Heterogeneous Risk Exposure and the Dynamics of Wealth Inequality

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Overview

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- Understand what shapes the evolution of the wealth distribution

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  - Empirical: portfolio heterogeneity + asset price movements ⇒ wealth *dynamics*

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- Propose a model for the dynamics of inequality based on *heterogeneous exposure to aggregate risk* in asset returns

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This paper:

- Propose a model for the dynamics of inequality based on *heterogeneous exposure to aggregate risk* in asset returns
  - Generate large and persistent movements in inequality
  - Rationalize the observed evolution of US top wealth shares
    - Realized returns happened to be favorable for the portfolios of the wealthy

Introduction

Some Relevant Facts

Fact 1: Large low-frequency movements in wealth inequality

- Top 10% wealth share increased by $\approx$ 10p.p. in 25 years

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- Poor $\rightarrow$ Safe; Middle-class $\rightarrow$ Housing; Wealthy $\rightarrow$ Equity

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Fact 3: Rates of return systematically vary

a. Across asset classes
b. Over time

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Q: Can facts 2 and 3 help us explain fact 1?

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Contribution

- Develop a model of wealth inequality based on *optimal* portfolio choice

▸ Literature

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Literature

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  - Households choose portfolio shares in bonds, housing, and equity.
  - Housing as investment asset and necessary good crucial for portfolios
    - Investment: portfolio shares in housing
    - Non-homotheticity: portfolio shares in equity

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- Replicate observed portfolio heterogeneity

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- Replicate observed portfolio heterogeneity
- Model matches both level and dynamics of inequality:
  1. Increasing returns to wealth amplify the level of inequality
  2. Households are differently exposed to fluctuations in returns

Literature

Introduction

Preview of results

Aggregate risk in asset returns

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   - Feeding realized returns replicates increase in top shares

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Preview of results

Aggregate risk in asset returns

1. Has large and persistent effects on inequality
2. Can explain rise in US top wealth shares:
   - Feeding realized returns replicates increase in top shares
   - Mainly driven by abnormal returns to equity

Outline

1. Model
2. Calibration
3. Results
4. Conclusion

Model

Setup

- Continuous time
- Households:
  - Die at constant rate ζ
  - Recursive preferences
  - Non-homothetic aggregator over consumption and housing
  - Trade financial assets (stocks and bonds) and illiquid housing
- Assets returns follow an exogenous process subject to aggregate shocks

Household

Preferences

Non-homothetic (addilog) intra-temporal utility:

$$u(c,n) = \left((1-\omega)\frac{1-\varepsilon_h^{-1}}{1-\varepsilon_c^{-1}} c^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + \omega n^{\frac{\varepsilon_h - 1}{\varepsilon_h}} \right)^{\frac{\varepsilon_h}{\varepsilon_h - 1}}$$

where n is consumption of housing services.

- Nests CES ($\varepsilon_h = \varepsilon_c$) and separable ($\varepsilon_h = \psi$) cases
- Non-homotheticity: $\varepsilon_h < \varepsilon_c \Rightarrow$ expenditure share of housing falls in total consumption (Wachter and Yogo 2010)

CEX - Housing Shares

Non-homotheticity

Implications

- **Poor** households have a larger share of consumption expenditure in housing:
  - Low EIS; high RRA
  - Heavily invest in bonds (illiquidity of housing makes it inadequate for consumption smoothing)

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  - Low EIS; high RRA
  - Heavily invest in bonds (illiquidity of housing makes it inadequate for consumption smoothing)
- **Rich** households on the other hand
  - High EIS; low RRA
  - Invest in stocks to reap the high return

CEX - Housing Shares

Household Problem

- Entry/Exit decision in both housing and equity markets

▸ Details

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- Households' individual states are $\mathbf{x} = (a, p, h, z)$:
  - Financial wealth a
  - Equity-market participation state p
  - Housing h (can also be rented)
  - Log earnings z
    - Follow an Ornstein-Uhlenbeck (AR-1) in logs

▸ Details

Distribution

Kolmogorov Forward Equation

PROPOSITION 1

The distribution of households over individual states, $g_t(\mathbf{x})$, solves the following KFE:

$$\mathrm{d}g_t(\mathbf{x}) = \left\{\mathcal{A}^* g_t(\mathbf{x}) + \zeta\left(\Psi(\mathbf{x}) - g_t(\mathbf{x})\right)\right\}\mathrm{d}t - \partial_{\mathbf{x}}\left\{\left[\boldsymbol{\sigma}_{\mathbf{x}}(\mathbf{x})\mathrm{d}\mathbf{W}_t\right] g_t(\mathbf{x})\right\}$$

where $\mathcal{A}^*$ is the adjoint of the HJB operator.

