

# Optimal Fiscal Policy in a Second-Best Climate Economy Model with Heterogeneous Agents

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# Motivation

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- Two key challenges for the 21<sup>st</sup> century: economic inequality and environmental degradation.
- Economists' favored solution: fiscal policy.
- If these two issues are linked, environmental and redistributive instruments should be determined jointly.
- Two important questions:
  - Do inequalities call for more/less stringent environmental policies?
  - Do environmental policies increase/decrease inequalities?

# What we do

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- Introduce a dynamic second-best climate-economy model with heterogeneous agents (HA).
  - Extension of [Barrage \(2019\)](#)'s representative agent's model.
- Solve Ramsey planner's problem to determine optimal linear taxes on labor, capital and pollution.
- Calibrate to Nordhaus' DICE model to study optimal carbon taxes.
- Examine several policy scenarios and multiple sources of households' heterogeneity.

→ Contributes to literature on 1) optimal pollution taxation and 2) household heterogeneity in environmental economics. [▶ See more](#)

# Model

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- **Households:** heterogeneous in productivity  $e_i$ , experience utility from consumption  $c$ , labor  $h$ , and the environment  $Z$ . [▶ See more](#)
- **Firms:** two sectors, final good uses energy produced in second sector. [▶ See more](#)
- **Abatement:** energy production pollutes, costly abatement  $\Theta(\mu, E)$ . [▶ See more](#)
- **Pollution:** stock depends on history of emissions. [▶ See more](#)
- **Government:** finances expenses  $G_t$  and transfers  $T_t$  using taxes on labor and capital income ( $\tau_H$  and  $\tau_K$ ), energy ( $\tau_I$ ), and pollution ( $\tau_E$ ). [▶ See more](#)

→ Can then define a competitive equilibrium and set the Ramsey planner problem. [▶ Definition](#)

# Ramsey problem

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Let  $\lambda \equiv \{\lambda_i\}$  be the planner's welfare weight. Ramsey planner problem:

$$\max_{\{c_t, H_{1,t}, H_{2,t}, K_{1,t}, K_{2,t}, E_t, Z_t, \mu_t\}_{t=0}^{\infty}, T, \varphi} \sum_{t,i} N_t \beta^t \pi_i \lambda_i u(c_{i,t}^m(c_t, h_t; \varphi), h_{i,t}^m(c_t, h_t; \varphi), Z_t)$$

subject to

$$U_{c,0}(R_0 N_0 a_{i,0} + T) \geq \sum_{t=0}^{\infty} N_t \beta^t \left( U_{c,t} c_{i,t}^m(c_t, h_t; \varphi) + U_{h,t} e_i h_{i,t}^m(c_t, h_t; \varphi) \right), \quad \forall i,$$

$$N_t c_t + G_t + K_{t+1} + \Theta_t(\mu_t, E_t) = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t) + (1 - \delta) K_t,$$

$$E_t = A_{2,t} G(K_{2,t}, H_{2,t}),$$

$$Z_t = J(S_0, E_0^M, \dots, E_t^M, \eta_0, \dots, \eta_t),$$

$$K_{1,t} + K_{2,t} = K_t,$$

$$H_{1,t} + H_{2,t} = N_t h_t.$$

First constraint: implementability condition, derived using method in

Werning (2007). [▶ See more](#)

# Optimal taxes

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- Optimal income taxes as in [Werning \(2007\)](#), optimal energy tax as in [Barrage \(2019\)](#). [▶ See more](#)
- First best pollution tax: equal to social cost of externality, Pigouvian principle:

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j}}{V_{c,t}} D_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M, t+j}.$$

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- Second best pollution tax, modified Pigouvian rule that accounts for marginal costs of public funds (MCF):

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j} + \text{cov}(\theta_i; IC_{c,i,t+j})}{V_{c,t} + \text{cov}(\theta_i; IC_{c,i,t})} D_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t} + \text{cov}(\theta_i; IC_{c,i,t})} \right) J_{E_t^M, t+j},$$

$$\text{with } \text{MCF}_t = 1 + \frac{\text{cov}(\theta_i; IC_{c,i,t})}{V_{c,t}}.$$

# Comparison with First Best

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$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j} + \text{cov}(\theta_i; IC_{c,i,t+j})}{V_{c,t} + \text{cov}(\theta_i; IC_{c,i,t})} D_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t} + \text{cov}(\theta_i; IC_{c,i,t})} \right) J_{E_t^M, t+j},$$

The 2<sup>nd</sup>-best tax may differ from 1<sup>st</sup>-best for three reasons:

- **Tax distortions:**
  - under some conditions, covariance always null;
  - main specification: fluctuates around 0. [▶ Plot MCF](#)
- **Distribution of individual allocations:**
  - $V_{c,t} = \sum_i \pi_i \lambda_i \frac{c_{i,t}}{c_t} u_{c,i,t}$ ;
  - inequalities reduce the tax iff IES < 1.
- Path of aggregate variables.



