the above inequality is indeed strict for any $\rho \in [0, 1]$.

In the end, we obtain:

Lemma 3: *A naïve agent strictly overestimates the utility from accepting any job offer.*

Moreover, if $d\,u_1(w)/d\,w > 0$ the optimal search strategy for either type is again given by a cutoff rule:

$$\mathbb{P}[w + \theta \geq \hat{R}_2] = \int_{\hat{R}_2 - w} \psi(\theta) d\theta$$

$$\mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w] = \int_{\hat{R}_2 - w} \theta\psi(\theta) d\theta \;/\; \int_{\hat{R}_2 - w} \psi(\theta) d\theta$$

$$u_1(w) = \big(w + \mathbb{E}[\theta]\big) + w \cdot \int_{\hat{R}_2 - w} \psi(\theta) d\theta + \int_{\hat{R}_2 - w} \theta\psi(\theta) d\theta + \big(\hat{v}_2 - \hat{c}_2\big) \cdot \int^{\hat{R}_2 - w} \psi(\theta) d\theta$$

Thus

$$
\begin{aligned}
d\,u_1(w)/d\,w &= 1 + \mathbb{P}[w+\theta \geq \hat{R}_2] + w \cdot \big[-\psi(\hat{R}_2 - w)(-1) \big] \\
&\quad + \big[-(\hat{R}_2 - w)\psi(\hat{R}_2 - w)(-1) \big] + \big(\hat{v}_2 - \hat{c}_2\big)\big[\psi(\hat{R}_2 - w)(-1)\big] \\
&= 1 + \mathbb{P}[w+\theta \geq \hat{R}_2] + \psi(\hat{R}_2 - w)\underbrace{\big[\hat{R}_2 - (\hat{v}_2 - \hat{c}_2)\big]}_{=0} > 0
\end{aligned}
$$

and like before, the optimal reservation wage R_1 satisfies:

$$
c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1)\big)\phi(w)dw
$$

Finally, we want to show that $R_1^N < R_1^S$.

Since

$$d\,u_1(w)/d\,w = 1 + \mathbb{P}[w + \theta \geq \hat{R}_2]$$

and $R_2^N > R_2^S$, we have $\frac{d\,u_1^N(w)}{d\,w} < \frac{d\,u_1^S(w)}{d\,w}$

Thus for any $w > R$:

$$u_1^N(w) - u_1^N(R) < u_1^S(w) - u_1^S(R)$$

Optimal cut-off rule implies:

$$c_1 = \int_{w \geq R_1^N} \left(u_1^N(w) - u_1^N(R_1^N)\right)\phi(w)dw = \int_{w \geq R_1^S} \left(u_1^S(w) - u_1^S(R_1^S)\right)\phi(w)dw$$

Now suppose that $R_1^N \geq R_1^S$. Then:

$$\int_{w \geq R_1^S} \left(u_1^S(w) - u_1^S(R_1^S)\right)\phi(w)dw =$$

$$= \int_{w \geq R_1^N} \left(u_1^S(w) - u_1^S(R_1^S)\right)\phi(w)dw \ + \ \int_{w=R_1^S}^{R_1^N} \left(u_1^S(w) - u_1^S(R_1^S)\right)\phi(w)dw \geq$$

$$\geq \int_{w \geq R_1^N} \left(u_1^S(w) - u_1^S(R_1^N)\right)\phi(w)dw \ + \ \int_{w=R_1^S}^{R_1^N} \left(u_1^S(w) - u_1^S(R_1^S)\right)\phi(w)dw \geq$$

$$\geq \int_{w \geq R_1^N} \left(u_1^S(w) - u_1^S(R_1^N)\right)\phi(w)dw >$$

$$> \int_{w \geq R_1^N} \left(u_1^N(w) - u_1^N(R_1^N)\right)\phi(w)dw$$

A contradiction. Thus, we have:

Lemma 4: *A naïve agent adopts a suboptimally low reservation wage when searching initially, that is* $R_1^N < R_1^S$.

Comparative statics on the naiveté wedge

1. Search costs
   - $c_2 \uparrow \implies$ WL $\uparrow$
   - The effect of $c_1 \uparrow$ depends on whether the cut-off R_1 is more sensitive to c_1 for naifs or for sophisticates.

1.1 Naifs overestimate the option value of searching on the job $\rightarrow R_1^N(c_1)$ flatter.
1.2 Sophisticates adopt a more picky search strategy initially $\rightarrow R_1^S(c_1)$ flatter.
   - Depending on which of these two effects dominates for a given distribution of offers, WL $\updownarrow$ in c_1...

