A Parsimonous Model of Idiosyncratic Income

Edmund Crawley, Federal Reserve Board

Martin B. Holm, University of Oslo

Håkon Tretvoll, Statistics Norway

24 August, 2022

Motivation

*[T]he key challenge for future work is to develop a specification for the **wage process** that is both **parsimonious** enough to be used as an input to incomplete-markets models, and rich enough to **account empirically for the covariance structure of wages in both levels and differences**.*

Heathcote, Perri, and Violante (2010, p. 40)

Motivation

The standard model:

$$y_t = p_t + \varepsilon_t + \theta \varepsilon_{t-1}$$
$$p_t = p_{t-1} + \nu_t$$

can be estimated using level moments, $Cov(y_t, y_s)$, or difference moments, $Cov(\Delta y_t, \Delta y_s)$.

But choice of moments gives different results:

	Level moments	Difference moments
σ^2_{perm}	0.004	0.011
σ^2_{tran}	0.032	0.020

i.e., the standard model is *misspecified*.

What We Do

Propose minimal changes to the standard model:

1. Shocks are spread uniformly through the year (time-aggregation problem)
2. Two types of transitory shocks (“bonus” and “passing”)

Still parsimonious: only one extra parameter to estimate

Main Results

1. We estimate the standard model on simulated data from proposed model:

Same misspecification ‘structure’ as in actual data

2. We estimate the proposed model in Norwegian data:

Similar parameter estimates irrespective of level or difference

Same for PSID

Outline

Data

Problem With the Standard Model

The Proposed Model

Simulation Results

Data Results

Conclusion

Data

Norwegian administrative data on annual income, 1971 – 2014

Main variable: log residualized pre-tax earnings (year, age, and education dummies)

Sample selection (Daly, Hryshko & Manovskii, 2022)

- males aged 35-50
- drop extreme income changes ($> 500\%$) and low income ($<$ approx USD 10,000)
- balanced panel

More than 500k individuals, in 27 cohorts, observed for 15 years each.

Outline

Data

Problem With the Standard Model

The Proposed Model

Simulation Results

Data Results

Conclusion

The Standard Model

Income process:

$$\begin{aligned} y_t &= p_t + \varepsilon_t + \theta\varepsilon_{t-1} \\ p_t &= p_{t-1} + \nu_t \end{aligned}$$

- p = permanent component, unit root
- ε = transitory component

Parameters to be estimated: $\sigma^2_{\nu_t}$, $\sigma^2_{\varepsilon_t}$, θ, $\sigma^2_{p_0}$.

Estimation Procedure

Compute empirical level or difference moments:

$$L_{t,s} = \frac{1}{N}\sum_{i=1}^{N} y_{i,t} y_{i,s} \qquad \text{or} \qquad D_{t,s} = \frac{1}{N}\sum_{i=1}^{N} \Delta y_{i,t} \Delta y_{i,s}$$

where $y_{i,t}$ is residualized log income and minimize the loss function:

$$\mathcal{L} = vech(D_{\text{data}} - D_{\text{model}})^T \Omega^{-1} vech(D_{\text{data}} - D_{\text{model}})$$

Where Ω is either

- identity matrix (**Identity**)
- optimal minimum distance weighting matrix (**Optimal**)
- optimal matrix along the diagonal (**Diagonal**)

Results: Estimating the Standard Model with Norwegian Data

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

1. Difference in permanent variance
2. Difference in transitory variance opposite of permanent variance
3. **Optimal**: ‘similar’ results irrespective of level or difference (Daly, Hryshko & Manovskii, 2022)

Outline

The Proposed Model

Two adjustments

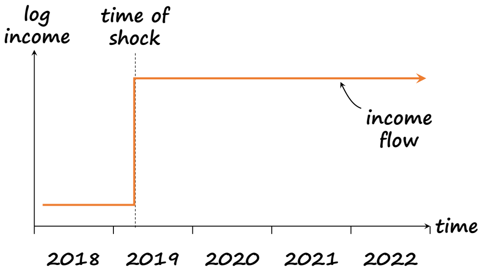
1. We allow shocks to happen throughout the year (in continuous time)

2. We allow for three types of income shocks

   1. Permanent Income Shocks
   2. ‘Bonus’ Income Shocks
   3. ‘Passing’ Income Shocks

   } Transitory Shocks (items 2 and 3)