# Calibration setting

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- We apply our framework to the taxation of carbon.
- Climate model: DICE 2016 ([Nordhaus, 2017](#)).
- Economic and fiscal model: calibrated to the U.S., scaled up to match global GDP and emissions.
- Thought experiment: optimal fiscal policy of the U.S. if they internalize externalities abroad and assume ROW behaves identically.

▶ Climate model

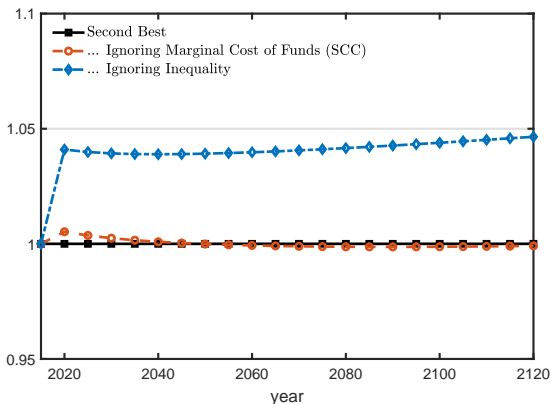
▶ Climate damages

▶ Households

▶ Production

▶ Government

# Carbon tax decomposition



▶ Link with MCF

▶ Plot tax levels

- Second best tax (black) almost equal to the SCC (red) → tax distortions do not justify significant deviations from Pigou.
- Without inequalities (blue), SCC  $\approx$  4% higher.
- With higher damages: similar figures. ▶ High damages scenario

# Comparison with climate-skeptic planner

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- How does carbon taxation affect the economy?
- We compare outcome of the optimal policy with policy of “climate-skeptic” planner.
- Sets policies optimally taking path of  $Z$  as given (exogenous climate change).

# Government budget adjustments

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	Revenue Source			Revenue Use		
	Labor	Capital	Carbon	Gov. Cons.	Transfer	Interest
No Carbon Tax	33.5%	0.6%	0.0%	17.2%	14.6%	2.3%
Optimal Carbon Tax	32.9%	0.6%	1.0%	17.1%	15.1%	2.3%
Change	<b>-0.6%</b>	0.0%	1.0%	-0.1%	<b>0.5%</b>	0.0%

*Note:* Numbers represent the present value of each component of the government budget constraint divided by the present value of GDP.

- Carbon tax revenue about equally split between increasing transfers and reducing labor income tax.

# Welfare gains

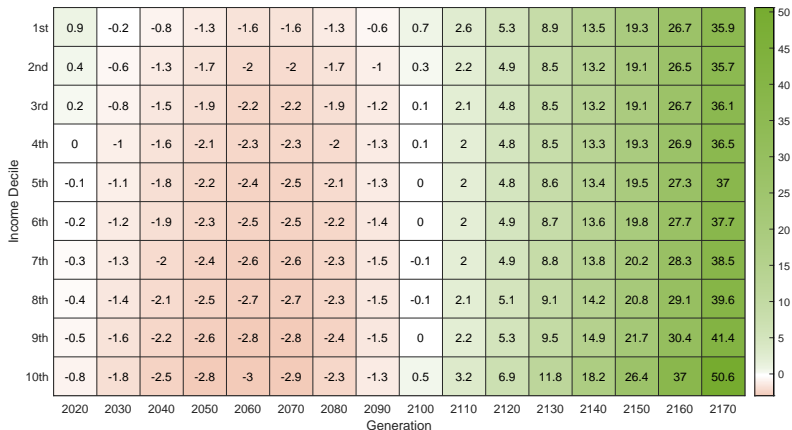


Figure: Period Welfare Gains (%)

- Negative and progressive welfare effects before 2100, positive and regressive after.

# Extensions

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We examine the following:

- Third best policies: fixed capital/labor tax
- Initial asset heterogeneity.
- Two goods, the most polluting being a necessity.
- Heterogeneous preferences for the dirtiest good, heterogeneous exposure to environmental damages.
- Alternative preferences for the planner.

