Single sourcing from a supplier with unknown efficiency and capacity

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Introduction

- Enormous amount of heterogeneity within same industry, on production technology or input costs (*see Baily, Hulten, Campbell, Bresnahan and Caves (1992) or Bartelsman and Drhymes (1998) amongst others*)
- Firms do not operate at the same scale or with the same efficiency (*see Röller (1990) or Van Biesebroeck (2003)*)
- *Single sourcing* is commonly used (even if risk management considerations should deter purchasers), as e.g. :
  - Administrative costs savings, larger attractiveness to supplier(s), better unit price, monopolized technology
  - Often used for indirect purchases, but not only
  - Suppliers' concentration reduces the choices

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- Each time a (downstream) firm procures an amount from its supplier, it faces a (upstream) supplier with
  - ▷ either a small cost/unit for a small scale of production (i.e. *soft capacity constrained*)
  - ▷ or able to produce at a large scale but at a higher cost/unit (i.e. *constant returns but less efficient at small scale*)

E.G. Capacities planned ahead of demand + purchase orders exhaust first the planned capacity of most efficient suppliers

⇒ Acquiring extra inputs to satisfy an order *above planned capacity* is generally *more costly* than planned unit cost

⇒ For a given procurement, *marginal cost is steeper for an efficient supplier* than an inefficient one

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Research question

- How should a buyer/retailer optimally purchase the product it resales from a single supplier with unknown cost, such that this supplier can be
  - ▷ either efficient for small output levels, but faces a steep marginal cost curve,
  - ▷ or is less efficient, but faces a flatter marginal cost curve,
  - ▷ or faces any combination of the two, such that *the steeper the marginal cost of production the smaller its intercept is*

⇔ *How to buy from a supplier when efficient ones are (soft) capacity constrained / less efficient ones are less constrained, without knowing the supplier true characteristics ?*

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Results overview

- *Over-ordering* or *under-ordering* is possible : retailer's order *above or below* the order of an informed monopoly
  - *Over-ordering* : asymmetric information makes the market *price lower* than the monopoly one (*absent storage/free disposal*)
  - *Under-ordering* : asymmetric information makes the market *price higher* than the monopoly one
- Distortions depend on the size of the market relative to the support of cost/type distribution
  - For an intermediate market size, *no distortion at the extremes* of the type distribution, *and for an interior type*
  - Single contract is offered to *a set of types interior to the support of types*

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- Supplier's rents depend on the demand size
  - ▷ For an intermediate market size, no rent for interior types *i.e. the supplier's rent is non monotonic in types*
- Under-ordering occurs in absence of double marginalization :
  - ▷ *in large or intermediate markets, retailers should order less than what a vertical monopoly would do, to reduce the incentives of non capacity constrained suppliers to lie*
- Despite marginal cost differences to produce this quantity, offering the same contract to a set of types is attractive :
  - ▷ *the cost of producing marginal and infra-marginal units compensate each other, and rents are nil*
  - ▷ *the dispersion of orders is smaller than the dispersion of marginal costs*

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Related literature

- *Results' driver :* countervailing incentives, without the Spence-Mirrlees condition, without a concave objective
- Lewis and Sappington (1989), Biglaiser and Mezzeti (1993), Maggi and Rodriguez-Clare (1995), and Jullien (2000)
  - ▷ *Countervailing incentives come from determinants of variable cost (not from participation constraint or from fixed cost)*
  - ▷ *Marginal costs do not rank identically across types as q increases (and cross once !)*
  - ▷ *To gain on the initial units of a batch*, capacity constrained efficient firms may pretend they can serve a larger quantity (*even if the last units of a batch are more costly to produce*)
  - ▷ *Even if they loose on the initial units of a batch*, unconstrained inefficient firms may pretend they can serve a smaller quantity (*to get paid at a higher price on the last unit of a batch*)

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  - **Rotations of marginal costs rule out global deviations**
  - Quantity ordered must be monotonic in supplier's type (*increasing when demand is large enough, and decreasing when demand is small enough*)
- Need to deal with non concavity of principal's objective
  - Consequence of the absence of Spence-Mirrlees condition, implying that monotonicity not always granted
  - *either "ironing" is needed (see Guesnerie and Laffont (1984)) or a condition which ensures monotonicity*
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The model (sketch)

- Downstream firm D procures q from upstream producer U (D *sells* q *but can't produce,* U *can't access the market*)
- Consumers inverse demand is linear in q

$$P(q) = \max\{a - bq, 0\} \quad \text{with } a > 0,\ b > 0. \tag{1}$$

- Disposal/storage prohibitive (q *entirely sold at* $P(q)$)
- *P/A model :* D offers a menu of binding contracts to U

$$\pi_D^e(\tilde{\theta}) = \mathrm{E}\left(P(q(\tilde{\theta}))q(\tilde{\theta}) - T(\tilde{\theta})\right) \tag{2}$$

and U's ex-post payoff is

$$\pi_U(q(\tilde{\theta});\theta) = T(\tilde{\theta}) - C(q(\tilde{\theta});\theta) \text{ for } \theta \in [0, \bar{c}] \tag{3}$$