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- **Implications:** in the presence of aggregate risk, the evolution of g_t will depend on:
  1. The specific path of shocks $\mathbf{W}_t$
  2. Exposure to aggregate risk (through $\boldsymbol{\sigma}_{\mathbf{x}}(\mathbf{x})$)

Calibration

Calibration

Strategy

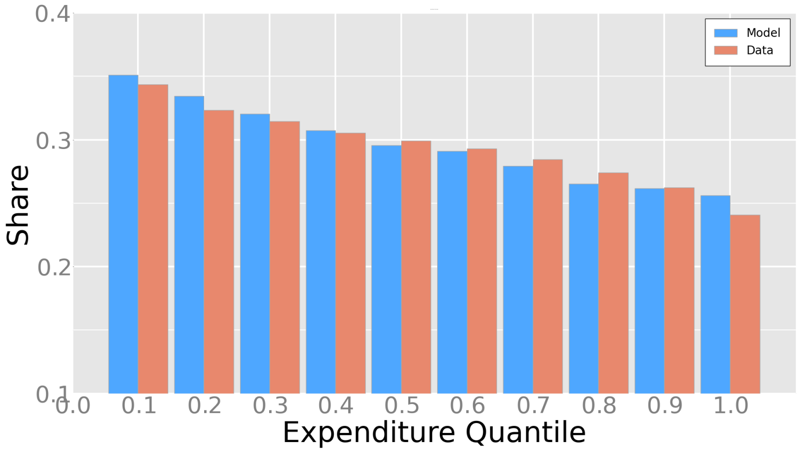
- Returns process directly from data
  - Return on bills and capital gains on housing from Jordà et al. (2019)
  - Equity returns from Kartashova (2014) (wgt. avg. of public and private)
- Use data on expenditure shares to pin down non-homotheticity
  - Target expenditure shares by expenditure decile
- Ask the model to match portfolio shares along wealth distribution
  - Target portfolio shares by wealth decile
  - Target avg. participation in equity and housing markets

Details

Model Match

Expenditure Shares

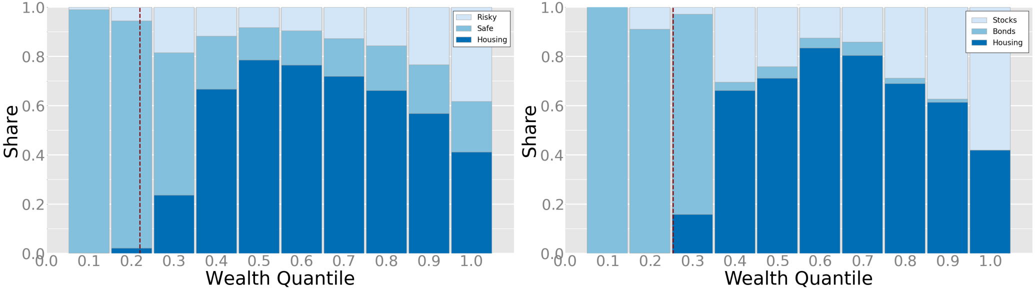
Figure 1: Expenditure share of housing



Model Match

Portfolio Shares

Figure 2: Portfolio shares



(a) Data (b) Model

- Matches main fact: bonds at the bottom, housing in the middle, equity at the top
- Missing some bond-holdings at the top

Data Definitions

Results

Results

Overview

- Model matches level of wealth inequality
  - Decomposition exercise shows that return heterogeneity accounts for 50%

▸ WI - Level

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  2. IRF to shocks in asset returns