## Conclusion – Key findings

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- **Theoretically**, second-best pollution tax is a modified Pigouvian rule that accounts for the marginal cost of funds (MCF)...
- ...which fluctuates around 1 in the optimal tax system, pushing the tax temporarily above/below the Pigouvian level.
- Inequalities matter: reduce Pigouvian tax iff IES below 1.
- **Quantitatively**, MCF plays a negligible role, but inequalities reduce the tax by 4% in the baseline.
- Carbon tax revenue optimally divided about equally between increasing transfers and reducing labor income taxes.
- Welfare effects from carbon taxation mostly negative and progressive in the 21<sup>st</sup> century, positive and regressive after.

# Supplementary Material



# Contribution – Optimal pollution tax

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- [Pigou \(1920\)](#): pollution should be taxed at its social cost.
- Double dividend literature (e.g. [Sandmo, 1975](#); [Bovenberg and de Mooij, 1994](#); [Bovenberg and van der Ploeg, 1994](#)): distortionary taxes should be adjusted (downward) to account for MCF.
- [Barrage \(2019\)](#): generalizes previous result to richer DGE framework linked to DICE. Again, MCF calls for lower taxes.
- Papers accounting for HA (e.g. [Kaplow, 2012](#); [Jacobs and de Mooij, 2015](#); [Jacobs and van der Ploeg, 2019](#)): second best tax should be Pigouvian.

**This paper:** uses [Werning \(2007\)](#) to extend [Barrage \(2019\)](#) to HA. Jointly studies inequality and environmental issues + micro-found tax distortions.

## Contribution – HA climate models

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- Large literature on heterogeneous financial burden from pollution taxation.
- Carbon tax alone regressive on the use side ([Pizer and Sexton, 2019](#)), ambiguous impact on the source side (e.g. [Rausch et al., 2011](#); [Fullerton and Monti, 2013](#); [Goulder et al., 2019](#); [Känzig, 2021](#)).
- Distribution of gains highly depends on revenue recycling (e.g. [Williams et al., 2015](#); [Fried et al., 2018](#); [Goulder et al., 2019](#)).
- Also depends on distribution of environmental benefits (between regions, [Hassler and Krusell, 2012](#); [Krusell and Smith, 2015](#); between generations, [Leach, 2009](#); [Kotlikoff et al., 2021](#)).

**This paper:** Jointly studies economic and environmental impacts from optimal pollution taxation over time and between HA.

# Households

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- Continuum of households  $i$  of size  $\pi_i$  and total population  $N_t$ , with preferences over consumption ( $c$ ), labor ( $h$ ), and the environment ( $Z$ ):

$$\sum_{t=0}^{\infty} \beta^t N_t u(c_{i,t}, h_{i,t}, Z_t).$$

- For simplicity, we assume strict separability between ( $c, h$ ) and  $Z$ .
- Agents differ in two ways:
  - labor productivity  $e_i$ ;
  - initial asset holdings  $a_{i,0}$ .
- Agent  $i$  budget constraint:

$$\sum_{t=0}^{\infty} p_t N_t \left( c_{i,t} - (1 - \tau_{H,t}) w_t e_i h_{i,t} \right) \leq R_0 N_0 a_{i,0} + T.$$

# Final-good sector

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- Final good sector produces output ( $Y$ ) from capital ( $K$ ), labor ( $H$ ), and energy ( $E$ ) with constant returns to scale

$$Y_{1,t} = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t).$$

- $D(Z_t)$ : production damages from environmental degradation  $Z$ .
- First order conditions of the firm:

$$r_t = (1 - D(Z_t)) A_{1,t} F_{K,t}$$

$$w_t = (1 - D(Z_t)) A_{1,t} F_{H,t}$$

$$p_{E,t} = (1 - D(Z_t)) A_{1,t} F_{E,t}$$

# Energy sector

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- Energy sector produces energy ( $E$ ) from capital ( $K$ ) and labor ( $H$ ) with constant returns to scale

$$E_t = A_{2,t}G(K_{2,t}, H_{2,t})$$

- Energy production generates emissions  $E_t^M = (1 - \mu_t)E_t$ , with  $\mu_t$  fraction of pollution abated at total costs  $\Theta_t(\mu_t, E_t)$ .
- With  $\tau_I$  and  $\tau_E$  the energy and emission taxes, profits are

$$\Pi_t = (p_{E,t} - \tau_{I,t}) E_t - \tau_{E,t}(1 - \mu_t) E_t - w_t H_{2,t} - r_t K_{2,t} - \Theta_t(\mu_t, E_t)$$