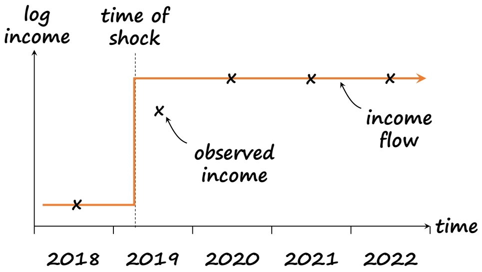
Shock Type 1: Permanent



(a) Income flow

(b) Covariance structure for difference moments

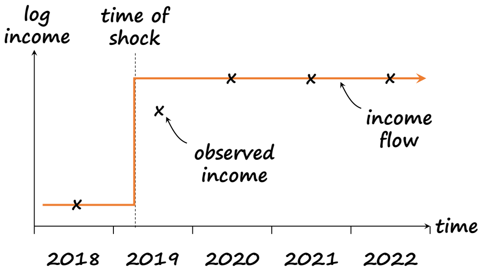
Shock Type 1: Permanent



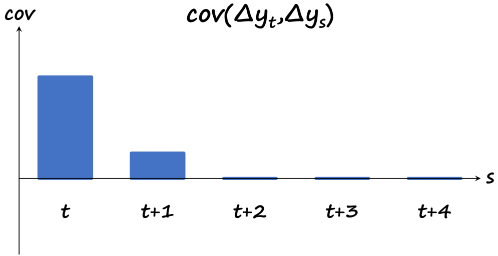
(a) Income flow

(b) Covariance structure for difference moments

Shock Type 1: Permanent

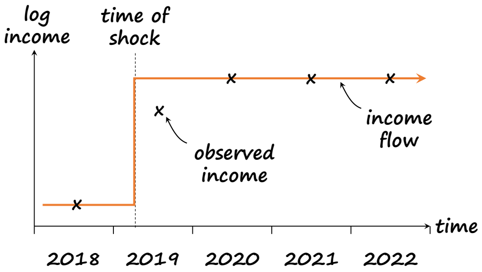


(a) Income flow

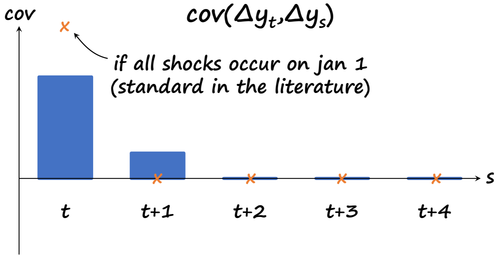


(b) Covariance structure for difference moments

Shock Type 1: Permanent

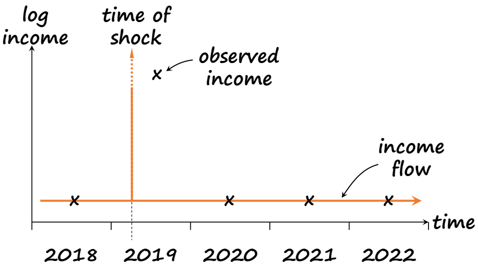


(a) Income flow

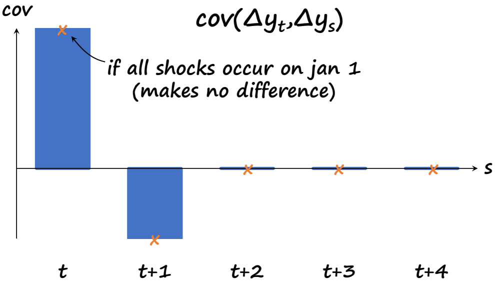


(b) Covariance structure for difference moments

Shock Type 2: Bonus

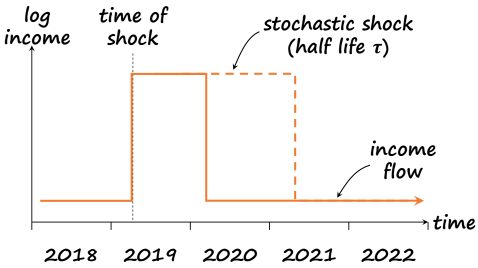


(a) Income flow



(b) Covariance structure for difference moments

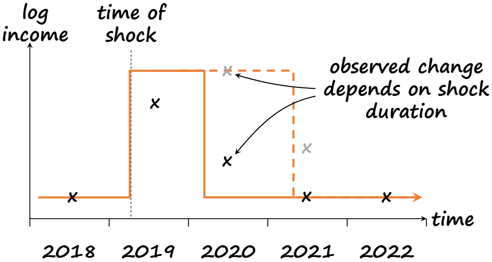
Shock Type 3: Passing



(a) Income flow

(b) Covariance structure for difference moments

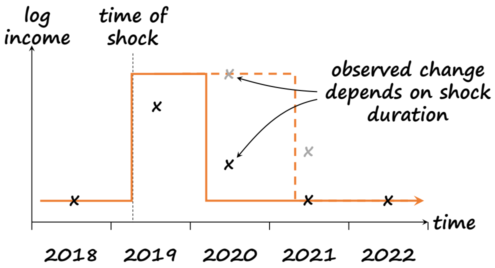