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- No fixed costs + convex variable cost

$$C(q;\theta) = \theta q + \frac{1}{2} d(\theta) q^2 \quad \text{with } \theta \geq 0 \tag{4}$$

where $d(\theta) = \bar{d}\left(1 - \frac{\theta}{\bar{c}}\right)$ decreases with θ

- θ unknown + function $d(\theta)$ known + θ distributed as

$$F(\theta) \in [0,1] \text{ and } f(\theta) \geq 0 \quad \text{for } \theta \in [0, \bar{c}], \tag{5}$$

▷ $F(\theta)$ s.t. $\frac{\partial}{\partial \theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0 \geq \frac{\partial}{\partial \theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)$ for $\theta \in [0, \bar{c}]$

- Industry profit : $\Pi(q;\theta) = P(q)q - C(q;\theta)$ with $\Pi_{qq} < 0$

▷ **first best** for $q^M(\theta)$ s.t. $\Pi_q(q^M(\theta);\theta) = 0$

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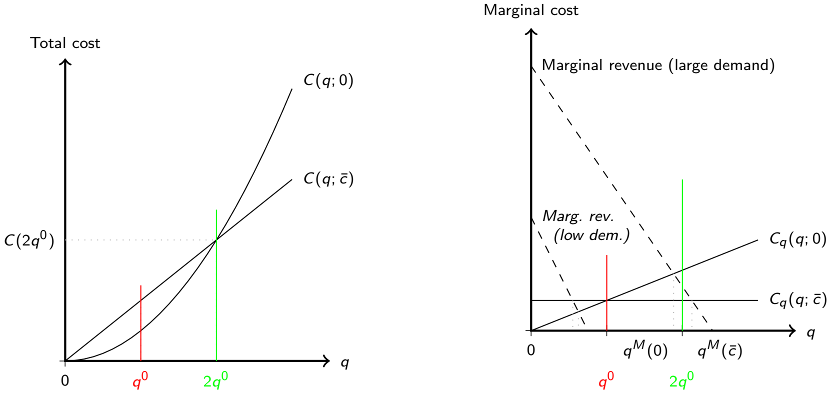
Graphical illustration



- All *marginal costs* are equal to each other at $q^0 = \frac{\bar{c}}{d}$ (*right*)
- All *total costs* are equal to each other at $2q^0$ and at 0 (*left*)

- "Rotation" consequences :
  - $C_\theta(q;\theta) = q - \frac{\bar{d}}{2\bar{c}}q^2 \geq 0$ if $q \leq 2q^0$, else negative
  - $C_{q\theta}(q;\theta) = 1 - \frac{\bar{d}}{\bar{c}}q \geq 0$ if $q \leq q^0 = \frac{\bar{c}}{\bar{d}}$, else negative
  - $C(2q^0;\theta) = C(2q^0;\theta') \equiv C(2q^0)$ for $\theta \neq \theta'$
  - Rotations imply that **q^0 is independent of θ**
  - *NB : Rotations of demand in Johnson and Myatt (2006) and Araujo and Moreira (2015)*
- Such changes in the rankings of marginal costs also occur with stepwise increasing marginal costs (*they must "cross" once*)

IR and IC constraints

- Individual rationality constraints

$$\pi_U(q(\theta);\theta) \geq 0 \text{ for all } \theta \in [0, \bar{c}] \qquad (IR_\theta)$$

- Incentive compatibility constraints

$$\pi_U(q(\theta);\theta) \geq \pi_U(q(\tilde{\theta});\theta) \text{ for } \tilde{\theta} \neq \theta \qquad (IC_\theta)$$

where supplier's U payoff has the following local properties

- Spence-Mirrlees condition not satisfied

$$\frac{\partial^2 \pi_U(q;\theta)}{\partial q \partial \theta} = -C_{q\theta}(q;\theta) > 0 \text{ if } q > q^0, \text{ else negative} \qquad (6)$$

- and U's profit not monotonic in θ

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Resolution (sketch) to construct $q^*(\theta)$

- Lewis and Sappington (1989) + Jullien (2000)
  - ▷ Countervailing incentives imply that IR constraints of types interior to $[0, \bar{c}]$ can bind
  - ▷ Virtual surplus (once local IC incorporated) must be rewritten to make this feature appear
- Virtual surplus not concave in q for all θ + virtual marginal surplus not monotonic with θ
  - ▷ Quantity ordered can hit an upper bound (*which exists for all demand functions*) depending on which IR constraints bind
  - ▷ Monotonicity of $q^*(\theta)$ must be granted
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  - ▷ *Rotations imply that there is no frontier in the graph* (θ, q) *along which global deviations must be checked*

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Downstream firm D relaxed problem