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- Model matches level of wealth inequality
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- Highlight the role of heterogeneous exposure for the dynamics of inequality:
  1. Ergodic distribution
  2. IRF to shocks in asset returns
  3. Feed the realized sequence of returns into the model

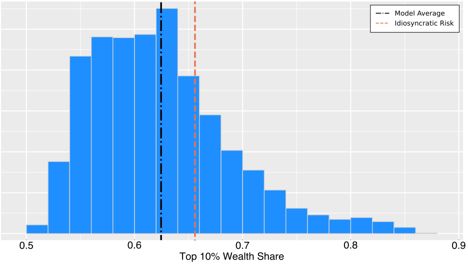
▸ WI - Level

Wealth Inequality

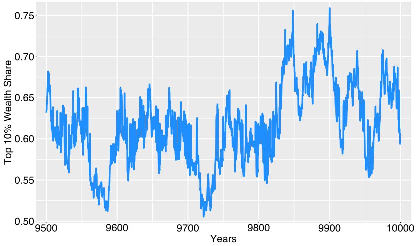
Distribution over time

- Plot top 10% wealth share along the ergodic distribution
  - Distribution is very disperse (st. dev. of top 10% share is 0.07)
  - Most of the time concentrated around mean, but long periods of high inequality

Figure 4: Wealth Inequality - Dynamics



(a) Ergodic Distribution



(b) Sample Path

▸ WI - Level

Wealth Inequality

Dynamics – Decomposition

- Compute IRF to a one-time 1% excess return in each asset
  - Equity shocks have a much larger, more persistent effect
    - A 1 s.d. shock to equity returns implies an increase in the top 10% share of 1.3 p.p.

Figure 6: IRF to 1% excess return

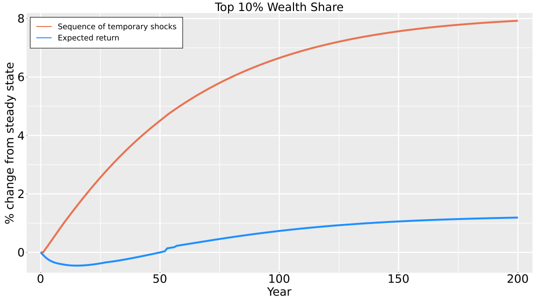
IRFs Temporary

Wealth Inequality

Dynamics – Return Changes

- Compute IRF to a 1% excess return in equity every period. Either:
  - Sequence of unexpected returns
  - Change in the equity premium

Figure 7: IRF to 1% excess return to equity



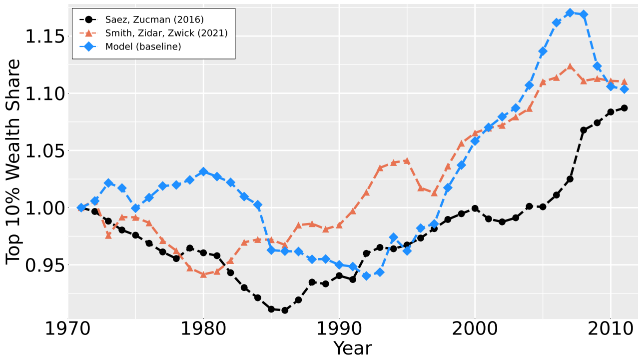
WI - Level

Wealth Inequality

Model vs. Data - Changes

- Feed the sequence of realized returns
  - The model generates all of the observed increase in wealth inequality

Figure 8: Top 10% wealth share – model and data



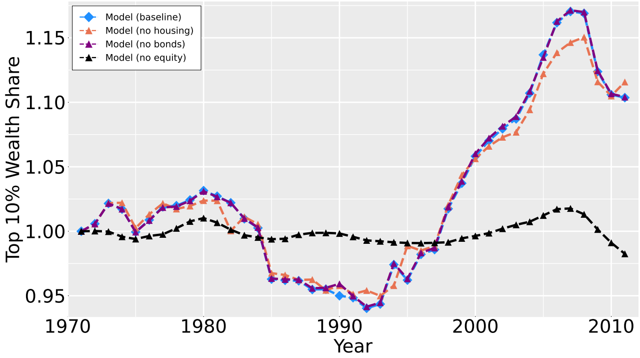
More

Wealth Inequality

Counterfactuals

- Keep returns to one asset at its historical average
  - Almost all of the increase was explained by returns to equity