Shock Type 3: Passing



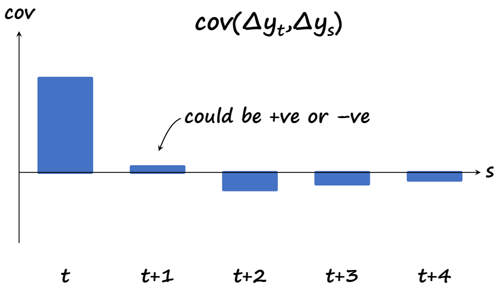
(a) Income flow

(b) Covariance structure for difference moments

Shock Type 3: Passing

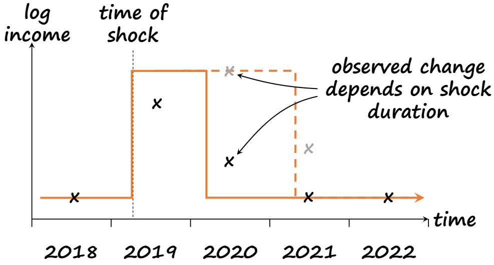


(a) Income flow

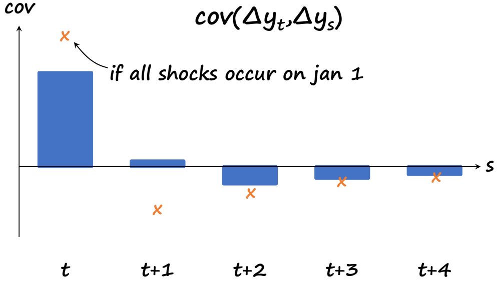


(b) Covariance structure for difference moments

Shock Type 3: Passing

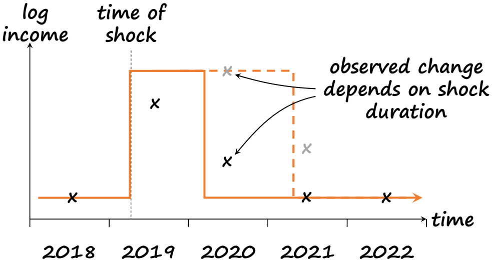


(a) Income flow

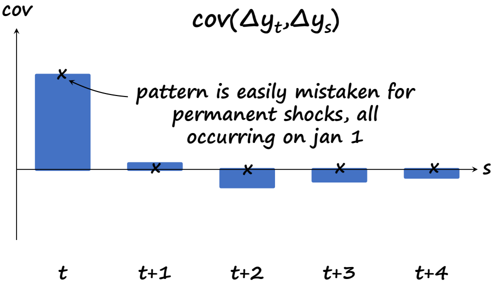


(b) Covariance structure for difference moments

Shock Type 3: Passing



(a) Income flow



(b) Covariance structure for difference moments

Mapping between standard and proposed model parameters

Parameter Description	Proposed	Standard
Permanent income variance	σ^2_{perm}	$\text{Var}(\nu)$
Transitory income variance	σ^2_{tran}	$(1+\theta^2)\text{Var}(\varepsilon)$
Half life of passing shock	τ	✗
Bonus fraction of σ^2_{tran}	b	✗
MA(1) transitory persistence	✗	θ
Initial permanent income variance	σ^2_{init}	$\text{Var}(p_0)$