Consider how the effects of naiveté vary across populations of agents with different initial search costs:

$$c_1 < c_1' < c_2$$

Naturally, $c_1 \uparrow \implies R_1 \downarrow$ because the RHS of

$$c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1)\big)\phi(w)dw$$

is strictly decreasing in R_1.

$$\begin{aligned} \frac{d\,\text{RHS}}{d\,R_1} &= -(u_1(R_1) - u_1(R_1))\phi(R_1) \ + \ \int_{w \geq R_1} -u'(R_1)\phi(w)dw \\ &= -u'(R_1) \cdot \mathbb{P}[w \geq R_1] < 0 \end{aligned}$$

Recall from above that

$$d\,u_1(w)/d\,w = 1 + \mathbb{P}[w + \theta \geq \hat{R}_2] > 0$$

and thus

$$\frac{d\,\mathsf{RHS}}{d\,R_1} = -\big(1 + \mathbb{P}[R_1 + \theta \geq \hat{R}_2]\big) \cdot \mathbb{P}[w \geq R_1]$$

Given that $R_1^S > R_1^N$ and $R_2^N > R_2^S$, the 'slope' of RHS can be either steeper or flatter for the naïve agents, depending on the underlying distributions for w and θ.

In particular, $|\frac{d\,\textsf{RHS}}{d\,R_1^S}| > |\frac{d\,\textsf{RHS}}{d\,R_1^N}| \iff$

$$\big(1+\mathbb{P}[\theta \geq R_2^S - R_1^S]\big) \;/\; \big(1+\mathbb{P}[\theta \geq R_2^N - R_1^N]\big) \quad > \quad \mathbb{P}[w \geq R_1^N] \;/\; \mathbb{P}[w \geq R_1^S]$$

where both sides of the inequality are > 1.

If the above is satisfied, then for $c_1' > c_1$, $(R_1^{S'} - R_1^{N'}) < (R_1^S - R_1^N)$ (and WL $\downarrow$).

- If the inequality is reversed, then $|\frac{d\,\textsf{RHS}}{d\,R_1^S}| < |\frac{d\,\textsf{RHS}}{d\,R_1^N}|$ (and WL $\uparrow$).
- R_2^S, R_2^N are invariant to changes in c_1.

Corollary 1: *If the following holds at the cutoff rules adopted by the sophisticated and the naïve agents, respectively:*

$$\left(1 + \mathbb{P}[\theta \geq R_2^S - R_1^S]\right) \,/\, \left(1 + \mathbb{P}[\theta \geq R_2^N - R_1^N]\right) \quad < (>) \quad \mathbb{P}[w \geq R_1^N] \,/\, \mathbb{P}[w \geq R_1^S]$$

then the welfare loss from naiveté is increasing (decreasing) in the initial search cost c_1.

2. Wage levels
   - $\phi^{\star}(w)$ first-order stochastically dominates $\phi(w) \implies$ WL $\uparrow$
     - Accepting inferior offers too quickly has larger negative consequences when the tail of the wage distribution is 'fatter'.

Under $\phi^{\star}(w)$, $R_1 \uparrow$ because the RHS of

$$c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1)\big)\phi(w)dw$$

is strictly greater under $\phi^{\star}(w)$ than $\phi(w)$.

For a fixed R_1, the increase in RHS caused by the shift to $\phi^\star(w)$ is more pronounced the steeper $u_1(\cdot)$.

We already know that $\frac{d\,u_1^N(w)}{d\,w} < \frac{d\,u_1^S(w)}{d\,w}$ and therefore:

$$\implies \;\; R_1^{S\star} - R_1^S > R_1^{N\star} - R_1^N \;\;\iff\;\; \left(R_1^{S\star} - R_1^{N\star}\right) > \left(R_1^S - R_1^N\right)$$

Corollary 2: *The welfare loss from naiveté is greater for agents who sample offers from a first-order stochastically dominant distribution of wages.*

The set of five inequalities reduces to:

1. $c_1 < (1-p)(w_2-w_1)+(1-p)(w_2-pw_1-(1-p)\mathbb{E}[w])-(1-p)p(\bar{b}-\mathbb{E}[b])+(1-p)^2c_2$
2. $c_1 < -p(w_2 - w_1) + p(\bar{b} - \underline{b})$
3. $c_1 \geq (1 - p^2)(w_2 - w_1)$
4. $c_2 < (1 - p)(w_2 - w_1) + p(\bar{b} - \underline{b})$
5. $c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$

The equal profits condition is:

$$w_2 = (1 - \lambda)y + \lambda w_1 + \lambda(1 - \tau)\bar{b} - (1 - \tau)\underline{b}$$

and the naifs' participation constraint simplifies to:

$$c_1 \ \leq \ 2/(2 - p)\mathbb{E}[w] + \mathbb{E}[b] + p/(2 - p)\bar{b}$$

For example, for:

- $\lambda = 0.30$,
- $c_1 = 3$,
- $c_2 = 7.5$,
- $y = 200$,
- $\underline{b} = 5$,
- $\bar{b} = 24.17$,
- $\tau = 0.25$,

there exists an equilibrium in which 90% of firms offer a compensation package $(w_1, \bar{b}) = (180.0, 24.17)$, 10% of firms offer a profit-equivalent compensation package $(w_2, \underline{b}) = (195.69, 5)$, and the search behaviour of sophisticated and naïve agents follows the rules described above.

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