- First order conditions:

$$r_t = (p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t}) A_{2,t} G_{K,t}$$

$$w_t = (p_{E,t} - \tau_{I,t} - \tau_{E,t}(1 - \mu_t) - \Theta_{E,t}) A_{2,t} G_{H,t}$$

$$\tau_{E,t} = \frac{\Theta_{\mu,t}}{E_t}$$

# Government

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- Government spending: exogenous expenses  $G_t$ , lump-sum transfers  $T_t$ .
- Government revenue: proportional income taxes on capital  $\tau_{K,t}$  and labor  $\tau_{H,t}$ , energy taxes  $\tau_{I,t}$ , emissions taxes  $\tau_{E,t}$ , and profit taxes  $\tau_{\pi,t}$ .
- Simplifying assumption: profits from energy sector (if any) taxed at confiscatory rate:  $\tau_{\pi,t} = 1$ .
- Government's intertemporal budget constraint

$$R_0 B_0 + T + \sum_t p_t G_t = \sum_t p_t \left( \tau_{H,t} w_t H_t + \tau_{K,t} (r_t - \delta) K_t + \tau_{I,t} E_t + \tau_{E,t} E_t^M + \Pi_t \right).$$

# Environmental degradation

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- Environmental variable affected by history of pollution emissions  $E_t^M$ , exogenous shifters  $\eta_t$ , and initial conditions  $S_0$ :

$$Z_t = J(S_0, E_0^M, \dots, E_t^M, \eta_0, \dots, \eta_t).$$

# Competitive equilibrium

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## Definition

Given a distribution of assets  $\{a_{i,0}\}$ , aggregate capital  $K_0$  and aggregate bond holdings  $B_0$ , a competitive equilibrium is a policy  $\{\tau_{H,t}, \tau_{K,t}, \tau_{I,t}, \tau_{E,t}, T_t\}_{t=0}^{\infty}$ , a price system  $\{p_t, w_t, r_t, p_{E,t}\}_{t=0}^{\infty}$  and an allocation  $\{(c_{i,t}, h_{i,t})_i, Z_t, E_t, K_{1,t}, K_{2,t}, K_{t+1}, H_{1,t}, H_{2,t}, H_t\}_{t=0}^{\infty}$  such that: (i) agents choose  $\{(c_{i,t}, h_{i,t})_i\}_{t=0}^{\infty}$  to maximize utility subject to their budget constraint taking policies and prices (that satisfy  $p_t = R_{t+1}p_{t+1}$ ) as given; (ii) firms maximize profits; (iii) the government's budget constraint holds; (iv) markets clear.

▶ Market clearing conditions

◀ Back



# Characterization problem

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- Linear tax rates: all agents face same MRS between consumption and leisure  $\rightarrow$  individual allocations efficient *given* aggregates.
- Following [Werning \(2007\)](#), denote by  $\varphi \equiv \{\varphi_i\}$  a set of *market* weights.
- Characterize individual allocations from aggregates by solving the following static sub-problem for each period  $t$ :

$$U(c_t, h_t, Z_t; \varphi) \equiv \max_{c_{i,t}, h_{i,t}} \sum_i \pi_i \varphi_i u(c_{i,t}, h_{i,t}, Z_t),$$

s.t.  $\sum_i \pi_i c_{i,t} = c_t$  and  $\sum_i \pi_i e_i h_{i,t} = h_t.$

- Obtain implementability conditions based on aggregates and market weights only. For all  $i$ ,

$$U_{c,0}(R_0 N_0 a_{i,0} + T) \geq \sum_{t=0}^{\infty} N_t \beta^t \left( U_{c,t} c_{i,t}^m(c_t, h_t; \varphi) + U_{h,t} e_i h_{i,t}^m(c_t, h_t; \varphi) \right).$$

▸ Derivation details

◀ Back

# Climate model

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- $S^{At}$ ,  $S^{Up}$ , and  $S^{Lo}$  represent **carbon concentration** in the atmosphere, upper oceans and biosphere, and deep oceans. They evolve according to:

$$S_{j,t} = b_{0,j}(E_t^M + E_t^{\text{land}}) + \sum_{i=1}^3 b_{i,j}S_{i,t-1},$$