Figure 9: Top 10% wealth share – counterfactuals

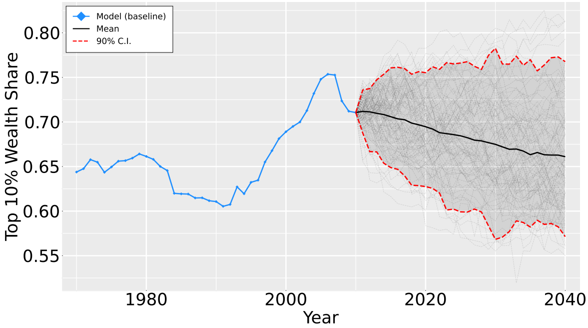


Wealth Inequality

Future Evolution

- Simulate 100 paths into the future
  - Inequality *slowly* reverts back to long-run average
  - Wide range of plausible realizations

Figure 10: Top 10% wealth share – Future



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Conclusion

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What I have done

- Model of portfolio choice consistent with observed behavior
  - Housing as a necessary good crucial to generate the correct equity shares
- Aggregate shocks in returns generates fluctuations in wealth inequality
  - Consistent with U.S. data; mostly driven by returns to equity

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Main takeaways:

- Heterogeneous exposure to aggregate risk crucial for inequality dynamics
- Increased inequality does not need structural changes

WI - Level

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Wealth Inequality and Asset Prices

- Asset returns are a fundamental determinant of wealth inequality
  - To understand inequality, need to understand prices

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Wealth Inequality and Asset Prices

- Asset returns are a fundamental determinant of wealth inequality
  - To understand inequality, need to understand prices
- Does wealth inequality matter for asset prices too?
- **Ongoing work**. Preliminary results suggest it does (under some conditions)
  - Increasing equity demand generates amplification from inequality to prices
  - Intuitively, a positive shock to dividend increases equity demand, which raises prices and further boosts demand (and inequality)

Introduction

Contribution to Literature

- **Portfolio Choice and Preference Heterogeneity:**
  - Meeuwis (2020), Wachter and Yogo (2010), Gomez (2019), and Vestman (2019)
  - Non-homotheticity: directly driven by housing, consistent with empirical evidence
- **Return Heterogeneity and Wealth Inequality:**
  - Theoretical: Benhabib and Bisin (2018), Gabaix et al. (2016), and Xavier (2020)
  - Empirical: Bach et al. (2020), Kuhn et al. (2020), and Martinez-Toledano (2020)
  - Endogenous portfolio heterogeneity and aggregate risk
- **Increased wealth inequality:**
  - Favilukis (2013), Hubmer et al. (2021), Greenwald et al. (2021), Gomez and Gouin-Bonenfant (2020), and Kacperczyk et al. (2019)
  - Focus on role of aggregate risk: *unexpected* return realizations

Contribution

Portfolio Shares

Data Definition

- Risky = Public Equity + Business Equity
  - Public Equity (total value of financal assets invested in stocks):
    - Directly held stocks
    - Stock mutual funds (includes proportion of mutual funds)
    - IRAs/Keoghs invested in stocks
    - Other managed assets with equity interest
    - Thrift-type retirement accounts invested in stocks
  - Business Equity (total value in which household has either active or non-active interest)
- Safe = Financial assets - Risky
  - Financial assets
    - Liquid assets; Certificates of deposit, Pooled investment funds
    - Stocks; Bonds; Savings bonds
    - Quasi-liquid assets; whole life insurance; other managed assets; other financial assets
- Housing = Primary residence + Other residential RE + Non-residential RE

Model

Details

- Returns are given by:

$$\mathrm{d}\mathbf{r}_t = \mathbf{r}\,\mathrm{d}t + \boldsymbol{\sigma}\,\mathrm{d}\mathbf{W}_t$$

  - $\mathbf{r}$: (r_B, r_S, r_H) expected returns
  - $\mathbf{W}_t$: $(W_{1,t}, W_{2,t}, W_{3,t})$ aggregate shocks
  - $\boldsymbol{\sigma}$: 3×3 matrix of sensitivities

