

# Coexistence of Money and Interest-Bearing Bonds

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July 15, 2022

# Agenda

- ① Introduction
- ② Model
- ③ Analysis
- ④ Other implications
- ⑤ Conclusion

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# Research question and motivation

## **Why should money and bonds coexist, and why may zero nominal interest rates be sub-optimal?**

Topical debate on the distribution of assets in an economy:

- Gained momentum during the zero lower bound (ZLB) episode.
- Monetary theory prescribes optimality of the ZLB—the Friedman rule.

An important question in monetary theory: Why do money and bonds coexist, i.e., how can a mere savings instrument be useful in a monetary economy?

# Contribution and key results

I construct and analyze a model of a monetary economy with:

- idiosyncratic shocks to agents' rate of time preference;
- information frictions rendering perfect insurance infeasible;
- bonds that can be traded for money once preferences are revealed.

Positive nominal rates imply a more efficient distribution of savings.

- Agents become constrained by their money holdings.
- Trade in financial markets arises.
- There is a net transfer of savings from impatient to patient agents.

When sufficiently many agents can trade in the financial market, the re-distributive effect dominates the negative in goods markets.

## Related literature

Coexistence puzzle:

- Kocherlakota (2003), Shi (2008), Andolfatto (2011).

Sub-optimality of the FR:

- Shi (1997), Aruoba, Rocheteau, and Waller (2007), Nosal (2011).

Heterogeneous time preferences in monetary economies:

- Boel and Camera (2006), Boel and Waller (2019), Van Buggenum and Uras (2022).

Secondary financial markets:

- Duffie, Garleanu, and Pedersen (2005), Berentsen, Camera, and Waller (2007), Li and Li (2013), Geromichalos and Herrenbrueck (2016).

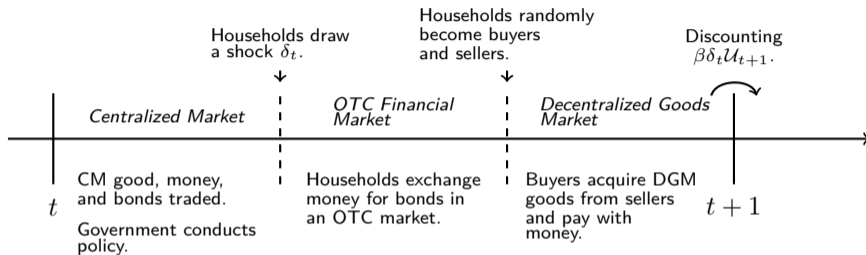
Heterogeneous asset valuations:

- Trejos and Wright (2016).

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# Core structure



- Time  $t$  is discrete and the horizon is infinite.
- Infinitely lived households and a government populate the economy.
- All goods are perfectly divisible and fully perishable.
- Government issues perfectly divisible fiat money and bonds.
- Agents trade in alternating markets as in Lagos and Wright (2005).



# Actors

Unit mass of households with preferences recursively described by

$$\mathcal{U}_t = U(y_t) - \bar{y}_t + u(q_t) - \bar{q}_t + \beta\delta_t\mathcal{U}_{t+1}, \quad \beta \in (0, 1).$$

- Consumption  $y$  and production  $\bar{y}$  of a CM good.
- Consumption  $q$  and production  $\bar{q}$  of a DGM good.
- Idiosyncratic  $\delta_t$ , i.i.d. with  $\delta^I < 1 < \delta^P$ ,  $\mathbb{P}\{\delta = \delta^i\} = \pi^i$ , and  $\mathbb{E}\{\delta\} = 1$ .

Government:

- Active only in the CM.
- Monopoly on money and nominal bond issuance.
- Can levy lump-sum taxes.
- Does not observe types.

# Centralized market

Competitive market for CM goods (numeraire), money (price  $\phi_t$ ), and discount bonds (price  $\psi_t$ ).

Households choose an optimal asset portfolio.

- CM good acts as transferable utility.
- Value function is linear in the real value of asset holdings:  
 $W_t(m, b) = m + b + \bar{W}_t$ . [▶ Details](#)
- Optimal portfolio choices are the same across all households.