- Atmospheric carbon concentration increases **radiative forcing**, *i.e.* the net radiation received by the earth:

$$\chi_t = \kappa(\ln(S_t^{At}/S_{1750}^{AT})/\ln(2)) + \chi_t^{\text{ex}}.$$

- **Mean temperature** of atmosphere ( $Z_t^{At}$ ) and deep oceans ( $Z_t^{Lo}$ ) determined by

$$Z_t^{At} = Z_{t-1}^{At} + \zeta_1(\chi_t - \zeta_2 Z_{t-1}^{At} - \zeta_3(Z_{t-1}^{At} - Z_{t-1}^{Lo})),$$

$$Z_t^{Lo} = Z_{t-1}^{Lo} + \zeta_4(Z_{t-1}^{At} - Z_{t-1}^{Lo}).$$

- All parameters taken from DICE 2016.

# Climate damages

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- We also model production damages as in DICE 2016, with

$$D(Z_t) = a_1 Z_t^{A_t} + a_2 (Z_t^{A_t})^{a_3}.$$

- We follow [Barrage \(2019\)](#) and split damages between production and utility: 74% of damages at +2.5°C assigned to output, 26% to utility.
- We also consider an alternative “high damages” scenario: same damage at current warming, but cubic instead of quadratic exponent.

# Households

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- We take the per period utility function from [Barrage \(2019\)](#)

$$u(c_i, h_i, Z) = \frac{(c_i(1 - \varsigma h_i)^\gamma)^{1-\sigma}}{1 - \sigma} + \frac{(1 + \alpha_0 Z^2)^{-(1-\sigma)}}{1 - \sigma}.$$

- We follow DICE and set  $\beta = 1/1.015$  and  $\sigma = 1.45$ .
- $\gamma$  and  $\varsigma$  set to match Frisch elasticity of 0.75 ([Chetty et al, 2011](#)) and average labor supply of 0.277 (Survey of Consumer Finances, 2013).
- Joint distribution of productivity and wealth computed from hourly wages and net worth data from SCF 2013. [▶ See table](#)
- Population growth: follow DICE. Population level: match U.S. GDP per capita.

# Production

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- Cobb-Douglas technology for both sectors

$$F(K_{1,t}, H_{1,t}, E_t) = K_{1,t}^\alpha H_{1,t}^{1-\alpha-\nu} E_t^\nu$$

with  $\alpha = 0.3$ , and  $\nu = 0.04$  (Golosov et al, 2014), and

$$G(K_{2,t}, H_{2,t}) = K_{2,t}^{1-\alpha_E} H_{2,t}^{\alpha_E}$$

with  $\alpha_E = 0.403$  (Barrage, 2019).

- Abatement cost function taken from DICE

$$\Theta(\mu_t, E_t) = c_{1,t} \mu_t^{c_2} E_t,$$

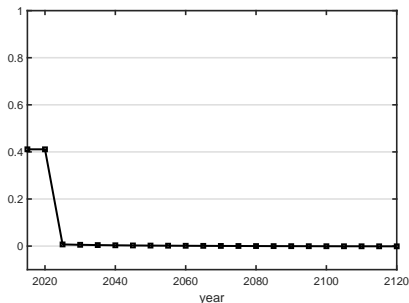
where  $c_{1,t} c_2 = P_t^{\text{backstop}}$  the backstop price calibrated as in DICE 2016.

# Government

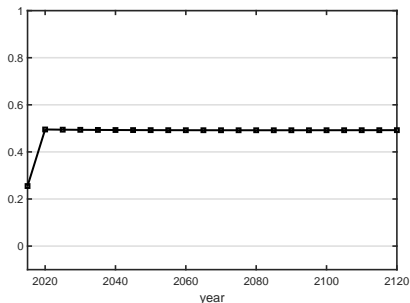
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- Tax rates on capital and labor income taken from [Trabandt and Uhlig \(2012\)](#). For the US:
  - capital income tax rate:  $\tau_K = 41.1\%$ ;
  - labor income tax rate:  $\tau_H = 25.5\%$ .
- As in [Barrage \(2019\)](#), set  $\tau_I = 0$ , but follow DICE 2016 to set  $\tau_E = \$2.01/\text{tCO}_2$  such that  $\mu_0 = 3\%$ .
- Debt (2011-15 average): difference between total liabilities and financial assets from the U.S. government's balance sheet:  $B_0/Y_{1,0} = 110\%$ .
- U.S. government expenses from IMF (2011-15 average): consumption is  $G_0^C/Y_{1,0} = 15.8\%$ , transfers are  $G_0^T/Y_{1,0} = 14.5\%$ .