Back

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- Entry/Exit decision in both housing and equity markets
  - Housing market frictions:
    - Entry/exit shock at rate λ^h, pay cost $\kappa^h(h)$ to buy/sell a house

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- Entry/Exit decision in both housing and equity markets
  - Housing market frictions:
    - Entry/exit shock at rate λ^h, pay cost $\kappa^h(h)$ to buy/sell a house
  - Equity market frictions:
    - Entry shock at rate λ_0^p, pay cost κ_0^p to enter the market
    - Exit shock at rate λ_1^p, pay cost κ_1^p to stay in the market

Simplified Problem

Equity Participation

- Abstract from housing, income risk and all other frictions
- Two participation states
  - State 0: can only invest in bonds; face entry shock at rate λ_0^p, can pay cost κ_0^p to enter the equity market
  - State 1: can invest in bonds and equity; face staying shock at rate λ_1^p, need to pay cost κ_1^p to stay in the equity market
- The household's HJB becomes:

$$\rho v_0(a) = \max_c u(c) + v_0'(a)(z + r_B a - c) + \\ + \lambda_0^p \left[\max\left\{v_1(a - \kappa_0^p), v_0(a)\right\} - v_0(a)\right]$$

$$\rho v_1(a) = \max_{c,\theta} u(c) + v_1'(a)(z + r_B a + (r_S - r_B)\theta a - c) + v_1''(a)\frac{(\theta a \sigma)^2}{2} + \\ + \lambda_1^p \left[\max\left\{v_1(a - \kappa_1^p), v_0(a)\right\} - v_1(a)\right]$$

Back

Simplified Problem

Housing Participation

- Abstract from equity markets and from both income- and return-risk
- Two participation states
  - Renter: own 0 housing and rent at rate r^h; face buying shock at rate λ^h, can pay cost $\kappa^h(h)$ to buy a house
  - Owner: face housing transaction costs and selling shock at rate λ^h, can pay cost $\kappa^h(h)$ to sell the house
- The household's HJB becomes:

$$\rho v(a,0) = \max_{c,n} \; u(c,n) + v_a(a,0)(z + r_B a - r^h n - c) +$$

$$+ \lambda^h \left[\max \left\{ \max_{a',h'} \; v(a',h'), v(a,0) \right\} - v(a,0) \right]$$

$$\text{s.t.} \quad a' + h' = a - \kappa^h(h')$$

$$\rho v(a,h) = \max_{c} \; u(c,\chi h) + v_a(a,h)(z + r_B a - c) + v_h(a,h) r_H h +$$

$$+ \lambda^h \left[\max \left\{ v(a,h), v\left(a + h - \kappa^h(h), 0\right) \right\} - v(a,h) \right]$$

Back

State variables

Evolution

$$
\begin{aligned}
da &= (z + r_B a + (r_S - r_B)\theta a - e + \mathbf{1}_{\{a<0\}}\kappa^b(1-\theta)a)dt \\
&\quad + (1-\theta)a\sigma_{1,1}dW_1 + \theta a(\sigma_{2,1}dW_1 + \sigma_{2,2}dW_2) \\
\frac{dh}{h} &= r_H dt + \sum_{i=1}^{3}\sigma_{3,i}dW_i \\
dz &= \eta_z(\bar{z} - z)\,dt + \sigma_z d\tilde{W}^z \\
a &\geq -\phi
\end{aligned}
$$

◂ Household

Hamilton Jacobi Bellman

The HJB equation is:

$$0 = \max \left\{ f(u(c,n),v) + \mathcal{A}v \right\}$$

where

$$\mathcal{A} = \mathbf{1}_{\{p=0\}} \left(\mathcal{L}^0 + \mathcal{P}^0 \right) + \mathbf{1}_{\{p=1\}} \left(\mathcal{L}^1 + \mathcal{P}^1 \right) + \mathbf{1}_{\{h=0\}} \mathcal{H}^0 + \mathbf{1}_{\{h\neq 0\}} \mathcal{H}^+ + \mathcal{Z} + \frac{\partial}{\partial t}$$