Government conducts policy:

- It controls money supply  $M_t$  and the face value of newly issued nominal bonds  $B_t$
- It levies real lump-sum taxes to satisfy its budget constraint

$$\tau_t = \phi_t(M_{t-1} + B_{t-1} - M_t) - \psi_t B_t.$$

# Decentralized goods markets I

Type-contingent value function

$$V_t^i(m, b) = \mathcal{L}^i(m) + \Delta_t^i + \beta\delta^i(m + b + \overline{W}_{t+1}), \quad i \in \{I, P\}.$$

$\Delta_t^i$  is value of becoming a seller and  $\mathcal{L}^i(m)$  captures the value of money as a payment instrument when becoming a buyer.

- $\hat{m}^i$  is a type-contingent satiation level rendering liquidity constraints slack.
- $\mathcal{L}$  is increasing until the satiation level:  $\mathcal{L}_m^i(m) \geq 0$ , with " $>$ " iff  $m < \hat{m}^i$ .
- $\mathcal{L}$  increases at a decreasing rate:  $\mathcal{L}_{mm}^i(m) < 0$  for  $m < \hat{m}^i$ .

## Assumption

*Impatient households need higher real money balances than patient households to have slack liquidity constraints;  $\hat{m}^I > \hat{m}^P$ .*

## Decentralized goods markets II

Social surplus of the money balances as a payment instrument is

$$\mathcal{L}^i(m)/\theta^i(m), \quad \text{where } \theta^i(m) \in (0, 1].$$

- $\theta^i(m)$  captures the household's share of social surplus.

### Assumption

*The social surplus  $\mathcal{L}^i(m)/\theta^i(m)$  is increasing in  $m$ .*

# OTC financial market

Matches between households with  $\delta^I$  and  $\delta^P$ , with transaction  $(l, a)$ .

$$\begin{aligned}\mathcal{F}_{IP} &= [\mathcal{L}^P(l + m) - \mathcal{L}^I(m)] + [\mathcal{L}^P(m - l) - \mathcal{L}^P(m)] \\ &\quad + \beta(\delta^P - \delta^I)(a - l) \\ \text{s.t. } &-b \leq a \leq b \quad \text{and} \quad -m \leq l \leq m.\end{aligned}$$

- Re-distribution of savings across heterogeneous agents.
- Focus on proportional bargaining with shares  $\alpha^I$  and  $\alpha^P$ .

Probability of finding a match with the opposite type is  $\eta^i$ .

- The indirect liquidity of bonds is captured by  $\omega = \pi^I \eta^I = \pi^P \eta^P$ .

Value of entering the OTC market given by the concave function

$$O_t^i(m, b) = \eta^i \alpha^i \mathcal{F}_{ij}(m, b; m', b') + \mathcal{L}^i(m) + \Delta_t^i + \beta \delta^i (m + b + \overline{W}_{t+1}).$$

# Symmetric equilibrium

## Definition

Given  $\{M_t, B_t\}_{t=0}^{\infty}$ , a symmetric equilibrium are CM portfolio choices and prices  $\{m_t, b_t, \phi_t, \psi_t\}_{t=0}^{\infty}$  such that for all  $t \geq 0$ :

- 1 Households maximize utility.
- 2 Markets clear;  $m_t = \phi_{t+1}M_t$  and  $b_t = \phi_{t+1}B_t$

Private surplus of an OTC<sub>t</sub> match is  $\mathcal{F}(m_t, b_t)$  and the externalities on sellers are captured by  $\mathcal{E}(m_t, b_t)$ . [▶ Details](#)

## Lemma

Utilitarian welfare satisfies the recursive relationship

$W_t = \mathcal{W}(m_t, b_t) + U(y^*) - y^* + \beta W_{t+1}$ , where

$$\mathcal{W}(m, b) = \omega[\mathcal{F}(m, b) + \mathcal{E}(m, b)] + \pi^I \frac{\mathcal{L}^I(m)}{\theta^I(m)} + \pi^P \frac{\mathcal{L}^P(m)}{\theta^P(m)}.$$

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# Stationary equilibria and DGM trade

Focus on stationary policies  $\langle \gamma, \mathcal{B} \rangle$  in a stationary equilibrium

- $\gamma$  is the growth rate of money supply,
- $\mathcal{B} = b/m$  is the bonds-to-money ratio.

Let  $i^f = (\gamma - \beta)/\beta$  denote the Fisher rate and  $i^b$  the nominal return on bonds.