# Optimal income taxes



(a) Optimal Capital-Income Taxes



(b) Optimal Labor-Income Taxes

- Optimal capital income tax quickly converges to 0.
- Optimal labor income tax quickly converges to 49%.
- With higher damages: same figures. [▶ High damages scenario](#)

# Optimal carbon tax

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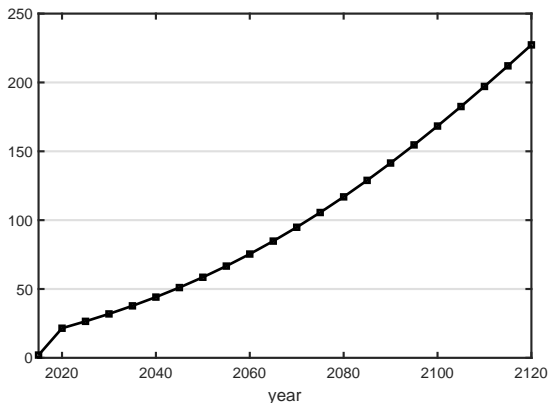


Figure: Optimal Carbon Taxes (\$/tCO<sub>2</sub>)

- Similar path to DICE. High damages scenario: close to 3 times higher.

▸ High damages scenario

◀ Back



# Market clearing conditions

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- Final good resource constraint:

$$N_t c_t + G_t + K_{t+1} + \Theta_t(\mu_t, E_t) = (1 - D(Z_t)) A_{1,t} F(K_{1,t}, H_{1,t}, E_t) + (1 - \delta) K_t.$$

- Energy resource constraint:

$$E_t = A_{2,t} G(K_{2,t}, H_{2,t}).$$

- Labor and capital market clearing:

$$K_{1,t} + K_{2,t} = K_t,$$

$$H_{1,t} + H_{2,t} = N_t h_t.$$

- Environmental constraint:

$$Z_t = J(S_0, E_0^M, \dots, E_t^M, \eta_0, \dots, \eta_t).$$

# Derivation implementability conditions

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- Household  $i$  budget constraint:

$$\sum_{t=0}^{\infty} p_t N_t \left( c_{i,t} - (1 - \tau_{H,t}) w_t e_i h_{i,t} \right) \leq R_0 N_0 a_{i,0} + T.$$

- Applying the envelope theorem to the characterization problem and using consumers' first order conditions:

$$\frac{U_{h,t}}{U_{c,t}} = \frac{u_{h,i,t}}{u_{c,i,t} e_i} = -w_t (1 - \tau_{H,t}),$$

$$\frac{U_{c,t}}{U_{c,0}} = \frac{u_{c,i,t}}{u_{c,i,0}} = \frac{p_t}{\beta^t}.$$

- Substituting:

$$U_{c,0} (R_0 N_0 a_{i,0} + T) \geq \sum_{t=0}^{\infty} N_t \beta^t \left( U_{c,t} c_{i,t}^m(c_t, h_t; \varphi) + U_{h,t} e_i h_{i,t}^m(c_t, h_t; \varphi) \right).$$

# Definitions

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We define the pseudo-utility function as

$$W(c_t, h_t, Z_t; \varphi, \theta, \lambda) \equiv V(c_t, h_t, Z_t; \varphi, \lambda) + \sum_i \pi_i \theta_i IC_i(c_t, h_t, \varphi),$$

with

$$V(c_t, h_t, Z_t; \varphi, \lambda) \equiv \sum_i \pi_i \lambda_i u(c_{i,t}^m(c_t, h_t; \varphi), h_{i,t}^m(c_t, h_t; \varphi), Z_t),$$

the aggregate utility based on the planner's weights,

$$IC_i(c_t, h_t, \varphi) \equiv U_{c,t} c_{i,t}^m(c_t, h_t; \varphi) + U_{h,t} e_i h_{i,t}^m(c_t, h_t; \varphi),$$

the difference between agent  $i$  consumption and labor income in period  $t$ , and  $MIC_{i,t} \equiv (\partial IC_{i,t} / \partial c_t)$ .