$$\mathcal{L}^0 v = \mu_a(\mathbf{x}) \frac{\partial}{\partial a} v$$

$$\mathcal{L}^1 v = \left(\mu_a(\mathbf{x}) \frac{\partial}{\partial a} + \sigma_a(\mathbf{x})^2 \frac{1}{2} \frac{\partial^2}{\partial a^2} + \rho_z \sigma_a(\mathbf{x}) \sigma_z(z) \frac{\partial}{\partial a \partial z} \right) v$$

$$\mathcal{P}^0 v = \lambda_0^p \left[\max \left\{ v(a - \kappa_0^p, h, z, 1-p), v(a,h,z,p) \right\} - v(a,h,z,p) \right]$$

$$\mathcal{P}^1 v = \lambda_1^p \left[\max \left\{ v(a - \kappa_1^p, h, z, p), v(a,h,z,1-p) \right\} - v(a,h,z,p) \right]$$

$$\mathcal{H}^0 v = \lambda_B^h \max \left\{ \max_{h'} v(a - h' - \kappa_B^h(h'), h', z, p) - v(a,0,z,p), 0 \right\}$$

$$\mathcal{H}^+ v = \lambda_S^h \max \left\{ v \left(a + h - \kappa_S^h(h), 0, z, p \right) - v(a,h,z,p), 0 \right\} + \mu_h(h) \frac{\partial}{\partial h} v(a,h,z,p)$$

$$\mathcal{Z} v = \left(\mu_z(z) \frac{\partial}{\partial z} + \sigma_z(z) \frac{1}{2} \frac{\partial^2}{\partial z^2} \right) v$$

KFE

Role of Aggregate Risk

A Special Case

- Briefly consider a (very) special case:
  - Two asset, one riskless, one risky
    - Housing only as a consumption asset ($\lambda^h = 0$)
  - No frictions
  - No income risk
  - No death
- Households' wealth evolves according to:

$$\frac{\mathrm{d}w}{w} = \underbrace{\left[r_B + (r_S - r_B)\theta(w) - \frac{e(w)}{w}\right]}_{\mu(w)} \mathrm{d}t + \underbrace{\theta(w)\sigma_S}_{\sigma(w)} \mathrm{d}W_t$$

KFE

Role of Aggregate Risk

Top Shares

Proposition 3

The share of wealth held by the top x-percent, $S_{x,t}$, evolve according to:

$$\mathrm{d}S_{x,t} = \frac{1}{\bar{W}_t}\left\{\int_{q_t}^{\infty}\mu(w)g_t(w)\,\mathrm{d}w - S_{x,t}\int_{-\infty}^{+\infty}\mu(w)g_t(w)\,\mathrm{d}w + \frac{1}{2}\sigma(q_t)^2 g_t(q_t)\right\}\mathrm{d}t + \\ + \frac{1}{\bar{W}_t}\left\{\int_{q_t}^{\infty}\sigma(w)g_t(w)\,\mathrm{d}w - S_{x,t}\int_{-\infty}^{+\infty}\sigma(w)g_t(w)\,\mathrm{d}w\right\}\mathrm{d}W_t$$

Corollary 3.1

If utility is time-separable and homothetic, $\varepsilon_h = \varepsilon_c$, portfolio shares are constant ($\sigma(w) \propto w$) and wealth shares $S_{x,t}$ are independent of aggregate shocks.