## Lemma

*All  $(m, b)$  can be implemented as a stationary equilibrium.*

- $m$  is continuous in  $\gamma$ .
- $\hat{m}^I \geq m \Leftrightarrow \gamma = \beta$  and  $\hat{m}^I < m \Leftrightarrow \gamma < \beta$ .
- $0 = i^b = i^f \Leftrightarrow \gamma = \beta$  and  $0 < i^b \leq i^f \Leftrightarrow \gamma > \beta$ .

Away from the FR,

- nominal rates are strictly positive;
- at least some liquidity constraints for impatient households are tight.



# Stationary equilibria and OTC trade

## Lemma

- $l = a = 0 \Leftrightarrow m \geq \hat{m}^I$ .
- *When  $(\hat{m}^I + \hat{m}^P)/2 \leq m < \hat{m}^I$  and  $b$  is sufficiently large, then  $a > l = \hat{m}^I - m$  so that  $l + m = \hat{m}^I$  and  $m - l \geq \hat{m}^P$ .*

At the FR, OTC trade vanishes.

For small deviations from the FR, impatient households are not yet satiated with liquidity.

Impatient households sell bonds at a discount to patient households, leading to

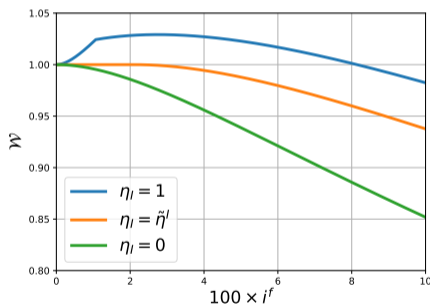
- a more efficient distribution of payment instruments;
- a more efficient distribution of savings instruments.

Mass  $\pi(1 - \eta^I)$  of households face binding liquidity constraints in the DGM.

# Sub-optimality of the Friedman rule

## Proposition

There exists an  $\tilde{\omega}$  such that the FR is sub-optimal iff  $\tilde{\omega} > \omega$  and  $b > 0$ .



If  $\omega > \tilde{\omega}$ , endogenous optimal coexistence of money and interest-bearing bonds.

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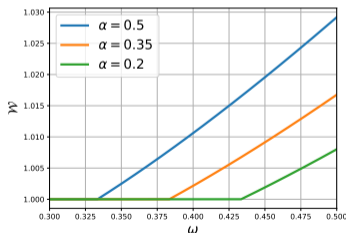
# Indirect liquidity

## Proposition

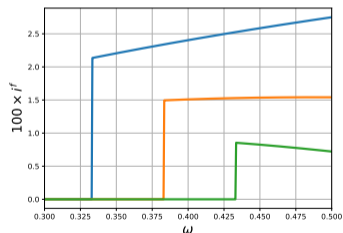
When policy is chosen optimally,

- welfare is strictly increasing in  $\omega$  if  $\omega \geq \tilde{\omega}$ ;
- welfare is independent of small changes in  $\omega$  if  $\omega < \tilde{\omega}$ .

The effect of  $\omega$  on optimal policy is theoretically ambiguous.



(a) Optimized welfare.



(b) Optimized nominal rate.

# Direct liquidity

Extended model with notes to study the direct liquidity of assets:

- Notes can be transacted in all OTC meetings.
- Notes can be transacted in a fraction  $\chi$  of DGM meetings.
- $\chi$  captures the direct liquidity of notes.

## Proposition

*In an economy with money and bonds, notes are inessential.*

## Proposition

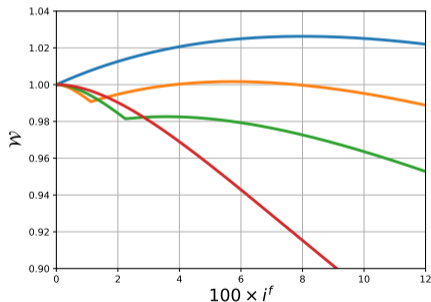
- *In an economy with money and notes, there exists a critical threshold  $\tilde{\omega}_\chi$  to render the FR sub-optimal.*
- *In an economy with money and notes, welfare is globally decreasing in  $\chi$ .*

▸ Details

# Walrasian financial market

Market clearing nominal rate  $l_t$ .

The possibility of a liquidity trap ( $l = 0$  while  $i^f > 0$ ) changes the qualitative effects of deviating from the FR.



The FR is still sub-optimal when the financial market is well-developed.