Outline

Data

Problem With the Standard Model

The Proposed Model

Simulation Results

Data Results

Conclusion

Simulation Exercise

Simulate 50k individuals using discretized (monthly) version of proposed model

Estimate standard model on the simulated data

Simulation Results

		Identity		Diagonal		Optimal	
Parameter	True Value	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.005	0.005	0.013	0.005	0.013	0.008	0.009
σ^2_{tran}	0.038	0.031	0.017	0.032	0.017	0.021	0.020
τ	2.0 years	✗	✗	✗	✗	✗	✗
b	0.40	✗	✗	✗	✗	✗	✗
θ	✗	0.50	0.10	0.53	0.10	0.18	0.16
σ^2_{init}	0.065	0.065	✗	0.065	✗	0.064	✗

Main results:

1. Same 'structure' of misspecification as when using data ▸ Data Results
2. **Optimal** gives similar parameter estimates regardless of moments (DHM, 2022)
3. Level moments with **Identity**/**Diagonal** perform well

Outline

Data

Problem With the Standard Model

The Proposed Model

Simulation Results

Data Results

Conclusion

Data Results

Exercises:

1. Results using Norwegian data

2. ~~Age-varying / time-varying results using Norwegian data~~

3. ~~Results using the PSID~~

4. ~~Time-varying results using the PSID~~

Norwegian Results

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
Panel A: Proposed Model						
σ^2_{perm}	0.003	0.005	0.003	0.005	0.003	0.004
σ^2_{tran}	0.039	0.038	0.038	0.039	0.039	0.036
τ	1.547	2.045	1.770	2.058	2.234	1.862
b	0.360	0.478	0.341	0.473	0.443	0.480
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗
Panel B: Standard Model						
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

Norwegian Results (No Bonus Shock)

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.003	0.011	0.003	0.011	0.005	0.007
σ^2_{tran}	0.036	0.021	0.036	0.021	0.023	0.022
τ	0.982	0.065	1.202	0.080	0.231	0.157
σ^2_{init}	0.064	✗	0.063	✗	0.060	✗

Norwegian Results (Persistence)

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
Panel A: Proposed Model						
σ^2_{perm}	0.004	0.006	0.004	0.006	0.003	0.005
σ^2_{tran}	0.038	0.034	0.041	0.035	0.039	0.033
τ	1.494	1.810	1.328	1.815	2.241	1.640
b	0.390	0.540	0.407	0.526	0.443	0.525
ρ	0.991	0.979	0.982	0.985	1.000	0.989
σ^2_{init}	0.063	✗	0.063	✗	0.059	✗
Panel B: Standard Model						
σ^2_{perm}	0.010	0.014	0.010	0.015	0.011	0.010
σ^2_{tran}	0.022	0.017	0.022	0.016	0.019	0.019
θ	0.216	0.026	0.233	0.000	0.088	0.092
ρ	0.923	0.894	0.921	0.809	0.926	0.952
σ^2_{init}	0.063	✗	0.062	✗	0.059	✗

Outline

Data

Problem With the Standard Model

The Proposed Model

Simulation Results

Data Results

Conclusion

Conclusion

We propose a new model of idiosyncratic income. Two changes to standard model:

- **Continuous time** to solve time aggregation problem
- **Three types of income shocks** to match data.

The model is **parsimonious** and has **no obvious misspecification** issues.

- Only one extra parameter to be estimated.
- Parameter estimates are similar across moments and weighting matrix applied.

For practitioners:

- Use our model!
- If you for some reason have to use the standard model: use level moments!

Standard Model Estimated in Data

Back

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

Simulation Results (Persistence)

‣ Back

Parameter	True Value	Identity Level	Identity Difference	Diagonal Level	Diagonal Difference	Optimal Level	Optimal Difference
σ^2_{perm}	0.005	0.008	0.015	0.009	0.015	0.013	0.013
σ^2_{tran}	0.038	0.026	0.015	0.027	0.015	0.017	0.017
τ	2.0 years	✗	✗	✗	✗	✗	✗
b	0.40	✗	✗	✗	✗	✗	✗
θ	✗	0.38	0.07	0.39	0.07	0.12	0.11
ρ	1.00	0.97	0.93	0.97	0.93	0.94	0.95
σ^2_{init}	0.065	0.062	✗	0.063	✗	0.064	✗