KFE

Yet Another Fact

Expenditure Shares

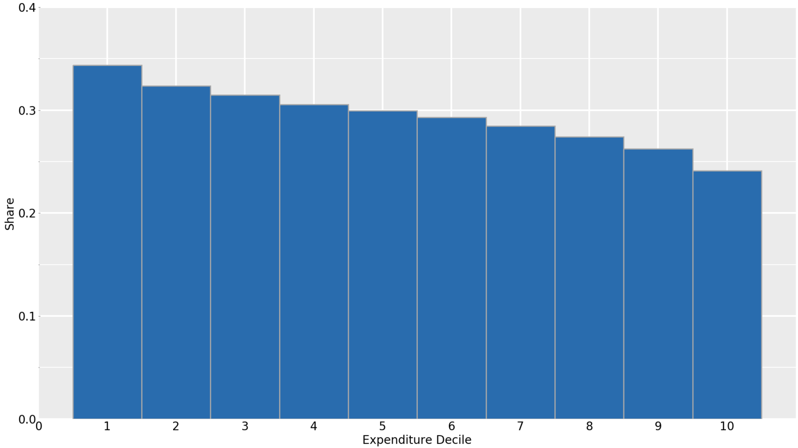

Figure 11: Housing expenditure shares by expenditure decile

Setup | Proposition 2 | Results

Calibration

Asset Returns

- Returns on bills and capital gains on housing from (Jordà et al. 2019)
  - Rental rate fixed at its historical average
  - Housing adjusted for leverage and cost of mortgages
- Equity returns from Kartashova (2014) (weighted average of public and private)

Table 1: Calibration - Returns

	B	S	H
r	0.019	0.111	0.003
$\boldsymbol{\sigma}$	0.022 -0.025 -0.006	- 0.095 0.03	- - 0.0533

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Calibration

Preferences

Table 2: Calibration - Preferences

PARAMETER	VALUE	TARGET
ρ	0.06	wealth-to-income ratio
ζ	0.022	avg. working life
γ	2	portfolio shares
χ	1.5	avg. homeownership rate
ω	0.31	avg. housing expenditure share
ε_h	0.75	expenditure shares
ε_c	0.91	avg. expenditure elasticity

- Directly target housing expenditure shares and elasticity
- Implied value of $\frac{1-\varepsilon_c^{-1}}{1-\varepsilon_h^{-1}}$ is 0.375:
  - RRA $\in$ (1.375, 2) with average 1.47
  - EIS $\in$ (0.61, 0.775) with average 0.7

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Calibration

Table 3: Calibration

Parameter	Value	Target	Source
Frictions			
ϕ	75% (avg. earnings)	median credit limit	Heathcote et al. (2020)
κ^b	0.06	fraction with $a = 0$	-
λ_0^p, λ_1^p	12, 365	-	-
κ_0^p	0.0	avg. participation	-
κ_1^p	1.5% (avg. earnings)	-	Vissing-Jørgensen (2002)
λ^h	5.2	avg. search time	Garriga and Hedlund (2020)
κ_0^h	5.0% (avg. earnings)	housing shares	-
κ_1^h	5.5%	-	Yao and Zhang (2005)
Earnings Process			
$\bar{z}$	-0.75	normalization	-
η_z	0.03	autocorrelation	-
σ_z	0.3	variance of earnings changes	-

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Wealth Inequality

Level

- Compare average inequality in model and data
  - Model averages are from 10,000 years simulation

Table 4: Wealth Inequality - Average

	Bottom 60%	Next 30%	Top 10%	Top 1%	**Gini**
SCF (1989-2019)	7.0%	22.7%	70.3%	33.9%	0.827
MODEL	6.0%	31.1%	62.9%	27.1%	0.759

Wealth Inequality

Level – Decomposition

- Solve model with exogenous portfolio shares and shut off each channel

Table 5: Wealth Inequality - Decomposition

	Parameters		Gini	
Model	Rates of return	Risk	Mean	Std. dev.
Baseline	$\mathbf{r}$	$\boldsymbol{\sigma}$	0.759	0.049
Earnings	-	-	0.662	-
No Ret. Heterogeneity, Homothetic	$r_j = 5.3\%$	$\sigma_{i,j} = 0$	0.684	-
Non-Homothetic Preferences	$r_j = 5.3\%$	$\sigma_{i,j} = 0$	0.711	-
Only Risk	$r_j = 5.3\%$	$\boldsymbol{\sigma}$	0.707	0.031
Only Ret. Heterogeneity	$\mathbf{r}$	$\sigma_{i,j} = 0$	0.746	-
All	$\mathbf{r}$	$\boldsymbol{\sigma}$	0.743	0.040

- Risk plays no role for the *level* of inequality
- Return heterogeneity alone increases Gini by 4 p.p.