- D max. π_D^e w.r.t. $(q(\theta), \pi_U(q(\theta);\theta))$ for all $\theta \in [0, \bar{c}]$

$$\pi_D^e = \int_0^{\bar{c}} \Pi(q(\theta);\theta) - \pi_U(q(\theta);\theta) dF(\theta) \tag{8}$$

$$\begin{aligned} \text{subject to } (IR)_\theta : &\quad \pi_U(q(\theta);\theta) \geq 0 \quad \forall \theta \in [0, \bar{c}] \\ (LIC)_\theta : &\quad \pi'_U(q(\theta);\theta) = -C_\theta(q(\theta);\theta) \quad \forall \theta \in [0, \bar{c}] \\ (MON) : &\quad q'(\theta) \leq 0 \text{ if } q \leq q^0 \text{ and } q'(\theta) \geq 0 \text{ if } q \geq q^0 \end{aligned}$$

- $\mu(\theta)$ multiplier of $(IR)_\theta$: *opportunity gain to reduce* π_U *to* 0
- $\mu(\theta)$ assumed to be integrable, $M(\theta) = \int_0^\theta \mu(t)dt$: *cumulated opportunity gain to reduce* π_U *to* 0 *for all types* $t = 0$ *to* $t = \theta$

Expected virtual surplus

- Expected virtual surplus (IPP from the Lagrangian of (8))

$$V_D^e = \int_0^{\bar{c}} \Pi(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_\theta(q(\theta);\theta) dF(\theta) \quad (9)$$

- point-wise optimization w.r.t. $q(\theta)$ for each θ gives

$$\Pi_q(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_{\theta q}(q(\theta);\theta) = 0 \quad (10)$$

$$\Pi_{qq}(q(\theta);\theta) - \frac{F(\theta) - M(\theta)}{f(\theta)} C_{\theta qq}(q(\theta);\theta) \leq 0. \quad (11)$$

- $M(\theta)$ behaves as a C.D.F. over $[0, \bar{c}]$ (possibly degenerated)
- Search for $(q^*(\theta), M^*(\theta))$ solving (10) and (11) for every θ

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Bounds on $q^*(\theta)$ for $M = 0$ and $M = 1$

- $\tilde{q}(\theta, 0)$ maximizes V_D^e for $M = 0$, $\tilde{q}(\theta, 1)$ for $M(\theta) = 1$
- $C_{\theta q} < 0 \Rightarrow \tilde{\mathbf{q}}(\theta, \mathbf{1}) \leq \mathbf{q}^{\mathbf{M}}(\theta) \leq \tilde{\mathbf{q}}(\theta, \mathbf{0})$ (equal at 0 and $\bar{c}$)

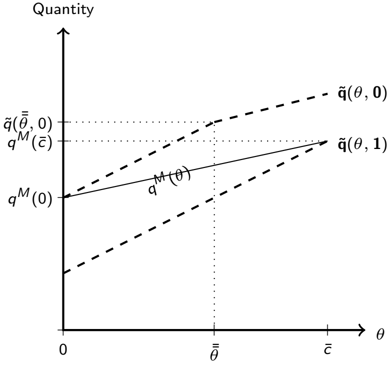

Figure – Bounds on $q^*(\theta)$ for $M = 1$ or $M = 0$ for all $\theta \in [0, \bar{c}]$

Determination of $q^*(\theta)$

How large market demand is determines where (and for which types) the IR constraints bind

- Only the IR of $\theta = \bar{c}$ binds on small markets
- Only the IR of $\theta = 0$ binds on large markets
- For intermediate demand, the IR of an interval of types bind, around θ^0 such that $q(\theta^0) = 2q^0$, which all produce $q(\theta^0)$ (*interval is endogenous*)
- *A single contract ordering $2q^0$ and reimbursing $C(2q^0)$ can be offered to any supplier without leaving a rent - bunching types is profitable for the retailer, at the loss of marginal efficiency*

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Intermediate demand : graphical illustration

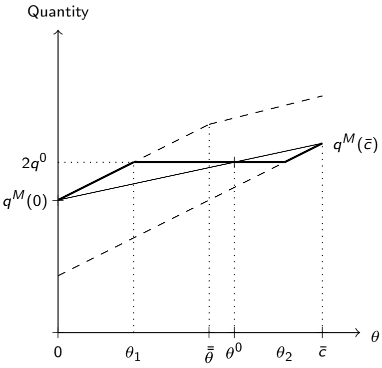

Figure – $C_q(2q^0;\bar{c}) \leq P(2q^0) + 2q^0P'(2q^0) \leq C_q(2q^0;0)$

Figure – Equilibrium when intermediate demand

Large demand : graphical illustration

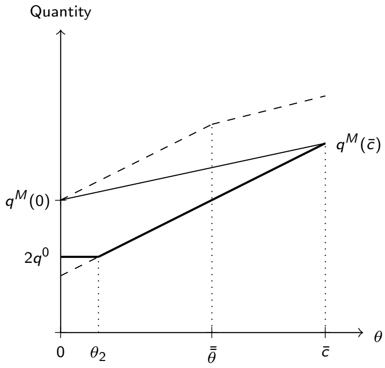

Figure – $P(2q^0) + 2q^0 P'(2q^0) > C_q(2q^0; 0)$

Figure – Equilibria when market demand is large