## Link with MCF

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Define the marginal cost of funds (MCF) as

$$\begin{aligned}\text{MCF}_t &\equiv \frac{\nu_{1,t}}{V_{c,t}} \\ &= \frac{W_{c,t}}{V_{c,t}} = 1 + \frac{\sum_i \pi_i \theta_i \text{MIC}_{i,t}}{V_{c,t}}.\end{aligned}$$

We can also write the ratio of MCFs as

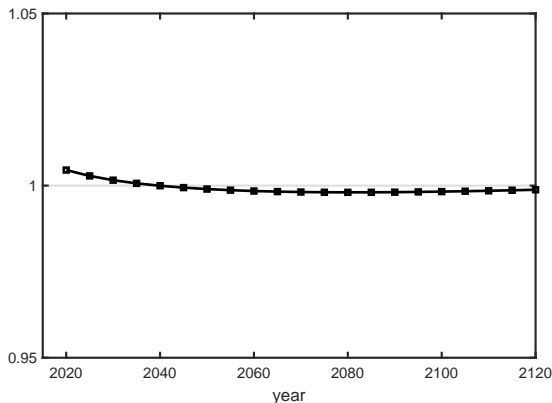
$$\frac{\text{MCF}_{t+j}}{\text{MCF}_t} = \prod_{k=1}^j \frac{R_{t+k}}{R_{t+k}^*}.$$

Denote  $\Delta_{t+s}$  the share of marginal production damages at  $t+s$  due to marginal change in emissions at  $t$ . Then

$$\tau_{E,t} = \sum_{j=0}^{\infty} \frac{\text{MCF}_{t+j}}{\text{MCF}_t} \Delta_{t+j} \tau_{E,t}^{\text{Pigou}, Y} \Big|_{SB} + \frac{\tau_{E,t}^{\text{Pigou}, U} \Big|_{SB}}{\text{MCF}_t}.$$

# Marginal Cost of Funds (MCF)

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- MCF is on average 1, small temporary deviations.
- Second best tax follows similar trajectory relative to Pigouvian level.

[◀ Back to formulas](#)

[◀ Back to graph](#)

# Optimal income tax formulas

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The general formulas (without functional form for utility) are

$$\tau_{H,t} = 1 - \frac{U_{h,t}}{U_{c,t}} \frac{W_{c,t}}{W_{h,t}},$$

and

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{W_{c,t+1}}{W_{c,t}} \frac{U_{c,t}}{U_{c,t+1}}.$$

# Optimal income tax formulas

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Using our functional form for utility we get

$$\tau_{H,t} = \frac{\Psi \varsigma (1 - \varsigma H_t)^{-1}}{\Phi + \Psi \varsigma (1 - \gamma (1 - \sigma)) (1 - \varsigma H_t)^{-1}},$$

and

$$\frac{R_{t+1}}{R_{t+1}^*} = \frac{\Phi - \Psi \varsigma \gamma (1 - \sigma) (1 - \varsigma H_{t+1})^{-1}}{\Phi - \Psi \varsigma \gamma (1 - \sigma) (1 - \varsigma H_t)^{-1}},$$

with

$$\Phi = \sum_j \pi_j \frac{\lambda_j}{\varphi_j} + (1 - (1 + \gamma)(1 - \sigma)) \text{cov}(\lambda_i / \varphi_i, \omega_i),$$
$$\Psi = - \frac{\text{cov}(\lambda_i / \varphi_i, e_i)}{\varsigma},$$

where  $\forall t, \omega_i = c_{i,t} / C_t$ .

# Productivity and asset distributions

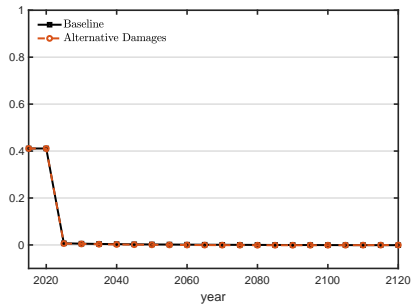
**Table:** Distribution of households hourly wages and net worth by productivity deciles (rows) and net worth deciles (columns), controlling for generational differences.