Results

Impulse Responses

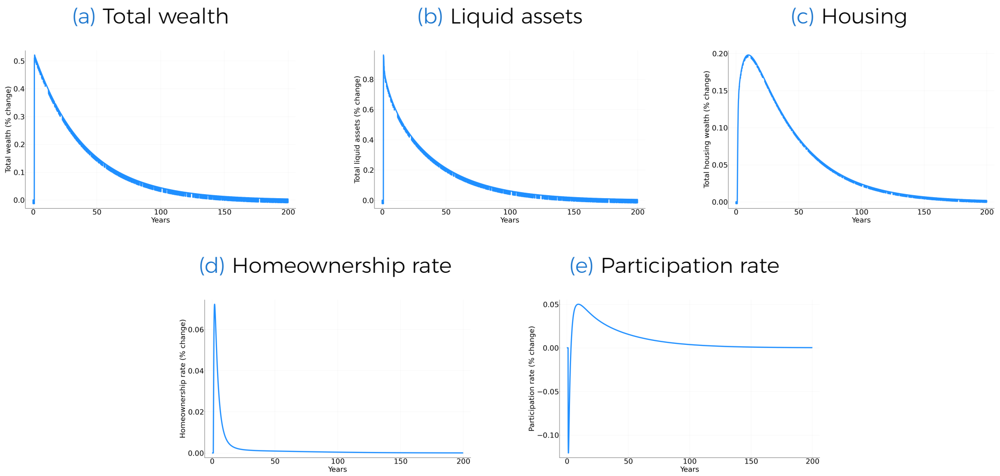
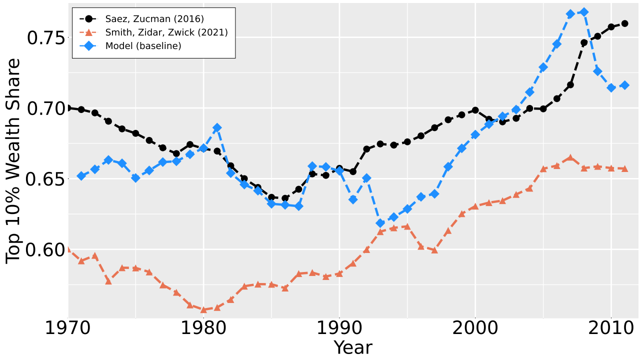


Figure 12: IRF to equity shock

Wealth Inequality

Model vs. Data - Levels

Figure 13: Top 10% wealth share – model and data



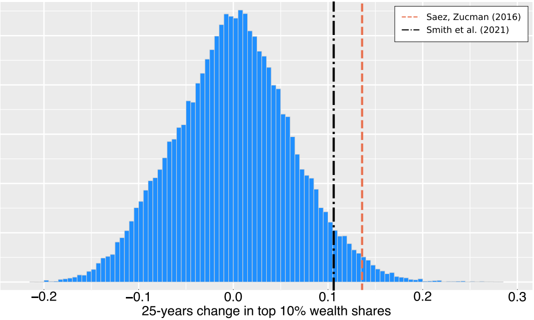
Results

Wealth Inequality

Dynamics – Changes

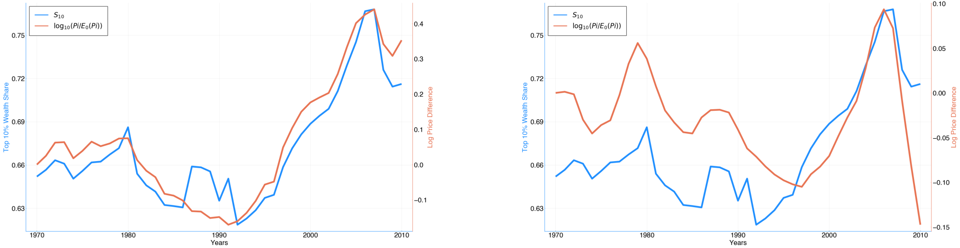
- Compute distribution of 25-years changes in inequality
  - $\mathbb{P}(\Delta_{25} S_{0.1} \geq 13.6\%) = 2.3\%$ for Saez and Zucman (2016) (3.5% conditional)
  - $\mathbb{P}(\Delta_{25} S_{0.1} \geq 10.6\%) = 6.1\%$ for Smith et al. (2021) (14.3% conditional)

Figure 14: Distribution of Changes



Price fluctuations

Figure 15: Wealth Inequality vs. Prices



(a) Equity Prices (b) Housing Prices

◀ Results