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# Conclusion

A theory that explains coexistence of money and interest-bearing bonds which incorporates:

- a role for the distribution of savings and payment instruments across HHs;
- optimally determined policies, abstracting from tax considerations.

Zero nominal rates maximize efficiency in goods markets but undermine financial markets' ability to provide insurance against preference shocks.



# Centralized market

Households choose the real amount of money and bonds they carry into the OTC.

$$W_t(m_{-1}, b_{-1}) = \max_{y, \bar{y}, m, b} \left\{ U(y) - \bar{y} + \pi^I O_t^I(m, b) + \pi^P O_t^P(m, b) \right\}$$

s.t.  $y + \tau_t + [\phi_t m + \psi_t b] / \phi_{t+1} \leq \bar{y}_t + m_{-1} + b_{-1}.$

- With  $y^* : U'(y^*) = 1$  sufficiently large, the non-negativity constraint on  $\bar{y}$  is slack so that

$$W_t(m_{-1}, b_{-1}) = \max_{\{m, b\} \geq 0} \left\{ -[\phi_t m + \psi_t b] / \phi_{t+1} + \sum_{i \in \{I, P\}} \pi^i O_t^i(m, b) \right\}$$

$+ m_{-1} + b_{-1} + U(y^*) - y^* - \tau_t.$

# Matches in the DGM

Surplus of a match between a buyer with  $\delta^i$  and a seller with  $\delta^j$ :

$$\mathcal{S}_{ij} = u(q) - q + \beta(\delta^j - \delta^i)p, \quad \text{s.t. } p \leq m_i.$$

Price protocol  $v_{ij} : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+^2$  maps  $q$  into  $p$ —the value of money, expressed in CM  $t + 1$  goods, transferred from the buyer to the seller.

$$q = \begin{cases} v_{ij}^{-1}(m_i) & \text{if } m_i < v_{ij}(\hat{q}_{ij}) \\ \hat{q}_{ij} & \text{if } m_i \geq v_{ij}(\hat{q}_{ij}) \end{cases}, \quad \hat{q}_{ij} : u'(\hat{q}_{ij}) = \beta\delta^I v'_{ij}(\hat{q}_{ij})$$

## Assumption

$u'(q)/v'(q)$  is decreasing in  $q$ .

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## Surplus of potentially becoming a seller

Surplus of potentially becoming a seller in the DGM— $\Delta_t^i$ —is given by

$$\Delta_t^i = \pi^I \int \frac{[1 - \theta^I(m')]\mathcal{L}^I(m)}{\theta^I(m')} dG_t(m'|\delta^I) \\ + \pi^P \int \frac{[1 - \theta^P(m')]\mathcal{L}^P(m)}{\theta^P(m')} dG_t(m'|\delta^P).$$

- $G_t(\cdot|\delta^j)$  is the conditional CDF of money holdings in  $DGM_t$ .

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# Welfare contribution of OTC matches

In a symmetric equilibrium, private surplus of an  $OTC_t$  match is

$$\mathcal{F}(m_t, b_t) = [\mathcal{L}^I(l_t + m_t) + \mathcal{L}^I(m_t)] + [\mathcal{L}^P(m_t - l_t) + \mathcal{L}^P(m_t)].$$

The effect on the surplus of sellers;

$$\mathcal{E}(m_t, b_t) = \left[ \frac{[1 - \theta^I(l_t + m_t)]\mathcal{L}^I(l_t + m_t)}{\theta^I(l_t + m_t)} - \frac{[1 - \theta^I(m_t)]\mathcal{L}^I(m_t)}{\theta^I(m_t)} \right] + \left[ \frac{[1 - \theta^P(m_t - l_t)]\mathcal{L}^P(m_t - l_t)}{\theta^P(m_t - l_t)} - \frac{[1 - \theta^P(m_t)]\mathcal{L}^P(m_t)}{\theta^P(m_t)} \right].$$

# Conditions

## Condition

*At the margin, liquid assets are more valuable for impatient than for patient agents:  $\mathcal{L}_m^I(m) \geq \mathcal{L}_m^m, \forall m \geq 0$ .*

## Condition

*Surplus of sellers in a DGM match is increasing in consumption by buyers:  $\mathcal{L}^I(m)[1 - \theta^i(m)]/\theta^i(m)$  is increasing in  $m$  for  $m \in [0, \hat{m}^i]$  and  $i \in \{I, P\}$ .*

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