		Net worth deciles										
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	Hourly wage
Productivity deciles	1st	-4.59e+04	-7.00e+03	1.22e+03	7.45e+03	1.79e+04	3.25e+04	6.44e+04	1.12e+05	2.18e+05	1.10e+06	6.44e+00
	2nd	-2.99e+04	-1.97e+03	4.89e+03	1.23e+04	2.50e+04	3.97e+04	6.46e+04	1.03e+05	1.83e+05	1.04e+06	1.11e+01
	3rd	-4.13e+04	-6.00e+03	3.72e+03	1.29e+04	2.76e+04	4.47e+04	7.69e+04	1.09e+05	2.01e+05	7.19e+05	1.42e+01
	4th	-4.56e+04	-2.65e+03	1.44e+04	3.31e+04	5.38e+04	7.48e+04	1.01e+05	1.50e+05	2.67e+05	7.64e+05	1.73e+01
	5th	-4.94e+04	-2.15e+03	1.55e+04	3.58e+04	6.72e+04	9.53e+04	1.40e+05	2.07e+05	2.98e+05	1.10e+06	2.05e+01
	6th	-3.82e+04	1.21e+04	3.94e+04	7.26e+04	1.14e+05	1.60e+05	2.13e+05	2.88e+05	4.60e+05	1.75e+06	2.41e+01
	7th	-2.41e+04	3.79e+04	6.75e+04	1.03e+05	1.54e+05	2.06e+05	2.63e+05	3.58e+05	5.32e+05	1.23e+06	2.86e+01
	8th	-2.93e+04	3.00e+04	7.10e+04	1.34e+05	2.11e+05	2.80e+05	3.90e+05	5.04e+05	6.94e+05	2.57e+06	3.48e+01
	9th	4.38e+03	6.86e+04	1.44e+05	2.11e+05	3.07e+05	4.20e+05	5.53e+05	7.45e+05	1.08e+06	3.50e+06	4.47e+01
	10th	-8.53e+04	1.40e+05	2.77e+05	4.43e+05	6.38e+05	8.55e+05	1.29e+06	2.14e+06	3.45e+06	1.00e+07	1.01e+02

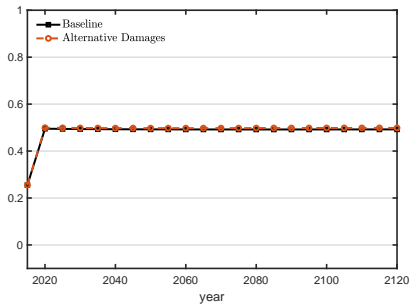


# Optimal income taxes alternative damages

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(a) Optimal Capital-Income Taxes



(b) Optimal Labor-Income Taxes

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# Optimal carbon tax alternative damages

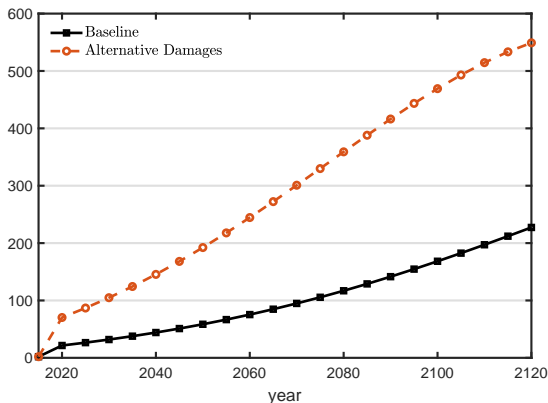
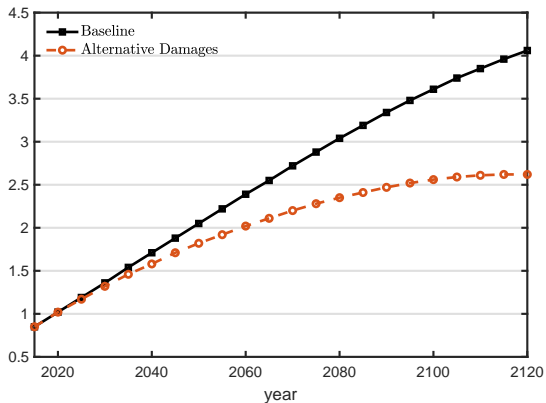


Figure: Optimal Carbon Taxes (\$/tCO<sub>2</sub>)

- Baseline peak temperature: +4.5°C
- High damages peak temperature: +2.5°C

# Temperature alternative damages

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# Tax formula

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- Second best tax:

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j} + \sum_i \pi_i \theta_i MIC_{i,t+j}}{V_{c,t} + \sum_i \pi_i \theta_i MIC_{i,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t} + \sum_i \pi_i \theta_i MIC_{i,t}} \right) J_{E_t^M, t+j}$$

- Second best tax ignoring MCF (SCC):

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M, t+j}$$

- Second best tax ignoring MCF and inequalities:

$$\tau_{E,t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{V_{c,t+j}}{V_{c,t}} D'_{t+j} A_{1,t+j} F_{t+j} - \frac{N_{t+j} V_{Z,t+j}}{V_{c,t}} \right) J_{E_t^M, t+j}$$

→ Same formula as before, different allocation.

# Carbon tax decomposition alternative damages

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